Universitat Autònoma de Barcelona

Essays on Digital Economics

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Doctoral Thesis

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To my brave family.

For their endless love, support, and encouragement.

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Introduction

Digital economics is the framework where agents engage in economic activities in a digital environment with improved technical abilities but limited or no social interaction. The activities in a setting with these distinctive characteristics create a compelling academic curiosity. Researchers of digital economics look at how technology affects the decision process of economic agents and economic activity in general. It is not necessary to develop an entirely new economic theory to understand the effects of digital technology. However, it calls for subtle modifications to represent the novel conditions introduced by technological innovation.

Asymmetry of information is a highly relevant phenomenon for digital economic activity due to the lack of social interaction and the widely anonymous nature of the internet. For instance, the participants of the digital platforms often have very little prior information on each other, generating uncertainty about the quality of the goods or services provided. The most commonly adopted solution by digital platforms is to provide a user review system to help consumers acquire information about sellers from past customers and discipline the sellers by publishing their customers' satisfaction. Hence, understanding the platform participants' decisions with a user review system comes a long way in understanding digital platforms.

In the first chapter "Seller Reputation with an Imperfect Review System," I investigate the platform participants' behavior and reputation incentive provided to the seller when customer satisfaction is revealed with some probability. I build a formal theoretical model with adverse selection and moral hazard and allow transactions with satisfied and unsatisfied customers to receive reviews with possibly distinct probabilities. These probabilities affect the type of Perfect Bayesian Equilibrium and the reputation incentive provided to the seller. I show that both sellers and customers are better off when the probability of receiving a review from a satisfied consumer increases. The sellers have an increased return for their effort, and buyers enjoy a higher utility from transactions. Thus, improving the reputation mechanism benefits everyone. On the contrary, an increase in the likelihood of receiving a review from an unsatisfied customer may decrease the profit. In this case, censoring negative reviews enables a higher profit with a lower effort. Therefore, the platform and the sellers may have an incentive to deteriorate the reputation mechanism, hurting the platform's customers.

Consumer data collection and analysis is another example of information exchange that became available with digital platforms. In this case, in contrast to the previous one, information flows from customers to firms. Consumer data enables firms to reveal consumer preferences and target them accordingly. In the second chapter "Product Market Outcomes with Targeted and Random Advertising," I explore the economic value of consumer data by modeling targeted and random advertising and how the market sides share it. I show that targeted advertising benefits everyone by enabling small markets, while random advertising does not provide enough income for the firm(s) to promote its products and creates entry barriers. In larger markets, consumers face a higher price with targeting than random advertising due to the passing on of advertising cost and partial market segmentation. Despite higher prices, targeting improves welfare thanks to eliminating unnecessary advertisements.

Thanks to the wide adoption of information technologies by commercial stores, most consumers now have the option of an online market in addition to their local brick-and-mortar shops. Therefore the local shops must consider the online competition in addition to the local one. In the third chapter "Local Market Equilibrium with Online Diffusion," I focus on the consequences of the presence of the online market on the price and profit of brick-and-mortar firms through price competition. I consider a model with heterogeneous products and agents who differ in their location and online search cost. I show that the offline price and profit decrease with online competition, whereas both the online and offline consumers are better off. In contrast to the original Hotelling result, the local firm's profit may decrease with the transportation cost if the online market penetration is strong. On the other hand, the online competition may lead to an increase in the offline price if it affects the local market structure change by driving away local competition.

Chapter 1

Seller Reputation with an Imperfect Review System

Abstract

I model the transactions that take place in an electronic marketplace where both adverse selection and moral hazard problems are present. A built-in review system is employed within the platform to ease the information problem and provide an incentive for the seller through reputation. Transactions with satisfied and unsatisfied consumers receive reviews with possibly distinct probabilities. These probabilities affect the type of Perfect Bayesian Equilibrium and the effort provided by the seller. I show that when the price is fixed, increasing the likelihood of a successful transaction being reviewed on the platform never decreases the seller's effort, with the possibility of a discrete jump. By contrast, increasing the likelihood of a failed transaction receiving a review may cause a discrete drop in the seller's effort. The seller's profit is non-monotonous in the probabilities of receiving reviews. Under endogenous price, the seller effort is continuous and non-monotonous in both parameters of the review system.

JEL L140, D82, L810, D83
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1.1 Introduction

The electronic marketplaces (platforms) for services and goods meet sellers and buyers around the world. We witness a great flourish of platforms that become the main venues for exchanging certain goods and services. The platforms' advancement mainly originates from advantages such as lower market costs for both sellers and buyers, 24/7 availability and trading possibilities without geographical concerns. However, these significant advantages do not come without any frictions. Often, the participants of the electronic marketplaces have very scarce prior information on each other, and the lack of information generates uncertainty about the quality of the goods or services provided.

The most common approach used to overcome the information problem in electronic marketplaces is to provide a built-in review system and incentivize sellers through *Seller Reputation*. In his comprehensive survey, Dellarocas (2003b) explains how these review systems emerged as a viable mechanism for fostering cooperation among strangers. Several papers indicate that the customers pay attention to online reviews and act upon them to make purchasing decisions (Chatterjee, 2001; Chevalier and Mayzlin, 2006; Senecal and Nantel, 2004).

Still, the built-in review systems may not (be able to) reveal the result of transactions in a platform entirely, and platforms with built-in review systems may not avoid inefficient transactions. Moreover, the literature on online reviews documents that positive reviews are strongly predominant. In their econometric results obtained with the longitudinal data from Amazon.com, Hu et al. (2009) report that all online product reviews follow a J-shaped distribution with more extreme positive (5-stars) than extreme negative (1-star) reviews on a 5-star scale, consistent with former reports by Eliashberg and Shugan (1997), Chevalier and Mayzlin (2006), and Liu (2006). Fradkin et al. (2014) and Zervas et al. (2015) confirm the biased distribution of the online reviews in their studies focused on the case of Airbnb.com, where individual homeowners provide accommodation service.

The distortion in the operation of a built-in review system can be explained by psychological and, more importantly, strategic factors. The abundance of positive reviews is linked to underlying phenomena such as herding behavior, under-reporting negative experiences, self-selection, strategic manipulation, or censoring of reviews. The presence of underlying strategic factors suggests that it is worth analyzing the outcomes in platforms where reviews may be posted with different likelihood conditional on the result of the transaction.

In this paper, I build on the mixed model of Bar-Isaac and Tadelis (2008) and model the transaction in an electronic marketplace which involves both adverse selection[1] and moral hazard[2] problems. The seller's unobserved characteristic, which plays a crucial role in determining the utility gained from the product or service, is represented by heterogeneous seller types. The uncertainty caused by the seller's unobserved action is represented by the costly effort provision of the seller. The effort provided by the seller, together with his type, determines the probability of a successful outcome, the only result which gives positive utility to the buyer. In this model, in contrast to Holmström (1999), effort acts as a complement rather than a substitute for type. Departing from Bar-Isaac and Tadelis (2008), I assume that the transaction outcome may not be reported for exogenous reasons, and the frequency of a review is conditional on the result of the transaction. Firstly, I go separately by assuming a fixed price model and then meet with their model by extending the discussion by endogenizing the price.

The marketplace of the model is active for two periods. At the beginning of each period, a buyer is matched to a seller of an unknown type, and she decides whether to purchase from the seller. She has a belief about the type of the seller. The buyer's expected utility is a function of her belief about the seller's type and the corresponding type's anticipated effort. I assume that the buyer's initial belief is high enough so that she always decides to buy in the first period.

The buyer observes the outcome of the transaction once she receives the product or service. The outcome becomes publicly available if a review regarding the transaction is posted on the platform, which provides some information on the seller's unobserved features. The second-period buyer's belief about the seller's type, which corresponds to the seller's reputation, is determined by the potential review of the previous transaction carried out by the seller and the belief about sellers' effort provision. The seller's reputation is formed with the buyer's belief about both the characteristic of the seller and the action taken by the seller in the previous period.

The imperfect revelation of the outcome of a transaction is represented in the model by the probabilistic post of a review on the platform's built-in review system. I assume that the reviewers are honest in the sense that a review always gives accurate information on a previous transaction. However, the probability of a review to be posted on the platform is given conditional on whether the transaction was successful or not by possibly distinct parameters.

[1]See Akerlof (1970).

[2]See Arrow (1970), Zeckhauser (1970), Spence and Zeckhauser (1971).

I provide the Perfect Bayesian Equilibria (PBE) of the model. I identify three types of PBE where the buyer decides to buy in the second period if she observes a high review, a high or no review, and a high, no, or a low review depending on the initial belief. Next, I perform comparative analysis for the probabilities of a good and bad outcome to be reported on the platform. I show that an increase in the probability of the transactions with a satisfied consumer being reviewed never decreases the seller's effort. It may even lead to a discrete jump of the effort upwards. However, an increase in the probability of the transactions with an unsatisfied consumer being reviewed may decrease the seller's effort.

The consumer surplus is very closely related to the seller's effort. The first-period consumer surplus is determined by how much effort the seller provides to build a reputation. So, it depends on how good the review system provides an incentive for the seller. In the second period, the consumer decides on the purchase decision based on the signals from the review system. The stronger the review system's signal, the higher the consumer surplus. Hence, the consumer surplus depends on how well the review system performs in providing information to the consumer about the seller's quality. An increase in the probability of the transactions with a satisfied consumer receiving a review does not harm consumer utility in either period. On the other hand, an increase in the likelihood of the transactions with an unsatisfied consumer receiving a review may harm consumer utility in both periods due to a discrete drop in the seller's effort, while a further increase has a positive effect that recovers the utility loss.

Comparative statics show that the seller benefits from an increase in the probability of a successful transaction being reviewed when the consumer requires to observe a positive review to agree on the transaction. The strategic action to influence the likelihood of positive reviews upwards is referred to as promotional reviews. The promotional reviews act as a positive externality by increasing both the effort and the profit of the seller. On the other hand, an increase in the likelihood of an unsuccessful transaction being reviewed can decrease the seller's profit. Hence, the seller may have an incentive to take a strategic action called censoring (negative reviews) to reduce the likelihood of a negative review being posted. I show that censoring negative reviews can decrease the seller's effort while increasing the profit. If the platform's interest is aligned with the seller, such as in the case of a business model based on commission fees, then a monopolist platform finds it beneficial to exercise both types of manipulation (promotional reviews and censoring) on the review system.

I also study a model where the seller sets the price at the beginning of each period. I identify a perfect Bayesian pooling equilibrium where both types of sellers set the same price. In this equilibrium, the seller is incentivized to build a reputation through the review system. At equilibrium, the effort is non-monotonic in the probability of a transaction receiving a review regardless of whether it is successful or unsuccessful. That is, the availability of information about past transactions does not always result in higher productivity.

The online platforms' reviewers are not generally economically rewarded (except on the platforms that build the business model solely on product/seller reviews). Still, it is entirely credible to consider that the reviewers have incentives to engage in such an effortful action. These incentives possibly work through social, psychological, or strategical phenomena. Approximately 65% of buyers leave feedback on eBay, which is a very high fraction (Tadelis, 2016). On the other hand, most of the time, users are not mandated to leave a review after a transaction. Although a review is taken as granted in theoretical models in the survey made by Bar-Isaac and Tadelis (2008), they also point out that further research is needed on the effect of the structure and design of the information spread on the reputational incentives. It appears that the frequency of feedback together with the design of the review mechanism is of vital importance in studying reputation and not only in the trading environment. In his paper on online social networks, Ellman (2017) compares the feedback mechanism designs where the rater gives feedback on a post with an exogenous probability.

When transactions are not always reviewed, it is not clear if the lack of review is because there was no transaction or the transaction was not reported. It is argued that if the transaction is not reported, the lack of review may be informative about the transaction rather than just a null signal considering the design of the review systems. Ellman (2017) shows that when the platform facilitates only positive feedback, then no feedback is a negative signal rather than being neutral in the online social network environment. In this stream but the trading environment, Nosko and Tadelis (2015) suggest a new quality measure they call effective percent positive (EPP), which is calculated by dividing the number of positive feedback transactions by the total number of transactions, thus penalizing sellers who are associated with more transactions for which the buyers left no review. The tenet of their approach is that silence is bad news.

Gaudeul and Jullien (2005) emphasize that one role of the intermediary is providing each side of the trade with information about the other side, in addition to putting potential trading partners into contact. Hence the intermediary is the likely party to be relied on to prevent a market failure and be trusted to tell the truth about the supplier's good. Indeed, when the platform's interest is not entirely in line with its marketplace's supply side, it is expected to provide objective advice to consumers with its reputation in concern. However, a monopoly intermediary may want to profit from its information on a supplier's product not by revealing it but by hiding it. The intermediary can achieve a higher profit, not necessarily by direct payment from the seller but a commission fee from a seller's transaction on the platform, which would not be realized in case of full disclosure to consumers. Dellarocas (2006) points out that the design of online reputation mechanisms can significantly benefit from the insights produced by decades of game theoretical research on the topic of reputation, while results need to be extended to take the unique new properties of online environments into account. Bar-Isaac and Tadelis (2008) summarize the theoretical background for seller reputation and provide intuitive implications on how well reputational concerns work in achieving efficient market outcomes in the case of electronic marketplaces. The seller reputation model with adverse selection and moral hazard problems of Bar-Isaac and Tadelis (2008) is modified with an imperfect revelation of the outcome of the transaction in this paper.

I start with the introduction of the model where the price is exogenous in the next section. The results are provided in three subsections in Section 3. The update of the buyer's belief is explained in the first part. Afterward, three Perfect Bayesian Equilibria of the model are identified, and a selection criterion is introduced in the case of multiple equilibria. The third part provides a comparative analysis using the selection criterion. Section 4 defines the manipulation of the review system in the current model and explains incentives for manipulation. Section 5 discusses the results in an alternative setting in which the price is endogenized by delegating the decision to the seller. Finally, concluding remarks are presented. All the proofs are in the Appendix.

1.2 Model

Consider a platform that is active for two periods. Linked to the platform, one seller can have at most one transaction in each period, and one buyer per period is interested in making at most one transaction. The good or service traded on the platform has a fixed price, p.

A transaction can result in a successful outcome that is worth 1 to a buyer or in an unsuccessful outcome that is worth 0. The probability of a successful transaction depends on both the seller's type and effort. The result of the transaction is known to the buyer after it has taken place and contingent contracts are not feasible.

In the first period, there is an early buyer that is matched to the seller. For the model to be meaningful, I will make an assumption that will ensure that a transaction occurs for this match. The buyer involved in the transaction in the first period will be referred to as the *first buyer*. Once the result of the transaction is observed by the first buyer, she is asked to evaluate the process via the platform's review system.

In the second period, the seller is matched to another buyer who observes the review of the first-period transaction if a review is posted and decides whether or not to purchase from the seller. The buyer in the second period will be referred to as the *second buyer* from this point on.

The seller can be either a 'good (g)' or a 'bad (b)' type, where the type of the seller is denoted by $\theta = g, b$, with $g, b \in (0, 1)$. At the beginning of each period t, for $t = 1, 2$, buyers have a belief that the seller is of good type, denoted by μ_1 and μ_2 respectively. That is, μ_t denotes the probability that the seller is of good type at time t according to the buyer's belief. The initial belief at the beginning of the first period, μ_1, is given exogenously. The belief in the second period, μ_2, is shaped by the information revealed (or not) about the first-period transaction.

The probability of a successful transaction is a function of the seller's type, θ, and effort e, and given by θe, where $e \in [\underline{e}, 1]$ with $\underline{e} > 0$. The effort provided by the g-type and the b-type sellers at period t will be denoted as e_t^g and e_t^b, respectively. Effort provision is costly for the seller and the cost is given by an increasing and convex cost function, $c(e)$ ($c' > 0$, $c'' > 0$ for $e > \underline{e}$ and $c'(\underline{e}) = 0$).

Buyers and sellers are risk-neutral. A transaction is performed if the buyer chooses to purchase at the given price level p. The buyers make their decision by considering their expected payoff. Hence, the buyer agrees to buy at period t if the following condition holds:

$$p \leq \mu_t g e_t^g + (1 - \mu_t) b e_t^b.$$

Assumption 1 guarantees that the first buyer always buys:

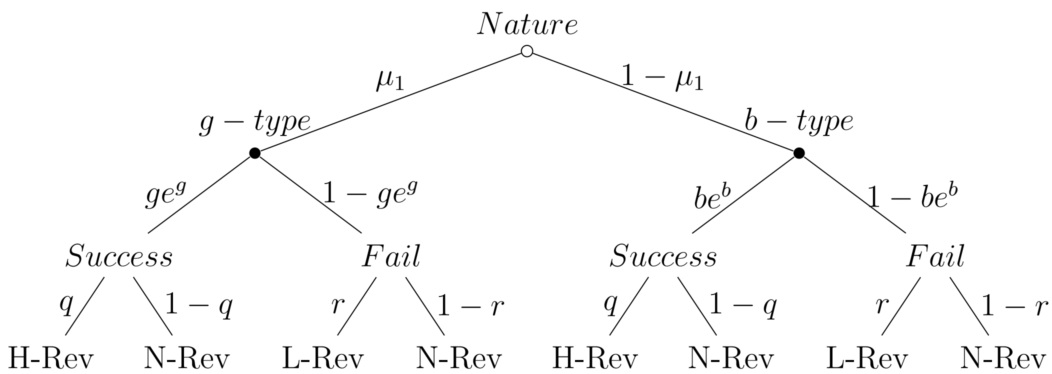


Figure 1.1: The Probability of Reviews After the 1st Period

Assumption 1:

$$p \leq \mu_1 g\underline{e} + (1-\mu_1)b\underline{e}. \tag{A1}$$

Therefore in the first period, the seller and the first buyer perform a transaction with a probability equal to 1. The seller decides the level of effort to be provided, either e_1^g or e_1^b, depending on her type.

The first buyer will observe the result of the transaction after it is realized, and she is asked to report the outcome of the transaction on the platform's review system. She is assumed to report the outcome truthfully. The successful outcomes are reported with High (H), and the unsuccessful outcomes are reported with Low (L) reviews. However, reviews are posted on the review system of the platform with some probability. A successful transaction is reported with probability q, whereas an unsuccessful transaction is reported with probability r. With different probabilities assigned to high and low reviews, the likelihood of revelation of information is allowed to be different if the customer is happy or not.[3] The third case with No (N) review emerges when the first period's transaction is not reported. Figure 1 depicts the probability tree.

In the second period, the outcome of the first-period transaction is available on the review system of the platform if a review is posted. Hence, the second buyer updates her beliefs upon observing the posted review. The belief μ_2 can take three

[3]Defining distinct parameters for positive and negative reviews enables us to study incentives for two kinds of strategic actions that the platform or the seller can carry out to manipulate the review system. The details will be discussed in Section 4.

possible values, μ_2^H, μ_2^L, and μ_2^N depending on whether the review was High, Low or there was no review at period 1. The decision of the second buyer on whether to buy or not depends on μ_2 and her expectation of the seller's effort provision. Given that it is the last period of the platform and the effort is costly, if the seller and the second buyer agree on the transaction, then both types of seller provide the minimum level of effort, that is, $e_2^g = e_2^b = \underline{e}$.

Therefore, there will be a transaction in the second period if the following condition holds:

$$p \leq \mu_2 g\underline{e} + (1-\mu_2)b\underline{e} = \mu_2\underline{e}(g-b) + b\underline{e}$$

which, after some rearrangement, gives the minimum level of belief required for a transaction to be performed at the second period:

$$\mu_2 \geq \underline{\mu} \equiv \frac{p - b\underline{e}}{\underline{e}(g-b)}.$$

For convenience, A1 is written with the new notation as:

$$\mu_1 \geq \underline{\mu}. \tag{A1'}$$

Finally, we assume that the exogenous price level is not too low, so that the expected payoff of the buyer is negative if the seller is known to be a b-type and provides the minimum level of effort:[4]

Assumption 2

$$p - b\underline{e} > 0. \tag{A2}$$

1.3 Results

The main results are provided in this section. I first describe how the belief of the second buyer is updated in the second period.

[4]See Section 5 for a discussion on the case when the price is set by the seller.

1.3.1 Update of Beliefs at the Second Period

For any possible pair of efforts (e_1^g, e_1^b) chosen by the two types of the seller at $t = 1$, each of the three signals (H, N, L) has a positive probability. Therefore the second buyer can always update the belief about the type of the seller using Bayes' rule. Although the seller's effort is not observable, the updated beliefs are given as a function of (e_1^g, e_1^b) with a slight abuse of notation, while in fact, they are the expectation of the buyer about the contingent first-period effort provided by the seller.

If there is a High-Review on the platform regarding the transaction at $t = 1$ (shown as H-Rev in Figure 1), that is, the transaction was successful and the review was posted on the platform, then the updated belief is:

$$\mu_2^H(e_1^g, e_1^b) = \frac{q\mu_1 g e_1^g}{q\mu_1 g e_1^g + q(1-\mu_1) b e_1^b} = \frac{\mu_1 g e_1^g}{\mu_1 g e_1^g + (1-\mu_1) b e_1^b}. \tag{1.1}$$

If there is a Low-Review on the platform regarding the transaction at $t = 1$ (shown as L-Rev in Figure 1), that is, the transaction was unsuccessful and the review was posted on the platform, then the updated belief is:

$$\mu_2^L(e_1^g, e_1^b) = \frac{r\mu_1(1 - g e_1^g)}{r\mu_1(1 - g e_1^g) + r(1-\mu_1)(1 - b e_1^b)} = \frac{\mu_1(1 - g e_1^g)}{\mu_1(1 - g e_1^g) + (1-\mu_1)(1 - b e_1^b)}. \tag{1.2}$$

Finally, if there is no review on the platform regarding the transaction realized in the first period (N-Rev in Figure 1), that is, the result of the transaction in the first period is not revealed, then the second buyer updates her belief as follows:

$$\mu_2^N = \frac{\mu_1[(1-q) g e_1^g + (1-r)(1 - g e_1^g)]}{\mu_1[(1-q) g e_1^g + (1-r)(1 - g e_1^g)] + (1-\mu_1)[(1-q) b e_1^b + (1-r)(1 - b e_1^b)]}. \tag{1.3}$$

I compare the updated beliefs to understand how do the reviews influence the second buyer's belief. For values $q, r \in (0, 1)$, the updated beliefs satisfy the following inequality:[5]

$$\mu_2^H(e^g, e^b) > \mu_2^N(e^g, e^b) > \mu_2^L(e^g, e^b) \quad \text{for} \quad e^g \geq e^b.$$

This is the verification that a good outcome generates a higher updated belief. In particular, the second buyer's belief is the highest if a High review is observed, whereas it is the lowest if a Low review is observed.

1.3.2 Main Results

Several types of equilibria can emerge depending on the behavior of the second buyer, who updates her belief about the seller being g-type according to the Bayes' rule, and the seller, who chooses the level of effort to provide in the first-period transaction.

Proposition 1.1 states the Perfect Bayesian Equilibria (PBE) that are sustained as a function of the value of the initial belief, μ_1. We state here the thresholds on μ_1 that appear in the proposition:

$$\begin{aligned}
\mu^I &\equiv \frac{[be_A^b(r-q)+1-r](p-b\underline{e})}{[be_A^b(r-q)+1-r](p-b\underline{e})+[ge_A^g(r-q)+1-r](g\underline{e}-p)}, \\
\mu^{II} &\equiv \frac{[be_B^b(r-q)+(1-r)](p-b\underline{e})}{[be_B^b(r-q)+(1-r)](p-b\underline{e})+[ge_B^g(r-q)+(1-r)](g\underline{e}-p)}, \\
\mu^{III} &\equiv \frac{(1-be_B^b)(p-b\underline{e})}{(1-be_B^b)(p-b\underline{e})+(1-ge_B^g)(g\underline{e}-p)}, \\
\mu^{IV} &\equiv \frac{(p-b\underline{e})(1-b\underline{e})}{(p-b\underline{e})(1-b\underline{e})+(1-g\underline{e})(\underline{e}g-p)}.
\end{aligned}$$

Proposition 1.1. *There exist three types of PBE depending on the initial belief of the buyers, μ_1. In all the equilibria, the second buyer's updated beliefs are μ_2^H, μ_2^L and μ_2^N. Moreover, both types of seller provide the minimum effort, $\underline{e}$ at $t = 2$. The first two types of equilibria are separating, whereas the last equilibrium is pooling:*

Type A. If $\mu_1 \leq \mu^I$ then there exists a PBE in which

> *(i) at $t = 1$, the g-type and b-type sellers choose, respectively: $e^g = e_A^g \equiv c'^{-1}(g(p - c(\underline{e})q)$ and $e^b = e_A^b \equiv c'^{-1}(b(p - c(\underline{e})q)$,*

[5] The Appendix provides the proof of the comparison among μ_2^H, μ_2^N, and μ_2^L for the case where $e^g \geq e^b$.

(ii) at $t = 2$, the second buyer buys only if there is a H-review posted.

Type B. If $\mu^{II} \leq \mu_1 \leq \mu^{III}$ then there exists a PBE in which

(i) at $t = 1$, the g-type and b-type sellers choose respectively: $e^g = e^g_B \equiv c'^{-1}(g(p - c(\underline{e})r)$ and $e^b = e^b_B \equiv c'^{-1}(b(p - c(\underline{e})r)$,

(ii) at $t = 2$, the second buyer buys if there is a H-review posted or no review at all.

Type C. If $\mu_1 \geq \mu^{IV}$ then there exists a PBE in which

(i) both types of seller provide the minimum effort at $t = 1$, i.e., $e^g = e^b = \underline{e}$,

(ii) at $t = 2$, the second buyer regardless of the review.

Each of the three types of equilibria exists for a certain interval of the first buyer's belief, μ_1. We now discuss the existence of an equilibrium for all $\mu_1 \in [\underline{\mu}, 1]$. Notice that the low end of the domain of μ_1 is covered by the Type-A equilibrium, which is supported if $\mu_1 \leq \mu^I$. The high end of the domain of μ_1 is covered by the Type-C equilibrium, which is supported if $\mu^{IV} \leq \mu_1$. In addition, I show in the Appendix that $\mu^I \geq \mu^{II}$ holds for any cost function. However it is not necessarily true that $\mu^{III} \geq \mu^{IV}$ for any cost function. The condition holds, for example, if $c(e) = ce^2$ with $c > 0$, as shown in the Appendix. From now on, the cost function is assumed to be such that $\mu^{III} \geq \mu^{IV}$, which ensures that there is at least one pure strategy equilibrium of the model for every value of $\mu_1 \in [\underline{\mu}, 1]$.

The Type-A equilibrium corresponds to a situation where the second buyer purchases from the seller only if the first period's review is a High review. This type of equilibrium exists if the second buyer needs a strong signal, i.e., a High review, to purchase from the seller. This happens when a priori, the buyer is not too optimistic about the seller's type; that is, the first buyer's belief (μ_1) are low. Recall that a High review is posted with probability q if the transaction in the first period is successful. Hence, the probability q is a crucial parameter in the effort provided in the first period by both types of sellers. An increase in the probability q increases the marginal contribution of the effort on the expected profit by improving the probability of a High review; hence, it positively impacts the seller's effort.

In the Type-B equilibrium, the second buyer engages in the transaction unless a negative review is reported in the first period. The second buyer does not need a signal that is as strong as in the previous type of equilibrium, which a higher initial belief should support. Indeed this type of equilibrium exists for higher levels of ex ante belief. Note that, in the Type-B equilibrium, the second buyer purchases from a seller who had a successful transaction in the first period regardless of the probability of the review being posted, since the review of such a transaction is either reported with a High review or not reported at all. Hence, the probability q does not play a role in the effort decision of the seller. However, the probability of a Low review being posted, r has a positive impact on the effort decision of the seller since it increases the likelihood that a failure is reported (in which case a transaction does not take place) and the marginal contribution of the effort on the expected profit of the seller.

In both of the previous separating equilibria, the seller's type has a significant role in his effort decision. The seller's type can be interpreted as the efficiency of the seller, for instance, the technology or skills available to the seller, which makes the marginal return of the effort different for the two types of the seller.

The third type of equilibrium (Type-C) exists when the buyer's initial belief is so high that reviews regarding the first-period transaction do not affect the second-period transaction. Both types of seller provide the minimum level of effort in the first period since the transaction in the second period is guaranteed.

Notice that the situation where the second buyer never purchases is not a PBE due to the assumption (A1) made earlier. It rules out the equilibrium where the buyer's initial belief is so low that even a High review is not sufficient for the updated belief of the second buyer to result in a transaction.

Given the intervals in Proposition 1.1, multiple equilibria may exist for some levels of initial belief, μ_1. Type-A and Type-B equilibria coexist for $\mu_1 \in [\mu^{II}, \mu^{I}]$. On the other hand, Type-B and Type-C can be sustained simultaneously for $\mu_1 \in [\mu^{IV}, \mu^{III}]$, provided that $\mu^{III} \geq \mu^{IV}$. Furthermore, $\mu^{I} > \mu^{IV}$ holds for levels of the probability q close to 1, which enables three equilibria to exist simultaneously for certain levels of μ_1.

The possibility of multiple equilibria generates ambiguity concerning the outcomes of the model. For the comparative statics covered in the following section, the definiteness of the equilibrium is favored. Therefore, a selection criterion is needed to select between the possible equilibria to ensure a unique equilibrium for every value of $\mu_1 \in [\underline{\mu}, 1]$. The following selection criterion serves this purpose:

Definition 1.1. *A PBE survives the selection criterion of High Effort PBE (HE PBE) if there does not exist any other PBE where the seller exerts higher effort. If* *the efforts are equal in the PBE compared, then the PBE with a higher probability of second-period transaction survives the selection criterion.*

The comparison of the seller's effort at PBE of the model depends on the probabilities q and r. In addition, the intervals where Type-A and Type-B equilibria exist are also given by functions of the probabilities q and r (in particular μ^{I} and μ^{II}). Taking these into account, Corollary 1.1 presents the HE PBE:

Corollary 1.1.

- *If $q \leq r$, then the HE PBE is*

$$\begin{array}{lll} \textit{Type-B} & \textit{for} & \underline{\mu} \leq \mu_1 \leq \mu^{III} \quad \textit{and} \\ \textit{Type-C} & \textit{for} & \mu^{III} < \mu_1 \leq 1. \end{array}$$

- *If $q > r$, then the HE PBE is*

$$\begin{array}{lll} \textit{Type-A} & \textit{for} & \underline{\mu} \leq \mu_1 \leq \mu^{I}, \\ \textit{Type-B} & \textit{for} & \mu^{I} < \mu_1 \leq \mu^{III} \quad \textit{if} \quad \mu^{I} < \mu^{III}, \\ \textit{Type-C} & \textit{for} & \max\{\mu^{I}, \mu^{III}\} < \mu_1 \leq 1. \end{array}$$

The refinement that I use to select among PBE looks for the PBE that leads to higher efforts. How about the efficiency of the PBE? The following definition states when a particular PBE is more efficient than another PBE:

Definition 1.2. *A PBE is said to be More Efficient than another PBE if the level of efforts provided by both types of the seller in the first PBE is closer to the first-best efficient level of effort than the effort levels provided in the second PBE. Moreover, if both types of seller's effort levels are equal in the two PBE compared, then the PBE where the second-period transaction is more likely is said to be More Efficient.*

At all three types of PBE of the model, the seller effort is lower than the first-best efficient effort. Therefore, the PBE with the higher seller effort is more efficient in terms of Definition 1.2. By the first part of the definition, the Type-A and the Type-B equilibria are more efficient than the Type-C equilibria, unless $e^g_A, e^g_B = \underline{e}$. Moreover, when Type-A and Type-B equilibrium coexist, Type-A equilibrium is more efficient than Type-B equilibrium for $q > r$ and vice versa.

