Evolving Macroeconomic Dynamics and Structural Change: Applications and Policy Implications

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to Laia

Macroeconometricians do four things: describe and summarize macroeconomic data, make macroeconomic forecasts, quantify what we do or do not know about the true structure of the macroeconomy, and advise (and sometimes become) macroeconomic policymakers.

Stock, J.H, and M.H. Watson, (2001).

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Foreword

Since the seminal contribution by Sims (1980), Structural Vector Autoregressions (SVAR) models have been assigned a central role for policy analysis. SVAR models are built on the idea, first introduced by Frisch (1933) and Slutzky (1937), that the economic system can be described as the summation of exogenous random disturbances which are propagated through some mechanisms called impulse response functions. Within this class of models, policy analysis coincides with impulse response functions analysis and allows the researcher to study the dynamic effects on macroeconomic variables of demand, technology, monetary policy shocks and any other type of shock of interest.

Standard SVAR models stand on the assumption that the mechanisms through which economic disturbances are propagated over time are constant. For instance, the response of the economic system to a technology shock is always the same, independently on the time period in which the shock occurs. In recent years a huge amount of contributions provided convincing evidence in favor of several changes in the structure of the economy of most industrialized economies (see e.g. Stock and Watson, 1996 among others). Changes are of different types: changes in the conduct of economic policies, changes in business cycle fluctuations, changes in the behavior of firms and consumers, institutional changes, etc. All such evidence stands in sharp contrast with the assumption that propagation mechanisms are constant over time. This may have two serious consequences. First, conclusions reached from policy analysis with standard VAR can be very misleading and distorted because potentially important changes in the transmission of economic shocks are a priori ruled out. Second, changes in the transmission mechanisms of structural shocks are completely hidden, and therefore the correct comprehension of the dynamics characterizing the economic system might be seriously undermined.

Time-Varying Coefficients Vector Autoregressions (TVC-VAR) represent a generalization of standard VAR models where the coefficients are allowed to evolve over time according to some process. Up to now, however, these models have been employed exclusively as reduced form models (see Cogley and Sargent, 2001) mainly for forecasting or to describe the statistical properties of economic time series. The purpose of this research is to extend structural dynamic analysis to TVC-VAR, focusing both on the theoretical foundations and the implications for economic policy design. The main motivation is that by allowing for changes in the structural relationships among macroeconomic variables, and therefore in the propagation mechanisms of structural shocks, important new implications for macroeconomic policy management may emerge and new and important insights can be provided to policy makers.

The present thesis is a collection of three separate essays, each corresponding to a chapter. Each essay represents an application of structural dynamic analysis within TVC-VAR models. In the first chapter, coauthored with Fabio Canova, we investigate the relationship between changes in output and inflation and monetary policy in the US. There are variations in the structural coefficients and in the variance of the structural shocks but only the latter are synchronized across equations. The policy rules in the 1970s and 1990s are similar as is the transmission of policy disturbances. Changes in inflation persistence are only partly explained by monetary policy changes. Variations in the systematic component of policy have limited effects on the dynamics of the system. Results are robust to alterations in the auxiliary assumptions.

In the second chapter, coauthored with Fabio Canova and Evi Pappa, we examine the dynamics of US output and inflation. We show that there are changes in the volatility of both variables and in the persistence of inflation. Technology shocks explain changes in output volatility, while a combination of technology, demand and monetary shocks explain variations in the persistence and volatility of inflation. We detect changes over time in the transmission of technology shocks and in the variance of technology and of monetary policy shocks.

In the third chapter we study whether hours worked rise or fall after a positive technology shock. According to the existing evidence it depends on whether they enter the VAR in levels (hours rise) or growth rates (hours fall). We argue that conflicting results may ultimately arise because important structural time variations in the US economy are a priori ruled out by empirical models. We identify technology shocks as the only shocks driving long-run labor productivity using postwar US quarterly data. We find that, under both specifications for hours (levels and growth rates), (i) hours fall, and (ii) technology shocks explain about 11-23% of total aggregate fluctuations giving rise to positive but small correlations between output and hours. Differences with respect to fixed coefficients VAR are due to instabilities in the relationship between labor productivity and levels of hours.

To conclude, the main contribution of the present research work as a whole is twofold. First, from an empirical perspective, we provide new evidence on some important macroeconomic issues of the US economy. In all the applications new interesting results, absent in standard VAR, emerge. Second, from a theoretical perspective, we contribute to develop new tools useful for policy analysis within the class of TVC-VAR models.

Chapter 1

Structural changes in the US economy: bad luck or bad policy?

1.1 Introduction

There is considerable evidence suggesting that the US economy has fundamentally changed over the last couple of decades. In particular, several authors have noted a marked decline in the volatility of real activity and in the volatility and persistence of inflation since the early 1980s (see e.g. Blanchard and Simon, 2000, McConnell and Perez Quiros, 2001, and Stock and Watson, 2003). What are the reasons behind such a decline? A stream of literature attributes these changes to alterations in the mechanisms through which exogenous shocks spread across sectors and propagate over time. Since the transmission mechanism depends on the structure of the economy, such a viewpoint implies that important characteristics, such as the behavior of consumers and firms or the preferences of policymakers, have changed over time. The recent literature has paid particular attention to monetary policy. Several studies, including Clarida, Gali and Gertler (2000), Cogley and Sargent (2001) (2005), Lubik and Schorfheide (2004), have argued that monetary policy was "loose" in fighting inflation in the 1970s but became more aggressive since the early 1980s and see in this change of attitude the reason for the observed changes in inflation and output. This view, however, is far from unanimous. For example, Bernanke and Mihov (1998), Leeper and Zha (2003), Orphanides (2004), find little evidence of significant changes in the policy rule used in the last 25-30 years while Hanson (2001) claims that the propagation of monetary shocks has been stable. Sims (2001) and Sims and Zha (2004) suggest that changes in the variance of exogenous shocks are responsible for the observed changes.

This controversy is not new. In the past rational expectations econometricians (e.g. Sargent, 1984) have argued that policy changes involving regime switches dramatically alter private agent decisions and, as a consequence, the dynamics of the macroeconomic variables, and searched for historical episodes supporting this view (see e.g. Sargent, 1999). VAR econometricians, on the other hand, often denied the empirical relevance of this argument suggesting that the systematic portion of monetary policy has rarely been altered and that policy changes are better characterized as random variations for the non-systematic part (Sims, 1982). This long standing debate now has been cast into the dual framework of "bad policy" (failure to adequately respond to inflationary pressure) vs. "bad luck" (shocks are drawn from a distribution whose moments vary over time) and new evidence has been collected thanks to the development of methods which allow to examine time variations in the structure of the economy and in the variance of the exogenous processes. Overall, and despite recent contributions, the role that monetary policy had in shaping the observed changes in the US economy is still open.

This paper provides novel evidence on this issue. Our framework of analysis is a time varying coefficients VAR model, similar to Cogley and Sargent (2001), where the coefficients evolve according to a nonlinear transition equation which puts zero probability on paths associated with explosive roots, and we use Markov Chain Monte Carlo (MCMC) methods to estimate the posterior distributions of the quantities of interest. Cogley and Sargent (2005) and Primicieri (2005) add to this framework a stochastic volatility model for the reduced form innovations. We also allow the variance of the forecast errors to vary over time but, as in Canova (1993), we do this in a simpler and more intuitive manner, which retains conditional linearity and links changes in the variance of the coefficients to changes in the variance of the forecast errors in an economically meaningful way. We identify structural disturbances via sign restrictions on dynamic response of certain variables to shocks. While we focus on monetary policy disturbances, the methodology is well suited to jointly identify multiple sources of structural disturbances (see e.g. Canova and De Nicolo', 2002). We choose to work with sign restrictions for two reasons. First, the contemporaneous zero restrictions conventionally used are often absent in those theoretical models one likes to use to guide the interpretation of the results. Second, while the restrictions we employ are robust to the parameterization, common to both flexible and sticky price models (see e.g. Gambetti et al., 2005) and independent of whether the economic environment delivers determinate or indeterminate solutions (see e.g. Lubik and Shorfheide, 2004), those imposed by zero type restrictions leave the system underidentified when indeterminacies emerge. This is important since one version of the bad policy hypothesis relies on the presence of indeterminacies in the earlier part of the sample.

The resulting structural system can be used to evaluate the magnitude of structural variations produced by changes in i) the systematic component of policy, ii) the propagation of policy shocks, iii) the variance of the structural disturbances and iv) the rest of the economy. Moreover, we can do this examining short and long run features of the estimated system. Both reduced form time varying coefficient and structural but constant coefficient approaches are unable to separate the relative importance of i)-iv) in accounting for the observed changes.

Contrary to the literature up to date, we construct posterior distributions which are consistent with the information available at each t. While such an approach complicates estimation considerably, it provides a more reliable measure of time variations present in the structural system and of the timing of the changes, if they exist. We innovate relative to the existing literature in another important dimension. Because time variations in the coefficients induce important non-linearities, standard statistics summarizing the dynamics in response to structural shocks are inappropriate. For example, since at each t the coefficient vector is perturbed by a shock, assuming that between $t+1$ and $t+k$ no shocks other than the monetary policy disturbance hit the system may give misleading results. To trace out the evolution of the economy in response to structural shocks, we employ a different concept of impulse response function, which shares similarities with those used in Koop, Pesaran and Potter (1996), Koop (1996), and Gallant, Rossi and Tauchen (1993). In particular, impulse responses are defined as the difference between two conditional expectations, differing in the arguments of their conditioning sets. The combined use of a robust identification scheme, of recursive analysis and of appropriately defined responses is crucial to deliver meaningful answers to the questions at stake.

Four main conclusions emerge from our investigation. First, as in Bernanke and Mihov (1998) and Leeper and Zha (2003), we find that excluding the Volker experiment, the monetary policy rule has been quite stable over time. Interestingly, point estimates of the coefficients obtained in the end of the 1990's are similar to those obtained in the late 1970's. Second, as in Sims and Zha (2004), we find posterior evidence of a decrease in the uncertainty surrounding the structural disturbances of the system but no synchronization in the timing of the changes in the variance of the shocks hitting various equations . Third, we show that the transmission of policy shocks has been very stable: both the shape and the persistence of output and inflation responses are very similar over time and quantitative differences statistically small. Fourth, we find that structural inflation persistence has statistically changed over time, that both monetary and non-monetary factors account for its magnitude and that the relative contribution of monetary policy shocks is increasing since the early 1980s.

We investigate, by way of counterfactuals, whether a more aggressive policy response to inflation would have made a difference for the dynamics of output and inflation. Such a stance would have reduced inflationary pressures and produced significant output costs in 1979, but produced no measurable inflation effects in the 1980s or 1990s and a perverse outcome in the 2000s. Hence, while the Fed could have had some room to improve economic performance at the end of the 1970s, altering the policy response to observable variables, it seems unlikely that such an alteration would have produced the changes observed in the US economy. Finally, we show that our results are robust to a number of changes in some auxiliary assumptions, in particular, the treatment of trends, the variables included in the VAR and to the identification procedure.

Overall, while the crudest version of the "bad policy" proposition has low posterior support, the evidence appears to be consistent both with more sophisticated versions of this proposition as well with the alternative "bad luck" hypothesis. To disentangle the two interpretations, a model in which preferences, technologies and the distributions of the shocks are allowed to change along with the preferences of the Fed is needed. While such a model is still too complex to be analyzed and estimated with existing tools, approximations of the type employed in Canova (2004), can shed important light on this issue.

The rest of the paper is organized as follows. Section 2 presents the reduced form model, describes our identification scheme and the approach used to obtain posterior distributions for the structural coefficients. Section 3 defines impulse response functions which are appropriate for our TVC-VAR model. Section 4 presents the results and Section 5 concludes. Two appendices describes the technical details involved in the computation of impulse responses and of posterior distributions.

1.2 The empirical model

Let y_t be a $n \times 1$ vector of time series with the representation

$$y_t = A_{0,t} + A_{1,t}y_{t-1} + A_{2,t}y_{t-2} + ... + A_{p,t}y_{t-p} + \varepsilon_t \tag{1.1}$$

where $A_{0,t}$ is a $n \times 1$ vector; $A_{i,t}$, are $n \times n$ matrices, $i = 1, ..., p$, and ε_t is a $n \times 1$ Gaussian white noise process with zero mean and covariance Σ_t. Let $A_t = [A_{0,t}, A_{1,t}...A_{p,t}]$, $x_t' = [1_n, y_{t-1}'...y_{t-p}']$, where 1_n is a row vector of ones of length n . Let $vec(\cdot)$ denote the stacking column operator and let $\theta_t = vec(A_t')$. Then (1.1) can be written as

$$y_t = X_t'\theta_t + \varepsilon_t \tag{1.2}$$

where $X_t' = (I_n \bigotimes x_t')$ is a $n \times (np+1)n$ matrix, I_n is a $n \times n$ identity matrix, and θ_t is a $(np+1)n \times 1$ vector. If we treat θ_t as a hidden state vector, equation (1.2) represents the observation equation of a state space model. We assume that θ_t evolves according to

$$p(\theta_t|\theta_{t-1}, \Omega_t) \propto \mathcal{I}(\theta_t)f(\theta_t|\theta_{t-1}, \Omega_t) \tag{1.3}$$

where $\mathcal{I}(\theta_t)$ is an indicator function discarding explosive paths of y_t. Such an indicator is necessary to make dynamic analysis sensible and, as we will see below, it is easy to implement numerically. We assume that $f(\theta_t|\theta_{t-1}, \Omega_t)$ can be represented as

$$\theta_t = \theta_{t-1} + u_t \tag{1.4}$$

where u_t is a $(np+1)n \times 1$ Gaussian white noise process with zero mean and covariance Ω_t. We select this simple specification because more general AR and/or mean reverting structures were always discarded in out-of-sample model selection exercises. We assume that $\Sigma_t = \Sigma \ \forall t$; that $corr(u_t, \varepsilon_t) = 0$, and that Ω_t is diagonal. At first sight, these assumptions may appear to be restrictive, but they are not. For example, the first assumption does not imply that the forecast errors are homoschedastic. In fact, substituting (1.4) into (1.2) we have that $y_t = X_t'\theta_{t-1} + v_t$ where $v_t = \varepsilon_t + X_t'u_t$. Hence, one-step ahead forecast errors have a time varying non-normal heteroschedastic structure even assuming $\Sigma_t = \Sigma$ and $\Omega_t = \Omega$. The assumed structure is appealing since it is coefficient variations that impart heteroschedastic movements to the variance of the forecasts errors (see Canova,1993, Sims and Zha, 2004, and Cogley and Sargent, 2005, have alternative specifications). The second assumption is standard but somewhat stronger and implies that the dynamics of the model are conditionally linear.

Sargent and Hansen (1998) showed how to relax this assumption by equivalently letting the innovations of the measurement equation to be serially correlated. Since in our setup ε_t is, by construction, a white noise process, the loss of information caused by imposing uncorrelation between the shocks is likely to be small. The third assumption implies that each element of θ_t evolves independently but it is irrelevant since structural coefficients are allowed to evolve in a correlated manner.

Let S be such that $\Sigma = SS'$. Let H_t be an orthonormal matrix, independent of ε_t, such that $H_t H_t' = I$ and let $J_t^{-1} = H_t' S^{-1}$. J_t is a particular decomposition of Σ which transforms (1.2) in two ways: it produces uncorrelated innovations (via the matrix S) and gives a structural interpretation to the equations of the system (via the matrix H_t). We have

$$y_t = A_{0,t} + \sum_j A_{j,t} y_{t-j} + J_t e_t \tag{1.5}$$

where $e_t = J_t^{-1}\varepsilon_t$ satisfies $E(e_t) = 0$, $E(e_t e_t') = I_n$. Equation (1.5) represents the class of "structural" representations of y_t we are interested in. For example, a standard Cholesky representation can be obtained setting S to be lower triangular and $H_t = I_n$ and more general patterns of zero restrictions result choosing S to be non-triangular and $H_t = I_n$. In this paper S is arbitrary and H_t implements interesting economic restrictions.

Letting $C_t = [J_t^{-1}A_{0t}, J_t^{-1}A_{1t}, \ldots, J_t^{-1}A_{pt}]$, and $\gamma_t = vec(C_t')$, (1.5) can be written as

$$J_t^{-1} y_t = X_t' \gamma_t + e_t \tag{1.6}$$

As in fixed coefficient VARs, there is a mapping between γ_t and θ_t since $\gamma_t = (J_t^{-1} \bigotimes I_{np})\theta_t$ where I_{np} is a $(np+1) \times (np+1)$ identity matrix. Whenever $\mathcal{I}(\theta_t) = 1$, we have

$$\gamma_t = (J_t^{-1} \bigotimes I_{np})(J_t^{-1} \bigotimes I_{np})^{-1} \gamma_{t-1} + \eta_t \tag{1.7}$$

where $\eta_t = (J_t^{-1} \bigotimes I_{np})u_t$, the vector of shocks to structural parameters, satisfies $E(\eta_t) = 0$, $E(\eta_t \eta_t') = E((J_t^{-1} \bigotimes I_{np})u_t u_t'(J_t^{-1} \bigotimes I_{np})')$. Hence, the vector of structural shocks $\xi_t' = [e_t', \eta_t']'$ is a white noise process with zero mean and covariance matrix

$$E\xi_t\xi_t' = \begin{bmatrix} I_n & 0 \\ 0 & E((J_t^{-1} \bigotimes I_{np})u_t u_t'(J_t^{-1} \bigotimes I_{np})') \end{bmatrix}$$

Since each element of γ_t depends on several u_{it} via the matrix J_t, shocks to structural parameters are no longer independent.

The structural model (1.6)-(1.7) contains two types of shocks: disturbances to the observations equations, e_t, and disturbances to structural parameters, η_t. While the formers have the usual interpretation, the latters are new. To understand their meaning, suppose that the $n-th$ equation of (1.6) is a monetary policy equation and suppose we summarized it by $\tilde{\gamma}_t = [\gamma_{(n-1)(np+1),t}, ..., \gamma_{n(np+1),t}]'$, which describes, say, how interest rates respond to the developments in the economy, and the policy shock $e_{n,t}$. Then, if variations in the parameters regulating preferences and technologies are of second order, an assumption commonly made in the literature, $\tilde{\gamma}_t$ captures changes in the preferences of the monetary authorities with respect to developments in the rest of the economy.

1.3 Impulse Responses

One question we would like to address is whether the transmission of monetary policy shocks has changed over time. In a fixed coefficient model, impulse response functions provide information on how the variables react to policy shocks. Impulse responses are typically computed as the difference between two realizations of $y_{i,t+k}$ which are identical up to time t, but one assumes that between $t+1$ and $t+k$ a shock in e_j occurs only at time $t+1$ and the other that no shocks take place at all dates between $t+1$ and $t+k, k = 1, 2, \dots,$. In a TVC model, responses computed this way are inadequate since they disregard the fact that between $t+1$ and $t+k$ the coefficients of the system may also change. Hence, meaningful impulse response functions ought to measure the effects of a shock in e_{jt+1} on y_{it+k}, allowing future shocks to the coefficients to be non-zero.

In order to understand the mechanics of impulse response functions in our setup let us rewrite model (1.1) in companion form

$$\mathbf{y}_t = \mu_t + \mathbf{A}_t \mathbf{y}_{t-1} + \epsilon_t$$

where $\mathbf{y}_t = [y_t' ... y_{t-p+1}']'$, $\epsilon_t = [\varepsilon_t' 0...0]'$ and $\mu_t = [A_{0,t}' 0...0]'$ are $np \times 1$ vectors and

$$\mathbf{A}_t = \begin{pmatrix} & A_t & \\ I_{n(p-1)} & & 0_{n(p-1),n} \end{pmatrix}$$

where $A_t = [A_{1,t} ... A_{p,t}]$ is an $n \times np$ matrix, $I_{n(p-1)}$ is an $n(p-1) \times n(p-1)$ identity matrix and $0_{n(p-1),n}$ is a $n(p-1) \times n$ matrix of zeros. Iterating k period forward and omitting for simplicity the constant term, we obtain

$$\mathbf{y}_{t+k} = \mathbf{A}_{t+k} ... \mathbf{A}_t \mathbf{y}_{t-1} + \mathbf{A}_{t+k} ... \mathbf{A}_{t+1} \epsilon_t + \mathbf{A}_{t+k} ... \mathbf{A}_{t+2} \epsilon_{t+1} + ... + \mathbf{A}_{t+k} \epsilon_{t+k-1} + \epsilon_{t+k}$$

Let $\mathcal{S}_{i,j}(M)$ be a selection function, a function which selects the first i rows and j columns of the matrix M. Taking as a benchmark case the case of no-shock occurrence, and assuming that coefficients and shocks ε_t are uncorrelated, the matrix of dynamic multiplier $\mathcal{S}_{n,n}(\mathbf{A}_{t+k} ... \mathbf{A}_{t+1})$ describes the effects of ε_t on y_{t+k}, while the effects associated to structural shocks can be derived from the relation $\varepsilon_t = J_t e_t$ and are given by $\mathcal{S}_{n,n}(\mathbf{A}_{t+k} ... \mathbf{A}_{t+1}) J_t$. Therefore impulse response functions to a shock e_t at horizon k are given by

$$IR(t, k) = \Psi_{t,k} J_t$$

where $\Psi_{t,k} = \mathcal{S}_{n,n}(\mathbf{A}_{t+k} ... \mathbf{A}_{t+1})$. Thus for each $t = 1, ..., T$ we have a path of impulse response defined by the sequence $\{\Psi_{t,k} J_t\}_{k=1}^{\tau}$. In this class of models impulse response functions are time-varying and they collapse to traditional impulse response functions only when autoregressive coefficients are constant.

A second complication is that in the TVC model is that we also have shocks to the systematic component of the monetary policy, that is shocks to the coefficients of the observed variables in the monetary policy rule. Unlike equations shocks, the effects of shocks to the coefficients produce highly non linear dynamics since they enter in a multiplicative way. In order to trace out the effects of a change in the monetary policy preferences to observed variables we proceed as follows. We define a realization of the impulse response functions at horizon k to a shocks η_{it} as the difference between two realizations, y^{δ}_{t+k} and y^{0}_{t+k}, which are identical except for the shock η_{it} which is equal to δ in the first and zero in the second. For instance, let us consider the univariate AR(1) process $y_t = a_t y_{t-1} + \varepsilon_t$, $a_t = a_{t-1} + u_t$. The contemporaneous effect of a shock u_t is given by $(a_{t-1}+\delta)y_{t-1} - (a_{t-1}+0)y_{t-1} = \delta y_{t-1}$ and at horizon one will be $(a_{t-1}+\delta+u_{t+1})y^{\delta}_t - (a_{t-1}+0+u_{t+1})y^{0}_t = \delta y^{\delta}_t + (a_{t+1} - u_{t+1})(y^{\delta}_t - y^{0}_t) = \delta y^{\delta}_t + (a_{t+1} - u_{t+1})\delta y_{t-1}$. Therefore impulse response functions to ηit are random variables that depend on the coefficients at time t, future shocks and, unlike impulse response functions to equation shocks, also on the value of the initial vector of time series[1].

1.4 Identification

In our setup, identifying structural shocks is equivalent to choosing H_t. As in Faust (1998), Canova and De Nicoló (2002) and Uhlig (2005), we select H_t so that the sign of the impulse response functions at $t+k, k = 1, 2, \ldots, K_1$ matches some theoretical restriction. In particular, we assume that a contractionary monetary policy shock must generate a non-positive effects on output, inflation and nominal balances and a non-negative effect on the interest rate for two quarters. (see Gambetti et al., 2005, for a class of DSGE models which robustly generates this set of restrictions).

We choose sign restrictions to identify shocks for two reasons. First, the contemporaneous zero restrictions conventionally used are often absent in those theoretical (DSGE) models economists like to use to guide the interpretation of VAR results. Second, a set zero restrictions which satisfies the standard order condition for identification, does not deliver an identified system in the case of indeterminacy (in this case, there are n+1 shocks). Sign restrictions do not suffer from this problem. Moreover, as shown by Lubik and Schorfheide (2004), a small scale version of the model used in Gambetti et al (2005) delivers the same qualitative implications we use as identifying both in determinate and in indeterminate scenarios. To implement sign restrictions we proceed as follows. (i) From the posterior distribution we draw θ_T and we iterate in the non linear evolution equation to compute $\theta^{T+\tau}_{T+1}$. (ii) We draw a realization of the variance covariance matrix of the reduced for shocks Σ and we compute its square root S. (iii) We draw a candidate for H as follows: we draw a $(n \times n)$ matrix X with each element having an independent standard normal distribution, and then we take its QR decomposition $X = QH$, with the diagonal of R normalized to be positive. H is uniformly distributed. (iv) We compute impulse response functions. (v) If sign restrictions are satisfied we collect the draw otherwise we discard it and go to point (i).

[1] As shown by Canova and Gambetti (2005), the so defined impulse response functions coincide with the difference between two conditional expectations, as in Gallant et al. (1996), Koop et al. (1996), where both information sets include the history of the data $(y_1, \ldots, y_t)$, the states $(\theta_1, \ldots, \theta_t)$, the structural parameters of the transition equation (which are function of J_t), all future shocks and they differ because in the former a shock of size δ is included while in the second it is not.

In the last section we report, as a robustness check, results obtained identifying policy shocks as the third element of a Cholesky system, i.e. we let the interest rate to react to output and inflation but assume that it has no effects within a quarter on these variables. Since we are interested in recovering the systematic and non-systematic part of monetary policy and in analyzing how the economy responds to their changes over time, we arbitrarily orthogonalize the other disturbances without giving them any structural interpretation.

1.5 Estimation

The model (1.6)-(1.7) is estimated using Bayesian methods. That is, having specified prior distributions for all the parameters of interest, we use data up to t to compute posterior distributions of the structural parameters and of continuous functions of them. Since our sample goes from 1960:1 to 2003:2, we initially estimate the model for the sample 1960:1-1977:3 and then reestimate it moving the terminal date by one quarter up to 2003:2.

Posterior distributions for the structural parameters are not available in a closed form. MCMC methods are used to simulate posterior sequences consistent with the information available up to time t. Estimation of reduced form TVC-VAR models with or without time variations in the variance of the shocks to the transition equation is now standard (see e.g. Cogley and Sargent, 2001): it requires treating the parameters which are time varying as a block in a Gibbs sampler algorithm. Therefore, at each t and in each Gibbs sampler cycle, one runs the Kalman filter and the Kalman smoother, conditional on the draw of the other time invariant parameters. In our setup the calculations are complicated by the fact that at each cycle, we need to obtain structural estimates of the time varying features of the model. This means that, in each cycle, we need to apply the identification scheme, discarding paths which are explosive and paths which do not satisfy the restrictions we impose. The computational costs are compounded because we need to run the Gibbs sampler more than a 100 times, one per sample we analyze. Convergence was checked using a CUMSUM statistic. The results we present are based on 10,000 draws for each t[2].

Because of the heavy notation involved in the construction of posterior distributions and the technicalities needed to produce draws from these posteriors, we present the details of the estimation approach in the appendix.

1.6 The Results

The data we use is taken from the FREDII data base of the Federal Reserve Bank of San Louis. In our basic exercise we use the log of (linearly) detrended real GDP, the log of first difference of GDP deflator, the log of (linearly) detrended M1 and the federal funds rate in that order. Systems containing other variables are analyzed in the next section.

We organize the presentation of the results around four general themes: (i) Do reduced form coefficients display significant variations? (ii) Are there synchronized changes in the

[2]Total computational time for each specification on a Pentium IV machine was about 100 hours.

structural coefficients and/or in the structural variances of the model? (iii) Are there changes in the propagation of monetary policy disturbance in the short and the long run? (iv) Would it have made a difference for macroeconomic performance if monetary policy were more aggressive in fighting inflation, in particular, at the end of the 1970's?

1.6.1 The evolution of reduced form coefficients

The first panel of figure 1 plots the evolution of the mean of the posterior distribution of the change in reduced form coefficients in each of the four equations. The first date corresponds to estimates obtained with the information available up to time 1977:3, the last one to estimates obtained with the information up to time 2003:2.

Several interesting aspects of the figure deserve some comments. First, consistent with the evidence of Sargent and Cogley (2001) and (2005) all equations display some coefficient variation. In terms of size, the money (third) and interest rate (fourth) equations are those with the largest changes, while variations in the coefficients of the inflation (second) equation are the smallest of all. Second, while changes appear to be stationary in nature, there are few coefficients which display a clear trend over time. For example, in the output (first) equation, the coefficient on the first lag of money is drifting downward from 0.6 in 1977 to essentially zero at the end of the sample; while in the money equation, the first lagged money coefficient is drifting upward from roughly zero in 1977 to about 0.9 in 2002.

Perhaps more importantly, there is little evidence of a once-and-for-all structural break in the coefficients of the output and inflation equation (i.e. coefficients do not jump at some date and stays there afterward). Third, the majority of the changes appear to be concentrated at the beginning of the sample. The 1979-1982 period is the one which displays the most radical variations; there is some coefficient drift up to 1986, and after that date variations appear to be random and small.

Finally, centered 68% posterior bands for the coefficients at the beginning (1977:3) and at the end of the sample (2003:2) overlap in many cases. Therefore, barring few relevant exceptions, instabilities appear to be associated with the Volker (1979-1982) experiment and the adjustments following it. Furthermore, they are temporary and mean reverting in nature.

Figure 2 reports the posterior mean drift of inflation and a posterior mean of inflation persistence obtained in the system. The mean drift of inflation tracks well the ups and downs of inflation over the period and the posterior mean of inflation persistence shows a dramatic decline at the beginning of the 1980's. Both of these patterns agree with those presented by Cogley and Sargent (2005), despite the fact that the VAR system differ in the number and kind of variables used. To go beyond the documentation of patterns of time variations in reduced form statistics and study whether monetary policy is responsible for the changes, we next examine the dynamics of structural coefficients.

1.6.2 Structural time variations

The second panel of figure 1 presents the evolution of the posterior mean of the changes in the lagged structural coefficients of each equation at each date in the sample. The first date corresponds again to estimates obtained with the information up to time 1977:3, the last one to estimates obtained with the information up to time 2003:2.

It is immediate to notice that changes in the structural coefficients are typically larger and more generalized than those in the reduced form coefficients. The output and the monetary policy equations are those displaying the largest absolute coefficient changes - these are up to 4 times as large as the largest absolute changes present in the other two equations - while the coefficients of the structural inflation equation are still the most stable ones. Furthermore, except for the money (demand) equation, most the variations are concentrated in the first part of the sample, are large in size, statistically and often economically significant.

More interestingly from our point of view, there is a pattern in the structure of time variations. The output equation displays two regimes of coefficient variations (one with high variations up to 1986 and one with low variations thereafter) and, within the high volatility regime, the largest coefficient variations occur in 1986. The inflation equation shows the largest coefficient changes up to 1982 and, barring few exceptions, a more stable pattern resulted since then. Finally, our identified monetary policy equation displays large and erratic coefficient changes up to 1986 and coefficients variation is considerably reduced after that. Since the timing of the variations in the structural coefficients of the output and inflation equations are somewhat asynchronous with those of the monetary policy equation, figure 1 casts some doubts on a causal interpretation of the observed changes running from changes in the policy equation to changes in the dynamics of output and inflation.

