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**OPTICAL SOLITONS IN QUADRATIC  
NONLINEAR MEDIA AND  
APPLICATIONS TO ALL-OPTICAL  
SWITCHING AND ROUTING DEVICES**

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## Part II

# Part II: Soliton Beam Steering, Pointing and Switching

The definition that can be found in a dictionary for the term *steer* describes it as the action of *direct the course of, guide*. As for wave propagation theory, *Beam Steering* refers to all studies regarding transverse, as related to the direction perpendicular to the propagation, displacements of the energy connected to a particular wave beam affecting the overall propagation direction of such beam. The issue is very common, e.g., in free space wave propagation where it is often related to the phenomena of diffraction arisen when a ray traverses areas in which the atmosphere experiences changes in its gas composition or density yielding different refractive indices which cause the progressive bending of the ray. Likewise, beam steering studies as applied to remote sensing have allowed to build electronically controlled scanners which eliminate the prehistoric antenna mechanical motion and have set the basis for a vast amount of different scanning techniques which take advantage of acoustooptical and totally optical effects among others.

Recently, with the success of the optical fiber as information transmitter, the telecommunications community is progressively becoming aware that in the future a more efficient handling of information should require optical techniques which offers a vast field for exploration. Optical beam steering has integrated device applications such as switching and routing in the future optical network as well as applications for the scanning and image processing. In this sense, spatial optical solitons seem to be specially well-suited due to their particle-like behavior.

Several research works have focussed on the study of the possibilities of using spatial  $\chi^{(3)}$  solitons for the design of efficient all-optical beam steering devices. Specifically, the cases of bright soliton interaction either with beams of the same [141]-[143] or different frequency [144]-[146] in self-focusing media as well as dark solitons in self-defocusing media which create the guide for a bright soliton [147] have been investigated for application to the routing of optical beams. In all configurations reported to date, a powerful beam is used to alter the trajectory of a weak one which has the drawback of the high powers that are needed to excite a spatial soliton in Kerr materials.

Transverse phase modulations have also been proposed as a means of beam deviation in nonlinear Kerr planar waveguides. The modulation can be accomplished in practice either by surface acoustic waves such that the parameters of the modulation serve as a control of the position of the soliton at the waveguide output [148]-[149] or spatial index gratings arranged in

a direction normal to the propagation of the light beams whereupon self power control should be the issue.

As for practical beam steering devices it is very promising the case of spatial solitons that exist in discrete structures such as nonlinear waveguide and fiber arrays [150]. These spatially localized nonlinear modes of the array originate from the balance between Kerr nonlinearity and linear transverse coupling and steering is achieved by imposing a linear phase variation across the initial transverse profile, which simply corresponds to tilting the input beam. The output fiber or waveguide selected by this tilted input beam depends both upon the initial phase tilt and upon the input energy which thus determines the degree of confinement and makes power induced spatial switching feasible.

This second part of the Thesis comprises along with a description of the basic concepts and tools related to optical beam steering problems, a detailed study of steering of  $\chi^{(2)}$  solitons focusing mainly on the possibilities for the control of such steering. The issues addressed in this part may be cast into two main categories: on one side there is the analysis of the possibilities for the steering control of the solitary waves existing in the absence of Poynting vector walk-off which in principle can propagate in any transverse direction inside the crystal within the limits of the paraxial and SVE approximations; and on the other side, there are the investigation of the effects and main mechanisms leading to the formation of solitons in the presence of walk-off, the actual calculation of the stationary profile shapes through numerical methods, the study of its main properties and characteristics, and the exploration of the possibilities to exploit these solitary waves to perform optical beam steering control.

Presentation of the usual tools for addressing beam steering problems is carried out in the first chapter of this part (chapter 4) while chapter 5 deals with the problem of finding traveling wave solutions when Poynting vector walk-off is present in the system, describing as well the basic properties of the soliton families found and the characteristics of the excitation of such solitons in planar waveguides with the aid of numerical beam propagation methods.

Chapter 6 is mainly devoted to explore the various possibilities for controlling the steering of beams in a quadratically nonlinear medium many of which are based in the particular dynamics undergone by the fields in these materials. In last section of this chapter the phenomenon of beam break-up into several solitons is analyzed and its potential for switching applications

discussed.