If $q = r$, the effort levels provided in both types of separating equilibria are equal. Then, Type-B equilibrium, which supports the transaction in the second period in case of no review is posted in addition to the case of a high review, is more efficient than Type-A equilibrium by the second part of Definition 1.2.

Proposition 1.2 shows the comparison of the PBE of Proposition 1.1 with respect to the definition of *More Efficient* PBE:

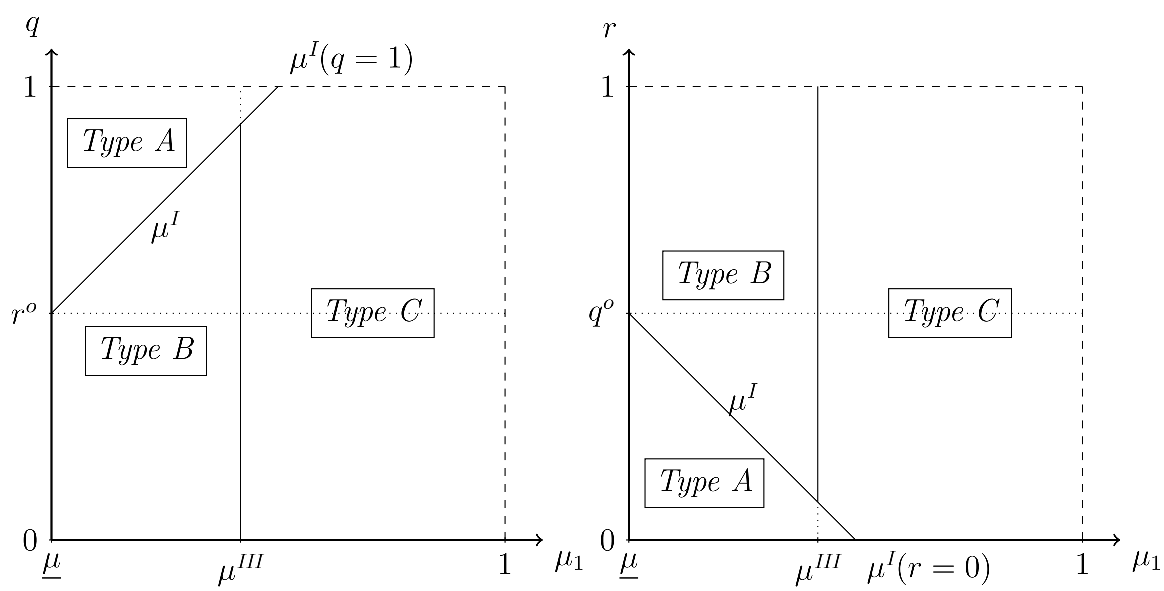
Proposition 1.2. *The ranking of the PBE in terms of efficiency is the following:*

(i) If $q \leq r$, then Type-B $>$ Type-A $>$ Type-C,

(ii) if $q > r$, then Type-A $>$ Type-B $>$ Type-C.

It follows by Corollary 1.1 and Proposition 1.1 that the *more efficient* PBE is selected by the selection criteria of HE PBE when multiple equilibria exist.

Figures 1.2a and 1.2b show the HE PBE for the domain of μ_1 with varying values of q and r, respectively.



(a) r^o is fixed (b) q^o is fixed

Figure 1.2: Type of High Effort Equilibria for Varying Values of μ_1, q and r.

As Figure 1.2 shows if the initial beliefs are not too high, particularly $\mu_1 < \mu^{III}$, then either Type-A equilibrium, where the second buyer only buys if there is a High review, or Type-B equilibrium, where the second buyer buys if there is a High review or there is No review, is the HE PBE. A higher initial belief in this region results in the Type-B equilibrium being more likely than the Type-A equilibrium. This is intuitive since the second buyer does not need a strong signal to purchase in the second period when the initial belief is higher. Moreover, if r^o is fixed and the probability of a successful transaction being reviewed, q, increases, the HE PBE switches to the Type-A equilibrium. This outcome can be explained by the second buyer's interpretation of the No review case: For values of q sufficiently high, a No review case is more likely to come from a failed transaction than from a successful one. Therefore, in the HE PBE, the second buyer engages in the transaction only if a High review is posted. Note that this effect plays a similar role if q^o is fixed and r decreases.

If a successful transaction is almost always reported (q approaches 1), then a No review is interpreted as an unsuccessful transaction. Similarly, if the probability of a failed transaction receiving a review approaches 0, then the No-review case strongly signals an unsuccessful transaction. In these extreme cases, the Type-B equilibrium can no longer exist, whereas the Type-A and Type-C equilibria still exist for some μ_1. Hence, for the interval $(\mu^{III}, \mu^{I}(q = 1)]$ the possible HE PBE are Type-A and Type-C equilibria. Note that a value of q close to 1 is required to sustain the Type-A equilibrium in this case.

Finally, high values of μ_1, such that $\mu_1 > \mu^{I}(q = 1)$, always result in the Type-C equilibrium. Type-C equilibrium exists when the buyer has a firm initial belief that the seller is g-type such that the second buyer always engages in the transaction regardless of the observed review and the probabilities of receiving a review.

1.3.3 Comparative Statics

In this section, I analyze the effect of changes in the probabilities of successful and failed transactions being reviewed, that is, changes in q and r, on the equilibrium effort level provided by the seller in the first period. The selection criteria of the HE PBE ensures a unique PBE for every value of μ_1, q, and r. Corollary 1.1 is used for the analysis in this entire subsection of Comparative Statics.

The Seller's Effort

First of all, I present the effect of the probabilities q and r on the effort level provided by the seller in each type of equilibrium in an isolated manner, regardless of whether the corresponding equilibrium is sustained as the HE PBE or not. After that, using these observations, together with the results in Corollary 1.1, I perform a general analysis.

The optimal decision of each type of seller on the effort provision is given in Proposition 1.1. In order to clearly identify the effects, I use the cost function: $c(e) = \frac{1}{2}e^2$ in this section. Hence the corresponding effort levels in the separating equilibria are:

$$\begin{aligned} e_A^g &= qg(p - c(\underline{e})) \quad \text{and} \quad e_A^b = qb(p - c(\underline{e})) \quad \text{in Type-A equilibrium,} \\ e_B^g &= rg(p - c(\underline{e})) \quad \text{and} \quad e_B^b = rb(p - c(\underline{e})) \quad \text{in Type-B equilibrium.} \end{aligned}$$

The effort levels provided by each type of seller in the first period in the Type-A equilibrium are increasing linearly with the probability q, whereas they are not affected by the probability r. Figure 1.3 shows the seller effort in Type-A equilibrium when it exists, that is, $q > q^A$ when r^o is fixed and $q > r^A$ when q^o is fixed (Notice that $q^A = r^o$ and $r^A = q^o$ when $\mu_1 = \underline{\mu}$).

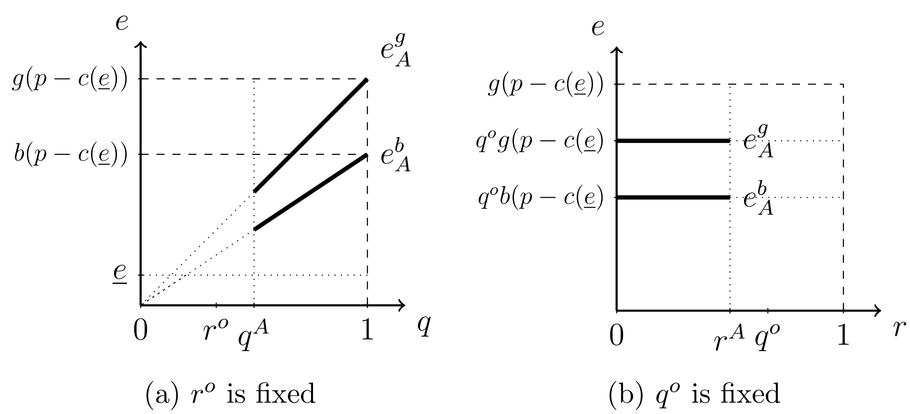


Figure 1.3: The Effort Levels in Type-A Equilibrium with Varying Values of q and r

The effort levels provided by each type of seller in the first period in the Type-B equilibrium are functions of the probability r, while they are not affected by probability q as shown in Figure 1.4.

Finally, the effort provided by each type of seller in Type-C equilibrium is equal to the minimum effort level regardless of the probabilities q and r.

Given the reaction of the seller's effort to the variations in the probabilities q and r in each type of equilibrium and Corollary 1.1, we can have a general comparative analysis. We start by fixing the value of μ_1 in a specific interval. The HE PBE for a given μ_1 is shown in Figure 1.2 as a function of q and r. The comparative statics

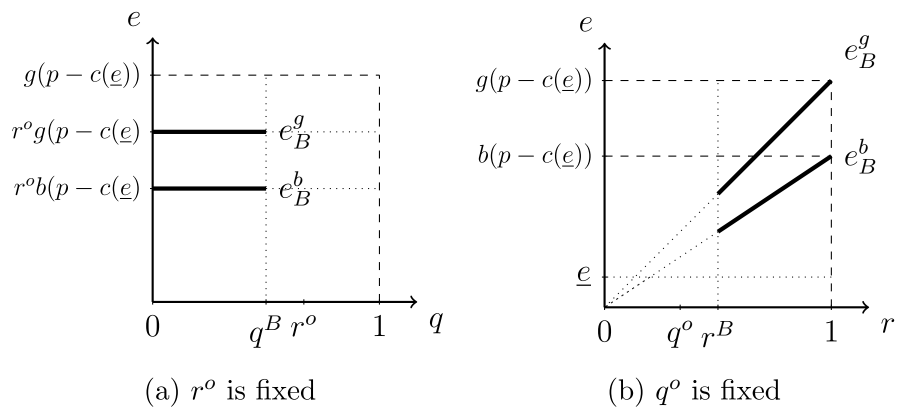


Figure 1.4: The Effort Levels in Type-B Equilibrium with Varying Values of q and r

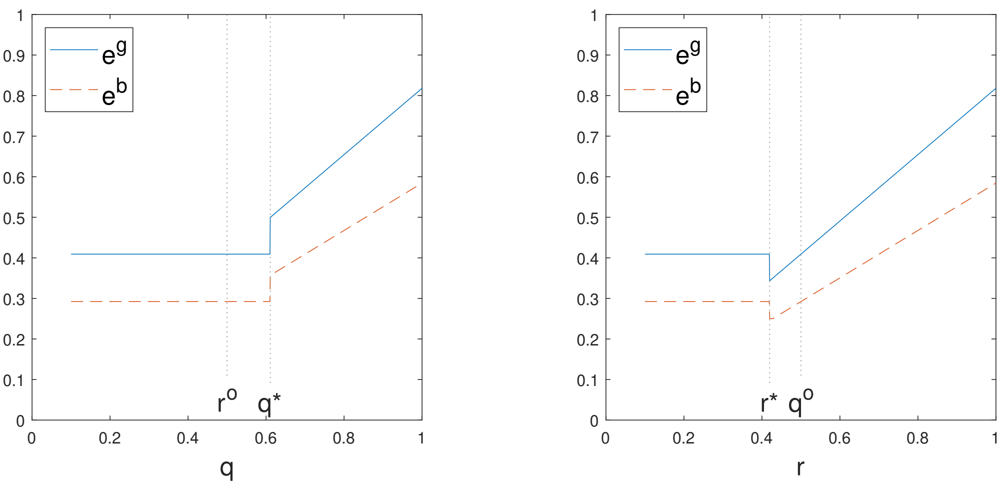
exercise is performed by changing q while r is fixed and then by changing r while q is fixed. In addition to the seller effort in a given equilibrium, changes in q and/or r also affect the boundaries of the intervals to be inspected.

The first interval of μ_1 to be inspected for the level of effort is $[\underline{\mu}, \mu^{III})$. Notice that the HE PBE for $q = r$ is the Type-B equilibrium for this interval. Let us denote $q^* \equiv q^*(r^o, \mu_1)$ the level of q where the HE PBE changes from Type-B to Type-A equilibria. It coincides with the lower bound where Type-A equilibrium exists ($q^* = q^A$). When we fix r^o and increase q, the Type-B equilibrium is the HE PBE until the threshold value q^*, after which the HE PBE is the Type-A equilibrium. Hence, as q is increased, the level of effort remains constant until $q = q^*$ and increases for $q > q^*$. Note that, the change of the HE PBE at the threshold q^* generates a discontinuity in the effort levels. Since $q^* > r^o$, e_A is higher than e_B at q^*, therefore when q is slightly increased the optimal decision of the seller performs an upwards jump. Furthermore, the threshold value q^* is positively affected by the increase in μ_1 in this interval. When $\mu_1 = \underline{\mu}$, the threshold value is $q^* = r^o$, so that there is no discontinuity in the effort. The analysis is illustrated in Figure 1.5a.[6] Starting from Figure 1.5, I use numerical exercises to illustrate the analysis.

The symmetric approach of fixing q^o and changing r has a similar effect on the effort level. Let us denote $r^* \equiv r^*(q^o, \mu_1)$ the threshold value of r where the HE PBE

[6]The discontinuity occurs because Type-A equilibrium does not exist for values of $q < q^*$, not because of the applied selection criteria. Selecting the equilibrium with higher effort guarantees the discontinuity to remain at the minimum possible level by selecting Type-A equilibrium for all values $q \geq q^*$. The threshold where the equilibrium changes, q^*, coincides with the lower bound of the interval where Type-A equilibrium exists. Therefore the discontinuity is robust to the applied selection criteria.

changes from Type-B to Type-A equilibrium. The level of effort is not affected by changes in r for $r < r^*$, while it is affected positively for $r > r^*$. Since r^* is different from q^o, there is a discontinuity at r^* where the HE PBE switches. In particular, since $r^* < q^o$, when r is slightly increased at r^* the optimal decision of the seller performs a jump down from $e_B(q^o)$ to $e_A(r)$. Note that the limit of r^* is equal to q^o as μ_1 goes to $\underline{\mu}$ and the discontinuity in the effort disappears at the limit when $\mu_1 = \underline{\mu}$. The analysis is illustrated in Figure 1.5b.[7]



(a) q^o is fixed

(b) r^o is fixed

Figure 1.5: The Effort Levels vs. the Probabilities q and r, for $\mu_1 \in (\underline{\mu}, \mu^{III})$ (Parameters: $q^o = 0.5$, $r^o = 0.5$, $g = 0.7$, $b = 0.5$, $\mu_1 = 0.51$, $e_{min} = 0.25$, $p = 0.15$).

The next interval where the analysis is performed is $\mu_1 \in [\mu^{III}, \mu^I(q = 1)]$ when r^o is fixed. In this interval, the Type-A equilibrium exists if the probability of a successful transaction receiving a review is sufficiently high. That is, the seller needs a strong incentive to provide effort and it is provided when there is less uncertainty that a successful transaction receives a review. On the other hand, the Type-C equilibrium exists regardless of the probabilities q and r. Hence, as given in Corollary 1.1, the Type-A is the HE PBE if q is sufficiently high, and the Type-C equilibrium is the HE PBE otherwise.

The corresponding interval is $[\mu^{III}, \mu^I(r = 0)]$ when q^o is fixed and r is varied. Similarly, the Type-A equilibrium exists if r is sufficiently low, where the seller is incentivized to provide effort proportional to its type to result in a successful trans-

[7]As before, the selection criteria results in the minimum level of discontinuity by selecting Type-A equilibrium when it exists ($r \leq r^*$).

action, the second buyer's belief is confirmed and she buys only after a High review. On the other hand Type-C equilibrium where the second buyer buys regardless of the review observed exists for all values of q and r in this interval and it is the HE PBE when r is not sufficiently low.

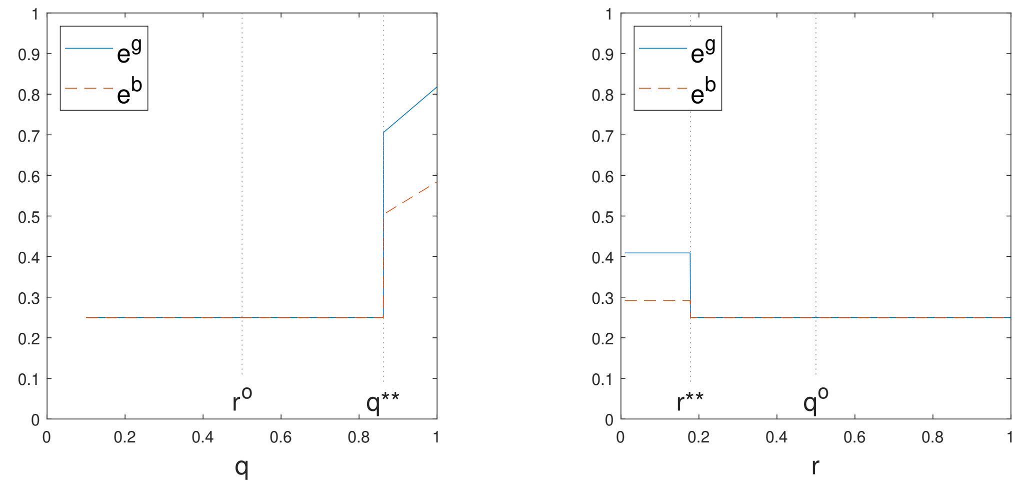
Let us denote $q^{**} \equiv q^{**}(r^o, \mu_1)$ and $r^{**} \equiv r^{**}(q^o, \mu_1)$ the threshold values of q and r, respectively for fixed r^o and q^o, where the HE PBE switches from Type-A to Type-C equilibrium. The threshold value at which the HE equilibrium changes is higher (lower) than the previous case when r^o (q^o) is fixed. In particular, the threshold value q^{**} is closer to 1 than q^* and r^{**} is closer to 0 than r^*.

If r^o is fixed, then the effort is at the minimum level as long as $q < q^{**}$, since the second transaction is guaranteed in the equilibrium regardless of the posted reviews. After the threshold is reached for q, the HE PBE switches to the Type-A equilibrium in which a High review is required for the second transaction to occur, and the level of effort jumps to e_A^g and e_A^b where it is increasing with q. On the other hand, if q^o is fixed, then the HE PBE requires a High review for a transaction in the second period and the corresponding effort levels are e_A^g and e_A^b for sufficiently small values of r such that $r \leq r^{**}$. In this case, the probability of a Low review being posted, r, has no role in the effort provision; therefore, the effort levels are constant with respect to changes in r. Once the probability r is higher than the threshold, the level of effort drops to the minimum level since the second transaction is guaranteed for each type of seller, regardless of the reviews posted. Figure 1.6 depicts the effort levels with respect to changes on q and r in this region.

The previous analysis shows that an increase in the probability of a High review never reduces the level of effort provided by the seller. Therefore, the quality of the goods or services within the platform improves with an increase in the likelihood of a successful transaction being reported.

On the other hand, changes in the probability of a low review being posted have opposite effects depending on the initial belief. In the lower end of the possible values of μ_1, an increase in r may lead to a discrete drop or an increase in the seller's effort.

A significant discontinuity is also observed in the mid-region of μ_1, as a slight increase in r may cause the effort to drop down to the minimum effort level. In particular, the Type-A equilibrium exists and is the HE PBE if r is sufficiently low. As r increases, the Type-A equilibrium fails to exist while Type-C equilibrium, where the second buyer buys regardless of the review observed, exists and is the HE PBE.



(a) q^o is fixed

(b) r^o is fixed

Figure 1.6: The Effort Levels vs. the Probabilities q and r, for $\mu_1 \in [\mu^{III}, \mu^I(q = 1)]$ (Parameters: $q^o = 0.5$, $r^o = 0.5$, $g = 0.7$, $b = 0.5$, $\mu_1 = 0.58$, $e_{min} = 0.25$, $p = 0.15$).

Consumer Surplus

A transaction with the seller is worth 1 to the consumer if it is satisfactory and 0 otherwise. So, the expected utility of the consumer equals to the expectation of a satisfactory transaction with the seller. The first consumer is assumed to purchase from the buyer and the probability of matching with a $g - type$ seller is given by the prior distribution. Hence, her surplus at the three PBE is the following:

$$
\begin{aligned}
CS_1^A &= \mu_1 g e_A^g + (1 - \mu_1) b e_A^b - p \\
CS_1^B &= \mu_1 g e_B^g + (1 - \mu_1) b e_B^b - p \\
CS_1^C &= \mu_1 g \underline{e} + (1 - \mu_1) b \underline{e} - p.
\end{aligned}
$$

The first-period consumer surplus is positively related to the seller's effort. Therefore the first consumer is directly affected by the incentive provided to the seller through the review system despite engaging with the seller prior to the appearance review: The higher the incentive for the seller to build a reputation the higher the consumer surplus in the first period.

The second consumer decides to purchase from the seller after observing the review. If she does, both types of sellers provide the minimum effort since there does not exist another period. Since the $g - type$ seller's productivity is higher, the consumer's utility is higher when she is matched to a $g-type$ seller. Therefore, the second consumer's utility is determined by the probability of matching with a $g-type$ seller. Let $prob(R)$ be the probability that the review R is observed at equilibrium, for $R \in H, N, L$. Then, the second-period consumer surplus at the three PBE is:

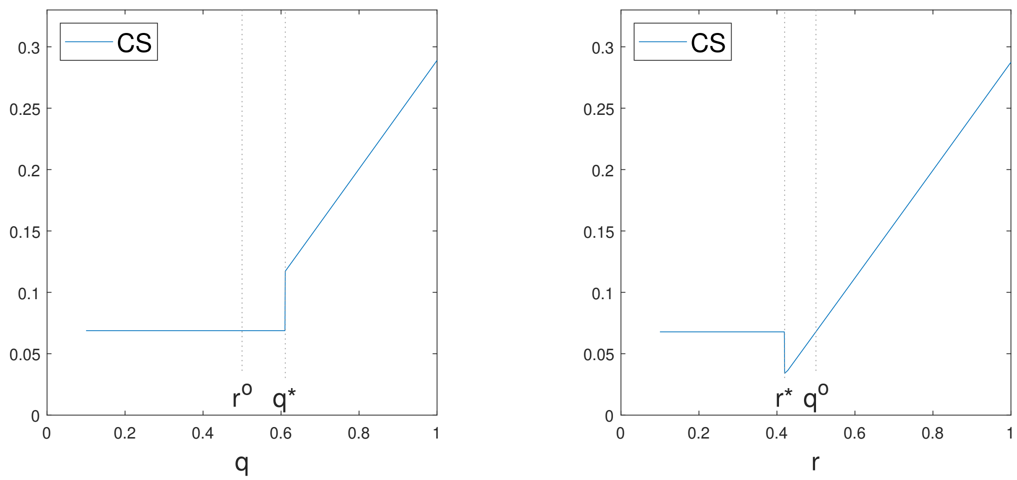
$$
\begin{aligned}
CS_2^A =& prob(H)^A[\mu_2^{H,A} g\underline{e} + (1-\mu_2^{H,A}) b\underline{e} - p] \\
CS_2^B =& prob(H)^B[\mu_2^{H,B} g\underline{e} + (1-\mu_2^{H,B}) b\underline{e} - p] + prob(N)^B[\mu_2^{N,B} g\underline{e} + (1-\mu_2^{N,B}) b\underline{e} - p] \\
CS_2^C =& prob(H)^B[\mu_2^{H,C} g\underline{e} + (1-\mu_2^{H,C}) b\underline{e}] + prob(N)^C[\mu_2^{N,C} g\underline{e} + (1-\mu_2^{N,C}) b\underline{e}] \\
& + prob(L)^C[\mu_2^{L,C} g\underline{e} + (1-\mu_2^{L,C}) b\underline{e}] - p.
\end{aligned}
$$

The second-period consumer surplus is higher when the review system provides more information about the type of seller. That is, the stronger the signal provided by the review system that the seller is $g-type$, the higher the consumer surplus. Note that, due to the complementary nature of the seller's type and effort (single crossing property), the signal becomes stronger with the incentive provided for the seller to exert effort.

Figure 1.7 illustrates the total consumer surplus (that is, the total surplus of the two consumers) with respect to the parameters of the review system. We observe that the consumer surplus is maximized when there is a perfect review system (all transactions are reviewed). In that case, the incentive provided for the seller is maximized, and the quality of the signal provided by the review system is the highest. Consumer surplus is not harmed by an increase in the probability of a successful transaction being reviewed. In contrast, an increase in the probability of a failed transaction being reviewed may lead to a discrete drop in the first-period consumer surplus. This drop in the consumer surplus can be quickly recovered.

The Seller's Profit

We now compute the seller's profit at the three PBE of the model. We start with the Type-C equilibrium, which requires a strong initial belief that the seller is of g-type, involves a combination of minimum effort exertion in the first period and a transaction with certainty in the second period. The profit of the seller in the Type-C equilibrium does not depend on the seller's type and effort, nor on the



(a) q^o is fixed

(b) r^o is fixed

Figure 1.7: Consumer Surplus vs. the Probabilities q and r, for $\mu_1 \in (\underline{\mu}, \mu^{III})$. (Parameters: $q^o = 0.5$, $r^o = 0.5$, $g = 0.7$, $b = 0.5$, $\mu_1 = 0.51$, $e_{min} = 0.25$, $p = 0.15$).

probabilities of the transaction being reviewed:

$$\Pi_C^S = 2(p - c(\underline{e})).$$

On the other hand, the profit of the seller under the Type-A and the Type-B equilibria are given as functions of the seller's type and effort provided in the first period and the probabilities of the transaction being reviewed. The profit of the seller of type θ in the Type-A equilibrium, $\Pi_A^{S(\theta)}$, is increasing in the probability of a successful transaction being reviewed, q, since the increase in expected profit in the second period dominates the increase in the cost due to the higher effort. Since there is a transaction in the second period only if there is a High review posted, the probability of an unsuccessful transaction being reviewed, r, does not appear in the profit function of the seller:

$$\Pi_A^{S(\theta)} = p - c(e_A^\theta) + \theta e_A^\theta q(p - c(\underline{e})).$$

The profit of the seller of type θ in the Type-B equilibrium, $\Pi_B^{S(\theta)}$, depends on the probability of an unsuccessful transaction being reviewed, r, and not on the probability q, simply because a successful transaction in the first period will be rewarded with a transaction in the second period regardless of the review being posted or not. Additionally, the seller will still be rewarded with a transaction in the second period with a probability $1-r$ even if the transaction in the first period is a failure:

$$\Pi_B^{S(\theta)} = p - c(e_B^\theta) + (1 - r + \theta r e_B^\theta)(p - c(\underline{e})).$$

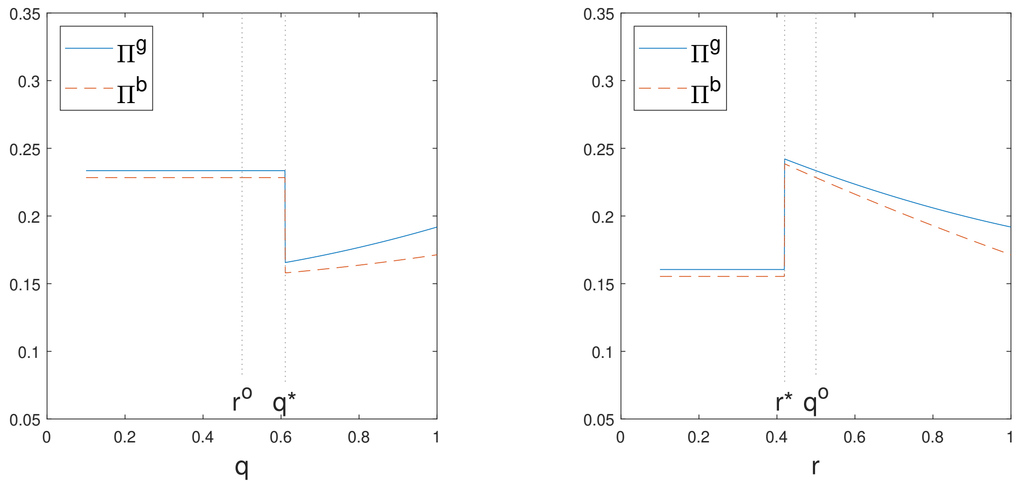
The direct effect of an increase in the probability of an unsuccessful transaction being reviewed (r) on the seller's profit in Type-B equilibrium works through the likelihood of a transaction in the second period and decreases the seller's profit. The indirect effect works through the effort provided by the seller since the effort is an increasing function of r in the Type-B equilibrium, and it can be decomposed into two parts: Firstly, the seller's profit decreases because the effort is costly. Secondly, more effort increases the likelihood of a transaction in the second period and thus increases the seller's profit. Combining the direct and the indirect effects, the profit of the seller in Type-B equilibrium decreases with the probability of an unsuccessful transaction being reviewed, r.[8]

Figure 1.8 combines the notion previously provided on the HE PBE and the seller profit in the equilibria. Recall that for a given initial belief that is not too high, particularly $\mu_1 < \mu^{III}$, either Type-A or Type-B equilibrium is the HE PBE. The dashed lines depict the seller's profit as a function of the probabilities of a successful and unsuccessful transaction being reviewed, respectively in Figure 1.8a and Figure 1.8b. The bold lines illustrate the seller profit considering the HE PBE for the given parameters.

Both graphs in Figure 1.8 show that the profit of the seller under the Type-B equilibrium is always higher than under the Type-A equilibrium. However, the Type-A equilibrium is the HE PBE when $q > q^*$. Figure 1.8a shows that the seller may suffer from a discrete profit's drop as q increases. It enjoys an increase in profits as q further increases; nevertheless, it does not recover the level of profit that would have been obtained with values of q closer to the fixed r^o. On the other hand, Figure 1.8b shows that decreasing r will increase the profit of the seller as long as the HE PBE is the Type-B equilibrium. However, a further decrease in r below r^* (further away from fixed q^o) results in a discrete drop in the seller's profit, which cannot be recovered unless it is reverted.

There exists a small interval for the initial belief of the first buyer in which either the Type-C or the Type-A equilibrium is the HE PBE ($\mu_1 \in [\mu^{III}, \mu^I(q=1)]$). In

[8] The cost function $c(e) = \frac{1}{2}e^2$, which is used in the comparative analysis, always results in a negative effect. However, a cost function with a rapidly increasing marginal cost might result in a profit increasing in r, if r is sufficiently high.