Figure 3 zooms in on the evolution of the coefficients of the monetary policy equation (which is normalized to be the last one of the system). Three facts stand out. First, posterior mean estimates of all contemporaneous coefficients are humped shaped: they significantly increase from 1979 to 1982 and smoothly decline afterwards. Second, although all contemporaneous coefficients are higher at the end than at the beginning of the sample, they are typically lower than the conventional wisdom would suggest. In particular, the contemporaneous inflation coefficient peaks at about 1.2 in 1982 and then declines to a low 0.3, on average, in the 1990s and this pattern is also shared by the two lagged inflation coefficients. In this sense, Alan Greenspan's regime was only marginally more effective than Arthur Burns's in insuring inflation stability: interest rate responses to inflation movements were barely more aggressive in the 1990s than they were in the 1970s. Note also that, again excluding the beginning of the 1980's, the estimated monetary policy rule displayed considerable stability, in line with the subsample evidence presented, e.g. by Bernanke and Mihov (1998). Since macroeconomic performance was considerably different in the two time periods, the size and characteristics of the shocks hitting the US economy in the two periods must have been different. We will elaborate on this issue later on.

Our estimated policy rule displays a six fold-increase in all contemporaneous and first lagged coefficients from 1979 to 1982. Interestingly, this increase is not limited to the inflation coefficients, but also involves output and the money coefficients. The high responsiveness of interest rates to economic conditions is consistent with the idea that by targeting monetary aggregates the Fed forced interest rates to jump to equilibrate a "fixed" money supply with a largely varying money demand - the period was characterized by a number of important financial innovations. The pervasive instability characterizing this period and the subsequent three years adjustments contrasts with the substantial stability of the coefficients of the monetary policy rule in the rest of the sample. Hence, excluding the "Volker experiment", the systematic component of monetary policy has hardly changed over time and if, any change must be noted, it is more toward a decline in the responsiveness of interest rates to economic conditions. This outcome is consistent with the "business as usual" characterization of monetary policy put forward by Leeper and Zha (2003) and with the time profile of the policy rule recursively estimated in a DSGE model (see Canova, 2004).

The evidence we have so far collected seems to give little credence to the crudest version of the "bad policy" hypothesis: there is no permanent increase in the inflation coefficient of the policy rule, nor clear evidence that the Taylor's principle was violated in the 1970's and satisfied afterwards. Both more sophisticated versions of the "bad policy" and the "back luck" hypotheses suggest that alterations in the distribution of the shocks hitting the economy are responsible for the improved macroeconomic outcome. In the former case, changes in the variance of policy shocks "caused" the observed changes; in the latter case, policy has little or nothing to do with the dynamics of output and inflation which are simply driven variations in the distributions of the shocks hitting the two equations.

Figure 4 presents some evidence on this issue. In the top panel we report the evolution of the posterior mean estimate of the variance of the structural forecast errors and, in the bottom panel, the variations produced by its heteroschedastic component, i.e the variations induced by product of the estimated innovations in the coefficient and the regressors of the model. Three features are of interest. First, the forecast error variance in three of the four equations is humped shaped: it shows a significant increase from 1979 to 1982 followed by a smooth decline. As it happened with structural coefficients, the posterior mean estimate of the variance of the shocks in the end of the sample is roughly similar in magnitude to the posterior mean estimate obtained in 1977. Second, the time profile of the changes in the forecast error variances of the output and the inflation equations are not synchronized with the variations in the forecast error variance of our estimated policy equation, which starts declining significantly after 1986. Third, the contribution of changes in the coefficients to the forecast error variance is much larger in the output and inflation equations than in the other two equations up to 1982 but similar after that date. Shocks to the model contribute most to the variability of the forecast error between 1979 and 1982 - they account for about 50% of the variance in the output and inflation equations - but their importance declined after 1982 and the decline is stronger in the inflation equation.

1.6.3 Changes in the propagation of monetary policy disturbances?

Figure 5 reports the posterior mean responses of output and inflation to identified monetary policy shocks in each date of the sample, for horizons running from 1 to 12 quarters. We do not report interest rate responses because they are similar over time and quite standard in shape and magnitude: after the initial impulse, the increase dissipates rather quickly and becomes insignificantly different from zero after the 3th quarter for each date in the sample.

The shape of both output and inflation responses is roughly unchanged over time. Output responses are U-shaped; a through response occurs after about 3 quarters and there is a smooth convergence to zero after that date. Inflation responses are also slightly U-shaped; the effect at the one quarter horizon is typically the largest, and responses smoothly converge toward zero afterwards.

There is a small quantitative difference in the mean responses over time. For output, the posterior mean of the instantaneous response is always centered around -0.15 and the size of the through responses at lag 3 varies in the range (-0.20,-0.05). For inflation, minor differences occur at lag one (posterior mean varies between -0.07 to -0.16) while in 1978 responses are more persistent than at all the other dates at horizons ranging from 3 to 8.

Differences in inflation responses are both statistically and economically small. The posterior 68% confidence band for the largest discrepancy (the one at lag 1) includes zero at almost all horizons and, if we exclude the initial three years, the time path of inflation responses is unchanged over time. The posterior 68% confidence band for the largest discrepancy in output responses (the one at lag 3) does at times exclude zero - the trough response in 1982 appear to be significantly deeper than the trough response in 1978 and 1979 and at some dates after 1992 - but differences are economically small: the maximum discrepancy in the cumulative output multiplier twelve quarters ahead is only 0.5%. In other words, a one percent increase in interest rates produced output responses which differ over time on average by 0.04% points at each horizon. Overall, the dynamics induced by monetary policy shocks are remarkably stable over time and, in agreement with the results of section 5.2, responses in the end of the 1990's look similar, in shape and size, to those in the end of the 1970's.

1.6.4 Inflation Dynamics and Monetary Policy

Cogley and Sargent (2001,2005) have examined measures of core inflation to establish their claim that monetary policy is responsible for the observed changes in inflation dynamics. They define core inflation as the persistent component of inflation, statistically measured by the zero frequency of the spectrum (that is, by the sum of all autocovariances of the estimated inflation process), and show i) persistence has substantially declined over time and ii) there is synchronicity between the changes in persistence and a narrative account of monetary policy changes. Pivetta and Reis (2004), using univariate conventional classical methods, dispute the first claim showing that differences over time in two measures of inflation persistence are statistically insignificant. Since our study has so far concentrated on short/medium run frequencies, we turn to investigate the longer run relationship between inflation and monetary policy. In particular, we are curious as to whether different frequencies of the spectrum carry different information and whether our basic conclusions on the role of monetary policy are altered.

Our analysis differs from existing ones in two important respects: we use output in place of unemployment in the estimated system; we measure persistence using the estimated structural model. While the first difference is minor, the second is not. In fact, thanks the orthogonality of the structural shocks and of the ordinates of the spectrum, we can not only to describe the evolution of the spectrum of inflation over time, but also directly measure of proportion of the spectral power at frequency zero due to monetary policy shocks and describe its evolution over time. From the structural MA representation of the system we have that $\pi_t = \sum_{i=1}^{n} \phi_{it}(\ell) e_{it}$, where e_{it} is orthogonal to e_{jt}. Hence the spectrum of inflation at Fourier frequencies ω is $S_\pi(\omega) = \frac{1}{2\pi} \sum_{i=1}^{n} \phi_{it}(\omega)|^2 \sigma_i^2$ and the component at frequency zero due to monetary policy shocks is $S_\pi^*(\omega = 0) = \frac{1}{2\pi} |\phi_{nt}(\omega = 0)|^2 \sigma_n^2$.

The top panel of figure 6 shows the time evolution of the posterior mean of the spectrum of inflation at the zero frequency and the contribution that monetary policy shocks had in shaping its changes. The estimate of the zero frequency displays an initial increase in 1978-1980 followed by a sharp decline the year after; since 1981 the estimated posterior mean of the zero frequency of the spectrum has been relatively stable (with the exclusion of 1991). The initial four fold jump and the following ten fold decrease are visually large and statistically significant. In fact, the bottom panel of figure 6 indicates that the 68% posterior band for the differences between the log spectrum in 1979 and 1996 (the date with the lowest estimates) does not include zero at the zero frequency. At all other frequencies, differences over time are negligible both in terms of size and shape. Hence, except for the zero frequency, the posterior distribution of the spectrum of inflation has also been relatively stable. What is the role of monetary policy shocks? The top panel of figure 6 indicates that the two graphs track each other reasonably well suggesting that, at least in terms of timing, monetary policy shocks are important in determining inflation persistence dynamics. Second, the contribution of monetary policy to inflation persistence varies over time: fluctuations are large and the percentage explained ranges from about 20 to about 75 percent. Interestingly, there is a significant trend increase since 1981. Third, there is a substantial portion of inflation persistence (roughly, 50 percent on average) which has nothing to do with monetary policy shocks. While the determination of the forces behind this large percentage is beyond the scope of this paper, one can conjecture that real and financial factors could account for these variations. As mentioned, the years between 1978 and 1982 were characterized by financial innovations and high nominal interest rate variability. The pattern present at the zero frequency over this period is consistent with these two features while the subsequent decline is consistent with the reduction of the volatility of interest rate disturbances shown in figure 4.

In conclusions, there is visual and statistical evidence of instabilities in the posterior mean of inflation persistence. Changes in the posterior mean of inflation persistence go hand in hand with changes in the contribution of monetary policy shocks. Perhaps more importantly, we find that the contribution of monetary policy shocks to variations in the posterior means of inflation persistence is smaller than expected, that factors other than monetary policy are crucial to understand its evolution over time, and that the relative contribution of monetary policy has increased since the early 1980s.

1.6.5 What if monetary policy would have been more aggressive?

It is common in the literature to argue, by mean of counterfactuals, that monetary policy failed to perform an inflation stabilization role in the 1970s (see e.g Clarida, Gali and Gertler, 2000, or Boivin and Giannoni, 2002) and that, had it followed a more aggressive stance against inflation, dramatic changes in the economic performance would have resulted. While exercises of this type are meaningful only in dynamic models with clearly stated microfundations, our structural setup allows us to approximate the ideal type of exercise without falling into standard Lucas-critique type of traps. In fact, to the extent that the monetary policy equation we have identified is structural, and given that we estimate posterior distributions which are consistent with the information available at each t, we can examine what would have happened if the policy response to inflation was significantly stronger, where by this we mean a (permanent) two standard deviations increased in the inflation coefficients above the estimated posterior mean. Figure 7 plots the percentage output and inflation changes from the value of the baseline year which would have been produced at selected dates in the sample. To interpret the numbers note, e.g., that the maximum inflation response in 1979 (-5 percent) correspond to a 1.0 point absolute decline in the annual inflation recorded at that date (which was around 19 percent) and that a 15 percent decline in 2003, at the annual rate of 2.5 percent, corresponds to an absolute fall of less than a 0.4 points.

A permanent more aggressive stance would have had important inflation effects in 1979, primarily in the medium run. However, at all dates in the 1980s and 1990s, the effect would have been statistically negligible. Interestingly, if such a policy were used in 2003, it would have produced a small but significant medium run increase in inflation. A tougher stance on inflation, however, is not painless: important output effects would have been generated. In 1979, the fall would have lasted about four years while the 7 percent fall recorded in 2003 would have lasted for quite a long time. The Phillips curve trade-off, measured here by the conditional correlation between output and inflation in response to the change, displays an interesting pattern: it is positive and significant in 1979, it is zero in 1983, and it is negative in 1992 and 2003, and at the last date it is statistically significant. While there are many reasons which can explain the change in the sign of the trade-off, a better control of inflation expectations and an improved credibility in the policy environment are clearly consistent with this pattern.

Overall, while there was room for stabilizing inflation in the end of the 1970, it is not clear that a tougher inflation stance would have been costless in terms of output. There is a sense in which the conventional view is right: being tough on inflation in the end of the 1970s would have produced a different macroeconomic outcome than in the end of the 1990s. However, the reasoning seems to be wrong: being tough on inflation is dangerous when the slope of the Phillips curve trade-off is different from the conventional one.

1.7 Robustness analysis

There is a number of specification choices we have made which may affect the results. In this section we analyze the sensitivity of our conclusions to variations in the identification method, in the treatment of trends, and in the variables included in the VAR. All the results we have presented so far have been produced identifying monetary policy shocks using sign restrictions on the dynamics of money, inflation and output. Would the pattern of time variations, the estimated policy rule and the time profile of impulse responses be altered if an alternative identification scheme was used? Figure 8 shows the evolution of the variance of the forecast errors and of output and inflation responses obtained identifying policy shocks with a Cholesky decomposition. Since here contemporaneous coefficients are time invariant, the evolution of structural coefficients reproduces the pattern of time variations present in the reduced form coefficients (they are simply multiplied by a constant). Therefore, the discussion of subsection 5.1 apply here without a change. Overall, the main conclusions we have derived are robust to this change: there are time variations in the coefficients but they are not synchronized across equations; the sum of the inflation coefficients in the policy equation is roughly the same in the end of the 1970s and of the 1990s; the evolution of the estimated forecast error variances reproduces the one present in figure 4; impulse responses are broadly similar across time. Clearly, there are changes in pattern of responses relative to our baseline case - inflation increases for at least a year after an interest rate shock. However, it is still true, that differences over time in the posterior mean of output and inflation responses are small and insignificant. Some feel uncomfortable with dynamic exercises conducted in a system where linearly detrended output and linearly detrended money are used. One argument against this choice is that after these transformations these two variables are still close to be integrated and are not necessarily cointegrated. Hence, the dynamics we trace out may be spurious. A second argument, put forward in Orphanides (2004), has to do with the fact that measures of the output gap obtained linearly filtering the data are plagued by measurement error. This measurement error is presumably reduced when output growth is employed. To verify whether arguments of this type alter our conclusions we have repeated estimation using the growth rate of output and of M1 in place of the detrended values of output and M1. A sample of the results appears in figure 9, where we plot the evolution of the posterior of the contemporaneous policy coefficients, of the variances of the forecasts errors and of the time profile of output and inflation in response to a policy shock, identified using sign restrictions. Once again, our basic conclusions remain unchanged. In particular, the variability of GDP and inflation forecast errors in the 1990's is about half what it was in the 1980's and 1970's; policy coefficients are stable; the transmission of policy shocks is stable and numerical difference emerge only in the response of inflation in the medium run, which is stronger at the beginning of the sample than at the end. We have also examined the sensitivity of our conclusions also to changes in the variables of the VAR. It is well known that small scale VAR models are appropriate only to the extent that omitted variables exert no influence on the dynamics of the included ones. A-priori it is hard to know what variables are more important and to check if our system effectively marginalized the influence of all relevant variables. We have therefore repeated our exercise substituting the unemployment rate to detrended output. Figure 10 reports the evolution of the posterior mean of the contemporaneous policy coefficients, of the variances of the forecast errors and of the responses of unemployment and inflation to a policy shock, identified via sign restrictions. Also in this case, our conclusions appear to be robust.

Finally, it is now common to examine monetary policy in empirical and theoretical models in which money play no role. We believe that such a practice is dangerous in a system like ours for two reasons. First, omission of money may cause identification problems (demand and supply of currency can not be disentangled). Second, money was a crucial ingredient in the considerations that shaped monetary policy decisions, at least up to the end of the 1980's. Commentators have argued that the inclusions of money in the policy rule may lead to an improper characterization of the policy decision of the Fed, especially during Greenspan's tenure. Figure 11 presents a sample of the results obtained with a trivariate system which excludes money. Our basic conclusions are robust also to this change. Interestingly, the posterior mean of the contemporaneous output coefficient in policy rule is counterintuitively negative and significant, suggesting that such a system could be misspecified.

1.8 Conclusions

This paper provides novel evidence on the contribution of monetary policy to the structural changes observed in the US economy over the last 30 years. We use a time varying structural VAR model to analyze the issues. Our exercise is truly recursive and, methodologically, we innovate on the existing literature in two important respects: we provide a sign scheme to identify structural shocks in a TVC model and a way to calculate impulse responses, which is coherent with the assumptions of the model. These three feature together allows us to assess how much time variation there is in the propagation of policy shocks, both in the short and in the long run, and to run counterfactuals to understand whether permanent changes in the systematic component of policy would have significantly altered macroeconomic performance.

We would like to emphasize four main conclusions of our investigation. First, excluding the 1979-1982 period, the posterior distribution of the policy coefficients has been relatively stable over time. Second, there is a clear trend decline in the posterior mean of the variability of the shocks hitting the economy but the changes observed in the output and inflation equations are unsynchronized with those present in the policy equation. Third, the posterior distribution of responses of output and inflation to policy shocks has been relatively stable over time, while changes in posterior distribution of inflation persistence appear to be partially related to changes in the contribution of policy shocks. Fourth, a more aggressive policy would have decreased inflation in the medium run in 1979 but not later. If this policy would have been implemented, output costs would have been large.

Since our results go against several preconceived notions present in the literature, it is important to highlight what are the features of our analysis which may be responsible for the differences. As repeatedly emphasized, our analysis uses a structural model, it is recursive and employs a definition of impulse responses which is consistent with the nature of the model we use. Previous studies which used the same level of econometric sophistication (such as Cogley and Sargent, 2001 and 2005) have concentrated on reduced form estimates and were forced to use the timing of the observed changes to infer the contribution of monetary policy to changes in output and inflation. Our approach allows not only informal tests but also to quantify a-posteriori the relationship between monetary policy, output and inflation dynamics. In studies where a semi-structural Cholesky based model is used, as in Primicieri (2005), the analysis is not recursive and the impulse responses are computed in the traditional way. Relative to earlier studies such as Bernanke and Mihov (1998), Hanson (2001) or Leeper and Zha (2003), which use subsample analyzes to characterize the changes over time in structural VARs, we are able to precisely track the evolution of the coefficients over time and produce a more complete and reliable picture of the relatively minor variations present in the monetary policy stance in the US.

Our results agree with those obtained recursively estimating a small scale DSGE model with Bayesian methods (see Canova, 2004) and contrast with those of Boivin and Giannoni (2002) who use an indirect inference principle to estimate the parameters of a DSGE model over two subsamples. We conjecture that identification problems could be responsible for the difference since the latter method has problems exploring flat objective functions. Finally, our results are consistent with those of Sims and Zha (2004), despite the fact that, in that paper, variations in both the coefficients and the variance are accounted for with a Markov switching methodology. Relative to their work, our analysis emphasizes that factors other than monetary policy could be more important in explaining the structural changes witnessed in the US economy and provides recursive impulse response analysis.

While the decline in the variance of the shocks hitting both the economy and the coefficients of its structural representation seems to suggest that exogenous reasons are responsible for the changes in the US economy, it is important to emphasize that our conclusions are consistent both with the analysis of McConnell and Perez Quiros (2001) and with the idea that a more transparent policy process has reduced the volatility of agent's expectations over time. It is therefore important to extend the current study, enlarging the number of variables included in the structural model, identifying other sources of shocks and disentangling possible factors which may be behind the decline in the volatility of structural shocks. Also, we have repeatedly mentioned that the monetary policy rule is similar in the 1970s and in the end of the 1990s. Why is it that inflation in the 1990s did not follow the same pattern as in the 1970s? What is the contribution of technological changes to this improved macroeconomic framework? We plan to study these and related issues in future work.

Figures

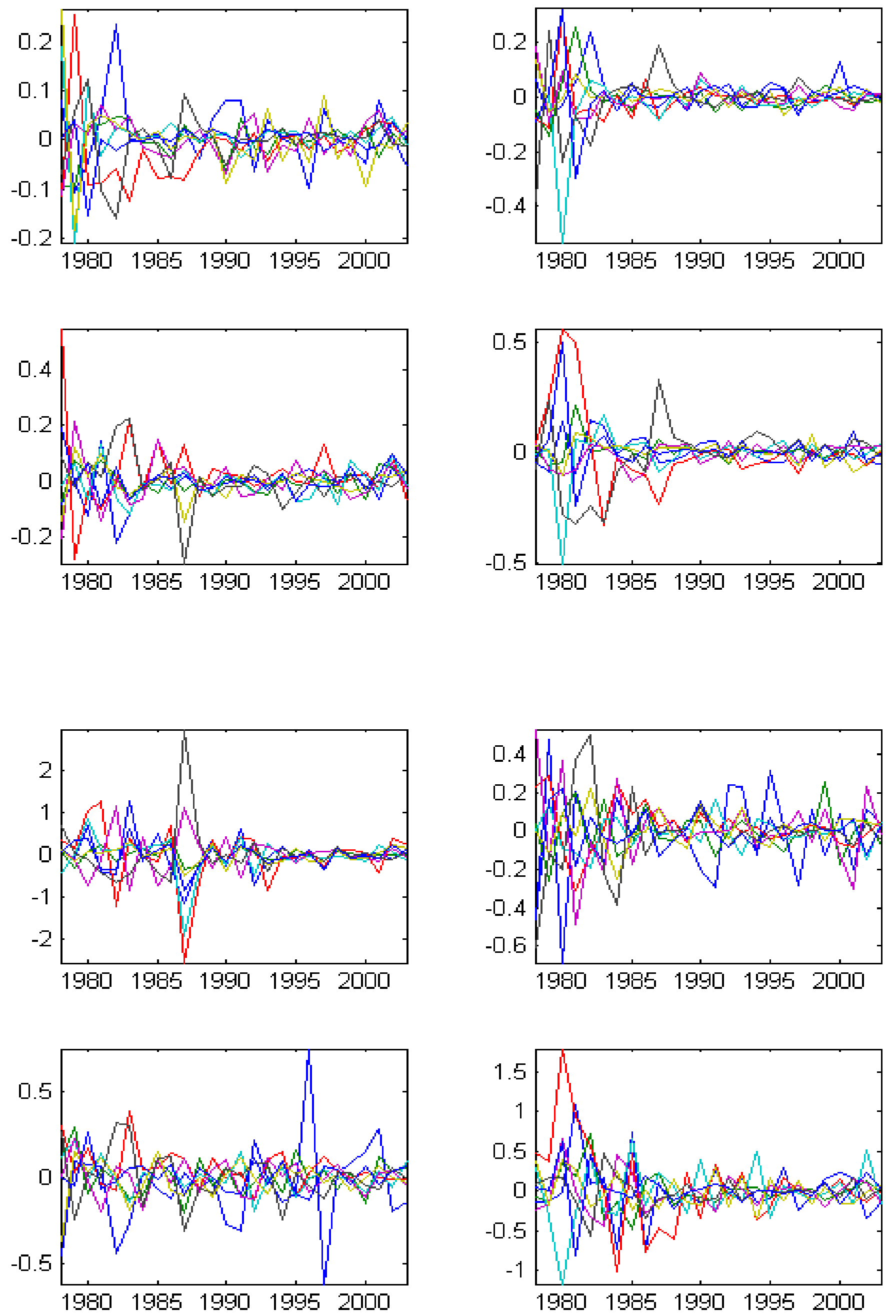


Figure 1: Mean changes: reduced form coefficients (top), structural coefficients (bottom). In clockwise direction GDP, inflation. money and interest rate.

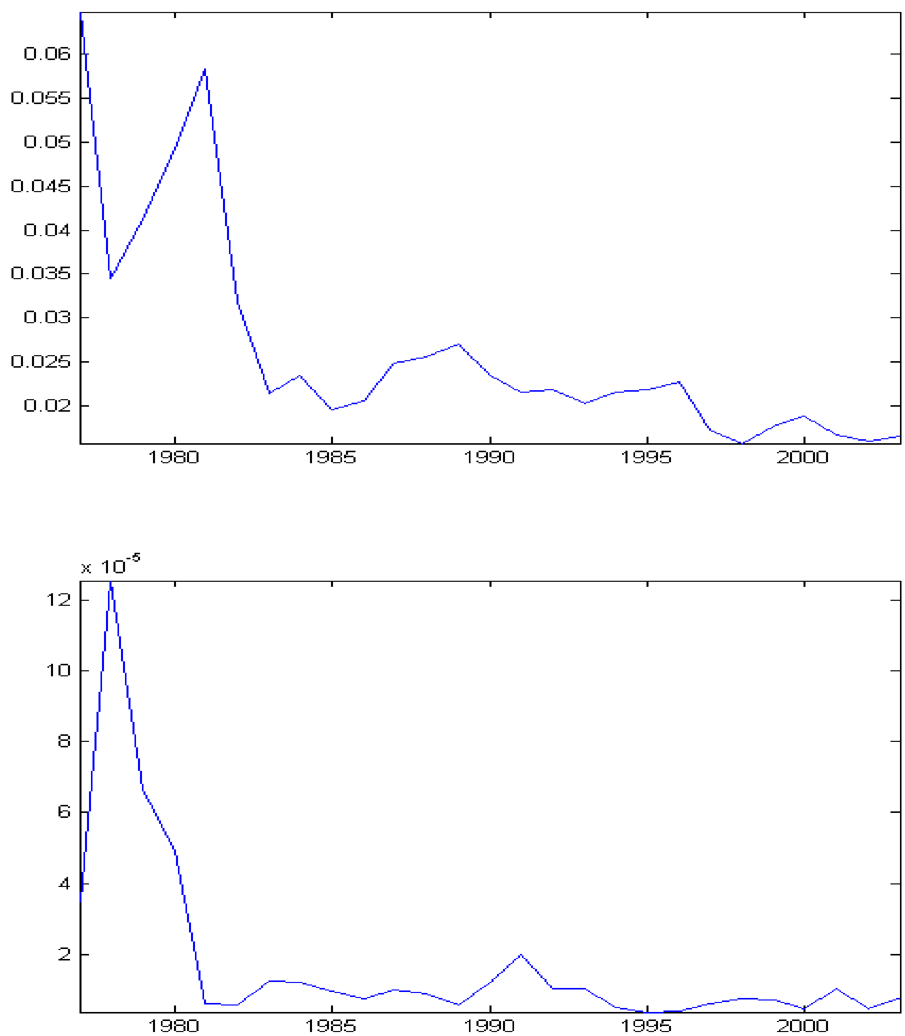


Figure 2: Reduced form mean inflation drift (top) and mean inflation persistence (bottom).

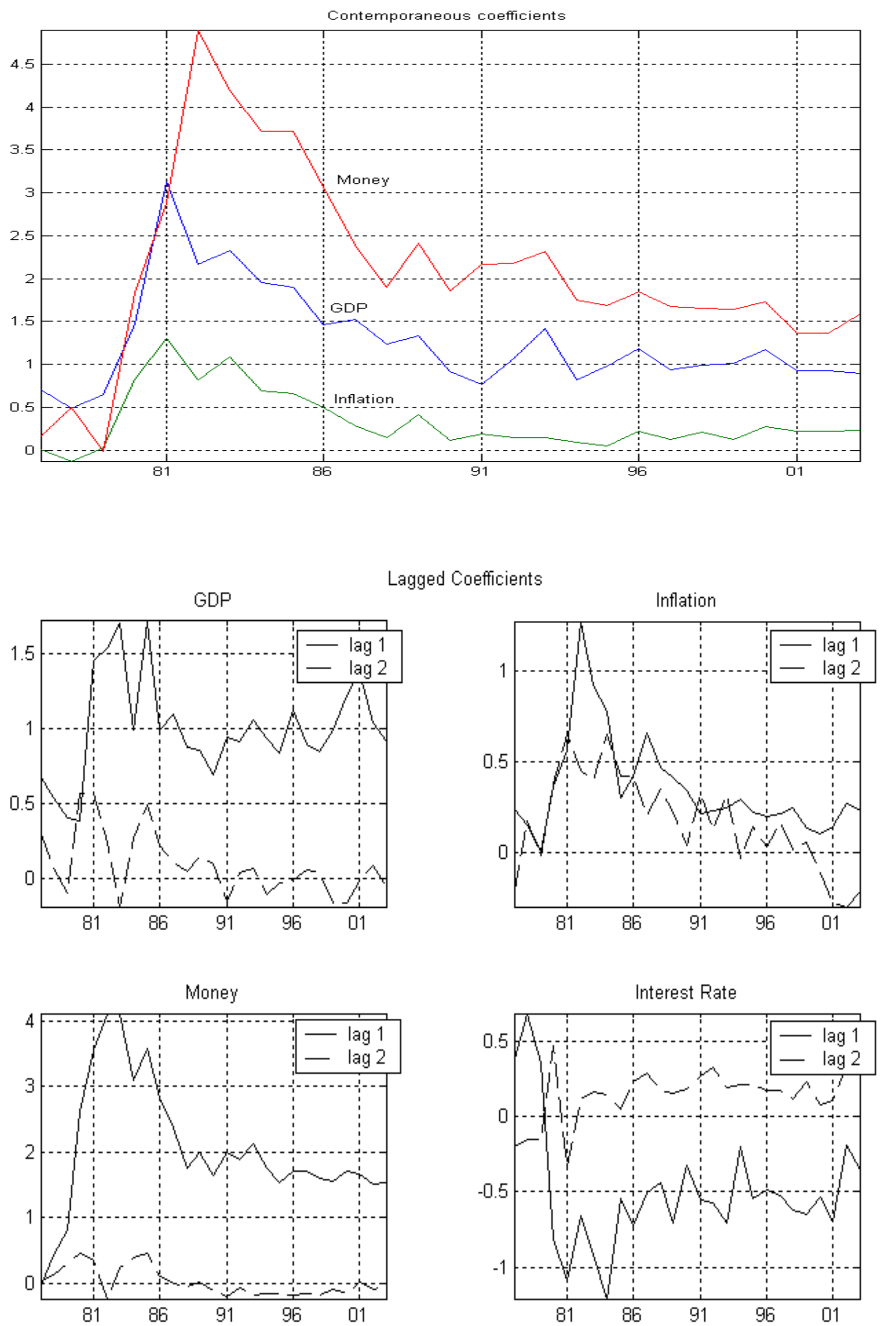
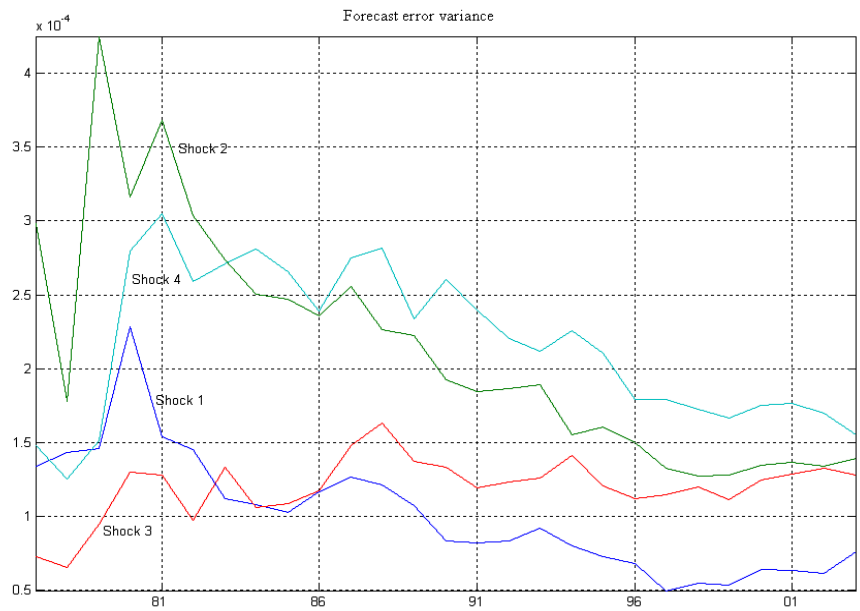


Figure 3: Structural coefficients, monetary policy equation.



