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Integrated Distribution Management Problems: An Optimization Approach

by

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Chapter 1

Introduction

The Council of Logistics Management[1] defines Logistics Management as "that part of Supply Chain Management that plans, implements, and controls the efficient, effective forward and reverse flow and storage of goods, services, and related information between the point of origin and the point of consumption in order to meet customers' requirements."

Logistics management, from this total systems viewpoint, is the means whereby the needs of customers are satisfied through the coordination of materials and information flows that extend from the marketplace through the firm and its operations and beyond to suppliers (Christopher 1998). The process of integration along the supply chain of a firm to its suppliers and customer is known as external integration.

However, effective logistic operations cannot exist without the high level of coor-

[1] http://www.clm1.org/

dination among various functional areas. This means that there is also a need for internal integration. At minimum, such coordination requires efficient and accurate communication among the different logistics functions. While purchasing, production, logistics and marketing functions have each been integrated within their individual processes, there has been less progress integrating between these areas (Johnson et. al. 1999).

The most demanding issues in Logistics today remain all about **integration processes** (Christopher 1998). From these processes, firms gain important competitive advantages and leading organizations are paying attention to these issues. There has been a growing recognition of the importance of the integration topic, not only inside the firm but also on a research level. However, most of what has been written about integration on logistics refers to the integration of logistics information, the next level is to extract value out of the integrated information, and use it to improve the planning. The advanced level of integration makes use of integrated decision models in which the concerns of the different functional areas are taken into account in the decision process. The present work embraces this key issue of integrating logistic processes both inside the logistics field and with other areas of the firm, from a strategic planning point of view.

Logistics as defined above covers a broad spectrum of activities and the integration process can be divided into two different areas, internal integration and external integration. Therefore, there are various directions in the study of integration processes.

In our case, we will concentrate our analysis on integrating decisions between three specific areas, transportation, inventory and customer service, under the umbrella of the distribution management field.

One of these areas is **Transportation**, which generates one of the largest logistics costs and, in many cases, accounts for a significant portion of the selling price of some products. Transportation physically moves products to markets that are geographically separated and provides added value to customers when the products arrive on time, undamaged and in the quantities required. In this way, transportation contributes to the level of customer service, which is one of the cornerstones of customers satisfaction. In general, the benefits of improved transportation, include greater vehicle utilization, high levels of customer service, lower transportation costs, reduced capital investments in equipment and better management decision making.

In leading organizations, **Customer Service** will increasingly become the key strategic issue. Customer Service concerns will be also covered along our work and can be defined as the collection of activities performed in filling orders and keeping customers happy, or creating in the customers' minds the perception of an organization that is easy to do business with. In many organizations, the marketing department is responsible for customer service and for the definition of customer service levels.

Inventory management is a logistics activity that can be a source of value to the firm. Management must have a true knowledge of inventory carrying costs to make decisions about customer service levels and logistics systems design, such as, the number and location of distribution centers, inventory levels, where to hold inventory and in which forms and transportation modes to use. Inventory decisions can be very complex and imply a trade-off between costs. For example, ordering in smaller quantities on a more frequent basis will reduce inventory investment, but, may result in higher ordering costs and increased transportation costs. It is necessary to compare the savings in inventory carrying costs with the increasing costs of ordering and transportation to determine how the decision to order affects profitability.

The goal of this thesis is the study of different logistics systems which integrate decisions in the logistics distribution network while searching for improving service levels and reducing costs. The **questions** we will try to answer are: Can efficiency be improved by the coordination of different areas of distribution management? and are there any alternative strategies for distribution that will allow the decision maker to make a better choice?

The main focus of this theses is to answer the above questions by solving three different distribution management problems. These problems reflect different distribution management situations, with specific characteristics, which have yet not been studied in the literature, as far as we know. Our study contributes to state of knowledge by constructing distribution management strategies that contribute to improve the performance of the Logistics functions and consequently the overall organization.

The first study explores three different distribution strategies: the first strategy corresponds to the classical vehicle routing problem; the second is a master route strategy with daily adaptations, and the third is a strategy that takes into account a cross-functional planning, between Marketing and Transportation, through a bi-objective model, the improvement in the customer service level, through customer relationship focus and the minimization of routing costs. All strategies are analyzed in a multi-period scenario. A metaheuristic based on Iterated Local Search (ILS) is used to solve the models and compare the strategies. Some examples are computed and results show that the three different strategies present good alternatives for distribution policies. The results also show that by integrating into the optimization model the objectives of different functional areas, better overall distribution policies can be achieved.

In the second study, we develop two models for an inventory system in which the distributor manages the inventory at the retailers' location. These type of systems correspond to the Vendor Managed Inventory (VMI) systems which are described in the literature. These systems are very common in many different types of industries, such as retailing and manufacturing. The objective of our models is to minimize total inventory cost for the distributor in a multi-period multi-retailer setting. The inventory systems include holding and stock-out costs for the first model. And, for the second model, an additional fixed setup cost is charged per delivery. We construct a computational experiment to analyze the model behavior and observe the impact of the characteristics of the model on the solutions.

Finally, in the third study, the need for integration in the Supply Chain Management (SCM) leads us to consider the coordination of two logistics planning functions: Transportation and Inventory. The coordination of these activities can be an extremely important source of competitive advantage in the Supply Chain Management. The battle for cost reduction can pass through the equilibrium of transportation versus inventory managing costs. A study of a specific case of an inventory-routing problem for a week planning period with two types of demand is conducted. The two types of demand come from two different types of customers: the vendor-managed inventory customers and the customer-managed inventory customers, with stochastic and known demand, respectively. A heuristic methodology, based on the ILS, is proposed to solve the Multi-Period Inventory Routing Problem with stochastic and deterministic demand.

The organization of this thesis is the following: In the next chapter, the first study is presented, describing and analyzing a group of three strategies for a distribution management problem. It starts by presenting each strategy and the correspondent model. Afterwards, a metaheuristic based on ILS is proposed for each strategy and a computational experiment is shown. In Chapter 3, the second study, based on two models for a VMI systems is presented. In this chapter, we also make a computational experiment. In Chapter 4, the Inventory Routing Problem (IRP) model of the third study is explained,with the correspondent heuristic methodology. In this chapter, again we conduct a computational experiment on a set of randomly generated examples. Finally, in Chapter 5, the main conclusions of the thesis are exposed.

Chapter 2

Strategies for an Integrated Distribution Problem

The growing number of problems that firms are facing nowadays with relation to the distribution of their products and services, has lead logistics to be of primary concern to many industries.

An important aspect of the logistics management task is to coordinate the activities of the traditional distribution functions together with purchasing, materials planning, manufacturing, marketing and often R&D. One important direction of the integration process is the cross functional planning, which consists of coordinating different areas inside the firm, allowing for cost reductions and improving service (see Christopher 1998).

The motivation of our work arises in this context of integration of logistics functions with other functions of the firm. In our case, we will focus our study in two key areas: the distribution management and marketing management.

On one hand, the importance of good distribution strategies in today's competitive markets can not be overstressed. In many industries, an important component of the distribution systems is the design of the routes of vehicles to serve their customer's demand. And, on the other hand, new trends in the supply-chain management are, as pointed out by some industry leaders, *"..better customer service...greater customer sophistication"* (Partyka and Hall 2000). Customer service is becoming more important, customers demand more than a product, they demand a product arriving on time, an easy ordering system or a just-in-time distribution.

In this work, we will study the decision making problems in distribution management related with the delivery of products to customers, on a given period of time, using a fleet of vehicles. The decisions on how to assign customers to drivers and to design the routes made by each vehicle constitute the Vehicle Routing Problem (VRP). Usually, the vehicle routing is responsibility of the Distribution department. However, since many drivers also perform commercial activities, the Marketing department has the objective of maintaining always the same delivery agent assigned to the same customer. How to balance these two, possible opposite, objectives is an interesting issue when the firm wants to implement integrated distribution processes.

To analyze the impact of integrating the two areas of the firm, Distribution and Marketing, we will explore three possible distribution strategies that reflect different policies in distribution in the organizations.

The first strategy, **Strategy** 1, has a distribution policy that minimizes distance. The appropriate model for this strategy is the classical VRP, which consists of designing the routes minimizing total routing cost, measured in distance units. However, this VRP objective is often object of criticism by the users and planners, since it does not take into consideration other concerns of the company, for example, customer service. The second strategy, **Strategy** 2, tries to implement a marketing policy. In a growing competitive environment many firms adopt strategies of tight relationships with their customers where loyalty and friendship play a key role, through the delivering agents. In this strategy, the routes are predefined such that one delivering agent is associated with a specific set of customers.

The third strategy, **Strategy** 3, is the one that considers both objectives, marketing and distribution at the same time, in an integrated manner.

The motivation to work on this VRP arises by the distribution problems faced by the food and beverage industry. In these industries the tendency is to have lower inventories and higher delivery frequencies. And also, to have drivers responsible for commercial activities such as: promotions or introduction of new products.

The present chapter is organized in the following way: In Section 2.1 we will present the three distribution strategies. In Section 2.2, a review of the VRPs is summarized. Then, we will describe the mathematical formulation of each strategy and the solution approach, based on ILS, to analyze those models. In section 2.5, we will present a computational experiment and analyze the results obtained. Finally, the conclusions of the work are presented.

2.1 The Distribution Strategies

The distribution strategies correspond to different situations and concerns inside the firm. By comparing them, we can analyze the effect that integrating two areas can have on the distribution policies. The objective of this analysis is to provide a set of possible alternative solutions to the decision maker. Whom, with the use of additional side information on each particular distribution problem, can make a good choice.

The strategies are settled in a planning horizon of a week, this is: five working days. The choice of this period is based on the need for a strategic perspective, we want to study the impact of a sequence of decisions on different objectives. As a consequence we need several periods to analyze the marketing effect and a week seems to be a reasonable choice since in many industries, the behavior of the orders for a customer have a week pattern (examples are the Beverage & Food industry). In any case, this assumption can be relaxed and the problem could be extended to more periods.

- **Strategy 1: Distance Minimization**

In this strategy the distribution policy is constructed based on routing cost. Cost reduction is one of the biggest concerns in transportation and distribution management, but not the only one as we will see later. We want to find the route for each of the vehicles that will pass through the demand points in such a way as to satisfy all the demand with the smallest travelling cost. The classical VRP considers only one period at a time and chooses the optimal routes for that period. Strategy 1 corresponds to the classical VRP repeated for each day of the planning horizon.

- **Strategy 2: Master Routes**

The second strategy is based on marketing principles, and the distribution strategy is based on service measures. An important source of value to the firm can be obtained from a close connection between firm and customers. This can be achieved through the development and investment on the relationship with its customers.

In this strategy, each driver will serve always the same customer. This marketing policy is giving emphasis to the personal relationship between drivers and customers as a way to improve customer service. In many industries, drivers are more than drivers, they also perform sales tasks, promotions and introduction of the new products.

One of the advantages pointed out to this customer relationship management policy is that it makes more difficult for a customer to switch to another provider. Since, a relationship requires an investment of time from both the customer and the provider, Simchi-Levi et al. (2003). These marketing strategies allow the firm to obtain more information on customer needs. And, at the same time, it becomes more easy to introduce new products, construct promotions and even speeds up the delivery process due to experience effects from both parts.

- **Strategy 3: Multi-Objective**

The third strategy is the integrated distribution management model, which consists of taking into account in the decision process the concerns of the Distribution department and Marketing department i.e. the reduction of transportation costs and the emphasis on the personal relationship between driver and customer. We propose a multi-objective model with two objectives, each objective corresponds to a different function. The first is the transportation and the second a marketing function. This strategy tries to include in the same model the objectives of the two previous strategies. The best solution for the transportation problem might not always be the best solution for the marketing objective, and vice versa. In some cases these two objectives may conflict and that is the main justification for a trade-off analysis between them. We need to find a solution (or several solutions) that integrate a marketing policy and route design policy.

2.2 Literature Review on VRP

The classical VRP model is behind the models for the three distribution strategies. Therefore, we make here a brief review of the classical VRP.

Vehicle routing decisions are extremely important within a company to maintain its competitiveness an allow it to best exploit the available resources and to distribute its products at the lowest possible cost.

Significant amount of research effort has been dedicated to VRP, see the survey articles on VRP by Laporte and Osman (1995), Laporte (1992), Desroisers et al. (1996), Bodin (1983), Cristofides et al. (1979) and Fisher (1995) and the book of Crainic and Laporte(1998). An extensive list of VRP research papers can be found on http://www.imm.dtu.dk/or/VRP_ref/vrp_1.html.

The most well known is a basic VRP which can be briefly defined in the following way: given a set of customers with known demand and location, define a set of routes, starting and finishing at one depot, that visits all customers with minimal cost.

The basic model of the vehicle routing problem considers a set of nodes, representing retailers or customers, at a known location, that must be served by one depot. Each node has a known demand. A set of vehicles, with equal capacity is available to serve the customers. The routes must start and finish at the depot. The objective is to define the set of routes to serve all customers with minimal cost.

For each pair of nodes, a fixed known *cost* is associated. We assume this cost matrix is symmetric and can represent a real cost, distance or time. The main constraints of the problem are that all the demand must be satisfied and the vehicles capacity can not be exceeded.

The basic VRP is a generalization of the Traveling Salesman Problem (TSP), where more than one vehicle is available, for TSP references see for example Lawer et al. (1985).

There are several formulations of the classical VRP in the literature, for some of these formulations see Fisher and Jaikumar (1978) (1981), Kulkarni and Bhave (1985), Gouveia (1995) and Toth and Vigo (2002).

The classical model of the VRP can be formulated as an integer linear programming and this is the formulation we will use along further chapters.

Consider the following data:

$I = 1, ..., n$ set of nodes, that correspond to the different locations of the customers, node 1 corresponds to the depot.

$K = 1, ..., m$ set of vehicles;

$Q =$ capacity of each vehicle;

$q_i =$ demand of customer i, $i = 1, ..., n$;

$c_{ij} =$ cost of going from i to j, $i = 1, ..., n$; $j = 1, ..., n$.

This formulation considers two types of variables:

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ visits customer } j \text{ immediatly after customer } i \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ik} = \begin{cases} 1, & \text{if customer } i \text{ is visited by vehicle } k \\ 0, & \text{otherwise} \end{cases}$$

The formulation of the problem is:

$$\text{Objective Function :}$$

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \sum_{k=1}^{m} x_{ijk} \tag{2.1}$$

$$\text{Subject to :}$$

$$\sum_{k=1}^{m} y_{ik} = 1, \qquad \forall\ i = 2, ..., n \tag{2.2}$$

$$\sum_{k=1}^{m} y_{1k} = m, \tag{2.3}$$

$$\sum_{i=2}^{n} q_i y_{ik} \leq Q, \qquad \forall\ k = 1, ..., m \tag{2.4}$$

$$\sum_{j=1}^{n} x_{ijk} = \sum_{j=1}^{n} x_{jik} = y_{ik},$$

$$\forall\ i = 2, ..., n; k = 1, ..., m \tag{2.5}$$

$$\sum_{j,i \in S} x_{ijk} \leq |S| - 1,$$

$$\forall\ S \text{ non-empty subset of } \{2, ..., n\}\,; k = 1, ..., m \tag{2.6}$$

$$x_{ijk} \in \{0, 1\}\,;\ y_{ik} \in \{0, 1\}\,,$$

$$\forall\ i = 1, ..., n; k = 1, ..., m \tag{2.7}$$

Constraint (2.2) ensures that each customer is visited by one vehicle only. Constraint (2.3) guarantees that all vehicles visit the depot. Constraint (2.4) represents the vehicle capacity constraint. For each vehicle k, we guarantee that the sum of the demand of the nodes that the vehicle covers is less or equal to its maximum capacity. Here we assume that none of the customers has a daily demand that exceeds Q. The constraint (2.5) ensures that if a vehicle visits a customer it also has to leave that customer.

Constraint (2.6) is the sub-tour elimination constraint. This constraint implies that the arcs selected contain no sub-cycles. It states that for every vehicle, the following holds: for every non-empty subset S of $\{2, .., n\}$, the number of arcs that are in the route of this vehicle, with both nodes belonging to S, has to be less or equal to the number of elements of S minus 1.

The last constraint, (2.7) defines the variables x and y as binary. The objective function is minimizing the total cost of the routes.

The TSP is a sub-problem of the VRP, the TSP belongs to the class of NP-hard (non-deterministic polynomial time) problems, and so does the basic VRP and extensions. This means that the computational complexity of the problem grows exponentially with its size, i.e., it grows exponentially with the number of customers.

In the next section we will describe the mathematical model of each distribution strategy.

2.3 The models for the distribution strategies

In this section we will present the mathematical models associated with the three distribution strategies. First of all, we describe the assumptions of the model.

The first assumption is that the firm is responsible for the distribution of its own products. Therefore, there are no questions of outsourcing to be handled. These firms face the pressures of a competitive market making them concerned both on consumer satisfaction and internal efficiency.

The classical VRP considers only one period and chooses the optimal routes for that period. Here we will introduce more periods by considering a week length of analysis. Each day we have a different set of customers to serve and quantities to deliver. The reduction of the inventory levels, and the increase in the frequency of orders is a tendency in many businesses to lower stock handling costs.

Other assumptions of the model are:

- All the demand is satisfied in the same day that it is required and not on any other day of the week.
- Only unload is done at each customer.
- The number of vehicles is fixed and there are no fixed costs associated with the use of the vehicles. They all have the same capacity. Moreover, the number of vehicles available is enough to satisfy all the demand.
- Another assumption is that the distance matrix between customers and between customers and warehouse is known and fixed. This matrix does not depend on the day or the quantity loaded.
- Each vehicle is assigned to a driver. And we consider that they work every day in the period in question.
- One vehicle can only be used once a day and the time it takes to deliver the

full capacity is less than a working day.

Next the model we will be presented in detail. The following data is considered in the mathematical formulation:

i, I = index and set of nodes, $I = 1, ..., n$ where 1 is the depot and 2 to n are the customers locations;

k, K = index and set of vehicles, $K = 1, ..., m$;

t, T = index and set of days which represent the period, $T = 1, ..., p$;

T_i = set of days where customer i has a demand that is greater than zero, $i = 2, ..., n$;

q_i^t = demand of customer i on day t, $i = 1, ..., n$ and $t = 1, ..., p$;

c_{ij} = the cost of going from i to j, this is a fixed matrix , $i = 1, ..., n$ and $j = 1, ..., n$;

Q = capacity of a vehicle.

The variables of the model are:

$$x_{ijk}^t = \begin{cases} 1, & \begin{array}{l} \text{if vehicle } k \text{ visits customer } j \\ \text{immediatly after customer } i \text{ on day } t \end{array} \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ik}^t = \begin{cases} 1, & \text{if customer } i \text{ is visited by vehicle } k \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

2.3.1 Strategy 1: Distance Minimization

The objective function minimizes routing costs, for all customers during the week period. This strategy corresponds to repeating a classical VRP for each day of the week.

The formulation of this objective will be the same as the one used for the classical model but with a new parameter, t, representing the day of the week.

$$\text{Objective function:} \qquad \text{Min} \sum_{t=1}^{p} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} c_{ij} x_{ijk}^t \tag{2.8}$$

2.3.2 Strategy 2: Master Routes

First we construct a set of routes, “master routes”, using the same VRP formulation as in Strategy 1.

This startegy and the model associated is very close to a common practice in several companies. It consists in: first defining “master routes” and, afterwards performing daily adjustments depending on the demand of the customer and on the capacity of the vehicle. To obtain the “master routes” we consider a VRP model, where all customers are in the input data and the demand of each customer depends on the average daily demand. To adjust the daily routes we consider other constraints such as capacity and number of vehicles.

So, the requirement that a customer will always be served by the same driver may have to be sacrificed but we will try to enforce this at least to the best customers. Therefore, the idea is, the better the customer the more interest we have in maintaining the same driver.

The mathematical formulation for this strategy is identical to the one of the classical VRP for one period presented in section 2.2, but, in this case all customers are considered for the "master routes".

2.3.3 Strategy 3: Multi-Objective

In this strategy we propose a multi-objective model with two objectives: minimizing of routing costs and minimization of service levels, that reflect the integrated strategies between Distribution and Marketing departments.

As far has we know there are no studies on routing problems with multiple periods and this marketing oriented objective function. Although there are some multi-objective VRP that consider other types of objectives. Examples of these are the papers by Hong and Park (1999) that consider the minimization of both the total vehicle travel time and the total customer waiting time, in a VRP with time windows constraints. Lee and Ueng (1999) developed an integer linear model that searches for the shortest travel path and balances driver's load simultaneously. The work is measured in terms of traveling and loading time. The objective function is the weighted sum of the above objectives. This second objective minimizes the difference between the working time of each vehicle and the working time of the vehicle with the shortest working time.

In most cases of multiple objectives it is unlikely that the problem is optimized by the same alternative parameter choices. Hence, some trade-off between the criteria is needed to ensure a satisfactory design.

In the multi-objective optimization an important relation is the dominance relation.

Let (z_1) and (z_2) be two solutions of a multi-objective problem with R objectives.

We say that:

Solution (z_1) dominates (z_2) if $z_{1r} \leq z_{2r}$ $\forall$ objectives r in $\{1, ..., R\}$ and $z_{1r} < z_{2r}$ for at least one r and $(z_1) \neq (z_2)$.

A feasible solution is efficient if it is non-dominated. Based on this concept we will optimize the two objective functions to find non-dominated solutions.

Ideally, we would like to find the solution that would be optimal for both objectives at the same time. In multi-objective programming this solution point rarely exists. So, we would like to find solutions that are closer to this ideal point.

Mathematically, all dominated solutions are equally acceptable, it is the decision maker, who is responsible for choosing the final solution. The decision maker is someone who has a deep knowledge of the problems, relationships and implications of each solution. The choice among these non-dominated solutions is determined by the decision maker's preferences among the multiple objective.

The two objective functions considered within the integrated strategy are:

Objective A: Minimizing Cost

The formulation of this objective will be the same as in (2.8), the one used for the model of strategy 1.

$$\text{Objective function A:} \qquad \text{Min} \sum_{t=1}^{p} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} c_{ij} x_{ijk}^{t}$$

Objective B: Marketing Objective

In terms of mathematical formulation, the second objective works as follows: For each customer we have a set of pairs of days with positive demand, T_i, for each pair of days g, h in T_i (with $g \neq h$) we want to minimize the difference in the assignment to a vehicle k. The objective is to minimize $\left|y_{ik}^{g} - y_{ik}^{h}\right|$.

The importance is given by the total demand in the period, therefore a weight is introduced by the total amount ordered by each customer. The objective function becomes:

$$\text{Min} \left\{ \sum_{i=1}^{n} \sum_{k=1}^{m} \sum_{\substack{g,h \in T_i \\ g<h}} \left[\left(\sum_{t=1}^{p} q_i \right) \times \left| y_{ik}^{g} - y_{ik}^{h} \right| \right] \right\}$$

The importance of a customer is measured in terms of sales. In some cases other measures could be used to classify the goodness of a customer, for example, frequency of orders, credit history, etc. This function is non-linear.