Finally, last chapter of this part (chapter 7) is concerned with some results in the two dimensional case, namely bulk light propagation in second order nonlinear media. Optical vortices which feature an spiral phase dislocation of the phase front will be shown to lead into the quadratic nonlinear crystal to beam break-up into several solitons whose number and characteristics are determined by input and system parameters.

# Chapter 4

## Basics

This Chapter presents in its first section the basic tools for addressing the problem of beam steering which will be applied to the specific problem of soliton beam steering in planar waveguides built into nonlinear quadratic media. Second section is aimed at easing the understanding of the phenomena that will be encountered in the analysis of the solitary waves existing in the presence of walk-off and in the search for similitudes, briefly revises the soliton dragging and soliton trapping concepts in birefringent fibers.

### 4.1 The energy centroid

A convenient way of approaching the beam steering problem is the establishment of a direct relation between the propagation direction or the angle of propagation with respect to a certain coordinate axis assumed to be the main component of the vector giving the propagation of the beam, say  $z$ , with the *position* of the beam in the transverse plane for each value of the propagation coordinate. That makes the beam steering problem analogous to the motion of a particle in a two- (2+1 case) or one- (1+1 case) dimensional space considering in the beam problem as the parameter of the motion the propagation variable  $\xi$  instead of time  $t$ . Since a wave beam is not actually a particle but rather though localized, a *distribution* of energy, a better approximation is obtained by considering the beam as a *system of particles* whereupon

by analogy with the *mass centroid* it is defined the *energy centroid* as follows

$$\vec{\sigma}(\xi) = \int_{S_\infty} \vec{s} \left| \vec{\mathcal{E}}(s, \xi) \right|^2 ds, \quad (4.1)$$

where  $\vec{\mathcal{E}}$  stands for either the electric or the magnetic fields defining the wave beam and hence  $\left| \vec{\mathcal{E}} \right|^2$  represents the intensity associated with the wave and  $S_\infty$  indicates integration over the transverse plane. The location of this energy centroid,  $\langle \vec{s} \rangle$ , indicates a point in the transverse plane where all the energy stored by the wave beam can be considered to gather so that it can be dealt with as a single particle moving in this transverse plane. According to that definition, when the position of the energy centroid is taken as the coordinates origin the intensities weighted average of the vectorial magnitudes giving the points where intensity is distributed must vanish, i.e.

$$\int_{S_\infty} \left( \vec{s} - \langle \vec{s} \rangle(\xi) \right) \left| \vec{\mathcal{E}}(s, \xi) \right|^2 ds = 0. \quad (4.2)$$

Using that, the location of the energy centroid can be calculated as

$$\langle \vec{s} \rangle(\xi) = \frac{\int_{S_\infty} \vec{s} \left| \vec{\mathcal{E}}(s, \xi) \right|^2 ds}{\int_{S_\infty} \left| \vec{\mathcal{E}}(s, \xi) \right|^2 ds} = \frac{\vec{\sigma}(\xi)}{I}, \quad (4.3)$$

where  $I$  denotes the total energy stored in the beam.

To evaluate the deviation suffered by the beam, one is interested in the *velocity* of this energy centroid in the transverse plane which will be referred to as *transverse velocity* of the beam. Mind that since in the analogy the parameter  $t$  of the motion is substituted by  $\xi$  propagation variable, the differentiation for obtaining the value of the transverse velocity of the beam must be with respect to  $\xi$  so that one obtains

$$\vec{v}(\xi) = \frac{d}{d\xi} \langle \vec{s} \rangle = \frac{d}{d\xi} \left( \frac{\vec{\sigma}}{I} \right). \quad (4.4)$$

Ideally the beam keeps its total intensity constant along propagation and therefore it is set

$$\vec{v} = \frac{1}{I} \frac{d}{d\xi} \vec{\sigma}. \quad (4.5)$$

In the specific problem treated here, one has to deal with two energy centroids corresponding to the two fields taking part in the interaction, namely

$$\vec{\sigma}_1 = \int_{S_\infty} \vec{s} |a_1|^2 ds, \quad \vec{\sigma}_2 = \int_{S_\infty} \vec{s} |a_2|^2 ds \quad (4.6)$$

each of these energy centroids may occupy different positions in the transverse plane given by

$$\langle \vec{s}_1 \rangle = \frac{\vec{\sigma}_1}{I_1}, \quad \langle \vec{s}_2 \rangle = \frac{\vec{\sigma}_2}{I_2}, \quad (4.7)$$

and travel at different transverse velocities, namely

$$v_1 = \frac{1}{I_1} \frac{d}{d\xi} \vec{\sigma}_1, \quad v_2 = \frac{1}{I_2} \frac{d}{d\xi} \vec{\sigma}_2. \quad (4.8)$$

