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Three essays on Business Groups discount in Chile: Theory and empirical study

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Rodrigo Hernán González González | PhD in Business

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UNIVERSITAT DE BARCELONA

A mi familia, especialmente a mis hijas

Trinidad y María de la Luz

por su invaluable apoyo,

y al Profesor Jordi Marti P. por su incondicional ayuda

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Section 1. Introduction

Being CEO of a holding company in Chile, not quoted in the Securities Market in Santiago de Chile, this doctoral candidate took the responsibility to restructure the business group in aspects such as legal, societal, financial and taxes.

This responsibility implied the formulation of a strategic development achieve a successful Initial Public Offering (IPO), with the main objective of transforming the business group in a public company, to be properly recognized and valued by the market, raising capital for its aggressive strategic plan, and providing liquidity to the shareholders.

The business group participated in several business sectors in Chile and Latin America, through public and nonpublic companies (subsidiaries)as well.

After restructuring the business group, and at the time to go for the IPO, I faced the natural problem of valuation of the business group. This implied several challenges:

a) Valuing our public subsidiaries and confronting such valuation with the market valuation at the time.

b) Valuing nonpublic subsidiaries, and

c) Providing good information to the securities market (profitability, growth opportunities, business risks, corporate governance, value resources, competitive advantages and so on) about the shares being offered, and the subsidiaries. For valuation purposes I developed suitable financial models following state of the art procedures.

During the process I went on a "road show" and, in particular, had to discuss with institutional investors, for example Pension Funds (whose managing companies are indisputable opinion leaders in the Chilean securities market). Their top managers firmly argued that the shares of a holding company should trade at a discount with respect to the value of the underlying companies' values (a phenomenon called holding discount). They argued that this kind of discount is present in all business groups traded in the securities markets around the world, and that to minimize the effect, and encourage a good stock trade, the business group should offer strong synergies and good and clear corporate governance. They also argued that the diversification the companies of the groups had, per se, it was not necessarily good for the business group.

Given the above, I was strongly incentivized to understand the holding discount from all its perspectives: Financial, Strategic (synergies, diversification), and structure. Then, this author formulated a growth and value adding strategy, based in Related Diversification and Value Resources.

Now, what is the importance of a holding discount? For a holding company raising capital in the securities markets, it has a direct implication: its shares are undervalued, and less capital than expected is raised (exactly the opposite to what is pretended when your objective is to finance projects and/or to prepay debt), and this capital is issued at a high cost.

On the other hand, for a shareholder buying discounted shares, it is expected that as subsidiaries companies improved their performance in their markets, their shares prices should increase, and as a consequence, the holding company's shares should be better valued.

For the holding company on its own, an improvement in (lower) discount is highly important because it signals to the market an efficient corporate governance, thus improving its reputation, generating more business opportunities, and as a consequence, improving its economic value.

According to Colpan et al. (2010) there are varied definitions of business group in the scholarly literature: Sociologist`s view of a collection of firms with a network conception bases on sociocultural ties; development economist`s view " groups are a form of big businesses distinguished chiefly by the technologically unrelated market portfolios characteristic of emerging economies; Finance scholars "tend to view business groups as a device used by controlling shareholders to disenfranchise and expropriate value from minority shareholders" mainly through the iconic pyramidal structure. Other definitions of business group come from the conventions of the regulatory system of a given country. A complete set of definitions to several countries are provided in the appendix to chapter of the cited handbook.

Along this thesis, I use the following definition of business group provided by the Chilean Superintendency of Securities: "Two or more listed companies that are controlled by the same shareholder (or group of shareholders), even when one of the companies is just an investment company holding shares of a single operating firm". This definition is utilized by Colpan et al. (2010) when describing the Chilean business groups. This definition accommodates, for purposes of this work, the term conglomerate, amply used in the literature as well.

Additionally, I will refer to the controlling firm of a business group as "holding company" or "holding", I will use the term "conglomeration" as the tendency of companies to belong to a business group, and Corporation as a business legal entity.

I define holding discount (or business group discount) as the difference between the economic (market) value of the holding company's equity, and the weighted sum of the economic or market values of the holdings in companies (downstream), adjusting for debt and cash in the parent company, among others. In the literature, holding discount is often referred to as conglomerate discount. For the purposes of this thesis, a business group or a conglomerate are indistinguishable.

An ample academic literature shows that the combined control of companies using holding companies permits the whole business group to obtain benefits of conglomeration through reputation, operational and financial synergies, and permits the holding company to appropriate the benefits obtained by the firms it controls according to the shareholding in each company.

In this study, I analyze the holding discount in business groups, independent of the detailed structure they have. All structures possess a controlling company, several firms under the same control, different verticality in its structure, and the nature of the controlling shareholders (for example family versus nonfamily control, institutional shareholders, government or private). All of them can suffer a holding discount, for several reasons, analyzed in the academic literature as conglomerate discount and in many cases as diversification discount.

The information available for the Chilean stocks market shows that a very important part of the total market capitalization, corresponds to this kind of companies.

A review of the information available for the Chilean stock market, shows that most of the economic value represented in the stock market is of companies that belong to a business group, mainly of a pyramidal structure. Lefort and Walker (2007) show that in Chile, 70% of the non-financial companies, and 90% of the equity value stock market in Chile are controlled by some of the business groups. In this iconic type of business group structure, present in all economies of the world, a shareholder at the top of the organizational chart, often a family, exercises control over a number of subsidiary companies through a chain of asset relations with varying degrees of divergence between control rights and rights to cash flows (Colpan et, 2010). On the other hand, these business groups are involved in one or several industrial sectors, have different structures, and different degrees of diversification. Additionally, an analysis of the valuation provided by market analysts to business groups in Chile shows that the companies called "holding Companies" or parent companies, are quoted at a discount in relation to the value of the underlying companies, i.e., those in which the group has some degree of ownership and control (Lema et al. 2007, Jara-Bertin et al. 2015).

A matter of interest of this study is: what justifies the existence and value of a business group? diversification should be understood as only one of the possible causes of the existence of business groups. The studies that make up this thesis further analyze the reasons for discounts or premiums in a business group in a pyramidal structure, including implicitly the so-called diversification discount.

Section 2. Theoretical and empirical framework

According to the academic literature, the discount or premium can be due to multiple causes. From a "macro" perspective, it is due to the poor development of the financial system and the quality of a country's institutions (La Porta, López-de-Silanes, and Shleifer, 1999). From a "micro" perspective, it is due to the lack of synergies, such as economies of scale, economies of scope, shared value resources (including reputation); an excessively levered capital structure (de Andrés, de la Fuente, and Velasco, 2016); risk diversification in a legal context of limited corporate liability (Grass, 2010); ownership and control structure of the companies. The latter can lead to agency problems between shareholders and management (vertical) and between controlling shareholders and minority shareholders (horizontal), while facilitating the extraction of private benefits by controlling shareholder (Holmen, and Högfeldt, 2009; Lefort and Walker, 2007). Other causes are the current legal regime on the structuring of companies; the relatively low quality of their corporate governance (Lee and Hooy, 2018; La Porta, López-de-Silanes, and Shleifer, 1999); the tax structure, especially in pyramid-type structures (Mindzak and Zeng, 2019); lack of activism of hedge funds (Kim, 2020); activism of Pension Fund Administrators, nature of the controlling shareholder, verticality of the conglomerate structure (Espinosa et al., 2018); Investor information specialization (Carpio and Guo, 2018); information imperfections, due to heterogeneity of agents' beliefs (Tong and Wei, 2018); the sentiment of market agents, existence of Noise Traders (Harper et al., 2017; Shleifer, 2000) and others discussed in Behavioral Finance[1] .

Empirical studies done in the United States, various European countries, and Japan, to both financial and non-financial firms, show that the business group discount exists (Berger, and Ofek, 1995; Lang, and Stulz, 1994), although because econometric technology, database quality, and multidisciplinary understanding of the phenomenon have improved, it has been demonstrated the existence of business group premium. That is, I found studies that show that conglomeration (diversification) decreases value, others that it does not have a significant impact, and others that conglomeration

1 This is in the field of organizational economics, especially Corporate Governance.

(diversification) adds value (Bood, 2001; Villalonga, 2000; 2002; 2003; Kaye and Yuwono, 2003). The reported discounts range from -15% (premium) to more than 25% (discount).

In the Chilean case, (and for developing countries) studies have analyzed the phenomenon of conglomeration and business groups, however, few studies directly analyze the economic effects of diversification or the reasons for the existence of business group discount (Espinosa et al., 2018). The evidence shows that business groups are discounted in the Chilean market, although the main agents do not seem to have a full understanding about why this happens and how it evolves over time. On the other hand, studies such as those of Valdés et al. (2007) and Lema et al. (2007) show discounts to some of the main Chilean business groups ranging from 7% to 32%[2]. Lastly, Jara-Bertin et al., (2015), and Espinosa et al. (2018), analyze the Chilean market and determine that there is indeed a significant discount due to diversification.

An interesting case is that of a company quoted in the New York Stock Exchange, with a long history in the United States of America (USA) -a country with good ratings in its Economy and Institutions- to which market analysts applied similar or higher discounts than to a communications conglomerate of the former Soviet Union listed in the London Stock Exchange (30%) (Teterevleva, and Busvine, 2007). Lately, holding discount has been challenged by international investors arbitrage recommendations TSI Wealth Daily Advice (March and October 2019).

Regarding Chilean business groups, in valuations made by market analysts for IPO's (initial public offerings), the sum of the parts duly adjusted, discounts are given beforehand in the order of 20% to find the value of the parent company[3].

The case of Chile is especially interesting. In Chile and Latin America there exist discount in business groups. All Latin American countries are, to some extent,

2 In 2003 the range was 15% to 60%.

3 From personal experience of the doctoral candidate, Institutional Investors declarations, not recorded, during the IPO of a business group.

emerging economies, and share several elements of the economies and institutions, so the lessons of the Chilean case may apply to the rest of the countries in the region.

The similarities with other emerging countries could indicate that the findings of the research could be interesting to those economies. On the other hand, Chile has particularities in comparison with other countries in the region, so understanding the phenomenon, its causes, and its main consequences, may also be useful for analyzing the particularities of other economies.

The academic literature has presented several arguments for there being discounts as varied and dissimilar as those mentioned above. For example, it has stated that the analyses are "cross-sectional" and do not faithfully capture the effects of time, of belonging to the industry, and of cause-effect relationships. It also refers to the omission of important variables, selection bias and non-captured endogenous effects. Finally, it mentions that performance measures are diverse, which makes it difficult to compare different results.

In this sense, the objective of this research will be to advance the understanding of the factors (their nature and their relation of causality, if possible) present in the discount of business groups, which is a stock market investor-level phenomenon. It will provide a critical vision contributing to the debate and understanding of the phenomena, either by perfecting the theoretical aspect or finding an unconsidered element in other studies.

2.1. Main theoretical contributions

Several branches of economic theory converge and contribute to the study of the aforementioned phenomena. In general, they are different perspectives analyzing common or at least analogous phenomena. They cover broad areas as corporate strategy, organizational economics, and corporate finance.

The following is a brief overview of the main theoretical contributions related to the phenomenon of business group discount.

2.1.1. Corporate Strategy

In relation to the corporate strategy and the decisions adopted in it, the criteria to be considered are presented (Collis and Montgomery 1997):

a) The scope of competition, in which business to be in, scope and scale of a business, competitive positioning, development of synergies between different businesses.
b) Organization structure, corporate systems, and processes such as form of organization, delegation of power, incentive structure, performance measurement, technology and innovation, transfer of assets and/or capabilities between existing businesses or target businesses, detecting, measuring value, investing, improving and leveraging strategic resources, particularly in the presence of externalities, expansion and growth.
c) Corporate Governance regarding the role and performance measurement of the general managers and attraction of new capital (role of the "market in corporate control", institutional investors, relations). Also considered are the structure and roles of Boards of Directors and forms of Control (voting rights and systems, restrictions on coalition formation, relations between majority and minority shareholders).

Extensive academic literature shows that the conglomeration responds to economic and managerial incentives. Among the former are responding to changes in demand, escaping from the life cycle of the sector, taking advantage of economies of scope, expanding market power, and capturing and maintaining competitive advantages, for direct financial reasons (development of an internal capital market to lower the cost of capital, diversification of risk and taking advantage of tax benefits). Among the latter, are the use of private benefits by the management, reduction of the risk of losing one's job, and managerial entrenchment that consists of adapting the company to one's own abilities.

However, the academic literature admits that along with the economic benefits of conglomeration, there are agency costs between shareholders and divisional managers, and between the latter and operational managers; costs of coordination, control, and incentives, costs of inefficient decision making, and information asymmetries; costs associated with the limited liability nature of corporations that encourage risky investments, increased administrative costs, and costs associated with losing focus on core activities when diversifying.

2.1.2. Organizational Economics

On the other hand, there is the microeconomic theory itself, which contributes in particular, with what is called "Organizational Economics". This is the name given to the theoretical body originally called the Theory of Firm, which includes the so-called Theory of Agency, Incentives and Property rights, Theory of Information, Theory of Contracts, Theory of Transaction Costs. Lastly, there is another more recent body of economic knowledge called "Institutional Economics" (Eggertson 1999).

Corporate strategy and organizational economics are bodies of knowledge that are intertwined; they attempt answering questions that, as stated above, are beyond the scope of this thesis. However, they can be analyzed later to delve into the causes of the phenomenon of conglomerate discount.

2.1.3. Financial Economics, Theory of Value and Corporate Finance

Lastly, there are the widely known theoretical bodies called Financial Economics and Theory of Value, that will be the basis of this thesis. Essentially these theoretical bodies will be analyzed in relation to Corporate Finance. In this discipline, all the necessary knowledge to analyze this thesis is gathered from Corporate Strategy (especially related to Corporate Governments), and Organizational Economics (especially agency problems). Thus, Corporate Finance contains in itself the necessary analytical and practical theoretical instruments for my purposes, which will be to measure, evaluate and empirically contrast the discount of business groups.

Thus, this study is focused on the phenomenon of "business group discount", synthesizing in Corporate Finance, the underlying theoretical aspects.

Regarding the discipline of Corporate Finance, which will be the basis for the study, the following phenomena will be present in this thesis, in different forms:

- Behavior of majority shareholders against minority shareholders (Private benefits of control): Corporate Governance (Tunneling literature)

- Existence of Synergies and Complementarities: Corporate Strategy
- Limited Liability regime in companies and its correlate, the Option to benefit or not from such regime: Corporate Finance (propping or bailout literature).

2.2. Main Results of the various research streams

The following are results taken from some studies of the relevant literature, which can serve as a basis for formulating work hypotheses.

Regarding the holding discount, referred in the literature as conglomerate discount as well, international studies show that this occurs at two levels: the investor level and at the corporation or business level.

Although this thesis will focus on the former, it is useful to know some of the factors that influence discount, that are mentioned in international studies. The studies show their positive contribution to the discount (+) or factors decreasing the discount (-), and (+/-) with unclear or divergent results between them.

The first group represents relations that will be addressed in a direct or indirect way, within the analysis of the thesis. They are present in the models of chapter 2 and 4

- Private control benefits (+).
- Conflicts of interest between majority and minority shareholders (+)
- Legal regime of corporations: Limited Liability and its correlate, the option to benefit or not from such regime (+/-)
- Benefits and costs of belonging to a business group: Existence of Synergies and Complementarities (-) but (+) when coupling with limited liability
- Corporate Leverage (-)
- Liquidity (-)
- Increase in debt capacity (+/-) and riskiness of debt (+)

The second group shows relations that are mentioned in the literature but that will not be analyzed in this thesis This group involves studies applied to the case of Chile not directly concerned with the discount of business groups, but rather study the benefits for a company of belonging or not to a business group, stating the following factors and their contribution sign (it is good (+), or it is not good (-) to the group).

As in international studies, this group shows relationships that are indicated in the Chilean academic literature that will not be within the analysis of the thesis.

- Concentration of Ownership and Control (+)
- Excess diversification (+)
- Excess of investment (+)
- Quality of Corporate Governance (-)
- Development of an internal capital market (-)
- Dispersion of risk between divisions (-)
- Cost of capital decrease by diversifying (-)
- Tax system for companies and individuals (+/-)
- Property and interlaced directories (+)
- Concentrated property (+ to neutral)
- Family membership (from + to neutral)
- Financial protection and development (-)
- Sharing financial risks (+)
- Development Corporate Governance Legislation (-)

It is interesting to note that, in all the empirical studies reviewed, with the exceptions stated below, quantitative relationships are determined between the variables that somehow measure the mentioned effects, but not the causal relationship, which cannot be analyzed herein due to insufficient data for the period empirically analyzed in chapter 3.

Local studies depart from the above by directly evaluating the effects of certain aspects of corporate governance and agency issues on the value of Chilean business groups. Along these lines, Lefort (2007) analyzes the effect of agency conflicts (measured as the non-coincidence between control rights versus the right to cash flows) between controlling and minority shareholders on dividend policy measured as the fraction of profits distributed (-). Lefort and Walker (2007) show that the market values more the business groups that have less agency conflicts (-), as in the previous study. Lefort and Urzúa (2007) analyze the impact of directors who are independent of the controller on the valuation of companies (+). In these three studies, the models control for the effects of endogeneity and inverse causation.

I can conclude from this first review of studies in Chile, that analyzing the phenomenon of business group discount is a highly interesting, and needs to be re-explored, field in Chile (Lema et al., 2007; Jara-Bertinet al., 2015).

Section 3. Research Methodology

After reviewing the theoretical contributions and the main results of the selected studies, I conclude two main things:

The first, is that business group discount is well documented in the literature for several countries, including Chile, utilizing an indirect method to calculate the discount, which is the imputation of values to the nonpublic subsidiaries of the business groups. Also, this discount appears to be stable along time on average.

The second is that many causes have been invoked to explain the business discount, for example too much diversification, too much investment, excess od indebtness, double taxation between holding company and subsidiaries, imperfections (such as taxes, poor corporate governance, agency problems (conflicts of interests between board of directors and shareholders, or within boards of directors and information asymmetries), and extraction of private benefits of control, in a legal system of limited liability of corporations.

Therefore, I propose making a theoretical-empirical study of the discount of business groups in Chile, attempting to find some of the main determinants in a context of efficient and competitive capital market, without the imperfections already mentioned, to see if discount can be modeled or predicted, consequently studying its predictability over time. Some interesting questions are raised below to carry out this research and the corresponding hypotheses will be formulated.

3.1. Questions and hypotheses

3.1.1. General

The analysis of the existing literature in so many related but different fields, can lead to the formulation of very interesting questions that have been deeply researched for many years in the economic, financial, and business literature worldwide[4]: In a business group, is the whole worth more or less than the sum of its parts, and why? Why do companies diversify? If business groups were to destroy value, why and how are they formed, why are pyramids structured? Is it because value is destroyed in the divisions, or are there other reasons? What have the business groups done to create value, and what have they stopped doing when they have destroyed value? What is the importance of institutional and corporate governance factors and why?

However, the objective of this study is focused to analyze a more specific issue, which is the holding discount, as defined previously, particularly in the Chilean case.

As a result, more specific questions are asked, some of which lead to the following five working questions:

1. Do Chilean business groups destroy economic value?
2. How does the different holding discounts evolve over time?
3. If holding discount exists in Chile, can it be modeled to make predictions?
4. What structural factors determine the holding company's accounting in the legal context of limited liability, and in the absence of stock market imperfections such as information asymmetries, agency problems, limited rationality of agents and others? Market imperfections lead to meny difficulties when designing empirical tests. This is the essence of this question.
5. Do factors such as private extraction of benefits by the controller, synergies, and externalities have an impact on the discount?

Studies of this phenomenon in Chile have insufficient data for my purposes, which are related to: measurement of intermediate or non-observable variables (many subsidiaries of holding companies are not traded in the market, which forces value imputations using comparable companies), endogenization and reverse causation.

This thesis will contribute by building a unique database to advance in the understanding of these problems, which will be a fundamental basis for the empirical study of the holding discount.

4 The literature developed in the field falls into several groups: a) on the diversification of companies and its impact on economic performance; b) on conglomeration and c) on conglomerate discount.

3.1.2. Tentative hypotheses for the Chilean case

As seen previously, the analyzed phenomenon is intimately related to other phenomena; so are the questions that arise when facing it. Given the difficulty of addressing the phenomena simultaneously, the thesis will address only the issues related to holding's discount from the corporate finance perspective. The tentative working hypotheses will therefore cover only the discount phenomenon, formulating hypotheses such as:

Hypothesis discussed in Chapter 2:

- Hypothesis H0: Null: business group discount exists, only when asymmetric information, agency costs or other imperfections, are considered, regardless of the legal system.

Hypothesis discussed in Chapter 3:

- Hypothesis H1: Null: there is not holding discount in Chilean business groups.
- Hypothesis H2 Null: holding discounts are not predictable. This hypothesis is subdivided in the following sub-hypotheses:
    - Hypothesis H2a. Null: holding discounts time series present unit roots, thus they are non-stationary, and behave as random walks (non-predictable)
    - Hypothesis H2b. Null: holding discounts do not present reversion to the mean, and thus are a Martingale (not predictable conditioned to all available information)
    - Hypothesis H2c. Null: holding company equity value and net asset value are cointegrated, thus present a long term stable linear relationship (in the long run given one, the other can be predicted with a reasonable confidence level)
    - Hypothesis H2d. Null: Chilean market is not affected by investor sentiment regarding the discounts. This implies a medium to high level of market efficiency, thus they are not correlated between them and behave as random walks as Efficient Market Hypothesis predicts (thus, discounts are non-predictable).

Hypothesis discussed in Chapter 4:

- Hypothesis H3 Null: The holding discount is positively related to the debt riskiness of the business group as reflected in the risk spread of bonds issued by the business group.

3.2. Methodology

3.2.1. General approach to the study

The empirical study will be of the hypothetical deductive type, in which possible explanations of the phenomenon will be selected in light of the exiting theory; work hypotheses will be formulated as well as theoretical models, which will allow the hypotheses to be contrasted with the available empirical evidence, as far as possible.

The study will have the following sequence for each main topic: literature review, construction of alternative explanations to the business group discount, formulation of contrastable hypotheses, and empirical contrasting where possible.

3.2.2. Methodological Aspects

The methods that have been used are the most adequate ones designed especially for what is being studied; it is how, theoretical models have been constructed in order to allow empirical contrasting.

This work will follow in general the following sequence: Literature revision, construction of alternative explanations of the business group discount, contrastable hypothesis formulation, and empirical contrasting where it is possible.

A general problem found in the Chilean case is that there is not enough data. One example is the difficulty finding similar "stand alone" companies to make direct comparisons, which in certain studies has led to the use of imputed values of comparable assumptions. Another example is the difficulty in accessing company-specific information that is not published in the information required by law, particularly for companies that are not traded on the stock market or companies that, being listed on the stock market, do not have much history.

The problems that will appear in the research are the specification and measurement of the different variables, as well as the lack of information as explained above. In particular, there is the problem of measuring the economic value of the companies belonging to a business group (assets and liabilities which implies making value imputations for a large number of subsidiaries, and consequently the "business group discount"); Damodaran (2006a) and (2006b) contain an extensive treatment on valuation of all types of assets, including intangibles and synergies. Nevertheless, in the groups selected in this thesis, more than 95% of the value of the subsidiaries is obtained directly from market prices[5], thus minimizing the imputation problem. Regarding liabilities, the values found in the market are in corporate bond transactions, which are not traded as often as shares are. So, there is a lack of data in the time series.[6]

During the development of this thesis, several approaches to analyze discount were modeled -although not shown here- to search for theoretical explanations in an environment of symmetrical information and rational expectations. The models were of a very simple structure, i.e., a holding company with a subsidiary, under various economic environments: Modigliani and Miller with and without tax, with no risk in debt, with risk in debt, with and without limited liability in corporations (using continuous and discrete option valuation methods). None of these models shed light on the emergence of a holding company discount.

There was only one important finding, which is studied in chapters 2 and 4. The possibility of the holding company discount emerging when it is admitted that the holding company has limited liability regarding its investment in a subsidiary. That is, when the latter enters into financial distress, or in bankruptcy, the holding company has the possibility to bail its subsidiary out. In other words, the holding company does not exercise the right to limited liability.

5 Valuation of conglomerates and the effects of diversification: "Stand alone" method of valuation of companies through calibration of M/B, FV/Eb, P/ U models developed for this purpose. Unpublished (in process) Working Paper Rodrigo González (2017).

6 In chapter 3 (empirical) and chapter 4, I have avoided using indirect valuation methods, based on imputation of values using comparable companies, and mixed methods (direct and indirect), selecting conglomerates with more than 95% of subsidiaries traded on the stock exchange.

Section 4. Research structure

4.1. Chapter 1: General

This thesis has four chapters: the introduction, a general description of the theoretical framework, the formulation of questions and hypotheses, and a description of the rest of the chapters. Finally, a summary chapter resumes the most relevant conclusions achieved and describes future research lines emerging from the results of previous chapters.

Chapter 1 shows the general overview, it makes a revision of the main literature regarding the holding discount (without distinguishing between business groups, pyramidal groups, conglomerates), outlines the theoretical framework, then addresses in chapters 2, 3 and 4, one or more of the five questions raised before, and the hypotheses already formulated.

The theoretical Chapter 2 addresses questions 3, 4 and 5, formulates hypothesis H0, and contrasts it through a sequence of theoretical models. In particular, models in chapters 2 intend to capture the effect of some of the issues already indicated in Introduction Section 2.2, such as not complying with limited liability rules, external benefits such as synergies, and reputation, indebtness and private extraction of benefits by the controlling shareholders.

Chapter 3, which is empirical, addresses questions 1, 2 and 3, formulates hypotheses, H1 and H2a, H2b, H2c, H2d, and contrasts them through various statistical procedures, with data from selected groups in the Chilean market.
The statistical analysis of selected Chilean business groups in chapter 3, overcomes the use of the imputation method to calculate discounts, and to some extend avoids analyzing too diversified business groups.

Chapter 4 is theoretical-empirical and addresses questions 4 and 5, feeds in some conclusions from the empirical literature, takes the general model from chapter two, and specifies it, and formulates hypothesis H3. In particular, some of the possible causes for the existence or absence of discounts, drawn from the analysis in chapter

1 and chapter 3, are modeled in two time-periods; these are synergies and externalities of the controlling shareholders as well companies themselves, and the extraction by the controlling shareholders of profits inherent in having control. The effects of these causes are analyzed separately, drawing the conclusions regarding the holding discount as well as shareholders conflicts.

The logical relationship of the chapters is shown in the figure 1:

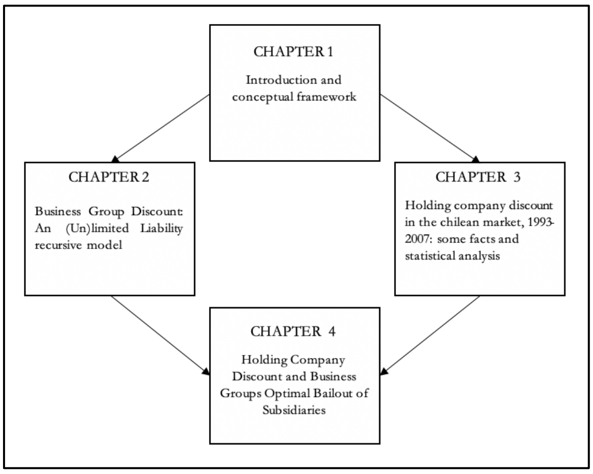

Figure 1. Research Structure

4.2. Chapter 2: Business Group discount: An (Un)limited Liability recursive model

The second chapter has been developed exclusively by the doctoral candidate, and is the development of a dynamic model, in the context of rational expectations, without market imperfections, in particular information asymmetry, aims to develop what finance literature has not explored in extent. That is, the option to a holding company to not follow the Limited Liability legal rules when a subsidiary is in financial distress, but to rescue it. If holding discount could be modeled withot need of imperfections, empirical testing would be highly simplified.

This legal rule is present in many countries as well as in Chile. The hypothesis of this chapter is that holding discount cannot exist, without recourse to asymmetric information or other imperfections, regardless of the legal system.

This work draws from repeated games and dynamic decisions theories. By analyzing the existing literature, very few models have been devised to explain the above-mentioned behavior in the way my models do.

An ample literature exists, mainly empirical, with respect to what is called holding discount (also called conglomerate discount). Many explanations have been provided, for example taxes issues, investor sentiment, information asymmetry, debt issues, liquidity, bad corporate governance, noise traders and constraints to short selling. For good but not exhaustive summaries refer to Rommens, Deloof and Jegers (2004), and Cornell and Liu (2001).

Some models have been devised, to explain the holding discount, on one hand, and propping behavior on the other, almost all of them relying on asymmetrical information and bounded rational investors (Berkovitch, Israel and Tolkowsky, 2000; Ammann and Verhofen, 2006; Almeida and Wolfenson, 2006).

In many countries, a legal provision for corporations exists, named Limited Liability of shareholders. It is actually a right but not an obligation to shareholders. In several cases, bailing out may convey not exercising this right in order to preserve some benefits.

The Bailout behavior, and the relationship between Tunneling (Controlling Shareholder extraction of private benefits from minority shareholders) and optimal rescue or propping (also called Bailout), is of interest because giving up the option to exercise Limited Liability rules is a rescue. This behavior is analyzed in several models (Cordella and Yeyati, 2003; Wilson, 2012; Kim, 2004; 2016; Friedman, Johnson, and Mitton, 2003; Riyanto and Toolsema, 2008; Lefort and Walker, 2007; Espinosa et al., 2018).

The potential Conflict between Controlling and minority Shareholders, is also vastly analyzed in the literature. Explanations are given invoking poor legal protection to minority shareholders. Poor legal protection augments incentives to majority shareholders to maintain holding benefits of control (private rent extraction, tunneling), by bailing out (propping) distresses subsidiaries (La Porta, López-de-Silanes, and, Shleifer, 1999; La Porta et al., 2002).

The first purpose of this paper is to fill the gap by providing a sequence of models that can explain under what circumstances a holding discount can exist, without recourse to asymmetric information issues or other imperfections. The models are designed using extensively the option of shareholders to give up following the Limited Liability ruling.

A secondary purpose is to visualize under what conditions, a bailing out behavior by controlling shareholders, is aligned with the interests of minority shareholders, in the holding company as well as in the affiliates, meaning that in order to maintain benefits, majority and minority shareholders should agree to bail out, instead of being in conflict, as literature suggests.

A series of recursive models is developed for a business group in which the levered holding company repeatedly faces the decision to bailout a distressed levered subsidiary in bad times. The models borrow from several sources to capture the recursive nature of the bailing out decision, identify the decision maker and other players involved, and what can be understood as an optimum decision in each stage.

Being business group discount (holding discount) an amply studied phenomena in the academia and business world, the purpose of this paper is to analyze the relationship between business group discount and limited Liability (LL). This is done in public corporations, specially focusing on the option of shareholders in corporation not to follow LL, thus follow an Unlimited Liability behavior (UL) called propping or bailing-out in the literature. I analyze the rationale for that behavior by means of simple but progressively more realistic corporate decisions models.

Contrary to models developed in the literature, designed to explain the holding discount under various market imperfections, my models rely in a context of perfect and efficient market, and are able to explain the existence of a holding discount under certain plausible conditions; this is a main contribution of this work. Moreover, I can also show that in certain conditions, majority shareholders and minority shareholders' interests will be aligned, and depending on the assumptions of the model, they can be in conflict, as existing literature prescribes.

In section 1, I outline the rest of the chapter. In Section 2, I analyze a simple model (Model 1) of a stand-alone company subject to Limited Liability rules (LL). This model serves to motivate the rest of the models and provides the basis to calculate the value (incentives) to the shareholders of following (UL).

In Model 1, I determine the conditions under which shareholders (represented by a representative shareholder) have incentives not to follow LL ruling (not to exercise the LL option), which is relevant in case of default, and instead they behave as if they were subject to Unlimited Liability (UL) and "rescue" the company.

Actually, under LL ruling, Shareholders have an option (the right but not the obligation) to comply with LL. The decision model is solved borrowing some concepts from Game Theory repeated games and Decision Theory, with infinite horizon, in which the players are the Representative Shareholder and Nature. In this model I assume there are no conflicts of interest (agency costs) between controlling and minority shareholders, and I find that under certain conditions, called Continuation Conditions, Shareholders will have the incentive to pay the costs of rescue the eventually distressed company.

In Section 3, I extend Model 1 to allow for a new shareholder to acquire a controlling stake in the stand-alone company (Model 2).

In Model 2, the new shareholder becomes a simple holding company ("Holding"), controlled by a single controlling shareholder; Holding has no controlling stakes in other companies nor other businesses. I allow for the existence of benefits of being a holding that accrue to controlling shareholders in one case (private benefits), and to all shareholders in other case.

Under this scenario I revise analysis of section 1, by introducing third and fourth players, Minority Shareholders at the affiliate and holding companies. These new players act as followers of majority shareholders (like in a Stackelberg game) in every period stage of the infinite game. If continuation condition hold, the optimal strategy of controlling shareholder is to rescue the distressed affiliate by issuing equity, and the optimal strategy of minority shareholders is not to concur to the equity call, showing the existence of conflicts of interest except in the case in which external benefits accrue to all shareholders. Depending on the nature of the external benefits, a holding discount may arise.

In Section 4, I show Model 3, extending Model 2, to allow the Holding to previously own controlling stakes in other companies as well as Debt; I revise and complement the analysis of the second section.

In Model 3, I allow for new debt to pay the costs of rescue of the distressed affiliate. In this case a holding discount arises, with higher probability (greater incentives to follow UL to controlling shareholder) the greater the amount of existing plus new debt. As in model 2, if external benefits are common to all shareholders, there will not be conflict of interests, but if benefits belong to controlling shareholder only, there will be conflict of interests. In Model 3 the probability of rescue in case of distress in the subsidiary are greater than in Model 2.

In Section 5, I show Model 4, as a special extension of Model 3. In this model I allow the holding company to pay the cost of rescue with existing cash, contrary to the debt case of model 3. As in models 2 and 3, I allow for the existence of external benefits, and I find that in case of rescue, a holding discount will arise, in a context of eventual conflict of interests between controlling and minority shareholders.

In Section 6, I explore several avenues for further research and some empirical implications are derived, without testing them empirically, which is not part of this work.

4.3. Chapter 3: Holding company discount in the Chilean market, 1993-2007: some facts and statistical analysis[7]

Chilean holding companies' stocks are traded in the Chilean stock market with a significant discount with respect to their Net Asset Value or NAV. This phenomenon in known in the financial markets as holding discount. A number of explanations has been deployed in the Literature (Lee, Shleifer and Thaler, 1991; Berger, and Ofek, 1995; Campa and Kedia, 2002; Rommens, Deloof, and Jegers, 2004; Kaye, and Yuwono, 2003; La Porta et al., 2000), nevertheless, the debate is far from being concluded.

My first null hypothesis H1 in this chapter, is that in the Chilean market holding discount doesn't exist, which proves to be rejected, by calculation of the discounts in Chile.

In Chile and other countries Investment Banks make buy/sell recommendations for the stocks of the holding company and their subsidiaries, based on the evolution of the discount with respect to its historical average. In short, they recommend arbitrage actions See Investments Security, Monitor Security (2008) and Valuation Report (2007), TSI wealth Daily Advice march and October (2019). Investment Banks argue that there is room to obtain significant abnormal results whit their strategies. This could be possible, if the discounts time series are stationary in the statistical sense and show mean reversion. In turn this would imply that discounts are somehow predictable. Gasbarro, Johnson, Zumwalt (2003) analyze mean reversion in closed

7 Chapter 3 is based in part on Galvez (2009), from which I extract the literature analysis, the descriptive analysis of Chilean holding companies discounts, and from which I use the unique data base constructed, for which the doctoral candidate, designed the methodology and directed its construction, as well as directed the selection of the holdings to analyze, and indicated some venues of analysis.

end funds' discounts. Closed end funds are financial structures that are very similar to holding structures.

Consequently, this chapter develops several complementary statistical tests, analyzes the time behavior of the discounts of several groups, establishes whether these discounts are predictable or not using well known statistical tests, and finally explores the efficiency of the Chilean stock market with the help of concepts extracted from behavioral finance.

Consequently, the second and main hypothesis, of this chapter, H2 Null is that the holding discounts are not predictable. This hypothesis is divided in four sub-hypotheses, H2a, H2b, H2c, H2d.

This work in this chapter possesses various dimensions of interest: In first place, the various business groups are selected from the list of 25 largest groups in Chile (Colpan et al, 2010) under a simple and clear selection criterion, this is that the business group structure be as simple as possible, and that more than 95% of the NAV is traded in the market. This criterion intends to eliminate two factors that are beyond analysis: diversification and imputation of NAV values. In total nine groups well described in Galvez (2009) are selected.

Second, the unique and proprietary data base for the nine selected groups, developed under my direction in Galvez (2009) is used. The time frame is 1993-2007. I considered the possibility of extending that time span but there were several factors that would difficult the comparison through time, specially changes in the stock market regulations, change to IFRS accounting in 2008, and the troubles imported from North American financial crisis that introduce enormous noises in the analysis.

Third, I analyze the efficiency of the Chilean stock market. The null hypothesis is that holding discounts are not predictable, thus the market would be informationally efficient, and arbitrage operations should be ineffective, impeding to obtain abnormal returns on average.

To analyze the efficiency of the Chilean stock market I deploy four hypotheses, H2a, H2b, H2c, H2d for the corresponding four statistical tests, aiming to determine the predictability of holding discounts. Tests are Unit Root Test (stationarity thus predictability); Variance Ratio Test (which permits to see whether a series shows mean reversion, hence predictability); Cointegration Test (permitting to determine if there is a long run stationary relationship in the statistical sense, between NAV an holding company's equity value); Comovement analysis (which analyses other aspect of the market efficiency dictated by the expanding theory of behavioral finance, in particular investor sentiment) that permits to see if prices of assets structurally unrelated move together over time, departing from the well-known random walk hypothesis.

The results obtained show that with a reasonable degree of confidence, Chilean market in no efficient in the period 1993-2007. I conclude from this that holding discounts are predictable with a significant degree of confidence.

The interest for the holding discounts itself has been present since the second half of the 1990s, mainly in the US, until the end of the first decade of 21st century in the existing literature. Notwithstanding during all these years, ample literature has been developed, most of it empirical, testing for the presence of several causes invoked to explain the holding discount and the somehow related discounts conglomerate discount and diversification discount (poor management, agency problems, ownership concentration, investor sentiments, wealth expropriation), in many countries including less developed countries like Chile. Recently the phenomenon has gained interest in the investor literature (TSI Whealth Daily Advice, 2019).

My interest is focused where the holding company or its subsidiaries are publicly traded firms. In this case, the market capitalization of the parent company tends to be lower than the sum of the share values of the subsidiaries it holds corrected for financial debt in the individual parent company. This can be summarized as the net asset value. This phenomenon is known as holding company discount. From the time series of the data, it can be observed that in Chile, the discount levels do not tend to be constant but rather display major fluctuations of up to forty or fifty percent of the corresponding net asset value. The main assumption of this analysis is that the discount levels although volatile, should return in a relative short period to their average levels or even to their long-term level. This implies that the time series describing discounts are stationary around a fixed mean or long run trend. How appropriate this assumption is, can be detected from an analysis of the results.

There are several explanations on the existence of holding discounts. One of them, the agency problems explanation, posits that the controlling company shareholder could obtain pecuniary and non-pecuniary benefits at the expense of the other (minority) shareholders. These are called Private Benefits of Control (Riyanto and Toolsema, 2008).

The objective of this study is to describe the discount of various holding companies in the Chilean market, focusing on simple structures (ideally one traded parent company and one traded subsidiary). There are no previous studies of a holding discount in Chile, been calculated directly from its definition, in the recent literature. There are several studies of company valuation, or dividend policy, or corporate governance effects in value, controlling for company's affiliation to business groups, and calculating the market value of non-listed companies, hence the net asset value, by means of imputing a value based on different multiples of comparable listed companies, obtaining an approximation to diversification discounts.

Two interesting studies are: Jara-Bertinet al. (2015) and Espinosa et al. (2018), in which they explore the effects of diversification on valuation, using imputed values[8] as explained before. In this study the holding discount is calculated directly in simple groups in which the value of non-listed affiliated companies represents less than 5% on average, thus avoiding the imputation problem.

8 This method is pervasive in the empirical literature.

The study is organized as follows: In Chapter 3, Section 2, Related Literature, I show the work pf several authors focusing on closed-end funds and the explanations provided on the discount is presented. However, it should be indicated that the case of closed-end funds is not identical to discount in the business groups of Chile, although it is what most resembles it the financial literature studied. Therefore, I include this literature for reference but not for explanatory purposes for the study, and I explore several explanations found in the literature for the holding discount. There are not many studies on holding company discount.

In Section 3, discounts in Chilean main business groups, the discounts in nine Chilean business groups are presented, explaining the Data Base Construction methodology, showing the results and analyzing the discount for the selected Chilean groups, and ending with the exposition of some facts analysis of some legal and financial issues.

In Section 4, Statistical Analysis, I propose a theory for the time behavior of discounts, and propose a general hypothesis H2, and four sub hypotheses H2a, H2b, H2c, H2d, preceding the 4 different but complementary tests I perform in this section, intending to understand whether the discounts evolve randomly, what it means are unpredictable: Unit root Analysis (tests for no stationarity), Variance Ratio Analysis (tests for mean reversion), Cointegration Analysis (tests for long run stable relationship between holding value and NAV), and Comovement Analysis (tests for the existence of inefficiency in the security market by searching for investor sentiment in the Chilean stock market related to the holding discounts).

In Section 5, I present conclusions. In Section 6, I explore several avenues for further research.

4.4. Chapter 4: Holding company discount and Business Groups Optimal Bailout of Subsidiaries[9]

In this chapter, written together with Professor Fernando Lefort G. we develop, based upon models of chapter 2, a simple two-times one-period model of the relationship between a holding company and its Subsidiary, that analyzes the option that controlling shareholders must optimally rescue the distressed subsidiary using financial resources of the holding company. Such a behavior is rational under several incentive structures originated by the existence of external benefits to the group, extraction of private benefits of control, and synergies, collectively called "benefits of conglomeration". Following the general models of chapter 2 we treat these benefits separately.

A growing body of literature in corporate governance and corporate strategy has shifted its focus away from the standard agency problem between managers and dispersed shareholders, and has, instead, looked closely into the relationship between minority and majority shareholders. In particular, it has been argued that business groups are prone to carrying inefficient investment and generating minority shareholder expropriation, especially when control is exercised through complex mechanisms such as pyramid schemes, crossholdings, and dual-class shares. In those cases, the agency problem is exacerbated because, on the one hand, ownership concentration insulates the controller from the market for corporate control, and on the other, control is executed by a shareholder that holds a relatively small fraction of the cash flow rights (Bebchuk, Cohen, and Ferrell, 2009; Lefort and Walker, 2007).

The combined control of several companies may also create operational synergies, which might be related to economies of scale and scope in product and factor markets, due to poor basic services like electricity, postal service, or others. In particular, internal capital markets, that is, the headquarters' collection and allocation

9 Early versions of this paper are published as submitted to Documentos de Trabajo #34, Facultad de Economía y Empresa Universidad Diego Portales (2011) and published in ResearchGate Network; it is listed but was not submitted to FMA Congress in Orlando 2015, and submitted but not presented to World Finance Conference Río de Janeiro (2012). This doctoral candidate keeps absolute responsibility of the empirical section of this chapter and keeps exclusive liability for it.

of funds to the different companies in a group could create value in a credit constrained world (Stein, 1997). Other financial synergies may arise because of the possibility for business groups to liquidate assets of specific firms in response to a general downturn (Shleifer and Vishny, 1997), and because of risk diversification that might be valuable to investors in economies with underdeveloped capital markets.

An interesting consequence of the existence of synergies and private benefits of controlling different companies through a business group is the creation of incentives that can trigger an opportunistic behavior consisting in bailing out distressed companies controlled by the group with financial resources belonging to other companies of the same business group. In the academic literature, this behavior has been labeled as propping or inverse tunneling (Riyanto and Toolsema, 2008).

Almeida and Wolfenzon (2006) have noticed that many business groups organized as pyramids have very minor separation between cash flow rights and voting rights. Common wisdom associates pyramids to a tool implemented to extract private benefits. However, the results of Almeida and Wolfenzon's model indicate that pyramids are the best way to finance the creation of new companies that yield relatively low-security benefits.

The results of our model and the penetrating presence of discounts in the holding company suggest that controlling shareholders of business groups are prone to share the financial costs of keeping the pyramid in place but are less willing to share the benefits of conglomeration with minority shareholders of the holding company.

Empirical international evidence shows that market capitalization of a holding company is usually less than the market capitalization of the investments it possesses, duly adjusted for debt in the holding company (Net Asset Value or NAV).

This empirical regularity, the "holding discount" is commonly attributed to low free float of the holding company's shares, tax inefficiencies, management costs in excess, inefficient investment, market inefficiency and market imperfections, investor sentiment, extraction of private benefits of control by the controlling shareholders.

In this work, we want to contribute to the literature on business groups by adding an overlooked empirical regularity to the complex issues that theoretical models on pyramid structures attempt to explain. Recently, several authors have drawn attention to the existence of a holding company discount in many business groups. Rommens, Deloof and Jegers (2004) show that holding companies, which play an important role in corporate finance in Belgium and in other Continental European countries, often trade at a discount to their estimated net asset value (NAV). Gálvez (2009) finds that in the Chilean case, even holding companies in which all assets are shares in traded companies and thus net asset values are calculated straightforwardly, trade at an average 30% gross discount over NAV.

Throughout our analysis, in a simple setting without taxes, full free float, no bankruptcy costs, ad information symmetry, we have shown that in the absence of external benefits (for example reputation), or synergies, or private benefits extraction, business groups will not have incentives to give up exercising its right to Limited Liability, what means not rescuing the distressed subsidiary. In such a context we would not observe holding discount.

However, under the different incentive structures already mentioned, it may result optimal to business groups to rescue a distressed subsidiary, and we will find discounts in the simple setting described.

General intuition is that when a subsidiary bankruptcy occurs, it may cause losses of the benefits of conglomeration in place, and the controlling shareholder will be eager to pay a cost to rescue the subsidiary, that will be shared with minority shareholders. In this sense, our conclusions complement Almeyda and Wolfenson (2006)'s conclusion because in both cases the controlling shareholder will use third party funds to gain access or to keep conglomeration benefits, as in Almeyda and Wolfenson (2006), and our model, respectively.

Almeida and Wolfenzon (2006) model predicts that business groups will find it optimal to structure pyramids in order to profit from their financing advantage only in situations where the model parameters predict the presence of holding company premiums.

Similar to our model, Almeida and Wolfenson (2006) model predicts that an unexpected acquisition of a new company into the pyramidal structure will cause a loss of value in the holding company, because this implies additional private benefits to the controlling shareholder, at the expense of the holding company's minority shareholders, which finance part of the acquisition.

In our model holding discounts or Premiums arise because NAV values do not consider conglomeration benefits shared with minority shareholders of the holding company, or the expected financial costs incurred by minority shareholders when the business group decides to rescue a distressed subsidiary.

Hence, if benefits of conglomeration to the controlling shareholders are not high enough, holding companies will leave the distressed subsidiary to go bankrupt, when minority shareholders share those benefits, a holding premium will be observed.

On the contrary, if conglomeration benefits are high enough, holding companies will rescue their distressed subsidiary, and if the part of the rescue financial cost belonging to minority shareholders exceeds their share in conglomeration benefits, then, a holding discount will be observed.

In sum, the existence of private benefits of control may induce the controlling shareholder to rescue its subsidiary, depending on the relative size of expected rescue benefits and costs.

In turn, the expected rescue benefits and costs depend, in our model, of the likelihood of bankruptcy, this is the probability of a "bad state" in the economy.

In our model we have configured an ideal world, in which we should expect that holding discounts increase (holding premia decrease) as the probability of bankruptcy of the subsidiary (i.e., of a bad state in the economy increases). We use the risk premium observed in the debt of the subsidiary and the holding company as a proxy of a bad state been likely to occur.

Therefore, all the rest equal, we can hypothesize, using hypothesis H3, a strong and positive correlation between the observed debt risk premium and holding discount[10,11].

Our empirical results are strongly aligned with this hypothesis, showing under several tests, a statistically significant correlation between both variables.

4.5. Chapter 5: Conclusions and avenues for further research

Chapter 5 compiles and summarizes the main conclusions and avenues for further research of chapters 2, 3 and 4.

The rest of this chapter paper is structured as follows: Section 2 discusses empirical evidence of holding discounts and relates it to the close-end fund literature.

Section 3, presents a model of optimal bailout of subsidiaries.

We model holding company discounts for a number of cases, depending on the nature of the benefits that the controlling shareholder can optimally derive from the pyramid, ranging from external benefits, to tunneling, to synergies. The decision to bail-out the distressed subsidiary and the welfare implications to minority shareholders given the existence of potential conflicts of interest are also analyzed in this section.

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10 No causation bas been possible to establish given the lack of sufficient data.

11 Additionally, the holding structures that show a greater separation of cash flows and control rights, should show, all the rest equal, greater discounts.

Section 4, summarizes the main results and conclusions of Sections 1 to 3.

Section 5, is the empirical section of this chapter. Develops and tests (at least partially) a hypothesis of a structural determinant of the holding discount.

Section 6, explores several avenues for further research.

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Chapter 2. Business Group discount: An (Un) limited Liability recursive model

Abstract

A series of recursive models is developed for a business group in which the levered holding company repeatedly faces the decision to bailout a distressed levered subsidiary in bad times. The conditions for this decision to be made, named exercising the "unlimited liability option" (in the literature this is a propping or bailing out behavior), are stated in several contexts through models of increasing complexity. Contrary to models developed in the literature, designed to explain the holding discount under asymmetrical information and other market imperfections, my models rely in a context of full and symmetrical information and rational investors, in the absence of taxes only, and explain the existence of a holding discount under certain plausible conditions; this is a main contribution of these models. Additionally, eventual conflicts of interest with minority shareholders in the holding company as well as in the subsidiary may arise, and the conditions are analyzed. Moreover, I can also show that in certain conditions, majority shareholders and minority shareholder's interests will be aligned, and depending on the assumptions of the model, they can be in conflict, as existing literature prescribes. Finally, a set of empirical implications is derived from the model.

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Section 1. Introduction

An ample literature exists, mainly empirical, with respect to what is called holding discount (also called conglomerate discount). Many explanations have been provided, for example taxes issues, investor sentiment, information asymmetry, debt issues, liquidity, bad corporate governance, noise traders and constraints to short selling. For good summaries refer to Rommens, Deloof and Jegers (2003), and Cornell and Liu (2001).

Rommens, Deloof and Jegers (2003), argue that the holding company discount can be explained by four possible reasons acting alone or in conjunction; one is that holding company joint control costs in excess of its benefits to subsidiaries destroy value at the holding level, other is that subsidiaries valuations (Net Asset Value or NAV) can be overestimated because lack of liquidity in some subsidiaries is not considered, other is because the existence of noise traders that by investing more in the holding company than in the subsidiaries, can create deviations in the true value of the holding company, and finally because the market anticipates that controlling shareholders extracts private benefits of control, and thus discounts the value of the holding company.

Cornell and Liu (2001) argue that the total (holding company) can be worth less than the parts (NAV) because of Noise Trader behavior misprices the subsidiaries at the holding level, and because the existence impediments to arbitrage. They neglect other explanations to the discount such as taxes, agency costs, liquidity issues and noise trader risk.

Some models have been devised, to explain the holding discount, on one hand, and propping behavior on the other, almost all of them relying on asymmetrical information and bounded rational investors:

Berkovitch, Israel and Torkosky (2000) design a three-date model with asymmetric information between managers and the market predicting that the decision to incorporate stand-alone firms can conclude in a holding discount or premium depending on the relative importance of growth opportunities and innovation opportunities that the stand-alone firms hold.

Ammann and Verhofen (2006), are able to explain the existence of a holding discount by considering a holding company as a portfolio of equities of the subsidiaries. Equity is modeled as the option (right but not obligation) to buy the assets of a company to debtholders. This Options approach to equity valuation is standard in Finance Literature, and relies on Limited Liability rules only, which are a crucial part of our model.

Almeida and Wolfenson's (2006) design a model that while intending to explain the rationale for the existence of business groups as pyramidal structures, explains the existence of a holding discount. Their model predicts that an unexpected acquisition of a new company into a pyramidal structure causes a loss of value in the holding company, because this implies additional private benefits to the controlling shareholder, at the expense of the holding company's minority shareholders, which finance part of the acquisition.