Considering a multi-period model is an essential point in our study since, objective B is not static, it measures decisions allong more than one period, it only makes sense in a continued time base.

In this integrated strategy the objective is to find a set of non-dominated solutions and give the decision maker the possibility to choose not only between strategies but also between solutions.

The constraints of the model for strategy 1 and 3 are:

$$\sum_{k=1}^{m} y_{ik}^{t} = 1,\ \forall\ i = 2, ..., n;\ t \in T_i \tag{2.9}$$

$$\sum_{k=1}^{m} y_{ik}^{t} = m,\ \forall\ t \in T;\ i = 1 \tag{2.10}$$

$$\sum_{i=2}^{n} q_i^t y_{ik}^t \leq Q,\ \forall\ k = 1, ..., m;\ t = 1, ..., p \tag{2.11}$$

$$\sum_{j=1}^{n} x_{ijk}^{t} = \sum_{j=1}^{n} x_{jik}^{t} = y_{ik}^{t},$$

$$\forall\ i = 2, ..., n; k = 1, ..., m; t = 1, ..., p \tag{2.12}$$

$$\sum_{k=1}^{m} y_{ik}^{t} = 0,\ \forall\ i = 2, ..., n \text{ and } t \notin T_i \tag{2.13}$$

$$\sum_{j,i \in S} x_{ijk}^{t} \leq |S| - 1,$$

$$\forall\ S \text{ non-empty subset of } \{2, ..., n\};$$

$$k = 1, ..., m;\ t = 1, ..., p \tag{2.14}$$

$$x_{ijk}^{t} \in \{0, 1\};\ y_{ik}^{t} \in \{0, 1\},$$

$$\forall\ i = 1, ..., n; k = 1, ..., m;\ t = 1, ..., p \tag{2.15}$$

Constraints (2.9) to (2.15) are similar to the ones in the basic model, but for each day of the period in question.

Constraint (2.9) ensures that in the days where the customers have a positive demand, that customer is visited by only one vehicle.

The constraint (2.10) forces that each day all vehicles go to the depot. Constraint (2.11) ensures that, the daily loading of a vehicle does not exceed its capacity.

Constraint (2.12) guarantees that if the vehicle enters a node, on day t, it also has to leave that node, on the same day.

Constraint (2.13) prohibits a vehicle to visit a customer on a day where he has zero demand.

Finally constraint (2.14) avoids sub-tours, but now not only for each vehicle but also for each day. The sub-tour elimination constraint represents an exponential number of constraints.

The last constraint (2.15) define all variables as binary.

For Strategy 2, the constrains of the model are the same as for VRP formulation in section 2.2, applied in this occasion to the “master routes”, only for one period.

2.4 Solution Approach

The above models are complex combinatorial optimization problems, classified as NP-hard[1]. The effort required to solve NP-hard problems increases exponentially

[1]NP-hard problems are the class of network and combinatorial problems for which no polynomial-bounded algorithm has yet been found.

with problem size. The approach for solving these type of problems optimally suffers from computational burden with problem size. As a result they require an heuristic methodology in order to solve them. Since our objective is to compare the three distribution strategies we will use the same solution technique for each strategy.

2.4.1 Heuristics method

A heuristic algorithm is a solution method that does not guarantee an optimal solution, but in general had a good level of performance in term of solution quality and convergence. Heuristics may be constructive (producing a single solution) or local search (starting from one given random solutions and moving iteratively to other nearby solutions) or a combination of both. Heuristics for VRP have been extensively studied. Cordeau et. el. (2002) summarize the most important classical and modern heuristics for the VRP. Laporte and Osman (1996) have a bibliography review on theory and application of metaheuristics.

Local search is the most powerful general approach for finding high quality solutions to hard combinatorial optimization problem in reasonable time. It is based on the iterative exploration of neighborhoods of solutions trying to improve the current solution by local changes. The type of local search that may be applied to a solution is defined by a neighborhood structure.

The most basic local search is the iterative improvement. It typically starts with an initial solution, generated randomly or by some constructive heuristic and tries to repeatedly improve a current solution by moves to better neighboring solutions. A major drawback of iterative improvement local search is that it may stop at very poor quality local minimum. To avoid the disadvantage of iterative improvement and, in particular, multiple descendent, we need to allow the local search to escape from local optima. To escape from stagnation at local optima is the main goal of methaheuristics. Metaheuristics are typically high level strategies, which guide an underlying, more problem specific heuristics, to increase their performance.

Our proposal is to use a metaheuristic algorithm that as proven to give quiet good results on other problems and is easy to implement and modify, adapting to different strategies, Iterated Local Search (ILS).

Iterated Local Search for the VRP

ILS algorithms have been applied successfully to a variety of combinatorial optimization problems. In some cases, these algorithms achieve extremely high performance and even constitute the current state-of-the-art metaheuristics, while in other cases the ILS approach is merely competitive with other metaheuristics. ILS has many of the desirable features of a metaheuristic: it is simple, easy to implement, robust, and highly effective.

ILS is a simple and generally applicable meta-heuristic which iteratively applies local search to modifications of the current search point, for more detailed information on ILS see Lourenço et. al. (2001), Lourenço et. al. (2002) and Stützle (1998).

At the start of the algorithm a local search is applied to some initial solution. Then, a main loop is repeated until a stopping criterion is satisfied. This main loop consists of a modification step ("perturbation"), which returns an intermediate solution corresponding to a modification of a previously found locally optimal solution. Next, local search is applied to yielding a locally optimal solution . An "acceptance criterion" then decides from which solution the search is continued by applying the next "perturbation". Both, the perturbation step and the acceptance test may be influenced by the search history. ILS is expected to perform better than if we just restart local search from a new randomly generated solutions.

The architecture of the ILS is as follows:

Architecture of the ILS algorithm

Procedure ILS:
s^0 = GenerateInitialSolution
s^* = LocalSearch(s^0)
Repeat
s' = Perturbation(s^*, history)
$s^{*\prime}$ = Local search(s')
s^* = Acceptance Criterion(s^*, $s^{*\prime}$, history)
Until termination condition met
End

The proposed ILS heuristics is based on the ILS metaheuristic developed by Stützle (1998) and Kunz (2000) to solve the classical VRP. The ILS used for the VRP is the following:

ILS for the VRP

Step 1. Savings Heuristic - Initial Solution
Step 2. ILS for TSP on each tour:
  Step 2.1. Local Search for TSP
  Step 2.2. Perturbation for TSP
  Step 2.3. Acceptance criterion
Step 3. ILS for the VRP
  Step 3.1. LS for the Assignment Problem
  Step 3.2. Perturbation for VRP
  Step 3.3 Acceptance Criterion
Step 4. ILS for the TSP on the new routes

We will now present the implementation of each step the above algorithm in more detail.

- **Savings Heuristic**

This is a greedy heuristic to construct an initial solution. It has been proved that starting from a random solution gives worst results (Stützle 1998). This savings heuristic gives us the tours to start the search, it was proposed by Clarke and Wright (1962).

- **ILS for the TSP**

On each of the tours obtained in the savings heuristic we apply an ILS. At this step of the algorithm we ignore any relation between routes.

LS for TSP

The LS used was a 2-opt, this is: on a tour 2 connections are exchanged as soon as all customers are tested with the others, we have a new constellation, since there

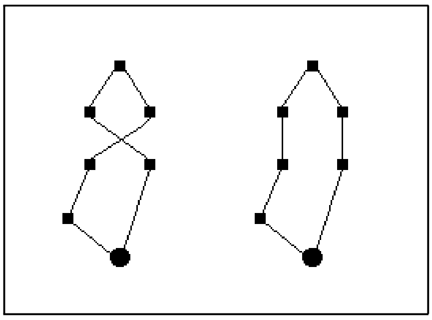


Figure 2.1: Example of a 2-opt move for the TSP.

are now new crusts that have not been checked, the process is repeated. Only when a complete run without improvements finishes, one has reached a 2-opt resolution.

Searching in a complete 2-opt would not be efficient. So, to reduce the search space, some techniques are introduced that faster the process and still arrive to good quality solutions: A list of candidates and "don't look bits". One "don't look bit" is associated with each node. Initially, all "don't look bits" = 0, if for a node no improving move can be found, then "don't look bit" is turned on (set to 1) and is not considered as a starting node in the next iteration. If an edge incident to a node is changed by a move, the node's "don't look bit" is turned off again - reduces to $O(n)$.

Perturbation (Kick-move) for TSP

On the local minimum that has been reached, we apply the kick-move and arrive to a new start solution. The goal here is to escape from local optima by applying perturbations to the current local minimum.

For the LS on the TSP we use "d*ouble bridge*", this perturbations cuts four edges,

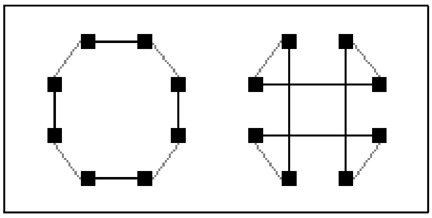


Figure 2.2: Example of a Double Bridge move.

and introduces 4 new ones.

Acceptance criterion

The acceptance criterion used at this step is better; this means that the new tour is accepted if it has a lower cost.

- **ILS for the VRP**

The ILS for the VRP is implemented considering the initial solution the routes obtained from the ILS of the TSP.

Local Search for the assignment problem

The local search for the VRP is a 2-opt and again a list of candidates and “don't look bits ” techniques are applied to restrict the search.

We have two possibilities for a 2-opt: A customer of a tour is postponed into another or a customer trades with another customer from another tour. First, if capacity restrictions allow and it reduces costs, a city is inserted in the tour. Only if it cannot be inserted, then we check if an exchange with another tour improves the solution.

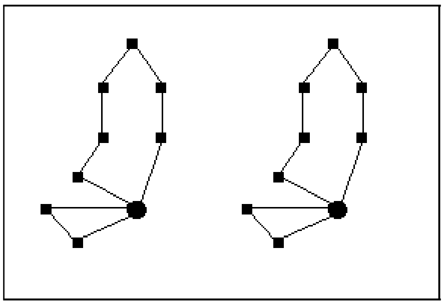


Figure 2.3: Example of a 2-opt move for the VRP.

The same techniques as in Local Search for the TSP are used: “don't look bit” and List of candidates.

Kick-moves

“Numb-crosser”: This perturbation consists in exchanges a group of customers from 2 tours. In this case, 1/3 of the customers of the tour are exchanged.

Acceptance criterion: Best, the same as the acceptance criterion for the TSP.

- **ILS for the TSP on the new routes**

Repeats the ILS procedure for the TSP.

The ILS for each strategy

The ILS for the VRP is now adapted to solve the 3 models for the different strategies. Next we will describe in detail th ILS for each of them.

Strategy 1: Since in this strategy we have a classical VRP model, for each day, we apply the ILS to find the best daily routes, according to the capacity of the vehicle and daily demand. The algorithm is repeated for several runs and for each day chooses the best solution.

Let L be the total number of loops:

Structure of the algorithm for Strategy 1

Step 1: Set $loop = 0$
Step 2: Set $day = 1$
Step 3. Savings Heuristic - Initial Solution
Step 4. ILS for TSP on each tour
Step 5. ILS for the VRP
Step 6. ILS for the TSP on the new routes
Step 7. Set $day = day + 1$; Repeat Step 3 to 6 until $day = 5$;
Step 8. Set $loop = loop + 1$
Step 9. Repeat Step 2 to 8 until $loop = L$

Strategy 2: In this strategy, we have considered a classical VRP model to obtain the "master routes", where all customers are taken into account using its average demand. Therefore, to obtain the master routes we apply an ILS.

Afterwards, the routes, for each day of the week, are obtained in the following way: exact the same routes are maintained for each day; eliminate from the routes the customers that have no demand on that day; if in any of the routes the capacity constraint is being violated, we pick up from the tour the customer that is considered less important and we delete it from this tour and insert it on another tour. This tour is chosen in such a way as to minimize routing costs.

Structure of the algorithm for Strategy 2

Step 1: For all customers do
  Step 1.1. Savings Heuristic - Initial Solution
  Step 1.2. ILS for TSP on each tour
  Step 1.3. ILS for the VRP
  Step 1.4. ILS for the TSP on the new routes
Step 2: Set $day = 1$
Step 3. For each tour eliminte customers with zero demand
Step 4: For each tour, if capacity constraints are violated remove customer with lowest total demand
Step 5: ILS for the TSP on the new routes
Step 6: Set $day = day + 1$
Step 7: Repeat Step 3 to 6 until $day = 5$;

Strategy 3: In this strategy we face a Multi-Objective Combinatorial Optimization Problem (MOCOP). Ehgott and Gandibleux (2000) provide an annotated bibliography on MOCOP.

Two main approaches can be found in the metaheuristics for the MOCOP: methods of local search in object space and population based methods. In the Local Search methods, we start from an initial solutions, and the procedure approximates a part of the non-dominated frontier corresponding to the search direction λ given. A local aggregation mechanism of the objectives, based on the weighted sum, produces the effect to focus the search on a part of the non-dominated frontier. The principle is repeated for several search directions. In the population based methods, all population contributes to the evolution process toward the non-dominated frontier. Here we will use the first approach, i.e. methods based on local search.

In this case, after having decided the routes for the first day, the program takes into consideration objectives B, through a wheighted function of both objectives. To do this, we calculate the effect of a move on the weighted function of the objectives. Then, in the acceptance criterion, a new solution is accepted if the weighted function has improved. The algorithm is repeated for several different sets of weights. All the non-dominated solutions are keep during the run of the algorithm.

A objective function Z is used as weighted function, which is a weighted sum of the single objectives A and B.

Let f_r be the single objective function of objective r,

$$Z = \sum_{r=1}^{2} w_r f_r \text{ and } \sum_{r=1}^{2} w_r = 1$$

The solution is very sensitive to the weights that have been defined. There is also the problem of having objectives with different variables and scales. In our case, for example we are adding costs and quantities.

Notation:

w_a weight for Objective A, with $0 \leq w_a \leq 1$;

w_b weight for Objective B, with $0 \leq w_b \leq 1$;

and

$w_a + w_b = 1$

$z = w_a(ObjectiveA) + w_b(ObjectiveB)$

Structure of the algorithm for Strategy 3

Step 1: Set $w_a = 1$ and $w_b = 0$
Step 2: Set $day = 1$
Step 3. Savings Heuristic - Initial Solution
Step 4. ILS for TSP on each tour
Step 5. ILS for the VRP
  Step 5.1. LS for the Assignment Problem
    Step 5.1.1 For each move calculate the effect on objective A and B
    Step 5.1.1 Accept only if the new z is smaller
  Step 5.2. Perturbation for VRP
  Step 5.3. Repeat Step 3.1
  Step 5.3 Acceptance Criterion
Step 6. ILS for the TSP on the new routes
Step 7. Set $day = day + 1$; Repeat Step 3 to 6 until $day = 5$;
Step 8. Set $w_a = w_a - 0.1$ and $w_b = w_b + 0.1$
Step 9. Repeat Step 2 to 8 until $w_a = 1$ and $wb = 0$

2.5 Computational experiment

The main objective of this experience is to evaluate the three strategies and analyze the effect of each objective in the solutions. With this purpose we have applied the above algorithms on several sets of randomly generated examples. The results are expressed in terms of the values of the objectives and total number of vehicles needed. For each strategy two values were calculated: the Routing Cost and the Marketing (or service) Value (these are the two objectives of strategy 3). The first is measured in distance and the second can be interpreted as the unit cost for the distributor for not serving a customer with the same salesman, working in a similar way as a penalty cost.

Next we will explain the data used and analyze some important results of this experiment.

2.5.1 The data

For the computational experiment, we have generated several sets of examples concerning the total number of customers $(50, 100, 200, 400)$. Also, we have examples with two types of demand (low variation and high variation) and two types of vehicles' capacity, high and low.

To obtain the demand, we have used a normal distribution with mean 50 and standard deviation 20, for the case demand has a high variation and a standard deviation of 5, for the examples with low variation. For each day, on average, 25% of the customers have zero demand. This implies that in a problem of 100 customers, with 5 days week, we will have around 375 deliveries to make.

The customers locations are uniformly generated in a 100×100 square with the depot located in the center with the coordinates $(50, 50)$.

Truck capacity is 300 for problems of sizes 50, 100, 200 and 700 for size 400. And we also ran cases with size 200 and capacity 500. In total we have studied 30 examples for each strategy, therefore we will consider 90 problems per run.

2.5.2 Analysis of the results

In this section we will present the results obtained for each example in terms of the objective functions values, number of vehicles used, non-dominated solutions and run times.

We can start illustrating the aim of the different strategies by looking at a small example with 2 days and a few customers: in Figure 2.4 we have the routes for two days, in strategy 1, 2 and 3. Strategy 1 has fewer and more efficient routes in terms of distance, Strategy 2 has more routes, but the routes are the same for each day. And, Strategy 3, has a solutions that is not completely efficient in terms of distance, but allowing for a better service level.

Table 2.1 and 2.2 show the results for each example and for each strategy. Strategy 1 tries to find lowest cost, strategy 2 the best service level and strategy 3 the set of non-dominated solutions with respect to the integrated strategy. We can observe that, as expected, strategy 1 will always give us the solution with the lowest objective A and the highest objective B when compared with strategy 2. In the strategy 2 we have much lower marketing values but the cost of routes increases significantly.

Concerning Strategy 3, we can say that, in almost every example, we can find more than one non-dominated solution and it would be the responsibility of the decision maker to decide on one of the alternatives. As can be seen from the results, Strategy 3 presents very good results for both objectives. The total routing cots is in general slightly higher than for Strategy 1, but, has a much lower value of objective

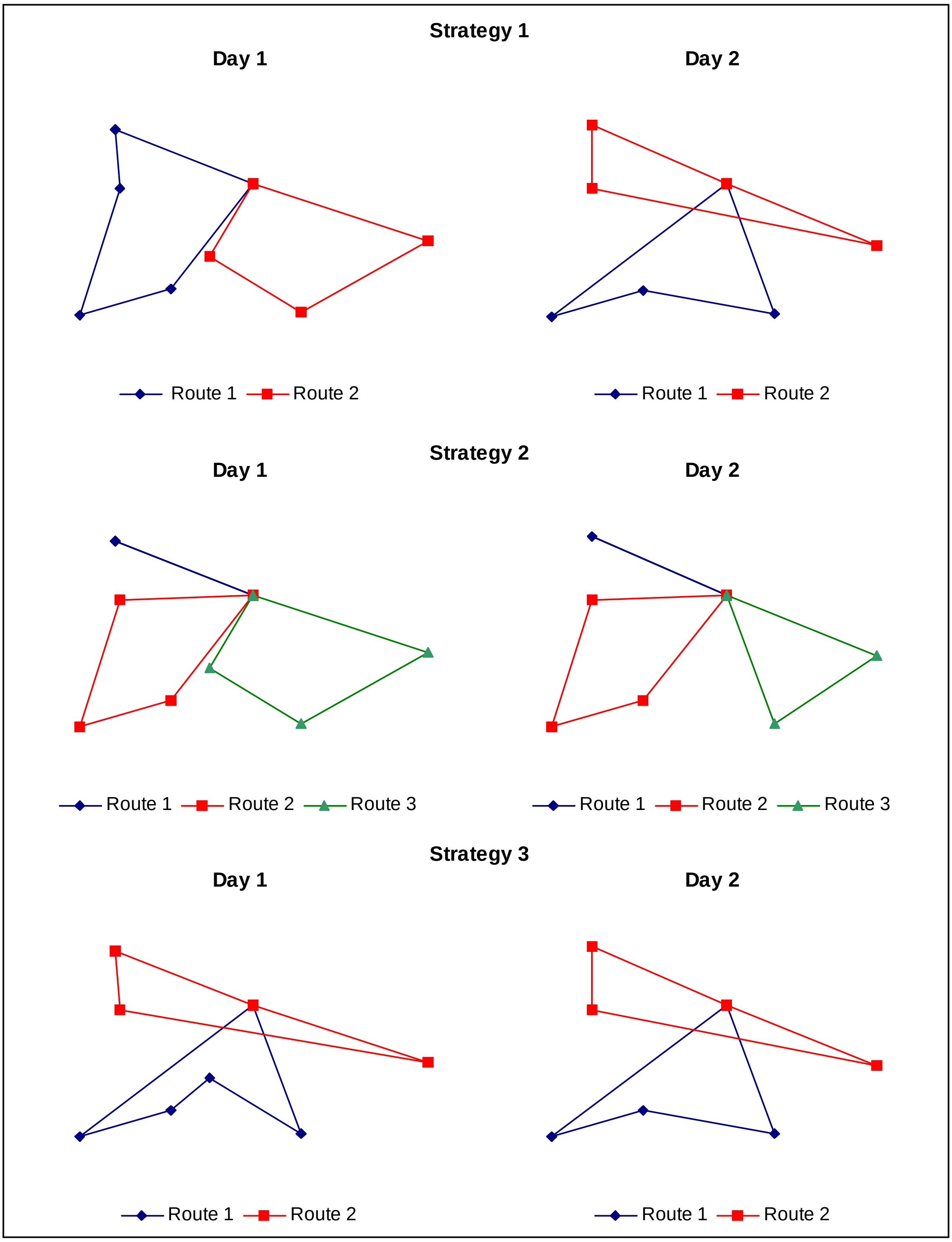


Figure 2.4: Routes for Strategy 1, 2 and 3.

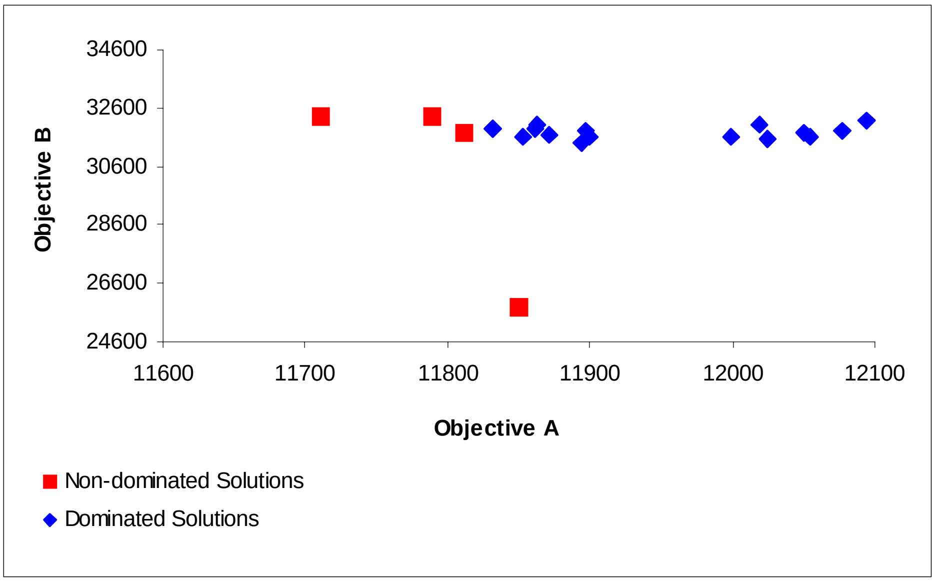


Figure 2.5: Example of a set of dominated and non-dominated solutions for Strategy 3.