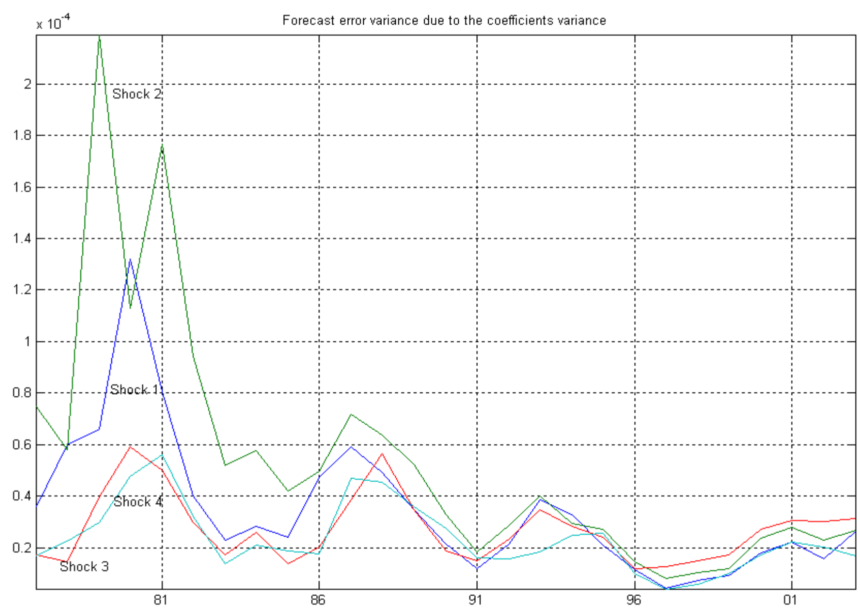
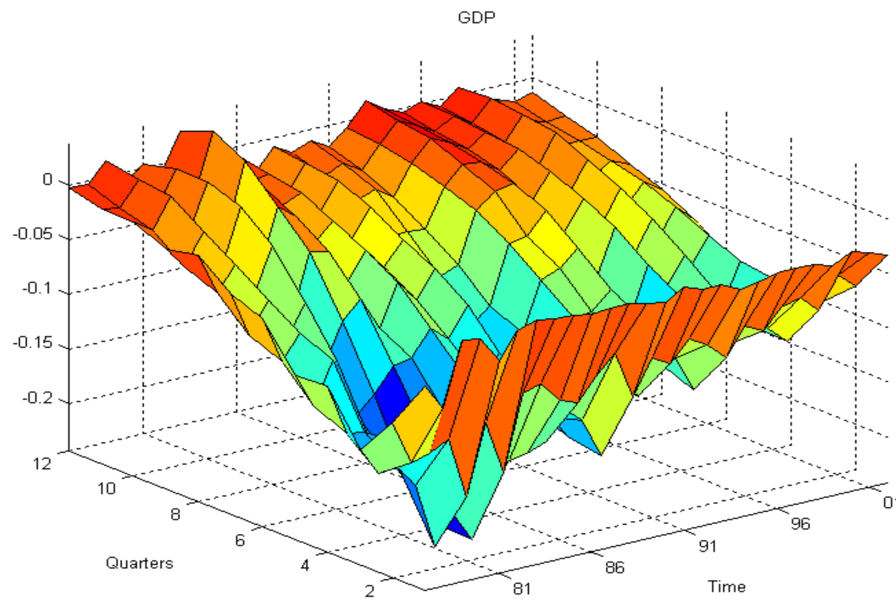


Figure 4: Forecast error variance



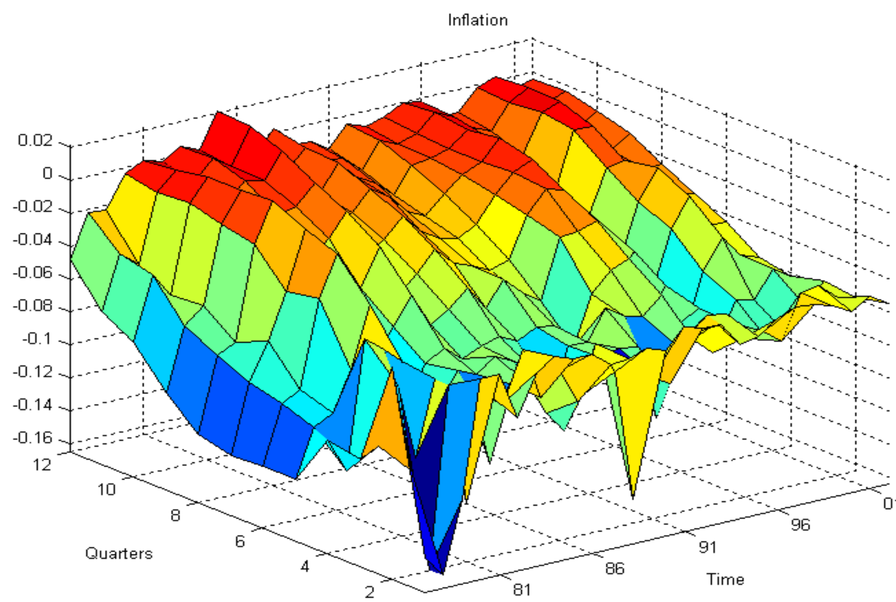


Figure 5: Structural impulse response to monetary policy shocks

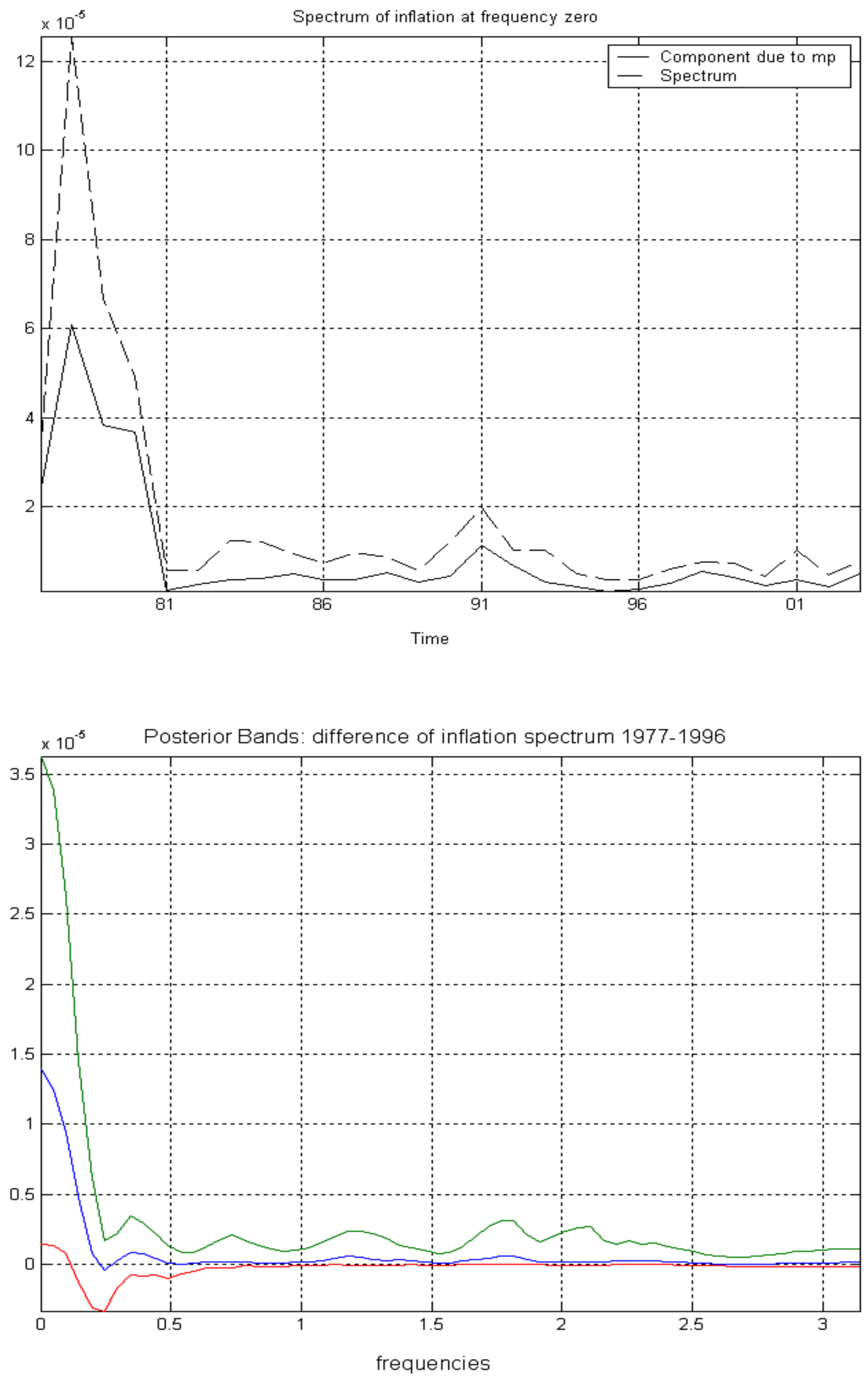


Figure 6: Inflation: persistence and spectrum.

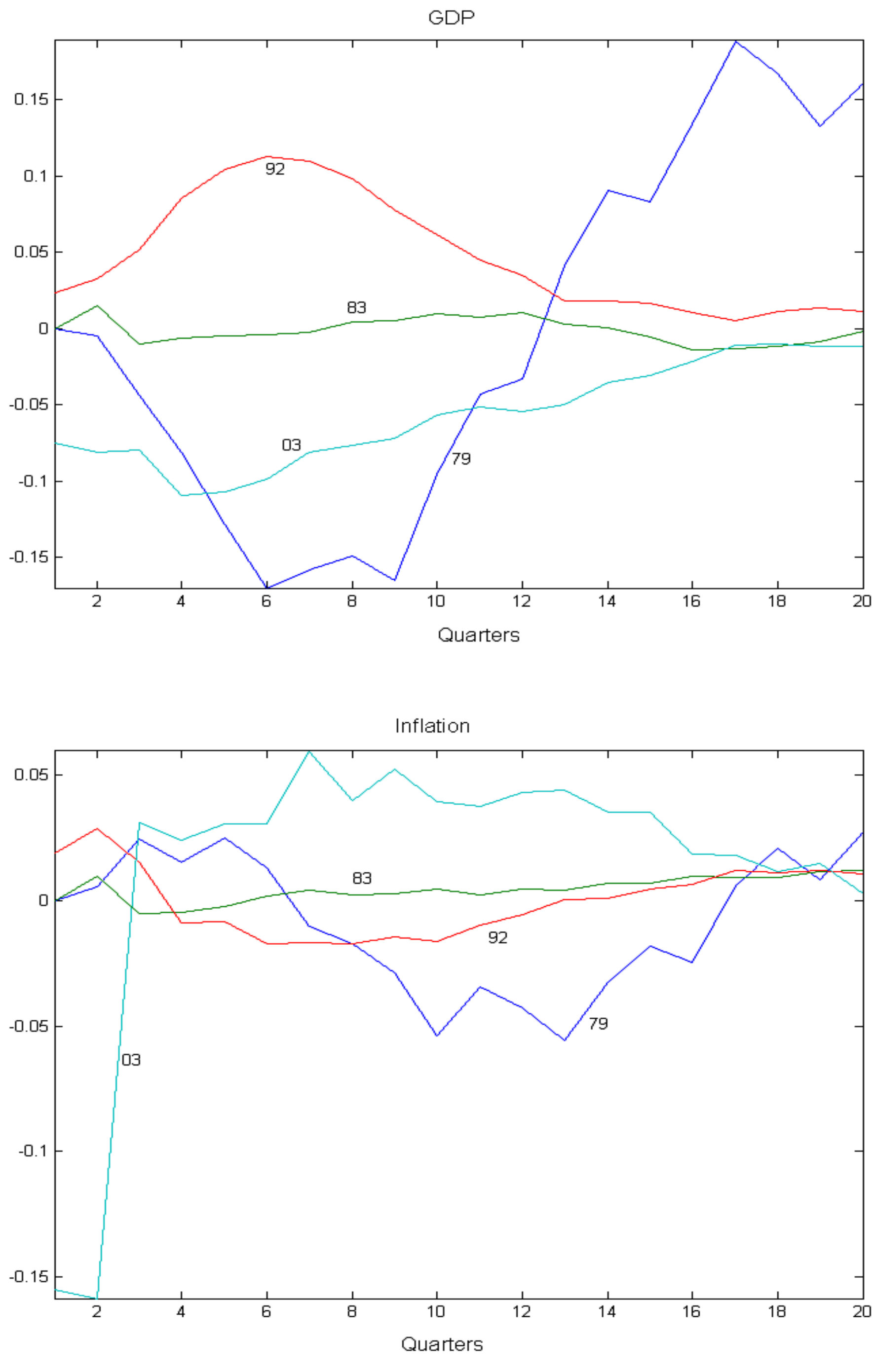


Figure 7: Impulse responses: more aggressive stance on inflation.

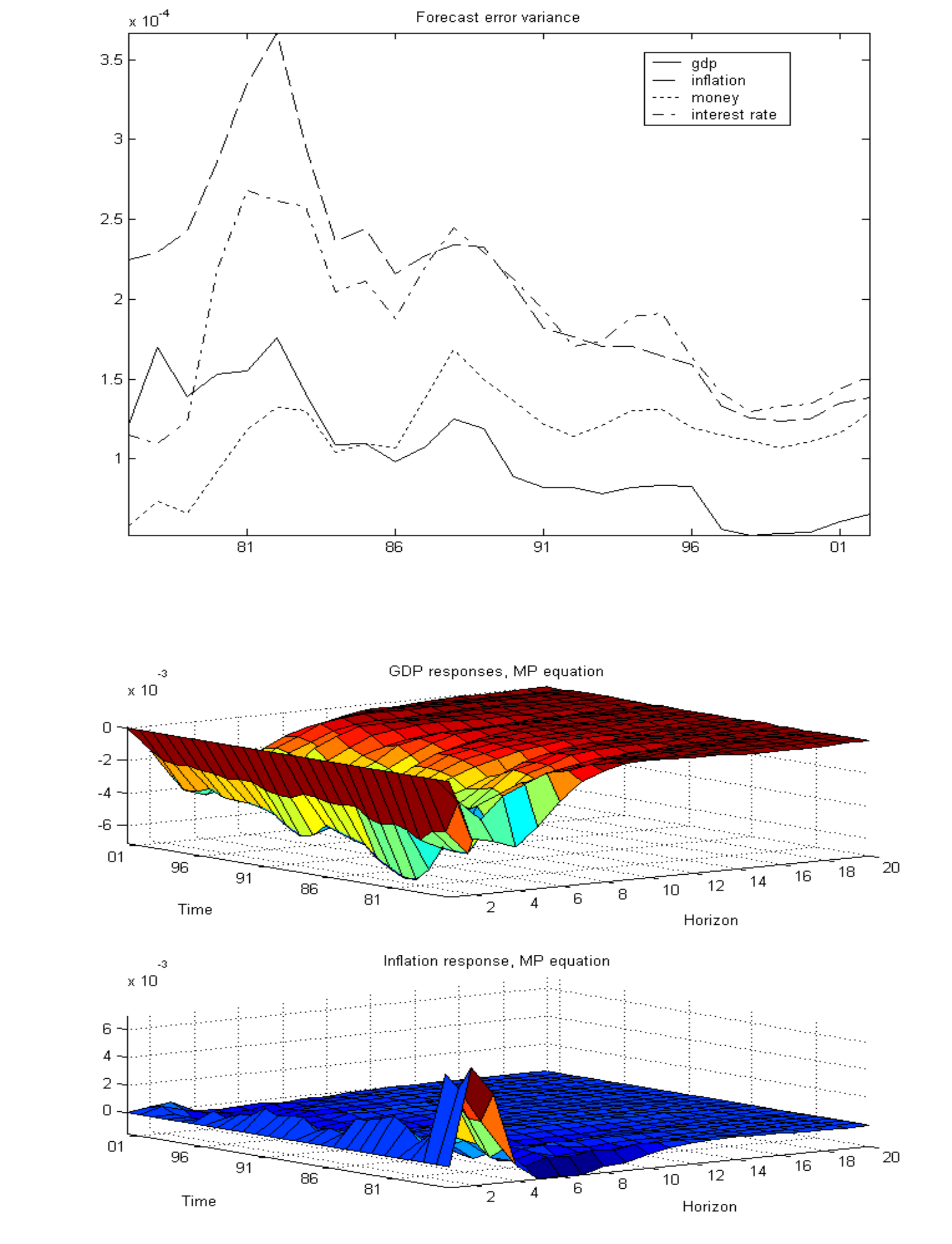


Figure 8: Forecast error variance and impulse response, Cholesky identification.

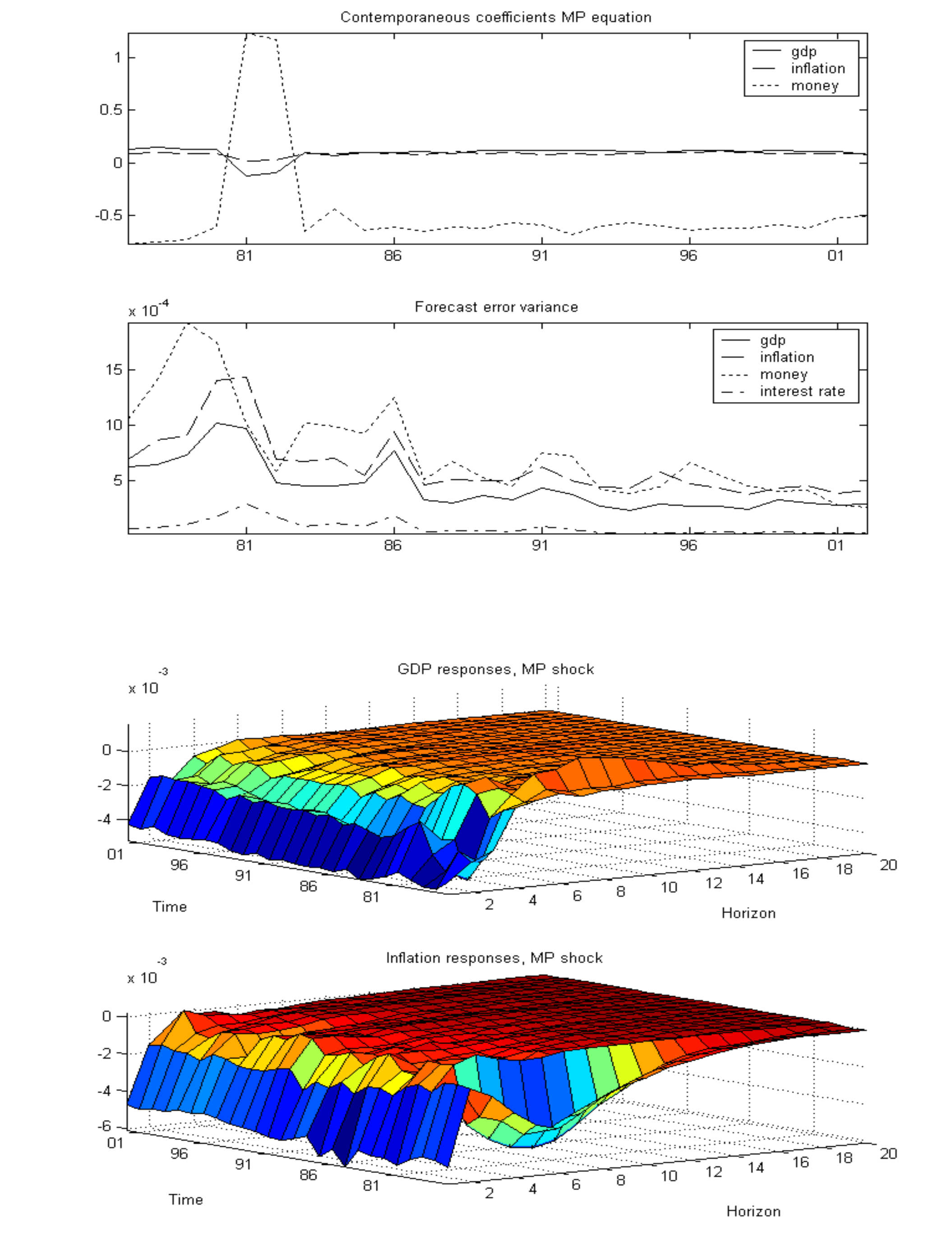


Figure 9: Contemporaneous coefficients, forecast error variance and impulse response functions, output and money in growth rates.

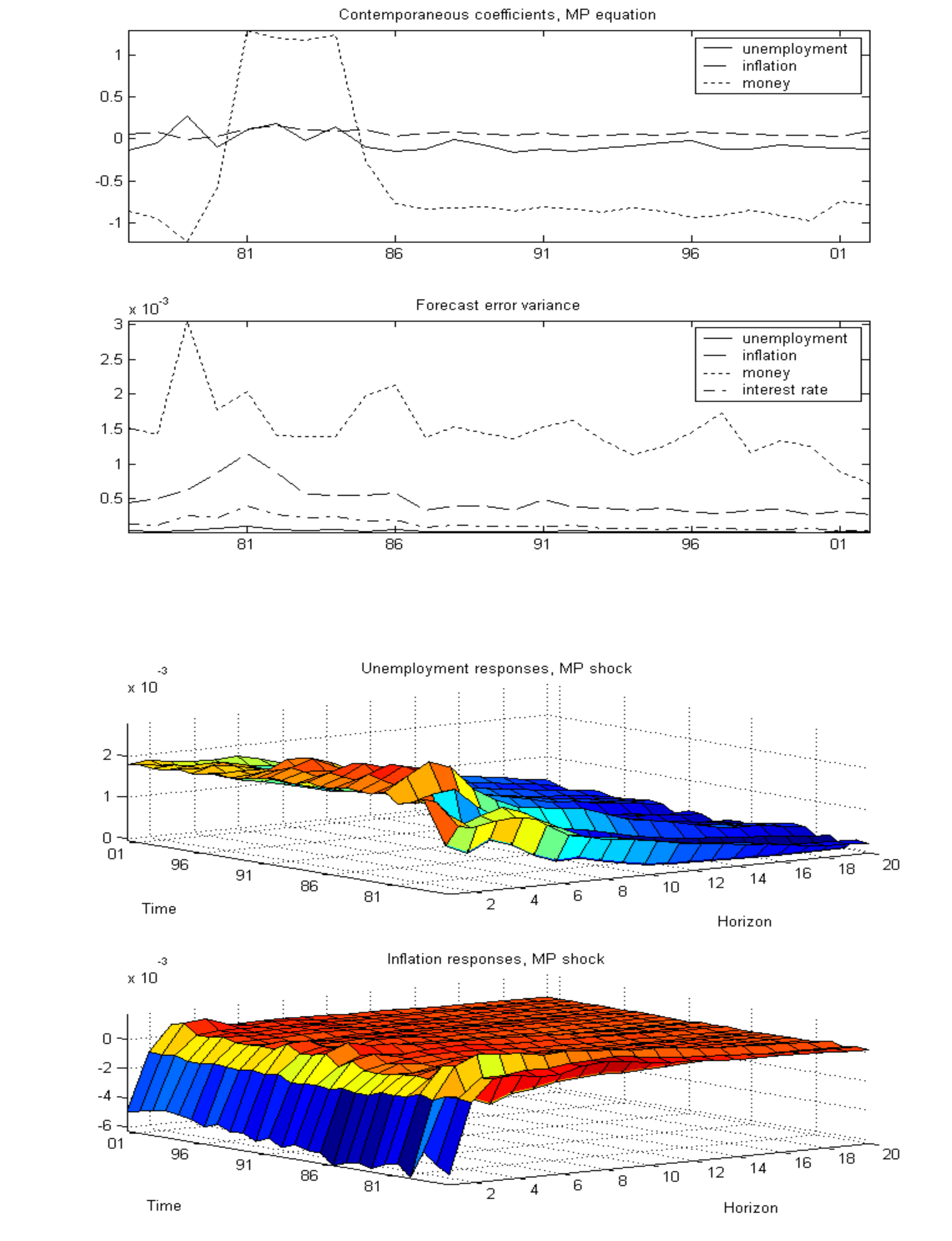


Figure 10: Contemporaneous coefficients, forecast error variance and impulse response functions, unemployment instead of output.

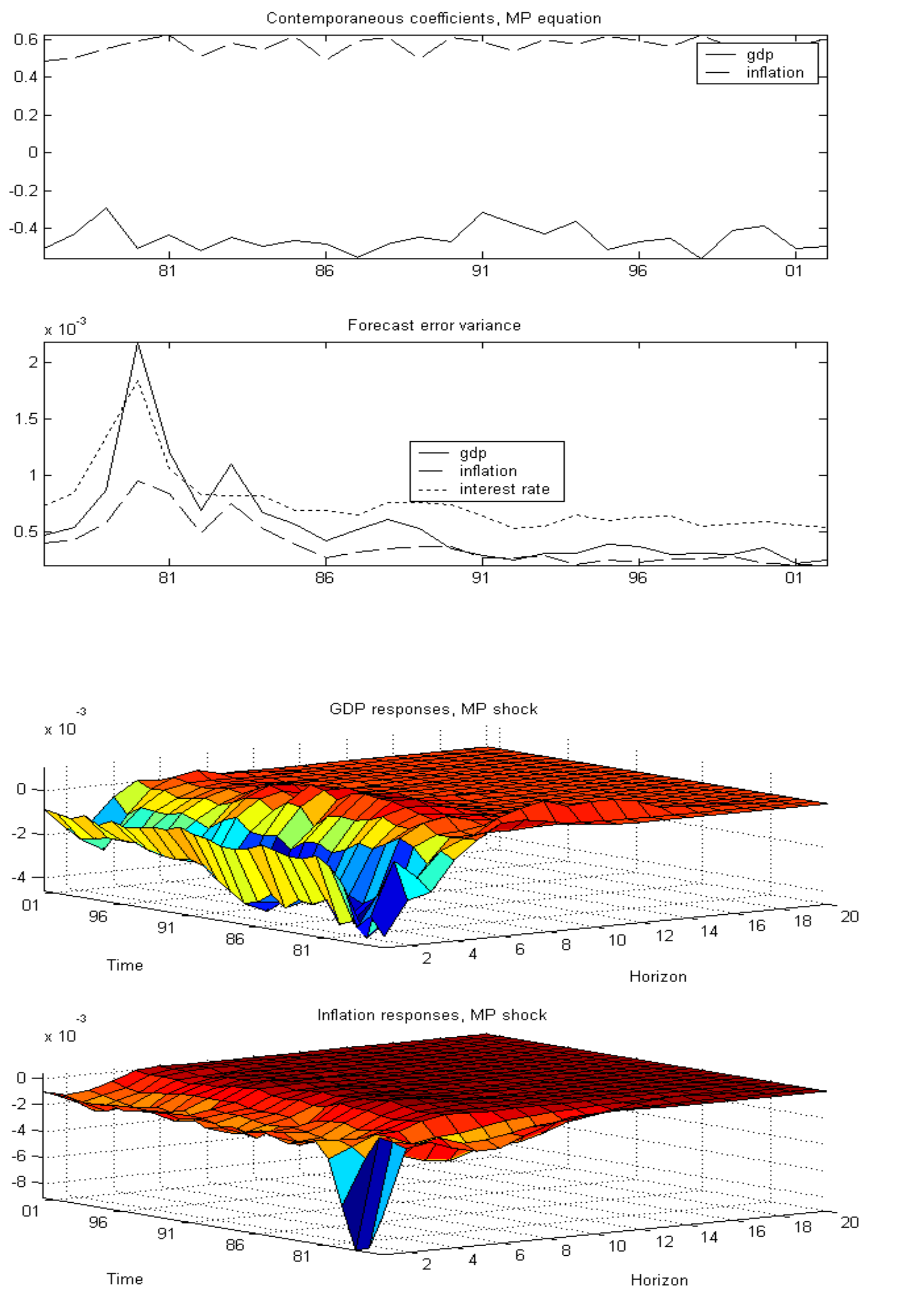


Figure 11: Contemporaneous coefficients, forecast error variance and impulse response functions, system without money.

Chapter 2

The structural dynamics of US output and inflation: what explains the changes?

2.1 Introduction

A growing amount of evidence suggests that the US economy has fundamentally changed over the last couple of decades. For example, Blanchard and Simon (2000), McConnell and Perez Quiros (2001), Sargent and Cogley (2001) and Stock and Watson (2003) have reported a marked decline in the volatility of real activity and inflation since the early 1980s and a reduction in the persistence of inflation over time. What causes these changes? The recent literature has paid particular attention to changes in policymakers' preferences. For example, Clarida, Gali and Gertler (2000), Cogley and Sargent (2001) and (2005), Boivin and Giannoni (2002), and Lubik and Schorfheide (2004) have argued that monetary policy was "loose" in fighting inflation in the 1970s but became more aggressive since the early 1980s. Leeper and Zha (2003), Sims and Zha (2004), Primiceri (2005), and Canova and Gambetti (2004) are critical of this view since they estimate a stable policy rule and find the transmission of policy shocks roughly unchanged over time.

There has been a resurgence of interest in the last few years in analyzing the dynamics induced by technology shocks, following the work of Gali (1999), Christiano, Eichenbaum and Vigfusson (2003), Uhlig (2004), Dedola and Neri (2004), Francis and Ramey (2005) and others. However, to the best of our knowledge, the link between structural changes and the way technology shocks are transmitted to the economy has not been made. This is a bit surprising given that the increase in productivity of the 1990s was to a large extent unexpected (see e.g. Gordon, 2003) and that it may have produced changes in the way firms and consumers responded to economic disturbances. Similarly, the way fiscal policy was conducted in the 1970s and the early 1980s differed considerably from the way it was conducted in the 1990s. For example, large deficits in the 1980s were turned into surpluses in the 1990s. Furthermore, benign neglect about the size of the public debt has been substituted by a keen awareness of the wealth effects and of the inflation consequences that large debts may have. Studying whether the dynamics induced by technology and fiscal shocks have changed over time may help to clarify which structural feature of the US economy has changed and whether variations in output and inflation dynamics reflect changes in the propagation mechanism or in the variance of the exogenous shocks.

This paper provides evidence on these issues investigating the contribution of technology, government expenditure and monetary disturbances to the changes in the volatility and in the persistence of US output and inflation. We employ a time varying coefficients VAR model (TVC-VAR), where coefficients evolve according to a nonlinear transition equation, which puts zero probability on paths associated with explosive roots, and the variance of the forecast errors is allowed to vary over time. As in Cogley and Sargent (2001,2005) we use Markov Chain Monte Carlo (MCMC) methods to estimate the posterior distributions of the quantities of interest. However, contrary to these authors, and as in Canova and Gambetti (2004), we analyze the time evolution of structural relationships. To do so, we identify structural disturbances which are allowed to have different features at different points in time. In particular, we permit time variations in the characteristics of the shocks, in their variance and in their transmission to the economy.

Our analysis is recursive. That is, we can construct posterior distributions for structural statistics, using the information available at that point in time. This complicates the computations significantly - a MCMC routine is needed at each t where the analysis is conducted - but provides a sharper picture of the time evolution of structural relationships. With this strategy our analysis becomes comparable with the one of Canova (2004), where a small scale DSGE model featuring three types of shocks with similar economic interpretations, is recursively estimated with MCMC methods. We identify structural disturbances using robust sign restrictions obtained from a DSGE model featuring monopolistic competitive firms, distorting taxes, utility yielding government expenditure, and rules describing fiscal and monetary policy actions, which encompasses RBC style and New-Keynesian style models as special cases. We construct robust restrictions allowing the parameters to vary within a range which is consistent with statistical evidence and economic considerations. We focus on sign restrictions for several reasons. First, magnitude restrictions typically depend on the parameterization while the sign restrictions we employ are less prone to such problem. Second, our model fails to deliver the full set of zero restrictions one would need to identify the three shocks with more conventional approaches. Third, the link between the theory and the empirical analysis is more direct, making the analysis transparent and inference stand on more solid ground.