(a) q^o is fixed

(b) r^o is fixed

Figure 1.8: Seller Profit at the HE PBE ($\mu_1 < \mu^{III}$).
(Parameters: $q^o = 0.5$, $r^o = 0.5$, $g = 0.7$, $b = 0.5$, $\mu_1 = 0.51$, $e_{min} = 0.25$, $p = 0.15$).

this interval, the seller's profit can be illustrated with a graph that is very similar to the graphs in Figure 1.8 with two exceptions: First, the threshold value for q that separates the Type-A and the Type-C equilibrium is higher, $q^{**} > q^*$, (and $r^{**} < r^*$). Second, the discrete jump in the seller's profit at the threshold value q^{**} is greater than at q^* since the seller's profit in Type-C equilibrium is greater than in Type-B equilibrium.

1.4 Incentives to Manipulate the Review System

The parameters of the review system in our model are exogenous and taken as given by the participants. However, numerous studies report the existence of manipulation of online reviews by vendors, producers, or monitoring third parties with a material interest in sales of the products/services.[9] In this section, we examine the potential incentives of the seller and the platform to manipulate the parameters of the review system.

The manipulation of the parameters of our model may be carried out with two strategic actions. The first one is to push up the probability of a successful transaction receiving a review, referred to as the 'promotional reviews'. The promotional reviews are mostly regarded as *fake* positive reviews in the literature (Mazylin, Dover

[9]See Mayzlin (2006), Dellarocas (2006), Hu, Liu and Sambamurthy (2011).

and Chevalier, 2012). Here, rather than posting a positive review regarding a fictitious transaction, it stands for increasing the probability of a successful transaction receiving a review above its natural level. The second strategic action is to decrease the probability of an unsuccessful transaction receiving a review below its natural level, known as ‘censorship of negative reviews’.[10]

In our model, the seller has somewhat limited power to influence the parameters compared to the platform. The platform manages the review system unless there is a third party to manage user reviews. In particular, the platform reserves the right to refuse to post any review submission for any reason. We investigate if the platform has an incentive to abuse its power.

The revenue stream options for platforms can be summarized in the following four models (or any combination of them): the commission model, the subscription model, the advertising model, and the service sales model (Schlie, Rheinboldt and Waesche, 2011). A recent statistical study by Tausher and Laudien (2018) reports that 72% of their sample set of 95 randomly picked platforms generate revenues from commission fees, another 22% collect subscription fees, 3% listing/bidding fees, and 2% advertising fees. We will consider that the platform in our model is generating revenues from either commission fees or advertising fees. The decision of whom to charge in the platform will not have indirect network effects in the current model since the seller and the buyers are fixed.

Consider a platform that charges a fixed commission fee of k per transaction (it can be considered a percentage if it is replaced by pk since the price is fixed). Let the seller that is active on the platform be of type θ with probability μ^θ. The profit of the platform based on the commission fees with its cost normalized to zero, Π^P, is given as follows under the three PBE:

$$\Pi_A^P = k(1 + \sum_{\theta=\{g,b\}} \mu^\theta \theta e^\theta q),$$

$$\Pi_B^P = k(1 + \sum_{\theta=\{g,b\}} \mu^\theta[\theta e^\theta + (1-\theta e^\theta)(1-r)]) \quad \text{and}$$

$$\Pi_C^P = 2k.$$

[10] We keep the truthfulness assumption of the model in the sense that a successful transaction does not receive a Low review and an unsuccessful transaction does not receive a High review.

The second components in the parenthesis of the profit function of the platform in Type-A and Type-B equilibria are simply the probability of a transaction being realized in the second period. The platform and the seller share almost perfectly aligned interests except for the cost of effort since the platform obtains revenue in the second period as long as the seller realizes a transaction. Therefore, we can apply the analysis of Figure 1.8 for the analysis of the platform's profit.

Figure 1.8a shows that when the probability of a successful transaction receiving a review is not very high compared to the probability of an unsuccessful transaction receiving a review, $q < q^*$, where $r^o < q^*$, the second buyer agrees to buy in case of no review in addition to the case of High review (Type-B). Hence, the profit of the platform is not affected by a change in q. On the other hand, if $q > q^*$, the buyer requires a High review to purchase (Type-A), and the platform can increase its profit by further pushing up q while r^o is fixed. The latter case provides the incentive for manipulating the review system with promotional reviews.

In Figure 1.8b we observe that if the HE PBE is the Type-A equilibrium, that is, $r < r^*$, where $r^* < q^o$, then the profit of the platform is not affected by a decrease in the probability of an unsuccessful transaction receiving a review, r. Moreover, the profits may exhibit a discrete jump upwards if r is increased towards q^o. On the other hand, if the HE PBE is the Type-B equilibrium, that is, $r > r^*$, then a decrease in r increases the profit of the platform as long as it does not fall below r^* and changes the HE PBE. Hence, the platform can increase its profit by censoring negative reviews.

Consider now that the platform is financed only by advertisements. In order to maximize the profit, the platform must host as many visitors (clicks) as possible by creating valuable information for them and eventually attracting advertisers with heavy traffic on the platform. In this case, the platform would prefer as many reviews as possible on the platform regardless of the suggestion. This situation corresponds to high probabilities for both a successful and an unsuccessful transaction receiving a review in our model. In the case of high values of q and r that are close to each other, the Type-B equilibrium is the HE PBE. In this scenario, the interest of the seller and the platform regarding the parameters of the review system conflict. In particular, the seller would benefit from censorship of negative reviews, while the platform would suffer from the loss of revenue due to limited information shared in its review system. Therefore, the platform has no incentive to censor negative reviews if its revenue is based on advertisements.

1.5 The Model with Endogenous Price

The platform's fixed price is a reasonable assumption in markets where enough sellers offer a similar good or service. However, in a market with few sellers, like for unique goods or services, it is more natural that the price is endogenous and that the seller has market power. In many platforms, the price decision is delegated to the seller without restrictions, occasionally with recommendations. In this section, we provide the results of the imperfect review system in a setting in which the seller sets the price.

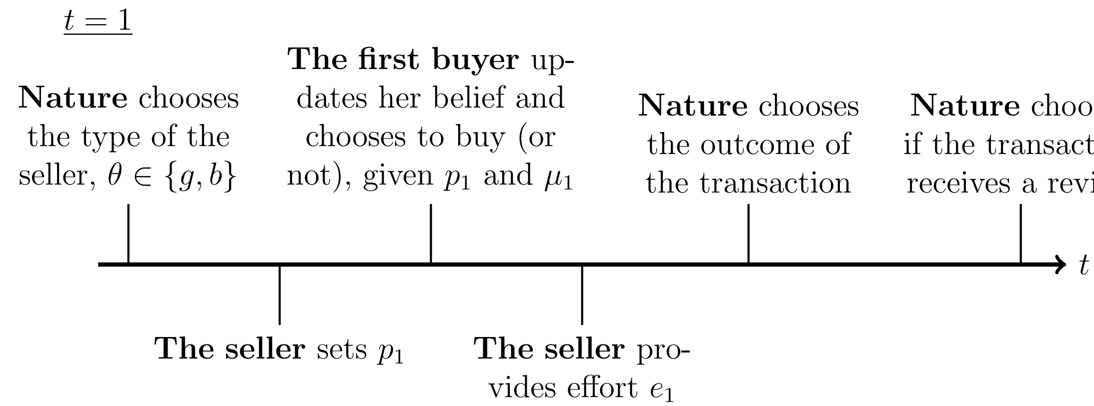
I keep the model of the previous sections except that now the price is endogenous. There is only one active seller on the platform; hence there is no competition effect on the price. Furthermore, the seller possesses private information on his productivity (type) and hence his effort level (action), whereas the initial belief of the buyers on the type of the seller is common information. The timeline of the model with endogenous price is shown in Figure 1.9.

We assume that there is no price commitment through time; that is, the price at each period is determined at the beginning of the period. Let $p_t \in [0, 1]$ be the price set by the seller for the service/good at period t. At any period, the highest price at which the buyer agrees to buy is given by a function of the belief of the buyer about the seller's type and the anticipated effort provision from the seller. The buyer engages in a transaction with the seller if

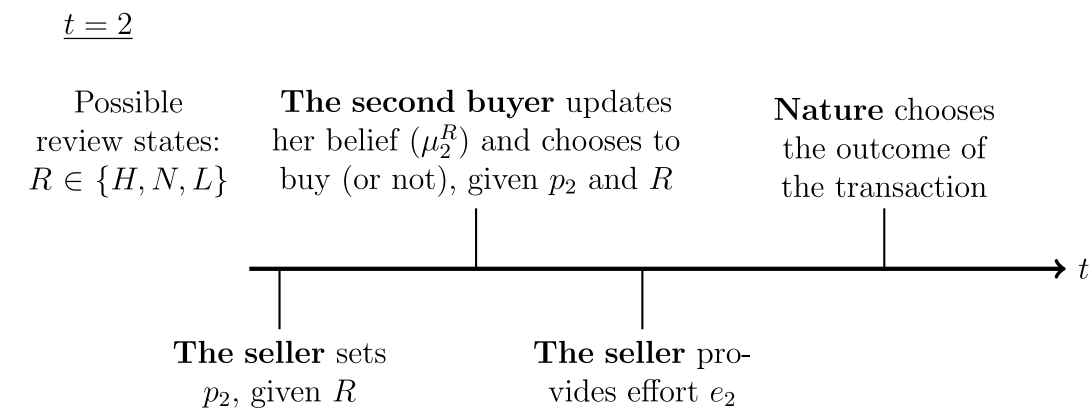
$$p_t \leq \mu_t g e_t^g + (1 - \mu_t) b e_t^b, \tag{1.4}$$

where μ_t is the buyer's belief at period t, after observing p_t (and the review of the previous transaction if $t = 2$).

The buyer has two sources of information when she updates her initial belief. The first source is the price set by the seller, which potentially can signal the type of the seller. The second source is only available in the second period when the review of the first-period transaction is available. If both types of seller set the same price in the first period, then the price is not informative about the seller's type, and the buyer cannot update her initial belief in the first period (pooling equilibrium). In this case, the review system is informative about the type of the seller and the buyer in the second period updates her belief after observing the review. Therefore, we draw our attention to the pooling equilibria in which the review system contributes to the mitigation of asymmetric information, and it is purposeful to study comparative



(a) Timeline of the first period



(b) Timeline of the second period

Figure 1.9: Timeline of the model with endogenous price

statics.[11]

Pooling Equilibria

In an equilibrium where both types of seller set the same price in the first period, the price is not an informative signal about the seller's type. Hence the buyer's belief remains unchanged upon observing the price in the first period. Let p_1, μ_0, and μ_1 be the price in the first period, the initial belief of the buyer, and the belief

[11]We note that a separating equilibrium in prices can not be sustained in the current model due to the assumption that both types of seller have the same marginal cost of effort. Therefore, the b-type seller can always imitate the g-type seller. In a separating equilibrium, the price set by the seller would perfectly signal his type, and the review system would have no role in revealing information about the type of the seller.

at $t = 1$ after observing the price, respectively. Then,

$$\mu_1(.|p_1) = \mu_0.$$

The second buyer observes the price set by the seller in the second period together with the review regarding the first-period transaction, $R \in \{H, N, L\}$. Let p_2 be the price in the second period, and μ_2^R be the second buyer's updated belief. Since the sellers pool their decision at every review outcome, the buyer updates her belief according to equations (1.1)-(1.3):

$$\mu_2(.|R, p_2) = \mu_2^R, \quad R \in \{H, N, L\}.$$

Proposition 1.3 describes a simple pooling equilibrium in which the price is equal to the buyer's valuation.[12]

Proposition 1.3. *The following strategies of the seller and the buyers constitute a perfect Bayesian pooling equilibrium:*

(i) The seller sets price p_1 and p_2 independent of his type, where

$$p_1 = \bar{p}_1 \equiv \mu_1 g e_1^g + (1 - \mu_1) b e_1^b, \tag{1.5}$$
$$p_2 = \bar{p}_2^R \equiv \mu_2^R g \underline{e} + (1 - \mu_2^R) b \underline{e} \quad for \quad R \in \{H, N, L\}, \tag{1.6}$$

and provides effort $e_1 = e_1^\theta$ according to his type while $e_2^\theta = \underline{e}$ is independent of his type, where

$$e_1^\theta = c'^{-1}\Big(q\theta(p_2^H - p_2^N) + r\theta(p_2^N - p_2^L)\Big). \tag{1.7}$$

(ii) The first buyer updates her initial belief as follows and buys from the seller if $p_1 \leq \bar{p}_1$:

$$\mu_1 = \begin{cases} \mu_0 & if \quad p_1 = \bar{p}_1 \\ 0 & otherwise \end{cases}$$

(iii) The second buyer updates her initial belief as follows and buys from the seller

[12] There exist other pooling equilibria where the price is lower than the valuation of the buyer.

if $p_2 \leq \bar{p}_2^R$*:*

$$\mu_2 = \begin{cases} \mu_2^R(e_1^g, e_1^b) & \textit{if} \quad p_2 = \bar{p}_2^R \quad \textit{for} \quad R \in \{H, N, L\} \\ 0 & \textit{otherwise}, \end{cases}$$

where $\mu_2^R(e_1^g, e_1^b)$ *are given in equations (1.1)-(1.3).*

For the analysis of the modified model, we focus on the pooling equilibrium in which the seller sets the price as high as possible, given that the seller agrees to purchase. Hence, for the rest of the analysis, the equilibrium price is equal to the upper bound given in equations (1.5) and (1.6). The price in the first period, $\bar{p}_1$, is based on the initial belief of the buyer, μ_1, and the effort of the seller. By contrast, the price of the second period is conditional on the review. Hence, there are three possible prices in the second period, p_2^H, p_2^L and p_2^N (provided that $p \geq c(\underline{e})$) given in equation (1.6).

Therefore the equilibrium is characterized by the following system of equations: The optimal effort decision of the sellers given by (1.7) and the three price equations conditional on the first-period review given by (1.6). This particular pooling PBE has certain similarities with the PBE of the original model, which makes room for comparison of the results of the two models.

The seller still has an incentive to provide effort to increase the profit margin of the second-period transaction. The marginal effort increases the chances of realizing a second transaction in the fixed price model, whereas it has two effects on the profit in the modified model. Firstly, the higher the effort, the higher the probability of a successful transaction, and hence the higher the likelihood of realizing the second transaction at a high price. Secondly, there is a negative feedback effect of effort. As both types of seller provide effort, the probability of a successful transaction increases for both types of seller, and reviews become less informative signals about the seller's type (the difference between the beliefs after a positive and a negative review decreases). This brings the endogenous price closer to each other after a positive and a negative review and diminishes the return of effort for both types of seller.

The direct effect of the price after No-review, p_2^N, on the optimal effort can be either positive or negative depending on the parameters of the review system. It is negative when it is more likely that no review results from a failed transaction $(q > r)$. In this case, the seller's best response effort decreases with p_2^N.

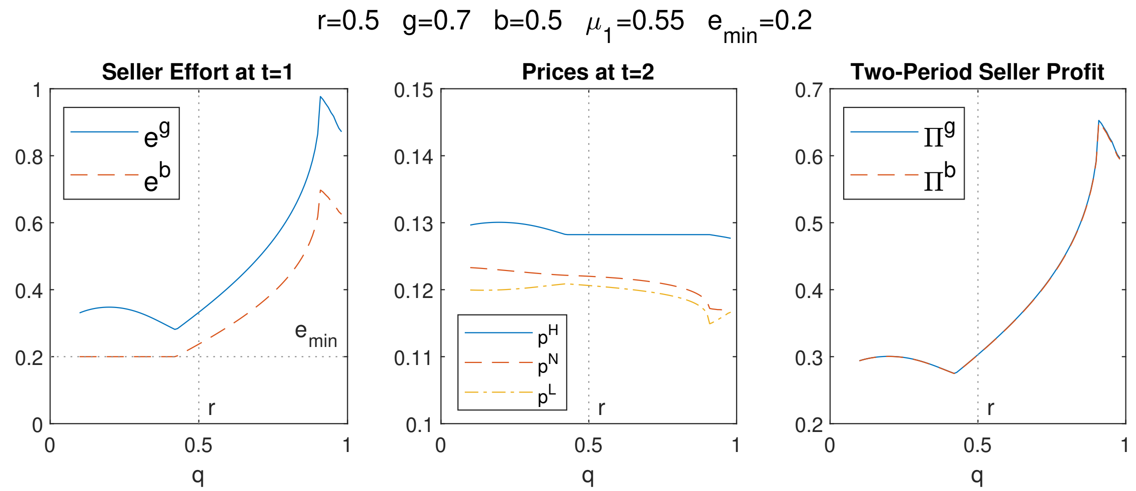


Figure 1.10: Comparative Statics of Equilibrium for Endogenous Price

The endogeneity of the price requires a solution of the system of equations that characterize the equilibrium and makes it analytically intractable. Therefore the results are provided with numerical analysis in this section.

Figure 1.10 provides comparative statics for the equilibrium, with the probability of a successful transaction being reviewed, q, on the horizontal axis, while the probability of a failed transaction being reviewed is kept constant at $r = 0.5$.

Unlike the fixed price model, when the price is endogenous the effects of a change in q on the seller's effort and profit are non-monotonic. We observe that for intermediate values of q, both types of seller are incentivized to provide more effort as q increases. In this region, both types of seller have interior solutions for the effort, and the return of the positive review dominates the negative review and the lack of review. When q is close to 1, the lack of review becomes equivalent to a negative review. The gain from charging p^N instead of p^L becomes insignificant. In addition, the feedback effect of effort through p^L becomes larger. That is, when both types of seller decrease effort, a negative review becomes a weaker signal that the seller is b-type since the success rate of g-type decreases, which increases p^L. Therefore the seller becomes less incentivized to exert effort and upgrade to a better review when q is very high.

For low values of q, the incentive mechanism of the review system is not sufficient for the b-type seller to exert effort, given his productivity. He cannot react optimally to different values of q; therefore, the feedback effect of his effort on the prices is limited.

The plot of profit in Figure 1.10 shows that the profits of the two types of seller are very similar. The increase in the expected second period profit for a g-type seller is almost entirely offset by the additional cost of effort in the first period. Therefore, the b-type seller can leech off of the g-type seller, and although the review system provides an incentive for sellers to exert effort, it does not result in a significant difference between the profits of the two types of seller.

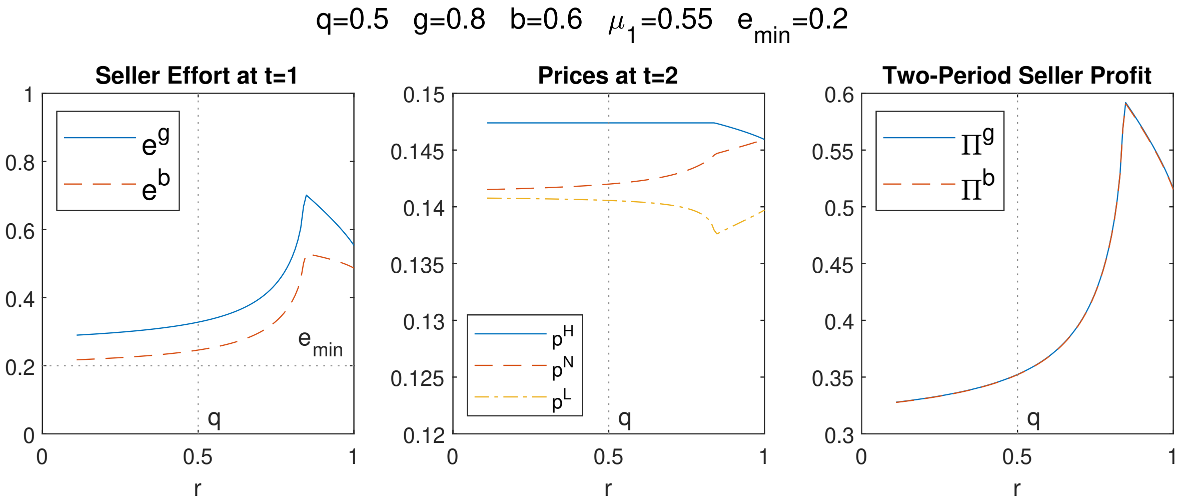


Figure 1.11: Comparative Statics of Equilibrium for Endogenous Price

The plots of Figure 1.11 present the effect of a change in the probability that a failed transaction is reviewed while the rest of the parameters are fixed. We observe that the seller's effort is non-monotonic in the probability, r. Both types of seller are incentivized to provide effort when the probability of a failed transaction receiving a review is low. Moreover, they increase the effort when r increases at the intermediate values. However, when a failed transaction is reported with a very high likelihood, a further increase harms the seller's incentive to provide effort. Hence, more information does not always give more motivation to the seller.

Both comparative statics show that both types of seller may prefer the imperfect review system. That is, the sellers may enjoy higher profits when $q, r < 1$ than when $q, r = 1$.[13] Hence, the seller (and the monopolist platform with revenues generated through commission fee as discussed in Section 4) may have an incentive to influence the parameters of the review system.

[13]This result does not require that the parameters q and r are independent. It is confirmed with the comparative statics when $q = r \in [0, 1]$, that is, the parameters q and r are perfectly correlated.

1.6 Conclusion

In this paper, I model the transactions in a platform with a built-in review system, where the buyer can post a review regarding the completed transaction she engaged in. Successful and unsuccessful transactions have distinct likelihoods to receive a review in the platform's review system. This feature of the model is a new aspect with respect to the literature and enables to perform a comparative analysis of the equilibrium behavior of the participants.

I show that increasing the probability of high reviews being posted never decreases the seller's effort. Sometimes it causes the seller's effort level to perform a discrete jump upwards. On the other hand, an increase in the probability of low reviews being posted has an opposite effect, occasionally causing the seller's effort to perform a discrete drop.

I also show that the seller may have incentives to employ two strategic manipulation actions: Promotional reviews and censorship of negative reviews. Furthermore, when the platform has a revenue stream based on the commission fees, the interests of the seller and the platform are aligned. So, the incentives to manipulate the review system are valid for the platform that has greater control over the review system. On the other hand, if the platform's revenue is based on advertisements, there is no incentive to censor negative reviews.

When the seller sets the price, the seller's effort is non-monotonic in both parameters of the review system. It is not always more productive to increase the information provided through the review system. In this case, sellers may be incentivized to provide more effort and obtain higher profit when there is a decrease in the probability that the transaction is reviewed.

Consumers of the digital marketplaces become aware of the characteristics of the marketplace only after getting familiar with it, and the ability to figure out specific characteristics is not the same for every consumer. The parameters of the review system are publicly available for consumers in our model. However, the parameters may not be observed perfectly and equally by every consumer as well. This is an important aspect to consider for future research.

Finally, the parameters of the review system of a platform can influence agents' participation on the platform. Moreover, in two-sided platforms, the number of agents on one side of the market affects an agent's value of participation in the platform on the other side (indirect network effects). The fixed number of the seller and buyers in the current model does not allow for capturing the effect of parameters of the review system through participation. Further research with endogenous participation of the agents will help understand the consequences of the changes in parameters of the review system in a more comprehensive way by accounting for indirect network effects.

Chapter 2

Product Market Outcomes with Targeted and Random Advertising

Abstract

This paper explores the product market outcomes with random and targeted advertising. With random advertising, the pricing and advertising strategies are independent. In contrast, pricing and targeted advertising strategies are correlated since the level of targeting changes the distribution of valuation of the consumers the firms face. As a result, firms pass part of the targeting cost to the consumers. Moreover, the firms partially segment the market by targeting their customer base. Hence, consumers face higher prices with targeting. Random advertising does not provide enough income for the firm(s) to promote their products and creates entry barriers in small markets. The market profit may be shared unevenly between ex-ante symmetric firms in contrast to targeted advertising. Targeting removes the barrier to entry and enables symmetric incentives to the firms in small markets. Despite higher prices, targeting technology improves the welfare of markets of all sizes thanks to the elimination of unnecessary advertisement.

JEL L840, M37, L13, D4

Keywords: targeted advertising, optimal pricing, segmentation

2.1 Introduction

The best practice of personal data privacy is one of the hottest debates of the decade. The progress of hardware and software technologies made big data collection, storage, and analysis feasible for data companies. The data companies keep investing in these technologies and engaging in aggressive data collection. The interest of giant tech companies strongly signals the large economic value of personal data.

One of the most common methods of capitalizing on personal data is advertising. The big tech companies act as an ad intermediary and target the consumers on behalf of firms according to consumers' revealed interest obtained by interpreting consumer data. Targeted advertising is very appealing for firms since it provides significantly higher returns to advertisers than traditional methods. On the other hand, data privacy advocates argue that the aggressive data collection may leave the consumers at the mercy of these tech companies.

This research explores the economic value of consumer data by modelling targeted and random advertising. We consider informative advertisement, that is, consumers learn about the existence, valuation, and price of the product through the advertisement.[1] Hence, we take a favourable view on advertising because it leads to more informed consumers and potentially induces more competition in the market.

In our model, the advertisement service is provided by an ad intermediary who has access to advertising venues and matches the firms with ad recipients. Under random advertisement, ad messages are sent uniformly to the consumers. This technology is available to the intermediary regardless of whether the consumer data is available. Targeted advertisement is possible when the consumer data is available to the ad intermediary and allows directing ad messages to the consumers who are more likely to be interested in the product. It allows the firms to access high valuation consumers for their product. We do not allow the firms to use consumer data for price discrimination.[2]

[1]We focus on display advertising, and consumers are assumed to have no information source to search for the products. Display advertising is the ad messages shown to consumers who are not searching for a product. It is estimated to constitute 55% of the ads expenditure worldwide in 2021 due to a rapid increase in social media advertising.

[2]Several reasons were influential in this decision. Firstly, price discrimination is not legal in most regulated markets. It is considered illegal to discriminate against consumers based on race, ethnicity, religion, or gender, which are characteristics that may affect product preferences/valuations. Price discrimination is banned in some parts of the world even if it is not

We study a monopoly and a potentially duopoly market with horizontally differentiated products. First we show that in small markets, the advertisement cost acts as a barrier for entry with random advertising. Given that the firms cannot distinguish among consumers, they are forced to send ads to low-valued consumers if they want to reach high-valued clients. Hence, advertisement is an expensive activity. On the other hand, targeting improves the effectiveness of the advertising by allowing firms to reach only the costumers that may be interested in their products. This leads to a decrease in the barrier for entry. Hence, both the firms and the consumers of small markets benefit from targeted advertising.

There is no entry problem in large markets since the firms can generate enough income to cover the costs of the random advertisement. Once they find it profitable to promote their product, firms consider the complete distribution of the consumer valuation. On the other hand, with targeting ability, the firms have the option to advertise only to the subset of consumers with a high valuation. Hence, they consider a trimmed version of the valuation distribution with a higher average. Moreover, if there are several firms, they compete only for a subset of the consumers since consumers have different tastes for the products. Hence, they partially segment the market by targeting their customer base, which provides a weaker incentive to undercut the opponent. Therefore, the price with targeted ads is higher than with random ads under both monopoly and duopoly.

With random advertisement, firms' pricing strategy does not depend on their advertising strategy since the valuation of the demand remains unchanged with random advertisement. In contrast, firms' targeted advertising strategy determines the targeted consumers' valuation. An increase in the marginal cost of targeted advertising leads to fewer advertised consumers, a higher average consumer valuation and a higher price. Hence, the price with targeting depends on the advertising cost. This partial pass-on effect leads to the excess price with targeted advertising.

In intermediate-sized markets, when there are more than one firm, the random advertisement may not provide enough income for two firms to make profits. In such a case, one firm advertises to the whole market with positive profit, while the

based on the above characteristics: "It is illegal if the seller sells the same goods in the same quantities at around the same times to different commercial customers, offering lower prices only to one or some of these customers" (Robinson-Patman Act, 1936).

Secondly, there is little evidence of personalized pricing or price discrimination (de Streel and Jacques, 2019). Last but not least, the value of consumer data is still a relevant question without price discrimination, and it is worth exploring its effects on product markets.

other does not advertise or advertises partially with zero profit. Hence, the ex-ante identical firms may have uneven incentives for promoting their products, resulting in an asymmetric duopoly. Targeting, on the other hand, always provides the firms with incentives proportional to their product's value and results in a symmetric duopoly.

The firms benefit from eliminating unnecessary advertising costs and targeting consumers with high valuations. Therefore, they prefer it to random advertising in all market settings and sizes. On the other hand, consumers in large markets face higher prices when targeted and prefer random advertising. Nevertheless, the market surplus is higher with targeted advertising in these markets. Overall, targeting technology improves welfare in markets of all sizes since profit gains dominate consumer surplus loss.

The value created by informative advertising in a heterogeneous product market is studied by Grossman and Shapiro (1984), showing that advertising expenditure is not wasteful compared to a homogeneous product market. Soberman (2004) shows that informative advertising increases consumers' information and leads to lower prices by increasing competition, although it depends on the level of product differentiation. In contrast to spatial models (a la Salop) used in these papers where consumers' preferences for alternatives are perfectly correlated, I employ independent heterogeneous valuations for alternative products. I show that advertising can be wasteful for heterogeneous product markets, but targeting offers a significant improvement. I also show that there is less informative advertising with targeting than random advertising, and in line with Soberman (2004), it leads to higher prices.

The notion of targeted advertising goes back to Grossman and Shapiro (1984). They consider it an effective way of decreasing advertising costs by directing the advertising messages to the consumers who are nearly positioned to the product's position in the product space. Iyer et al. (2004) model targeted advertising in the following way: A segment of consumers has strong preferences over the seller's product and does not engage in the price comparison. Targeted advertising enables the sellers to target this segment as a whole. They show that targeting resolves the wasteful advertising expenditure and increases the firms' profits without price discrimination. Acquisiti et al. (2021) consider that firms can target consumers through the intermediary advertisement platform and perform a welfare analysis considering consumers, sellers, and the ad intermediary under different consumer information regimes. Their framework has binary valuations for heterogeneous products, and firms compete for ad spots rather than the product market. Similarly, the sellers hire an intermediary ad service here, but it is not a strategic player. In contrast to Acquisiti et al. (2021), consumers have continuous valuations for the products, and sellers engage in price competition and promotion, but ad spots are not exclusive. Our targeting model, which enables firms to inform the consumers with a high valuation for their product, is closest to Grossman and Shapiro (1984).

In the next section, we introduce the model. Section 3 provides the analysis for a monopoly, and Section 4 for the potential duopoly market structures. Section 5 provides discussion for two extensions. Finally, Section 6 concludes. All the proofs are in the Appendix.

2.2 The Model

In this section, we set up a model of advertising with heterogeneous consumer preferences for the product(s). The consumers' information about the product and the market relies solely on the advertisement, as in Butters (1977). There is no other information source available for the consumer other than the advertisement, and the consumers do not actively search for the product. An advertisement conveys a message with the (uniform) price and actual characteristics of the product so that the recipient can infer her net valuation of the product and decide whether to purchase it or not.

