Essays on Inflation Dynamics and Monetary Policy in Currency Areas

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To my father Michele,
my mother Antonella,
and my sister Claudia

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Abstract

This thesis extends the basic New Keynesian (NK) model to answer two questions. How should monetary policy be designed in heterogeneous currency areas? What is the effect of competitive pressures on the inflation dynamics? The first chapter analyzes the monetary policy design in currency areas in which countries display different degrees of external openness. Such heterogeneity implies that the optimal policy plan exhibits a stronger motive for the currency area exchange rate stabilization in order to dampen inflation differentials. The second chapter studies the design of targeting rules in currency areas with country-specific cost-push shocks that have different volatilities. The third chapter estimates a NK Phillips curve derived from a model with endogenous firm entry in which the number of active firms is inversely related to their desired markup. It quantifies the effect of the desired markup fluctuations on the pass-through of real marginal cost.

Resumen

Esta tesis extiende el modelo estándar Neo Keynesiano con el propósito de contestar dos preguntas: ¿cómo debe ser diseñada la política monetaria en uniones monetarias heterogéneas? y ¿cuál es el efecto de presiones competitivas en la dinámica de la inflación? El primer capítulo analiza el diseño de política monetaria en uniones monetarias en las cuales los países miembros muestran diferentes grados de apertura externa. Esta heterogeneidad implica que el plan de la política óptimo muestra una inclinación muy fuerte por la estabilización del tipo de cambio, con el objetivo de disminuir los diferenciales de inflación. El segundo capítulo estudia el diseño de reglas de metas en una unión monetaria con choques idiosincráticos *cost-push* que tienen diferentes volatilidades. El tercer capítulo estima un curva de Phillips Neo Keynesiana derivada de un modelo con entrada endógena de firmas, en el cual el número de firmas activas está inversamente relacionado con el markup deseado. Se cuantifica el efecto de las fluctuaciones del markup deseado en el *pass-through* de los costes marginales reales.

Foreword

This thesis contributes to the development of the New Keynesian framework by adapting and extending the basic models in order to answer two questions: how should monetary policy be designed in a currency area in which members countries display some heterogeneity? What is the effect of competitive pressures on the inflation dynamics? The thesis is composed of three chapters. The first two chapters explores whether the presence of heterogeneity across member countries of a currency area should affect the conduct of the monetary policy. I consider two cases: differences in the structural parameters of the model, namely the degree of trade openness toward countries outside the monetary union[1], and the existence of idiosyncratic shocks that influences the member countries inflation process (i.e. cost-push shocks) with different stochastic properties. The third chapter extends the standard New Keynesian model to introduce endogenous fluctuations of the desired markup (that is the one that firms are willing to charge on marginal cost absent nominal rigidities) and quantifies their effect on the inflation dynamics process of the United States.

Ten years ago several European countries have decided to abandon their national currency to join the Euro currency area. Such unique historical episode has stimulated a strong debate on the likely and observed effects of such change in the monetary regime. A large part of the literature aims at describing the main features of the monetary transmission mechanism of the Euro area and the differences across countries.[2] Contemporaneously, a small strand of the literature has started to set up models that, building on the most recent modelling features of the New Open Economy macroeconomics, encompasses the salient elements of a currency area economy composed by different countries in order to study the normative implications of several kind of heterogeneity (see Benigno (2004), Galì and Monacelli (2008) and Ferrero (2009)). The first two chapters contribute to this literature. I document some relevant dimensions

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[1] I use the terms currency area and monetary union interchangeably throughout the thesis.

[2] For instance the papers, that are the results of the research of the Monetary Transmission Network coordinated by the ECB, focus on the empirical implications of cross-country heterogeneities for the monetary policy transmission mechanism (see Angeloni et al. (2003) for a comprehensive survey).

of heterogeneity in the Euro area and I incorporate them in a coherent model for the currency area in order to derive the prescriptions for the monetary policy design.

The setup of the European Monetary Union (EMU) induced a considerable increase of the intra trade volumes of the member countries, due to the advantages of having the same unit of accounts and no exchange rate risk. Despite this, the member countries of the EMU have continued displaying different degrees of external trade openness due to the historical and geographical linkages with external trade partners.

In chapter 1, titled *External Trade and Monetary Policy in a Currency Area*, I document this stylized fact and I study the implications for the optimal monetary policy of this type of heterogeneity across currency area member countries. I incorporate the differences in the external trade openness in a model for a currency area composed of two regions. One region is more open to trade with a third country outside the area than the other. When regions are heterogeneous, the relative competitiveness across regions influences the extent to which shocks are transmitted to the area-wide inflation and output gap. Using the utility-based loss function for the currency area, I derive the optimal monetary policy in the case in which the regions are homogeneous and in the more realistic case in which they are not. I then compare the outcome of these policies in terms of responses to the main (area-wide and idiosyncratic) shocks. According to my findings and under a plausible calibration for the EMU, the optimal policy plan exhibits a stronger motive for the currency area exchange rate stabilization when regions are heterogeneous in the degree of external openness. Moreover, it is shown that it is welfare-improving to forgo some area-wide inflation stabilization to dampen inflation differentials.

Furthermore, since the adoption of the common currency, the EMU member countries experienced different volatilities of the national inflation and output. In a currency area monetary policy cannot be a valid candidate to explain this cross-country heterogeneity. Instead, the observed pattern of the macroeconomic volatility can be the result of structural differences in the monetary transmission mechanism, of idiosyncratic shocks or a combination of both factors. While Benigno (2004) and Benigno and Lopez-Salido (2006) study the monetary policy design in the presence of structural differences of the inflation dynamics process, I concentrate on differences

of the volatility of the cost-push shocks.

In chapter 2, titled *Targeting Rules in a Monetary Union with Heterogeneous Cost-Push Shocks*, I analyze the design of policy rules in a monetary union with country-specific cost-push shocks. I build a model of a currency area in which one country has a more volatile idiosyncratic cost-push shock. The main finding is that, when the central bank cannot implement the full commitment optimal policy, one can improve on the HICP inflation targeting rule (i.e. a rule in which the weights to the two countries' inflations are equal to their economic size) by choosing the weights on countries' inflations that minimize the utility-based loss function. I quantify the welfare gains associated to the "optimally weighted" inflation targeting rule relative to the HICP inflation targeting rule.

The last part of the thesis focus on the determinants of the inflation dynamics. The New Keynesian Phillips curve links the dynamics of aggregate inflation to the one of current and future real marginal costs. The markup that firms are willing to impose on the unit cost is constant over time and does not play a role in explaining inflation fluctuations.

The chapter 3, titled *Firm Entry, Competitive Pressures and the U.S. Inflation Dynamics*, extends the basic New Keynesian framework in order to allow for endogenous time-varying fluctuations in the firms' market power. In the model the entry of firms is endogenous and the total number of existing firms is inversely related to the market power. In such framework, the inflation dynamics depends not only on the marginal cost but also on the level of competitiveness in the market. The implied New Keynesian Phillips curve is estimated and allows to study the effect of competitive pressures on the inflation dynamics. By taking into account the number of competitors, the pass-through of real marginal cost on inflation is separately identifiable from the effect of endogenous desired markup fluctuations. Estimates with US data suggest that the effect of real marginal cost on inflation is stronger than what found in the empirical test of the standard model. The estimated elasticity of the desired markup to the number of firms implies that an increase of 10% in the number of active firms would lower annual inflation by 1.4% in the short run.

Contents

Chapter 1

External Trade and Monetary Policy in a Currency Area

1.1   Introduction

With the creation of the European Monetary Union (EMU) the member countries agreed to relinquish individual control on both the nominal interest rate and the nominal exchange rate. Some empirical studies confirm that the establishment of the currency area, by irrevocably fixing the exchange rates across member countries thereby eliminating the related risk in foreign transactions, stimulated the intra area trade.[1] Notwithstanding, there is no evidence of diversion of trade away from non-members countries (see Micco et al. (2003)). We observe instead that some of the EMU countries have strong trade linkages with countries not participating to the euro area mainly for historical or geographical reasons, while others show a pattern of trade that is more oriented toward intra area goods. About ten years after the onset of the EMU and despite the ever increasing trade integration, member countries continue to display heterogeneity in the degree of openness toward countries that are

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[1]One of the most recent estimates by Frankel (2008) found that the euro boosted intra-Eurozone trade by around 10 to 20% on average, in line with what shown in previous studies (e.g. Baldwin (2006)).

outside the currency area.[2]

The heterogeneity along this dimension is relevant for the setting of the monetary policy stance in a currency area as the transmission mechanism of external shocks may differ across regions and monetary policy may become less effective in controlling inflation and stabilizing the output gap. The works of Honohan and Lane (2003 and 2004) show that, due to differences in the degree of external openness, exchange rate movements have had an impact on inflation differentials within EMU, following the adoption of the common currency.

The paper sets up a model for an open currency area composed of countries that have different trade linkages with the rest of the world. After deriving the microfounded welfare measure, it then solves for the optimal monetary policy from a currency area point of view. Under a plausible calibration for the EMU it analyzes how the degree of heterogeneity in the external dimension affects the optimal responses of the macroeconomic variables to area-wide and asymmetric shocks .

The adoption of a common currency by European countries has spawned a large body of works on monetary policy in currency areas. The common driver of these works has been the fact that the economies of EMU member countries display some structural differences. A strand of literature has been devoted to studying and documenting these differences. For instance, since the euro area member countries displays large and persistent inflation differentials in contrast with other currency areas (in particular the US)[3], several efforts have been undertaken in order to understand the sources of this dispersion.[4]

Building on these facts, a second strand of literature has focused on the optimal design of monetary and fiscal policy in currency areas. Benigno (2004) and Benigno and Lopez-Salido (2006) studies the implications of, respectively, heterogeneous de-

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[2]Ireland, the Netherlands and Belgium stand out in this respect. Between 1997 and 2006, the average degree of external openness (measured either as the external trade volume over GDP or as the external imports over GDP) of Belgium, Ireland and the Netherlands are about 3 times higher than the correspondent ratios of the remaining euro area countries. Furthermore, this is a rather stable fact over the sample considered.

[3]See ECB (2003) for a careful analysis of different measures of inflation dispersion and a comparison with the US.

[4]See, among others, Angeloni and Ehrmann (2004) and Rabanal (2009).

grees of stickiness and different degree of inflation persistence among regions for the optimal target of inflation in a currency area. Galì and Monacelli (2008) and Ferrero (2009) consider the role of independent fiscal policies in a currency area in which regions share the same structural features but are hit by asymmetric technology shocks.

The main contribution of this paper is to build in the external trade linkages with the rest of the world in a standard model for a currency area and to study the effect of the cross-countries dissimilarities on the openness dimension on the optimal monetary policy design. The main issue is not whether the EMU is an optimal currency area, but to what extent, given the institutional setting, the monetary policy prescriptions that are valid for a homogeneous economy can be applied to a heterogeneous economy.

The model shares most of its features with the New Open Economy macroeconomics literature. The parameter for the degree of external openness coincides with the preferences of the households in each region for the good produced by the rest of the world. Symmetrically, the same parameter indicates the preferences of the rest of the world for the goods produced in each of the two regions.

When there is heterogeneity on the degree of external openness, the dynamics of the area-wide inflation and output gap are affected by the relative competitiveness between regions. Thus to fully describe the behavior of the area-wide economy the fluctuations of price differentials must be taken into account. The intuition for this result is the following. In an open currency area, shifts in the terms of trade between the monetary union and the rest of the world (hereafter *external* terms of trade) affect the demand for the goods produced in the area and thus its output gap and inflation. As to changes in the relative price between the goods produced in the two regions (hereafter *internal* terms of trade), their effects depend on the rest of the world preferences for the goods produced in the two regions (i.e. the heterogeneity as defined above). If the rest of the world is indifferent between consuming the goods coming from one or the other region of the monetary union, price differentials among regions induce the rest of the world to substitute out the more expensive good with the cheaper one and the aggregate demand for currency area goods is not affected by them. If instead the rest of the world has a stronger preference for the good produced by one of the two regions, this substitution is not perfect and the fluctuations of

the internal terms of trade matter for the aggregate demand of currency area goods coming from the rest of the world.

After deriving a quadratic utility-based loss function for the currency area, I solve for the full commitment optimal monetary policy from a timeless perspective. Several studies (e.g.De Paoli (2009) andFaia and Monacelli (2008)) on the optimal monetary policy in an open economy have shown that, once one departs from a special parametrization,[5] the strict domestic inflation stabilization is no longer the first-best policy and a partial stabilization of the exchange rate is desirable. This result holds in the case of an homogeneous currency area open to external trade. The paper shows that, in the presence of heterogeneity, the extent to which exchange rate should be stabilized is reinforced and higher volatility of area-wide inflation is prescribed.

In response to an area-wide technology shock, for instance, the optimal response in a homogeneous economy is to partially accommodate the shock by reducing the nominal interest rate in order to keep area-wide inflation and output gap stable. This policy generates a depreciation of the monetary union currency that boosts the external demand of currency area goods. When the regions of the currency area are heterogeneous, the shock generates higher inflation in the more open country.[6] A depreciation of the nominal exchange rate would amplify such inflation differentials, as the more open region comes across stronger inflationary pressures, causing a deadweight loss in the utility-based welfare. It is welfare improving to reduce the nominal interest rate by less in order to attenuate the depreciation of the monetary union currency. This implies that higher fluctuations of the area-wide inflation are allowed in order to partially dampen those of the inflation differentials caused by the shock.

The result enlightens an important point. It is commonly believed that in a currency area the centralized monetary authority has the ability and finds it optimal

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[5]Namely, when both the elasticity of intertemporal substitution of consumption and the intratemporal elasticity of substitution between domestic and foreign goods are different from one (seeCorsetti and Pesenti (2001)).

[6]In the flexible price equilibrium the internal terms of trade increases because the gains of external competitiveness due to the positive area-wide technology shock generate a higher demand of the good produced in the more open region, inducing an adjustment of the relative price. Due to price stickiness however the gap between the actual and the natural terms of trade becomes negative and inflationary pressures arise in the more open region.

to react only to aggregate disturbances while region-specific shocks can be stabilized only through other policy instruments, as for instance national fiscal policies (see Galì and Monacelli (2008) and Ferrero (2009)). The paper shows that in a heterogeneous currency area the central bank, by internalizing that the transmission mechanism of monetary policy impulses differs across regions, is able to generate differential macroeconomic response across regions. Furthermore, the optimal policy plan prescribes to make active use of this channel to influence inflation differentials since the aggregate welfare measure for the currency area depends also on them (see Benigno (2004)).

The rest of the paper is organized as follows. Section 1.2 illustrates some stylized facts about the external openness of EMU countries and describes the way in which it is incorporated in the model. Section 1.3 sets up the model for the currency area. Section 1.4 analyzes the effects of heterogeneity on the dynamics of the model. Section 1.5 derives and illustrates the optimal monetary policy plan. Section 1.6 concludes. The appendix A illustrates the details of the solution of the model, the derivation of the approximated welfare measure and the equations that characterize the optimal plan.

1.2 The external openness of EMU countries

Figure 1.1 plots the degree of total openness against the degree of external openness of 11 of the EMU member countries.[7] Both indicators are measured as the ratio of imports plus exports over GDP. The degree of total openness refers to trade flows with all countries whereas the degree of external openness refers to trade flows with countries that are not in the euro area.[8]

According to figure 1.1, the countries that are more open to external trade are also those that are overall more open. Furthermore, one can distinguish three groups of countries. One includes countries that have a relatively low degree of external and

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[7]Luxembourg, Slovenia, Cyprus, Malta and Slovakia are excluded.

[8]The average degree of total openness across EMU members from 1997 to 2006 is 0.86 and the average degree of external openness over the same period is 0.34. The cross-country dispersions of total and external degree of openness, as measured by the relative standard deviation, are respectively 47% and 54%.

Figure 1.1: Degrees of total and external openness
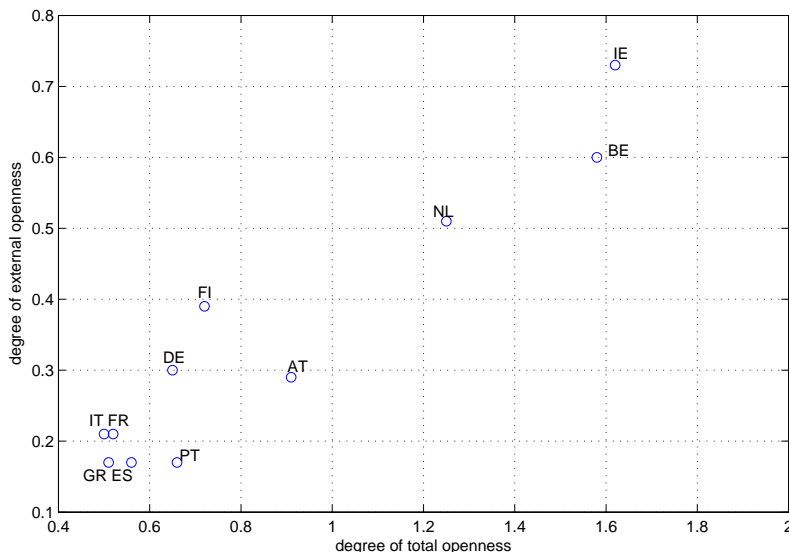
Notes: Average of quarterly data between 1997 and 2006. The degree of total and external openness is constructed as the ratio of the imports plus exports, respectively toward all trading partners and toward countries that are not members of the euro area, over the GDP. Source: Eurostat.

total openness, namely France, Greece, Italy, Portugal and Spain. Another group includes countries with a relatively high degree of external and total openness, namely Belgium, Ireland and the Netherlands. Finally, there is a third group of countries (Austria, Finland and Germany) that are in an intermediate position between the two above. This pattern has been quite stable over the last decade and a similar one can be observed if one measures the degrees of openness as (total and external) imports over GDP (see figure 1.2, panel (a)). In the composition of trade there is some dispersion across countries as well (see figure 1.2, panel (b)).

I introduce these features of the data in the model by assuming that inhabitants of the two regions of the currency area differ in their preferences toward the goods produced by the rest of the world (*row*) outside the area. The consumption bundles of citizens of regions 1 and 2 are given respectively by

$$C_t^1 = C_{1,t}^{1-\alpha-\omega+\varepsilon} C_{2,t}^{\alpha} C_{row,t}^{\omega-\varepsilon} \tag{1.1}$$

$$C_t^2 = C_{2,t}^{1-\alpha-\omega-\varepsilon} C_{1,t}^{\alpha} C_{row,t}^{\omega+\varepsilon} \tag{1.2}$$

Figure 1.2: Other measures of the external openness of EMU countries
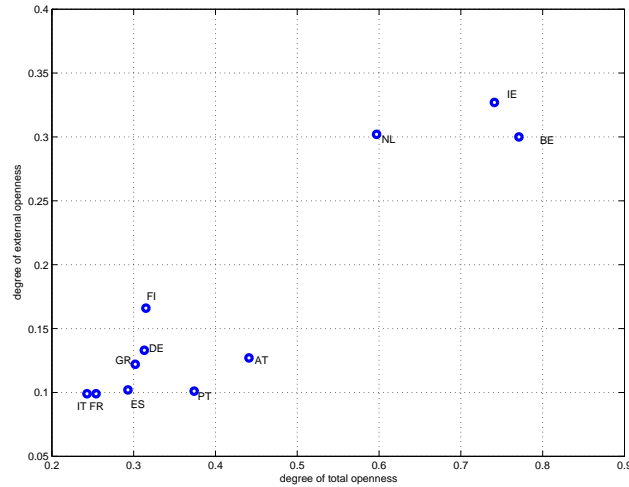
(a) Total and external imports over the GDP)
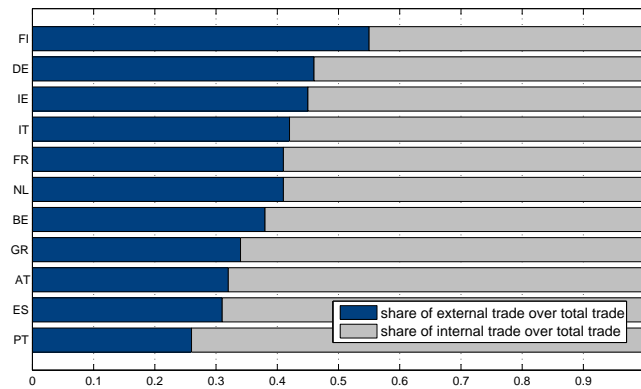
(b) External trade over total trade

Notes: Average of quarterly data between 1997 and 2006. Source: Eurostat.

where $C_{i,t}$ is the consumption good produced in region $i = 1$, 2 and *row*. This specification of preferences implies that the consumption basket of each region in the currency area includes, beyond the good domestically produced, goods produced in the other region of the currency area and in the rest of the world.[9]

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[9]In this way, the degree of external and total openness is modeled as a structural feature of the

The parameters α, $(\omega - \varepsilon)$ and $(\omega + \varepsilon)$ can be thought of as degrees of openness toward each trading partner. In fact, the steady state solution of the model described in the next section implies that those are the shares of imported consumption goods in total consumption. Thus, $\alpha + \omega - \varepsilon$ and $\alpha + \omega + \varepsilon$ are the degrees of total openness of regions 1 and 2 respectively.

The proposed assumption on the composition of the consumption bundles in the two regions captures the fact that, inside the EMU, more open countries are also the ones that are more open to external trade. The region that is more open to external trade (region 2) is also the one that has a lower degree of home bias (i.e. a higher degree of overall openness) and viceversa. The parameter α that indicates the trade inside the currency area is constant across countries. This assumption is motivated by the need to keep the model simple by shutting down one possible source of asymmetry.[10] Indeed the paper focuses on the effects of asymmetry in the degree of exposure to shocks originating outside the monetary union.

In a trade-balanced steady state, the chosen specification implies that the parameters $(\omega - \varepsilon)$ and $(\omega + \varepsilon)$ indicate also the fraction of the goods produced respectively in region 1 and 2 demanded from the rest of the world. Under the chosen specification, region 2 is more open to external trade than region 1, not only because its consumption basket is more oriented to foreignly produced goods, but also because the rest of the world demands more of its good. This is a way to capture the fact that some countries in the EMU have, for historical or geographical reasons, closer linkages with partners outside the currency area.

The parameter ε is an indicator of the heterogeneity in the preference toward the good *row*. When $\varepsilon = 0$, the two regions have identical preferences toward the good produced outside the currency area and symmetric preferences for their own produced good. By setting $\varepsilon \neq 0$, the two regions diverge and the model is able to capture the heterogeneity described in figure 1.1.

To introduce heterogeneity in the external dimension of EMU countries, an alter-

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regions. This is what we observe since the pattern of figure 1.1 is quite stable over the period from 1997 and 2006.

[10]Andrès et al. (2008) allows for differences in the degree of openness toward the other member countries in a model for a closed currency area.

native modeling strategy would have been the following

$$
\begin{aligned}
C_t^1 &= C_{1,t}^{(1-\alpha)} C_{2,t}^{\alpha(\omega+\varepsilon)} C_{row,t}^{\alpha(1-\omega-\varepsilon)} \\
C_t^2 &= C_{2,t}^{(1-\alpha)} C_{1,t}^{\alpha(\omega-\varepsilon)} C_{row,t}^{\alpha(1-\omega+\varepsilon)}.
\end{aligned}
$$

where now α indicates the degree of overall openness and $\alpha(1 - \omega \pm \varepsilon)$ the degree of external openness. In this case it is assumed that the two regions differ in their patterns of trade but not in the degree of total openness. This assumption, while consistent with the data showed in figure 1.2, is at odds with the data about the degree of total openness showed in figure 1.1. The assumption of preferences in (1.1) and (1.2) is consistent not only with the facts depicted in figure 1.1 but also with the existence of heterogeneity in the composition of the trade structure.[11]

1.3 The model

The model presented in this section is a slight modification of the New Keynesian framework for open economies.[12] The world is composed of a currency area (U) and the rest of the world (ROW). The currency area is formed by two regions, called thereafter region 1 and 2. Both regions are inhabited by a continuum of mass one of households and they have the same size. The two regions of the monetary union are explicitly modeled while the rest of the world is described by an exogenous stochastic process on the vector of the variables of interest.

1.3.1 Households

The representative household of each region $i = 1, 2$ has the following lifetime utility function

$$
E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \right] \tag{1.3}
$$

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[11] According to preferences (1.1) and (1.2), the share of trade with the rest of the world on the total trade is given by $\frac{\omega \pm \varepsilon}{\alpha + \omega \pm \varepsilon}$ and is therefore influenced by the degree of heterogeneity in trade, ε.

[12] See Lane (2001) for a survey on the so-called New Open Economy macroeconomics.

where N_t^i are the hours of labor supplied by the agent living in region i and C_t^i is a bundle of consumption goods coming from the two regions of the currency area and from the rest of the world. The bundles for the two regions are defined as in equations (1.1) and (1.2). The Cobb-Douglas specification of preferences implies that the elasticity of substitution among goods is one.

The goods $C_{1,t}$, $C_{2,t}$ and $C_{row,t}$ are CES aggregates of differentiated varieties of each good and, for $i = 1, 2$ and row, they are defined as follows[13]

$$C_{i,t} \equiv \left[\int_0^1 c_{i,t}(h)^{(\theta-1)/\theta} dh \right]^{\theta/(\theta-1)}$$

where h is a generic variety of the differentiated good i and $\theta > 1$ is the elasticity of substitution among varieties.

The representative household living in country i maximizes (1.3) subject to a sequence of budget constraints of the form

$$P_{i,t}^c C_t^i + E_t Q_{t,t+1} D_{t+1}^i \leq D_t^i + W_t^i N_t^i + T_t^i \tag{1.4}$$

where $P_{i,t}^c$ is the price index of the consumption bundle relevant for welfare in region i expressed in the currency of the monetary union; D_{t+1}^i is the nominal payoff in $t + 1$ of the portfolio of state-contingent claims held at the end of period t; $Q_{t,t+1}$ is the stochastic discount factor for one period ahead nominal payoff relevant to the households living in country i; W_t^i is the nominal wage and T_t^i are lump-sum transfers. I assume that markets are complete both at the domestic and international level.

As it is common in the literature, the household problem can be solved in two steps. The first step implies solving for the optimal allocation of expenditure across the bundle of goods produced in different countries and across the varieties of each

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[13]Hereafter the subscript indicates the provenance of the good.

bundle. The solution yields to the following demand functions for $i, j = 1, 2$

$$c^i_{j,t}(h) = \left(\frac{p_{j,t}(h)}{P_{j,t}}\right)^{-\theta} C^i_{j,t}$$

$$C^i_{j,t} = \alpha \left(\frac{P^i_t}{P_{j,t}}\right) C^i_t \quad for \ j \neq i$$

$$C^1_{1,t} = (1 - \alpha - \omega + \varepsilon) \left(\frac{P^c_{1,t}}{P_{1,t}}\right) C^1_t$$

$$C^2_{2,t} = (1 - \alpha - \omega - \varepsilon) \left(\frac{P^c_{2,t}}{P_{2,t}}\right) C^2_t.$$

Analogously, the demands for the goods imported from the rest of the world by households of the two regions are:

$$C^1_{row,t} = (\omega - \varepsilon) \left(\frac{P^c_{1,t}}{P^*_{row,t} S_t}\right) C^1_t$$

$$C^2_{row,t} = (\omega + \varepsilon) \left(\frac{P^c_{2,t}}{P^*_{row,t} S_t}\right) C^2_t$$

where S_t is the nominal exchange rate between the currency of the monetary union and that of the rest of the world and the asterisk on a variable indicates that it is expressed in the currency of the rest of the world. Given these demand functions one can define the following producer price (PPI) and consumer price (CPI) indices for the region $i = 1, 2$ of the monetary union

$$P_{i,t} = \left(\int_0^1 p_{i,t}(h)^{1-\theta} \, dh\right)^{1/(1-\theta)} \tag{1.5}$$

$$P^c_{1,t} = P^{(1-\alpha-\omega+\varepsilon)}_{1,t} P^\alpha_{2,t} \left(P^*_{row,t} S_t\right)^{(\omega-\varepsilon)} \tag{1.6}$$

$$P^c_{2,t} = P^{(1-\alpha-\omega-\varepsilon)}_{2,t} P^\alpha_{1,t} \left(P^*_{row,t} S_t\right)^{(\omega+\varepsilon)}. \tag{1.7}$$

The second step involves the household optimal choice of the hours worked and of the intertemporal allocation of consumption. The first order conditions for the representative agent of country $i = 1, 2$ are

$$(C^i_t)^\sigma (N^i_t)^\varphi = \frac{W^i_t}{P^c_{i,t}} \tag{1.8}$$

$$\beta \left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\sigma} \left(\frac{P^c_{i,t}}{P^c_{i,t+1}}\right) = Q_{t,t+1}. \tag{1.9}$$

By taking expectations conditional on the information available at time t on both sides of (1.9) I obtain the following Euler equation

$$\beta R_t E_t \left\{ \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{P_{i,t}^c}{P_{i,t+1}^c} \right) \right\} = 1 \tag{1.10}$$

where $R_t = 1/E_t Q_{t,t+1}$ is the return on a riskless bond in the currency of the monetary union. Under the assumption of complete markets, combining the Euler equations of each region and assuming that the initial debt holdings are equal across regions, one obtains the following risk sharing condition

$$C_t^1 = \left(\frac{P_{2,t}^c}{P_{1,t}^c} \right)^{1/\sigma} C_t^2 \quad ; \quad C_t^i = \left(\frac{P_{row,t}^c S_t}{P_{i,t}^c} \right)^{1/\sigma} C_t^{row} \quad for \ i = 1, 2. \tag{1.11}$$

If $\sigma = 1$, i.e. the utility is logarithmic in consumption, the risk sharing is such that the consumption expenditure in the three countries is the same at each time t and the model does not exhibit a foreign asset dynamics. Even in this special parametric case, however, risk sharing is not perfect. In fact, the existence of home bias in consumption implies that purchasing power parity (PPP) does not hold.