Because time variations in the coefficients induce important non-linearities, standard response analysis is inappropriate. For example, since at each t the coefficient vector is perturbed by a structural shock, assuming that between $t+1$ and $t+k$ no shocks other than the disturbance under consideration hit the system may give misleading conclusions. To trace out the evolution of the economy when perturbed by structural shocks, we define impulse responses as the difference between two conditional expectations, differing in the arguments of their conditioning sets. Such a definition reduces to the standard one when coefficients are constant, allows us to condition on the history of the data and of the parameters, and permits the size and the sign of certain shocks to matter for the dynamics of the model (see e.g. Canova and Gambetti, 2004).

Our results are as follows. First, while there is evidence of structural variations in both the volatility of output and inflation and in the persistence of inflation, our posterior analysis fails to detect significant changes because of large posterior standard errors. Second, the three structural shocks we identify explain between 50 and 65 percent of the variability of output and inflation on average across frequencies for every date in the sample: technology shocks account for the largest portion of output variability at frequency zero and, on average, across frequencies, while real demand and monetary shocks account for the bulk of inflation variability at frequency zero and, on average, across frequencies. Variations in inflation persistence are due to a decline in the relative contribution of real demand and technology shocks while changes in output and inflation volatility are accounted for by all three shocks, with the contribution of technology shocks showing the largest time variations. Third, there are important variations in the transmission of technology shocks and significant changes in the variances of technology and monetary policy shocks. Finally, technology shocks always imply positive contemporaneous comovements of hours and productivity but the correlation turns negative after a few lags.

In sum, consistent with McConnell and Perez Quiros (2001) and Gordon (2003), our analysis attributes to variations in the magnitude and the transmission of technology shocks an important role in explaining changes in output volatility. It also suggests that variations in the magnitude of both technology and monetary shocks and the transmission of technology shocks are important in explaining changes in the volatility and in the persistence of inflation. Therefore, it complements those of Sims and Zha (2004), Primiceri (2005) and Gambetti and Canova (2004), who only examined the role of monetary policy shocks.

The rest of the paper is organized as follows. The next section describes the empirical model. Section 3 presents a DSGE model which produces the restrictions used to identify structural shocks. Section 4 briefly deals with estimation - all technical details are confined to the appendix. Section 5 presents the results and section 6 concludes.

2.2 The empirical model

Let y_t be a 5×1 vector of time series including real output, hours, inflation and the federal funds rate and M1 with the representation

$$y_t = A_{0,t} + A_{1,t}t + A_{2,t}y_{t-1} + A_{3,t}y_{t-2} + ... + A_{p+1,t}y_{t-p} + \varepsilon_t \tag{2.1}$$

where $A_{0,t}, A_{1,t}$ are a 5×1 vectors; $A_{i,t}$, are 5×5 matrices, $i = 2, ..., p+1$, and ε_t is a 5×1 Gaussian white noise process with zero mean and covariance Σ_t. Letting $A_t = [A_{0,t}, A_{1,t}, A_{2,t}...A_{p+1,t}]$, $x_t' = [1_5, 1_5 * t, y_{t-1}'...y_{t-p}']$, where 1_5 is a row vector of ones of length 5, $vec(\cdot)$ denotes the stacking column operator and $\theta_t = vec(A_t')$, rewrite (1) as

$$y_t = X_t'\theta_t + \varepsilon_t \tag{2.2}$$

where $X_t' = (I_5 \bigotimes x_t')$ is a $5 \times (5p+2)5$ matrix, I_5 is a 5×5 identity matrix, and θ_t is a $(5p+2)5 \times 1$ vector. We assume that θ_t evolves according to

$$p(\theta_t|\theta_{t-1}, \Omega_t) \propto \mathcal{I}(\theta_t) f(\theta_t|\theta_{t-1}, \Omega_t) \tag{2.3}$$

where $\mathcal{I}(\theta_t)$ discards explosive paths of y_t and $f(\theta_t|\theta_{t-1}, \Omega_t)$ is represented as

$$\theta_t = \theta_{t-1} + u_t \tag{2.4}$$

where u_t is a $(5p+2)5 \times 1$ Gaussian white noise process with zero mean and covariance Ω_t. We select this specification because more general AR and/or mean reverting structures were always discarded in out-of-sample model selection exercises. We assume that $corr(u_t, \varepsilon_t) = 0$, and that Ω_t is diagonal. The first assumption implies conditional linear responses to changes in ε_t, while the second is made for computational ease - structural coefficients are allowed to change in a correlated fashion. Our model implies that forecast errors are non-normal and heteroschedastic even when $\Sigma_t = \Sigma$ and $\Omega_t = \Omega$. In fact, substituting (4) into (2) we have that $y_t = X_t'\theta_{t-1} + v_t$, where $v_t = \varepsilon_t + X_t' u_t$. Such a structure is appealing since whatever alters coefficients also imparts heteroschedastic movements to the variance of the forecasts errors. Since also Ω_t is allowed to vary over time, the model permits various forms of stochastic volatility in the forecast errors of the model (see Sims and Zha, 2004, and Cogley and Sargent, 2005, for alternative specifications).

Let S_t be a square root of Σ_t, i.e., $\Sigma_t = S_t S_t'$, let H_t be an orthonormal matrix, independent of ε_t, such that $H_t H_t' = I$ and set $J_t^{-1} = H_t' S_t^{-1}$. J_t is a particular decomposition of Σ_t which transforms (2) in two ways: it produces uncorrelated innovations (via the matrix S_t) and it gives a structural interpretation to the equations of the system (via the matrix H_t). Premultiplying y_t by J_t^{-1} we obtain

$$J_t^{-1} y_t = J_t^{-1} A_{0,t} + J_t^{-1} A_{1,t} t + \sum_j J_t^{-1} A_{j+1,t} y_{t-j} + e_t \tag{2.5}$$

where $e_t = J_t^{-1}\varepsilon_t$ satisfies: $E(e_t) = 0$, $E(e_t e_t') = I_5$. Equation (2.5) represents the class of "structural" representations of interest: for example, a Cholesky system is obtained choosing $S_t = S$ to be lower triangular matrix and $H_t = I_5$, and more general patterns, with non-recursive zero restrictions, result choosing $S_t = S$ to be non-triangular and $H_t = I_5$. In this paper, since S_t is an arbitrary square root matrix, identifying structural shocks is equivalent to choosing H_t. We select it so that the sign of the responses at $t+k, k = 1, 2, \ldots, K_1,\ K_1$ fixed, matches the robust model-based sign restrictions presented in the next section. We choose sign restrictions to identify structural shocks for three reasons. First, magnitude restrictions typically depend on the parameterization of the model while the sign restrictions we employ are less prone to such problem. Second, our model fails to deliver the full set of zero restrictions one would need to identify the three shocks of interest with more conventional approaches. Third, as it would be clear from the next section, the link between the theory and the empirical analysis is more direct.

Letting $C_t = [J_t^{-1} A_{0t}, \ldots, J_t^{-1} A_{p+1t}]$, and $\gamma_t = vec(C_t')$, (2.5) can be written as

$$J_t^{-1} y_t = X_t' \gamma_t + e_t \tag{2.6}$$

As in fixed coefficient VARs there is a mapping between the structural coefficients γ_t and the reduced form coefficients θ_t since $\gamma_t = (J_t^{-1} \bigotimes I_{5p)})\theta_t$. Whenever $\mathcal{I}(\theta_t) = 1$, we have

$$\gamma_t = (J_t^{-1} \bigotimes I_{np})(J_t^{-1} \bigotimes I_{np})^{-1}\gamma_{t-1} + \eta_t \tag{2.7}$$

where $\eta_t = (J_t^{-1} \bigotimes I_{5p})u_t$ satisfies $E(\eta_t) = 0$, $E(\eta_t\eta_t') = E((J_t^{-1} \bigotimes I_{5p})u_t u_t'(J_t^{-1} \bigotimes I_{5p})')$. Since each element of γ_t depends on several u_{it} via the matrix J_t, shocks to structural parameters are no longer independent. Note that the (6)-(7) contain two types of structural shocks: VAR disturbances, e_t, and structural parameters disturbances, η_t. This latter type of shocks will not be dealt with here and is analyzed in details in Canova and Gambetti (2004).

To study the transmission of disturbances in a fixed coefficient model one typically employs impulse responses. Impulse responses are generally computed as the difference between two realizations of $y_{i,t+k}$ which are identical up to time t, but one assumes that between t and $t+k$ a shock in the j-th component of e_{t+k} occurs only at time t, and the other that no shocks take place at all dates between t and $t+k, k = 1, 2, \ldots,$.

In a TVC model, responses computed this way disregard the fact that structural coefficients may also change. Hence, meaningful response functions ought to measure the effects of a shock in e_{jt} on y_{it+k}, allowing future shocks to the structural coefficients to be non-zero. Let us rewrite the model in companion form

$$\mathbf{y}_t = \mu_t + \mathbf{A}_t\mathbf{y}_{t-1} + \epsilon_t$$

where $\mathbf{y}_t = [y_t' ... y_{t-p+1}']'$, $\epsilon_t = [\varepsilon_t' 0 ... 0]'$ and $\mu_t = [A_{0,t}' 0 ... 0]'$ are $np \times 1$ vectors and

$$\mathbf{A}_t = \begin{pmatrix} & A_t \\ I_{n(p-1)} & 0_{n(p-1),n} \end{pmatrix}$$

where $A_t = [A_{1,t} ... A_{p,t}]$ is an $n \times np$ matrix, $I_{n(p-1)}$ is an $n(p-1) \times n(p-1)$ identity matrix and $0_{n(p-1),n}$ is a $n(p-1) \times n$ matrix of zeros. Iterating k period forward and omitting for simplicity the constant term, we obtain

$$\mathbf{y}_{t+k} = \mathbf{A}_{t+k} ... \mathbf{A}_t\mathbf{y}_{t-1} + \mathbf{A}_{t+k} ... \mathbf{A}_{t+1}\epsilon_t + \mathbf{A}_{t+k} ... \mathbf{A}_{t+2}\epsilon_{t+1} + ... + \mathbf{A}_{t+k}\epsilon_{t+k-1} + \epsilon_{t+k}$$

Let $\mathcal{S}_{i,j}(M)$ be a selection function, a function which selects the first i rows and j columns of the matrix M. Taking as a benchmark case the case of no-shock occurrence, and assuming that coefficients and shocks ε_t are uncorrelated, the matrix of dynamic multiplier $\mathcal{S}_{n,n}(\mathbf{A}_{t+k} ... \mathbf{A}_{t+1})$ describes the effects of ε_t on y_{t+k}, while the effects associated to structural shocks can be derived from the relation $\varepsilon_t = J_t e_t$ and are given by $\mathcal{S}_{n,n}(\mathbf{A}_{t+k} ... \mathbf{A}_{t+1})J_t$. Therefore impulse response functions to a shock e_t at horizon k are given by

$$IR(t, k) = \Psi_{t,k} J_t$$

where $\Psi_{t,k} = \mathcal{S}_{n,n}(\mathbf{A}_{t+k} ... \mathbf{A}_{t+1})$. Thus for each $t = 1, ..., T$ we have a path of impulse response defined by the sequence $\{\Psi_{t,k} J_t\}_{k=1}^{\tau}$.

2.3 The identification restrictions

The restrictions we use to identify the shocks come from a general equilibrium model that encompasses flexible price RBC and New-Keynesian sticky price setups as special cases. The restrictions we consider are robust, in the sense that there are generated by a wide range of parameterizations and for alternative specifications of the policy rules. We use a subset of the large number of model's predictions and, as in Canova (2002), we focus only on qualitative (sign) restrictions, as opposed to quantitative (magnitude) restrictions, to identify shocks. While it is relatively easy to find robust sign restrictions, magnitude restrictions are typically fragile and depend on the exact parameterization of the model.

The economy is a simplified version of the one in Pappa (2004). It features a representative household, a continuum of firms, a monetary authority and a fiscal authority which consumes goods that may yield utility for the households.

2.3.1 Households

Households derive utility from private, C_t^p, and public consumption, C_t^g, leisure, $1 - N_t$ and real balances $\frac{M_t}{p_t}$. They maximize $E_0 \sum_{t=0}^{\infty} \beta^t \frac{[(aC_t^{p\frac{\varsigma-1}{\varsigma}} + (1-a)C_t^{g\frac{\varsigma-1}{\varsigma}})^{\frac{\theta_n \varsigma}{\varsigma-1}}(1-N_t)^{1-\theta_n}]^{1-\sigma}-1}{1-\sigma} + \frac{1}{1-\theta_M}(\frac{M_t}{p_t})^{1-\theta_M}$ choosing sequences for private consumption, hours, capital to be used next period K_{t+1}, nominal state-contingent bonds, D_{t+1}, nominal balances and government bonds, B_{t+1}. Here $0 < \beta < 1$ is the subjective discount factor and $\sigma > 0$ a risk aversion parameter. Public consumption is exogenous from the point of view of households. The degree of substitutability between private and public consumption is regulated by $0 < \varsigma \leq \infty$; $0 < a \leq 1$ controls the share of public and private goods in consumption: when $a = 1$, public consumption is useless from private agents' point of view. $\vartheta_M > 0$ regulates the elasticity of money demand. Household time is normalized to one at each t. We assume $C_t^p = \left[\int_0^1 C_{it}^p(i)^{\frac{\lambda-1}{\lambda}} di\right]^{\frac{\lambda}{\lambda-1}}$; $C_t^g = \left[\int_0^1 C_{it}^g(i)^{\frac{\lambda-1}{\lambda}} di\right]^{\frac{\lambda}{\lambda-1}}$ and $\lambda > 0$ measures the elasticity of substitution between types of goods. The sequence of budget constraints is:

$$\begin{aligned} P_t(C_t^p + I_t) + E_t\{Q_{t,t+1}D_{t+1}\} + R_t^{-1}B_{t+1} + M_{t+1} &\leq \\ (1-\tau^l)P_t w_t N_t + [r_t - \tau^k(r_t - \delta)]P_t K_t + D_t + B_t - T_t P_t + M_t + \Xi_t & \end{aligned} \tag{2.8}$$

where $(1-\tau^l)P_t w_t N_t$, is the after tax nominal labor income, $[r_t - \tau^k(r_t - \delta)]P_t K_t$ is the after tax nominal capital income (allowing for depreciation), Ξ_t are nominal profits distributed by firms (which are owned by consumers), and $T_t P_t$ are nominal lump-sum taxes. We assume complete private financial markets: D_{t+1} are holdings of state-contingent nominal bonds, paying one unit of currency in period $t+1$ if a specified state is realized, and $Q_{t,t+1}$ is their period-t price. Finally, R_t is the gross return on a one period government bond B_t. With the disposable income the household purchases consumption goods, C_t^p, capital goods, I_t, and assets. Capital accumulates according to:

$$K_{t+1} = I_t + (1-\delta)K_t - \nu\left(\frac{K_{t+1}}{K_t}\right)K_t \tag{2.9}$$

where $0 < \delta < 1$ is a constant depreciation rate, $\nu\left(\frac{K_{t+1}}{K_t}\right) = \frac{b}{2}\left[\frac{K_{t+1}-(1-\delta)K_t}{K_t} - \delta\right]^2$ and $b \geq 0$ determines the size of the adjustment costs. Since households own and supply capital to the firms, they bear the adjustment costs.

2.3.2 Firms

A firm j produces output according to the constant returns to scale production function:

$$Y_{tj} = (Z_t N_{tj})^{1-\alpha}(K_{tj})^{\alpha} \tag{2.10}$$

where K_{tj} and N_{tj} are capital and labor inputs and Z_t is an aggregate technology shock.

We assume perfectly competitive in the input markets[1]: firms minimize costs choosing private inputs and taking wages and the rental rate of capital as given. Since firms are identical, they all choose the same amount of inputs and cost minimization implies

$$\frac{K_{tj}}{N_{tj}} = \frac{\alpha}{(1-\alpha)}\frac{w_t}{r_t} \quad \forall j \tag{2.11}$$

Equation (13) and the production function imply that (nominal) marginal costs are:

$$MC_t = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} Z_t^{\alpha-1} w_t^{1-\alpha} r_t^{\alpha} P_t \tag{2.12}$$

In the goods market firms are monopolistic competitors. The strategy used to set prices depends on whether they are sticky or flexible. In the former case, each domestic producer is allowed to reset her price with a constant probability, $(1-\gamma)$, independently of the time elapsed since the last adjustment. When a producer receives a signal, she chooses her new price, P^*_{tj}, to maximize $E_t \sum\limits_{k=0}^{\infty} \gamma^k Q_{t+k+1,t+k}(P^*_{tj} - MC_{t+kj})Y_{t+kj}$ subject to the demand curve $Y_{t+kj} = (\frac{P^*_{tj}}{P_{t+k}})^{-\lambda} Y_{t+k}$. Optimization implies

$$\sum_{k=0}^{\infty} \gamma^k E_t\{Q_{t+k+1,t+k} Y_{t+kj}(P^*_{tj} - \frac{\lambda}{\lambda-1}\frac{1}{1-\tau^{\lambda}} MC_{t+k})\} = 0 \tag{2.13}$$

where $\tau^{\lambda} = -(\lambda-1)^{-1}$ is a subsidy that, in equilibrium, eliminates the monopolistic competitive distortion . Given the pricing assumption, the aggregate price index is

$$P_t = [\gamma P_{t-1}^{1-\lambda} + (1-\gamma) P_t^{*1-\lambda}]^{\frac{1}{1-\lambda}} \tag{2.14}$$

When firms can reset the price at each t, prices become flexible and:

$$P_t = \frac{\lambda}{\lambda-1}\frac{1}{1-\tau^{\lambda}} MC_t, \quad \forall t \tag{2.15}$$

[1]The robust restrictions we emphasize below are independent of the presence of frictions in labor markets such as sticky wages or labor unions.

2.3.3 Fiscal and Monetary Policy

Government's income consists of seigniorage, tax revenues minus subsidies to the firms and proceeds from new debt issue. The government budget constraint is:

$$P_t C_t^g + \tau^\lambda P_t Y_t - \tau^l w_t P_t N_t - \tau^k (r_t - \delta^p) P_t K_t - P_t T_t + B_t + M_t = R_t^{-1} B_{t+1} + M_{t+1} \quad (2.16)$$

We treat tax rates on labor and capital income parametrically; assume that the government takes market prices, hours and capital as given, and that B_t endogenously adjusts to ensure that the budget constraint is satisfied. In order to guarantee a non-explosive solution for debt (see e.g., Leeper 1991), we assume a tax rule of the form:

$$\frac{T_t}{T^{ss}} = [(\frac{B_t}{Y_t})/(\frac{B^{ss}}{Y^{ss}}))]^{\phi_b} \quad (2.17)$$

where the superscript ss indicates steady states. Finally, there is an independent monetary authority which sets the nominal interest rate according to the rule:

$$\frac{R_t}{R^{ss}} = (\frac{\pi_t}{\pi^{ss}})^{\phi_\pi} u_t \quad (2.18)$$

where π_t is current inflation, and u_t is a monetary policy shock. Given this rule, the authority stands ready to supply nominal balances that the private sector demands.

There are two types of aggregate constraints. First, labor supply must equate labor employed by the private firms. Second, aggregate production must equate the demand for goods from the private and public sector, that is $Y_t = C_t^p + I_t + C_t^g$.

We assume that the three exogenous processes $S_t = [Z_t, C_t^g, u_t]'$, evolve according to

$$\log(S_t) = (I_4 - \varrho)\log(\overline{S}) + \varrho \log(S_{t-1}) + V_t \quad (2.19)$$

where I_3 is a 3×3 identity matrix, ϱ is a 3×3 diagonal matrix with all the roots less than one in modulus, $\overline{S}$ is the mean of S and the 3×1 innovation vector V_t is a zero-mean, white noise process [2]. Let $\mathcal{A} = (\mathcal{A}^1, \mathcal{A}^2)$ represent the vector of parameters of the model.

2.3.4 Deriving the identification restrictions

Figure 1 presents responses to impulses in the three shocks when the parameters are allowed to vary within the ranges listed in table 1. To be precise, each box reports 68% of the 10000 paths generated randomly drawing $\mathcal{A}_j,\ j = 1, 2, \ldots$ independently from a uniform distribution covering the range appearing in table 1. The first column of figure 1 represents responses to technology shocks, the second responses to government expenditure shocks, and the third responses to monetary shocks. Since our VAR includes output, hours, inflation, nominal rate and money, we only plot the responses of these variables.

Few words regarding the assumed ranges are in order. First, we decompose the parameter vector in two components: $\mathcal{A}^1$ includes the parameters held fixed to a particular value

[2] A previous version of the paper allowed also for government investment and government employment disturbances. Since the sign restrictions we emphasize are the same for shocks to government consumption, government investment and government employment, the last two type of shocks are currently omitted.

because of steady state considerations, while in $\mathcal{A}^2$ are the parameters which are allowed to vary. In $\mathcal{A}^1$ we have the discount factor, set so that the annual real interest rate equals 4%, and the debt ratio,$(\frac{B}{Y})^{SS}$, which is selected to match the average US debt to GDP ratio.

The intervals for the other parameters are centered around standard values and the ranges are selected to contain existing estimates, values assumed in calibration exercises or chosen to satisfy theoretical considerations. For example, the range for the *risk aversion parameter* σ includes the values typically used in RBC (σ from 0.5 to 2), and New-Keynesian models (σ from 1 to 6). Theoretical considerations suggest that the share of public goods in total consumption, $1 - a$, should be low (since the private wealth effects following fiscal shocks crucially depend on this parameter) and the chosen range reflect this concern. The range for ς allows for both complementarities and substitutabilities between private and public goods. The parameters θ_n, θ_M regulate the labor supply and money demand elasticity and the chosen ranges cover well the range of existing estimates. The ranges for the *capital share* in production, α, the *capital adjustment costs parameter*, b, and the *depreciation of capital*, δ include standard values. The ranges for labor and capital income tax parameters (τ^l, τ^k) cover the values of interest to policymakers and those for the expenditure ratio $(\frac{C^g}{Y})$ match the cross sectional range of values for US states. The range for the *degree of price stickiness* γ is wide and covers cases where prices are very sticky and almost flexible.

Finally, the *coefficient on inflation* ϕ_π, and the *coefficient on debt* ϕ_b in the policy rules control whether equilibria are determinate or not. The ranges we have selected imply that fiscal policy is passive and monetary policy is active, in the terminology of Leeper (1991); this insure determinacy of the equilibria and implies that our analysis neglects equilibria of the types considered in Lubik and Schorfheide (2004). Therefore, the interpretation of our monetary policy shocks is different from theirs. Considering active fiscal policy and passive monetary policy leave the qualitative features of the responses unchanged.

The model produces several robust sign implications in responses to the shocks. For example, a persistent technological disturbance increases output, decreases inflation, nominal rates and nominal balances and the sign of the response is independent of the horizon. Furthermore, when government consumption expenditure increase, output, hours, inflation, nominal interest rates and nominal balances all increase, while surprise decreases in the interest rate increase output, hours, inflation and nominal balances. Note, in particular, that these patterns obtain for a wide range of values of the elasticity of substitution between private and public goods, the strength of the reaction of interest rates and taxes to inflation and debt and the degree of price stickiness.

The identification restrictions used are summarized in table 2. Note that the dynamics of hours (and labor productivity) are unrestricted in all cases.

There are many ways of implementing sign restrictions. The results we present are obtained using an acceptance sampling scheme where draws that jointly satisfy the restrictions for all three shocks are kept and draws that do not are discarded. Tim Cogley pointed out to us that, since the bands in figure 1 do not insure that some parameter combinations may fail to satisfy the restrictions, an importance sampling scheme, which gives positive but different weights to different types of draws, could be more appropriate. In general, since identification restrictions make the prior for the reduced form parameters informative, one may want to analyze how sensitive are the conclusions we reach to these choices. We have tried few alternatives to implement an importance sampling scheme. First, we have weighted draws in proportion to the number of horizons at which restrictions are satisfied. Thus, if we impose restrictions at three horizons, we give weight $0.5/n_1$ to draws that satisfy restrictions at all horizons, weight $0.33/n_2$ to draws that satisfy restrictions at two horizons, and weight $0.17/n_3$ to draws that satisfy restrictions at one horizon, $n_1+n_2+n_3=n$, where n is the total number of draws. Second, we have weighted the draws satisfying all the restrictions by $0.68/n_1$ and draws which do not satisfy all the restrictions by $0.32/n_2$, $n_1+n_2=n$. The results we present are qualitatively independent of the scheme used to weight draws even though, quantitatively, some conclusions become more or less significant. An appendix available at www.econ.upf.edu/crei/people/canova, contains the results obtained with these alternatives weighting schemes.

Since the sign restrictions we use are robust to the horizon, we are free to choose how many responses we wish to restrict. However, there is an important trade-off to be considered, since the smaller is the number of restrictions, the larger is the number of draws consistent with the restrictions but, potentially, the weaker is the link between the model and the empirical analysis. As the number of restricted responses increases, we tight up the empirical analysis to the model more firmly, but the number of draws satisfying the restrictions may drop dramatically, making estimates of standard errors inaccurate. Since the relationship between number of restrictions and number of accepted draws is highly nonlinear, there is no straightforward way to optimize this trade-off. We present results obtained imposing restrictions at two horizons (0 and 1) since this choice seems to account for both concerns.

2.4 Estimation

The model (2.6)-(2.7) is estimated using Bayesian methods. That is, having specified prior distributions for all the parameters of interest, we use data up to t to compute posterior distributions of the structural parameters and of continuous functions of them. Since our sample goes from 1960:1 to 2003:2, we initially estimate the model for the period 1960:1-1970:2 and then reestimate it 33 times moving the terminal date by one year up to 2003:2

Our estimation approach proceeds in two steps. First, we characterize the (truncated) posterior distribution of the reduced form parameters. Second, given these posteriors and the identification restrictions, we construct posteriors for the structural parameters. Unfortunately, posterior distributions for the structural parameters are not available in a closed form. Therefore, MCMC methods are used to simulate posterior sequences consistent with the information available up to time t. Construction of the truncated posterior for reduced form parameters is relatively standard (see e.g. Cogley and Sargent 2005): it requires treating the parameters which are time varying as a block in a Gibbs sampler algorithm. Hence, at each t and in each Gibbs sampler cycle, one runs the Kalman filter and the simulation smoother, conditional on the draw of the other time invariant parameters, and discard paths for the coefficients generating explosive time series for the endogenous variables. These standard calculations are complicated in our setup by the fact that at each cycle, we need to obtain structural estimates of the time varying features of the model and that we need to run an MCMC routine for each t. This means that, in each cycle and each t, we also need to discard paths which do not satisfy the restrictions. Convergence was checked using a CUMSUM statistic. The results we present are based on 20,000 draws for each t - of these, after the non-explosive and the identification filters are used, about 200 are kept for inference. The methodology used to construct posterior distributions for the unknowns is contained, together with the prior specifications, in the appendix. The data comes from the FREDII data base of the Federal Reserve Bank of St. Louis and consists of GDP (GDPC1), GDP deflator inflation (ΔGDPDEF), the Federal funds rate (FEDFUNDS), hours of all persons in the non-farm business sector (HOANBS) and M1 (M1SL) - in parenthesis are the mnemonic used by FREDII. Four lags of each variable are used in the estimation.

2.5 The Results

2.5.1 The dynamics of structural volatility and persistence

Figure 2,3 and 4 present respectively the posterior estimates of the structural spectrum of output and inflation for dates ranging from 1970:1 to 2003:2, the median and the 68% central posterior bands for structural persistence (figure 3) and for structural volatility (figure 4) of output and inflation. Persistence is measured by the height of the spectrum at frequency zero; volatility by the value of the cumulative spectrum. Few interesting features are worth commenting upon. First, the structural spectrum of inflation is relatively stable over time, except for the zero frequency. Therefore, changes in structural inflation volatility are closely associated with changes in its structural persistence. The spectrum of output is also relatively stable over time at almost all frequencies. However, variations in structural volatility are primarily linked to structural variations occurring in the frequencies corresponding to three to five years cycles ($\omega \in [0.314, 0.52]$).

Second, inflation persistence shows a marked hump-shaped pattern: it displays a five fold increase in 1973-1974 and then again in 1977-1978, it drops dramatically after that date, and since 1982 the posterior distribution of inflation persistence displays marginal variations. The size of the drop is economically large: from its peak value, the median persistence in the 1990s is about 66 percent smaller. On the contrary, variations over time in the posterior distribution of output persistence are relatively small. Third, as expected from previous discussion, the dynamics of the posterior structural inflation volatility reflect those of the posterior of structural inflation persistence. On the other hand, the median of the posterior distribution of output volatility declines by roughly 25 percent from the beginning to the end of the sample.

Since posterior standard errors are large, even remarkable changes, like those displayed by the posterior median of inflation persistence, or of output volatility, turn out to be statistically insignificant. This outcome is consistent with the univariate, reduced form evidence presented by Pivetta and Reis (2004) and their classical statistical analysis and casts some doubts on inference derived analyzing the evolution of the mean (or the median) of these statistics. What features of our approach could be responsible for these large posterior standard errors? We singled out three possibilities. First, it could be that some parameter draws are more consistent than others with the sign restrictions. If these draws imply larger volatility in the coefficients, it could be that the estimated variance of the error in the law of motion of the coefficients is larger for accepted than rejected draws. This turns out not to be the case: the two variances are statistically indistinguishable. As a further check, we have computed posterior standard errors using a non-structural Cholesky decomposition and results are unchanged. Second, figure 2,3 and 4 are constructed using recursive analysis. Therefore, our estimates contains less information than those produced using estimates of the parameters obtained from the full sample. Although standard errors are reduced when full sample estimates are considered, the pattern of changes is qualitative unaltered. Third, since our spectral estimates are constructed allowing future coefficients to be random, it could be that this uncertainty is responsible for the large standard errors we report. We have therefore repeated the computations averaging out future shocks to the coefficients and found that posterior standard errors are smaller by about 25 percent. Hence, even changing a few features in our estimation approach, we would not be able to confidently claim that the observed changes in output and inflation persistence and volatility are statistically large.

In summary, three points can be made. First, while there is visual evidence of a decline in the median estimates of output and inflation volatility, the case for evolving volatility is considerably reduced once posterior standard errors are taken into account. This evidence should be contrasted with the one obtained with univariate, in-sample, reduced form methods, for example McConnell and Perez Quiros (2001) or Stock and Watson (2003), which overwhelmingly suggest the presence of a significant structural break in the variability of the two series. Second, when structural, recursive, multivariate analysis is used, the case for evolving posterior distributions of persistence measures is also far weaker. Consistent with the evidence contained in Cogley and Sargent (2001) and (2005), the posterior median of inflation persistence shows a declining trend but posterior uncertainty is sufficiently large to make time differences irrelevant. The posterior distribution of output persistence, on the other hand, displays neither breaks nor evolving dynamics. Third, leaving aside issues of statistical significance, the timing of the changes in persistences and volatilities does not appear to be synchronized. Hence, contrary to what it is commonly perceived, it is unlikely that a single explanation accounts for the observed variations in output and inflation dynamics.

2.5.2 What drives variations in structural volatility and in persistence?

Recall that our structural model has implications for three types of disturbances, roughly speaking, supply, real demand and monetary shocks. Therefore, we can identify at most three of the five structural shocks driving the VAR. This means that there will be a residual capturing unexplained variations in output and inflation volatility and persistence.