Almeida and Wolfenson's (2010) test this theoretical finding in the Korean market and observe that an important fact for the discount to exist is the acquisition of a subsidiary being made with founds internally kept in the holding company (see Sections 3 and 4). The discount is because the minority shareholders anticipate the acquisition that is done with a loss for them.

In many countries, a legal provision for corporations exists, named Limited Liability of shareholders. It is actually a right but not an obligation to shareholders. In several cases, bailing out may convey not exercising this right in order to preserve benefits.

The Bailout behavior, and the relationship between Tunneling (Controlling Shareholder extraction of private benefits from minority shareholders) and optimal rescue or propping (also called Bailout), is of our interest because giving up the option to excersise Limited Liability rules is a rescue. This behavior is analyzed by Cordella and Yeyati (2003) who model the optimal policy of a bank to bailout distressed borrowers, based on contract theory and moral hazard issues. Wilson (2012) and Kim (2016) analyze the efficiency of government bailouts of banks. They design a three date game models that balance, in a context of information asymmetries and moral hazard, the costs of distress with the costs of bailing out private banks.

Kim (2004) designs a dynamic continuous time model to analyze some implications of bailing out to by government, finding that such a behavior exacerbates moral hazard problems, existent in their model.

Friedman et al. (2003) designs a simple two date model, with asymmetric information between an entrepreneur (controlling shareholder) and external investors, suggesting that issuing debt to finance a new project can credibly commit the controlling shareholder to propping, which can imply that an indebted firm has more incentives to give up the limited liability ruling. In their model, controlling shareholder may choose whether to bail out a distressed firm, depending on the deepness of the low cycle of the economy, and on the magnitude of the private benefits of control they extract from their firms, against the interests of minority shareholders.

Riyanto and Toolsema (2008), design a model to explain conglomeration in the presence of tunneling and propping, and find plausible conditions under which conglomeration structure is preferred to stand alone firms, advancing that tunneling must not be the only reason to conglomerate. Myopic minority shareholders (do not foresee tunneling effects), and incentives to controlling shareholders to eventually propping the distressed subsidiary, are also needed. In their model there is a clear conflict of interests between controlling and minority shareholders when tunneling, but given tunneling, interests are aligned at the time of propping.

The potential Conflicts between Controlling and minority Shareholders, is also vastly analyzed in the literature. Explanations are given invoking poor legal protection to minority shareholders (La Porta et al., 1999; 2002). Poor legal protection augments incentives to majority shareholders to maintain holding benefits of control (private rent extraction, tunneling), by bailing out (propping) distresses subsidiaries.

Friedman et al. (2003), show that propping behavior by controlling shareholder (negative tunneling), benefits minority shareholders, in a stand-alone firm.

Kim (2004) analyzes the conglomeration problem, in a context of moral hazard between the companies and banks, without agency problems between controlling and minority shareholders, and find that poor firms tend to conglomerate to maximize bailout probability.

Peng et al. (2011), test empirically Friedman (2003)' model, and find that it matches what is observed in China. Also, the find that when valuing the firms, the market reacts favorably to related transactions announcements when the firm is in poor financial health, and the opposite when the firm is in good financial health.

Lefort and Walker (2007) analyze, for the case of Chile an emerging economy, the effect of agency problems between controlling a minority shareholder on the valuation of companies. They use as a proxy to agency costs, the degree of coincidence between cash flow and control rights. The less coincidence, greater chances of minority shareholders expropriation. The find that in Chile, the market penalizes agency problems, in the sense that the less coincidence of rights, the more the penalization in market value (measured by market values corresponding to transactions of minority shareholders). They also declare that "business groups are not necessarily bad to minority shareholders". precisely because, given the relative imperfections of the market, minority shareholders have access to investment opportunities, stability, and protection.

In the same vein, Espinosa et al. (2018) report a negative relation between the largest shareholder ownership and firm value, and that separation between control rights and cash flows rights is also negatively related to firm value.

The first purpose of this paper is to fill the gap by providing a sequence of models that can explain under what circumstances a holding discount can exist, without recourse to asymmetric information issues or other imperfections. The models are designed using extensively the option of shareholders to give up following the Limited Liability ruling.

Therefore, I state the main hypothesis of this chapter, H0: holding discount exists, only when asymmetric information, agency costs or other imperfections, are considered, regardless of the legal system.

A secondary purpose is to visualize under what conditions, a bailing out behavior by controlling shareholders, is aligned with the interests of minority shareholders, in the holding company as well as in the affiliates, meaning that in order to maintain benefits, majority and minority shareholders should agree to bail out, instead of being in conflict, as literature suggests.

The model borrows from several sources to capture the dynamic and recursive nature of the bailing out decision, identify the decision maker and other players involved, and what can be understood as an optimum decision in each stage. Stuart (2008) presents a similar model to highlight the value of a recursive decision to a Venturer and takes the steady state dynamic programing solution to show that a Venturer will prefer to insist in a project when failed in the first stage given the chances to gain in the future. Almeida and Philipon (2007, 2008) use a similar framework to derive a formula to calculate the real cost of distress and hence to correct the standard yields calculated for bonds using historical probabilities of default. Finally, Friedman (1971) and Abreu (1988) provide sound basis to estimate equilibrium y infinitely repeated games. to capture the dynamic and recursive nature of the bailing out decision, identify the decision maker and other players involved, and what can be understood as an optimum decision in each stage.

Limited Liability (LL) is a legal rule for corporations, used all over the world. On the other hand, business group discount is an amply studied phenomena in the academia and business world. The purpose of this paper is to analyze the relationship between business group discount and limited Liability (LL) in corporations, specially focusing on the option of shareholders in corporation not to follow LL, thus follow an Unlimited Liability behavior (UL) called propping or bailing-out in the literature. I analyze the rationale for that behavior by means of simple but progressively more realistic corporate decisions models.

In Section 2, I analyze a simple model (Model 1) of a stand-alone company subject to Limited Liability rules (LL). This model serves to motivate the rest of the models and provides the basis to calculate the value (incentives) to the shareholders of following (UL). In Model 1 I determine the conditions under which shareholders (represented by a representative shareholder) have incentives not to follow LL ruling (not to exercise the LL option), which is relevant in case of default, and instead they behave as if they were subject to Unlimited Liability (UL) and "rescue" the company. Actually, under LL ruling, Shareholders have an option (the right but not the obligation) to comply with LL. The decision model is solved borrowing some concepts from Game Theory repeated games and Decision Theory, with infinite horizon, in which the players are: The Representative Shareholder and Nature. In this model I assume there are no conflicts of interest (agency costs) between controlling and minority shareholders, and I find that under certain conditions, called Continuation Conditions, Shareholders will have the incentive to pay the costs to rescue the eventually distressed company.

In this section holding discount is not applicable, thus no reference to hypothesis H0 is made.

In Section 3, I extend Model 1 to allow for a new shareholder to acquire a controlling stake in the stand-alone company (Model 2). The new shareholder becomes a simple holding company ("Holding"), controlled by a single controlling shareholder; Holding has no controlling stakes in other companies nor other businesses. I allow for the existence of benefits of being a holding that accrue to controlling shareholders in one case (private benefits), and to all shareholders in other case. Under this scenario I revise analysis of section 1, by introducing third and fourth players, Minority Shareholders at the affiliate and holding companies. These new players act as followers of majority shareholders (Stackelberg game) in every period stage of the infinite game. If continuation condition hold, the optimal strategy of controlling shareholder is to rescue the distressed affiliate by issuing equity, and the optimal strategy of minority shareholders is not to concur to the equity call, showing the existence of conflicts of interest except in the case in which external benefits accrue to all shareholders. Depending on the nature of the external benefits, a holding discount may arise.

Thus, in the conditions of model 2, hypothesis H0 cannot be rejected up to this point, even in the legal ruling of Limited Liability.

In Section 4, I show Model 3, extending Model 2, to allow the Holding to previously own controlling stakes in other companies as well as Debt; I revise and complement the analysis of the second section. In Model 3 I allow for new debt to pay the costs of rescue of the distressed affiliate. In this case a holding discount arises, with higher probability (greater incentives to follow UL to controlling shareholder) the greater the amount of existing plus new debt. As in model 2, if external benefits are common to all shareholders, there will not be conflict of interests, but if benefits belong to controlling shareholder only, there will be conflict of interests. In Model 3 the probability of rescue in case of distress in the subsidiary are greater than in Model 2
Thus, in the conditions of model 3, main hypothesis H0 can be rejected with a reasonable degree of confidence, in the legal ruling of Limited Liability and certain plausible conditions.

In Section 5, I show Model 4, as a special extension of Model 3. In this model I allow the holding company to pay the cost of rescue with existing cash, contrary to the debt case of model 3. As in models 2 and 3, I allow for the existence of external benefits, and find that in case of rescue, a holding discount will arise, in a context of eventual conflict of interests between controlling and minority shareholders.

Thus, in the conditions of model 4, main hypothesis H0 can be rejected with a reasonable degree of confidence, in the legal ruling of Limited Liability and certain plausible conditions.

In Section 6, I outline several avenues for further research using some relations derived, without empirical testing which is not part of this work.

Section 2. A Stand-Alone Company (Motivation to Un Limited Liability behavior)

2.1. General Setting, the Assets story

The company is represented by a Productive Asset, subject to uncertainty, in such a way that every period the true asset value (state) is revealed, and immediately the assets "regenerate" and a new "one period story" begins.

Uncertainty is represented by 2 states of nature, up and down: States are labeled $"u" \text{ with probability } (q) \quad \text{and } "d" \text{ with probability } (1-q)$. True state is revealed next period $(t=1)$ and Asset can pay V_u *or* V_d which are the state dependent realizations of Asset each period (I will refer to a nature state revelation as a move or play by nature).

Then a new one period story starts, and so does for infinite periods. I assume that probabilities of "up" and "down" states as well as V_u *and* V_d remain constant forever. All relevant information is common knowledge. Figure 2 below shows the Asset story:

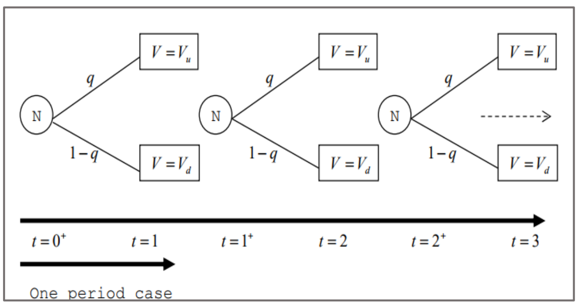

Figure 2. Asset evolution general setting

Mathematically, Assets may be viewed as following a Markov process with the following transition matrix:

State	V up	V down
V up	q	$(1-q)$
V down	q	$(1-q)$

This particular transition matrix has the property that it keeps unchanged after N transitions (N periods), regardless of N,

$$\begin{pmatrix} q & 1-q \\ q & 1-q \end{pmatrix}^N = \begin{pmatrix} q & 1-q \\ q & 1-q \end{pmatrix}$$

similar to a static process.

I conclude this section affirming that I have a reasonable model for valuing assets.

2.2. Preliminaries: Modeling the Limited and Unlimited Liability Cases in a static model

I assume the assets are financed by Shareholder's equity and Debt.

Definitions:

$D_{1,s}$: Debt to be reimbursed to debtholders in state s.

$S_{1,s}$: Shareholder's value at the end of this story in state s.

B_0: Expected present value of Debt

S_0: Expected present value of Equity

V_u: Asset value if nature reveals state u

V_d: Asset value if nature reveals state d

V_0: Expected present Value of Assets

R_a: one period discount factor reflecting asset's (undiversifiable) risk. $R_a = (1 + r_a)$

R_e: one period discount factor reflecting equity's (undiversifiable) risk. $R_e = (1 + r_e)$

R_b: one period discount factor reflecting Debt's (undiversifiable) risk. $R_b = (1 + r_b)$

$E(V_1) = q \cdot V_u + (1-q) \cdot V_d$: expected one period asset realization value

Agents are Expected Present Value of monetary cash flows maximizers[12], there is a competitive security market to which shareholders and debt holders have free access and set values. There are no taxes, no information asymmetries, and no bankruptcy costs or other market imperfections[13]. In order to make the case feasible I will assume that $V_u > D_1$; and to make the case interesting, I assume that $V_d < D_1$.

Under "Limited Liability rules", and previous assumptions, I can write:

2.1
$$\begin{cases} \textit{Asset Values} \\ V_{1,u} = V_u \\ V_{1,d} = V_d \\ V_0 = E_0(V, t=0) = \left(qV(s=u) + (1-q)V(s=d)\right)R_a^{-1} = \left(qV_u + (1-q)V_d\right)R_a^{-1} \end{cases}$$

2.2
$$\begin{cases} \textit{Equity Values} \\ S_{1,s} = \max(0, V_s - D_1) \\ \Rightarrow S_{1,u} = \max(0, V_u - D_1) \Rightarrow S_{1,u} = V_u - D_1 \\ \Rightarrow S_{1,d} = \max(0, V_d - D_1) \Rightarrow S_{1,d} = 0 \\ S_0 = q(V_u - D_1)R_e^{-1} \end{cases}$$

2.3.
$$\begin{cases} \textit{Debt Values} \\ B_{1,s} = \min(V_s, D_1) \\ \Rightarrow B_{1,u} = \min(V_u, D_1) \Rightarrow D_1 \\ \Rightarrow B_{1,d} = \min(V_s, D_1) \Rightarrow V_d \\ B_0 = \left(qD_1 + (1-q)V_d\right)R_b^{-1} \end{cases}$$

12 One possibility is agents are risk neutral. Other view is that separation between production and consumption decisions holds.

13 discount rates can be derived using MM assumptions

Under "Unlimited Liability behavior", and with the previous assumptions I can write:

$$2.4 \quad \begin{cases} \textit{Asset Values} \\ V_{1,u} = V_u \\ V_{1,d} = V_d \\ V_0 = \left(qV_u + (1-q)V_d\right)R_a^{-1} \end{cases}$$

$$2.5 \quad \begin{cases} \textit{Equity Values} \\ S_{1,s} = V_s - D_1 \\ \Rightarrow S_{1,u} = V_u - D_1 \\ \Rightarrow S_{1,d} = V_d - D_1 < 0 \\ S_0 = \left(q(V_u - D_1) + (1-q)(V_d - D_1)\right)R_e^{-1} < S_0 \quad \textit{under Limited Liability} \end{cases}$$

$$2.6 \quad \begin{cases} \textit{Debt Values} \\ B_{1,s} = D_1 \\ \Rightarrow B_{1,u} = D_1 = B_{1,d} \\ B_0 = \left(qD_1 + (1-q)D_1\right)R_b^{-1} = D_1 R^{-1}{}_{rf} > D_1 R_b^{-1} > B_0 \quad \textit{under Limited Liability} \\ \qquad \text{where } r_f \text{ means risk free} \end{cases}$$

Note that in the case of unlimited Liability, Debt becomes "risk-free", increasing its value (with respect to LL) at the expense of Equity value which decreases in the same amount[14].

The reason for this is that in the down state, the bad case, the shareholder simply returns the assets to debt holders in case of LL thereby ending with no equity, with some probability. On the contrary, if the shareholder behaves as UL, will end with a loss with the same probability.

14 discount factors may differ between LL and UL situations. The analysis is postponed and is not relevant to the present note.

The following graphs in figure 3 depict the situation for equity holders and bond holders:

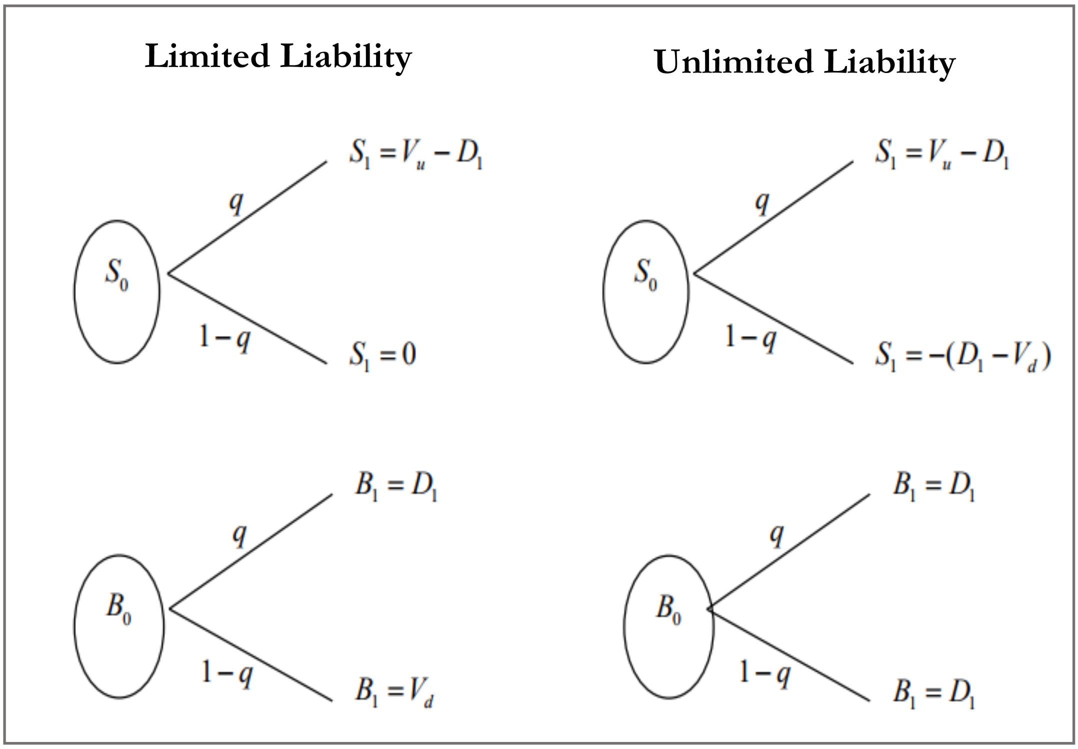

Figure 3. Asset, Equityholders and Bondholders position, with and without Limited Liability

2.3. The complete (multi period) model, Model 1

In section 3.3.1 I first show a "one stage decision model". Then in section 3.3.2, I extend one stage to "multistage decision model". The timing of the model is the following:

In $t=0$, a representative shareholder who previously owns the assets of the stand-alone firm, sells the assets to a creditor in exchange for debt funds. The debt must be repaid one period forward. Shareholder holds a call option on the assets with a strike price equal to D_1, and holds a second option which is not to comply with LL ruling.

In $t=1$ nature determines the value of assets:

$$V_u\ (probability\ q), or\ V_d\ (probability\ 1-q).$$

If,

$$V = V_u,$$

the shareholder exercises his option, and cashes-out, with profit:

$$V_u - D_1.$$

If

$$V = V_d,$$

the shareholder decides if she quits (LL), with 0 profit, or if she continues to the next stage (UL). In this later case, the shareholder will pay a cost c to have the right to continue[15]. In this particular case,

$$c = D_1 - V_d,$$

nature will reveal its true state one period after.

In $t = 1^+$, assets regenerate, the shareholder sells the assets to a creditor for the same amount as before, D_1, and one period later $t=2$ nature determines the value of assets:

$$V_u\ (probability\ q), or\ V_d\ (probability\ 1-q).$$

If,

$$V = V_u,$$

shareholder exercises his option, and cashes-out, with profit,

$$V_u - D_1.\ \text{If}\ V = V_d,$$

the shareholder decides if she quits, with 0 profit, or if she continues to the next stage. In this later case, the shareholder will pay a cost c to have the right to continue. In this particular case,

$$c = D_1 - V_d.$$

15 Note that our assumptions imply that if $V_d - D_1 > 0$ shareholder will cash out immediately and end the game.

Nature plays in $t = 0, 1^+, 2^+, \ldots\ldots$; decisions are made in $t = 1, 2, 3, \ldots\ldots$ and this continues until infinity[16]. I assume that the capital structure is given exogenously.

To solve the dynamic model, and determine the optimal strategy for the shareholder, I must first analyze a "one stage" representative model, starting in decision node at $t = 1$, and then I extend it through time.

Figure 4 below, describes the dynamic or extensive and recursive model:

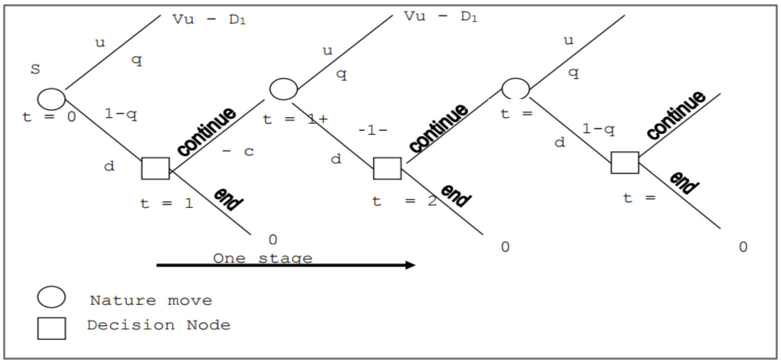

Figure 4. Model 1. Dynamic, Extensive, and Recursive Model

From the previous figure I can identify a "one stage decision" (from $t = 1 \;\; to \;\; t = 2$, or $t = 2 \;\; to \;\; t = 3$) which must be analyzed previous to the complete dynamic solution.

2.3.1. One Stage Decision model

In node *t = 0*, the shareholder owns equity worth S_0 (to be determined); if nature plays *"u"* she cashes-out and the story ends, if nature plays *"d"* the shareholder faces decision node *t = 1*. In this decision node $t = 1$, the shareholder has two alternative actions: to quit or to continue. I now calculate the expected value of these actions:

$$E^1(continue\ one\ more\ time) = V^1_{cont} = -c + \frac{q \cdot (V_u - D_1) + (1-q) \cdot 0}{R_e}, with \;\; c = D_1 - V_d$$

$$E^1(end) = V^1_{end} = 0$$

shareholder's Value at this decision node is given by,

16 This is an essential condition of this model.

$$V_1^+ = \max(V_{cont}^1, V_{end}^1).$$

First assume that,

$$-c + \frac{q \cdot (V_u - D_1)}{R_e} \leq 0.$$

Then,

$$V_1^+ = \max(V_{cont}^1, V_{end}^1) = 0.$$

This implies that the optimal action is to abandon (end) in node *t = 1*, in other words, since the beginning, given the assumptions on the time evolution of assets.

Now assume that,

$$-c + \frac{q \cdot (V_u - D_1)}{R_e} = -(D_1 - V_d) + \frac{q \cdot (V_u - D_1)}{R_e} > 0 \cdot$$

Then,

$$2.7 \quad V_1^+ = \max(V_{cont}^1, V_{end}^1) = -(D_1 - V_d) + \frac{q \cdot (V_u - D_1)}{R_e} = \left(-R_e \cdot (D_1 - V_d) + q \cdot (V_u - D_1)\right) R_e^{-1}.$$

This implies that the optimal action, given she is in node t=1, is to continue at least for one more stage; if nature plays "u" shareholder cashes-out and the story ends; if nature plays "d". The shareholder faces again the decision to end or continue, in *t=2*.

Now, decision faced in $t = 2$ is exactly the same faced in $t = 1$. Moreover, even the shareholder has spent $D_1 - V_d$; given that this cost is sunk[17], it is not pertinent to the decision whether to continue to a next stage.

With this reasoning, if the conditions of the problem remain unchanged, the optimal decision if nature plays *"u"* will be to cash-out, and if nature plays *"d"* to keep paying and going on.

17 I can assume that shareholders have deep pockets enough to fund a sufficiently big number of continuation payments, without financial problems. Nevertheless, as seen below, the expected present value of continuation payments, is of the same order of magnitude of one period payment.

From the previous analysis, I characterize the condition under which the shareholder will optimally continue game to infinity:

2.8 Continuation Condition (CC): $\dfrac{q\cdot(V_u - D_1)}{R_e} > (D_1 - V_d) \quad 0<q<1; \quad V_d < D_1 < V_u \cdot$

If game ended at this point (for example shortsighted agents), I could write a more complete version of equation (2.2) as:

$$\textit{Equity Values}: \quad CC_1^+ \quad : \quad (V_d - D_1 < 0)$$

$$S_{1,u} = \max(0, V_u - D_1) = V_u - D_1$$

$$S_{1,d} = \max(V_1^+;0) = \max\left(\max(V_{cont}^1, V_{end}^1);0\right) = \max\left(\underbrace{-(D_1 - V_d) + \frac{q\cdot(V_u - D_1)}{R_e}}_{>0 \ \textit{if Continuation Condition holds}};0\right)$$

Then,

$$S_0^{+1} = \left(q(V_u - D_1) + (1-q)\cdot\max\left(V_1^+;0\right)\right)\cdot R_e^{-1}$$

2.9

$$S_0^{+1} = \frac{E(V_1) - D_1}{R_e} + (1-q)\cdot q\cdot(V_u - D_1) \qquad \textit{iif complying with (} \ \textit{(2.8)}$$

It must be noted that equity present expected value S_0, is greater than the corresponding LL value, given that the option the equity holder owns (not to enforce LL but to continue at least for one more period) has a positive value if Continuation Condition holds.

At this point it is convenient to show the difference between LL and UL behavior in relation with fundamental parameters of the model as shown in figure 5 below:

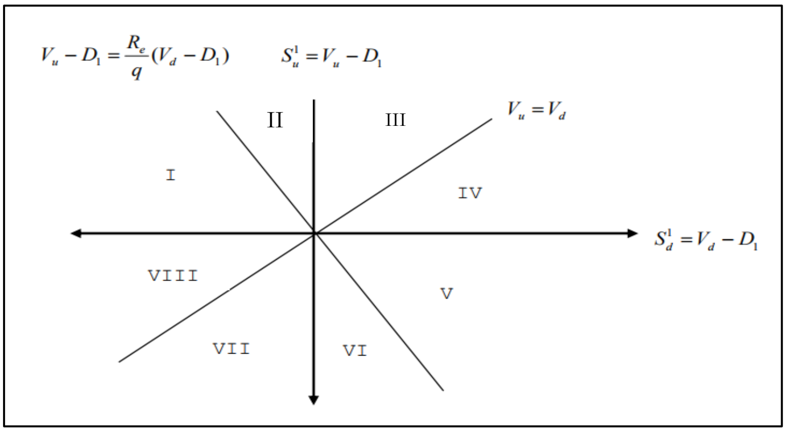

Figure 5. Graphic analysis of LL and UL Behavior

The following relations divide the parameter space in regions I to VIII:

1. S_u^1 versus S_d^1 in vertical and horizontal axes respectively. The feasibility condition,

$$V_u - D_1 > 0,$$

eliminates regions V to VIII.

Under LL,

$$S_d^1 \geq 0,$$

which eliminates regions I and II, thus leaving regions III and IV as the only feasible ones under that rule.

2. Line:

$$V_u = V_d,$$

which is the lower frontier to the relationship between V_u and V_d. A standard constraint imposes,

$$V_u > V_d,$$

thus, eliminating region IV.

3. Line:

$$V_u - D_1 = \frac{R_e}{q}(V_d - D_1),$$

represents the lower frontier of Continuation Condition, (2.8) that can be written as:

$$V_u - D_1 > \frac{R_e}{q}(V_d - D_1).$$

Given that in my model, under UL there are not constraints imposed to S_d^1, the feasible region under this ruling together with constraints 1 and 2, are II and III. In this way I show that under UL I add region II to the feasible regions in the parameter space, thus increasing the number of cases in which it is possible to add value following UL with respect to standard equity under LL rules. From the one stage decision model, I have learned that in case CC holds, the expected value to the shareholder to enter in one more stage, in case nature plays "*d*" is:

$$\text{2.9a} \quad V_c^{+1} = \frac{q \cdot (V_u - D_1)}{R_e} - (D_1 - V_d),$$

as the decision to continue is expected to be made permanently in case nature always plays "*d*", the total Continuation Value will be at least equal to V_c^{+1}.

2.3.2. Complete (multistage) Decision model

To analyze the multistage decision model, let V_c denote the Complete Continuation Expected Value, if CC in equation 2.8 holds and shareholder is faced to a decision (in case nature plays "*d*"). I make the assumptions that V_u, V_d will remain unchanged, beliefs represented by q are not revised (or if revised remain the same), and all information is common knowledge to shareholder and creditors.

To compute the Complete Continuation Expected Value V_c, I exploit the infinite recursive nature of the decision model as follows in figure 6:

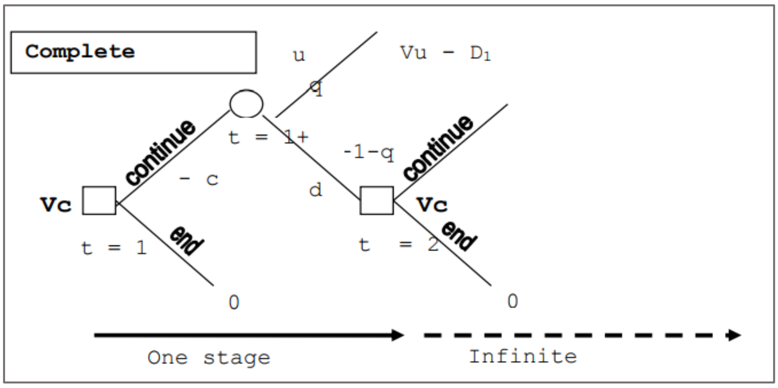

Figure 6. Model 2. Multistage Decision Model

In case nature plays *"d"*, I obtain the following:

$$V_c = \max(V_{end}, -c + \frac{q(V_u - D_1) + (1-q)V_c)}{R_e}).$$

I know that if CC (eq.2.8) does not hold, then $V_{end} = 0$.

On the contrary if CC holds, then,

$$V_c = -c + \frac{q(V_u - D_1) + (1-q)V_c}{R_e},$$

from which I obtain:

$$V_c = \frac{q(V_u - D_1) - R_e(D_1 - V_d)}{q + r_e} = \underbrace{V_c^{+1}}_{>0}(\underbrace{\frac{R_e}{r_e + q}}_{>1}) > V_c^{+1} \quad 2.10$$

where V_c^{+1} is defined in equation 1. 2.9a

Now, stepping back to $t = 0$ (see figures "Extensive Form" and "multistage decision model"), I can compute the expected value of the shareholder's equity position at t=0 in cases Continuation and No Continuation are optimum respectively:

No Continuation is Optimum decision:

$$
2.11 \quad \begin{aligned} & S_0^{nc} = \frac{q(V_u - D_1) + (1-q)V_c}{R_e} = \frac{q(V_u - D_1)}{R_e} \\ & iff: \frac{q \cdot (V_u - D_1)}{R_e} \le (D_1 - V_d), \quad 0 < q < 1; \quad and \quad V_d < D_1 < V_u \end{aligned}
$$

$continuation\ value = 0 \Rightarrow optimal\ strategy\ is\ cash-out\ at\ the\ beggining \equiv no\ continuation: nc$

Continuation is Optimum decision:

$$
2.12 \quad \begin{aligned} & S_0^{c} = \frac{q(V_u - D_1) + (1-q)V_c}{R_e} = \frac{q(V_u - D_1) + (1-q)\left(\dfrac{q(V_u - D_1) - R_e(D_1 - V_d)}{q + r_e}\right)}{R_e} \\ & S_0^{c} = \frac{q(V_u - D_1) + (1-q)(V_d - D_1)}{r_e + q} = \frac{E(V_1) - D_1}{r_e + q} \\ & iff: \frac{q \cdot (V_u - D_1)}{R_e} > (D_1 - V_d) \quad and \quad 0 < q < 1; \quad V_d < D_1 < V_u \end{aligned}
$$

$continuation\ value > 0\ or\ optimal\ strategy\ is\ cash-out\ if\ nature\ plays\ "u"\ and$
$continuation\ at\ every\ stage\ if\ nature\ plays\ "d".$

The last result imposes another practical condition: in order for this model to make sense to shareholders and creditors, the expected present value of equity at *t=0* must be positive, i.e., $E(V_1) - D_1 > 0$ (the company is not in default), given that the continuation condition holds.

I can see, that under plausible parameters:

$$
S_0^c(eq.2.12) > S_0^{+1}(eq.2.9) > S_0(eq.2.2)
$$

Implying that the continuous possibility to rescue the company in the bad state, adds value to the shareholders.

Refer to Appendix for an extended derivation of the model.

2.3.3. Conclusions

The main specific conclusions of section 2.3 are:

- This can be stated as follows: there will be an optimal decision to follow upon default, for all the shareholders (represented by a Representative Shareholder), and their related:
  a) Optimal Continuation decision,
  b) Expected Value and Expected Present Equity, and
  c) Debt values, all depending on the asset values in different states of nature, their probabilities, debt level, discount rates and information efficiency (I assume common knowledge).
- Equity present expected value under UL option, $S_0{}^c$, is greater than, or equal to, the corresponding LL value, given that the Option the equity holder owns (not to enforce LL but to continue at least for one more period) has a positive value if Continuation Condition in equation (2.8) holds.
- In case of default in a given period, the "One period continuation value" V_c^{+1}, is less than the continuous and permanent (upon realization of state "d") continuation value V_c. This can be seen comparing equations (2.7) with equation (2.9b).

$$V_c - V_1^+ = \frac{q(V_u - D_1) - R_e(D_1 - V_d)}{q + r_e} - \frac{\left(-R_e \cdot (D_1 - V_d) + q \cdot (V_u - D_1)\right)}{R_e}$$
$$= q(V_u - D_1) \cdot (\frac{1}{q + r_e} - \frac{1}{R_e}) - R_e(D_1 - V_d) \cdot (\frac{1}{q + r_e} - \frac{1}{R_e})$$
$$but: \frac{1}{q + r_e} - \frac{1}{R_e} = \frac{1 + r_e - q - r_e}{(q + r_e)R_e} = \frac{1 - q}{(q + r_e)R_e} > 0$$
$$then: V_c - V_1^+ = \frac{\left(q(V_u - D_1) - R_e(D_1 - V_d)\right) \cdot (1 - q)}{(q + r_e)R_e} > 0$$
$$given\ that\ eq.(1.7): \left(-R_e \cdot (D_1 - V_d) + q \cdot (V_u - D_1)\right) R_e^{-1} > 0$$

- In case conditions stated in (2.8) and conclusion 3 holds, then the following must be true:

$$S_0^{nc} < S_0^{+1} < S_0^c,$$

With,

$$S_0^{nc} = \frac{q(V_u - D_1)}{R_e}, \; S_0^{+1} = \frac{E(V_1) - D_1}{R_e} + (1 - q) \cdot q \cdot (V_u - D_1), \; S_0^c = \frac{E(V_1) - D_1}{r_e + q}.$$

- The value of the "Option to Continue" in case of default is:

$$\frac{E(V_1) - D_1}{r_e + q} - \frac{q(V_u - D_1)}{R_e} = \frac{(1 - q) \cdot V_1^+}{R_e \cdot (r_e + q)} > 0 \text{ iff } \mathrm{V}_1^+ > 0 \Leftrightarrow CC\ holds$$

- If Continuation Condition holds, then the optimal decision to all shareholders will be to follow UL, the no conflicts are expected.

Model 1 Summary:

Optimal strategy for the Representative Shareholder:

1. (*nc*): If Continuation Condition (CC) doesn't hold: Cash Out and end in the first node.

2. (*c*): If Continuation Condition (CC) Holds: if nature plays *"d"* continue for a next period and pay the corresponding cost. If nature plays *"u"*, cash out and end.

Conditions:

1. Continuation Condition (in case of default) is: $q \cdot (V_u - D_1)/R_e > (D_1 - V_d)$
2. Parameter mandatory conditions: $V_u, V_d > 0; \quad 0 < q, r_a, r_e, r_b < 1$
3. Parameter "convenience" conditions: $V_d < D_1 < V_u; \;\; E(V) - D_1 > 0$

Valuations:

$$I)\;\; S_{nc}^0 = \frac{q(V_u - D_1)}{R_e};\;\; B_{nc}^0 = \frac{\left(qD_1 + (1-q)V_d\right)}{R_b};\;\; V_{nc}^0 = \frac{q(V_u - D_1)}{R_e} + \frac{\left(qD_1 + (1-q)V_d\right)}{R_b}$$

$$II)\;\; S_c^0 = \frac{E(V_1) - D_1}{r_e + q};\;\; B_c^0 = D_0;\;\; V_c^0 = \frac{E(V_1) - D_1}{r_e + q} + D_0$$

If the company is in condition I (II), Stand Alone Continuation Condition (CC) doesn't hold (holds), and according to the assumptions of the model, CC will never (always) hold.

Section 3. Introducing Conflicts of Interest between Controlling and Minority Shareholders the Unlevered Holding Case

3.1. Introduction

In Section 2 I have derived several conclusions for a stand-alone company, whose shareholders face the rescue decision, have no conflicts of interest, in the sense that their optimal strategy is the same for a controlling as well as a minority shareholder (representative shareholder). The company has a predefined capital structure, and all relevant information is common knowledge.

In this section I let the stand-alone company of Section 2 now to be an affiliate company of an Unlevered holding company, becoming Model 2. Else I introduce the analysis of possible conflicts between controlling shareholders and minority shareholders when the formers exert their right to rescue the affiliate company.

The general procedure is the following: establishing the Acquisition Approach to be the case; first in sub section 3.2.1.a) I establish the conditions under which the controlling shareholder will force a rescue of the distressed Affiliate calling for new equity, and minority shareholders concur to the equity call, and in sub section 3.2.1.b) derive formulas for prices and valuation in the case minority shareholders concur to the equity calls (I call this case the Naïve model 2). Then I go one step forward, sub section 3.2.1.c) and analyze the best strategy or the best response of minority shareholders at Holding Co. as well as in the Affiliate. In the final part, sub section 3.2.1.c3) I analyze the more realistic model 2 in which minority shareholders do not concur to equity calls, and I calculate what I call the realistic valuation Schedule. In all the sub sections I derive conclusions regarding the possibility of a holding discount, and the presence or not of conflicts of interest between controlling and minority shareholders.

Finally, in sub section 3.3 I summarize all the cases and their respective conclusions.

3.2. The Unlevered Holding case: Model 2

I extend Stand Alone case to a holding company owning and controlling an Affiliate and no additional operations. Affiliate has same "fundamentals" as the stand-alone company of Model 1.

First, I assume that CC from model 1 doesn´t hold, so in case of default, unless benefits idiosyncratic to CS exist (to be defined later), that counterbalance the net cost of rescue, the optimum strategy for the representative shareholder in Affiliate is not to rescue whenever state of nature is "d".

In this condition, if a rescue strategy implemented in model 2 by issuing equity is followed (which is common knowledge, and for that reason can be anticipated), it will be done pursuing the interests of CS and eventually against the interest of minority shareholders. This fact must be recognized in the initial and final equity prices.

As in Model 1, an optimal investment (disinvesting) strategy for controlling shareholder will be determined, and valuations calculated; additionally, I will explore the impact the optimum strategy of CS may have on Affiliate as well Holding companies' valuation. A change in relative valuations to both, controlling and minority shareholders produced by certain strategy will be interpreted as a holding discount or Premium with respect to initial conditions.

I solve Model 2 using two different approaches that should deliver the same conclusions:

- Starting point is a Holding Already in Place: a holding Company already owning an affiliate; this model is generalized in Model 3, and in the Appendix; alternatively,
- Starting point is a holding company possesses no assets nor liabilities; in certain point in time, acquisition of an affiliate (stand alone in Model 1) takes place.

Key issues in both approaches are:

a) Controlling Shareholder´s (CS) strategy in case of default of the subsidiary
b) Before and After acquisition equilibrium equity prices
c) Minority shareholders at both Affiliate and Holding companies' best response to CS strategy

3.2.1. "Acquisition" Approach

Suppose a holding company (the "Holding"), having a controlling shareholder (CS) with share $m: (0,5 < m < 1)$, acquires from existing shareholders a controlling position $f: (0,5 < f < 1)$ of a previously stand-alone company (now the "Affiliate"), conforming a business group.

According to Chilean law, when a shareholder takes control of a public company, a Public Offer must be carried on through the stock market. If the controlling stake exceeds $f \geq 2/3$ of the outstanding shares, an offer to buy the rest of the shares must be done to all shareholders in the same conditions. Pricing is also upper bounded by law to 15% over the average market price of previous 3 to 6 months.

The assumptions:

- Public Offer price of shares issued in the holding company as well as in the Affiliate, to fund the acquisition, will be the fair (stand-alone) price S_{nc}^{0}, or LL stock price (this will be the case because CC does not hold); and market knows that before Affiliate becomes an acquisition target by a potential "Rescuer" Holding. Additionally, I assume no control premium is paid.
- Holding has no other state dependent Investments or operations and has no Debt (unlevered Holding). In this way, after acquisition CS controls both the Holding and the Affiliate, with $m \cdot f$% cash flows and voting rights in the Affiliate.
- Controlling shareholder (CS) derives certain specific benefits M per period[18] for having (or keeping) the company as part of the Group even if it Affiliate enters default. This means that it if the Affiliate goes into default in any period (nature plays *"d"*), CS will face the decision whether to continue or not, evaluating all benefits and costs relevant to CS.
- If best strategy to CS is to Continue (recue), he calls for equity at affiliate´s level, as well as at Holding´s level. Minority shareholders may concur (from here on the Naïve Case) or not concur (the realistic Case).

First, I study the case in which all minority shareholders concur to the equity call (the Naive Model 2). Finally, in Realistic Model 2, I correct the Naive Model by introducing the best strategy or best response to minority shareholders, which is not to concur.

The (Naive) decision tree for CS shown in figure 7, becomes now:

18 Nature of M will be analyzed later. It can correspond to private benefits of control already in place, or some kind of operational synergies between both companies. Refer to Lefort and González "optimal bailout of subsidiaries", chapter 4 of this Thesis.

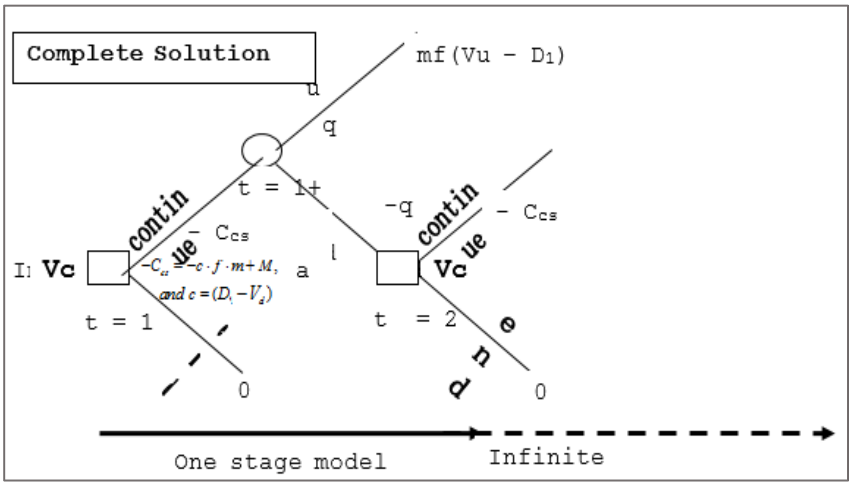

Figure 7. Model 3. Naive Decision Tree

In which,

$$-C_{cs} = -c \cdot f \cdot m + M,$$
$$and\ c = (D_1 - V_d) \text{ as in Model 1}$$

Solution to this model is to follow the procedure derived for the "stand alone case". I must first derive the one stage continuation strategy, and then, the optimal strategy for CS.

<u>3.2.1a) Optimal Strategy to CS in case all minority shareholders follow him:</u>

One stage Decision Model:

Continuation Condition (CC_{cs}) *will* now be: $\dfrac{f \cdot m \cdot q \cdot (V_u - D_1)}{R_e} - C_{cs} > 0$

$$\Rightarrow \frac{f \cdot m \cdot q \cdot (V_u - D_1)}{R_e} - (c \cdot f \cdot m - M) > 0 \quad with \quad c = (D_1 - V_d) > 0$$

$$3.1 \quad \frac{f \cdot q \cdot (V_u - D_1)}{R_e} - c \cdot f + M^* > 0 \quad with \quad \frac{q \cdot (V_u - D_1)}{R_e} < c; \quad M^* = \frac{M}{m}$$

Analysis:

Continuation Condition CC_{CS} is analogous to Model 1´s CC, differing in the existence of M, the specific benefit to CS. If M=0, then CC is the same in both models. When M>0, I can see that in spite of been against interests of minority shareholders, it may be beneficial to CS to adopt the continuation strategy, detrimental to minority shareholders, but beneficial to CS. Given M, CS´s benefit increases as m and f decrease, because costs of continuation are shared with minority shareholders of both companies.

As an example, suppose that (as in the stand-alone case) $m = f = 1$. Then the existence of M lowers the hurdle to decide to continue (facilitates the decision to continue by CS). Letting $m < 1 \quad and \quad f = 1$ facilitates the decision even more.

Complete (multistage) solution:

The complete extended solution of the model depicted in the previous graph implies that if CC_{cs} holds, I will have:

$$V_c^{cs} = -C_{cs} + \frac{q(V_u - D_1) + (1-q)V_c^{cs}}{R_e}$$

$$\Rightarrow V_c^{cs} = \left(\frac{R_e}{q + r_e}\right) \cdot M + \frac{f \cdot m}{(q + r_e)} \cdot \underbrace{\left(q \cdot (V_u - D_1) + R_e (V_d - D_1)\right)}_{<0 \text{ if CC doesn't hold}}$$

$$3.2 \quad V_c^{cs} = \left(\frac{R_e}{q + r_e}\right) \cdot M + \frac{f \cdot m}{(q + r_e)} \cdot \left(E(V_1) - D_1\right) - f \cdot m \cdot (D_1 - V_d)$$

Continuation as Optimal Strategy can be decided to favor CS against the interests of the minority shareholders in the Affiliate if $V_c^{cs} > 0$ even if CC (from model 1) doesn't hold.

Note that *CCcs* poses the following lower bound to M:

$$M > f \cdot m \cdot \left(\frac{(r_e + q)}{R_e}(D_1 - V_d) - \frac{\left(E(V_1) - D_1\right)}{R_e} \right), \quad \text{or}$$

$$3.3 \qquad M > \frac{f \cdot m}{R_e} \cdot \left(D_1 (1 - (r_e + q)) - \left(E(V_1) - V_d (r_e + q)\right)\right)$$

<u>3.2.1b) Prices and Valuations:</u>

If market anticipates that best strategy to CS is to Continue, *in agreement with minority shareholders* (CC_{CS} holds), Affiliate´s equity value will change to $S_0^{c,CC}(CC\ holds) > S_0^{nc}$ after information about acquisition is revealed. Recall the Stand-Alone Case, and assume that CC from model 1 holds; then I have:

$$\frac{q\cdot(V_u-D_1)}{R_e}>(D_1-V_d);$$

$$V_c^{+1}=\frac{q\cdot(V_u-D_1)}{R_e}-(D_1-V_d)>0;\ V_c=-c+\frac{q(V_u-D_1)+(1-q)V_c}{R_e};$$

$$\text{then } V_c=\frac{q(V_u-D_1)-R_e(D_1-V_d)}{q+r_e},$$

and shareholder's value will be:

$$3.4 \quad S_0^{c,CC}=\frac{q(V_u-D_1)+(1-q)(V_d-D_1)}{r_e+q}=\frac{E(V_1)-D_1}{r_e+q}>S_{nc}^0=\frac{q(V_u-D_1)}{R_e} \quad \textit{iff CC} \text{ model 1 } \textit{holds}\cdot$$

Nevertheless, the case of interest is that CS forces Continuation against the interests of minority shareholders (CC doesn´t hold). In this case, Affiliate´s equity value will be:

$$S_0^{c,noCC}(CC\ doesn't\ hold)<S_0^{nc}.$$

Recall again the Stand-Alone Case, but now impose CC doesn't hold; then I have:

$$\frac{q\cdot(V_u-D_1)}{R_e}<(D_1-V_d);\ V_c^{+1}=\frac{q\cdot(V_u-D_1)}{R_e}-(D_1-V_d)<0;$$

$$V_c=-c+\frac{q(V_u-D_1)+(1-q)V_c}{R_e}$$

$$\text{And } V_c=\frac{q(V_u-D_1)-R_e(D_1-V_d)}{q+r_e}.$$

Shareholder's value in that case is:

$$3.5 \quad S_0^{c,noCC}=\frac{q(V_u-D_1)+(1-q)(V_d-D_1)}{r_e+q}=\frac{E(V_1)-D_1}{r_e+q}<S_{nc}^0=\frac{q(V_u-D_1)}{R_e}(CC\ doesn't\ hold).$$

Base (Naive) Case Valuation Summary. Naïve Model 2:

The following schedule shows a summary of valuations to CS and minority shareholders, in the (Naive) assumptions:

1. Funds needed to acquire the Affiliate are raised as "new equity" in Holding and correspondingly in Affiliate as well. Call is forced by CS.
2. Holding is unlevered (no net debt) and has no assets, before acquisition.

3. If *CCcs* holds (cases 3, 4, C, D), all agents know that in certain circumstances a rescue will take place, and value the companies accordingly.
4. Staying minority shareholders at affiliate level are not able to capture any extra value when selling their controlling position *f*.
5. When CS forces a rescue of, and calls for new equity in Affiliate, calls for new equity in the Holding Co., and minority shareholders in both companies will concur.

When CC doesn't hold, and CC_{cs} holds, all shareholders in the Affiliate will suffer a loss after acquisition. In that case, and anticipating the loss, selling minority shareholders include this loss in their sale price of the controlling stake.

Moreover, staying minority shareholders in Affiliate, and Holding´s minority shareholders, will also suffer a loss after acquisition by the same cause. The issue is whether or not they get compensated by their loss, and when.

Case summary schedule and results (CC doesn´t hold, no compensation to staying minority shareholders):

Table 1 compares equity positions of majority and minority shareholders, before and after the acquisition, given that CC does not hold, and in cases CC_{cs} holds, and CC_{cs} doesn't.

Table 1: Equity positions of majority and minority shareholders

Equity Value	New Majority Shareholder (at affiliate level)	New Minority Shareholder (affiliate level)	Majority Shareholder (at Holding level)	Majority Shareholder (at Holding level)
Before Acquisition	1	2	A	B
After Acquisition				
CC_{cs} holds	3	4	C	D
CC_{cs} doesn't hold	5	6	E	F

At Affiliate Level:

$$1:0$$

$$2: S_{nc}^{0} = \frac{q \cdot (V_u - D_1)}{R_e}$$

$$3: \frac{f \cdot \left(E(V_1) - D_1\right)}{r_e + q}$$

$$4: \frac{(1-f) \cdot \left(E(V_1) - D_1\right)}{r_e + q}$$

$$5: \frac{f \cdot q \cdot (V_u - D_1)}{R_e}$$

$$6: \frac{(1-f) \cdot q \cdot (V_u - D_1)}{R_e}$$

At holding company Level:

$$A:0$$

$$B:0$$

$$C: \frac{m \cdot f \cdot \left(E(V_1) - D_1\right)}{r_e + q} + \frac{(1-q)M}{r_a + q}$$ [19]

$$D: \frac{(1-m) \cdot f \cdot \left(E(V_1) - D_1\right)}{r_e + q}$$

$$E: \frac{f \cdot m \cdot q \cdot (V_u - D_1)}{R_e}$$

$$F: \frac{(1-m) \cdot f \cdot q \cdot (V_u - D_1)}{R_e}$$

$Naive\ Holding\ Discount(\$): market\ value\ Holding - NAV\ \ Holding$ (minority shareholders valuation)

Continuation: $(D)\frac{f \cdot \left(E(V_1) - D_1\right)}{r_e + q} - (4)\frac{f \cdot \left(E(V_1) - D_1\right)}{r_e + q} = 0$

No continuation: $(F)\frac{f \cdot q \cdot (V_u - D_1)}{R_e} - (6)\frac{f \cdot q \cdot (V_u - D_1)}{R_e} = 0$

Now I calculate: i) Change in the net economic position of shareholders before and after acquisition of the controlling stake in the Affiliate by the Holding; and ii) Relative change in position of the minority shareholders[20].

[19] $(1-q) \cdot M/(r_a + q)$ is the expected present value of specific benefits to CS.

20 I assume that market price is driven by minority shareholders valuations, which is the case in real world

i. Change in net economic position of shareholders.