There is a continuum of consumers with unit mass. The valuation of any consumer i for the product k is uniformly distributed, $v_{ik} \sim U[0, V]$. The distribution is common information. The consumers have unit demand, and they learn their valuation for the product when its ad is received. The surplus enjoyed by consumer i who purchases product k is given by: $u_{ik} = v_{ik} - p_k$ where p_k is the price of the product k.

The firm's marginal production cost is assumed to be 0 for simplicity. Since the consumers have no prior information about the product, they must be informed with advertising. Knowing the distribution of the consumer valuation for his product, the seller decides on the extent of the advertisement and the price for the product simultaneously, which enables an informative advertisement. The firm cannot price discriminate the consumers with respect to their valuation.

Advertising Technology

The advertisement is provided by a third party who has access to advertisement venues. The ad intermediary receives an ad request from the seller and handles the transmission of the ad message to the consumers. An ad order consists of two components: The fraction of the consumers who will receive the ad messages, $\phi \in [0, 1]$, and the product's information to be advertised, namely price and characteristics. The ad intermediary handles the ad service without strategic objectives[3] and charges the firm a fee that is linear in the fraction of consumers who will receive the ad message, ϕ:

$$A(\phi) = a\phi.$$

To focus on the interesting cases and eliminate those in which the firm never promotes the product, the marginal cost of advertising is assumed to be lower than the maximum valuation of the product:

$$V > a.$$

There are two extreme versions of the advertisement technology. The first one is called *Random Advertisement*, where the ad messages are sent randomly to the consumers without any discrimination. Random advertisement does not require the acquisition of consumer data. Consumer i with valuation v_i who receive the ad buys the product $v_i \geq p$. Taking into account the distribution of v_i, the demand generated by the random advertisement sent to a fraction ϕ of the consumers, given the price of the product p, for any $p \leq V$, is equal to:

$$D(p, \phi) = \phi(\frac{V - p}{V}), \tag{2.1}$$

and shown in Figure 2.1.

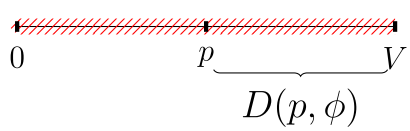


Figure 2.1: Demand generated by ϕ fraction of random advertising, given $p \leq V$.

[3] In Section 2.5.2 we endogenize the advertising cost where we consider the entry strategy of an intermediary with targeting technology to an established (random) advertisement market.

The second advertisement technology is called *Targeted Advertisement*, where the ad messages are sent, prioritizing the consumers with the highest valuation for the product. The targeted advertising is available if the ad intermediary can acquire consumer data such that the valuation of the consumers for the product is predicted given the product characteristics. The prediction technology of the ad intermediary is assumed to be very advanced so that the valuation of the consumers is predicted correctly.

With targeted advertising, the ϕ fraction of the consumers who will receive the ad messages corresponds to a threshold valuation for the product such that all consumers with a higher valuation than the threshold will receive the ad message, while others will not. Let $\underline{v}$ be the valuation of the threshold consumer. Given the uniform distribution of the consumer valuation, $\underline{v}(\phi) = (V - \phi V)$ or $\phi = 1 - \underline{v}/V$. A consumer acquires the product if she receives the ad, that is, $v_i \geq (\phi)$, and his surplus is not negative, that is, $v_i - p \geq 0$. Hence, the demand generated by sending targeted advertising to a fraction ϕ of the consumers, given $p \leq V$, is the following:

$$D(p, \phi) = \begin{cases} \phi & \text{if} \quad \underline{v}(\phi) \geq p \\ \frac{V-p}{V} & \text{if} \quad \underline{v}(\phi) < p. \end{cases} \tag{2.2}$$

Figure 2.2 shows the demand generated by sending targeted advertising to a ϕ fraction of the consumers, when $\underline{v}(\phi) \geq p$:

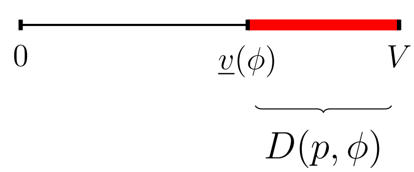


Figure 2.2: Demand generated by ϕ fraction of targeted advertising, given $p \leq \underline{v}(\phi)$.

In the following sections, the market outcomes of the two extreme advertising cases are analyzed separately for two market structures. The ad setting is indicated with a subscript and the market structure is indicated with superscript. For instance, p_R^M and p_T^M stand for monopoly price with random advertising and monopoly price with targeted advertising, respectively.

2.3 Optimal Strategy for a Monopolist

We first consider a market where a monopolist produces and advertises a single product. It decides on the extent of advertisement and the uniform price for the product simultaneously.

2.3.1 Random Ads

We assume here that the monopolist contracts with an intermediary who has access to random advertisement technology. Given the expression for the demand $D(p, \phi)$ (see eq. (2.1)), the monopolist's problem is:

$$\max_{p,\phi} \quad \phi p(1 - \frac{p}{V}) - a\phi \quad \text{s.t.} \quad p \in [0, V], \quad \phi \in [0, 1].$$

The optimal price is independent of the level of advertisement:

$$p_R^M = \frac{V}{2}. \tag{2.3}$$

It is also easy to check that the optimal advertisement is either to promote the product to the whole market or none:[4]

$$\phi_R^M = \begin{cases} 0 & \text{if} \quad \frac{V}{4} < a \\ 1 & \text{if} \quad \frac{V}{4} \geq a. \end{cases} \tag{2.4}$$

The ratio of the product's valuation to the advertisement cost, V/a, gives the product's market size. If the product's valuation is low relative to the advertisement cost, that is, $V < 4a$, then the market is small, and it is not worthwhile for the firm to advertise the product. The monopolist has no incentive to promote the product. In this situation a market with potential welfare gains fails due to costly advertisements to inform consumers.

If $V \geq 4a$, that is, the market is large, then the monopolist obtains enough

[4]We assume that the firm advertises to the whole market if it is indifferent.

income from advertising the product and has an incentive to promote it to the whole market. Since the set of randomly advertised consumers is the whole market, the monopolist sets the same price as in complete information. The profit of the monopolist, the consumer surplus, and the total surplus are the following:

$$\Pi_R^M = \frac{V}{4} - a \tag{2.5}$$

$$CS_R^M = \frac{V}{8} \tag{2.6}$$

$$TS_R^M = \frac{3V}{8} - a. \tag{2.7}$$

2.3.2 Targeted Ads

We now consider that the monopolist contracts with an intermediary who has access to the targeted advertisement technology. Recall that when a fraction ϕ of the consumer is targeted, the consumer with the lowest valuation to be targeted is given by: $\underline{v}(\phi) = (V - \phi V)$. Then, the demand for the product when $p \leq V$ given in (2.2) can be written as the following:

$$D(p, \phi) = \frac{V - \max\{(\phi), p\}}{V}.$$

Optimal Targeting: The optimal targeted advertisement necessarily leads to target all the consumers who are willing to purchase the product given the price. That is, if the firm chooses the price p, then the indifferent consumer must be the consumer with the lowest valuation to be targeted, that is, $\underline{v}(p) = p$. In terms of the fraction of consumers to be advertised, the optimal targeting must satisfy $\phi(p) = (1 - p/V)$.

Any level of targeting different from $\phi(p)$ is sub-optimal for the seller. If the seller targets more consumers given the price, then there is a waste of advertisement expenditure. The targeted consumers who are unwilling to purchase the product given the price do not have any demand for the product, and the seller can cut advertisement costs by not targeting these consumers. On the other hand, if the seller targets fewer consumers, then the demand generated is equal to the targeted consumers. However, given the valuation of targeted consumers, the seller can still generate the same demand if he increases the price up to and generates a higher profit.

Given the optimal targeting for any possible price, the monopolist's problem is the following:

$$\max_{p} \qquad p(1-\frac{p}{V}) - a(1-\frac{p}{V}) \quad \text{s.t.} \quad p \in [0, V].$$

The optimal price and targeting are:

$$p_T^M = \frac{V+a}{2} \tag{2.8}$$

$$\phi_T^M = \frac{V-a}{2V}. \tag{2.9}$$

The profit of the monopolist, the consumer surplus, and the total surplus at equilibrium are the following:

$$\Pi_T^M = \frac{(V-a)^2}{4V} \tag{2.10}$$

$$CS_T^M = \frac{(V-a)^2}{8V} \tag{2.11}$$

$$TS_T^M = \frac{3(V-a)^2}{8V}. \tag{2.12}$$

With targeted advertising, the price depends on the marginal advertising cost. That is, the seller passes part of the ad cost to the consumers through the price. This is due to the optimal targeting rule. The higher the ad cost, the fewer the consumers who are targeted, the higher the average valuation of the advertised consumers, and the higher the seller charges for the product. This contrasts with the situation with random advertising, where the price is independent of the ad cost since the advertising strategy does not affect the informed consumers' valuation. As the marginal return is positive, in that case the seller advertises to as many consumers as possible.

2.3.3 The Comparison of the Market Outcomes

This section compares the market outcomes when the monopolist has access to random versus targeted advertisement through the intermediary.

Proposition 2.1 (whose proof is immediate) provides the comparison of market outcomes with random and targeted advertising.

Proposition 2.1. *The relation between the outcomes of the market when the monopolist produces and advertises a single product with random advertisement (R) and targeted advertising (T) is the following:*

i If $V < 4a$*,*

$$\begin{aligned} \Pi_T^M &> \Pi_R^M = 0 \\ CS_T^M &> CS_R^M = 0 \\ TS_T^M &> TS_R^M = 0. \end{aligned}$$

ii If $V \geq 4a$*,*

$$\begin{aligned} p_T^M &> p_R^M \\ \Pi_T^M &> \Pi_R^M \\ CS_T^M &< CS_R^M \\ TS_T^M &> TS_R^M. \end{aligned}$$

Random advertisement does not provide the firm with an incentive to promote the product in small markets ($V < 4a$), and the market fails. Traditional advertising cost is similar to a fixed cost for entry and may create a barrier for entry in small markets. In contrast, targeted advertising does not require large markets and removes the barrier for entry in small markets. Therefore, the targeting technology improves welfare by creating a market and benefiting both the firms and the consumers of the small markets.

In large markets ($V \geq 4a$), random advertising allows the firm to promote its product to the whole market. In these markets, the price with targeted advertising is higher than with random advertising. By targeting high valuation consumers, the firm can trim the lower end of the distribution of the consumer valuation and generate demand with a higher average value than the whole market.

The targeted advertising increases the firm's profit regardless of the market size. In large markets where random advertising provides a benchmark, targeted advertising improves the firm's profit. The firm's ad expenditure has a higher return with targeted advertising since it is not wasted on the consumers who are not interested in the product. Second, the firm sets a higher price for the generated demand with a higher average valuation. Despite the decrease in revenue, the lower advertisement cost dominates the firm's balance sheet. Therefore, the monopolist prefers targeting to random advertising.

Consumers benefit from targeted advertising in small markets since it removes the fixed cost of entry. However, they face a higher price in large markets with targeted advertising. The monopolist has an incentive to lower the price under random advertisement because once the (sunk) advertisement cost is paid, consumers of every valuation (not only those of high valuation) are potential buyers. Therefore, in established markets, consumers prefer random advertising where they are not targeted because of their valuation of the product (even with a uniform market price).

Targeting technology creates value and improves total surplus compared to traditional advertising in small and large (established) markets. In small markets, eliminating fixed entry costs due to advertising benefits both the firm and the consumers. In large markets, the increase in profit dominates the loss in consumer surplus with the introduction of targeted advertising.

Let us mention that the incentive provided to the firm for entry with targeting technology extends to the promotion of additional differentiated products to the market. That is, in addition to favouring entry, targeting technology also favours product variety. A multi-product-monopolist is provided with an incentive to promote multiple products to the market with targeted advertising regardless of the size of the market. In comparison, random advertisement requires larger markets to provide an incentive for promoting multiple products. See Appendix B for the analysis of the market with a multi-product-monopolist.[5]

2.4 Equilibrium Strategies for a Duopoly

In this section we study the competition between two firms. We assume that there are two firms in the market, $j \in \{A, B\}$, and each seller produces a single, horizontally differentiated product. The unit mass of consumers' valuations for the products are given by $v_i = (v_A, v_B)$ and have a joint uniform distribution on $[0, v] \times [0, v]$ with

[5] Appendix B provides the market outcomes where a monopolist produces and advertises two horizontally differentiated products.

density $\frac{1}{V^2}$. The firms have to inform the consumers by hiring advertisement services. Given the advertisement decision of the firms, the consumers can be shown zero, one, or two advertisements.

The consumers who observe one advertisement purchase the product if their net utility is positive. Those who observe two advertisements purchase the product that provides a higher net utility, given it is positive.[6]

2.4.1 Random Ads

We consider now that the firms can hire random advertisement. They decide on the price of their product and the frequency of the advertisement simultaneously.

We first calculate the demand for the two products. Let ϕ_j and p_j be the fraction of consumers who receive the advertisement and the price of product $j = A, B$, respectively. Without loss of generality, assume $p_A \geq p_B$.

Figure 2.3 shows the demand for good A. It can be divided into three parts: The first part (1) represents the consumers with $v_A \geq p_A$ and $v_B < p_B$. They buy good A if they receive its ad, but they would not buy good B at the price p_B. The second part (2) represents the consumers who prefer good A over good B if they receive the ad for good A ($v_A - p_A \geq v_B - p_B \geq 0$), regardless of receiving an ad for good B. The third part (3) represents the consumers who buy good A if they receive the advertisement for good A but not for good B.

Hence the demand of product A can be written as the sum of three parts:

$$\begin{aligned} D_A(p_A, \phi_A; p_B, \phi_B) = \frac{\phi_A}{V^2} \Big[& \underbrace{(V - p_A)p_B}_{(1)} + \underbrace{\frac{1}{2}(V - p_A)(V - p_A)}_{(2)} \\ & + (1 - \phi_B) \underbrace{[\frac{1}{2}(V - p_A)(V - p_A) + (V - p_A)(p_A - p_B)]}_{(3)} \Big]. \end{aligned}$$

The demand of product B is obtained similarly:

[6]The consumer is assumed to choose good A over B if she is indifferent.

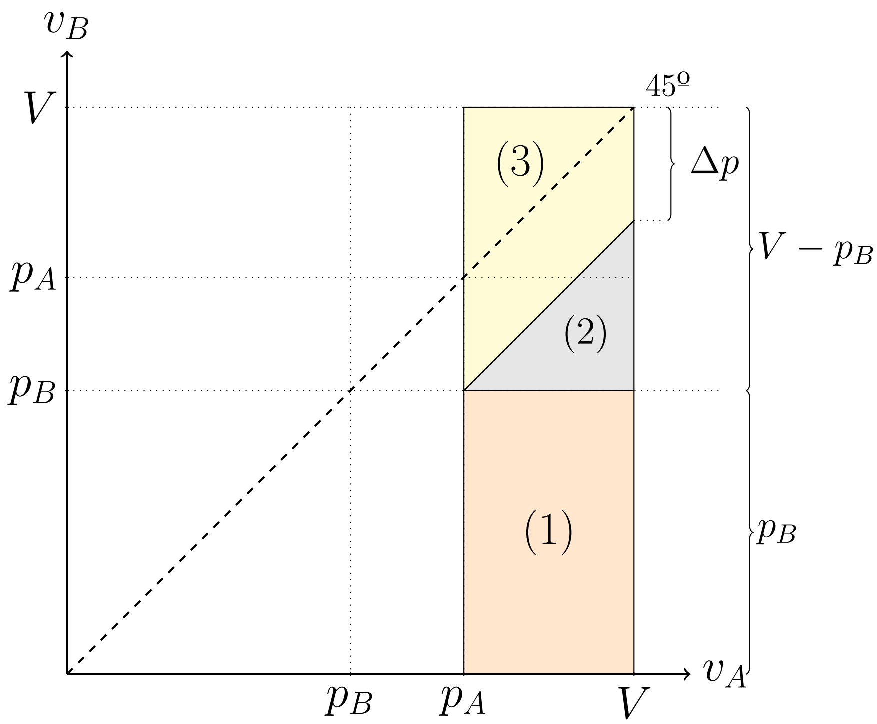


Figure 2.3: Demand generated for good A

$$D_B(p_B, \phi_B; p_A \phi_A) = \frac{\phi_B}{V^2}\Big[\underbrace{(V-p_B)p_A}_{(1')} + \underbrace{[\frac{1}{2}(V-p_A)(V-p_A) + (V-p_A)(p_A-p_B)]}_{(2')}$$

$$+ (1-\phi_A)\underbrace{\frac{1}{2}(V-p_A)(V-p_A)}_{(3')}\Big].$$

Given the demands, each firm chooses price and the level of advertisement to maximize its profits, taking into account the rival's strategy. The equilibrium behavior is characterized in Proposition 2.2.

Proposition 2.2. *If the duopolistic firms have access to random advertisement, the equilibrium is the following:*

i If $V < 4a$, neither seller advertises their product.

ii If $4a \leq V \leq 5.23a$, one seller, for instance, firm A fully advertises the product *and the other firm does not advertise to any consumers. The equilibrium is:*

$$p_A = V/2, \quad \phi_A = 1, \quad \phi_B = 0.$$

iii *If* $5.23a \leq V \leq 5.83a$*, one seller, for instance, firm A fully advertises the product, while the other partially advertises. The equilibrium is:*

$$\begin{aligned}
p_A &= V - \sqrt{2V}\sqrt{V - 2\sqrt{aV}} \\
p_B &= \sqrt{aV} \\
\phi_A &= 1 \\
\phi_B &= \frac{8V(a+V) + 5V\sqrt{2V}\sqrt{V-2\sqrt{aV}} - \sqrt{aV}(25V - 12\sqrt{2V}\sqrt{V-2\sqrt{aV}})}{2V^2 - \sqrt{aV}(4V + 16a - 7\sqrt{aV})}.
\end{aligned}$$

iv *If* $5.83a \leq V$*, both sellers fully advertise to the market with*

$$\begin{aligned}
p_A = p_B = p_R^D &= (\sqrt{2} - 1)V \\
\phi_A = \phi_B = \phi_R^D &= 1.
\end{aligned}$$

Figure 2.4 illustrates the market equilibrium described in Proposition 2.2. We observe that, the number of active firms and their activity in the market increase with the market size when the firms send random advertisement.

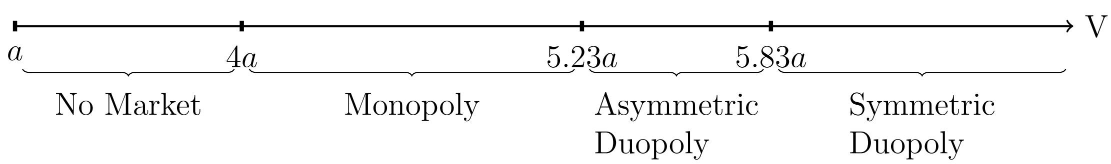


Figure 2.4: Potential duopoly setting with random advertisement with respect to market size.

As it is the case for a monopoly, in small markets ($V < 4a$), neither firm has an incentive to promote its product with random advertising. The reason is the same as before: Promoting the product does not generate enough income for the firm to cover the advertisement cost.

If $4a \leq V \leq 5.23a$, the average value of the product provides enough income for one seller to advertise and sell its product. However, there is only space for only one firm in the market and the second firm does not find it worthwhile to pay the advertising cost. Thus, the active firm acts as a monopolist.

In larger markets ($V \geq 5.23a$), both firms have an incentive to promote their products with random advertisement. In particular, if $5.23a \leq V \leq 5.83a$, the equilibrium is that one firm promotes its product to the whole market and the other to a part of the market. Hence, despite the ex-ante symmetry of the firms, there exists an asymmetric duopoly market. On the other hand, if the market is large enough ($V \geq 5.83a$), both firms generate profit by promoting their product to the whole market; hence, there exists a symmetric duopoly with full advertisement.

In the asymmetric duopoly (that exists if $5.23a \leq V \leq 5.83$), the firm which advertises to the whole market operates with positive profit. On the other hand, the firm which advertises to a part of the market operates at zero profit. Hence, despite the firms are symmetric in all priors, there may exist an equilibrium where the profit generated in the market is collected by only one firm.[7]

In the symmetric equilibrium which exists in large markets ($V \geq 5.83a$) the firms share the market profit equally. The profit, the consumer surplus, and the total surplus are the following:[7]

$$\begin{aligned}
\Pi_R^D &= V(3 - 2\sqrt{2}) - a \\
CS_R^D &= \frac{2V}{3}(\sqrt{2} - 1) \\
TS_R^D &= \frac{2V}{3}(8 - 5\sqrt{2}) - 2a.
\end{aligned}$$

2.4.2 Targeted Ads

We now consider that the firms hire targeted advertisement and set the price and level of the advertisement simultaneously.

By being able to target the consumers with high valuation for their product, firms can differentiate their advertisement recipients and are less affected by the opponent's strategy. Since firms generate revenue from their own consumer segments and share a small customer base, they have symmetric incentives. Therefore, in contrast to random advertising, ex-ante symmetry always results in ex-post symmetry.

[7]See Appendix for calculations.

Proposition 2.3 characterizes a symmetric equilibrium with targeted advertising and duopolistic firms.

Proposition 2.3. *Let duopolistic firms with differentiated products have access to targeted advertisement. There exists a symmetric equilibrium where both sellers target all consumers who have positive net utility for their product given the price, that is,* $_A(\phi_A) = p_A$ *and* $_B(\phi_B) = p_B$. *Furthermore, the firms set their price equal to* $p_A = p_B = p_T^D$, *where*

$$p_T^D = \sqrt{2V(V+a)} - V.$$

When duopolistic firms have access to targeted advertising, they both have an incentive to promote their products given $V > a$. This condition is the same as for a monopoly, that is, the targeted advertisement gives the competing firms an incentive to enter as strong as in a monopoly. It favours entry even in small markets, contrary to the random advertisement. By being able to target consumers with the highest valuation for their product, firms can differentiate their promotion strategy and compete only for the subset of the consumers whose valuation is large for both firms instead of competing for the whole market.

As in a monopoly, a part of the advertising cost is passed to the consumers. Moreover, given the firms compete only for a fraction of the consumers, the price is more sensitive to the advertising cost. Indeed, the pass-on effect is stronger in a duopoly compared to a monopoly:

$$\frac{\partial p_T^D}{\partial a} > \frac{p_T^M}{\partial a}.$$

The profit of the sellers, the consumer surplus, and the total surplus when the sellers have access to targeted advertisement are the following:

$$\begin{aligned}
\Pi_T^D &= 3V + a - 2\sqrt{2V(V+a)} \\
CS_T^D &= \frac{(2V - \sqrt{2V(V+a)})^2(V + \sqrt{2V(V+a)})}{3V^2} \\
TS_T^D &= \frac{2}{3V}\left(8V^2 - (5V-a)\sqrt{2V(V+a)}\right).
\end{aligned}$$

2.4.3 The Comparison of the Market Outcomes

In this section we provide the comparison of the duopoly market outcomes when the two sellers with horizontally differentiated products can hire random advertisement versus targeted advertisement.

We start with the comparison of price under the two settings and illustrate the comparison in Figure 2.5. The price with targeted advertising is higher than with random advertising, regardless of the number of active sellers with random advertisement. In particular, if $4a \leq V \leq 5.23a$, only one seller advertises with random advertisement and acts as a monopolist. Yet, the duopolistic market price with targeted advertisement is higher than that of a monopolist with random advertisement. Hence, we observe that the ability to access the consumers with the highest valuation for their product allows firms to segment the market partially and focus on their segments. The firms compete for the small part of the demand. that may like both products.

In large markets ($V \geq 5.83a$), the price with the two types of advertisement compares as follows: p_T^D approaches p_R^D as the market size increases (V approaches ∞) or advertising cost, a approaches zero. That is, in very large markets where advertising cost is negligible, price is the same with the two types of advertisement. In other markets, the price difference under the two advertisement settings can be measured by the (partial) pass-on of advertising cost, and it diminishes as the market size increases.

Let $\mathbf{\Pi_T^D}$ and $\mathbf{\Pi_R^D}$ be the total profit of the sellers with targeted and random advertisement, respectively. Then:

$$\mathbf{\Pi_T^D} = 2\Pi_T^D$$

$$\mathbf{\Pi_R^D} = \begin{cases} \Pi_R^M & \text{if} \quad 4a \leq V \leq 5.23a \\ \Pi_R^{AD} = \Pi_{R,A}^{AD} & \text{if} \quad 5.23a \leq V \leq 5.83a \\ 2\Pi_R^D & \text{if} \quad 5.83a \leq V, \end{cases}$$

where $\Pi_{R,A}^{AD}$ is the profit of the firm A, which advertises to the whole market.

The consumer surplus, CS_T^D and CS_R^D, provided in Section 2.4.1 and 2.4.2 are

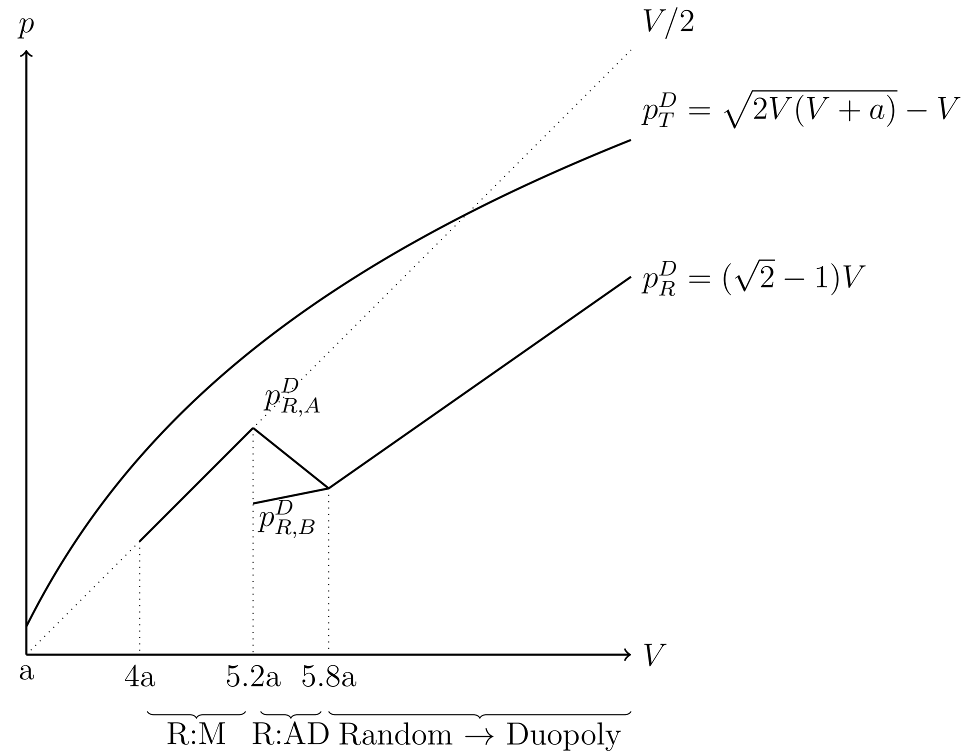


Figure 2.5: Product Price in a Duopoly Market
p_T^D: Price set by both sellers with targeted advertisement,
$p_{R,A}^D$: Price set by the seller which advertises to the whole market with random advertisement,
$p_{R,B}^D$: Price set by the seller which advertises to a part of the market with random advertisement,
p_R^D: Price set by the sellers when both advertise to the whole market with random advertisement.

the total consumer surplus of the consumers who purchase from firms A and B:[8]

$$CS^D = CS_A^D + CS_B^D.$$

Proposition 2.4 provides the comparison of the profit, consumer surplus, and total surplus:

Proposition 2.4. *The comparison of the duopoly market outcomes with random versus targeted advertising is the following:*

[8] The consumer surplus of consumers who purchase from firm B is zero if only one firm advertises its product with random advertisement.

i If $a < V < 4a$, then neither firm advertises its product with random advertising, whereas both firms advertise their products with targeted advertising.

$$\mathbf{\Pi_T^D} > \mathbf{\Pi_R^D} = 0$$
$$CS_T^D > CS_R^D = 0$$
$$TS_T^D > TS_R^D = 0.$$

ii If $4a \leq V \leq 5.23a$, one firm advertises its product to the whole market while the other firm does not advertise to any consumers with random advertising, whereas both firms advertise their products with targeted advertising.

$$p_T^D > p_R^D = p_R^M$$
$$\mathbf{\Pi_T^D} > \mathbf{\Pi_R^D} = \Pi_R^M$$
$$CS_T^D > CS_R^D = CS_R^M$$
$$TS_T^D > TS_R^D = TS_R^M.$$

iii If $5.23a \leq V \leq 5.83a$, one firms advertises its product to the whole market while the other firm advertises to a partition of the market with random advertising (AD: Asymmetric Duopoly), whereas both firms advertise their products with targeted advertising.

$$p_T^D > p_{R,A}^{AD} > p_{R,B}^{AD}$$
$$\mathbf{\Pi_T^D} > \mathbf{\Pi_R^D}$$
$$CS_T^D > CS_R^{AD} \quad \text{if} \quad V < V^*$$
$$CS_T^D \leq CS_R^{AD} \quad \text{if} \quad V \geq V^*$$
$$TS_T^D > TS_R^{AD},$$

where $V = V^*$ satisfies: $CS_T^D = CS_R^{AD}$.[9]

iv If $5.83a \leq V$, both firms advertise their product to whole market with random

[9]See Appendix B.1 for the derivation of V^*.

advertising, whereas both firms advertise their products with targeted advertising:

$$p_T^D > p_R^D$$
$$\mathbf{\Pi_T^D} > \mathbf{\Pi_R^D}$$
$$CS_T^D < CS_R^D$$
$$TS_T^D > TS_R^D.$$

Figure 2.6 summarizes the comparison of outcomes in a market with two sellers with random and targeted advertising.[10] Targeting technology provides the duopolistic firms with an incentive to enter regardless of the market size, while random advertising fails to support the market if it is small.

a	$4a$	$5.23a$	$5.83a$ V
Market operates only with Targeted Ads (Duopoly)	Random → M	Random → AD	Random → D
	Targeted → D	Targeted → D	Targeted → D
	$p_T^D > p_R^M$	$p_T^D > p_R^{AD}$	$p_T^D > p_R^D$
	$\mathbf{\Pi_T^D} > \mathbf{\Pi_R^D}$	$\mathbf{\Pi_T^D} > \mathbf{\Pi_R^D}$	$\mathbf{\Pi_T^D} > \mathbf{\Pi_R^D}$
	$CS_T^D > CS_R^M$	$CS_T^D \gtrless CS_R^{AD}$	$CS_T^D < CS_R^D$
	$TS_T^D > TS_R^M$	$TS_T^D > TS_R^{AD}$	$TS_T^D > TS_R^D$

Figure 2.6: The comparison of the market outcomes in a Duopoly

The firms' profits with targeted advertising are higher than with random advertising, regardless of the market size. With targeting, firms access the consumers with the highest valuation for their product and feel less pressure to undercut the opponent's price, practically segmenting the market. Moreover, the advertisement cost is lower with targeted advertising since fewer adverts are sent compared to random advertising.

