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Mercedes Teruel Carrizosa
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ANNEX IV

ECONOMETRIC METHODOLOGY

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ANNEX IV

ECONOMETRIC METHODOLOGY

1. INTRODUCTION

1.1. The Model

For each cross section (individual) *i=1,2,...N* and each time period (time) t=1,2,...T,

$$Y_{it} = X_{it}\beta_{it} + \varepsilon_{it}$$

Let $\beta_{it} = \beta$ and assume $\varepsilon_{it} = u_i + v_t + e_{it}$ where u_i represents the individual or cross section difference in intercept and v_t is the time difference in intercept. Two-ways analysis includes both time and individual effects. For simplicity, we further assume $v_t = 0$. That is, there is no time effect. In other words, only the *one-way* individual effects will be analyzed in the following.

The component e_{it} is a classical error term, with zero mean, homogeneous variance, and there is no serial correlation and no contemporary correlation. Also, e_{it} is uncorrelated with the regressors X_{it}. That is,

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$$\begin{cases} E(e_{it}) = 0 \\ E(e_{it}^2) = \sigma_e^2 \\ E(e_{it}e_{i\tau}) = 0, \quad for \quad t \neq \tau \\ E(e_{it}e_{jt}) = 0, \quad for \quad i \neq j \\ E(X_{it}e_{it}) = 0 \end{cases}$$

1.2. Fixed Effects Model

Assume that the error component u_i, the individual difference, is *fixed* or *nonstochastic* (but it varies across individuals). Thus, the model error is simply $\varepsilon_{it} = e_{it}$. The model is expressed as:

$$Y_{it} = (X_{it}\beta + u_i) + e_{it}$$

where u_i is interpreted as the *change* in the intercept. Therefore the *individual effect* is defined as u_i plus the intercept.

1.3. Random Effects Model

Assume that the error component u_i, the individual difference, is *random* and satisfies the following assumptions:

$$\begin{cases} E(u_i) = 0 \\ E(u_i) = \sigma_u^2 \quad (\text{homoscedasticity}) \\ E(u_iu_j) = 0 \quad for \quad i \neq j \quad (no \quad cross-section \quad correlation) \\ E(u_ie_{it}) = E(u_ie_{jt}) = 0 \quad (independent \quad from \quad each \quad e_{it} \quad or \quad e_{jt}) \end{cases}$$

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Then, the model error is $\varepsilon_{it} = u_i + e_{it}$ with the following structure:

$$\begin{cases} E(\varepsilon_{it}) = E(u_i + e_{it}) = 0 \\ E(\varepsilon_{it}^2) = E\left[(u_i + e_{it})^2\right] = \sigma_u^2 + \sigma_e^2 \\ E(\varepsilon_{it}\varepsilon_{i\tau}) = E\left[(u_i + e_{it})(u_i + e_{i\tau})\right] = \sigma_u^2, \quad for \quad t \neq \tau \\ E(\varepsilon_{it}\varepsilon_{jt}) = E\left[(u_i + e_{it})(u_j + e_{jt})\right] = 0, \quad for \quad i \neq j \end{cases}$$

In other words, for each cross section "*i*", the variance covariance matrix of the model error $\varepsilon_i = [\varepsilon_{i1}, \varepsilon_{i2}, ..., \varepsilon_{iT}]'$ is the following TxT matrix:

$$\sum = \begin{bmatrix} \sigma_e^2 + \sigma_u^2 & \sigma_u^2 & \cdots & \sigma_u^2 \\ \sigma_u^2 & \sigma_e^2 + \sigma_u^2 & \cdots & \sigma_u^2 \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \sigma_u^2 & \sigma_u^2 & \cdots & \sigma_e^2 + \sigma_u^2 \end{bmatrix} = \sigma_e^2 I + \sigma_u^2$$

Let $\boldsymbol{\varepsilon}$ be a NT-element vector of the stacked errors $\varepsilon_1, \varepsilon_2, ..., \varepsilon_N$, $\boldsymbol{\varepsilon} = [\varepsilon_1, \varepsilon_2, ..., \varepsilon_N]'$, then $E(\boldsymbol{\varepsilon}) = 0$ and $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \Sigma \otimes I$, where I is an NxN identity matrix and Σ is the TxT variance-covariance matrix defined above.