At Affiliate Level:

Majority shareholder at Affiliate gets *f %* of the Affiliate, paying the pre-acquisition (stand-alone) valuation or price. His net position at this level is:

$$3.6\quad \frac{f\cdot\left(E(V_1)-D_1\right)}{r_e+q}-\$\left(\frac{f\cdot q\cdot(V_u-D_1)}{R_e}\right)=f\cdot\frac{1-q}{r_e+q}\underbrace{\left(-(D_1-V_d)+\frac{q\cdot(V_u-D_1)}{R_e}\right)}_{CC\ does\ not\ hold.....<0}<0\cdot$$

Staying Minority shareholders keep *(1-f) %* of the Affiliate, obtaining for themselves the pre-acquisition (stand-alone) price for their share. Their net position at this level is:

$$3.7\quad \frac{(1-f)\cdot\left(E(V_1)-D_1\right)}{r_e+q}+\$\left(\frac{(1-f)\cdot q\cdot(V_u-D_1)}{R_e}\right)<case\ 2\cdot$$

At Holding Level:

Majority shareholder gets *mf %* of the Affiliate, plus the expected present value of the Per Period Benefit *M*, paying his share of the pre-acquisition (stand-alone) price to all shareholders. Assuming that funds needed to acquire the Affiliate are raised as "new equity" in Holding, and that all shareholders concur, majority shareholder's net position at this level is:

$$3.8\quad \frac{m\cdot f\cdot\left(E(V_1)-D_1\right)}{r_e+q}+\frac{(1-q)M}{r_a+q}-\$\left(\frac{m\cdot f\cdot q\cdot(V_u-D_1)}{R_e}\right),$$

$$\frac{m\cdot f\cdot\left(E(V_1)-D_1\right)}{r_e+q}+\frac{(1-q)M}{r_a+q}-\$\left(\frac{m\cdot q\cdot(V_u-D_1)}{R_e}\right).$$

Minority shareholders get (1-*m)f %* of the Affiliate, paying their share of the pre-acquisition (stand-alone) price paid to all shareholders at Affiliate. Their net position at this level is:

$$3.9\quad \frac{(1-m)\cdot f\cdot\left(E(V_1)-D_1\right)}{r_e+q}-\$\left(\frac{(1-m)\cdot f\cdot q\cdot(V_u-D_1)}{R_e}\right)<case\ B,$$

$$\frac{(1-m)\cdot f\cdot\left(E(V_1)-D_1\right)}{r_e+q}-\$\left(\frac{(1-m)\cdot q\cdot(V_u-D_1)}{R_e}\right)\ll case\ B\cdot$$

ii. Relative change in Equity position of Minority Shareholders

At Affiliate Level:

If *CCcs* doesn't hold (and *CC* doesn't hold), the optimal strategy of CS will be to cash out since the beginning. Minority shareholders at the Affiliate will hold $(1-f)\cdot q\cdot(V_u-D_1)/R_e$.

On the other hand, if *CCcs* holds, the optimal strategy for CS will be to continue, forcing minority shareholders at the Affiliate to incur in a loss; their holdings in this case will be $(1-f)\cdot\left(E(V_1)-D_1\right)/(r_e+q)$. Total monetary loss to minority shareholders will be:

$$Aff.Min.Sh\ Loss=(1-f)\cdot\left(E(V_1)-D_1\right)\big/_{r_e+q}-(1-f)\cdot q\cdot(V_u-D_1)\big/_{R_e},$$

3.10 $$Aff.Min.Sh\ Loss=(1-f)\cdot\frac{1-q}{r_e+q}\underbrace{\left(-(D_1-V_d)+\frac{q\cdot(V_u-D_1)}{R_e}\right)}_{CC\ does\ not\ hold\,.....<0}<0\cdot$$

The corresponding price discount will be:

$$1-\left(\frac{(1-f)\cdot\left(E(V_1)-D_1\right)}{r_e+q}\right)/\left(\frac{(1-f)\cdot q\cdot(V_u-D_1)}{R_e}\right),$$

3.11 $$=1-\frac{\left(E(V_1)-D_1\right)}{q\cdot(V_u-D_1)}\cdot\frac{R_e}{r_e+q}<1.$$

At holding company Level:

If *CCcs* doesn't hold (and *CC* doesn't hold), the optimal strategy of CS will be to cash out Affiliate since the beginning; minority shareholders at Holding will hold:

$$(1-m)\cdot f\cdot q\cdot(V_u-D_1)/R_e.$$

On the other hand, if *CCcs* holds, the optimal strategy for CS will be to continue, forcing minority shareholders at the Affiliate to incur in a loss; in this case their holdings will be:

$$(1-m)\cdot f\cdot\left(E(V_1)-D_1\right)/(r_e+q).$$

The total monetary loss to minority shareholders in Holding will be:

$$Hold.Min.Sh\ Loss=\frac{(1-m)\cdot f\cdot\left(E(V_1)-D_1\right)}{r_e+q}-\frac{(1-m)\cdot f\cdot q\cdot(V_u-D_1)}{R_e},$$

$$3.12\quad Hold.Min.Sh\ Loss=(1-m)\cdot f\cdot\frac{1-q}{r_e+q}\underbrace{\left(-(D_1-V_d)+\frac{q\cdot(V_u-D_1)}{R_e}\right)}_{CC\ does\ not\ hold\ .....<0}<0.$$

The corresponding price discount will be:

$$1-\left(\frac{(1-m)\cdot f\cdot\left(E(V_1)-D_1\right)}{r_e+q}\right)/\left(\frac{(1-m)\cdot f\cdot q\cdot(V_u-D_1)}{R_e}\right)=1-\frac{\left(E(V_1)-D_1\right)}{q\cdot(V_u-D_1)}\cdot\frac{R_e}{r_e+q}<1.$$

This is exactly the same relative price discount suffered by minority shareholders at the Affiliate level. Although both the affiliate and holding companies suffer a loss.

I conclude this section that in Naive Model 2, where no conflicts of interest arise, there will be no "holding discount" (relative discount), given that the loss in value is equal for both in percentage terms because Holding has no other operation (Assets and liabilities).

3.2.1c) Best Strategy or best response of Minority Shareholders to the equity call

Given the optimum strategy to CS is to continue when CC doesn´t hold and CCcs holds, I need to check whether the Naive minority shareholders response (to concur to the equity call, corresponds to a Nash equilibrium.

To do so, I analyze the position of shareholders at Affiliate in case minority shareholders concur or do not concur. Taking advantage of the infinite nature of the game, I state that in any node when nature plays, equity value will be the same.

Assume it is:

$$S_0^{c(NoCC)} = S.$$

Then, Capital Call:

$$\Delta E = (D_1 - V_d);$$

Let:

$$\alpha = \frac{\Delta E}{S} = \frac{(D_1 - V_d)}{S},$$

be the proportion of equity value that must be raised to "rescue" Affiliate at any node in which nature plays "d".

In cases (3, 4, C, D), in any node nature plays "*d*", Affiliate´s minority shareholders will have the chance not to concur to the equity call, thereby diluting their position in the company. But the economic loss will take place regardless of that. If minority shareholders do not concur, the Affiliate´s controlling shareholder (Holding) will support the full cost of rescue, increasing the loss at the Holding level. This loss will impact both CS and the minority shareholder in their respective sharing in Holding. Same analysis can be done at the Holding level. Minority shareholders will have the right not to concur to the equity call, and if they decided to concur, and if the minority shareholders in Affiliate did the same, they could decide not to concur if minority shareholders in the Affiliate did not concur. In this final case, an extreme case appears which is equivalent to consider that *f=m=1* for rescue (marginal) purposes.

3.2.1c.1.) The case for minoritarian shareholders at Affiliate.

i. If all shareholders at Affiliate concur in node 1:

Equity Change (as a fraction of S)

	Investment	*Cap.Gain / Loss*	*Total Call* 1	*Total Call* $n-1$
CS	$-f \cdot \alpha$	0	$-f \cdot \alpha$	$-f \cdot \alpha$
min*ority*	$-(1-f) \cdot \alpha$	0	$-(1-f) \cdot \alpha$	$-(1-f) \cdot \alpha$

Shareholding

	before call	after call 1	after Call $n-1$
CS	f	$f\cdot(1+\alpha)/(1+\alpha)=f$	f
minority	$(1-f)$	$(1-f)\cdot(1+\alpha)/(1+\alpha)=(1-f)$	$(1-f)$

ii. If Minority shareholders at Affiliate don´t concur in node 1:

Equity Change (as a fraction of S)

	Investment	Cap.Gain / Loss	Total Call 1
CS	$-\alpha$	$\alpha\cdot(1-f)/(1+\alpha)$	$-\alpha\cdot(f+\alpha)/(1+\alpha)$
minority	0	$-\alpha\cdot(1-f)/(1+\alpha)$	$-\alpha\cdot(1-f)/(1+\alpha)$

Shareholding

	before call	after call 1	.... after call $n-1$
CS	f	$f/(1+\alpha)+\alpha/(1+\alpha)$	$f/(1+\alpha)^{n-1}+\alpha\cdot\sum_{j=1}^{j=n-1}1/(1+\alpha)^j$
minority	$(1-f)$	$(1-f)/(1+\alpha)$	$(1-f)/(1+\alpha)^{n-1}$

Every period, minority shareholder dilutes her position more and more as seen in the last schedule.

I can analyze the decision whether to concur or not to minority shareholders at Affiliate by comparing total variation of her position in period *n-1* given she is already in period *n-2:*

1. First, if she concurs, invests $-(1-f)\cdot\alpha\cdot S$ to maintain its position $-(1-f)\cdot S$ without capital loss.
2. Now if she doesn´t concur, invests nothing but experiments a capital loss given by $=S\cdot\left\{(1-f)/(1+\alpha)^{n-1}-(1-f)/(1+\alpha)^{n-2}\right\}=-\alpha\cdot(1-f)/(1+\alpha)^{n-1}\cdot S$
3. Not to concur is preferable to minority shareholder if $-\alpha\cdot(1-f)/(1+\alpha)^{n-1}\cdot S>-(1-f)\cdot\alpha\cdot S$, or $(1-f)/(1+\alpha)^{n-1}\cdot S<(1-f)\cdot S\Leftrightarrow 1/(1+\alpha)^{n-1}<1$, which is true for every period and $\alpha>0$.

From above I can see that:

1. Equity Change for both shareholders, in both cases, is negative (both lose value), reflecting the payment to be done to Debtholders to recuperate Affiliate´s Asset ownership, α

2. Minority shareholders experiment less equity loss when they do not concur to equity call than when they do concur. They prefer not to concur.
3. Even CS has a loss in equity when minority shareholders do not concur (their best response), CS best strategy can still be to force continuation depending on the idiosyncratic benefit M maintained.

Given that Debt holders are always paid, their debt is risk-free, and being Assets unchanged, total equity remains unchanged through time periods. A transfer of value takes place from CS to minority shareholders. By switching from "concurring" (Naive case) to "not concurring" in period *"j"*, minority shareholders gain, with respect to that case,

$$CSLoss_j = S\cdot\alpha\cdot(1-f)\cdot\left[1-1/(1+\alpha)^j\right] = S\cdot\alpha\cdot(1-f) - S\cdot\alpha\cdot(1-f)/(1+\alpha)^j \; ,$$

in that period, exactly the same CS loses that period.

$CSLoss_j$ is composed by two terms: the first represents the savings in cash flow by not concurring to the equity call in that period. The second represents the loss in equity value at the company because by no concurring, shareholder is diluting its position whatever the value of Affiliate´s equity is.

Conclusion here is, that minority shareholders at the Affiliate will always prefer not to concur to the equity call (rescue), regardless the market equity price *S*.

Depending on M[21], and despite the loss suffered with respect to the "Naive" case, CS will prefer to Continue. In that way the equilibrium strategy of the repeated game will be: *"CS to continue if Nature plays "d" – Minority Shareholders at the Affiliate do not concur to the corresponding equity call"*.

I can calculate the expected present value of the Value Transfer from CS to minority Shareholder, which is to be afforded entirely by Holding´s shareholders, without changing Affiliate´s equity value, as:

21 Minimum hurdle M increases respect to the Naive Case M in the amount of extra loss causes by affiliate´s minority shareholder best strategy.

$Event_\tau$	$\Pr obability_\tau$	$\Pr esent\, Value_\tau$	$E(\mathrm{P}VCSLoss_\tau)$
1 u	$(1-q)^0 q = q$	0	0
2 du	$(1-q)^1 q$	$CSLoss_1 \cdot R_e^{-1}$	$(1-q)^1 q \cdot (CSLoss_1 \cdot R_e^{-1})$
3 ddu	$(1-q)^2 q$	$CSLoss_1 \cdot R_e^{-1} + CSLoss_2 \cdot R_e^{-2}$	$(1-q)^2 q \cdot (CSLoss_1 \cdot R_e^{-1} + CSLoss_2 \cdot R_e^{-2})$
.			
n $\underbrace{dddd..ddd}_{n-1\, times}u$	$(1-q)^{n-1} q$	$\sum_{\tau=1}^{\tau=n-1} CSLoss_\tau \cdot R_e^{-\tau}$	$(1-q)^{n-1} q \left(\sum_{\tau=1}^{\tau=n-1} CSLoss_\tau \cdot R_e^{-\tau} \right)$
	$\sum_{t=1}^{t=\infty} (1-q)^t q = 1$		$\sum_{\tau=1}^{\tau=\infty} E(\mathrm{P}VCSLoss_\tau) = Exp.Loss\, Value = CSL_0^{Aff}$

From above, I have:

$$CSL_0^{Aff} = \lim_{n\to\infty} \sum_{j=1}^{j=n} (1-q)^{j-1} q \left(\sum_{\tau=1}^{\tau=j-1} CSLoss_\tau R_e^{-\tau} \right),$$

developing, solving, and simplifying, I obtain:

$$CSL_0^{Aff} = S \cdot a \cdot (1-f) \cdot (1-q)^2 \cdot \underbrace{\left(\frac{1}{R_e - 1 + q} - \frac{1}{A \cdot R_e - 1 + q} \right)}_{net\ present\ value\ factor:\ NPVF > 0}, \quad R_e = (1 + r_e),\ and\ A = (1 + a)$$

Note that:

$$NPVF > 0 \Leftrightarrow \alpha > o \text{ (when "capital calls to rescue strategy" takes place)} ,$$

CSL_0^{Aff} is composed by two terms: the first represents the expected present value of savings in cash flow by not concurring to the equity call in that period. The second represents the expected present value of the loss in equity value at the company because by no concurring, shareholder is diluting its position whatever the value of Affiliate´s equity is.

3.2.1c.2.) The case for minority shareholders at Holding

i). If all shareholders at Holding concur in node 1:

Equity Change (as a fraction of S)

	$Investment$	$Cap.Gain / Loss$	$Total\ Call\ 1$	$Total\ Call\ n-1$
CS	$-m \cdot f \cdot \alpha$	0	$-m \cdot f \cdot \alpha$	$-m \cdot f \cdot \alpha$
$\min ority$	$-(1-m) \cdot f \cdot \alpha$	0	$-(1-m) \cdot f \cdot \alpha$	$-(1-m) \cdot f \cdot \alpha$

Shareholding

	before call	after call 1	after Call $n-1$
CS	$m \cdot f$	$m \cdot f \cdot (1+\alpha \cdot f)/(1+\alpha \cdot f) = m \cdot f$	$m \cdot f$
minority	$(1-m) \cdot f$	$(1-m) \cdot f \cdot (1+\alpha \cdot f)/(1+\alpha \cdot f) = (1-m) \cdot f$	$(1-m) \cdot f$

ii) If Minority shareholders at Holding don´t concur in node 1:

Equity Change (as a fraction of S)

	Investment	Cap.Gain / Loss	Total Call 1
CS	$-\alpha \cdot f$	$\alpha \cdot f \cdot (1-m)/(1+\alpha \cdot f)$	$-\alpha \cdot f \cdot (m+\alpha \cdot f)/(1+\alpha \cdot f)$
minority	0	$-\alpha \cdot f \cdot (1-m)/(1+\alpha \cdot f)$	$-\alpha \cdot f \cdot (1-m)/(1+\alpha \cdot f)$

Shareholding

	before call	after call 1	after call $n-1$
CS	m	$m/(1+\alpha \cdot f) + \alpha \cdot f/(1+\alpha \cdot f)$	$m/(1+\alpha \cdot f)^{n-1} + \alpha \cdot f \cdot \sum_{j=1}^{j=n-1} 1/(1+\alpha \cdot f)^j$
minority	$(1-m)$	$(1-m)/(1+\alpha \cdot f)$	$(1-m)/(1+\alpha \cdot f)^{n-1}$

Every period, minority shareholder dilutes her position more and more as seen in the last schedule.

I can analyze the decision whether to concur or not to minority shareholders at Holding by comparing total variation of her position in period *n-1* given she is already in period *n-2:*

1. First, if she concurs, invests $-(1-m) \cdot \alpha \cdot f \cdot S$ to maintain its position $-(1-m) \cdot f \cdot S$ without capital loss.
2. Now if she doesn´t concur, invests nothing but experiments a capital loss given by $= f \cdot S \cdot \{(1-m)/(1+\alpha \cdot f)^{n-1} - (1-m)/(1+\alpha \cdot f)^{n-2}\} = -\alpha \cdot f \cdot (1-m)/(1+\alpha \cdot f)^{n-1} \cdot S$ Not to concur is preferable to minority shareholder if $-\alpha \cdot f \cdot (1-m)/(1+\alpha \cdot f)^{n-1} \cdot S > -(1-m) \cdot \alpha \cdot f \cdot S$, or $(1-m)/(1+\alpha \cdot f)^{n-1} \cdot S < (1-m) \cdot S$ $\Leftrightarrow 1/(1+\alpha \cdot f)^{n-1} < 1$, which is true for every period and $\alpha \cdot f > 0$.

From above I can see that:

1. Equity Change for both shareholders, in both cases, is negative (they both lose value), reflecting the payment to be done to Debtholders to recuperate Affiliate´s Asset ownership, $\alpha \cdot f$.

2. Minority shareholders experiment less equity loss when they do not concur to equity call than when they do concur. Minority shareholders at the holding company won´t concur to the equity call.
3. Even CS has a loss in equity when minority shareholders do not concur (their best response), CS best strategy can still be to force continuation depending on the idiosyncratic benefit M maintained.

Similar to Affiliate case, a transfer of value takes place from CS to minority shareholders at Holding. By switching from "concurring" (Naive case) to "not concurring" in period *"j",* Holding´s minority shareholders gain, with respect to that case,

$$CSLoss_j^H = S \cdot \alpha \cdot f \cdot (1-m) \cdot \left[1 - 1/(1+\alpha \cdot f)^j \right]$$

$$= S \cdot \alpha \cdot f \cdot (1-m) - S \cdot \alpha \cdot f \cdot (1-m)/(1+\alpha \cdot f)^j \, ,$$

in that period, exactly the same CS loses that period.

$CSLoss_j^H$ is composed by two terms: the first represents the savings in cash flow by not concurring to the equity call in that period. The second represents the loss in equity value at the company because by no concurring, shareholder is diluting its position whatever the value of Holding´s equity is.

The conclusion of section 3.2.1 is that minority shareholders in the Holding Co. will always prefer not to concur to the equity call to rescue the affiliate, regardless the market equity price *S*.

Depending on M[22], and in spite of the loss suffered with respect to the "Naive" case, CS will prefer to Continue. In that way the equilibrium strategy of "the repeated game" will be *"CS to continue if Nature plays "d" – Minority Shareholders at the Holding do not concur to the corresponding equity call".*

22 Minimum hurdle M increases respect to the Naive Case M in the amount of extra loss causes by affiliate´s minority shareholder best strategy

Again, I can calculate the expected present value of the Value Transfer from CS to minority Shareholder, which is to be supported entirely by Holding´s shareholders, without changing Affiliate´s equity value, as:

$Event_\tau$	$\Pr obability_\tau$	$\Pr esent\ Value_\tau$	$E(\mathrm{P}VCSLoss_\tau)$
1 u	$(1-q)^0 q = q$	0	0
2 du	$(1-q)^1 q$	$CSLoss_1^H \cdot R_e^{-1}$	$(1-q)^1 q \cdot (CSLoss_1^H \cdot R_e^{-1})$
3 ddu	$(1-q)^2 q$	$CSLoss_1^H \cdot R_e^{-1} + CSLoss_2^H \cdot R_e^{-2}$	$(1-q)^2 q \cdot (CSLoss_1^H \cdot R_e^{-1} + CSLoss_2^H \cdot R_e^{-2})$
.			
n $\underbrace{dddd..ddd}_{n-1\ times}u$	$(1-q)^{n-1} q$	$\sum_{\tau=1}^{\tau=n-1} CSLoss_\tau^H \cdot R_e^{-\tau}$	$(1-q)^{n-1} q \left(\sum_{\tau=1}^{\tau=n-1} CSLoss_\tau^H \cdot R_e^{-\tau} \right)$
	$\sum_{t=1}^{t=\infty} (1-q)^t q = 1$		$\sum_{\tau=1}^{\tau=\infty} E(\mathrm{P}VCSLoss_\tau^H) = Exp.Loss\ Value = CSL_0^H$

From above, I have:

$$CSL_0^H = \lim_{n\to\infty} \sum_{j=1}^{j=n} (1-q)^{j-1} q \cdot \left(\sum_{\tau=1}^{\tau=j-1} CSLoss_\tau^H \cdot R_e^{-\tau} \right),$$

developing, solving and simplifying, I obtain:

$$CSL_0^H = S \cdot \alpha \cdot f \cdot (1-m) \cdot (1-q)^2 \cdot \underbrace{\left(\frac{1}{R_e - 1 + q} - \frac{1}{A^H \cdot R_e - 1 + q} \right)}_{net\ present\ value\ factor:\ NPVF>0}, \quad R_e = (1+r_e),\ and\ A^H = (1+\alpha \cdot f).$$

Note that,

$$NPVF > 0 \Leftrightarrow \alpha > o \text{ (when "capital calls to rescue strategy" takes place)} .$$

CSL_0^H is composed by two terms: the first represents the expected present value of savings in cash flow by not concurring to the equity call in that period. The second represents the expected present value of the loss in equity value at the company because by no concurring, shareholder is diluting its position whatever the value of Holding´s equity is.

3.2.1c.3.) The Realistic Valuation Schedule:

With the adoption of minority shareholder´s best strategies, it is straightforward to change to a more Realistic Valuation Schedule starting from and comparing with The "Naive" Valuation Schedule:

At Affiliate Level:

$$3: \frac{f \cdot \left(E(V_1) - D_1\right)}{r_e + q} - CSL_0^{Aff}$$

$$4: \frac{(1-f) \cdot \left(E(V_1) - D_1\right)}{r_e + q} + CSL_0^{Aff}$$

$$5: \frac{f \cdot q \cdot (V_u - D_1)}{R_e}$$

$$6: \frac{(1-f) \cdot q \cdot (V_u - D_1)}{R_e}$$

At holding company Level:

$$C: \frac{m \cdot f \cdot \left(E(V_1) - D_1\right)}{r_e + q} + \frac{(1-q)M}{r_a + q} - m \cdot CSL_0^{Aff} - CSL_0^{H}$$

$$D: \frac{(1-m) \cdot f \cdot \left(E(V_1) - D_1\right)}{r_e + q} - (1-m) \cdot CSL_0^{Aff} + CSL_0^{H}$$

$$E: \frac{f \cdot m \cdot q \cdot (V_u - D_1)}{R_e}$$

$$F: \frac{(1-m) \cdot f \cdot q \cdot (V_u - D_1)}{R_e}$$

i. Relative Values discount (a holding discount)

a. Affiliate discount

In the Naive model:

$$0 < 1 - \frac{(1-f) \cdot \left(E(V_1) - D_1\right)}{(1-f) \cdot q \cdot (V_u - D_1)} \left(\frac{R_e}{r_e + q} \right) < 1$$

In the realistic model:

$$1 - \frac{(1-f) \cdot \left(E(V_1) - D_1\right) + CSL_0^{Aff}}{(1-f) \cdot q \cdot (V_u - D_1)} \left(\frac{R_e}{r_e + q} \right),$$

which is smaller than Naive discount given the "recuperation" of value, CSL_0^{Aff}, due to minority shareholder´s best strategy; and it and can be greater or less than zero (discount or premium) depending on the relative size of CSL_0^{Aff}.

b. holding company discount (equity value at Holding divided into Net Asset Value from Affiliate)

In the Naive model:

$$1-\left(\frac{(1-m)\cdot f\cdot\left(E(V_1)-D_1\right)}{r_e+q}\right)/\left(\frac{(1-m)\cdot f\cdot q\cdot(V_u-D_1)}{R_e}\right)=1-\frac{(1-m)\cdot f\cdot\left(E(V_1)-D_1\right)}{(1-m)\cdot f\cdot q\cdot(V_u-D_1)}\cdot\frac{R_e}{r_e+q}<1.$$

In the realistic model:

$$1-\left(\frac{(1-m)\cdot f\cdot\left(E(V_1)-D_1\right)}{r_e+q}-\left((1-m)\cdot CSL_0^{Aff}-CSL_0^{H}\right)\right)/\left(\frac{(1-m)\cdot f\cdot q\cdot(V_u-D_1)}{R_e}\right).$$

In fact, Realistic holding company discount is due to:

$$\left((1-m)\cdot CSL_0^{Aff}-CSL_0^{H}\right),$$

the "Net Loss" to CS: It corresponds to Naive discount, minus the %loss supported by minority shareholders in Holding due to best strategy of minority shareholders in Affiliate, plus the %value transferred from CS to minority shareholders at Holding.

The first effect increases, and the second effect diminishes the discount. If both effects cancel each other, holding company discount is zero.

c. holding discount = holding company discount/Affiliate discount

In the Naive Model: as shown previously, no holding discount arises.

In the realistic model: holding company discount/Affiliate:

$$3.13\quad \frac{1-\left(\frac{(1-m)\cdot f\cdot\left(E(V_1)-D_1\right)}{r_e+q}-\left((1-m)\cdot CSL_0^{Aff}-CSL_0^{H}\right)\right)/\left(\frac{(1-m)\cdot f\cdot q\cdot(V_u-D_1)}{R_e}\right)}{1-\frac{(1-f)\cdot\left(E(V_1)-D_1\right)+CSL_0^{Aff}}{(1-f)\cdot q\cdot(V_u-D_1)}\left(\frac{R_e}{r_e+q}\right)}$$

In this case a Relative discount or holding discount may arise, because from (3.13):

- On the one hand Affiliate discount is less (even it can become a premium) than in the Naive case.
- On the other hand, holding discount may be greater than in the Naive model depending on $NetLoss = (1-m)\cdot CSL_0^{Aff} - CSL_0^H$.

For example,

- In case $CSL_0^{Aff} = 0$ and $CSL_0^H = 0$, I return to Naive case.
- In case $CSL_0^{Aff} > 0$ and $CSL_0^H = 0$, I obtain holding discount.
- In case $CSL_0^{Aff} > 0$ and $CSL_0^H > 0$, I can have either Holding Premium or discount depending on the relative size of $(1-m)\cdot CSL_0^{Aff}$ and CSL_0^H.
- In a particular case in which $m = f$, a holding discount exists whenever $CSL_0^{Aff} > -\left((1-m)\cdot CSL_0^{Aff} - CSL_0^H\right) = -(1-m)\cdot CSL_0^{Aff} + CSL_0^H$
- $\Rightarrow CSL_0^H < \left(1+(1-m)\right)\cdot CSL_0^{Aff}$.

ii. Analysis of Loss Elements CSL_0^{Aff} and CSL_0^H

First, I have shown that Affiliate minority shareholder`s losses always exist provided CC doesn´t hold and minority shareholders follow their optimum strategy.
Second, I have shown holding company discount in the realistic model may exist and differs from the Naive discount in the term:

$$-(1-m)\cdot CSL_0^{Aff} + CSL_0^H \quad in\ which$$

$$CSL_0^{Aff} = S\cdot\alpha\cdot(1-f)\cdot(1-q)^2\cdot\left(\frac{1}{R_e-1+q} - \frac{1}{A\cdot R_e-1+q}\right)$$

$$CSL_0^H = S\cdot\alpha\cdot f\cdot(1-m)\cdot(1-q)^2\cdot\left(\frac{1}{R_e-1+q} - \frac{1}{A^H\cdot R_e-1+q}\right)$$

$$NetLoss = (1-m)\cdot CSL_0^{Aff} - CSL_0^H =$$

$$= S\cdot\alpha\cdot(1-q)^2\cdot(1-m)\cdot\left((1-f)\cdot\left(\frac{1}{R_e-1+q} - \frac{1}{A\cdot R_e-1+q}\right) - f\cdot\left(\frac{1}{R_e-1+q} - \frac{1}{A^H\cdot R_e-1+q}\right)\right)$$

Net Loss is positive if,

$$(1-f)\cdot\left(\frac{1}{R_e-1+q} - \frac{1}{A\cdot R_e-1+q}\right) - f\cdot\left(\frac{1}{R_e-1+q} - \frac{1}{A^H\cdot R_e-1+q}\right) > 0$$

Which is not necessarily true, because, given structural parameters, it depends on the relative size of the equity call α, and the initial controlling share of Holding in Affiliate.

I can distinguish the following special cases:

1. Minority shareholders don't care of capital loss (dilution) but do care on cash outlays.

In this case I have:

$$CSL_0^{Aff}(cash) = S \cdot \alpha \cdot (1-f) \cdot (1-q)^2 \cdot \left(\frac{1}{R_e - 1 + q} \right)$$

$$CSL_0^{H}(cash) = S \cdot \alpha \cdot f \cdot (1-m) \cdot (1-q)^2 \cdot \left(\frac{1}{R_e - 1 + q} \right)$$

$$NetLoss = (1-m) \cdot CSL_0^{Aff} - CSL_0^{H} =$$
$$= S \cdot \alpha \cdot (1-q)^2 \cdot (1-m) \cdot \left((1-f) \cdot \left(\frac{1}{R_e - 1 + q} \right) - f \cdot \left(\frac{1}{R_e - 1 + q} \right) \right)$$

Which is unambiguously negative loss (Net Premium) when $f > 0,5$ which is the case of interest, when Holding´s controls Affiliate.

Then the possibility of a holding discount appears only when minority shareholders care about all their stakes in the companies.

2. Private Benefits M accrue not only to CS but also to Holding´s minority shareholders.

In case M, the idiosyncratic benefit to CS, also benefits minority shareholders at Holding[23], then their best strategy will be the same as CS thus they concur to equity call; in the model this makes $CSL_0^H = 0$, and produces an unambiguous holding discount:

$$CSL_0^{Aff} = S \cdot \alpha \cdot (1-f) \cdot (1-q)^2 \cdot \left(\frac{1}{R_e - 1 + q} - \frac{1}{A \cdot R_e - 1 + q} \right); \quad CSL_0^H = 0$$

$$NetLoss = (1-m) \cdot CSL_0^{Aff} = S \cdot \alpha \cdot (1-q)^2 \cdot (1-m) \cdot (1-f) \cdot \left(\frac{1}{R_e - 1 + q} - \frac{1}{A \cdot R_e - 1 + q} \right) > 0$$

23 This can be the case when holding directly extracts private benefits from Affiliate, or when synergies are present.

Net Loss is positive (holding discount) if:

$$(1-f)\cdot\left(\frac{1}{R_e-1+q}-\frac{1}{A\cdot R_e-1+q}\right)>0,$$

which is true if $\alpha>0$.

Finally,

I can restate the condition M, the idiosyncratic benefit to CS must comply in order to make Continuation Strategy valuable to CS.

Recall the Naive case, equation:

$$3.3 \quad M_{naive}>\frac{f\cdot m}{R_e}\cdot\left(D_1\,(1-(r_e+q))-\left(E(V_1)-V_d\,(r_e+q)\right)\right)\cdot$$

Using Realistic Valuation formulae, I can say that:

$$3.14 \qquad M>M_{naive}+(m\cdot CSL_0^{Aff}-CSL_0^{H})\cdot(r_e+q)\cdot$$

3.3. Summing up and concluding Model 2

Under the widely used legal rule of "limited liability", shareholders of a levered company have the right to give up the company to debtholders in case of default. Nevertheless, and at the same time, the shareholders have the option (not the obligation) not to comply with that rule.

3.3.1. From Model 1

I have shown that under certain conditions, it will be optimum to shareholders of a Stand-Alone company to exercise that option, what means, to rescue the company in case of default, and keep running it. In valuation terms, the option has a value that can be easily calculated.

An essential condition to obtain the previous results, is that shareholders as well as debt holders are long-sighted. In my model this means that decisions are made evaluating all the future long-term prospects of the company, but not only the short term.

In fact, if evaluation is done on the short term (static model), it will always be optimum to all shareholders comply with the Limited Liability rule. On the contrary, under long term evaluation (dynamic model), the possibility to exercise the option to continue will arise.

3.3.2. From Model 2

Now, if I allow the stand-alone company to belong to a business group (become an Affiliate, and thereby introducing conflicts of interests between controlling and minority shareholders), I have shown that even, if from all Affiliate´s shareholders point of view, conditions are such that it is not optimum to exercise the option (rescue the Affiliate and continue), and provided certain additional conditions, it may be valuable to Controlling Shareholders to do so, making minority shareholders suffer a loss, thus a holding discount.

In fact, majority shareholders also suffer a loss in their equity stakes, but that loss is counterweighted by some benefits idiosyncratic to controlling shareholders: M.
Model 2 has been useful to go one step beyond the stand-alone case, in analyzing the implications of a Controlling shareholder forcing not to follow LL rules because by doing so, derives some benefits that accrue only to the controller but not to the minority shareholders.

By means of a somewhat "artificial" model, I have shown that when a stand-alone company becomes an acquisition target (Affiliate) of another company (Holding), and given certain circumstances, controlling shareholder has incentives to rescue the affiliate from a default against minority shareholder's interests; if that is the case both companies should suffer an equity value loss (the counterparty being affiliate´s debt value increase), and moreover, a holding discount may arise.

Model 2 assumes this effect to happen in the moment of the (unanticipated) acquisition; the key is that value losses will occur any way. But there are several ways to rationalize them. In Model 1, Stand Alone Company, there will be some probability that in event of default, the company will be rescued, favoring all shareholders. That probability depends on structural parameters as well as the level of indebtedness of the affiliate, and the state of the economy.

When the company is an affiliate, model 2, the probability of rescue may increase with respect to the stand-alone probability[24], because there are some private benefits M to controlling Shareholders that make the rescue a best strategy for them in circumstances that differ from the stand-alone case.

This latter probability depends on the controlling stakes of the Controlling Shareholder in each company, the level of private benefits derived from forcing a rescue in case of default, the "size" of the rescue, whether these benefits can be shared with minority shareholders at Holding, on structural parameters, the level of indebtedness of the affiliate, and the state of the economy.

Moreover, although probability of rescue may increase, the hurdle benefit M increases as well. Finally, if M benefits all shareholders, a holding discount will exist, and there will be no conflicts of interest between controlling and minority shareholders.
I conclude section 3 stating that in the conditions of model 2, main hypothesis H0 cannot be rejected up to this point with a reasonable degree of confidence, in the legal ruling of Limited Liability.

Section 4. Introducing a Levered Holding, and Conflicts of Interest between Controlling Shareholders and Minority Shareholders, Model 3

The general procedure is the following: establishing the Acquisition Approach raisin debt funds in Holding and calling for equity in Affiliate to be the case.

The procedure in this section is the following: In subsection 4.1 I state the continuation condition for this case, as well as several assumptions needed. In subsection 4.2 I show the pre-acquisition and post-acquisition valuations for Holding

24 As seen in the text, there are some cases in which it increases unambiguously.

and Affiliate. In subsection 4.3 I analyze the decision model for the Controlling Shareholder whether to comply with Limited Liability or with Unlimited Liability, ending with the complete multistage decision conditions in 4.3.1.b.ii, under several scenarios. In subsection 4.4 I focus on what I consider the most interesting case, Affiliate falls into default, the holding company not, in the naïve assumption that minority shareholders at the affiliate concur to the corresponding equity call. In subsection 4.5 I analyze the continuation condition of the case considered in 3.4, and I compare with the same from model 2. In subsection 4.6 I summarize and conclude.

4.1. The Levered Holding Case Model 3

4.1.1. Preliminaries

So far I have analyzed the problem following a progressively difficult path: First a Stand-Alone Company (Model 1); second, a stand-alone company owned by a holding company with no other investments and no debt (Model 2). The acquisition of the affiliate company has been done with equity.

Now, I introduce the possibility of the holding company to be levered, and to make the model more complete, to own other investments (Model 3). The acquisition is done with 100% Debt, which may be possible if the company has other investments.

4.1.2. Continuation Conditions for controlling shareholder

1. Stand-Alone *(CC)*, 2 cases: *CC* (holds or does not hold in case state of nature is "d")
2. No levered Holding (CC_{cs}), 4 cases: CC_{cs} (holds or not in case state of nature is "d") crossed with Affiliate´s, *CC* (holds or not in case state of nature is "d"). I have focused on the case *CC* doesn't hold (for Stand Alone and Affiliate), thus leaving 2 interesting cases.
3. Levered Holding CC_{cs}^{H}, for each of 4 scenarios to be defined later, given that *CC* doesn't hold, I have the same 2 cases as in point 2.

4.1.3. Several assumptions made, in order to keep things simple.

1. In Model 1, when state of nature is "u", no default exists in Stand Alone; when state of nature is "d", company is in default and can be rescued or not (*CC*).

2. In Model 2, when state of nature is "u", no default exists in Affiliate; when state of nature is "d", Affiliate company is in default and can be rescued or not by controlling shareholder (the same for Affiliate as well as Holding), (*CCcs*).
3. In Model 3, I will assume the same as in Model 2 for Affiliate. For Holding before acquisition, I assume that when state of nature is "u", no default exists, and when state of nature is "d", Holding pre-acquisition will not be necessarily in default. Controlling shareholder will have the option to Continue if state of nature is "d" because: I) both companies are in default at the same time, or just because either Affiliate is in default not producing Holding´s default, or Holding is in default regardless of Affiliate´s condition.
4. As in Model 2, benefits M exists.
5. I develop the Naïve Model 3. Where Naïve means that in any equity call needed to acquire the distressed Affiliate, minority shareholders will concur.

4.2. Naïve Model 3

4.2.1. Preacquisition holding company

Let:

$$F^H(s) = \sum_{i \in Holding} f_i \cdot S_i(s)$$

for each state of nature *(s=u or s=d)* denote pre-acquisition proportional equity stakes in a number of previously acquired affiliates companies for. The company has pre-acquisition net Debt $D_0^{H,p}$ and $D_1^{H,p}$ one period ahead in which superscript "p" stands for "pre-acquisition" and "0" and "1" refer to present and end of next period. Then:

$$S_1^{H,p}(u) = F_1^H(u) - D_1^{H,p} \geq 0 \text{ and } S_1^{H,p}(d) = F_1^H(d) - D_1^{H,p} <,= or > 0 .$$

The following condition assures Holding is not in default previous to acquisition:

$$S_1^{H,p} = \underset{s}{E}\left(S_1^{H,p}(s)\right) = \underset{s}{E}\left(F_1^H(s) - D_1^{H,p}\right) > 0 \cdot$$

4.2.2. Pre-acquisition and Post acquisition Affiliate Company

$$S_1^{aff}(u) = V_u - D_1^{aff} \geq 0 \text{ and } S_1^{aff}(d) = V_d - D_1^{aff} < 0$$ [25];

Funds needed to acquire the Affiliate are (previously) raised as "Debt" at Holding: the sequence is the following: First, a new debt is contracted, and proceeds are held as cash; second, a controlling position (f) of the affiliate company is acquired. From

25 later I return to the assumption that NoCC, $q \cdot (V_u - D_1) / R_e < (D_1 - V_d); 0 < q < 1; V_d < D_1 < V_u$, which means that in case state of nature is "d", optimal strategy to a representative shareholder of the affiliate will be not to continue.

Model 2 I have that Debt amount equals the stand-alone price, and minority shareholders have the same share preacquisition and post-acquisition:

$$4.1 \quad \$(D^{H}_{0,acq}) = \$\left(\frac{f \cdot q \cdot (V_u - D^{aff}_1)}{R_e}\right); \quad D^{H}_{1,acq} = D^{H}_{0,acq} \cdot R_b$$

4.2.3. Post-acquisition holding company

$$S^H_1(u) = S^{H,p}_1(u) + f \cdot (V_u - D^{aff}_1) - D^H_1 = F^H_1(u) - D^H_1 + f \cdot (V_u - D^{aff}_1),$$

in which,

$$D^H_1 = D^{H,p}_1 + D^H_{1,acq},$$

is total Holding´s Debt. My assumptions imply that

$$S^H_1(u) > 0.$$

To calculate $S^H_1(d)$ I need to check as in Model 2, whether in that case the optimal strategy for CS will be to abandon, or to pay Affiliates Debt (in case only Affiliate is in default) or Holding and Affiliates Debts in case a simultaneous default occurs in which cases the corresponding continuation values should be calculated. This is analyzed as the Decision model for the Controlling Shareholder CS.

4.3. Decision Model for CS of the Levered Holding

I describe two cases, and 4 scenarios for each case: For all the following holds:

$$S^H_1(u) = F^H_1(u) - D^H_1 + f \cdot (V_u - D^{aff}_1).$$

4.3.1. Cases

I) CS complies with LL rules

II) CS doesn't necessarily comply with LL rules

4.3.2. Scenarios

Scenario 1) Affiliate company is in default, holding company is in default (d-d)

$$S^H_1(d) = F^H_1(d) - D^H_1 + f \cdot (V_d - D^{aff}_1)$$
$$(V_d - D^{aff}_1) < 0$$
$$S^H_1(d) < 0 \Rightarrow F^H_1(d) - D^H_1 < -f \cdot (V_d - D^{aff}_1)$$

Scenario 2) Affiliate Company is in default, holding company is not in default: sector (d-n) in figure 8:

$$S_1^H(d) = F_1^H(d) - D_1^H + f \cdot (V_d - D_1^{aff})$$
$$(V_d - D_1^{aff}) < 0$$
$$S_1^H(d) > 0 \Rightarrow F_1^H(d) - D_1^H > -f \cdot (V_d - D_1^{aff})$$

Scenario 3) Affiliate Company is not in default, holding company is in default (n-d)

$$S_1^H(d) = F_1^H(d) - D_1^H + f \cdot (V_d - D_1^{aff})$$
$$(V_d - D_1^{aff}) > 0$$
$$S_1^H(d) < 0 \Rightarrow F_1^H(d) - D_1^H < -f \cdot (V_d - D_1^{aff})$$

Scenario 4) Affiliate Company is not in default, holding company is not in default (n-n)

$$S_1^H(d) = F_1^H(d) - D_1^H + f \cdot (V_d - D_1^{aff})$$
$$(V_d - D_1^{aff}) > 0$$
$$S_1^H(d) > 0 \Rightarrow F_1^H(d) - D_1^H > -f \cdot (V_d - D_1^{aff})$$

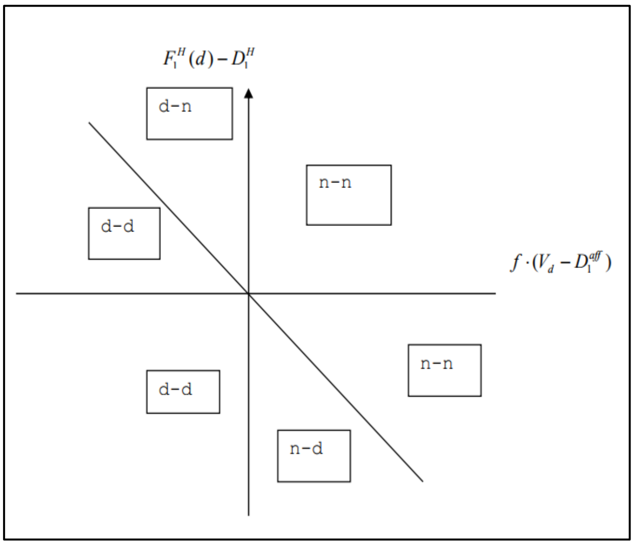

Figure 8. Scenarios for CS of the Levered Holding

4.3.3. Expected Equity Values

Case I: CS complies with LL rules at any event

Affiliate and Holding:

$$S_1^{aff}(u) = \max(0; V_u - D_1^{aff})$$
$$S_1^{aff}(d) = \max(0; V_d - D_1^{aff})$$
$$E(S_1^{aff}) = q \cdot S_1^{aff}(u) + (1-q) \cdot S_1^{aff}(d)$$
$$S_1^{H}(u) = \max(0; F_1^{H}(u) - D_1^{H} + f \cdot (V_u - D_1^{aff}))$$
$$S_1^{H}(d) = \max(0; F_1^{H}(d) - D_1^{H} + f \cdot (V_d - D_1^{aff}))$$
$$E(S_1^{H}) = q \cdot S_1^{H}(u) + (1-q) \cdot S_1^{H}(d)$$

State dependent values:

Affiliate

$$S_1^{aff}(u) = (V_u - D_1^{aff})$$
$$1: S_1^{aff}(d) = 0$$
$$2: S_1^{aff}(d) = 0$$
$$3: S_1^{aff}(d) = (V_d - D_1^{aff})$$
$$4: S_1^{aff}(d) = (V_d - D_1^{aff})$$

Holding

$$S_1^{H}(u) = F_1^{H}(u) - D_1^{H} + f \cdot (V_u - D_1^{aff})$$
$$1: S_1^{H}(d) = 0$$
$$2: S_1^{H}(d) = F_1^{H}(d) - D_1^{H} + f \cdot (V_d - D_1^{aff})$$
$$3: S_1^{H}(d) = 0$$
$$4: S_1^{H}(d) = F_1^{H}(d) - D_1^{H} + f \cdot (V_d - D_1^{aff})$$

Equity Expected Values:

Affiliate

$$E(S_1^{aff}) = q \cdot S_1^{aff}(u) + (1-q) \cdot S_1^{aff}(d)$$
$$1: E(S_1^{aff}) = q \cdot (V_u - D_1^{aff})$$
$$2: E(S_1^{aff}) = q \cdot (V_u - D_1^{aff})$$
$$3: E(S_1^{aff}) = q \cdot (V_u - D_1^{aff}) + (1-q) \cdot (V_d - D_1^{aff})$$
$$4: E(S_1^{aff}) = q \cdot (V_u - D_1^{aff}) + (1-q) \cdot (V_d - D_1^{aff})$$

Holding

$$E(S_1^H) = q \cdot S_1^H(u) + (1-q) \cdot S_1^H(d)$$
$$1: E(S_1^H) = q \cdot \left(F_1^H(u) - D_1^H + f \cdot (V_u - D_1^{aff})\right)$$
$$2: E(S_1^H) = q \cdot \left(F_1^H(u) - D_1^H + f \cdot (V_u - D_1^{aff})\right) + (1-q)\left(F_1^H(u) - D_1^H\right)$$
$$3: E(S_1^H) = q \cdot \left(F_1^H(u) - D_1^H + f \cdot (V_u - D_1^{aff})\right) + (1-q) \cdot f \cdot (V_d - D_1^{aff})$$
$$4: E(S_1^H) = q \cdot \left(F_1^H(u) - D_1^H + f \cdot (V_u - D_1^{aff})\right) + (1-q) \cdot \left(F_1^H(d) - D_1^H + f \cdot (V_d - D_1^{aff})\right)$$

<u>Case II: CS doesn't comply with LL rules (goes for UL)</u>

As in Model 2, all calculations that follow, represent a Naive case in which all minority shareholders, at Affiliate and Holding, concur to equity calls, fact that will be introduced at the end of section 3.

The actual procedure followed in the respective companies is as follows: First, Affiliate is in default, an CS at Holding decide to rescue the Affiliate by raising debt in de holding company. At the Affiliate level this means an equity call which will be subscribed by the controller (Holding), and in the Naïve Case, by the minority shareholders at Affiliate as well, keeping their previous shareholding.

Figure 9 below, shows the relevant decision tree for the controlling shareholder:

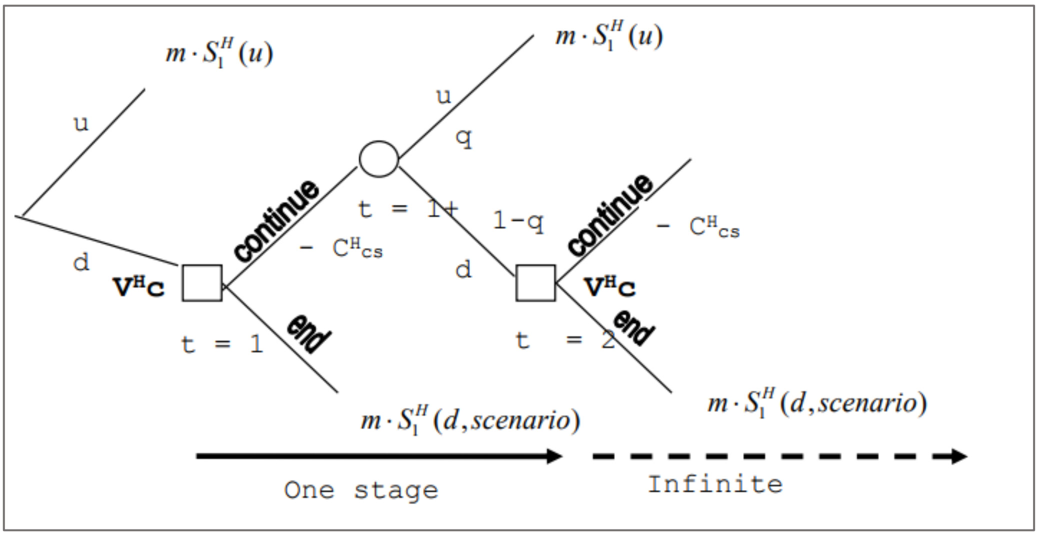

Figure 9. Relevant decision tree for the controlling shareholder

V_c^H corresponds to the continuation value at the Holding Level, condition labeled; CC_{cs}^H (which includes but is not limited to the affiliate level); $-C_{cs}^H = -c_{aff} \cdot f \cdot m - c_H \cdot m + M$, and $c_{aff}{=}c_{aff}(scenario)$ and $c_H = c_H(scenario)$.

The following schedules show valuations of each variable-scenario:

Scenario	c_{aff}	c_H	$S_1^H(d)$
1	$-(V_d - D_1^{aff})$	$-\left(F_1^H(d) - D_1^H\right)$	0
2	$-(V_d - D_1^{aff})$	0	$\left(F_1^H(d) - D_1^H\right)$
3	0	$-\left(F_1^H(d) - D_1^H + f \cdot (V_d - D_1^{aff})\right)$	0
4	0	0	$\left(F_1^H(d) - D_1^H + f \cdot (V_d - D_1^{aff})\right)$

Scenario	$S_1^H(u)$	$E\left(S_1^H\right)$
1	$\left(F_1^H(u) - D_1^H\right) + f \cdot \left(V_u - D_1^{aff}\right)$	$q \cdot \left(\left(F_1^H(u) - D_1^H\right) + f \cdot \left(V_u - D_1^{aff}\right)\right)$
2	$\left(F_1^H(u) - D_1^H\right) + f \cdot \left(V_u - D_1^{aff}\right)$	$\left(E\left(F_1^H\right) - D_1^H\right) + q \cdot f \cdot \left(V_u - D_1^{aff}\right)$
3	$\left(F_1^H(u) - D_1^H\right) + f \cdot \left(V_u - D_1^{aff}\right)$	$q \cdot \left(\left(F_1^H(u) - D_1^H\right) + f \cdot \left(V_u - D_1^{aff}\right)\right)$
4	$\left(F_1^H(u) - D_1^H\right) + f \cdot \left(V_u - D_1^{aff}\right)$	$\left(E\left(F_1^H\right) - D_1^H\right) + f \cdot \left(E(V) - D_1^{aff}\right)$

Note that $c_{aff,}\, c_H, S_1^H(d)$ are allowed to be positive, zero (intuitive), or negative (counter intuitive) costs and exit values, depending on the parameters as well as the scenario I analyze.