B. When compared to the solutions of Strategy 2, the solutions of Strategy 3 have much lower routing costs and slightly higher values for objective B. These results represent a good trade-off between both objectives and, together with the possibility of obtaining several non-dominated solutions, leads us to the conclusion that Strategy 3 can be an excellent approach to solve this integrated distribution problem.

In Figure 2.5, we can see the set of all solutions obtained for 22 iterations for an example of 50 customers. The blue squares correspond to the dominated solutions and the red squares correspond to non-dominated solutions.

The number of vehicles needed for each solution strategy also varies and this is reflected in the total distance cost. In Table 2.3, we can observe these differences.

		Strategy 1		Strategy 2		Startegy 3	
N	Example	a	b	a	b	a	b
50	1	14344,15	36840	17018,93	3306	14510,34	32327
						14344,15	36840
	2	13589,47	38538	16304,18	4590	13755,11	35826
						13697,92	38148
						13589,47	38538
						13680,72	38278
	3	13225,97	40284	15744,37	4997	13289,63	36178
						13228,96	40284
						13278,06	40140
	4	11710,85	32341	15597,20	5950	11850,30	25753
						11811,55	31776
						11710,85	32341
						11788,70	32289
	5	13452,19	38084	16832,33	3012	13541,62	30733
						13462,15	38426
						13473,14	38198
						13485,40	37925
100	6	22970,35	75096	28356,94	8368	23240,29	64967
						23033,34	75012
						22971,54	75096
						23162,88	74404
						23153,67	74601
	7	21999,94	73497	27755,47	7209	22045,08	61244
						22041,77	73416
						21999,94	73497
	8	21839,25	74834	26873,79	10264	21907,99	63623
						21839,25	74834
	9	17062,82	70744	24874,82	5045	17236,27	59365
						17036,71	70838
						17220,50	69385
						17167,73	70579
	10	22383,01	76252	27892,61	5987	22632,56	63871
						22467,32	76110
						22493,46	75632
						22462,16	76252
200	11	40438,91	152536	52738,09	19418	40682,93	132320
						40438,91	152536
						40484,59	152292
	12	40420,20	152564	52089,76	9967	40757,39	134118
						40420,20	152564
	13	38484,12	151187	49881,28	19803	38498,67	131916
						38423,13	150677
	14	38481,27	148056	47837,33	9999	38681,92	129944
						38710,30	147051
						38731,63	147035
						38601,73	148056
						38502,60	148186
						38581,57	148113
						38624,37	147863
						38457,64	148254
						38649,82	147718
	15	40047,94	130399	50744,95	15778	40047,94	130399
						40028,26	151554

Table 2.1: Routing Cost (a) and Marketing Cost (b) for Strategy 1, 2 and 3.

		Strategy 1		*Strategy 2*		*Strategy 3*	
N	***Example***	***a***	***b***	***a***	***b***	***a***	***b***
200	16	28589,23	152040	35849,68	8375	28883,37	132744
(cap=500)						28782,42	151361
						28780,13	151569
						28686,55	151883
						28551,02	152164
						28630,92	152040
	17	28675,59	150844	34171,21	2096	28814,46	130891
						28711,49	150542
						28704,59	150844
	18	27350,06	149576	33919,55	9715	27511,86	130867
						27474,54	149315
						27255,56	150108
						27503,72	149261
						27397,52	149435
						27371,87	149576
	19	27674,99	147608	33632,54	9637	27676,81	128579
						27613,74	147754
	20	27836,23	152388	34002,28	9320	28035,45	130763
						27896,09	151456
						27824,33	151509
						27990,09	151355
400	21	40013,37	304388	49637,54	9607	40120,91	260814
						40101,29	304184
						40023,38	304388
	22	39758,16	308956	47691,43	9401	39961,71	270449
						39845,06	308604
						39842,24	308956
	23	39576,79	300057	47635,00	6954	39658,17	255775
						39584,53	299568
	24	39949,22	302980	47978,58	9525	40022,79	266906
						39956,93	303057
						39970,21	302980
	25	39552,16	299976	46682,13	9538	39593,31	264664
						39583,24	299976
50	26	12606,09	37756	11465,43	2032	12870,39	33258
(low stdev)						12716,22	37124
						12606,09	37756
						12867,78	36802
	27	13752,04	37759	11913,72	0	13861,01	34403
						13715,64	37757
						13855,72	37428
	28	12723,78	39888	11096,46	0	12820,78	33940
						12781,62	39888
	29	12468,30	32983	11225,91	0	12571,04	25854
						12489,20	32330
	30	12687,26	36396	10831,83	0	12765,22	31287
						12657,08	36504
						12685,23	36352
						12747,45	36043

Table 2.2: Routing Cost (a) and Marketing Cost (b) for Strategy 1, 2 and 3.

The master routes approach has always much higher number of vehicles a week, this happens due to the procedure of the routes design, they are constructed considering all customers, and then for each day eliminating the ones with no demand. When constructing the "master routes" we have used the daily average demand of each customer. Nonetheless, if we had chosen higher values for the demand, associated with each customer, the higher the number of vehicle used in the "master routes" and the lower the marketing objective.

Comparing the number of vehicles from Strategy 1 and Strategy 3 we observe that, on average, Strategy 3 has the same or a higher number of vehicles, see Table 2.3, this is due to the existence of the second objective, that introduces a preference for service rather than distance. To have a better service we need to sacrifice the routing efficiency and this can pass through the use of an additional vehicle.

In terms of running time, the importance of the results can be overstressed. The first and third strategies are the ones that take more time to compute. But, since we are referring to strategic planning, it does not seem inefficient for a firm, with a network of 400 customer, to spend one hour on a strategic planning task for a week. Take into consideration that in many cases the week planning pattern is repeated in the next weeks. Table 2.4 summarizes running times. For Strategy 1 and 3 we have done the same number of iterations. The magnitude of the difference in run time of strategy 2 is due to the fact that we only run once the VRP, for the master case. On the other two strategies we have to run VRP for each day of the panning period

N	*Strategy 1*	*Strategy 2*	*Strategy 3*
50	36	44	36
100	68	84	68
200	135	167	135
200(cap=500)	80	99	81
400	114	141	114
50 (low stdev)	36	26	36

Table 2.3: Average number of vehicles needed per week, for Strategy 1, 2 and 3.

several times.

Finally, in Table 2.5 we show the results of other versions of algorithm for Strategy 3. In version 2, we have introduced more iterations for each weight. And, in version 3 we have done more iterations for the ILS for each day, and kept the same number of iterations per weight. From the results we can conclude that by allowing more running time, the algorithm of version 2, on average, gives more non-dominated solutions in 3 of the 5 problems. In version 3, on average the number of non-dominated solutions is smaller than in the other versions but we are able to improve the solutions, when comparing with version 1 and 2.

N	Strategy 1	Strategy 2	Strategy 3
50	146,19	2,32	142,40
100	383,95	6,24	384,38
200	1116,73	17,09	1076,41
200(cap=500)	1497,82	21,13	1409,40
400	3199,07	29,07	3088,93
50 (low stdev)	152,59	3,05	149,16

Table 2.4: Average run time in seconds, per problem size, for Strategy 1, 2 and 3.

2.6 Summary and Conclusions

In this chapter we have explored different distribution strategies to analyze an integrated distribution problem. The strategies cover a week planning horizon and reflect different ways of looking at the distribution problem. The first strategy is the classical VRP approach, which reflects only transportation cost: for each day of the planning horizon the routes are designed minimizing routing costs. The second strategy is a more customer oriented strategy based on customer relationship management principle, where "master routes" are constructed to ensure a marketing policy where each customer is always served by the same driver. The third strategy is a multi-objective combinatorial optimization problem with two objectives: minimizing cost and improving customer service. This third strategy results from the integration of

	Startegy 3, version 1		Startegy 3, version 2		Startegy 3, version 3	
Examples	**a**	**b**	**a**	**b**	**a**	**b**
1	14510,34	32327	14510,34	32327	14372,34	32327
	14344,15	36840	14376,34	36570	14251,39	36840
			14318,05	36840		
Run Time	151,60		369,65		406,61	
2	13755,11	35826	13755,11	35826	13657,49	35826
	13697,92	38148	13697,92	38148	13631,61	38148
	13589,47	38538	13680,72	38278		
	13680,72	38278	13644,05	38376		
			13589,47	38538		
Run Time	141,22		358,72		386,46	
3	13289,63	36178	13289,63	36178	13235,85	35668
	13228,96	40284	13224,70	40284	13229,69	40148
	13278,06	40140			13183,31	40284
Run Time	150,23		378,33		398,76	
4	11850,30	25753	11850,30	25753	11812,68	25393
	11811,55	31776	11805,80	31416	12126,23	31416
	11710,85	32341	11751,11	32062		
	11788,70	32289	11710,85	32341		
Run Time	123,64		311,10		329,89	
5	13541,62	30733	13541,62	30733	13515,69	30336
	13462,15	38426	13485,40	37925	13441,13	36998
	13473,14	38198	13452,19	38084	13364,65	38084
	13485,40	37925				
Run Time	145,33		368,83		397,50	

Table 2.5: Routing Cost (a) and Marketing Cost (b) for the non dominated solutions of Strategy 3, for 3 different versions of the algorithm.

the two other strategies and brings together two important areas in many industries: Distribution and Marketing. The idea was to compare this new approach with the other two strategies.

For each of the above strategy we have presented a mathematical model and a heuristic procedure, based on the ILS, to solve the problems. Then, the three algorithms were applied to a set of randomly generated instances. The main conclusion is that the multi-objective model gives several non-dominated solutions, that can be seen as a good balance between optimizing the transportation cost or customer service. The decision maker has to choose the solution which meets better his business needs, since cost minimization is not the only concern in distribution management.

There are several possible extensions of this work, one is in the area of the metaheuristics and here it would be interesting to develop multi-objective population based metaheuristic to solve the multi-objective model and to perform a comparison with the current approach. The second extension would be to include other objectives that reflect different business needs, as for example, the one of balancing the routes. This is particularly interesting if we assume that driver's remuneration is related with truck loading. In this case, we would be studying the integration decisions with the Human Resources department.

Chapter 3

Multi-Period Vendor Managed Inventory Systems

Inventory systems vary throughout many dimensions, such as: the activities involved, parts studied, degrees of structural complexity, flows (information flows versus product flows) and the time horizon of the problem (one period, and finite and infinite horizons). See Zipkin (2000) for an overview of inventory systems

We can make a distinction between models with deterministic elements and stochastic elements. There are also some studies on how the introduction of inventory management technologies affects the inventory movements in some sectors, an example is the study by Worthington (1998).

The Vendor Managed Inventory (VMI) system is a particular type of inventory management system that can be defined as a production/distribution and inventory control system where the manufacturer or the retailer makes the replenishment decisions for the consuming organization. This means the vendor monitors the buyers inventory levels (physically or via electronic messaging) and makes periodic resupply decisions regarding order quantities shipping and timing, Waller et al. (1999).

Examples of industries where VMI policies are being used include: the petrochemical industry (gas stations), the grocery industry (supermarkets), the soft drink industry (vending machines) and the automotive industry (parts and components). There are several firm that have been adopting a type of a VMI system: Wal-Mart, Kmart, Dillard department stores, JCPenny are among the early adopters of VMI. But also in Hospital Materials Management, VMI achieved higher penetration, Dong and Xu (2002). Another example of a VMI system is the Campbell Soup continuous replenishment program, Cachon and Fisher (1997).

Many advantages have been pointed out by several authors on the use of VMI systems: with VMI, greater coordination supports the supplier's need for smoother production without sacrificing the buyers service and stock objectives. Transportation costs are reduced, as well as truckload management, since the distributors do not respond automatically to orders as they are received, they can hold orders and decide when to execute them. Another attractive option pointed to VMI is a more efficient route planning.

Improving coordination of replenishment orders and deliveries across multiple customers helps to improve service by, for example, changing critical deliveries. Most of the inventory reduction achieved with VMI can be attributed to the more frequent inventory reviews, order intervals and deliveries.

In our particular case, we are studying an inventory management system, with multiple and finite periods. The demand is stochastic, and the system has a single product and multiple-retailers. There is one warehouse or distributor that delivers a single product to multiple-retailers. A particular element is the week planning period, with no possibility of observing stock levels during that period, and at least one delivery during the planning period. The objective is to decide the delivery quantities and the delivery days for a set of vendor managed points such that the total inventory costs are minimized.

The organization of this chapter is the following: In Section 3.1 a literature review on VMI will be briefly summarized. Then, on Section 3.2, we will present two Multi-Period Vendor Managed Inventory (MPVMI) models. The first model does not consider any setup cost and the second model includes this additional cost. In Section 3.3, we will perform a computational experiment on some examples. Finally, some conclusions and further research are drawn in Section 3.4.

3.1 Literature review

There exists a considerable amount of literature regarding inventory management problems and control systems. However, in this section we will focus our literature review on the work that has been done about VMI systems.

The VMI systems affect the supply chain, Dong and Xu (2002) evaluates this effect and states that VMI always leads to a higher buyer's profit, but the supplier's profit varies. And, these benefits are obtainable only in a fully integrated Supply Chain.

Many references study the benefits of a VMI system over other inventory management systems. Waller et al. (1999) studies the effect of the VMI in several environments. He states that, in this relationship, buyers relinquish control of key re-supply decisions and sometimes even transfer financial responsibility for the inventory to the supplier. The arrangement transfers the burden of asset management from the consuming organization to the vendor, who, may be obliged to meet specific customer service goal (usually some sort of in stock target). Some advantages of VMI are pointed, for example, reducing costs for each partner, reducing demand volatility, mitigating uncertainty of demand and solving the dilemma of conflicting performance measures. With VMI the frequency of replenishment is usually increased from monthly to weekly (or even daily), which benefits both sides. The vendor can make replenishment decisions and the buyer transfer inventory responsibilities.

Emigh (1999) presents an overview of VMI and its growing use throughout the retail industry. The author states that VMI is popular throughout the supply chain because it lets companies shift responsibility for inventory management to vendor in order to reduce overhead. Typically the manufacturer takes a daily review of inventory by pulling down EDI files from the distributor. The manufacturer then uses the inventory data to put together an anticipated order for the distributor. The apparel industry and hospital supplies, have been using non-automated VMI for decades, supermarkets have taken longer than department stores. VMI practitioners range from food manufacturers like Kraft Inc. to chain stores such as Wal-Mart stores to industries that use VMI automation like car and paper manufacturing industries.

Harrington (1996) describes the difference between VMI and consignment selling. Consignment selling allows manufacturers to place inventory at retailer's location, with the retailer never actually owning the product. Consignment selling is similar to VMI but differs in one key area. In traditional VMI, the retailer still owns the inventory; the manufacturer simply manages it. Consignment selling is the next step, the manufacturer owns the inventory and the retailer takes a percentage for providing shelf space and customers. In our work will refer our model as a VMI system although we will not make any distinction between who owns the inventory.

Andel (1996), suggests that many manufacturers are more adept to handling inventory than the retailers they supply. In a VMI relationship, vendors receive withdrawal and current balance information from the retailer and the vendor can arrange their shipments, build their loads, and cut their purchase orders to optimize their transportation and inventory requirements.

Dong and Xu (2002) evaluates how VMI affects a supply channel. VMI always leads to higher buyer's profit, but the supplier's profit varies. VMI is an effective supply chain strategy that can realize many of the benefits obtainable only in a fully integrated supply chain.

Aichlymayr (2000) shows some benefits of VMI and collaborative planning, forecasting and replenishment.

Other authors have studied the information sharing issue at VMI systems. Chueng and Lee (2002), considers a supply serving multiple retailers located in a close proximity, in his paper he examines the benefit of using customer demand information. Two types of Supply Chains are considered that are often linked to VMI programs: the first uses information on the retailers' inventory position to coordinate shipments from the supplier to enjoy economies of scale; the second uses information for unloading the shipments to the retailers to rebalance their stocking positions.

Finally, some authors have worked on modeling VMI and optimizing replenishment policies. Disney and Towill (2002) defines VMI has a production/distribution and inventory control system where stock positions and demand rates are known across more than 1-echelon of the supply chain. VMI comes in many different forms described by terms such as Synchronized Consumer Response, Continuous Replenishment Programs, Efficient Consumer Response, and Rapid Replenishment, Collaborative Planning, Forecasting and Replenishment, Centralized Inventory Management. In his paper, the system is designed to minimize inventory-holding costs and adaptation related costs covering the need to ramp production up and down to meet perceived needs. In this work the system is designed for various different ratios of production adaptation costs and inventory holding costs and presents a decision support system that allows tuning the VMI system.

Fry et al. (2001) models a type of VMI agreement where optimal replenishment and production policies for a supplier are found to be up-to-policies. The conclusion is that VMI performs better than a Retailer Managed Inventory (RMI) system (where the retailer managed inventory), in many settings but can perform worse in others, depending on the scenario and the contract parameters. For example, VMI performs better if outsourcing is very expensive or variance is high.

The VMI can also provide benefits by allowing the vendor to coordinate production and delivery, particularly in the case of multiple retailers, the following authors have studied this situation: Cheung and Lee (2002); Campbell et al. (1999) studied a VMI policy where the objective was to minimize the average daily distribution cost during the planning period without causing stock-outs at any of the customers' location; Kleywest et al. (2000), formulates an inventory routing problem for a VMI system where the supplier can measure the inventory level and decisions are made daily; Çetinkaya and Lee (2000) present an analytical model for coordinating inventory and transportation decisions in a VMI system where the vendor can hold the orders.

Our work can be included in this last group of literature. The basic idea is to develop a model for a VMI system with the characteristics of a week planning period, stochastic demand, unobservable stock levels and with a minimum of one visit per week.

The motivation of this work is to respond to a strategic need and a growing tendency for planning deliveries and coordinating strategies within an integrated supply chain. Given a set of inventory managing costs we want to design the best delivery strategy taking into consideration the entire planning horizon. Even if, in practice, you may adapt your deliveries to customers and orders, you can obtain substantial gains by planning your inventory needs in advance. Another motivation for this work was based on the advantages that seem to exist when using a VMI system, in the coordination of the inventory management area with other management areas at the operational and strategic level, such as distribution, scheduling or location.

In the next section we will present the two models of VMI system.

3.2 The MPVMI Models

Our model consists of a VMI system with one supplier and multiple retailers. The distributor has the responsibility to decide when to visit the retailer's locations and how much to deliver. This particular model has the objective of designing the deliveries for a planning period, for a five day week period. One of the characteristics of the VMI systems is that the distributor is responsible for managing the inventory at the retailer point. He would like to minimize inventory handling costs while avoiding stock-out situations.

In a stock-out situation, a customer arrives at the selling point and there is no unit of product available at the site. This situation leads the distributor to incur a stock-out cost. These costs can be seen in two different ways: the cost of an emergency delivery to be able to serve the customer or an opportunity cost of a lost sale (in this case, assuming the customer leaves the retailers point and goes elsewhere). In both cases there is a strong cost, higher than the cost of handling an item on the shelves. So, one assumption of the model is that stock-out cost is always bigger than the holding cost and both are costs applicable per unit of product per day.

Next, we will list other assumptions of the model.

3.2.1 Assumptions of the model

- A week planning period is considered. We want to analyze a delivery strategy, we could have chosen two weeks or even one month but we have considered that one week is a common frequency in many industries and it is reasonable to plan for a week.

- There are no handling stock costs at the warehouse, and we assume that there is enough amount of product (unlimited capacity). Keeping stock at the warehouse is always relatively cheep when comparing with the cost of keeping stock at retailers location, due to economies of scale and dispersion of locations. We will consider this cost as being part of production costs.

- Retailers have stochastic demand, but the distribution function of the demand for each customer and for each day is known.

- At the retailers location there are inventory handling costs and a cost for stock out, payed by the distributor.

- Each location is visited at least once a week. There is a periodic need for visiting the location for many possible reasons like: control reasons, checking stocks, shelves positions, promotion controls or other marketing activities. This assumption, in a way, reflects the cross-functional integration within a firm.

- The stock is only observed at the beginning of the planning period, and the decisions are made for the all planning horizon, independently of what occurs during the week. So, if demand exceeds the amount available there is a stock-out cost. There are no emergency deliveries or changes in the delivery plan due to the arrival of information. This does not mean that in practice these situations do not occur. But, we are analyzing a strategy for the best delivery policy with the information that we have, to minimize expected costs.

- The holding and stock-out costs only depend on quantities and not on retailers. They are both applied per unit of product per day and are the same for all retailers.

- We will consider two models: On the first model, there is no delivery cost, the only costs considered are the holding and stock-out cost at the retailers point. On the second model, there is an additional fixed setup cost per delivery made.

- The setup cost is a delivery cost per visit. This cost varies by retailers location and by day, but does not depend on the quantities delivered.

3.2.2 The MPVMI model with no setup cost

In the first model, we consider that there is no cost associated with the delivery of the product to the retailer. The only costs considered are the costs of managing the inventory at the retailers point and the cost of a stock-out situation. The distributor has to decide, for each day, how much to deliver and to which retailers, minimizing total costs. Delivering higher quantities increases the holding cost but decreases the possibilities of a stock-out.

The costs of the problem are:

- Inventory handling costs (holding costs) per unit of product at the retailers point, per day;
- Stock out costs at the retailers point, per unit lost, per day;

The decisions to make are the following:

- Decide, for each day, which points will be visited;
- How much to deliver at these points on each day.

Objective function:

Objective is to minimize the expected total cost at the end of the week;

Min Weekly cost = Inventory holding cost + Stock-out cost.

Notation:

n = number of retailers, indexed from 1 to n;

P = number of periods (in this case, 5 periods, from 0 to 4);

h = inventory carrying cost per unit;

s = shortage cost per unit;

β_{pi} = initial inventory at location i on day p, $i = 1, ...n$; $p = 1, ..., P$ and β_{0i} is known;

$F_{ip}(.)$ = cumulative distribution function of the one period demand at retailer's location i, for day p, $i = 1, ...n$ and $p = 1, ..., P$;

f_{ip} = distribution function of the one period demand at retailer's location i, for day p, $i = 1, ...n$ and $p = 1, ..., P$;

t_{ip} = random variable representing the demand of retailer i on day p, $i = 1, ...n$ and $p = 1, ..., P$;

Decision Variables:

$$w_{ip} = \text{amount delivered to retailer } i \text{ on day } p$$

$$y_{ip} = \begin{cases} 1 & \text{if} \quad \text{retailer } i \text{ is visited on day } p \\ 0 & \text{if} \quad \text{otherwise} \end{cases}$$

Inventory cost:

$$\sum_{i \in B} \sum_{p} \left[\begin{array}{l} h \sum\limits_{t_{ip}=0}^{\beta_{ip}+w_{ip}} (\beta_{ip} + w_{ip} - t_{ip}) f_{ip}(t_{ip}) dt_{ip} \\ +s \sum\limits_{\beta_{ip}+w_{ip}}^{\infty} (t_{ip} - \beta_{ip} - w_{ip}) f_{ip}(t_{ip}) dt_{ip} \end{array} \right] \tag{3.1}$$

For each day and for each retailer, the inventory cost is equal to the the sum of the expected inventory holding and stock-out costs.