2. Model Estimation

2.1. Fixed Effects Model

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Consider the model as follows:

$$Y_{it} = \left(X_{it}\beta + u_i\right) + \varepsilon_{it} \quad \left(i = 1,2,\ldots,N; t = 1,2,\ldots,T\right)$$

Let $Y_i = [Y_{i1}, Y_{i2}, \ldots, Y_{iT}]'$, $X_i = [X_{i1}, X_{i2}, \ldots, X_{iT}]'$, and $\varepsilon_i = [\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{iT}]'$, then the pooled (stacked) model is

$$\begin{bmatrix} Y_1 \\ Y_2 \\ : \\ Y_N \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ : \\ X_N \end{bmatrix} \beta + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ : \\ \varepsilon_N \end{bmatrix}$$

or, $Y = X\beta + \varepsilon$

- **Dummy Variables Approach**

  For each "*i*", define NT×1 vector D_i with the element:

$$D_{ij} = \begin{cases} 1 \ \ if \ \ (i-1) \times T + 1 \leq J \leq i \times T \\ 0 \ \ otherwise \end{cases}$$

  Then $\mathbf{D} = [D_1, D_2, \ldots, D_{N-1}]$ is NT×(N-1) matrix of N-1 dummy variables. Ordinary least squares can be used to estimate the model with dummy variables as follows:

$$Y = X\beta + D\delta + \varepsilon$$

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Since **X** includes a constant term, one less dummy variables are included for estimation and the estimated δ measures the individual *change* from the intercept.

- **Deviation Approach**

Let $Y_i^m = \frac{\left(\sum_{t=1,2,\ldots,T} Y_{it}\right)}{T}$, $X_i^m = \frac{\left(\sum_{t=1,2,\ldots,T} X_{it}\right)}{T}$, and $e_i^m = \frac{\left(\sum_{t=1,2,\ldots,T} e_{it}\right)}{T}$.

Then the *within* estimates of the model can be obtained by estimating the mean deviation model:

$$\left(Y_{it} - Y_i^m\right) = \left(X_{it} - X_i^m\right)\beta + \left(e_{it} - e_i^m\right)$$

Or, equivalently

$$Y_{it} = X_{it}\beta + \left(Y_i^m - X_i^m\beta\right) + \left(e_{it} - e_i^m\right)$$

Note that the constant term drops out due to mean deviation transformation. Therefore, the estimated *individual effects* of the model is $u_i = Y_i^m - X_i^m\beta$. The variance-covariance matrix of individual effects is estimated as follows:

$$Var(u_i) = \frac{v}{T} + X_i^m\left[Var(\beta)\right]X_i^{m'}$$

where v is the estimated variance of the mean deviation regression corrected for the degree of freedom NT-N-K (instead of NT-K). That is,

$$v = \frac{\Sigma_{i=1,2,\ldots,N}\Sigma_{t=1,2,\ldots,T}\left(e_{it} - e_i^m\right)^2}{\left(NT - N - K\right)}$$

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Note that K is the number of explanatory variables not counting the constant term.

It may be of interest to estimate the *between* parameters of the model by estimating

$$Y_i^m = X_i^m \beta + u_i + e_i^m$$

which is related to the estimated *individual effects* from the *within* estimates.

- **Testing for Fixed Effects**

Based on the dummy variable approach, this is a Wald F-test for the joint significance of the parameters associated with dummy variables representing the individual effects. If the null hypothesis δ = 0 can not be rejected, then there is no fixed effects in the model.

Based on the deviation approach, the equivalent test statistic is computed from the restricted (pooled model) and unrestricted (mean deviation model) sum of squared residuals. That is,

$$\frac{\dfrac{RSS_R - RSS_U}{N-1}}{\dfrac{RSS_U}{NT-N-K}} \approx F\left(N-1, NT-N-K\right)$$

2.2. Random Effects Model

Recall the pooled model for estimation

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$$Y = X\beta + \varepsilon$$

where $\boldsymbol{\varepsilon} = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N]'$, $\varepsilon_i = [\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{iT}]'$, and the random error components $\varepsilon_{it} = u_i + e_{it}$. By assumptions, $E(\boldsymbol{\varepsilon}) = 0$, and $E(\boldsymbol{\varepsilon\varepsilon}') = \Sigma \otimes I$. The Generalized Least Squares estimates of β is

$$\beta = \left[X'\left(\Sigma^{-1} \otimes I\right)X\right]^{-1} X'\left(\Sigma^{-1} \otimes I\right)Y$$

Since Σ^{-1} can be derived from the estimated variance components σ^2_e and σ^2_u, in practice the model is estimated using the following *partial deviation approach*.