Optimum strategy for CS will maximize:

$$m \cdot E(S_1^H) = q \cdot m \cdot S_1^H(u) + (1-q) \cdot V_c^H \ .$$

i. The One Stage (Short Sighted) Decision

As in Models 1 and 2, I first derive the "one stage decision":

4.2 $$V_{c,1}^H(scenario) = \max\left(\underbrace{m \cdot S_1^H(d, scenario)}_{end}; \underbrace{-C_{cs}^H(scenario) + \frac{q \cdot m \cdot S_1^H(u) + (1-q) \cdot m \cdot S_1^H(d, scenario)}{R_e}}_{continue}\right)$$

From here I derive the Continuation Condition:

$$-C_{cs}^{H}(scenario)+\frac{q\cdot m\cdot S_1^H(u)+(1-q)\cdot m\cdot S_1^H(d,scenario)}{R_e}>m\cdot S_1^H(d,scenario),$$

or

$$CC_{cs,1}^{H}:M>m\cdot\left(S_1^H(d,scenario)+c_{aff}\cdot f+c_H-\frac{E(S_1^H)}{R_e}\right), \tag{4.3}$$

in which c_{aff}, and c_H also depend on each scenario.

If condition $CC_{cs,1}^{H}$ holds, then it will prove beneficial to CS to continue at least for one more period if s=d at the first decision node.

ii. The Complete (Multi Stage) solution

As in previous models, if CS´s one stage decision is to continue, then I analyze the dynamic decision. As before:

$$V_c^H(scenario)=\max\left(m\cdot S_1^H(d,scenario),-C_{cs}^H+\frac{m\cdot q\cdot S_1^H(u)+(1-q)V_c^H)}{R_e}\right)$$

$$\Rightarrow V_c^H(scenario)=\max\left(m\cdot S_1^H(d,scenario),\frac{\left(-c_{aff}\cdot f\cdot m-c_H\cdot m+M\right)\cdot R_e+q\cdot m\cdot S_1^H(u)}{r_e+q}\right)$$

CS´s optimal strategy will be to continue if:

$$\frac{\left(-c_{aff}\cdot f\cdot m-c_H\cdot m+M\right)\cdot R_e+q\cdot m\cdot S_1^H(u)}{r_e+q}-m\cdot S_1^H(d,scenario)>0$$

$$\Rightarrow\left(-c_{aff}\cdot f\cdot m-c_H\cdot m+M\right)\cdot R_e+q\cdot m\cdot S_1^H(u)-(r_e+q)\cdot S_1^H\cdot m\cdot(d,scenario)>0$$

$$\Rightarrow M\cdot R_e>(r_e+q)\cdot m\cdot S_1^H(d,scenario)+\left(c_{aff}\cdot f\cdot m+c_H\cdot m\right)\cdot R_e-q\cdot m\cdot S_1^H(u)$$

$$\Rightarrow M\cdot R_e>(r_e+1-1+q)\cdot m\cdot S_1^H(d,scenario)+\left(c_{aff}\cdot f\cdot m+c_H\cdot m\right)\cdot R_e-q\cdot m\cdot S_1^H(u)$$

Giving:

$$V_c^H(scenario)>0\Rightarrow CC_{cs}^H:M>m\cdot\left(S_1^H(d,scenario)+c_{aff}\cdot f+c_H-\frac{E(S_1^H)}{R_e}\right),$$

which is the same one stage continuation condition $CC_{cs,1}^{H}$.

Finally,

$$E\left(S_1^H(scenario)\right)=q\cdot S_1^H(u)+(1-q)\cdot\max\left(S_1^H(d,scenario),V_c^H(scenario)\right),$$

or,

4.4 $$E\left(S_1^H(scenario)\right) = q \cdot S_1^H(u) + (1-q)\cdot \max\left(S_1^H(d, scenario), \frac{-C_{cs}^H \cdot R_e + q \cdot S_1^H(u)}{r_e + q}\right).$$

At this point I verify if imposing Model 2´s conditions to scenario 2 (d-n), I obtain the same conditions of Model 2 as a particular case:

Scenario	c_{aff}	c_H	$S_1^H(d)$
2	$-(V_d - D_1^{aff})$	0	$m\cdot\left(F_1^H(d) - D_1^H\right)$

Model 2:

$$V_c^{cs} = \left(\frac{R}{q+r_e}\right)\cdot M + \frac{f\cdot m}{(q+r_e)}\cdot\left(E(V_1) - D_1\right) - f\cdot m\cdot(D_1 - V_d)$$

Model 3:

$$V_C^H : \frac{\left(-c_{aff}\cdot f\cdot m - c_H\cdot m + M\right)\cdot R_e + q\cdot m\cdot S_1^H(u)}{r_e+q} - m\cdot S_1^H(d, scenario)$$

$$V_C^H : \frac{M\cdot R_e}{r_e+q} - \frac{\left(c_{aff}\cdot f\cdot m + c_H\cdot m\right)\cdot R_e}{r_e+q} + \frac{q\cdot m\cdot S_1^H(u)}{r_e+q} - m\cdot S_1^H(d, scenario)$$

$$V_C^H : \frac{M\cdot R_e}{r_e+q} + \frac{(V_d - D_1)\cdot f\cdot m\cdot R_e}{r_e+q} + \frac{q\cdot m\cdot f\cdot(V_u - D_1)}{r_e+q}$$

$$= \frac{M\cdot R_e}{r_e+q} + \frac{(V_d - D_1)\cdot f\cdot m\cdot(r_e+q)}{r_e+q} + \frac{(V_d - D_1)\cdot f\cdot m\cdot(1-q)}{r_e+q} + \frac{q\cdot m\cdot f\cdot(V_u - D_1)}{r_e+q}$$

$$V_C^H = \frac{M\cdot R_e}{r_e+q} + \frac{(E(V_1) - D_1)\cdot f\cdot m}{r_e+q} - (D_1 - V_d)\cdot f\cdot m = V_c^{cs} \qquad \text{q.e.d.}$$

Many scenarios can be analyzed with the generalized Model 3. Notwithstanding I will concentrate on Case II, Scenario 2 (d-n), Affiliate is in default and Holding is not, but contrary to Model 2, Holding has previous investments and indebtedness (that can even be raised immediately prior to the rescue operation: In the next subsection I focus on one scenario which is more interesting. Controlling Shareholder decides (eventually forces) the Rescue (Case II), Affiliate is in default but not Holding (Scenario 2, d-n in figure 8) and minority shareholders concur to the equity call in the Affiliate (Naïve case).

4.4. Analysis of Model 3, Case II, Scenario 2, (Naive Case)

Analogous to Model 2, I compare equity positions of majority and minority shareholders, before and after the acquisition, given that CC (Model 1) doesn't hold, and in both cases: CC_{cs}^{H} holds and CC_{cs}^{H} doesn't hold. (table 2):

4.4.1. Case summary (II-2), schedule and results:

Table 2: Case summary

Equity Value	New Majority Shareholder (at affiliate level)	New Minority Shareholder (affiliate level)	Majority Shareholder (at Holding level)	Majority Shareholder (at Holding level)
Before Acquisition	1	2	A	B
After Acquisition				
CC_{cs}^{H} holds	3	4	C	D
CC_{cs}^{H} doesn't hold	5	6	E	F

At Affiliate Level:

$$1:0$$

$$2: S_{nc}^{0} = \frac{q\cdot(V_u - D_1)}{R_e}$$

$$3: \frac{f\cdot\left(E(V_1) - D_1\right)}{r_e + q}$$

$$4: \frac{(1-f)\cdot\left(E(V_1) - D_1\right)}{r_e + q}$$

$$5: \frac{f\cdot q\cdot(V_u - D_1)}{R_e}$$

$$6: \frac{(1-f)\cdot q\cdot(V_u - D_1)}{R_e}$$

At holding company Level:

$$A:0$$

$$B:0$$

$$C: \frac{m}{r_e+q}(E(F_1^H)-D_1^H)+\frac{m\cdot f}{r_e+q}\left(E(V_1)-D_1^{aff}\right)+M\cdot\frac{(1-q)}{r_e+q}$$

$$D: \frac{(1-m)}{r_e+q}(E(F_1^H)-D_1^H)+\frac{(1-m)\cdot f}{r_e+q}\left(E(V_1)-D_1^{aff}\right)$$

$$E: \frac{m\cdot\left(E(F_1^H)-D_1^H\right)+q\cdot m\cdot f\cdot(V_u-D_1^{aff})}{R_e}$$

$$F: \frac{(1-m)\cdot\left(E(F_1^H)-D_1^H\right)+q\cdot(1-m)\cdot f\cdot(V_u-D_1^{aff})}{R_e}$$

Now, I calculate the holding discount as a relative discount, with the same procedures used for Model 2:

$$Hold.Discount = 1-\left(\frac{\frac{(1-m)}{r_e+q}(E(F_1^H)-D_1^H)+\frac{(1-m)\cdot f}{r_e+q}\left(E(V_1)-D_1^{aff}\right)}{\frac{(1-m)\cdot\left(E(F_1^H)-D_1^H\right)+q\cdot(1-m)\cdot f\cdot(V_u-D_1^{aff})}{R_e}}\right)\Bigg/ 1-\frac{\frac{(1-f)\cdot\left(E(V_1)-D_1\right)}{r_e+q}}{\frac{(1-f)\cdot q\cdot(V_u-D_1)}{R_e}},$$

$$= \ 1-\left(\frac{(E(F_1^H)-D_1^H)+f\cdot\left(E(V_1)-D_1^{aff}\right)}{\left(E(F_1^H)-D_1^H\right)+q\cdot f\cdot(V_u-D_1^{aff})}\cdot\left(\frac{R_e}{r_e+q}\right)\right)\Bigg/ 1-\left(\frac{\left(E(V_1)-D_1\right)}{q\cdot(V_u-D_1)}\left(\frac{R_e}{r_e+q}\right)\right).$$

It can be very verified that, in this Naive model of holding discount of a previous operating holding company, instead of a holding discount, a Holding Premium will exist, whenever previous to acquisition (raising equity as in Model 2) Holding has a positive equity value represented by $E(F_1^H)-D_1^H>0$.

Nevertheless, if Acquisition is done by raising debt $D_{1,acq}^H$ as indicated in the beginning of section 4, and previously no other operations exist, so that in the model $E(F_1^H)-D_1^H<0$, a *holding discount will exist*, depending on the level of debt raised.

4.5. Comparative analysis of CS Continuation Condition in Models 2 and Naïve Model 3 several scenarios

Finally, in this sub section I compare both lower bounds for M, for Model 2 (unlevered holding) and Model 3 (Levered Holding), to analyze the impact of holding´s Leverage on the incentives for CS to rescue Affiliate in case it defaults.

From Model 2, I established that for CS the optimal strategy (if state of nature is "d") would be to continue if:

$$4.6 \quad M^2 > f \cdot m \cdot \left(\frac{(r_e + q)}{R_e}(D_1^{aff} - V_d) - \frac{\left(E(V_1) - D_1^{aff}\right)}{R_e} \right).$$

On the other hand, from Model 3 I established that for CS the optimal strategy (if state of nature is "d") would be to continue (and rescue Affiliate) if:

$$4.7 \quad M^3 > m \cdot \left(S_1^H(d, scen) + c_{aff}(scen) \cdot f + c_H(scen) - \frac{E\left(S_1^H(scen)\right)}{R_e} \right)$$

To deploy M^3 I use values for each variable scenario to obtain the following:

$$\mathit{Scenario} \qquad \underline{M^3} = m \cdot \left(S_1^H(d, scen) + c_{aff}(scen) \cdot f + c_H(scen) - \frac{E\left(S_1^H(scen)\right)}{R_e} \right)$$

$$1: m \cdot \left[-\left(F_1^H(d) - D_1^H\right) - f \cdot (V_d - D_1^{aff}) - \frac{q}{R_e}\left(\left(F_1^H(u) - D_1^H\right) + f \cdot (V_u - D_1^{aff})\right) \right]$$

$$= m \cdot \left[D_{1,acq}^H (R_e + q) - \left\{ \left(E(F_1^H) - D_1^{H,p}\right) + f \cdot \left(E(V_1) - D_1^{aff}\right) + (r_e + q)\left(F_1^H(d) - D_1^H\right) + f \cdot (V_d - D_1^{aff}) \right\} \right]$$

$$2: m \cdot \left[\left(F_1^H(d) - D_1^H\right) + f \cdot (D_1^{aff} - V_d) - \frac{1}{R_e}\left(\left(E(F_1^H) - D_1^{H,p}\right) + f \cdot q \cdot (V_u - D_1^{aff})\right) \right]$$

$$= m \cdot \left[D_{1,acq}^H \left(1 + \frac{1}{R_e}\right) - \left\{ \frac{\left(E(F_1^H) - D_1^{H,p}\right)}{R_e} + f \cdot \left(E(V_1) - D_1^{aff}\right) + \left(F_1^H(d) - D_1^H\right) + f \cdot (r_e + q) \cdot (V_d - D_1^{aff}) \right\} \right]$$

$$\mathit{Scenario} \qquad \underline{M^3} = m \cdot \left(S_1^H(d, scen) + c_{aff}(scen) \cdot f + c_H(scen) - \frac{E\left(S_1^H(scen)\right)}{R_e} \right)$$

$$3: m \cdot \left[-\left(F_1^H(d) - D_1^H\right) - f \cdot (V_d - D_1^{aff}) - \frac{q}{R_e}\left(\left(F_1^H(u) - D_1^H\right) + f \cdot (V_u - D_1^{aff})\right) \right]$$

$$= m \cdot \left[D_{1,acq}^H (R_e + q) - \left\{ \left(E(F_1^H) - D_1^{H,p}\right) + f \cdot \left(E(V_1) - D_1^{aff}\right) + (r_e + q)\left(F_1^H(d) - D_1^H\right) + f \cdot (V_d - D_1^{aff}) \right\} \right]$$

$$4: m \cdot \left[\left(F_1^H(d) - D_1^H\right) + f \cdot (V_d - D_1^{aff}) - \frac{1}{R_e}\left(\left(E(F_1^H) - D_1^H\right) + f \cdot (E(V_1) - D_1^{aff})\right) \right]$$

$$= m \left[S_1^H(d) - \frac{E(S_1^H)}{R_e} \right]$$

Thus, comparing Model 2 with Model 3 (d-n) I have:

$$\underline{M}^2 = f \cdot m \cdot \left(\frac{(r_e + q)}{R_e}(D_1^{aff} - V_d) - \frac{\left(E(V_1) - D_1^{aff}\right)}{R_e} \right)$$

4.8
$$\underline{M}^3 = m \cdot \left[D_{1,acq}^H (1 + \frac{1}{R_e}) - \left\{ \frac{\left(E(F_1^H) - D_1^{H,p}\right)}{R_e} + f \cdot \left(E(V_1) - D_1^{aff}\right) + \left(F_1^H(d) - D_1^{H,p}\right) + f \cdot (r_e + q) \cdot (V_d - D_1^{aff}) \right\} \right]$$

$$\underline{M}^3 = \underline{R_e \cdot M^2} + m \cdot D_{1,acq}^H (1 + \frac{1}{R_e}) - m\left[\left(E(F_1^H) - D_1^{H,p}\right) + \left(F_1^H(d) - D_1^{H,p}\right) \right]$$

4.6. Summing Up and Conclusions Model 3

In Models 2 and 3 I have been able to show that in order for a holding discount to exist, in the Acquisition version of the story, the way to raise the funds for acquisition matters as matters the reaction of minority shareholders to equity calls, in other words, the existence of conflicts of interest matter.

In fact, in Naive Model 2 acquisition is done by raising equity and no holding discount exists, except when minority shareholders do not concur to equity calls (their best response to CS). On the contrary, in Naive Model 3, if I allow acquisition to be made with debt, a *holding discount exists by this last fact*, even when I consider that minority shareholders do not follow their best strategy. This conclusion is consistent with Almeida and Wolfenson (2006) and Almeida et al. (2010).

I hypothesize that if I can add to Naive Model 3 the more realistic assumption (not developed in section 3) that minority shareholders do not concur to equity call (as I did in Model 2), I add conditions that facilitate but not ensure the arising of a holding discount.

Finally, again I have found more situations or scenarios in which a holding discount may arise, but at the same time I have shown that once more, the hurdle idiosyncratic benefit to CS, *M,* increases as shown in previous paragraphs. The process of increasing the hurdle *M* starts in Naive Model 2, to Realistic Model 2, (eventually Naive Model 3), to Realistic Model 3.

Thus, in the conditions of model 3, main hypothesis H0 can be rejected with a reasonable degree of confidence, in the legal ruling of Limited Liability and certain plausible conditions.

Section 5. The Levered Holding Case Model 4, acquisition is done with preexisting cash in Holding. (Special case of Model 3: "Holding Already in Place" Approach)

In this case, if it proves to be the best strategy, CS forces continuation for one time, using cash previously gathered at holding company. In 5.1 I indicate valuations of model 4. In 5.2 I state the Decision Model for CS of the Levered Holding. In 5.3 I show the Valuation Schedule, for the Naive Case. In 5.4 I summarize and conclude Model 4. Finally, in 5.5, I summarize the findings of Models 1 to 4.

5.1. Valuations

Affiliate Company:

As usual, $S_1^{aff}(u) = V_u - D_1^{aff} \geq 0$ and $S_1^{aff}(d) = V_d - D_1^{aff} < 0$[26].

holding company:

Apart from share in Affiliate, holding company owns only certain amount of cash $Cash(\$)$, and no debts, so

26 later I return to the assumption that NoCC, $q \cdot (V_u - D_1) / R_e < (D_1 - V_d); 0 < q < 1; V_d < D_1 < V_u$, which means that in case state of nature is "d", optimal strategy to a representative shareholder of the affiliate will be not to continue.

$$F_1^H(s) - D_1^H = Cash(\$)$$
$$D_1^H = 0$$
$$S_1^H(u) = Cash(\$) + f \cdot (V_u - D_1^{aff})$$
$$S_1^H(d) = ?$$

If CS´s optimal strategy is to rescue Affiliate in case of default, funds needed will be taken from $Cash(\$)$. My assumptions imply that $S_1^H(u) > 0$.

To calculate $S_1^H(d)$ I need to check, as in Model 2, whether in that case the optimal strategy for CS will be to abandon, or to pay Affiliate´s Debt (only Affiliate can be in default) in which case the corresponding continuation value should be calculated. This is analyzed as the Decision model for the Controlling Shareholder CS.

5.2. Decision Model for CS of the Levered Holding

I describe two cases, and 4 scenarios for each case. For all the following holds:

$$S_1^H(u) = Cash(\$) + f \cdot (V_u - D_1^{aff})$$

Cases:

I) CS complies with LL rules.
II) CS doesn't necessarily comply with LL rules.

The only Scenario in this particular case is: 2) Affiliate Company is in default, holding company is not in default (d-n). Figure 10 depicts the different cases:

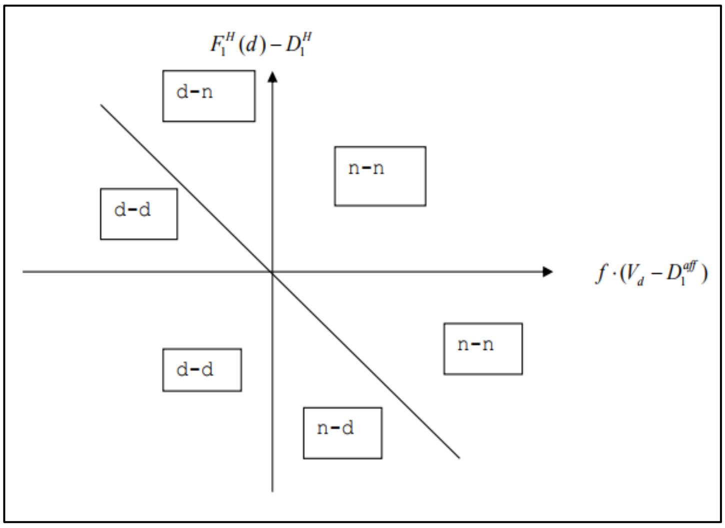

Figure 10. Scenarios of behavior for the controlling shareholding of the levered Holding

Expected Equity Values:

Affiliate

$$S_1^{aff}(u)=\max(0;V_u-D_1^{aff})$$
$$S_1^{aff}(d)=\max(0;V_d-D_1^{aff})$$
$$E(S_1^{aff})=q\cdot S_1^{aff}(u)+(1-q)\cdot S_1^{aff}(d)$$

Holding

$$S_1^H(u)=\max(0;F_1^H(u)-D_1^H+f\cdot(V_u-D_1^{aff}))$$
$$S_1^H(d)=\max(0;F_1^H(d)-D_1^H+f\cdot(V_d-D_1^{aff}))$$
$$E(S_1^H)=q\cdot S_1^H(u)+(1-q)\cdot S_1^H(d)$$

5.2.1.Case I: CS complies with LL rules at any event

State dependent values:

Affiliate

$$S_1^{aff}(u)=(V_u-D_1^{aff})$$
$$2:S_1^{aff}(d)=0$$

Holding

$$S_1^H(u)=Cash(\$)+f\cdot(V_u-D_1^{aff})$$
$$2:S_1^H(d)=Cash(\$)+f\cdot(V_d-D_1^{aff})$$

Equity Expected Values:

Affiliate

$$E(S_1^{aff})=q\cdot S_1^{aff}(u)+(1-q)\cdot S_1^{aff}(d)$$
$$2:E(S_1^{aff})=q\cdot(V_u-D_1^{aff})$$

Holding

$$E(S_1^H)=q\cdot S_1^H(u)+(1-q)\cdot S_1^H(d)$$
$$2:E(S_1^H)=q\cdot\left(Cash(\$)+f\cdot(V_u-D_1^{aff})\right)+(1-q)\cdot Cash(\$)=Cash(\$)+q\cdot f\cdot(V_u-D_1^{aff})$$

5.2.2.Case II: CS doesn't comply with LL rules

In case of default of Affiliate, and if CS´s best strategy is to rescue, funds need not to be raised since they are already in holding company. Additionally, I assume the naïve case for minority shareholders at Affiliate: They keep their shareholdings percentages. Figure 11 below, shows the relevant decision tree to Controlling Shareholder.

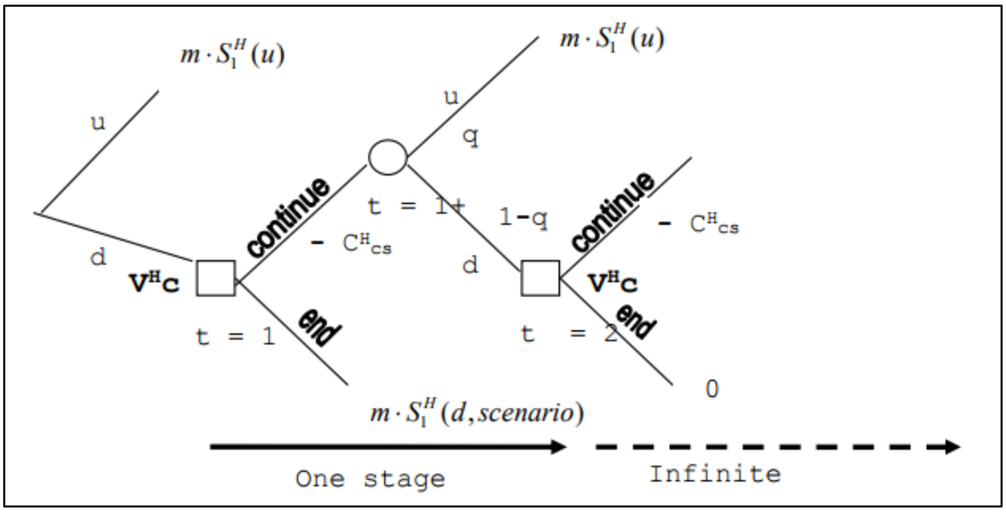

Figure 11. Relevant decision tree to Controlling Shareholder.

V_c^H corresponds to the continuation value to CS at the Holding Level, condition labeled CC_{cs}^H (which includes but is not limited to the affiliate level);

$$-C_{cs}^H = -c_{aff} \cdot f \cdot m - c_H \cdot m + M \,,$$

and:

Scenario	c_{aff}	c_H	$S_1^H(d)$
2	$-(V_d - D_1^{aff})$	0	$Cash(\$)$

Scenario	$S_1^H(u)$	$E\left(S_1^H\right)$
2	$Cash(\$) + f \cdot \left(V_u - D_1^{aff}\right)$	$Cash(\$) + q \cdot f \cdot \left(V_u - D_1^{aff}\right)$

Optimum strategy for CS will maximize:

$$m \cdot E(S_1^H) = q \cdot m \cdot S_1^H(u) + (1-q) \cdot V_c^H \,.$$

a. One Stage Solution:

A main simplifying assumption is that in this model, I end story in period 1, because no cash is available to follow on. Therefore, I do not analyze the multistage solution.

$$V_{c,1}^{H}(scen2)=\max\left(\underbrace{m\cdot S_{1}^{H}(d,scen2)}_{end};\underbrace{-C_{cs}^{H}(scen2)+\frac{q\cdot m\cdot S_{1}^{H}(u)+(1-q)\cdot m\cdot 0}{R_e}}_{continue}\right)$$

$$V_{c,1}^{H}(scen2)=\max\left(m\cdot Cash(\$);-\left(D_{1}^{aff}-V_{d}\right)\cdot f\cdot m+M\right)+\frac{q\cdot m\cdot f\cdot\left(V_u-D_1^{aff}\right)+m\cdot 0}{R_e}\Bigg)$$

As Cash held in holding company, exactly matches the one-time cost of rescue,

$$Cash(\$)=f\cdot(D_1^{Aff}-V_d).$$

Then I have:

$$V_{c,1}^{H}(scen2)=\max\left(m\cdot Cash(\$);-Cash(\$)\cdot m+M+\frac{q\cdot m\cdot f\cdot\left(V_u-D_1^{aff}\right)}{R_e}\right)$$

From which I derive CS Continuation Condition:

$$CC_{cs,1}^{H}:M>m\cdot Cash(\$)-\frac{q\cdot m\cdot f\cdot\left(V_u-D_1^{aff}\right)}{R_e}\quad\Rightarrow\frac{M}{m}>(D_1^{Aff}-V_d)-\frac{q\cdot f\cdot\left(V_u-D_1^{aff}\right)}{R_e}.$$

If condition $CC_{cs,1}^{H}$ holds, holding company rescues for one-time Affiliate and cashes out or leave. In terms of my model, this means that:

$$V_c^H=V_{c,1}^H=-Cash(\$)\cdot m+M+\frac{q\cdot m\cdot f\cdot\left(V_u-D_1^{aff}\right)}{R_e}$$

and,

$$S_1^H(cont)=q\cdot f\cdot\left(V_u-D_1^{aff}\right)+(1-q)\cdot\left(-Cash(\$)+\frac{q\cdot f\cdot\left(V_u-D_1^{aff}\right)}{R_e}\right)+(1-q)\cdot\frac{M}{m}\quad to\ CS$$

and

$$S_1^H(cont)=q\cdot f\cdot\left(V_u-D_1^{aff}\right)+(1-q)\cdot\left(-Cash(\$)+\frac{q\cdot f\cdot\left(V_u-D_1^{aff}\right)}{R_e}\right)\quad to\ \min Shh.$$

Now I recall that continuation condition to minority shareholders does not hold:

$$\frac{q\cdot(V_u-D_1)}{R_e}<(D_1^{Aff}-V_d)=\frac{Cash(\$)}{f},$$

and compare $S_1^H(cont)\ vs.\ S_1^H(no\ cont)$:

$$S_1^H(no\ cont) = q \cdot f \cdot \left(V_u - D_1^{aff}\right)$$

$$S_1^H(cont) = S_1^H(cont) = q \cdot f \cdot \left(V_u - D_1^{aff}\right) + (1-q) \cdot \left(-Cash(\$) + \frac{q \cdot f \cdot \left(V_u - D_1^{aff}\right)}{R_e}\right)$$

$$S_1^H(no\ cont) - S_1^H(cont) = q \cdot f \cdot \left(V_u - D_1^{aff}\right) - q \cdot f \cdot \left(V_u - D_1^{aff}\right) - (1-q) \cdot \left(-Cash(\$) + \frac{q \cdot f \cdot \left(V_u - D_1^{aff}\right)}{R_e}\right)$$

$$S_1^H(no\ cont) - S_1^H(cont) = -(1-q) \cdot \underbrace{\left(-Cash(\$) + \frac{q \cdot f \cdot \left(V_u - D_1^{aff}\right)}{R_e}\right)}_{<0 \text{ if CC at affiliate doesn't hold}} > 0$$

$$S_1^H(no\ cont) - S_1^H(cont) = -(1-q) \cdot V_{cont}^{Aff} = C^{Aff} Loss > 0$$

$$\Rightarrow S_1^H(cont) = q \cdot f \cdot \left(V_u - D_1^{aff}\right) - C^{Aff} Loss$$

I conclude as in Models 2 and 3, that by forcing affiliate´s rescue eventually against interests of minority shareholders at affiliate, there will exist a value loss to minority shareholders at affiliate, an also to affiliate´s CS (Holding), but this later loss is compensated by idiosyncratic benefit *M*.

5.2. Summary Valuation Schedule, for the Naive Case

In this subsection I allow for minority shareholders to keep their shareholding percentages in Affiliate. Analogous to Model 2, I compare equity positions of majority and minority shareholders, before and after the acquisition, given that CC (Model 1) doesn't hold, and in both cases: CC_{cs}^H holds and CC_{cs}^H doesn't hold (table 3).

Table 3: Case summary (II-2), schedule and results

Equity Value	New Majority Shareholder (at affiliate level)	New Minority Shareholder (affiliate level)	Majority Shareholder (at Holding level)	Majority Shareholder (at Holding level)
Before Acquisition	1	2	A	B
After Acquisition				
CC_{cs}^H holds	3	4	C	D
CC_{cs}^H doesn't hold	5	6	E	F

At Affiliate Level:

$$1:0$$

$$2: S_{nc}^{0} = \frac{q \cdot (V_u - D_1)}{R_e}$$

$$3: f \cdot \left(\frac{q \cdot (V_u - D_1)}{R_e} - C^{Aff} Loss \right)$$

$$4: (1-f) \cdot \left(\frac{q \cdot (V_u - D_1)}{R_e} - C^{Aff} Loss \right)$$

$$5: \frac{f \cdot q \cdot (V_u - D_1)}{R_e}$$

$$6: \frac{(1-f) \cdot q \cdot (V_u - D_1)}{R_e}$$

At holding company Level:

$$A:0$$

$$B:0$$

$$C: m \cdot (1-f) \cdot \left(\frac{q \cdot (V_u - D_1)}{R_e} - C^{Aff} Loss \right) + (1-q) \cdot \frac{M}{R_e}$$

$$D: (1-m) \cdot (1-f) \cdot \left(\frac{q \cdot (V_u - D_1)}{R_e} - C^{Aff} Loss \right)$$

$$E: m \cdot (1-f) \cdot \left(\frac{q \cdot (V_u - D_1)}{R_e} \right) + m \cdot Cash(\$)$$

$$F: (1-m) \cdot (1-f) \cdot \left(\frac{q \cdot (V_u - D_1)}{R_e} \right) + (1-m) \cdot Cash(\$)$$

Now, I calculate the holding discount as a relative discount, with the same procedures used for Model 2:

$$HD = 1 - \left(\frac{(1-f) \cdot \left(\frac{q \cdot (V_u - D_1)}{R_e} - C^{Aff} Loss \right)}{(1-f) \cdot \left(\frac{q \cdot (V_u - D_1)}{R_e} \right) + Cash(\$)} \right) \Bigg/ 1 - \left(\frac{\left(\frac{q \cdot (V_u - D_1)}{R_e} - C^{Aff} Loss \right)}{\frac{q \cdot (V_u - D_1)}{R_e}} \right)$$

5.4. Summary and conclusions Model 4

In previous section, it can be verified that, in this naïve (no conflict of interest) model of holding discount of a previous operating holding company, provided that cash to rescue affiliate is already in the company, a *holding discount arises unambiguously*. In terms of Model 2, this model is equivalent to have a naive strategy of minority shareholders at affiliate, and a "forced" naive strategy by minority shareholders at Holding. This result is consistent with Almeida and Wolfenson (2006) and Almeida et al. (2010).

As in model 3, I hypothesize that if minority shareholders follow their best strategy, (conflicts of interest) the possibilities for a holding discount to arise are increased. Thus, in the conditions of model 4, main hypothesis H0 is rejected, with a reasonable degree of confidence, in the legal ruling of Limited Liability and certain plausible conditions.

5.5. Summary of findings of Models 1 to 4

I Table 4 I present a summary schedule with the main characteristics of each model, and whether in each model a holding discount arises, as well as the existence of conflict of interest between Majority and minority shareholders:

Table 4: Summary schedule models 1 to 4

Model	Description	holding discount when propping	Conflicts Controller - Minority	Incentives to Controller to Bail out subsidiary	Private Benefits to Controller
1	Stand-Alone Firm	Not applicable	No	Yes, if Continuation Condition (CC) holds	No
2	Unlevered holding company, rescue of distressed Affiliate is made by equity call	No if minority shareholders concur to the equity call (naïve case); may arise if minority shareholders don't concur (best response); yes, if benefits M go to all shareholders	Yes, if M is private to controller, no if M is common to all shareholders	Yes	Yes (M)
3	Levered holding company, rescue of distressed Affiliate is made by issuing debt	Yes, depending on the level of new debt. Probability of rescue in case of distress is higher than in model 2	Yes, if M is private to controller, no if M is common to all shareholders	Yes	Yes (M)
4	Special case of model 3. holding company rescues the distressed Affiliate using existing cash in the holding company	Yes, in the Naïve Case (shown), and in case minority shareholders do not concur, I hypothesize it will exist with greater probability than in the naïve case	Yes	Yes	Yes (M)

Section 6. Summary and conclusions

In this chapter, I developed a series of recursive models for a business group in which the levered holding company repeatedly faces the decision to bailout a distressed levered subsidiary in bad times. I have stated the conditions for this decision to be made, named exercising the "unlimited liability option" (in the literature this is a propping or bailing out behavior), in several contexts of increasing complexity. This increasing complexity is reflected in models 1 to 4, the former stating the structural basis of the rest of the models, the latter is an extension of model 3.

Model 1 correspondes to a Stand-Alone Firm. Model 2 analyzes an unlevered holding company, that when decided by the controlling shareholder, rescues rhe distressed Affiliate making an equity call. Model 3 analyzes a levered holding company, that when decided by the controlling shareholder rescues rhe distressed affiliate by issuing debt. Model 4 is a special case of model 3, in which holding company rescues the distressed affiliate using existing cash in the holding company. Specific characteristics of the models developed, can be foun in section 5.5, Table 4.

Contrary to models developed in the literature, designed to explain the holding discount under asymmetrical information and other market imperfections, my models rely in a context of full and symmetrical information and rational investors, in the absence of taxes only, and explain the existence of a holding discount under certain plausible conditions; this is a main contribution of this chapter.

The analysis starts by formulating Hypothesis H0: holding discount exists, only when asymmetric information, agency costs or other imperfections, are considered regardless of the legal system. This hypothesis is contrasted against models 2,3 and 4, sequentially.

Additionally, eventual conflicts of interest with minority shareholders in the holding company may arise. Moreover, I can also show that in certain conditions, majority shareholders and minority shareholder's interests will be aligned when the bailout decision is faced, and depending on the assumptions of the models, they can be in conflict, as existing literature prescribes.

Finally, I derive a set of empirical implications from the models.

Section 7. Avenues for further research

In practice, what I can observe in the marketplace is a time series of observable variables for a number of given business groups already conformed, and their stock traded since the IPO. I do not observe all the variables that affect the continuation conditions, the losses or price discounts. What I can infer from the model, are variations in losses (discounts) caused by variations in observed variables that alter the continuation conditions.

The chain of causality is as follows: $\partial(Obs) \Rightarrow \partial(NotObs) \Rightarrow \partial CC \Rightarrow \partial \Pr ice$, where "$\partial$" means change in a variable while keeping all the rest unchanged, "Obs" are exogenous observed variables firm specific or market conditions, "NotObs" are variables that are in my models, and are functions of observed variables, CC means continuation conditions and price would be holding premium or holding discount.

The questions I can propose can be derived from the following facts:

1. When Controlling shareholder follows the LL rules, I will not observe premium or discount in the context of no market imperfections, no information asymmetries.
2. It follows that a holding discount may arise when the distressed affiliate is rescued, which implies a cost to minority shareholders, with some probability of rescue or occurrence. The concurrence of a favorable continuation conditions for the Controlling Shareholder (*CCcs*), as well as the probability of distress (1-q), and the cost of rescue are crucial. The holding discount can be regarded as the expected loss to minority shareholders, which has embedded the probability of default and the cost of rescue.

Then, the most important variables that I can monitor and include in a model are:

a) Continuation Conditions, mainly indebtness of the subsidiary, and private benefits of control which are not observed in the market.

b) Probability of default. This can be proxied by some variable that reflects that probability in the market place, for example the risk classification, or the observed spread of loans existing in the holding company and if possible in the affiliate company. Refer to Chapter 4, "Optimal Bailout of subsidiaries" as a good example of this.
c) Cost of rescue that will be borne by minority shareholders.

As it can be seen in the rest of Section 7, If I had the necessary public information to work with (which is not the case), may empirical tests can be done, involving time series of the discount for a specific holding, and a cross section (panel) of several holdings, to make conclusions more robust. The main counter force is the availability of public information.

Let us see several models that can be advanced in the form of panel:

a. Continuation Conditions

Controlling Shareholder forces rescue when this condition is positive:

$$CC_{cs}: \frac{q \cdot (V_u - D_1)}{R_e} - c \cdot f \cdot m + M = \frac{q \cdot (V_u - D_1)}{R_e} - (D_1 - V_d) \cdot f \cdot m + M \cdot$$

Stand-alone Company representative shareholder benefits from rescue when positive is:

$$CC: \frac{q \cdot (V_u - D_1)}{R_e} - (D_1 - V_d) \cdot$$

An answer that can be advanced is, for example: if CC gets more positive, the greater the probability of rescue, given a state of nature, and the greater the discount
Other hypothesis can be: the worst the state of the economy (1-q increases) the greater the probability of rescue.

Then, how do I answer those questions? Starting from an initial operating point, were $CS_{cs}\ can\ be >, =, or > 0$, but $CC < 0$, I can evaluate the changes of *CCcs* and CC due to a change in the relevant variables. In this way I can write:

$$CC_{cs} = CC_{cs}(f^-, m^-, D_1^-, R_e^-, M^+, q^+, V_u^+, V_d^+)$$
$$CC = CC(D_1^-, R_e^-, q^+, V_u^+, V_d^+)$$

This expression can be read as follows: the likelihood of observing a price reduction on the affiliate as well as the holding companies, increases as variables labeled with "+" increase, and decreases when variables labeled with "-" increase.

b. Shareholder´s Losses (CS forcing vs. CS not forcing)

Majority, affiliate (MajA):

$$LMajA = f \cdot \frac{1-q}{r_e+q}\left(-(D_1-V_d)+\frac{q\cdot(V_u-D_1)}{R_e}\right) = f\cdot\frac{1-q}{r_e+q}\cdot CC(D_1^-,R_e^-,q^+,V_u^+,V_d^+)$$
$$= LMajA\left(f^+,q^-,CC^+(D_1^-,R_e^-,q^+,V_u^+,V_d^+)\right)$$

Minority, affiliate (MinA):

$$LMinA = (1-f)\cdot\frac{1-q}{r_e+q}\cdot CC(D_1^-,R_e^-,q^+,V_u^+,V_d^+)$$
$$= LMinA\left(f^-,q^-,CC^+(D_1^-,R_e^-,q^+,V_u^+,V_d^+)\right)$$

Majority, holding (MajH):

$$LMajH = m\cdot f\cdot\frac{1-q}{r_e+q}\cdot CC(D_1^-,R_e^-,q^+,V_u^+,V_d^+)$$
$$= LMajH\left(m^+,f^+,q^-,CC^+(D_1^-,R_e^-,q^+,V_u^+,V_d^+)\right)$$

Minority, holding (MinH):

$$LMinH = (1-m)\cdot f\cdot\frac{1-q}{r_e+q}\cdot CC(D_1^-,R_e^-,q^+,V_u^+,V_d^+)$$
$$= \left(m^-,f^+,q^-,CC^+(D_1^-,R_e^-,q^+,V_u^+,V_d^+)\right)$$

c. Price discount (minority shareholders valuation (CS forcing vs. CS not forcing)

Affiliate company (PdA):

$$1-\frac{\left(E(V_1)-D_1\right)}{q\cdot(V_u-D_1)}\cdot\frac{R_e}{r_e+q}$$

holding company (PdH):

$$1-\left(\sum_i f_i\cdot S_i - D_0^H + \frac{(1-m)\cdot\left(E(V_1)-D_1\right)}{r_e+q}\right)\Big/\left(\sum_i f_i\cdot S_i - D_0^H + \frac{(1-m)\cdot f\cdot q\cdot(V_u-D_1)}{R_e}\right)$$

Appendix

In this appendix I derive in extended form the expressions for Equity, Debt and Asset values, when majority shareholders have the option not to comply with Limited Liability, and minority shareholders following the optimal strategy. This can give us some light regarding the value of not following limited liability regulation, in other words of following what I have called Unlimited Liability. In finance literature it is well known the equity of a company subject to Limited Liability can be regarded as a call option that shareholders own, to purchase the assets of the company to debtholders. This option is valuable. Here I contribute to the valuation to another option, option to go UL.

First, I must recognize the sequential nature of events as in the next graph.

Second, I calculate the cash flows to shareholder, debt holder and asset holders (shareholder and debt holder); Third I evaluate the probability of all independent and possible events; and finally, we, calculate the expected present value to each stakeholder. Figure 12 below shows the evolution of Equity, Debt and Assets:

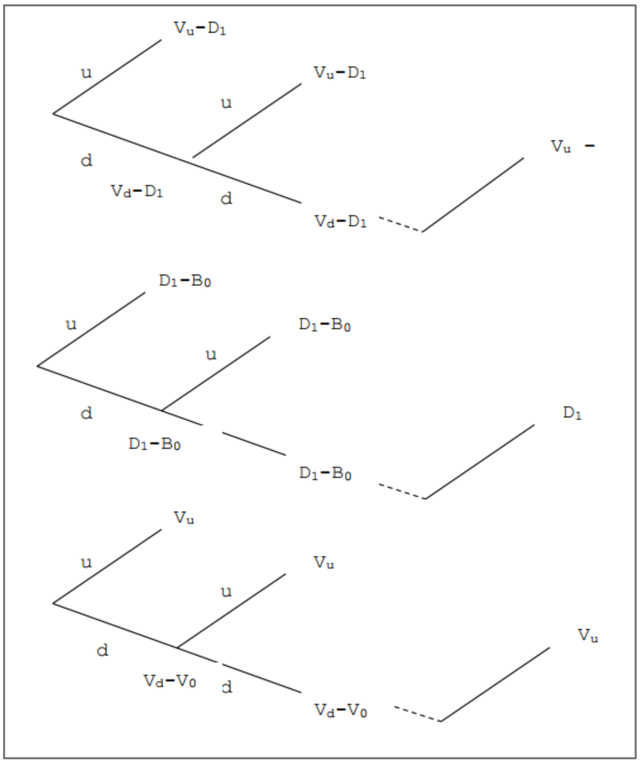

Figure 12. Evolution of Equity, Debt and Assets

In the previous graph, cash flows to shareholders, debt holders and asset holders are shown respectively at nodes "*t=0*" , "*t=1*" and the final node of the sequence. The independent events are "u", "du", "ddu"…with probabilities p(u), p(du), p(ddu)…

I first calculate Asset value, second Debt Value, and finally, as a residual Value, equity value.

As I have described, Asset realizes value at the end of each period, an at the same moment, if nature plays "d", Asset which in virtue of rescue in case of default (continuation), belongs to shareholders, is reinvested for one more period. At the same time, At the end of each period Debt proceeds are repaid to debt holders, who immediately lend again for a next period.

Assets:

Applying the same Equity procedure, Asset will be worth V_u or V_d if nature plays $"u"$ or $"d"$ respectively. At the beginning of any period, except the last (infinity), assets are reinvested at their economic value. To discount cash flows, I use R_a instead of R_e . If game ends at node *n*, it cannot end at *n-1* nor *n+1*. I then have a map of disjoint events each with its own probability of occurrence, and as the events are disjoint the probabilities must add to 1.

	$Event_t$	$\Pr obability_t$	$\Pr esent\ Value_t$	$E(\mathrm{P}VV_t)$
1	u	$(1-q)^0 q = q$	$V_u R_a^{-1}$	$qV_u R_a^{-1}$
2	du	$(1-q)^1 q$	$V_u R_a^{-2} + (V_d - V_0) R_a^{-1}$	$(1-q)^1 q(V_u R_a^{-2} + (V_d - V_0) R_a^{-1})$
3	ddu	$(1-q)^2 q$	$V_u R_a^{-3} + (V_d - V_0) R_a^{-1} + (V_d - V_0) R_a^{-2}$	$(1-q)^2 q(V_u R_a^{-3} + (V_d - V_0) R_a^{-1} + (V_d - V_0) R_a^{-2})$
.				
n	$\underbrace{dddd..ddd}_{n-1\ times}u$	$(1-q)^{n-1} q$	$V_u R_a^{-n} + (V_d - V_0)\sum_{t=1}^{t=n-1} R_a^{-t}$	$(1-q)^{n-1} q\left(V_u R_a^{-n} + (V_d - V_0)\sum_{t=1}^{t=n-1} R_a^{-t}\right)$
		$\sum_{t=1}^{t=\infty}(1-q)^t q = 1$		$\sum_{t=1}^{t=\infty} E(\mathrm{P}VV_t) = Asset\ Value = V_0$

From above, I can state:

$$V_0 = \lim_{n\to\infty} \sum_{j=1}^{j=n} (1-q)^{j-1} q \left(V_u R_a^{-j} + (V_d - V_0) \sum_{\tau=1}^{\tau=j-1} R_a^{-\tau} \right)$$

Simplifying the previous expression, I finally obtain:

$$V_0 = \frac{qV_u}{r_a + q} + \frac{(1-q)(V_d - V_0)}{r_a + q} \Rightarrow V_0 = \frac{E(V_1)}{R_a}$$

in case CC holds.[27]

Debt (standard calculation):

In case CC holds, it can be verified that following the optimal strategy implies to pay Debt every period and node the game reaches in any state of nature, implicating that Debt is risk-free.

As in every node asset reveals its value, and a new story begins, the new story in every node is: shareholder pays the Debt, recuperates the ownership of assets, and contracts the same debt again (sell the assets to debt holders while keeping a one period option) and waits for the next period.

Cash flows to Debt holders are then:

$$\begin{array}{lll}
t=0 & -D_0 & = -D_0 \\
t=1 & +D_1 - D_0 = D_0(1-r_b) - D_0 & = r_b D_0 \\
t=2 & +D_1 - D_0 = D_0(1-r_b) - D_0 & = r_b D_0 \\
t=3 & & = r_b D_0 \\
. & & \\
t=n & & = +D_1
\end{array}$$

It can be easily checked that the present value of the above cash flow stream discounted at the risk-free rate r_b starting in $t=1$, equals D_0 for all "n".

27 Note that the expected present value of a constant stream $V_d - V_0$ if state of nature is "d", and starting in period 1 is $(1-q)\cdot(V_d - V_0)/(r_e + q)$, applicable to any value in the same conditions.

As seen before, in case CC doesn't hold, debt will be risky, and

$$B_0 = \left(qD_1 + (1-q)V_d\right)R_b^{-1},$$

(same as one stage limited liability case).

Debt Expected present Value Calculation:

Applying the same Asset procedure, Debt will be worth D_1 if nature plays "u" or "d" respectively. At the beginning of each period, except for the last, debtholders provide Debt to shareholders. To discount cash flows, I use R_b.

	$Event_t$	$\Pr obability_t$	$\Pr esent\ Value_t$	$E(\mathrm{P}VV_t)$
1	u	$(1-q)^0 q = q$	$D_1 R_b^{-1}$	$qD_1 R_b^{-1}$
2	du	$(1-q)^1 q$	$D_1 R_b^{-2} + (D_1 - B_0)\cdot R_b^{-1}$	$(1-q)^1 q(D_1 R_b^{-2} + (D_1 - B_0)\cdot R_b^{-1})$
3	ddu	$(1-q)^2 q$	$D_1 R_b^{-3} + (D_1 - B_0)\cdot R_b^{-1} + (D_1 - B_0)\cdot R_b^{-2}$	$(1-q)^2 q(D_1 R_b^{-3} + (D_1 - B_0)\cdot R_b^{-1} + (D_1 - B_0)\cdot R_b^{-2})$
.				
n	$\underbrace{dddd..ddd}_{n-1\ times}u$	$(1-q)^{n-1} q$	$D_1 R_b^{-n} + (D_1 - B_0)\cdot \sum_{t=1}^{t=n-1} R_b^{-t}$	$(1-q)^{n-1} q\left(D_1 R_b^{-n} + (D_1 - B_0)\cdot \sum_{t=1}^{t=n-1} R_b^{-t}\right)$
		$\sum_{t=1}^{t=\infty}(1-q)^t q = 1$		$\sum_{t=1}^{t=\infty} E(\mathrm{P}VV_t) = Asset\ Value = V_0$

From above, I can state:

$$B_0 = \lim_{n\to\infty} \sum_{j=1}^{j=n} (1-q)^{j-1} q\left(D_1 R_b^{-j} + (D_1 - B_0)\cdot \sum_{\tau=1}^{\tau=j-1} R_b^{-\tau}\right)$$

Simplifying I obtain:

$$B_0 = \frac{q(1+r_b)D_0}{r_b+q} + \frac{(1-q)\cdot(D_1 - B_0)}{r_b+q} \Rightarrow B_0 = \frac{D_1}{R_b} = D_0$$

in case CC holds.

Equity (S_c^0):

I have shown that:

$$S_c^0 = \frac{q(V_u - D_1) + (1-q)(V_d - D_1)}{r_e + q}$$

$$iff: \frac{q\cdot(V_u - D_1)}{R_e} > (D_1 - V_d) \quad and \quad 0 < q < 1; \quad V_d < D_1 < V_u$$

With:

$$V_c = \frac{q(V_u - D_1) - R_e(D_1 - V_d)}{q + r_e}$$

I calculate Equity in two ways: the first, is a backwards solution (from the future to the present) replicating the sequence of decisions the shareholder makes. The second is calculating the expected equity value in the same way I have done with assets.