For each period, if demand exceeds the initial stock of that period plus the quantity delivered, there is a cost s per unit exceeded. Otherwise, if the demand is smaller than the quantity delivered and the initial stock, the vendor incurs in a holding cost h, per unit of end stock.

This expression applies for the case where demand is a discrete variable. Since the representation of the probability distribution is difficult to find, particularly when demand ranges over a large number of possible values, the discrete random variable is often approximated by a continuous random variable (Hillier and Lieberman 1995). Furthermore, when demand ranges from over a large number of possible values, this approximation will generally yield a nearly exact value of the optimal amount. In addition, when discrete demand is used, the resulting expression may become slightly more difficult to solve analytically. So, we will approximate the demand as a continuous random variable and at the end we approximate the solution to the closest integer.

Let $I_{ip}(w_{ip})$ represent the inventory managing cost of location i on day p.

$$\sum_i \sum_p I_{ip}(w_{ip})$$
$$= \sum_i \sum_p \left[\begin{array}{c} h \int_0^{\beta_{ip}+w_{ip}} (\beta_{ip} + w_{ip} - t_{ip}) f_{ip}(t_{ip}) dt_{ip} \\ +s \int_{\beta_{ip}+w_{ip}}^{\infty} (t_{ip} - \beta_{ip} - w_{ip}) f_{ip}(t_{ip}) dt_{ip} \end{array} \right]$$

The problem can be stated as follows:

$$\text{Min} \sum_i \sum_p I_{ip}(w_{ip}) \tag{3.2}$$

Subject to:

$$w_{ip} \leq M \times y_{ip}, \qquad \forall\ i,\ p \tag{3.3}$$

$$1 \leq \sum_p y_{ip}, \qquad \forall\ i \tag{3.4}$$

$$w_{ip} \geq 0, \qquad \forall\ i,\ p \tag{3.5}$$

$$y_{ip} \in \{0, 1\} \qquad \forall\ i,\ p \tag{3.6}$$

The meaning of the above constraints is:

(3.3) If the quantity to be delivered is positive, than that location has to be visited on that day.

(3.4) The VMI retailers are visited at least once a week.

(3.5) The variable w_{ip}, that represents the quantity delivered to a retailer on a given day is always greater or equal to 0.

(3.6) This constraint defines y_{ip} as binary.

At this point, we have an additional problem with the initial inventory: The initial stock in only observed at the beginning of the planning horizon. Then, we have to decide the delivery days and quantities without considering any information of the stocks during the period. We observe the initial inventory of each retailer at the beginning of period 0, so the initial stock of the future periods is also a random variable, since it depends on demand of all the previous periods.

At each period, if there is a stock-out, a cost is incurred and the initial stock of the following period will be zero. There is an interdependence between all periods, the initial inventory for each period is:

$$\begin{aligned}
\beta_{i0} &= u_i > 0, \text{ observed} \\
\beta_{i1} &= Max\left\{0, u_i + w_{i0} - t_{i0}\right\} \\
\beta_{i2} &= Max\left\{0, \beta_{i1} + w_{i1} - t_{i1}\right\} = Max\left\{0, Max\left\{0, u_i + w_{i0} - t_{i0}\right\} + w_{i1} - t_{i1}\right\} \\
\beta_{i3} &= Max\left\{0, \beta_{i2} + w_{i2} - t_{i2}\right\} = \\
&= Max\left\{0, Max\left\{\begin{array}{c} 0, Max\left\{0, u_i + w_{i0} - t_{i0}\right\} + \\ w_{i1} - t_{i1} \end{array}\right\} + w_{i2} - t_{i2}\right\} \\
\beta_{i4} &= Max\left\{0, \beta_{i3} + w_{i3} - t_{i3}\right\} = \\
&= Max\left\{0, Max\left\{\begin{array}{c} 0, Max\left\{\begin{array}{c} 0, Max\left\{0, u_i + w_{i0} - t_{i0}\right\} \\ +w_{i1} - t_{i1} \end{array}\right\} \\ +w_{i2} - t_{i2} \end{array}\right\} + w_{i3} - t_{i3}\right\}
\end{aligned}$$

So, the initial stock of period p will be a function of the initial stock at period 0 and of the quantities delivered and demand of all previous periods.

$$\beta_{ip}(u_i, w_{i0}, ..., w_{ip-1}, t_{i0}, ...t_{ip-1})$$

In this model we will assumed a stochastic demand that follows an exponential distribution known by the distributor, with the form:

$$f(t_{ip}) = a_{ip}e^{-a_{ip}t_{ip}}$$

with the average demand $1/a_{ip}$. There are many different statistical distribtuions to choose from when working with inventory control. The exponential distribution function is commonly used to represent the demand variation in inventory models. See Lidke and Malstrom (1987) and Snyder (1984).

It makes sense that in some cases demand varies from day to day and from retailer to retailer, we have considered a demand function parameter for each retailer and for each day.

We need to calculate the inventory cost for each period. So, for each period we need to consider all the possible scenarios of the previous periods initial inventory. This is, for period 0, we know the initial inventory, for period 1 we need to consider two possibilities: stock-out at the end of period zero (zero initial inventory at period 1) and positive stock at the end of period 0 (initial stock at period 1 positive). Extending this for all the periods we obtain the complete inventory cost function.

$$h\int_0^{\beta_{ip}+w_{ip}}(\beta_{ip}+w_{ip}-t_{ip})f_{ip}(t_{ip})dt_{ip}+s\int_{\beta_{ip}+w_{ip}}^{\infty}(t_{ip}-\beta_{ip}-w_{ip})f_{ip}(t_{ip})dt_{ip}$$

We will use the Heaviside function to simplify the expression,

substituing:

$$1(t_{ip}<\beta_{ip}+w_{ip})=\begin{cases}1 \quad if \quad t_{ip}-\beta_{ip}-w_{ip}<0\\ 0 \quad if \quad \text{otherwise}\end{cases}$$

becomes:

$$\int_0^{\infty}\begin{bmatrix} h\times 1(t_{ip}<\beta_{ip}+w_{ip})(\beta_{ip}+w_{ip}-t_{ip}) \\ +s\times 1(t_{ip}>\beta_{ip}+w_{ip})(t_{ip}-\beta_{ip}-w_{ip})\end{bmatrix} f(t_{ip})dt_{ip}$$

For simplicity, in the next sections we will derive the expressions for one retailer only, then add for all retailers.

Since obtaining the expression for the inventory cost function is complex we will first obtain the expressions for the inventory cost expression for each period separately, the complete cost function is the sum of the inventory cost of each day.

For the first period, the inventory cost, I_0, is:

Period 0:

$$\begin{aligned} I_0 &= \int_0^{\infty}\begin{bmatrix} h*1(u+w_0-t_0)*(u+w_0-t_0) \\ -s*1(-u-w_0+t_0)*(u+w_0-t_0)\end{bmatrix} f(t_0)dt_0 \\ &= exp(-a_0*(u+w_0))*(s+h)/a_0+h*(a_0*u+a_0*w_0-1)/a_0 \end{aligned}$$

I_0 is the inventory cost in period 0. It is equal to a holding cost of $h*(u+w_0-t_0)$ if $u+w_0>t_0$ and a shortage cost of $s*(t_0-u-w_0)$ if $u+w_0<t_0$.

For period 1 we will have to consider the 2 possible outcomes of period 0.

Period 1:

$$I_1 = \int_0^{\infty}\int_0^{\infty} \left[\begin{array}{c} 1(u+w_0-t_0)*(h*1(u+w_0+w_1-t_0-t_1) \\ *(u+w_0+w_1-t_0-t_1) \\ -s*1(-u-w_0-w_1+t_0+t_1) \\ *(u+w_0+w_1-t_0-t_1)) \\ +1(-u-w_0+t_0)*(h*1(w_1-t_1)*(w_1-t_1) \\ -s*1(-w_1+t_1)*(w_1-t_1)) \end{array} \right] f(t_0)f(t_1)dt_0dt_1$$

$$\begin{aligned} I_1 &= ((\alpha_1*u+\alpha_1*w_0+1)/\alpha_1*h+(\alpha_1*u+\alpha_1*w_0 \\ &\quad +1)/\alpha_1*s)*exp(-\alpha_1*(u+w_0+w_1)) \\ &\quad +1/\alpha_1*h*exp(-\alpha_1*(u+w_0)) \\ &\quad +(-2+\alpha_1*w_1+\alpha_1*u+\alpha_1*w_0)/\alpha_1*h \end{aligned}$$

I_1 is the inventory cost of period 1. It is the inventory cost considering two possibilities:

- Positive stock in period 0:

$$1(u+w_0-t_0)*(h*1(u+w_0+w_1-t_0-t_1)*(u+w_0+w_1-t_0-t_1)$$

$$-s*1(-u-w_0-w_1+t_0+t_1)*(u+w_0+w_1-t_0-t_1))$$

- Stock-out in period 0:

$$1(-u - w_0 + t_0) * (h * 1(w_1 - t_1) * (w_1 - t_1) - s * 1(-w_1 + t_1) * (w_1 - t_1))$$

For the third period we would have to consider 4 possibilities, for the fourth period 8 and 16 for the fifth.

The analytical expression for the inventory cost function of all the periods is in Appendix A.

The total inventory cost will be the sum of the cost of each period:

$$\sum_{p=0}^{4} I_p$$

The inventory cost function is a non-linear function. The first order derivatives of the inventory cost function are also non-linear functions. And, we also have the constraint that the w's can not assume negative values. So, to minimize the function we will use the Gauss-Newton's approximation methods (Bertsekas 1995).

The result obtained after solving this model is a delivery policy which implies that the distributor visits a location almost every day. In some cases, if initial stock is sufficiently high, then the best solution will be to deliver after this stock has ran out. Nonetheless, it makes sense to include a cost per delivery that might change the above delivery policy. This cost, in a way, reflects the transportation cost. This is what we will consider in the next model.

3.2.3 The model with setup cost

Consider now that there is an additional cost, a setup cost per delivery. This setup cost is a fixed charge, independent of the quantity delivered, that will vary from retailer to retailer and eventually from day to day and is applied whenever a deliver is made to a retailer. It does not depend on the quantities delivered, it corresponds to the cost of preparing the order, transportation cost and delivery cost.

The model will again minimize total cost, but this time it includes holding and stock-out costs per unit of product per day and the setup cost per day and per retailer visited.

The setup cost varies from retailer and day, this assumption is justified when looking at the setup cost as a transportation cost. Customer far away from the warehouse or with more difficult access (for example: downtown areas) have associated a higher delivery cost.

Delivery costs might also vary by day, looking again in terms of transportation and analyzing the group of retailers to be served, a retailer away from the warehouse but closer to other group of retailer being visited has a lower cost than if he is isolated. Another aspect is the number of routes, a retailer served on a given day might be included in a tour which is costly than if it is included on another day.

Let us now introduce the setup cost in the inventory function:

z_{ip} = setup cost of delivering to retailer i on day p, with $i = 1, ...n$ and $p = 1, ..., P$;

The inventory cost function with setup cost, (IS), is:

$$IS = \sum_i \sum_p \begin{bmatrix} 1(w_{ip} < 0) \times z_{ip} \\ +h \int_0^{\beta_{ip}+w_{ip}} (\beta_{ip} + w_{ip} - t_{ip}) f_{ip}(t_{ip}) dt_{ip} \\ +s \int_{\beta_{ip}+w_{ip}}^{\infty} (t_{ip} - \beta_{ip} - w_{ip}) f_{ip}(t_{ip}) dt_{ip} \end{bmatrix}$$

with

$$1(w_{ip} > 0) = \begin{cases} 1 \quad if \quad w_{ip} > 0 \\ 0 \quad if \quad otherwise \end{cases}$$

Let us start by analyzing the inventory cost for only one retailer i and one period p:

$$\begin{array}{ll} z_{ip} + I(w^*_{ip}) & if \quad w^*_{ip} > 0 \\ I(w^*_{ip}) & if \quad w^*_{ip} = 0 \end{array}$$

To solve this problem:

First, let w^*_{ip} be the delivering quantity that Minimizes I_{ip}

If we deliver, it becomes: $I(w^*_{ip}) + z_{ip}$

So we have to compare $I(w_{ip} = 0)$ with $I(w^*_{ip}) + z_{ip}$

Let r^*_{ip} be the value that equals $I(w_{ip} = 0) = I(w^*_{ip}) + z_{ip}$

Then the optimal cost will be:

$$\begin{array}{ll} z + I(w^*_{ip}) & if \quad \beta_{ip} \le r^*_{ip} \\ I(w_{ip} = 0) & if \quad \beta_{ip} > r^*_{ip} \end{array}$$

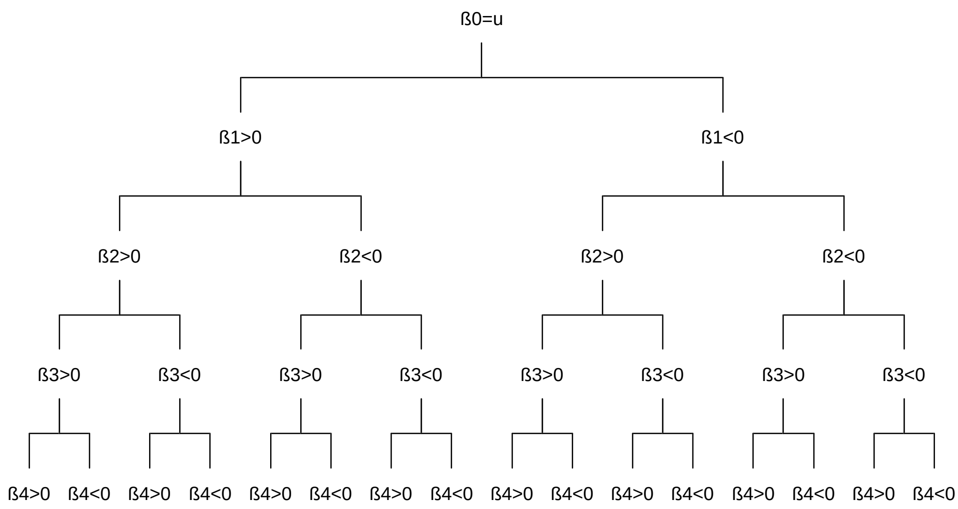


Figure 3.1: Decision tree for the MPVMI model with setup cost.

If we try to extend this problem to more than one period the decision tree will increase, see Figure 3.1 for the decision tree of all the planning horizon.

To solve this problem becomes a complex task. The solution we propose is to analyze the best delivery quantities for all the possible combinations of delivery days and choose the solution with minimum total costs (inventory + stock-out + setup costs).

So, for each retailer given the setup cost for each day of the week, we calculate the best delivery quantities for all the possible combinations of days.

We do not consider the case when there is no delivery since one of the assumptions is that at least once a week the retailer has to be visited.

The procedure to solve the problem is the following :

- Minimize the inventory cost for each combination of delivery days.

- Obtain the best delivery quantities for that combination.

- Calculate the total inventory cost by adding to the inventory cost the corresponding setup-cost.

- Choose the combination and quantities that gives the best solution (minimum total inventory cost).

- Repeat the procedure for each retailer.

Another point is the constraint that imposes only positive values for the deliveries, which transforms the problem into a constrained minimization problem.

The solution to the first model, with no setup cost corresponds to the combination where we considered that is possible to delivery every day of the planning period (5 days). The second model, with the setup costs, chooses the best solution from all the possible combinations, including delivering every day.

In the next section, we will describe the computational experiment that we have done on several examples, and show the main results.

3.3 Computational experiment

In this section, we will present a numerical study on the two MPVMI models with and without setup costs. We will refer to Model 1 (M1) for the MPVMI with no setup cost and Model 2 (M2) to the MPVMI with setup cost.

The objective of this experiment is to compare the solutions of the two models in terms of costs and delivery days.

Results show that optimal solutions depend on the type of demand and on the setup cost. When introducing a setup cost, the average number of delivery days decreases while total costs increase.

Next, we will explain the data used and analyze some important results of this experiment.

3.3.1 The data

For the computational experiment we have generated several instances that can be divided in different groups according to their size, measured by the number of retailers $(10, 50, 100, 200)$, type of setup cost (equal every day versus different for each day, and low versus high) and type of demand (same every day and different every day).

For the demand data we have generated the distribution function parameter α_{ip}. So, for each retailer i and day p we have generated a value for α_{ip}. These values were randomly chosen from a Normal distribution with mean 50 and standard deviation 20 between retailers. For instances with demand varying from day to day, the mean was the same as before, but the standard deviation between days was equal to 2.

The initial stock for each retailer was generated by a random uniform distribution between 0 and 50. Finally, the setup cost was generated for each retailer and for each day of the week, following a uniform distribution between 10 and 200 for high setup cost and between 5 and 50 for low setup cost. The total number of cases analyzed was 160.

3.3.2 Analysis of the results

We can start by analyzing and comparing the Inventory Cost. In Table 3.1, we have the Inventory Cost for M1 and M2, with and without the setup cost[1], separated by size and type of setup cost. The cost in this table is an average of all instances with the same size.

From Table 3.1 we can observe that M2 has always an Inventory Cost (free of the setup cost) greater than M1. However, M1 and M2 are closer to each other when the setup cost is low. These results were expected since if we are using an additional cost in the decision process the solution will be worst. The opposite happens when looking at both models when adding the correspondent setup cost, M1 will have always the highest total cost.

In Table 3.2, the instances were separated into two groups: the group with a setup cost that is equal every day and a group with setup cost that differs each day.

The results show that, when setup cost differs from day to day, then the best solution of M2 has a higher cost than when the setup cost is equal every day. This

[1]Although we have assumed that in M1 there are no setup costs, we have added to M1 the setup cost that would be applied if we were delivering on the days proposed by M1. This is done so that we can compare both models.

N	*Setup*	*M1*	*M1+setup*	*M2*	*M2+setup*
10	HS	6211,60	11039,55	7193,79	8270,99
	LS	6211,60	7335,95	6318,67	6989,07
50	HS	30304,26	55249,71	35309,20	40974,90
	LS	30304,26	36798,16	31431,37	34983,87
100	HS	60588,02	110789,07	70499,63	81778,17
	LS	60588,02	73643,97	62803,86	69832,36
200	HS	120429,09	222111,27	140718,38	163254,89
	LS	120429,09	146841,40	125169,50	139334,50

Table 3.1: Average Inventory Costs for M1 and M2 for Low (LS) and High (HS) setup costs.

N	***Low Setup***	***M1***	***M1+setup***	***M2***	***M2+setup***
10	=	6211,60	7314,10	6298,71	6997,01
	≠	6211,60	7357,80	6338,63	6981,13
50	=	30304,26	36592,86	31176,13	34944,93
	≠	30304,26	37003,46	31686,60	35022,80
100	=	60588,02	73531,22	62465,64	70004,54
	≠	60588,02	73756,72	63142,08	69660,18
200	=	120429,09	146875,40	124546,33	139814,12
	≠	120429,09	146807,40	125792,68	138854,89
N	***High Setup***	***M1***	***M1+setup***	***M2***	***M2+setup***
10	=	6211,60	10994,00	7142,08	8441,98
	≠	6211,60	11085,10	7245,50	8100,00
50	=	30304,26	54572,06	34918,82	41780,72
	≠	30304,26	55927,36	35699,58	40169,08
100	=	60588,02	110704,02	70171,28	83733,48
	≠	60588,02	110874,12	70827,97	79822,86
200	=	120429,09	223041,82	140034,83	167567,55
	≠	120429,09	221180,72	141401,92	158942,23

Table 3.2: Average inventory cost for M1 and M2 with equal setup cost (=) every day versus different setup cost ($\neq$) for each day.

N	Demand	M1	M1+setup	M2	M2+setup
10	=	6494,42	9512,12	7128,76	8060,41
	≠	5928,78	8863,38	6383,69	7199,64
50	=	29787,64	45563,24	33150,73	37696,18
	≠	30820,88	46484,63	33589,84	38262,59
100	=	60830,22	92653,16	67667,79	76941,63
	≠	60345,83	91779,88	65635,70	74668,90
200	=	121550,58	185917,46	135340,07	153995,22
	≠	119307,60	183035,21	130547,82	148594,18

Table 3.3: Average Inventory Cost for M1 and M2 with equal demand (=) every day versus different demand (≠) for each day.

can be explained by the higher influence setup cost plays on solutions, if it varies, solutions adjust more to these differences.

We have also analyzed and separated the instances by the type of demand. In Table 3.3, we have the inventory cost for M1 and M2 with equal demand every day and different demand for each day. The same relation applies for the difference between M1 and M2. This difference is smaller when demand varies from day to day. This is, the model leads to lower costs by playing with the variations on the expected demand.

In Table 3.4, we can see the average number of delivery days in the solutions of M1 and M2. The first observation is that the number of delivery days does not depend on

N	M1	M2
10	4,70	2,41
50	4,76	2,37
100	4,63	2,23
200	4,78	2,32

Table 3.4: Average number of delivery days per week in M1 and M2.

the size of the problem (retailer have an inventory cost that is independent of other retailers). As expected M1 has high delivery days in the planning period, on average $4,7$ days per week (in 5 days). When introducing a setup cost in the decision process (M2), the number of days a retailer is visited during the planning horizon falls to an average of $2,3$ days a week.

And, if we analyze the two instances separated by high and low setup cost (see Table 3.5), in M2, the average number of delivery days goes from an average of 3 days per week for the Low setup cost case to an average of 1,6 delivery days for the case of high setup costs. There is a strong relation between the magnitude of the setup cost and the number of delivery days per week in the solution.

The relation between the two models can also be observed in Figure 3.2. In this graph we have the inventory cost for M1 and M2 with and without setup cost, for a

N	Setup	M1	M2
10	HS	4,70	1,58
	LS	4,70	3,25
50	HS	4,76	1,64
	LS	4,76	3,10
100	HS	4,64	1,74
	LS	4,62	2,72
200	HS	4,78	1,61
	LS	4,78	3,03

Table 3.5: Average number of delivery days in M1 and M2 for the cases of Low (LS) and High (HS) setup cost.

set of 20 retailers with equal demand and low and equal setup cost. As can be seen, the inventory cost of M1 is always the lowest if free of setup costs and the highest costs if we add the correspondent setup cost.