When a solitary wave is formed the two fields lock to each other through the parametric interaction and evolve as a unity, hence  $v_1 = v_2 = v$  and using that  $I_1 + I_2 = I$  and the above expressions for the energy centroid velocity one arrives to

$$\frac{1}{v_1} \frac{d}{d\xi} \vec{\sigma}_1 + \frac{1}{v_2} \frac{d}{d\xi} \vec{\sigma}_2 = I \longrightarrow v = \frac{1}{I} \left( \frac{d}{d\xi} \vec{\sigma}_1 + \frac{d}{d\xi} \vec{\sigma}_2 \right), \quad (4.9)$$

whereupon a total energy centroid may be defined as follows

$$\vec{\sigma} = \int_{S_\infty} \vec{s} \left( |a_1|^2 + |a_2|^2 \right) ds, \quad (4.10)$$

which is physically relevant once the locking has taken place and the two waves are self-trapped traveling with the same transverse velocity. Using the above definition the transverse velocity of the solitary wave is expressed

$$v = \frac{1}{I} \frac{d}{d\xi} \vec{\sigma}. \quad (4.11)$$

The governing equations serve to explicitly show the effects contributing to endow the solitary wave with a certain transverse velocity. For simplicity the equations for the 1+1 case are used but the result can be shown to hold equally for 2+1 configurations following an

analogous derivation. Evaluate

$$\frac{d}{d\xi} \vec{\sigma} = \int s \left( a_{1\xi} a_1^* + a_1 a_{1\xi}^* + a_{2\xi} a_2^* + a_2 a_{2\xi}^* \right) ds. \quad (4.12)$$

using for  $a_{1\xi}$ ,  $a_{2\xi}$  the expression given by the governing equations (2.94) to arrive through assumption that the fields and its derivatives when  $|s| \rightarrow \infty$  tend to zero faster than  $|s|$  itself tends to infinity, to the nice result

$$\frac{d}{d\xi} \vec{\sigma} = \frac{jr}{2} \int (a_1^* a_{1s} - a_1 a_{1s}^*) ds + \frac{j\alpha}{2} \int (a_2^* a_{2s} - a_2 a_{2s}^*) ds - \delta I_2. \quad (4.13)$$

The first two terms are reminiscent of the expression of the momentum, allowing to write

$$\frac{d}{d\xi} \vec{\sigma} = \vec{\mathcal{J}} - \delta \widehat{\delta} I_2 - (2\alpha + 1) \vec{\mathcal{J}}_2 - (r + 1) \vec{\mathcal{J}}_1, \quad (4.14)$$

where to extend the result to 2+1 configurations the vectorial nature of the walk-off parameter has been included.

For the spatial case  $r$  is exactly set to  $r = -1$  so that the last term vanishes, whereas  $\alpha$  as depending on the ratio of linear propagation constants at both frequencies strongly depends upon the dispersion relation in the material and is only in the phase-matching case exactly set to  $\alpha = -0.5$  thus allowing for exact cancellation of the third term. In most other practical cases  $\alpha \simeq -0.5$ , so that  $2\alpha + 1$  is on the order of  $10^{-4}$  or smaller and the following can be considered a very good approximation

$$\frac{d}{d\xi} \vec{\sigma} \simeq \vec{\mathcal{J}} - \delta \widehat{\delta} I_2, \quad (4.15)$$

which yields for the transverse velocity of the soliton

$$v \simeq \frac{\vec{\mathcal{J}}}{I} - \delta \widehat{\delta} \frac{I_2}{I}. \quad (4.16)$$