Backwards solution:

First, I start from a final hypothetical decision node $t = n$, the derive V_n, then V_1^n, and then I take the expression to the limit $n \to \infty$ to obtain $V_1 \ (if\ CC\ holds) = V_c$:

$$
\begin{aligned}
V_n &= -c + \frac{1}{\mathrm{R}_e}\left(q(V_u - D_1) + (1-q)(V_d - D_1)\right) = -c + \frac{1}{\mathrm{R}_e} E_n(V_1 - D_1) \\
V_{n-1} &= -c + \frac{1}{\mathrm{R}_e}\left(q(V_u - D_1) + (1-q)V_n\right) = -c + \frac{1}{\mathrm{R}_e}\left(q(V_u - D_1) + (1-q)\left(-c + \frac{1}{\mathrm{R}_e} E_n(V_1 - D_1)\right)\right) \\
V_{n-2} &= -c + \frac{1}{\mathrm{R}_e}\left(q(V_u - D_1) + (1-q)V_{n-1}\right) = \\
&= -c + \frac{1}{\mathrm{R}_e}\left(q(V_u - D_1) + (1-q)\left(-c + \frac{1}{\mathrm{R}_e}\left(q(V_u - D_1) + (1-q)\left(-c + \frac{1}{\mathrm{R}_e} E_n(V_1 - D_1)\right)\right)\right)\right)
\end{aligned}
$$

with this iterated substitution procedure, I obtain:

$$
V_{n-t} = -c + \frac{q}{\mathrm{R}_e}(V_u - D_1)\sum_{\tau=0}^{\tau=t} a^\tau + (V_d - D_1)(1-q)a^t \quad with\ a = \frac{(1-q)}{R_e} < 1
$$

$now:$

$$
\begin{aligned}
& V_1 = V_{n-t}\ with\ t = n-1 \\
& V_1^n = -c + \frac{q}{\mathrm{R}_e}(V_u - D_1)\sum_{\tau=0}^{\tau=n-1} a^\tau + (V_d - D_1)(1-q)a^{n-1} \\
& V_c = \underset{\lim n\to\infty}{V_1^n} = -c + \frac{q}{\mathrm{R}_e}(V_u - D_1)\frac{1}{1-a} + 0 = -c + \frac{q}{\mathrm{R}_e}(V_u - D_1)\left(\frac{R_e}{r_e + q}\right)
\end{aligned}
$$

$but: \quad c = (D_1 - V_d),\ then:$

$$
V_c = \left(q(V_u - D_1) - R_e(D_1 - V_d)\right)\left(\frac{1}{r_e + q}\right)
$$

from which we obtain in node $\mathrm{t} = 0$:

$$
S_c^0 = \frac{q(V_u - D_1) + (1-q)V_c}{R_e} = \frac{q(V_u - D_1) + (1-q)\left(\dfrac{q(V_u - D_1) - R_e(D_1 - V_d)}{q + r_e}\right)}{R_e}
$$

$$
S_c^0 = \frac{q(V_u - D_1) + (1-q)(V_d - D_1)}{r_e + q} = \frac{E(V_1) - D_1}{r_e + q} \quad \text{in case CC holds.}
$$

This value is greater to the standard one-shot equity Value:

$$
S_c = \frac{E(V_1) - D_1}{R_e}.
$$

Expected direct value:

To calculate Expected equity value under this optimal strategy in case CC holds, I must first recognize that at every node, Equity will be worth $V_u - D_1$ or $V_d - D_1$ if nature plays "u" or "d" respectively. In the first case, the game will end at that node. In the second case the game will continue until the first case appears and ends then. The different events can be seen in the following sequence:

	$Event_t$	$\Pr obability_t$	$\Pr esent\ Value_t$	$E(\Pr VE_t)$
1	u	$(1-q)^0 q = q$	$E_u R_e^{-1}$	$qE_u R_e^{-1}$
2	du	$(1-q)^1 q$	$E_u R_e^{-2} + E_d R_e^{-1}$	$(1-q)^1 q(E_u R_e^{-2} + E_d R_e^{-1})$
3	ddu	$(1-q)^2 q$	$E_u R_e^{-3} + E_d R_e^{-1} + E_d R_e^{-2}$	$(1-q)^2 q(E_u R_e^{-3} + E_d R_e^{-1} + E_d R_e^{-2})$
.				
n	$\underbrace{dddd..ddd}_{n-1\ times}u$	$(1-q)^{n-1} q$	$E_u R_e^{-n} + E_d \sum_{t=1}^{t=n-1} R_e^{-t}$	$(1-q)^{n-1} q\left(E_u R_e^{-n} + E_d \sum_{t=1}^{t=n-1} R_e^{-t} \right)$
		$\sum_{t=1}^{t=\infty}(1-q)^t q = 1$		$\sum_{t=1}^{t=\infty} E(\Pr VE_t) = Exp.Equity\ Value = S_0$

From above, I have:

$$S_0 = \lim_{n\to\infty} \sum_{j=1}^{j=n}(1-q)^{j-1} q\left(E_u R_e^{-j} + E_d \sum_{\tau=1}^{\tau=j-1} R_e^{-\tau} \right)$$

Replacing $E_u = V_u - D_1 \quad and \quad E_d = V_d - D_1$ solving and simplifying, I obtain:

$$S_0 = \frac{q\cdot\left(V_u - D_1\right)}{r_e + q} + \frac{(1-q)\cdot\left(V_d - D_1\right)}{r_e + q} = \frac{E(V_1) - D_1}{r_e + q}$$

in case CC holds, and as before, $S_0^c = \dfrac{q(V_u - D_1)}{R_e}$ in case CC doesn't hold (same as one stage LL case).

Conservation of value implies:

$$V_0 = S_0 + B_0 = \frac{q(V_u - D_1)}{r_e + q} + \frac{(1-q)(V_d - D_1)}{r_e + q} + B_0 = \frac{E(V_1) - (1+r_b)\cdot D_0}{r_e + q} + D_0 = \frac{E(V_1)}{1 + r_a}$$ [28]

So, in case CC doesn't hold,

$$V_0 = S_0 + B_0 = \frac{q(V_u - D_1)}{R_e} + \frac{\left(qD_1 + (1-q)V_d\right)}{R_b},$$

(same as one stage limited liability case).

---

28 This identity provides a consistent way to calculate rates of return for assets, debt and equity

Finally, the value of the option to go UL is calculated as follows:

If CC holds we have $S_0^{cch} = \dfrac{E(V_1) - D_1}{r_e + q}$

If CC does not hold we have $S_0^{ccnh} = \dfrac{q(V_u - D_1)}{R_e}$

The value of the option to go UL is then

$$\frac{E(V_1) - D_1}{r_e + q} - \frac{q(V_u - D_1)}{R_e} = \frac{qV_u + (1-q)V_d - D_1}{r_e + q} - \frac{qV_u - qD_1}{R_e}$$

$$= \frac{\left(qV_u + (1-q)V_d - D_1\right)R_e - (r_e + q)\left(qV_u - qD\right)_1}{(r_e + q)R_e}$$

If we simplify making $r_e \approx 0$ the UL option is worth and $q^2 \approx 0$

$$\frac{\left(qV_u + (1-q)V_d - D_1\right) - q\left(qV_u - qD_1\right)}{q} \approx \frac{\left(qV_u + (1-q)V_d - D_1\right)}{q} = \frac{E(V_1) - D_1}{q}$$

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Chapter 3. Holding company discount in the Chilean market, 1993-2007: some facts and statistical analysis

Abstract

Holding companies in Chile are traded at significant discounts compared to their net asset value. This phenomenon is known in financial markets as the holding company discount. A considerable number of explanations has been quoted to clarify it, but the debate about holding company discount is far from being solved. In Chile, investments banks recommend the purchase of the holding company stock or the underlying shares from the study of the difference between the current level of discount and its historic mean, arguing that there is some space to obtain significant excess returns. This could only be possible if discount time series are in fact stationary and mean reverting, in other words, the discounts are predictable. By performing several statistic tests, I explore the time series behavior of the discounts, check for predictability and explore for what can be understood as inefficient market due to behavioral finance issues among other causes. In the empirical section, several hypotheses are developed, relating the non-predictability to a random walk behavior of the discounts, and mixed results are obtained.

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Section 1. Introduction and hypotheses formulation[29]

The interest for the holding discount itself has been present since the second half of the 1990s, mainly in the US, until the end of the first decade of 21st. century in the existing literature. Notwithstanding during all these years, ample literature has been developed, most of it empirical, testing for the presence of several causes invoqued to explain the holding discount and the somehow related Diversification discount (poor management, agency problems, ownership concentration, investor sentiments, wealth expropriation), in many countries including less developed countries like Chile. Recently the phenomenon has gained interest in the investor literature (TSI whealth Daily Advise, 2019).

In some developed countries, company property is normally spread among many shareholders. It is not common to find one dominant shareholder with a significantly higher shareholding than the others. This kind of company was analyzed by Berle and Means. In 1932, they studied large companies in the U.S., and found that their equity capital was divided among many shareholders, with each possessing a shareholding too small to control the decisions of company management. The literature maintains that this kind of company is found in countries where the legal protection for shareholders is well established, forwarding their interests and protecting them from eventual expropriation.

Meanwhile, in countries where the legal protection is not well established, ownership tends to be concentrated among few shareholders. In this case, large companies form business groups, face the imperfections of local financial markets, and exploit potential financial and operational synergies, developing the so called "internal capital markets". In Chile, there are several holding companies linking groups of companies in different industrial sectors. Several holding companies have a parent company that

29 This paper has built in part upon an unpublished (2009) Applied Master´s Thesis of Ignacio Galvez from Pontificia Universidad Católica de Chile Business School, in which I acted as assistant director of the thesis, with director Fernando Lefort. I had a main participation in the construction of the data base, the selection of relevant literature, and the selection of pertinent statistical tests; I have changed the focus of the work from designing trading strategies to take advantage of market inefficiency, to determining properly the degree of market inefficiency for which I have included a complete, new section which search for investor sentiment (behavioral finance explanation for market inefficiencies). I have performed all the tests again as well as new tests, for three frequencies of data, stating and testing specific hypotheses, and obtaining our own conclusions. Descriptive sections and a revised literature review in Galvez (2009) are presented with some improvements and more focused on the hypotheses of the work.

is mainly an investment vehicle. Their function is simply to acquire shares in their subsidiaries and hold them in order to exercise control. The interest of this work is focused where the holding company or its subsidiaries are publicly traded firms. In this case, the market capitalization of the parent company tends to be lower than the sum of the share values of the subsidiaries it holds corrected for financial debt in the individual parent company. This can be summarized as the net asset value. This phenomenon is known as holding company discount. It is observed that in Chile, the discount levels do not tend to be constant but rather display major fluctuations of up to forty or fifty percent of the corresponding net asset value. Investment banks, main actors in the security markets, make trade recommendations on discount movements in the short term. The recommendation is generally to buy shares of the parent company and sell shares of the subsidiary when the discount is above the average of the last two or three years. On the other hand, when the discount is below the average, the subsidiary's shares are recommended. The main assumption of this analysis is that the discount levels although volatile, should return in a relative short period to their average levels or even to their long-term level. The assumption behind this is that the time series describing discounts are stationary around a fixed mean or long run trend. How appropriate this assumption is, can be detected from an analysis of the results.

There are several hypotheses on the existence of holding discounts. One of them, the hypothesis of agency problems posits that the controlling company shareholder could obtain pecuniary and non-pecuniary benefits at the expense of the other (minority) shareholders. These are called Private Benefits of Control (Riyanto, and Toolsema, (2008). It could be by drawing gains from the company or embarking on projects that are detrimental to the interests of the minority shareholders, being itself the least affected party in the event of poor results. Thus, the possibility arises for expropriation of value from minority shareholders. holding company discount seems to contradict this traditional hypothesis. If the controlling company shareholder of a subsidiary is a holding company operating only as an investment vehicle, it would be expected that the minority shareholders of the subsidiary would be more likely to be expropriated than the minority shareholders of the holding company. This should lead to a premium in the shares of the parent company rather than a discount.

However, the empirical evidence shows that discounts are most recurrent. Some explanations have been devised but the literature is not abundant on holding company discount. There are studies on discount by diversification in U.S. conglomerates, but there is no clear explanation on discount in groups with a simple structure, where there is no diversification of activities. Other related hypotheses arise when comparing "holding discounts" of closed end mutual funds with those of closed end mutual funds. Fortunately, there is sufficient research this phenomenon in the U.S.: discount in the "closed-end mutual funds". Closed-end mutual funds issue shares that are traded on the Stock Exchange and invest in a portfolio of shares just like open-end-funds, but these shares cannot be redeemed in advance as the shares of open-end funds are. An investor with shares in the close end fund must find a buyer to whom he/she can transfer all the shares he/she holds, instead of redeeming his/her part directly with the fund at the respective quota value. While an open-end mutual fund has to buy or sell its own shares at the net asset value, closed-end funds are traded at market prices that may at times differ significantly from their net asset value. The literature explores this phenomenon and helps to explain the persistence of relatively high discount over time, although volatile, in most funds.

The objective of this study is to describe the behavior and characteristics of the discount of various holding companies in the Chilean market, focusing on simple structures (ideally 1 traded parent company and one traded subsidiary). There are no previous studies of a holding discount in Chile in the recent literature. Several studies can be found of company valuation, or dividend policy, or corporate governance effects in value, controlling for company's affiliation to business groups, and calculating the market value of non-listed companies, hence the net asset value, by means of imputing a value based on different multiples of comparable listed companies, obtaining an approximation to diversification discounts. Two interesting studies are: Jara-Bertin et al. (2015) and Espinosa et al. (2018), in which they explore the effects of diversification on valuation, using imputed values[30] as explained before. In this study the holding discount are calculated directly using market data, for simple

30 This method is pervasive in the literature

groups in which the value of non-listed affiliated companies represents less than 5% on average, thus avoiding the imputation problem.

First, holding discounts for a selected group of simple business group (selected from the list of 25 largest groups in Chile, in Colpan et al. (2010) chapter 14, business groups in Chile) are calculated in this direct way. With these discounts already calculated, the next step is to study their evolution in the sample period and make a statistical analysis. In recent decades, some legislative efforts have been to increase minority shareholder protection. An example is the Stock Public Offering Law of 2000 that aimed to reduce the concentration of ownership, obtain a greater protection for minority shareholders and the supply of better-quality information and enhance market transparency. It would be interesting to analyze if there was some effect on discounts, particularly a generalized reduction for all or some of the groups because of the above-mentioned law. A visual inspection of Figures 20 and 21 in section 4.5, does not show a significantly different behavior of the discounts since 2001 (middle of the time range), compared with the previous years. The agency problems could also play a significant role in the existence of holding discounts. In a capital market with relatively low investor protection like Chile, the response has been a higher concentration of property through majority and controlling company shareholders. Partial control is achieved with the corresponding vertical agency problem, but a probability arises for the expropriation of minority shareholders value, creating a horizontal agency problem.

1.1. Hypothesis Formulation

With all background developed in this section, I formulate two main hypotheses:

a) Regarding the existence of holding discounts even without diversification effects,

Hypothesis H1 Null: there is not holding discount in at least some Chilean business groups.

b) Regarding the predictability of holding discounts, provided Hypothesis H1 is rejected:

Hypothesis 2 Null: holding discounts are not predictable.

- Hypothesis H2a. Null : holding discounts present unit roots, an thus are non-stationary series, and behave as random walks (non predictability).
- Hypothesis H2b. Null : holding discounts do not present reversion to the mean, and thus are a Martingale (no predictability conditioned to all available information)
- Hypothesis H2c.Null : Holding price value and net asset value are cointegrated, thus present a long term stable linear relationship (in the long run given one, the other can be estimated with a reasonable confidence level).
- Hypothesis H2d. Null : Chilean market is not affected by investor sentiment regarding the discounts. This implies a medium to high level of market efficiency, thus they are not correlated between them and behave as random walks as Efficient Market Hypothesis predicts (thus, discounts are non-predictable).

The study is organized as follows:

In Section 2, Related Literature, I present the related literature focusing on closed-end funds and the explanations provided on the discount. However, it should be indicated that the case of closed-end funds is not identical to discount in the business groups of Chile, although it is what most resembles it the financial literature studied. Therefore, I include this literature for reference but not for explanatory purposes for the study, and I explore several explanations found in the literature for the holding discount. There are not many studies on holding company discount.

In Section 3, discounts in Chilean main conglomerates, the discounts of in 9 Chilean business groups are presented, explaining the Data Base Construction methodology, showing the results and analyzing the discount for the selected Chilean conglomerates or groups, and ending with the exposition of some facts analysis of some legal and financial issues.

In Section 4, Statistical Analysis, I propose a theory for the time behavior of discounts, and propose a general hypothesis, hypothesis H2, and four sub hypotheses, H2a, H2b, H2c, H2d, preceding 4 different but complementary tests I perform in this section, intending to understand whether the discounts evolve randomly, what it means are unpredictable: Unit root Analysis (tests for no stationarity), Variance Ratio Analysis (tests for mean reversion), Cointegration Analysis (tests for long run stable relationship between holding value and NAV), and Comovement Analysis (tests for the existence of inefficiency in the security market by searching for investor sentiment in the Chilean stock market related to the holding discounts).

In Section 5, I present conclusions, and in section 6, I explore some avenues for further research.

Section 2. Related Literature

2.1. On the holding company discount and its possible explanations

holding company discount is normally defined as the percentage difference between the stock market capitalization of the parent company[31]. And the net asset value (NAV) of a holding company (in the case of a holding company where the parent company is only an investment vehicle, this would be the sum of its investments in the shares of its subsidiaries).

One of the explanations extensively used for holding discount has been the diversification of the business groups. When a holding company participates in a specific sector and decides to take control of other companies in a different sector, I refer to a holding company that diversifies its activities. Supposedly it could be a sector in which it can exploit some advantages it possesses in its original sector, although in various cases the holding company enters completely different areas.

A common view is that diversified holding companies are not attractive to investors. The main arguments against the existence of holding companies are the following: Diversification does not offer any advantage to investors because they can diversify on their own. at a much lower cost, the structure of holding companies is inefficient and does not focus on what matters; the performance of the less profitable activities engaged in, hinder the performance of the more profitable activities. The market interprets this diversification negatively, as something that distracts a holding

31 holding discount can also be defined as a monetary difference. Our approach here is to define holding discount as a percentual difference

company from its main activity. This argument indicates that the holding company is misusing its available cash by investing in developing other businesses, which would lead the market to punish the stock price of the holding company.

Campa and Kedia (2002) analyze the diversification penalty, (being one of the first to analyze empirically the diversification effect on value (Berger, and Ofek, 1995), and emphasize that the discount is due to specific characteristics of the holding company more than to diversification itself. There is insufficient empirical evidence to state that diversification destroys value, but there is a strong negative correlation between company value and the decision to diversify. This correlation comes from specific factors of these companies that affect their decision to diversify and their market value. Firms that operate inefficiently compared to other companies have lower opportunity costs when targeting resources at other activities, and therefore seek to diversify. Campa and Kedia (2002) claim that the discount of a diversified company is already explained by its inefficiency prior to the diversification process.

Campa and Kedia (2002), also argue that the existence of agency costs between managers and shareholders may encourage the search for value destroying diversification. Managers with the greatest private returns are most inclined towards diversification activities that may lead to conflicts of interest with shareholders. However, much of the discount is explained by the prior to diversification agency costs. If this is not considered, the destruction of value through diversification is being overestimated. The authors conclude that it is necessary to control for particular aspects of the holding company and not attribute the entire discount to diversification.

Chandler (1977) on the contrary, argues that diversification offers more benefits than costs when economies of scale in administration can be exploited. Lewellen (1971) indicates that the debt capacity increases with diversification. One of the main benefits would be the creation of internal capital markets, as in Stulz (1990) and Stein (1997). Through this, the surplus of one company can alleviate the financial deficit of another. Additionally, there are companies with unique management capacities that they want to exploit, seeking diversification opportunities. This process is long and expensive.

In addition, in many cases, conglomerate managers think that they are the best equipped to make a company grow. This is why they sometimes hold on to some companies in order to give them the best growth prospects, when in fact they should be looking for potential buyers who are better able to achieve that growth.

Lins and Servaes (2002) compare the value of diversified and undiversified companies in emerging countries. studied over 1000 companies in 7 emerging countries in 1995. In these markets, there are greater market imperfections such as asymmetric information and limited access to capital markets for example than in developed countries. The authors indicate that diversification could be beneficial for the development of an internal capital market. However, in diversified companies the minority shareholders can be expropriated more easily, producing a lower company valuation.

Specifically, in the Chilean case I can hypothesize that diversification does not exclusively explain the discount. This is because of the existence of "pure" holding companies, with one controlling company parent company and a single subsidiary, as the holding structures analyzed in this work. There is no diversification in these cases, but there is a significant discount as it is shown in the corresponding sections. But there are more possible explanation for the holding discount.

Rommens, Deloof and Jegers (2003), indicate several other possible explanations for the holding discount 1) If the costs of the holding company exceed its benefits then it destroys value; 2) The net asset value of the holding company may be overestimated as an estimate of the value of the holding company. This may occur because the NAV does not consider the lack of liquidity of some assets; 3) Noise traders (or irrational traders according to the classification used in the literature) could invest much more in the shares of the parent company than in its subsidiaries, distancing its value from its fundamentals and leading to an underpricing of the parent company, and 4) The discount may be due to possible extraction of private benefits of control. Regarding the fourth reason, according to the authors, the pyramidal structures observed in some countries allow the owner of the companies at the top to keep control of the other companies with only a small right over the cashflows. If investors expect possible conflicts due to the attitude of the controlling company, they would pay less for the parent company stock.

In Rommens, Deloof and Jegers (2003), holding company discount is compared to the discount of closed-end mutual funds, a good proxy to a holding structure, to be analyzed below. There is a difference between both: holding companies aim to control the companies they invest in while closed-end funds aim to get returns buying or selling the assets they hold. When explaining the discount, the authors make a difference between divergent and non-divergent factors. The former is those that affect the value of the parent company without producing changes in the subsidiaries or having a different effect.

Rommens, Deloof and Jegers (2003) indicate that in contrast to closed-end funds, the holding company constantly monitors the subsidiaries in which it has shareholdings. It can create value for the shareholders of the parent company and the subsidiaries. However, the monetary compensation of the holding company management is an additional cost, greater than that of closed-end funds, since the latter do not exhaustively monitor their companies. If these costs were excessive, then it would explain part of the discount. The second point applies in conglomerates with investments in some closed subsidiaries. However, this is not the case in some of the purest conglomerates of this work, where there is only one parent company and a subsidiary, both listed on stock exchanges. If I consider that the parent company has shares of subsidiaries that are not sufficiently traded, this lack of liquidity should be captured in the share price. Therefore, the market should correct its price and would no longer be overestimating the value of the assets. The authors mention that different kinds of investors could be investing in the shares of the parent company and of the subsidiaries.

Kaye and Yuwono (2003) defend the existence of holding company structures and claim that they are far from disappearing from the scene. Kaye and Yuwono analyze many conglomerates globally and find that those that avoid practices that sacrifice value in favor of growth, best succeed in maintaining an adequate level of value creation for shareholders.

According to Kaye and Yuwono (2003), the best results are obtained by groups that focus more on the financial synergies and on the development of an internal capital market than on operational or strategic synergies. In a study of 88 public companies with a market value of over US$ 500 million, from several industrialized and developing countries. According to the authors the successful groups are those whose management is concerned with the right financial focus on autonomous businesses. The senior management of these groups is characterized by a business spirit, by designing work teams and monitoring them, as well as being concerned with a good allocation of financial resources between the businesses. In some cases, groups are characterized by having a decentralized management structure. Each unit is managed with sufficient autonomy to get the most out of the business, avoiding an overly hierarchical structure. Another practice of these groups is to offer a share in the equity of the company to the management of each unit, to strengthen their interest in the company's performance and to share the risk. All in all, conglomerates that avoid the typical quest for growth at any cost tend to perform best. The prejudice against conglomerates is justifiable in several cases but should not be applied to all the groups.

2.2. Discount in closed end mutual funds as the best proxy of a holding structure

Several explanations can be put in place for the holding discount, from the analysis of the closed end funds. They are as varied as Investor Sentiment, Taxes, Liquidity, Noise traders, market inefficiencies.

Closed End Funds (mutual funds that cannot be redeemed in advance) discounts are a phenomenon very much investigated in the financial literature. They are of interest in this study, because Closed End Funds are the investment structure that best resembles the holding companies. Good references are Boudreaux (1973) and Brauer (1984). Closed end Funds are similar to "open-end funds" in which they hold shares in national scale stock exchanges. The difference lies in how they are redeemed. Since they issue a fixed number of tradable shares, a person holding a part of a Closed end Funds must find another investor to buy it instead of redeeming it directly with the fund at the corresponding quota value as in the open-ended funds. Interestingly, close end funds , depending on the stage of the fund's life cycle, generally trade in the U.S. at a discount on their net asset value. Moreover, they are rarely leveraged. Though they generally trade at a discount, they can also have premiums over their NAV.

Boudreaux (1973) indicates the explanations available at that time on the discount, although they seem insufficient given the duration over time and the variability of the discount. They can be summarized as follows: 1) Transaction and administrative costs, 2) Effect of portfolio diversification, 3) Irrationality or market inefficiency.
It is claimed that the costs in the first argument are too small compared to the discounts. Brickley et al. (1991) also find that these costs are too small and stable compared to the scale and variability of the discounts. Malkiel (1977) also finds that they are not significant. The inefficiency argument is uncertain given the persistence of the differences of the Net Asset Values (NAV). For a definition of NAV see section 3.

Fund managers typically seek undervalued shares in order to generate excess returns. Boudreaux claims that if the market thinks that the managers cannot achieve it and undertake too many costs trying to do so, there will be a discount.

Boudreaux (1973) includes an additional hypothesis: the variations in this difference are related to the expectations of the market on changes to the original portfolio. Good changes are associated to a premium, and bad changes to a discount. The larger the expected change, the larger the discount or premium. This change should be positively correlated to the absolute value of the difference between the net asset value and the market value of the fund. Its results match this hypothesis, but only included

13 closed-end funds that only invested in shares of companies listed between 1960 and 1970.

Lee et al. (1991) mention some of the classic reasons for the closed end funds discount. They seek to explain the existence of a 4-stage cycle in all closed-end funds. The first is when the funds are issued, upon which they tend to trade at a premium of approximately 10%. After around a hundred days from the beginning of trading, a discount emerges. They much more commonly remain trading at a discount than at a premium. As long as they remain "closed end" mutual funds, the discount is persistent and highly variable, which implies that it is never a constant fraction of the net asset value. Various studies find that the discount in these mutual funds tends to have a reversion to the mean. This offers opportunities to investors that take long positions in shares of funds when the discount attains a fairly high level. Lee, Shleiffer and Thaler (1991) criticize the theory of agency costs as a determinant of discount. As in most studies, they consider these costs to be too small. Malkiel (1977) claims that some funds target much of their resources at investment in assets with restricted sales. In this case, the fund acquires these assets with the commitment to hold them for a period of time. Once this time period elapses, it can resell the assets. Given its illiquidity, its market value is not a good indicator of possible sale value. Malkiel (1977) concludes that the higher the proportion of these assets out of the total fund portfolio, the higher the discount at which the fund will trade. However, the theory of lack of liquidity is promptly rejected by Lee et al. (1991) since most closed-end funds mostly invest in liquid assets. If they had other kinds of assets, they would not be a significant proportion of the total fund portfolio.

Brickley et al. (1991) link the discount of "closed-end mutual funds" with the taxes payable by the investor. They claim that part of the discount may be due to the timing of this payment. The argument is as follows: when an investor acquires "closed-end fund" shares, the investor acquires an option on the payment of their taxes (tax timing option), but not on the underlying assets. U.S. Tax laws do not allow the transfer of tax benefits to investors, which reduces the value of the tax-timing option compared to a direct holding of shares in the fund portfolio. They also argue that the fund share price is much more stable than that of the underlying shares, since the fund is more diversified. Given that the value of an option increases with the volatility of the underlying asset, the value of the tax-timing option of the fund is lower than that on the assets inside the fund. This view is also defended by Schill and Zhou (2001) who attribute part of the holding company discount to the lower value of the tax-timing option. The existence of the discount could the result of a bad estimate of the true net asset value of the fund. However, Brickley et al. (1991) analyze this view. They find that for 16 closed-end funds, the market value rose after the fund opening, liquidation or merger. This would indicate that discounts are not the result of calculation errors: if that were the case, there would not be an increase in the price after a restructuring of the fund.

Investors have certain possibilities when it comes to setting off some losses or gains in terms of their tax obligations. This phenomenon is analyzed by Malkiel (1977) and is also mentioned in Brickley et al. (1991). When an investor acquires part of a fund with unrealizes losses, it may have a tax benefit that would justify the existence of a premium in the fund's shares. On the contrary, when a fund has some unrealized gains, the buyer is acquiring a tax liability that will have to pay at some point of time, which could lead the fund to trade at a discount. Some papers that offer evidence against tax theory are Brauer (1984) and Brickley and Schallheim (1985). They show that once it stops being "closed-end" (process of open-ending) the price of the funds increases to the NAV, instead of the NAV dropping to the value of the fund. Malkiel (1977) finds that the tax liabilities cannot represent more than a 6% discount. In effect, it studies the influence of different variables on observed discount. An implication of taxation theory is that the discount should increase when the market rises. In effect, one should expect that in a bull market, the total of unrealized gains should increase, which would lead to a greater tax liability and a greater discount. However, Lee et al. (1991) do not support this idea, arguing that it goes contrary to evidence found in Brauer (1984) and for the most this effect accounts for less than 6% of the total average discount reported of around 25%.

Lee et al. (1991) indicate that closed-end funds are mainly acquired and held by individual investors. They link discounts with individual investor "sentiment", in other words, expectations with respect to the future performance of shares, differing from the sentiment of investors in underlying shares of the fund. For comparison, they investigate if in the shares of small companies, the main investors are also individual and determine if sentiment affects both instruments equally. In other words, the changes in the discounts should be correlated to the returns of small firms. "Investor sentiment" refers to expectations on the return of assets not guaranteed by their fundamentals. "Sentiment" or its expectations with respect to the market is a risk that reflected in prices. It could be reflected in greater optimism or pessimism with respect to the future of the funds. Clearly this change in the perception of investors has no sure or objective foundations, but since what part of the expectations is reasonable and what part is exaggerated cannot be perfectly determined, it can produce a change in the demand of the funds. Thus, the greater optimism on the performance of the funds would produce a premium or lower discount. According to Lee et al. (1991), investors in closed-end funds are in general, individuals and few, in contrast to investors in fund shares that are generally institutional. Thus, the risk of having an investment in closed-end funds has two components: the future performance and risk that change the perceptions of poorly informed investors or "noise traders". In general, there should be a discount compared to shares within the portfolio that are held by a much larger proportion of investors. A more detailed explanation of "investor sentiment" is found in Section 4.5.

According to Black (1986), there are two kinds of investors: those who are informed and rational, and those who are irrational and poorly informed that trade based on rumors and not on analysis. Both would be necessary in the market to ensure a minimum level of transactions and liquidity, but the problem is that the noise distances the stock price from its fundamentals. Black (1986) agrees with the findings of Lee et al. (1991) on the clientele of closed-end funds: those who invest in the shares of these funds or of the parent companies of holding companies are mainly individuals. According to Black, they would be the investors that use noise a lot given that they do not have access to the same information as institutional investors, who would be the rational informed investors.

2.3 Holding discount, Debt and Dividends, in a Modigliani and Miller's world with taxes

More than three decades ago Miller (1988) published on the famous propositions of Franco Modigliani and Merton Miller thirty years after: Modigliani and Miller (1958), (1963). The propositions shed an interesting light on this study. According to their first proposition, in a world with no taxes, no transaction costs and no possibilities of arbitrage, the value of a leveraged company should be the same as that of a company that operates with 100% of equity. Once some assumptions are made more flexible, and the existence of taxes is allowed, then the financing structure became relevant to the value of the firm. Thanks to the possibility of reducing taxes through debt interest servicing, the company acquired a tax shield that allowed it to increase its value above that of a non-leveraged company by the same quantity as the tax shield.

Modigliani and Miller concluded that in a world with taxes, with non-risky debt, the optimal financing structure is 100% debt in order to maximize the tax shield.

Now, if the assumption is made more flexible and risky debt is used, then there is a certain level of optimal debt in which company value can be maximized. Beyond that point, the cost of capital starts to increase which leads to a decreasing firm value.

In Chilean conglomerates, there are many cases where the debt level is a significant part of the balance sheet of the holding company. The tax structure of Chile during the time span of the study, is in theory neutral given that the corporate taxes are credited at the personal level. The effect of this structure is that increasing the debt level should not produce any impact on the firm value. However, depending on the nature of the investor, the existence of debt in the parent company could add value given the scope for a tax shield. In practice, something similar to the theory of Modigliani-Miller with taxes could occur, in which the investors would not remain indifferent to the leveraging of the parent company. An increase in value would lead to greater flows for shareholders and therefore these could be willing to pay a premium to acquire part of the property of the parent company. This could occur among some institutional investors that in practice would not be completely indifferent to the higher debt level in the parent company. Focusing on the technique for calculating the discount shown below, it can be observed that the financial debt level of the parent company can have a significant influence. Instead of working with discounts, quotients could be used as well. The discount is then defined as 1-Quotient: $1-C$, so the quotient instead of the discount can be analyzed. It would be like looking at the other side of the same coin.

$$C = \left(P_m \cdot Q_m \right) / \left(\left(\sum P_i \cdot Q_i \right) - DF \right)$$[32]

Where $C =$ *Quotient*,

$P_m =$ *Stock price of the holding company*

$Q_m =$ *Quantity of shares of the holding company*

$P_i =$ *Stock price of subsidiary i*

$Q_i =$ *Quantity of shares of subsidiary i held by the parent company*

$DF =$ *Quantity of financial debt of the holding company.*

Supposedly, It could be said that increasing debt should be indifferent for investors. A higher debt level would lower the denominator of this quotient, but it would be pertinent to assume that the company would be riskier or could face problems paying its debts or dividends. This should lead to a reduction in the stock price of the parent company. If the increase in financial debt leads to a reduction in the price of the parent company such that the quotient (or percentage that represents the value of the parent company out of the total net asset value) is invariable, there should be no effect on the holding company discount.

This conclusion has been verified by us by when constructing financial models of a holding company and one subsidiary, in which the subsidiary as well as the parent

32 For a detailed description of rhe holding discount uses in this work, refer to Section 3.

company were modelled in a standard way (unlimited responsibility), and also considering the equity of a company as an option the shareholders have to purchase the Firm (assets) to the debtholders (limited liability), producing an option on an option, i.e. a compound option. These models and the results are not shown in this work.

Recalling the conclusions of Miller and Modigliani (1961), dividend policies should not have any influence on the wealth level of the shareholders. Supposedly, investment decisions are given, so increased dividends should lead to lower capital earnings and retained earnings, being irrelevant to total wealth. However, signaling theory indicates that an increase in dividend payments is a sign of greater future returns. This would lead to an increase in the share price of companies that start to pay higher dividends. Lately, dividend policies have generated great interest analyzed together with agency problems between controlling company shareholders of companies and minority shareholders. Horizontal agency problems become important when deciding on investing in shares. La Porta et al. (2000) explain the theory of investor preferences to receive more dividends since if the controlling company shareholders prefer to keep profits in the company, it is highly likely that they will use those funds in investments that destroy value or for their own personal gain.

It is necessary to determine what is understood by "insiders" in a company, since its meaning varies between countries. For example, in countries where property is less concentrated (as in the Berle and Means company), the main "insiders" would be management, while in countries where property is concentrated among a few shareholders, they would in turn be the "insiders". In any case, the affected party would be the minority shareholders. In countries where the legal protection for minority shareholders is limited, dividends would be a way to offer a protection from controlling company shareholders.

According to the Chilean Laws, all public joint-stock companies must distribute at least 30% of their profits as dividends in each fiscal year unless agreed by the unanimity of the shareholders. There are cases in which parent companies receive higher dividends from their subsidiaries than they are legally enforced to, which gives them some degree of flexibility in the management of these cash flows. The dividend policies of the parent company can influence the discount. If the parent company only distributes the minimum of 30%, while it receives much more than 30% from its subsidiaries, investing in the parent company could be less attractive, and hence command a discount. This is an interesting matter, but it is out of the scope of this work.

Section 3. Discounts in the Chilean selected Business Groups

In this section a direct procedure to calculate the ratio of Holding value to net asset values, or NAVs (holding discount if less than one, or premium if greater than one) and the percentual difference as well, is presented for nine Chilean selected groups, and find evidence of strong discounts in all the nine groups as explained below using daily, weekly, and monthly data. This occurs for all the time frame used, except for two groups that for a short period of time present holding premium (negative discounts). This can be clearly seen in appendix 3 to this chapter.

Hypothesis H1 is strongly rejected as seen below.

3.1. Database Construction Methodology and calculation of the discount

The groups charts are shown in Appendixes 1 and 2, and the main statistics for the selected groups as explained below, are shown in Appendix 3. The methodology of the study can be summarized as follows:

Firstly, groups in which 95% or more of the value of the investments in related companies are investments in public companies are selected. This percentage is calculated with the detail of Investments in related companies at December 31, 2007 and is reviewed for the previous quarters. If in some period, this percentage drops below this level, it would not be pertinent to calculate the discount. This occurs in the case of the Inversiones Pampa Calichera company from the year 2000 and back. This study does not attempt to value nonpublic companies since having market prices for parent companies and subsidiaries is what seems most appropriate. The percentage of public companies owned by each selected group is shown in Appendix 2.

However, there are some cases in which the parent company controls public companies through nonpublic companies. These companies are generally investment companies that have a shareholding in public companies (as in the case of the Luksic group for example, in which Quiñenco controls companies such as CCU and Banco de Chile through the companies Inversiones y Rentas S.A. and LQ Inversiones Financieras S.A. respectively). I focus on cases where the shareholding of the parent company is direct to the subsidiary.

Cases in which the controlling company is only an investment vehicle and has no operations itself are included. Daily, weekly and monthly discounts based on prices quotes are calculated, and for balance sheet data public quarterly information from the financial statements of the companies (FECU, official standardized format for financial statements in Chile) is used.

3.1.1. Decomposition of the quotient

<u>a) Alternative nº1:</u>

Once the groups are chosen with the data of 12/2007, the corresponding quarterly data of the 4 quarterly FECU per year of each year are used. To calculate the discount, information is needed on the market capitalization of the parent company and the net asset value. The latter is the sum of the investments in public joint-stock companies less the long run financial debt of the parent company. The quantity of shares of the subsidiaries being held by the parent company and their market price are collected. The financial debt of the parent company for each quarter is also needed. This value is subtracted from the total investment and the result is the denominator of the quotient. The numerator corresponds to the market capitalization of the parent company's equity. To obtain this the quantity of shares issued by the parent company and their market prices is needed too.

Thus, the quotient is as follows:

$$C = \left(P_m \cdot Q_m \right) \; / \; \left(\left(\Sigma\, P_i \cdot Q_i \right) - DF \right)$$

$$\begin{aligned}
&\textit{Where } C = \textit{Quotient},\\
&\quad P_m = \textit{Stock price of the holding company}\\
&\quad Q_m = \textit{Quantity of shares of the holding company}\\
&\quad P_i = \textit{Stock price of subsidiary } i\\
&\quad Q_i = \textit{Quantity of shares of subsidiary } i \textit{ held by the parent company}\\
&\quad DF = \textit{Quantity of financial debt of the holding company.}
\end{aligned}$$

The items included in financial debt are the following: Long term financial liabilities include Liabilities with Banks and Financial Institutions, Liabilities with bond holders, various long-term creditors, and other long-term liabilities (FECUs indicate that liabilities should be classified according to their degree of exigibility. Sometimes, when a subsidiary of the parent company has negative results, the negative Proportional Value (VPP) is entered into this account.

Short term liabilities that are part of the financial debt such are liabilities with banks and financial institutions, short term portion of a long-term liability, short term portion of bonds, in general, part of long-term liabilities that are due within a year, and notes.

<u>b) Alternative nº2:</u>

Alternatively, an estimate of the equity value can be made in another way. The idea of the quotient is to put in the numerator the equity value calculated through the product between the number of shares of the parent company and their price. In the denominator, the estimated equity value is included using the value of the investment in subsidiaries at market prices. Decomposing the accounting basic identity, an alternative expression of the equity value is obtained:

$$\textit{Assets} - \textit{Liabilities} = \textit{Book Equity}$$

Decomposing assets,

$$\textit{Book Equity} = \textit{Other assets} + \textit{IER} + \textit{Higher value} + \textit{Lower value} + \textit{IOS} - \textit{Liabilities}$$

Where: *IER*= Investment in related firms, *IOS*= Investment in other companies, *Higher(lower) value* corresponds to the deficit (excess) of the registered value of an investment compared with its acquisition value. Accounting principles allow to amortize these values for a certain number of years.

In the case of the Inversiones Tricahue group, the shareholding of the parent company in the Eléctrica Pehuenche Company is below 5% of total shares of the company, so its proportional equity value falls within Investment in Other companies. Then,

$$Other\ assets\ -\ Liabilities\ = BookEquity - \ IER\ -\ Higher\ value\ -\ Lower\ value - IOS$$

Then, the equity value can be expressed as:

$$Economic\ Value\ of\ Equity = \ Other\ assets\ +\ Economic\ value\ of\ the\ investment\ -\ Liabilities$$

From the previous equation,

$$Other\ assets\ -\ Liabilities\ = BookEquity - \ IER\ -\ MayV. - MenV.\ -\ IOS$$

In Chilean standards, Higher (lower) values are called Mayor (menor) valor. abbreviating them as MayV and MinV respectively.

Therefore,

$$\begin{aligned}&Economic\ Equity\ Value = \ Economic\ value\ of\ investment\ in\ subsidiaries\\&+ BookEquity - IER - May.V - MenV. - IOS.\end{aligned}$$

In other words,

$$Economic\ Equity\ = EVIS\ +\ Book\ equity\ -\ IER\ -\ May\ V.\ -\ Men\ V.\ -\ IOS$$

All the various liabilities and assets of the representative accounts of investments in shares of other companies, are included to calculate the quotient. In effect, the latter equation will be the denominator of the quotient. In the numerator, is the market value of equity, expressed as the product between the number of shares of the parent company and its market price. In other words,

$$C = \left(Pm * Qm \right) / \left(\left(\sum Pi * Qi \right) + Book\ equity - IER - MayV - Men.V - IOS \right)$$

When working with discounts, the adjustment D= (1-C) must be done with the previous calculation.

In the following sections the Main characteristics and methodological issues related to the proprietary data base are described.

3.1.2. Data Base Characteristics

Database is constructed from 01/01/1993 to 31/12/2007, and for descriptive statistics to 30/6/2009. The actual time period covered for the 9 selected groups is shown in Appendix 4. There are cases in which the parent companies went public after 31/03/1991. In those cases, the starting date of the data is the publication date of the first FECU (public financial statements) in which the company was established as a public joint-stock company. For example, the quantity of data included in the Iam-Aguas Andinas group is far lower than the rest given that the first day stock trading of Iam was November 18, 2005. As explained before, groups whose investments in public companies drop below 95% of total investments at some moment in time are excluded.

The “purest” or simplest cases are those in which the parent company is an investment company and controls a single subsidiary. There are various cases: Campos-Iansa; Tricahue-Pehuenche, Inversiones Aguas Metropolitanas-Aguas Andinas, Pacífico V Region – Pucobre, Oro Blanco-Pampa Calichera-SQM.

The aim is to obtain discounts for each working day from the databases. Consideration must be done to the fact that public information of financial statements are delays with respect to stock market information.

The market prices recorded in the databases were obtained through the Santiago Stock Exchange. They refer to the trading of shares without adjustment for rights or inflation. In fact, they precisely match the prices given by the Economática Data Base without adjusting for rights.

To obtain daily discount data and given that shares are generally not traded on all working days of the year, the closing prices of the last trading day is used if they did not trade in a particular day.

For daily discounts, some assumptions are made, for example much of the data correspond to fictional prices, given that there was no trading then. It is assumed that the price is maintained constant during those days, but the calculated discount is not real. To solve this problem, weekly discounts are calculated with the closing prices of all Fridays. An analogous procedure is followed with monthly data, giving less frequent but more reliable data.

All the stock prices and debt values are entered at an exact date, without correcting data for inflation. All data are expressed in nominal terms.

3.2. Discount in the selected Chilean Business Groups

Chilean business groups are mostly organized as holding companies, sometimes pyramidal, where the top tier corresponds to the holding company. In most cases, this level corresponds to the investment company, with no business activity other than managing the investment in its subsidiaries. The cases of interest are those in which both the investment company and its subsidiary(s) are listed. This provides an estimate of the value of the parent company's assets. By subtracting its financial debt, an estimate of the equity value is obtained. By comparing this value to the stock market capitalization of the parent company, the holding company discount or premium (less frequent, but existent in some periods) can be calculated.

Some business groups have a fairly simple structure: just a holding company and a subsidiary (whose shares are traded in the domestic market). The analysis centers in this kind of structure. It is the most interesting because it allows to see if there is a discount even when there is no diversification of assets of the parent company.

3.2.1. Brief description of the groups

The structure of the selected Chilean business groups is presented in Appendix 1. The percentages and number of shares indicated in the subsidiary's blocks correspond to the shareholding of the holding company in the subsidiary. In addition, the percentage representing the investment in shares out of total assets in the balance sheet of investment companies is presented in Appendix 2.

Group 1: Tricahue Investment Company in shares in the Pehuenche Electric Power generation firm is its main business. In effect, Tricahue holds nearly 3% of the shares of that firm. By December 31, 2007, 84% of Tricahue's assets corresponds to its shares in Pehuenche. The rest corresponds to current assets, specifically several debtors. Inversiones Tricahue is not the controlling company of Pehuenche (controller is Empresa Nacional de Electricidad S.A. with 92.65% of total shares).

Group 2: Another group with a simple "pure" structure is the Campos Chilenos S.A. Investment company. It holds shares in Iansa. By December 2007, the investment in related companies was 84% of Campos total Assets. The rest corresponds almost exclusively to current assets. Campos is a public joint-stock company controlled indirectly by the British company EDandF Man Holdings Limited, which is also the indirect controller of Iansa with a shareholding of 28.01%. EDandF Man Holdings Limited controls Campos through subsidiaries like the Dutch company EDandF Man Holdings B and Sofpac BV. It indirectly holds 100% of the capital of these firms, giving a consolidated indirect shareholding of 28.01% of Iansa. The parent company of this group, EDandF Man Holdings Limited does not have a controlling shareholder.

Group 3: Inversiones Aguas Metropolitanas is another interesting case. At 31 December 2007, practically 100% of its assets was its investment in Aguas Andinas, including the excess value paid in the acquisition. By December 31, 2008, its main shareholders were Inversiones Aguas Metropolitanas (50.1%), Corporación de Fomento de la Producción, CORFO, a Chilean State Company (35.0%) and Pension

Fund Managing Companies, AFPs (10.1%). The company is part of group Aguas, a set companies related to the sanitary industry.

Group 4: Pacífico V Region is an investment company that mainly holds shares of Pucobre. 63% of its total assets corresponds to that investment. The rest corresponds to current assets that are mainly Accounts Receivable from related firms. Excluding this account from total assets, the IER is very close to the total assets of Pacífico. The quantity of shares of Pucobre held by Pacífico is by 31/12/2007, represented 83.6% of the property of Pucobre.

Groups 5 and 6: In Soquimich group, there are two subgroups that are interesting to analyze. They are Oro Blanco-Pampa Calichera and Pampa Calichera-Compañía Química y Minera de Chile (Soquimich or SQM). A feature of this group is that it is structured in cascades. In effect, the only company of the group with an operational business line is Compañía Química y Minera de Chile which is in the bottom tier of the structure. By December 2007, Pampa Calichera investment company held 26% of SQM. The controlling company of Pampa Calichera is Oro Blanco Investment Company that held 68.73% of Calichera. The controlling company of Pampa Calichera is Oro Blanco Investment Company, which at the end of 2007 held 78.7% of the shares of the company. This company is controlled by Norte Grande S.A. and it in turn is controlled by Compañía de Inversiones SQYA, where Julio Ponce Lerou is a main shareholder. Therefore, the generation of income in these companies depends directly on the amount of dividends from Soquimich (SQM).

Groups 7 and 8: Another pair of holding companies that are interesting are Empresas Navieras S.A. of "Urenda" group and Marítima de Inversiones S.A., Marinsa, of "Claro" group. Although these are not groups with the simple structure one parent company and subsidiary, they do have shareholdings in public companies exceeding 95% of their investments in other firms, which complies with the selection criteria. Moreover, both Marinsa and Naviera are parent companies that serve only as investment vehicles.

Group 9: Another holding company with a relatively simple structure is Antarchile S.A. In effect, it has a direct shareholding in 3 subsidiaries, Copec, Pesquera Eperva and Pesquera Iquique Guanaye. These are 3 listed firms, and the sum of the shareholdings of Antarchile in them easily exceeds 95% of total investments, thus complying with the selection criteria. Antarchile controls Copec, Eperva and Iquique Guanaye fisheries. Investment in Copec represented 97.6% of investments in related companies and 90.2% of total assets of Antarchile33. Through this company, Antarchile controls 99% of Celulosa Arauco y Constitución SA, Compañia de Petróleos de Chile (Copec SA) and Abastible; it also holds 39.8% of Metrogas SA, 25% of Eléctrica Guacolda, and 81.9% of Pesquera Iquique Guanaye.

Antarchile directly possesses 17.4% of Pesquera Iquique Guanaye SA and 18.4% of Pesquera Eperva SA. It also has stakes in other companies such as Astilleros Arica SA, Sigma SA and Services Corporativos Sercor SA but the value of these investments is a very small part of total assets. In addition, it has 9.58% of Colbún SA electricity Generation Company. The assets of Eperva are mainly investments in other companies: 46.36% of Corpesca SA, 10% of Servicios Corporativos SerCor SA, and 5% of Sigma SA. Iquique-Guanaye Company is also structured as a holding investment company. It participates in the fishing industry through its subsidiary SouthPacific Corp S.A. and its related company Corpesca SA.

3.2.2. Businesses and industrial sectors of the companies analyzed.

Group 1: Tricahue Investment company and Eléctrica Pehuenche company: Tricahue company is an investment company that invests in shares of Pehuenche. As of December 31, 2007, Tricahue held 2.91% of Pehuenche's shares. Tricahue's financial results depend directly on those of Pehuenche and its generation of dividends. Its only business is the buying and selling of Pehuenche Electric Power company shares. The main business of Pehuenche is the generation, transmission and distribution of electric energy and is controlled by Endesa Chile. Pehuenche owns and operates three hydroelectric plants in the Maule river basin. The first plant, Pehuenche, was

33 Figures obtained from the company Balance Sheet in the records of the SVS at 31 December 2008.

completed 1991 and is the second largest hydroelectric power station in Chile, after Ralco. The second power plant is Curillinque, which entered in operations in 1994. Finally, the third power plant is Loma Alta, which entered in operations in 1997. The power plants use the waters of the Maule and Melado rivers for their production.

Group 2: Sociedad de Inversiones Campos Chilenos-Iansa: Campos Chilenos S.A. Investment company is the controlling company of Iansa, with a direct and total shareholding of 45.13% of the company. The sole business line was initially defined as investment in the shares of Industria Azucarera Nacional S.A., Iansa S.A. afterwards extended to allow for a diversification of activities. However, the only business of the company continues to be its shareholding in Iansa. Iansa is an agroindustrial holding company, specialized in the production of quality natural foods. Its core business in the last fifty years has been sugar production. It also operates in the fruit concentrates sector, frozen vegetables and fruits, agricultural supplies and feedstuffs for cattle and horses. During 2007, it added the production of vegetable oils and petfood. In its plants in Peru, it processes tomato and frozen vegetable products.[34]

Group3: Inversiones Aguas Metropolitanas (IAM)- Aguas Andinas: Inversiones Aguas Metropolitanas is the holding controlling company of Aguas Andinas through a shareholding of 50.1% in the company, at 31 of December 2008. Aguas Andinas together with its subsidiaries make up the largest water utilities group of Chile and one of the largest in Latin America. IAM is a public traded company. The company limited its business line to "investment in shares of Aguas Andinas and the provision of all kinds of consulting services related to the transfer of technology and know-how, technical support, businesses and project management, specially related to the management and operation of water management businesses."[35] Compañía General de Aguas Barcelona, Agbar, owning 56.6% of IAM´s stocks is the only controlling company shareholder. Aguas Andinas provides water management services to the city of Santiago and nearby areas. It serves a population of around seven million

34 Source: company's web page.

35 Company´s Annual report 2007: Inversiones Aguas Metropolitanas

inhabitants. It produces and distributes drinking water, captures, treats and disposes of the wastewater together with other related services.[36] The services of production, distribution of drinking water, capture, treatment, and disposal of waste water are carried out by Aguas Andinas, Aguas Cordillera, Aguas Manquehue and Aguas Los Dominicos. The services related to the treatment of industrial waste, sale of equipment and laboratory analysis are carried out by unregulated subsidiaries.

Group 4: Pacífico V Región-Pucobre: Pacífico V Región company main business is investment in shares of Pucobre company. Its main business line is "buying, selling, importing, exporting, production, and distribution, either directly or indirectly, of all kinds of goods related to mining, fishing and fish farming, food, construction and real estate, agriculture, agroindustry, transport, pharmaceutical products, marine, metallurgy, energy and telecommunications and logistic services. The main business of the company is the extraction, treatment and sale of copper through its shareholding of 83.62% in Compañía Minera Punta del Cobre S.A., Pucobre. This company represents 64% of Pacífico´s total assets. The rest corresponds to investment in other related companies, current assets "Documents and Accounts Receivable from Related Firms" other assets "Documents and Accounts Receivable from Related companies Long Term, adding 99.84% of total assets. Pucobre, the main subsidiary of Pacífico V Region is a company specialized in the exploitation of medium scale copper deposits that adds value to the minerals obtained through processing in its own facilities located in the third region, the north of Chile. Those plants produce Copper Concentrate for the smelters and copper cathodes for export. The company produces concentrates with 29% copper, with gold and silver content. These are acquired by national companies that pass it through pyrometallurgical processes, thereby obtaining high purity electro refined copper for export. By processing copper oxidation minerals, Pucobre produces high purity cathodes with a copper content of 99.999%. These cathodes are exported to America, Europa and Asia."