Before discussing in details sources of structural volatility and persistence, we would like to note that our identification exercise was quite successful. Our three structural shocks explain between 50 and 65 percent of the variability of output and inflation on average across frequencies for every date we consider. We believe this magnitude is remarkable, given our analysis has completely disregarded e.g. labor supply or investment specific shocks, which Chang and Schorfheide (2004) and Fisher (2003) have shown to be important in explaining output (and potentially inflation) fluctuations. On average over time, technology shocks explain 25% of inflation variability and about 15% of output variability, demand shocks about 17% of inflation variability and 25% of output variability, and monetary shocks about 14% of inflation variability and 12% of output variability. When we look at specific frequencies, we find that technology shocks exercise their largest impact on inflation variability at business cycle and high frequencies (mean contribution is about 28%) while their largest impact on output variability is at low frequencies (mean contribution is about 17%). On the other hand, the two demand shocks explain the largest portion of inflation variability at low frequencies (roughly, 20% for real demand shock and 17% for monetary shocks) and have their largest explanatory power for output fluctuations at business cycle frequencies (roughly, 25% for demand shocks and 17% for monetary shocks).

Given that the spectrum at frequency ω is uncorrelated with the spectrum at frequency ω', when ω and ω' are Fourier frequencies, it is easy to compute the relative contribution of each of the three structural shocks to changes in the volatility and in the persistence of output and inflation. In fact, disregarding the constant and the trend, the (time varying) structural MA representation is $y_{it} = \sum_{j=1}^{5} \mathcal{B}_{jt}(\ell) e_{jt}$ where e_{it} is orthogonal to $e_{i't}$, $i' \neq i, i = 1, \ldots, 5$. Since structural shocks are independent, the (local) spectrum of y_{it} at frequency ω can be written as $S_{y_i}(\omega(t) = \sum_{j=1}^{5} |\mathcal{B}_{jt}(\omega)|^2 S_{e_j}(\omega)(t)$. Therefore, the fraction of the persistence in y_{it} due to structural shock j is $S^j_{y_i}(\omega = 0)(t) = \frac{\mathcal{B}_{jt}(omega=0)|^2 S_{e_j}(\omega=0)(t)}{S_{y_i}(\omega=0)(t)}$ and the fraction of the volatility in y_{it} due to structural shock j is $\sum_\omega S^j_{y_i}(\omega)(t)$. Intuitively, these measures are comparable to variance decomposition shares. Variance decomposition shares inform us on the relative contribution of different shocks at various forecasting horizons. The measures we propose evaluate the contribution of structural shock j to the variability of y_{it} at either one frequency or at all frequencies.

We divide the discussion of the results into two parts. First, we examine the contribution of monetary policy shocks to the variations presented in figure 2,3 and 4. The large number of papers studying this issue and the consequent discussion that followed the original contribution of Clarida, Gali and Gertler (2000) justify our focus. Second, we assess the role of the two other shocks in accounting for the observed changes.

It is useful to recall that if the conventional wisdom is correct, the decline observed in the median of output and inflation volatility and inflation persistence should be largely explained by a decline in the contribution of monetary shocks to these statistics. Figure 5, which reports the median and the 68% posterior bands for the percentage of the persistence of output and inflation explained by the three shocks, and figure 6, which reports the same statistics for the volatility of output and inflation, tell a different story. For example, the share of inflation and output persistence attributable to monetary shocks displays an increasing trend and the median contribution at the end of the sample is about 30 percent larger than in the 1970s. Also, the contribution of monetary policy shocks to output and inflation volatility is roughly constant over time.

Several authors have attempted to relate changes in inflation persistence with changes in the stance of monetary policy (see e.g. Cogley and Sargent, 2001, or Benati, 2005), or to the way monetary shocks are transmitted to the economy (see e.g. Leeper and Zha, 2003, or Sims and Zha, 2004). Contrary to the views of many policymakers, our results suggest that monetary policy could not have been a major factor behind the observed declines in inflation persistence, and that other shocks may have played a larger role. Similarly, the claim that the increased stability observed in the US economy since the mid 1980s, is a result of a more conservative monetary policy actions appears to be in contrast with the empirical evidence we present: the decline in output and inflation volatility is only partially explained by monetary policy and other sources of disturbances appear to have contributed to the decline.

The percentage of the persistence of output and inflation explained by real demand and supply shocks fluctuates around a constant mean value. Hence, these two shocks are equally responsible for the decline in inflation persistence we have observed since 1980s. Interestingly, the peak in inflation persistence in the early 1970s is attributed by our identification scheme to technology shocks while the one in the mid-late 1970s is attributed to demand disturbances. Furthermore, it appears that the sluggishness in the changes in inflation persistence is due to a very slow change in the contribution of technology shocks.

The relative contribution of real demand shocks to output and inflation volatility is relatively stable over time suggesting that the decline in inflation volatility since the beginning of the 1980s is due to a proportional decline in the contribution of these shocks. Finally, the mean contribution of technology shocks to output volatility declines over time and the mean contribution to inflation volatility shows first a downward jump in the mid of the 1970s and then upward jump in the end of the 1970s.

In sum, while the decline in inflation persistence seems to be largely due to a decline in the contribution of real demand and technology shocks, the fall in output and inflation volatility is attributed by our identification scheme to all three structural shocks, with the contribution of technology shocks showing the largest variations over time.

2.5.3 Time Varying Transmission?

Since the relative contribution of a shock varies because its relative variance at frequency ω (i.e. $\frac{S_{e_j}(\omega)(t)}{S_{y_i}(\omega)(t)}$) changes, or because its transmission mechanism (i.e. $|\mathcal{B}_{jt}(\omega)|^2$) changes, we need to disentangle the two sources of variations to explain the somewhat surprising set of results we obtain. In Figure 9 we plot the median responses of output and inflation to the three structural shocks. Since we normalize the impulse to be the same in every period, the evolution of these responses over time gives us an idea of the changes in the transmission of shocks in isolation from the changes in the distribution of the shocks (i.e. we trace out time variations in $\mathcal{B}_{jt}$.

Few striking features of the figure are worth discussing. First, the pattern of responses to the three structural shocks is qualitatively similar over time. Second, there are quantitative changes in the magnitude of some responses. For example, the peak response of output to technology shocks changes location and size and the through response of inflation to demand shocks changes location over time. The most stable responses appear to be those to monetary shocks: the shape, the size and the location of output and inflation peak and through responses are very similar over time. Third, real demand shocks appear to produce the largest displacements of the two variables followed by technology and monetary shocks. Fourth, the largest relative changes in the transmission appear to be associated with output responses to technology shocks. For example, the magnitude of contemporaneous responses is 50% larger in the 1990s than in was in the 1970s.

Hence, while the qualitative features of the transmission of structural shocks are similar over time, changes in the quantitative features, involving the magnitude of the responses and, at times, the location of the peak/through are present. Interestingly, while responses to monetary disturbances appear to be similar over time, the transmission of technology disturbances shows important changes.

2.5.4 Time Varying volatility of the structural shocks?

To examine whether there have been significant changes in the relative distribution of the structural shocks hitting the economy, we plot the time profile of the estimated posterior median of their volatility in figure 10. Real demand shocks are those associated with the first structural equation (normalized on output), supply shocks are those associated with the second structural equation (normalized on inflation) and the monetary policy shocks are those associated with the third structural equation (normalized on the nominal rate).

Overall, the volatility of supply and of the monetary policy disturbances has declined over time. However, while the decline is smoother for the former, it is much more abrupt for the latter, where a drop of 15% in the late 1970s is evident. The volatility of demand shocks is higher on average than for the other two shocks and, except for late 1980s and the late 1990s, it is relatively similar across time. Interestingly, the decline in the volatility of technology and monetary policy shocks terminates by the early 1980s and since then no changes are detected.

The decline in the volatility of monetary policy shocks of the late 1970s appears to precede the one found by Sims and Zha (2004). However, differences can be reconciled if one takes into account different estimation techniques and the different ways in which these volatilities are computed (recursive vs. smoothed estimates). Several authors have argued that there is very little evidence that the monetary policy rule and the transmission of monetary policy shocks have changed over time. Instead, they have suggested that drops in the volatility of monetary policy shocks could be responsible for the fall in the variability of output and inflation. Our results are consistent with these view but also suggest that the contribution of technology shocks to the changes observed in the US economy is non-negligible. The sharp increase and rapid decline in the variability of reduced form output and inflation forecast errors observed at the end of the 1970s is due, in part, to variations in the distribution from which technology shocks are drawn.

2.5.5 The dynamics of hours and labor productivity

Although somewhat unrelated to the main scope of the paper, our estimated structural system allows us to also discuss a controversial issue which has been at the center of attention in the macroeconomic literature since work by Gali (1999), Christiano, et. al. (2003), Uhlig (2003), Dedola and Neri (2004) and others: the dynamics of hours and productivity in response to technology shocks. The empirical evidence on this issue is at best mixed, it appears that under some identification and some data transformations (in particular, identification via long run restrictions and variables in the VAR in growth rates) technology disturbances increase labor productivity and decrease hours while with other identifications and other data transformations (in particular, hours in log level and identification based on short or medium run restrictions) both labor productivity and hours increase.

The dynamics of hours and labor productivity are thought to provide important information about sources of business cycle dynamics. In fact, a negative response of hours to technology disturbances is considered by some to be inconsistent with RBC-flexible price based explanations of business cycles (a point disputed e.g. by Francis and Ramey, 2005). In a basic RBC model, in fact, technology shocks act as a supply shifter and therefore have positive effects on hours, output and productivity, unless they induce considerable wealth effects. On the other hand, in a basic sticky price model without capital, technology shocks act as labor demand shifters. Therefore, regardless of the nature of the technological disturbance, firms experience a decline in their marginal costs but, because price are sticky, aggregate demand increases less than proportionally than the increase in output making hours decline. These qualitative patterns are present in the model we have presented in section 3: when prices are flexible and the policy rules appropriately chosen, technology disturbances imply robust positive contemporaneous hours responses; when prices are sticky, the contemporaneous response of hours is mostly negative.

Our estimated structural model allows us to investigate two interesting questions related to this the issue. First, what are the dynamics of hours and labor productivity when sign restrictions derived from a general model are used to identify technology shocks? It is well known, at least since Faust and Leeper (1997), that long run restrictions are only weakly identifying the objects of interest and that they are vacuous in near-integrated systems, despite the fact that the time series pattern of integrated and near-integrated systems can hardly be distinguished with finite stretches of data. Since model based robust sign restrictions offer a viable alternative, void to a large extent of these problems, they can be used to sharpen our understanding of the effects of technology shocks in near-integrated systems. Second, is there any evidence that the responses of hours to technology shocks displays a time varying pattern? In other words, could it be that the contemporaneous response of hours changes sign as the sample changes?

Figure 11 indicates that the contemporaneous response of hours and productivity to technology shocks is positive at all dates. Interestingly, the response of hours is humped shaped, with the peak occurring after 2 or 3 quarters and this, combined with a smoothly declining output responses, implies that labor productivity becomes negative after some periods. While the results are consistent with a RBC-flexible price explanation of the propagation of technology shocks, one should also stress that the technology shocks we recover are not necessarily permanent and that permanent shocks in the model may deliver sign restrictions different from those we use. Hence, although are conclusions are fully comparable with the evidence in Ulhig (2004), Dedola and Neri (2004), or Peersman and Straub (2005), they do not necessarily disproof the idea that permanent technological improvements may induce a decline in hours worked.

There are quantitative variations in the responses of hours and productivity over time, but the sign of the responses is the same at every date in the sample. Therefore, the mixed results found in the literature can not be due to time variations in the response of hours. Note also that, consistent with both RBC and sticky price models, hours positively comove with output in response to both demand shocks. However, the magnitude of the changes is such that in response to demand shocks labor productivity responds positively instantaneously but turns negative afterwards, while in response to monetary policy shocks labor productivity responses are instantaneously negative and the sign of the responses changes with the horizon of the analysis.

2.6 Conclusions

In this paper we examined structural sources of output and inflation volatility and persistence and attempted to draw some conclusions about the causes of the variations experienced in the US economy over the last 30 years. There has been a healthy discussion in the literature on this issue, thanks to the work of Clarida, Gali and Gertler (2000), Cogley and Sargent (2001,2005), Boivin and Giannoni, (2002), Leeper and Zha (2003), Sims and Zha (2004), Lubik and Schorfheide (2004), Primiceri (2005) and Canova and Gambetti (2004) among others, and although opinions differ, remarkable methodological improvements occurred trying to study questions having to do with time variations in structure of the economy and in the distributions of the shocks.

In this paper, we contribute to advance the technical frontiers estimating a structural time varying coefficient VAR model; identifying a number of structural shocks using sign restrictions derived from a general DSGE model; providing recursive analysis, consistent with information available at each point in time; and using frequency domain tools to address questions concerning time variations in persistence and volatility. In our opinion, the paper also contributes to advance our understanding of the cause of the observed variations in output and inflation. In particular, we show that while there are time variations in both the volatility of output and inflation and in the persistence of inflation, differences are statistically insignificant. Standard errors are larger than in other studies for two reasons: our recursive analysis makes them depend on the information available at each t; shocks to future parameters are not averaged out.

We show that output has become less volatile because the contribution of technology shocks has declined over time and that changes in the persistence and the volatility of inflation are jointly explained by changes in the contribution of technology, real demand and monetary policy shocks. Furthermore, we find that there are changes in the transmission of technology shocks and that the variance of both technology and monetary policy shocks has declined over time. We also provide novel evidence on the effects of technology shocks on labor market variables. In our estimated system, technology shocks robustly imply positive contemporaneous comovements of hours and labor productivity, even though the correlation between the two variables turns negative after a few lags.

All in all, our results question the conventional wisdom which attributes changes in the dynamic properties of output and inflation to monetary policy, and instead indicates that variations in both the magnitude and the transmission of technology shocks are an important vehicle to explain observed variations. Therefore, our conclusions are consistent with those of McConnell and Perez Quiros (2001) and Gordon (2003) and those of Sims and Zha (2004), Canova and Gambetti (2004) and Primiceri (2005).

Few words of caution are important to put our results in the correct perspective. First, by construction, our analysis excludes the possibility that in one period of history the monetary policy rule produced indeterminate equilibria. Therefore, our analysis differ from the one of Lubik and Schorfheide (2004), even though it points out that we can account for a large portion of the observed variations without the need to resort to sunspot explanations. Second, while the decline in the volatility of the shocks is consistent with exogenous explanations of the changes in output and inflation dynamics, such a pattern is also consistent with explanations which give policy actions some role. For example, if monetary policy had a better control of inflation expectations over the last 20 years and no measure of inflation expectations is included in the VAR, such an effect may show up as a reduction of the variance of the shocks.

Clearly, much work still needs to be done. We think it would be particularly useful to try to identify other structural shocks, for example, labor supply or investment specific shocks, and examine their relative contribution to changes in output and inflation volatility and persistence. It would also be interesting to study in details what are the technology shocks we have extracted, how do they correlate with what economists think are technological sources of disturbances and whether they proxy for missing variables or shocks. The model has implications for a number of variables which are excluded from the empirical analysis. Enlarging the size of our VAR could provide additional evidence on the reasonableness of the structural disturbances we have extracted. Finally, while much of the evidence is available for the US, very few exercises have looked at other countries or compared sources of output and inflation volatility and persistence across countries.

Tables

Table 1: Parameter values or ranges

β	discount factor	0.991
$(B/Y)^{ss}$	steady state debt to output ratio	1.2
σ	risk aversion coefficient	[0.5,6.0]
$1-a$	share of public goods in consumption	[0.0,0.15]
ς	elasticity of substitution public/private goods	[0.5,3.0]
θ_n	preference parameter	[0.1,0.9]
b	adjustment cost parameter	[0.1,10]
δ	capital depreciation rate	[0.013,0.05]
α	capital share	[0.2,0.4]
τ^l	average labor tax rate	[0,0.3]
τ^k	average capital tax rate	[0,0.2]
$(C^g/Y)^{ss}$	steady state C^g/Y ratio	[0.07,0.12]
γ	degree of price stickiness	[0.2,0.85]
ϕ_π	Taylor's coefficient	[1.1,2.0]
ϕ_b	coefficient on debt rule	[1.05, 4.05]
λ	elasticity of substitution between varieties	[7.0,8.0]
θ_M	elasticity of money demand	[1.0,10]
ρ_Z	persistence of Z_t shock	[0.8,0.95]
ρ_{C_g}	persistence of C_t^g shock	[0.6,0.9]
ρ_u	persistence of u_t^R shock	[0.7,0.9]

Table 2: Identification restrictions

	Output	Inflation	Interest rate	Money
Technology	≥ 0	≤ 0		
Government	≥ 0	≥ 0	≥ 0	≥ 0
Monetary	≥ 0	≥ 0	≤ 0	≥ 0

Figures

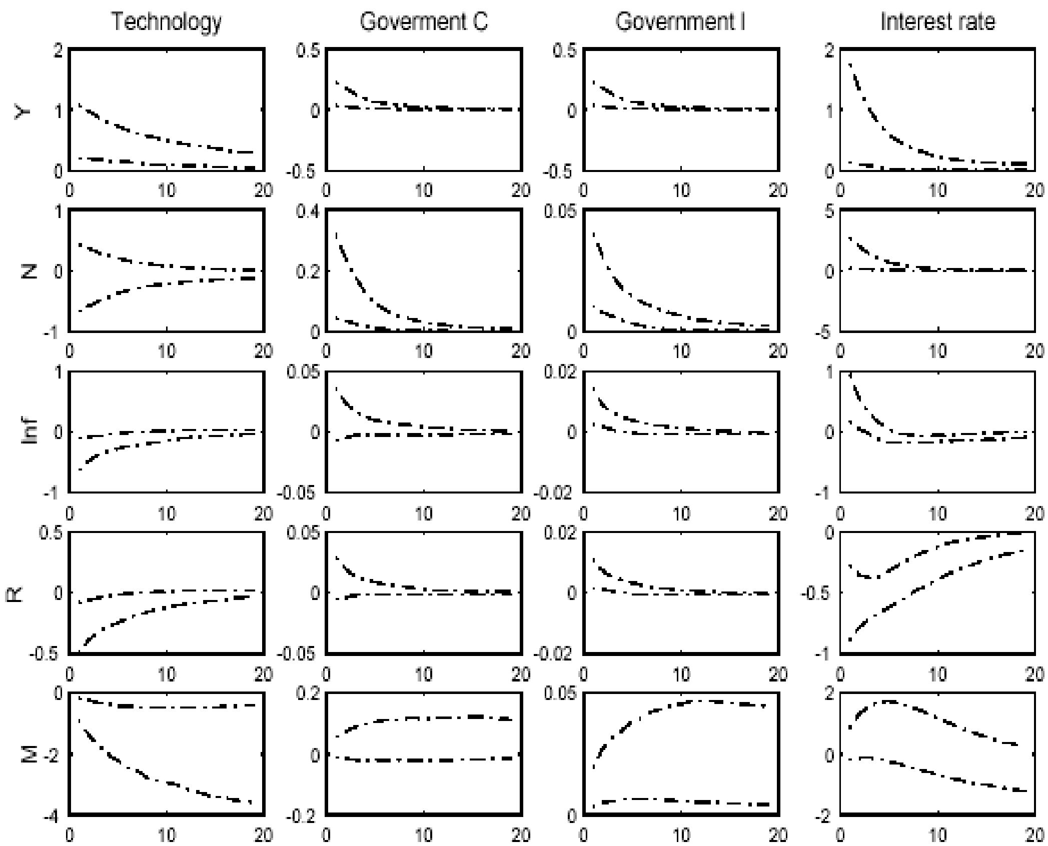


Figure 1: responses to shocks in the model.

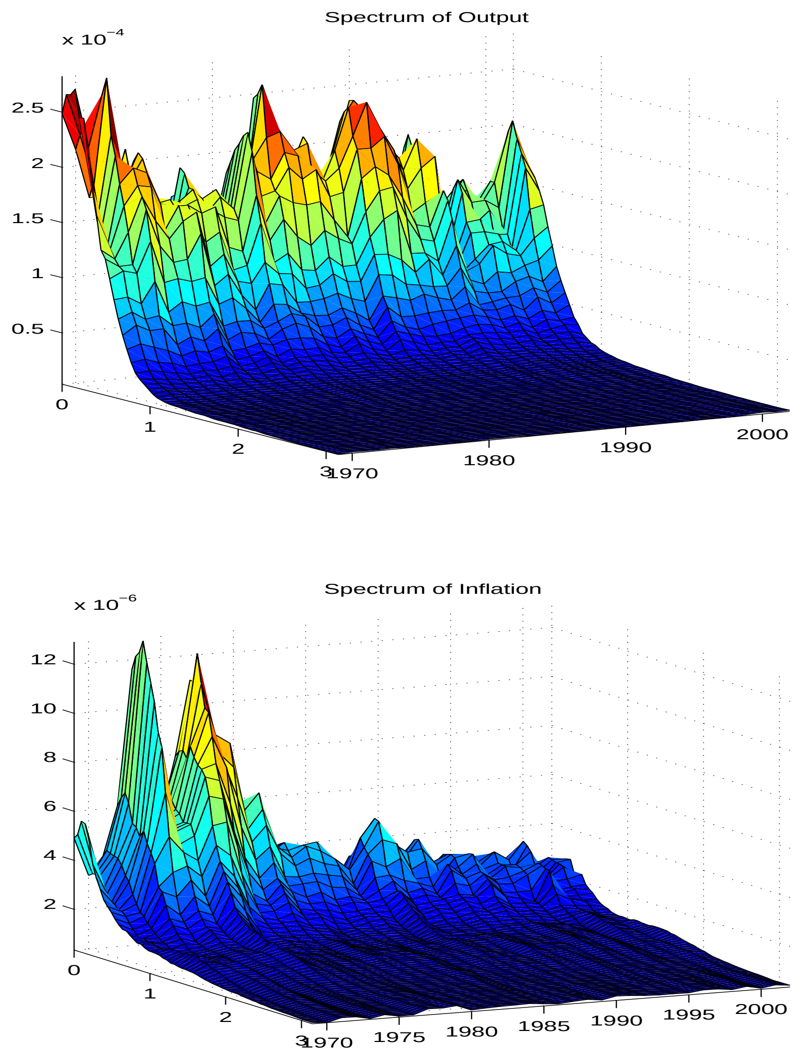


Figure 2: spectra of output (top) and inflation (bottom).

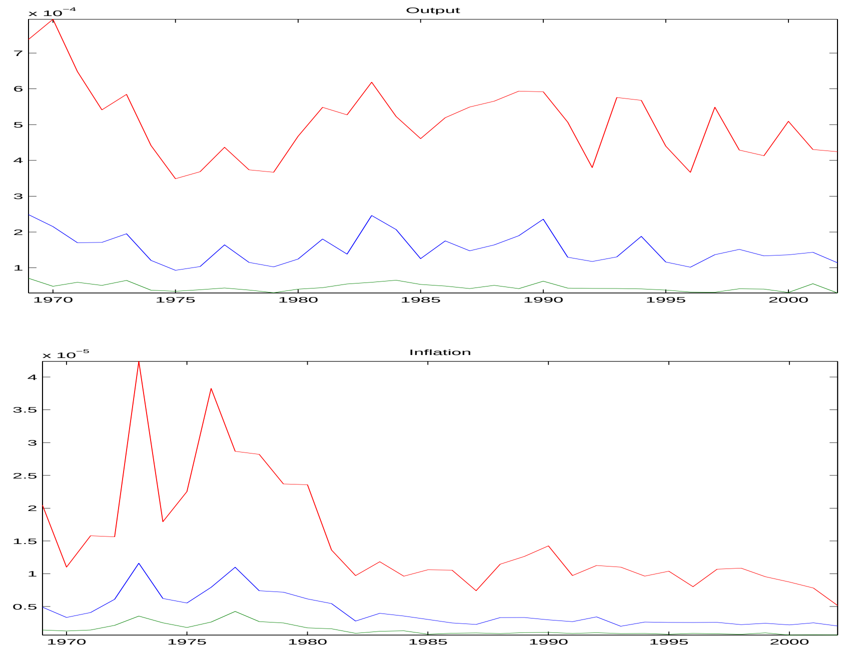


Figure 3: Persistence of output (top) and inflation (bottom).

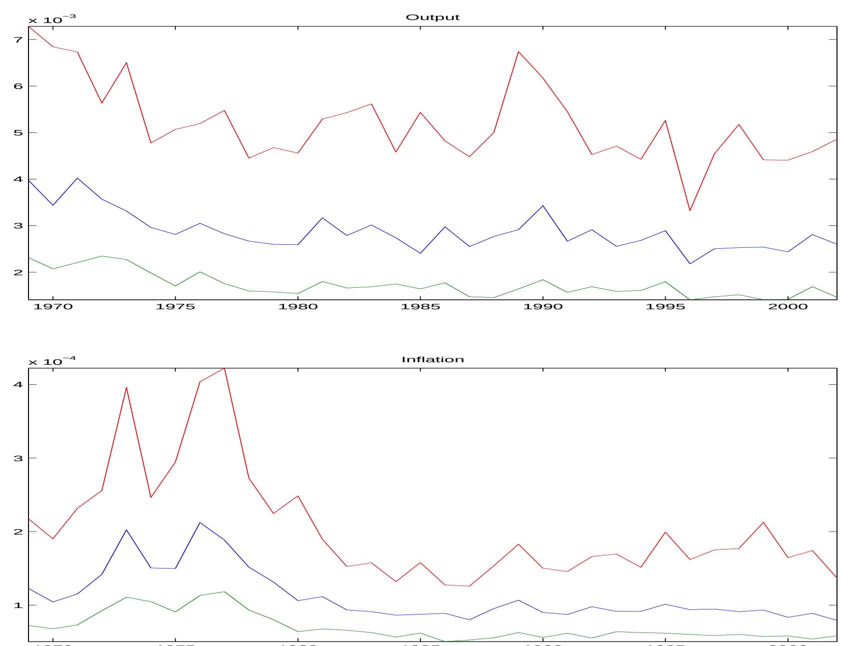


Figure 4: Volatility of output (top) and inflation (bottom).

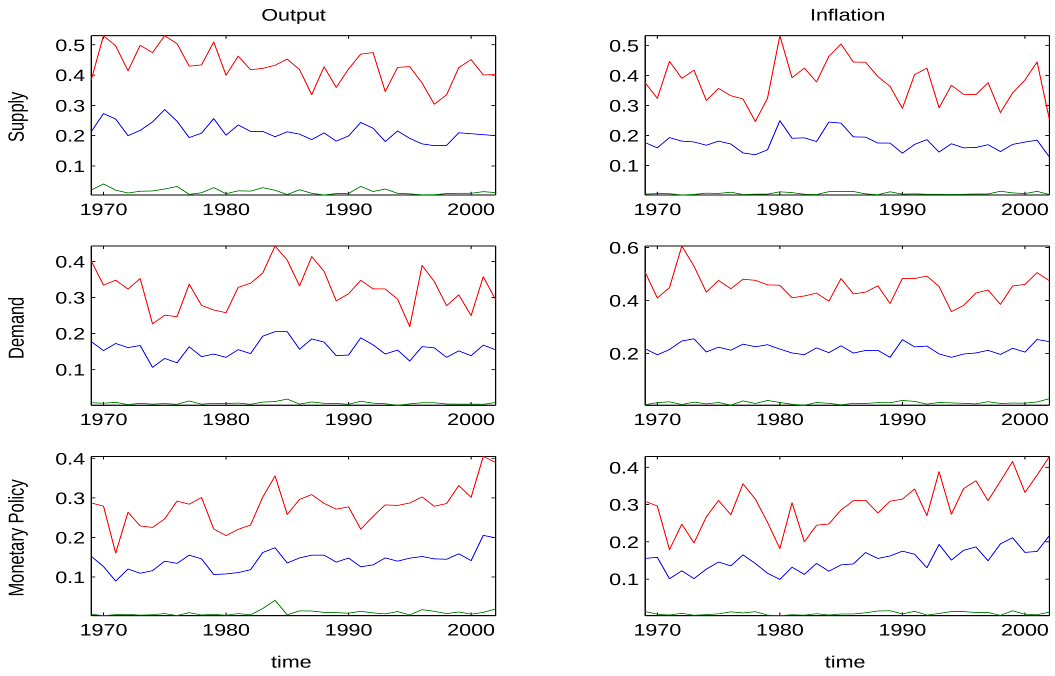


Figure 5: Persistence shares.

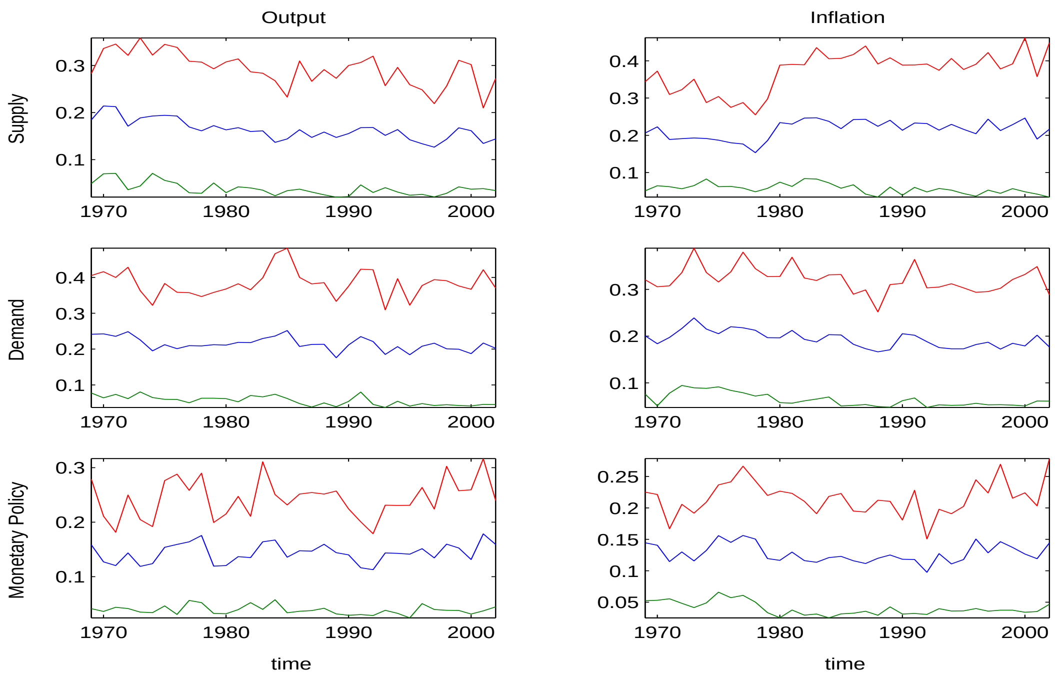


Figure 6: Volatility shares.