Targeted advertising enables segmenting the market and provides both firms with a substantial incentive to enter by enabling them to aim at their demand and alleviate the pressure of competition. Recall that if $4a \leq V \leq 5.23a$, the seller acts as a monopolist with random advertisement since the second firm does not find it profitable to promote its product. In these markets, the firms obtain higher profit

[10] D: Symmetric Duopoly, AD: Asymmetric Duopoly, M: Monopoly.

with targeted than with random advertisement even if the first technology leads to a duopoly whereas the second leads to a monopoly:

$$\Pi_T^D > \Pi_R^M \quad \text{if} \quad 5.23a \geq V \geq 4a.$$

Consumers prefer targeted advertising in small markets where random advertisement does not favour entry (even for one firm) despite higher prices because more consumers enjoy the products in a duopoly. In large markets where a duopoly can exist with random advertising, consumers prefer random to targeted advertising due to lower prices.

The total surplus is higher with targeted advertising in all market sizes. In small markets, both the firms and consumers benefit from entry-favouring targeting technology. In large markets already established with random advertising, targeting improves total welfare since the increase in profit dominates the decrease in consumer surplus.

2.5 Extensions

In this section, we discuss two extensions of the model proposed in this paper.

2.5.1 Different Costs for Random and Targeted Advertising

In the previous sections, we considered an established advertising market for both types of advertising and assumed the same marginal cost for random and targeted advertising. However, since targeting is a new technology, fewer intermediaries may provide this service than random advertising. This section assumes that the targeted advertising is costlier than random advertising and discuss the case of monopoly.

Let a^T and a^R be the marginal cost of targeted and random advertising, with $a^T > a^R$. As we have seen in Section 2.3.1, there is no market under random advertisement if $V < 4a^R$, hence, we focus here in markets with $V > 4a^R$.

It is easy to check that the firms in these large markets prefer random advertising to targeted advertising if the latter is too costly. In particular,

$$a^T > V - \sqrt{V(V - 4a^R)} \quad \implies \quad \Pi_R^M > \Pi_T^M.$$

The threshold separating the preference between random and targeted advertisement decreases in the market size, V. Hence, the firms in large markets are more likely to prefer random to the targeted advertisement. We may have coexistence of two ad settings, where firms prefer targeted advertising in small markets and random advertising in large markets.

In terms of total welfare, we have:

$$a^T > V - \sqrt{\frac{3V^2 - 8Va^R}{3}} \quad \implies \quad TS_R^M > TS_T^M.$$

The threshold that separates the random and targeted advertisements in terms of total welfare is lower than that for the monopoly profit. Hence, total welfare may suffer from the introduction of targeted advertising for a range of parameters where the firm prefers targeted but total welfare is higher under random advertisement.

2.5.2 Entry of an Intermediary with Targeting Technology

The ad intermediary and the cost of advertising are assumed to be exogenous in the current model. However, if there is a unique intermediary with access to both consumer data and targeted advertisement technology, then he is an important strategic actor in this market. He can set the price of the targeted advertisement. This extension enables a more comprehensive calculation of the total surplus by including the profit of the ad intermediary.

Consider that an intermediary with targeted advertisement technology can enter the advertisement market where only random advertisement service is provided at the competitive price by incurring a fixed cost equal to F.

Let a and a^T be the competitive marginal cost of random advertisement and the marginal cost of targeted advertisement set by the entrant, respectively.

Recall from Section 2.3.1 that, if $4a > V > a$, the monopolist does not advertise the product with random advertisement. Therefore, the entrant is the only provider of advertisement in these markets. Given the optimal targeting strategy of the monopolist is $\phi_T^M = \frac{V-a^T}{2V}$, the entrant's problem is the following:

$$\max_{a^T} \quad \frac{a^T(V-a^T)}{2V} \quad \text{s.t.} \quad a^T \leq V.$$

The intermediary's revenue is maximized at $a^T = V/2$. The intermediary enters the market if $V \geq 8F$.

In large markets ($V \geq 4a$), random advertisement allows the monopolist to advertise its product to the whole market. It is profitable for the monopolist to hire targeted advertisement instead of random advertisement as long as following condition holds:

$$\Pi_T^M \geq \Pi_R^M \iff V - \sqrt{V(V-4a)} \geq a^T.$$

Therefore, $\bar{a}^T = V - \sqrt{V(V-4a)}$ is the highest marginal cost of targeted advertisement the monopolist is willing to pay.

The intermediary solves the following constrained problem:

$$\max_{a^T} \quad \frac{a^T(V-a^T)}{2V} \quad \text{s.t.} \quad a^T \leq \bar{a}^T.$$

The intermediary's revenue is maximized at:

$$a^T = \begin{cases} V/2 & \text{if} \quad V \leq \frac{16a}{3} \\ \bar{a}^T & \text{if} \quad V > \frac{16a}{3}. \end{cases}$$

The intermediary enters if $F \leq V/8$ for $V \in [a, \frac{16a}{3}]$ and if $F \leq \frac{1}{2}(\sqrt{V(V-4a)} - (V-4a))$ for $V \in [\frac{16a}{3}, \infty)$.

Next, we compute the welfare. Let Market Surplus (MS) be the profit of the firm and the consumer surplus and Total Surplus (ΣS) be the Market surplus plus the profit of the intermediary. If the intermediary enters, the market outcomes are the following:

$$
\begin{array}{llll}
MS_T = \dfrac{3V}{32} & \Sigma S_T = \dfrac{7V}{32} - F & \text{if} & V \leq \dfrac{16a}{3} \\
MS_T = 3(\dfrac{V}{4} - a) & \Sigma S_T = \dfrac{V}{4} - a + \dfrac{\sqrt{V(V-4a)}}{2} - F & \text{if} & V > \dfrac{16a}{3}.
\end{array}
$$

The comparison of the market outcomes with (T) and without (R) the entrant is summarized below:

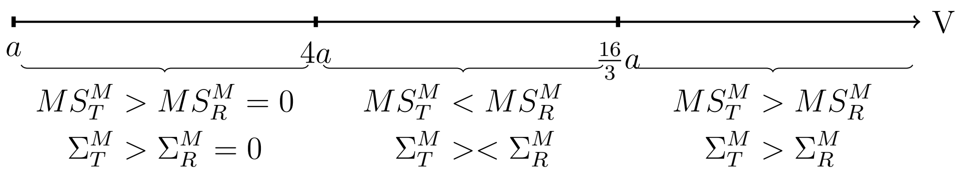


Figure 2.7: The comparison of the market outcomes of a single-product monopoly with competitive random ads and entry of targeted ads

The entrant's targeting technology improves the market surplus in small markets by facilitating entry. In large markets, the entrant has to provide a better incentive to the monopolist than random advertisement does. Therefore the entrant's relative price for advertising is lower than it is in smaller markets. Despite a higher marginal advertising cost of targeting, the market surplus improves for the large markets compared to random advertising. In medium-sized markets, the entrant charges a revenue-maximizing price for targeting since it enables a higher profit for the monopolist than random advertising does. These markets are worse off with targeting due to high targeting costs for the firm and higher product prices for the consumers.

2.6 Conclusion

In the markets that rely on the informative advertisement, traditional methods fail to provide an efficient way of informing consumers. In small markets where the firms do not find it worthwhile to promote their product with random advertisement, the advertisement requirement may act as a barrier for entry. In contrast, targeted advertising favours entry by enabling the firms to access high-valued consumers and improving the return of the firms' advertisement costs. Moreover, targeting allows the firms to access the product's own segment and differentiate the strategies from the competitor's products. Hence, the incentive for entry extends to product variety by the monopolist or another firm. Both the firms and the consumers enjoy a positive surplus with targeted advertising in small markets.

For products with large markets, the firms find it profitable to promote their products even with random advertising. Still, their profit and price with targeted advertising are higher than under random advertising in monopoly and duopoly settings.

Finally, in the medium-sized markets, random advertising leads to an equilibrium where only one firm promotes its product while targeting technology enables a duopoly. However, even in this case, the efficiency of the advertisement expenditure and the access to their own segment of consumers with targeting provide the duopolistic firms with a higher profit than what a monopolist collects with random advertising.

We find that the welfare in all market sizes is improved when firms can hire targeted ads. All market participants benefit from targeted ads in small markets, which would fail with random advertisements. Though, consumers in large markets where random advertisement allows competition do not prefer to be targeted due to the higher equilibrium prices with targeted ads. The welfare in these markets is still improved since the firms' gains are higher than the consumer surplus loss.

We have assumed in this paper that consumers do not suffer any disutility from receiving an ad, even if they are not interested in the product. Considering the potential negative effects of informative advertising is an important extension outside the scope of the current paper.

Chapter 3

Local Market Equilibrium with Online Market Diffusion

Abstract

Most consumers now have the option of an online market in addition to their local brick-and-mortar retailers. Hence, when developing their strategy to compete for the demand, the local firms must simultaneously consider the competition against the online market and local rivals. In this project, I study local firms' price strategy with the two dimensions of the competition in a framework I build on Hotelling's linear city model. I show that online diffusion benefits both the online and offline consumers, while the offline firms generate lower profits as long as the offline market structure does not change. In contrast to the classic result, the transportation cost decreases the offline profit if the online diffusion is strong. Moreover, strong online competition may drive away some local competition and lead to an increase in offline price, which hurts captive offline consumers.

JEL L810, D83, L13, D43
Keywords: online shopping, optimal search, Hotelling model

3.1 Introduction

The internet is no longer an optional communication medium in today's society; it is a vital requirement for every household worldwide. Commercial activities have taken their share of the time spent online. Consumers enjoy visiting virtual shops from the comfort of their homes (also due to safety reasons following the Covid-19 pandemic). Additionally, the enhancements in information technologies facilitated the presence of retailers on the internet by decreasing the entry costs for a wide range of industries. Therefore, consumers can find any product from many other online suppliers besides their local ones.

Consumers who require a product that is available in both online and offline environments have two options. They can go to the local shop and choose the best product that fits their interest upon observing the available products, or they can search and buy it online. Hence, sellers of two environments with comparative advantages over each other compete for the consumers' demand.

In this chapter, I focus on the consequences of the presence of a highly competitive online market on the price and profit of brick-and-mortar firms and consumer surplus. I consider a model with heterogeneous products and agents who differ in their location and online search cost.

The conventional stores are located at the edge of the Hotelling's linear city model and compete simultaneously against each other and the online firms. The model facilitates the analysis of the short-term response of the local firms to the online competition.

Online retailers have certain cost gains, such as saving from showroom and workers costs, so they can offer lower prices and obtain an advantage in price competition against conventional retailers. Moreover, the online consumer saves from the transportation cost to visit the store. On the other hand, it is a very demanding search process to look for a product online by sampling it one by one. The search process can be complicated without feeling the product (and a piece of professional advice which is crucial in some instances). Hence, the cost of online search to the consumer plays a vital role for online consumers.

Even if we assume that a product's features are perfectly available with virtual inspection, it requires particular technical abilities and experience to reveal them. The consumers have different levels of technical competency and experience in this activity. While experienced internet users can perform online searches easily and quickly, others may find it very difficult, highly time-consuming, and even impossible. Therefore, the cost of searching online search differs significantly across consumers in a market.

The online consumers are assumed to perform sequential search. It is one of the most natural methods to represent consumers' online search behavior since searchers dedicate their attention to sampling one product at a time. An online consumer who adopts the sequential search rule decides on the minimum acceptable utility level as a threshold. Then, she samples the products/prices one by one until a sample is at least as good as this threshold and stops searching.

On the other hand, the local shop provides the opportunity to have a sense of the product and assistance in the process of the product search. Hence, offline consumers can direct their search and reveal the product features much more quickly. Therefore, the search cost is negligible once the consumer travels to the store. However, she incurs a transportation cost to visit the store.

In this model, a share of the consumers in the market have positive and heterogeneous valuations from buying online. This fraction of consumers measures the online market penetration in the extensive margin while the online price measures the intensive margin. I show that the offline price and profit decrease with both measures of online competition. On the other hand, the offline price increases with the transportation cost. Since the consumers who are distant from local shops are more likely to leave local shopping for online one, the local firm's demand is more concentrated to the consumers who are close to them, over those the firms have more market power. In contrast to the original Hotelling result, the local firm's profit may decrease with the transportation cost if the online market penetration is strong.

Both online and offline consumers benefit from an increase in online market penetration, as long as the offline market does not change. However, online market penetration may substantially affect the offline market and lead to a structural change. This can happen if the online competition drives away a local firm by stealing the local market's business. In this case, the surviving local firm sets a higher price and generates a higher profit thanks to the removed local competition by the online one. The consumers who can switch to the online market enjoy a higher surplus. However, the captive offline consumers who do not have an online option are left with a less competitive local price and a much lower surplus.

The consequences of the expansion of e-commerce have attracted significant attention from researchers and policymakers as it truly deserves. There is a growing literature of theoretical papers on the competition between online and offline sellers. Balasubramanian (1998) bases his framework on the circular city model of Salop (1979) and studies price competition between conventional retailers and a direct channel (mail order) as a precursor of the literature on e-commerce. The direct presence is so strong in Balasubramanian (1998) that each retailer competes only against the remotely located direct marketer. Loginova (2009) considers heterogeneous agents who need conventional stores to learn their preferences. She shows that the entry of virtual retailers may cause a price increase and decrease social welfare by enabling physical shops to single out high valuation consumers.

Goldmanis et al. (2010) focus on the effects of the diffusion of e-commerce, interpreting it as a reduction in consumers' price searches. They show that it reallocates market shares from high-cost to low-cost producers on the supply-side. Madden and Pezzino (2011) modify Salop's circular city model with an online supplier of a homogeneous good at the center. They show that the standard Salop result of more firms than the socially optimal might be reversed. Guo and Lai (2017) model the competition between one online retailer and heterogeneous conventional retailers located on the linear city model of Hotelling (1929) and provide the short-term (price) and long-term (location) equilibrium response of the conventional retailers.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the strategies of the consumers and firms. Next, the market equilibrium for a duopolistic local market and comparative statics are provided. Finally, an analysis of a local monopoly, a possible market structure that results from high online competition, is presented. All the proofs are in the Appendix.

3.2 The Model

Consider a market of horizontally differentiated goods that are available both in the local and the online market and with features that require inspection either virtually or physically to be revealed by the consumer. Several examples of goods and features are the design or size of an apparel, compatibility of a tool or a hobby object, a flavor of a nutrition good. Some features of goods may be harder to confirm virtually because of their nature. We assume that a continuum of horizontally differentiated goods, k, are uniformly distributed on a circular space with a circumference of length equal to 2. The density of the distribution is $\frac{1}{2}$. A priori, no particular good is better than another.

The consumers have perfect information on the product space and the distribution of the goods and wish to buy one unit of the good. Each consumer has a personal taste, k^*, the most preferred good on the product space. There is a continuum of consumer tastes uniformly distributed on the circular product space.

Consumers enjoy a common utility equal to 1 from consuming their most preferred variety. The utility of consuming a unit of good decreases as it is further located from the consumer's taste in the product space. Let $d = |k^* - k|$ be the difference between a given consumer taste k^* and a given product, k. The best match to a given taste is located at the exact point with $d = 0$, and the worst match is located at a distance equal to $d = \max|k^* - k| = 1$ from the consumer's taste. Figure 1 shows the product space, a consumer taste k^*, a product k, and their difference d.

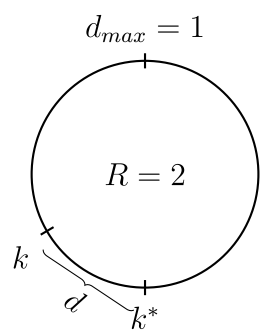


Figure 3.1: Product space with consumer k^*, product k, and their difference d.

The utility from consuming one unit of good is a linearly decreasing function of the difference, d:

$$u(d) = 1 - d.$$

The consumers can learn about the type of the good and buy it once they observe it. They have two options to access the goods. They can go to the local shop and physically observe the products (offline), or they can search online and virtually observe the products. Consumers are risk-neutral and maximize their expected utility.

The offline consumer must travel to the shop's location to access the products. The consumers' location is distributed uniformly on a line of length one following the linear city model of Hotelling (1929). They incur transportation cost τ per unit length of distance. Therefore, traveling to a physical shop located at x distance from the consumer costs her τx.

The offline shops are assumed to have unconstrained capacity; therefore, all goods in the product space are available at the shop. Furthermore, with the ability to see, feel, and try the goods on top of the professional assistance of the seller if required, the consumer can find her most preferred good, $d = 0$, and obtains one unit of utility. Therefore, net utility of an offline consumer who buys the good from a local shop located at a distance x is the following:

$$u^{off}(x) = 1 - p - x\tau, \tag{3.1}$$

where p is the price of the good at the shop.

The online consumer must access the webpage of the product in order to observe the type of the good and purchase it. A single online search provides this information perfectly about the sampled product. One online search costs γ_i to the consumer i and she knows her search cost γ_i. The set of consumers is distributed according to the continuous distribution function, f_γ, in the interval $[0, 1]$. Moreover, the distribution f_γ is independent of the distribution of the consumer's location.

Online consumers perform sequential search, that is, they draw one good at a time, observe it, and decide to draw another good or stop searching.[1] They sample the goods randomly and there is no learning through the search process; that is, the per search cost γ_i remains constant as the number of searches increases.

Let d_n be the difference of the best sampled product from an online consumer's taste after n online searches. The net utility of a consumer who buys the online good after n searches is the following:

$$u^{on}(\gamma) = 1 - d_n - p - n\gamma. \tag{3.2}$$

There are two firms in the offline market, located at the extremes of the linear city of length 1. Local firms have the marginal cost of production c and compete in price.

[1] Optimal Statistical Decisions, DeGroot (1970).

The number of active sellers online is assumed to be very high since once a seller is available online, it becomes the supplier of the good without location constraints. On the other hand, the introduction of new technologies on price comparison and availability of the price information of the product of online sellers decreases the search cost of price information significantly. Once the search cost of price is zero while the search cost for product information is positive, a Bertrand-type competition is induced at the equilibrium in the market for heterogeneous products as the number of active sellers is very high (Bakos, 1997).

The online consumers search for the information of the product while the price information is available at zero cost[2]; therefore, the online market is assumed to be operating at the competitive price p_{on} equal to the online firm's marginal cost of production ($c_{on} \leq c$).

In this market, consumers are heterogeneous regarding their distance to the local shops (location) and online search efficiency (online search cost). Price of the good in both online and offline markets are common knowledge. Consumers decide to buy the good from their local shop or online considering their expected net utility, given the prices, their distance to the local shops and their search cost.

The consumer characteristics are common knowledge for duopolistic local firms. They compete in price, given the online price and the distribution of consumer location and search cost.

The next section starts by presenting the strategies of the consumers and local firms. Next, Proposition 3.1 provides the offline price and profit at equilibrium. Finally, we provide comparative statics and discuss the effects of online competition on the local market.

3.3 Results for the Duopolistic Offline Market

3.3.1 Consumer Strategy

If the consumer shops offline, then she decides which local shop to buy from. Let p_1 and p_2 be the price of Firm 1 and Firm 2, respectively. Denote $U^{F1}(x)$ and $U^{F2}(x)$

[2]For example through price comparison websites.

the net utility of the consumer located at x distance from Firm 1, from each firm:

$$U^{F1}(x) = 1 - p_1 - \tau x$$
$$U^{F2}(x) = 1 - p_2 - \tau(1 - x).$$

It the consumer shops online, then she searches for the type of good that fits best to her taste. She is in a very similar situation as the person looking for a 5-centimeter needle to sew a button in a haystack of 1000 needles, with each having a different size varying between 3 cm to 10 cm.

The consumers use the sequential search rule; hence, she decides to either stop or perform one more search after each sampling. The product space is a continuum; therefore, the set of products to be sampled from is the same for each search. The optimal strategy for the consumer is to define a constant threshold level since a threshold is optimal for every period once it is optimal at any given time. Hence, given their per search cost γ and the price of the good, the consumers must decide on the threshold, which is the maximum difference they can tolerate between their taste and the good, and continue searching until they sample a good that is at least as good as their threshold level.

A consumer's most preferred variety is located at zero distance from her taste, whereas her least preferred good has a difference equal to one. Due to the symmetric nature of the product space and distributions, the difference $\tilde{d}$ of a randomly sampled variety has a uniform distribution for all consumers: $\tilde{d} \sim U[0, 1]$.

Let $\bar{d}$ be the threshold that the consumer chooses as the stopping rule for the sequential search. The valuation of the consumer who uses this threshold is the following:

$$\mathbb{E}\left[\, u(d) \mid \bar{d}\, \right] = \left(\int_0^{\bar{d}} u(d) f(d) dd + (1 - \bar{d})(\mathbb{E}[\bar{d}] - \gamma) \right) - \gamma.$$

The first part of the expression represents the utility from stopping the search. The consumer stops searching if she samples a product that is good enough ($d \leq \bar{d}$). The expected valuation of stopping the search is calculated by integrating the probability of sampling a good enough product times the utility from its consumption. If the sampled product is not acceptable, she continues to search by incurring the search cost again, which happens with a probability equal to $1 - F_d(\bar{d}) = 1 - \bar{d}$.

The expected utility from $\bar{d}$ is obtained by rearranging the above valuation:

$$\mathbb{E}\left[\, u(d) \mid \bar{d} \,\right] = \frac{\int_0^{\bar{d}} u(d) f(d) dd - (2-d)\gamma}{d}.$$

The optimal strategy d^* which maximizes the expected utility of the consumer must satisfy following condition:

$$u(d^*) + \gamma = \mathbb{E}\left[\, u(d) \mid d^* \,\right].$$

Given that $u(d) = 1 - d$, the optimal strategy d^* and the expected utility of the consumer with per search cost γ for the online search is the following:

$$\begin{aligned} d^* =& 2\sqrt{\gamma}, \\ \mathbb{E}\left[\, u(d) \mid \bar{d} = d^* \,\right] =& 1 + \gamma - 2\sqrt{\gamma}. \end{aligned}$$

The optimal strategy for the sequential search rule is affordable to the consumers who do not have too high per search cost. In particular, the consumers with $\gamma < 1/4$ can maximize their utility with an interior optimal.

The consumers with $\gamma > 1/4$ set the threshold d equal to 1, which is equivalent to searching only once. Since the expected utility from the result of searching once is $\mathbb{E}[u(d) \mid \bar{d} = 1] = 1/2 - \gamma$, the consumers with per search cost $1/4 < \gamma < 1/2$ indeed search only once if they choose to purchase online.

The consumers with search cost higher than $1/2$ cannot afford to search online at all, since their expected utility from online purchase is lower. These consumers have zero expected utility from the online market and are the 'captive consumers' of the offline market.

We denote by $v(\gamma)$ the expected utility of the consumer with per online search cost γ. We say that $v(\gamma)$ is her *online valuation*. Therefore,

$$v(\gamma) = \begin{cases} 1 + \gamma - 2\sqrt{\gamma} & \text{if} \quad \gamma \in [0, {}^1\!/\!_4] \\ \frac{1}{2} - \gamma & \text{if} \quad \gamma \in [{}^1\!/\!_4, {}^1\!/\!_2] \\ 0 & \text{if} \quad \gamma \in [{}^1\!/\!_2, 1]. \end{cases}$$

The consumers with online search cost $\gamma > 1/2$ have zero online valuation, and it does not depend on their search cost. The mass of these consumers is $1-F_\gamma(1/2)$. Let $s=1-F_\gamma(1/2)$ be the fraction of the captive consumers and $1-s$ the fraction that has online valuation between 0 and 1.

For the set of consumers with positive online valuation, we assume for simplicity that v_i has a uniform distribution on the interval $[0,1]$ with a density equal to $1-s$. Hence, the online valuation is a mixed (a mixture of both discrete and continuous) random variable with the following distribution:[3]

$$f_v = \begin{cases} 1-s & \text{if} \quad v \in (0,1], \\ s & \text{if} \quad v = 0, \\ 0 & \text{o.w.} \end{cases} \tag{3.3}$$

3.3.2 The Consumer Decision: Online or Offline Market

Consumer i is identified by the pair consisting of her location on the linear city and online valuation, (x,v). She maximizes her payoff by purchasing the good online or offline, given the prices in each market.

A fraction $1-s$ of consumers with $v \in (0,1]$ can potentially choose to purchase the good from either the online or the offline market. They make their decision by comparing their expected utility from the online market and the two local firms in the offline market. Let p_{on}, p_1 and p_2 be the prices of the good at the online market, local Firm 1 and local Firm 2, respectively. Then, consumer i, who is located at $x_i \in [0,1]$ and has online valuation v_i, compares the following payoffs:

$$U_i^{on} = v_i - p_{on} \tag{3.4}$$
$$U_i^{F1} = 1 - p_1 - \tau x_i \tag{3.5}$$
$$U_i^{F2} = 1 - p_2 - \tau(1-x_i). \tag{3.6}$$

On the other hand, a fraction s of consumers have zero online valuation. They only consider buying from their local market, and compare U^{F1} and U^{F2}.

[3]See Appendix B for a distribution of the search cost, f_γ, that gives s fraction of consumers are captive and $1-s$ fraction have an online valuation distributed uniformly in the interval $[0,1]$.

3.3.3 The Firms' Strategy

There is a very high number of firms in the online market. As mentioned above, the online market is operating at the competitive price, p_{on} equal to the online firm's marginal cost of production. The online price is not affected by the local market's price since it is already down to the marginal cost.

A brick-and-mortar firm has two directions of competition, the local market and the online market.

The consumers with too high search cost to search online select the best out of the local firms. The indifferent consumer between the two brick-and-mortar firms has equal utility from the firms, $U_i^{F1} = 1 - p_1 - \tau x = U_i^{F2} = 1 - p_2 - \tau(1-x)$. The indifferent consumer is characterized in the same manner of the original model of Hotelling (1928). Let $\bar{x}$ be the location of the indifferent consumer between Firm 1 and Firm 2. Then,

$$\bar{x} = \frac{p_2 - p_1}{2\tau} + \frac{1}{2}. \tag{3.7}$$

The offline consumers who are located at the left of the indifferent consumer ($x \leq \bar{x}$) purchase from Firm 1 and the rest purchase from Firm 2.

We focus on the case of a covered offline market, where all captive consumers purchase from one of the local firms. This is possible if the captive consumers who pay the highest transportation cost have non-negative utility. Given symmetric firms, the consumers located at $x = \frac{1}{2}$ incur the largest transportation cost. Hence, market coverage requires the following:

$$U^{F1}|_{x=\frac{1}{2}} = U^{F2}|_{x=\frac{1}{2}} \geq 0 \iff 1 - p - \frac{\tau}{2} \geq 0.$$

Assumption 1: The transportation cost, τ, and the local firms' marginal cost of production, c, are assumed to be not too high:[4]

$$1 - c - \frac{3\tau}{2} \geq 0. \tag{A1}$$

The $1-s$ fraction of consumers can potentially buy from a local firm or the online market. The indifferent consumer between the online market and the brick-and-mortar Firm 1 has equal expected utility from these two options, $U_i^{on} = U_i^{F1}$.

[4]It will be later shown that Assumption (A1) ensures the market is covered at equilibrium.

Hence, the indifferent consumer is given by her online valuation and location. Let $\bar{v}^I$ be the online valuation of the indifferent consumer at x:

$$\bar{v}^I = 1 - p_1 + p_{on} - \tau x. \tag{3.8}$$

Given the prices of the two markets and the transportation cost, the online valuation acts as the consumer's reservation utility compared to an offline purchase. Note that the indifferent consumer's online valuation decreases as the consumer is located further away from the local firm. That is, the consumers further away from the local shop are more likely to purchase online.

Since the online valuation of the non-captive consumers is positive, complete market coverage is achieved when captive offline demand is covered. Figure 3.2 shows that all the consumers who prefer the online market to the local one has positive online valuation when the offline market is covered, that is, $\bar{v}^I \geq U^{F1} \geq 0$.

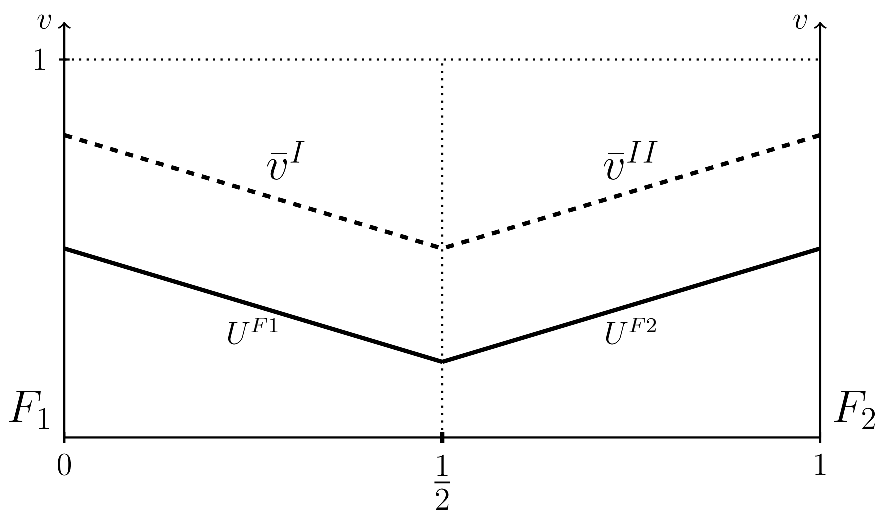


Figure 3.2: Online and Offline Valuation with Indifferent Consumer at $x = \frac{1}{2}$.

The indifferent consumers between the online market and the offline firms give the demand of the offline firm as follows: The consumers who are located to the left of $\bar{x}$ prefer Firm 1 to Firm 2, and vice versa. At any given location, consumers with online valuation higher than $\bar{v}(x)$ prefer the online market to their best local option. The consumers with lower online valuation, buy from their most preferred local firm.

The market coverage condition implies $\bar{v}(x) \in [0, 1]$ for all $x \in [0, 1]$ since $c \geq$

$p_{on} \geq 0$. The shaded area in Figure 3.3, shows the demand of the brick-and-mortar Firm 1 in the non-captive segment, where the online valuation of the consumers is represented at the vertical axis on top of the linear city of length 1.

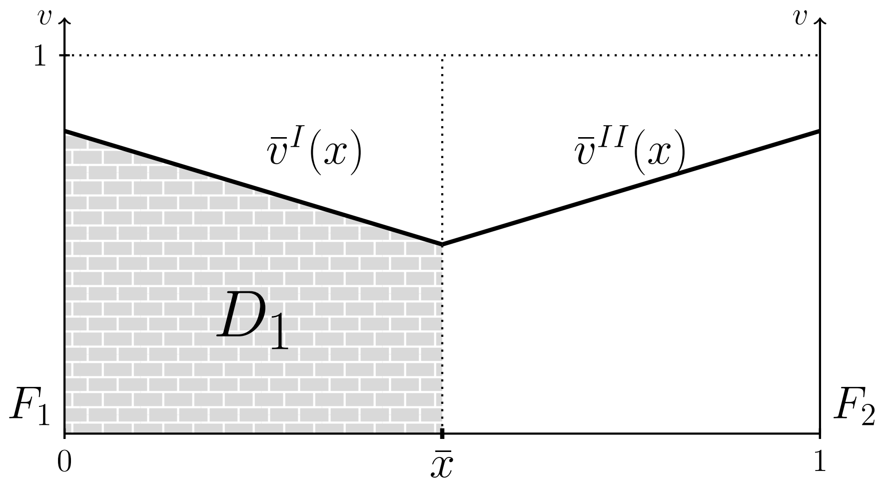


Figure 3.3: The Demand for the Brick-and-Mortar Firm 1 in the Non-Captive Segment.