Moreover, under the assumption of complete financial markets the equilibrium price of the foreign riskless bond in terms of the monetary union's currency is equal to $(R_t^*)^{-1} S_t = E_t Q_{t,t+1} S_{t+1}$. Combining it with the equilibrium price of the riskless bond in the currency area (that is $R_t = 1/E_t Q_{t,t+1}$), one obtains the uncovered interest parity (UIP) condition

$$E_t \left\{ Q_{t,t+1} \left[R_t - R_t^* \left(\frac{S_{t+1}}{S_t} \right) \right] \right\} = 0. \tag{1.12}$$

1.3.2   Firms

In each region there is a continuum of mass 1 of firms. Each firm produces a variety of the differentiated good in a regime of monopolistic competition. If a firm is located in region 1, she can produce only a variety of the consumption good produced in that region ($C_{1,t}$). The technology available to produce the variety h is linear in the labor input, i.e.

$$Y_{i,t}(h) = A_{i,t} N_{it}(h)$$

where $A_{i,t}$ is a technology shock that is region-specific.

Each firm chooses the price that maximizes her profit taking as given the demand of her variety. The demands of the variety h of the goods produced in the two regions of the currency area are given by

$$
\begin{aligned}
y_{1,t}(h) &= \left(\frac{p_{1,t}(h)}{P_{1,t}}\right)^{-\theta}\left[(1-\alpha-\omega+\varepsilon)\left(\frac{P_{1,t}^c}{P_{1,t}}\right)C_t^1 + \alpha\left(\frac{P_{2,t}^c}{P_{1,t}}\right)C_t^2 + \right. \\
&\quad \left. +(\omega-\varepsilon)\left(\frac{P_{row,t}^c S_t}{P_{1,t}}\right)C_t^{row}\right] \tag{1.13} \\
y_{2,t}(h) &= \left(\frac{p_{2,t}(h)}{P_{2,t}}\right)^{-\theta}\left[(1-\alpha-\omega-\varepsilon)\left(\frac{P_{2,t}^c}{P_{2,t}}\right)C_t^2 + \alpha\left(\frac{P_{1,t}^c}{P_{2,t}}\right)C_t^1 + \right. \\
&\quad \left. +(\omega+\varepsilon)\left(\frac{P_{row,t}^c S_t}{P_{2,t}}\right)C_t^{row}\right] \tag{1.14}
\end{aligned}
$$

However, price stickiness à la Calvo implies that firms are not able to change their price whenever they want. Each period there is a probability δ, independent of time, that the price cannot be changed. This implies that the optimal price is decided in a forward-looking manner, as a markup charged over the expected value of future discounted marginal costs. The log-linear expression for the optimal new price is[14]

$$
p_{i,t}^{new} = \mu + (1-\beta\delta)\sum_{k=0}^{\infty}(\beta\delta)^k E_t\left\{rmc_{i,t+k} + p_{i,t}\right\}.
$$

where μ is the markup and depends on the substitutability among varieties (i.e. $\mu \equiv \log\left(\frac{\theta}{\theta-1}\right)$) and $rmc_{i,t+k}$ are the (log deviation) of the real marginal costs, defined in the next section.

1.3.3 Equilibrium

The labor market is competitive and its clearing condition implies that firms' real marginal costs for $i = 1, 2$ are

$$
RMC_{i,t} = (1-\tau)(C_t^i)^\sigma(N_t^i)^\varphi \frac{P_{i,t}^c}{P_{i,t}A_{i,t}}. \tag{1.15}
$$

---

[14]Variables written in lower case letters are log deviations from the deterministic steady state of that variable.

where τ is a subsidy to the labor supply and it is assumed to be the same across regions of the currency area. Before deriving the clearing condition of the market for goods in each region, one must first aggregate across all varieties both the demand and the supply side. The index for aggregate output in region i is defined as $Y_{i,t} \equiv \left[\int_0^1 y_{i,t}(h)^{(\theta-1)/\theta} dh \right]^{\theta/(\theta-1)}$. Aggregating over all varieties h in equations (1.13) and (1.14), I obtain the following aggregate demand function for the goods 1 and 2

$$
\begin{aligned}
Y_{1,t} &= (1 - \alpha - \omega + \varepsilon) \left(\frac{P_{1,t}^c}{P_{1,t}} \right) C_t^1 + \alpha \left(\frac{P_{2,t}^c}{P_{1,t}} \right) C_t^2 + \\
&\quad + (\omega - \varepsilon) \left(\frac{P_{row,t}^c S_t}{P_{1,t}} \right) C_t^{row} \quad\quad\quad\quad (1.16) \\
Y_{2,t} &= (1 - \alpha - \omega - \varepsilon) \left(\frac{P_{2,t}^c}{P_{2,t}} \right) C_t^2 + \alpha \left(\frac{P_{1,t}^c}{P_{1,t}} \right) C_t^2 + \\
&\quad + (\omega + \varepsilon) \left(\frac{P_{row,t}^c S_t}{P_{2,t}} \right) C_t^{row} \quad\quad\quad\quad (1.17)
\end{aligned}
$$

To obtain the aggregate production function I make use of the index of aggregate employment defined by Galì and Monacelli (2005): $N_t^i \equiv \int_0^1 N_t^i(h) dh = \frac{Y_{i,t} Z_{i,t}}{A_{i,t}}$ where $Z_{i,t} \equiv \int_0^1 \frac{y_{i,t}(h)}{Y_{i,t}} dh$. They show that equilibrium variations of $z_{i,t} \equiv \log Z_{i,t}$ around the perfect foresight steady state are of second order and can thus be ignored in a first order approximation. The loglinear expression for the aggregate production function is then

$$
y_{i,t} = a_{i,t} + n_{i,t}.
$$

1.3.4   Solution of the model

In this section I propose a solution of the model described above that has three features. First, it is a log linear solution. The optimality conditions of the firms and the households in each region together with the market clearing conditions are log linearized around the trade-balanced deterministic steady state. In this steady state the three countries share the same level of consumption, output and hours. Whenever ε is different from zero, the only difference among the steady state equilibrium of each country is the composition of their consumption bundles.[15]

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[15]It can be shown that both the terms of trade, the one between the two regions of the currency area and the one between the currency area and the rest of the world are uniquely pinned down in

Second, instead of obtaining the dynamics of the macroeconomic variables of each of the two regions of the currency area, I rewrite the model so as to have a solution for the dynamics of area-wide variables and a solution for the differentials between the two regions. I obtain the area-wide variables by aggregating those of the two regions weighted by their relative size and the differentials by comparing the variables in the more open region with those in the more closed one. In other terms, considering the generic variable x, I define the area-wide corresponding variable as $x_u \equiv mx_1 + (1 - m)x_2$, where m is the size of the region 1, and the differentials as $x_r \equiv x_2 - x_1$. The model has therefore two blocks: the area-wide and the differentials block. Solving the model in this way allows me to shed light on what is the main contribution of introducing heterogeneity in an otherwise standard framework of a homogeneous currency area. In the benchmark case I assume that the regions have equal size.

Third, I solve the model in the case of fully flexible prices and sticky prices. As it is standard in the literature I then rewrite it in terms of the gaps between these two equilibria to highlight the effects of nominal rigidities.

Henceforth I use the following definitions. The *internal terms of trade* is the relative price of the good produced in the more open region of the currency area and the one produced in the less open one, i.e. $T^{in} \equiv \frac{P_2}{P_1}$. The *external terms of trade* is the relative price between the good produced in the rest of the world and the aggregate of the goods produced in the currency area, i.e. $T^{ex} \equiv \frac{SP_{row}}{P_u}$. Analogously, the *internal real exchange rate* is the ratio of the CPIs of the two regions, i.e. $Q^{in} \equiv \frac{P_2^c}{P_1^c}$. The *external real exchange rate* is the ratio of CPIs of the rest of the world and the aggregate of the currency area, i.e. $Q^{ex} \equiv \frac{SP_{row}^c}{P_u^c}$. The internal terms of trade and real exchange rate can be thought of as measures of the producer and consumer price level differentials, respectively.

As mentioned above, the rest of the world is not explicitly modeled and fluctuations in its macroeconomic variables are specified as exogenous stochastic processes. In this section I report the equations that describe the dynamics of the small-scaled version of the model. The reader should refer to the appendix A.1 for the algebraic

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the perfect foresight steady state.

derivations behind them.

The equations of the area-wide block

By combining the aggregate demands for the two goods produced in the currency area (equations (1.16) and (1.17)) and using the risk sharing conditions (1.11), I obtain the following aggregate demand for the goods produced in the currency area

$$y_{u,t} = \frac{1}{\sigma_\omega} t_t^{ex} + y_{row,t} - \sigma_\varepsilon t_t^{in} \tag{1.18}$$

where $y_{row,t}$ is the output of the rest of the world. The parameter σ_ε and σ_ω are combinations of the structural parameters that affect the openness dimension of the economy $(\alpha, \omega, \varepsilon)$, the inverse of the intertemporal elasticity of substitution (σ) and the size of the regions, which is assumed to be equal to 0.5 throughout the paper (see appendix A.1).

Equation (1.18) is similar to the aggregate demand equation of the open economy of Galí and Monacelli (2005). Since the currency area economy is open to the rest of the world and given the assumption of market completeness, a change in the rest of the world output $(y_{row,t})$ affects the demand for currency area goods. Analogously, an increase of the external terms of trade enhances the competitiveness of the monetary union goods, thus boosting the aggregate demand for them. When there is no heterogeneity in the preferences, that is $\varepsilon = 0$, the parameter σ_ε is equal to zero.[16] In this case the rest of the world is indifferent between the goods produced in the two regions and the average level of competitiveness of the currency area goods (t_t^{ex}) is sufficient to determine the aggregate demand for the area goods. When there is heterogeneity in preferences, that is $\varepsilon \neq 0$, the last term in (1.18) is different from zero and the price dispersion across regions matters. If, *ceteris paribus*, the goods produced by the more open region becomes more competitive (i.e. t_t^{in} decreases), the demand of currency area goods coming from the rest of the world increases.

As regards the supply side of the area-wide economy, log linearizing equation

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[16]In this case, it also results that $\sigma_\omega = \frac{\sigma}{\omega\sigma + (\sigma-1)(1-\omega)\omega + (1-\omega)}$ and the currency area economy behaves exactly as the small open economy of Galí and Monacelli (2005).

(1.15) for $i = 1, 2$, substituting for the risk sharing conditions (1.11) and aggregating across regions of the currency area, I obtain the following expression for the area-wide real marginal costs

$$rmc_{u,t} = -\nu + (\sigma_\omega + \varphi)y_{u,t} + (\sigma - \sigma_\omega)y_{row,t} + \sigma_\omega\sigma_\varepsilon t_t^{in} - (1 + \varphi)a_{u,t}. \tag{1.19}$$

where $\nu = -log(1-\tau)$. Since the currency area is an open economy, the real marginal costs are affected not only by the demand for domestic goods $(y_{u,t})$ and by the domestic productivity level $(a_{u,t})$, but also by the world aggregate demand $(y_{row,t})$. Increases of the domestic and international demand call for a higher labor supply that can be provided by households at higher real wages, driving up the real marginal cost.

In addition, when $\varepsilon \neq 0$, the internal terms of trade matters as well. In fact, in an open economy the real marginal costs depend also on the ratio between the consumer price level and the producer price level. This is because the real wage that matters for firms is deflated with the producer price index, while the one relevant for households is deflated with the consumer price index. If the two economies have the same degree of home bias the area-wide real marginal cost, aggregated from the two regions real marginal costs, is affected by the gap between the union CPI and the union PPI which can be rewritten in terms of the external terms of trade.[17] When countries are heterogeneous in the degree of home bias (or external trade openness) the gap between the currency area CPI and the PPI does not depend only on the external terms of trade but also on the internal terms of trade.

Given the expression for the real marginal costs, the natural level of output, that is the level of output when prices are fully flexible, is given by[18]

$$\tilde{y}_{u,t} = \frac{\nu - \mu}{(\sigma_\omega + \varphi)} - \frac{\sigma - \sigma_\omega}{(\sigma_\omega + \varphi)}y_{row,t} - \frac{\sigma_\omega\sigma_\varepsilon}{(\sigma_\omega + \varphi)}\tilde{t}_t^{in} + \frac{(1 + \varphi)}{(\sigma_\omega + \varphi)}a_{u,t}. \tag{1.20}$$

Linearizing and aggregating across regions the firms' price optimality conditions imply that the inflation dynamics is described by the following New Keynesian Phillips

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[17]Using equation (1.18) the external terms of trade can then be rewritten in terms of domestic and foreign demand, as it is done in equation (1.19).

[18]The variables that are expressed at their natural level are indicated by a tilde ˜.

curve[19]

$$\pi_{u,t} = \beta E_t \pi_{u,t+1} + \lambda rmc_{u,t}$$

where $\lambda = \frac{(1-\beta\delta)(1-\delta)}{\delta}$. Substituting for the real marginal costs I have

$$\pi_{u,t} = \beta E_t \pi_{u,t+1} + \kappa_x^u x_{u,t} + \kappa_T^u (t_t^{in} - \tilde{t}_t^{in}) \qquad (1.21)$$

where $x_{u,t}$ is the output gap (i.e. $x_{u,t} \equiv y_{u,t} - \tilde{y}_{u,t}$) and the parameters are given by

$$\kappa_x^u \equiv \lambda(\sigma_\omega + \varphi) > 0$$
$$\kappa_T^u \equiv \lambda\sigma_\omega\sigma_\varepsilon > 0$$

In the absence of heterogeneity, the last term of (1.21) drops and the dynamics of the currency area inflation is described as the standard one of an open economy. When preferences are heterogeneous, inflationary or deflationary pressures arise also in the case in which the relative price of the currency area goods is not at its natural level.[20] A positive internal terms of trade gap indicates that the price of good 1, the one that is preferred by most of the currency area citizens, is lower than it should be in the flexible price equilibrium. This brings about an inefficient reallocation of resources from region 2 toward region 1, generating an increase in the aggregate demand for currency area goods (beyond the one captured by the area-wide output gap term) that drives up inflation. The opening of an internal terms of trade gap implies that there is a trade off between the output gap and the inflation stabilization in the currency area and that the complete stabilization of inflation is not equivalent to the stabilization of the gap between the actual and the natural output level. Such trade off arises endogenously after any misalignments of the price differentials from its natural level $\tilde{t}_t^{in}$.

From the household intertemporal first order conditions I obtain the following IS curve

$$x_{u,t} = E_t x_{u,t+1} + \sigma_\varepsilon E_t \Delta(t_{t+1}^{in} - \tilde{t}_{t+1}^{in}) - \frac{1}{\sigma_\omega}(r_t - E_t \pi_{u,t+1} - \widetilde{rr}_{u,t}) \qquad (1.22)$$

---

[19]For a complete derivation see Appendix B of Galì and Monacelli (2005).

[20]Notice that, in a first order approximation, the only relevant relative price in the currency area is the internal terms of trade.

where $\widetilde{rr}_{u,t}$ is the natural rate of interest, which is a function of the primitive shocks in the currency area economy

$$\widetilde{rr}_{u,t} \equiv \phi_{y_{row}} E_t \Delta y_{row,t+1} + \phi_{t^{in}} E_t \Delta \tilde{t}^{in}_{t+1} + \phi_{a_u} E_t \Delta a_{u,t+1}.$$

where $\phi_{y_{row}}$, $\phi_{t^{in}}$ and ϕ_{a_u} are defined in the appendix A.1. Interestingly, the internal terms of trade gap plays also a role in the aggregate demand side of the currency area economy.[21]

The equations of the differentials block

The dynamics of the inflation differentials in the currency area are given by

$$\pi_{r,t} = \beta E_t \pi_{r,t+1} + \lambda rmc_{r,t}.$$

After solving for the difference of the real marginal costs in the more and less open region, the inflation differentials are given by the following equation

$$\pi_{r,t} = \beta E_t \pi_{r,t+1} + \kappa^r_T (t^{in}_t - \tilde{t}^{in}_t) + \kappa^r_x x_{u,t}. \tag{1.23}$$

where

$$\kappa^r_x \equiv \lambda \xi_{ex} \sigma_\omega > 0$$
$$\kappa^r_T \equiv \lambda \left(\xi_{in} + \xi_{ex} \sigma_\omega \sigma_\varepsilon \right) < 0$$

As specified in the appendix A.1, the parameter ξ_{ex} is equal to zero when $\varepsilon = 0$. Hence, heterogeneity implies that the area-wide variables, namely the output gap, affect the dynamics of the differentials.

Benigno (2004) points out the disconnection of the terms of trade dynamics from the area-wide block of the model in a currency area with homogenous regions hit by asymmetric shocks and the inability for the authority that controls the area-wide nominal interest rate to affect the differentials. This is a feature of the proposed model when $\varepsilon = 0$. Instead, when $\varepsilon \neq 0$, the area-wide and difference blocks of the model cannot be solved separately.

---

[21]The determinants of natural internal terms of trade are defined in the next section.

When prices are fully flexible, the internal terms of trade is affected by asymmetric shocks (i.e. $a_{r,t}$) and, whenever there is heterogeneity across regions, by the area-wide shocks (i.e. $a_{u,t}$ and $y_{row,t}$). The natural internal terms of trade is thus

$$\tilde{t}_t^{in} = \gamma_{y_{row}} y_{row,t} + \gamma_{a_u} a_{u,t} - \gamma_{a_r} a_{r,t}. \tag{1.24}$$

where $\gamma_{y_{row}}$, γ_{a_u} and γ_{a_r} are defined in the appendix A.1.

The block of the model that describes the behavior of the differentials within the currency area is closed by specifying the dynamics of the internal terms of trade gap. From the definition of the internal terms of trade (i.e. $t_t^{in} \equiv p_{2,t} - p_{1,t}$) this is given by

$$t_t^{in} - \tilde{t}_t^{in} = t_{t-1}^{in} - \tilde{t}_{t-1}^{in} + \pi_{r,t} - (\tilde{t}_t^{in} - \tilde{t}_{t-1}^{in}). \tag{1.25}$$

1.4   The effects of heterogeneity

This section analyzes quantitatively the effect of heterogeneity on the dynamics of the area-wide inflation and output gap, the inflation differentials and the terms of trade in the model set up above. I first describe the calibration for the EMU and then I study the transmission mechanism of a symmetric technology shock under an *ad hoc* monetary policy rule.

1.4.1   Calibration

The time period is the quarter. The value for the time-discount parameter β is set equal to 0.99 so that the steady state real interest rate is 4% in annual terms. The inverse of the intertemporal elasticity of substitution σ is set equal to 2. There is no clear evidence on what should be the value of this parameter. However, in studying the effects of external developments on the home economy in the case of complete markets, one wants to depart from the case of logarithmic utility (i.e. $\sigma = 1$) because it implies that the external influences on the home economy are shut down and the open economy is isomorphic to the closed one.[22] The parameter φ represents the

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[22]See Clarida et al. (2001)).

inverse of the labor supply elasticity and, as it is common in the real business cycle literature, it is set to be equal to 3. The elasticity of substitution between varieties of each differentiated good (θ) is equal across countries and it is set so that the steady state mark up is 1.2. Therefore, θ equals to 6. I calibrate the Calvo parameter δ, that indicates the degree of price stickiness using the standard value of 0.75. This implies an average duration of the price contract of 4 quarters.

The parameters for the degrees of openness that need to be calibrated are α, ω and ε. The first one is the degree of internal openness and it is calibrated as the average of the GDP share of imports from other members of the union across 11 European countries.[23] In fact, there is heterogeneity across member countries also in this parameter. However, I restrict this parameter to be the same across countries to insulate the effect of heterogeneity in the degree of external openness for the reasons explained in section 1.2. Hence, I set $\alpha = 0.25$. The parameter for external openness ω is set equal to the average of the GDP share of the external import across 11 member countries of the euro area, that is $\omega = 0.17$. The parameter that measures heterogeneity ε takes the values of 0 and 0.14. The former represents the case of homogeneous regions whereas the latter is set so that $\omega + \varepsilon$ is the average degree of external openness across the more open countries (the Netherlands, Belgium and Ireland).

There are three sources of exogenous fluctuations in the model: the aggregate technology shock $a_{u,t}$, the asymmetric technology shock $a_{r,t}$ and the external shock $y_{row,t}$. I assume that their stochastic processes are all AR(1). To calibrate the parameters of the external shock process I use, as a proxy for the world aggregate demand, the log of the world demand variable of the Area Wide Model dataset (Fagan et al. (2001)) and I fit the following univariate trend-stationary process [24]

$$y_{row,t} = 0.60 + 0.0003t + 0.94 y_{row,t-1} + \epsilon_{row,t}.$$

The standard deviation of the innovations is $\sigma_{y_{row}} = 0.005859$. In order to calibrate the shocks for the area-wide and differentials of productivity, I consider the primitive

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[23]Luxembourg, Slovenia, Cyprus, Malta and Slovakia are excluded.
[24]I used quarterly data from 1970:1 until 2005:4.

technology shocks in the two regions and I specify them as

$$a_{1,t} = \rho a_{1,t-1} + \epsilon_{1,t}$$

$$a_{2,t} = \rho a_{2,t-1} + \epsilon_{2,t}.$$

The persistence parameter ρ and the standard deviation of the innovation σ_ϵ are the same across regions and equal respectively to 0.94 and 0.0061, based on the evidence of Smets and Wouters (2003) for the entire euro area. Moreover the shocks are uncorrelated across regions (i.e. $corr(\epsilon_{1,t}, \epsilon_{2,t}) = 0$). Hence, the processes for $a_{u,t} \equiv m a_{1,t} + (1-m)a_{2,t}$ and $a_{r,t} \equiv a_{2,t} - a_{1,t}$ have the following specification

$$a_{u,t} = \rho a_{u,t-1} + \epsilon_{u,t}$$

$$a_{r,t} = \rho a_{r,t-1} + \epsilon_{r,t}$$

where the variance of the innovations are $\sigma_u^2 = \left[m^2 + (1-m)^2\right]\sigma_\varepsilon^2$ and $\sigma_r^2 = 2\sigma_\epsilon^2$.

1.4.2 The transmission mechanism of area-wide technology shocks

To highlight the effect of heterogeneity in the transmission mechanism of an exogenous disturbance and its interaction with nominal rigidities, I consider the responses to an iid area-wide technology shock under a monetary policy regime of strict inflation targeting. The monetary policy is represented by the following targeting rule

$$\pi_{u,t} = 0.$$

Figure 1.3 displays the responses of the area-wide and differentials variables to a positive one percent symmetric shock in technology (a_u) in the case of homogeneous (solid line) and heterogeneous regions (starred line).

A positive technology shock induces a decline in the natural interest rate of the currency area. In order to keep inflation stable the monetary authority fully accommodates such decline by decreasing the nominal interest rate. If the rest of the world does not respond to the shock, such monetary policy response generates a depreciation of the nominal exchange rate on impact and an increase of the external terms of trade.

Figure 1.3: IRFs to a positive area-wide technology shock under strict inflation targeting
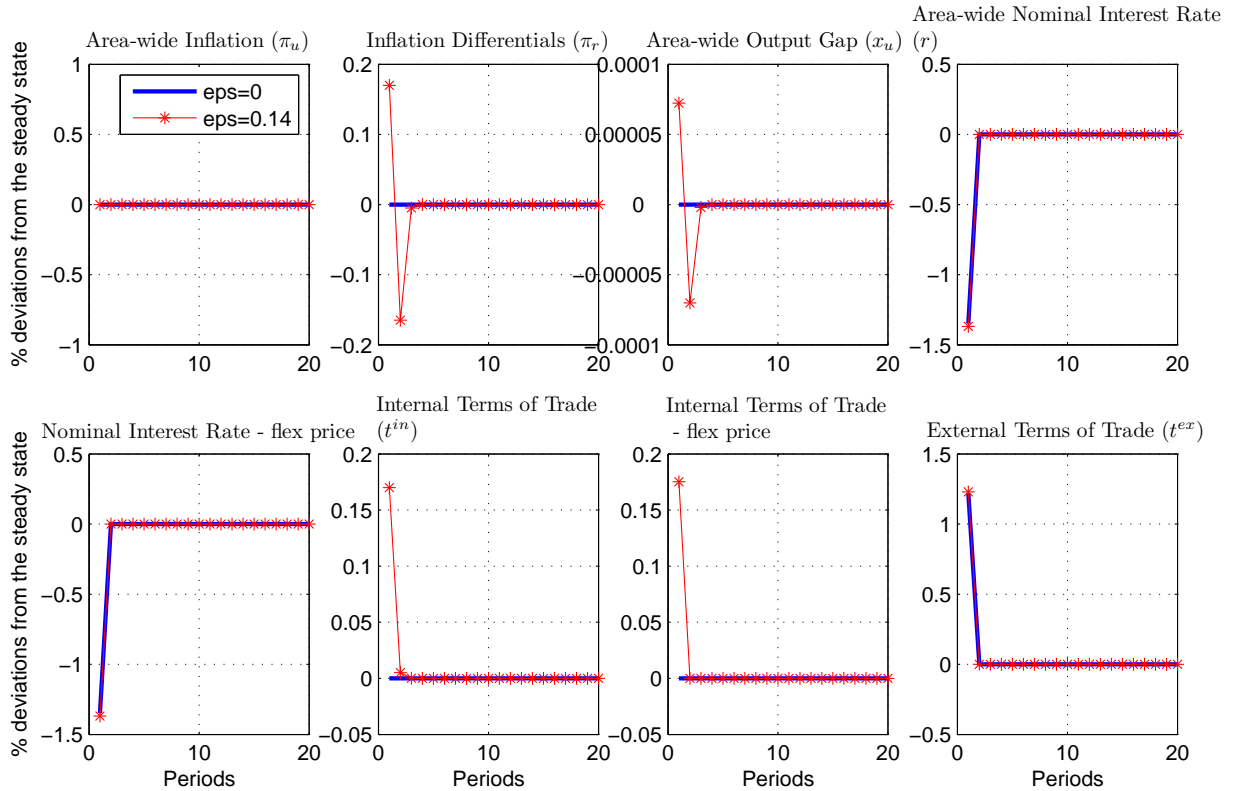
Notes: The shock is equal to 1% st dev of the area-wide technology. The solid line is the homogeneity case ($\varepsilon = 0$); the starred line the heterogeneity one ($\varepsilon = 0.14$).

When the regions of the currency area are homogeneous (i.e. $\varepsilon = 0$) the dynamics of the model is standard. Under the policy of strict inflation stabilization, the central bank adjusts the nominal interest rate so that the area-wide inflation remain stable. This policy results in closing the gap between the actual and natural output level. The currency area goods are more competitive following the exogenous rise in productivity and the depreciation of the nominal exchange rate induced by the monetary policy response. Moreover, the internal terms of trade is not affected by the area-wide technology shock and no inflation differentials are generated.

When the regions of the currency area are differently exposed to the rest of the

world (i.e. $\varepsilon \neq 0$), the nominal depreciation generated by the monetary policy response has a different impact on the two regions. The demand of the goods produced by the more open region (good 2) increases more than the one of the other goods produced in the area. The different demand pressures induce an adjustment of the relative price of the goods in the two regions, that is an increase in the internal terms of trade.

The dynamics of the inflation differentials is driven by the gap between the actual internal terms of trade and its flexible price level and by the area-wide output gap. As pointed out by Aoki (2001) and Benigno (2004), the complete price stabilization induced by the policy rule considered here has the effect of inducing a sluggish response of the actual internal terms of trade.[25] The internal terms of trade gap is thus negative on impact and positive in subsequent periods. A negative internal terms of trade gap indicates that, due to nominal rigidities, the relative price of the goods produced in the more open region is lower than in the flexible price equilibrium. This generates inflationary pressures on that region and thus an inflation differential. The increase of the area-wide output gap in the quarter of the shock exacerbates the effect on the inflation differentials. Inheriting the sluggishness of the internal terms of trade, the inflation differentials are slightly more persistent than the shock that generates them.[26]

1.5   Optimal monetary policy

I compute and analyze the solution of the model presented above under a policy that maximizes the welfare of the currency area households. I obtain a microfounded measure of welfare by taking a second order approximation of the utilities of the

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[25] Aoki (2001) and Benigno (2004) focus on changes in the natural level of the relative price (or internal terms of trade) caused by asymmetric shocks across countries (or sectors). Here such changes are brought about by the heterogeneous transmission mechanism of a symmetric shock.