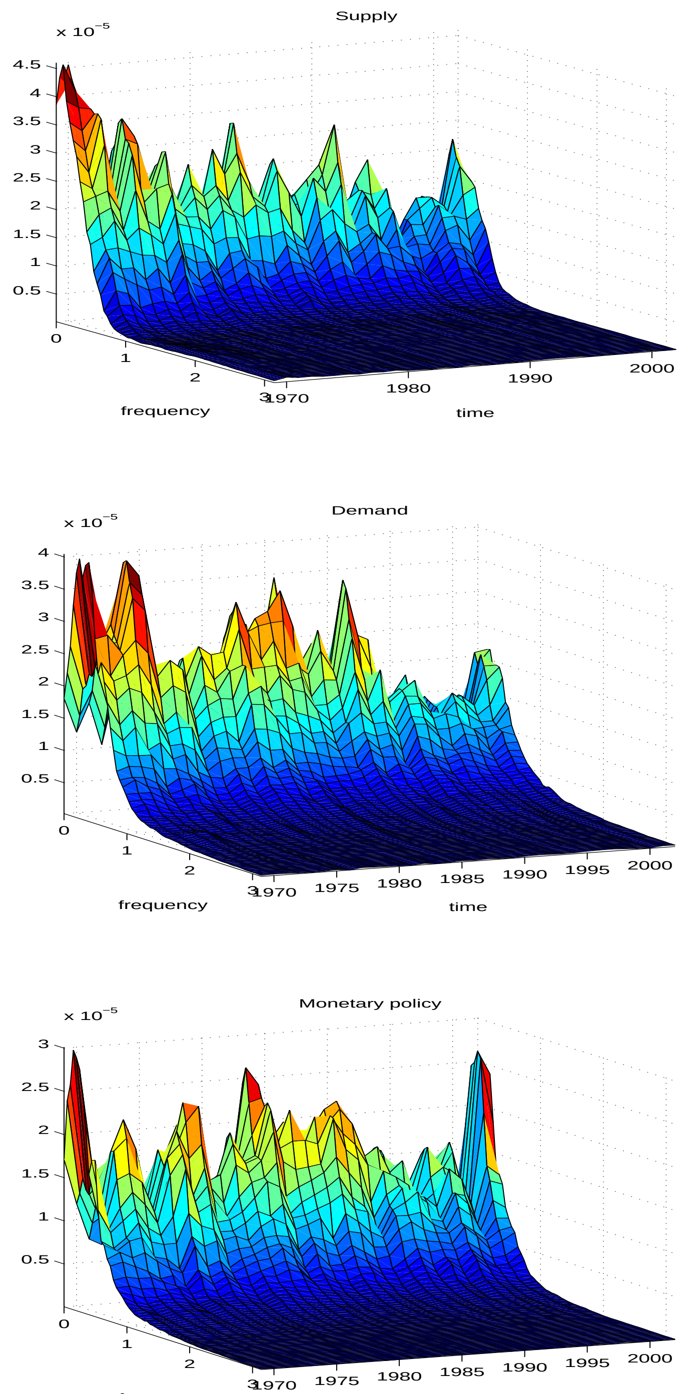


Figure 7: Contribution of different shocks to output spectrum

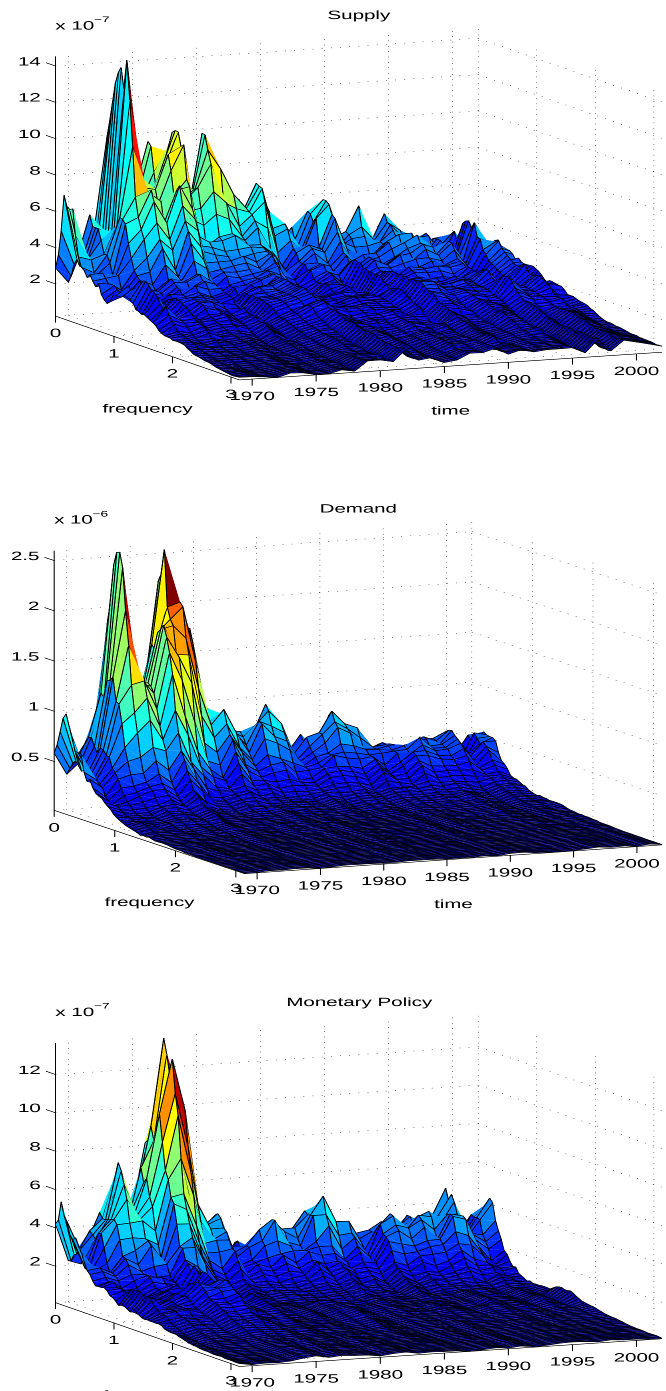


Figure 8: Contribution of different shocks to inflation spectrum

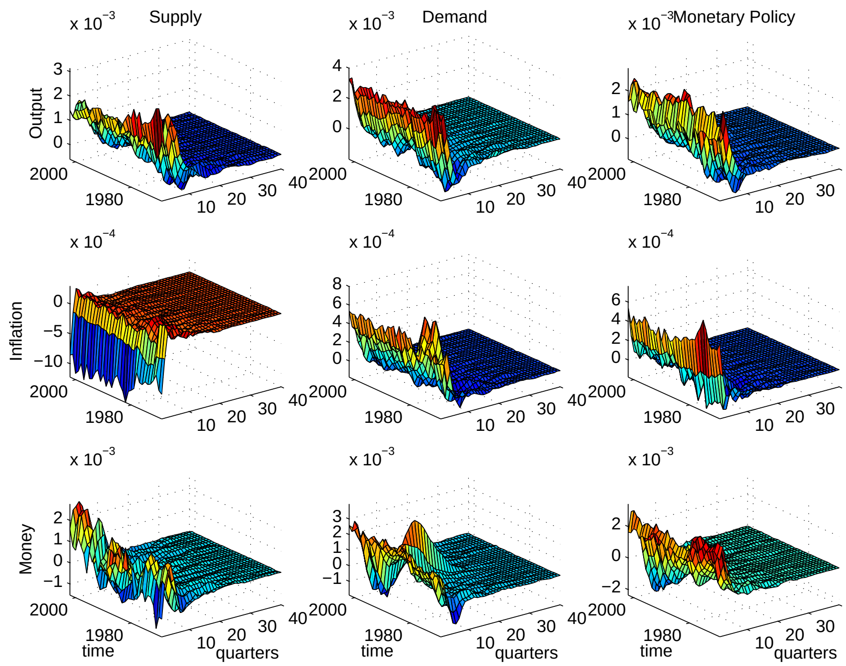


Figure 9: Output, Inflation and Money responses.

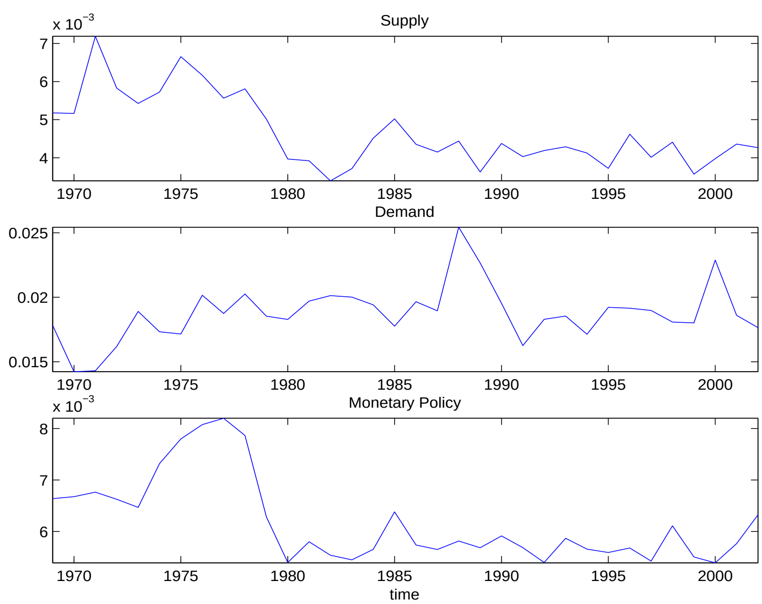


Figure 10: Structural shocks variances.

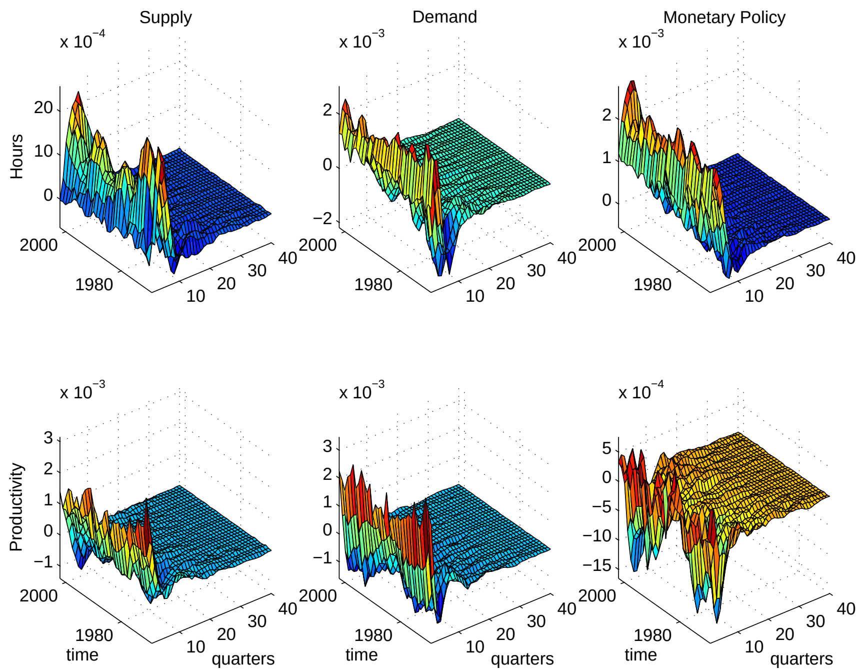


Figure 11: Hours and productivity responses.

Chapter 3

Technology shocks and the response of hours worked: time-varying dynamics matter

3.1 Introduction

The short-run dynamics of hours worked following a positive technology shock have an essential role in assessing competing theories of the business cycle. Standard versions of Real Business Cycle (RBC) models (see e.g. Prescott, 1986) predict that hours must increase: an improvement in technology raises marginal productivity of labor and the labor demand which, with an upward sloping supply, implies a rise in hours worked[1]. On the other hand, other theories of the business cycle, like models embodying nominal rigidities (see e.g. Gali, 1999) or RBC models with habits formation and capital adjustment cost (see e.g. Francis and Ramey, 2003) predict that hours fall. The sign of the response of hours has very important implications for the role of technology shocks in explaining aggregate fluctuations. Actually a shock that fails in generating a strong positive correlation between output and hours can hardly be considered one of the main forces driving business cycles.

In recent years an interesting and intense debate on whether, in the data, hours rise or fall after a positive technology shock has emerged. Implicitly, the contention is whether the standard RBC paradigm can correctly describe the business cycle and whether technology shocks can be considered important sources of economic fluctuations. Gali (1999), using reduced form vector autoregressions augmented with the restriction that technology shock is the only shock driving long-run labor productivity, finds that hours fall. Moreover technology shocks can account just for a very small part of total fluctuations in output and hours worked at the business cycles frequencies. The author interprets all this as compelling evidence against the RBC paradigm. Similar conclusions are reached, through different approaches, by Basu Fernald and Kimball (2004), Francis and Ramey (2003) and Francis,

[1]Under standard calibrations, such a mechanism arise no matter when the technology shock is modeled as a persistent stationary AR or a random walk.

Owyang and Theodorou (2003), Pesavento and Rossi, (2004) and Shea (1998). The reaction to this growing consensus came soon. Christiano Eichenbaum and Vigfusson (2004) (CEV henceforth), using a similar reduced form vector autoregressions representation and the same identifying restriction, replicate the exercise by Gali (1999) and they find the opposite result: as predicted by standard RBC models, hours persistently rise[2] Evidence in line with the CEV conclusions is provided in the works by Dedola and Neri (2004), Fisher (2005), Peersman and Straub (2003, 2005) and Uhlig (2001).

Why are the results of Gali and CEV so different? The reason is in the different specification for the time series of hours worked used in the VAR. Gali, arguing that hours are difference stationary, uses growth rates[3]. On the contrary CEV, justifying their choice with an encompassing argument, specify hours in levels. The whole debate is nowadays at a standstill because of such a specification controversy[4]: the response of hours to a positive technology shock depends on whether they are specified in levels (hours rise) or growth rates (hours fall). Consequently much effort has been spent in trying to justify from a statistical and economic point of view each of the two specifications. CEV show that the levels specification can easily explain the growth rates specification while the converse is not true. On the other hand Gali (2005) provides empirical and theoretical evidence in favor of the nonstationarity of hours worked across industrialized countries.

This paper contributes to the debate from a completely different perspective. We investigate the effects of technology shocks on hours worked in the US using Bayesian Vector Autoregressions with drifting coefficients, thus allowing for general forms of time variations and structural changes. The basic idea of the paper comes from the simple consideration that despite the different treatment of hours worked, all empirical models used in previous contributions stand on the assumption that model coefficients are constant over time. Although standard in VAR literature, such an assumption seems to be very strong when the analysis is run over a sample of fifty years and mainly when variables describing the labor market are included. Actually important changes in labor market trends, like changes in the composition of hours worked or participation rates, and in the US economy in general, like changes in the central bank anti-inflationary preferences[5] or changes in labor productivity trends[6], have been extensively documented in literature. Moreover these changes seem to

[2]Chari Kehoe and McGrattan (2005) call into question the VAR approach as a useful guide to assess the relevance of theoretical models. They show, using simulated data, that VAR analysis would imply a fall of hours when the underlying theoretical model predicts a positive response after a technology shock. However Christiano Eichenbaum and Vigfusson (2005) show that the Chari Kehoe and McGrattan model is a case of little empirical relevance since it is strongly rejected by the data. On the contrary when models with higher posterior support are employed, VAR predict the right responses. Similar findings emerge in the works by Erceg, Guerrieri and Gust (2004) and Francis, Owyang and Roush (2005).

[3]The same results emerge when hours are detrended using quadratic trends.

[4]See Whelan (2004) for a detailed review and a study of the robustness of the results to alternative specifications.

[5]See among others Boivin and Giannoni (2002a, 2002b), Clarida, Gali and Gertler, (2000), Cogley and Sargent, (2001, 2003) documenting a change in the response of monetary authorities to inflation after 1979.

[6]See Brainard and Perry, (2001), Kahn and Rich, (2003) and Roberts, (2000) documenting that two big changes in labor productivity took place in early 70's and again during the mid 90's.

be relevant for the transmission mechanisms of technology shocks since in many works (see Gali, Lopez-Salido and Valles 2003, GLV henceforth, CEV, Fisher 2005, and Fernald 2005) it clearly emerges that results are highly sensitive to the sample or subsamples considered in the analysis. In this paper we argue that conflicting results may ultimately arise simply because some of these changes are a priori ruled out by previous empirical models. Specifically, differences in the results that are apparently due to a different treatment of hours worked may simply originate from a more fundamental misspecification arising from the too strong assumption of model coefficients constancy. Actually we show that once one allows for changes in the US economy whether hours should be specified in levels or growth rate becomes of secondary importance since competing specifications yield the same answer: at least until mid 90's hours persistently fall.

This paper addresses the following questions. What are the effects of technology shocks on hours worked and what is the importance of technology shocks in explaining aggregate fluctuations when time variations are accounted for? Can we reach robust conclusions by allowing for time variations in the US economy? To address these questions we augment the reduced form model, which is almost identical to the one originally proposed by Cogley and Sargent (2001), with the same restriction as in Gali (1999) and CEV that technology shocks are the only shocks driving labor productivity in the long-run and we use both specifications for hours worked, levels and growth rates. Given that the specification is identical to the one used in literature, our analysis can concentrate on differences directly attributable to coefficients time variations. To conduct dynamic analysis we use conditional impulse response functions, that is we condition on all out-of-sample coefficients being equal to the end-of-sample coefficients. This is motivated, on the one hand, by the fact this is the best forecast whenever coefficients evolve according to a random walk. On the other hand, under such a definition, impulse response functions display useful long-run properties. The model is estimated using Bayesian MCMC methods: we use the Gibbs sampling algorithm augmented with a rejection sampling to generate draws from the posterior distributions of the objects of interest. We check the robustness of the results to alternative end-of-sample dates and alternative identification schemes and eventually we extend the model in order to consider also investment-specific technology shocks.

Our main findings can be summarized as follows. (i) Hours fall under *both* specifications, levels and first differences. (ii) The impact effect is more pronounced and significantly different from zero only before 1990. For the level specification also the degree of persistency of the response substantially reduces over-time. (iii) Differences with respect to fixed coefficients VAR are due to instabilities in the relationship between labor productivity and the levels of hours worked. (iv) Technology shocks generate positive but small correlations between output and hours at the business cycles frequencies and the portion of output variance explained by technology shocks over the business cycles is about 11-25%. When, in addition to aggregate sector-neutral shocks, also investment-specific technology shocks are considered the percentages relative to technology shocks as a whole raise up to 39-53%. (v) Results are robust to alternative identifying restrictions. (vi) The response of monetary policy to technology shocks has changed over time but this does not seem to affect the response of hours worked.

The paper is organized as follows: section 2 revisits the evidence from fixed coefficients VARs; section 3 describes the empirical model; section 4 discusses main results; section 5 explains the differences between time-varying and fixed coefficients VARs; section 6 provides some structural interpretations of the results; section 7 assesses the robustness of the results to various alternatives; section 8 concludes.

3.2 Revisiting the Evidence from Fixed Coefficients SVARs

Let y_t be a $n \times 1$ vector of time series with the following VAR representation

$$A(L)y_t = \varepsilon_t \tag{3.1}$$

where L is the lag operator, $A(L) = I - A_1L - A_2L^2 - ... - A_pL^p$, A_i are $n \times n$ matrices of coefficients and ε_t is a $n \times 1$ Gaussian white noise process with zero mean and covariance Σ. If the roots of $A(L)$ in modulus are outside the unit circle, y_t admits the following MA representation of infinite order

$$y_t = B(L)\varepsilon_t \tag{3.2}$$

where $B(L) = A(L)^{-1}$. Let S be the unique lower triangular matrix such that $SS' = B(1)\Sigma B(1)'$ where $B(1) = I + B_1 + B_2 + ...$ and let $K = B(1)^{-1}S$. We can rewrite (3.2) in terms of orthogonal shocks

$$y_t = C(L)e_t$$

where $e_t = K^{-1}\varepsilon$ and $C(L) = B(L)K$. If labor productivity growth is ordered first in the vector y_t, then the first shock, e_{1t}, is the technology shock identified by the restriction that is the only shock affecting long-run productivity.

Figure 1 plots the impulse response functions of per capita hours to a technology shock from a bivariate VAR with labor productivity growth and hours worked. Top and bottom panels refer to the specification with hours in first differences and levels respectively. When specified in first differences, hours persistently and significantly decline. On the contrary, in levels, the response is positive, significant and hump-shaped, reaching its maximum after two years after the shock. Here the terms of the controversy clearly emerge: when hours are specified in growth rates they persistently decline while in levels they persistently increase.

To motivate our interest in time variations let us consider what happens when we repeat the analysis for the subsamples considered in GLV and Fisher (2005), 1954:III-1979:IV and 1982:III-2003:IV, and corresponding to the presumed breaks in the monetary policy conduct. Instabilities are evident for the levels specification (Figure 2): in the second subsample the response is positive while in the first it becomes negative. On the other hand, in growth rates, results appear to be more robust since in both subsamples hours reduce. The lack of robustness of results is not limited to the two subsamples considered above. For instance Fernald (2005) shows that if one takes into account potential shifts in trend productivity, specifically the slow-down in 1973 and pick-up in mid 90's hours worked fall for both specifications in all the subsamples. Perhaps the most striking result is that if the analysis had been done ten years before the paper by CEV, say at the very beginning of the 90's, no debate would have emerged. Actually if we exclude from the analysis the last ten years (1994-2003), hours fall under both specifications. All these findings are hard to explain unless we admit the possibility that when we use the whole sample we are mixing periods in which the structural features characterizing the US economy are different. This, we believe, strongly suggests that the link between structural changes and the propagation mechanisms of technology shocks and the way the formers may have influenced the seconds deserves further investigations.

3.3 The Empirical Model

We use a Bayesian Vector Autoregression where the coefficients are allowed to smoothly drift over-time to describe the evolution of the US economy. Several reasons drive our choice. First, time variations and structural changes may be important. Second, there can be various features of the US economy that have changed and they should be considered simultaneously rather than separately. Third, we believe that changes in macroeconomic relationships suggest more evolution rather than sudden breaks[7]. Fourth, our model represents a generalization of fixed coefficients VAR and includes this as a special case.

3.3.1 VAR Representation

Let y_t be a $n \times 1$ vector of time series which admits the following reduced form VAR representation

$$y_t = A_{0,t} + A_{1,t}y_{t-1} + A_{2,t}y_{t-2} + ... + A_{p,t}y_{t-p} + \varepsilon_t \tag{3.3}$$

where $A_{0,t}$ is an $n \times 1$ vector of time-varying intercepts, $A_{i,t}$, for $i = 1, ..., p$, are $n \times n$ matrices of time-varying coefficients[8] and ε_t is a $n \times 1$ Gaussian white noise process with zero mean and covariance Σ. Let K_t be any, possibly time varying, nonsingular matrix such that $K_t K_t' = \Sigma$. Rewriting the model in terms of orthogonal shocks we have

$$y_t = A_{0,t} + A_{1,t}y_{t-1} + A_{2,t}y_{t-2} + ... + A_{p,t}y_{t-p} + K_t e_t \tag{3.4}$$

where $e_t = K_t^{-1}\varepsilon_t$ is a Gaussian white noise process with zero mean and covariance the identity matrix I_n. Equation (3.4) represents the class of structural representations of the vector of time series and each particular matrix K_t defines a particular representation of y_t.

3.3.2 Dynamics

Model dynamics are summarized in the mechanisms through which shocks spread over time. Impulse response functions measure the effects of a shock on future time series relative to some benchmark case. Equation (3.3) has the following companion form

$$\mathbf{y}_t = \mu_t + \mathbf{A}_t \mathbf{y}_{t-1} + \epsilon_t$$

[7]We do not claim that breaks from period to period are unlikely to occur but rather we argue that most of macroeconomic changes, in particular those related to the labor market, take place in a gradual way.

[8]The fixed coefficients VAR is a special case of the model in which $A_{i,t} = A_i$ for all i and t.

where $\mathbf{y}_t = [y_t' ... y_{t-p+1}']'$, $\epsilon_t = [\varepsilon_t' 0 ... 0]'$ and $\mu_t = [A_{0,t}' 0 ... 0]'$ are $np \times 1$ vectors and

$$\mathbf{A}_t = \begin{pmatrix} & A_t & \\ I_{n(p-1)} & & 0_{n(p-1),n} \end{pmatrix}$$

where $A_t = [A_{1,t} ... A_{p,t}]$ is an $n \times np$ matrix, $I_{n(p-1)}$ is an $n(p-1) \times n(p-1)$ identity matrix and $0_{n(p-1),n}$ is a $n(p-1) \times n$ matrix of zeros. Iterating k period forward and omitting for simplicity the constant term, we obtain

$$\mathbf{y}_{t+k} = \mathbf{A}_{t+k} ... \mathbf{A}_t \mathbf{y}_{t-1} + \mathbf{A}_{t+k} ... \mathbf{A}_{t+1} \epsilon_t + \mathbf{A}_{t+k} ... \mathbf{A}_{t+2} \epsilon_{t+1} + ... + \mathbf{A}_{t+k} \epsilon_{t+k-1} + \epsilon_{t+k}$$

Let $\mathcal{S}_{i,j}(M)$ be a selection function, a function which selects the first i rows and j columns of the matrix M. Taking as a benchmark case the case of no-shock occurrence, and assuming that coefficients and shocks ε_t are uncorrelated, the matrix of dynamic multiplier $\mathcal{S}_{n,n}(\mathbf{A}_{t+k} ... \mathbf{A}_{t+1})$ describes the effects of ε_t on y_{t+k}, while the effects associated to structural shocks can be derived from the relation $\varepsilon_t = K_t e_t$ and are given by $\mathcal{S}_{n,n}(\mathbf{A}_{t+k} ... \mathbf{A}_{t+1}) K_t$. Few important features of the impulse response functions in our setup need to be highlighted. First, the effects of the shocks depend on future coefficients: unlike in the fixed coefficients case, here propagation mechanisms are subject to future changes in the structure of the economy. Second, the effects of a shock for the same k but different t may vary over time, both because at each time period we can associate a particular reduced form dynamic multiplier, and because the identifying matrix, K_t, may change over time. Third, data provide information about model dynamics up to the end of the sample date, T, because posterior information is available only for VAR coefficients up to that date. Thus in order to study dynamics after T some forecast of future VAR coefficients is needed. To construct impulse response functions we assume $\mathbf{A}_{T+j} = \mathbf{A}_T$ for all $j = 1, 2, ...$. Three reasons motivate our choice. First, we want to use all the information contained in the data. Second, $\mathbf{A}_T$ represents the best forecast of $\mathbf{A}_{T+j}$ whenever coefficients evolve according to a random walk. Third, impulse response functions, under this assumption, have useful long-run properties[9]. Formally impulse response functions of a shock occurring at time t at horizon k are given by

$$IR(t,k) = B_{t,k} K_t$$

where

$$B_{t,k} = \begin{cases} \mathcal{S}_{n,n}(\mathbf{A}_{t+k} ... \mathbf{A}_{t+1}) & \text{if} \quad t+k < T \\ \mathcal{S}_{n,n}(\mathbf{A}_T^{t+k-T} \mathbf{A}_T ... \mathbf{A}_{t+1}) & \text{if} \quad t+k \geq T \end{cases}$$

Thus for each $t = 1, ..., T$ we have a path of impulse response defined by the sequence $\{B_{t,k} K_t\}_{k=1}^{\infty}$ and cumulated impulse response functions $\{\tilde{B}_{t,k} K_t\}_{k=1}^{\infty}$ where $\tilde{B}_{t,k} = \sum_{j=1}^{k} B_{t,k}$. First note that, as in the fixed coefficients case, if all the eigenvalues of any realization of $\mathbf{A}_T$

[9] Other alternatives are available. For instance, Canova and Gambetti (2004), in a similar Bayesian approach, consider the effects of the shock under all the possible realizations of future coefficients for some finite horizon of interest. This implies drawing future coefficients from the prior density conditional to a draw for coefficients up to time T from the posterior. Actually, while useful for finite horizons, such an approach creates non-trivial complications for infinite horizons since available necessary and sufficient conditions for the convergence of $\sum_{i=1}^{k} \prod_{j=1}^{i} \mathbf{A}_{t+j} ... \mathbf{A}_{t+1}$ are too restrictive for our purposes.

are smaller than one in modulus impulse response functions converge pointwise (see appendix). In particular the limit of cumulated impulse response functions will be varying over time, depending on t. Second, the speed of convergence and thus the long-run cumulated effects will depend on the end-of-sample date coefficients. We use the last available time period observation in order to maximize the quantity of information from the data, but in the empirical part we will investigate the sensitivity of our results to different end-of-sample dates[10], that is we will end the sample at arbitrary dates different from T.

3.3.3 Identification

In order to identify the model and recover the representation of y_t in terms of structural shocks we should, in general, fix for all $t = 1, ...T$ a particular matrix K_t. Since our focus is only on technology shocks we only partially identify the model, that is we only fix a column of K_t without attempting to identify all the other shocks. The restriction we use is the same as in Gali (1999) and CEV: the technology shock is the only shock affecting long-run labor productivity[11]. For each $t = 1, ...T$, let S_t be the unique lower triangular matrix such that $S_t S_t' = \tilde{B}_{t,\infty} \Sigma \tilde{B}_{t,\infty}'$. We set

$$K_t = \tilde{B}_{t,\infty}^{-1} S_t$$

Thus the path of structural impulse response functions for each $t = 1, ...T$ will be given by

$$IR(t, k) = B_{t,k} \tilde{B}_{t,\infty}^{-1} S_t, \quad k = 1, 2, ...$$

If, as in the fixed coefficients case, labor productivity is ordered first, the first shock e_{1t} is the technology shocks[12].

3.3.4 Specifications and Estimation

A State-Space Representation

In order to understand model estimation it is useful to rewrite the model in a state space form. Let $A_t = [A_{0,t}, A_{1,t} ... A_{p,t}]$, $x_t' = [1_n, y_{t-1}' ... y_{t-p}']$, where 1_n is a row vector of ones of length n , let $vec(\cdot)$ denote the stacking column operator and let $\theta_t = vec(A_t')$, $\theta^T = [\theta_1' ... \theta_T']'$ and $y^T = [y_1' ... y_T']'$. Then (3.3) can be written as

$$y_t = X_t' \theta_t + \varepsilon_t \tag{3.5}$$

[10]Another feasible alternative would be to study local dynamics, i.e assuming that all the coefficients are constant from the period in which the shock occurs. In this case however a lot of in-sample information would not be used and for this reason we do not purse this strategy.

[11]It is clear that it is the only shock affecting long-run labor productivity among the shocks in e_t. In fact in our model potentially shock to coefficients could affect variables at long-run horizons, but in this case they would have a different interpretation, since they would affect permanently the growth rates of labor productivity.

[12]We do not attempt to identify the others $n-1$ shocks and we simply fix them using an atheoretical recursive long-run ordering among the other variables. However it is important to stress that such an ordering does not affect the dynamics of the so identified technology shock, in fact it can be showed that by changing the ordering of the other variables the responses of all variables to technology shocks remain unchanged.

where $X_t' = (I_n \bigotimes x_t')$ is a $n \times (np+1)n$ matrix, I_n is a $n \times n$ identity matrix, and θ_t is a $(np+1)n \times 1$ vector. Treating θ_t as a hidden state vector, equation (3.5) represents the observation equation of a state space model. Let $f(\cdot)$ be a normal density and let us assume that $f(\theta_{t+1}|\theta_t, \phi)$ can be represented as

$$\theta_{t+1} = F\theta_t + u_{t+1}$$

where u_t is a $(np+1)n \times 1$ Gaussian white noise process independent of ε_t with zero mean and covariance Ω[13], $\phi = \{\Omega, \Sigma\}$ and F is a diagonal matrix of constant coefficients. We assume that θ_t evolves according to

$$p(\theta_{t+1}|\theta_t, \phi) \propto \mathcal{I}(\theta_{t+1}) f(\theta_{t+1}|\theta_t, \phi) \tag{3.6}$$

where $\mathcal{I}(\theta_{t+1})$ is an indicator function assuming value zero if roots of the associated VAR polynomial are outside or on the unit circle and one otherwise. In other words the function discards path of θ_t whenever the associated VAR polynomial roots are unstable. Such a restriction ensures convergence of impulse response functions and then makes the above discussed identification scheme implementable since it cuts the support of the distribution in correspondence of draws with unit or explosive roots[14]. Equation (3.6) represents the conditional prior for θ_t. We assume that $F_{jj} = 1$ if the coefficient is associated to lagged variables or equal to 0.999 for the time-varying intercept terms. The first assumption yields random walk coefficients for lagged variables provided that the roots restriction is satisfied. On the other hand, we assume that the intercept term evolve according to a very highly persistent but stationary process. This is needed since a random walk process for the intercept term would signify infinite prior variance. Except for the restriction on the unit root and the assumption of stationarity of the time-varying intercept term the above state space representation is identical to the one originally proposed by Cogley and Sargent (2001).