- **Partial Deviation Approach**
  1. From the dummy variable approach for estimating the fixed effect model, the estimated variance σ^2_e is obtained.
  2. Assuming the randomness of u_i, estimate the *between* parameters of the model:

$$Y_i^m = X_i^m \beta + \left(u_i + e_i^m\right)$$

where the error structure of $u_i + e^m_i$ satisfies:

$$\begin{cases} E(u_i + e_i^m) = 0 \\ E\left(\left(u_i + e_i^m\right)^2\right) = \sigma_u^2 + \dfrac{\sigma_e^2}{T} \\ E\left(\left(u_i + e_i^m\right)\left(u_j + e_j^m\right)\right) = 0, \quad for \quad i \neq j \end{cases}$$

Let $v = \sigma_e^2$ and $v_1 = T\sigma_u^2 + \sigma_e^2$. Define $w = 1 - \left(\dfrac{v}{v_1}\right)^{1/2}$.

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3. Using w to transform (partial deviations) all the data series as follows:

$$Y_{it}^{*} = Y_{it} - wY_{i}^{m}$$

$$X_{it}^{*} = X_{it} - wX_{i}^{m}$$

Then the model for estimation is:

$$Y_{it}^{*} = X_{it}^{*}\beta + \varepsilon_{it}^{*}$$

where $\varepsilon_{it}^{*} = (1-w)u_{i} + e_{it} - we_{i}^{m}$

Or, equivalently

$$Y_{it} = X_{it}\beta + w\left(Y_{i}^{m} - X_{i}^{m}\beta\right) + \varepsilon_{it}^{*}$$

It is easy to validate that

$$\begin{cases} E(\varepsilon_{it}^{*}) = 0 \\ E(\varepsilon_{it}^{*2}) = \sigma_{e}^{2} \\ E(\varepsilon_{it}^{*}\varepsilon_{i\tau}^{*}) = 0 \quad for \quad t \neq \tau \\ E(\varepsilon_{it}^{*}\varepsilon_{jt}^{*}) = 0 \quad for \quad i \neq j \end{cases}$$

The least squares estimate of $[w(Y_{i}^{m} - X_{i}^{m}\beta)]$ is interpreted as the *change* of individual effects.

- **Testing for Random Effects**

To test for *no* correlation relationship of the error terms $u_i + e_{it}$ and $u_i + e_{i\tau}$, the following Breusch-Pagan LM test statistic based on the

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estimated residuals of the restricted (pooled) model, ε_{it} (i=1,2,...N, t=1,2,...,T), is distributed as Chi-square with *one* degree of freedom:

$$LM = \frac{NT}{2(T-1)}\left(\frac{\sum_{i=1,2,\ldots N}\left(\sum_{t=1,2,\ldots T}\varepsilon_{it}\right)^2}{\sum_{i=1,2,\ldots N}\sum_{t=1,2,\ldots T}\varepsilon_{it}^2}-1\right)^2$$

$$= \frac{NT}{2(T-1)}\left(\frac{\sum_{i=1,2,\ldots N}\left(T\varepsilon_i^m\right)^2}{\sum_{i=1,2,\ldots N}\sum_{t=1,2,\ldots T}\varepsilon_{it}^2}-1\right)^2$$

Note that $\varepsilon_i^m = \frac{\sum_{t=1,2,\ldots,T}\varepsilon_{it}}{T}$.

2.3. Hausman's Test for Fixed or Random Effects

Let b_{fixed} be the estimated slope parameters of the fixed effects model (using dummy variable approach), and b_{random} be the estimated slope parameters of the random effects model. Moreover, Var(b_{fixed}) and Var(b_{random}) are the corresponding estimated variance-covariance matrix, respectively. Hausman's test for *no* difference of these two sets of parameters is a Chi-square test in which the degree of freedom corresponds to the number of slope parameters. The test statistic is defined as follows:

$$H = (b_{random}-b_{fixed})'[Var(b_{random})-Var(b_{fixed})]^{-1}(b_{random}-b_{fixed})$$

3. Extensions

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3.1. Unbalanced Panel Data

Panels in which the group sizes (time periods) differ across groups (individuals) are not unusual in empirical panel data analysis. These panels are called *unbalanced panels*. Estimation for fixed effects and random effects models discussed above must be modified to reflect the structure of unbalanced panels. Modify the dummy variable or deviation approach for estimating the fixed effects with unbalanced panel data is straightforward. However, for the random effects model, by allowing unequal group sizes, there presents the problem of groupwise heteroscedasticity.