[26] Whenever there is heterogeneity across regions, the sluggishness of the internal terms of trade is transmitted to the area-wide variables. The effect is, however, quantitatively small in a two-region economy. Carlstrom et al. (2006) show that differences in the degree of price stickiness or asymmetric sectoral monetary policy responses generate a relative price dynamics that affects the inflation and the output gap in aggregate. In this case, asymmetric responses to area-wide shocks, due to heterogeneity in preferences, is what generates a terms of trade dynamics.

currency area citizens. The optimal monetary policy problem takes the form of a standard linear-quadratic optimization problem as it is common in the literature.[27] In this section I describe the distortions of the market allocation with respect to the Pareto efficient one, the welfare measure adopted and some moments that characterize the optimal policy plan, namely the impulse response functions after an area-wide and an asymmetric technology shock.

1.5.1 Flexible price equilibrium and the Pareto efficient allocation

In the open currency area described above three distortions move the market allocation away from the Pareto efficient one. (i) The firms market power lowers the output relative to the one in the efficient equilibrium; (ii) the presence of nominal rigidities alter the relative prices, therefore causing a misallocation of resources across the varieties of the differentiated goods; (iii) the market equilibrium allocation does not take into account the incentive to distort the terms of trade with the rest of the world in a way beneficial to the currency area citizens.[28] The social planner of the currency area, in fact, would tend to increase domestic consumers' purchasing power internationally. An improvement in the external terms of trade induces the currency area citizens to consume more of the imported good, reducing the hours worked without diminishing their overall level of consumption. Heterogeneity *per se* does not introduce a distortion in the economy. It is the stickiness of prices coupled with the fact that the nominal exchange rate across regions is irrevocably fixed that generates an additional distortion in the economy: the sluggish adjustment of the relative price of the goods produced in the two regions after a shock implies a misallocation of the resources.[29]

Galì and Monacelli (2005) show the optimality of strict domestic inflation stabilization for a special parametrization under which both the intertemporal and the intratemporal elasticities of substitution are equal to unity. In this case a constant

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[27]See Benigno and Woodford (2006).

[28]See Corsetti and Pesenti (2001) and De Paoli (2009) among others.

[29]Aoki (2001) and Benigno (2004) highlight such distortion.

subsidy to monopolistic competitive firms in steady state is sufficient to guarantee the coincidence between the flexible price allocation and the efficient one. When the subsidy is implemented, the only distortion left in the economy is price stickiness; setting inflation to zero delivers the Pareto efficient allocation. In fact, in this special case the economy behaves as if it is closed. Faia and Monacelli (2008) and De Paoli (2009) relax these assumptions and show that, in the more realistic case in which the three distortions mentioned above are in place, the flexible price allocation is not optimal and the optimal policy prescribes some inflation volatility, together with a partial stabilization of the exchange rate.

Since this paper studies how the openness dimension affects the monetary policy design in a currency area it is important to consider a calibration such that the currency area economy is not isomorphic to a closed one. For this reason in the benchmark calibration the intertemporal elasticity of substitution is different from one. In this case, even when the labor subsidy is in place, the policy maker can improve the welfare of the currency area citizens achieved under the flexible price equilibrium allocation by taking into account that it has the ability to manipulate the external terms of trade. This result holds independently from the degree of heterogeneity across regions.

1.5.2 The currency area welfare function

This subsection describes the welfare measure for the currency area economy, which is based on the discounted sum of the future utility flows of the households of both regions of the monetary union. For region $i = 1, 2$ the utility flows are given by

$$w_t^i = U(C_t^i, N_t^i) \equiv \left[\frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \right].$$

Depending on the region in which they live, there are two types of agents in the currency area. As monetary policy is decided at an area-wide level, the relevant welfare function must be only one and a criterion to aggregate the utility functions of each type of agent is needed. I consider an utilitarian social welfare function. Among the aggregation functions, this specification gives less weight to the heterogeneity

across regions and it implies that the currency area welfare function is a weighted average of the welfare of each region, where the weights are equal to the sizes of the regions, that is

$$W^u \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[m w_t^1 + (1-m) w_t^2 \right].$$

In order to have the optimal monetary policy problem in the standard linear-quadratic form I take a second order Taylor expansion of the utility flows of each region around the optimal deterministic steady state. Then I aggregate across regions and, after substituting for the linear terms using the second order approximation of the model's equilibrium condition, I obtain a purely quadratic approximation of the currency area welfare function. The objective of the currency area benevolent monetary authority can be written as follows

$$-E_0 \sum_{t=0}^{\infty} \beta^t L_t^u$$

where L_t^u is a measure of the deadweight losses:[30]

$$
L_t^u = \left\{ \Phi_{\pi_u} \pi_{u,t}^2 + \Phi_{x_u} \left(y_{u,t} - \tilde{\tilde{y}}_{u,t} \right)^2 + \Phi_{t^{in}} (t_t^{in} - \tilde{\tilde{t}}_t^{in})^2 + \Phi_{\pi_r} \pi_{r,t}^2 + \right. \tag{1.26}
$$

$$
\left. + \Phi_{x_u, t^{in}} \left(y_{u,t} - \tilde{\tilde{y}}_{u,t} \right) (t_t^{in} - \tilde{\tilde{t}}_t^{in}) \right\} + t.i.p. + o(\|\xi\|^3)
$$

where a double tilde on a variable indicates its efficient level, $t.i.p.$ stands for terms independent from policy and $o(\|\xi\|^3)$ contains all terms that are of order higher than two in the given bound for the vector of shocks $\xi \equiv [y_{row} \ \ a_u \ \ a_r]'$. The deadweight losses are generated by any deviation of the actual variables from their efficient level. The parameters Φs are functions of the structural parameters of the model and their values under the baseline calibration of section 1.4.1 are reported in the table 1.

When $\varepsilon = 0$, the loss function in (1.26) has the same terms as the one obtained in Benigno (2004). In particular, compared with the loss function of a closed economy, there are two additional quadratic terms beyond the ones on inflation and output gap: the internal terms of trade gap and the inflation differentials. The deviation of the internal terms of trade from its efficient level generates losses as resources are

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[30]Details on the derivation of this function are in the appendix A.2.

Table 1.1: The coefficients of the utility-based loss function under the benchmark calibration

ε	**0**	**0.14**
Φ_{π_u}	69.9	69.9
Φ_{x_u}	3.54	3.52
Φ_{π_r}	17.5	17.5
$\Phi_{t^{in}}$	0.07	0.07
$\Phi_{x_u,t^{in}}$	0	-0.14

inefficiently moved from one region to the other. As explained above, the internal terms of trade does not adjust immediately to region-specific fluctuations because there is price stickiness and the nominal exchange rate is irrevocably fixed and this brings about a cost at the area-wide level.

Moreover, equation (1.26) features not only a term that penalizes the fluctuations in area-wide inflation, but also one that penalizes fluctuations in the inflation differentials. As shown by Woodford (2003, ch. 6) the presence of nominal rigidities in the form of Calvo price stickiness generates a motive for inflation stabilization. Since we assumed that such distortion in the market equilibrium is present in the production of goods in both regions, implementing the efficient allocation requires that inflation of the domestic goods produced in each region is stabilized. A loss function that penalizes only the aggregate currency area inflation does not guarantee that the volatilities of the national inflations are minimized. The requirement that is needed is that both the area-wide inflation and the differentials across regions are stabilized.[31]

When $\varepsilon \neq 0$, the negative comovement between the area-wide output gap and the internal terms of trade gap gives rise to a deadweight loss ($\Phi_{x_u,t^{in}}$ is zero when the $\varepsilon = 0$ and negative when $\varepsilon > 0$). A positive internal terms of trade gap indicates a competitive advantage of the country that is less oriented to produce goods for the rest of the world market. This would induce a decline in the demand for currency area goods coming from the rest of the world and thus of the aggregate demand

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[31]Notice that in equation (1.26) I could have written $\Phi_{\pi_u}\left[m\pi_{1,t}^2 + (1-m)\pi_{2,t}^2\right]$ instead of $\Phi_{\pi_u}\left[\pi_{u,t}^2 + m(1-m)\pi_{r,t}^2\right]$.

of currency area goods (see equation (1.18)). Therefore, when this is coupled with a negative area-wide output gap the efficiency cost of the shift of the work effort toward the less open region is stronger.

### 1.5.3 The IRFs to an area-wide technology shock under the optimal plan

Figure 1.4 describes the impulse response functions to an area-wide technology shock under the optimal policy plan, comparing the case in which regions are homogeneous ($\varepsilon = 0$) with the one in which regions are heterogeneous ($\varepsilon = 0.14$).[32]

An area-wide technology shock increases the relative competitiveness of the currency area goods with respect to those produced in the rest of the world. The optimal policy accommodates such increase in competitiveness by a nominal exchange rate depreciation and an increase of the external terms of trade t_t^{ex}. Due to the external terms of trade externality however the social planner wants to partially dampen the response of the exchange rate, therefore the area-wide inflation is not perfectly stabilized after the shock. When regions are homogeneous, an area-wide shock affects equally all member countries of the area; thus the internal terms of trade gap and the inflation differential do not move.

When regions are differently open toward the rest of the world, the more open region (region 2) benefits more from the gain in competitiveness as the rest of the world prefers its good to the one produced in the other region. Hence, in the flexible price equilibrium, the higher demand of good 2 leads to an increase in its relative price. In the sticky price equilibrium, the internal terms of trade gap cannot adjust immediately and a negative terms of trade gap opens. This induces higher inflationary pressures in the more open region and therefore an increase in inflation differentials.

The optimal policy has to balance the objective of stabilizing the area-wide output gap and inflation with the one of stabilizing the inflation differentials and the internal terms of trade. It thus prefers to attenuate the depreciation of the nominal exchange rate relative to the homogeneous case since such depreciation would amplify the

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[32]The equations that fully describe the optimal policy plan are specified in the appendix A.3.

Figure 1.4: IRFs to a positive area-wide technology shock under the optimal policy
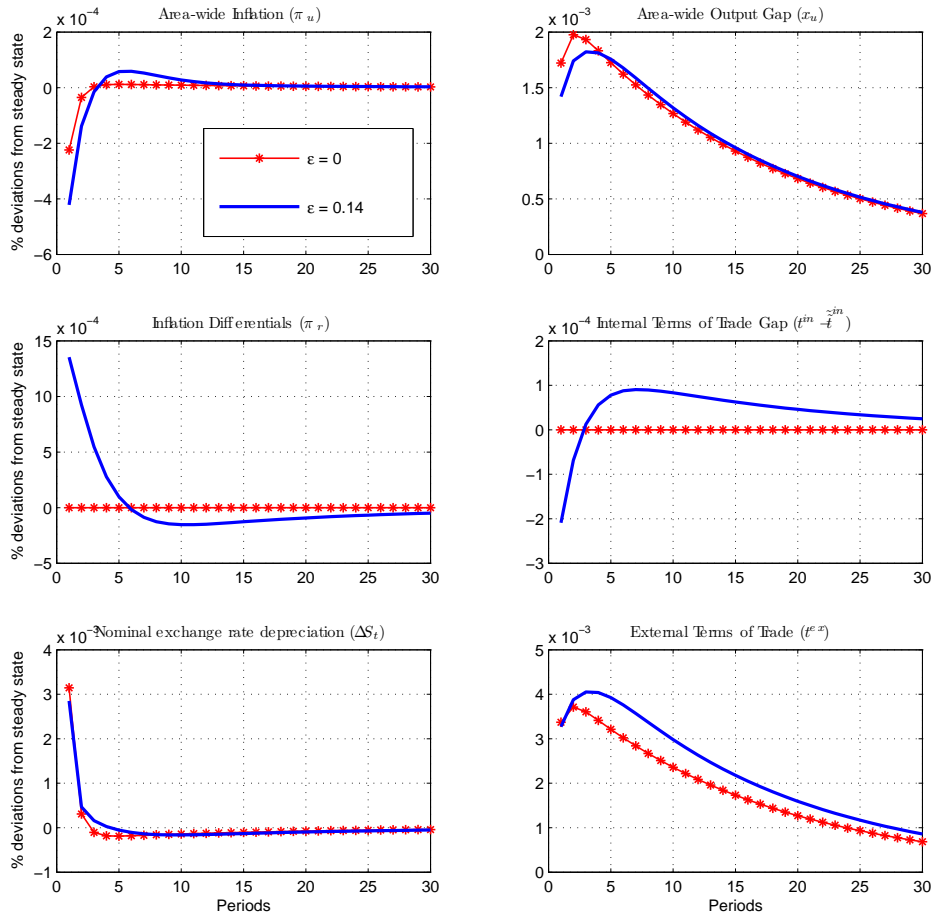
Notes: The shock is equal to 1% st dev of the area-wide technology. The starred line is the homogeneity case ($\varepsilon = 0$); the solid line the heterogeneity one ($\varepsilon = 0.14$).

inflation differentials across regions. This implies that the area-wide inflation display a larger volatility when regions are heterogeneous. The presence of heterogeneity, thus, induces a stronger motive for exchange rate stabilization and goes against the complete stabilization of inflation.

Figure 1.5 displays the responses of the nominal interest rate, after each of the shocks of the model, that is implied by the optimal policy plan. After a positive

technology shock the nominal interest rate reacts more on impact when regions are heterogeneous than in the homogeneous case; afterward it decreases more sharply in the homogeneous case in order to allow for a stronger depreciation in the monetary union currency. The opposite happens when a shock coming from the rest of the world hits the currency area economy.

Figure 1.5: The IRFs of the nominal interest rate under the optimal policy
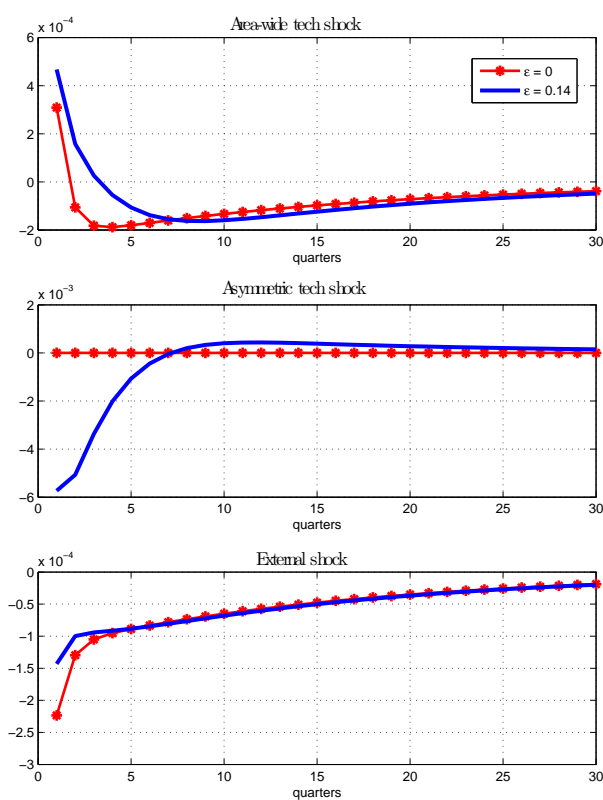
Notes: The starred line is the homogeneity case ($\varepsilon = 0$); the solid line the heterogeneity one ($\varepsilon = 0.14$).

Figure 1.6 shows the responses of the main macroeconomic variables to an area-wide technology shock under a policy of strict inflation stabilization and under the optimal policy when regions are heterogeneous.[33] In this case the complete stabilization of the area-wide inflation does not imply the stabilization of the welfare-relevant

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[33]The strict inflation stabilization policy is described by the targeting rule $\pi_{u,t} = 0$.

output gap. The heterogenous transmission mechanism across regions induces fluctuations in the distance of the internal terms of trade from its efficient level, which in turn affect the relationship between area-wide inflation and output gap as described by the New Keynesian Phillips curve. Compared to the optimal policy, a strict inflation targeting rule induces a stronger response of the nominal exchange rate of the currency area and generates higher inflation differentials. Both these variables are instead partially stabilized under the optimal policy.

The relatively considerable weight, under the benchmark calibration, on the inflation differentials in the objective of the currency area central bank induces the social planner to care about inflation differentials stabilization. However it is the interaction between the area-wide and the difference block of the model introduced by the presence of heterogeneity that allows the central bank to affect them. The optimal policy plan internalizes that with heterogeneity the transmission mechanism of monetary policy impulses differs across regions and makes active use of it in order to influence the differentials.

1.5.4 The IRFs to an asymmetric technology shock under the optimal plan

Figure 1.7 displays the impulse response functions to an asymmetric technology shock $(a_{r,t})$. It can be interpreted as a shock in the efficiency level of the production of one good of the currency area relative to the other. In particular, a positive variation of $a_{r,t}$ implies a gain in competitiveness in the more open region (region 2) relative to the more closed one (region 1).

Following the shock, a positive internal terms of trade gap opens because the presence of price stickiness implies that the relative price across regions does not adjust immediately. This gap affects the inflation differentials: inflation is higher in region 1 because the distortion in the relative price caused by the stickiness inefficiently boosts the demand of good 1.

When the regions are homogenous, the area-wide variables are not affected by the shock and only the relative prices and quantities adjust. The differential block of the

Figure 1.6: The IRFs to a positive area-wide technology shock under different policy regimes
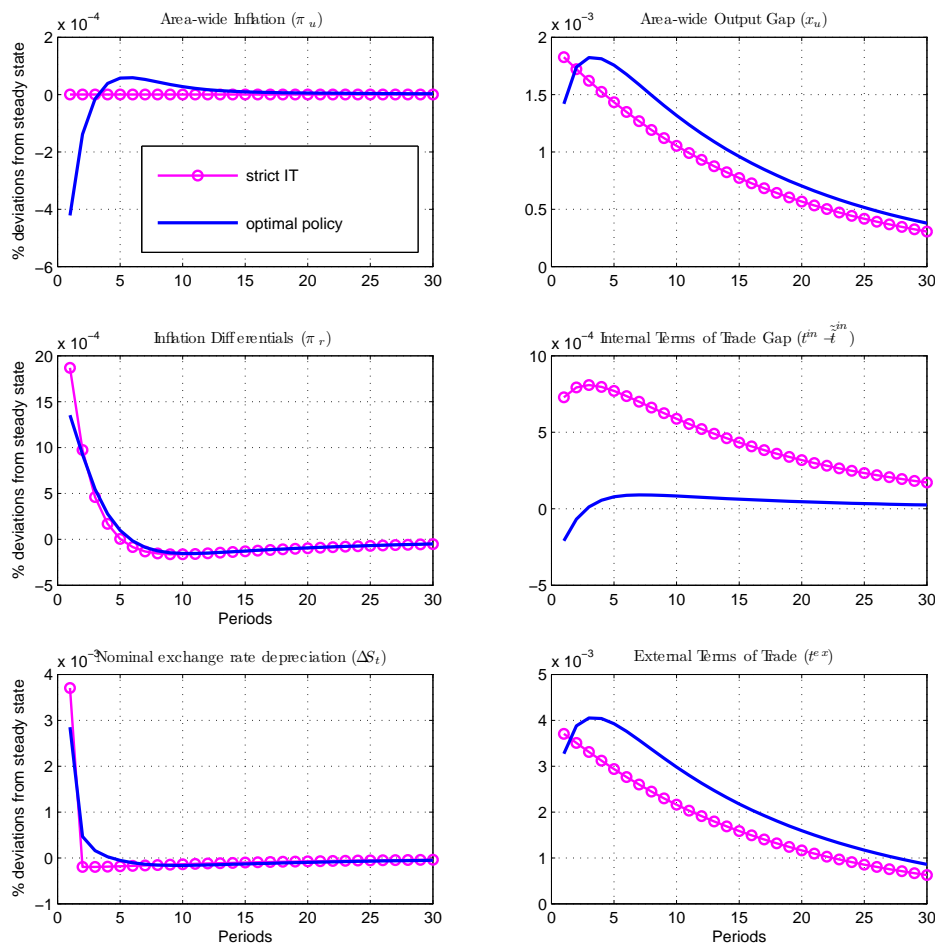
Notes: The IRFs are obtained for a calibration of $\varepsilon = 0.14$ under the optimal policy (solid line) and the strict inflation targeting (IT) regime (starred line)

model is disconnected from the area-wide one and monetary policy, having as the only instrument the currency union nominal interest rate, cannot undo the distortion in the internal terms of trade generated by nominal rigidities. When, instead, $\varepsilon \neq 0$ the area-wide inflation and output gap are affected. Since sizable inflation differentials generate welfare losses, the central bank raises the output gap under the optimal

Figure 1.7: The IRFs to an asymmetric technology shock under the optimal policy plan
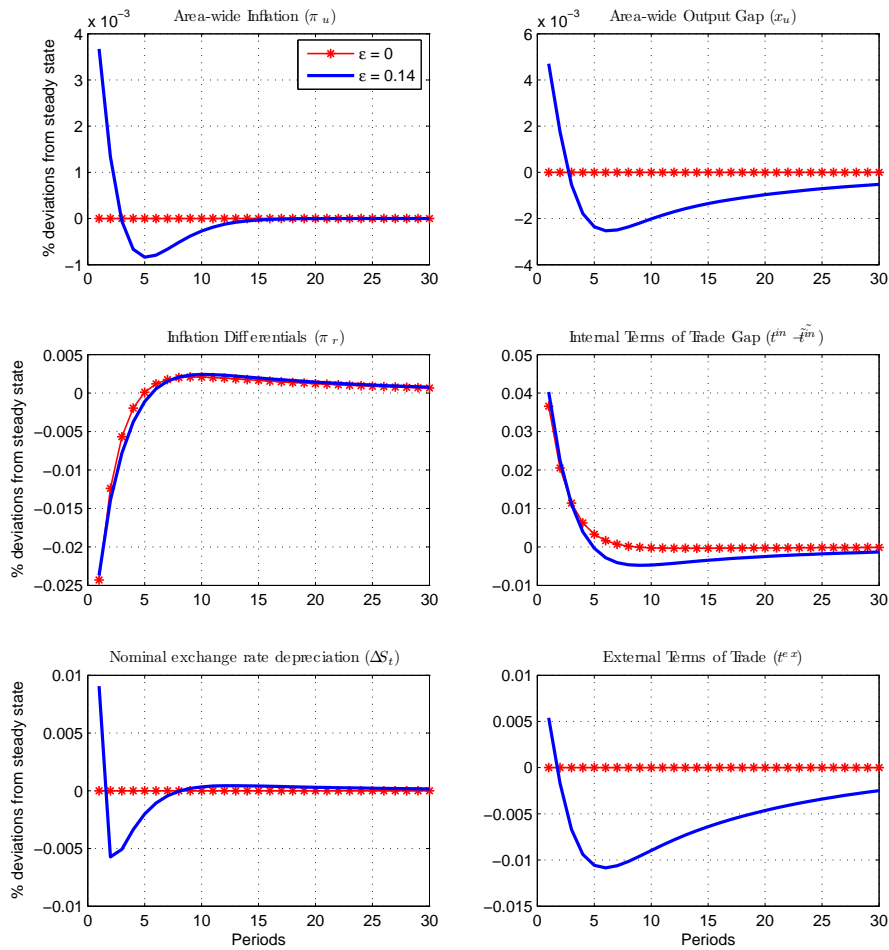
Notes: The asymmetric shock is an exogenous increase in the relative productivity of the more open region; the starred line is the homogeneity case ($\varepsilon = 0$); the solid line the heterogeneity one ($\varepsilon = 0.14$).

policy in order to partially mitigate the inflation differentials (see equation (1.23)). This in turn affects the area-wide inflation that it is slightly increasing after the shock.

1.6   Conclusions

The degree of external openness is an important element that shapes the transmission mechanism of global shocks in the countries of the EMU. Evidence shows that the member countries are different along this dimension.

The paper highlights the effect of the heterogeneity on external trade openness among the regions of a currency area in the optimal design of monetary policy. It builds a framework for an open currency area economy composed of two regions that have different preferences toward the goods produced by the rest of the world outside the currency area. After obtaining a welfare measure that is consistent with the model and microfounded, the optimal monetary policy from the currency area viewpoint is derived.

When the currency area regions have different degree of external openness, both the transmission mechanism of the shocks and the microfounded welfare function are different from the baseline case of a homogeneous currency area. Heterogeneity in external trade generates a stronger motive for exchange rate stabilization in the optimal policy plan. Accordingly, more inflation volatility is advisable. For example, after a positive area-wide technology shock the central bank should not accommodate perfectly the shock and should decrease the nominal interest rate by less than what it is predicted by the optimal policy for a homogeneous currency area. In this way the union nominal exchange rate depreciation is lower and the inflation differentials, generated by the different exposure of the two regions toward the rest of the world, is partially alleviated.

The results of the paper suggest that some degree of heterogeneity across regions is sufficient for the cross-region dispersion to influence the area-wide variables dynamics. The optimal monetary policy plan takes into account the structural differences across regions and balances the welfare losses coming from area-wide inflation and output gap with those coming from the inflation differentials.

Chapter 2

Targeting Rules in a Monetary Union with Heterogeneous Cost-Push Shocks

2.1 Introduction

The cost-push shock is commonly defined as an inefficient exogenous shift of the relationship between the inflation and the real activity. Most of the models that are widely used for macroeconomic policy analysis introduce this type of shock in order to generate a trade off between the inflation and the output gap stabilization.

This paper analyzes the design of targeting rules in a monetary union with country-specific cost-push shocks.[1] In general, the presence of a time-varying shift in the aggregate supply relation complicates the task of central banks forcing them to choose the optimal balance between the stabilization of inflation and real activity. In a monetary union in which the severity of such trade off varies across countries the central bank has to decide whether and how to take into account such heterogeneity.

The European Monetary Union (EMU) member countries experienced different

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[1]As defined in Woodford (2003), a targeting rule is a commitment to adjust the nominal interest-rate instrument as necessary so that inflation and the other variables possibly specified in the target criterion are consistent with the central banks target.

volatilities of the national inflation and output in the first ten years of the common currency, as shown in figure 2.1.[2] In a currency area monetary policy cannot be a valid candidate to explain this cross-country heterogeneity; the observed pattern of the macroeconomic volatility can be the result of structural differences in the monetary transmission mechanism, of idiosyncratic shocks or a combination of both factors.

Figure 2.1: The trade off across Euro area countries.
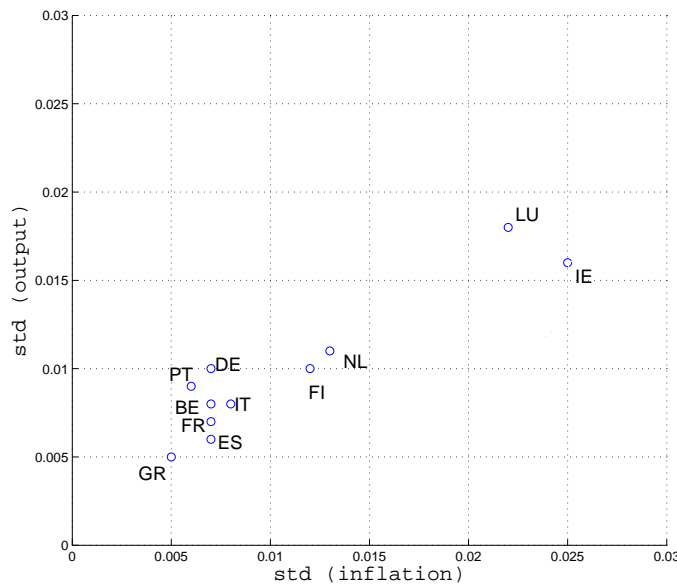
Notes: Sample period: 1999:Q1-2008:Q3. Output is the hp-filtered real GDP; inflation is the log differences of the GDP deflator. Source: OECD Economic Outlook.

Benigno (2004) and Benigno and Lopez-Salido (2006) show that, when the degree of nominal rigidities and the price setting mechanism differ across member countries of a currency area, as it is the case in the Euro area, it is welfare-improving to take into account such heterogeneities in the definition of the target criterion of the monetary union central bank. While they consider structural differences across economies, I

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[2]The standard deviations of inflation and output in Ireland, for instance, are respectively about five and three times higher than the ones in Greece and about three times higher than the average standard deviations in 11 of the Euro area countries.

concentrate on the differences in the stochastic properties of the cost-push shocks, that is the exogenous disturbances to the inflation dynamics. This exercise is important for two reasons. First, evidence from the estimation of dynamic stochastic general equilibrium model, as the one provided by Smets and Wouters (2003), suggests that in the historical decomposition of the inflation time series the cost-push shock accounts for most of the observed volatility.[3] Second, the ability of econometric analysis to tell apart the effects of different cost-push shock volatilities and of different structural parameters on the observed inflation dynamics is limited by the short sample of the data on national inflation processes after the EMU setup. The work of Benigno (2004) analyzes only the normative implications of different degrees of price rigidities. It is useful to complement his results with the analysis of the normative consequences of heterogeneous cost-push shock.

I set up a model for a monetary union composed by two regions, or group of countries. The economy of the currency area features monopolistic competition, nominal rigidities in the form of Calvo price stickiness and country-specific cost-push shocks. The regions are homogeneous ex ante: they share the same value of the structural parameters but differ in the stochastic features of the shocks. In particular, in our benchmark case the variance of the cost-push shock in one region is higher than in the other.

I compute the microfounded welfare loss by approximating the monetary union representative household's utility function and I compare the welfare losses under different policy regimes: (i) an inflation targeting rule in which the target criterion is the aggregate of member countries' inflations weighted by their economic size (henceforth, HICP inflation targeting rule); (ii) the optimal plan under full commitment; (iii) an "optimally weighted" inflation targeting rule in which the weights to countries' inflation minimize the welfare loss function. I consider the regime (i) as the approximate description of the objective that is implicitly defined in the European Central Bank statute and the regime (ii) as a benchmark case, since it delivers the

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[3]I do not investigate here whether such shocks are a truly exogenous source of fluctuations or they are the result of a misspecification of the inflation dynamics in the model. I only acknowledge their importance in explaining part of the inflation volatility at the current state of art.

first best allocation.