Estimation Strategy

Estimation is done in two steps. First, we characterize the unrestricted posterior distribution $p_u(\theta^T, \phi|y^T)$. Second, we discard the draws that do not satisfy the restrictions on the VAR polynomial roots. Since the posterior distribution is not available in closed form, we simulate it using MCMC methods. Specifically, the first step is done using the Gibbs sampling algorithm where the time-varying parameters and the hyperparameters are treated as two different blocks, while the second is done by applying a rejection sampling to the unrestricted posterior distribution. Because of the heavy notation and the technicalities involved with

[13] We estimate the model under different assumption on Ω: diagonal, block-diagonal with the block corresponding to the coefficients of the same equation and for the bivariate case we also specify it as full matrix. While the degree of time variation depends on the specific assumptions main results are roughly unchanged. Furthermore independently on the particular specification, structural coefficients are always allowed to evolve in a correlated manner (see Canova and Gambetti, 2004).

[14] The restriction on the VAR polynomial roots makes the model locally stationary at each point in time, which does not imply global stationarity.

the construction of posterior distributions we defer the details of the estimation to the appendix. Once the posterior distribution is available, we draw a path for the states and the variances, we identify the technology shock and we compute the associated structural impulse response functions. After having computed a sufficiently large number of draws inference is implemented by taking the mean and 68% confidence band.

Specifications and Data

We use a bivariate VAR including labor productivity growth and per capita hours worked, and a four variables VAR in which the interest rate and inflation are added (the $R\pi$-specification henceforth). The bivariate VAR is important since it is the benchmark specification from which the debate originates. On the other hand, VARs that include more variables are important both because it is of interest to study the effects of technology shocks also on other macroeconomic variables, and because the results may change compared to the bivariate specification.

We use quarterly US data spanning from 1954:IV to 2003:III taken from the FRED II data base of the Federal Reserve Bank of San Louis. We initially estimate the model for the sample 1954:IV-1966:IV using fixed coefficients VAR to calibrate prior parameters and then reestimate it from 1967:I up to 2003:III. The variables used are the following: the first differences of the logs of labor productivity in the non-farm business sector (OPHNFB); the first difference of the logs of the GDP deflator (GDPDEF); the federal funds rate (FEDFUNDS); per capita hours are defined as hours worked (HOANBS) divided by the non-institutional population over 16 (CNP16OV). We use both growth rates of hours worked and the levels in logs.

3.4 Results

3.4.1 Impulse Response Functions

For each quarter we collect the posterior mean[15] of the impulse response functions for horizons up to 20 quarters. All 3D IRF are plotted using the following convention: on the x-axis there are quarters after the shock, on the y-axis there are the time periods, from 1967:I up to 2003:II and the z-axis there is the value of the response.

Bivariate VARs

Figure 3 displays the response of per capita hours worked (level specification in the bottom panel and the growth rates in the top panel) to a positive technology shock in the bivariate VAR. No matter the specification used, levels or growth rates, before early 90's the response of hours worked is negative and significant on impact. It is also quite persistent, lasting on average about one year and reaching the minimum at about two or three quarters after the shock. In the levels specification the degree of persistency gradually reduces from mid 80's.

[15] Results are very similar when the medians are considered instead of the means.

Starting from mid 90's, the response becomes positive on impact, although not significant, and hump shaped. On the other hand in the growth rates specification the response is always negative and after mid 80's also permanent. While generating slightly different dynamics in the last part of the sample, until early 90's both empirical specifications point to a persistent reduction of hours.

Larger VARs

Figure 4 displays the response of per capita hours worked (level specification in the bottom panel and the growth rates in the top panel) to a positive technology shock in the $R\pi$ VAR. Figure 5 focuses on the posterior mean of the impact effect with 68% confidence bands. The mean response of hours at all dates and for both specifications is negative on impact although significant only until mid 90's. As in the bivariate case, in the levels specification the response is much more persistent and pronounced before mid 80's while after that date the degree of persistency tends to reduce. A similar pattern concerns the size of the response. At the end of the 90's the response is about one fifth of the response during the 70's and the reversion to the pre shock level is completed after one year, while in the first part of the sample it occurs after two or more years. The response for the growth rate specification is almost identical to the one in the bivariate case, it is negative at al horizon and permanent after mid 80's. In sum, two important results arise. First, no matter the specification for hours worked, until early 90's technology shocks are contractionary, hours worked fall. Second the response on impact displays a break dated early 90's: before that date it is very pronounced and statistically different from zero while after it is much smaller and not significant.

Figure 6, 7 and 8 display the response of labor productivity and output and inflation respectively for the levels specification[16]. Labor productivity and output increase on impact, the former increasing more than the second because of the reduction in the labor input. At few quarters after the shock, both responses begin to climb to their new steady state level. Notice that, consistently with the response of hours, the response of output in the levels specification is smaller on impact in the first part of the sample and it takes more quarters to reach the new long-run level. Interestingly, at all dates, the impact effect of labor productivity is smaller that the long-run effect. Hence technology shocks appear to spread gradually or, at least, they affect both labor productivity and output gradually. It should be stressed that while the same finding emerges in the fixed coefficients case with hours worked in growth rates, in the level specification the response of labor productivity is substantially different (see CEV): when hours enters in levels the impact effect of labor productivity generally overshoots its new steady state. So that, labor productivity gradually decline to the new long-run equilibrium instead of increasing to it. Thus,when one takes into account time variations, not only the dynamics of hours change compared to fixed coefficients VAR, but also those of labor productivity. Inflation falls on impact and for few quarters after

[16]We omit impulse response functions for first differences specifications, available upon request, since are very similar.

the shock at all dates. The response of inflation is much more persistent before 1980 than after, in particular before 1980 the response is hump shaped reaching the minimum after one year, while after it steadily reduces after a large initial effect. The result suggests that technology shocks could have contributed substantially to the changes in terms of volatility and persistence of inflation after mid 80's confirming results by Canova, Gambetti and Pappa (2005).

3.4.2 Technology Shocks and the Business Cycle

Are technology shocks important for business cycles? Are technology shocks responsible for the pattern of output and employment fluctuations associated with the business cycle? The empirical framework we use allows us to address these questions by decomposing historical fluctuations in output, labor productivity and hours into a technology and a non-technology component. From the posterior distribution we draw realizations for structural coefficients and for each realization we collect the particular realization of structural shocks. Then using only the estimated technology shocks and the structural coefficients we compute the predicted time series for output, labor productivity and hours worked. Using a bandpass filter, we extract from the resulting series the component associated with business cycle frequencies and we compute correlations and variances. We repeat the same exercise for the non-technology component. We perform the analysis using both the levels and the growth rates specification for hours worked.

Table 1 reports the results for the technology shock. Point estimate of the correlation between output and hours attributable to technology shocks is 0.76 in the bivariate and 0.55 in the multivariate case when hours are specified in levels. Only in the bivariate case the correlation generated by technology shocks is similar to the correlation arising in actual data and it is entirely attributable to the dynamics arising in the last ten years of the sample. Correlations reduce substantially when hours are specified in first differences. In this case they are 0.46 in the bivariate and 0.28 in the larger VAR. On the other hand, non-technology shocks produce correlations between output and hours which are of the order of about 0.9. The picture is even more clear if we look at the portion of explained variance. In the levels specification technology shocks account for about 15-28% of the hours variance and 14-25% of the output variance, while in first difference they are even smaller, 9-15% and 11-14% respectively. This means that the non-technology component account for at least the 75% of cyclical output fluctuations.

By investigating the pattern of output fluctuations and the component associated with technology shocks under various specifications two robust facts emerge. First, the amplitude of total output fluctuations substantially reduces over time, particularly starting from mid 80's. Second, the size of fluctuations due to technology shocks are roughly constant over time. This has two main implications. First, technology shocks can hardly be considered the main cause of the changes observed in the US business cycles in terms of size of fluctuations. Second, because fluctuations associated to technology shocks are roughly constant while those associated to the non-technology component reduce over time, this means that contribution of technology shocks must have increased after mid 80's.

A new interesting feature emerges in our framework. The non-technology component includes two elements: the non-technology shocks $(e_{2t,}...e_{4t})$, and a second part resulting from shocks in the time-varying intercept term propagated by the stochastically time-varying coefficients. Adding the portion of variance explained by technology and non-technology shocks a portion of output variance of about 5-15%, depending on the particular specification, is left unexplained. This means that even if no such shocks occur, nonetheless we could observe fluctuations in output and hours accounting for about the 5-15% of the variance of actual output fluctuations and generating correlations of about 0.8-0.9. This finding is clearly ruled out in fixed coefficients. However when the linear structure is replaced by a non-linear one in which non-linearity comes from stochastically varying coefficients, multiplicative disturbances and shocks to the intercept term play a role in shaping US business cycles fluctuations. This evidence is consistent with the idea that transition dynamics arising from changes in trends or means are gradual instead of abrupt and they generate substantial movements in output and hours which are recognizable at the business cycles frequencies.

3.4.3 Testing Time-Variations

We perform two types of test: the first is an informal test on the rate of drift of the reduced form coefficients, while the second is based on posterior intervals for the differences in impulse response functions. Recall that Ω represents the variance of the shocks in the unrestricted law of motion of the coefficients. As shown in the appendix, we calibrated the prior scale matrix, Ω_0, so that a priori there is a high probability of small changes in the coefficients. In all the specifications the posterior distribution of $tr(\Omega)$ shifts to the right of $tr(\Omega_0)$, with a 80-90% of posterior mass concentrated on values higher than $tr(\Omega_0)$. This means that the data are shifting the distribution toward a region of higher, compared to our prior, coefficients time variations. In other words data seem to favor a specification in which coefficients are varying over time. Figure 9 exemplifies the result for the bivariate case with hours in levels: the trace of the prior scale matrix, $tr(\Omega_0)$ (the segment) lays on the left tail of the posterior histogram of $tr(\Omega)$.

The second test is a simple posterior interval test. The idea is to test whether the responses are different over the sample. Let $\bar{t}$ be some fixed date. For all $t = 1, ..., \bar{t} - 1, \bar{t} + 1, ..., T$ we draw from the posterior distribution of the impulse response functions to characterize the posterior of

$$D(t, \bar{t}, k) = IR_{2,1}(t, k) - IR_{2,1}(\bar{t}, k)$$

which is the difference between the response of hours at time t and $\bar{t}$ at lag k to a technology shock. We take a posterior interval centered at the posterior mean of $D(t, \bar{t}, k)$ and we check whether the zero is included. In case of no significant time-variations we should find that zero is included in the interval for all t. In the levels specification we set $\bar{t}$ =1998:III[17]. For

[17] We choose $\bar{t}$ to be the date in which the 68% lower bound for the impact effect is higher. That is the date in which is more probable to find differences with the responses at other dates.

$k = 0$, we find that there are 42 dates, concentrated between 1972 and 1981, for which the difference is significantly different from zero. At such dates the posterior probability of the impact effect to be smaller than the impact effect in 1998:II is on average about 0.9. For $k = 4$, one year after the shock, there are 4 dates, between 1978 and 1979, for which the response is different from zero. For the bivariate case numbers are very similar: we find 36 dates for k=0 and 4 for k=4 in which the differences are significant. For the growth rates specifications we choose 2003:III. In this case we do not find significant differences in the responses, probably because the high uncertainty surrounding the response after early 90's makes the confidence band for $D(t, \bar{t}, k)$ extremely wide.

3.5 Fixed vs. Time-Varying Coefficients VARs

We compare our findings with those arising from standard VAR. In order to make the comparison clear and simple we limit the attention to the bivariate specification.

3.5.1 What Explains the Differences?

Once time variations are allowed for, hours significantly reduce on impact at least until mid 90's also when specified in levels. Why do results change with respect to the fixed coefficients case? The goal here is to investigate what are the reduced form coefficients responsible for the switch in sign of the response. We proceed as follows: first we divide all the reduced form coefficients in four blocks, each corresponding to the coefficients of the lags of the same variable in one equation; second, we set all the coefficients belonging to the same block constant and equal to the corresponding fixed coefficient estimates; third, we draw from the posterior for all the remaining time-varying coefficients and we compute the implied impulse response functions; we repeat this procedure for all the blocks. The switch from negative to positive occurs when the block corresponding to hours worked in the labor productivity equation is set to be constant over time. In this case the implied impulse response functions at all dates are positive and hump-shaped (see Figure 10), whereas when the other coefficients are replaced the resulting dynamics are roughly unchanged, in particular the sign of the response is unaffected. Moreover, time variations in the response of hours completely disappear, the impact effect being nearly constant over the whole sample. Therefore, such coefficients not only account for the switch of the sign, but they also seem to drive time variations in the response of hours.

Figure 11 focuses on both the fixed and time-varying estimates of the coefficients of the lags of hours worked in the labor productivity equation. Few features are worth noting. First, all the time-varying estimates, apart the coefficient for lag one which is roughly constant over-time, display the same pattern. They are U-shaped with a clear upward trend starting from mid 80's and crossing fixed coefficients estimates at some date around the end of the 80's (for lag 2) and the beginning of the 90's (for lag 3 and the sum of lagged coefficient). Second, the long-run coefficient, the sum of lagged coefficients, seems to be the most important, from a quantitative point of view, in tracking time variations in the response of hours since it exactly matches the pattern of variations in the impact effect. Interestingly we find that the correlation between this coefficient and the impact effect is 0.9. Third, fixed coefficients estimates resemble a sort of weighted average of the time-varying estimates in which the weight attributed to the last part of the sample is higher than that attributed to the first part. This is probably due to the sharp and synchronized increase in labor productivity growth and per capita hours worked starting from early 90s. This is consistent with the finding that by running the analysis with fixed coefficients and hours in levels excluding from the sample the last ten years, hours fall.

As an additional check we re-estimate the model constraining the intercept term to be constant over time while letting all other coefficients vary. This is an important exercise since, as shown by Fernald (2004), once one allows for trend breaks in fixed coefficients VAR, hours fall also in levels. The reason, he claims, is that short-run dynamics are dominated by a non causal low frequencies correlation between labor productivity growth and the levels of hours worked attributable to synchronized changes in the means of the two series. Therefore, it could be that these, rather than changes in VAR coefficients, are the true responsible for the negative response of hours. If actually trend changes are responsible for the switch, by constraining them we should observe a rise of hours. We find that hours reduce and the response both in terms of persistence and size is nearly identical to the benchmark case. Thus, although probably important, changes in trend labor productivity do not seems to be the main, or at least the only, factor affecting dynamics of hours worked.

3.5.2 Encompassing Fixed Coefficients Specifications

CEV show that when the true model is the VAR with hours in levels and the analysis is performed using hours in growth rates, hours fall. The converse is not true: when the growth rates is the true model and hours are specified in levels again hours reduce. Therefore, they argue that the specification with hours in levels is more plausible since it can explain also the results of the misspecified model while the growth rates specification does not. Here, using a similar approach we investigate whether our model can encompass fixed coefficients VARs. Specifically we study whether, by running the analysis with fixed coefficients and data generated by the time varying-coefficients model, we can replicate the two basics facts: hours fall in growth rates and increase in levels. We proceed as follows. We assume that the time varying coefficients VAR is the true model and we set all the coefficients at their posterior mean values. Using the *true* model we generate 500 new time series data for labor productivity growth and hours worked. Then for each new vector of time series we estimate the response of hours to technology shocks under fixed coefficients using both specifications, levels and growth rates. Finally we average over the 500 responses.

Figure 12 displays the results when the time varying coefficients VAR with hours in growth rates is assumed to be the true model. Solid and dotted lines represent the responses of hours, specified in levels and growth rates respectively, estimated with fixed coefficients VARs and actual data. Starred lines, solid and dotted, represent the same responses but arising with simulated data and averaged over the 500 realizations. When the true model is the time varying coefficients VAR with hours in growth rates hours decline under both specifications. This means that, while easily explaining the Gali's results, the model fails in explaining the CEV's results, since the response of hours is negative instead of being positive. Figure 13 displays the same responses but when the time varying coefficients VAR with hours in levels is assumed to be the true model. In this case hours reduce when specified in first differences and increase when specified in levels, exactly as with actual data. The model encompasses both fixed coefficients specifications, since the misspecified VAR exactly reproduces the results of Gali and CEV. This means that using fixed coefficients VAR and hours in levels we would conclude that hours increase while the true model implies a significant decline in hours for most of the sample period. Under this encompassing criterion the time-varying levels specification seems to perform better than the growth rates one since it is able to explain all the results previously found in literature.

3.6 Structural Explanations for the Dynamics of Hours Worked

3.6.1 Explaining the Decline of Hours

There exist basically two classes of structural explanations of why hours worked may fall after a positive technology shock. The first relies on the presence of some frictions in the economy, while the second relies on frictionless models in which technology generates large wealth effects. Here we investigate whether theoretical predictions match our empirical findings.

Nominal vs. Real Frictions

A first explanation of the decline of hours relies on the presence of sticky prices and a not completely accommodative monetary policy. The intuition originally provided by Gali (1999) is the following. Consider an economy where in equilibrium output equals real balances, prices are set in advance and the monetary policy follows a simple money rule[18]. If, in response of a positive technology shock, monetary policy is not sufficiently accommodative and aggregate demand expands less than the increase due to the technological improvement, then employment must fall in order to keep supply and demand in the goods market in equilibrium[19]. A second explanation relies on the presence of some real rigidities. Francis and Ramey (2001) propose a modification of the standard RBC model which can potentially account for the reductions of hours after a technological improvement. The basic ingredients are habit formation in consumption and capital adjustment costs. The authors show that the response of consumption and investment is much more sluggish than in the standard case because consumers prefer not to change consumption by too much and investment is

[18]Similar mechanisms generate from more complete dynamic models in which price predeterminacy is substituted with a Calvo-type random price adjustment, see e.g. King and Wolman (1996).

[19]While monetary policy is a crucial ingredients for such an explanation, it should be stressed that a money target rule is not a necessary condition to generate the fall in hours worked; actually some authors (Basu, 1998, Gali and Rabanal, 2004) show that sticky prices model with more realistic policy rules, like a Taylor rule, are still able to generate the decline in hours.

made expensive by the capital adjustment costs. Thus if the resulting increase in output is smaller than the increase in productivity hours must fall.

The two explanations have, as stressed by Francis and Ramey (2004), very different implications in terms of the response of real wages. In the sticky price model real wages either fall or at most increase by little on impact and then they gradually converge to a new higher steady state level. On the other hand the habit formation adjustment costs model predicts that real wages immediately rise overshooting the long-run level slightly. We reestimate the model adding the real wage. The resulting response of wages closely track from a qualitative point of view that of productivity. Specifically real wages slightly increase on impact and then gradually rise until reaching the new long-run level. While in sharp contrast with the predictions of the model embedding real rigidities, the behavior of wages appears to be roughly consistent with sticky prices models.

Wealth Effect and Slow Technological Change

In an important paper, Campbell (1994) showed that technology improvement may generate "perverse effects" on labor inputs. Contrary to common wisdom, in a RBC model a persistent and permanent negative technology shock (what the author calls a "productivity slowdown") may actually increase hours worked for some quarters. The reason is that, due to its slow diffusion, the shock triggers a large wealth effect that dominates the substitution effect in the short-run and makes consumers to substitute leisure for work. The increase in hours can be so sustained that output can rise in the very short-run. By reversing the sign of the shock, the above mechanism could explain why a technological improvement may actually reduce, instead of raising, hours worked. A similar mechanism emerges in the recent works by Linde (2004) and Rotemberg (2000).

One of the main implications of the dynamics arising from those models is that both consumption and consumption-to-output ratio must increase on impact. The former increases because of the wealth effect, while the second increases because investment reduces since agents anticipate that marginal productivity of capital will be higher in the future. In order to assess the relevance of this explanation we estimate the model adding consumption and investigating the response of hours and both consumption and consumption-to-output ratio. As in previous specification hours fall on impact and the dynamics are almost unchanged. Consumption rises on impact over all the sample although the response is not significantly different from zero except for few year at the end of the 90s. On the contrary consumption-to-output declines on impact for all the dates and until mid 80s the response is also significantly different from zero. The sign of the response of consumption-to-output ratio is at odds with the predictions of RBC models with slow technological changes. Therefore while it cannot be excluded that large wealth effects stand behind the reduction of hours worked, such effects do not seem to be generated by technological progress diffusing slowly throughout the economy.

3.6.2 Explaining Time Variations: the Fed's Time-Varying Response

From an econometric point of view the reduction in magnitude of the response of hours on impact seems to depend to a large extent on changes in the reduced form coefficients in the labor productivity equation. Nevertheless there could be several possible structural explanations for this pattern. As mentioned earlier, in a sticky prices model the response of hours worked crucially depends on the monetary policy conduct. The more expansionary is the monetary policy after the technological improvement, the smaller is the decline of hour worked because the higher is the expansion in the aggregate demand. Therefore, changes in monetary policy could explain why the response of hours has changed over time. Some authors (see e.g. Orphanides and GLV) argue that before 1979 monetary authorities had a less aggressive stance against inflation and were giving more importance to output stabilization. Due to mismeasurements of potential output, movements in interest rate overshooted those prescribed by the optimal rule and policies adopted before 1979 turned out to be overstabilizing. Such a conjecture could explain why the fall of hours worked is more pronounced before mid 80's than after[20].

Our framework allows us to study whether changes in the response of hours depend on shifts in monetary policy preferences. We estimate a simple Taylor rule in which the interest rate responds only to contemporaneous inflation and output growth[21]

$$i_t = a_t \pi_t^{tech} + b_t \Delta y_t^{tech} + \varepsilon_t \tag{3.7}$$

where i_t is the federal funds rate and π_t^{tech}, Δy_t^{tech} are respectively the component of inflation and output growth associated to technology shocks[22]. The previous explanation would hold if the coefficient on output is high before mid 1980 than after. Figure 14 displays the two coefficients, a_t b_t, along with the 68% confidence bands. First, consistently with a large amount of evidence in empirical literature, we find that monetary policy stance becomes more aggressive against inflation from early 80's, the coefficients raising from about 1 during the 70's up to 2.5-3 during the 80's. However, differently from what is argued by the majority of works, and consistently with a growing stream of literature (see e.g., Bernanke and Mihov, 1998, Canova, 2004, Canova and Gambetti, 2004, Primiceri, 2005, Sims, 2001, and Sims and Zha, 2004) the change does not represent a permanent break. Interestingly around 1992 the coefficient reduces again, being about 1.4, and in 2001 is not significantly different from the 70's level, around 1. Second, the coefficient on output is almost constant over all the sample, in particular we do not find a significant change after mid 80's. Hence the result is hardly consistent with a primary role of monetary policy in shaping changes in the transmission of technology shocks[23].

[20] Similar results can arise in a framework where monetary authorities are learning, see Lansing (2000).

[21] An alternative strategy would be to compute the ratio between the response of the interest rate and inflation and output growth. We do not follow this strategy because in that case we would not control for the other variables.

[22] Note that by construction the regressors exogenous and orthogonal to the residuals justifying the Kalman Filter estimation.

[23] The same conclusion is reached by looking at the response of the real interest rate. Actually we do not

3.7 Robustness and Extensions

We perform a number of robustness checks. We first check whether our results are robust to the choice of the end-of-sample date and definitions of IRF and second whether alternative identification schemes give qualitatively similar results. In addition, we extend the model to consider also investment-specific technology shocks.

3.7.1 Alternative End-of-Sample Dates and IRF Definitions

The definition of impulse response functions used in the paper has a potential drawback when identification is achieved with long run restrictions. Structural short-run dynamics for each date in the sample depend on the end-of-sample coefficients matrix $\mathbf{A}_T$. We choose as end-of sample date the last available observation for the data in order to maximize the available information. However, it could be that different choices of T yield different results, in particular for the last part of the sample. Therefore, we cut the sample at arbitrary dates, we choose two and four years before the last available observation and we run the analysis using $\mathbf{A}_{T-8}$ and $\mathbf{A}_{T-16}$. As expected small quantitative differences emerge mainly for the quarters in the last part of the sample. However our main conclusions are very robust. First, the response of hours worked is still negative at all horizons with shapes almost identical to those resulting from the benchmark case. Second, the size of the impact effect is reducing over time in absolute value, in particular the response of hours is not significantly different from zero after early 90's.

We also check if results are robust using a different definition of impulse response functions. Specifically, we sample future coefficients from the prior density conditional to a draw from the posterior for the in-sample-coefficients. In so doing we take into account future coefficients variation. Clearly, we have to discard the draws which yield impulse response functions which do not satisfy some convergence criterion. Also in this case, mean impulse response are almost identical to the benchmark case.

3.7.2 Sign Restrictions

Recently, a number of papers questioned the validity of the conclusions drawn using long-run restrictions (see e.g. Uhlig, 2003). Some authors (Francis, Owyang and Theodorou, 2003, Dedola and Neri, 2004, and Peersman and Straub, 2004, 2005), taking a radically different approach, suggest to use inequalities restrictions directly derived from DSGE models, in the spirit of the restrictions originally proposed by Canova and De Nicolo (200) and Uhlig (2005). Here we check the robustness of results when long-run identifying restrictions are replaced with sign restrictions. We take as identifying restrictions a set of sign inequalities which are robust under different specifications of the technology process. Specifically we assume that a positive technology shock (i) does not raise inflation and the interest rate for three quarters and (ii) does not decrease labor productivity for 40 quarters after the

find evidence that the real interest rate reacts more before 1979 than after. Nevertheless we do find a clearly declining trend in the response of the real rate but only starting from early 90's and lasting until 2000.

shock. We leave all the other shocks unidentified. We use the same draws for reduced form coefficients used under long-run restrictions and the implementation of the restrictions is identical as Canova Gambetti and Pappa (2005). Again per capita hours worked fall after a positive technology shock. The impact effect is negative, the responses reach their minimal level between the first and second quarter after the shock and then they begin to climb back toward the pre-shock level and after between one and two years the responses become positive. Responses are qualitatively similar to those found under long-run restrictions while time variation seem to be relatively limited: in fact responses are roughly similar at all dates.

3.7.3 Investment-Specific Technology Shocks

Greenwood, Hercowitz and Krussel (2000) (GHK henceforth) put forward a version of the RBC model in which the main source of technological progress is not of the aggregate sector neutral kind as we identified but rather is specific to the investment sector. Using a calibrated version of the model, the authors find, that investment-specific technology shocks explain about 30% of output fluctuations. Similarly, Fisher (2005) through VAR analysis finds that unlike neutral shocks investment specific technological change contribute for about 40-60% to aggregate fluctuations. We investigate how results change when also investment-specific technology shocks are considered in the analysis. We estimate the TVC-BVAR using, in the following order, real price of investment, labor productivity and per capita hours worked. Following the identification scheme proposed by Fisher (2005), we assume that (i) neutral and investment-specific technology shocks are the only shocks affecting long run labor productivity and that (ii) investment-specific technology shocks the only shock affecting long run real price of investment. Using the previous recursive long run scheme, the first shock will be the investment-specific and the second the sector-neutral shock. Differently from the benchmark case here both shock may affect long run labor productivity.

Unlike the case of neutral technological progress, hours increase at all dates after an investment-specific technology shocks and except for some quarters around early 80's the response is particularly persistent and hump-shaped. Furthermore, in response to neutral technology shocks hours fall in both specifications. Table 2 documents the contribution of the two types of technology shocks to aggregate fluctuations and the implied correlations among variables. Panel A refers to the levels specification, panel B to the first difference specification. The two technology shocks together explain about 39-53% of the total volatility of output ad hours worked at business cycles frequencies, depending on the particular specification. In particular neutral technology shocks, as in the benchmark case, account for about 10-20% while investment-specific for about 20-30% of the total variability at the business cycles fluctuations for both variables. Interestingly investment-specific shocks generate a high correlation between output and hours, about 0.8-0.9, which is similar to the one found in actual data, while correlation generated by neutral shocks are similar to the previous case, about 0.5-0.6. When also investment-specific shocks are included in the analysis the importance of technology shocks on the whole in explaining aggregate fluctuations is remarkably increased. On the other hand, results for neutral technology found previously are confirmed here.

3.7.4 Sensitivity to the Choice of Variables

Finally we check whether results are sensitive to the choice of variables. CEV argue that it is important, at least in fixed coefficients VARs, to include consumption-to-output and investment-to-output ratio. Taking their suggestion we estimate the model using a different specification including labor productivity growth rates, hours consumption-to-output and investment-to-output ratio. Results using the new specification are qualitatively very similar to previous results. In the growth rates specification hours reduce persistently at all dates. In the levels specification the pattern of the response of hours worked is almost identical to the bivariate case. The response is negative, particularly persistent and significant on impact until mid 90's. From mid 90's the mean response turns positive and humped shaped but not significantly different from zero on impact. Table 3 displays the implied correlations and percentages of variances explained by technology shocks. When hours are specified in levels technology shocks generate a correlation between output and hours of 0.79 and explain about the 38% of the total output variance. Numbers are slightly higher than in the benchmark specifications. In the growth rates specification technology shocks generate a correlation between output and hours of 0.52 and the percentage of explained output variance is about 17%. Also under the new specification main conclusions are confirmed.

3.8 Conclusions

The response of hours worked to technological improvements is a key issue in assessing the relevance of different theoretical characterizations of the business cycle. From the point of view of the empirical research, evidence in favor of both a decline and a rise of hours worked emerges. Results crucially depend on how the time series for hours worked is specified in the VAR. In this paper we argue that conflicting results may arise because important time variations and structural changes the US economy underwent during the postwar period are a priori ruled out by standard models. In other words, we argue that differences in the results depending on the particular specification for hours worked may simply originate from a more fundamental misspecification arising from the too strong assumption of model coefficients constancy. We investigate the effects of technology shocks on hours worked using a Bayesian Vector Autoregression with drifting coefficients augmented with the same standard restriction used in the literature, that is the technology shock is the only shock affecting long-run labor productivity.

Time-varying dynamics matter. Once time variations are allowed for, competing empirical specifications (levels and growth rates) yield similar results: hours fall at least until mid 90s. The decline is particularly pronounced and statistically different from zero until early 90's, while after that date hours are less responsive to technology shocks. We argue that the differences between fixed and time-varying coefficients are due to instabilities in the coefficients of hours worked in the labor productivity equation. Other findings complement our main result. Aggregate sector neutral technology shocks of the kind emphasized by RBC proponents can hardly be considered the only force driving business cycles since they can only explain about 11-25% of the total output variance. Nevertheless when also investment-specific technology shocks are considered, the percentage of output variance accounted for by technology shocks as a whole is remarkably increased.

The decline of hours worked is in line with models of nominal rigidities or with RBC models in which technology generates large wealth effects. However while the negative sign of the response has reliable structural explanations, time variations in the size of the response are left unexplained. Actually changes in the monetary policy conduct are not able to account for the reduction in absolute value of the impact effect on hours. So why are technology shocks less and less contractionary beginning from early 90's? We leave the answer to this question to future investigations.