Let D_1^{1-s} be the demand of the local Firm 1 from the non-captive segment of the market. It is computed by integrating the consumers who have lower valuation than $\bar{v}$ in the interval $[0, \bar{x}]$:

$$D_1^{1-s} = \int_0^{\bar{x}} F_v(\bar{v}^I) dx.$$

On the other hand, the demand of the local Firm 1 from the captive consumers is $D_1^s = s\bar{x}$. Therefore, the total demand for Firm 1 is the following:

$$D_1 = \int_0^{\bar{x}} F_v(\bar{v}^I) dx + s\bar{x}, \tag{3.9}$$

where $\bar{x}$ and $\bar{v}^I$ are given by equations (3.7) and (3.8).

Given that non-captive consumers with $v \in (0, 1]$ have a cumulative distribution function $F_v = (1-s)v$, D_1 can be written as follows:

$$D_1(p_1; p_2, p_{on}) = (\frac{p_2 - p_1}{2\tau} + \frac{1}{2})\Big(1 - \frac{(1-s)}{4}(3p_1 - 4p_{on} + p_2 + \tau)\Big).$$

The demand for Firm 2 is computed similarly.

The two offline firms choose price taking into account the competition of their competing offline firm and the existence of the competitive online market. Proposition 3.1 states the equilibrium price that results from this competition.

Proposition 3.1. *At equilibrium, the duopolistic local firms set the price* $p_1 = p_2 = p^*$ *and generate profit equal to* $\Pi_1 = \Pi_2 = \Pi^*$*:*

$$p^* = \frac{1}{2}\left(\frac{1}{1-s} + p_{on} + c + \frac{3\tau}{2} - \sqrt{\Delta}\right) \tag{3.10}$$

$$\Pi^* = \frac{1}{8}\left(\frac{1}{1-s} + p_{on} - c + \frac{3\tau}{2} - \sqrt{\Delta}\right)\left(1 - (1-s)\left(c - p_{on} + 2\tau - \sqrt{\Delta}\right)\right), \tag{3.11}$$

where

$$\Delta = \left(\frac{1}{1-s} + p_{on} - c - \frac{\tau}{2}\right)^2 + 3\tau^2.$$

Proof. See Appendix A. □

The local firms compete simultaneously with each other and the online market. The parameter s gives the market size without the online competition, and the parameter p_{on} represents how strong the online competition is for those consumers who can buy online. On the other hand, the transportation cost τ increases the local market power against local competition but strengthens online competition since it makes offline shopping costlier.

We note that if the online market does not serve the local demand or everyone have zero valuation for the online goods, the current model converges to Hotelling. Indeed, the equilibrium price and profit obtained by taking the limit of (3.10) and (3.11) as s approaches 1 is:

$$\lim_{s\to 1} p^* = c + \tau \quad \text{and} \quad \lim_{s\to 1} \Pi^* = \tau/2,$$

which corresponds to the classic equilibrium in a Hotelling model.

Without the online market, the local firms have locally monopolistic power in their neighborhoods. With the online market, some consumers have a cheaper online option. Therefore, local firms have to cut prices not to lose too much demand to the online market. Moreover, the online option gets relatively cheaper as consumers are located farther away from the local shops, and it is harder for the shops to recover the distant demand by reducing the price. Corollary 3.1 states the comparative statics exercise of the price, demand, profit and consumer surplus with respect to the parameters s, p_{on}, τ.

Corollary 3.1. *The local market price, the local firms' demand and profit increase with the online market price p_{on} and the fraction of captive consumers s. The price is also increasing in the transportation cost τ, while the demand is decreasing. The profit may be decreasing or increasing in τ, depending on the size of the captive market. Consumer surplus is decreasing in all three parameters.*

Proof. See Appendix A.[5] □

There are two effects in force influencing the local market price through the online competition. As the online market price p_{on} increases, fewer consumers consider it a viable option. The online competition softens, and it pushes up the offline price. On the other hand, the offline demand to be shared gets larger. The local competition gets tougher and slows down the increase in prices. The former effect dominates the latter since local firms have market power over the consumers who switch to the offline market and are located in their neighborhood. Therefore, the local price increases with p_{on}. Similarly, the larger the offline captive consumers, the softer the online competition. Hence, the local price also increases with the fraction of consumers s with zero valuation for online goods.

An increase in the online price results in an increase in the offline price and some consumers switching to the offline market since the increase in the offline price is slower. Therefore, the profit of the local firms increases with the online price. Likewise, an increase in the size of the captive offline consumers increases the offline price. However, it does not result in a greater demand shift from the offline market to the online one in the segment of consumers who compare the two markets. Hence, the profit of local firms increases with the fraction of captive offline consumers.

The higher the transportation cost, the more market power the offline firms have locally. However, this power is challenged by the online market since the non-captive consumers have the online market option regardless of their location. Hence,

[5] The change in the local firms' demand and profit with respect to s is shown with numerical simulations.

the online market slows down the increase in the offline price. The offline demand decreases because of the increase in both the transportation cost and the offline price. If the online penetration is high, that is, there is no captive offline consumers, the local firm's profit is decreasing in the transportation cost. On the other hand, when the offline share of the local firm's profit is high, the profit increases with the transportation cost.

The online and offline markets cover the demand collectively at equilibrium; all consumers buy either from their local firm or the online market. Therefore, the consumer surplus decreases with the parameters that increase the offline price. Suppose the offline price increase is due to an increase in captive consumers or the transportation cost. In that case, some non-captive consumers switch to the online market. However, they would enjoy a lower surplus than they had if it was not for the offline price increase. The consumers who stay offline, whether they are captive or compare the two markets, have a lower surplus with an increase in the price. On the other hand, if the online price leads to an increase in the offline price, some consumers switch to the offline market since the price increase is slower there. Still, those who switch obtain a lower surplus than they would have from the online market if it were not for the online price increase.

Now, suppose the online price increases. A part of the non-captive consumers will continue to prefer the local market. Together with the captive consumers, this market size remains constant with an increase in the online price. The total surplus in the original Hotelling model is constant with the price, since the market size is fixed. Similarly, the total surplus of the offline consumers who did not switch from the online market is constant. Those who switch from the online market to the local one and those who keep buying online have lower surplus. The local market on the other hand, generate additional total profit from the new customers. These consumers did not generate any profit for the online firms. Therefore, the total surplus, at least that of the local market, may increase with an increase in the online price if the additional profit dominates the loss of consumer surplus.

3.4 A Discussion on the Monopolistic Offline Market as a Result of Online Competition

The previous sections provide an analysis on the intensive margin, assuming the effects of the online competition on the local market does not lead to a change in the duopolistic market structure. In this section, we look at the potential effects in the extensive margin, when the online competition disrupts the market structure.

The brick-and-mortar firms often incur some fixed costs to run the business. Let F be the fixed cost of production for an offline firm. As the online competition gets tougher, there may not be enough profit generated in the local market for both offline firms to cover the fixed cost of production. In this section, we investigate the equilibrium when only one firm remains in the offline market, that is, there is a monopoly offline market.

There exists a monopoly offline market if:

$$\Pi^*_M > F > \Pi^*,$$

where Π^*_M is the profit of a monopoly offline firm at equilibrium.

We assume here that the marginal cost of production and the transportation cost is not too high, so that the market will be covered at equilibrium.[6]

$$1 - c - 2\tau \geq 0. \tag{A1'}$$

The offline firm serves the whole captive market if it is covered and competes with the online market for the non-captive consumers. Its demand is the following:

$$D^M(p^M; p_{on}) = \int_0^1 F_v(\bar{v}^I)dx + s, \tag{3.12}$$

where $\bar{v} = 1 + p_{on} - p^M - \tau x$.

The monopoly sets the price $p^M = p^*_M$ at equilibrium,

$$p^*_M = \begin{cases} \frac{1}{2}(\frac{1}{1-s} + p_{on} + c - \frac{\tau}{2}) & \text{if} \quad s \leq s' \\ 1 - \tau & \text{if} \quad s > s', \end{cases} \tag{3.13}$$

[6](A1') is slightly stricter than (A1) and ensures that there exist an offline consumer at every point of the linear city.

and generates profit Π_M^*:

$$\Pi_M^* = \begin{cases} \frac{1}{4}(\frac{1}{1-s} + p_{on} - c - \frac{\tau}{2})(1 - (1-s)(c + \frac{\tau}{2} - p_{on})) & \text{if} \quad s \leq s' \\ \frac{1}{4}\left(\frac{1}{1-s} + p_{on} - \frac{\tau}{2} - c\right)^2 & \text{if} \quad s > s'. \end{cases} \tag{3.14}$$

where s' is the maximum rate of captive offline consumers for the online market to influence the price and profit of an offline market that is covered by a monopoly:

$$s' = \frac{1 - \frac{3\tau}{2} - p_{on} - c}{2 - \frac{3\tau}{2} - p_{on} - c}.$$

In an offline monopoly, the captive consumers are served only by the local firm. Therefore, the captive market may constitute a significant share of the offline firm as it gets larger. In that case, it is optimal for the local firm to set the price regardless of the online market.

Previous section shows that, in a duopoly the offline price and profit decrease with increased online penetration through lower online price. This also holds for a monopoly. However, as the online price drops, one of the local firms leaves the market because the duopoly profits are lower than F and the offline price is set by the surviving monopoly. In this case, the offline price actually increases as the offline price decreases. Figure 3.4 shows how the offline price and the profit of the surviving local firm changes if the online competition leads to a structural change in the local market.

A local firm may benefit from a stronger online competition if it drives away the local one, conditional on being the surviving firm. In this case, a part of the non-captive consumers switch to the online market and enjoy higher surplus. However, the captive offline consumers and those with low online valuation have a lower surplus due to the monopoly price.

3.5 Conclusion

The online market diffusion has reached significant levels in highly digitalized markets. The online market brings extra competition to the local markets and potentially significantly affects local market equilibrium. Given that the local market structure is not changed, I show that both the online and offline consumers bene-

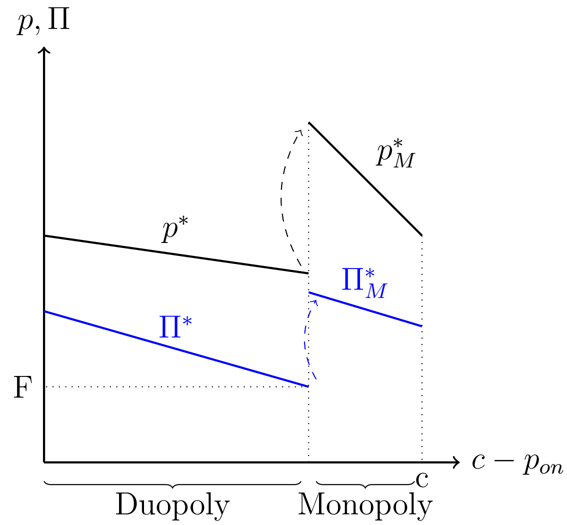


Figure 3.4: Local Price and Profit with Online Price Competition in case of a Market Structure Change

fit from online diffusion thanks to lower offline prices, while local firms generate a lower profit. The transportation cost hurts consumer surplus because of a higher offline price, but it does not necessarily increase the local profit in case of high online penetration.

Online competition may lead to structural changes in the local markets by grabbing the local market profit and leading some local firms to leave the market. In this case, the offline price increases with the online competition. While the consumers with online options are better off, the captive offline consumers suffer significantly from the higher prices due to a lack of local competition.

The local firms need to develop strategies to survive the simultaneous competition with the online and offline markets. One strategy to reconsider is differentiation. It is more likely for consumers who are distant from local shops to go online. Hence, it is not optimal for local firms to fully differentiate from each other when there is online competition. The location choice of the firms with the online market is an intriguing topic for further research. Less differentiated local firms are closer to the socially optimal locations than extreme differentiation since it results in less welfare loss to transportation. On the other hand, the size of the local market shrinks, generating less surplus.

Bibliography

Akerlof, G. (1970). The market for 'lemons': qualitative uncertainty and the market mechanism. *Quarterly Journal of Economics*, 89, 488-500.

Arrow, K. J. (1970). *Essays in the theory of risk-bearing*. North-Holland Publishing Company.

Bakos, Y. (1997). Reducing buyer search costs: Implications for electronic marketplaces. *Management Science*, 43(12), 1676–1692.

Balasubramanian, S. (1998). Mail versus mall: A strategic analysis of competition between direct marketers and conventional retailers. *Marketing Science*, 17(3), 181–195.

Bar-Isaac, H., & Tadelis, S. (2008). Seller reputation. *Foundations and Trends in Microeconomics*, 4(4), 273-351.

Baye, M. R., & Scholten, P. (2004). Price dispersion in the small and in the large: Evidence from an internet price comparison site. *The Journal of Industrial Economics*, 52(4), 463–496.

Bergemann, D., & Bonatti, A. (2011). Targeting in advertising markets: implications for offline versus online media. *The RAND Journal of Economics*, 42(3), 417-443.

Braghieri, L. (2019). Targeted advertising and price discrimination in intermediated online markets. Available at SSRN: http://dx.doi.org/10.2139/ssrn.3072692.

Brown, J., & Goolsbee, A. (2002). Does the internet make markets more competitive? evidence from the life insurance industry. *Journal of Political Economy*, 110(3), 481–507.

Brynjolfsson, E. & Smith, M. D. (2000). Frictionless commerce? a comparison of internet and conventional retailers. *Management Science*, 46(4), 563–585.

Chatterjee, P. (2001). Online reviews: Do consumers Use them?. *Advances in Consumer Research*, 28(1), 129-133.

Chevalier, J. & Mayzlin, D. (2006). The effect of word of mouth on sales: Online book reviews. *Journal of Marketing Research*, 43(3).

Choi, H., Mela, C. F., Balseiro, S. R., & Leary, A. (2020). Online display advertising markets: A literature review and future directions. *Information Systems Research*, 31(2), 556-575.

Clay, K., Krishnan, R. & Wolff, E. (2001). Prices and price dispersion on the web: Evidence from the online book industry. *The Journal of Industrial Economics*, 49(4), 521–539.

d'Aspremont, C., Gabszewicz, J. J. & Thisse, J. F. (1979). On hotelling's "stability in competition". *Econometrica*, 47(5), 1145–1150.

DeGroot, M. H. (2005). *Sequential Sampling.* John Wiley Sons, Ltd.

Dellarocas, C. (2003). The digitization of word of mouth: Promise and challenges of online feedback mechanisms. *Management Science*, 49(10), 1407-1424.

Dellarocas, C. (2005). Reputation mechanism design in online trading environments with pure moral hazard. *Information Systems Research*, 16(2), 209-230.

Dellarocas, C. (2006). Reputation mechanisms. *Handbook on Economics and Information Systems.* Elsevier Publishing.

De streel, A., & Jacques, F. (2019). Personalised pricing and EU law.

Economides, N. (1984). The principle of minimum differentiation revisited. *European Economic Review*, 24(3), 345–368.

Eliashberg, J., & Shugan, S. (1997). Film critics: Influencers or predictors?. *Journal of Marketing*, 61(2), 68-78.

Ellman, M. (2017). Online social networks: Approval by design. *Working Papers 17-18 NET Institute.*

Evans, D., S. (2009). The online advertising industry: Economics, evolution, and privacy. *Journal of economic perspectives*, 23(3), 37-60.

Fradkin, A., Grewal, E., Holtz, D., & Pearson, M. (2014). Reporting bias and reciprocity in online reviews: Evidence from field experiments on Airbnb. *Proceedings of the Sixteenth ACM Conference on Economics and Computation*, 641-641. ACM.

Goldmanis, M., Hortaçsu, A., Syverson, C. & Emre, Ö. (2010). E-commerce and the market structure of retail industries. *The Economic Journal*, 120(545), 651–682.

Grossman, G. M., & Shapiro, C. (1984). Informative advertising with differentiated products. *The Review of Economic Studies*, 51(1), 63-81.

Guo, W. C. & Lai, F. C. (2017). Prices, locations and welfare when an online retailer competes with heterogeneous brick-and-mortar retailers. *The Journal of Industrial Economics*, 65(2), 439–468.

Johnson, J. P. (2013). Targeted advertising and advertising avoidance. *The RAND Journal of Economics*, 44(1), 128-144.

Gaudeul, A. & Jullien, B. (2005). E-Commerce, two-sided markets and infomediation. *IDEI Working Paper*, 380.

Hotelling, H. (1929). Stability in competition. *The Economic Journal*, 39(153), 41–57.

Holmström, B. (1999). Managerial incentive problems: A dynamic perspective. *The Review of Economic Studies*, 66(1), 169-182.

Hu, N., Pavlou, P., & Zhang, J. (2009). Why do online product reviews have a J-shaped distribution? overcoming biases in online word-of-mouth communication. *Communications of the ACM*, 52(10), 144-147.

Hu, N., Liu, L., & Sambamurthy, V. (2011). Fraud detection in online consumer reviews. *Decision Support Systems*, 50(3), 614-626.

Iyer, G., Soberman, D., & Villas-Boas, J. M. (2005). The targeting of advertising. *Marketing Science*, 24(3), 461-476.

Liu, Y. (2006). Word-of-mouth for movies: Its dynamics and impact on box office revenue. *Journal of Marketing*, 70(3), 74-89.

Loginova, O. (2009). Real and virtual competition. *The Journal of Industrial Economics*, 57(2), 319–342.

Madden, P. & Pezzino, M. (2011). Oligopoly on a salop circle with centre. *The B.E. Journal of Economic Analysis & Policy*, 11(1), 1–30.

Marotta, V., Wu, Y., Zhang, K., & Acquisti, A. (2021). The welfare impact of targeted advertising technologies. *Information Systems Research.*

Mayzlin, D. (2006). Promotional chat on the internet. *Marketing Science*, 25(2), 155-163.

Mayzlin, D., Dover, Y. & Chevalier, J. (2014). Promotional reviews: An empirical investigation of online review manipulation. *American Economic Review*, 104(8), 2421-55.

Morgan, P. & Manning, R. (1985). Optimal search. Econometrica, 53(4), 923–944.

Morton, F. S., Zettelmeyer, F. & Silva-Risso, J. (2001). Internet car retailing. *The Journal of Industrial Economics*, 49(4), 501–519.

Nosko, C., & Tadelis, S. (2015). *The Limits of reputation in platform markets: An empirical analysis and field experiment* (National Bureau of Economic Research Working paper No. 20830). https://doi.org/10.3386/w20830

Salop, S. C. (1979). Monopolistic competition with outside goods. *The Bell Journal of Economics*, 10(1), 141–156.

Senecal, S., & Nantel, J. (2004). The influence of online product recommendations on consumers' online choices. *Journal of Retailing*, 80, 159–169.

Schlie, E., Rheinboldt, J., & Waesche, N. (2011). *Simply seven: seven ways to create a sustainable internet business*. Springer.

Soberman, D. A. (2004). Research note: Additional learning and implications on the role of informative advertising. *Management Science* 50(2), 1744-1750.

Spence, M., & Zeckhauser, R. (1971). Insurance, information, and individual action. *The American Economic Review*, 61(2), 380-387.

Tadelis, S. (2016). Reputation and feedback systems in online platform markets. *Annual Review of Economics*, 8, 321-340.

Täuscher, K., & Laudien, S. M. (2018). Understanding platform business models: A mixed methods study of marketplaces. *European Management Journal*, 36(3), 319-329.

Zeckhauser, R. (1970). Medical insurance: A case study of the trade-off between risk spreading and appropriate incentives. *Journal of Economic Theory*, 2, 10-26.

Zervas, G., Proserpio, D., & Byers, J. (2015). A first look at online reputation on airbnb, where every stay is above average. Available at SSRN: http://dx.doi.org/10.2139/ssrn.2554500

Appendix A

Seller Reputation with an Imperfect Review System

A.1 Proofs

Proof of Proposition 1.1. We look for the pure-strategy equilibria of the model by considering all of the cases that can emerge as a function of the behavior of the second buyer. For this purpose, we examine all possible combinations of reviews which may lead the second buyer to engage in a transaction with the seller. There are 8 possible combination of reviews: $\varnothing$ - no trade, (H,N,L), (H), (H,N), (N), (L), (N,L), (H,L). For each case, we assume that both types of seller expect a trade in the second period only if the specific case of reviews are posted and optimize their effort decision accordingly. Then the second buyer updates her belief according to the optimal effort by each type of seller. Finally, we check if the beliefs are confirmed; that is, if the updated beliefs of the second buyer indeed result in a trade in the second period, so that the corresponding case is a pure-strategy perfect Bayesian Nash equilibrium.

(I) If the seller expects that there will not be a second-period transaction regardless of the first period outcome, then he chooses the level of effort according to the following program:

$$\max_e p - c(e).$$

Hence both types of seller provide the minimum effort level.

Given that $e^g = e^b = \underline{e}$, we have: $\mu_2^H(\underline{e},\underline{e}) > \mu_2^N(\underline{e},\underline{e}) > \mu_2^L(\underline{e},\underline{e})$. Finally, the second buyer will indeed not purchase even under the high review if and only if $\underline{\mu} \geq \mu_2^H(\underline{e},\underline{e})$, which boils down to following condition:

$$\mu_1 \leq \frac{b(p-b\underline{e})}{b(p-b\underline{e})+g(g\underline{e}-p)}.$$

However, this condition does not hold under the assumption A1: $\mu_1 \geq \underline{\mu} = \frac{p-b\underline{e}}{\underline{e}(g-b)}$.
Therefore, there does not exist an equilibrium of this type.

(II) **(H)** If the seller expects a transaction in the second period only if the review of the first period is posted as a High review, then he chooses the level of effort according to the following program:

$$\max_e p - c(e) + \theta e q(p - c(\underline{e}))$$

$$\Longrightarrow \quad e_A^g = c'^{-1}(gq(p-c(\underline{e}))) \quad \text{and} \quad e_A^b = c'^{-1}(bq(p-c(\underline{e}))).$$

The second buyer indeed transacts only in case of a High review if the following conditions hold: $\mu_2^H \geq \underline{\mu}$, $\mu_2^N, \mu_2^L \leq \underline{\mu}$.

The first condition holds if and only if:

$$\mu_2^H(e_A^g, e_A^b) > \underline{\mu} \iff \mu_1 > \frac{be_A^b(p-b\underline{e})}{be_A^b(p-b\underline{e})+ge_A^g(g\underline{e}-p)}.$$

which is true when: $\mu_1 \geq \underline{\mu} = \frac{p-b\underline{e}}{\underline{e}(g-b)}$.
Given that $\mu_2^N > \mu_2^L$ for $q, r \in (0,1)$, the second and the third conditions hold if and only if:

$$\underline{\mu} \geq \mu_2^N(e_A^g, e_A^b)$$

which gives the following condition:

$$\iff \mu_1 \leq \mu^I = \frac{[be_A^b(r-q)+1-r](p-b\underline{e})}{[be_A^b(r-q)+1-r](p-b\underline{e})+[ge_A^g(r-q)+1-r](g\underline{e}-p)}.$$

Therefore, there exists a Type-A equilibrium if and only if $\mu_1 \leq \mu^I$.

(III) **(H,N)** If the seller expects a transaction in the second period in case the review of the first period is a H-review (H) or there is no review (N) posted, then he chooses the level of effort according to the following program:

$$\max_e p - c(e) + [\theta e + (1-\theta e)(1-r)](p - c(\underline{e}))$$

$$\implies e^g_B = c'^{-1}(gr(p - c(\underline{e}))) \quad \text{and} \quad e^b_B = c'^{-1}(br(p - c(\underline{e}))).$$

The second buyer indeed purchases in case of a high review or no review if the following conditions hold: $\mu_2^H, \mu_2^N \geq \underline{\mu}$, $\mu_2^L \leq \underline{\mu}$.
Given that $\mu_2^H > \mu_2^N$ for $q, r \in (0,1)$, the first and the second condition hold if and only if:

$$\mu_2^N(e^g_B, e^b_B) \geq \underline{\mu}$$

which boils down to the following condition:

$$\iff \mu_1 \geq \mu^{II} = \frac{[be^b_A(r-q) + (1-r)](p - b\underline{e})}{[be^b_A(r-q) + (1-r)](p - b\underline{e}) + [ge^g_A(r-q) + (1-r)](g\underline{e} - p)}.$$

The third condition holds if and only if:

$$\underline{\mu} \geq \mu_2^L(e^g_B, e^b_B) \iff \mu_1 \leq \mu^{III} = \frac{(1 - be^b_A)(p - b\underline{e})}{(1 - be^b_A)(p - b\underline{e}) + (1 - ge^g_A)(g\underline{e} - p)}.$$

Therefore there exist a Type-B equilibrium if and only if $\mu^{II} \leq \mu_1 \leq \mu^{III}$.

(IV) **(H,N,L)** If the seller expects that a transaction in the second period will take place independently of the first period outcome (that is, High (H), No(N) and Low(L) reviews result in trade in the second period), then he chooses the level of effort according to the following program:

$$\max_e p - c(e) + p - c(\underline{e}).$$

Hence, both types of seller provide the minimum effort level.

Given that $e^g = e^b = \underline{e}$, we have: $\mu_2^H(\underline{e}, \underline{e}) > \mu_2^N(\underline{e}, \underline{e}) > \mu_2^L(\underline{e}, \underline{e})$. Finally, the second buyer will indeed transact with the seller even under the low review if and only if $\mu_2^L(\underline{e}, \underline{e}) \geq \underline{\mu}$, which boils down to the following condition:

$$\mu_1 \geq \mu^{IV} = \frac{(p - b\underline{e})(1 - b\underline{e})}{(p - b\underline{e})(1 - b\underline{e}) + (1 - g\underline{e})(\underline{e}g - p)}.$$

Therefore, there exists a Type-C equilibrium if and only if $\mu_1 \geq \mu^{IV}$.

(V) **(N)** If the seller expects a transaction in the second period only if there is no review regarding the first period, then he chooses the level of effort according to the following program:

$$\max_e p - c(e) + [\theta e(1 - q) + (1 - \theta e)(1 - r)](p - c(\underline{e}))$$

$$\implies e^g = c'^{-1}(g(r - q)(p - c(\underline{e}))) \quad \text{and} \quad e^b = c'^{-1}(b(r - q)(p - c(\underline{e}))).$$

Hence, there are two cases for the level of efforts provided by the two type of sellers:

(i) If $r > q$, then $e^g > e^b$.

(ii) If $r < q$, then $e^g = e^b = \underline{e}$.

Given that $e^g \geq e^b$, the following inequality holds in either case: $\mu_2^H > \mu_2^L$. Note that, the second buyer indeed pruchases only in case of no review is posted if the following conditions hold: $\mu_2^N \geq \underline{\mu}$ and $\mu_2^H, \mu_2^L \leq \underline{\mu}$. However, since $\mu_2^H(e^g, e^b) > \mu_2^N(e^g, e^b)$ for $q, r \in (0, 1)$, the first condition implies $\mu_2^H > \underline{\mu}$, which is a contradiction to the second condition that complies with the second buyer's behavior in this case. At equilibrium, the second buyer does not purchase only in case of no review.

(VI) **(L)** If the seller expects a transaction in the second period only if the review of the first period is posted as Low review, then he chooses the level of effort according to the following program:

$$\max_e p - c(e) + (1 - \theta e)r(p - c(\underline{e}))$$

$$\implies e^g = c'^{-1}(-gr(p - c(\underline{e}))) \quad \text{and} \quad e^b = c'^{-1}(-br(p - c(\underline{e}))).$$

Since the marginal cost cannot be negative, both types of seller provide the minimum effort. The second buyer indeed transacts only in case of low review is posted if the following conditions hold: $\mu_2^L \geq \underline{\mu}$, and $\mu_2^H, \mu_2^N \leq \underline{\mu}$. Given $e^g = e^b = \underline{e}$, recall that $\mu_2^H(\underline{e}, \underline{e}) > \mu_2^N(\underline{e}, \underline{e}) > \mu_2^L(\underline{e}, \underline{e})$, which implies the first condition contradicts the second and third condition that complies with the second buyer's behavior in this case. At equilibrium, the second buyer does not purchase only in case of Low review.

(VII) **(N,L)** If the seller expects a transaction in the second period in case the review of first period is a Low review (L) or there is no review (N) posted, then he chooses the level of effort according to the following program:

$$\max_e p - c(e) + [\theta e(1-q) + (1-\theta e)](p - c(\underline{e}))$$

$$\implies e^g = c'^{-1}(-gq(p - c(\underline{e}))) \quad \text{and} \quad e^b = c'^{-1}(-bq(p - c(\underline{e}))).$$

As in the previous case, both types of seller choose the minimum effort.

The second buyer indeed purchases in case of a Low review or no review is posted if the following conditions hold: $\mu_2^N, \mu_2^L \geq \underline{\mu}$, and $\mu_2^H \leq \underline{\mu}$. Again given $e^g = e^b = \underline{e}$, we have $\mu_2^H(\underline{e}, \underline{e}) > \mu_2^N(\underline{e}, \underline{e}) > \mu_2^L(\underline{e}, \underline{e})$, which implies that the first and the second conditions contradict the third condition that complies with the second buyer's behavior in this case. In equilibrium, the second buyer does not purchase if a low review or no review is posted. There is no equilibrium in this case, either.

(VIII) **(H,L)** If the seller expects a transaction in the second period in case the review of the first period is a High (H) or a Low review (L), then he chooses the effort according to the following program:

$$\max_e p - c(e) + [\theta e q + (1-\theta e) r](p - c(\underline{e}))$$

$$\implies e^g = c'^{-1}(g(q-r)(p - c(\underline{e}))) \quad \text{and} \quad e^b = c'^{-1}(b(q-r)(p - c(\underline{e}))).$$

Hence, there are two cases for level of efforts provided by the two type of sellers:

(i) If $q > r$, then $e^g > e^b$.

(ii) If $q < r$, then $e^g = e^b = \underline{e}$.

Given that $e^g \geq e^b$, the following inequality holds in either case: $\mu_2^H > \mu_2^L$. The second buyer indeed transacts in case of a High review or a Low review is posted if the following conditions hold: $\mu_2^H, \mu_2^L \geq \underline{\mu}$ and $\mu_2^N \leq \underline{\mu}$. However, since $\mu_2^L(e^g, e^b) < \mu_2^N(e^g, e^b)$ for $q, r \in (0,1)$, the second condition implies $\mu_2^N > \underline{\mu}$, which is a contradiction to the third condition that complies with the second buyer's behavior in this case. At equilibrium, the second buyer does not transact in case a High review or Low review is posted.