If the central bank cannot implement the full commitment optimal plan and has to rely on a targeting rule, under our specific assumptions about the cost-push shock volatilities, one can improve on the HICP inflation targeting rule by optimally choosing the weights on countries' inflations. The paper shows that, when the standard deviation of the cost-push shock innovation in one country is lower than the one in the other country, the central bank should attach a lower weight to the inflation of the latter country in the optimal inflation index to be targeted. The welfare gains of switching from the HICP rule to the "optimally weighted" inflation targeting rule, computed by means of the microfounded loss function, are around 50% when the volatility in one country is about half of the one in the rest of the currency area and they become more substantial when the differences in the shock variances are larger.

The volatility of the cost-push shocks affects the efficient policy frontier, that is the outcome achievable by the first best policy in terms of inflation and output gap stabilization. Thus, *ceteris paribus*, under the fully optimal policy plan the unconditional variance of inflation is higher in the country whose idiosyncratic cost-push shock is more volatile. In a second best environment, a symmetric targeting rule that assigns equal weights to the two countries' inflation when the volatilities of the cost-push shocks are different does not replicate such feature of the state-contingent optimal policy plan. Allowing for asymmetry in the weights, as it is done in the optimal inflation index, helps to approach the first best.

The rest of the paper is organized as follows: section 2.2 sets up the model and the objective of the central bank; section 2.3 studies the design of the monetary policy rule; section 2.4 concludes.

2.2   The Model

The monetary union is populated by a continuum of mass one of agents. Agents $j \in (0, s)$ live in the home country (H), agents $j \in (s, 1)$ live in the foreign country (F). The union representative agent j has preferences described by the following lifetime

expected utility

$$U^j = E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t^j) - v(y_t^j, z_t) \right] \tag{2.1}$$

where $\beta \in (0,1)$ is the discount factor, u is increasing and concave and v is increasing and convex. Henceforth, the index j denotes a variable that is agent-specific, while the index i will indicate a variable that is country-specific, i.e. $i = H, F$. The parameter s represents the size of the home country relative to the whole union. C_t^j is a bundle of two consumption goods, H and F, produced in the two country of the currency area and aggregated in the following way

$$C_t^j \equiv \frac{(C_{H,t}^j)^s (C_{F,t}^j)^{1-s}}{s^s (1-s)^{1-s}}$$

implying that the elasticity of substitution between these two types of goods is 1. Good H (good F) can be produced only by households living in the home (foreign) country. $C_{H,t}^j$ and $C_{F,t}^j$ are Dixit-Stiglitz indices of a continuum of brands produced respectively by country H and F

$$C_{H,t}^j \equiv \left[\left(\frac{1}{s} \right)^{1/\theta_t^H} \int_0^s c^j(h)^{(\theta_t^H - 1)/\theta_t^H} dh \right]^{\theta_t^H/(\theta_t^H - 1)}$$

$$C_{F,t}^j \equiv \left[\left(\frac{1}{1-s} \right)^{1/\theta_t^F} \int_s^1 c^j(f)^{(\theta_t^F - 1)/\theta_t^F} df \right]^{\theta_t^F/(\theta_t^F - 1)}$$

I assume that the population size of a country coincides with its economic size and they are represented by the parameters s and $1-s$, respectively in the home and foreign country. Following Steinsson (2003) and Smets and Wouters (2003), I assume that the elasticities of substitution among different brands (i.e. θ_t^H and θ_t^F for the home and foreign country, respectively) are stochastic. The variety of brands produced in each country and their substitutability are varying over time, implying that the market power in the production of both goods changes over time and across countries. They have the same mean θ and $\theta_t^H, \theta_t^F > 1 \ \forall \ t$.[4]

I refer to h (f) as the brand produced in country home (foreign) and belonging to the good of type H (F). Correspondingly, the monetary union price index is the

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[4]Such markup shock is assumed to be aggregate at a country level, i.e. it affects in the same way all producers in the country.

following

$$P_t \equiv (P_{H,t})^s (P_{F,t})^{1-s}$$

where $P_{H,t}$ and $P_{F,t}$ are the Dixit-Stiglitz aggregator of the brand prices, $p_t(h)$ and $p_t(f)$, as follows

$$P_{H,t} = \left[\frac{1}{s} \int_0^s p_t(h)^{1-\theta_t^H} dh \right]^{(1-\theta_t^H)-1} , \qquad P_{H,t} = \left[\frac{1}{1-s} \int_s^1 p_t(f)^{1-\theta_t^F} df \right]^{(1-\theta_t^F)-1} .$$

The relative price, or terms of trade, is defined as $T_t = \frac{P_{F,t}}{P_{H,t}}$. There is no home bias in consumption and the total share of expenditure in the H type of goods is equal to s for all the agents living in the monetary union, while that in the F type of goods is $(1-s)$. Assuming that there are neither investment nor government expenditure, the demands of the two types of goods coming from all inhabitants of the monetary union are

$$Y_{H,t} = sT_t^{1-s}Y_t \tag{2.2}$$

$$Y_{F,t} = (1-s)T_t^{-s}Y_t \tag{2.3}$$

where Y_t is the monetary union aggregate output. The demands for each brand are equal to

$$y_t(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta_t^H} T_t^{1-s}Y_t \tag{2.4}$$

$$y_t(f) = \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\theta_t^F} T_t^{-s}Y_t \tag{2.5}$$

I assume that markets are complete and the initial debt holding is the same for all households living in the monetary union. This implies perfect risk sharing among the members of the monetary union. The households budget constraint is standard

$$P_tC_t^j + E_tQ_{t,t+1}^j D_{t+1}^j \leq D_t^j + p_t(j)y_t(j) + T_t^i$$

where D_{t+1}^j is the nominal payoff in $t+1$ of the portfolio of state-contingent claims held at the end of period t; $Q_{t,t+1}$ is the stochastic discount factor for one period ahead

nominal payoff relevant to the households living in country i and T_t^i are country-specific lump-sum transfers. Maximizing the lifetime utility (2.1) subject to the budget constraint yields the following Euler equation for all households

$$u_c(C_t^j) = \beta(1 + R_t)E_t \left\{ u_c(C_{t+1}^j) \frac{P_t}{P_{t+1}} \right\} \tag{2.6}$$

where C_t^j is the total consumption and R_t is the return on a riskless bond.

Each household is at the same time a consumer and a producer of a brand: household j living in the home country produces a different brand h of the H good type, while household j living in the foreign country produces a different brand f of the F good type. The production of a brand has a cost in terms of utility of $v(y_t(j), z_t)$ for $j \in (0, 1)$. The function $v(\cdot)$ depends both on the level of production $y_t(j)$ and on an area-wide shock in preferences z_t that can be interpreted as a technology shock.

Producers are monopolistic competitors and they set the price of their brand taking the demand as given. The same price is charged to consumers of both countries. There are nominal rigidities in the form of Calvo price stickiness. At each period the producer is allowed to change its price with a probability $(1 - \alpha)$. Differently from Benigno (2004), I set the parameter α to be equal across countries, since I focus on a different type of heterogeneity. The price of a brand is optimally chosen by maximizing the expected value of the discounted future profits, conditional on not having the chance of changing the price in the subsequent periods, under the constraint of the demand function of that brand $y_{t,t+k}(j)$. The first order condition of this problem indicates that the producer sets a price that is the expected value of discounted present and future markups over the correspondent marginal costs:

$$p_t(h)^* = \frac{E_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k \theta_{t+k}^H v_y \left(y_{t,t+k}(h); z_{t+k} \right) y_{t,t+k}(h) \right]}{E_t \left[\sum_{k=0}^{\infty} (\alpha\beta)^k \frac{u_c(C_{t+k}^H)}{P_{t+k}} (\theta_{t+k}^H - 1) y_{t,t+k}(h) \right]} \tag{2.7}$$

A similar expression holds for the optimal price of the brand f, $p_t(f)^*$, produced by households in the foreign country. Equation (2.7) differs from the usual optimal price setting condition because there is uncertainty not only about the marginal costs but also about future markups. The price indices in the two countries evolve as follows

$$P_{i,t} = \left[\alpha P_{i,t-1}^{1-\theta_t^i} + (1 - \alpha) p_t^*(i)^{1-\theta_t^i} \right]^{(1-\theta_t^i)^{-1}} \qquad i = H, F. \tag{2.8}$$

2.2.1 The Loglinearized Model

In this subsection I loglinearize the equilibrium conditions of the model around the steady state, defining a loglinearized variable as $\hat{x} \equiv \ln(x - \overline{x})$ where $\overline{x}$ is the deterministic steady state level of x. In order to evaluate different policies relative to the Pareto efficient allocation, it is useful to write the equations of the model in terms of the welfare-relevant gaps. This implies solving the model also in the theoretical case in which there are no distortions.

Heterogeneity between the two countries is introduced by assuming that they have different time-varying distortions. The presence of monopolistic competition implies that the output is below its efficient level. If the two economies are equally distorted in the steady state, the terms of trade, absent any shock, is equal to one and the steady state output is $\overline{Y}_H = \overline{Y}_F = \overline{Y}$.

The assumption of time varying elasticities of substitution implies that home and foreign markups fluctuates stochastically around a common mean $\mu = \frac{\theta}{\theta-1}$. Once one allows for these variations there are three relevant concepts of output: the efficient output, the natural output and the actual output. The first one implies no distortions whatsoever in the economy and is implicitly defined by

$$\frac{v_y\left(Y^e_{H,t}; z_t\right)}{u_c\left(Y^e_t\right)} = \frac{P_{H,t}}{P_t} = (T^e)^{s-1} \tag{2.9}$$

for the home country and by

$$\frac{v_y\left(Y^e_{Ft}; z_t\right)}{u_c\left(Y^e_t\right)} = \frac{P_{F,t}}{P_t} = (T^e)^{-s} \tag{2.10}$$

for the foreign country. The superscript e indicates the efficient level of the variable. Producers are setting the price equal to the marginal cost at each time. In the efficient equilibrium there are no country-specific shocks, therefore the terms of trade is constant. Furthermore, given that the economies of the member countries are assumed to be identical ex ante such terms of trade is equal to one.[5]

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[5]Consistently also the output levels are the same across countries. If I assumed idiosyncratic productivity shocks, the changes of the relative competitiveness of the goods produced in the two countries would generate fluctuations of the efficient terms of trade and differentials in the output levels (see Benigno (2004)).

The second output concept, the natural output, is affected by just one distortion, that is the presence of monopolistic competition among producers, while prices are fully flexible. Their levels are defined by

$$\mu_{H,t}\frac{v_y\left(Y_{H,t}^n; z_t\right)}{u_c\left(Y_t^n\right)} = \frac{P_{H,t}}{P_t} = (T_t^n)^{s-1} \tag{2.11}$$

$$\mu_{F,t}\frac{v_y\left(Y_{F,t}^n; z_t\right)}{u_c\left(Y_t^n\right)} = \frac{P_{F,t}}{P_t} = (T_t^n)^{-s}. \tag{2.12}$$

The superscript n indicates the natural level of the variable. The natural terms of trade fluctuates because affected by the country-specific markup shocks.

Finally, the actual output is affected not only by the distortion generated by the presence of monopolistic competition but also by the staggered setting of prices that influences the demand and the production of the two types of goods. As a consequence of the price stickiness, the actual terms of trade is not immediately adjusting to the different shocks in the two countries, therefore creating a further distortion in the allocation of resources.

In the absence of markup fluctuations, one can write the model in terms of the gap between actual and natural levels of the variables. The difference between the efficient and the natural level of output would be a constant and it can be taken as an expansion parameter and therefore disregarded in a policy analysis exercise concerned with stabilization issues. Having the gap between the natural and efficient output oscillating over time implies that policy should concentrate on stabilizing the deviations of the actual output from its efficient level.

The efficient equilibrium

By loglinearizing (2.9) and (2.10) around their steady state values one obtains

$$-\sigma^{-1}\hat{Y}_t^{H,e} = \omega\left[\hat{Y}_t^{H,e} - \overline{a}_t\right] \tag{2.13}$$

$$-\sigma^{-1}\hat{Y}_t^{F,e} = \omega\left[\hat{Y}_t^{F,e} - \overline{a}_t\right] \tag{2.14}$$

where $\overline{a}_t \equiv \frac{v_{yz}}{v_{yy}\overline{Y}}z_t$, $\omega \equiv \frac{v_{yy}}{v_y}\overline{Y}$ and $\sigma^{-1} \equiv -\frac{u_{cc}}{u_c}\overline{Y}$. Equations (2.13) and (2.14) show that the efficient output is the same for all countries. The efficient output in the

currency area is thus

$$\hat{Y}_t^e = \frac{\omega}{\sigma^{-1} + \omega}\overline{a}_t. \tag{2.15}$$

The output fluctuates with the area-wide productivity shock, which is the only driver of the business cycle in the Pareto efficient equilibrium of the model. Loglinearization of (2.6) gives us the efficient interest rate of the economy

$$\hat{R}_t^e = \sigma^{-1} E_t \left\{ \hat{Y}_{t+1}^e \right\} - \hat{Y}_t^e. \tag{2.16}$$

The sticky prices equilibrium

The loglinearization of the first order condition (2.7) for the producer of h and f together with the equation (2.8) for the evolution of the price index leads to the so-called New Keynesian Phillips curves (NKPC, henceforth)[6]

$$\pi_t^H = (1 - s)\kappa_T \hat{T}_t + \kappa_y(\hat{Y}_t - \hat{Y}_t^e) + \beta E_t \pi_{t+1}^H + u_t^H \tag{2.17}$$

$$\pi_t^F = -s\kappa_T \hat{T}_t + \kappa_y(\hat{Y}_t - \hat{Y}_t^e) + \beta E_t \pi_{t+1}^F + u_t^F \tag{2.18}$$

The parameters κ_T and κ_y are functions of the structural parameters of the model and are equal across countries. These equations differ from the standard one because of a terms of trade effect in the inflation dynamics: a level of the terms of trade higher than the one in the efficient equilibrium (which is equal to one at all times) generates inflationary pressures in the home country and deflationary pressures in the foreign country. The terms u_t^H and u_t^F are the so-called cost-push shocks. They are inefficient supply shocks that shift the dynamic relationship between the monetary union output gap and country's inflation amplifying or reducing the existing trade off for the stabilization of both.[7] They are defined as follows

$$u_t^i \equiv \frac{(1 - \alpha\beta)(1 - \alpha)}{\alpha(1 + \omega\theta)(1 - \theta)}\hat{\theta}_t^i \qquad i = H, F. \tag{2.19}$$

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[6]The algebraic details are in the appendix B.1.

[7]Compared to the standard closed economy version of the NKPC, here the sluggish adjustment of the terms of trade to asymmetric shocks creates a trade off between inflation and output gap stabilization.

Loglinearizing (2.6) and subtracting equation (2.16) from it, one obtains the aggregate demand equation

$$\hat{Y}_t - \hat{Y}_t^e = E_t \left\{ \hat{Y}_{t+1} - \hat{Y}_{t+1}^e \right\} + \sigma \left[(\hat{R}_t - \hat{R}_t^e) - E_t \left\{ \pi_{t+1} \right\} \right] \tag{2.20}$$

where π_t is the monetary union inflation defined as $\pi_t \equiv s\pi_t^H + (1 - s)\pi_t^F$.

From equations (2.2) and (2.3) I have the following market clearing conditions for the good markets

$$\hat{Y}_{H,t} = (1 - s)\hat{T}_t + \hat{Y}_t \tag{2.21}$$

$$\hat{Y}_{F,t} = -s\hat{T}_t + \hat{Y}_t \tag{2.22}$$

In order to close the model I need an equation that establishes the link between home and foreign inflation with the terms of trade, that is the terms of trade identity, $T_t = \frac{P_{F,t}}{P_{H,t}} \frac{P_{H,t-1}}{P_{F,t-1}} \frac{P_{F,t-1}}{P_{H,t-1}}$, which, once loglinearized, is equal to

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H. \tag{2.23}$$

Equations (2.17), (2.18), (2.20), (2.21), (2.22) and (2.23) describe the equilibrium in sticky prices. To solve for the dynamics of output gap, countries' inflations, the terms of trade and the interest rate one needs to specify the way in which the central bank sets the monetary policy.

2.2.2 The objective of the central bank

The welfare criterion for the currency area central bank is derived from the utility of the union representative household. By taking a second order Taylor approximation of the representative agent utility function one obtains the following loss function[8]

$$W_t = -\Omega \sum_{t=0}^{\infty} \beta^t L_t \qquad where \tag{2.24}$$

$$L_t = s(\pi_t^H)^2 + (1 - s)(\pi_t^F)^2 + \theta^{-1}\kappa_y x_t^2 + s(1 - s)\theta^{-1}\kappa_T \hat{T}_t^2$$
$$+ t.i.p. + o(\|\xi_t\|^3)$$

$$\Omega \equiv \frac{1}{2} u_c \overline{Y} \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} \theta(1 + \theta\omega).$$

[8]Details about the calculations are in the Appendix D of Benigno (2001).

where $x_t \equiv \hat{Y}_t - \hat{Y}_t^e$ is the welfare relevant output gap, the term $t.i.p.$ includes all terms that are independent from the policy, ξ_t is a vector that includes all random disturbances in the model and $o(\|\xi_t\|^3)$ includes all terms that are of order higher than two in the bound $\|\xi_t\|$ on the amplitude of the shocks considered in the approximation. Equation (2.24) indicates that losses in the monetary union average welfare level are caused by the gap between actual and efficient union output, the presence of inflation in both countries and the gap between actual and efficient terms of trade. The weights in the loss function to each of these terms are functions of the structural parameters of the model.

2.3   The design of monetary policy

The task of the central bank of a currency area is challenging. Monetary policy faces a conflict due to the interaction of the stickiness of prices and the inertia of the terms of trade adjustments to asymmetric shocks. The complete removal of the first distortion goes against the closure of the terms of trade gap when there are asymmetric shocks. Targeting inflation implies a low variability of prices and therefore a sluggish adjustment of relative prices to such asymmetric shocks. This in turn brings about an inefficient reaction of output to changes in the economic conditions.

I concentrate on a particular type of heterogeneity that concerns the dynamic of inflation. I assume that the economies share the same values for all structural parameters of the model except for the variances of the countries' cost-push shocks u_t^H and u_t^F. In this section I analyze whether the ratio of the two standard deviations, σ_H/σ_F, matters in the design of monetary policy in the case in which the optimal plan is not implementable. In order to do this, I compare the dynamics and the deadweight losses of the monetary union economy under three kinds of policies: (i) the HICP inflation targeting, (ii) the optimal plan under full commitment, that is the first best, and (iii) the "optimally weighted" inflation targeting. Before analyzing each of them, I describe the calibration of the model and the assumptions about the shocks.

2.3.1 Calibration

The values assigned to the parameters of the model are indicated in table 2.1. These

Table 2.1: Calibrated parameters

parameter	*description*	*value*
β	discount factor	0.99
θ	elasticity of substitution	7.66
ω	elasticity of v_y wrt y	0.67
σ	risk aversion	1.66
α	degree of stickiness	0.75
s	economic/population size of H	0.5

are standard values in the business cycle literature and, with the exception of α, they are chosen following Benigno (2004), in which a similar model is calibrated for the Euro area. The calibration of the elasticity of substitution among differentiated goods implies an average desired markup over the marginal cost equal to 15%. I assume that countries are symmetric in their economic size and s is set equal to 0.5. The parameter α, that indicates the probability of not having the chance of changing the price, has been calibrated following Beetsma and Jensen (2004) and implies that the average duration of the price contracts is one year across all countries in the monetary union.

I assume that the cost-push shocks follow an AR(1) process

$$u_{H,t} = \rho_H u_{H,t-1} + \varepsilon_{H,t} \tag{2.25}$$

$$u_{F,t} = \rho_F u_{F,t-1} + \varepsilon_{F,t}. \tag{2.26}$$

The persistence parameters ρ_H and ρ_F have the same value of 0.6, while the standard deviations σ_H and σ_F of the innovations ($\varepsilon_{H,t}$ and $\varepsilon_{F,t}$) of the cost-push shocks differ. The other stochastic source of fluctuations in the model is the area-wide productivity shocks, z_t, which follows an AR(1) stochastic process

$$z_t = \rho_z z_{t-1} + \eta_t \tag{2.27}$$

where ρ_z is assumed equal to 0.2 and the standard deviation of the innovation σ_z is equal to 0.01. The innovations $\varepsilon_{i,t}$ and η_t are orthogonal.

2.3.2 An "optimally weighted" inflation targeting rule

I consider the case in which the currency area monetary authority cannot implement the full commitment optimal plan but has to adopt a targeting rule. This assumption is not unrealistic since the state-contingent optimal plan might not be easily implemented and it could be difficult to be understood by the private sector. Commitment to a targeting rule represents a feasible alternative which is straightforward to communicate. On the other hand, the disadvantage of conducting monetary policy through simple policy rules is that these cannot always achieve the first best equilibrium.[9]

Within the broad set of policy rule families I concentrate on strict inflation targeting rules. I do not consider a flexible targeting rule, in which the target criterion involves not only the projected path of inflation but also the output gap, for two reasons. First, output gap is not observable and its measurement is subject to a great deal of uncertainty. Second, a strict inflation targeting rule is similar to the rule that is implicitly defined in the European Central Bank statute.

The strict inflation targeting rule is defined as follows

$$\pi_t \equiv \gamma \pi_t^H + (1 - \gamma)\pi_t^F = \overline{\pi}. \tag{2.28}$$

where γ is the weight on the inflation in the home country, $(1 - \gamma)$ is the one on the inflation in the other country and $\bar{\pi}$ is the target that the currency area monetary authority aims to achieve. I call HICP inflation targeting rule the special case of the rule described above in which the weights to countries' inflation are equal to the size of the country[10]

$$\pi_t \equiv s\pi_t^H + (1 - s)\pi_t^F = \overline{\pi}. \tag{2.29}$$

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[9]Giannoni and Woodford (2003) illustrates a method to derive an optimal target criterion that implements the state contingent optimal policy plan and that is robust to any stochastic specification of the shocks. In this paper I analyze suboptimal monetary policy rules that might not be robust to some features of the shocks.

[10]Throughout I assume that the inflation target $\bar{\pi}$ is equal to zero.

The optimal monetary policy plan in a linear-quadratic framework, as the one considered in this paper, is certainty equivalent. As such it is robust to the specification of the stochastic features of the exogenous disturbances. Thus the fact that the volatility of the cost-push shocks are different across country does not invalidate the optimal policy prescriptions in the symmetric case. However, it is shown that in a second best environment the outcome of a symmetric policy rule in terms of welfare is not robust to different specification of the stochastic process of the shocks.

Other things being equal, I show that the optimal weights γ and $(1 - \gamma)$ depend on the ratio $\frac{\sigma_H}{\sigma_F}$.[11] In particular, the higher is the volatility of the cost-push shock in one country relative to the other the lower should be the weight on that country inflation in the target criterion of the currency area central bank. Figure 2.2 displays the welfare losses as a function of the weight γ on the home inflation in the targeting rule in the case of equal variance of the cost-push shocks (panel (a)) and in the case of different variances (panel (b)). In the first (symmetric) case the HICP and the optimally weighted rules coincide. In the second case, in which the standard deviation of the innovation of the home cost-push shock is almost the double of the one of the correspondent innovations in the foreign country, the optimal γ is lower than the one of the symmetric case.

Table 2.2 shows the optimal $\gamma's$ for different values of the ratio of the standard deviations. Such ratio has been varied so that the standard deviation of the area-wide cost-push shock, obtained through the aggregation of the member countries' cost-push shocks, is kept constant. Figure 2.3 illustrates the relationship between the ratio of the standard deviations of the cost-push shocks and the optimal γ. One can notice that there is a nonlinear relationship between the two variables and that the optimal γ lies approximately in the interval 0.25-0.8.

The intuition for this result can be obtained by looking at the impulse response functions (IRF) to a cost-push shocks in the home country.[12] Figure 2.4 illustrates the

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[11]The optimal $\gamma's$ are computed numerically in the following way. I first simulate the time series of the system of equations (2.17), (2.18), (2.23) and the rule (2.28), where γ is chosen inside a grid of length 500. I compute the associated loss for each point in the grid using the unconditional expected value of (2.24), $E[W_t]$, and I choose the γ that minimize it.

[12]I solve for the dynamics of π_t^H, π_t^F, x_t and $\hat{T}_t$ jointly. The dynamics of the interest rate can be

Figure 2.2: The losses as a function of the weight on π^H in the targeting rule. Panel (a): equal variances of the countries cost-push shocks. Panel (b): different variances
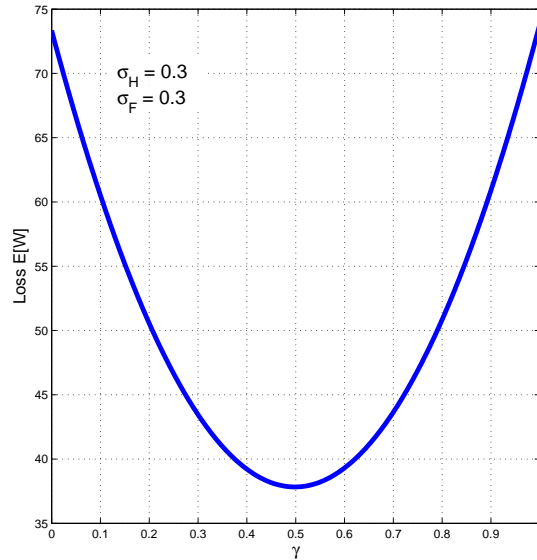
(a)
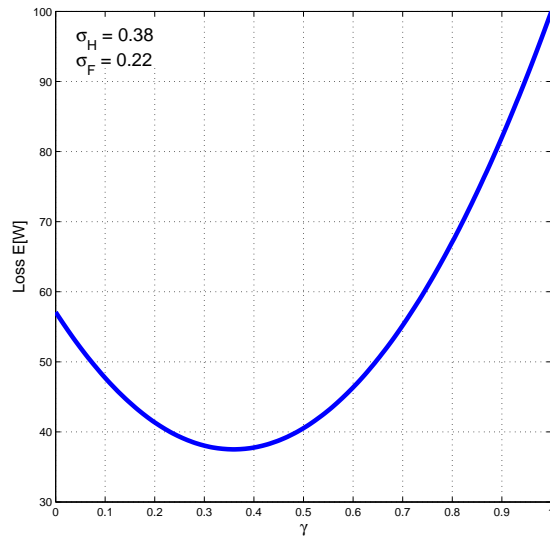
(b)

Figure 2.3: The optimal gamma for different ratios of the cost-push shock standard deviations
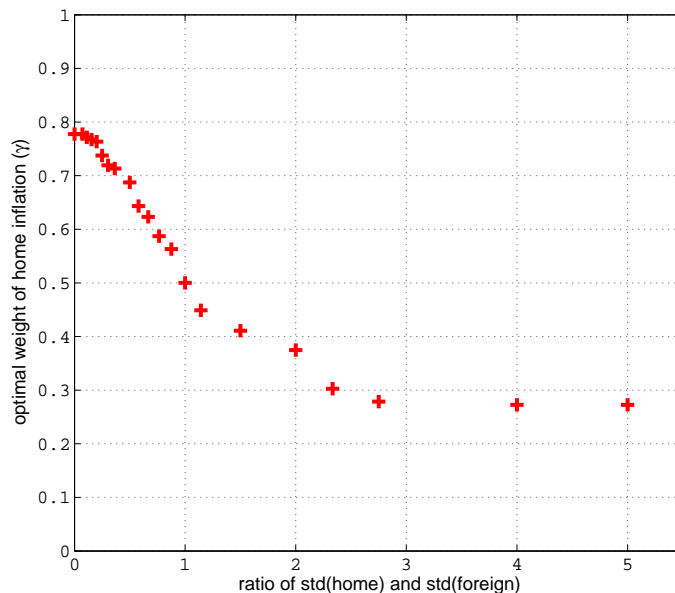
IRFs under three different policies: the full-commitment optimal plan, the "optimally weighted" inflation targeting rule and the HICP inflation targeting rule.[13] After a cost-push shock originated in the home country the optimal policy plan allows for an asymmetric response of the inflations in the two countries which moderates the losses of the inefficient supply shock in terms of the output gap relative to the other policies. The magnitude of the increase of the home country inflation is higher than the one of the decrease in the other country inflation. Thus the country where the shock is originated displays a more volatile response of inflation.[14] If the unconditional volatility of the cost-push shock in the home country is higher, the optimal policy

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obtained by plugging that solution into equation (2.20). Since there is no concern for interest rate stabilization in the objective of the central bank I abstract from any considerations about them in what follows.

[13]The first order conditions of the minimization of the loss function that characterize the full commitment optimal plan are in the appendix B.2.

[14]The response of the terms of trade to the home cost-push shock is invariant to the choice of the policy regime. In fact, as showed by Benigno (2004), in a currency area the evolution of the terms of trade is insulated from the monetary policy when the two economies are identical ex ante.

prescribes a more volatile inflation in that country in order to limit the deflationary effect in the other country and the negative spillover effects on the currency area output gap. A targeting rule that assign asymmetric weights to the inflation of the two countries in the target criterion mimics such feature of the optimal policy.