3.2. Random Coefficients Model

For each cross section "*i*"*=1,2,...,N*, the model is written as:

$$Y_i = X_i \beta_i + e_i$$
$$\beta_i = \beta + \upsilon_i$$

where $Y_i = [Y_{i1}, Y_{i2}, \ldots, Y_{iT}]'$, $X_i = [X_{i1}, X_{i2}, \ldots, X_{iT}]'$, and $\varepsilon_i = [\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{iT}]'$. We note that not only the intercept but also the slope parameters are random across individuals. The assumptions of the model are:

$$\begin{cases} E(\varepsilon_i) = 0_{N\times 1} \\ Var(\varepsilon_i) = E(\varepsilon_i \varepsilon_i') = \sigma_i^2 I_{N\times N} \\ Cov(\varepsilon_i, \varepsilon_j) = 0_{N\times N}, \quad i \neq j \end{cases}$$

and

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$$\begin{cases} E(\upsilon_i) = 0_{K\times 1} \\ Var(\upsilon_i) = E(u_i u_i') = \Gamma_{K\times K} \\ Cov(\upsilon_i, \upsilon_j) = 0_{K\times K}, \quad i \neq j \\ Cov(\upsilon_i, \varepsilon_i) = 0_{K\times 1} \end{cases}$$

The model for estimation is

$$Y_i = X_i\beta + (X_i\upsilon_i + \varepsilon_i), \ or$$

$$Y_i = X_i\beta + \omega_i \quad where \quad \omega_i = (X_i\upsilon_i + \varepsilon_i), \quad and$$

$$\begin{cases} E(\omega_i) = 0_{N\times 1} \\ Var(\omega_i) = E(\omega_i\omega_i') = E\left(X_i\upsilon\upsilon' X_i' + X_i\upsilon_i\varepsilon + \varepsilon_i\upsilon_i' X_i + \varepsilon_i\varepsilon_i'\right) = \sigma_i^2 I_{N\times N} + X_i\Gamma X_i' = \Omega_i \end{cases}$$

The stacked (pooled) model is

$$Y = X\beta + \omega$$

where $\omega = [\omega_1,\dots,\omega_N]'$, and

$$E(\omega) = 0_{NTx1}$$

$$Var(\omega) = E(ww') = V = \begin{bmatrix} \Omega_1 & 0 & .. & 0 \\ 0 & \Omega_2 & .. & 0 \\ : & : & : & \\ 0 & 0 & .. & \Omega_N \end{bmatrix}$$

GLS is used to estimate the model. That is,

$$b^* = (X'V^{-1}X)^{-1}X'V^{-1}Y$$

$$Var(b^*) = (X'V^{-1}X)^{-1}$$

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The computation is based on the following steps (Swamy, 1971):

1. For each regression equation *i*, $Y_i = X_i \beta_i + \varepsilon_i$, obtain the OLS estimator of β_i:

   $$b_i = (X_i' X_i)^{-1} X_i' Y_i$$
   $$Var(b_i) = (X_i' X_i)^{-1} (X_i' \Omega_i X_i)(X_i' X_i)^{-1} = \sigma_i^2 (X_i' X_i)^{-1} + \Gamma = V_i + \Gamma$$

   (Taking account of heteroscedasticity, where $V_i = \sigma_i^2 (X_i' X_i)^{-1}$)

   Note that σ_i^2 is estimated by $s^2_i = e_i'e_i/(N-K)$, where $e_i = Y_i - X_i b_i$.

   Then, $V_i = s_i^2 (X_i' X_i)^{-1}$.

2. For the random coeffcients equation, $\beta_i = \beta + \upsilon_i$, the variance of b_i (estimator of β_i) is estimated by

   $$\frac{\sum_{i=1,\ldots,G} \left(b_i - b^m\right)\left(b_i - b^m\right)'}{(G-1)} = \frac{\sum_{i=1,\ldots,G} \left(b_i b_i' - G b^m b'^m\right)}{(G-1)},$$

   where $b^m = \frac{\sum_{i=1,\ldots,G} b_i}{G}$.

   Therefore, $\Gamma = \frac{\Sigma_{i=1,\ldots,G}\left(b_i b_i' - G b_m b_m'\right)}{(G-1)} - \frac{\Sigma_{i=1,\ldots,G} V_i}{G}$.