□

Proof of Proposition 1.3. Given subgame perfection, the analysis starts from the last period and builds by backward induction. If the transaction takes place in the last period, then both types of seller provide minimum effort in the second period.

On the other hand, the second buyer has information to update her belief about the type of the seller. In the pooling equilibria, the price set by the seller is not informative since both types of seller set the same price. However, the review of the previous transaction includes information regarding the type of the seller. Consider the second buyer updates her belief according to Bayes' Rule and $\mu_2^R(e_1^g, e_1^b)$ is the updated belief (given by equations 1.1-1.3) and $\bar{p}_2^R$ is the highest price at which the second buyer agrees to buy after observing review R:

$$\mu_2 = \begin{cases} \mu_2^H(e_1^g, e_1^b) & \text{if} \quad R = H \quad \& \quad p_2 = \bar{p}_2^H \\ \mu_2^N(e_1^g, e_1^b) & \text{if} \quad R = N \quad \& \quad p_2 = \bar{p}_2^N \\ \mu_2^L(e_1^g, e_1^b) & \text{if} \quad R = L \quad \& \quad p_2 = \bar{p}_2^L \\ 0 & \text{otherwise.} \end{cases}$$

where e_1^g and e_1^b are the beliefs of the buyer about the effort provided by the seller if he is g-type and the b-type seller, respectively. That is, the second buyer expects the price to be set to the best response of the seller for each possible state of review.

Given the belief of the second buyer, it is in the seller's best interest to set the price equal to $\bar{p}^R$ following a review R.

Carrying on with backward induction, in the first period, the seller chooses the optimal effort if the first buyer agrees to buy. While the effort is costly, it can increase the second-period profit of the seller through the review system and the second buyer's valuation. Hence, the seller chooses his first-period effort according to the following program, taking the prices as given:

$$\max_{e_1} \underbrace{p_1 - c(e_1)}_{\substack{\text{First Period}\\ \text{Profit}}} + \underbrace{\theta q e_1 p^H + r(1-\theta e)p^L + \Big((1-q)\theta e_1 + (1-r)(1-\theta e_1)\Big)p^N - c(\underline{e})}_{\text{Expected Second Period Profit}}$$

where θ denotes the type of the seller. The first order condition yields:

$$e_1^\theta = c'^{-1}\Big(q\theta(p^H - p^N) + r\theta(p^N - p^L)\Big).$$

The first buyer has no informative signal since both types of seller sets the same price in a pooling equilibrium. She keeps the initial belief as long as the price is set to her valuation $\bar{p}_1$. Given the optimal effort of the seller, it is the seller's best interest to set $p_1 = \bar{p}_1$ and any other price is a probability-zero event for the first buyer. Hence, the following beliefs are Bayesian:

$$\mu_1 = \begin{cases} \mu_0 & \text{if} \quad p_1 = \bar{p}_1 \\ 0 & \text{otherwise.} \end{cases}$$

Finally, given the belief of the first buyer, the optimal price strategy of both types of seller is to set the price equal to the valuation of the first buyer, $\bar{p}_1$.

$\square$

Comparison of the Updated Beliefs: μ_2^H, μ_2^N and μ_2^L. Firstly, we compare the updated beliefs with High and Low reviews:

$$\mu_2^H(e^g, e^b) \gtrless \mu_2^L(e^g, e^b) \iff \frac{\mu_1 g e^g}{\mu_1 g e^g + (1-\mu_1) b e^b} \gtrless \frac{\mu_1(1-ge^g)}{\mu_1(1-ge^g) + (1-\mu_1)(1-be^b)}$$

$$\iff \frac{\mu_1 g e^g}{(1-\mu_1) b e^b} \gtrless \frac{\mu_1(1-ge^g)}{(1-\mu_1)(1-be^b)} \iff ge^g - gbe^g e^b \gtrless be^b - gbe^g e^b$$

$$\iff ge^g \gtrless be^b.$$

Therefore, since $g > b$, we have the following:

$$\mu_2^H(e^g, e^b) > \mu_2^L(e^g, e^b) \quad \text{for} \quad e^g \geq e^b.$$

Secondly, while $\mu_2^H(e^g, e^b)$ and $\mu_2^L(e^g, e^b)$ do not change with the probabilities q and r, $\mu_2^N(e^g, e^b)$ satisfies the following properties:

$$\frac{\partial \mu_2^N(e^g, e^b)}{\partial q} < 0 \qquad \text{and} \qquad \frac{\partial \mu_2^N(e^g, e^b)}{\partial r} > 0.$$

Moreover, at its maximum value ($q = 0$, $r = 1$) we have $\mu_2^H(e^g, e^b) = \mu_2^N(e^g, e^b)$ and at its minimum value ($q = 1$, $r = 0$) we have $\mu_2^L(e^g, e^b) = \mu_2^N(e^g, e^b)$. Therefore, for $q, r \in (0, 1)$ we obtain the following relation by combining the two conditions:

$$\mu_2^H(e^g, e^b) > \mu_2^N(e^g, e^b) > \mu_2^L(e^g, e^b) \quad \text{for} \quad e^g \geq e^b.$$

□

Existence of Equilibrium. We pairwise compare the boundary values of the equilibria in Proposition 1.1.

For the comparison between μ^I and μ^{II}, notice first that for $q = r$ we have $\mu^I = \mu^{II} = \frac{p - b\underline{e}}{\underline{e}(g-b)} = \underline{\mu}$. Next, we check the marginal effect of q and r on μ^I and μ^{II}:

$$\frac{\partial \mu^I}{\partial q} = \frac{(g\underline{e} - p)(p - b\underline{e})(1 - r)(ge_A^g - be_A^b)}{C^2} > 0 \quad \text{and}$$

$$\frac{\partial \mu^{II}}{\partial q} = \frac{(g\underline{e} - p)(p - b\underline{e})(1 - r)(ge_B^g - be_B^b)}{C^2} > 0$$

where $C = [be_A^b(r - q) + 1 - r](p - b\underline{e}) + [ge_A^g(r - q) + 1 - r](g\underline{e} - p)$. Therefore we have:

$$\frac{\partial \mu^I}{\partial q} \gtrless \frac{\partial \mu^{II}}{\partial q} \iff (ge_A^g - be_A^b) \gtrless (ge_B^g - be_B^b)$$

which is equivalent to the following (for any increasing and convex cost function):

$$\frac{\partial \mu^I}{\partial q} \gtreqless \frac{\partial \mu^{II}}{\partial q} \iff q \gtreqless r.$$

Given that $\mu^I = \mu^{II} = \underline{\mu}$ for $q = r$, for a fixed r we can see that:

- If q increases from $q = r$, both μ^I and μ^{II} increase. Moreover $\frac{\partial \mu^I}{\partial q} > \frac{\partial \mu^{II}}{\partial q}$. Therefore $\mu^I > \mu^{II}$.
- If q decreases from $q = r$, both μ^I and μ^{II} decrease. Moreover $\frac{\partial \mu^I}{\partial q} < \frac{\partial \mu^{II}}{\partial q}$. Therefore $\mu^I > \mu^{II}$.

Hence, we conclude $\mu^I \leq \mu^{II}$ in general.

Next, we compare μ^{IV} and μ^{III}:

$$\mu^{IV} \gtreqless \mu^{III} \iff (1 - b\underline{e})(1 - ge_B^g) \gtreqless (1 - be_B^b)(1 - g\underline{e}).$$

If the cost function is $c(e) = ce^2$ with $c > 0$, we obtain the optimal effort level of both types of the seller as following:

$$e_B^g = \frac{gr(p - c(\underline{e}))}{2c} \quad \text{and} \quad e_B^b = \frac{br(p - c(\underline{e}))}{2c}.$$

After we plug the efforts into the comparison and rearrange, we get the following conditions:

$$\begin{aligned}
(1 - b\underline{e})(1 - g^2r(p - c(\underline{e}))/2c) &\gtreqless (1 - b^2r(p - c(\underline{e}))/2c)(1 - g\underline{e}) && \iff \\
1 - g^2r(p - c(\underline{e}))/2c - b\underline{e} + g^2br\underline{e}(p - c(\underline{e}))/2c &\gtreqless \\
&\quad 1 - b^2r(p - c(\underline{e}))/2c - g\underline{e} + gb^2r\underline{e}(p - c(\underline{e}))/2c && \iff \\
g\underline{e} - b\underline{e} &\gtreqless \frac{r(p - c(\underline{e}))}{2c}[\underline{e}gb(b - g) + (g^2 - b^2)] && \iff \\
\underline{e}(g - b) &\gtreqless \frac{e_B^g}{g}(g - b)[g + b - gb\underline{e}] && \iff \\
\underline{e} &\gtreqless \frac{e_B^g}{g}(g + b - gb\underline{e}) && \iff \\
g\underline{e} &\gtreqless ge_B^g + e_B^g b(1 - g\underline{e}).
\end{aligned}$$

Since $e_B^g \geq \underline{e}$ and $e_B^g b(1 - g\underline{e}) \geq 0$ we have $\mu^{IV} \leq \mu^{III}$.

Above results guarantee that there is at least one PBE for every $\mu_1 \in [\underline{\mu}, 1]$ with the cost function $c(e) = ce^2$ where $c > 0$. $\square$

Appendix B

Product Market Outcomes with Targeted and Random Advertising

B.1 Proofs

Proof of Proposition 2.2: Without loss of generality, consider an equilibrium where $p_A \geq p_B$. Given (p_B, ϕ_B), the seller A's problem is the following:

$$\max_{p_A, \phi_A} \quad p_A\left(\phi_A(V-p_A)p_B + \phi_A(1-\phi_B)(V-p_A)(V-p_B) + \phi_A\phi_B(V-p_A)\frac{(V-p_A)}{2}\right) - a\phi_A.$$

The F.O.C.'s of the problem are:

$$\begin{aligned}
\frac{\partial \Pi_A}{\partial p_A} &= \frac{\phi_A}{2V^2}\left(3p_A^2\phi_B - 2p_A(2V + 2\phi_B p_B + c\phi_B) + V(2V + 2\phi_B p_B - V\phi_B)\right) \\
\frac{\partial \Pi_A}{\partial \phi_A} &= \frac{p_A(V-p_A)[V(2-\phi_B) + 2\phi_B p_B - p_A\phi_B]}{2V^2} - a.
\end{aligned}$$

Similarly, the Seller B's problem is the following:

$$\begin{aligned}
\max_{p_B, \phi_B} \quad p_B\Bigg(&\phi_B(V-p_B)p_A + \phi_B(1-\phi_A)(V-p_B)(V-p_A) \\
&+\phi_A\phi_B(V-p_A)\frac{(V+p_A-2p_B)}{2}\Bigg) - a\phi_B.
\end{aligned}$$

The F.O.C.'s of the problem are:

$$
\begin{aligned}
\frac{\partial \Pi_B}{\partial p_B} &= \frac{\phi_B}{2V^2}\left(V^2(2-\phi_A) - 4Vp_B - \phi_A p_A^2 + 2V\phi_A p_A\right) \\
\frac{\partial \Pi_B}{\partial \phi_B} &= \frac{p_B[V^2(2-\phi_A) - \phi_A p_A^2 + 2V(p_A\phi_A - p_B)]}{2V^2} - a.
\end{aligned}
$$

We note that a seller's marginal return of advertising does not change with own advertising.

(i) We first analyze the possibility of an equilibrium where both sellers advertise to all of the consumers, that is, $\phi_A = \phi_B = 1$. In this case, the FOC's with respect to p_A and p_B lead to a symmetric equilibrium in prices with $p_A = p_B = p^*$, where

$$
p^* = (\sqrt{2} - 1)V.
$$

It is profitable for each seller to advertise his product to the whole market at equilibrium when the rival advertises to the whole market under the following conditions:

$$
\begin{aligned}
\frac{\partial \Pi_A}{\partial \phi_A}|_{p_A=p_B=p^*,\phi_B=1} \geq 0 &\iff V \geq \frac{1}{3-2\sqrt{2}}a \\
\frac{\partial \Pi_B}{\partial \phi_B}|_{p_A=p_B=p^*,\phi_A=1} \geq 0 &\iff V \geq \frac{1}{3-2\sqrt{2}}a,
\end{aligned}
$$

Moreover, if $V \geq \frac{1}{3-2\sqrt{2}}a$, then the profit of each firm is positive since:

$$
\Pi_{A,B}^R \geq 0 \iff V \geq \frac{1}{3-2\sqrt{2}}a \approx 5.83a
$$

(ii) We consider now a potential equilibrium where one seller does not advertise his product to any consumer, $\phi_B = 0$. Then firm A is a monopolist. We can use the analysis in Section 2.3.1 to state that if $V \geq 4a$, then $\phi_A = 1$ and $p_A = V/2$; otherwise firm A also does not enter the market.

We check when it is optimal for the firm B not to enter the market when firm A fully advertises its product at monopoly price:

$$\left.\frac{\partial \Pi}{\partial p_B}\right|_{p_A=\frac{v}{2},\phi_A=1} = 0 \implies p_B = \frac{7V}{16}$$
$$\left.\frac{\partial \Pi}{\partial \phi_B}\right|_{p_A=\frac{v}{2},\phi_A=1,p_B=\frac{7V}{16}} \leq 0 \iff V \leq \frac{256}{49}a \approx 5,23a.$$

(iii) A third type of equilibrium may emerge where $\phi_A = 1$ and $\phi_B \in (0, 1)$. The following set of conditions need to be satisfied at the equilibrium:

$$\left.\frac{\partial \Pi_A}{\partial p_A}\right|_{\phi_A=1} = 0, \quad \left.\frac{\partial \Pi_B}{\partial p_B}\right|_{\phi_A=1} = 0, \quad \left.\frac{\partial \Pi_B}{\partial \phi_B}\right|_{\phi_A=1} = 0.$$

The solution to the above set of equations is the following:

$$\begin{aligned} p_A^* &= V - \sqrt{2V}\sqrt{V - 2\sqrt{aV}} \\ p_B^* &= \sqrt{aV} \\ \phi_B^* &= \frac{8V(a+V) + 5V\sqrt{2V}\sqrt{V - 2\sqrt{aV}} - \sqrt{aV}(25V - 12\sqrt{2V}\sqrt{V - 2\sqrt{aV}})}{2V^2 - \sqrt{aV}(4V + 16a - 7\sqrt{aV})}. \end{aligned} \tag{B.1}$$

If $V \in (5.23a, 5.83a)$, the solution satisfies the following: $p_A^* \in (0, V)$, $p_B^* \in (0, V)$, and $\phi_B^* \in (0, 1)$. Moreover, $\phi_B^*|_{V=5.23a} = 0$ and $\phi_B^*|_{V=5.83a} = 1$.

Given the best responses in (B.1) and $\phi_A = 1$, the firm A has no incentive to reduce the advertising since marginal return to advertising is positive if $V \in (5.23a, 5.83a)$:

$$\left.\frac{\partial \Pi_A}{\partial \phi_A}\right|_{\text{Eq.(B.1)},\phi_A=1} > 0 \qquad \text{for} \quad V \in (5.23a, 5.83a).$$

□

Proof. Proof of the expressions for the market outcomes in an asymmetric duopoly with random ads.

We provide the profit of the firms and the consumer surplus in the asymmetric duopoly equilibrium with random advertisement.

The demand for good A which is fully advertised to the market is given by two parts: The consumers who prefer good A regardless of receiving the ad for good B

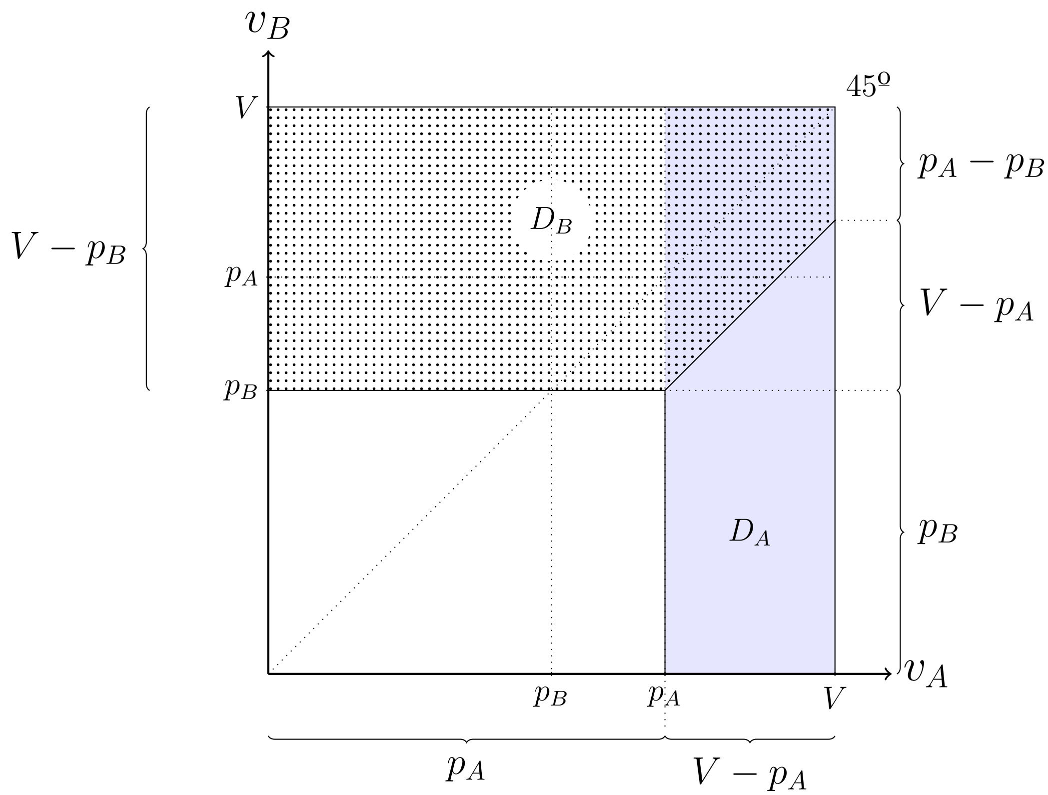


Figure B.1: Demand for goods in the asymmetric duopoly, $p_A \geq p_B$

and the consumers who buy good A since they receive only the ad for good A (who would prefer good B if they receive the ad for it):

$$D_A^{AD} = \frac{(V - p_A + 2p_B)(V - p_A)}{2V^2} + \frac{(1 - \phi_B)(V - p_A)(V + p_A - 2p_B)}{2V^2}$$

The demand for good B is the following:

$$D_B^{AD} = \frac{\phi_B}{V^2}\left(V(V - p_B) - \frac{(V - p_A)^2}{2}\right)$$

The profit of the firm B is equal to zero at the asymmetric duopoly, because $\frac{\partial \Pi}{\partial \phi_B}$ is independent of ϕ_B and $\frac{\partial \Pi}{\partial \phi_B}$ evaluated at the optimum is zero. Hence, Π_B is equal to the profit without advertisement which is zero.

The profit of the firm A in the asymmetric duopoly is the following:

$$\Pi_A^{AD} = p_A\bigg(\frac{(V-p_A+2p_B)(V-p_A)}{2V^2} + \frac{(1-\phi_B)(V-p_A)(V+p_A-2p_B)}{2V^2}\bigg) - a$$

,

Since the competing firm's incentive to advertise the product increases with the product's value, Π_A^{AD} is decreasing in V, given the advertising cost a, and it has the following values at the extremes of the interval where the asymmetric duopoly exists:

$$\begin{aligned}
\Pi_A^{AD} &= \Pi_A^{1M} && \text{if} \quad V = 5.23a \\
\Pi_A^{AD} &= 0 && \text{if} \quad V = 5.83a
\end{aligned}$$

The consumer surplus is found for the consumer who buy good A and good B, separately. The consumer surplus due to good A is given as follows:

$$\begin{aligned}
CS_A^{AD} &= \frac{1}{V^2}\bigg(\int_{p_A}^{V}\int_{0}^{p_B}(\tilde{v}_A - p_A)d\tilde{v}_B d\tilde{v}_A + \int_{p_A}^{V}\int_{p_B}^{\tilde{v}_A-p_A+p_B}(\tilde{v}_A - p_A)d\tilde{v}_B d\tilde{v}_A \\
&\quad + (1-\phi_B)\int_{p_A}^{V}\int_{\tilde{v}_A-p_A+p_B}^{V}(\tilde{v}_A - p_A)d\tilde{v}_B d\tilde{v}_A\bigg) \\
&= \frac{(V-p_A)^2}{6V^2}\bigg((2V+3p_B-2p_A) + (1-\phi_B)(V+2p_A-3p_B)\bigg)
\end{aligned}$$

The consumer surplus of buyers of good B is given as follows:

$$\begin{aligned}
CS_B^{AD} &= \phi_B\bigg(CS_B^{1M} - \frac{1}{V^2}\int_{p_B}^{V-p_A+p_B}\int_{\tilde{v}_B-p_B+p_A}^{V}(\tilde{v}_B - p_B)d\tilde{v}_A d\tilde{v}_B\bigg) \\
&= \phi_B\bigg(\frac{(V-p_B)^2}{2V} - \frac{(V-p_A)^3}{6V^2}\bigg)
\end{aligned}$$

The consumer surplus of buyers of both goods is increasing in V, given the advertising cost a. It has following values at the extremes of the interval:

$$
\begin{aligned}
CS^{AD} &= CS^{1M} && \text{if} \quad V = 5.23a \\
CS^{AD} &= CS^{D} && \text{if} \quad V = 5.83a
\end{aligned}
$$

□

Proof. Proof for the expression of the consumer surplus with Two Goods and Symmetric Price

The surplus of the buyers of the two goods are equal when the price of the two goods are the same. The consumer surplus of the buyers who buy good A can be calculated by two parts as shown in Figure B.2. Hence the total consumer surplus is given as follows:

$$
\begin{aligned}
CS &= 2(CS_I + CS_{II}) \\
&= 2\left(\frac{1}{V^2}\int_p^V \int_0^p (\tilde{v}_A - p)d\tilde{v}_B d\tilde{v}_A + \frac{1}{V^2}\int_p^V \int_p^{\tilde{v}_A} (\tilde{v}_A - p)d\tilde{v}_B d\tilde{v}_A\right) \\
&= 2\left(\frac{p(V-p)^2}{2V^2} + \frac{(V-p)^3}{3V^2}\right) \\
&= \frac{(V-p)^2(p+2V)}{3V^2}
\end{aligned}
$$

□

Proof. Proof of Proposition 2.3:

It is shown in Section 2.3.2 that in a monopoly market it is optimal for the seller to target all consumers with positive net utility given the price. Assume that the sellers in a duopoly market adopt the same strategy of targeting consumers:

$$
\underline{v}_A = p_A \quad \text{and} \quad \underline{v}_B = p_B. \tag{A2}
$$

Without loss of generality, consider an equilibrium where $p_A \leq p_B$. Given the targeting assumption (A2), the Seller A's problem is the following:

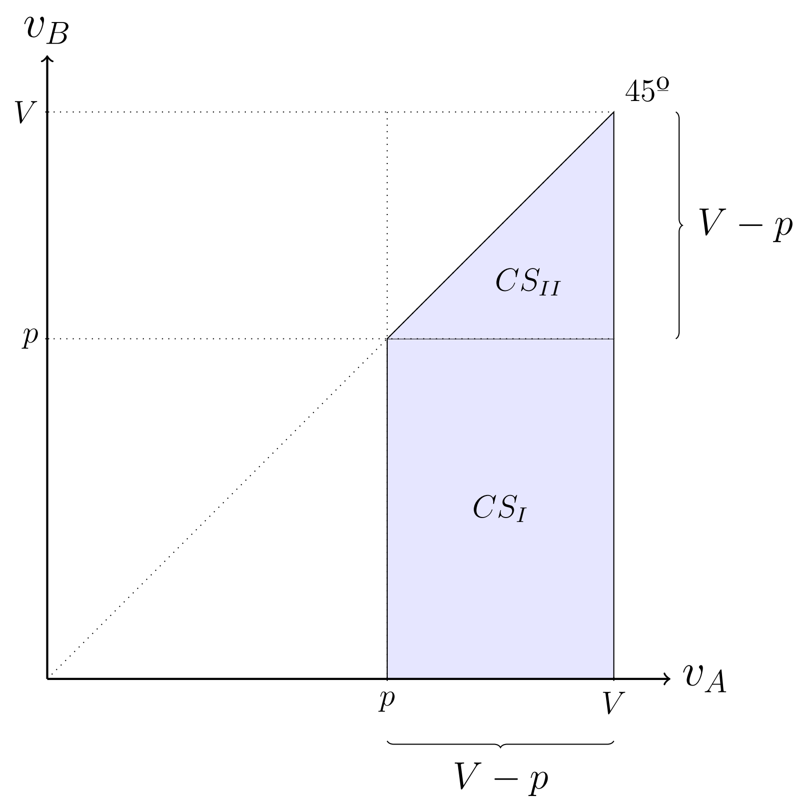


Figure B.2: Consumer Surplus with two goods, $p_A = p_B = p$

$$\max_{p_A} \quad \frac{p_A}{V^2}\left[(V-p_A)p_B + \frac{1}{2}(V+p_B-2p_A)(V-p_B)\right] - a(1-\frac{p_A}{V}).$$

Then, the best response of the Seller A is:

$$p_A(p_B) = \frac{V^2 + 2Vp_B + 2aV - p_B^2}{4V}.$$

Similarly, the Seller B's problem is the following:

$$\max_{p_B} \quad \frac{p_B}{V^2}\left[(v-p_B)p_A + \frac{1}{2}(v-p_B)^2\right] - a(1-\frac{p_B}{V}).$$

And the best response of Seller B is:

$$p_B(p_A) = \frac{1}{3}\left(2V + 2p_A \pm \sqrt{4p_A^2 - 6aV + 2p_AV + V^2}\right).$$

There exists a symmetric solution to the best response functions of the sellers, $p_A = p_B = p^*$, where

$$p^* = \sqrt{2V(V+a)} - V.$$

We confirm that the price set by the 2-product monopolist is higher than the price set by duopoly sellers, given $V > a$:

$$\begin{aligned}
p_T^{2M} > p_T^D \quad &\iff \quad \frac{\sqrt{3V(V+2a)}}{3} > \sqrt{2V^2 + 2aV} - V \\
&\iff 3V + \sqrt{3V(V+2a)} > 3\sqrt{2}\sqrt{V^2 + aV} \\
&\iff 9V^2 + 6V\sqrt{3V(V+2a)} + 3V^2 + 6aV > 18V^2 + 18aV \\
&\iff 3V\sqrt{3V(V+2a)} > 3V^2 + 6aV \\
&\iff \sqrt{3V(V+2a)} > V + 2a \\
&\iff \sqrt{3V} > \sqrt{V+2a} \\
&\iff V > a.
\end{aligned}$$

We now show that this is indeed an equilibrium without the targeting assumption (A2). Suppose the Seller A sets targeting parameter different than the price. We know that it is not optimal to set $p_A >_A$. Hence, we consider $p_A \leq_A$.

Given $p_B =_B= p^*$, two cases may arise:

(I) $\underline{v}_B = p_B = p^* \leq p_A$ and

(II) $p_A \leq \underline{v}_B = p_B = p^*$.

CASE (I):

The Lagrangian of the constrained optimization problem of Seller A given $p_B =_B$, is the following:

$$\mathcal{L}(p_A, \underline{v}_A, \lambda) = \frac{p_A}{2V^2}(V - \underline{v}_A)(V + \underline{v}_A - 2p_A + 2p_B) - a(1 - \frac{\underline{v}_A}{V}) - \lambda(p_A - \underline{v}_A).$$

If $\lambda > 0$, then $p_A =_A$ must be true. The best response of Seller A is identical to the best response obtained with the targeting assumption (A2).

If $\lambda = 0$,

$$\frac{\partial \mathcal{L}}{\partial_A} = \frac{p_A}{2V^2}\Big(- V -_A + 2p_A - 2p_B + V -_A \Big) + \frac{a}{V} = 0.$$

The marginal effect of targeting given the price is the following:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial_A} &= \frac{p_A}{2V^2}\Big(- V -_A + 2p_A - 2p_B + V -_A \Big) + \frac{a}{V} \\ &= \frac{(p_A^2 - p_{AA} - p_A p_B + aV)}{V^2}. \end{aligned}$$

Now we show that the above derivative is negative. First, consider the first two terms in the parenthesis:

$$p_A^2 - p_{AA} \leq 0 \quad \text{since} \quad _A \geq p_A.$$

Second, consider the last two terms in the parenthesis:

$$-p_A p_B + aV \leq -p_B^2 + aV \quad \text{since} \quad p_A \geq p_B > 0.$$

And the right hand side of the above inequality is negative, given $p_B = \sqrt{2V(a+V)} - V$:

$$p_B^2 > aV \iff (V-a)^2 > 0.$$

Hence,

$$-p_A p_B + aV < 0.$$

By the previous result, for all price p_A, the seller can have a higher profit by decreasing $_A$ (targeting more consumers) until $p_A =_A$ under the constraint $p_A \leq_A$. Hence, $\lambda = 0$ also implies the targeting assumption (A2) holds.

CASE (II): $p_A \leq \underline{v}_B = p_B = p^*$.

The Lagrangian of the constrained optimization problem of Seller A given $p_B =_B$, is the following:

$$\mathcal{L}(p_A, \underline{v}_A, \lambda) = \frac{p_A}{V^2}[(V - \underline{v}_A)V - \frac{1}{2}(V -_A - p_B + p_A)^2] - a(1 - \frac{\underline{v}_A}{V}) - \lambda(p_A - \underline{v}_A).$$

If $\lambda = 0$, then the marginal effect of the price is positive:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial p_A} > 0 &\iff (V - \underline{v}_A)V > \frac{1}{2}(V -_A - p_B + p_A)^2 + p_A(V -_A - p_B + p_A) \\ &\iff (V - \underline{v}_A)V > (V -_A - p_B + p_A)(\frac{V -_A - p_B + 3p_A}{2}). \end{aligned}$$

Consider the first terms of the each side of the above inequality:

$$(V - \underline{v}_A) \geq (V -_A - p_B + p_A) \quad \text{since} \quad p_B \geq p_A \geq 0.$$

Next, the second terms of the each side of the inequality satisfies the following:

$$\begin{aligned} &V > \frac{(V -_A - p_B + 3p_A)}{2} \\ &V > 3p_A -_A - p_B \\ &V > (p_A -_A) + (p_A - p_B) + p_A \end{aligned}$$

since either $V > p_A$ or $_A > p_A$ or $p_B > p_A$.

Hence, it is always profitable to increase the price given any level of targeting. That is, $p_A =_A$ must be true under the constraint $p_A \leq_A$. Therefore, $\lambda = 0$ implies the targeting assumption (A2) must hold.

If the seller sets a price lower than the opponent and targets fewer consumers than the consumers with positive net utility for own product, he can always increase his profit by increasing the price. That is, the revenue loss due to competition is recovered with the higher markup.