36 Source: web page Santiago Stock Exchange, www.bolsadesantiago.com

Groups 5 and 6: Soquimich Group: Oro Blanco-Pampa Calichera-Soquimich: Soquimich group has a cascade company structure. The core of the group is Sociedad Química y Minera de Chile S.A. (SQM), the only company with an operational business activity of the group. The rest are investment companies that hold shares of the companies in the level below them. Therefore, practically the only asset of Compañía de Inversiones Pampa Calichera are the shares of SQM. Soquimich produces and sells specialized plant nutrients, iodine and lithium. It has a low cost production strategy based on large-scale high quality natural resources, know-how and technological development in production processes, logistical infrastructure, high production volumes, an international commercial network, and synergies from the production of a wide range of products. The raw materials are obtained from deposits in the Atacama Desert in the first and second regions of Chile.

Group 7: Urenda Group: Empresas Navieras: Urenda group includes the parent company companies Navieras S.A., an investment public company, controlling its direct subsidiaries: Company Chilena of Navegation Interoceanica S.A. (CCNI), Agencias Universales S.A: (Agunsa) and Port Cabo Froward (Froward). Through these subsidiaries, it has shareholding in other more than 80 business related firms, both national and foreign. With these investments, companies Navieras is in the business of global marine transport, shipping agency, cargo services, representations of transport operators, both marine and air, investments in port and air infrastructure, land equipment and fleets, land transport, warehousing and distribution of products and, in general, in all areas related to cargo transport and services, for both import and export. Compañía Chilena de Navegation Interoceanica S.A. Is a cargo shipping company with operations around the world, which also engages in land links and transfers. It is a Chilean company, part of the Navieras holding company. Together with the other subsidiaries, it forms part of a chain of cargo transport services. The company operates three main types of cargo: Containers (for dry, refrigerated or frozen cargo), bulk cargo (copper, forestry products, project cargo), and Vehicles (automobiles, trucks, buses and machinery). Agencias Universales S.A. Is a port services company operating in Latin America. Agunsa developed from being a shipping agency to being active in various transport sectors. For example, it participates in the airport business in Chile as promoter and main shareholder of SCL Terminal Aéreo Santiago S.A. It also provides tugboat services in Chile and Ecuador through CPT Agencia Marítima S.A. and has logistics and cargo distribution businesses, operating one of the largest warehouses and distribution centers in Santiago. Finally, Puerto Cabo Froward company provides services of docking, reception and bulk storage, carraige, loading and offloading, logistics and cargo distribution in Coronel and Calbuco terminals. It has a mechanized transport system with variable speed and reversible conveyor belts, for loading and offloading of solid bulk. The industrial port of Calbuco also does offloading of liquid bulk through a system of pumps and transport ducts, stored in tanks for subsequent distribution.

Grupo 8: Claro Subgroup: Marinsa - Sudamericana-Elecmetal: Claro group includes a sub-group comprised of investment company Marinsa, Compañia Sudamericana de Vapores and Elecmetal. Marinsa´s business is investment in real estate and properties tangible or intangible, urban or rural. Investment in movable intangible goods, such as shares, bonds, debentures, rights in companies, promissory notes, bills of exchange, certificates of deposit, negotiable documents, transferable securities, and commercial paper in general, issued by the State, by public institutions or private firms, either national or foreign". Marinsa businesses are mainly in the financial area through term deposits in Banks and acquisitions of shares of already existing companies such as in Sudamericana de Vapores subsidiary and Electro Metalúrgica company. The business line of the Company is mainly financial and based on investments, and it does not possess property or equipment or industrial machinery. Its main asset is shares of Sudamericana de Vapores and Elecmetal. Compañía Sudamericana de Vapores is the main asset of Marinsa. It operates in container ship transport, transport of liquid and solid bulk cargos, refrigerated cargos and automobile transport. Its subsidiary Sudamericana Agencias Aéreas and Marítimas S.A. provides port services and land logistics in several Latin American ports. Marítima de Inversiones also has investments in Elecmetal. This company designs and produces steel parts that serve to improve the processes of earth moving, shredding, and milling of minerales of its clients. In these processes, the mechanical properties of the products such as resistance to wear and impact are essential. Elecmetal produces spare parts for spinning, cone, jaw, and impact shredders and others. It also produces coatings in steel and soft irons for semi autogenous mills, bar and ball mills, as well as cast steel parts and spares for earth movement.[37]

Group 9: Angelini Group: Antarchile – Copec – Pesquera Iquique Guanaye – Pesquera Eperva: One of the largest business groups in Chile is the Angelini group. Through Antarchile, the main holding company of the group, it holds investments in industrial, forestry, fisheries and energy firms. Copec SA operates in the energy and natural resource sectors. The company website indicates that the company operates in the distribution of liquid fuels, liquid gas, natural gas and electric generation. In the natural resources area, it holds shares in the forestry, fisheries and mining industries. Pesquera Eperva SA is an investment company whose main assets are related to the fisheries industry in Chile. One of its main investments is Corpesca SA, which is located in the north of Chile, is dedicated to the extraction of pelagic fish and the production and sale of fishmeal and fish oil, in northern Chile.

3.3. Description of the discounts in these groups

This section presents the data corresponding to the discounts between the stock market capitalization of the parent company and the net asset value. Appendix 1 shows the simplified structure of the groups; Appendix 3 shows discount´s descriptive statistics for all groups in their own sample as well as in the common sample. Appendix 5 shows the graphs for the selected groups discount, showing levels, trends and moving averages, as well as relevant prices in log scale.

The first cases correspond to three groups with a very simple structure, one parent company and one subsidiary: Inversiones Tricahue-Pehuenche, Group 1, Campos-Iansa, Group 2, and Pacífico V–Pucobre, Group 4. A quick glance at these figures reveals the immense volatility of these discounts. It is hard to justify the large variations between quarters given the very similar price variations. The price of the

37 Source: Santiago Stock Exchange.

company whose business line is to be an investment vehicle (Campos and Tricahue) closely follows the price of the subsidiary.

Next Campos Chilenos, Group 2, is owner of 45% in Iansa, public Company. In this case long swings in the market prices are observed, accompanied with a highly volatile discount. In this group discounts appear to follow a cyclical behavior, with cycles of around two years, ranging from 40% to -20% to -25%. This is a very interesting phenomenon which cause is beyond the scope of this work.

The next case analyzed corresponds to Inversiones Aguas Metropolitanas, Investment Company in shares of Aguas Andinas, Group 3. The period analyzed is extremely short since IAM started to trade in November 2005. Daily data on the discount are included.

For Pacífico V Region group and Pucobre, Group 4, the period analyzed starts on 31 of August 1999, since prior to that date the share of investments in related companies corresponding to public companies (in this case, the Pucobre company) was below 95%. In spite of the selection criteria used to when construct the databases, the period prior to this date is not excluded for completeness reasons. Information of the FECU provided by the Superintendencia de Valores y Seguros at 30 of June 2002, shows that in the Annual Shareholder Meeting of the company Punta del Cobre, a stock split, 100 to 1 was agreed. On other side, Pacífico V Region company launched a Public Offering to acquire 100% of its subsidiary Pucobre during the third quarter of 2002. After that offer, it attained 83.6% of the property at 30 of September 2002. The problem arose on 30 of June 2002. According to the information provided by Economatica, the decision on the split of shares in Pucobre was made on 20 of May 2002, but the definitive issue of the new shares for each original one only occurred on 31 of August 2002.

In the case of Oro Blanco-Pampa Calichera sub-group, Group 5, the period studied is longer, although the discount cannot be calculated continuously between 1997 and 2000. The number of shares of Pampa Calichera held by Oro Blanco changes substantially between these years. Including the data of the early years would lead to unprecise results, since the precise the number of shares in each quarter is not known. Thus, data of the early years are not included. In the same Soquimich group, the controlling company of the companies mentioned before is Norte Grande. It is also an investment company and nearly all its assets correspond to Oro Blanco shares. It also has shareholdings in other closed firms, but they are much smaller than its investment in Oro Blanco. When calculating the discount between the years 1995 and 2007, a large difference can be found compared to the other groups.

The next case corresponds to Pampa Calichera-Soquimich sub-group, Group 6. The period of analysis starts at 30/10/2000 given that prior to that date, the part corresponding to SQM, the only public firm, was less than 95% of total investments. After that date, that percentage is exceeded in all cases. Prices for the series B shares of SQM prior to 30 of September 2002, since the parent company Pampa Calichera did not have shares of that series prior to that date.

The following case is the Urenda group, Group 7, The parent company is Empresas Navieras, which has shareholdings in 3 public firms, Interoceanica, Agunsa, and Froward. The analysis starts on 31/12/1996. As in the case of Marinsa, group 8 below, the price of the parent Compañías Navieras, closely follows the changes in the price of Interoceanica and Agunsa. Appendix 5 compares the price of Naviera and Interoceanica and of Naviera and Agunsa. There is an exceptional drop in the discount at the end of 2004 in this group. In effect, in August that year there is a sharp drop, from discounts of 50% to 20% to some weeks later having premiums of 30 a 40% instead of discounts. This phenomenon will help to explain the difficulty to reject the hypothesis of random behavior of the discount series in the statistical analysis. Accordingly, the discount should be stationary, allowing some fluctuations in the equilibrium level, but it should not distance itself permanently from a stable mean and variance. What could occur in this group is a structural change in the discount of Naviera since a drop in the series starts on August 2004 (from 59.2% to 28.7% between this date and September, reaching a premium of 43.6% on October 2004 and then becoming relatively stable with an average discount of 5%

approximately). Then there is less volatility from the end of 2004 to the end of the sample. There are no greater movements in most variables of the discount calculations, such as the number of shares of the Naviera investment company or in the quantity of shares it has in Interocean, Agunsa and Froward. The explanation may be found in the figures of price variations in Navieras, Interoceanica and Agunsa. Between 20 of August 2004 and 24 of December 2004, the price of Naviera increased 644% (measured according to the formula: (Final Value - Initial Value) / Initial Value, in other words its variation percentage) while Interoceanica increased 322% and Agunsa only 64%. This led to an increase of 319% in the stock market value of Naviera while the stock market value of its investment with the corresponding adjustments only increased 118% in same period. The low increase in the price of Agunsa is crucial in the drop in the discount since the proportion of Agunsa in Naviera's total investment at 20 of August 2004 was 56.22%, while that of Interoceanica was 26.48%. At 24 of December 2004, when the discount reached its lowest level of -37.22% (or a premium of 37.2%) the proportion of Agunsa was 41.9% and of Interoceanica 50.8%. In other words, before the date of the drop, the shares in Agunsa were the largest asset of Naviera and with such a low growth in price compared to the Naviera investment company, the discount had such a sharp drop.
In sub-groups of the Claro group: the Marítima de Inversiones, Group 8, with shares in Sudamericana de Vapores and Elecmetal as shown in appendix 5, it would be interesting to explore why the discount changes so much if the prices of both shares move so closely.

In Antarchile, Group 9 the discount from December 2000 can be calculated because a lack of relevant information before that time. By December 2000, Antarchile had investments in Compañías Copec, Sudamericana de Vapores and the Iquique Guanaye and Eperva fisheries companies. Prior to that date, the investment in Copec was indirect: Antarchile held shares in other investment companies such as Inversiones Socoroma, Inversiones y Desarrollo Los Andes and Inversiones Lascar and through the first two it has a shareholding in Copec. These two companies, Socoroma, and Los Andes are not public, and it was not possible to determine the number of shares that each possessed in Copec prior to December 2000.

3.4. Some Facts and Analysis

3.4.1. On the tax reform of the year 2001

On November 2001, Law 19.768 entered in force, introducing a Capital Gains Tax Exemption. This law could have had an impact on the valuation of the shares of the main Chilean holding companies. However, the literature indicates that a tax change like this would affect both the shares of the parent company and of the subsidiaries; there would be a positive effect on the price of both, so it would be a non-divergent factor: it would affect both shares equally.

The discount variations do not indicate a significant change after 2001. In fact, for some groups it can be observed an increase in the discount level. However, interestingly, there is a significant reduction in the variability of the discounts for nearly all the groups after that date. If these companies made the tax payment on the capital gains, there should have been a reduction in the discount level without returning to the discount levels prior to that date. In some cases, the discount dropped but then returned to the high discount levels in previous periods. Given that the earnings obtained are tax exempt, if the investment company used this tax benefit, it could have been expected that the discount should not have returned to high levels. This leads to assume that the taxation theory on capital unrealized gains would not have a large impact on the discount at least in the groups analyzed in the Chilean market. A significant part of the discount cannot be attributed to the tax's issues.

3.4.2. On the Liquidity of holding company and underlying shares

An explanation usually found of the holding company discount.as well as the Closed end funds discount, relates to the liquidity of the shares. Supposedly, holding company shares are generally less liquid than the underlying shares. This makes investing in the investment or holding company less attractive; therefore, the preference is to invest in the underlying shares. However, this argument is not conclusive since the direction of causality is not clear: the existence of a holding company discount could favor the investment in the underlying share over that in the investment company. In other words, given that there is a discount, the shares of the holding company are less liquid. In any case, an analysis of the liquidity of both shares was made for the groups chosen. The liquidity indicator was created from the data of the total quarterly transactions of shares divided by the total number of shares issued. Visually, there is no clear pattern showing that, when the holding company shares become less liquid than the underlying shares, that there is an increase in the discount. The same intuitive conclusion can be extracted analyzing the 9 selected groups[38].

3.4.3. On the Buy-Sell Recommendations by Investment Banks

In Chilean financial market, various investment banks have made buy-sell recommendations based on the holding company discount of some Chilean groups. In a study as of February 2007[39] the analysts find that the discounts of some groups are above their average o long term level. Generally, the average discount of the two years prior to the study are used. When the discount of the moment is below its historical average, the underlying share is prioritized over the shares of the parent company. This assumes that the discount should return to its trend level, synonymous of a reversion to the mean in the short term. In a report as of 11 of April 2008[40], the variation in discount level of various holding companies is analyzed with respect to the average of the previous three years. Deviating from that value would be enough to estimate that there is an opportunity to buy or sell assuming that the discount should return to its trend level. These opportunities do not really consider the fundamentals of the subsidiaries of the parent company or the underlying assets.

Other consideration of the study of Banco Security in explaining the discount focuses on the discussion of the debt level of the parent company. They mention that one of the benefits of a holding company structure is that it has a higher debt capacity by being more diversified and thus having a lower risk. However, if the debt level of the parent company is excessive, the capacity to pay dividends drops. In this case, it could be more advisable to have shares of the subsidiary.

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38 See Appendix 6

39 Informe de Valorización (Valuation Report), Inversiones Security Holding companies as of February 2007.

40 Inversiones Security, Monitor Security: Holding companies. Banco Security report made by Mauricio Ibarra, 11 of April 2008.

Section 4. Statistical Analysis

4.1. Introduction and underlying Theory

Does it make sense to make recommendations, as some banks actually do, based on how far the discount is relative to the average? An analysis of the discounts figures reveals that the behavior of the series of the discount is highly volatile. In general, for the data after 2001 and 2002, the discount level is more stable but if I take the total sample, it seems very risky to focus only on the average level of the discount.

If I assume that stock prices are random walks as Efficient Markets Hypothesis says, I should expect holding discount to be also random walk as the following analysis shows.

First, I must note that a discount is the quotient of two supposedly random variables, market capitalization in the holding company (X)/NAV(Y).

1) If the price of the holding company, X, and NAV (a weighted average of stock prices plus some noise), Y, are supposed to be random variables with a lognormal distribution. This means that $\ln(X)$ and $\ln(Y)$ are normally distributed. Now, $\ln(\frac{X}{Y})$ = $\ln(X)-\ln(Y)=Z$. I assume that $\ln(X)$ and $\ln(Y)$ have means $\mu_X \; and \; \mu_Y$, variances $\sigma_X^2 \; and \; \sigma_y^2$, and covariance σ_{XY} (equal to zero if X and Y are independent) and are jointly normally distributed. The difference Z is then normally distributed with mean $\mu_Z = \mu_X - \mu_Y$, and variance $\sigma_Z^2 = \sigma_X^2 + \sigma_X^2 - 2\sigma_{XY}$.

Now I let $\frac{X}{Y} = \mathrm{e}^Z$, meaning that X/Y is lognormally distributed as well, with parameters $\mu_Z \; and \; \sigma_Z^2$. The relationship between the mean and variance of a lognormal variable and the mean and variance of the corresponding normal variable is: $\mu\left(X/Y\right)=\mu\left(e^Z\right)=e^{\left\{\mu_Z+1/2\sigma_Z^2\right\}}$; $\sigma^2\left(X/Y\right)=\sigma^2\left(e^Z\right)=e^{\left\{2\mu_Z+2\sigma_Z^2\right\}}-e^{\left\{2\mu_Z+\sigma_Z^2\right\}}$.

Although the case shown above is a particular case, what I am seeing is that the discount (X/Y) preserves the probability distributions of X and Y. This is a ground to my further analyses.

2) Time behavior of the quotient of two standard lognormal random walk variables.

- Assuming that prices and NAV are independent random walk variables, I have performed the following experiment: I have simulated two independent random walk variables, X and Y, computed their quotient X/Y, i.e. the discount, and executed the two procedures I use in this work to analyze the evolution of a discount time series: Unit Root Test (Augmented Dickey-Fuller Test) (if a stochastic process, a random variable, possesses a unit root, the best forecast for the next period conditional to all existing information to the moment, is today's value; see section 4.2 for technical details), and Variance Ratio Test (see section 4.3 for technical details) ; the later tests for mean reversion (if a stochastic process, a random variable's behavior shows mean reversion, this implies that the variable tends to a predictable mean value in the long run. Else a martingale, is a stochastic process that has not mean reversion, and again, the best forecast for the next period conditional to all existing information to the moment, is today's value) versus random walk. Both tests show, for several random variable runs, that the simulated discount time series, based on (pseudo) random walks of their components, cannot be rejected to be random walk, or martingale, and has not mean reversion, so is not a stationary series. This can be seen in the test result of a representative random variables run. It is important to notice that, in five out of 50 runs, the VR test showed that the quotient series cannot be rejected to be a martingale and does not present mean reversion, but random walk hypothesis was rejected using the AFD test. Actually, it can be demonstrated that the quotient of two normally distributed variables displays a Cauchy distribution, similar to de normal distribution but skewed to the right. The simulation procedure is therefore one step forward to assume that the quotient is random walk (normally distributed around a known value). Table 5, Figure 13 and Table 6 below, show the results of a sample run for Variance Ratio Analysis (VR) and Unit root Analysis respectively.

Table 5: Variance Ratio Analysis for model simulation of randomness of the quotient of two independent random variables, sample run

Joint Tests	Value	df	Probability
Max |z| (at period 4)*	0.952174	499	0.8114

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	1.027208	0.041709	0.652337	0.5142
4	1.074417	0.078155	0.952174	0.3410
8	1.071512	0.128233	0.557674	0.5771
16	1.038121	0.194501	0.195992	0.8446

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = 0.276553106212)

Period	Variance	Var. Ratio	Obs.
1	79.6784	--	499
2	81.8463	1.02721	498
4	85.6078	1.07442	496
8	85.3764	1.07151	492
16	82.7158	1.03812	484

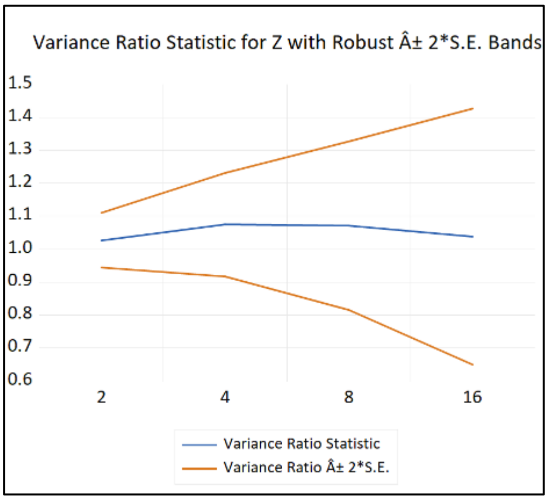

Figure 13. Variance Ratio Analysis for model simulation of randomness of the quotient of two independent random variables

Table 6: Unit Root Test Analysis for model simulation of randomness of the quotient of two independent random variables, sample run

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.639990	0.8542
Test critical values:	1% level	-2.569604	
	5% level	-1.941459	
	10% level	-1.616273	

*MacKinnon (1996) one-sided p-values.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Z(-1)	0.000256	0.000399	0.639990	0.5225

R-squared	-0.000139	Mean dependent var	0.276553
Adjusted R-squared	-0.000139	S.D. dependent var	8.926275
S.E. of regression	8.926896	Akaike info criterion	7.218017
Sum squared resid	39685.36	Schwarz criterion	7.226459
Log likelihood	-1799.895	Hannan-Quinn criter.	7.221330
Durbin-Watson stat	1.952100		

- Now I assume that Prices in the holding company, and NAV, are dependent lognormal standard variables, meaning that there is a correlation coefficient greater than zero between them. This aligns with the common sense in that holding price reflects fully, or at least partially, NAV value. In this case, running the experiment a number of times, with a correlation coefficient of 0,95 between numerator variable and denominator variable, which is high enough for my purposes, all the Variance Ratio Tests performed show that the affirmation that the quotient is a Martingale, cannot be rejected at 5%, 10% significance levels. Thus, discount would be a martingale. On the other hand, accompanying standard Dickey-Fuller tests, indicate that the affirmation that the quotient has a unit root, cannot be rejected at 5%, 10% significance levels. This result is aligned with the Variance Ratio Tests and keeps the essence of what I am looking for, to test the null hypothesis H2, that the holding discount is unpredictable. Table 7, Figure 14 and Table 8 below, show the results of a sample run for Variance Ratio Analysis (VR) and Unit root Analysis respectively.

Table 7: Variance Ratio Analysis for model simulation of randomness of the quotient of two dependent random variables, sample run

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	0.904186	499	0.8383

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	1.042195	0.046666	0.904186	0.3659
4	1.005035	0.085733	0.058734	0.9532
8	0.880783	0.133890	-0.890407	0.3732
16	0.831000	0.195717	-0.863492	0.3879

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Test Details (Mean = 0.0460921843687)

Period	Variance	Var. Ratio	Obs.
1	4.42157	--	499
2	4.60813	1.04219	498
4	4.44383	1.00504	496
8	3.89444	0.88078	492
16	3.67432	0.83100	484

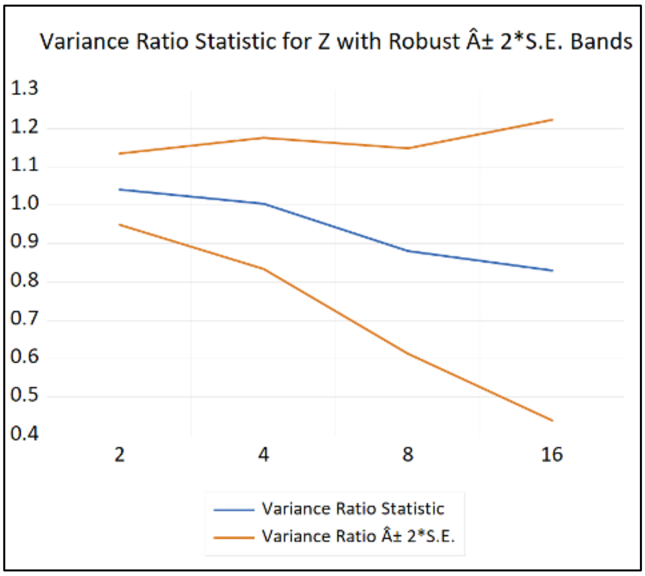

Figure 14. Variance Ratio Analysis for model simulation of randomness of the quotient of two dependent random variables

Table 8: Unit Root Test Analysis for model simulation of randomness of the quotient of two dependent random variables, sample run

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.464664	0.8147
Test critical values:	1% level	-2.569604	
	5% level	-1.941459	
	10% level	-1.616273	

*MacKinnon (1996) one-sided p-values.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Z(-1)	4.31E-05	9.27E-05	0.464664	0.6424

R-squared	-0.000048	Mean dependent var	0.046092
Adjusted R-squared	-0.000048	S.D. dependent var	2.102752
S.E. of regression	2.102802	Akaike info criterion	4.326421
Sum squared resid	2202.045	Schwarz criterion	4.334863
Log likelihood	-1078.442	Hannan-Quinn criter.	4.329734
Durbin-Watson stat	1.916029		

The above analysis means that in first place I should perform two types of tests to the holding discounts: A direct test of stationarity (non unit root) of discount Series, the standard Augmented Dickey Fuller Test, and on the other hand de Variance Ratio Test, which tests whether a time series shows a mean reversion behavior, i.e.: is it a martingale (a symmetric random walk for my purposes) or not? I expect both tests, applied to daily, weekly, and monthly discounts series to give analogous results.

If some series show to be stationary, or alternatively displays a mean reversion behavior, I am entitled to accept that stock market shows some degree of inefficiency, hence predictability, and in such cases, banks recommendations could make sense given that they would be exploiting such inefficiencies.

To test for nonrandom walk behavior or stationarity (which means in some way predictability) of the discount, I perform four classes of tests: I investigate whether there in stationarity in the discounts series using Unit Root Tests (section 4.2), then I investigate whether there is mean reversion of the discounts using Variance Ratio analysis (section 4.3), I continue searching for a stationary relationship between discounts of different groups, through Cointegration tests (section 4.4), and finally, I investigate following Shleifer (2000), if there is some association or comovement between various discount series, showing market inefficiency caused by Investor Sentiment.

4.2. Unit Root Testing (Augmented Dickey Fuller or ADF Test)[41]

In the present Section, I present the main results of testing for stationarity (unit root and predictability) of the discounts for the nine selected groups and for daily, weekly, monthly discounts series.

4.2.1. The test model

Consider a simple auto Regressive integrated order 1 process, AR(1):

$$4.1.\quad y_t = \rho y_{t-1} + x_t' \delta + \varepsilon_t$$

where x_t are optional exogenous regressors which may consist of constant, or a constant and trend, and ρ and δ are parameters to be estimated, and the ε_t are assumed to be white noise.

If $|\rho| \geq 1$, y is a nonstationary series and the variance of y increases with time and approaches infinity. If $|\rho| < 1$, y is a (trend-) stationary series. Thus, the hypothesis of

41 The statistical procedures as well as their descriptions, are extracted from Eviews9SVR user´s manual. EviewsR is the statistical package used in this work.

(trend-) stationarity can be evaluated by testing whether the absolute value of ρ is strictly less than one.

Our unit root testing, tests the null hypothesis $H_0 : \rho = 1$ against the one-sided alternative $H_0 : \rho < 1$, using the Augmented Dickey-Fuller (ADF) Test.

The standard DF test is carried out by estimating equation (1) after subtracting y_{t-1} from both sides of the equation:

$$4.2. \quad d(y_t) = \alpha y_{t-1} + x_t' \delta + \varepsilon_t$$

where $d(y_t) = y_t - y_{t-1}$, and $\alpha = \rho - 1$. The null and alternative hypotheses may be written as,

$$4.3. \quad \begin{aligned} H_0 &: \alpha = 0 \\ H_1 &: \alpha < 0 \end{aligned}$$

and evaluated using the conventional t - ratio for α :

$$4.4. \quad t_\alpha = \frac{\hat{\alpha}}{\text{standard error}(\hat{\alpha})}$$

where $\hat{\alpha}$ is the estimate of α, and $\text{standard error}(\hat{\alpha})$ is the coefficient standard error.

Dickey and Fuller (1979) show that under the null hypothesis of a unit root, this statistic does not follow the conventional Student's t-distribution, and they derive asymptotic results and simulate critical values for various test and sample sizes.

More recently, MacKinnon (1991, 1996) implements a much larger set of simulations than those tabulated by Dickey and Fuller. In addition, MacKinnon estimates response surfaces for the simulation results, permitting the calculation of Dickey-Fuller critical values and p - values for arbitrary sample sizes. The more recent MacKinnon critical value calculations are used in constructing test output.

The simple Dickey-Fuller unit root test described above is valid only if the series is an AR(1) (auto regressive of order one) process. If the series is correlated at higher order lags, the assumption of white noise disturbances ε_t is violated. The Augmented Dickey-Fuller (ADF) test constructs a parametric correction for higher-order correlation by assuming that the y series follows an AR(p) process and adding p lagged difference terms of the dependent variable y to the right hand side of the test regression:

4.5. $$d(y_t) = \alpha y_{t-1} + x_t'\delta + \beta_1 d(y_{t-1}) + \beta_2 d(y_{t-2}) + \ldots \beta_p d(y_{t-p}) + \upsilon_t.$$

This augmented specification (5) is then used to test (3) using the -ratio (4). An important result obtained by Fuller is that the asymptotic distribution of the t-ratio for α is independent of the number of lagged first differences included in the ADF regression. Moreover, while the assumption that y_t follows an autoregressive (AR) process may seem restrictive, Said and Dickey (1984) demonstrate that the ADF test is asymptotically valid in the presence of a moving average (MA) component, provided that sufficient lagged difference terms are included in the test regression.

The ADF tests performed, use no exogenous variables, but include trend and constant. The number of lags is selected automatically by EviewsR using the Schwartz information Criterion.

I carry out the Augmented Dickey-Fuller test, in levels and I see if it is possible to reject the null hypothesis of non-stationarity in the series. I present the t statistic of the test carried out and the value p. Compared with the critical values in each case, I can see if there is stationarity in the series. Table 9 presents the results obtained. If null hypothesis is not rejected, the series would not be stationary so its behavior over time would be stochastic and impossible to predict. To reject this hypothesis, it is necessary that the value of the t statistic be lower (more negative) than the critical values for different confidence levels.

4.2.2. Results

Table 9 shows the results for the ADF Test carried all over the discounts series. The null hypothesis H2a is that the series have a unit root (so they are non-stationary and thus unpredictable).

Table 9: Results of ADF test on the discount series

#	Group Name	Frequency	"t" statistic	probability	Critical Values (significance level) 1%	5%	10%	Null Rejection	
1	**Antarchile**	Daily	-4,339051	0,0027	-3,963192	-3,412328	-3,128101	Reject at all levels	
		Weekly	-3,809733	0,0170	-3,982920	-3,421950	-3,133796	cannot reject at 1%, reject rest	unit root
		Monthly	-3,905092	0,0152	-4,050509	-3,454471	-3,152909	cannot reject at 1%, reject rest	unit root
2	**Calichera**	Daily	-3,492413	0,0404	-3,963189	-3,412327	-3,128100	cannot reject at 1%, reject rest	unit root
		Weekly	-3,124885	0,1020	-3,982920	-3,421950	-3,133796	cannot reject at any level	unit root
		Monthly	-3,542795	0,0401	-4,048682	-3,453601	-3,152400	cannot reject at 1%, reject rest	unit root
3	**Campos**	Daily	-4,266617	0,0035	-3,960494	-3,411007	-3,127318	Reject at all levels	
		Weekly	-4,127408	0,0060	-3,969876	-3,415595	-3,130037	Reject at all levels	
		Monthly	-3,851597	0,0159	-4,005318	-3,432799	-3,140195	cannot reject at 1%, reject rest	unit root
4	**IAM**	Daily	-3,124016	0,1017	-3,975632	-3,418403	-3,131699	cannot reject at any level	unit root
		Weekly	-3,397364	0,0570	-4,044415	-3,451568	-3,151211	cannot reject at 5%	unit root at 5%
		Monthly	-4,234523	0,0091	-4,198503	-3,523623	-3,192902	Reject at all levels	
5	**Marinsa**	Daily	-3,871394	0,0133	-3,960491	-3,411006	-3,127317	cannot reject at 1%, reject rest	unit root
		Weekly	-3,691364	0,0234	-3,969860	-3,415588	-3,130033	cannot reject at 1%, reject rest	unit root
		Monthly	-3,455652	0,0472	-4,005318	-3,432799	-3,140195	cannot reject at 1%, reject rest	unit root
6	**Naviera**	Daily	-2,237960	0,4676	-3,961509	-3,411504	-3,127613	cannot reject at any level	unit root
		Weekly	-2,467708	0,3442	-3,974677	-3,417937	-3,131423	cannot reject at any level	unit root
		Monthly	-2,241138	0,4629	-4,023042	-3,441330	-3,145211	cannot reject at any level	unit root
7	**Oro Blanco**	Daily	-3,772520	0,0182	-3,963453	-3,412456	-3,128177	cannot reject at 1%, reject rest	unit root
		Weekly	-3,028704	0,1258	-3,984195	-3,422569	-3,134162	cannot reject at any level	unit root
		Monthly	-4,118243	0,0082	-4,053392	-3,455842	-3,153710	Reject at all levels	
8	**Pacífico**	Daily	-2,592201	0,2840	-3,962444	-3,411962	-3,127884	cannot reject at any level	unit root
		Weekly	-2,574131	0,2925	-3,979295	-3,420187	-3,132754	cannot reject at any level	unit root
		Monthly	-2,857273	0,1804	-4,038365	-3,448681	-3,149521	cannot reject at any level	unit root
9	**Tricahue**	Daily	-5,902128	0,0000	-3,960669	-3,411093	-3,127369	Reject at all levels	
		Weekly	-5,828790	0,0000	-3,970708	-3,416001	-3,130278	Reject at all levels	
		Monthly	-5,685657	0,0000	-4,008428	-3,434299	-3,141079	Reject at all levels	

Note: Unit Root Test Augmented Dickey Fuller Test; Null Hypothesis: time series has a unit root; Prob: MacKinnon (1996) one sided p-values; Exogenous variables: Constant and Linear Deterministic Trend.

4.2.3. Analysis

According to the previous results, I cannot reject the null hypothesis H2a, of unit root (predictability for my purposes) at 99% confidence level or 1% significance, for almost all the groups and frequencies, except for Antarchile-daily, Campos-daily and weekly, IAM-monthly, Oro Blanco-Monthly and Tricahue daily, weekly and monthly. There are 3 out of 9 groups whose unit root test allows the null hypothesis H2a to be rejected using daily discounts: Antarchile, Campos and Tricahue; there are 2 out of 9 groups whose unit root test allows the null hypothesis H2a to be rejected using weekly discounts: Campos and Tricahue; there are 3 out of 9 groups whose unit root test permits the null hypothesis H2a to be rejected using monthly discounts: IAM, Oro Blanco and Tricahue.

If I accept a 95% confidence level, thus a 5% probability of error type I, and the power of the test is greater, conclusions change dramatically. For almost all groups and frequencies I reject the null hypothesis H2a of unit root (no predictability). In fact, for daily discounts, for 3 of 9 groups, IAM, Naviera and Pacífico, null hypothesis H2a cannot be rejected. For weekly discounts, for 5 o 9 groups, Calichera, IAM, Naviera, Oro Blanco and Pacífico, null hypothesis H2a cannot be rejected. For monthly discounts, for 3 of 9 groups, Marinsa, Naviera and Pacífico, null hypothesis H2a cannot be rejected.

4.2.4. Conclusions

The results suggest that using a 1% significance level, the majority of the groups (6 out of 9) present unit root behavior, for my purposes unpredictability of the discounts, regardless the frequency of the data, thus showing some degree of market inefficiency. Now, using 5% or 10% significance levels, the conclusions are the opposite.

When I use monthly data up to second quarter 2009 (instead fourth quarter 2007) conclusions are the same for that frequency.

My conclusion from the non-stationarity tests, is that there is a weak to strong evidence of stationarity in the discount for the nine Chilean groups under analysis, hence the chances that the discount is predictable are not negligible.

Hypothesis H2a is therefore weakly to strongly rejected at 5% significance level.

The ADF tests must be complemented with the rest of the tests of section 4.

4.3. Variance Ratio Test[42]

As said before, the question of whether asset prices, or holding discounts are predictable, has long been the subject of considerable interest. One popular approach to answering this question, the Lo and MacKinlay (1988, 1989) overlapping variance ratio test, examines the martingale behavior, hence the predictability of time series data by comparing variances of differences of the data calculated over different intervals.

I use this analysis to test the existence of persistence in the discounts or a possible reversion to the mean after a while. The daily discount figures show a reversion to the mean in most groups. What would be significant would be to observe how long the persistence lasts and when a regression starts. I perform the test outlined in section 4.3.1 below.

As a reference, Walker and Lefort (2002) carry out a variance ratios test for the changes in interest rates in Chile using the Campbell, Lo and Mackinlay methodology. With this procedure, they analyze the changes in the short- and long-term interest rates and find a significant positive autocorrelation in the changes in short term interest rates, until the second month approximately and until the fifth month in the changes in long term rates. For longer time intervals, the variance ratios drop below 1. In short term rates, they can be significantly lower than 1 indicating a fast reversion

42 The statistical procedures as well as their descriptions, are extracted from Eviews9SVR user´s manual. EviewsR is the statistical package used in this work.

to the mean around the sixth month. There is also a reversion to the mean for long term rates albeit significantly more gradual.

4.3.1. The Test Model

If I assume the data follow a random walk, the variance of a q-period ahead difference should be q times the variance of the one-period difference.

1) $$\frac{\hat{\sigma}^2(q)}{q \cdot \hat{\sigma}^2(1)} = 1$$

Evaluating the empirical evidence for or against this restriction is the basis of the variance ratio test. I use the Lo and MacKinlay variance ratio test for homoscedastic and heteroscedastic random walks for discounts original data, using unbiased variances, heteroscedastic robust S.E., and using the asymptotic normal distribution (Lo and MacKinlay, 1988) to evaluate statistical significance of the Null Hypothesis H2c: The series under analysis is a random walk.

The Statistic Procedure for Variance Ratio Analysis is the following: Suppose having the time series $\{Y_t\} = (Y_0, Y_1, Y_2, \ldots Y_T)$ satisfying:

2) $$d(Y_t) = \mu + \varepsilon_t$$

where μ is an arbitrary drift parameter. The key properties of a random walk that I would like to test are $E(\varepsilon_t) = 0$ for all t and $\mathrm{E}(\varepsilon_t \varepsilon_{t-j}) = 0$ for any positive j.

The Basic Test Statistic is as follows: Lo and MacKinlay (1988) formulate two test statistics for the random walk properties that are applicable under different sets of null hypothesis assumptions about ε_t : Lo and MacKinlay first make the strong assumption that ε_t are independent and identically distributed (i.i.d.) Gaussian with variance σ^2 (the normality assumption is not strictly necessary). Lo and MacKinlay term this the homoscedastic random walk hypothesis; others refer to this as the i.i.d. null.

Alternatively, Lo and MacKinlay outline a heteroscedastic random walk hypothesis where they weaken the i.i.d. assumption and allow for more general forms of conditional heteroscedasticity and dependence. This hypothesis is sometimes termed the martingale null, since it offers a set of sufficient (but not necessary), conditions for ε_t to be a martingale difference sequence (m.d.s.).

One may define estimators for the mean of first difference and the scaled variance of the q-the difference:

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{t=T}(Y_t - Y_{t-1})$$

3) $$\hat{\sigma}^2(q) = \frac{1}{Tq}\sum_{t=1}^{t=T}(Y_t - Y_{t-q} - q\hat{\mu})^2$$

and the corresponding variance ratio

4) $$VR(q) = \frac{\hat{\sigma}^2(q)}{\hat{\sigma}^2(1)}$$

The variance ratio under the null hypothesis of random walk should converge to average one.

The variance ratio estimators may be adjusted for bias, as suggested by Lo and MacKinlay, by replacing T in equation (2) with $T - q + 1$ in the no-drift case, or with $(T - q + 1)(1 - q/T)$ in the drift case.

Lo and MacKinlay show that the variance ratio z-statistic:

5) $$z(q) = \frac{(VR(q) - 1)}{\sqrt{\hat{s}^2(q)}}$$

is asymptotically $N(0,1)$ for appropriate choice of estimator $\hat{s}^2(q)$.

Under the i.i.d. hypothesis which I use in this work, I have the estimator

6) $$\hat{s}^2(q) = \frac{2(2q-1)(q-1)}{3qT}$$

Under the martingale difference sequence (m.d.s.) assumption I may use the estimator

7) $$\hat{s}^2(q) = \sum_{j=1}^{j=q-1} \left(\frac{2(q-j)^2}{q} \right) \cdot \hat{\delta}_j$$

where

8) $$\hat{\delta}_j = \frac{\sum_{t=j+1}^{T} (y_{t-j} - \hat{\mu})^2 (y_t - \hat{\mu})^2}{\left\{ \sum_{t=j+1}^{T} (y_{t-j} - \hat{\mu})^2 \right\}^2}$$

Testing the null hypothesis: To test if the series is a random walk implies defining several intervals q and check for each one that the statistics (5) is out of the interval (-1.96;1.96) or far enough of Zero (low p), in which later case the null hypothesis of random walk must be rejected with 95% confidence. The intervals are selected according to the frequency of the data tested, so is different when testing daily, weekly or monthly data.

4.3.2. Variance Ratio Test results on the discounts

In this section, I present the Variance Ratio Test results for the 9 groups, using the three frequency data series of discount constructed for each group. Table 10 presents the main results, and Figure 14 shows the graphs of the Variance Ratio versus with time frame. In this case if the graph evolves around one, I should reject the hypothesis H2b of martingale behavior.

Table 10: Variance ratio tests results

Group #	Group Name	Frequency	Max !Z!	Time period at Max Z	probability	Reject Hypothesis Critical value(significance level) at 5% : 1,96	
					Joint Test Main Results		
1	**Antarchile**	Daily	3,773049	2	0,0008	Reject Null Hypothesis	
		Weekly	2,215959	8	0,1498	cannot reject Null	martingale
		Monthly	1,774731	2	0,2709	cannot reject Null	martingale
2	**Calichera**	Daily	2,131023	50	0,1828	cannot reject Null	martingale
		Weekly	1,799543	16	0,3610	cannot reject Null	martingale
		Monthly	2,174053	4	0,1136	cannot reject Null	martingale
3	**Campos**	Daily	4,357764	8	0,0001	Reject Null Hypothesis	
		Weekly	3,704268	2	0,0013	Reject Null Hypothesis	
		Monthly	1,148035	16	0,6852	cannot reject Null	martingale
4	**IAM**	Daily	1,642425	50	0,4703	cannot reject Null	martingale
		Weekly	1,575211	8	0,5202	cannot reject Null	martingale
		Monthly	2,080085	2	0,1418	cannot reject Null	martingale
5	**Marinsa**	Daily	2,843396	50	0,0265	Reject Null Hypothesis	
		Weekly	2,860799	4	0,0251	Reject Null Hypothesis	
		Monthly	1,913704	4	0,2047	cannot reject Null	martingale
6	**Naviera**	Daily	1,722696	8	0,4129	cannot reject Null	martingale
		Weekly	0,664752	100	0,9855	cannot reject Null	martingale
		Monthly	0,599108	16	0,9587	cannot reject Null	martingale
7	**Oro Blanco**	Daily	5,388205	4	0,0000	Reject Null Hypothesis	
		Weekly	4,160802	2	0,0002	Reject Null Hypothesis	
		Monthly	2,932157	2	0,0134	Reject Null Hypothesis	
8	**Pacífico**	Daily	1,578249	100	0,5179	cannot reject Null	martingale
		Weekly	1,806578	16	0,3565	cannot reject Null	martingale
		Monthly	1,104956	16	0,7147	cannot reject Null	martingale
9	**Tricahue**	Daily	2,883510	50	0,0234	Reject Null Hypothesis	
		Weekly	2,867603	16	0,0246	Reject Null Hypothesis	
		Monthly	2,662697	4	0,0306	Reject Null Hypothesis	

Note: Variance Ratio Test Lo-MacKinlay overlapping variance ratio test; Null Hypothesis: time series is a martingale or random walk; Heteroskedasticity robust standard error estimates; Number of Lags Used: Daily 2-4-8-16-50-100, Weekly 2-4-8-16-50-100, Monthly 2-4-8-16.

To visualize the criterium used to test the hypothesis H2b, I look at the VR of each series, central plus and minus two standard errors. If the null hypothesis represented by level 1,0 horizontal line, falls out of the VR plus minus two standard errors, I robustly reject the null hypothesis with a 95% confidence; otherwise, I cannot reject the Null. Figure 15, shows as an example two cases: Antarchile daily for which Null has been rejected, and Calichera daily for which I cannot reject the Null.

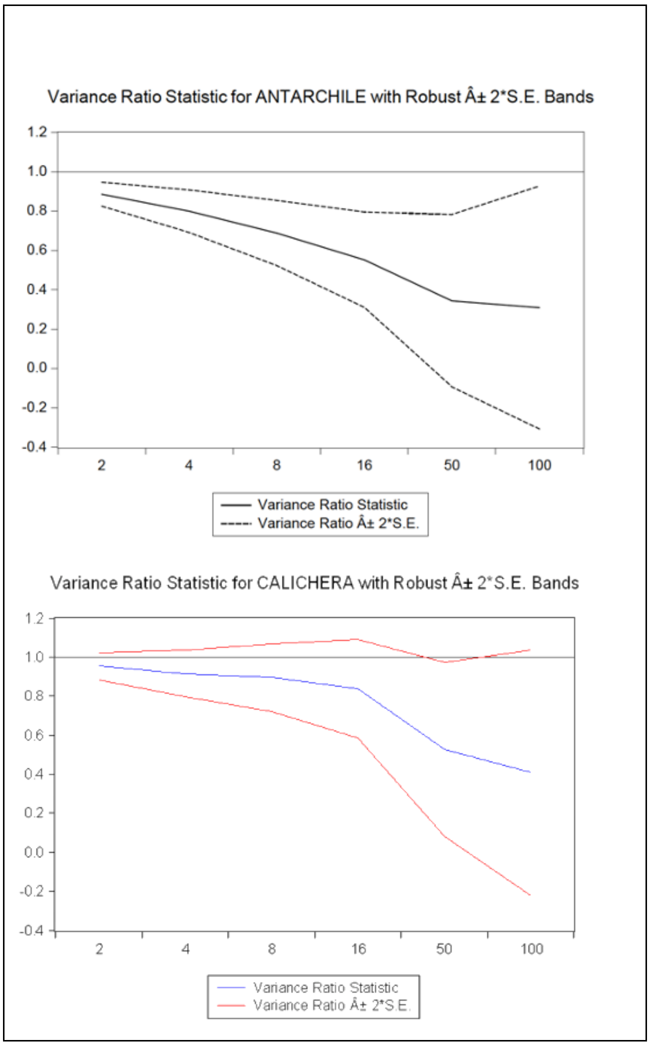

Figure 15. Example Variance Ratio Test for discounts, calculated VR statistic

Figure 16 show graphically the Variance Test for the discounts of the nine groups, for the three frequencies under analysis: daily, weekly and monthly:

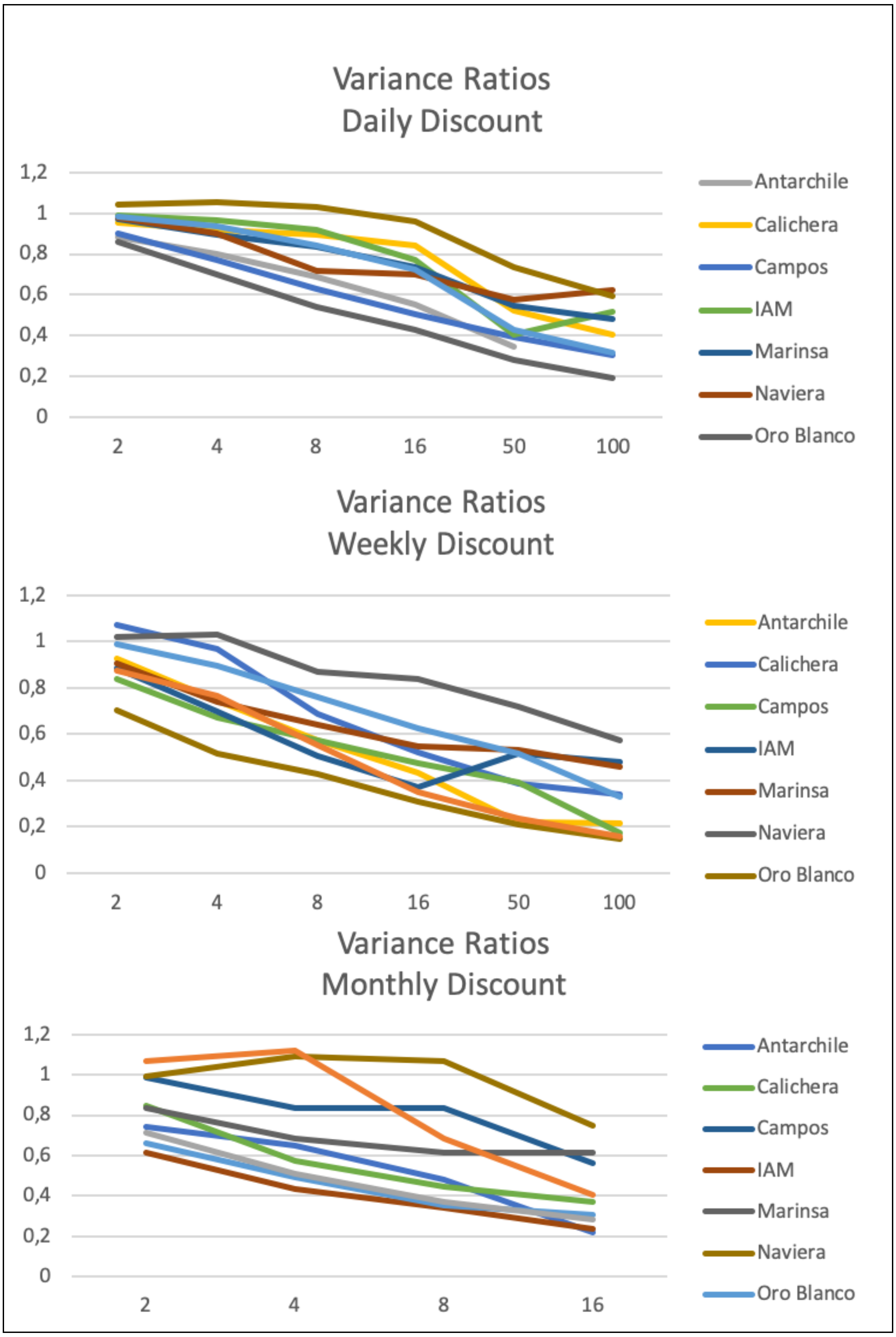

Figure 16. Variance Ratio Test for discounts, calculated VR statistic

4.3.3. Analysis

Form Table 10 it can be observed that I reject the null hypothesis H2b of martingale behavior, for 11 of the pairs group-frequencies. excepting for IAM, Naviera and Pacífico, for the rest of the groups null hypothesis H2b there is rejection in one or more frequency: Antarchile-daily, Campos daily and weekly, Marinsa daily and weekly, Oro Blanco daily, weekly and monthly, and Tricahue daily, weekly and monthly. For 5 out of 9 groups I reject the martingale hypothesis H2b using daily data: Antarchile, Campos, Marinsa, Oro Blanco and Tricahue. For 4 out of nine groups I reject the martingale hypothesis H2b using weekly data: Campos, Marinsa, Oro Blanco and Tricahue. For 2 out of 9 groups I reject the random walk hypothesis H2b using monthly data: Oro Blanco and Tricahue.

4.3.4. Conclusions

The results of the Variance Ratio Tests suggest that the minority of the groups (2 of 9) present martingale behavior, for my purposes unpredictability of the discounts, showing a higher degree of market inefficiency, thus predictability, than that showed with the Unit root tests. The results of the VR test are against the null hypothesis H2b using high frequency data (daily and weekly) but are more in favor of the null hypothesis H2b if I use monthly data.

Therefore, hypothesis H2b is weakly to strongly rejected at a 5% significance level.
When I use monthly data up to second quarter 2009 (instead fourth quarter 2007) conclusions are the same for that frequency. This is consistent with theory developed previously.

4.4. Cointegration Analysis[43]

In the previous sections, 4.2 and 4.3, I analyze random walk or non-predictability properties of the discount time series. The non-stationarity of the series allows us to carry out a deeper analysis: cointegration of two variables. The usefulness of this analysis is that it presents the behavior of adjustment to long term equilibrium

43 See Engel Granger (1987)

between two variables that do not have a stationary behavior. Gasbarro, Johnson and Zumwalt (2003) argue that cointegration analysis determines the degree with which two economic time series move towards an equilibrium relationship and allows variations from this equilibrium in the short term but not in the long term. Thus, if two cointegrated series are not in equilibrium, the trend with which they move towards it can be modeled using an error correction method (ECM). I affirm that neither the stock price of an investment company nor the underlying shares are stationary series. However, the series of returns of the shares prices that have a stationary process are obtained when the first difference of these series is calculated. Lefort (2004) analyzes these time series properties and states that "non-stationary processes of this kind are referred to as integrated processes. The integration order of these processes determines the number of times that the original process needs to be differentiated to reach a stationary process. The cases of stock prices are order 1 integrated process, since the original process is not stationary, but stationary is obtained after getting the first difference". Therefore, the series I work with are order 1 integrated process. The discount series are no more than a linear combination of two integrated variables, so I can use the Engle-Granger cointegration model.