Table 2.2: The optimal weight on the inflation target criterion and the welfare gains from the "optimally weighted" inflation targeting rule

σ_H	σ_F	$\frac{\sigma_H}{\sigma_F}$	γ^*	$D~\%$
0.5	0.1	5	0.27	98.5%
0.4	0.2	2	0.37	81.8%
0.38	0.22	1.73	0.37	70%
0.36	0.24	1.5	0.41	65.8%
0.3	0.3	1	0.5	-
0.28	0.32	0.88	0.54	49.2%
0.24	0.36	0.67	0.63	53.3%
0.22	0.38	0.58	0.68	66.5%
0.2	0.4	0.5	0.69	79.2%
0.1	0.5	0.2	0.76	98%

Figure 2.5 shows that, under the optimal policy plan, the unconditional standard deviation of the inflation in the two countries is different depending on the relative volatility of the inefficient shocks. An inflation targeting rule that gives a lower weight to the inflation in the more volatile economy delivers a profile of the unconditional standard deviations of the two countries' inflation under different volatilities of the cost-push shocks that is closer to the one displayed in the optimal policy plan.

In table 2.2 I quantify the welfare gains coming from the adoption of an asymmetric inflation target. In the last column it is shown the percentage reduction of the welfare losses when switching from the HIPC inflation targeting rule to the "optimally weighted" inflation targeting rule. I normalize the loss variations considering the minimum loss achievable, the one arising when the full commitment optimal policy plan is implemented, as in Benigno (2004)

$$D = \frac{E[W^{HICP}] - E[W]}{E[W^{HICP}] - E[W^{OPT}]} 100$$

Figure 2.4: The IRFs to a cost-push shock in the home country
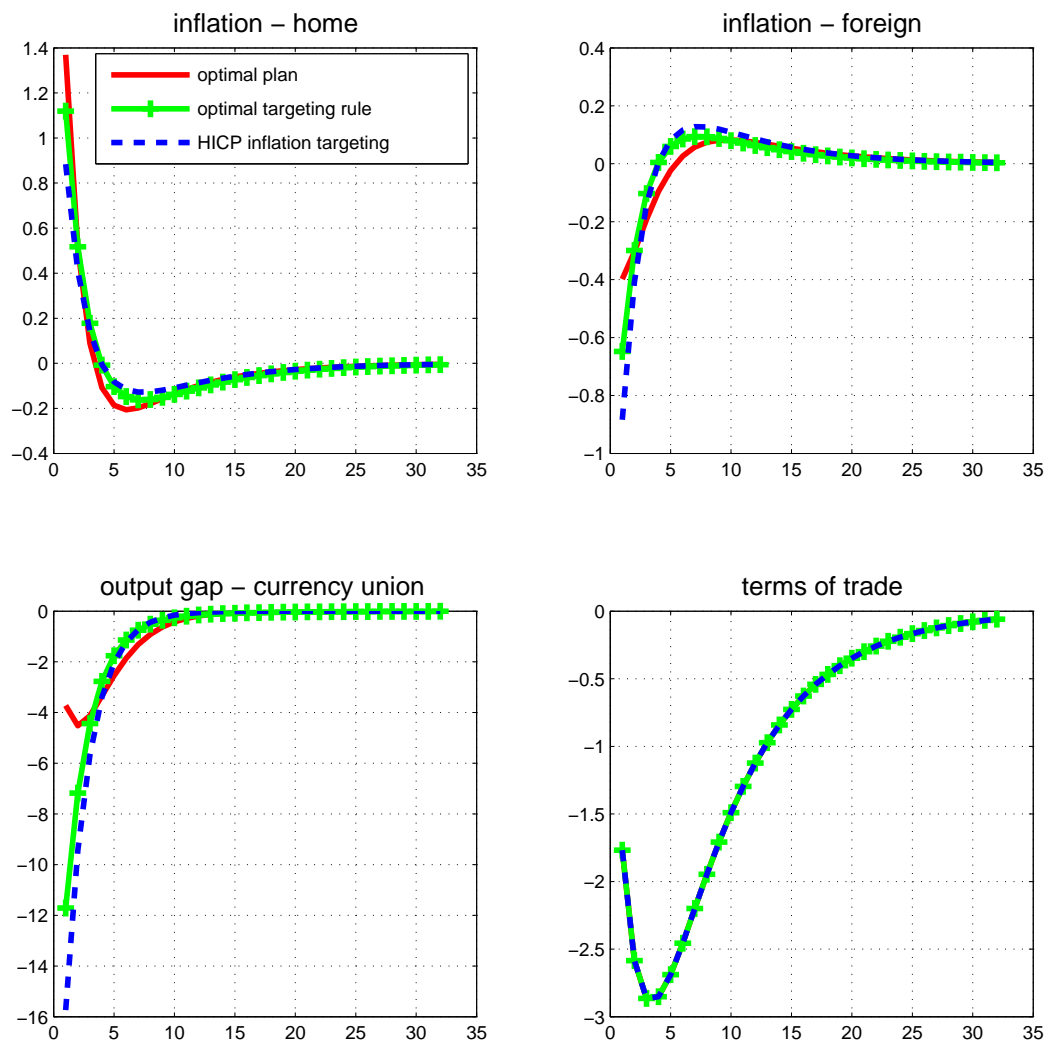
Notes: The standard deviations of the cost-push shocks are $\sigma_H = 0.38$ and $\sigma_F = 0.22$. The under the optimal policy plan, under the optimally weighted inflation targeting rule and under the HICP targeting rule

where $E[W]$ is the loss under the "optimally weighted" inflation targeting, $E[W^{HICP}]$ is the loss under the HICP inflation targeting rule and $E[W^{OPT}]$ is the minimum value achievable by the loss function.

Figure 2.5: The unconditional standard deviations of π^H and π^F under different
ratio of the home and foreign cost-push shock standard deviations
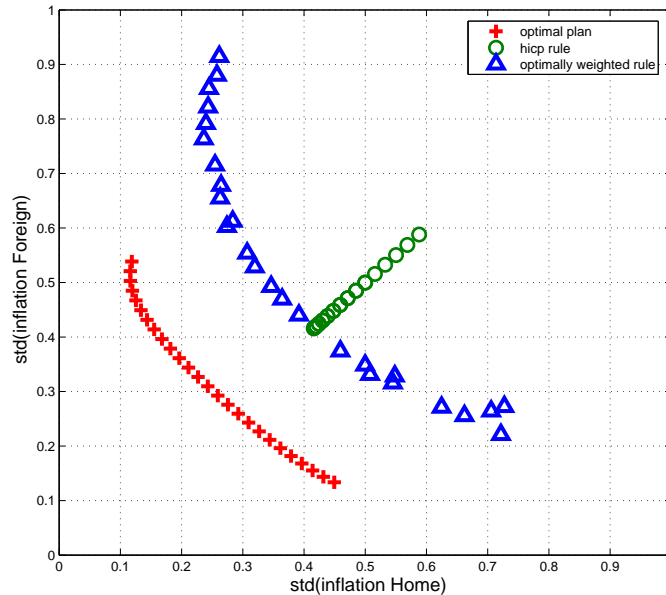
The benefits of such "optimally weighted" compared with the HICP one, are of
about 50% reduction of the deadweight losses when the cost-push shock standard
deviations are slightly different across countries and they become more relevant, up
to 98%, when the shocks are more heterogeneous.

I consider some changes in the benchmark framework in order to check whether
the prescription of asymmetric weights of inflation on the target criterion is a robust
one. The results are reported in table 2.3. In the benchmark case I have assumed that
the unconditional volatility of the cost-push shocks differ across countries because the
standard deviations of their innovations are different. When instead the persistence
of the shocks is heterogeneous, the HICP inflation targeting rule delivers a welfare
loss that is similar to the one of the "optimally weighted" inflation targeting rule.
This suggests that the persistence of the cost-push shock is not a relevant source
of heterogeneity for the design of targeting rules. When the cost-push shocks are
positively or negatively correlated across countries the prescriptions about the optimal

Table 2.3: The optimal weight on the inflation target under different cases

	benchmark	equal stochastic properties	different persis-tence	positively corre-lated shocks	negatively corre-lated shocks
σ_H	0.38	0.3	0.3	0.38	0.38
σ_F	0.22	0.3	0.3	0.22	0.22
ρ_H	0.6	0.6	0.75	0.6	0.6
ρ_F	0.6	0.6	0.26	0.6	0.6
$\sigma_{H,F}$	0	0	0	0.5	-0.5
γ^*	**0.37**	**0.5**	**0.47**	**0.22**	**0.48**

weights on the target criterion differ; in particular the weight on the inflation of the more volatile country is lower (higher) than in the benchmark case when the correlation is positive (negative).

2.4 Conclusions

In a monetary union cross-country heterogeneity in the stochastic features of the country-specific cost-push shocks is relevant in the design of targeting rules in a second best environment. In particular, I show that in the case in which the volatility of the cost-push shock in one country is higher than in the rest of the currency area, the monetary authority can improve on the HICP inflation targeting rule (i.e. a rule with a target criterion equal to the weighted average of countries' inflations based on their economic size) by choosing an "optimally weighted" inflation targeting rule.

Using the microfounded welfare measure function I quantify that the percentage reduction in losses when switching from the HICP inflation targeting rule to the "optimally weighted" one is of about 50% when the standard deviation of the cost-push shock is the double of the one in the other country. This result suggests that more research should be pursued in order to identify the sources of the cross-country heterogeneity in the macroeconomic trade off as it affects the design of monetary

policy rules.

Chapter 3

Firm Entry, Competitive Pressures and the U.S. Inflation Dynamics

3.1 Introduction

> *"The relationship between marginal cost, properly measured, and prices also depends on the markups that firms can impose. One important open question is the degree to which variation over time in average markups may be obscuring the empirical link between prices and labor costs. [...] A consensus on the role of changing markups on the inflation process remains elusive."* - Fed Chairman Ben Bernanke (June 9, 2008)

The New Keynesian Phillips curve (henceforth, NKPC), that has become the benchmark description of the inflation dynamics during the last decade, is derived from a model of monopolistic competition and nominal rigidities and it links the fluctuations of the average real marginal costs to those of aggregate inflation. It assumes that the so-called desired markup, that is the one that firms are willing to charge on unit costs absent nominal rigidities, is constant over time and that markups fluctuate because nominal frictions induce a sluggish response of prices to changing economic conditions.

This paper extends the standard New Keynesian framework by introducing time-varying competitive pressures generated by the entry of new firms. The endogenous fluctuations of the number of firms that operate in the economy induce changes in

firms' desired markup and eventually affect the inflation dynamics. I estimate this variant of the NKPC, similar to the one obtained by Bilbiie et al. (2007), and I find that, when taking into account the competitive pressures coming from the entry of new firms, the Phillips curve is not as flat as empirical test of the standard New Keynesian model have suggested. When one omits the number of firms from the estimation, the pass-through of real marginal cost on inflation is not separately identifiable from the effect of the desired markup on inflation and the estimates of it are downward biased. The paper also provides a structural estimate of the elasticity of the desired markup to changes in the number of firms.

Contrary to statistical (reduced form) models of the inflation dynamics, the NKPC has the appealing feature of having sound microfoundations. When confronting the data, however, a considerable uncertainty surrounds the determinants of the inflation dynamics and the pass-through of real marginal costs.[1] The estimates of Galí and Gertler (1999) and Sbordone (2002), using real unit labor costs in the U.S. to measure real marginal costs, differ from those of Rudd and Whelan (2005) suggesting opposite results. While the former studies found a significant role for the measured real marginal cost, the latter argues that such variable fails to be the appropriate measure of inflationary pressures both from a theoretical and empirical point of view. Moreover, the estimates of the pass-through of the real marginal cost in the Phillips curve based on aggregate data have implications for the structural parameters, such as the degree of nominal rigidities, which are at odds with evidence on micro data for the same parameters (i.e. the estimated coefficient is often too low compared to the one that would be consistent with micro evidence on the frequency of price adjustments). I claim that by taking into account endogenous fluctuations in the desired markup, the estimates of the effect of real marginal costs on inflation are higher and more precise.[2]

The assumption of a constant desired markup is usually relaxed in more recent

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[1]One important point that is at stake in the debate is the extent to which forward and backward-looking components of inflation are relevant to explain the current one. This aspect, though extremely relevant, is not the strict focus of this paper.

[2]Galí and Gertler (1999) already suggested that with a countercyclical desired markup the implied slope of the Phillips curve would be higher.

models. Steinsson (2003), among many others, introduces exogenous fluctuations in the elasticity of substitution among differentiated goods which result in a markup (or cost-push) shock. In this case, variations of the desired markup enter as a residual of the inflation dynamics equation. Eichenbaum and Fisher (2007) introduce endogenous fluctuations in the desired markup assuming Kimball (1995) preferences and estimate the implied NKPC. The parameters that are relevant for the effect of competitiveness on inflation cannot be disentangled from the ones that pertain to nominal rigidities as they all enter in the coefficient of the real marginal costs. While in these works the quantitative impact of an endogenous time-varying desired markup on the inflation dynamics cannot be computed, this paper contributes to the literature since it identifies the effect of market power changes on the inflation dynamics by choosing a proxy for the competitive pressures in the market.

Measuring the fluctuations of the desired markup at an aggregate level is an ambitious task due to the lack of aggregate data and the diversity of industries and market structures that are present in the economy. I focus on the changes in the desired markup that are produced by fluctuations in the number of active firms in the economy. This is a rough measure of the degree of competitiveness in the market and it might be not comprehensive of all possible factors that affect market competition. However the industrial organization literature has pointed out that the relationship between market power and the number of competitors is quite robust across a broad range of industries (see e.g. Bresnahan and Reiss (1991) and Campbell and Hopenayn (2005). Oliveira Martins et al. (1996), focusing on a number of manufacturing sectors across OECD countries, find that markups tend to be lower for sectors with a high number of firms.[3] Furthermore aggregate data on the entry of firms are available for the U.S. economy for a sufficiently large span of time and display volatility at the business cycle frequency.

The proposed framework builds on the standard new Keynesian model of monopolistic competition and price stickiness. The entry decision and the number of firms, however, are determined endogenously as in the work of Floetotto and Jaimovich

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[3]Up to my knowledge there is no time series evidence of this relationship.

(2008). There is a finite number of firms that produce differentiated goods in a regime of monopolistic competition. Firms set their price by taking as given the conditional demand of their own good and the prices set by the other competitors in the market. The nominal rigidities are modeled as in Rotemberg (1982); thus firms must pay a cost when they want to change their price. The log-linear solution of the model entails an inflation dynamics equation that has the same reduced form as the forward-looking NKPC derived from a model with Calvo price stickiness but it features an additional term on the number of firms in the market, the proxy for the desired markup. Recently several contributions in the real business cycle literature emphasized the importance of taking into account endogenous firm entry. Bilbiie et al. (2007) are among the first to introduce nominal rigidities into this kind of models. They also obtain a NKPC that depends on the real marginal costs and an extra term on the number of producers (varieties, in their interpretation).

I estimate the NKPC using the present-value approach, originally used in the empirical finance literature by Campbell and Shiller (1987), computing the expectations of future real marginal costs and number of firms with the projections of a VAR. Following Guerrieri et al. (2008) I estimate jointly the inflation dynamics and the VAR parameters. I use the law of motion of the number of firms implied by the model to reconstruct data on the total number of firms from the data on new incorporations in the U.S. I focus on the estimation of two parameters: the pass-through of real marginal costs on inflation and the elasticity of the desired markup to changes in the number of firms.

The main results are the following. (i) The elasticity of the desired markup with respect to the number of firms is significantly different from zero and it implies that a theoretical increase of 10% in the number of active firms would lower annual inflation by 1.4 percentage points in the short run. (ii) The point estimate of the coefficient of real marginal costs on inflation is found to be 70% higher than in the standard case with a constant desired markup. In fact when the number of active firms and the real marginal costs are positively correlated, the increase of the latter would come with a rise of the former that, through the implied decrease of the desired markup, has a negative impact on inflation. If one omits the number of firms from the estimation of

the inflation dynamics process, the estimates of the impact of the real marginal costs on inflation are biased downward. The model with endogenous firm entry is then calibrated with some of the estimated parameters and the responses to a positive technology shock and to an expansionary monetary policy shock are compared to the ones of the benchmark NK model. While the transmission of the technology shock is affected by the presence of firm entry, as previously found by Bilbiie et al. (2005), the implications of endogenous desired markup for the monetary transmission mechanism are almost negligible.

The rest of the paper is organized as follows. Section 3.2 lays out the model and derives the NKPC. Section 3.3 describes the data, some empirical issues in the measurement of the real marginal costs as well as the number of active firms and the econometric methodology. Section 3.4 presents the estimation results and some robustness checks. Section 3.5 illustrates the implications of these results for the response to a positive technology and a tightening monetary policy shock. Section 3.6 concludes.

3.2   The Model

In this section I lay out a very basic model for a closed economy that features monopolistic competition and free entry together with price rigidities à la Rotemberg (1982). Building on Floetotto and Jaimovich (2008) the number of operating firms is an endogenous variable that causes fluctuations in the desired markup at business cycle frequencies.

3.2.1   Households

The representative household has preferences described by the following lifetime utility function

$$U(C_t, L_t) \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left\{ log C_t - \frac{L_t^{1+\xi}}{1+\xi} \right\}. \tag{3.1}$$

The consumption that enters in the utility function is a bundle of many differentiated consumption goods aggregated as follows

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\omega-1}{\omega}} dj \right]^{\frac{\omega}{\omega-1}}$$

(3.2)

where $\omega > 1$ is the elasticity of substitution among them. The representative household maximizes its lifetime utility subject to the following intertemporal budget constraint written in nominal terms

$$B_{t+1} + P_t C_t + a_t v_t P_t N_t \leq B_t(1+i_{t-1}) + W_t L_t + a_{t-1} v_t P_t (1-\delta) N_{t-1} + a_{t-1} \Pi_{t-1} (1-\delta) N_{t-1} P_{t-1}$$

where B_t is a risk-free nominal bond and W_t is the nominal wage paid on hours worked. Households own a share a_t of each of the N_t firms with value v_t that are operating at time t. Each firm distributes as a dividend the entire profits earned at time t, Π_t. Every period t each firm faces an exogenous probability δ of exiting the market.

The solution of the expenditure minimization problem yields the following demand of consumption for each good j

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\omega} C_t$$

where P_t is the consumer price index

$$P_t = \left[\int_0^1 P_t(j)^{1-\omega} dj \right]^{\frac{1}{1-\omega}}.$$

At each point in time households decide how many hours they want to work and their holdings of bonds and shares. The first order conditions with respect to L_t, B_{t+1} and a_t are

$$\frac{W_t}{P_t} = L_t^\xi C_t \tag{3.3}$$

$$1 = \beta(1+i_t) E_t \left[\frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right] \tag{3.4}$$

$$v_t = \beta(1-\delta) E_t \left[\frac{C_t}{C_{t+1}} (v_{t+1} + \Pi_t) \right] \tag{3.5}$$

Equation (3.3) describes the labor supply of the household. The labor market is perfectly competitive; thus the household supplies hours so that the marginal rate of substitution between leisure and consumption equals the real wage. Equations (3.4) and (3.5) are the asset pricing equations of the nominal bond and the firms' shares respectively. Future firms' shares must be discounted taking into account that firms may exit the market.

3.2.2 Firms

The production in the economy occurs in two layers. The economy has a continuum of mass one of industries indexed by j. Each industry produces a differentiated good, $Y_t(j)$. The industry goods are imperfect substitutes for the consumers with an elasticity of substitution equal to $\omega > 1$. Inside each industry, a finite number of firms, indexed by i, supplies differentiated intermediate goods. The output of industry $j \in (0,1)$ is a Dixit-Stiglitz aggregate of the production of all firms in the industry and it is defined as follows

$$Y_t(j) = N_t(j)^{-\frac{1}{\tau_t - 1}} \left[\sum_{i=1}^{N_t(j)} x_t(i,j)^{\frac{\tau_t - 1}{\tau_t}} \right]^{\frac{\tau_t}{\tau_t - 1}} \tag{3.6}$$

where $x_t(i,j)$ is the output of firm i in industry j and $N_t(j)$ is the number of active firms in the industry.[4] Each firm in industry j produces a differentiated good and τ_t is the elasticity of substitution among them. I assume that such elasticity is stochastic and that its variance is such that in all possible states τ_t is higher than one, implying imperfect substitutability among goods.[5] The price index of industry j is defined as

$$P_t(j) = N_t(j)^{\frac{1}{\tau_t - 1}} \left[\sum_{i=1}^{N_t(j)} p_t(i,j)^{1-\tau_t} \right]^{\frac{1}{1-\tau_t}}.$$

---

[4]The first term of the right hand side of equation (3.6) offsets the "love for variety" effect that is present in the Dixit-Stiglitz index of aggregation. See Bilbiie et al. (2007) and Fujiwara (2007) for an analysis of the implications of taking into account a time-varying number of available varieties.

[5]As explained by Steinsson (2003) this is a shortcut to introduce a cost-push shock, that is exogenous fluctuations in the desired markup. The model I propose has also endogenous fluctuations in the market power. As it will be clearer later I need to introduce exogenous markup fluctuations to estimate the Phillips curve.

The intermediate good firms in each industry operate in a regime of monopolistic competition. As in the original work of Dixit and Stiglitz (1977), each producer has a finite number of competitors and chooses the price in order to maximize its profit given the conditional demand for its own good and the price set by its $N_t(j)-1$ competitors. Firms thus take into account that their price decisions have a nonnegligible weight in the market.

The production of good $x_t(i,j)$ requires firm-specific labor with the following technology

$$x_t(i,j) = Z_t \ell_t(i,j)$$

where Z_t is an economy-wide technology shock whose stochastic process is given by

$$log Z_t = \rho_z log Z_{t-1} + \epsilon_t^z.$$

The conditional demand of the variety i of the good produced by industry j is the following

$$
\begin{aligned}
x_t(i,j) &= \left(\frac{p_t(i,j)}{P_t(j)}\right)^{-\tau_t} \frac{Y_t(j)}{N_t(j)} & (3.7) \\
&= \left(\frac{p_t(i,j)}{P_t(j)}\right)^{-\tau_t} \left(\frac{P_t(j)}{P_t}\right)^{-\omega} \frac{Y_t}{N_t(j)}. & (3.8)
\end{aligned}
$$

When the elasticity of substitution among varieties (τ) differ from the one among industries (ω), the demand of the variety $x_t(i,j)$ depends both on the price of the variety relative to the industry price index and on the relative price of the industry.

Each firm i of industry j faces a convex price adjustment cost

$$AC_t(i,j) = \frac{\theta}{2} \left[\frac{p_t(i,j)}{p_{t-1}(i,j)} - \lambda \pi_{t-1} - (1-\lambda)\right]^2.$$

Firms must pay this cost, proportional to their sales, if they want to change the price.[6] When λ is equal to zero, the producer has to pay a fixed cost for whatever

---

[6]For simplicity I assume that also newly entered firms pay a cost to change their price so that all firms are symmetric. Bilbiie et al. (2007) consider the possibility that new entrants in their first period choose their price as if there were no nominal rigidities. They show that the dynamic responses to shocks are almost identical to the benchmark ones.

change in the price he is willing to implement. When $\lambda > 0$, such cost is paid only when the desired change in price is different from the one implied by a fraction λ of the previous period inflation.[7] The real profits at time t and for the generic firm (i,j) are the following

$$\Pi_t(i,j) = x_t(i,j) \left[1 - \frac{W_t}{P_t} Z_t^{-1} - AC_t(i,j) \right]. \tag{3.9}$$

The value of a firm is given by the expected value of the discounted future stream of profits

$$v_t(i,j) \equiv E_t \sum_{s=t}^{\infty} Q_{t,s} \Pi_s(i,j) \tag{3.10}$$

where $Q_{t,s} = [\beta(1-\delta)]^{s-t} \frac{U_{c,s}}{U_{c,t}} \frac{P_t}{P_s}$.

The price setting problem is thus to maximize (3.10) subject to (3.8). I solve this problem under the assumption of symmetry both across firms belonging to the same industry and across industries. This implies that the optimal price, the price adjustment cost and the number of competitors are the same across industries. The first order condition is

$$(1 - \tau_t) + (\tau_t - \omega)N_t^{-1} + \frac{W_t}{Z_t}p_t^{-1} \left[\tau_t - (\tau_t - \omega)N_t^{-1} \right] -$$

$$-\theta(\pi_t - \lambda\pi_{t-1})(1 + \pi_t) - \frac{\theta}{2}(\pi_t - \lambda\pi_{t-1})^2 \left[1 - \tau_t + (\tau_t - \omega)N_t^{-1} \right] +$$

$$+\beta(1-\delta)E_t \left\{ \frac{C_t}{C_{t+1}} \theta(\pi_{t+1} - \lambda\pi_t)(1 + \pi_{t+1}) \frac{x_{t+1}}{x_t} \right\} = 0.$$

Rearranging the terms it implies the following price for all firms

$$p_t = \mu_t MC_t$$

where the nominal marginal costs MC_t are

$$MC_t \equiv Z_t^{-1} W_t$$

---

[7]This is a shortcut to introduce indexation in a way similar to Christiano et al. (2005) but in a Rotemberg (1982) framework.

and the actual markup charged over marginal costs is

$$\mu_t = \frac{\tau_t - (\tau_t - \omega)N_t^{-1}}{\left[1 - \frac{\theta}{2}(\pi_t - \lambda\pi_{t-1})^2\right]\left[\tau_t - 1 - (\tau_t - \omega)N_t^{-1}\right] + \theta\Omega} \tag{3.11}$$

$$\Omega \equiv (\pi_t - \lambda\pi_{t-1})(1 + \pi_t) - \beta(1 - \delta)E_t\left[(1 + \pi_{t+1})(\pi_{t+1} - \lambda\pi_t)\frac{C_t}{C_{t+1}}\frac{x_{t+1}}{x_t}\right]$$

where I defined inflation as $\pi_t = \frac{p_t}{p_{t-1}} - 1$.

When nominal rigidities are absent (i.e. when $\theta = 0$), firms charge their desired markup on the nominal marginal costs. Differently from the standard New Keynesian framework, the desired markup is time-varying and it depends inversely on the number of competitors in the market[8]

$$\mu_t^n = \frac{\tau_t - (\tau_t - \omega)(N_t^n)^{-1}}{\tau_t - 1 - (\tau_t - \omega)(N_t^n)^{-1}}. \tag{3.12}$$

Conditional on the number of competitors in the market, the price elasticity of demand faced by each competitor is defined as follows

$$\varepsilon(N_t^n) = \tau_t - (\tau_t - \omega)\frac{1}{N_t^n} \tag{3.13}$$

Even if the variance of τ_t is equal to zero, the price elasticity is time-varying and so it is the desired markup. When $N \to \infty$, the price elasticity of demand is equal to τ_t and the desired markup does not depend on the fluctuations of the number of firms as in the standard model.

Consistently with the findings of Broda and Weinstein (2007), according to which the price elasticities of demand are higher for more disaggregated goods[9], I consider the case in which $\omega < \tau$, that is the elasticity of substitution among the goods produced by different industries is lower than the elasticity of substitution among the varieties of the goods in each industry. In this case an increase in the number of firms increases the price elasticity of demand.[10]

---

[8]The n suffix indicates the flexible price ("natural") equilibrium.

[9]One could think to the two layers of production as levels of statistical aggregation.

[10]The price elasticity of demand depends not only on the elasticity of substitution among varieties (τ) but also on the extent to which the producer of a variety by changing his price is able to affect the

3.2.3 Entry

There is free entry in the market. At each point in time a large group of prospective entrepreneurs decides whether to create a new firm and enter the market. They compare the discounted stream of future profits with the entry cost Ψ. Differently from Bilbiie et al. (2007), the entry cost is paid in terms of output units instead of effective labor units. The free entry condition is thus

$$v_t = \Psi.$$

All firms, from the period subsequent to incorporation, face an exogenous probability δ of exiting the market. The number of firms in the economy evolves according to the following law of motion

$$N_t = (1 - \delta)N_{t-1} + N_t^e. \tag{3.14}$$

3.2.4 Equilibrium, aggregate accounting conditions and monetary policy

The aggregate output of firms is allocated to consumption and to pay the entry and the price adjustment cost. Thus the following accounting equation must hold

$$Y_t = Z_t N_t l_t = C_t + \Psi N_t^e + AC_t. \tag{3.15}$$

In a symmetric equilibrium the output of each firm is given by $x_t = \frac{Y_t}{N_t}$.

The equilibrium condition for nominal bonds and the firms' shares in the economy are $B_t = 0$ and $a_t = 1$ for each time t. The aggregate accounting equation, derived from the aggregation of the household's budget constraint is given by

$$\frac{W_t}{P_t}L_t + \Pi_{t-1}(1 - \delta)N_{t-1}(1 + \pi_t)^{-1} = C_t + v_t N_t^e.$$

---

price index of the industry. A decline in the firm's market share (i.e. an increase in the number of firms) would decrease the price elasticity of demand only in the theoretical case in which an increase in the price of a variety leads to a substitution away from the industry, which the variety belongs to, that is stronger than the substitution across varieties (i.e. $\omega > \tau$).