Figures

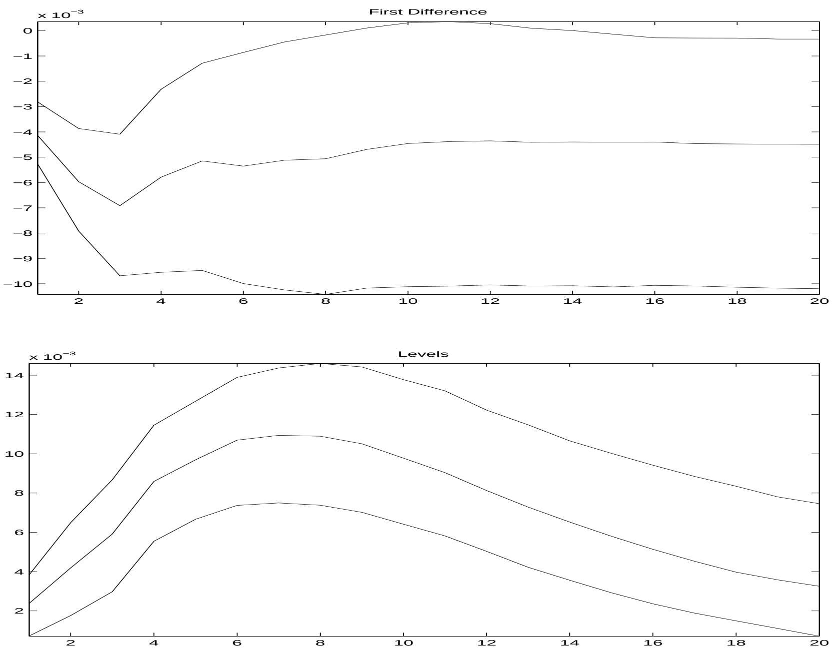


Figure 1:Effects of technology shocks on hours worked (first differences and levels specification in top and bottom panel respectively) in the bivariate VAR, full-sample.

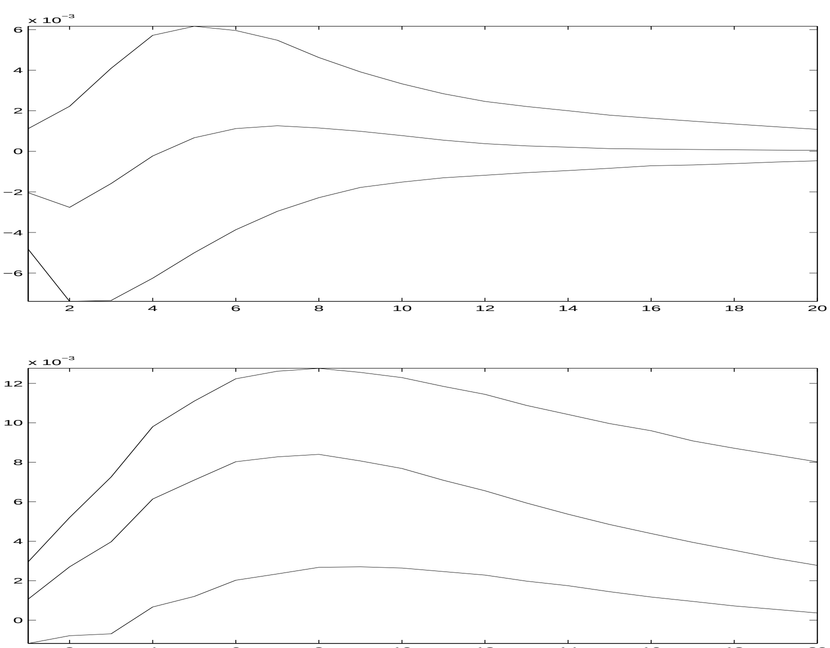


Figure 2: Effects of technology shocks on hours worked (levels) in two subsamples: 1954:III-1979:IV in the top panel, 1982:III-2003:IV in the bottom panel.

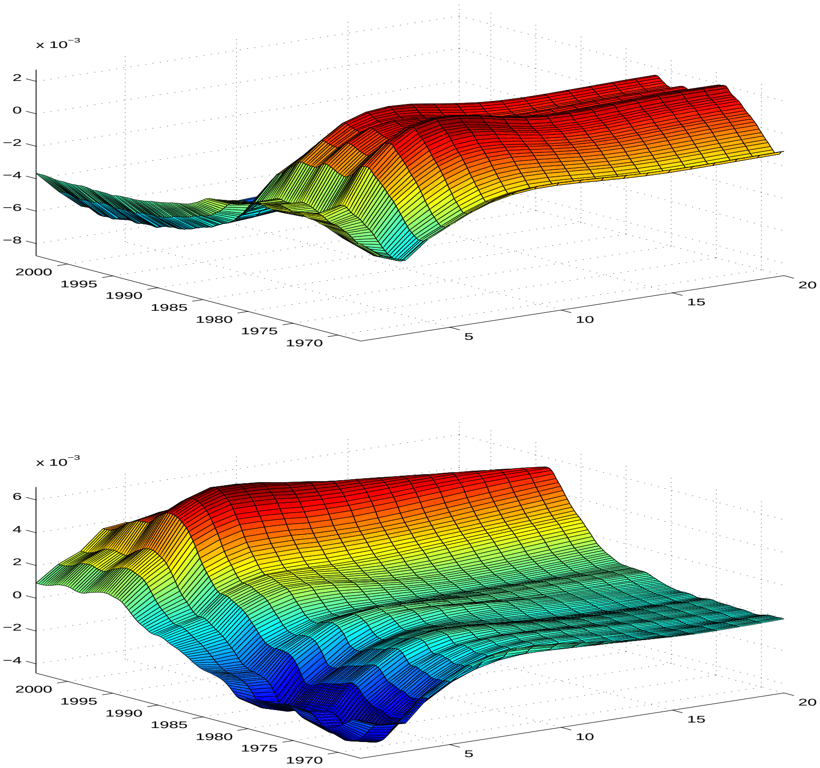


Figure 3: Response of hours worked to a technology shock in the bivariate VAR: top panel first difference specification, bottom panel levels specification.

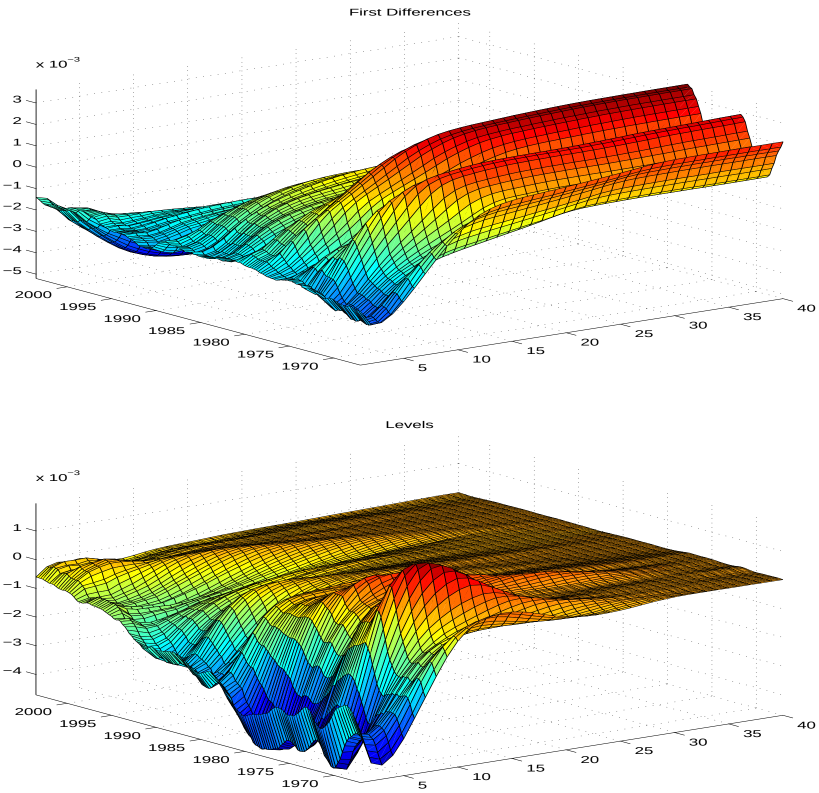


Figure 4: Response of hours worked to a technology shock in the $R\pi$ VAR: top panel first difference specification, bottom panel levels specification.

Figure 5: Impact effects of technology shock on hours worked in the $R\pi$ VAR, posterior median and 68% confidence bands. Top panel first difference specification, bottom panel levels specification.

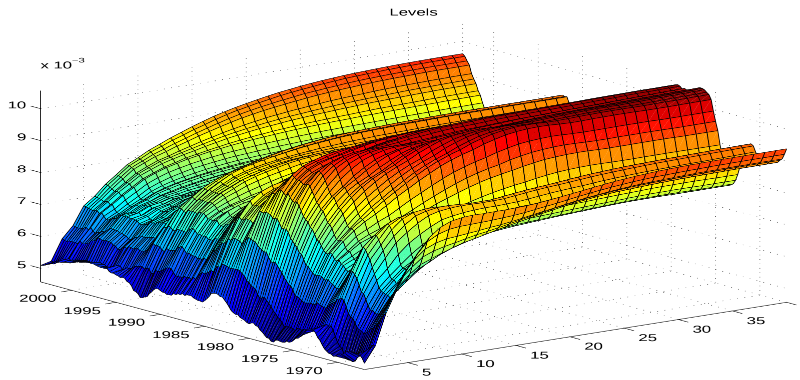


Figure 6: Response of labor productivity to a technology shock in the $R\pi$ VAR, levels specification.

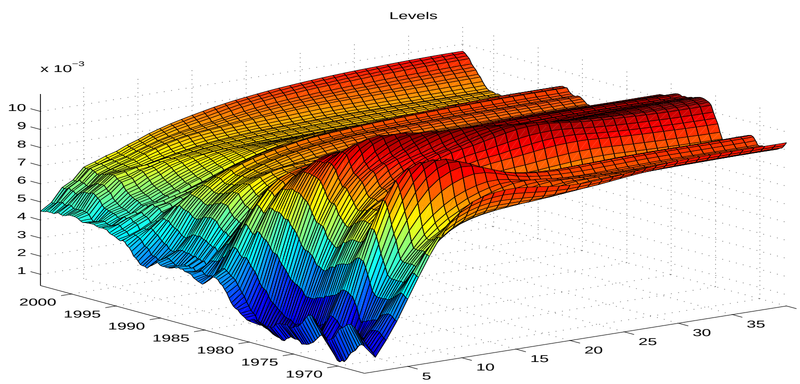


Figure 7: Impulse response functions of output to a technology shock in the $R\pi$ VAR, levels specification.

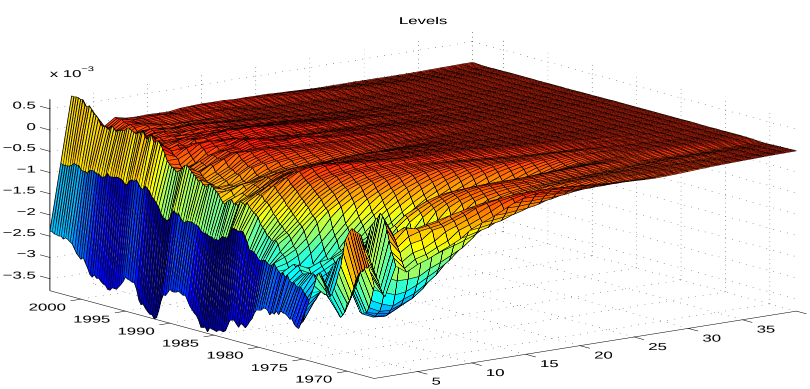


Figure 8: Impulse response functions of inflation to a technology shock in the $R\pi$ VAR, levels specification.

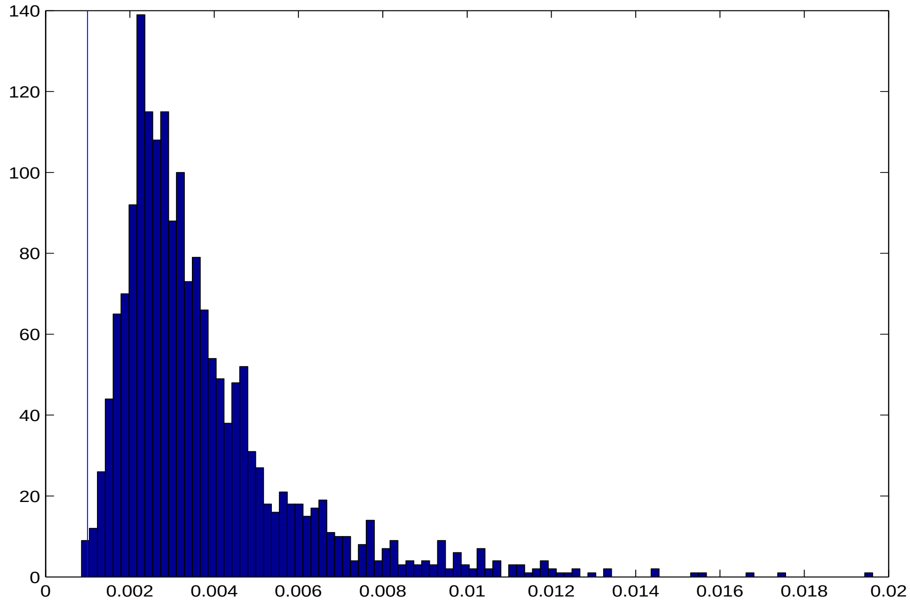


Figure 9: Trace of the posterior (histogram) and prior (segment) variance matrix of the coefficients.

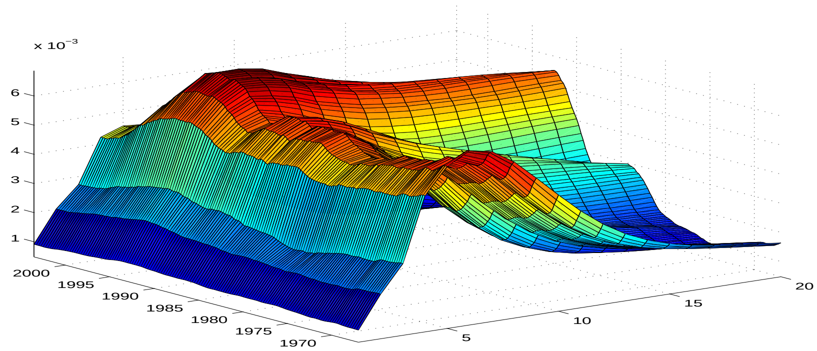


Figure 10: Top panel impulse response of levels of hours with coefficients of hours in the labor productivity equation replaced, bivariate VAr with hours in levels.

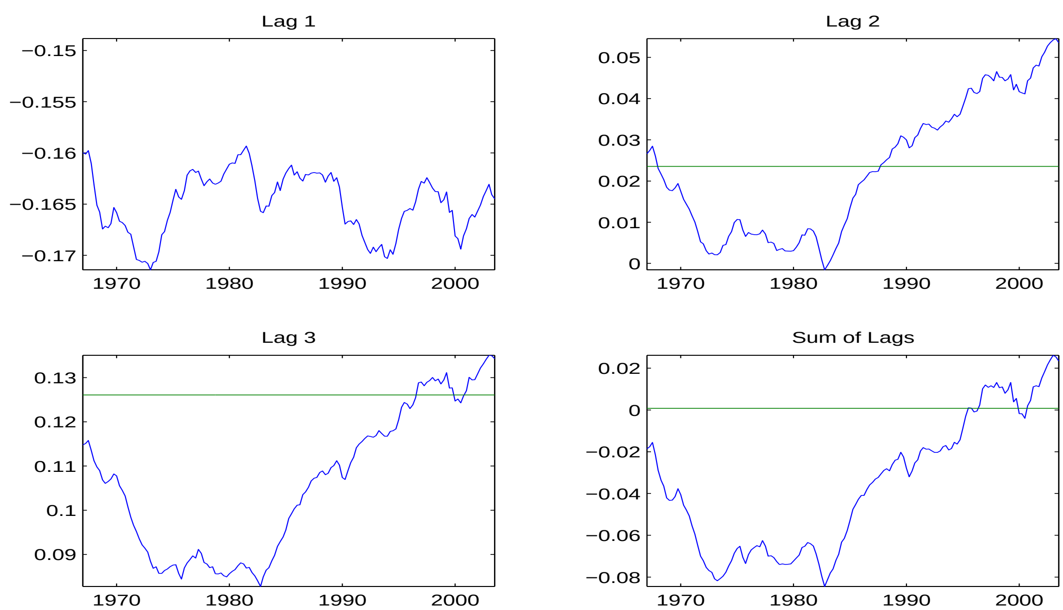


Figure 11: Estimates of lagged coefficients of hours worked in the labor productivity equation in the fixed and time-varying coefficients VAR.

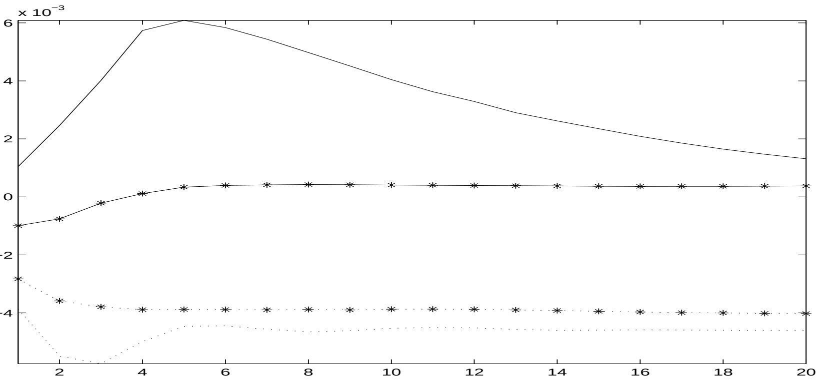


Figure 12: Encompasing test for the growth rates specificaion. Dotted line: response of hours in the growth rates specification using real data. Solid line: response of hours in the levels specification using real data. Dotted starred line: response of hours in the growth rates specification using data generated by the time varying model with hours in growth rates. Solid starred line: response of hours in the levels specification using data generated by the time varying model with hours in growth rates.

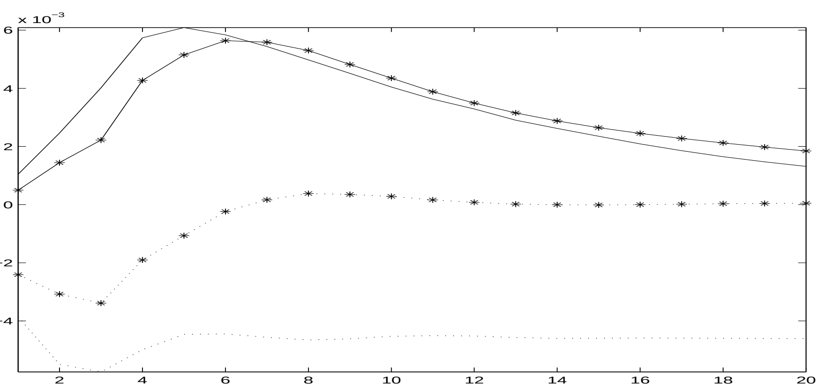


Figure 13: Encompasing test for the levels specificaion. Dotted line: response of hours in the growth rates specification using real data. Solid line: response of hours in the levels specification using real data. Dotted starred line: response of hours in the growth rates specification using data generated by the time varying model with hours in levels. Solid starred line: response of hours in the levels specification using data generated by the time varying model with hours in levels.

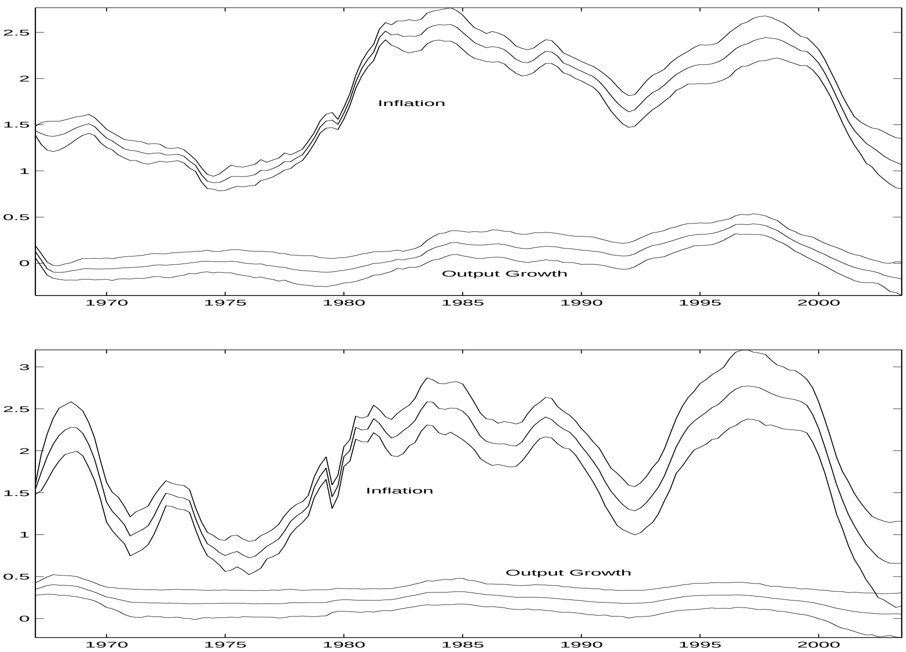


Figure 14: Central banks preferences: coefficients on inflation and output growth in levels (top panel) and growth rate (bottom panel) specifications.

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Appendix

A Estimation

Priors

We assume θ_0, Σ and Ω to be independent. We specify the following prior distributions[24]

$$\begin{aligned} p(\theta_0) &= N(\bar{\theta}, \bar{P}) \\ p(\Sigma) &= IW(\Sigma_0^{-1}, \nu_0) \\ p(\Omega) &= IW(\Omega_0^{-1}, \nu_0) \end{aligned}$$

The joint prior is

$$\begin{aligned} p(\theta^T, \phi) &= p(\theta^T|\phi)p(\phi) \\ &\propto \mathcal{I}(\theta^T) f(\theta^T|\phi) p(\theta_0) p(\Sigma) p(\Omega) \end{aligned}$$

where $\mathcal{I}(\theta^T) = \prod_{t=0}^{T} \mathcal{I}(\theta_t)$ and $f(\theta^T|\phi) = f(\theta_0|\phi) \prod_{t=0}^{T-1} f(\theta_{t+1}|\theta_t, \phi)$.

Posterior Density

The posterior density, $p(\theta^T, \phi|y^T)$ can be decomposed as

$$p(\theta^T, \phi|y^T) \propto p(y^T|\theta^T, \phi) p(\theta^T, \phi)$$

where the first term of the right hand side is the likelihood and the second the joint posterior density. Conditional to the states up to time T and the hyperparameters the measurement equation is linear with Gaussian innovation, thus the conditional likelihood is Gaussian. The second term can be splitted into a conditional and a marginal density thus we have

$$\begin{aligned} p(\theta^T, \phi|y^T) &\propto f(y^T|\theta^T, \phi) p(\theta^T|\phi) p(\phi) \\ &\propto \mathcal{I}(\theta^T) \left[f(y^T|\theta^T, \phi) f(\theta^T|\phi) p(\phi) \right] \end{aligned}$$

[24]For the block diagonal and diagonal specification we use respectively $p(\Omega_i) = IW(\Omega_{i0}^{-1}, \nu_0)$, where i refers to the $i - th$ equation and $p(\Omega_{ii}) = IG(\frac{1}{2}, \frac{\Omega_{ii0}}{2})$.

where from the first to the second line we used the restricted prior distribution for the states. Note that the term in brackets is the posterior without the restriction on autoregressive coefficients. Thus our posterior distribution is proportional to the unrestricted posterior density, p_u,

$$p(\theta^T, \phi|y^T) \propto \mathcal{I}(\theta^T) p_u(\theta^T, \phi|y^T)$$

This is particularly convenient since we can first characterize the unrestricted posterior and then perform the rejection sampling (see below) to collect the draws satisfying the restriction.

Drawing from the posterior of reduced form parameters

The Gibbs Sampler we use to compute the posterior for the reduced form parameters iterate on two steps. The implementation is identical to Cogley and Sargent (2001).

- Step 1: States given hyperparameters

Conditional on y^T, ϕ, the unrestricted posterior of the states is normal and $p_u(\theta^T|y^T, \phi) = f(\theta_T|y^T, \phi) \prod_{t=1}^{T-1} f(\theta_t|\theta_{t+1}, y^t, \phi)$. All densities on the right end side are Gaussian they their conditional means and variances can be computed using the Kalman backward filter. Let $\theta_{t|t} \equiv E(\theta_t|y^t, \phi); P_{t|t-1} \equiv Var(\theta_t|y^{t-1}, \phi); P_{t|t} \equiv Var(\theta_t|y^t, \phi)$. Given $P_{0|0}$, $\theta_{0|0}$, Ω and Σ, we compute Kalman filter recursions

$$\begin{aligned}
\theta_{t|t-1} &= F\theta_{t-1|t-1} \\
P_{t|t-1} &= FP_{t-1|t-1}F' + \Omega \\
K_t &= (P_{t|t-1}X_t)(X_t'P_{t|t-1}X_t + \Sigma)^{-1} \\
\theta_{t|t} &= \theta_{t|t-1} + K_t(y_t - X_t'\theta_{t-1|t-1}) \\
P_{t|t} &= P_{t|t-1} - K_t(X_t'P_{t|t-1})
\end{aligned}$$

The last iteration gives $\theta_{T|T}$ and $P_{T|T}$ which are the conditional means and variance of $f(\theta_t|y^T, \phi)$. Hence $f(\theta_T|y^T, \phi) = N(\theta_{T|T}, P_{T|T})$. The other $T-1$ densities can be computed using the backward recursions

$$\begin{aligned}
\theta_{t|t+1} &= \theta_{t|t} + P_{t|t}F'P_{t+1|t}^{-1}(\theta_{t+1} - F\theta_{t|t}) \\
P_{t|t+1} &= P_{t|t} - P_{t|t}F'P_{t+1|t}^{-1}FP_{t|t}
\end{aligned}$$

where $\theta_{t|t+1} \equiv E(\theta_t|\theta_{t+1}, y^t, \phi)$ and $P_{t|t+1} \equiv Var(\theta_t|\theta_{t+1}, y^t, \phi)$ are the conditional means and variances of the remaining terms in $p_u(\theta^T|y^T, \phi)$. Thus $f(\theta_t|\theta_{t+1}, y^t, \phi) = N(\theta_{t|t+1}, P_{t|t+1})$. Therefore, to sample θ^T from the conditional posterior we proceed backward, sampling θ_T from $N(\theta_{T|T}, P_{T|T})$ and θ^t from $N(\theta_{t|t+1}, P_{t|t+1})$ for all $t < T$.

• Step 2: Hyperparameters given states

Since (Σ, Ω) are independent, we can sample them separately. Conditional on the states and the data ε_t and u_t are observable and Gaussian. Combining a Gaussian likelihood with an inverse-Wishart prior results in an inverse-Wishart posterior, so that

$$\begin{aligned} p(\Sigma|\theta^T, y^T) &= IW(\Sigma_1^{-1}, \nu_1) \\ p(\Omega|\theta^T, y^T) &= IW(\Omega_1^{-1}, \nu_1) \end{aligned}$$

where $\Sigma_1 = \Sigma_0 + \sum_{t=1}^T \varepsilon_t \varepsilon_t'$, $\Omega_1 = \Omega_0 + \sum_{t=1}^T u_t u_t'$, $\nu_1 = \nu_0 + T$[25].

Under regularity conditions and after a burn-in period, iterations on these two steps produce draw from $p_u(\theta^T, \Sigma, \Omega|y^T)$. We have constructed CUMSUM graphs to check for convergence and found that the chain had converged roughly after 2000 draws for each date in the sample. The densities for the parameters obtained with the remaining draws are well behaved and none is multimodal. We keeping one every four of the remaining 18000 draws and discard all the draws generating non convergent impulse response functions. The autocorrelation function of the 2000 draws which are left is somewhat persistent.

The Rejection Sampling

This second step ensures that posterior density puts zero probability to draws which do not satisfy the restriction on impulse response functions convergence[26]. The implementation of the rejection sampling is very similar to those in Cogley and Sargent (2001). First we need a candidate density $g(\theta^T, \phi)$, satisfying three properties: i) must be non negative and well defined for all (θ^T, ϕ) for which $p(\theta^T, \phi|Y^T) > 0$; ii) it must have finite integral; iii) the importance ratio $R(\theta^T, \phi)$ must have an upperbound Z

$$R(\theta^T, \phi) = \frac{p(\theta^T, \phi|Y^T)}{g(\theta^T, \phi)} \leq Z < \infty$$

where

$$p(\theta^T, \phi|Y^T) = \frac{\mathcal{I}(\theta^T) p_u(\theta^T, \phi|y^T)}{\int\int \mathcal{I}(\theta^T) p_u(\theta^T, \phi|y^T) d\theta^T d\phi}$$

A natural candidate density is the unrestricted posterior $p_u(\theta^T, \phi|y^T)$ because is a probability density, integrates to one and it is non-negative and it is defined for all (θ^T, ϕ). Moreover we have

$$R(\theta^T, \phi) \leq \frac{1}{\int\int \mathcal{I}(\theta^T) p_u(\theta^T, \phi|y^T) d\theta^T d\phi} = Z$$

[25] For the block diagonal and diagonal specification we have $p(\Omega_{ii}|\theta^T, y^T) = IG(\frac{T+1}{2}, \frac{(T+1)\Omega_{ii1}}{2})$ where $\Omega_{ii1} = \Omega_{ii0} + \sum_{t=1}^T u_{it}^2$ and when block-diagonal $p(\Omega_i|\theta^T, y^T) = IW(\Omega_{i1}^{-1}, \nu_1)$, $\Omega_{i1} = \Omega_{i0} + \Omega_{iT}$ and $\Omega_{iT} = \sum_{t=1}^T u_t^i u_t^{i\prime}$ where u_t^i is the vector of shocks in the coefficients of equation i.

[26] See Gellman, Carlin, Stern and Rubin (1995).

and Z is finite if the probability of a draw with associated convergent impulse response functions from the unrestricted posterior, the denominator, is non-zero. First we draw a trial (θ_i^T, ϕ_i) from the unrestricted posterior, second we accept it with probability $\frac{R(\theta^T,\phi)}{Z} = \mathcal{I}(\theta^T)$ that is with probability one if it satisfies restrictions or zeros if it does not.

B Impulse response functions

Convergence in chapter 3

Consider any $\tau < T$. Long-run impulse response and cumulated impulse response functions are given respectively by the limits

$$\lim_{k\to\infty} \mathbf{A}_T^k \mathbf{A}_T ... \mathbf{A}_{\tau+1}$$

$$\lim_{k\to\infty} \mathcal{A}_\tau + (I + \sum_{j=1}^{k} \mathbf{A}_T^j)\mathcal{B}_\tau$$

where $\mathcal{A}_\tau = I + \mathbf{A}_{\tau+1} + \mathbf{A}_{\tau+2}\mathbf{A}_{\tau+1} + ... + \mathbf{A}_{T-1}\mathbf{A}_{T-2}...\mathbf{A}_{\tau+2}\mathbf{A}_{\tau+1}$ and $\mathcal{B}_\tau = \mathbf{A}_T\mathbf{A}_{T-1}...\mathbf{A}_{\tau+2}\mathbf{A}_{\tau+1}$. If for any realization of $\mathbf{A}_T$ the largest eigenvalue is smaller than one in absolute value then impulse response converge pointwise to zero while long-run cumulated impulse response converge pointwise to $\mathcal{A}_\tau + (I - \mathbf{A}_T)^{-1}\mathcal{B}_\tau$. This comes from

$$\lim_{k\to\infty} \mathbf{A}_T^k = 0$$

$$\lim_{k\to\infty} (I + \sum_{j=1}^{k} \mathbf{A}_T^j) = (I - \mathbf{A}_T)^{-1}$$