Cointegration analysis will be done as follows: I take the discount series of each group, and test for cointegration of the stock price with its corresponding NAV (Net Asset Value). If the analysis shows that Stock price and Nav have a long run stationary relationship, then this will indicate that the respective discount shows a reversion to some mean, and thus are predictable.

Before explaining the model, I indicate that the series I use correspond to the prices of investment companies and the series corresponding to the net asset value of these companies. To work with these series, it is necessary to calculate its natural logarithm and work with those values.

If a linear combination of two variables is stationary, then the residuals of the regression between the two variables are stationary and the variables are cointegrated. The error term of this regression can be combined with a regression in differences between the variables to estimate the long-term relationship and the speed of the reversion to the mean.

The aim of this section is to analyze, from a direct perspective, the behavior of the holding discount for the nine selected Business groups.

4.4.1. The test model.

Given that the discount is defined as:

$$Discount = 1 - quotient = 1 - \frac{Holding\ company\ Market\ Value}{Net\ Asset\ Value},$$

I will try to find a direct stable (stationary) and long-term linear relation between holding company Market Value (MV) and its corresponding Net Asset Value (NAV), using the concept of Cointegration.

The analysis will be performed using daily discount series, and instead of using MV an NAV directly, I use ln(MV) and ln(NAV). This latter transformation is especially useful to eliminate eventual stochastic trends in the series and does not alter results. I intend to find a linear a relation such as:

$$(1)\quad \ln(MV_t) = a_a + b_0 \ln(NAV_t) + \varepsilon_t \qquad with: \varepsilon_t \sim N(0,\sigma^2), \mathrm{cov}(\varepsilon_t, \varepsilon_{t-1}) = 0,$$

and the error term is stationary or $\varepsilon_t = I(0)$, or integrated order 0. In this case I say that $\ln(MV), \ln(NAV) \sim I(1,1)$.

In economic time series is common to find spurious relations due to the fact that both series are integrated order 1 or greater, $X_t = I(1)$. The error term obtained in these conditions is non-stationary (variance increasing with time span) and self-correlated. In summary, relations obtained typically show a very high R2, very low Durbin Watson statistic, and not consistent[44] coefficients. Nevertheless, it is possible that two $I(1)$ series have a stationary linear relationship; in this special case I say that both series are cointegrated order 1.

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44 A consistent estimator is that which converges to the population value as the sample size grows

The method to find if two series are cointegrated, is due to Engle and Granger 1981, 1987 and incorporates the idea of estimating an error correction model (ECM).

The main idea of the ECM is the following: Assume I have two variables, Yt and Xt, both $X_t, Y_t = I(1)$ this is, both series possess a unit root, and this means that the first difference of the variables is stationary or $d(X_t), d(Y_t) = I(0)$. If I regress Yt on Xt I obtain:

$$(2)\quad Y_t = b_1 + b_2 X_t + \varepsilon_t$$

probably with the undesired properties of spurious regressions described above. Given that $X_t, Y_t = I(1)$, then $d(X_t), d(Y_t) = I(0)$, this is both first differenced series will be stationary. The following model,

$$(3)\quad d(Y)_t = a_1 + a_2 d(X_t) + d(\varepsilon_t)$$

will denote a proper stationary linear relation where a_1, a_2 are correct estimates of the true parameters, and no spurious relations problems are present. The problem is that this stationary relation shows the influence of the increment in the independent variable, on the increment of the dependent variable, but does not give any information of the levels of the variables. It is therefore a good short-term relationship.

What I am looking for, is a long-term relation between Y_t and X_t of the form

(4) $\hat{Y} = \hat{b}_1 + \hat{b}_2 \hat{X}$,

but what I have so far obtained is a short-term relation, equation (3), between the variables. If I estimate equation (3) and project it over the whole set of data, I will obtain a relation that connects long term to short term.

(5) $\hat{Y}_t = \hat{b}_1 + \hat{b}_2 \hat{X}_t + \hat{\varepsilon}_t$, and

(6) $\hat{\varepsilon}_t = Y_t - \hat{b}_1 - \hat{b}_2 X_t, and\ \ \hat{\varepsilon}_{t-1} = Y_{t-1} - \hat{b}_1 - \hat{b}_2 X_{t-1}$

The key insight here is that if Y_t and X_t are cointegrated, i.e. there is a long term linear and stationary relation between them, then $\hat{\varepsilon}_t = I(0)$, the error term is stationary.

Then, if both variables are cointegrated, I can write an Error Correction Model (ECM) that connects short term with long term:

(7) $d(Y_t) = \gamma_0 + \gamma_1 d(X_t) + \gamma_2 \varepsilon_{t-1} + Y_t$.

which by construction should be stationary since connects a stationary short-term relation with a stationary long run relation.

The Engle Granger procedure I follow is the following[45]:

a) Given that two I(0) series are stationary, a linear stationary relation between them always exists; a I(0) series will never be cointegrated with a I(1) series; and that two I(1) series may have a long term stationary linear relation, i.e. may be cointegrated, I first test for the integration order of ln(MV) and ln(NAV) series. This is done using the standard Augmented Dickey Fuller Test of Unit Root. I expect all series to be I(1).

b) When I detect that both ln(MV) and ln(NAV) both have a Unit Root, I then estimate the long term linear relation of equation (4), using standard OLS, of ln(MV) on ln(NAV). I expect a stationary error term if and only if both series are cointegrated. I also expect to find the spurious equation problems described before.

$$\ln(MV_t) = \hat{b}_1 + \hat{b}_2 \ln(NAV_t) + \hat{\varepsilon}_t$$

c) Next I estimate the error term in equation (5) and test for its integration properties, using the Augmented Dickey Fuller Test of Unit Roots. I expect to find that the error term is I(0), if both series are cointegrated.

$$\hat{\varepsilon}_t = \ln(MV_t) - \hat{b}_1 - \hat{b}_2 \ln(NAV_t)$$

45 I closely follow detailed proceduras by Asteriou, D. and Hall, S. (2016).

d) In the case both variables are cointegrated, I then estimate the error correction Model using standard OLS, and check that as predicted, the auto correlation of error is not present. I also expect to see the error correction term coefficient, γ_2 , to be negative, acting as a true correction from short term to long term

$$d(\ln(MV_t)) = \gamma_0 + \gamma_1 d(\ln(NAV_t)) + \gamma_2 \varepsilon_{t-1} + \ln(MV_t) + \mu_t$$

e) I then check for the presence of Unit Root in the error term of ECM and expect to find that the error is stationary, I(0), using the Standard ADF test of unit root.

$$\mu_t = d(\ln(MV_t)) - \gamma_0 - \gamma_1 d(\ln(NAV_t)) - \gamma_2 \varepsilon_{t-1} - \ln(MV_t)$$

f) I finally judge for the quality of fit of the ECM., and calculate what is called the reverting or converging period, $\frac{1}{\gamma_2}$ as the number of days (I work with daily series) in which short term dependent value converges to long term value when an increment in the independent value occurs.

4.4.2. Results

Next I follow the steps of the procedure outlined above. In figure 17 I show the graphs of ln(MV) and ln(NAV) for the nine groups, undated, meaning that they all start at observation one:

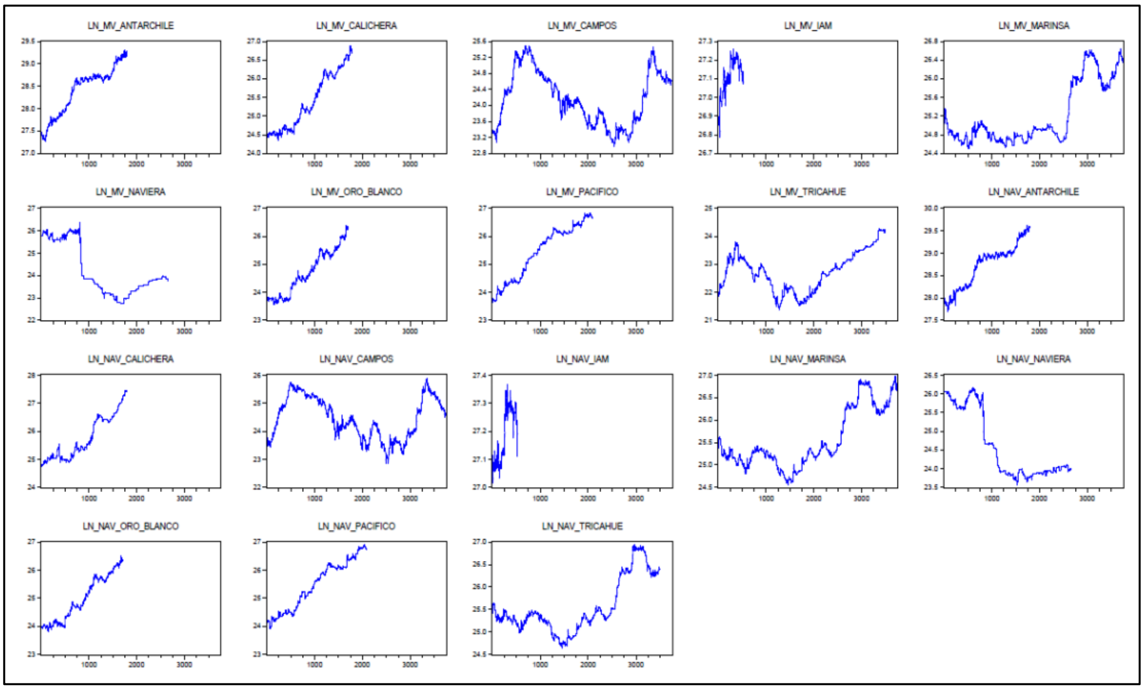


Figure 17. Graphs of ln(market Value) and ln(Net Asset Value) for the nine groups

4.4.3 Step Procedure and Analysis (Engle Granger)

Now I follow the Engle Granger procedure following steps a) to f), and show intermediate results as well:

<u>Step a):</u>

First, I test using the standard Augmented Dickey and Fuller (ADF) test for the presence of unit root in the ln(MV) and ln(NAV). In Tables 11 and 12, I present the results for the dependent variable ln(MV) and for the independent variable ln(NAV) respectively are shown.

As I can see, as expected, for both sets of series and for the nine business groups in each series, it is impossible to reject this null hypothesis of unit root of the residuals, inspecting the "t" statistics for the one sided ADF test.

Table 11: ADF Unit Root Test for natural logs of market value of holding companies

Variable	ADF test satisfice	Prob.	Test Critical Value			Reject/ Non Reject
			1%	5%	10%	
(LN_MV_Antarchile)	-0,87	0,3818	-3,433811	-2,862955	-2,567571	non reject
(LN_MV_Calichera)	0,21	0,8313	-3,433809	-2,862955	-2,567570	non reject
(LN_MV_Campos)	-1,48	0,1373	-3,431912	-2,862115	-2,567120	non reject
(LN_MV_IAM)	-2,33	0,0199	-3,442601	-2,866836	-2,569652	non reject
(LN_MV_Marinsa)	0,02	0,9840	-3,431912	-2,862115	-2,567120	non reject
(LN_MV_Naviera)	-1,39	0,1645	-3,432627	-2,862432	-2,567290	non reject
(LN_MV_Oro Blanco)	0,32	0,7425	-3,433989	-2,863034	-2,567613	non reject
(LN_MV_Pacífico)	-1,45	0,1452	-3,433288	-2,862724	-2,567447	non reject
(LN_MV_Tricahue)	-0,44	0,6553	-3,432038	-2,862171	-2,567150	non reject

Note: Exogenous: Constant; Null Hypothesis: Variable has a unit root.

Table 12: ADF Unit Root Test for natural logs of Net Asset (NAV) Values of holding companies investments

Variable	ADF test statistic	Prob.	Test Critical Value			Reject/ Non Reject
			1%	5%	10%	
(LN_NAV_Antarchile)	-0,44	0,6579	-3,433811	-2,862955	-2,567571	accept
(LN_NAV_Calichera)	0,76	0,4422	-3,433809	-2,862955	-2,567570	accept
(LN_NAV_Campos)	-1,68	0,0915	-3,431912	-2,862115	-2,567120	accept
(LN_NAV_IAM)	-1,48	0,1384	-3,442578	-2,866826	-2,569646	accept
(LN_NAV_Marinsa)	-0,21	0,8311	-3,431913	-2,862116	-2,567120	accept
(LN_NAV_Naviera)	-1,58	0,1130	-3,432619	-2,862428	-2,567288	accept
(LN_NAV_Oro Blanco)	0,14	0,8874	-3,433989	-2,863034	-2,567613	accept
(LN_NAV_Pacífico)	-0,53	0,5918	-3,433288	-2,862724	-2,567447	accept
(LN_NAV_Tricahue)	-0,38	0,6980	-3,432038	-2,862171	-2,567150	accept

Note: ADF Unit Root Test; Exogenous: Constant; Null Hypothesis: Variable has a unit root.

I now test for the order of integration of the same two sets, using the ADF test on the first difference of the series. The results are shown in table 13 and 14 and show the robust result that all the series are integrated order 0, so I can proceed with step b).

Table 13: ADF test for the first difference of ln(Market Value)

Variable	ADF test statistic	Prob.	Test Critical Value			Reject/ Non Reject
			1%	5%	10%	
D(LN_MV_Antarchile)	-37,51	0,0000	-3,438110	-2,862955	-2,567571	Reject
D(LN_MV_Calichera)	-42,77	0,0000	-3,433811	-2,862955	-2,567571	Reject
D(LN_MV_Campos)	-54,10	0,0000	-3,431912	-2,862115	-2,567120	Reject
D(LN_MV_IAM)	-20,45	0,0000	-3,442601	-2,866836	-2,569652	Reject
D(LN_MV_Marinsa)	-57,56	0,0000	-3,431912	-2,862115	-5,567120	Reject
D(LN_MV_Naviera)	-13,75	0,0000	-3,432627	-2,567290	-2,567920	Reject
D(LN_MV_Oro Blanco)	-41,64	0,0000	-3,433991	-2,863035	-2,567613	Reject
D(LN_MV_Pacífico)	-27,42	0,0000	-3,433288	-2,862724	-2,567447	Reject
D(LN_MV_Tricahue)	-37,55	0,0000	-3,432037	-2,862171	-2,567149	Reject

Note: ADF Unit Root Test; Exogenous: Constant; Null Hypothesis: Variable has a unit root.

Table 14: ADF Unit Root Test for the first difference of natural logs of Net Asset Values of holding companies investments

Variable	ADF test statistic	Prob.	Test Critical Value			Reject/ Non Reject
			1%	5%	10%	
D(LN_NAV_Antarchile)	-37,47	0,0000	-3,433811	-2,862955	-2,567571	Reject
D(LN_NAV_Calichera)	-41,11	0,0000	-3,433811	-2,862955	-2,567571	Reject
D(LN_NAV_Campos)	-57,09	0,0000	-3,431912	-2,862115	-2,567120	Reject
D(LN_NAV_IAM)	-22,15	0,0000	-3,442601	-2,866836	-2,569652	Reject
D(LN_NAV_Marinsa)	-26,35	0,0000	-3,431913	-2,862116	-2,567120	Reject
D(LN_NAV_Naviera)	-43,98	0,0000	-3,432619	-2,862428	-2,567228	Reject
D(LN_NAV_Oro Blanco)	-41,43	0,0000	-3,433991	-2,863035	-2,567613	Reject
D(LN_NAV_Pacífico)	-28,52	0,0000	-3,433288	-2,862724	-2,567447	Reject
D(LN_NAV_Tricahue)	-25,27	0,0000	-3,432038	-2,862171	-2,567150	Reject

Note: ADF Unit Root Test; Exogenous: Constant; Null Hypothesis: Variable has a unit root.

<u>Step b):</u>

I now estimate a long run linear relationship between ln(MV) and ln(NAV) for all the business groups given the results in step a). The results are shown in table 15, and I can see that the spurious relations problems (High correlation coefficient and low Durbin Watson Statistics) are present, as expected.

By inspection of table 15, I can observe, that the slope coefficient shows as economic theory predicts, a positive value in all cases. The magnitude of the slope, deserves to be analyzed: If there is no discount or discount is constant without error, the slope coefficient and intercept should be as follows:

$$Discount = 1 - quotient = 1 - \frac{Holding\ company\ Market\ Value}{Net\ Asset\ Value}$$

$$Discount = 0 \Rightarrow \frac{Holding\ company\ Market\ Value}{Net\ Asset\ Value} = 1$$

$$\Rightarrow \ln(MV) - \ln(NAV) = 0 : \text{slope} = 1; \text{ intercept} = 0$$

$$Discount = \text{constant} \Rightarrow \frac{Holding\ company\ Market\ Value}{Net\ Asset\ Value} = k; -1 \leq k \leq +1$$

$$\Rightarrow \ln(MV) - \ln(NAV) = \ln(k): \text{ intercept possitive when premium and negative when discount}$$

Table 15: Long run linear relationship between ln(MV) and ln(NAV)

Variable	# observ.	R^2	DW statistic	Coef. Indep.	Coef. Constant	t stat. indep	t statistic const.
Antarchile	1787	0,9869	0,04703	1,0718	-2,3715	368,076	-28,3527
Calichera	1787	0,9625	0,02836	0,9594	0,6823	214,056	5,9075
Campos	3744	0,9608	0,03923	0,8897	2,4116	303,055	33,5608
IAM	526	0,6726	0,0606	0,8915	2,8692	32,816	3,8842
Marinsa	3744	0,9436	0,0161	0,9655	0,5368	250,254	5,4442
Naviera	2659	0,9454	0,0139	1,2271	-5,9713	214,506	-42,4325
Oro Blanco	1703	0,9892	0,0728	0,9883	0,0268	394,7461	0,4294
Pacífico	2088	0,9862	0,0072	1,0681	-1,7585	387,1229	-25,0132
Tricahue	3492	0,5658	0,0026	0,8893	0,0227	67,4506	0,0676

Note: Method: Least Squares: Dependent: LN_MV_Variable; Independent and Constant: LN_NAV_variable, Constant.

In table 15, I can see that the slopes range from 1,22 to 0,88 with an average of 0,994. The Wald tests for coefficient restrictions (results not shown) have been ran to test whether the slope coefficients are statistically different from 1,0; the results are that it is not possible to reject this hypothesis in the nine cases.

Step c):

Having obtained the proposed long term linear relation between ln(MV) and ln(NAV), I obtained the residual series and tested the null hypothesis that the residuals have a unit root, using the ADF Test. The results are shown in table 16.

Table 16: ADF test for the residual of long run linear relationship between ln(MV) and ln(NAV)

Variable	ADF test statistic	Prob.	Test Critical Value 1%	Test Critical Value 5%	Test Critical Value 10%	Reject/ Non Reject
(RES_LR_Antarchile)	-4,63	0,0000	-3,434188	-2,862955	-2,567571	Reject ***
(RES_LR_Calichera)	-3,34	0,0008	-3,433809	-2,862955	-2,567570	Reject **
(RES_LR_Campos)	-4,13	0,0000	-3,431913	-2,862116	-2,567120	Reject ***
(RES_LR_IAM)	-3,21	0,0014	-3,442578	-2,866826	-2,569646	Reject **
(RES_LR_Marinsa)	-3,87	0,0001	-3,431911	-2,862115	-2,567120	Reject ***
(RES_LR_Naviera)	-2,89	0,0038	-3,432618	-2,862428	-2,567287	Reject **
(RES_LR_Oro Blanco)	-3,82	0,0001	-3,433995	-2,863037	-2,567614	Reject ***
(RES_LR_Pacífico)	-2,98	0,0028	-3,433285	-2,862723	-2,567446	Reject **
(RES_LR_Tricahue)	-1,82	0,0686	-3,432037	-2,862171	-2,567149	non reject

Note: ADF Unit Root Test; Exogenous: Constant; Null Hypothesis: Variable has a unit root (non stationary)

The following Figure 18 shows the residual graphs obtained from the previous regressions:

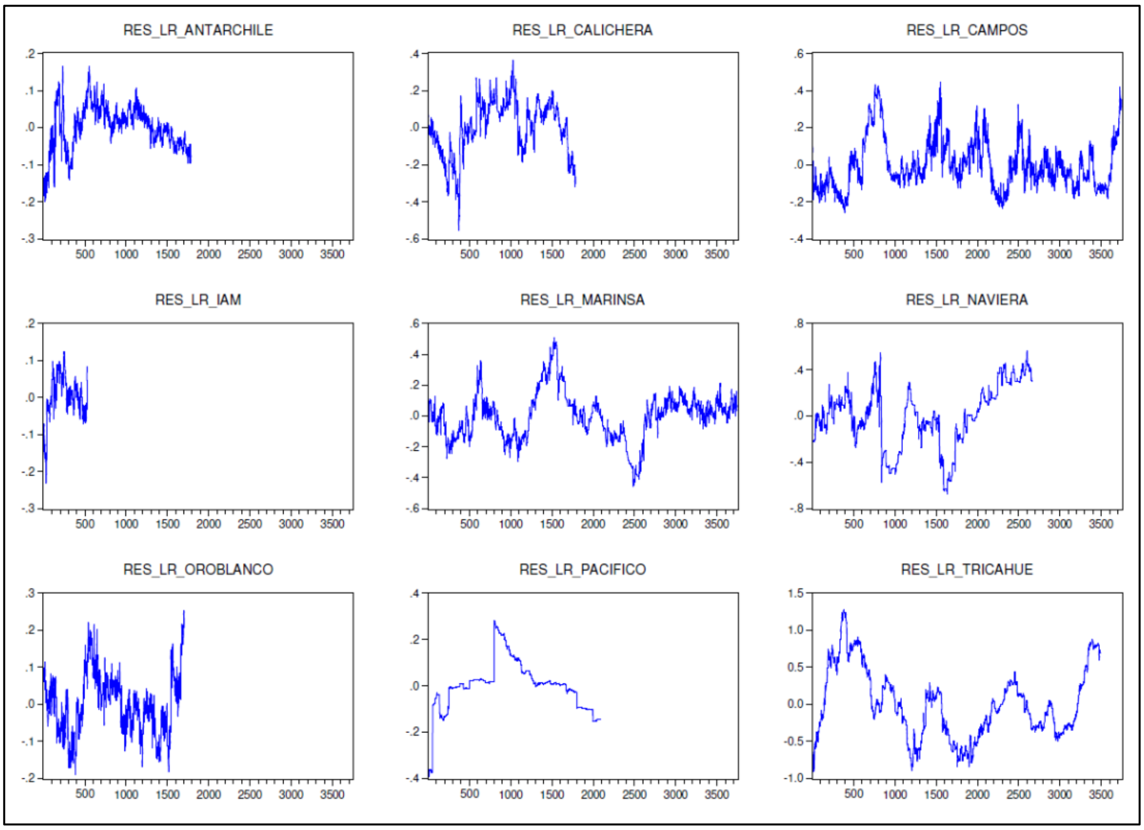


Figure 18. Residuals of the long run regression between ln(MV) and ln(NAV)

I can see from the graphs that error do not appear at an intuitive level to be stationary in all the cases. Notwithstanding, Table 16 shows that the null hypothesis of unit root in the error term of the long term proposed linear relation, is rejected at 95% or 99%

in all cases except one, Grupo Tricahue, which means that I can continue to step d) with eight of the nine groups.

<u>Step d):</u>

In this step I formulate the ECM for the eight remaining groups (all except Tricahue). The results are shown in table 17 and 18.

First, I can observe that, as expected, the autocorrelation of residuals is not present, which can be checked by inspecting Durbin Watson Statistic which, in all the cases is sensibly close to 2,0 a standard benchmark for no auto correlation of residuals.

Second, I can find that, as theory predicts, and as expected, the coefficient of the lagged error term of the long run linear relationship, is negative, for all groups except Pacífico.

Third, I can observe that the correlation of all regressions is not important, indicating that other variables are missing. The only case in which the correlation coefficient is important, is in Pacífico group, which has the error term with the wrong sign.

Table 18 shows the statistical significance of the variables: the lagged error coefficient is highly significant for all groups except for Pacífico, the constant term is significant in Campos, IAM and Marinsa, the short term component, the first difference of ln(MV) and ln(NAV), is significant for the eight groups, and the level variable ln(MV) is significant for Campos, IAM and Marinsa.

According to these results, I keep as candidates for Cointegration, the seven Groups, being the best three, Campos, IAM and Marinsa.

Table 17: Error Correction Model out from Engle Granger Procedure, for 8 candidate groups

Dependent D(LN_MV_variable)	# observ.	R^2	DW statistic	Constant	D(LN_NAV_var)	RES_LR_N_var(-1)	LN_MV_var
Antarchile	1786	0,3540	2,0985	-0,0019	0,5354	-0,0153	0,0001
Calichera	1786	0,0333	2,0534	-0,0214	0,1625	-0,0075	0,0009
Campos	3743	0,2694	2,0163	-0,0357	0,4128	-0,0205	0,0015
IAM	525	0,1131	1,7903	-0,3843	0,3481	-0,0363	0,0142
Marinsa	3743	0,0733	1,9751	-0,0286	0,2869	-0,0086	0,0011
Naviera	2658	0,0586	1,8953	-0,0093	0,3903	-0,0055	0,0004
Oro Blanco	1702	0,3044	2,1838	-0,0275	0,6738	-0,0316	0,0011
Pacífico	2087	0,7319	1,9233	0,0097	0,7627	0,0023	-0,0004

Note: Method: Least Squares; Dependent: D(LN_MV_variable); Independent: Constant-D(LN_NAV_variable) -RES-LR-LN_variable -LN_MV_variable.

Table 18: "t" statistics of the Error Correction Model out from Engle Granger Procedure, for 8 candidate groups

Dependent D(LN_MV_variable)	t stat Const	t stat D( )	t stat RES	t stat LN_MV
Antarchile	-0,1470	31,2268	-3,7011	0,1893
Calichera	-1,4210	7,4593	-2,4377	1,4916
Campos	-2,7579	36,3311	-7,5688	2,7757
IAM	-2,1624	7,7404	-3,1611	2,1648
Marinsa	-2,6624	16,4448	-4,7987	2,6868
Naviera	-0,8188	12,6420	-2,7221	0,7804
Oro Blanco	-1,7196	26,9439	-5,0503	1,7536
Pacífico	2,0868	75,3418	1,4577	-1,9879

Note: Method: Least Squares; Dependent: D(LN_MV_variable); Independent: Constant-D(LN_NAV_variable) -RES-LR-LN_variable -LN_MV_variable.

Step e):

Finally, I check the results of the ECM by testing the presence of a unit root in the residual. I expect to find no evidence of Unit root if the regressions go according to the ECM. The results are shown in table 19. The graph of these ECM residuals is shown in the following figure 19:

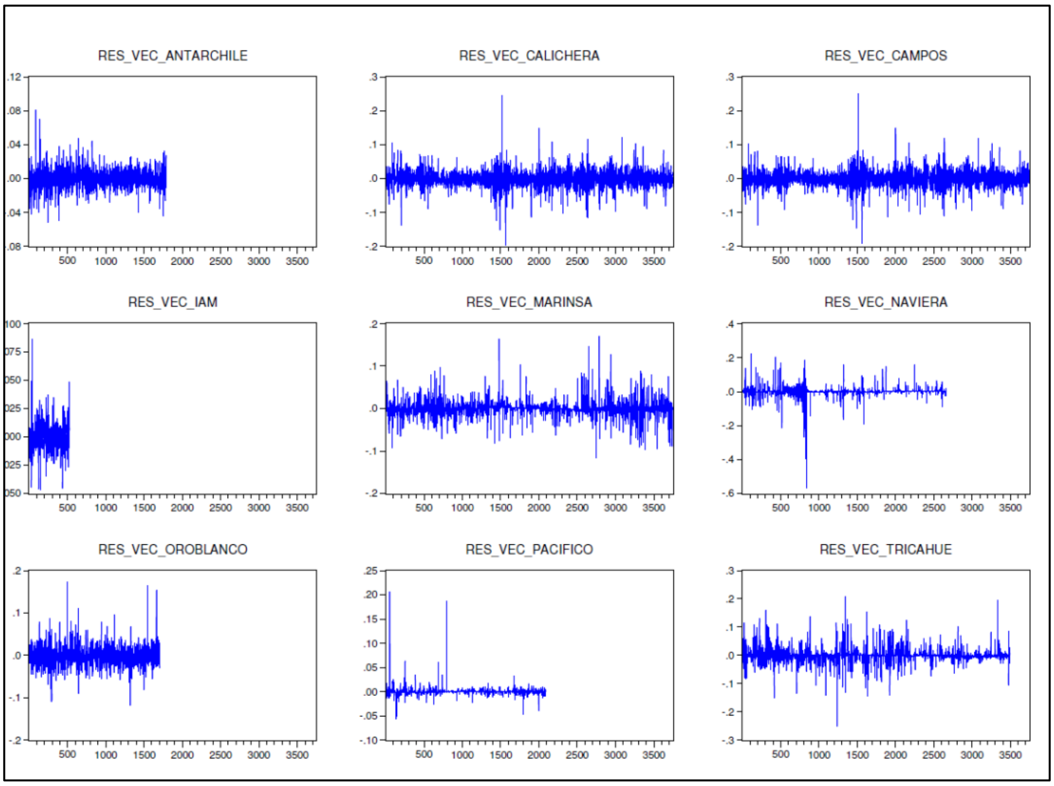


Figure 19. ECM residuals

I can see from the graphs, as first inspection, that all the residuals of the ECM seem to be stationary, or better, white noise. Of course, there is some evidence of not constant variance, and some extreme points that are nor predicted by the normal distribution constant variance zero mean white noise. As I can see in table 19, the residual for the ECM are stationary, this is equivalent to say that I reject the null hypothesis of unit root in all the seven remaining cases.

Table 19: results of ECM testing the presence of a unit root in the residual

Note: ADF Unit Root Test; Exogenous: Constant; Null Hypothesis: Variable has a unit root.

Variable	ADF test satistic	Prob.	Test Critical Value			Reject
			1%	5%	10%	
(RES_ECM_Antarchile)	-44,39	0,0000	-3,433811	-2,862955	-2,567571	***
(RES_ECM _Calichera)	-62,02	0,0000	-3,431912	-2,862115	-2,567120	***
(RES_ECM Campos)	-61,70	0,0000	-3,431912	-2,862115	-2,567120	***
(RES_ECM _IAM)	-20,56	0,0000	-3,442601	-2,866836	-2,569652	***
(RES_ECM Marinsa)	-60,44	0,0000	-3,431912	-2,862115	-2,567120	***
(RES_ECM Naviera)	-14,14	0,0000	-3,432627	-2,862432	-2,567290	***
(RES_ECM Oro Blanco)	-45,23	0,0000	-3,433991	-2,863035	-2,567613	***

Step f):

Finally, I intend to extract an overall assessment of the quality of the EC Model. To this point I have to accept that there is evidence of cointegration present in the groups. The discounts, although moving very randomly, appear to revert to a long-term value. What Cointegration means, is that the discounts of some groups seem to have a long-term logarithmic level, but it is important to say, that this result can be endogenous given that the recommendation of the analysts, to price Holding and NAV shares in a way that reflects an average of 20% discount by a default, no evidence statement, could be leading the observed results. What I can observe is that the evidence of long run stability is not present in all business groups, and arbitrage analysis must be studied case by case.

The reversion period measured in working days is the inverse of the absolute value of the coefficient of the error term in the ECM model. This is shown in the following schedule:

Table 20: Expected days for the discount to revert to the average according to the ECM

Group	Error term coeff	Expected days Revertion to LR relation
Antarchile	-0,0153	65
Calichera	-0,0075	134
Campos	-0,0205	49
IAM	-0,0363	28
Marinsa	-0,0086	116
Naviera	-0,0055	183
Oro Blanco	-0,0316	32

Further studies intending to correct for missing explanatory variables, and heteroscedasticity can be performed, and are not part of the scope of this work.

4.4.4. Conclusions

It is not simple to accept that there is strong or robust evidence for cointegration; I should be prepared to accept the conclusions with a 90% or even 95% confidence level. But I can state that there is at least weak evidence in favor of the cointegration hypothesis H2c. Nevertheless, the Engle Granger procedure gives us lights in this analysis. I have followed the procedure step by step, and I can conclude that there is some linear long run relationship between ln(MV) and ln(NAV) in seven of the nine groups analyzed. Hence there appears to be some degree of predictability. Whether the traders in the stock market can benefit of these relations is a matter of their ability to afford risks in short selling for the sufficient time.

Thus, hypothesis H2c is not rejected.

Given that I have found weak evidence of cointegrations of Market Value and Net Asset Value for a majority of the groups analyzed, in a further research I could find the direction of causality, this means answering the following question: does a change in NAV cause a change in MV (bottom-up approach) or does a change in MV causes a change in NAV (top-down approach. This phenomenon could be explained by arbitrage operations intended to capitalize a long run or steady state discount level.

4.5. Comovement Analysis

A final analysis of the groups discount time series intends to understand whether the different series move together with the other series. If this is found to some extent, it will mean that there is an eventual imperfection in the stock market, that otherwise states that prices follow random walks. Here I extend the concept to assume that EMH (Efficient Market Hypothesis) theory predicts that discounts follow random walk processes.

The idea comes from Andrei Shleifer, who develops a model of Investor Sentiment (a specification of the phenomena grouped under "Behavioral Economics").

According to Shleifer, "In the United States, smaller capitalization stocks, as well as other stocks held and traded predominantly by individual investors, are likely to be influenced by the same sentiment. The conceptually most important implication of the model is that it suggests that contrary to the basic notion of efficient markets, there will be a comovement in the prices of securities that are fundamentally unrelated to each other, solely because they are traded by similar investors, and therefore influenced by similar sentiment. The detection of comovement would contradict the basic principle of efficient markets that security prices should not move when there is no news since comovement represents a response to demand changes that are unlikely to be related to fundamental news for fundamentally unrelated securities. Methodologically this is a crucial implication of noise trader theory".

4.5.1. The Model of Investor Sentiment

I closely follow and borrow from Shleifer´s model, developed by DeLong et al. 2000. The basic model is a "stripped-down overlapping generations model with two-period-lived agents (Samuelson 1958)", without first period consumption, bequests, or labor supply issues. The economy contains two assets that pay identical dividends. One of the assets, the riskless asset, pays a fixed real dividend r. Riskless asset is in perfectly elastic supply and its price is always fixed at one. The dividend r paid on riskless asset is thus the riskless rate. The other asset, the risky asset, always pays the same fixed real dividend r as the riskless asset. But the risky asset is not in elastic supply: it is in fixed quantity, normalized at one unit. The price of risky asset in period t is denoted P_t. If the price of each asset were equal to the net present value of its future dividends (rational expectations), then both assets would be perfect substitutes and would sell for the same price of one in all periods. But this is not how the price of the risky asset is determined in the presence of noise traders.

The model assumes there are two types of agents that make portfolio decisions between both types of assets when young and sell their portfolio holdings when old to the new young. Types are Smart Money (SM), in proportion $1-\mu$, which correctly perceive the distribution of returns of the risky asset, i.e., act under rational expectations; and Noise Traders (NT), in proportion μ, that misperceive the true distribution of future risky asset price and can be "optimistic" or "pessimistic". Let the average misperception of NT be ρ^*, and denote ρ_t the current misperception. Then I have:

- $\rho_t \sim N(\rho^*, \sigma_p{}^2)$.

where

- ${}_t\sigma^2_{P_{t+1}} = E_t\left\{\left[P_{t+1} - E_t(P_{t+1})\right]^2\right\}$

is the one period forward looking variance of P_{t+1} .

Agents maximize a standard exponential utility of Wealth (risky wealth) with constant risk aversion

- γ: $U = -e^{-2\gamma W}$;

Making standard assumptions of portfolio theory (returns of the risky assets are normally distributed) this is equivalent to maximize

- $u = E(W) - \gamma\sigma^2_W$

Solving the model recursively and in steady state, for the present price level in t, of the risky asset, the current price is

- $P_t = 1 + \dfrac{\mu(\rho_t - \rho^*)}{1+r} + \dfrac{\mu\rho^*}{r} - \dfrac{2\gamma({}_t\sigma^2_{t+1})}{r}$

In steady state Shleifer obtains

- ${}_t\sigma^2_{P_{t+1}} = \sigma^2_{P_{t+1}} = \dfrac{\mu^2\sigma^2_P}{(1+r)^2}$

And finally, the steady state price function, which depends only on exogenous parameters and on public information about misperceptions of NT traders is:

- $P_t = 1 + \dfrac{\mu(\rho_t - \rho^*)}{1+r} + \dfrac{\mu\rho^*}{r} - \dfrac{2\gamma\mu^2{\sigma_p}^2}{r(1+r)^2}$

Assume P_t is the Market Value of the closed end fund, and the safe asset is its corresponding NAV. If the distribution of Noise traders misperception converges to a mass point at cero, then the price of the risky asset will be the fundamental price of 1. Now, letting $\rho_t - \rho^* = 0$ and $\rho^* = 0$ (no average misperception), I see that in the absence of NT misperceptions, the fund will trade at $P_t < 1$, i.e. at a discount, because of inherent noise trader risk which is assumed to be systematic, that means priced in equilibrium. Keeping $\rho^* = 0$, I can see that risky asset´s price will be even less than one when there is a pessimistic current misperception. And if I let $\rho_t = \theta\rho_{t-1}$ with θ<1 , i.e., misperception is an auto regressive process (as one may presume), I can find several periods of continuous misperceptions before they come back to zero. Now, if I assume that the misperception in produced by investor sentiment, it will be common to several closed end funds. Given that the noise trader risk is systematic, I can expect to see all funds moving together. This means that P_t for different funds will co move, meaning a statistically significant correlation among them. Finally, note that

- $P_t - P_{t-1} = \dfrac{\mu(\rho_t - \rho_{t-1})}{1+r}$,

meaning that for a common series of misperception, the first difference of prices (discounts) of several funds should also be correlated.

4.5.2. Comovement Statistical Analysis[46]*, Testing for Comovement*

Shleifer (2000) tests the model with monthly and annual data of 87 closed end funds in the United States for the period 1960-1986 and finds evidence that the implications of his model are present in the data. According to the Investor Sentiment Model, the levels as well the changes in the discount should be correlated. The test consists in calculating the pairwise correlation coefficient between discounts and checking for the significance of the correlation checking the corresponding "p" values of a two-sided test.

I test for the nine selected holding companies for the period 1993-2007 using daily discounts. This has several departures from Shleifer´s model: First, his model considers that the NAV of the closed end founds is riskless (its dividends are non-stochastic), and only the price of the shares of the closed end founds is risky. In the

46 The statistical procedures as well as their descriptions, are extracted from Eviews9SVR user´s manual. EviewsR is the statistical package used in this work.

holding companies, both, Market Value and NAV are risky. Second, I use daily data which have, naturally, more noise than the weakly or even monthly data. Figure 20 shows the daily discounts for the nine groups:

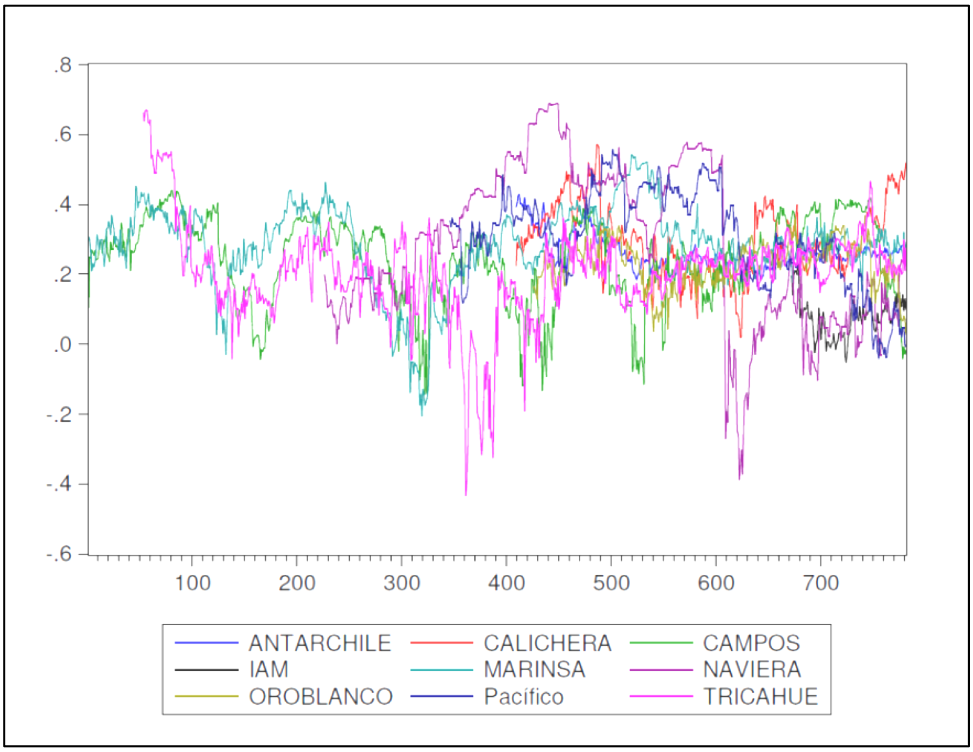


Figure 20. Daily discounts for the nine groups

In order to better visualize and detect possible comovements, as well as testing for comovement I have smoothed the series using Hodrick and Prescott Filter methodology.

<u>Hodrick and Prescott Filtering:</u>

Prescott filter is a two-sided linear filter that computes the smoothed series s from a non-smoothed series y by minimizing the variance of y around s, subject to a penalty that The Hodrick-Prescott Filter (1997) is a smoothing method widely used in macroeconomics to obtain a smooth estimate of the long-term trend component of a series. Hodrick- constrains the second difference of s . That is, the Hodrick-Prescott filter chooses s to minimize:

$$\sum_{t=1}^{t=T}(y_t - s_t)^2 + \lambda \sum_{t=2}^{t=T-1}((s_{t+1} - s_t) - (s_t - s_{t-1}))^2 .$$

The penalty parameter λ controls the smoothness of the series. The larger the λ the smoother the variance. As λ increases to infinity, s approaches to a linear trend.

The original criteria suggested by Hodrick-Prescott to fix λ are the following:

$$\lambda = 100 \text{ for annual data}$$
$$\lambda = 1.600 \text{ for quarterly data}$$
$$\lambda = 14.400 \text{ for monthly data}$$

Ravn and Uhlig (2002) suggest what is called the frequency power rule (number of periods per year divided by four, raised to the power of two, and multiplied by 1600. This is the rule I have followed in this procedure, using 261 working days per year. The smoothed series for the nine groups is shown in Figure 21. The discounts series have men named NNN_HPTREND.

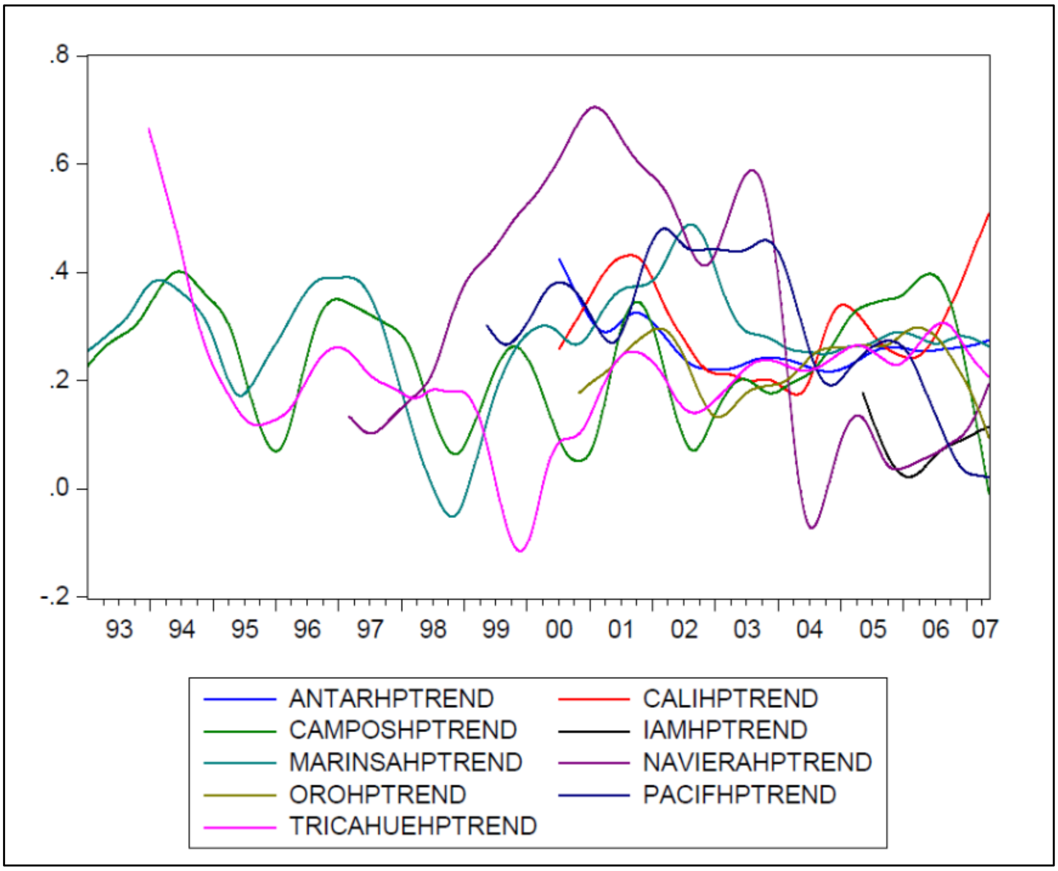


Figure 21. Daily discount series smoothed with HodrickandPrescott smoothing method

The previous smoothing procedure has permitted two important things: First, it is possible to visualize some comovement (that must be tested statistically), specially, if I sub-divide the time span in two portions: first portion, from 1993 to 2000, and second portion, from 2000 to 2007. This is the complete procedure I will follow, through selective search.

<u>Comovement Statistical Analysis[47]:</u>

The comovement analysis I have performed is a standard Covariance / Correlation Analysis.

The covariance analysis may be used to obtain different measures of association (covariances and correlations) and associated test statistics for several time series. In this analysis I compute measures of association of the ordinary centered type (Pearson Product Moment), using balanced designs, without weighting individual observations, and without the use of other conditioning variables.

In calculating Ordinary Centered Covariances, the sums-of-squared cross-products (SSCP) are computed using

$$SSCP(X,Y) = \sum_{i=1}^{n} (X_i - \hat{\mu}_X)(Y_i - \hat{\mu}_Y)$$

where μ_X, μ_Y are the estimates of the means of X, Y.

The covariances are computed by dividing the SSCP by the number of observations without a degrees-of-freedom correction (no conditioning variables):

$$\hat{\sigma}(X,Y) = \frac{\sum_i (X_i - \hat{\mu}_X)(Y_i - \hat{\mu}_Y)}{n-k} \quad \textit{with } k = 0, \text{no degrees of freedom correction}$$

where n is the number of observations associated with the observed X, Y, pairs, and k is a degree-of freedom adjustment term set to =0.

47 The statistical procedures as well as their descriptions, are extracted from Eviews9SVR user´s manual. EviewsR is the statistical package used in this work.

The correlation between the variables X and Y is computed from the following expression:

$$\hat{\rho}(X,Y) = \frac{\hat{\sigma}(X,Y)}{\sqrt{\hat{\sigma}(X,X)\cdot\hat{\sigma}(Y,Y)}}.$$

The test statistics and associated *p-value*s reported are for testing the null hypothesis H2d of no existence of investor sentiment, or no comovement, which I operationalize saying that a single correlation coefficient is equal to zero.

For ordinary Pearson and Spearman correlations, the *t-statistic* is computed as

$$t = \frac{r\sqrt{n-k-1}}{\sqrt{1-r^2}}$$

where r is the estimated correlation, and k is the number of conditioning variables. The *p-value* is obtained from a *t-distribution* with $n-k-1$ degrees of freedom. In this case of centered non-partial correlations with no conditioning variables, $k=1$, so the degrees of freedom are $n-2$.

4.5.3 Results and Analysis

Period, 1993-2000:

There are only data for 4 groups: Campos, Marinsa, Naviera and Tricahue. When I run the Covariance Analysis for the smoothed series, I obtained the results shown in Table 21.

Table 21: Correlation Matrix for the trended daily discount series 1993-2000

Correlation	**CAMPOS1993**	**MARINSA1993**	**NAVIERA1993**	**TRICAHUE1993**
CAMPOS1993	1,000000			
MARINSA1993	0,920203	1,000000		
NAVIERA1993	-0,384395	-0,131159	1,000000	
TRICAHUE1993	-0,046851	-0,237267	-0,859842	1,000000
T- statistic	**CAMPOS1993**	**MARINSA1993**	**NAVIERA1993**	**TRICAHUE1993**
CAMPOS1993	-------			
MARINSA1993	66,61559	--------		
NAVIERA1993	-11,79926	-3,749076	--------	
TRICAHUE1993	-1,329089	-6,921146	-47,72314	--------
Probability	**CAMPOS1993**	**MARINSA1993**	**NAVIERA1993**	**TRICAHUE1993**
CAMPOS1993	-------			
MARINSA1993	0,0000	------		

NAVIERA1993	0,0000	0,0002	--------	
TRICAHUE1993	0,1842	0,0000	0,0000	------
Cases	**CAMPOS1993**	**MARINSA1993**	**NAVIERA1993**	**TRICAHUE1993**
CAMPOS1993	805			
MARINSA1993	805	805		
NAVIERA1993	805	805	805	
TRICAHUE1993	805	805	805	805

I can observe a strong positive association between Campos and Marinsa in the levels of discount, is found. R2=0,92; p=0,000 (correlation is statistically different from 0). The rest of the possible associations are negative (Naviera and Tricahe) or statistically insignificant.

To check if the smoothing procedure distorts the results, I run the same test for the raw series obtaining the results shown in Table 22.

Table 22: Correlation Matrix for the daily discount series 1993-2000

Correlation	**CAMPOS1993**	**MARINSA1993**	**NAVIERA1993**	**TRICAHUE1993**
	1,000000			
MARINSA1993	0,799848	1,000000		
NAVIERA1993	-0,303783	-0,161694	1,000000	
TRICAHUE1993	-0,084477	-0,161998	-0,648714	1,000000
T- statistic	**CAMPOS1993**	**MARINSA1993**	**NAVIERA1993**	**TRICAHUE1993**
CAMPOS1993	-------			
MARINSA1993	37,76311	--------		
NAVIERA1993	-9,035392	-4,643050	--------	
TRICAHUE1993	-2,402435	-4,652018	-24,15506	--------
Probability	**CAMPOS1993**	**MARINSA1993**	**NAVIERA1993**	**TRICAHUE1993**
CAMPOS1993	-------			
MARINSA1993	0,0000	------		
NAVIERA1993	0,0000	0,0002	--------	
TRICAHUE1993	0,0165	0,0000	0,0000	------
Cases	**CAMPOS1993**	**MARINSA1993**	**NAVIERA1993**	**TRICAHUE1993**
CAMPOS1993	805			
MARINSA1993	805	805		
NAVIERA1993	805	805	805	
TRICAHUE1993	805	805	805	805

As I can see, results with the raw data are very similar those obtained with the smoothed data.

I conclude that I find a strong association between Campos and Marinsa for the period 1993-2000, weakly rejecting null hypothesis H2d.

The final test to this first period refers to the second prediction of the Comovement Model, Shleifer (2000), that is whether there is correlation or not between the first difference of the discounts. I run the Covariance Test to the smoothed first difference series and obtained the results shown in Table 23.