The labor market clears when the hours demanded for the production by all firms are equal to the hours that households are willing to supply:

$$N_t l_t = L_t.$$

Finally, the conduct of monetary policy is described by a very standard Taylor rule

$$i_t = \frac{1}{\beta} - 1 + \phi_\pi \pi_t + i_t^*.$$

The i^* is an unexpected deviation from the interest rate path implied by the Taylor rule and it can be interpreted as a monetary policy shock.

3.2.5   Markups and the New Keynesian Phillips curve

Substituting into (3.11) the equilibrium condition (3.15) the actual markup is given by

$$\mu_t = \frac{\tau_t - (\tau_t - \omega)N_t^{-1}}{\left[1 - \frac{\theta}{2}(\pi_t - \lambda\pi_{t-1})^2\right]\left[\tau_t - 1 - (\tau_t - \omega)N_t^{-1}\right] + \theta\bar{\Omega}} \tag{3.16}$$

$$\bar{\Omega} \equiv (\pi_t - \lambda\pi_{t-1})(1 + \pi_t) - \beta(1-\delta)E_t\left[(1 + \pi_{t+1})(\pi_{t+1} - \lambda\pi_t)\frac{N_t}{N_{t+1}}\frac{1 - \frac{\theta}{2}(\pi_t - \lambda\pi_{t-1})^2}{1 - \frac{\theta}{2}(\pi_{t+1} - \lambda\pi_t)^2}\right]$$

For the moment I restrict the parameter on inflation indexation λ to be equal to zero. By log-linearizing equation (3.16) it results that the fluctuations in the actual markup are caused by the changes in the desired markup, which are driven by variations in the number of firms and by exogenous changes of the elasticity of substitution, and by the nominal rigidities that imply sluggish adjustment of prices after any perturbation of the steady state equilibrium. After log-linearization,[11]

$$\hat{\mu}_t = \underbrace{\frac{\theta}{\varepsilon - 1}\left[\beta(1-\delta)E_t\pi_{t+1} - \pi_t\right]}_{\textit{nominal rigidities}} - \underbrace{\frac{\tau - \varepsilon}{(\varepsilon - 1)\varepsilon}\hat{n}_t}_{\textit{endogenous desired markup}} + \underbrace{\frac{\tau(N^{-1} - 1)}{(\varepsilon - 1)\varepsilon}\hat{\tau}_t}_{\textit{exogenous desired markup}}$$

$$\tag{3.17}$$

---

[11]Henceforth I write $\varepsilon = \varepsilon(\bar{N})$ for shortness, where $\varepsilon(\cdot)$ is defined in equation (3.13); furthermore τ without a time subscript indicates the steady state level of the elasticity of substitution.

Rearranging the terms, I obtain the usual form of the NKPC

$$\pi_t = \beta(1-\delta)E_t\pi_{t+1} + \kappa\widehat{rmc}_t - \kappa\eta\hat{n}_t + u_t \tag{3.18}$$

where $\kappa \equiv \frac{\varepsilon-1}{\theta}$ and $u_t \equiv \frac{\tau(N^{-1}-1)}{(\varepsilon-1)\varepsilon}\hat{\tau}_t$. The parameter η is the elasticity of the desired markup with respect to the fluctuations in the number of active firms and it is defined as follows

$$\eta \equiv \frac{\partial\mu^n(N)}{\partial N}\frac{N}{\mu^n(N)} = \frac{\tau-\varepsilon}{(\varepsilon-1)\varepsilon}.$$

When this elasticity is equal to zero, that is the desired markup does not fluctuate in response to changes in the number of active firms, the model implies a standard NKPC.

In the case of indexation, i.e. $\lambda > 0$, I obtain the so-called hybrid NKPC that features a backward-looking term on inflation

$$\pi_t = \frac{\beta(1-\delta)}{1+\beta(1-\delta)\lambda}E_t\pi_{t+1} + \frac{\lambda}{1+\beta(1-\delta)\lambda}\pi_{t-1} + \frac{\kappa}{1+\beta(1-\delta)\lambda}\widehat{rmc}_t - \frac{\eta\kappa}{1+\beta(1-\delta)\lambda}\hat{n}_t + u_t. \tag{3.19}$$

Henceforth I consider as a benchmark for the empirical exercise equation (3.18). However I also estimate equation (3.19) to check the robustness of the results as many studies have found a significant role for lagged inflation in the inflation dynamics.

3.3 Empirical issues

This section discusses some of the issues that are related to the estimation of the NKPC derived in section 3.2.5. I first examine how to measure the real marginal costs and the number of firms in the data. Then I present the empirical methodology with which I intend to proceed with the estimation.

3.3.1   Data and measurement issues

In order to estimate the NKPC implied by the model, some issues about the measurement of the real marginal costs and of the number of firms must be discussed. In fact, real marginal costs are not directly observable in the data. The model, however, provides some conditions under which they can be constructed from available data. It has been shown that with a Cobb-Douglas technology the real marginal costs are proportional to the real unit labor costs.

$$
\begin{aligned}
RMC_t &= \frac{W_t}{P_t} Z_t^{-1} \\
&= \frac{W_t L_t}{Y_t P_t} \equiv S_t.
\end{aligned}
$$

S_t is an observable variable, that is the real unit labor costs.[12]

For what concerns the measurement of the number of firms, only data on the number of newly entered firms (i.e. new incorporations) are available at quarterly frequency and for a sufficiently large span of time.[13] Data on business failures are available only for a subset of industries[14] and thus they are not comparable with those on new incorporations. Furthermore the business failures series has a discontinuity in 1984 and, up to my knowledge, reconstructed data are available only at annual frequency (see Naples and Arifaj (1997)). To overcome the lack of data I proceed in two different ways. Using some conditions of the model, I rewrite the NKPC so that it features the number of new entrants as a driving force of inflation instead of the stock of active firms (see section 3.2.3). This will be the equation on which are based the benchmark estimates of the paper. Furthermore, to conduct some robustness exercises, I reconstruct data on the total number of firms in a way that is consistent with the model. From equation (3.14), that is the dynamic evolution of business population, using the data on new incorporations and calibrating the parameter for

---

[12]I abstract from capital accumulation. However one can show that in a model with capital accumulation real marginal costs are the same up to a first order approximation.

[13]These data are in the Survey of Current Business published by the BEA. They are collected by the Dun & Bradstreet corporation and are available until 1998. An index of net business formation is also available for the same period.

[14]Namely the commercial and industrial sectors.

the probability of a firm of exiting the market, δ, I can retrieve the data on the stock of existing firms. As initial condition I use the number of firms that were active in December 1947.[15] A plausible calibration of δ can be derived using the annual data on business failures.[16] In the sample period of 1988-1998, on average 12% of the total number of firms active in the U.S. economy failed each year. This implies a calibration of 0.03 for δ. The business failures are countercyclical, as documented by Bilbiie et al. (2005). My model, by considering a constant fraction of firms exiting the market, undervalues the impact of $\hat{n}_t$ on the business cycle.

As a measure of inflation I use the log differences of the GDP deflator for two reasons. First of all, most of the literature estimate the NKPC using the changes in the GDP deflator as a measure of inflation, thus results are more easily comparable. Secondly, the GDP deflator reflects the prices of all domestically produced goods, excluding import prices, thus it is the appropriate inflation measure for the closed economy described in the model. Appendix C.1 illustrates the data source and shows the plots of the real marginal cost and the constructed data for the number of firms.

Solving forward the NKPC (3.18) I have

$$\pi_t = \kappa E_t \sum_{j=0}^{\infty} [\beta(1-\delta)]^j \widehat{rmc}_{t+j} - \kappa \eta E_t \sum_{j=0}^{\infty} [\beta(1-\delta)]^j \hat{n}_{t+j} + u_t. \tag{3.20}$$

According to the model, current inflation should be correlated with leads of both the log deviations from steady state of the real marginal costs and of the number of firms. Figure 3.1 shows the cross correlograms of inflation with real marginal costs and the number of firms.

The maximum correlation of inflation with the measured real marginal costs and the number of firms is obtained for leads of both variables as predicted by the model. Regarding the sign of the comovements, these unconditional correlations are not too informative and the estimation of the inflation dynamics equation is needed to carry out a meaningful comparison with the model's predictions.

---

[15]This information is available from the Economic Report of the President of 1948, available through the Federal Reserve of Saint Louis (FRED) database.

[16]The U.S. Small Business Administration has detailed data on the total number of firms and the business demography. However, the observations are annual and they start starting from 1988.

Figure 3.1: Dynamic correlation of inflation and leads and lags of real marginal costs and the number of firms.
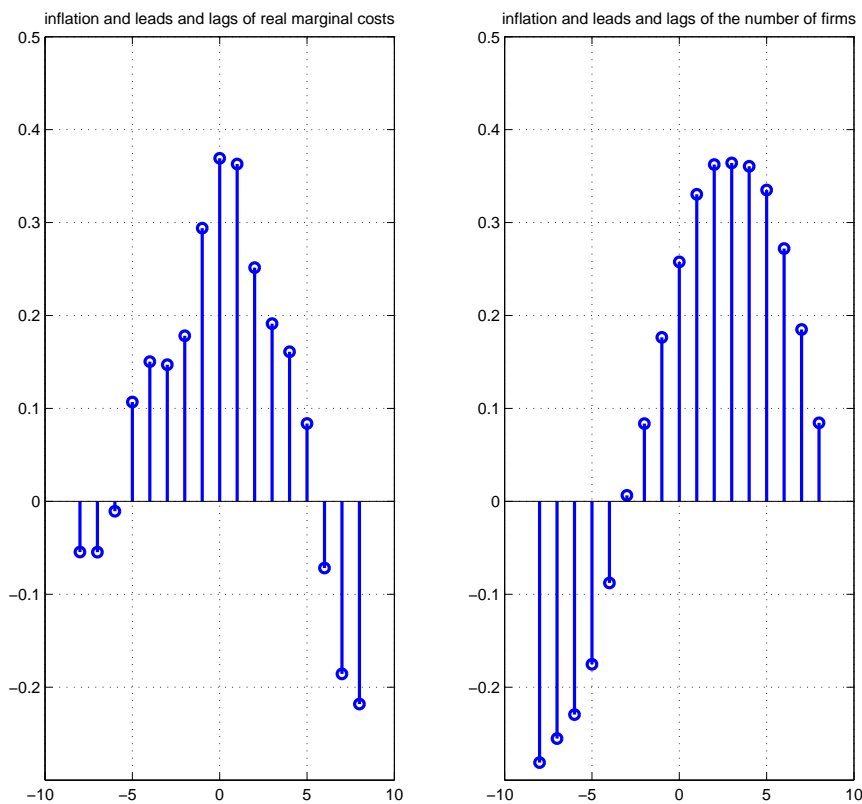
Notes: Data are in logs and they have been HP-filtered. Sample period: 1960q1-1998q4.

Table 3.1 reports the unconditional correlations of the relevant series. It shows that the reconstructed series for the number of firms is procyclical and that it is positively correlated with my measure of the real marginal costs.

3.3.2   Empirical methodology

The estimation of the NKPC, as equation (3.18), is challenging since it involves the measurement of expectations. The empirical methodologies that have been used so

Table 3.1: Correlations

	Real unit labor costs	*Number of firms*	*Real GDP*
Real unit labor costs	1.000 (—)		
Number of firms	0.381 (0.000)	1.000 (—)	
Real GDP	-0.287 (0.000)	0.250 (0.002)	1.000 (—)

Notes: Data are in logs and they have been HP-filtered. Sample period: 1960q1-1998q4. In brackets the p-value of the t-statistics test on the significance of the correlation.

far have been strongly debated (see the Journal of Monetary Economics issue of 2005 on the econometrics of the new Keynesian price equation). The approaches, proposed respectively by Galí and Gertler (1999) and Sbordone (2002), handle the expectation term in the inflation dynamics equation differently. The former exploits the rational expectations hypothesis to have an orthogonality condition to estimate the NKPC with GMM; the latter estimates the closed form solution of the NKPC using a two-step procedure that involves obtaining the projections of the relevant variables from a VAR and using a distance estimator to find the values of the structural parameters.

As discussed in the previous section, data on the number of active firms are not available and one has to make some assumptions in order to reconstruct them from the available data on new entry of firms. However, working with the closed-form solution and using the dynamics of the number of firms of the model in equation (3.14), I can rewrite equation (3.20) so that the NKPC depends on the number of new entrants instead of the stock of existing firms.

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j (1-\delta)^j E_t \widehat{rmc}_{t+j} - \frac{\kappa \eta \delta}{1 - \beta(1-\delta)^2} \left[\sum_{h=1}^{\infty} (1-\delta)^h \hat{n}_{t-h}^e + \sum_{j=0}^{\infty} \beta^j (1-\delta)^j E_t \hat{n}_{t+j}^e \right] + u_t.$$

$$(3.21)$$

I estimate the closed-form equation above in the same spirit as Sbordone (2002). However instead of using her two-step procedure, that is estimate the VAR first and then use a simple minimum distance criterion to find the relevant parameters of the NKPC, I estimate jointly all the parameters as in Guerrieri et al. (2008). This is a one step-procedure that allows to take better into account the uncertainty around the estimates of the NKPC parameters.

As in Sbordone (2002) I construct the expectations of future real marginal cost and new incorporations from the projections of a VAR. The VAR is specified as follows

$$x_t = A x_{t-1} + \epsilon_t \tag{3.22}$$

where $x_t = [\widehat{rmc}_t \ \hat{n}_t^e]'$. After having incorporated such assumptions on the representation of the real marginal costs and the new incorporations series into equation (3.21), I jointly estimate with GMM the system composed by (3.22) and the following equation

$$\pi_t = \kappa e_1'[I - \beta(1-\delta)A]^{-1}x_t - \frac{\delta\kappa\eta}{1 - \beta(1-\delta)^2}\left\{e_2'[I - \beta(1-\delta)A]^{-1}x_t + \sum_{h=1}^{L}(1-\delta)^h e_2' x_{t-h}\right\} + u_t \tag{3.23}$$

where e_1' and e_2' are vectors that select the real marginal cost and the new incorporations respectively. Appendix C.2 shows the algebraic details.

To deal with the backward-looking terms in the LHS of the equation above I have to arbitrarily truncate the infinite sum of past realizations of the new incorporations. In the benchmark specification I set L equal to 3. I then perform a check on the robustness of the estimates to this truncation.[17] As pointed out by Guerrieri et al. (2008) this estimation methodology requires to model an error in equation (3.21), which, in this case, is represented by an iid markup shock (i.e. $u_t \equiv \frac{\tau(N^{-1}-1)}{(\varepsilon-1)\varepsilon}\hat{\tau}_t$). As a benchmark set of instruments I use two lags of inflation, real GDP, wage inflation, real marginal costs. All variables, except inflation and wage inflation, are in logs

---

[17]This truncation implies that the uncertainty around the estimates is higher since we are using a constructed regressor for the number of firms. However since the standard errors of the estimates are very similar across different choices of the truncation parameter L, we conclude that the additional uncertainty is small.

and detrended with a polynominal of third order.[18] I do not consider as instruments the lagged values of the new incorporations, since they could bias the estimates given their direct impact on inflation according to the theoretical specification of the NKPC. Following Eichenbaum and Fisher (2007) I also present results for a different set of instruments that include current and five lags of the monetary policy and technology shocks as identified in Altig et al. (2005).

In the above equation I identify only two parameters: the pass-through of real marginal cost on inflation, κ, and the elasticity of the desired markup with respect to the number of firms, η. I need to calibrate the exit rate of firms δ and the discount factor β. I consider a value of 0.03 for δ, as explained in section 3.3.1, and I set $\beta = 0.99$ as it is usual in the business cycle literature. I thus estimate the parameters κ and η together with the VAR companion matrix. Unfortunately, the steady state level of the price elasticity of demand, ε, and the curvature of the price adjustment function, θ, are not separately identifiable. My estimation results are thus comparable only with the reduced form parameter that measures the slope of the Phillips curve obtained in previous contributions of the literature.

The positive comovement between the real marginal costs and the number of firms (see table 3.1) suggests that if one omits the number of firms from the standard NKPC the estimates of the coefficient on real marginal costs is smaller and it might not be significant. This is because the increase in real marginal costs would come with a rise in the number of entrants that decreases the desired markup, partially compensating the inflationary pressures of the increasing real marginal costs. By explicitly considering the fluctuations of the markup that comes from those changes in competitiveness due to new entrants, one can disentangle the effect of real marginal costs on the inflation dynamics from the one of decreasing market power.

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[18]Table C.1 illustrates the data sources.

3.4   Estimation results

Table 3.2 shows the estimates of the NKPC in the case of endogenous firm entry and in the standard case. As it is shown in the first column, the coefficient on real marginal cost fluctuations (κ) and the elasticity of the desired markup with respect to the number of market participants (η) are both significantly different from zero.[19] The estimate of κ suggests that measured real marginal costs are a relevant driving force of the inflation dynamics as found by Galí and Gertler (1999) and Sbordone (2002). The estimate of η indicates that, when the number of firms that are active in the economy increases, the market power of firms declines as predicted by the model. The benchmark estimate for η implies that a theoretical 10% increase of the number of firms from the steady state would bring down the desired markup by about 12%; this, in turn, would lower annual inflation of about 1.4% in the short run.

The point estimate of the real marginal cost pass-through is almost 70% higher than what obtained by estimating with the same methodology the standard curve without endogenous firm entry. As explained above, omitting the fluctuations of the desired markup from the empirical specification of the inflation dynamics generates a downward bias in the estimates of the coefficient on real marginal costs. Such bias however is relatively small in size. The value of κ estimated in the standard case lies in the confidence interval of the estimates of the inflation dynamics curve with firm entry. This paper is not the first one that provides evidence which suggests that the NKPC is not as flat as baseline estimations seem to imply. Imbs et al. (2007) show that heterogeneity in pricing behavior matters for assessing the driving forces of inflation. Not accounting for it generates a downward bias in the estimation of the coefficient of aggregate marginal costs on aggregate inflation. Küster et al. (2007) found that if shocks to the price markup are persistent and negatively correlated with the real unit labor costs the estimated pass-through of measured marginal costs into inflation is limited, even if prices are fairly flexible. Here endogenous fluctuations in the desired markup, due to entry and exit of firms, imply that the estimated pass-

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[19]I computed Newey-West corrected standard errors with 12 lags since the estimated residuals show some autocorrelation under both the standard and the endogenous firm entry specification.

Table 3.2: Estimates of the Phillips curve with firm entry

	with firm entry	*standard*	*with firm entry*	*standard*
κ	0.03	0.018	0.055	0.051
	(0.007)	(0.006)	(0.010)	(0.014)
η	1.172	-	0.786	-
	(0.221)	-	(0.296)	-
VAR estimates:				
a_{11}	0.953	0.949	0.919	0.887
	(0.015)	(0.024)	(0.053)	(0.041)
a_{12}	0.014	-	0.021	-
	(0.003)	-	(0.009)	-
a_{22}	0.927	-	0.846	-
	(0.011)	-	(0.044)	-
a_{21}	-0.219	-	-0.783	-
	(0.052)	-	(0.252)	-
J-statistic	12.84	10.39	12.1	9.87
	[0.30]	[0.32]	[0.36]	[0.54]
g_{min}	27.18	74.87	0.39	1.77
crit. values	16.80	15.18	4.75	4.75
Instrument set	**A**	**A**	**B**	**B**

Notes: Sample: 1960q1-1998q4. Standard errors are in brackets. P-values are in square brackets. The covariance matrix has been computed with a 12-lag Newey-West estimator. g_{min} is the Cragg-Donald statistics and the critical values are those of table 5.1 of Stock and Yogo (2005). Instrument set **A** includes two lags of inflation, real marginal costs, detrended real GDP and wage inflation. Instrument set **B** includes current and five lags of identified monetary policy and technology shocks.

through is higher. The intuition behind this result is the same as the one of Küster et al. (2007) but an economically more sound explanation of the cost-push shock usually added in the inflation equation is provided.[20]

In order to check whether the chosen set of instruments is sufficiently correlated with independent variables, table 3.2 also reports the Cragg-Donald statistics. Compared with the critical values computed by Stock and Yogo (2005) I can reject the null hypothesis of instrument weakness. In order to test for the validity of the overi-

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[20]This shock, even in large-scale estimated model, plays an important role in explaining inflation. In an estimated model for the euro area, Smets and Wouters (2003) found that it accounts for at least 50% of the volatility of inflation.

dentifying restrictions the J statistics is reported. It indicates that such restrictions are satisfied. I also estimate the same system with a different set of instruments that includes the identified current and lagged monetary policy and technology shocks. The results are more or less in line with those obtained with the benchmark set of instruments, though the elasticity of the desired markup is somewhat lower and the marginal cost pass-through higher. In this case the null hypothesis for the validity of overidentifying restrictions is accepted, however the Cragg-Donald statistics suggests that these instruments are weak. For this reason, from now on I consider estimates based on the benchmark set of instruments.

The estimation of equation (3.21) required to make some assumptions about the calibration of the average rate of firm exit, the number of lags at which the new incorporations backward-looking terms are truncated and the specification of the VAR to produce forecasts of the real marginal costs and the new incorporations. Table 3.3 shows the results of the estimated NKPC parameters when such assumptions are modified. The benchmark estimates are generally robust to such changes. The forecasting VAR used in the estimates of the last column of table 3.3 is restricted so that the lags of the real marginal costs do not affect the new incorporations. Such restrictions are justified by the fact that the corresponding coefficient is not significant in the benchmark estimates.

Table 3.4 reports the results of other robustness checks. In the first two columns I show the estimates of the NKPC in two subsample periods. I split the sample at the year 1984 since much of the literature on the Great Moderation in the U.S. economy agrees in recognizing it as a relevant breakdate. The results are approximately similar to those found in the estimates over the whole sample. In particular the increase of the estimated marginal cost coefficient when taking into account the firm entry dynamics is of the same order of what found in the benchmark case. The third column of the table displays the estimates in the case in which a different detrending procedure, namely a fourth order polynomial trend, is applied to the real marginal cost and the new incorporations series.[21]

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[21]Detrending with a linear and quadratic trend does not eliminate the unit root in the new incorporations series and the GMM estimation procedure requires stationary time series.

Table 3.3: Robustness checks

		Truncation lags			*Calibration of δ*		*Forecasting VAR*
		$L = 1$	$L = 2$	$L = 4$	$\delta = 0.01$	$\delta = 0.04$	
κ	*firm entry*	0.022 (0.006)	0.022 (0.007)	0.025 (0.006)	0.028 (0.007)	0.031 (0.007)	0.023 (0.005)
	standard	0.018 (0.006)	0.018 (0.006)	0.018 (0.006)	0.014 (0.006)	0.02 (0.006)	0.018 (0.006)
η		3.51 (0.621)	1.75 (0.357)	1.19 (0.248)	1.17 (0.201)	1.00 (0.188)	1.50 (0.326)
J	*firm entry*	12.32 [0.26]	12.66 [0.24]	12.14 [0.28]	12.88 [0.23]	12.82 [0.23]	12.60 [0.34]
	standard	10.39 [0.32]	10.39 [0.32]	10.39 [0.32]	10.39 [0.24]	10.39 [0.24]	10.39 [0.32]

Notes: Standard errors are in brackets. P-values are in square brackets. The covariance matrix has been computed with a 12-lag Newey-West estimator. J is the J-statistics for overidentifying restrictions.

Table 3.4: Robustness checks - Alternative samples and detrending

		Subsample stability		*Detrending*
		1960q1-1983q4	1984q1-1998q4	fourth order polynomial
κ	*firm entry*	0.074 (0.011)	0.012 (0.002)	0.037 (0.008)
	standard	0.033 (0.007)	0.007 (0.003)	0.017 (0.006)
η		1.09 (0.123)	1.92 (0.376)	1.15 (0.185)
obs		96	60	156
J stat	*firm entry*	8.53 [0.58]	6.26 [0.79]	12.78 [0.17]
	standard	8.17 [0.42]	5.80 [0.67]	10.17 [0.18]

Notes: Standard errors are in brackets. P-values in square brackets. The covariance matrix has been computed with a 12-lag Newey-West estimator.

Table 3.5: Robustness checks - Alternative econometric methodologies

	Reduced form estimates		FIML	
	with firm entry	*standard*	*with firm entry*	*standard*
κ	0.018	0.006	0.031	0.023
	(0.009)	(0.008)	(0.009)	(0.007)
κ_n	-0.011	–	–	–
	(0.004)	–	–	–
$\beta(1-\delta)$	0.980	0.909	–	–
	(0.031)	(0.029)	–	–
η	–	–	1.155	–
	–	–	(0.451)	–
g_{min}	91.99	119.39	–	–
crit vals	17.8	16.8	–	–

Notes: Standard errors are in brackets. The covariance matrix has been computed with a 12-lag Newey-West estimator. The specification for the reduced form estimates is $\pi_t = \beta(1-\delta)E_t\pi_{t+1} + \kappa\widehat{rmc}_t + \kappa_n\hat{n}_t$. The instrument set include 4 lags of inflation and 2 lags of the real marginal costs, the number of firms, the wage inflation and the detrended real GDP.

I also check whether the results are robust to the econometric methodology that has been considered so far. Instead of considering the closed form of the NKPC, I also estimate equation (3.18). The results are in table 3.5. I used the series for the number of firms constructed in the way explained in section 3.3.1. Parameters have been estimated with GMM, which in this linear case coincide with the two-stage least square estimator. The set of instruments include: four lags of inflation and two lags of wage inflation, the output gap, the real marginal costs and the number of firms. The data for my measure of real marginal costs and the number of firms have been detrended with a third order polynomial. The estimates above imply that η is significant and equals 0.6. Since GMM have been criticized on the grounds of delivering poor small sample properties when instruments are weakly identified, I consider also the estimation of the system of equations (3.21) and (3.22) using a full information approach. The system of equations is estimated with maximum likelihood. The results are in line with those produced by a limited information approach (see the second column of table 3.5).

Table 3.6 presents the estimates of the hybrid version of the NKPC above, namely

Table 3.6: Estimates of the hybrid New Keynesian Phillips curve

	with firm entry	*standard*
κ	0.002	0.001
	(0.001)	(0.001)
η	0.937	-
	(0.468)	-
λ	0.316	0.341
	(0.026)	(0.021)
J-stat	11.630	9.650
	[0.39]	[0.38]

Notes: Standard errors are in brackets. P-values are in square brackets. The covariance matrix has been computed with a 12-lag Newey-West estimator.

the one in which firms pay a cost in order to change their price by more or less than a fraction λ of the previous period inflation. The current inflation dynamics in this case depend also on lagged inflation as evidenced in equation (3.19) derived above.[22] In this case neglecting endogenous firm entry implies that the estimates of κ are not significant, while they are significant, though lower than the benchmark one, when the new incorporations series is taken into account. The parameter η is significant and it has a value in line with the one of the benchmark estimates. The parameter λ that indicates the degree of indexation to past inflation of firms is significant and it has a similar magnitude in the standard model and in the one with firm entry.

3.5 The model's responses to technology and monetary policy shocks

In this section, I analyze the changes in the transmission mechanism of technology and monetary policy shocks when considering the endogenous firm entry. To this end, I use the estimates obtained in the previous section and I calibrate the parameters that I cannot back out from the empirical results. Then, I plot the impulse response

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[22]The closed form of the hybrid NKPC that is estimated is equation (C.3) in the appendix C.2.

functions of the model described above and the one of the standard model of constant desired markup. While the transmission of the technology shock is affected by the presence of firm entry, as previously found by Bilbiie et al. (2005), the implications of endogenous desired markup for the monetary transmission mechanism are almost negligible.

The calibration of the model is illustrated in table 3.7. The inverse elasticity of

Table 3.7: Calibrated parameter values

β	0.99	Subjective discount factor
δ	0.03	Exit rate of firms
φ	3	Inverse of the labor supply elasticity
θ	165	Degree of price stickiness
τ	17	Elasticity of substitution across goods
ω	2.001	Elasticity of substitution across sectors
Ψ	2.424	Entry cost
ρ_z	0.9	Persistence of the technology shock
ψ_π	1.5	Taylor rule coefficient

labor supply ξ is set to 3 and the discount factor β to 0.99. The average exit rate δ is computed from annual data on firms' death, as explained in section 3.3.1, and it is equal to 3%. The degree of price stickiness θ can be inferred from the estimates presented in the previous section. Given that κ is estimated to be equal to 0.03 and assuming a steady state markup of 30%, the price elasticity of demand in steady state is equal to 4.33. Knowing that $\kappa = \frac{\varepsilon-1}{\theta}$, I set θ equal to 165, a somewhat higher value than the one estimated in the context of a full-fledged DSGE model with maximum likelihood by Ireland (2001). The parameter τ is chosen to match the estimated elasticity of desired markup to changes in the number of competitors η. I estimate $\eta = 1.1$; using the ε specified above, I set τ equal to 17. I then choose ω equal to 2 in order to have the steady state number of firms consistent with a steady state markup μ of 1.3. The entry cost Ψ is determined in the steady state equilibrium. The persistence parameter of the technology shock is calibrated to 0.9. The monetary policy response to inflation, that is the Taylor rule coefficient ψ_π, is set to 1.5 in order to guarantee the uniqueness of the equilibrium.