A simple as well as enlightening expression featuring in the absence of walk-off an analogous behavior than for single particle motion and linear propagation problems. Since in this case the moving entity is made up of two different elements, namely the fundamental and the second harmonic locked together, control of the transverse velocity may be performed through either of

them through endowment of an initial linear momentum which through the dynamics and after some of it is lost into radiation, will be communicated to the solitary wave. The linear momentum may be obtained through spatial transverse phase modulations of the input beams following  $\vec{\mathcal{J}} = (1/2) \int (2U_1^2 \phi_{1,s} + U_2^2 \phi_{2,s}) ds$  where the fields have been expressed as  $a = U \exp(j\phi)$  with  $U$  and  $\phi$  being real functions. The usefulness of such an effect for the solitons steering control in setups of practical relevance is assessed in section 6.2 and 6.3, which consider tilted input beams and input phase gratings respectively.

The second term in (4.16) accounts for the asymmetry introduced into the system by the Poynting vector walk-off and shows how the velocity of the energy centroid is governed by the second harmonic power flow which is not a conserved quantity of the system resulting in high dependence of the transverse velocity on the dynamics of the soliton excitation in terms of the input signals such that the steering of the beam may be controlled through the intensities and beam shapes at both frequencies, its relative phase or the wavevector mismatch value. Furthermore the walk-off term points out the dragging effect that the extraordinary second harmonic wave has on the fundamental. The possibilities to exploit this effect for steering control are investigated in section 6.1.

#### 4.1.1 Type II configurations

In type II configurations three are the interacting waves, and hence three the energy centroids to consider. Using the same variables as in (2.96) they read

$$\vec{\sigma}_1 = \int_{S_\infty} \vec{s} |a_1|^2 ds, \quad \vec{\sigma}_2 = \int_{S_\infty} \vec{s} |a_2|^2 ds, \quad \vec{\sigma}_3 = \int_{S_\infty} \vec{s} |a_3|^2 ds, \quad (4.17)$$

while for its velocities it is obtained

$$v_1 = \frac{1}{I_1} \frac{d}{d\xi} \vec{\sigma}_1, \quad v_2 = \frac{1}{I_2} \frac{d}{d\xi} \vec{\sigma}_2, \quad v_3 = \frac{1}{I_3} \frac{d}{d\xi} \vec{\sigma}_3. \quad (4.18)$$

Recalling that a conserved quantity in the system, i.e. total energy is

$$I = \frac{1}{2} (I_1 + I_2) + I_3 \quad (4.19)$$

the velocity of the solitary wave appearing as a result of the parametric interaction reads

$$\frac{1}{2v_1} \frac{d}{d\xi} \vec{\sigma}_1 + \frac{1}{2v_2} \frac{d}{d\xi} \vec{\sigma}_2 + \frac{1}{v_3} \frac{d}{d\xi} \vec{\sigma}_3 = I \longrightarrow v = \frac{1}{I} \left( \frac{1}{2} \left( \frac{d}{d\xi} \vec{\sigma}_1 + \frac{d}{d\xi} \vec{\sigma}_2 \right) + \frac{d}{d\xi} \vec{\sigma}_3 \right). \quad (4.20)$$

Following the same steps than in previous section for the type II system one is led to

$$\frac{d}{d\xi} \vec{\sigma} = \vec{\mathcal{J}} - \delta_2 I_2 - \delta_3 I_3 - (r_2 + 1) \vec{\mathcal{J}}_2 - (2r_3 + 1) \vec{\mathcal{J}}_3 - (r_1 + 1) \vec{\mathcal{J}}_1. \quad (4.21)$$

The last term exactly vanishes for the spatial beam propagation case whereas the fourth and fifth depend upon the ratios between the linear phase constants at the fundamental frequency for the ordinary and extraordinary waves and between the fundamental ordinary wave and the second harmonic extraordinary wave respectively, i.e.  $r_2 = -k_1/k_2$ ,  $r_3 = -k_1/k_3$  and are subject to the material and setup conditions. In most practical cases  $r_2 \simeq -1$ ,  $r_3 \simeq -0.5$ , hence one may approximately write

$$\frac{d}{d\xi} \vec{\sigma} = \vec{\mathcal{J}} - \delta_2 I_2 - \delta_3 I_3, \quad (4.22)$$

where the first term represents the particle-like behavior which in this case allows the velocity of the solitary wave to be controlled by any of the three input fields thus providing a richer variety of setups for steering control by input spatial phase modulations. As one could expect two different walk-off parameters corresponding to two extraordinary waves taking part in the interaction cause the transverse velocity to depend upon the intensities contained into each of the corresponding extraordinary waves thus supplying an additional degree of freedom when the soliton excitation conditions are meant to be used for steering control. Perhaps the more relevant issue stemming from expression (4.22) is the fact that the phenomena of dragging in this type of wave interaction can be controlled though the power launched into each of the two fundamental inputs with orthogonal polarizations whereupon great beam deviation can be achieved carefree of relative phases.