If $\lambda > 0$, then $p_A =_A$. $\frac{\partial \mathcal{L}}{\partial p_A} = 0 \implies$

$$\begin{aligned} \lambda &= \frac{1}{V^2}\left[V(V - p_A) - \frac{(V - p_A - p_B + p_A)^2}{2} - p_A(V - p_A - p_B + p_A)\right] \\ \lambda &= \frac{1}{V^2}\left[V(V - p_A) - \frac{(V - p_B)^2}{2} - p_A(V - p_B)\right]. \end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial_A} = 0 \implies \lambda = \frac{1}{V^2}\left(p_A p_B - aV\right).$$

$$p_A = \frac{V + 2p_B + 2a}{4} - \frac{p_B^2}{4V}.$$

Given the opponent sets $p_B = p^* = \sqrt{2V(a+V)} - V$, the best response is to set $p_A^* = p^*$:

$$p_A(p_B = p^*) = \sqrt{2V(a+V)} - V = p^*.$$

$\square$

Proof. Proposition 2.4

All the comparisons in the proposition are easy to check except the comparison between the consumer surplus in the asymmetric duopoly with random ads and duopoly with targeted ads. We have the following properties:

$$CS_R^{AD}|_{V=5.23a} < CS_T^D|_{V=5.23a}$$
$$CS_R^{AD}|_{V=5.83a} > CS_T^D|_{V=5.83a}.$$

where both are increasing in the value of the product, given the advertising cost:

$$\frac{\partial CS_R^{AD}}{\partial V} > 0 \quad \text{if} \quad V \in [5.23a, 5.83a]$$
$$\frac{\partial CS_T^{D}}{\partial V} > 0 \quad \text{if} \quad V \in [5.23a, 5.83a].$$

Therefore, there exists a $V^* \in [5.23a, 5.83a]$ such that $CS_R^{AD}|_{V=V^*} = CS_T^D|_{V=V^*}$ and

$$CS_R^{AD} < CS_T^D \quad \text{if} \quad V < V^*$$
$$CS_R^{AD} \geq CS_T^D \quad \text{if} \quad V \geq V^*$$

$\square$

B.2 Multi-Product Monopoly

Here we consider a market where a monopolist produces and advertises two horizontally differentiated products, $k \in \{A, B\}$. The seller's problem is to maximize the total profit by setting the price and the level of advertisement for the two products simultaneously. Given the advertisement decision of the seller, a consumer can be shown zero, one, or two product advertisements.

There is a continuum of consumers with unit mass and heterogeneous valuations for the products. Each consumer is interested in acquiring at most one product. The consumer's valuations for the products are given by $v_i = (v_A, v_B)$ and have a joint uniform distribution on $[0, v] \times [0, v]$. There is still a unit mass of consumers distributed in $[0, v] \times [0, v]$; hence, the density is $\frac{1}{V^2}$.

The consumers who observe one advertisement purchase the product if their net utility is positive. Those who observe two advertisements purchase the product that provides a higher net utility, given it is positive.[1]

Random Ads

First, we consider the setting where the available technology only allows for random advertisement. Let ϕ_k and p_k be the fraction of consumers who receive the advertisement and the price of product $k = A, B$, respectively. Without loss of generality, assume $p_A \geq p_B$.

The demand for the products A and B, $D_A(p_A, p_B, \phi_A, \phi_B)$ and $D_B(p_A, p_B, \phi_A, \phi_B)$, are the same as for the duopoly in Section 2.4.1.

Once we have the demands as a function of the prices and the advertisement strategy, we can analyze the monopolist's problem. Its objective is to maximize the total profit from the two products:

$$\begin{aligned} \max_{p_A, p_B, \phi_A, \phi_B} \quad & p_A D_A(p_A, p_B, \phi_A, \phi_B) + p_B D_B(p_A, p_B, \phi_A, \phi_B) - a\phi_A - a\phi_B \\ \text{s.t.} \quad & V \geq p_A \geq p_B \geq 0 \\ & \phi_A, \phi_B \in [0, 1]. \end{aligned}$$

The solution to this problem requires the optimality conditions for the price and advertising of both products to hold. We modify this problem following the results

[1]The consumer is assumed to choose good A over B if she is indifferent.

in section 2.3.1 and assume there is no partial advertising strategy strictly better for the monopolist.[2] Indeed, numerical exercises of the monopolist's problem show that the optimal advertising of a product is a binary decision. That is, the monopolist's optimal decision of advertising is either to promote the product to the whole market or not promote at all. Hence, we limit the set of advertising of a product such that $\phi_A, \phi_B \in \{0, 1\}$.

Proposition B.1 provides the monopolist's optimal decision with a binary advertisement strategy.[3]

Proposition B.1. *The monopolist with two horizontally differentiated products sets the following price and advertising of the products when random advertising is available:*

i If $V \leq 4a$, the monopolist does not advertise neither product, $\phi_A = \phi_B = 0$.

ii If $4a \leq V \leq 7.4a$, the monopolist fully advertises only one product, $\phi_k = 1$, ($\phi_{-k} = 0$), with the price equal to p_R^M, where[4]

$$p_R^M = \frac{V}{2}.$$

iii If $V \geq 7.4a$, the monopolist fully advertises both products, $\phi_A = \phi_B = 1$, with the same price equal to p_R^{2M}, where

$$p_R^{2M} = \frac{V}{\sqrt{3}}.$$

Proof. Proof of Proposition B.1: Multi-Product Monopoly with Random Advertising The monopolist's problem is to maximize the total profit from the two products. First, consider the monopolist's problem allowing for partial advertisement of the

[2]The optimality conditions are not analytically tractable with partial advertisement of the products, that is, $\phi_A, \phi_B \in (0, 1)$ and the solution of this problem is not available.

[3]See Appendix B.2 for proof of the modified problem.

[4]M: 1 single-product Monopoly, 2M: 2-product Monopoly.

products:

$$\max_{p_A,p_B,\phi_A,\phi_B} \quad p_A D_A(p_A,p_B,\phi_A,\phi_B) + p_B D_B(p_A,p_B,\phi_A,\phi_B) - a\phi_A - a\phi_B$$
$$\text{s.t} \quad V \geq p_A \geq p_B \geq 0$$
$$\phi_A,\phi_B \in [0,1].$$

The F.O.C.s of the problem are:

$$\frac{\partial \Pi}{\partial p_A} = \frac{\phi_A}{V^2}\bigg((1+\phi_B)(V-p_A)p_B + (V-p_A)(1-\phi_B)(V-p_B) + \frac{\phi_B(V-p_A)^2}{2} - Vp_A + \phi_B p_A(p_A-p_B)\bigg)$$
$$\frac{\partial \Pi}{\partial p_B} = \frac{\phi_B}{V^2}\bigg((V-p_B)p_A + (1-\phi_A)(V-p_A)(V-p_B) + \phi_A(V-p_A)\frac{(V+3p_A-2p_B)}{2} - Vp_B\bigg).$$

$$\frac{\partial \Pi}{\partial \phi_A} = \frac{p_A(V-p_A)[p_B+(1-\phi_B)(V-p_B)]}{V^2} + \frac{\phi_B(V-p_A)^2(p_A-p_B)}{2V^2} - a$$
$$\frac{\partial \Pi}{\partial \phi_B} = \frac{p_B(V-p_B)[p_A+(1-\phi_A)(V-p_A)]}{V^2} - \frac{\phi_A(V-p_A)(V+p_A-2p_B)(p_A-p_B)}{2V^2} - a.$$

If the advertising decision is assumed to be a binary decision as the numerical exercises of the monopolist's problem suggest, that is, the monopolist either promotes a product to the whole market or none ($\phi_A, \phi_B \in \{0,1\}$), then there are 3 potential cases:

$\phi_A = \phi_B = 1$	Case 1
$\phi_A = 1 \quad \phi_B = 0 \quad$ (or $\quad \phi_A = 0 \quad \phi_B = 1$)	Case 2
$\phi_A = \phi_B = 0$	Case 3

Consider Case 1. If the monopolist decides $\phi_A = \phi_B = 1$, then the optimal price for the products are $p_A = p_B = p_R^{2M}$ where

$$p_R^{2M} = \frac{V}{\sqrt{3}}.$$

At this price, the monopolist is indeed incentivized to promote product A given that the product B is promoted to the whole market (and similarly for product B) if the following condition holds:

$$\left.\frac{\partial \Pi}{\partial \phi_A}\right|_{p_A=p_B=p_R^{2M},\phi_B=1} > 0 \iff V > \frac{9}{3-\sqrt{3}}a \approx 7.1a.$$

Hence, Case 1 is a candidate for optimal decision of the monopolist for $V > 7.1a$.

Next consider cases 2 where $\phi_B = 0$. This is identical to the situation where the monopolist only promotes product A. As we show in Section 2.3.1, the optimal decision is $\phi_A = 1$ and $p_A = V/2$ if $V \geq 4a$ and $\phi_A = 0$ if $V < 4a$. Hence, Case 2 dominates Cases 3 if $V > 4a$. Moreover, $\phi_B = 0$ is optimal if the following holds:[5]

$$\left.\frac{\partial \Pi}{\partial \phi_B}\right|_{\phi_A=1,p_A=V/2,\phi_B=0,p_B=\frac{7-\sqrt{13}}{6}V} \leq 0 \iff V \leq \frac{216}{13\sqrt{13}-19}a \approx 7.75a.$$

Finally, We show that it is optimal to promote the two products to the whole market rather than promoting a single product to the whole market (Case 1 dominates Case 2), under following condition:

$$\Pi\Big|_{\substack{\mathrm{p}_A = p_B = \frac{V}{\sqrt{3}} \\ \phi_A=\phi_B=1}} > \Pi\Big|_{\substack{\mathrm{p}_A = \frac{V}{2} \\ \phi_A = 1 \\ \phi_B = 0}} \iff V > \frac{36}{8\sqrt{3}-9}a \approx 7.4a.$$

The market structure, the monopolist's optimal decision for the prices and random advertising of the products with two products are summarized below:[6]

[5]The optimal price of product B is higher than $p_A = V/2$ and is given by the following best response function which is obtained by interchanging the products A and B of the original problem where we assume $p_A \geq p_B$:

$$p_B(p_A, \phi_A) = \left.\frac{2V + 3\phi_A p_A - \sqrt{(2V+3\phi_A p_A)^2 - 3\phi_A((2-\phi_A)V^2 + 4V\phi_A p_A)}}{3\phi_A}\right|_{\phi_A=1,p_A=V/2}$$

$$p_B = \frac{7-\sqrt{13}}{6}V > p_A = V/2.$$

[6]2M: 2-product monopoly, 1M: 1-product monopoly, NA: Not Active

$$\begin{array}{llll} \textbf{2M:} & \phi_A = 1, p_A = p_R^{2M} \quad \phi_B = 1, p_B = p_R^{2M} & \text{if} & V \geq 7.4a, \\ \textbf{1M:} & \phi_k = 1, p_k = p_R^{1M} \quad \phi_{-k} = 0 & \text{if} & 7.4a \geq V \geq 4a, \\ \textbf{NA:} & \phi_A = \phi_B = 0 & \text{if} & 4a > V. \end{array} \tag{B.2}$$

□

With random advertising, the monopolist prefers not to promote the second product unless the product's market is large enough. This is because the demand generated by the second product's promotion consists of two types of consumers. The first type is the consumer who is interested in the second product but not in the first (new demand). The second group is the consumers who are interested in both products but prefer the second one to the first. Notice that the second type of consumer is already captured with the promotion of the first product. Because of the second type of consumers, promoting the second product has a demand stealing effect from the first product. If the product's average value is not high enough, then the demand stealing effect dominates the new demand generated with the second product's promotion and the monopolist prefers to promote only one of the products. For products with large markets, the demand generated by the first type of consumers dominates the demand stealing effect and the monopolist promotes both porducts.

For intermediate average values of the product (i.e. $7.4a \geq V \geq 4a$), the monopolist promotes only one of the products. The profit of the monopolist, the consumer surplus, and the total surplus for this market are given by equations (2.5)-(2.7).

If the product is valuable enough for the monopolist to promote both products (i.e. $V \geq 7.4a$), the market outcomes are given as follows:

$$\Pi_R^{2M} = 2(\frac{V}{3\sqrt{3}} - a) \tag{B.3}$$

$$CS_R^{2M} = \frac{2V(9 - 4\sqrt{3})}{27} \tag{B.4}$$

$$TS_R^{2M} = \frac{2V(9 - \sqrt{3})}{27} - 2a. \tag{B.5}$$

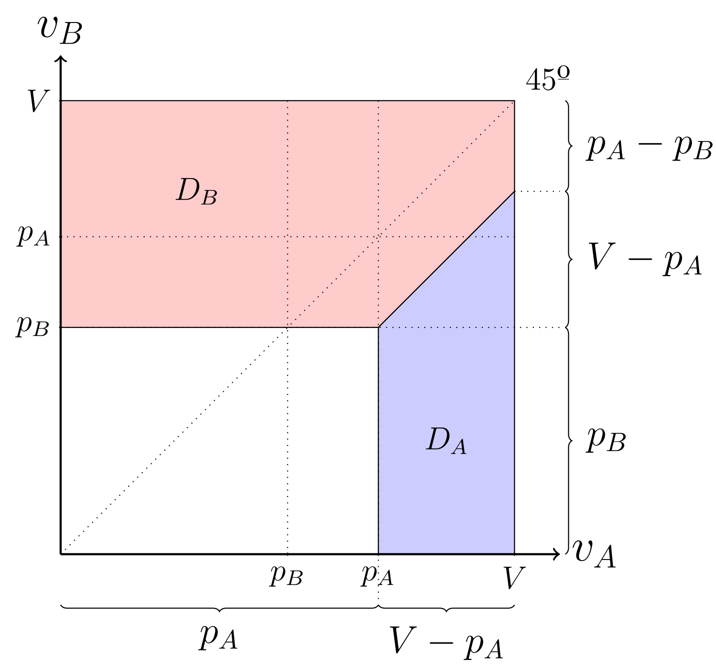


Figure B.3: Demand for two-product monopoly with targeted ads, $p_A \geq p_B$

Targeted Ads

When the monopolist has access to targeted advertisement, the technology allows the firm to promote a product to the consumers with valuations higher than a threshold. There is no exclusivity; that is, a consumer can be targeted for both products if her valuation is higher than the given thresholds for both of them. The firm can set the price and advertisement of two products independently. Let p_k be the price of product k and $\underline{v}_k(p_k)$ the threshold for targeted advertising of product k, where $k \in \{A, B\}$.

The monopolist is assumed to follow the optimal targeting rule obtained in Section 2.3.2 and advertise the product to all of the consumers with positive net utility given its price:

$$\underline{v}_k(p_k) = p_k.$$

The monopolist's problem is to maximize the total profit from the two products by simultaneously setting the price for the products. Without loss of generality, assume $p_A \geq p_B$. Figure B.3 shows the demand generated with targeted ads.

The monopolist's maximization problem with the optimal targeting rule is the following:

$$\begin{aligned}
\max_{p_A, p_B} \quad & \frac{p_A}{2V^2}\Big[(V-p_A)(V-p_A+2p_B)\Big] - a(1-\frac{p_A}{V}) \\
+ \quad & \frac{p_B}{2V^2}\Big[2p_A(V-p_B)+(V+p_A-2p_B)(V-p_A)\Big] - a(1-\frac{p_B}{V}) \\
& \textbf{s.t.} \quad V \geq p_A \geq p_B \geq 0.
\end{aligned}$$

Proposition B.2 characterizes the solution to the monopolist's problem.

Proposition B.2. *Let a monopoly with two products have access to targeted advertisement. If the monopolist targets all consumers who have positive net utility for the product given its price, that is,* $_A(\phi_A) = p_A$ *and* $_B(\phi_B) = p_B$*, then it is optimal to set the price of the products equal to* $p_A = p_B = p_T^{2M}$*, where*

$$p_T^{2M} = \frac{\sqrt{3(V^2+2aV)}}{3}.$$

The fraction of consumers to be advertised for the products are $\phi_A = \phi_B = \phi_T^{2M}$, where

$$\phi_T^{2M} = (1-\frac{p_T^{2M}}{V}) = \frac{3V-\sqrt{3(V^2+2aV)}}{3V}.$$

Notice that the advertising cost, a, is partly passed to the consumer through the price with targeted advertising. This pass-on effect occurs because the firm's pricing strategy is correlated with the optimal advertising strategy. The higher the advertising cost, the fewer consumers are targeted by the firm. As fewer consumers are targeted, the targeted consumers' average valuation increases; therefore, the firm's optimal price increases. On the contrary, the advertising strategy does not change the advertised consumers' average valuation and does not affect the optimal price with random advertising. Due to the pass-on effect under targeted advertising, the price is higher than with random advertising. As a result, fewer consumers are advertised, and fewer transactions are realized with targeted advertising.

The targeting technology minimizes the effect of stealing demand from own product since it allows the firm access the consumers with the highest valuation for each product. Therefore, the product variety does not require high-valued products. The monopolist prefers promoting two products to promoting one product given $V > a$. The profit of the monopolist, the consumer surplus, and the total surplus with targeted advertising are the following:

$$\Pi_T^{2M} = \frac{2}{9V}\left((V+2a)\sqrt{3V(V+2a)} - 9aV\right) \tag{B.6}$$

$$CS_T^{2M} = \frac{2}{27}\left(9V + \sqrt{3V(V+2a)}(\frac{a}{V} - 4)\right) \tag{B.7}$$

$$TS_T^{2M} = \frac{2}{27}\left(9V + \sqrt{3V(V+2a)}(\frac{7a}{V} - 1)\right) - 2a. \tag{B.8}$$

The Comparison of the Market Outcomes

This section compares the market outcomes when the two-product-monopolist has access to random versus targeted advertisement technology through the intermediary. Proposition B.3 (whose proof is immediate) provides the comparison of the market outcomes.

Proposition B.3. *The comparison of the multi-product-monopoly market outcomes with random and targeted advertising is the following:*

i If $a < V < 4a$, the monopolist does not advertise either product with random advertising, whereas he advertises both products to the targeted consumers with targeted advertising.

$$\Pi_T^{2M} > \Pi_R^M = 0$$
$$CS_T^{2M} > CS_R^M = 0$$
$$TS_T^{2M} > TS_R^M = 0.$$

ii If $4a \leq V \leq 7.4a$, the monopolist advertises only one product to the whole market and does not advertise the other with random advertising, whereas he advertises *both products to the targeted consumers with targeted advertising.*

$$
\begin{aligned}
p_T^{2M} &> p_R^{M} \\
\Pi_T^{2M} &> \Pi_R^{M} \\
CS_T^{2M} &< CS_R^{M} \\
TS_T^{2M} &> TS_R^{M}.
\end{aligned}
$$

iii If $V > 7.4a$, the monopolist advertises both products to the whole market with random advertising and to the targeted consumers with targeted advertising.

$$
\begin{aligned}
p_T^{2M} &> p_R^{2M} \\
\Pi_T^{2M} &> \Pi_R^{2M} \\
CS_T^{2M} &< CS_R^{2M} \\
TS_T^{2M} &> TS_R^{2M}.
\end{aligned}
$$

Figure B.4 visualizes the comparison of the multi-product monopoly market outcomes with random and targeted advertisement provided in Proposition B.3. The first observation is that the random advertisement does not lead to a market with product variety unless the product is quite valuable ($V > 7.4a$). In contrast, targeted advertising enables product variety for products of all market size.

In small markets ($V \in [a, 4a]$), the targeted advertising benefits both the firm and the consumers since it activates market that fails to function with random advertising.

In large markets ($V > 7.4a$) both types of advertising favor product variety. Consumers prefer random advertising due to lower prices while the monopolist prefer targeting. The two-product monopoly price with random ads is higher than the one-product monopoly price with random ads but not as high as with targeted ads ($p_T^{2M} > p_R^{2M} > p_R^{1M}$).

In medium sized markets ($V \in [4a, 7.4a]$), random advertising allows for promotion of only one product while targeting allows for product variety due to facilitated entry. The price with targeted advertising is higher than with random advertising. First, the monopolist can set a higher price for the products when there is two products to capture the demand. Second, there exists a pass-on effect of advertising cost with targeted advertising, which does not exist with random advertising. Hence, the consumers who purchase in both ad settings enjoy more surplus with random advertising. On the other hand, the consumers with a high valuation for the second product are better off with the targeted advertising because of product variety.

Although the demand generated for one product is lower with targeted advertising, the total demand for the two products exceeds the demand generated for one product with random advertising. Let $(D_A + D_B)_T$ be the total demand generated with targeted advertising of two products and D_R^M be the total demand with random advertising of a single product. Given $V \in [4a, 7.4a]$, the total number of transactions is higher with targeted advertising:

$$\underbrace{\frac{(V - p_T^{2M})(V + p_T^{2M})}{V^2}}_{(D_A+D_B)_T} > \underbrace{\frac{(V - p_R^M)}{V}}_{D_R^M} \quad \iff \quad v > 4a.$$

More consumers purchase the product at a higher price with targeting than random advertising in these markets. Overall, the consumers are better off with lower prices under random advertisement since loss from higher prices dominates the gain from product variety with targeted advertising. That is, product variety benefits the firms and not the consumers with targeted advertising.

The firm enjoys a higher profit with targeted advertising, with higher revenue and lower advertising cost. The profit increase dominates the decrease in consumer surplus, resulting in a higher market surplus with targeted advertising.

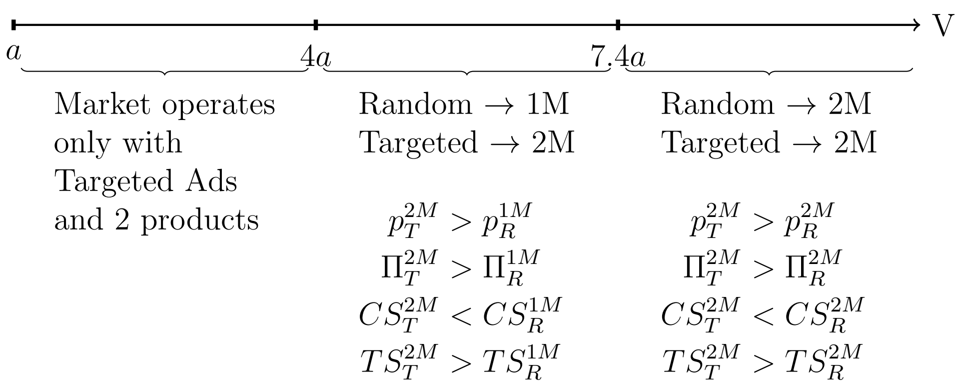


Figure B.4: The comparison of the market outcomes of a 2-product monopoly with random and targeted ads

Appendix C

Local Market Equilibrium with Online Market Diffusion

C.1 Proofs

Proof of Proposition 3.1: Let the $D_1(p_1; p_2, p_{on})$ be the demand for Firm 1 given the price of the local opponent, p_2, and the online price, p_{on}. Then, the Firm 1's profit maximization problem is the following:

$$\max_{p_1}(p_1 - c)D_1(p_1; p_2, p_{on}).$$

The first order condition of Firm 1's problem is:

$$(\frac{p_2 - p_1}{2\tau} + \frac{1}{2})\Big(1 - \frac{(1-s)}{4}(3p_1 - 4p_{on} + p_2 + \tau)\Big) - \frac{3}{4}(p_1 - c)(\frac{p_2 - p_1}{2\tau} + \frac{1}{2}) = 0. \tag{C.1}$$

Since the two offline firms are symmetric, the first order condition of Firm 2 is equal to (C.1), given p_1 and p_{on}. Therefore, the offline market price and the profit for each local firm are the following:

$$p = \frac{1}{2}\left(\frac{1}{1-s} + p_{on} + c + \frac{3\tau}{2} - \sqrt{\Delta}\right)$$
$$\Pi = \frac{1}{8}\left(\frac{1}{1-s} + p_{on} - c + \frac{3\tau}{2} - \sqrt{\Delta}\right)\left(1 - (1-s)\left(c - p_{on} + 2\tau - \sqrt{\Delta}\right)\right),$$

where

$$\Delta = \left(\frac{1}{1-s} + p_{on} - c - \frac{\tau}{2}\right)^2 + 3\tau^2.$$

□

Proof of Corollary 3.1: Let D_1^* be the demand for local Firm 1 at equilibrium:

$$D_1^*(p^*) = \frac{1}{2}\Big(1 - (1-s)(p^* - p_{on} + \frac{\tau}{4})\Big)$$

The comparative statics exercises for the offline market equilibrium are the following:

$$\begin{aligned}
\frac{\partial p^*}{\partial p_{on}} &= \frac{1}{2}\Big(1 - \frac{1}{\sqrt{\Delta}}(\frac{1}{1-s} + p_{on} - c - \frac{\tau}{2})\Big) \\
&= \frac{1}{2\sqrt{\Delta}}(\sqrt{(\frac{1}{1-s} + p_{on} - c - \frac{\tau}{2})^2 + 3\tau^2} - (\frac{1}{1-s} + p_{on} - c - \frac{\tau}{2})) \geq 0. \\
\frac{\partial D_1^*}{\partial p_{on}} &= \frac{1}{2}(1-s)(1 - \frac{\partial p^*}{\partial p_{on}}) \geq 0.
\end{aligned}$$

Hence, the Π^* increases with the online price, p_{on}.

$$\begin{aligned}
\frac{\partial p^*}{\partial s} &= \frac{1}{2}\Big(\frac{1}{(1-s)^2} - \frac{(\frac{1}{1-s} + p_{on} - c - \frac{\tau}{2})}{\sqrt{\Delta}(1-s)^2}\Big) \\
&= \frac{1}{2\sqrt{\Delta}(1-s)^2}(\sqrt{(\frac{1}{1-s} + p_{on} - c - \frac{\tau}{2})^2 + 3\tau^2} - (\frac{1}{1-s} + p_{on} - c - \frac{\tau}{2})) \geq 0.
\end{aligned}$$

The changes in the demand and profit of the local firm with a share of captive offline consumers s are shown by numerical simulation with the set of parameters given below for (s, p_{on}, c, τ):

$$\frac{\partial D_1^*}{\partial s} \geq 0 \quad \frac{\partial \Pi_1^*}{\partial s} \geq 0 \quad \text{for} \quad s \in (0,1), \tau \in (0,1), p_{on} \in [c, 0), c \in (1, p_{on}],$$

and assumption (A1): $1 - c - \frac{3\tau}{2} \geq 0$.

$$\frac{\partial p^*}{\partial \tau} = \frac{1}{4\sqrt{\Delta}}\Big(3(\sqrt{\Delta} - 2\tau) + (\frac{1}{1-s} + p_{on} - c - \frac{\tau}{2})\Big).$$

By assumption (A1), $\sqrt{\Delta} - 2\tau \geq 0$. Therefore,

$$\frac{\partial p^*}{\partial \tau} \geq 0.$$

The demand for the offline firm decreases with the transportation cost τ:

$$\frac{\partial D_1^*}{\partial \tau} = -\frac{1}{2}(1-s)(\frac{1}{4} + \frac{\partial p}{\partial \tau}) \leq 0.$$

The change in the profit of the offline firm with respect to the transportation cost τ could be positive or negative:

$$\frac{\partial \Pi_1^*}{\partial \tau} = \frac{1}{2}\Big(\frac{\partial p^*}{\partial \tau} - (1-s)(p^* - p_{on} + \frac{\tau}{4}) - (p^* - c)(1-s)(\frac{\partial p^*}{\partial \tau} + \frac{\tau}{4})\Big).$$

It is increasing with the size of the captive market s:

$$\frac{\partial \Pi_1^*}{\partial \tau \partial s} = (p^* - p_{on} + \frac{\tau}{4}) - (p^* - c)(\frac{\partial p^*}{\partial \tau} + \frac{\tau}{4}) \geq 0.$$

Moreover, it can be negative when the online penetration is strong with a very small fraction captive consumers s. For instance,

$$\frac{\partial \Pi_1^*}{\partial \tau} = \begin{cases} -0.011 & \text{if} \quad s = 0.02 \quad c = 0.4 \quad p_{on} = 0.1 \quad \tau = 0.4 \\ +0.02 & \text{if} \quad s = 0.20 \quad c = 0.4 \quad p_{on} = 0.1 \quad \tau = 0.4. \end{cases}$$

□

C.2 Distribution of the Online Valuation and Search Cost

Firstly, the consumers with the search cost $\gamma < 1/4$ can optimally perform sequential search. Therefore, the online valuation of the consumer is given by:

$$v_i(\gamma_i) = 1 + \gamma_i - 2\sqrt{\gamma_i} \quad \text{if} \quad \gamma \in [0, 1/4]. \tag{C.2}$$

Notice that $v(\gamma) \in [1/4, 1]$ and decreasing in (γ).

Let the search cost has the following piece-wise distribution function:

$$f_\gamma = (1-s)\left(\frac{1-\sqrt{\gamma}}{\sqrt{\gamma}}\right) \qquad \text{if} \quad \gamma \in [0, {}^1\!/_4]. \tag{C.3}$$

By using equations (C.2) and (C.3) and transforming the random variable, we obtain the the distribution of the online valuation search cost:

$$f_v = 1 - s \qquad \text{if} \quad v \in [{}^1\!/_4, 1]. \tag{C.4}$$

Secondly, the consumers with search cost $\gamma \in [{}^1\!/_4, {}^1\!/_2]$ afford to search only once and have the following online valuation:

$$v_i(\gamma) = 1/2 - \gamma_i \quad \text{if} \quad \gamma \in [{}^1\!/_4, {}^1\!/_2]. \tag{C.5}$$

These consumers have an online valuation in $v \in [0, {}^1\!/_4]$ and decreasing in γ.

Let the search cost have the following piece-wise distribution function:

$$f_\gamma = (1-s) \qquad \text{if} \quad \gamma \in [{}^1\!/_4, {}^1\!/_2]. \tag{C.6}$$

By using equations (C.5) and (C.6) and transformation of the random variables, we get:

$$f_v = 1 - s \qquad \text{if} \quad v \in [0, {}^1\!/_4]. \tag{C.7}$$

Finally, the consumers with $\gamma > \frac{1}{2}$ have zero online valuation. Let γ has the following piece-wise distribution function:

$$f_\gamma = (1-s) \qquad \text{if} \quad \gamma \in [{}^1\!/_2, {}^1\!/_2 + {}^s\!/_{(1-s)}]. \tag{C.8}$$

Integrating f_γ in the above support gives a fraction s consumers with zero online valuation. Therefore,

$$f_v = s \qquad \text{if} \quad v = 0. \tag{C.9}$$

Combining the equations (C.4), (C.7) and (C.9) gives the mixed variable online valuation in equation (3.3).

Hence, the distribution for the online valuation in equation (3.3) corresponds to the following underlying distribution for the search cost, f_γ:

$$f_\gamma = \begin{cases} (1-s)\left(\frac{1-\sqrt{\gamma}}{\sqrt{\gamma}}\right) & \text{if} \quad \gamma \in [0, {}^1\!/\!_4] \\ (1-s) & \text{if} \quad \gamma \in [{}^1\!/\!_4, {}^1\!/\!_2 + {}^s\!/\!_{(1-s)}] \\ 0 & \text{o/w.} \end{cases}$$