Figure 3.2 shows the impulse response functions to a one standard deviation positive technology shock in a standard New Keynesian model with Rotemberg price adjustment cost and in the model presented above with endogenous firm entry.
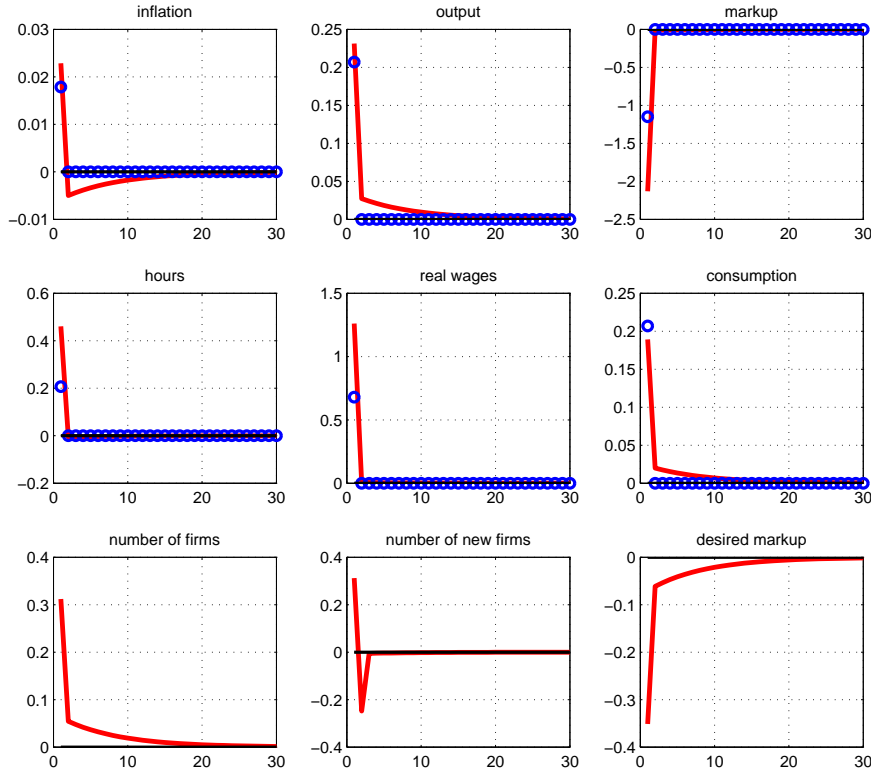
Figure 3.2: Impulse response functions to a 1% st. dev. positive technology shock
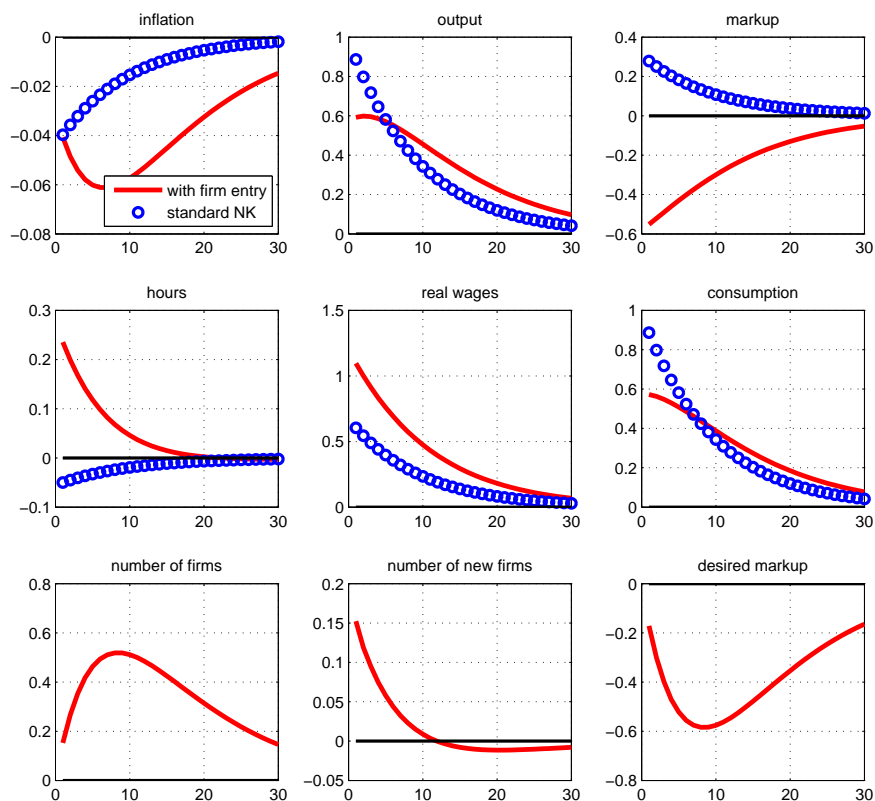
Notes: The dotted blue lines are the responses of each variables in the standard NK model; the solid red lines are those in the model with endogenous firm entry

A positive technology shock boosts output and consumption. In the model with endogenous firm entry output increases both at the intensive and extensive margin on impact. After the shock, it becomes more profitable for firms to enter the market and the total number of firms increases. Real wages increase since the producers must offer a higher wage in order to induce households to work. Interestingly, the model predicts that total hours increase after an improvement in the labor productivity.

Figure 3.3: Impulse response functions to a iid 1% st. dev. expansionary monetary policy shock

Notes: The dotted blue lines are the responses of each variables in the standard NK model; the solid red lines are those in the model with endogenous firm entry

Such increase is at the extensive margin, while hours per firm decline (not shown in the figure) as in the basic NK model. This suggests that some of the evidence found on the positive response of hours to a technology shock (see Christiano et al. (2003)) is still consistent with the existence of nominal rigidities once we endogenize the dynamics of firm entry.

The response of markups is dramatically different from the baseline model. Conditional on a technology shock, the actual markups are countercylical in my model while they are procyclical in the baseline NK model. In the model presented above,

the drop in the desired markup generated by the entry of new firms when technology improves drives down the actual markup. While in the standard NK model the real marginal costs decline because of the increased productivity, in the model with firm entry the rise of real wages offsets the downward effect of the technology shock on the real marginal costs. Inflation nevertheless declines because of the competitiveness pressures coming from the increased number of firms in the market.

Figure 3.3 displays the impulse response functions to an expansionary monetary policy shock. The decrease of the real interest rate implies an increase of output, hours worked and consumption. The model predicts that loosening monetary policy implies an expansion of the entry of new firms.[23] The inflation increases on impact due to the rise of real wages, which offsets the decline of the desired markup. From the period after the shock the continued increase of the number of firms brings the inflation below its steady state level.

3.6   Conclusions

Although the inflation dynamics has been thoroughly studied, there is still some uncertainty surrounding the determinants of inflation. In particular, it is not clear what is the role of competitive pressures on the inflation fluctuations and whether they can affect the empirical link between prices and labor costs. This paper confirms that real marginal costs are a relevant driving force of the inflation process as previously found by Galí and Gertler (1999) and Sbordone (2002). Moreover it finds that the competitive pressures due to the entry of new firms in the economy affect the inflation dynamics and that neglecting the effect of changes in the desired markup at business cycle frequency results in underestimating the importance of the unit labor costs. In a model of monopolistic competition, price stickiness and free entry in which the desired markup fluctuates in response to changes in the number of active firms in the economy, the implied NKPC inflation depends not only on the marginal cost, but also on the number of firms, a proxy for the changes in market power. The curve is empirically

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[23]This is in line with evidence found by Lewis (2009), who shows that a monetary policy contraction induces a drop in the number of new firms in the medium term.

tested using the methodology of Guerrieri et al. (2008). The pass-through of the real marginal costs on aggregate inflation is higher than in baseline estimates in which the number of firms is not considered. Furthermore the elasticity of the desired markup with respect to the number of firms in the market is estimated to be significantly different from zero. Using the estimation results, I calibrate the general equilibrium model to compare the responses to a technology and monetary policy shock with those of a basic New Keynesian model. Endogenous time-varying markups affect the transmission mechanism of a technology shocks, while it affects almost negligibly the transmission of monetary policy impulses. Interestingly, the model predicts that total hours increase after an improvement in the labor productivity. Such increase is due to the extensive margin, while hours per firm decline as in the basic NK model. This suggests that some of the evidence found on the positive response of hours to a technology shock (see Christiano et al. (2003)) is still consistent with the existence of nominal rigidities once we endogenize the dynamics of firm entry.

Appendix A

External Trade and Monetary Policy in a Currency Area

A.1 The (Sum and Difference) Solution of the Model

In this appendix I derive in detail the solution of the model in terms of area-wide and differentials variables (that is the equations described in section 1.3.4). I consider the case in which the two regions of the currency area have equal size (i.e. $m = 0.5$). I loglinearized the model around the trade-balanced deterministic steady state. For a generic variable x I define the area-wide corresponding variable as $x_u \equiv mx_1 + (1 - m)x_2$ and the differentials as $x_r \equiv x_2 - x_1$. As specified above, I assume that the rest of the world is a large country relative to the currency area (specifically, the amount of goods produced in the currency area and consumed by the rest of the world is negligible in its basket of consumption, thus one can say $C^{row} = Y_{row}$).

A.1.1 Derivation of equation (1.18)

Consider first the aggregate demand of goods produced in region 1 (equation (1.16))

$$Y_{1,t} = (1 - \alpha - \omega + \varepsilon) \left(\frac{P_{1,t}}{P_{1,t}^c}\right)^{-1} C_t^1 + \alpha \left(\frac{P_{1,t}}{P_{2,t}^c}\right)^{-1} C_t^2 + (\omega - \varepsilon) \left(\frac{P_{1,t}}{P_{row,t}^c S_t}\right)^{-1} C_t^{row}.$$

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Using the risk sharing conditions (1.11) to substitute for the consumption of region 2 and of the rest of the world, I obtain

$$Y_{1,t} = \left(\frac{P_{1,t}}{P_{1,t}^c}\right)^{-1} C_t^1 \left[(1 - \alpha - \omega + \varepsilon) + \alpha \left(\frac{P_{1,t}^c}{P_{2,t}^c}\right)^{\frac{1}{\sigma}-1} + (\omega - \varepsilon) \left(\frac{P_{1,t}^c}{P_{row,t}^c S_t}\right)^{\frac{1}{\sigma}-1}\right].$$

Loglinearizing around the steady state I obtain

$$y_{1,t} = \left[\alpha + \frac{1}{2}(\omega - \varepsilon)\right] t_t^{in} + (\omega - \varepsilon)t_t^{ex} + c_t^1 + \frac{\sigma - 1}{\sigma} \left[\alpha q_t^{in} + (\omega - \varepsilon)q_t^{ex}\right] \qquad \text{(A.1)}$$

where I used the following definitions for the internal and external terms of trade and real exchange rates

$$
\begin{aligned}
t^{in} &= p_2 - p_1 \\
t^{ex} &= p_{row} + s - p_u \\
q^{in} &= p_2^c - p_1^c \\
q^{ex} &= p_{row}^c + s - p_u^c.
\end{aligned}
$$

Using (1.6) and (1.7) I derived the (log linearized) relationships between the (internal and external) terms of trade and the real exchange rates

$$
\begin{aligned}
q_t^{in} &= (1 - 2\alpha - \omega)t_t^{in} + 2\varepsilon t_t^{ex} & \text{(A.2)} \\
q_t^{ex} &= \frac{\varepsilon}{2}t_t^{in} + (1 - \omega)t_t^{ex}. & \text{(A.3)}
\end{aligned}
$$

Substituting (A.2) and (A.3) into the aggregate demand of goods produced in region 1 I have

$$y_{1,t} = \left[\alpha\Phi + \frac{1}{2}(\omega - \varepsilon)\Gamma_1\right] t_t^{in} + (\omega - \varepsilon)\Gamma_1 t_t^{ex} + c_t^1. \qquad \text{(A.4)}$$

Analogously, the aggregate demand for goods produced in region 2 is

$$y_{2,t} = -\left[\alpha\Phi + \frac{1}{2}(\omega + \varepsilon)\Gamma_2\right] t_t^{in} + (\omega + \varepsilon)\Gamma_2 t_t^{ex} + c_t^2. \qquad \text{(A.5)}$$

where

$$
\begin{aligned}
\Phi &\equiv 1 + \frac{\sigma - 1}{\sigma}(1 - 2\alpha - 2\omega) \\
\Gamma_1 &\equiv 1 + \frac{\sigma - 1}{\sigma}\left(\frac{2\alpha\varepsilon}{\omega - \varepsilon} + 1 - \omega + \varepsilon\right) \\
\Gamma_2 &\equiv 1 + \frac{\sigma - 1}{\sigma}\left(1 - \omega - \varepsilon - \frac{2\alpha\varepsilon}{\omega + \varepsilon}\right).
\end{aligned}
$$

Notice that when $\varepsilon = 0$, $\Gamma_1 = \Gamma_2 = 1 + \frac{\sigma-1}{\sigma}(1-\omega)$. When the utility is logarithmic (i.e. $\sigma = 1$), $\Gamma_1 = \Gamma_2 = \Phi = 1$. Aggregating over the regions of the currency area, the aggregate demand of the currency area goods is

$$y_{u,t} = \frac{1}{4}[(\omega - \varepsilon)\Gamma_1 - (\omega + \varepsilon)\Gamma_2]t_t^{in} + \frac{1}{2}[(\omega - \varepsilon)\Gamma_1 + (\omega + \varepsilon)\Gamma_2]t_t^{ex} + c_t^u. \qquad (A.6)$$

Using (1.11), the following risk sharing condition between the currency area and the rest of the world holds

$$C_t^u = (Q_t^{ex})^{1/\sigma} C_t^{row}.$$

Loglinearizing and using (A.2) and (A.3), one obtains

$$c_t^u = \frac{1}{\sigma}\left[(1-\omega)t_t^{ex} + \frac{\varepsilon}{2}t_t^{in}\right] + c_t^{row} \qquad (A.7)$$

which, substituted into (A.6), yields to equation (1.18) of the text

$$y_{u,t} = \frac{1}{\sigma_\omega}t_t^{ex} + y_{row,t} - \sigma_\varepsilon t_t^{in}$$

where

$$\sigma_\omega^{-1} = \frac{1}{2}\left[(\omega - \varepsilon)\Gamma_1 + (\omega + \varepsilon)\Gamma_2 + \frac{2(1-\omega)}{\sigma}\right]$$

$$\sigma_\varepsilon = \frac{1}{4}\left[(\omega + \varepsilon)\Gamma_2 - (\omega - \varepsilon)\Gamma_1 + \frac{2\varepsilon}{\sigma}\right].$$

When $\sigma = 1$ (i.e. log utility in consumption) the parameter σ_ω is equal to one and σ_ε is equal to zero independently of ε.[1]

A.1.2 Derivation of equation (1.19)

From the labor market clearing condition (1.15) for both regions of the currency area using the production function and the definition of the internal and external terms of trade, I obtain the following log linear expressions for the real marginal costs in region 1 and 2

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[1]This is the special case in which an open economy is isomorphic to a closed economy. See Clarida et al. (2001).

$$rmc_{1,t} = -\nu + \sigma c_t^1 + \varphi y_{1,t} + \left[\alpha + \frac{1}{2}(\omega - \varepsilon)\right] t_t^{in} + \qquad \text{(A.8)}$$
$$+ (\omega - \varepsilon) t_t^{ex} - (1 + \varphi) a_{1,t}$$

$$rmc_{2,t} = -\nu + \sigma c_t^2 + \varphi y_{2,t} - \left[\alpha + \frac{1}{2}(\omega + \varepsilon)\right] t_t^{in} + \qquad \text{(A.9)}$$
$$+ (\omega + \varepsilon) t_t^{ex} - (1 + \varphi) a_{2,t}.$$

Aggregating over the regions of the currency area the area-wide real marginal costs are

$$rmc_{u,t} = -\nu + \sigma c_t^u + \varphi y_{u,t} - \frac{\varepsilon}{2} t_t^{in} + \omega t_t^{ex} - (1 + \varphi) a_{u,t}.$$

Using the risk sharing condition (A.7) and substituting t_t^{ex} using (1.18), I obtain equation (1.19) of the text, that is

$$rmc_{u,t} = -\nu + (\sigma_\omega + \varphi) y_{u,t} + (\sigma - \sigma_\omega) y_{row,t} + \sigma_\omega \sigma_\varepsilon t_t^{in} - (1 + \varphi) a_{u,t}.$$

A.1.3   Derivation of equation (1.22)

From the intertemporal first order conditions of the households living in the currency area I have the following (area-wide) Euler equation

$$c_t^u = E_t c_{t+1}^u - \frac{1}{\sigma} \left(r_t - E_t \pi_{u,t+1}^c\right).$$

Using (1.6) and (1.7) and the definition of the terms of trade, the following is the relationship between the CPI and the PPI inflation in the currency area

$$\pi_{u,t}^c = \pi_{u,t} + \omega \Delta t_t^{ex} - \frac{\varepsilon}{2} \Delta t_t^{in}.$$

Substituting the last expression and (A.6) into the area-wide Euler equation above, I have

$$y_{u,t} = E_t y_{u,t+1} + \sigma_\varepsilon E_t \Delta t_{t+1}^{in} - \Theta E_t \Delta t_{t+1}^{ex} - \frac{1}{\sigma} \left(r_t - E_t \pi_{u,t+1}\right),$$

where $\Theta \equiv \frac{1}{2} \left[(\omega - \varepsilon) \Gamma_1 + (\omega + \varepsilon) \Gamma_2 - \frac{2\omega}{\sigma}\right]$. Using equation (1.18) to substitute for t_t^{ex} and rearranging the terms, I obtain

$$y_{u,t} = E_t y_{u,t+1} + \sigma_\varepsilon E_t \Delta t_{t+1}^{in} - \frac{1}{\sigma_\omega} \left(r_t - E_t \pi_{u,t+1}\right) + \Theta \sigma E_t \Delta y_{row,t+1}.$$

Adding and subtracting the natural rate of the currency area output (1.20) and rearranging terms, I can rewrite the IS equation in terms of the output gap and the real interest rate gap as in equation (1.22), that is

$$x_{u,t} = E_t x_{u,t+1} + \sigma_\varepsilon E_t \Delta(t^{in}_{t+1} - \tilde{t}^{in}_{t+1}) - \frac{1}{\sigma_\omega}\left(r_t - E_t \pi_{u,t+1} - \widetilde{rr}_{u,t}\right).$$

The natural rate of the real interest rate is defined as

$$\widetilde{rr}_{u,t} \equiv \phi_{y^*} E_t \Delta y^*_{t+1} + \phi_{t^{in}} E_t \Delta \tilde{t}^{in}_{t+1} + \phi_{a_u} E_t \Delta a_{u,t+1}$$

where the coefficients are defined as

$$\begin{aligned} \phi_{y_{row}} &\equiv \sigma_\omega \left(\Theta\sigma - \frac{\sigma - \sigma_\omega}{\sigma_\omega + \varphi}\right) \\ \phi_{t^{in}} &\equiv +\frac{\sigma_\omega \sigma_\varepsilon \varphi}{\sigma_\omega + \varphi} \\ \phi_{a_u} &\equiv \frac{\sigma_\omega(1 + \varphi)}{\sigma_\omega + \varphi}. \end{aligned}$$

A.1.4   Derivation of equation (1.23) and (1.24)

In order to obtain the equation that describes the dynamics of the inflation differentials between the two regions of the currency area, I derive the real marginal costs differentials. Subtracting (A.8) from (A.9) I have the differentials of real marginal costs

$$rmc_{r,t} = \sigma c^r_t + \varphi y_{r,t} - (2\alpha + \omega)\, t^{in}_t + 2\varepsilon t^{ex}_t - (1 + \varphi)a_{r,t}. \tag{A.10}$$

Subtracting (A.4) from (A.5), I have the following expression for the differentials of output in the two regions

$$y_{r,t} = -\left[2\alpha\Phi + \frac{1}{2}(\omega + \varepsilon)\Gamma_2 + \frac{1}{2}(\omega - \varepsilon)\Gamma_1\right] t^{in}_t + \left[(\omega + \varepsilon)\Gamma_2 - (\omega - \varepsilon)\Gamma_1\right] t^{ex}_t + c_{r,t}.$$

The risk sharing condition (1.11) together with (A.2) and (A.3) implies that

$$c^r_t = -\frac{1}{\sigma}(1 - 2\alpha - \omega)t^{in}_t - \frac{2\varepsilon}{\sigma}t^{ex}_t.$$

Combining the last two equations with (A.10), I obtain

$$rmc_{r,t} = \xi_{in}t^{in}_t + \xi_{ex}t^{ex}_t - (1 + \varphi)a_{r,t}$$

where

$$\xi_{in} \equiv -\frac{\sigma + \varphi}{\sigma}(1 - 2\alpha - \omega) - \varphi\left[2\alpha\Phi + \frac{1}{2}(\omega + \varepsilon)\Gamma_2 + \frac{1}{2}(\omega - \varepsilon)\Gamma_1\right] - (2\alpha + \omega)$$

$$\xi_{ex} \equiv -\frac{\sigma + \varphi}{\sigma}2\varepsilon + \varphi\left[(\omega + \varepsilon)\Gamma_2 - (\omega - \varepsilon)\Gamma_1\right] + 2\varepsilon.$$

Notice that when $\varepsilon = 0$, $\xi_{ex} = 0$ implying that the external competitiveness of the currency area toward the rest of the world is not affecting the real marginal cost differentials inside the union. Substituting t_t^{ex} using equation (1.18), I have

$$rmc_{r,t} = (\xi_{in} + \xi_{ex}\sigma_\omega\sigma_\varepsilon)\, t_t^{in} + \xi_{ex}\sigma_\omega y_{u,t} - \xi_{ex}\sigma_\omega y_{row,t} - (1 + \varphi)a_{r,t}. \qquad (A.11)$$

In the flexible price equilibrium the real marginal costs are equal to the opposite of the mark up in both regions. Since the monopolistic distortion is assumed to be the same across regions of the currency area, the differential of the real marginal costs is equal to zero when prices are fully flexible. Making use of this fact, by setting equation (A.11) equal to zero I obtain the expression for the internal terms of trade that is in (1.24), that is

$$\tilde{t}_t^{in} = \gamma_{y_{row}}y_{row,t} + \gamma_{a_u}a_{u,t} - \gamma_{a_r}a_{r,t}$$

where

$$\gamma_{y_{row}} \equiv \frac{\xi_{ex}\sigma_\omega(\sigma + \varphi)}{\xi_{in}(\sigma_\omega + \varphi) + \varphi\xi_{ex}\sigma_\omega\sigma_\varepsilon}$$

$$\gamma_{a_u} \equiv -\frac{(1 + \varphi)\xi_{ex}\sigma_\omega}{\xi_{in}(\sigma_\omega + \varphi) + \xi_{ex}\varphi\sigma_\omega\sigma_\varepsilon}$$

$$\gamma_{a_r} \equiv \frac{(1 + \varphi)(\sigma_\omega + \varphi)}{\xi_{in}(\sigma_\omega + \varphi) + \xi_{ex}\varphi\sigma_\omega\sigma_\varepsilon}.$$

The New Keynesian Phillips curve for the inflation differentials is given by

$$\pi_{r,t} = \beta E_t\pi_{r,t+1} + \lambda rmc_{r,t}.$$

Thus, using (A.11) and (1.24), I obtain equation (1.23) of the text.

A.2 Derivation of the approximated welfare loss function

The utility of the representative agent in each region i of the currency area is the following

$$U(C_t^i, N_t^i) \equiv \frac{(C_t^i)^{1-\sigma}}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi}$$

Taking a second order Taylor expansion around the optimal steady state I have

$$W_{i,t} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \bar{C}_i^{1-\sigma} \left[c_{i,t} + \frac{(1-\sigma)}{2} c_{i,t}^2 \right] - \bar{N}_i^{1+\varphi} \left[y_{i,t} + z_{i,t} + \frac{(1+\varphi)}{2} y_{i,t}^2 \right] \right\} + \\ + t.i.p. + o(\|\xi\|^3)$$

where in $t.i.p.$ are collected all terms that are independent from the monetary policy and in $o(\|\xi\|^3)$ the terms that are of order higher than two. To substitute for the hours worked $n_{i,t}$ I used the production function aggregated for all varieties produced in the same country. That is, in log linear terms, $y_{i,t} = n_{i,t} + a_{i,t} - z_{i,t}$.

I define the welfare function for the whole currency area as the weighted average of the welfare measures in the two regions. The weights are given by the size of each region. Among the possible aggregation function, the specification of an utilitarian social welfare function is the one that gives less weight to the heterogeneity across regions. Thus

$$W_{u,t} \equiv m W_{1,t} + (1-m) W_{2,t}$$

For now I am assuming that the two regions have equal size ($m = 0.5$).

Galì and Monacelli (2005) showed that $z_{i,t} = \frac{\theta}{2} var_j(\pi_{i,t}) = \frac{\theta}{2\lambda} \pi_{i,t}^2$. Using this result, the currency area approximated welfare function can be rewritten as follows[2]

$$W_{u,t} = \bar{C}^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \left[w_y' y_t - \frac{1}{2} y_t' W_y y_t - y_t' W_\xi \xi_t - \frac{1}{2} \pi_t' W_\pi \pi_t \right] + t.i.p. + o(\|\xi\|^3)$$

$$(A.12)$$

---

[2] A bar on a variable indicates that it is at the steady state level. I define $\Phi \equiv \frac{\bar{N}^{1+\varphi}}{\bar{C}^{1-\sigma}}$ such that $(1-\Phi)\bar{Y}$ is the output distorted steady state.

where

$$y_t' = \begin{bmatrix} y_{u,t} & c_{u,t} & t_t^{in} & t_t^{ex} & q_t^{in} & q_t^{ex} & y_{r,t} & c_{r,t} \end{bmatrix}$$

$$\xi_t' = \begin{bmatrix} y_{row,t} & a_{u,t} & a_{r,t} \end{bmatrix}$$

$$\pi_t' = \begin{bmatrix} \pi_{u,t} & \pi_{r,t} \end{bmatrix}$$

$$\mathbf{w_y'} = \begin{bmatrix} -\frac{1}{\Phi} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{W_y} = \begin{bmatrix} \frac{(1+\varphi)}{\Phi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (1-\sigma) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{(1+\varphi)}{\Phi}m(1-m) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-\sigma)m(1-m) \end{bmatrix}$$

$$\mathbf{W_\xi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{W_\pi} = \begin{bmatrix} \frac{\theta}{\lambda\Phi} & 0 \\ 0 & m(1-m)\frac{\theta}{\lambda\Phi} \end{bmatrix}$$

In order to get the appropriate welfare approximation I need to eliminate the linear terms $w_y' y_t$ in the second order approximation (A.12). Following Benigno and Woodford (2006), I can do it using the second order Taylor expansion of some of the equilibrium conditions of the model. I consider the following equilibrium conditions:
a) The aggregate demand of goods produced in the currency area (i.e. the weighted

average of equations (1.16) and (1.17)) ; **b)** The relative demand of goods produced in the two regions of the currency area (i.e. the difference between (1.17) and (1.16)) ; **c)** The risk sharing condition between the currency area and the rest of the world (i.e. $C_{u,t} = (Q_t^{ex})^{\frac{1}{\sigma}} C_{row,t}$); **d)** The risk sharing condition between the regions inside the currency area (i.e. $C_{r,t} = (Q_t^{in})^{-\frac{1}{\sigma}}$); **e)** The definition of the real exchange rate of the currency area with the rest of the world (i.e. $Q_t^{ex} = (T_t^{ex})^{1-\omega}(T_t^{in})^{\frac{\varepsilon}{2}}$); **f)** The definition of the real exchange rate between the regions of the currency area (i.e. $Q_t^{in} = (T_t^{in})^{1-2\alpha-\omega}(T_t^{ex})^{2\varepsilon}$); **g)** The labor market clearing condition aggregated for all regions in the currency area (i.e. the weighted average of (1.15) for $i = 1, 2$); **h)** The labor market clearing condition in differences between regions in the currency area (i.e. the difference between (1.15) when $i = 2$ and when $i = 1$).