Finally, concerning the run time of the algorithm, in Table 3.6, we see that it is proportional to the size of the problem (number of retailers), this result is due to independence of the inventory cost function among retailers. The run time, in these particular models, does not have major implications since it takes an average of 0,38 seconds per retailer to solve a week planning problem.

The objective of the above numerical experiment is twofold: In the first place analyze the two models in terms of inventory cost and number of delivery days.

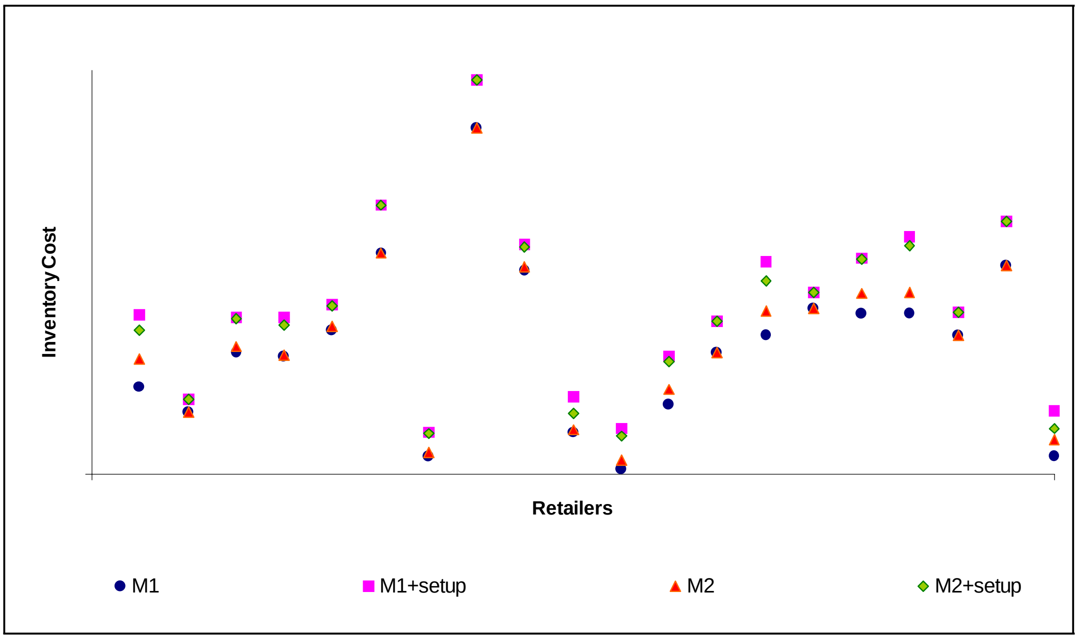


Figure 3.2: Inventory Costs for M1 and M2 for a group of 20 Retailers.

N	***Run time in seconds***
10	4,03
50	19,32
100	38,26
200	76,59

Table 3.6: Run time in seconds for M1+M2.

Secondly, to give an insight on the effect the delivery frequency can have in terms of costs. There is a clear trade-off between delivery days and inventory costs and the decision maker might evaluate what is the effect, for example, in terms of marketing or other control policies, of not visiting a retailer on a given day.

3.4 Summary and Conclusions

Inside the trend to a wider cooperation and integration of logistics processes is an initiative called the VMI system. In a VMI system the supplier makes the inventory decisions for the consuming organization.

These type of systems have gained importance during the last few years. The introduction of technologies for information sharing and coordination has contributed for the success of those policies in many industries. Nonetheless, there are still gaps to fill in the research in the area of modeling these agreements and using algorithms to strategically plan the inventory managing systems.

The motivation of this work arrives from the need to strategically respond to the growing tendency for planning deliveries and coordinating strategies within an integrated suplly chain. And also, from the advantages that seem to exist when using a VMI system, in the coordination of the inventory management area with other management areas at the operational and strategic level, such as distribution, scheduling or location.

This work presents two inventory models: the MPVMI with no setup cost and the

MPVMI with setup costs. In these models it is the responsibility of the distributor to manage the inventory at the retailers' location. The distributor does not know the demand of the product and only observes the initial stock at the beginning of the planning period. He has to decide how much and when to deliver to each of its retailers while trying to minimize total inventory costs. We have done a computational experiment and the results show that, by introducing a setup cost, the number of deliveries per planning period are reduced while holding plus stock-out costs increase. The impact of this increment depends on the type of demand and level of setup cost used.

However, when comparing both models adding the setupcost we can observe that the model M2 holds better results. This observation leads us to conclude that developing integrated decision models will result in a better global decision process. So, our models gain interest in the above setting if we consider them as tools for the coordination with other areas in the Supply Chain. One example is to consider the setup cost as a transportation cost and to use this model to manage and define policies for a large scope of the supply chain management, in this particular case the transportation and inventory management. This is commonly known as the inventory-routing model and will be the subject of the next chapter: design a model for the integration of this VMI system with other transportation issues, such as the routing of vehicles for the distribution the products to retailers.

Chapter 4

Multi-Period Inventory-Routing Model with two types of customers

In many industries, the logistic planning functions of transportation and inventory play an important role and integrating these two areas may lead to significant gains and more competitive distribution strategies. The coordination of these activities can be an extremely important source of competitive advantage in the supply chain management. The battle for cost reduction can pass through the equilibrium of transportation costs versus inventory managing costs.

Both the Vehicle Routing and the Inventory Management problems have been extensively studied separately and there exists a vast amount of literature on these areas. However, when looking at the two problems together, the amount of work found is much less. Many models have been proposed for inventory problems with no routing decisions considered and many studies exists on vehicle routing problems in which no inventory management is mentioned.

The problem that considers Vehicle Routing and Inventory Management decisions together is known as the Inventory Routing Problem (IRP). The main objective in the IRP is to design the set of routes and delivery quantities that minimize transportation cost while controlling inventory costs. Considering these two problems in an integrated manner can reduce total costs.

The model we propose is a Multi-Period Inventory-Routing with two types of customers: the Vendor-Managed Inventory customers and the Customer Managed Inventory (CMI) customers. The VMI customers have a random demand and the distributor manages the stock at the VMI customer location. The CMI have fixed demand and the distributor faces no inventory costs associated with these customers.

The objective is to determine the routes for a week planning period and the quantities delivered to the VMI points, minimizing total transportation plus inventory costs.

The motivation of this work can be found on the advantages that can be obtained when integrating decision processes along the supply chain. In this particular case, trying to reduce total costs through the coordination of decisions between distribution management and inventory management areas. Although there exist some literature on the integration issue, none of them addresses the specific case of this model: two types of customers, a week minimum visit and no information on the inventory levels during the period.

This chapter is organized in the following way: First we make a review on some relevant IRP literature. Then, we expose our **Multi-Period Inventory-Routing Problem with Stochastic and Deterministic Demand** (MPIRP-SDD) model in detail. In section 4.3, the solution method, based on the Iterated Local Search is presented. In section 4.4, we show the results of a computational experiment. And finally, we draw some conclusions and further research.

4.1 Literature review

Ferdergruen and Simchi-Levi (1995) make a good summary on inventory-routing problems. These authors divided the IRP models into two variants: the single period model and the infinite horizon model.

Baita et al. (1998) also present a review on dynamic routing and inventory. These problems are characterized by having a dynamic environment. Repeated decisions have to be taken at different times within some time horizon and, earlier decisions influence later decisions.

We will now present a review on IRP literature relevant to our study, separating the existing work into Finite and Infinite Horizon Models.

4.1.1 Finite Horizon

For the single period inventory routing model, Ferdergruen and Zipkin (1984) addressed the problem of allocating a scarce resource, available at the central depot, among several locations, each with random demand while planning the deliveries using a fleet of vehicles. At the beginning of the period the initial inventory is reported to the depot. This information is used to determine the allocation of the available product, for the next day, among the locations. At the same time the assignment of customers to vehicles and their routes are determined. After deliveries are made, the demands occur and inventory carrying and shortage costs are incurred at each location proportional to the end of the period inventory level. In this model it is possible to choose not to visit some of the locations.

Dror and Ball (1987), decompose the multi-period problem into series of single period problem. They studied the case where, in each time interval, only customers, who will reach their safety stock level during this interval, are serviced. Trudeau and Dror (1992) solve the problem for stochastic demand.

Bard et al. (1998) present a decomposition scheme for solving inventory routing problems with satellite facilities, in which a central depot must restock a subset of customers on an intermittent basis. In this setting, the customer demand is not known with certainty. A unique aspect is the presence of satellite facilities where vehicles can be reloaded and customer deliveries continue until the closing time is reached. Jaillet et al. (2002) present an incremental cost approximation to be used in a rolling horizon framework for the above problem of minimizing total expected annual delivery costs.

Campbell et al. (1999) presents an inventory-routing problem and an optimization based approach for its solution. In the 1^{st} phase they decide which customers receive a delivery on each day of the planning period and decide on the size of deliveries and on the 2^{nd} phase the actual delivery routes and schedules for each day. They consider a small set of delivery routes, constructed using a cluster and vehicles are allowed to make multiple trips per day. The objective is to minimize the average daily distribution cost during the planning period without causing stockouts. There is no inventory holding costs or stockouts costs included in this model.

In the literature we find IRP with specific characteristics, such as considering a limited amount of product at the warehouse, see Chien et al. (1989). They address the problem of distributing a limited amount of inventory among customers using a fleet of vehicles so as to maximize profit. The problem is to decide how to allocate its available inventory to the different customers and they assume that the warehouse does not have enough supply to satisfy each customer maximum demand. In order to determine which customers must be served and the amount to supply to each selected customer, we need routing cost information so that the marginal profit (revenue minus delivery cost) for each customer can be accurately computed. The delivery cost for each customer depends on the vehicle routes, which in turn requires information about customer selection and the mount of inventory allocated.

Some research can also be found on the Direct Deliveries Strategies[1] in IRPs. Burns et al. (1985) developed an analytical method for minimizing the costs of distributing freight by truck from a supplier to many customers. They derive formulas for transportation and inventory costs, and determine the optimal trade-off between these costs. The paper analyses and compares two distribution strategies Direct Shipping (Direct Delivery) and Peddling[2]. For Direct Shipping, the optimal shipment size is given by Economic Order Quantity (EOQ) model while for Peddling, the optimal shipment size is a full truck. There is no VRP, they consider a Minimum Path model inside each region.

Bertazzi et al. (2002) study a multi-period model with deterministic demand in which a set of products is shipped from a common supplier to several retailers. A retailer can be visited several times during the time horizon. A shipping policy consists of determining for each delivery time instant the set of retailers to visit, the quantity of each product to ship to each retailer and the route of the vehicle. The inventory policy is an Order Up-to Level policy[3]. They investigate the case of a single product and a single vehicle.

[1] Direct Delivery Strategies consists in shipping separate loads for each customer.

[2] A peddling strategy consists in dispatching trucks that deliver items to more than one customer.

[3] A Order Up-to Level policy is a policy where every time a retailer is visited, the quantity of each product delivered is such that the maximum level of inventory is reached.

4.1.2 Infinite Horizon

In the Infinite Horizon IRP, Anily and Ferdergruen (1990) developed a model of a one warehouse multi-retailer system with vehicle routing costs. The objective is to determine the feasible replenishment strategies minimizing long run average transportation and inventory costs. A replenishment strategy specifies a collection of regions covering all outlets. If an outlet belongs to several regions a specific fraction of its sales is assigned to each of these regions. Each time, one of the outlets in a given region, receives a delivery; this delivery is made by a vehicle that visits all other outlets in the region as well (in an efficient sequence or route). These authors allow regions to overlap. See Hall (1991) and Anily and Ferdergruen (1991) for more comments on this model.

Anily and Federgruen (1993) extended their analysis on the one warehouse multiple retailer system with vehicle routing costs, to the case where inventories may be kept in the warehouse.

Another work on the Infinite Horizon IRP is the one by Barnes-Schuster and Bassok (1997). These authors studied the situation where retailers face random demands from known distributions functions. Ordered goods arrive at depot and are allocated and delivered to the retailers. Retailers see demands, report to the depot and are charged inventory holding and shortage costs; depot decides whether or not to place the order for retailer i. The question that the authors answer is: when it will be effective for the depot to use Direct Shipping and Order Up-to Level rounded to full trucks as its sole ordering policy. Effectiveness is defined as the ratio of the long run average cost per period of the policy at hand to a lower bound on the long run average cost over all possible policies.

Chan et al. (1998), consider a distribution system consisting of a single warehouse and many geographically dispersed retailers. The objective is to determine an inventory policy and a routing strategy such that each retailer can meet its demands and the long-run average transportation and inventory costs are minimized. This paper studies the Zero-Inventory Ordering policy[4] and the Fixed Partition Policies[5].

Also, Kleyweght et al. (2002), studied the problem of determining optimal policies for the distribution of a single product from a single supplier to multiple customers. The objective is to maximize expected discounted value, incorporating sales revenues, production costs, transportation costs, inventory holding costs and shortage penalties, over an infinite horizon. They study the special case of the Direct Deliveries.

In Berman and Larson (2001), the objective is to adjust dynamically the amount of product provided on scene to each customer so as to minimize total expected costs, comprising costs of earliness, lateness, product shortfall, and returning to depot non-empty. This problem can be encountered within an "industrial gases context".

Following the above classification, our work can be included in the Finite Horizon and multi-period group. The model has infinite capacity at the warehouse, distributor

[4]Zero Inventory Policies, are policies under which a retailer is replenished if and only if its inventory is down to zero.

[5]Fixed Partitioning Policies, partitions the set of retailers into a number of regions, such that each region is served separatly and independently from all other regions, whenever a retailer in a set is visited by a vehicle, all other retailers in the set are visited as well.

faces both stockout and holding costs, initial stock is only known at the beginning of the first period and each customer is visited at least once in the planning period. This model distances from the above literature on another aspect: the incorporation in the routing problem of both CMI points and VMI points.

4.2 The MPIRP-SDD

In this MPIRP-SDD model we consider two types of customers: the Vendor-Managed Inventory (VMI) customers and the Customer Managed Inventory (CMI) customers. The VMI customers have a random demand with a known distribution function for each period in the planning horizon. The distributor manages the stock at the VMI customer location and is responsible for the inventory cost incurred on these locations. For this group, it is the responsibility of the distributor to decide how much to deliver and when. The distributor has two types of costs related to these points: the holding cost (i.e. the cost of having inventory at these points, this cost is per unit and per period); and a stock out cost (i.e. cost per unit not sold).

The CMI have fixed demand that has to be fully satisfied on a specific day and the distributor faces no inventory costs associated with these customers. The CMI customers place an order to the distributor, some time in advance, to be delivered on an agreed date. These customers decide the quantity and delivery days and the distributor has no responsibility on the inventory they posses.

The objective is to determine the routes for a week planning period (composed of five working days) and the quantities delivered at the VMI points, minimizing total transportation plus inventory costs. The choice of a week planning period is motivated by two aspects: the strategic perspective of the model, our objective is to design a distribution policy in advance; and for control reasons, there is a minimum frequency of visiting a customer, we assume that it is at least once in the planning period. We have to decide when to visit the VMI customers and how much to deliver each time we visit them. The costs of the model include traveling cost, inventory managing costs associated with the VMI customers and a fixed cost of using a vehicle.

Although there exist some literature on the integration issue, none of them addresses the specific case of this model: two types of customers, a week minimum visit and no information on the inventory levels during the period.

The motivation of this work is based on the need for coordination within the supply chain management. In this case, we are trying to coordinate decisions from the distribution management area with decisions from the inventory management area. The aim of this model is, by coordinating strategies in different areas of the SCM try to reduce total costs. In other words, define an integrated inventory-routing strategy that proves to be more efficient than a non-integrated inventory routing strategy (solving both problems independently).

So, based on this idea, the objective of this model will be to design the routes and the delivering quantities in such a way has to minimize total cost. The decisions will be based on the assumptions explained below.

4.2.1 Assumptions of the model

The model tries to define the best routes for all customers and the best delivering quantities for the VMI customers. The first assumption is that we have two sets of customers and the VMI customers are visited at least once a week. Another important aspect is that we only know the initial inventory at the beginning of the first period, and the decisions are taken for the all week, independently of what occurs during the week.

The assumptions of the model are.

- Set of customers with well known geographical locations.
- Week period delivery system is considered (five working days).
- The CMI customers have a demand that is fixed, that is, at the beginning of the period we know the demand and, this exact amount has to be delivered at a specific day.
- There are no inventory costs to be managed at the CMI sites.
- There are no handling stock costs at the depot, and we assume that there is enough amount of product at the depot (unlimited capacity).
- VMI customers have stochastic demand, the quantities to deliver depend on the expected demand.

- For the VMI customers, demand probability function is known and varies by customer and from day to day.
- At the VMI points, there are inventory holding costs and a cost for stock out.
- The vehicles have a fixed capacity.
- The highest demand is always smaller than a vehicle capacity.
- There is no fixed number of vehicles but a high fixed cost for the use of a vehicle.
- Each customer can only be visited once a day and is visited at least once a week.
- The holding and stock out costs only depend on quantities and not on customers. And the stock out cost is always bigger than the holding cost.

4.2.2 The MPIRP-SDD

The costs of the problem are:

- Transportation cost between locations;
- Stock out and inventory costs for the VMI customers, per unit of product and per day;
- Fixed cost per vehicle used.

The decisions to be made are the following:

- Decide the routes for each day of the week, for each vehicle;
- Decide the number of vehicles needed each day of the period;
- Decide for each day which of the VMI customers will be included and where in the tour;
- How much to deliver to the VMI customers, on each day.

Objective function:

Objective is to minimize the expected cost at the end of the week:

$$\begin{aligned}\text{Min Weekly cost} &= \text{transportation cost} + \\ &\quad \text{inventory holding cost} + \\ &\quad \text{stockout cost} + \\ &\quad \text{fixed cost for the use of vehicles}\end{aligned}$$

Notation:

A = set of CMI customers.

B = set of VMI customers.

n = number of customers, indexed from 1 to n; index 0 denotes the central depot.

Q = capacity of a vehicle.

P = number of periods (in this case, $P = 5$ periods, from 0 to 4).

c_{ij} = cost of direct travel from location i to location j.

$F_{ip}(.)$ = cumulative distribution function of the demand of one period, in location i, with $i \in B$ for day p.

f_{ip} = distribution function of the one period demand for customer i and day p, $i = 1, ...n$ and $p = 1, ..., P$;

h = inventory holding cost per unit and per day.

s = shortage cost per unit and per day.

β_i^p = initial inventory at location i on day p.

d_i^p = demand of customer i on day p with $i \in A$.

T_i = set of days where i has a positive demand, with $i \in A$.

C = fixed cost per vehicle used per day.

K = maximum number of available vehicles.

Variables:

$$x_{ijk}^p = \begin{cases} 1, & \begin{array}{l} \text{if vehicle } k \text{ travels directly} \\ \text{from customer } i \text{ to customer } j \text{ on day } p; \end{array} \\ 0, & \text{otherwise.} \end{cases}$$

$$y_{ik}^{p} = \begin{cases} 1, & \text{if customer } i \text{ is assigned to vehicle } k \text{ on day } p; \\ 0, & \text{otherwise.} \end{cases}$$

$$w_{ik}^{p} = \text{amount delivered to location } i \text{ on day } p \text{, by vehicle } k.$$

$$K_p = \text{number of vehicles needed on day } p.$$

Transportation cost:

$$\sum_{i,j,k} c_{ij} x_{ijk}^{p}$$

Inventory cost:

Consider

$$\begin{aligned} w_i^p &= \sum_{k=1}^{K} w_{ik}^{p} \\ w_i^p &= \text{amount delivered to location } i \text{ on day } p \end{aligned}$$

$$\sum_{i \in B} \sum_{p=0}^{P-1} \left[h \sum_{0}^{\beta_i^p + w_i^p} (\beta_i^p + w_i^p - t_{ip}) f_{ip}(t_{ip}) dt_{ip} + s \sum_{\beta_i^p + w_{ip}}^{\infty} (t_{ip} - \beta_i^p - w_i^p) f_{ip}(t_{ip}) dt_{ip} \right]$$

Fixed cost per vehicle used:

$$C \times \sum_{p=0}^{P-1} K_p$$

Let $I_{ip}(w_i^p)$ represent the inventory managing cost of VMI location i on day p.

The problem can be stated as follows:

$$\text{Min} \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{K}\sum_{p=0}^{P=1} c_{ij}x_{ijk}^{p} + \sum_{i\in B}\sum_{p=0}^{P=1} I_{ip}(w_i^p) + \sum_{p=0}^{P=1} K_p * C \tag{4.1}$$

Subject to:

$$\sum_{k=1}^{K} y_{ik}^{p} = 1, \qquad \forall\, i \in A, \qquad \forall\, p \in T_i \tag{4.2}$$

$$\sum_{k=1}^{K_p} y_{0k}^{p} = K_p, \qquad \forall\, p = 0, ..., P-1 \tag{4.3}$$

$$\sum_{i\in A} d_i^p y_{ik}^p + \sum_{i\in B} w_{ik}^p \le Q, \qquad \forall\, p = 0, ..., P-1 \qquad k = 0, ..., K \tag{4.4}$$

$$\sum_{j=0}^{n} x_{ijk}^{p} = \sum_{j=0}^{n} x_{jik}^{p} = y_{ik}^{p}, \;\forall\, i = 1, ..., n;\; k = 1, ..., K_p;\; p = 0, ..., P-1 \tag{4.5}$$

$$\sum_{j,i\in S} x_{ijk}^{p} \le |S| - 1, \;\forall\, S \text{ non-empty subset of } \{1, ..., n\}$$

$$k = 1, ..., K;\; p = 0, ..., P-1 \tag{4.6}$$

$$1 \le \sum_{k=1}^{K}\sum_{p=0}^{P-1} y_{ik}^{p}, \qquad \forall\, i \in B \tag{4.7}$$

$$\sum_{k=1}^{K} y_{ik}^{p} \le 1, \qquad \forall\, i \in B \qquad p = 0, ..., P-1 \tag{4.8}$$

$$y_{ik}^{p} \le w_{ik}^{p} \le y_{ik}^{p} M, \qquad \forall\, i \in B \qquad p = 0, ..., P-1; \qquad k = 1, ..., K; \tag{4.9}$$

$$w_{ik}^{p} \ge 0, \qquad i \in B \qquad p = 0, ..., P-1 \tag{4.10}$$

$$x_{ijk}^{p} \in \{0, 1\}\,;\; y_{ik}^{p} \in \{0, 1\}\,\forall\, i = 0, ..., n; k = 1, ..., K;\; p = 0, ..., P-1 \tag{4.11}$$

The meaning of the above constraints is:

(4.2) For the CMI, in days where the customer have a positive demand, that customer is visited by only one vehicle.

(4.3) The second constraint forces that each day all vehicles go to the depot.

(4.4) This constraint ensures that the daily loading of a vehicle does not exceed its capacity.

(4.5) This constraint guarantees that if the vehicle enters a node, on day p, it also has to leave that node, on the same day.