   Concerning the possibility that Γ may be nonpositive definite, we use

   $$\Gamma = \frac{\Sigma_{i=1,\ldots,G}\left(b_i b_i' - G b_m b_m'\right)}{(G-1)}.$$

3. Write the GLS estimator of b as:

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$$
\begin{aligned}
b^* &= \left(X'V^{-1}X\right)^{-1} X'V^{-1}Y \\
&= \left[\Sigma_{i=1,2,\ldots,G} X_i'\Omega_i X_i\right]^{-1}\left[\Sigma_{i=1,2,\ldots,G} X_i'\Omega_i Y_i\right] \\
&= \left[\Sigma_{i=1,2,\ldots,G} X_i'\Omega_i X_i\right]^{-1}\left[\Sigma_{i=1,2,\ldots,G} X_i'\Omega_i X_i b_i\right] \\
&= \left[\Sigma_{i=1,2,\ldots,G} (\Gamma+V_i)^{-1}\right]^{-1}\left[(\Gamma+V_i)^{-1} b_i\right] \\
&= \Sigma_{i=1,2,\ldots,G} W_i b_i \quad where \quad W_i = \left[\Sigma_{i=1,2,\ldots,G} (\Gamma+V_i)^{-1}\right]^{-1}\left[(\Gamma+V_i)^{-1}\right]
\end{aligned}
$$

Similarly,

$$
Var(b^*) = \left(X'V^{-1}X\right)^{-1} = \left[\Sigma_{i=1,2,\ldots,G} (\Gamma+V_i)^{-1}\right]^{-1}
$$

The individual parameter vectors may be predicted as follows:

$$
b_i^* = (\Gamma+V_i)^{-1}\left[\Gamma^{-1}b^* + V_i^{-1}b_i\right] = A_i b^* + (I-A_i)b_i
$$

where $A_i = (\Gamma+V_i)^{-1}\Gamma^{-1}$.

$$
Var(b_i^*) = \begin{bmatrix} A_i & I-A_i \end{bmatrix} \begin{bmatrix} \sum_{i=1,\ldots,G} W_i(\Gamma+V_i)W_i' & W_i(\Gamma+V_i) \\ (\Gamma+V_i)W_i' & (\Gamma+V_i) \end{bmatrix}^{-1} \begin{bmatrix} A_i \\ I-A_i \end{bmatrix}
$$

3.3. Seemingly Unrelated System Model

Consider a more general specification of the model:

$$
Y_{it} = X_{it}\beta_i + \varepsilon_{it} \quad (i = 1,2,\ldots,N;\ t = 1,2,\ldots,T)
$$

Let $Y_i = [Y_{i1},Y_{i2},\ldots,Y_{iT}]'$, $X_i = [X_{i1},X_{i2},\ldots,X_{iT}]'$, and $\varepsilon_i = [\varepsilon_{i1},\varepsilon_{i2},\ldots,\varepsilon_{iT}]'$, the stacked N equations (T observations each) system is $\boldsymbol{Y = X\beta + \varepsilon}$, or

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$$\begin{bmatrix} Y_1 \\ Y_2 \\ : \\ Y_N \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \cdot\cdot & 0 \\ 0 & X_2 & \cdot\cdot & 0 \\ : & : & & : \\ 0 & 0 & \cdot\cdot & X_N \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ : \\ \beta_N \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ : \\ \varepsilon_N \end{bmatrix}$$

Notice that not only the intercept but also the slope terms of the estimated parameters are different across individuals. The error structure of the model is summarized as follows:

$$\begin{cases} E(\varepsilon) = 0 \\ E(X'\varepsilon) = 0 \\ E(\varepsilon\varepsilon') = \Sigma \otimes I \end{cases}$$

where $\Sigma = [\sigma_{ij}, i,j=1,2,...N]$ is the NxN variance-covariance matrix and I is a TxT identity matrix. Notice that contemporary correlation across individuals is assumed although there is no time serial correlation. The error structure of this model is different than that of random effects model described above.

The model is estimated using techniques for systems of regression equations.

The system estimation techniques such as 3SLS and FIML should be used for parameter estimation. It is called the Seemingly Unrelated Regression Estimation (SURE) in the current context. Denote $\boldsymbol{b}$ and S as the estimated $\boldsymbol{\beta}$ and S, respectively. Then,

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$$b = \left[X'\left(S^{-1} \otimes I\right)X\right]^{-1} X'\left(S^{-1} \otimes I\right)Y$$

$$Var(b) = \left[X'\left(S^{-1} \otimes I\right)X\right]^{-1}, \ and$$

$$S = \frac{ee'}{T}, \ where \ \ e = Y - Xb \ \ is \ \ the \ \ estimated \ \ error \ \ \varepsilon.$$