a) **Demand of area-wide goods:**

$$\sum_{t=0}^{\infty} \beta^t [a_y' y_t + \frac{1}{2} y_t' A_y y_t - y_t' A_\xi \xi_t] + o(\|\xi\|^3) = 0 \tag{A.13}$$

$$\mathbf{a_y'} = \left[\begin{array}{cccccccc} -1 & 1 & -\frac{\varepsilon}{2} & \omega - \frac{\varepsilon}{2}\frac{(\sigma-1)}{\sigma} & \omega\frac{(\sigma-1)}{\sigma} & 0 & 0 \end{array} \right]$$

$$\mathbf{A_y} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x_{5,5} & x_{5,6} & 0 & 0 \\ 0 & 0 & 0 & 0 & x_{6,5} & x_{6,6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where

$$x_{5,5} = -\frac{1}{4}\frac{(\sigma-1)^2}{\sigma^2}\left(4\alpha^2 + \omega^2 + \varepsilon^2 + 4\alpha\omega - \omega - \alpha\right) + \frac{\varepsilon}{2}(\sigma-1)$$

$$x_{5,6} = x_{6,5} = \frac{1}{2}\frac{(\sigma-1)^2}{\sigma^2}\varepsilon(-1 + 2\alpha + 2\omega)$$

$$x_{6,6} = -\frac{(\sigma-1)^2}{\sigma^2}(\omega^2 + \varepsilon^2) - \frac{(\sigma-1)}{\sigma^2}\omega$$

$$\mathbf{A}_\xi = 0$$

b) **Demand differentials:**

$$\sum_{t=0}^{\infty} \beta^t [b'_y y_t + \frac{1}{2}y'_t B_y y_t - y'_t B_\xi \xi_t] + o(\|\xi\|^3) = 0 \tag{A.14}$$

$$\mathbf{b'_y} = \begin{bmatrix} 0 & 0 & -2\alpha - \omega & 2\varepsilon & \frac{(\sigma-1)}{\sigma}(2\alpha + \omega) & 2\frac{(\sigma-1)}{\sigma}\varepsilon & -1 & 1 \end{bmatrix}$$

$$\mathbf{B_y} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2}\frac{(\sigma-1)^2}{\sigma^2}\varepsilon\left(4\alpha + 2\omega - 1\right) & \frac{(\sigma-1)^2}{\sigma^2}(-\omega + 2\alpha\omega + \omega^2 + \varepsilon^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(\sigma-1)^2}{\sigma^2}\left(-\omega + 2\alpha\omega + \omega^2 + \varepsilon^2\right) & -\frac{(\sigma-1)^2}{\sigma^2}4\omega - \frac{2(\sigma-1)\varepsilon}{\sigma^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_\xi = 0$$

c) **Risk sharing - Currency area vs rest of the world**

$$\sum_{t=0}^{\infty} \beta^t [c'_y y_t + \frac{1}{2}y'_t C_y y_t - y'_t C_\xi \xi_t] + o(\|\xi\|^3) = 0 \tag{A.15}$$

$$\mathbf{c'_y} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & \frac{1}{\sigma} & 0 & 0 \end{bmatrix}$$

$$\mathbf{C_y} = 0$$

$$\mathbf{C_\xi} = 0$$

d) Risk sharing - region 1 vs region 2 of the Currency area

$$\sum_{t=0}^{\infty} \beta^t [d'_y y_t + \frac{1}{2} y'_t D_y y_t - y'_t D_\xi \xi_t] + o(\|\xi\|^3) = 0 \tag{A.16}$$

$$\mathbf{d'_y} = \begin{bmatrix} 0 & 0 & 0 & 0 & -\frac{1}{\sigma} & 0 & 0 & -1 \end{bmatrix}$$

$$\mathbf{D_y} = 0$$

$$\mathbf{D_\xi} = 0$$

e) Real exchange rate definition - Extra currency area

$$\sum_{t=0}^{\infty} \beta^t [e'_y y_t + \frac{1}{2} y'_t E_y y_t - y'_t E_\xi \xi_t] + o(\|\xi\|^3) = 0 \tag{A.17}$$

$$\mathbf{e'_y} = \begin{bmatrix} 0 & 0 & \frac{\varepsilon}{2} & (1-\omega) & 0 & -1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{E_y} = 0$$

$$\mathbf{E_\xi} = 0$$

f) Real exchange rate definition - Intra currency area

$$\sum_{t=0}^{\infty} \beta^t [f'_y y_t + \frac{1}{2} y'_t F_y y_t - y'_t F_\xi \xi_t] + o(\|\xi\|^3) = 0 \tag{A.18}$$

$$\mathbf{f'_y} = \begin{bmatrix} 0 & 0 & (1-2\alpha-\omega) & 2\varepsilon & -1 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{F_y} = 0$$

$$\mathbf{F}_\xi = 0$$

g) **Labor market clearing - aggregate currency area**

$$\sum_{t=0}^{\infty} \beta^t [g_y' y_t + \frac{1}{2} y_t' G_y y_t - y_t' G_\xi \xi_t] + o(\|\xi\|^3) = 0 \qquad (A.19)$$

$$\mathbf{g_y'} = \begin{bmatrix} \varphi & \sigma & -\frac{\varepsilon}{2} & \omega & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{G_y} = 0$$

$$\mathbf{G}_\xi = \begin{bmatrix} 0 & -(1+\varphi)\varphi & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

h) **Labor market clearing - differences between regions**

$$\sum_{t=0}^{\infty} \beta^t [h_y' y_t + \frac{1}{2} y_t' H_y y_t - y_t' H_\xi \xi_t] + o(\|\xi\|^3) = 0 \qquad (A.20)$$

$$\mathbf{h_y'} = \begin{bmatrix} 0 & 0 & -2\alpha - \omega & 2\varepsilon & 0 & 0 & \varphi & \sigma \end{bmatrix}$$

$$\mathbf{H_y} = 0$$

$$\mathbf{H}_\xi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -(1+\varphi)\varphi & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Eliminating the linear term in (A.12) implies finding the vector $\mathbf{Lx}$ such that

$$\mathbf{w_y} = \begin{bmatrix} a_y & b_y & c_y & d_y & e_y & f_y & g_y & h_y \end{bmatrix} \mathbf{Lx}$$

where Lx has dimension 8 by 1.

I can therefore rewrite equation (A.12) as

$$E_0 \sum_{t=0}^\infty \beta^t [\frac{1}{2} y_t' L_y y_t + y_t' L_\xi \xi_t + \frac{1}{2} L_\pi \pi_t^2] + t.i.p. + o(\|\xi\|^3) \tag{A.21}$$

where

$$L_y = W_y + A_y Lx_1 + B_y Lx_2$$
$$L_\xi = W_\xi + G_\xi Lx_7 + H_\xi Lx_8$$
$$L_\pi = W_\pi$$

In order to write the welfare in terms of the currency area output, the output differentials in the two regions, the internal and external terms of trade, the currency area inflation and the regions' inflation differentials, I construct the mapping N such that

$$y_t' = N[y_{u,t} \; y_{r,t} \; t_t^{in} \; t_t^{ex}] + N_\xi \xi_t'$$

where

$$
\mathbf{N} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & \frac{\varepsilon}{2\sigma} & \frac{(1-\omega)}{\sigma} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & (1-2\alpha-\omega) & 2\varepsilon \\
0 & 0 & \frac{\varepsilon}{2} & (1-\omega) \\
0 & 1 & 0 & 0 \\
0 & 0 & -\frac{(1-2\alpha-\omega)}{\sigma} & \frac{2\varepsilon}{\sigma}
\end{bmatrix}
$$

and

$$
\mathbf{N}_\xi =
\begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

I can substitute for $y_{R,t}$ using the following mapping

$$
\begin{bmatrix}
y_{u,t} \\
y_{r,t} \\
t_t^{in} \\
t_t^{ex}
\end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\varphi}\left[\left(1-\frac{1}{\sigma}\right)(2\alpha+\omega)\sigma^{-1}\right] & -\frac{1}{\varphi}\left(1-\frac{1}{\sigma}\right)2\varepsilon \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
y_{u,t} & t_t^{in} & t_t^{ex}
\end{bmatrix}
+ M_\xi \xi_t'
$$

where

$$
\mathbf{M}_\xi =
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & -\frac{1+\varphi}{\phi} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.
$$

In order to reduce further the number of variables, one can write

$$
\begin{bmatrix} y_{u,t} \\ t_t^{in} \\ t_t^{ex} \end{bmatrix} = Q \begin{bmatrix} y_{u,t} & t_t^{in} \end{bmatrix} + Q_\xi \xi_t'
$$

where

$$
\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \sigma_\omega & -\sigma_\omega \sigma_\varepsilon \end{bmatrix}
$$

and

$$
\mathbf{Q}_\xi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\sigma_\omega & 0 & 0 \end{bmatrix}
$$

The new coefficients are in the matrix

$$
\begin{aligned}
H &= Q'M'N'L_y NMQ \\
H_\xi &= Q'M'N'L_y \left(NMQ_\xi + NM_\xi + N_\xi \right) + Q'M'N'L_\xi.
\end{aligned}
$$

Thus, I can rewrite the loss function (A.21) as

$$
L_{t_0}^u = E_{t_0} \sum_{t=t_0}^\infty \beta^t \left\{ \frac{1}{2}[y_{u,t} \ t_t^{in}] H [y_{u,t} \ t_t^{in}]' + [y_{u,t} \ t_t^{in}] H_\xi [y_{row,t} \ a_{u,t} \ a_{r,t}]' + \frac{1}{2}\pi_t' L_\pi \pi_t \right\} + t.i.p. + o(\|\xi\|^3)
$$

The period loss function, written in terms of gap of the variables from their efficient level, is given by

$$
\begin{aligned}
L_t^u &= \left\{ \Phi_{\pi_u} \pi_{u,t}^2 + \Phi_{x_u}(y_{u,t} - \tilde{\tilde{y}}_{u,t})^2 + \Phi_{t^{in}}(t_t^{in} - \tilde{\tilde{t}}_t^{in})^2 + \Phi_{\pi_r} \pi_{r,t}^2 + \right. \\
&\quad \left. + \Phi_{x_u,t^{in}}(y_{u,t} - \tilde{\tilde{y}}_{u,t})(t_t^{in} - \tilde{\tilde{t}}_t^{in}) \right\} + t.i.p. + o(\|\xi\|^3)
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi_{x_u} &= H(1,1) \\
\Phi_{t^{in}} &= H(2,2) \\
\Phi_{x_u,t^{in}} &= [H(1,2) + H(2,1)] \\
\Phi_{\pi_u} &= W_\pi(1,1) \\
\Phi_{\pi_r} &= W_\pi(2,2)
\end{aligned}
$$

The efficient equilibrium level of the area-wide output and the internal terms of trade is given by the following equations

$$
\begin{aligned}
\tilde{\tilde{y}}_{u,t} &= \frac{1}{H(1,1)} \left\{ \frac{1}{2}[H(1,2) + H(2,1)]\tilde{\tilde{t}}_t^{in} - H_\xi(1,1)y_{row,t} - H_\xi(1,2)a_{u,t} - H_\xi(1,3)a_{r,t} \right\} \\
\tilde{\tilde{t}}_t^{in} &= \frac{1}{H(2,2)} \left\{ \frac{1}{2}[H(1,2) + H(2,1)]\tilde{\tilde{y}}_{u,t} - H_\xi(2,1)y_{row,t} - H_\xi(2,2)a_{u,t} - H_\xi(2,3)a_{r,t} \right\}
\end{aligned}
$$

In the special case of $\sigma = 1$ and $\varepsilon = 0$ the coefficients are the following

$$
\begin{aligned}
\Phi_{x_u} &= \frac{(1 + \varphi)}{\Phi} \\
\Phi_{t^{in}} &= m(1 - m)\frac{(1 + \varphi)}{\Phi} \\
\Phi_{x_u,t^{in}} &= 0
\end{aligned}
$$

These coefficients are equal to those of Galì and Monacelli (2005) with the exception of the internal terms of trade that is a result of aggregating the welfare functions of the two regions.

A.3   The Optimal Policy Plan

The benevolent monetary authority of the currency area solves the following linear quadratic problem

$$
\max E_0 \sum_{t=0}^{\infty} \beta^t L_t^u
$$

subject to

$$\pi_{u,t} = \beta E_t \pi_{u,t+1} + \kappa_x^u \left(y_{u,t} - \tilde{\tilde{y}}_{u,t} \right) + \kappa_T^u \left(t_t^{in} - \tilde{\tilde{t}}_t^{in} \right) - \kappa_x^u \left(\tilde{y}_{u,t} - \tilde{\tilde{y}}_{u,t} \right) - \kappa_T^u (\tilde{t}_t^{in} - \tilde{\tilde{t}}_t^{in})$$

$$\pi_{r,t} = \beta E_t \pi_{r,t+1} + \kappa_x^r \left(y_{u,t} - \tilde{\tilde{y}}_{u,t} \right) + \kappa_T^r \left(t_t^{in} - \tilde{\tilde{t}}_t^{in} \right) - \kappa_x^r \left(\tilde{y}_{u,t} - \tilde{\tilde{y}}_{u,t} \right) - \kappa_T^r (\tilde{t}_t^{in} - \tilde{\tilde{t}}_t^{in})$$

$$t_t^{in} - \tilde{\tilde{t}}_t^{in} = t_{t-1}^{in} - \tilde{\tilde{t}}_{t-1}^{in} + \pi_{r,t} - (\tilde{\tilde{t}}_t^{in} - \tilde{\tilde{t}}_{t-1}^{in})$$

Calling $\lambda_{1,t}$, $\lambda_{3,t}$ and $\lambda_{3,t}$ the Lagrange multipliers associated with the three constraints above, the following are the first order conditions with respect to $\pi_{u,t}$, $\pi_{r,t}$, $x_{u,t}$ and $(t_t^{in} - \tilde{t}_t^{in})$ of the full commitment optimal policy problem.

$$\Phi_{\pi_u} \pi_{u,t} + \lambda_{1,t} - \lambda_{1,t-1} = 0 \qquad (A.22)$$

$$\Phi_{x_u} \left(y_{u,t} - \tilde{\tilde{y}}_{u,t} \right) + \Phi_{x_u,t^{in}} \left(t_t^{in} - \tilde{\tilde{t}}_t^{in} \right) - \lambda_{1,t} \kappa_x^u - \lambda_{2,t} \kappa_x^r = 0 \qquad (A.23)$$

$$\Phi_{\pi_r} \pi_{r,t} + \lambda_{2,t} - \lambda_{2,t-1} - \lambda_{3,t} = 0 \qquad (A.24)$$

$$\Phi_{t^{in}} \left(t_t^{in} - \tilde{\tilde{t}}_t^{in} \right) +$$

$$+\Phi_{x_u,t^{in}} \left(y_{u,t} - \tilde{\tilde{y}}_{u,t} \right) - \lambda_{1,t} \kappa_T^u - \lambda_{2,t} \kappa_T^r + \lambda_{3,t} - \beta \lambda_{3,t+1} = 0 \qquad (A.25)$$

Equations (A.22)-(A.25), the structural equations of the model (equations (1.21), (1.23) and (1.25)), i.e. the first order conditions with respect to the Lagrange multipliers, together with the stochastic processes specified for the three exogenous variables, $a_{u,t}$, $a_{r,t}$ and $y_{row,t}$, fully describe the dynamics of the currency area economy under the optimal policy plan.

Appendix B

Targeting Rules in a Monetary Union with Heterogeneous Cost-Push Shocks

B.1 The New Keynesian Phillips Curves

In this appendix we show how to derive the New Keynesian Phillips curves in equations (2.17) and (2.18). We derive only the equation for the home country; the derivation for the foreign one follows the same lines.

The first order equation for the optimal price setting (equation (2.7)) can be rewritten as follows

$$
\begin{aligned}
0 \;=\; & E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left\{ \left[u_c(C_{t+k})(\theta_{t+k}^H - 1)\frac{p_t(h)^*}{P_{H,t}}T_{t+k}^{s-1} \right. \right. \\
& \left. \left. -\theta_{t+k}^H v_y\left(y_{t,t+k}(h); z_{t+k}\right) \right] y_{t,t+k}(h) \right\}
\end{aligned}
\tag{B.1}
$$

We first loglinearize the demand of the brand

$$
y_{t,t+k}(h) = \left(\frac{p_t(h)}{P_{H,t+k}} \right)^{-\theta_{t+k}^H} T_{t+k}^{1-s} Y_{t+k}
\tag{B.2}
$$

which gives

$$
\hat{y}_{t,t+k}(h) = -\theta(\hat{p}_t(h) - \hat{P}_{H,t+k}) + (1-s)\hat{T}_{t+k} + \hat{Y}_{t+k}
$$

where the hatted variables are log-deviations from their steady state. Let's define $\hat{p}_{t,t+k} = \hat{p}_t(h) - \hat{P}_{H,t+k}$, then one obtains

$$\hat{y}_{t,t+k}(h) = -\theta\hat{p}_{t,t+k} + (1-s)\hat{T}_{t+k} + \hat{Y}_{t+k}. \tag{B.3}$$

Plugging this into (B.1) and using the market clearing condition in the good market, $C_t = Y_t$, we get

$$
\begin{aligned}
0 &= E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left\{\hat{p}_{t,t+k} - (1-s)\hat{T}_{t+k} - \sigma^{-1}\hat{Y}_{t,t+k} + \frac{1}{1-\theta}\hat{\theta}_{t+k}^H \right.\\
&\qquad\left. -\omega\left[-\theta\hat{p}_{t,t+k} + (1-s)\hat{T}_{t+k} + \hat{Y}_{t+k} - \overline{a}_{t+k}\right]\right\}
\end{aligned}
$$

where

$$\overline{a}_t \equiv \frac{v_{yz}}{v_{yy}\overline{Y}}z_t, \qquad \omega \equiv \frac{v_{yy}}{v_y}\overline{Y}, \qquad \sigma^{-1} \equiv -\frac{u_{cc}}{u_c}\overline{Y}.$$

and $\overline{Y}$ is the steady state level of output.

Knowing that $\hat{p}_{t,t+k} = \hat{p}_{t,t} - \sum_{h=1}^{k}\pi_{t+h}^H$ we can write

$$
\begin{aligned}
\frac{\hat{p}_{t,t}}{1-\alpha\beta} &= E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left\{\frac{1+\omega}{1+\omega\theta}(1-s)\hat{T}_{t+k} + \frac{\sigma^{-1}+\omega}{1+\omega\theta}\hat{Y}_{t+k}\right. \tag{B.4}\\
&\qquad\left. +\frac{1}{(1-\theta)(1+\omega\theta)}\hat{\theta}_{t+k}^H - \frac{\omega}{1+\omega\theta}\overline{a}_{t+k}\right\} + E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left\{\sum_{h=1}^{k}\pi_{t+h}^H\right\}.
\end{aligned}
$$

We can write (2.8) in the following way

$$1 = \alpha\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{1-\theta_t^H} + (1-\alpha)\left(\frac{p_t(h)}{P_{H,t}}\right)^{1-\theta_t^H}$$

so that once loglinearized is equal to $\hat{p}_{t,t} = \frac{\alpha}{1-\alpha}\pi_t^H$. Substituting this expression into (B.4), we can write

$$
\begin{aligned}
\frac{\pi_t^H}{1-\alpha\beta}\frac{\alpha}{1-\alpha} &= E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left\{\frac{1+\omega}{1+\omega\theta}(1-s)\hat{T}_{t+k} + \frac{\sigma^{-1}+\omega}{1+\omega\theta}\hat{Y}_{t+k}\right.\\
&\qquad\left. +\frac{1}{(1-\theta)(1+\omega\theta)}\hat{\theta}_{t+k}^H - \frac{\omega}{1+\omega\theta}\overline{a}_{t+k}\right\} + E_t\sum_{k=0}^{\infty}(\alpha\beta)^k\frac{\pi_{t+k}^H}{1-\alpha\beta}
\end{aligned}
$$

and subsequently the recursive expression

$$\pi_t^H = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}\left[\frac{1+\omega}{1+\omega\theta}(1-s)\hat{T}_t + \frac{\sigma^{-1}+\omega}{1+\omega\theta}\hat{Y}_t + \frac{1}{(1-\theta)(1+\omega\theta)}\hat{\theta}_t^H\right.$$
$$\left.-\frac{\omega}{1+\omega\theta}\overline{a}_t\right] + \beta E_t\pi_{t+1}^H. \tag{B.5}$$

From the solution of the model in the efficient equilibrium we know that

$$\hat{Y}_t^e = \frac{\omega}{\sigma^{-1}+\omega}\overline{a}_t$$
$$\hat{T}_t^e = 0$$

from which it follows that $-\omega\overline{a}_t = -(\sigma^{-1}+\omega)\hat{Y}_t^e$. Substituting this equivalence into (B.5) we obtain

$$\pi_t^H = \kappa_y(\hat{Y}_t - \hat{Y}_t^e) + (1-s)\kappa_T\hat{T}_t + \beta E_t\pi_{t+1}^H + u_t^H \tag{B.6}$$

where

$$\kappa_Y \equiv \frac{(1-\alpha\beta)(1-\alpha)(\sigma^{-1}+\omega)}{\alpha(1+\omega\theta)}; \quad \kappa_T \equiv \frac{(1-\alpha\beta)(1-\alpha)(1+\omega)}{\alpha(1+\omega\theta)} \tag{B.7}$$

and the cost-push shock in the home country is defined as follows

$$u_t^H \equiv \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\omega\theta)(1-\theta)}\hat{\theta}_t^H \tag{B.8}$$

B.2   The Full Commitment Optimal Policy

We derive now the system of equations that describes the dynamic of the inflations, the output gap and the terms of trade under the full commitment optimal plan. This comes from the solution of the maximization of (2.24) subject to the structural equations (2.17), (2.18) and (2.23). The Lagrangean of the problem is given by

$$\mathcal{L} = -\Omega E_0 \sum_{t=0}^{\infty}\beta^t\left\{s(\pi_t^H)^2 + (1-s)(\pi_t^F)^2 + \theta^{-1}\kappa_y x_t^2 + s(1-s)\theta^{-1}\kappa_T(\hat{T}_t - \hat{T}_t^e)^2\right.$$
$$+\phi_{1t}\left[\pi_t^H - (1-s)\kappa_T(\hat{T}_t - \hat{T}_t^e) - \kappa_y x_t - \beta E_t\pi_{t+1}^H - u_t^H\right]$$
$$+\phi_{2t}\left[\pi_t^F + s\kappa_T(\hat{T}_t - \hat{T}_t^e) - \kappa_y x_t - \beta E_t\pi_{t+1}^F - u_t^F\right]$$
$$+ \phi_{3t}\left[\hat{T}_t - \hat{T}_{t-1} - \pi_t^F + \pi_t^H+\right]\right\}$$

Taking the first order conditions with respect to π_t^H, π_t^F, x_t and $\hat{T}_t$ we obtain

$$
\begin{array}{rcl}
x_t & : & \theta^{-1}x_t - \phi_{1t} - \phi_{2t} = 0 \\[4pt]
\pi_t^H & : & s\pi_t^H + \phi_{1t} + \phi_{3t} - \phi_{1t-1} = 0 \\[4pt]
\pi_t^F & : & (1-s)\pi_t^F + \phi_{2t} - \phi_{3t} - \phi_{2t-1} = 0 \\[4pt]
\hat{T}_t & : & s(1-s)\theta^{-1}\kappa_T\hat{T}_t - (1-s)\kappa_T\phi_{1t} + s\kappa_T\phi_{2t} + \phi_{3t} - \beta E_t\phi_{3t+1} = 0.
\end{array}
$$

These first order conditions together with the constraints (2.17), (2.18) and (2.23) determine jointly the dynamics of $\pi_t^H, \pi_t^F, x_t, \hat{T}_t$ and of the three Lagrange multipliers ϕ_{1t}, ϕ_{2t} and ϕ_{3t}.

Appendix C

Firm Entry, Competitive Pressures and the U.S. Inflation Dynamics

C.1   The data

Table C.1: Data sources

	Name	*Explanation*	*Source*
P_t	Price level	GDP deflator	FRED
Y_t	Output	Real GDP	FRED
S_t	Real unit labor costs	Nonfarm business sector unit labor cost divided by the implicit price deflator	FRED
x_t	Output per firms	Real GDP over the constructed number of firms	-
N_t^e	New incorporations	Number of new firms created	Dun & Bradstreet, BEA
W_t	Nominal wages	Nominal hourly compensation in the nonfarm business	FRED

Figure C.1: The real marginal costs, the number of firms and the real GDP.
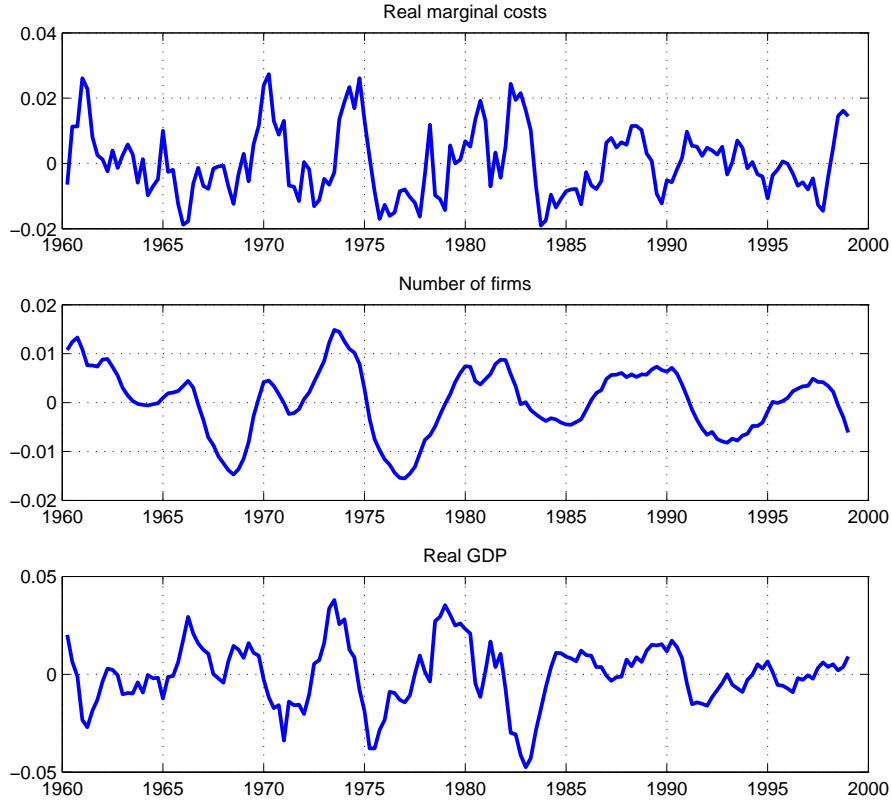
C.2 Derivation of the closed form of the NKPC for estimation

The inflation and the dynamics of the number of firms are represented, in log linear terms, by the following equations

$$\pi_t = \beta(1-\delta)E_t\pi_{t+1} + \kappa\widehat{rmc}_t - \kappa\eta\hat{n}_t + u_t \tag{C.1}$$

$$\hat{n}_t = (1-\delta)\hat{n}_{t-1} + \delta\hat{n}_t^e \tag{C.2}$$

Solving forward both equations and plugging (C.2) into (C.1) I obtain

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j (1-\delta)^j E_t \widehat{rmc}_{t+j} - \delta\kappa\eta \sum_{j=0}^{\infty} \beta^j (1-\delta)^j E_t \sum_{h=0}^{\infty} (1-\delta)^h \hat{n}^e_{t+j-h} + u_t$$

By rearranging terms I obtain

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j (1-\delta)^j E_t \widehat{rmc}_{t+j} - \frac{\delta\kappa\eta}{1-\beta(1-\delta)^2} \left[\sum_{h=1}^{\infty} (1-\delta)^h \hat{n}^e_{t-h} + \sum_{j=0}^{\infty} \beta^j (1-\delta)^j E_t \hat{n}^e_{t+j} \right] + u_t$$

Using the VAR defined on (3.22) to compute the expectations of real marginal costs and new incorporations I obtain

$$\pi_t = \kappa e'_1 [I-\beta(1-\delta)A]^{-1} x_t - \frac{\delta\kappa\eta}{1-\beta(1-\delta)^2} \left\{ e'_2 [I-\beta(1-\delta)A]^{-1} x_t + \sum_{h=1}^{L} (1-\delta)^h e'_2 x_{t-h} \right\} + u_t$$

where e'_1 and e'_2 are vectors that select respectively the first and second element of the vector x_t, I is a conformable identity matrix and A is the companion matrix of the forecasting VAR.

Accordingly, the expectation term on the real marginal cost and the new incorporations are given by

$$E_t \widehat{rmc}_{t+j} = \frac{[1-\beta(1-\delta)a_{22}] \widehat{rmc}_t + \beta(1-\delta)a_{12}\hat{n}^e_t}{1-\beta(1-\delta)(a_{11}+a_{22}) + \beta^2(1-\delta)^2(a_{11}a_{22}-a_{12}a_{21})}$$

$$E_t \hat{n}^e_{t+j} = \frac{[1-\beta(1-\delta)a_{11}] \hat{n}^e_t + \beta(1-\delta)a_{21}\widehat{rmc}_t}{1-\beta(1-\delta)(a_{11}+a_{22}) + \beta^2(1-\delta)^2(a_{11}a_{22}-a_{12}a_{21})}.$$

Considering an hybrid Phillips curve that includes lagged inflation as a determinant of the current inflation the closed form of the inflation dynamics that incorporates the projections of the VAR is given by

$$\pi_t = \gamma_1 \pi_{t-1} + \kappa \frac{1}{\gamma_2 \beta(1-\delta)} e'_1 [1-\beta(1-\delta)A]^{-1} x_t - \tag{C.3}$$

$$\frac{\delta\kappa\eta}{1-\beta(1-\delta)^2} \frac{1}{\gamma_2 \beta(1-\delta)} \left\{ e'_2 [I-\beta(1-\delta)A]^{-1} x_t + \sum_{h=1}^{L} (1-\delta)^h e'_2 x_{t-h} \right\} + u_t$$

γ_1 and γ_2 are respectively the stable and unstable roots of the second order difference equation in (3.19). They are equal to

$$
\begin{aligned}
\gamma_1 &= \frac{1 - \sqrt{1 - 4\beta(1-\delta)\lambda}}{2\beta(1-\delta)} \\
\gamma_2 &= \frac{1 + \sqrt{1 - 4\beta(1-\delta)\lambda}}{2\beta(1-\delta)}.
\end{aligned}
$$

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