(4.6) Avoids sub-tours, for each vehicle for each day. The sub-tour elimination constraint represents an exponential number of constraints. The problem with this constraint is that its number grows exponentially with n.

(4.7) The VMI customers are visited at least once a week.

(4.8) Each day, there can only be at most one vehicle visiting the VMI customers.

(4.9) If the amount delivered, by vehicle k, to a VMI i on day p is positive then, vehicle k has to visit that location on that day. M represents a very big value.

(4.10) The variable w_{ip}, representing the quantity delivered to a random demand customer on a given day is always greater or equal to 0.

(4.11) This constraint define the variables x^p_{ijk} and y^p_{ik} as binary.

The inventory cost only applies for the VMI customers and is equal to: the expected inventory holding cost, if the initial stock plus the quantity delivered is less than the demand, or a stockout cost otherwise. This expression applies for the case where demand is a discrete variable.

Since the representation of the probability distribution is difficult to find, particularly when demand ranges over a large number of possible values, the discrete random variable is often approximated by a continuous random variable. Furthermore, when demand ranges from over a large number of possible values, this approximation will generally yield a nearly exact value of the optimal amount. In addition, when discrete demand is used, the resulting expression becomes more difficult to solve analytically.

So, from now on we will consider the demand of the VMI customers as a continuous random variable:

$$\begin{aligned} & \sum_{i \in B} \sum_{p=0}^{P-1} I_{ip}(w_i^p) \\ = & \sum_i \sum_p \left[\begin{array}{c} h \int_0^{\beta_i^p + w_i^p} (\beta_i^p + w_i^p - t) f_{ip}(t) dt \\ +s \int_{\beta_i^p + w_i^p}^{\infty} (t - \beta_i^p - w_i^p) f_{ip}(t) dt \end{array} \right] \end{aligned} \tag{4.12}$$

The above inventory cost function (4.12) only works if the initial inventory of each period is known in advance, or observed. Since we are planning for several periods in advance we do not know the demand on each period. The initial inventory is then also a random variable that depends on the demand of all the previous periods and on quantities delivered in all the previous periods.

We will assume that the only quantity we can observe is the initial inventory of each customer at the beginning of period 0.

At each period, if there is a stockout, a cost is incurred and the initial stock of the following period will be zero.

Then, the initial inventory for each period is:

$$\begin{aligned}
\beta_i^0 &= u_i > 0 \\
\beta_i^1 &= Max\left\{0, u_i + w_i^0 - t_i^0\right\} \\
\beta_{i2} &= Max\left\{0, \beta_i^1 + w_i^1 - t_i^1\right\} = \\
& Max\left\{0, Max\left\{0, u_i + w_i^0 - t_i^0\right\} + w_i^1 - t_i^1\right\} \\
\beta_{i3} &= Max\left\{0, \beta_i^2 + w_i^2 - t_i^2\right\} = \\
& Max\left\{0, Max\left\{\begin{array}{c} 0, Max\left\{0, u_i + w_i^0 - t_i^0\right\} + \\ w_i^1 - t_i^1 \end{array}\right\} + w_i^2 - t_i^2\right\} \\
\beta_{i4} &= Max\left\{0, \beta_i^3 + w_i^3 - t_i^3\right\} = \\
&= Max\left\{\begin{array}{c} 0, Max\left\{0, Max\left\{\begin{array}{c} 0, Max\left\{0, u_i + w_i^0 - t_i^0\right\} \\ + w_i^1 - t_i^1 \end{array}\right\} + w_i^2 - t_i^2\right\} \\ + w_i^3 - t_i^3 \end{array}\right\}
\end{aligned}$$

So, the initial stock of period p will be a function of the initial stock of period 0, of the quantities delivered and demand of all previous periods.

$$\beta_i^p(u_i, w_i^0, ..., w_i^{p-1}, t_i^0, ...t_i^{p-1})$$

Assume from now on that the demand follows an exponential distribution function with the form:

$$f(t) = a_i e^{-a_i t} \tag{4.13}$$

There are many different statistical distribtuions to choose from when working with inventory control. The exponential distribution function is commonly used to represent the demand variation in inventory models. See Lidke and Malstrom (1987) and Snyder (1984).

We need to calculate the inventory cost for each period. So, for each period we need to consider all the possible scenarios of the previous periods initial inventory. This is, for period 0, we know the initial inventory, for period 1 we need to consider two possibilities: stock-out at the end of period zero (zero initial inventory at period 1) and positive stock at the end of period 0 (initial stock at period one positive). Then we extend this for the second, third and fourth periods for all possible combinations.

The expression of the inventory cost function for each customer and each period can be seen in detail in Section 3.2.2.

4.3 Heuristics Solution Method for the MPIRP-SDD

The limitations of available computational techniques make it impractical to try to solve this problem directly for all but very small instances. The structure of the problem argues for some type of decomposition. As mentioned there are two subproblems embedded in the IRP. The first is to decide the delivery day and the quantity, the second involves routing decisions.

Our decomposition scheme for the IRP is outlined in the following steps:

- Step 1: Obtain an initial solution where the inventory problem is solved separately without considering any delivery cost.
- Step 2: For each day in the planning horizon, try to find a good feasible solution by solving a VRP.
- Step 3: Calculate an approximation of the VMI customers delivery cost.
- Step 4: Determine the new quantities and delivery days for the VMI customers, taking into account the setup cost.
- Step 5: For each day in the planning horizon, try to find a good feasible solution by solving a VRP.
- Step 6: Repeat steps 3 and 4 until a satisfied solutions is found.

Next, we will explain in more detail each step of the heuristics for the MPIRP-SDD.

Step 1: The Initial solution - The inventory problem

In the first step of the algorithm we will solve the inventory problem alone. Considering that there are no transportation costs to be handled. The cost function (4.12) only works if the initial inventory of each period is known in advance. However, one of the assumptions of the model is that the stock at the VMI locations can only be observed at the beginning of the planning period. Therefore, the initial inventory is also a random variable that depends on the demand of all the previous periods and on quantities delivered in all the previous periods.

We need to calculate, for each VMI customer, the inventory cost for each period.

For period zero, we observe the initial stock and incur in holding cost if expected demand exceeds the initial inventory plus the quantity delivered or a stockout cost otherwise. If there is a stockout, in the next period the initial stock will be zero.

For the other periods (period 1 to 4), we have to consider all the possible scenarios of the previous periods. For period 1, for example, we need to consider two possibilities: stock-out in period zero (zero initial inventory at period 1) and positive stock at the end of period 0 (initial stock at period one positive). For each VMI customer i:

$$\text{Period 1:} \begin{cases} \beta^1 & = & 0 \\ \beta^1 & > & 0 \end{cases}$$

For period 2 we need to consider 4 scenarios:

$$\text{Period 2:} \begin{cases} \beta^1 & = & 0 & \text{and} & \beta^2 & = & 0 \\ \beta^1 & = & 0 & \text{and} & \beta^2 & > & 0 \\ \beta^1 & > & 0 & \text{and} & \beta^2 & = & 0 \\ \beta^1 & > & 0 & \text{and} & \beta^2 & > & 0 \end{cases}$$

Then we extend this for period 3 and 4 following the same reasoning.

This inventory problem, is different from other inventory problems that can be found in the literature. Since, in this particular case we have a planning horizon, 5 days, for which we need to plan the delivery quantities. Also, we do not known what is the initial stock at each day for each customer. We have to decide how often and how much to deliver. Since we do not allow for negative quantities, we have a non-linear constrained minimization problem.

This initial solution is obtained by solving the inventory model for each customer. The inventory cost function results in a complex integral with 5 integrals. This expression is minimizes using the Gauss-Newton method, with the average demand as a starting point. The objective is to minimize inventory costs (holding + stock-out costs). The best solution is to deliver almost every day to the VMI customers, since there is no setup cost associated with the deliveries.

In summary, in step 1 we calculate the optimal quantities to deliver on each day of the week to each VMI customer. We assume an initial solution for the first day and an exponential distribution function for the demand of each customer on each day. For the detailed inventory cost function analysis and solution method refer to Section 3.2.

Step 2: The VRP

After step 1, we known the quantities and the days to delivery to each customer. Therefore, we have to solve a VRP each day to obtain the best routes. To solve the VRP each day, we consider an ILS for all customers on that specific day and their respective quantities. After, we calculate the total cost: transportation cost and inventory cost associated with the VMI customers.

Due to the complexity of the VRP problems (NP-hard) we need to develop an heuristic. Our proposal is to use an heuristic algorithm that as proven to give quiet good results on other problems and is easy to implement and modify, the ILS. For more details on this metaheuristic see Section 2.4.1.

Structure of the algorithm for the VRP

Step 2.1: Set $day = 1$
Step 2.2. Savings Heuristic - Initial Solution
Step 2.3. ILS for TSP on each tour
Step 2.4. ILS for the VRP
Step 2.5. ILS for the TSP on the new routes
Step 2.6. Set $day = day + 1$; Repeat Step 3 to 6 until $day = 5$;

At the end of 2, we have obtained an initial solution for the MPIRP-SDD model based on a sub-optimization of the two existing problems: the routing and the inventory problem. Now, Step 3 to 6 consists in developing an heuristic method that improves this initial solution.

Step 3: Inventory and transportation - The setup cost approach

When considering the inventory problem separately from the transportation problem, the best solution is to deliver frequently, every day or almost every day, however, this implies higher transportation costs. Our objective is to balance the delivery costs with the inventory cost. One way to do this is by considering a setup cost: a cost per delivery made to a VMI customer. This setup cost only applies to these set of customers since, the CMI customers have to be visited on a specified day and no changes are allowed on these customer's orders. If a VMI customer is visited on day p, then there is a fixed cost associated with this customer, on this day.

We will obtain the setup cost by calculating the approximate cost of serving a VMI customer on a specific day, this is, the cost of including a customer i on tour j on day p. This is an approximation since the real value would have to consider the relationship between the delivering decisions of all the other customers and the best corresponding routes.

It is logical to think that if a customer is close to a group of customers that are visited on day p, than we have a lower cost of delivering that customer on that day. This is, the setup cost of customer i will depend on the location of this customer with respect to other customers in the route.

This cost represents the cost associated with delivering to a customer on a day p, and will only depend on distance. This setup cost will be calculated in the following way:

g_i^p = cost of going to customer i on day p.

θ = a fixed setup weight per distance unit.

Let j and k be the previous and successor customer in the route of customer i on day p, then.

$$g_i^p = (c_{ji} + c_{ik} - c_{jk}) \times \theta \tag{4.14}$$

At Step 3 we have calculated the setup cost for each VMI customer on each day.

Step 4: New deliveries

Now, we have an inventory model with a setup cost. For each VMI customer we need to redefine the optimal deliveries. For a more detail discription of this inventory model with setup costs see Section 3.2.3.

The procedure can be summarized as follows:

- Consider all possible combinations of delivery days.
- Solve the inventory problem for each combination.
- Add the corresponding setup cost in each solution.(obtained in Step 3).
- Choose, for each VMI customer, the solution with the lowest total cost (inventory + setup).

At this stage we obtain the new set of delivery days and quantities.

Step 5: New VRP

For each day in the planning horizon, and using the new delivering quantities from Step 4, find a good feasible solution by solving a VRP (repeating Step 2).

Step 6: Repeat

Repeat steps 3 to 6 until a satisfied solution is found, or until a certain number of iterations have been performed.

Next, we will present a computational experiment on the application of this method to the MPIRP-SDD.

4.4 Computational Results

In this section we will present a computational study that assess the impact of integrating transportation and inventory. To analyse this impact it is interesting to compare two solutions: The integrated solution and the non-integrated solution. This non-integrated solution is characterized by the separability of the two problems: the inventory and the routing are independent, first we solve the inventory problem and use the solution to decide the routes. The solution of the integrated problem is the one obtained after the running of the algorithm presented in the previous section.

The objective is to compare the two solutions (the integrated versus the non-integrated solution) obtained when solving the two different problems (Inventory and Routing) and analyzing the impact of considering the problems in an integrated form.

There are two ways in which we will orient this analysis: the first is to compare total costs; and the second is to make a multi-objective approach. In this second perspective the inventory cost and the routing cost are the two objectives that we would like to minimize. Instead of choosing the best total cost we will consider all non-dominated solutions obtained from the algorithm and would be the responsibility of the decision maker to choose.

Next, we will explain the data used and analyze some important results of this experiment.

4.4.1 The data

For the computational experiment we have generated several sets of examples, each group with different characteristics concerning: total number of customers $(100, 200, 400)$; percentage of VMI customers (10% and 50%); type of demand (equal every day, different each day) and setup cost parameter (high setup cost θ=100 per distance unit and low setup cost, $\theta = 10$ per distance unit).

For the VMI customers, we have used a demand parameter α_{ip} (for each customer i and day p) that follows a normal distribution with mean 50 and standard deviation 20 for each customer. In the cases where demand is different every day the standard deviation between days used was 5.

The initial stock for each customer was generated by a random uniform distribution between 0 and 50.

The results were obtained considering 8 iterations. The stockout cost used was twice the holding cost, in this experiment $h = 2$ and $s = 4$. There is a fixed charge per vehicle used per day: $C = 200$.

4.4.2 Analysis of the results

We can start by looking at a few examples in terms of the solutions obtained at each iteration, see Figure 4.1.

The non-integrated solution corresponds to the initial solution obtained at the end of step 2 of the algorithm. By continuing the algorithm (step 3 to 6) and exploring other VMI delivery strategies, the new inventory cost increases but, allows some savings in terms of routing. In Figure 4.1, iteration 0 corresponds to the non-integrated solution while the other iterations 1 to 7 correspond to integrated solutions. In these two particular examples, the best solution would be found at iterations 1. However, supposing that a higher preference was given to reduce routing costs, then the best solution would be at iteration 7 for example A and at iteration 2, for example B.

In Table 4.1, we present the average total cost improvemet for each problem size. This average Cost Reduction was calculated by comparing, for each group of examples, the best solution for the non-integrated (i.e. the initial solution obtained at the end of step 2) versus integrated case (i.e. the best solution in terms of total cost, obtained by the algorithm). The average savings, for instances of 100 customers with 10% VMI customers, when integrating routing and inventory is $1,42\%$ of the total cost.

In Figure 4.2, we can see the trade-off between Inventory cost and Routing cost. This corresponds to viewing the problem has a multi-objective problem, where all the set of non-dominated solution are of interest for the decision maker to choose the best delivering and routing strategy. In example C we would have three non-dominated solutions (the three circles in the graph). In example D, only two solutions would be non-dominated (the two circles in the graph).

We also compared the non-integrated solution with the best solution in terms of routing cost. Table 4.2 considers the trade-off between inventory and routing

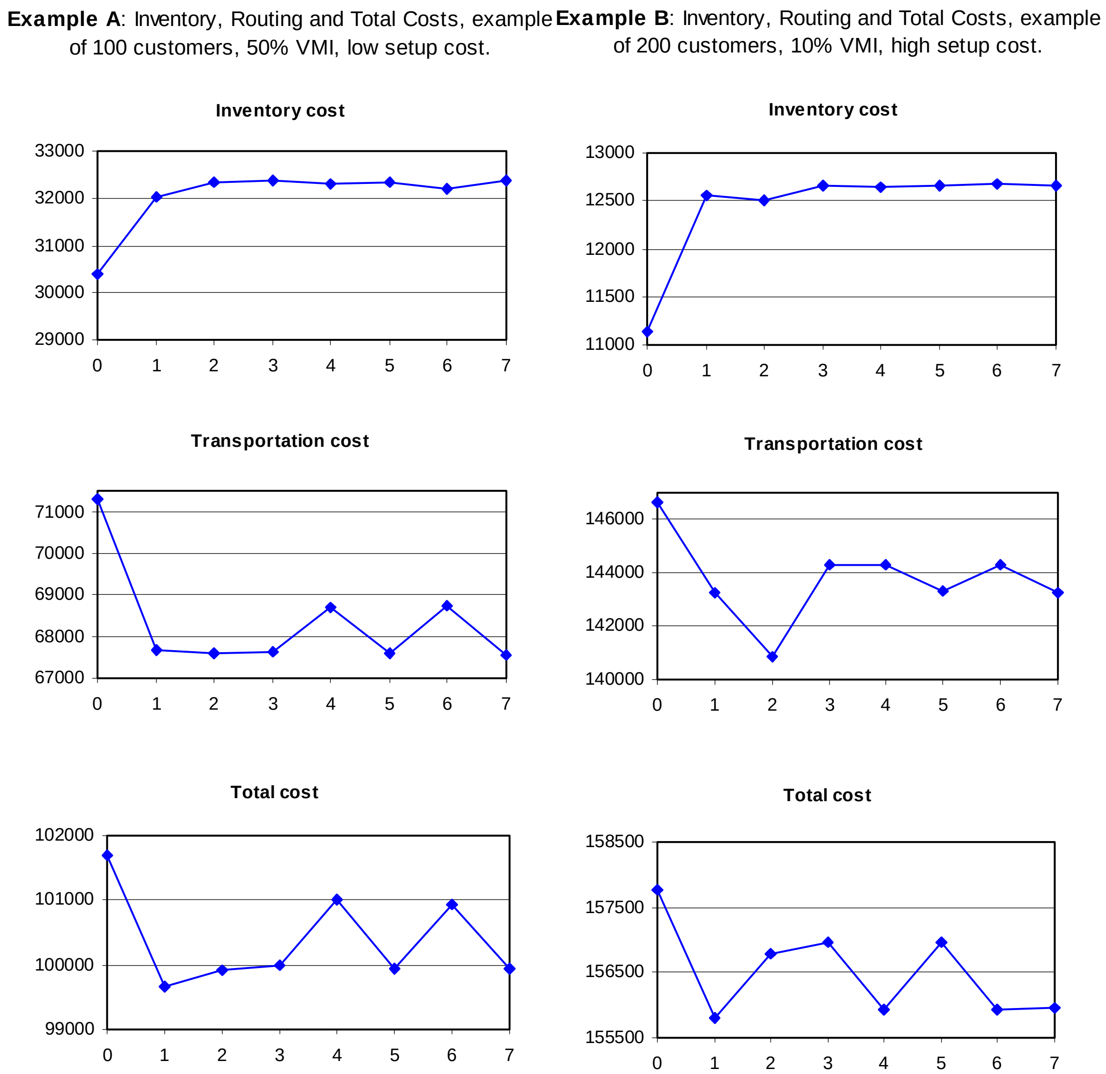


Figure 4.1: Examples of Inventory, Routing and Total Costs at each iteration.

Problem Size	Number VMI	Average Total Cost Reduction
100	10	1,42%
	50	0,99%
200	20	0,94%
	100	2,09%
400	40	0,26%
	200	0,06%

Table 4.1: Average Total Cost Reduction for each group size.

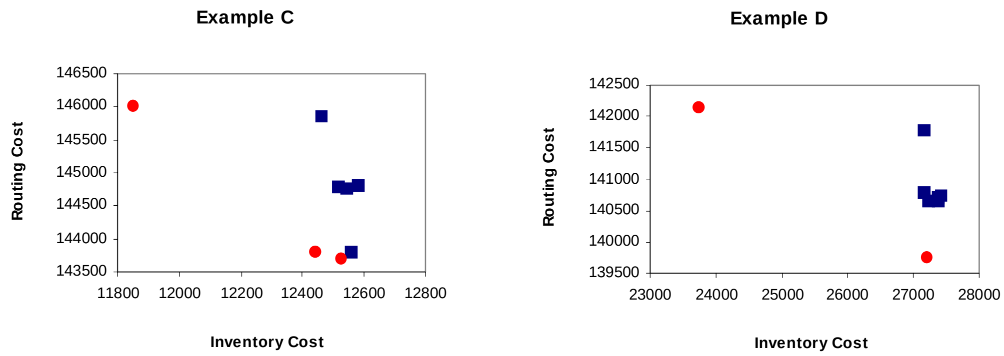


Figure 4.2: Trade-off between Inventory and Transportation cost.

Problem Size	Number VMI	High setup cost parameter				Low setup cost parameter			
		Equal demand		Different demand		Equal demand		Different demand	
		ip_cost	vrp_cost	ip_cost	vrp_cost	ip_cost	vrp_cost	ip_cost	vrp_cost
100	10	-18,71%	2,77%	-22,09%	3,28%	-6,98%	1,24%	-7,73%	1,75%
	50	-21,40%	4,95%	-18,67%	3,77%	-5,66%	4,57%	-6,04%	4,99%
200	20	-15,97%	2,59%	-14,88%	2,02%	-4,61%	1,19%	-2,85%	1,09%
	100	-18,35%	12,33%	-17,17%	12,25%	-4,23%	3,23%	-4,33%	3,43%
400	40	-14,92%	2,27%	-13,87%	2,09%	-2,17%	0,51%	-2,26%	0,79%
	200	-15,31%	11,10%	-15,10%	10,86%	-2,72%	2,33%	-2,45%	2,30%

Table 4.2: Trade-off between inventory and routing costs.

costs when going from the solution of the non-integrated case to the integrated one. The results are shown in terms of average variation in routing and inventory: the inventory costs increase and the routing costs decrease, for the solutions of each case see appendix C.

The best solution in terms of transportation cost always implies an increase in inventory cost. The difference in the magnitude of these variations is justified on one hand by the values chosen for the inventory costs and transportation cost. And, on the other hand, by the existence of other customers, the CMI customers. These customers have fixed delivery days and quantities: The higher is the percentage of CMI customers the less we can reduce routing costs by concentrating deliveries (the impact on total routing cost is less visible).

In terms of delivery days, when optimizing separately we obtain an average of $4,68$ delivery days a week, for the VMI customers, while for the integrated case, this average falls to 3, 49 delivery days a week.

Another analysis that can be done is in terms of vehicles needed. When we integrate transportation and inventory, this implies that the delivery frequency is reduced and we are able to reduce the number of routes. Assuming that the distributor pays a fixed charge per use of a vehicle then, by reducing the number of routes at the end of the week the distributor is able to reduce routing costs. Table 4.3, shows for each group of examples, the average reduction in the number of vehicles needed per week. The higher is the percentage of VMI customers, the more we can reduce the number of vehicles needed. This reduction is higher when the setup cost parameter is high. A higher setup cost parameter means that solutions with fewer deliveries are preferable. For example, in the group with 100 customers 10 of them VMI, for the integrated solution, the total number of vehicles needed per week reduces on average 2, 81% and 1, 73% for the cases with high and low setup cost parameter respectively.

In terms of run time, Table 4.4 summarizes the average number of running time per problem, measured in seconds for the integrated solutions.

The above results show that the distributor who has VMI customers gains from considering inventory and routing in an integrated manner, and this gain can be seen in terms of total cost reduction. The best delivery strategy will be to deliver very frequently which implicates higher transportation costs. When including the cost of delivering, the solution has few delivery days and transportation costs are reduced. The magnitude of this improvement depends on the problem size, on the proportion

Problem Size	Number VMI	Reduction N. of vehicles	
		High setup cost parameter	Low setup cost parameter
100	10	-2,81%	-1,73%
	50	-12,34%	-4,67%
200	20	-2,55%	-1,65%
	100	-12,02%	-3,19%
400	40	-2,08%	-0,99%
	200	-10,65%	-2,14%

Table 4.3: Reduction in the total number of vehicles.