## 4.2 Soliton trapping and soliton dragging

Right after the question about whether solitary wave propagation, as the numerical simulations seem to confirm is possible inside the  $\chi^{(2)}$  crystal, it comes that about which mechanisms should this solitary wave propagation rely on when, taking the type I wave interaction as an example, ordinary fundamental and extraordinary second harmonic waves tend to propagate in different directions. Once again it is useful to resort to the thoroughly and widely studied  $\chi^{(3)}$  solitary wave propagation problem for concepts which properly accounting for the differences can be adapted to the  $\chi^{(2)}$  case. Thus, from a formal point of view an analogy may be established between the phenomena of fundamental and second harmonic beam locking inside the  $\chi^{(2)}$  crystal and the soliton trapping and dragging phenomena occurring between pulses launched into each of the orthogonal modes of a birefringent fiber having different group velocities, a case studied by Menyuk [151], [152], and that has been exploited to build routing and switching devices in the temporal domain [76].

The pulse propagation in a birefringent optical fiber is described by the coupled nonlinear Schrödinger equations (CNLSE), which as derived using the standard soliton normalizations [151], [152], yield

$$\begin{aligned}
 -j(u_z + \delta u_t) &= \frac{1}{2}u_{tt} + |u|^2 u + \frac{2}{3}|v|^2 u + \frac{1}{3}v^2 u^* \exp(-jR\delta z), \\
 -j(v_z - \delta v_t) &= \frac{1}{2}v_{tt} + |v|^2 v + \frac{2}{3}|u|^2 v + \frac{1}{3}u^2 v^* \exp(jR\delta z),
 \end{aligned} \tag{4.23}$$

where  $u$  and  $v$  are the envelope functions along the two axes and  $\delta$  characterizes the group velocity walk-off among them. The left-hand side of the equations represents the pulse walk-off due to birefringence in a normalized coordinate system traveling at the mean velocity between the axes while the terms on the right-hand side correspond respectively to group-velocity dispersion, self-phase modulation, cross-phase modulation and coherent four-wave-mixing or phase conjugation. This latter coherence term is responsible for the polarization instability displayed by low-birefringence fibers but it has proven not to be essential for the soliton-dragging and trapping phenomena. As Menyuk shows through numerical examples [151], [152] for most relevant experimental conditions this coherent term is rapidly oscillating with zero average so that

the CNLSE reduces to

$$\begin{aligned} -j(u_z + \delta u_t) &= \frac{1}{2}u_{tt} + |u|^2 u + \frac{2}{3}|v|^2 u, \\ -j(v_z - \delta v_t) &= \frac{1}{2}v_{tt} + |v|^2 v + \frac{2}{3}|u|^2 v. \end{aligned} \tag{4.24}$$

The numerical experiments carried out by Menyuk showed evidence of the time shift undergone by either of the two orthogonal modes due to cross-phase modulation with the other mode, a phenomenon which was generally called *soliton dragging*. The effect is known as *soliton trapping* in the specific case in which both modes have comparable amplitudes so that they experience respective shifts such that the faster mode is delayed so to ‘wait’ for the slower one which in turn is speeded up through the phase modulation thus being able to catch the slowed down faster mode whereupon the net result is that the two modes travel together as a unit.

In spite of the many differences displayed with the  $\chi^{(2)}$  beam locking phenomenon, these two concepts as it shall be seen, prove useful in interpreting many of the properties of walking solitons and the mechanisms of its formation and evolution. A crucial difference though is the fact that in (4.24) no power exchange between the two polarizations occurs, opposite to (4.23) and the  $\chi^{(2)}$  case where energy transfer does occur. Also notice that in (4.24) a soliton can exist in each polarization state separately whereas in the  $\chi^{(2)}$  system both waves are required to form a soliton.