Problem Size	Number VMI	Average Run Time in seconds
100	10	110,73
	50	113,88
200	20	465,72
	100	376,39
400	40	1783,901
	200	1490,591

Table 4.4: Average run time in seconds, per problem size.

of VMI customers in the problem and also on the unit costs chosen for both problems. It is also interesting to consider the problem in a multi-objective perspective. For this case, analyze the set of non-dominated solutions and it becomes the responsibility of the decision maker to choose the best solution based on a given inventory strategy and a delivering plan associated.

4.5 Summary and Conclusions

The logistic planning functions of transportation and inventory play an important role in may industries and integrating these two areas may lead to significant gains and more competitive distribution strategies. The movement towards more integrated processes cannot ignore these tow key logistic fields.

In this chapter, we present a Multi-Period Inventory Routing Problem with Stochastic and Deterministic Demand (MPIRP-SDD). We have considered the particular case of a distribution firm that has to decide on its distribution and inventory strategies. This firm has two types of customers, the VMI and CMI customers and decisions have to be made on the quantities delivered and days of visit to the VMI customers, and also in relation with the route planning for the complete set of customers for a week planning period. The additional assumption of only observing stock levels at the beginning of the planning period brings more complexity into the model. The objective of this model is to design an integrated Inventory-Routing strategy for a distributor that has to manage inventory and transportation costs of their VMI customers. Considering the inventory and transportation management in an integrated mode can yield to a better performance. As far as we know, there no studies on the IRP with these characteristics. This model can be applied in many distribution processes: for example, in the retailing industry for suppliers of supermarkets and department stores.

An heuristic approach, based on the ILS was constructed to solve this problem. The heuristic has 6 steps, on the first step an initial solution for the inventory problem is obtained. Then, for each day in the planning horizon, tries to find a good feasible solution by solving a VRP. On step 3, an approximation of the VMI customers delivery cost is calculated and based on the results the new quantities and delivery days for the VMI customers are obtained. The process is repeated on the new routes until a satisfied solution is found.

A computational study was done to analyze the impact of integrating Inventory and Routing: the results show that cost reductions are obtained when considering inventory and routing in an integrated manner. The degree of this improvement depends on the problem size, on the proportion of VMI customers in the problem and also on the unit costs chosen for both problems. Given the relationship between the inventory and transportation costs, the decision maker can decide how much of the deliveries to the VMI customers to concentrate.

We are considering future extensions of this work: One includes the analysis of the case where demand faces a distribution function different than the one we have assumed in our work, for example, the Normal distribution. Another interesting extension is to measure the setup costs, in Step 3 of the algorithm, in a dynamic way, this is, taking into consideration not only the precessor and successor customers in the tour but a global effect on the week plan. Finally, we would also like to develop a multi-objective model and solution method to solve the MPIRP-SDD.

Chapter 5

Conclusion

The previous decision models are related with integration issues in Distribution Management and, from their analysis, we can conclude that given a specific logistics structure, a firm can improve their distribution strategy by taking into consideration a diversity of concerns and joint them together in an integrated decision policy. Moreover, this effect can be found both in the cases which integrate decisions from other business areas, such as marketing policies, and in the cases in which the integration process happens along several logistics functions, such as inventory and transportation.

In the first study we have explored different distribution strategies to analyze an integrated distribution problem. The first strategy is the classical VRP approach, which reflects only transportation cost. The second strategy is a customer oriented strategy based on customer relationship management principles. The third strategy is a multi-objective combinatorial optimization problem with two objectives: minimizing cost and improving customer service. This third strategy results from the integration of the two other strategies and brings together two important areas in many industries: Distribution and Marketing. The main conclusion is that the multi-objective model gives several non-dominated solutions, that can be seen as a good balance between optimizing the transportation cost and the customer service.

The second work presents two inventory models: the MPVMI with no setup cost and the MPVMI with setup costs. The distributor has to decide how much and when to deliver to each of its customers while trying to minimize total inventory costs. Results show that, when introducing a setup cost, the number of deliveries per planning period are reduced while inventory costs (holding and stockout cost) increase. The impact of this trade-off depends on the type of demand and level of setup cost used. However, when comparing both models adding the setupcost we can observe that the model M2 holds better results. This observation leads us to conclude that developing integrated decision models will result in a better global decision process.

In the last study, we presented a Multi-Period Inventory Routing Problem with Stochastic and Deterministic Demand, we have considered the particular case of a distribution firm that has to decide on its distribution and inventory strategies. The objective of this model is to design an integrated inventory routing strategy for a firm that has two types of customers, the VMI and CMI customers and decisions have to be made on the quantities delivered and days of visit to the VMI customers, and also in relation with the route planning for the complete set of customers for a week planning period. A numerical study was done to analyze the impact of integrating Inventory and Routing: the results show that cost reductions are obtained when considering inventory and routing in an integrated manner. Given the relationship between the inventory and transportation costs, the decision maker can decide how much of the deliveries to the VMI customers to concentrate.

The three different studies are an important contribution to the field of integrated distribution management, since they cover three specific logistic systems whose characteristics have not been studied before, as far as we know.

Efficiency can be improved by the coordination of different areas of distribution management and there are alternative strategies for distribution that will allow the decision maker to make a better choices.

As mentioned before, the importance of the integration process within a firm or a Supply Chain is an important issue both in management and at research level. This integration can be seen from two perspectives: the information integration and the decision process integration. In this second perspective, the integrated decision models play and will continue to play an important role in the logistics field.

As future research, the models and approaches done in this thesis can be extended to other areas within the firm or outside the firm within its Supply Chain.

Bibliography

[1] Aichlymayr, M. (2000). "DC mart: Who manages inventory in a value chain?" Tranportation & Distribution 41(10): 60-66.

[2] Andel, T. (1996). "Manage Inventory, Own Information." Tranportation & Distribution: 54-58.

[3] Anily, S. and Federgruen, A. (1990). "One warehouse multiple retailer system with vehicle routing costs." Management Science 36(1): 92-114.

[4] Anily, S. and Federgruen, A. (1991). "Rejoider to "Comments on one-warehouse multiple retailer systems with vehicle routing costs"." Management Science 37(11): 1497-1499.

[5] Anily, S. and Federgruen, A. (1993). "Two-Echelon distribution systems with vehicle routing costs and central inventories." Operations Research 41(1): 37-47.

[6] Baita, F., Ukovich, W., Pesenti, R. and Favaretto, D. (1998). "Dynamic routing-and-inventory problems: a review." Transportation Research A 32(8): 585-598.

[7] Bard, J., Huang, L., Jaillet, P. and Dror, M. (1998). "A decomposition approach to the inventory routing problem with satellite facilities." Transportation Science 32(2): 189-203.

[8] Barnes-Schuster, D. and Bassok, Y. (1997). "Direct shipping and the dynamic single-depot/multi-retailer inventory system." European Journal of Operations Research 101: 509-518.

[9] Berman, O. and Larson, R.C. (2001). "Deliveries in an Inventory/Routing Problem Using Stochastic Dynamic Programming." Transportation Science 35(2): 192-213.

[10] Bertazzi, L., Paletta, G. and Speranza, M.G. (2002). "Deterministic order-up-to level policies in an inventory routing problem." Transportation Science 36(1): 119-132.

[11] Bertsekas, D. P. (1995). Non-Linear Programming. Belmont Massachusettes, Athena Scientific.

[12] Bodin, L. (1983). "Routing and Scheduling of Vehicles and Crews, The state of the art." Computers & Operations Research 10: 63-211.

[13] Burns, L.D., Hall, R.W., Blumenfeld, D.E. and Daganzo, C.F. (1985). "Distribution Strategies that Minimize Transportation and Inventory Costs." Operations Research 33(3): 469-490.

[14] Cachon, G.P. and Fisher, M. (1997). “Campbell Soup’s continuous replenishment program: evaluation and enhanced inventory decision rules.” Production and Operations Management 6(3): 266-276.

[15] Campbell, A., Clarke, L. and Savelsbergh (1999). “An inventory routing problem.” working paper.

[16] Çetinkaya, S. and Lee, C. (2000). ”Stock Replenishment and Shipment Scheduling for Vendor_Managed Inventory Systems.” Management Science 46(2): 217-232.

[17] Chan, L.M.A., Federgruen, A. and Simchi-Levi, D. (1998). “Probabilistic analysis and practical algorithms for inventory-routing models.” Operations Research 46(1): 96-106.

[18] Cheung, K.L. and Lee, H.L. (2002). “The inventory Benefit of shipment Coordination and stock rebalancing in a supply chain.” Management Science 48(2): 300-306.

[19] Chien, W., Balakrishnan, A. and Wong, R. (1989). “An integrated inventory allocation and vehicle routing problem.” Transportation Science 23(2): 67-76.

[20] Christofides, N., Mingozzi, A. and Toth, P. (1979). The Vehicle Routing problem. Combinatorial Optimization. N. Christofides, A. Mingozzi, P. Toth and Saudi. New York, John Wiley & Sons: 315-338.

[21] Christopher, M. (1998). Logistics and Supply Chain Management: Strategies for reducing cost and improving service, Financial Times, Prentice Hall.

[22] Clarke, G. and J. W. Wright (1964). “Scheduling of vehicles from a central depot to a number of delivery points.” Operations Research 12: 568-581.

[23] Cordeau, J.F., Laporte, G., Potvin, J.Y. and Semet, F. (2002). ”A guide to vehicle routing heuristics.” Journal of the Operational Research Society 53: 512-522.

[24] Crainic, T. G. and G. Laporte (1998). Fleet Management and Logistics. Centre for Research on Transportation, Kluwer Academic Publishers.

[25] Desrosiers, J. ,Dumas, Y., Solomon, M.M., and Soumis, F. (1996), “The routing and Scheduling” in Networks and Distribution, Handbooks in OR and MS, North-Holland, Amsterdam.

[26] Disney, S.M. and Towill, D.T. (2002). “A procedure for the optimization of the dynamic response of a Vendor Managed Inventory system.” Computers & Industrial Engineering 43: 27-58.

[27] Dong, Y. and Xu, K. (2002). “A supply chain model of vendor managed inventory.” tranportation Research. Part E 38: 75-95.

[28] Dror, M. and Ball, M. (1987). “Inventory/Routing: Reduction from an Annual to a Short-Period Problem.” Naval Research Logistics 34: 891-905.

[29] Ehrgott, M. and X. Gandibleux (2000). An Annotated Bibliography of Multiobjective Combinatorial Optimization, Universitat Kaiserslautern.

[30] Emigh, J. (1999). "Vendor Managed Inventory." Computers World: 52.

[31] Federgruen, A. and Simchi-Levi, D. (1995). "Analysis of Vehicle Routing and Inventory-Routing Problems". Network Routing. M. O. Ball, Magnanti, T.L., Monma, C.L., Nemhauser, G.L. Amsterdam, Elsevier Science - North Holland. 8: 297-373.

[32] Federgruen, A. and Zipkin, P. (1984). "A combined vehicle routing and inventory allocation problem." Operations Research 32(5): 1019-1037.

[33] Fisher, M. L. (1995). Vehicle Routing. Network Routing. M.O.Ball, T. L. Magnanti, C. L. Monma and G. L. Nemhauser. Amsterdam, North-Holland. 8: 1-33.

[34] Fisher, M.L., Jaikumar (1978), "A decomposition algorithm for large-scale vehicle routing", report 78-11-05, Department of decision Sciences, The Wharton School, University of Pennsylvania, Philadelphia [12;2,3].

[35] Fisher, M.L., Jaikumar (1981), "A generalized assignment heuristic for vehicle routing", *Networks* 11, 109-124[12:2,4].

[36] Fry, M.J., Kapuscinski, R. and Olsen, T.L. (2001). "Coodinating Production and Delivery Under a (z, Z)-Type Vendor-Managed Inventory Contract." Manufacturing & Service Operations Management 3(2): 151-173.

[37] Gouveia, L. (1995), "A result on projection for the Vehicle Routing Problem", *European Journal of Operational Research* 85, 610-624.

[38] Hall, R.W. (1991). "Comments on one warehouse multiple retailer system with vehicle routing costs." Management Science 37(11): 1496-1497.

[39] Harrington, L. (1996). "Consignment selling: Trend or another World?" Tranportation & Distribution 37(6): 45.

[40] Hillier, F.S. and Lieberman, G.J. (1995). Introduction to Operations Research, McGraw-Hill International Series.

[41] Hong, S.-C. and Y.-B. Park (1999). "A heuristic for bi-objective vehicle routing with time windows constraints." International Journal of Production Economics 62: 249-258.

[42] Jaillet, P., Bard, J.F., Huang, L. and Dror, M. (2002). "Delivery costs approximations for inventory routing problems in a Rolling Horizon Framework." Transportation Science 36(3): 292-300.

[43] Johnson, J., Wood, D., Wardlow, D.L. and Murphy, P.R. (1999). Contemporary Logistics, Prentice Hall.

[44] Kleywegt, A.J., Nori, V.S. and Savelsbergh, M.W.P. (1998). "A computational approach for the inventory routing problem." working paper.

[45] Kleywegt, A.J., Nori, V.S. and Savelsbergh, M.W.P. (2002). "The Stochastic Inventory Routing Problem with Direct Deliveries." Transportation Science 36(1): 94-118.

[46] Kulkarni, R. V. and Bhave, P.R.(1985). "Integer programming formulations of vehicle routing problems." European Journal of Operational Research 20: 58-67.

[47] Kunz, C. (2000). Iterierte Lokale Suche ur das Vehicle-Routing Problem, TU Damstadt.

[48] Laporte, G. (1992). "The Vehicle Routing Problem: An overview of exact and approximate algorithms." European Journal of Operational Research 59: 345-358.

[49] Laporte, G. and Osman, I.H. (1995), "Routing problems: A bibliography", *Annals of Operations Research.*

[50] Laporte, G. and Osman, I. H. (1996). "Metaheuristics: A bibliography." Annals of Operations Research 63: 513-623.

[51] Lawer, E.L., Lenstra, J.K., Rinnooy Kan, A.H.G. and Shmoys, D.B. (1985), "The Traveling Salesman Problem", Wiley & Sons Ltd.

[52] Lee, T. and Ueng, J. (1999), "A study of vehicle routing problems with load-balancing", *International Journal of Physical Distribution & Logistics Management,* Vol.29, No.10, 646-658.

[53] Lidke, S. L. and E. M. Malstrom (1987). "A recursive computer algorithm fo determining joint probability inventory distributions." Computers & Industrial Engineering 12(2): 105-116.

[54] Lourenco, H., Martin, O. and Stützle T. (2001), "A Beginner's Introduction to Iterated Local Search.", In Proceedings of MIC 2001, pages 1-6, Porto, Portugal, July 2001.

[55] Lourenco, H., Martin, O. and Stützle T. (2002), "Iterated Local Search.", In F. Glover and G. Kochenberger, editors, Handbook of Metaheuristics, pages 321-353, Kluwer Academic Publishers, Norwell, MA.

[56] Partyka, J. G. and R. W. Hall (2000). "On the Road to Service". OR/MS Today(August 2000): 26-35.

[57] Simchi-Levi, D., Kaminsky, P. and Simchi-Levi, E. (2003). Designing and Managing the Supply Chain: Concepts, Strategies and Case Studies, MacGraw-Hill, Irwin.

[58] Snyder, R. D. (1984). "Inventory control with gamma probability distribution." European Journal of Operational Research 17(3): 373-381.

[59] Stützle, T. G. (1998). Local Search Algorithms for Combinatorial Problems - Analysis, Improvements, and New Applications. Department of Computer Science, Darmstadt University of Technology.

[60] Toth, P. and D. Vigo (2002). “Models, relaxations and exact approaches for the capacitated vehicle routing problem.” Discrete Applied Mathematics 123: 487-512.

[61] Trudeau, P. and Dror, M. (1992). “Stochastic Inventory Routing: Route Design with Stockouts and Route Failures.” Transportation Science 26(3): 171-184.

[62] Waller, M., Johnson, M.E. and Davis, T. (1999). “Vendor-managed inventory in the retail supply chain.” Journal of Business Logistics 20(1): 183-203.

[63] Worthington, P.R. (1998). “The increasing importance of retailers' inventories.” Economic Prespectives 22(3): 2-13.

[64] Zipkin, P.H. (2000). Foundations of Inventory Management, McGraw-Hill International Editions.

Appendix A

Inventory cost function for each period

For one retailer:

Period 0: $I_0 = exp(-a_0 * (u + w_0)) * (s + h)/a_0 + h * (a_0 * u + a_0 * w_0 - 1)/a_0$

Period 1: $I_1 = ((\alpha_1 * u + \alpha_1 * w_0 + 1)/\alpha_1 * h + (\alpha_1 * u + \alpha_1 * w_0 + 1)/\alpha_1 * s) * exp(-\alpha_1 * (u + w_0 + w_1)) + 1/\alpha_1 * h * exp(-\alpha_1 * (u + w_0)) + (-2 + \alpha_1 * w_1 + \alpha_1 * u + \alpha_1 * w_0)/\alpha_1 * h$

Period 2: $I_2 = 1/2(2 + 2\alpha_2 u + 2\alpha_2 w_0)/\alpha_2 hexp(-\alpha_2(u + w_0 + w_1)) + exp(-\alpha_2(u + w_0))/\alpha_2 h + (1/2(2 + \alpha_2^2 u^2 + 2\alpha_2^2 w_1 u + 2\alpha_2^2 u w_0 + 2\alpha_2 u + 2\alpha_2 w_1 + \alpha_2^2 w_0^2 + 2\alpha_2 w_0 + 2\alpha_2^2 w_1 w_0)/\alpha_2 h + 1/2(2 + \alpha_2^2 u^2 + 2\alpha_2^2 w_1 u + 2\alpha_2^2 u w_0 + 2\alpha_2 u + 2\alpha_2 w_1 + \alpha_2^2 w_0^2$

$+2\alpha_2 w_0 + 2\alpha_2^2 w_1 w_0)/\alpha_2 s) exp(-\alpha_2(u + w_0 + w_1 + ww2)) + 1/2(-6 + 2\alpha_2 u + 2\alpha_2 w_1 + 2\alpha_2 ww2 + 2\alpha_2 w_0)/\alpha_2 h$

Period 3: $I_3 = 1/6(6\alpha_3^2 u w_0 + 6\alpha_3^2 w_1 w_0 + 6\alpha_3^2 w_1 u + 6\alpha_3 w_1 + 3\alpha_3^2 w_0^2 + 6\alpha_3 u +$

$6\alpha_3 w_0+3\alpha_3^2 u^2+6)/\alpha_3 hexp(-(u+w_0+w_1+w_2)\alpha_3)+1/6(6+6\alpha_3 u+6\alpha_3 w_0)/\alpha_3 hexp(-(u+ w_0+w_1)\alpha_3)+(1/6(6\alpha_3 u+6\alpha_3 w_0+6+6\alpha_3 w_2+6\alpha_3 w_1+6\alpha_3^2 w_2 w_1+3\alpha_3^2 w_1^2+3\alpha_3^2 u^2+ 3\alpha_3^2 w_0^2+6\alpha_3^3 w_2 u w_0+3\alpha_3^3 w_1^2 u+3\alpha_3^3 w_0 u^2+3\alpha_3^3 w_1 w_0^2+6\alpha_3^2 w_2 u+3\alpha_3^3 w_1 u^2+\alpha_3^3 w_0^3+ 3\alpha_3^3 w_0^2 u+3\alpha_3^3 w_2 w_0^2+6\alpha_3^3 w_1 u w_0+\alpha_3^3 u^3+6\alpha_3^3 w_2 w_1 w_0+6\alpha_3^3 w_2 w_1 u+6\alpha_3^2 w_2 w_0+3\alpha_3^3 w_2 u^2+ 3\alpha_3^3 w_1^2 w_0+6\alpha_3^2 w_1 u+6\alpha_3^2 w_1 w_0+6\alpha_3^2 u w_0)/\alpha_3 h+1/6(6\alpha_3 u+6\alpha_3 w_0+6+6\alpha_3 w_2+ 6\alpha_3 w_1+6\alpha_3^2 w_2 w_1+3\alpha_3^2 w_1^2+3\alpha_3^2 u^2+3\alpha_3^2 w_0^2+6\alpha_3^3 w_2 u w_0+3\alpha_3^3 w_1^2 u+3\alpha_3^3 w_0 u^2+ 3\alpha_3^3 w_1 w_0^2+6\alpha_3^2 w_2 u+3\alpha_3^3 w_1 u^2+\alpha_3^3 w_0^3+3\alpha_3^3 w_0^2 u+3\alpha_3^3 w_2 w_0^2+6\alpha_3^3 w_1 u w_0+\alpha_3^3 u^3+ 6\alpha_3^3 w_2 w_1 w_0+6\alpha_3^3 w_2 w_1 u+6\alpha_3^2 w_2 w_0+3\alpha_3^3 w_2 u+3\alpha_3^3 w_1^2 w_0+6\alpha_3^2 w_1 u+6\alpha_3^2 w_1 w_0+ 6\alpha_3^2 u w_0)/\alpha_3 s)exp(-\alpha_3(u+w_0$

$+w_1+w_2+w_3))+exp(-\alpha_3(u+w_0))/\alpha_3 h+1/6(6\alpha_3 u+6\alpha_3 w_0+6\alpha_3 w_3-24+ 6\alpha_3 w_2+6\alpha_3 w_1)/\alpha_3 h$

Period 4: $I_4=((\alpha_4^2 u w_0+\alpha_4^2 w_2 u+\alpha_4^2 w_3 u+\alpha_4^2 w_3 w_0+1+\alpha_4^2 w_1 u+\alpha_4^2 w_1 w_0+ \alpha_4^2 w_2 w_0+\alpha_4 w_3+\alpha_4 w_1+\alpha_4 w_0+\alpha_4 u+\alpha_4^3 w_2 u w_0+\alpha_4^3 w_3 w_1 w_0+\alpha_4^3 w_3 w_1 u+\alpha_4^3 w_3 w_2 w_0+ \alpha_4^3 w_3 u w_0+\alpha_4^3 w_1 u w_0+\alpha_4^4 w_3 w_1 w_0 u+\alpha_4^4 w_3 w_2 u w_0+\alpha_4 w_2+\alpha_4^2 w_3 w_2+\alpha_4^3 w_2 w_1 w_0+ \alpha_4^3 w_2 w_1 u+\alpha_4^3 w_3 w_2 u+\alpha_4^2 w_2 w_1+\alpha_4^2 w_3 w_1+\alpha_4^3 w_3 w_2 w_1+\alpha_4^4 w_2 w_1 w_0 u+\alpha_4^4 w_3 w_2 w_1 u+ \alpha_4^4 w_3 w_2 w_1 w_0+1/2(\alpha_4^2 u+\alpha_4^3 w_1 u+\alpha_4^3 w_2 w_0^2+\alpha_4^3 w_0^2 u+\alpha_4^3 w_1^2 u+\alpha_4^3 w_2^2 w_0+\alpha_4^3 w_3 w_0^2+ \alpha_4^3 w_1 w_0^2+\alpha_4^3 w_1^2 w_0+\alpha_4^3 w_2^2 w_1$

$+\alpha_4^3 w_2^2 u+\alpha_4^3 w_3 u+\alpha_4^3 w_2 u+\alpha_4^3 w_0 u+\alpha_4^3 w_2 w_1^2+\alpha_4^4 w_1 u w_0+\alpha_4^4 w_3 w_0^2 u+\alpha_4^4 w_3 w_1 u+ \alpha_4^4 w_2^2 w_1 w_0+\alpha_4^4 w_2^2 w_1 u+\alpha_4^4 w_3 w_2 w_0^2+\alpha_4^4 w_3 w_1 w_0^2+\alpha_4^3 w_3 w_1^2+\alpha_4^2 w_1^2+\alpha_4^2 w_2^2+\alpha_4^4 w_2 w_1^2 w_0+ \alpha_4^4 w_2^2 w_0 u+\alpha_4^4 w_2 w_1 u+\alpha_4^4 w_2 w_1 w_0^2+\alpha_4^4 w_3 w_1^2 u+\alpha_4^4 w_1^2 u w_0+\alpha_4^4 w_3 u w_0+\alpha_4^4 w_1 u w_0^2+ \alpha_4^4 w_2 w_1^2 u+\alpha_4^4 w_2 u w_0^2+\alpha_4^4 w_2 u w_0+\alpha_4^4 w_3 w_1^2 w_0+\alpha_4^4 w_3 w_2 u+\alpha_4^2 w_0^2)+1/4(\alpha_4^4 w_1^2 u+$

$\alpha_4^4 w_2^2 w_0^2 + \alpha_4^4 w_2^2 u + \alpha_4^4 u w_0^2 + \alpha_4^4 w_1^2 w_0^2)$

$+1/6(\alpha_4^4 u^3 w_0 + \alpha_4^4 u w_0^3 + \alpha_4^4 w_1 u^3 + \alpha_4^4 w_3 u^3 + \alpha_4^4 w_1^3 u + \alpha_4^3 w_0^3 + \alpha_4^3 u^3 + \alpha_4^4 w_1^3 w_0 + \alpha_4^4 w_2 u^3 + \alpha_4^4 w_2 w_0^3 + \alpha_4^4 w_3 w_0^3 + \alpha_4^4 w_1 w_0^3 + \alpha_4^3 w_1^3) + 1/24(\alpha_4^4 w_0^4 + 1/24 \alpha_4^4 u^4))/\alpha_4 h + 1/24(\alpha_4^4 w_0^4 + \alpha_4^4 u^4 + 24 + 12(\alpha_4^2 u + \alpha_4^3 w_1 u + \alpha_4^3 w_2 w_0^2 + \alpha_4^3 w_0^2 u + \alpha_4^3 w_1^2 u + \alpha_4^3 w_2^2 w_0 + \alpha_4^3 w_3 w_0^2 + \alpha_4^3 w_1 w_0^2 + \alpha_4^3 w_1^2 w_0 + \alpha_4^3 w_2^2 u + \alpha_4^3 w_3 u + \alpha_4^3 w_2 u + \alpha_4^3 w_0 u + \alpha_4^4 w_2^2 w_1 w_0 + \alpha_4^4 w_2^2 w_1 u + \alpha_4^3 w_2^2 w_1 + \alpha_4^3 w_2 w_1^2 + \alpha_4^4 w_1 u w_0 + \alpha_4^4 w_3 w_0^2 u + \alpha_4^4 w_3 w_1 u + \alpha_4^4 w_3 w_2 w_0^2 + \alpha_4^4 w_3 w_1 w_0^2 + \alpha_4^3 w_3 w_1^2 + \alpha_4^2 w_1^2 + \alpha_4^2 w_2^2 + \alpha_4^4 w_2 w_1^2 w_0$

$+\alpha_4^4 w_2^2 w_0 u + \alpha_4^4 w_2 w_1 u + \alpha_4^4 w_2 w_1 w_0^2 + \alpha_4^4 w_3 w_1^2 u + \alpha_4^4 w_1^2 u w_0 + \alpha_4^4 w_3 u w_0 + \alpha_4^4 w_1 u w_0^2 + \alpha_4^4 w_2 w_1^2 u + \alpha_4^4 w_2 u w_0^2 + \alpha_4^4 w_2 u w_0 + \alpha_4^4 w_3 w_1^2 w_0 + \alpha_4^4 w_3 w_2 u + \alpha_4^2 w_0^2) + 4(\alpha_4^3 w_0^3 + \alpha_4^3 u^3 + \alpha_4^4 u w_0^3 + \alpha_4^4 w_1 u^3 + \alpha_4^4 w_3 u^3 + \alpha_4^4 w_1^3 u + \alpha_4^4 w_1^3 w_0 + \alpha_4^4 w_2 u^3 + \alpha_4^4 w_2 w_0^3 + \alpha_4^4 w_3 w_0^3 + \alpha_4^4 u^3 w_0 + \alpha_4^4 w_1 w_0^3 + \alpha_4^3 w_1^3) + 24(\alpha_4^2 w_1 u + \alpha_4^2 w_1 w_0 + \alpha_4^2 u w_0 + \alpha_4^2 w_2 u + \alpha_4^2 w_3 u + \alpha_4^2 w_3 w_0 + \alpha_4^2 w_2 w_0 + \alpha_4 w_3 + \alpha_4 w_1 + \alpha_4 w_0$

$+\alpha_4 u + \alpha_4^3 w_2 u w_0 + \alpha_4^3 w_3 w_1 w_0 + \alpha_4^3 w_3 w_1 u + \alpha_4^3 w_3 w_2 w_0 + \alpha_4^3 w_3 u w_0 + \alpha_4^3 w_1 u w_0 + \alpha_4^3 w_2 w_1 w_0 + \alpha_4^3 w_2 w_1 u + \alpha_4^3 w_3 w_2 u + \alpha_4^2 w_2 w_1 + \alpha_4^2 w_3 w_1 + \alpha_4^3 w_3 w_2 w_1 + \alpha_4^4 w_3 w_2 u w_0 + \alpha_4 w_2 + \alpha_4^2 w_3 w_2 + \alpha_4^4 w_2 w_1 w_0 u + \alpha_4^4 w_3 w_2 w_1 u + \alpha_4^4 w_3 w_2 w_1 w_0 + \alpha_4^4 w_3 w_1 w_0 u) + 6(\alpha_4^4 w_1^2 u + \alpha_4^4 w_2^2 w_0^2 + \alpha_4^4 w_2^2 u + \alpha_4^4 u w_0^2 + \alpha_4^4 w_1^2 w_0^2))/\alpha_4 s) exp(-\alpha_4(u + w_0 + w_1 + w_2 + w_3 + w_4)) + 1/\alpha_4 exp(-\alpha_4(u + w_0))h + 1/24(24 + 24\alpha_4 u + 24\alpha_4 w_0)/\alpha_4 h exp(-(u + w_0 + w_1)\alpha_4) + 1/24(24(\alpha_4 w_1 + \alpha_4^2 w_1 u + \alpha_4 w_0 + \alpha_4 u$

$+1 + \alpha_4^2 w_1 w_0 + \alpha_4^2 u w_0) + 12(\alpha_4^2 u + \alpha_4^2 w_0^2))/\alpha_4 h exp(-(u + w_0 + w_1 + w_2)\alpha_4) + 1/24(12(\alpha_4^2 u + \alpha_4^3 w_1 u + \alpha_4^3 w_2 w_0^2 + \alpha_4^3 w_0^2 u \alpha_4^3 w_1^2 u + \alpha_4^3 w_1 w_0^2 + \alpha_4^3 w_1^2 w_0 + \alpha_4^3 w_2 u + \alpha_4^3 w_0 u + \alpha_4^2 w_1^2 + \alpha_4^2 w_0^2) + 4(\alpha_4^3 w_0^3 + \alpha_4^3 u^3) + 24(1 + \alpha_4^2 w_1 u + \alpha_4^2 w_1 w_0 + \alpha_4^2 u w_0 + \alpha_4^2 w_2 u + \alpha_4^2 w_2 w_0 + \alpha_4 w_1 + \alpha_4 w_0 + \alpha_4 u + \alpha_4^3 w_2 u w_0 + \alpha_4^3 w_1 u w_0 + \alpha_4^3 w_2 w_1 w_0 + \alpha_4^3 w_2 w_1 u + \alpha_4^2 w_2 w_1) +$

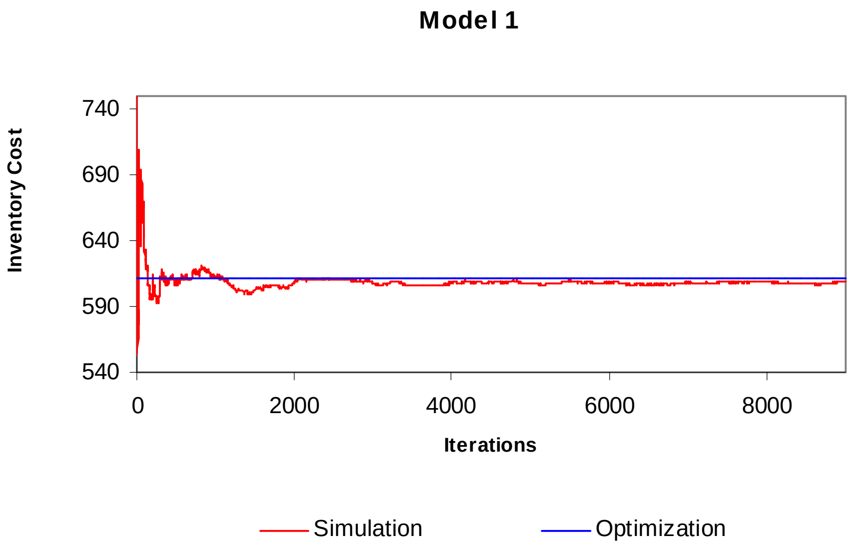
$$24\alpha_4 w_2)/\alpha_4 hexp(-(u+w_0+w_1+w_2+w_3)\alpha_4)+(\alpha_4 w_3+\alpha_4 w_1+\alpha_4 w_0+\alpha_4 u+\alpha_4 w_2+\alpha_4 w_4-5)/\alpha_4 h$$

Appendix B

Simulation

The inventory cost function is a complex expression. To verify if the expression was obtained in a correct form, we have ran a simulation.

For M1 and M2, we have picked up a retailer and solve the optimization problem. Additionaly, we have simulated the demand with the same behaviour (same mean and same standard deviation) for 10000 iterations and compared the solution obtained by optimization with the simulated one. The simualtion converges to the optimal solution, the slightly difference is due to the conversion from continuous to dicrete delivery quantities.



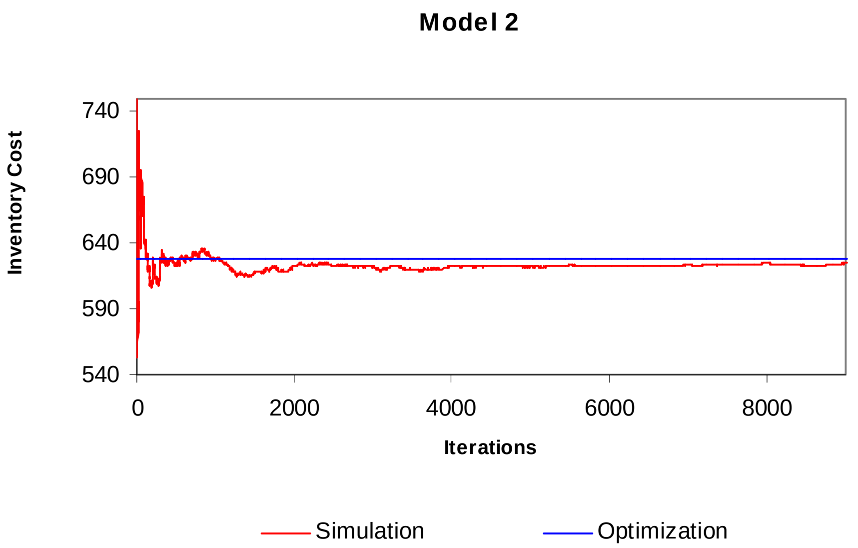


Figure B.1: Simulation versus Optimization, for M1 and M2, for one customer.

Appendix C

Computational experiment

Results from computational experiment of Chapter 4.

High setup cost parameter						
	Solution without integration			*Best solution in terms of routing cost*		
	ip_cost	***vrp_cost***	***irp cost***	***ip_cost***	***vrp_cost***	***irp cost***
1	6498,22	30628,63	37126,86	7861,85	29828,59	37690,43
2	6355,31	31375,62	37730,93	7434,59	30262,58	37697,17
3	5243,26	30977,93	36221,19	6381,84	30459,46	36841,30
4	7121,96	29147,56	36269,52	8711,73	28427,08	37138,81
5	5467,93	31655,90	37123,83	6098,23	30532,67	36630,91
6	5799,68	30833,62	36633,31	7001,11	29609,45	36610,57
7	5875,08	31035,80	36910,88	7174,82	29948,35	37123,17
8	4881,30	31509,59	36390,89	6125,71	30384,86	36510,57
9	6371,38	28611,75	34983,13	7677,00	27876,12	35553,12
10	5940,05	30928,22	36868,28	7223,96	30071,99	37295,95
11	29461,91	134669,51	164131,42	35229,64	128655,93	163885,57
12	26378,18	137100,31	163478,48	31618,04	131219,73	162837,77
13	26649,86	137002,51	163652,37	32837,95	128848,28	161686,23
14	30342,85	131775,51	162118,36	37047,23	123413,53	160460,77
15	28903,85	134073,11	162976,96	35336,77	129087,49	164424,26
16	30467,18	126200,44	156667,62	36374,30	121183,70	157558,00
17	25354,41	128423,79	153778,20	29423,74	124898,44	154322,18
18	29855,08	127208,54	157063,61	35623,64	121325,79	156949,43
19	31942,34	122568,69	154511,03	38293,11	116961,38	155254,49
20	29788,85	125960,32	155749,17	35364,07	122279,06	157643,13
21	11851,10	146009,14	157860,24	13609,64	141526,73	155136,37
22	11620,89	148817,97	160438,86	13591,28	146417,56	160008,85
23	10988,98	148994,77	159983,76	13166,90	143583,57	156750,47
24	13520,39	144356,64	157877,03	15510,37	138884,68	154395,05
25	10896,98	145440,85	156337,83	12368,91	144215,66	156584,57
26	11136,77	146623,41	157760,18	12563,53	143234,13	155797,66
27	12040,65	150948,77	162989,41	13690,65	148648,29	162338,94
28	10678,55	146053,11	156731,66	12292,63	141718,82	154011,45
29	12319,38	141799,48	154118,86	14570,24	138363,20	152933,44
30	12003,82	146401,55	158405,36	13743,45	145150,91	158894,37

Table C.1: Trade-off between inventory and routing costs.

High setup cost parameter						
	Solution without integration			***Best solution in terms of routing cost***		
	ip_cost	***vrp_cost***	***irp cost***	***ip_cost***	***vrp_cost***	***irp cost***
31	58674,63	133829,29	192503,93	69005,01	119925,65	188930,66
32	59889,80	137499,71	197389,52	71463,35	119506,11	190969,46
33	58313,88	136085,59	194399,48	69508,10	118919,19	188427,29
34	59371,39	132244,21	191615,60	71085,14	113293,53	184378,67
35	60901,65	137730,66	198632,32	70592,22	122290,03	192882,25
36	63463,89	138273,92	201737,81	75402,09	120275,84	195677,92
37	57826,86	135862,69	193689,54	66186,96	121223,22	187410,18
38	55050,70	131415,18	186465,88	64829,78	116630,75	181460,52
39	62090,84	133501,33	195592,17	72921,35	116603,68	189525,04
40	58096,64	133894,36	191991,00	68180,87	115780,98	183961,85
41	23405,30	145807,66	169212,97	26920,70	142360,28	169280,98
42	25022,88	146285,91	171308,79	28592,83	143859,95	172452,78
43	22712,34	141635,34	164347,68	25752,70	138121,55	163874,26
44	25192,47	144722,15	169914,62	29463,98	141190,34	170654,32
45	22747,68	141185,46	163933,14	26155,48	137765,47	163920,95
46	24299,35	147304,68	171604,03	28150,52	142829,40	170979,92
47	21002,83	145300,65	166303,48	23441,20	141974,75	165415,95
48	23747,97	142127,97	165875,94	27215,04	139737,67	166952,71
49	25225,42	147922,02	173147,44	28565,95	145433,92	173999,87
50	23659,68	141671,12	165330,80	26983,41	139201,34	166184,75
51	122100,85	136956,93	259057,78	141602,91	120886,17	262489,07
52	116837,25	135295,62	252132,87	133106,36	121465,77	254572,13
53	119174,04	134272,18	253446,21	138728,67	119115,67	257844,34
54	120608,52	137250,82	257859,35	138362,39	121179,12	259541,51
55	117370,26	132365,88	249736,14	135575,56	118380,05	253955,61
56	120053,16	140289,70	260342,86	138177,69	125344,54	263522,22
57	118380,38	135432,22	253812,61	136176,59	120430,08	256606,68
58	122992,87	136707,75	259700,62	142040,55	121633,15	263673,70
59	120800,61	133394,11	254194,72	139646,92	118506,20	258153,12
60	119347,77	133058,10	252405,86	136401,03	119234,27	255635,30

Table C.2: Trade-off between inventory and routing costs.

Low setup cost parameter						
	Solution without integration			***Best solution in terms of routing cost***		
	ip_cost	***vrp_cost***	***irp cost***	***ip_cost***	***vrp_cost***	***irp cost***
1	6498,22	111428,63	117926,86	7105,70	111246,53	118352,23
2	6355,31	115375,62	121730,93	6654,56	115301,78	121956,34
3	5243,26	114977,93	120221,19	5672,49	112649,15	118321,64
4	7121,96	106747,56	113869,52	7743,07	105621,18	113364,25
5	5467,93	116455,90	121923,83	5683,68	113094,02	118777,70
6	5799,68	112433,62	118233,31	6105,84	110355,38	116461,22
7	5875,08	115035,80	120910,88	6244,89	111829,06	118073,95
8	4881,30	116309,59	121190,89	5413,04	113104,59	118517,63
9	6371,38	105411,75	111783,13	6764,85	105070,48	111835,33
10	5940,05	113328,22	119268,28	6535,36	112132,27	118667,62
11	29461,91	69315,38	98777,29	30616,47	66929,56	97546,03
12	26378,18	68389,74	94767,91	28143,20	64831,80	92975,00
13	26649,86	69186,54	95836,40	28313,92	66614,39	94928,31
14	30342,85	66902,88	97245,73	32169,24	62389,47	94558,71
15	28903,85	68358,42	97262,27	30465,54	65804,42	96269,96
16	30389,25	71295,01	101684,26	32386,09	67559,76	99945,86
17	25354,41	65223,79	90578,20	26807,99	63753,90	90561,89
18	29855,08	73608,54	103463,61	31592,56	69142,17	100734,73
19	31942,34	68168,69	100111,03	33698,29	64788,00	98486,29
20	29788,85	69960,32	99749,17	31746,63	65460,39	97207,02
21	11851,10	146009,14	157860,24	12524,70	143691,04	156215,74
22	11620,89	148817,97	160438,86	12011,40	148643,22	160654,62
23	10988,98	148994,77	159983,76	11638,76	146923,80	158562,56
24	13520,39	144356,64	157877,03	14090,26	141249,13	155339,39
25	10896,98	145440,85	156337,83	11319,79	144435,96	155755,75
26	11136,77	146623,41	157760,18	11543,56	143247,09	154790,65
27	12040,65	150948,77	162989,41	12312,05	150897,17	163209,22
28	10678,55	146053,11	156731,66	10934,04	143835,58	154769,62
29	12319,38	141799,48	154118,86	12687,48	139528,32	152215,80
30	12003,82	146401,55	158405,36	12359,90	146397,36	158757,26

Table C.3: Trade-off between inventory and routing costs.

Low setup cost parameter						
	Solution without integration			***Best solution in terms of routing cost***		
	ip_cost	***vrp_cost***	***irp cost***	***ip_cost***	***vrp_cost***	***irp cost***
31	58674,63	133829,29	192503,93	61569,35	130357,64	191926,99
32	59889,80	137499,71	197389,52	62306,07	132912,07	195218,13
33	58313,88	136085,59	194399,48	61337,93	132281,43	193619,36
34	59371,39	132244,21	191615,60	61547,91	126663,45	188211,36
35	60901,65	137730,66	198632,32	62923,54	133313,04	196236,59
36	63463,89	138273,92	201737,81	66238,09	135706,20	201944,30
37	57826,86	135862,69	193689,54	60573,85	129060,16	189634,00
38	55050,70	131415,18	186465,88	57207,93	127862,95	185070,88
39	62090,84	133501,33	195592,17	64709,21	128985,01	193694,21
40	58096,64	133894,36	191991,00	60646,84	128258,76	188905,61
41	23405,30	145807,66	169212,97	24040,76	144638,47	168679,23
42	25022,88	146285,91	171308,79	25763,50	146098,80	171862,30
43	22712,34	141635,34	164347,68	23341,04	139409,23	162750,27
44	25192,47	144722,15	169914,62	25795,77	144621,35	170417,12
45	22747,68	141185,46	163933,14	22747,68	141185,46	163933,14
46	24299,35	147304,68	171604,03	24811,48	144170,32	168981,80
47	21002,83	145300,65	166303,48	21578,16	144106,24	165684,40
48	23747,97	142127,97	165875,94	24334,98	141941,15	166276,13
49	25225,42	147922,02	173147,44	25735,83	146767,94	172503,77
50	23659,68	141671,12	165330,80	24121,99	141571,93	165693,92
51	122100,85	136956,93	259057,78	125263,46	134423,73	259687,19
52	116837,25	135295,62	252132,87	119849,04	131709,03	251558,07
53	119174,04	134272,18	253446,21	123186,39	130697,41	253883,80
54	120608,52	137250,82	257859,35	123494,64	133836,59	257331,23
55	117370,26	132365,88	249736,14	120507,41	129714,36	250221,77
56	120053,16	140289,70	260342,86	123388,88	136659,86	260048,74
57	118380,38	135432,22	253812,61	120873,64	132857,53	253731,17
58	122992,87	136707,75	259700,62	125877,36	133182,79	259060,15
59	120800,61	133394,11	254194,72	123769,99	130972,43	254742,43
60	119347,77	133058,10	252405,86	122403,42	129607,21	252010,64

Table C.4: Trade-off between inventory and routing costs.