Table 23: Correlation matrix for the first difference of the trended daily discount series 1993-2000

Correlation	**DCAMPOS1993**	**DMARINSA1993**	**DNAVIERA1993**	**DTRICAHUE1993**
DCAMPOS1993	1,000000			
DMARINSA1993	0,999977	1,000000		
DNAVIERA1993	-0,940350	-0,940277	1,000000	
DTRICAHUE1993	0,673202	0,669784	-0,789943	1,000000
T- statistic	**DCAMPOS1993**	**DMARINSA1993**	**DNAVIERA1993**	**DTRICAHUE1993**
DCAMPOS1993	-------			
DMARINSA1993	1155, 073	--------		
DNAVIERA1993	-21,58779	-21,57347	--------	
DTRICAHUE1993	7,110430	7,044835	-10,06173	--------
Probability	**DCAMPOS1993**	**DMARINSA1993**	**DNAVIERA1993**	**DTRICAHUE1993**
DCAMPOS1993	-------			
DMARINSA1993	0,0000	------		
DNAVIERA1993	0,0000	0,0000	--------	
DTRICAHUE1993	0,0000	0,0000	0,0000	------
Cases	**DCAMPOS1993**	**DMARINSA1993**	**DNAVIERA1993**	**DTRICAHUE1993**
DCAMPOS1993	63			
DMARINSA1993	63	63		
DNAVIERA1993	63	63	63	
DTRICAHUE1993	63	63	63	63

As I can see, from the above, there is also a strong association (and statistically different from 0) between the smoothed series of the first difference for Groups Campos and Marinsa. This result together with the results of tests in the discount levels, permits us to infer that Campos and Marinsa have followed the implications of Shleifer´s model. They appear to be subject to a common factor (eventually investor sentiment) in levels and in first difference for the period 1993-2000.

Thus for 2 of four fundamentally unrelated groups, hypothesis H2d is rejected for the period 1993 to 2000.

<u>Period 2000-2007:</u>

For this period, I have data for the nine groups for a total of 1702 observations. Notwithstanding, when IAM group is included, I can test for only 526 observations (the last two years of the series).

In the following tables, I first show covariance analysis for the smoothed discount levels for the nine groups (2006-2007) including IAM (Table 24). Then I show the Covariance test for the smoothed first difference of the nine series (Table 25). I then continue by showing the same two tables, in this case for eight groups, all groups excepting IAM Table 26 and Table 27.

Table 24: Correlation matrix for nine groups trended Level 2000-2007

Correlation	ANTAR	CALICH	CAMPOS	IAM2000	MARINS	NAVIE	OROBLAN
ANTARCHILE2000	1,000000						
CALICHERA2000	0,712125	1,000000					
CAMPOS2000	-0,801863	-0,936080	1,000000				
IAM2000	-0,016265	0,586870	-0,490695	1,000000			
MARINSA2000	0,138021	-0,393537	0,252666	-0,660085	1,000000		
NAVIERA2000	0,453791	0,914268	-0,868190	0,795419	-0,636538	1,000000	
OROBLANCO2000	-0,795707	-0,976254	0,981903	-0,552163	0,305038	-0,889501	1,000000
PACIFICO2000	-0,607503	-0,865813	0,678725	-0,313049	0,339992	-0,680970	0,764030
TRICAHUE2000	-0,499669	-0,235737	0,549468	-0,075881	-0,322940	-0,239978	0,420227
t-statistic	**ANTAR**	**CALICH**	**CAMPOS**	**IAM2000**	**MARINS**	**NAVIE**	**OROBLAN**
ANTARCHILE2000	-------						
CALICHERA2000	23,21949	-------					
CAMPOS2000	-30,72001	-60,91122	-------				
IAM2000	-0,372363	16,59182	-12,89123	-------			
MARINSA2000	3,189982	-9,799167	5,977753	-20,11476	-------		
NAVIERA2000	11,65711	51,66139	-40,04943	30,04320	-18,89284	-------	
OROBLANCO2000	-30,0782	-103,1591	118,6834	-15,16019	7,332078	-44,56142	-------
PACIFICO2000	-17,50734	-39,60953	21,15587	-7,545280	8,275786	-21,28617	27,107811
TRICAHUE2000	-13,20450	-5,552758	15,05409	-1,742025	-7,810961	-5658704	10,60087
Probability	**ANTAR**	**CALICH**	**CAMPOS**	**IAM2000**	**MARINS**	**NAVIE**	**OROBLAN**
ANTARCHILE2000	-------						
CALICHERA2000	0,0000	-------					
CAMPOS2000	0,0000	0,0000	-------				
IAM2000	0,7098	0,0000	0,0000	-------			
MARINSA2000	0,0015	0,0000	0,0000	0,0000	-------		
NAVIERA2000	0,0000	0,0000	0,0000	0,0000	0,0000	-------	
OROBLANCO2000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	-------
PACIFICO2000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
TRICAHUE2000	0,0000	0,0000	0,0000	0,0821	0,0000	0,0000	0,0000
Cases	**ANTAR**	**CALICH**	**CAMPOS**	**IAM2000**	**MARINS**	**NAVIE**	**OROBLAN**
ANTARCHILE2000	526						
CALICHERA2000	526	526					
CAMPOS2000	526	526	526				
IAM2000	526	526	526	526			
MARINSA2000	526	526	526	526	526		
NAVIERA2000	526	526	526	526	526	526	
OROBLANCO2000	526	526	526	526	526	526	526
PACIFICO2000	526	526	526	526	526	526	526
TRICAHUE2000	526	526	526	526	526	526	526

Table 25: Correlation matrix for nine groups trended first difference 2000-2007

Correlation	DANTAR	DCALICH	DCAMPOS	DIAM2000	DMARINS	DOROBLAN	DPACIF
DANTARCHILE2000	1,000000						
DCALICHERA2000	-0,076026	1,000000					
DCAMPOS2000	-0,317300	-0,823039	1,000000				
DIAM2000	-0,471729	0,852129	-0,434590	1,000000			
DMARINSA2000	0,283837	-0,449666	0,261587	-0,649256	1,000000		
DOROBLANCO2000	-0,374813	-0,874814	0,939156	-0,544283	0,303186	1,000000	
DPACIFICO2000	0,668911	-0,472907	-0,092458	-0,804465	0,296738	0,077513	1,000000
DTRICAHUE2000	-0,596990	-0,318556	0,754448	0,207840	-0,321990	0,605381	-0,558993
DNAVIERA2000	-0,261321	0,873519	-0,715121	0,840673	-0,72557	-0,709344	-0,406627
s-statistic	**DANTAR**	**DCALICH**	**DCAMPOS**	**DIAM2000**	**DMARINS**	**DOROBLAN**	**DPACIF**
DANTARCHILE2000	------						
DCALICHERA2000	-1,743698	------					
DCAMPOS2000	-7,651806	-33,13667	------				
DIAM2000	-12,23492	37,23737	-11,03532	------			
DMARINSA2000	6,769543	-11,51313	6,19811	-19,52217	------		
DOROBLANCO2000	-9,245682	-41,29613	62,52776	-14,83762	7,276082	------	
DPACIFICO2000	20,57930	-12,27425	-2,123526	-30,97215	7,106221	1,778011	------
DTRICAHUE2000	-17,01801	-7,685501	26,28682	4,859250	-7,777872	17,39410	-15,41745
DNAVIERA2000	-6,191329	41,03746	-23,39660	35,50131	-24,11192	-23,01460	-10,17874
Probability	**DANTAR**	**DCALICH**	**DCAMPOS**	**DIAM2000**	**DMARINS**	**DOROBLAN**	**DPACIF**
DANTARCHILE2000	------						
DCALICHERA2000	0,0818	------					
DCAMPOS2000	0,0000	0,0000	------				
DIAM2000	0,0000	0,0000	0,0000	------			
DMARINSA2000	0,0000	0,0000	0,0000	0,0000	------		
DOROBLANCO2000	0,0000	0,0000	0,0000	0,0000	0,0000	------	
DPACIFICO2000	0,0000	0,0000	0,0342	0,0000	0,0000	0,0760	------
DTRICAHUE2000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
DNAVIERA2000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

From the two previous schedules, that show correlations of the discounts trended levels and the first difference in the discount level, for the last two years of the sample, I can observe strong associations in the cases of Antarchile- Calichera; Antarchile – Naviera; Calichera-IAM; Calichera-Naviera; Campos-Oro Blanco; Campos-Pacífico; Campos-Tricahue; IAM-Naviera; Oro Blanco-Pacífico.

For the first differences I can find strong association in the cases of: Antarchile-Pacífico; Calichera-IAM; Calichera-Naviera; Campos-Oro Blanco; Campos-Tricahue; IAM-Naviera; Oro Blanco-Tricahue.

To find evidence of comovement, discount levels as well as first difference must show strong association, pairwise. This is the case for: Calichera- IAM , Calichera-Naviera; Campos-Oro Blanco, Campos-Tricahue; IAM-Naviera. This permits us to infer that the five previous pair of groups have followed the implications of Shleifer´s model. They appear to be subject to common factors (eventually investor sentiment) in levels and in first difference for the period 2005-2007, thus rejecting again hypothesis H2d of no comovement.

Now, I proceed to test for comovement between eight groups (the original nine groups less IAM), for which I have seven years of daily observations. The results are shown in Table 26:

Table 26: Correlation matrix for eight groups trended Level 2000-2007

Correlation	**ANTAR**	**CALICH**	**CAMPOS**	**MARINS**	**NAVIE**	**OROBLAN**	**PACIFIC**
ANTARCHILE2000	1,000000						
CALICHERA2000	0,696073	1,000000					
CAMPOS2000	0,093866	0,045642	1,000000				
MARINSA2000	0,148800	0,095060	-0,360911	1,000000			
NAVIERA2000	0,495144	0,,211792	-0,462356	0,576009	1,000000		
OROBLANCO2000	0,246640	0,138439	0,760675	-0,057686	-0,311233	1,000000	
PACIFICO2000	-0,041754	-0,462287	-0,332890	0,584160	0,651892	-0,132765	1,000000
TRICAHUE2000	-0,006170	0,155879	0,830645	-0,571053	-0,523138	0,491765	-0,563118
t-statistic	**ANTAR**	**CALICH**	**CAMPOS**	**MARINS**	**NAVIE**	**OROBLAN**	**PACIFIC**
ANTARCHILE2000	------						
CALICHERA2000	39,98532	------					
CAMPOS2000	3,888482	1,884403	------				
MARINSA2000	6,206087	3,938427	-15,96087	------			
NAVIERA2000	23,50490	8,937751	-21,50578	29,06186	------		
OROBLANCO2000	10,49650	5,765177	48,33005	-2,383116	-13,50707	------	
PACIFICO2000	-1,723564	-21,50166	-14,55986	29,68393	35,45522	-5,524560	------
TRICAHUE2000	-0,254493	6,508506	61,52720	-28,69002	-25,31641	23,29310	-28,10437
Probability	**ANTAR**	**CALICH**	**CAMPOS**	**MARINS**	**NAVIE**	**OROBLAN**	**PACIFIC**
ANTARCHILE2000	------						
CALICHERA2000	0,0000	------					
CAMPOS2000	0,0001	0,0597	------				
MARINSA2000	0,0000	0,0001	0,0000	------			
NAVIERA2000	0,0000	0,0000	0,0000	0,0000	------		
OROBLANCO2000	0,0000	0,0000	0,0000	0,0173	0,0000	------	
PACIFICO2000	0,0850	0,0000	0,0000	0,0000	0,0000	0,0000	------
TRICAHUE2000	0,7991	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Table 27: Correlation matrix for eight groups trended first difference 2000-2007

Correlation	DANTAR	DCALICH	DCAMPOS	DMARINS	DOROBLAN	DPACIF	DTRICAH
DANTARCHILE2000	1,000000						
DCALICHERA2000	0,150594	1,000000					
DCAMPOS2000	0,212139	-0,013260	1,000000				
DMARINSA2000	-0,387511	-0,030050	-0,152032	1,000000			
DOROBLANCO2000	0,049695	-0,098617	0,639768	0,068727	1,000000		
DPACIFICO2000	0,297607	-0,431984	0,011504	0,009822	0,092946	1,000000	
DTRICAHUE2000	0,098438	0,260590	0,832311	-0,192289	0,509987	-0,119248	1,000000
DNAVIERA2000	0,149070	0,398669	-0,048803	-0,169932	-0,175245	0,255970	0,263663
statistic	**DANTAR**	**DCALICH**	**DCAMPOS**	**DMARINS**	**DOROBLAN**	**DPACIF**	**DTRICAH**
DANTARCHILE2000	------						
DCALICHERA2000	6,280779	------					
DCAMPOS2000	8,950417	-0,546781	------				
DMARINSA2000	-17,33172	-1,239565	-6,342143	------			
DOROBLANCO2000	2,051496	-4,086009	34,32137	2,840397	------		
DPACIFICO2000	12,85302	-19,74887	0,474362	0,405005	3,848923	------	
DTRICAHUE2000	4,078504	11,12892	61,90963	-8,079046	24,44514	-4,952052	------
DNAVIERA2000	6,215749	17,92349	-2,014608	-7,109866	-7,339130	10,91762	11,26988
Probability	**DANTAR**	**DCALICH**	**DCAMPOS**	**DMARINS**	**DOROBLAN**	**DPACIF**	**DTRICAH**
DANTARCHILE2000	------						
DCALICHERA2000	0,0000	------					
DCAMPOS2000	0,0000	0,5846	------				
DMARINSA2000	0,0000	0,2153	0,0000	------			
DOROBLANCO2000	0,0404	0,0000	0,0000	0,0046	------		
DPACIFICO2000	0,0000	0,0000	0,6353	0,6855	0,0001	------	
DTRICAHUE2000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	------
DNAVIERA2000	0,0000	0,0000	0,0441	0,0000	0,0000	0,0000	0,0000

From the two previous tables 26 and 27, that show correlations of the discounts trended levels and the first difference in the discount level, for the last two years of the sample, I can observe strong associations in the cases of: Antarchile-Calichera, Antarchile-Naviera, Campos-Oro Blanco, Campos-Tricahue, Marinsa-Naviera, Marinsa-Pacífico, Naviera-Pacífico and Oro Blanco-Tricahue.

For the first differences I can find weak (correlation coefficient positive but less than 0.5) association in the cases of: Antarchile-Calichera, Antarchile-Campos, Antarchile-Pacífico, Antarchile-Naviera, Calichera-Tricahue, Calichera-Naviera, Pacífico-Naviera, Tricahue-Naviera; and strong association between Campos-Oro Blanco, Campos-Tricahue and Oro Blanco-Tricahue.

In this case I find conditions to accept rather weak evidence of comovement: discount levels as well as first difference must show strong correlation in levels, and weak correlation in first difference, pairwise. This is the case for three pairs of groups: Antarchile-Calichera; Antarchile-Naviera; Naviera-Pacífico.

Finally, I use the strong conditions to find evidence of comovement: discount levels as well as first difference must show strong correlation in levels, and in first difference, pairwise. This is the case for the next three pairs of groups: Campos-Oro Blanco; Campos-Tricahue; Oro Blanco-Tricahue.

This permits us to weakly infer that the six previous pair of groups have followed the implications of Shleifer´s model in a weak or strong sense. They appear to be subject to common factors (eventually investor sentiment) in levels and in first difference for the period 2000-2007, and I again reject the no comovement hypothesis H2d.

4.5.4. Conclusions

From the comovement analysis for the groups discount series I cannot conclude that there is no comovement between some of the series pairwise. The analysis has closely followed Shleifer (2000), except for the fact that I have used daily detrended data instead of monthly or annual data. At the same time, I cannot conclude that there is comovement in the sense of Shleifer because the associations between all the groups together do not exist. I can observe for several combinations of structurally and fundamentally nonrelated groups, of course, but not for all the combinations available.

What is interesting is that in every time period I have analyzed, I have found that, locally, the investor sentiment appears to exist, implying a market inefficiency could be present, thus predictability, that eventually can be exploited by arbitrageurs. Again, the issue of endogeneity is present.

Finally, it is interesting to note that as explained in the introduction to this chapter, several statistical tests were made trying to find an association between holding discounts and several macroeconomic variables proxies to expectations, without results. This implies that investor sentiment can be driven by causes different to general market expectations.

Section 5. Summary and Conclusions

The aim of this work has been in several dimensions: In first place, I have analyzed several Chilean business groups using simple criteria for selection, that is the group must be as simple as possible, in order to eliminate two factors, I did not intend to analyze: the diversification factor, and the non-public companies' valuation issue. With this I select a set of 9 business groups, which are fully described.

In second place, I have used a unique and proprietary data base for the nine selected Chilean groups constructed under my direction and permitted to calculate very reliable groups discounts on three frequencies: daily, weekly and monthly. In the appendixes I document the standard statistics for the discounts. The discounts show high averages and high volatility.

Third, I test for stock market inefficiency. Or null hypothesis H2 is that the market is efficient, hence the discount is not predictable. To test my hypothesis, I use four sub hypotheses and four complementary methods, whose main objective is to check whether the discounts can be predicted in some way. If that were the case, stock market would present inefficiency, and main actors in the stock market would be correct in arbitraging on the discounts.

The four methods are: Unit Root Tests (test for stationarity -for my purposes predictability- of a time series); Variance Ratio Tests (test for the presence of a mean reversion of a time series, or martingale behavior (hence predictability again); Cointegration Analysis (tests for a long run stationary relationship between the two components of the discount: Market Capitalization of the holding company (Market Value or MV) and its Net Asset Value (NAV); Comovement Analysis (that tests for another aspect of the Efficient Market Hypothesis, that is the presence of investor sentiment, which makes that the stocks, or in this case the discount, of structurally and fundamentally unrelated businesses move together, departing from the random walk hypothesis.

The results of these tests show that there is some degree of inefficiency in the Chilean stock market during the period 1993-2007.

The Unit root tests show that six out of nine groups present random walk behavior regardless of the frequency of data. This must be considered as a not concluding evidence against non-predictability, according to my null hypothesis H2a.

Variance ratio tests go more strongly against my hypothesis H2b. The VR results suggest that only the minority of the groups present martingale behavior, for my purposes unpredictability of the discounts, showing a higher degree of market inefficiency than that showed with the Unit root tests. The results of the VR test permit me to reject the null hypothesis H2b using high frequency data (daily and weekly) but are more in favor -although still against- of the null hypothesis H2b if I use monthly data.

Cointegration analysis results suggest that with a at least 90% confidence level, I have at least one long run stationary relationship between the lnMarket Value and the lnNav, for almost all the groups, which represent the discount, showing reversion periods of several months. I cannot conclude that this kind of information is enough to enter in arbitrage operations, but I can conclude that there is at least weak evidence in favor of market inefficiency, this means not been able to reject hypothesis H2c.

Finally, the comovement analysis is less conclusive against my null hypothesis H2d; there is some evidence of comovement, and if I accept the underlying theory, of investor sentiment, but is far from been a general phenomenon in the time span of analysis.

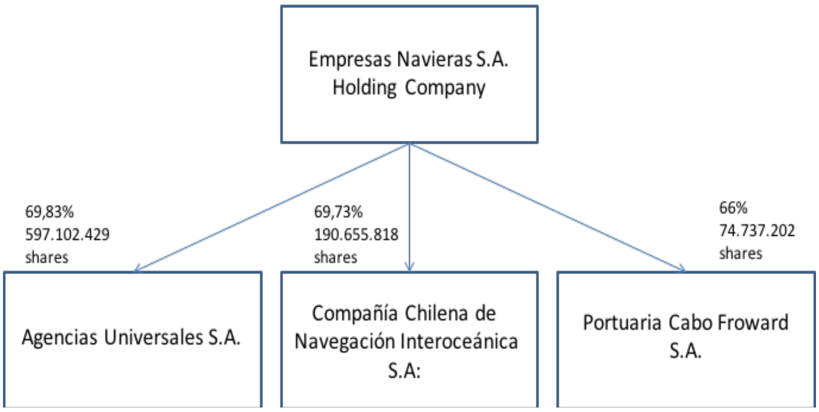
Section 6. Avenues for further research

From an economic perspective, not high-developed countries, like Chile, had not databases containing enough information to contrast several models constructed in high-developed countries. In this thesis I had the opportunity to organize information about the Chilean stock market and accounting information about the firms traded in it, in a robust data base. Using this organized information, I have worked to contrast the hypothesis shown in this chapter. If I had a new opportunity, or if the Chilean financial authorities develop a new database with the relevant information for the nine selected groups, analyzed in this chapter, for the recent years, I have the will, and the tools to expand the conclusions, and if it possible to extend this work to other groups. All the statistical analysis could be done in a new temporal horizon, confirming o contradicting the conclusions of this chapter along the time.

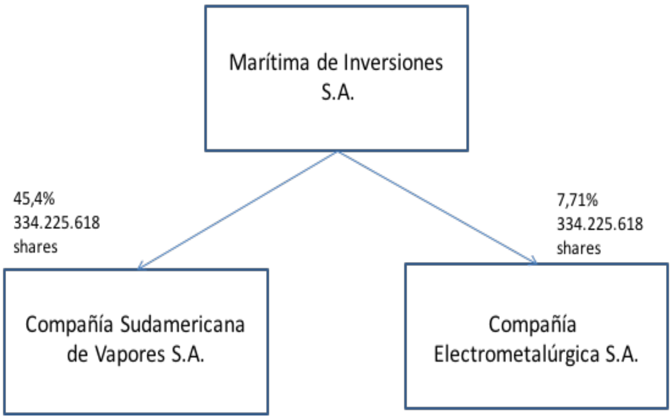
Appendixes

Appendix 1: Structure of Selected Chilean Business Groups

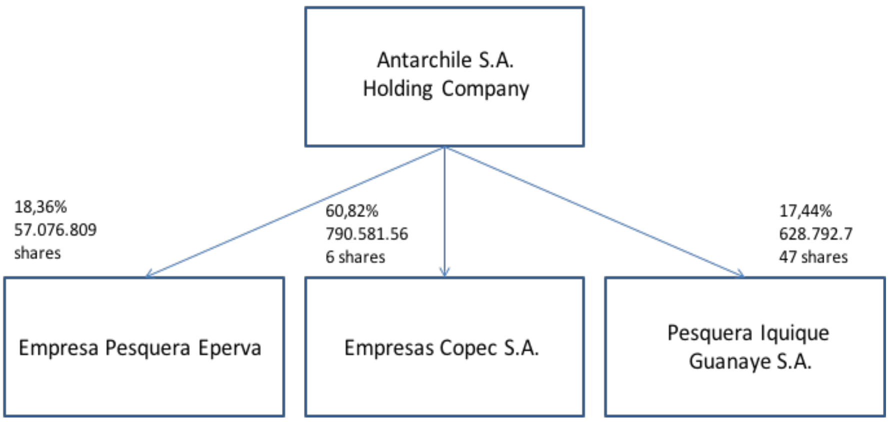
Group 7



Gruoup 8



Group 9



Appendix 2: Percentage of the value of investments in shares out of total assets of the investment firms.

The criterion used to select the investment companies was that 95% or more of the investments in related companies should be in stock market listed shares. Another significant element is the proportion or percentage of these investments out of the total assets of the investment company. This percentage in calculated for each quarter. A percentage close to 100% as possible is desirable. If that is the case, then the company chosen practically only has investments in shares of other companies and practically does not have any other activity apart from managing its investments. In some cases, other accounts of current assets are included since they become a significant part of total assets. Under those circumstances, the percentage of the value of investment out of total assets is below 90% but is due only to an increase in a current asset account. In the discount calculation methodology, all the various assets of IER (investment in related companies) and Investment in Other companies are totalized, and all the liabilities are subtracted to obtain the Net Asset Value. Therefore, the NAV already includes the effect of an increase in a current asset and does not bias the discount. The results of the groups studied are presented here below:

Period	Quarter	Group: investmens/total assets								
		Antarchile	Campos	Naviera	Iam	Marinsa	Pacifico v	Oro blanco	Calichera	Tricahue
1993	I									
	II									
	III									
	IV		98,85%			98,76%				95,89%
1994	I		98,97%			99,37%				98,80%
	II		97,96%			99,47%				99,75%
	III		99,35%			99,33%				96,17%
	IV		99,53%			98,99%				95,53%
1995	I		89,48%			99,70%				99,72%
	II		87,64%			99,09%				99,87%
	III		93,76%			96,77%				93,11%
	IV		99,78%			97,48%				91,30%
1996	I		99,67%			98,46%				95,99%
	II		97,28%			98,86%				96,97%
	III		97,81%			98,72%				96,02%
	IV		98,32%			98,64%				94,49%
1997	I		99,76%	97,51%		99,23%				88,67%

Perio d	Quarte r	Group: investmens/total assets								
		Antarchile	Campos	Naviera	Iam	Marinsa	Pacificov	Oro blanco	Calichera	Tricahue
	II		98,40%	93,38%		98,08%				98,07%
	III		99,70%	94,53%		97,69%				95,37%
	IV		99,56%	97,58%		98,27%				96,08%
1998	I		99,49%	98,80%		96,52%				96,25%
	II		99,28%	98,01%		98,98%				99,59%
	III		99,44%	97,87%		98,87%				99,81%
	IV		97,27%	98,28%		98,74%				99,91%
1999	I		99,15%	98,54%		99,23%	45,10%	96,58%	91,57%	99,91%
	II		90,97%	98,38%		98,92%	42,64%	96,69%	84,97%	99,91%
	III		98,88%	98,76%		98,79%	51,61%	97,72%	85,20%	99,95%
	IV		97,72%	99,01%		98,49%	49,89%	97,50%	86,10%	99,97%
2000	I		98,85%	99,32%		98,49%	69,75%	97,40%	87,94%	99,97%
	II		99,04%	97,66%		98,15%	79,86%	95,64%	81,58%	99,95%
	III	98,50%	99,06%	99,22%		97,82%	80,53%	89,10%	79,52%	99,95%
	IV	97,10%	96,76%	99,73%		98,86%	82,70%	91,25%	79,12%	99,97%
2001	I	97,70%	96,76%	99,65%		99,68%	80,22%	92,74%	86,00%	99,44%
	II	98,60%	96,71%	99,71%		98,93%	84,44%	95,78%	83,94%	99,65%
	III	99,20%	96,83%	99,44%		98,53%	84,77%	96,01%	83,36%	99,47%
	IV	99,20%	96,44%	98,46%		99,48%	85,93%	97,54%	86,06%	99,71%
2002	I	99,40%	96,39%	98,67%		99,69%	84,70%	97,60%	88,47%	99,16%
	II	98,50%	97,23%	98,82%		99,65%	87,30%	98,41%	76,40%	99,71%
	III	98,80%	97,28%	97,77%		98,93%	91,46%	98,95%	85,06%	97,53%
	IV	97,40%	97,22%	97,50%		99,06%	91,65%	98,82%	97,33%	95,21%
2003	I	97,70%	97,29%	98,95%		99,55%	88,27%	98,92%	97,57%	98,20%
	II	98,00%	98,71%	98,95%		98,71%	91,69%	98,55%	95,66%	93,51%
	III	98,10%	98,93%	99,41%		98,37%	91,91%	98,72%	96,99%	89,56%
	IV	96,20%	99,33%	99,32%		98,74%	94,43%	98,83%	96,31%	92,19%
2004	I	96,60%	99,36%	99,47%		99,30%	95,34%	98,98%	96,47%	89,59%
	II	97,20%	99,53%	98,48%		98,68%	95,97%	98,86%	95,69%	96,33%
	III	99,70%	99,55%	99,13%		98,21%	91,64%	99,09%	95,95%	99,52%
	IV	99,10%	99,51%	99,45%		97,66%	83,11%	98,97%	96,44%	99,08%
2005	I	92,20%	98,84%	99,63%		99,20%	95,75%	98,61%	93,84%	99,91%
	II	94,10%	99,11%	97,97%		98,45%	95,15%	96,17%	82,92%	95,02%
	III	94,30%	99,17%	96,22%	99,45%	93,89%	93,09%	97,23%	91,22%	92,63%
	IV	93,50%	96,53%	99,16%	99,41%	99,20%	87,44%	97,27%	91,38%	93,03%
2006	I	94,10%	97,27%	99,53%	99,42%	99,79%	95,61%	97,51%	91,77%	90,40%
	II	96,40%	99,16%	98,75%	99,37%	99,55%	87,77%	97,79%	91,23%	94,57%
	III	99,70%	99,21%	99,15%	96,48%	99,52%	57,34%	98,68%	98,26%	92,67%
	IV	98,10%	99,25%	98,96%	99,59%	99,64%	60,99%	96,41%	96,48%	92,44%
2007	I	98,00%	99,31%	99,15%	99,83%	99,64%	75,38%	99,32%	95,71%	99,93%

Perio d	Quarte r	Group: investmens/total assets								
		Antarchile	Campos	Naviera	Iam	Marinsa	Pacificov	Oro blanco	Calichera	Tricahue
	II	99,50%	98,24%	98,42%	95,81%	99,60%	56,57%	99,27%	93,11%	92,16%
	III	99,60%	98,34%	95,84%	96,78%	99,61%	57,52%	96,12%	73,21%	89,47%
	IV	97,70%	82,69%	96,68%	99,83%	99,71%	62,79%	96,13%	72,94%	84,16%
MEAN		97,50%	97,60%	98,40%	98,60%	98,80%	79,20%	97,20%	89,00%	96,30%
STD DEV		2,01%	3,12%	1,34%	1,57%	0,97%	16,28%	2,24%	7,25%	3,82%

Appendix 3: Descriptive Statistics

Statistics own sample

Group		Daily discount					
#	**Name**	# Observ	min	Max	Mean	std.dev	CV
1	Tricahue	3492	-0,433687	0,700888	0,202700	0,134264	0,66
2	Campos	3744	-0,244863	0,452418	0,238420	0,123192	0,52
3	IAM	526	-0,052519	0,259501	0,075852	0,053949	0,71
4	Pacífico	2088	-0,040716	0,560094	0,306713	0,136377	0,44
5	Oro Blanco	1703	0,028230	0,368035	0,230054	0,064884	0,28
6	Calichera	1787	-0,007218	0,597520	0,296905	0,101261	0,34
7	Naviera	2659	-0,425389	0,740949	0,325915	0,249575	0,77
8	Marinsa	3744	-0,217054	0,550147	0,282065	0,116965	0,41
9	Antarchile	1787	0,158698	0,429145	0,263108	0,049079	0,19

Group		Weekly discount					
#	**Name**	# Observ	min	Max	Mean	std.dev	CV
1	Tricahue	729	-0,433687	0,670721	0,203002	0,133014	0,66
2	Campos	781	-0,143408	0,439492	0,239316	0,122369	0,51
3	IAM	110	-0,052519	0,257004	0,075911	0,056340	0,74
4	Pacífico	436	-0,040716	0,557868	0,306531	0,136139	0,44
5	Oro Blanco	356	0,028230	0,368035	0,229557	0,064380	0,28
6	Calichera	373	0,019551	0,571371	0,297750	0,101774	0,34
7	Naviera	556	-0,387561	0,690119	0,285194	0,223485	0,78
8	Marinsa	781	-0,206713	0,541968	0,283109	0,117044	0,41
9	Antarchile	373	0,183487	0,428609	0,263279	0,048433	0,18

Group		Monthly discount					
#	**Name**	# Observ	min	Max	Mean	std.dev	CV
1	Tricahue	186	-0,288030	0,646399	0,201794	0,125012	0,62
2	Campos	198	-0,159563	0,434199	0,243138	0,126749	0,52
3	IAM	42	-0,011238	0,257004	0,091752	0,055624	0,61
4	Pacífico	119	-0,473310	0,542788	0,250243	0,199783	0,80
5	Oro Blanco	100	0,046349	0,358692	0,213077	0,075263	0,35
6	Calichera	105	0,065897	0,545494	0,305778	0,102446	0,34
7	Naviera	145	-0,271319	0,690119	0,287320	0,208325	0,73

Group		Monthly discount					
#	**Name**	# Observ	min	Max	Mean	std.dev	CV
8	Marinsa	198	-0,174687	0,528531	0,277085	0,115511	0,42
9	Antarchile	103	0,185605	0,419816	0,263797	0,049484	0,19

Statistics common sample

Group		Daily discount				
#	Name	# Observ	min	max	Mean	std.dev
1	Tricahue	526	0,145916	0,466679	0,257911	0,056776
2	Campos	526	-0,136392	0,420416	0,307828	0,116365
3	IAM	526	-0,052519	0,259501	0,075852	0,053949
4	Pacífico	526	-0,040716	0,346936	0,161031	0,110522
5	Oro Blanco	526	0,028230	0,368035	0,247629	0,070196
6	Calichera	526	0,175638	0,526642	0,316704	0,088049
7	Naviera	526	-0,223044	0,298536	0,080522	0,085623
8	Marinsa	526	0,144901	0,363008	0,276411	0,036609
9	Antarchile	526	0,228141	0,293667	0,260420	0,013131

Group		Weekly discount				
#	Name	# Observ	min	max	Mean	std.dev
1	Tricahue	110	0,147097	0,466679	0,259169	0,058652
2	Campos	110	-0,041034	0,420018	0,308619	0,113880
3	IAM	110	-0,052519	0,257004	0,075911	0,056340
4	Pacífico	110	-0,040716	0,319003	0,160654	0,110165
5	Oro Blanco	110	0,028230	0,368035	0,247619	0,069420
6	Calichera	110	0,179801	0,519583	0,31621	0,087684
7	Naviera	110	-0,103301	0,292564	0,079110	0,085344
8	Marinsa	110	0,153719	0,363008	0,278517	0,036814
9	Antarchile	110	0,235676	0,292989	0,260780	0,013164

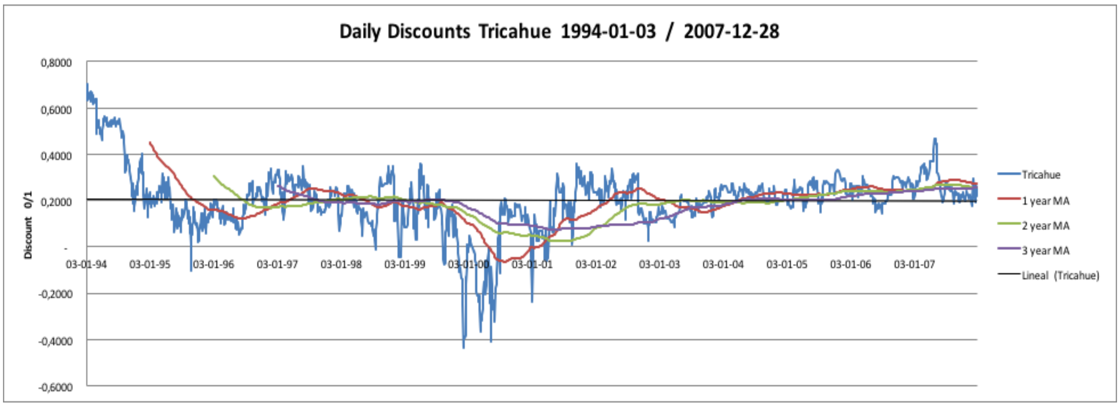
Group		Monthly discount				
#	Name	# Observ	Min	max	Mean	std.dev
1	Tricahue	42	0,134842	0,466501	0,230927	0,065505
2	Campos	42	-0,056143	0,434199	0,296740	0,129886
3	IAM	42	-0,011238	0,257004	0,091752	0,055624
4	Pacífico	42	-0,473310	0,346936	0,066412	0,202604
5	Oro Blanco	42	0,046349	0,344123	0,196952	0,088933
6	Calichera	42	0,190022	0,545494	0,336058	0,092025
7	Naviera	42	-0,076251	0,397280	0,165166	0,143134
8	Marinsa	42	0,124946	0,365734	0,268394	0,054072
9	Antarchile	42	0,193333	0,298164	0,256454	0,020806

Appendix 4: Comprehensive Data span for Group discounts (beyond 2007 used for descriptive statistics and monthly frequency teste only)

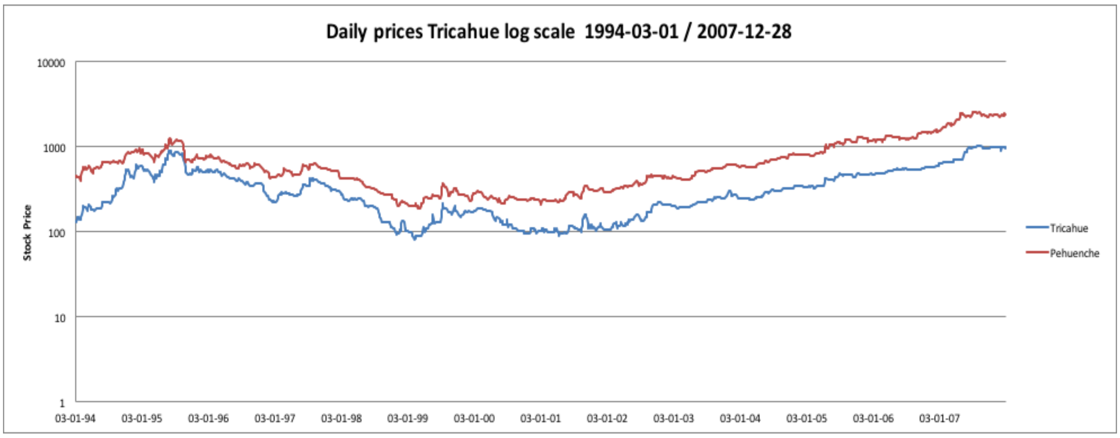
Group		**Companies**	Start	End
#	Nombre	Nombre		
1	Tricahue	Inversiones Tricahue S.A.	ene-94	jun-09
		Empresa Eléctrica Pehuenche S.A.	ene-94	jun-09
2	Campos	Sociedad de Inversiones Campos Chilenos S.A.	ene-93	jun-09
		Empresas Iansa S.A.	ene-93	jun-09
3	IAM	Inversiones Aguas Metropolitanas S.A.	nov-05	abr-09
		Aguas Andinas S.A.	nov-05	abr-09
4	Pacífico	Pacífico V Región S.A.	ago-99	jun-09
		Sociedad Punta del Cobre S.A.	ago-99	jun-09
5	Oro Blanco	Sociedad de Inversiones Oro Blanco S.A.	mar-01	jun-09
		Sociedad de Inversiones Pampa Calichera S.A.	mar-01	jun-09
6	Calichera	Sociedad de Inversiones Pampa Calichera S.A.	oct-00	jun-09
		Sociedad Química y Minera de Chile S.A.	oct-00	jun-09
7	Naviera	Empresas Navieras S.A.	abr-97	jun-09
		Agencias Universales S.A.	abr-97	jun-09
		Compañía Chilena de Navegación Interoceánica S.A.	abr-97	jun-09
		Portuaria Cabo Froward S.A.	abr-97	jun-09
8	Marinsa	Marítima de Inversiones S.A.	ene-93	jun-09
		Compañía Sudamericana de Vapores S.A.	ene-93	jun-09
		Compañía Electro Metalúrgica S.A	ene-93	jun-09
9	Antarchile	Antarchile S.A.	oct-00	abr-09
		Empresa Pesquera Eperva	oct-00	abr-09
		Empresas Copec S.A.	oct-00	abr-09
		Pesquera iquique Guanaye S.A.	oct-00	abr-09

Appendix 5: discount between the stock market capitalization of the investment companies, and Net Asset Value

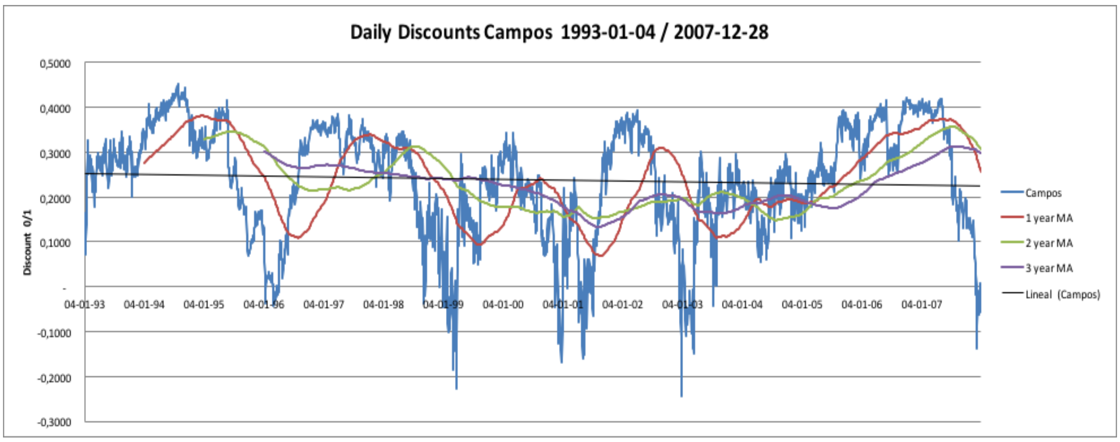
Grupo 1: Tricahue



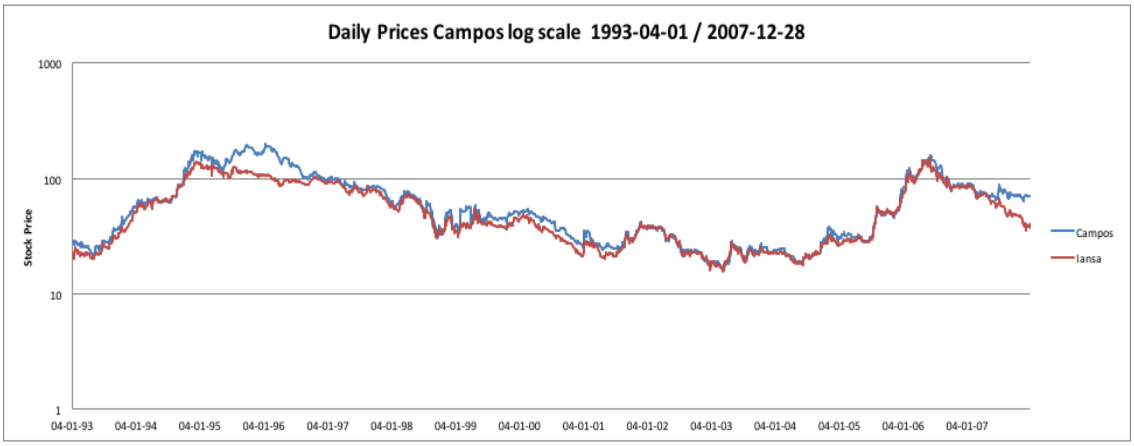
Grupo 1: Tricahue



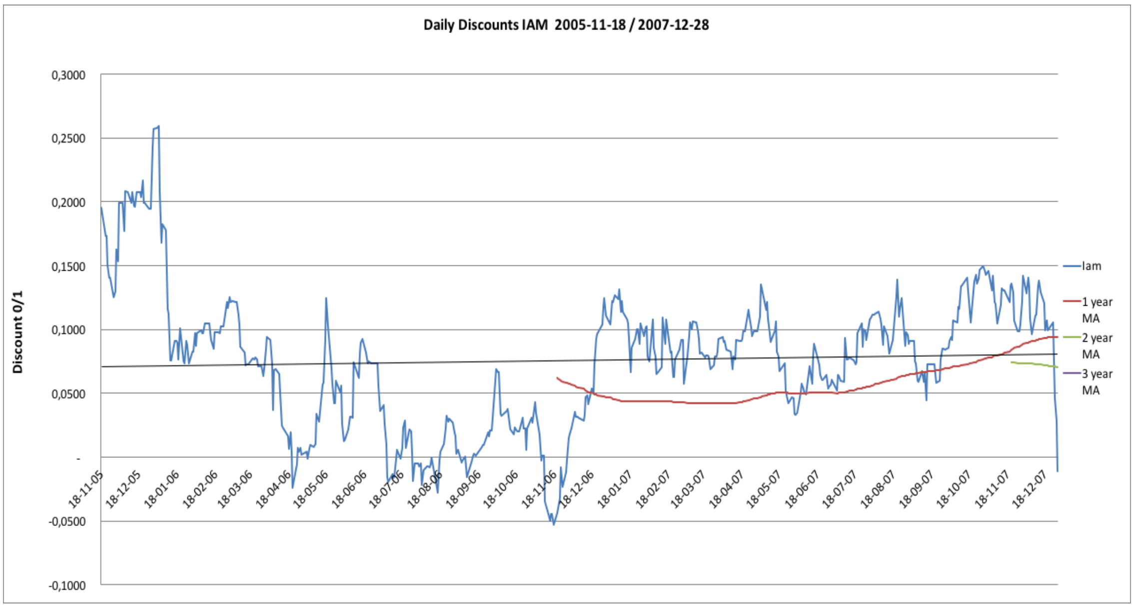
Grupo 2: Campos



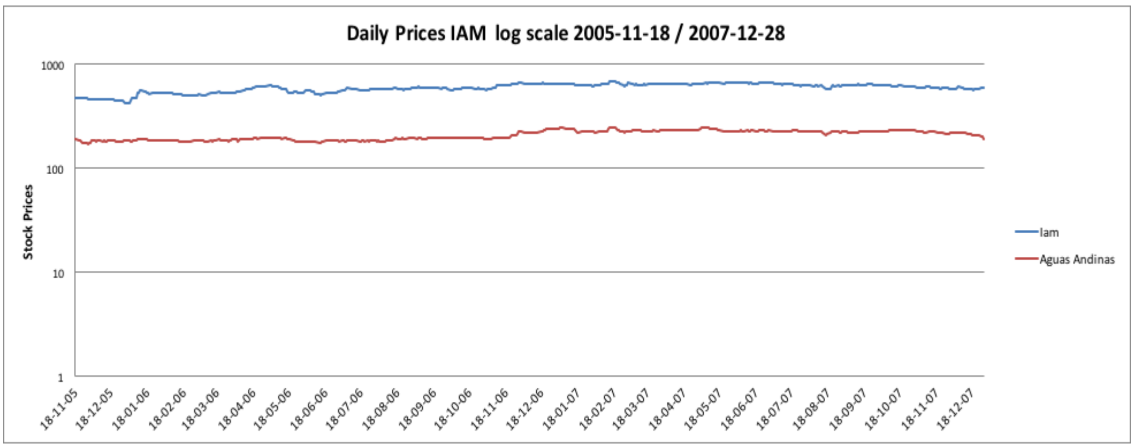
Grupo 2: Campos



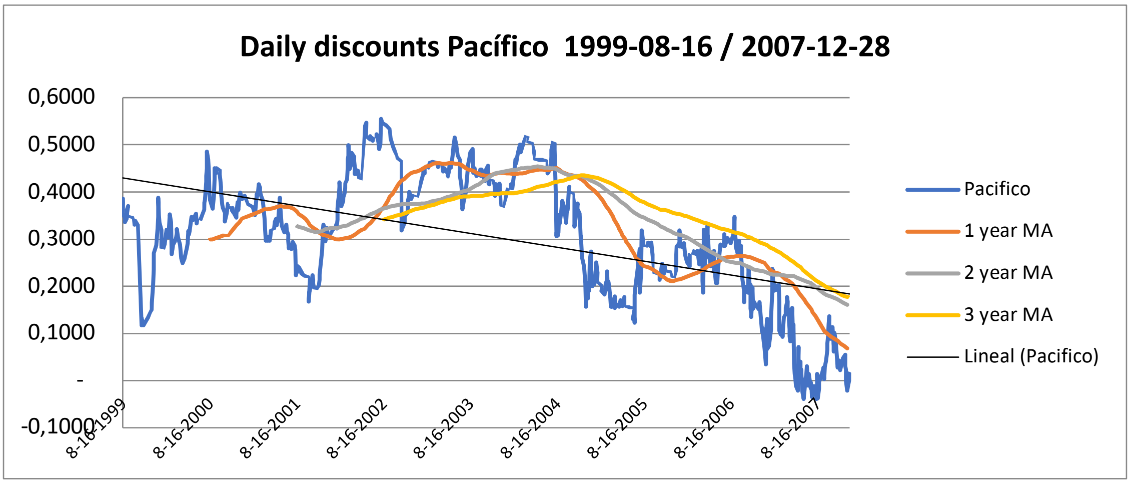
Grupo 3: IAM



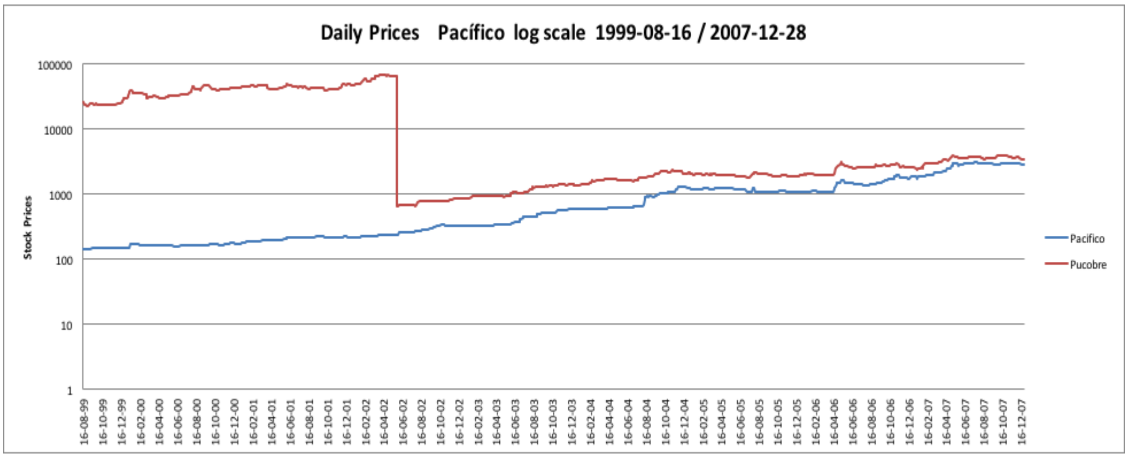
Grupo 3: IAM



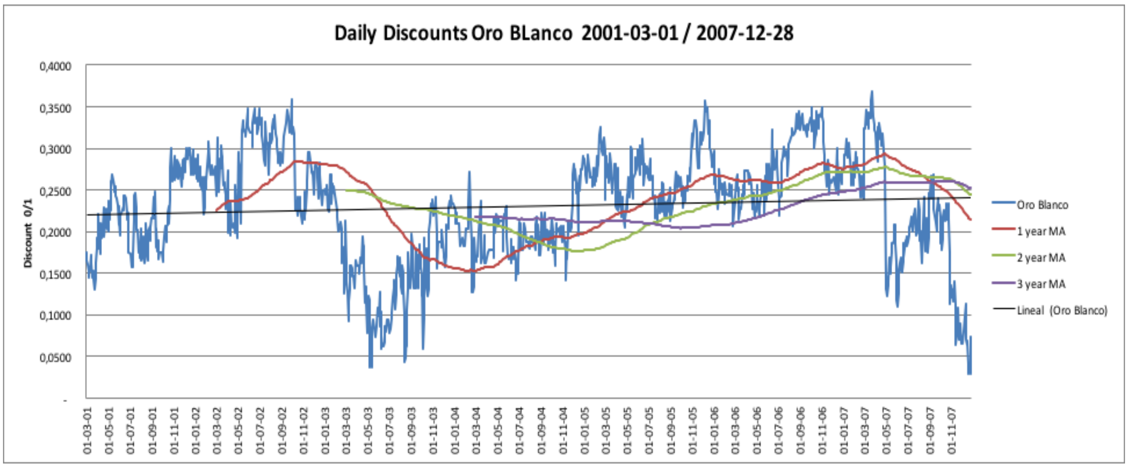
Grupo 4 :Pacífico



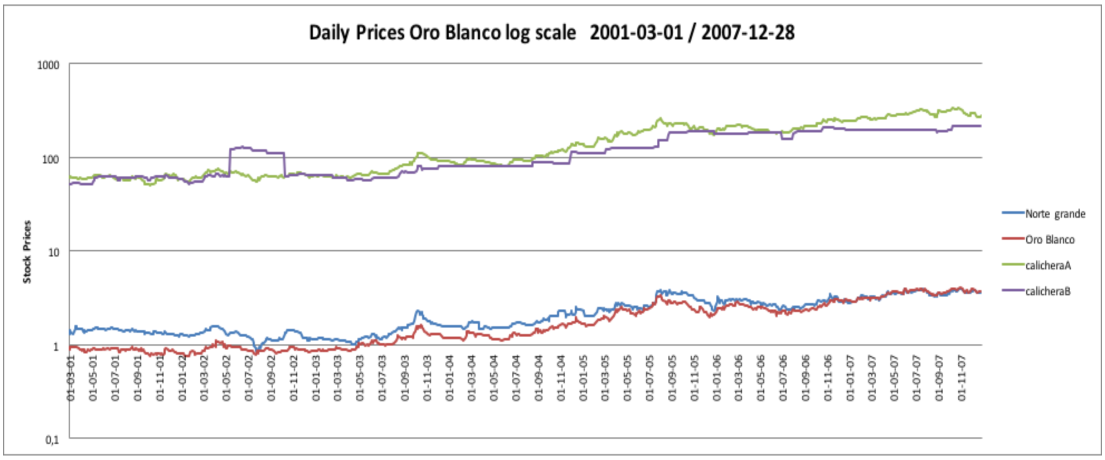
Grupo 4: Pacífico



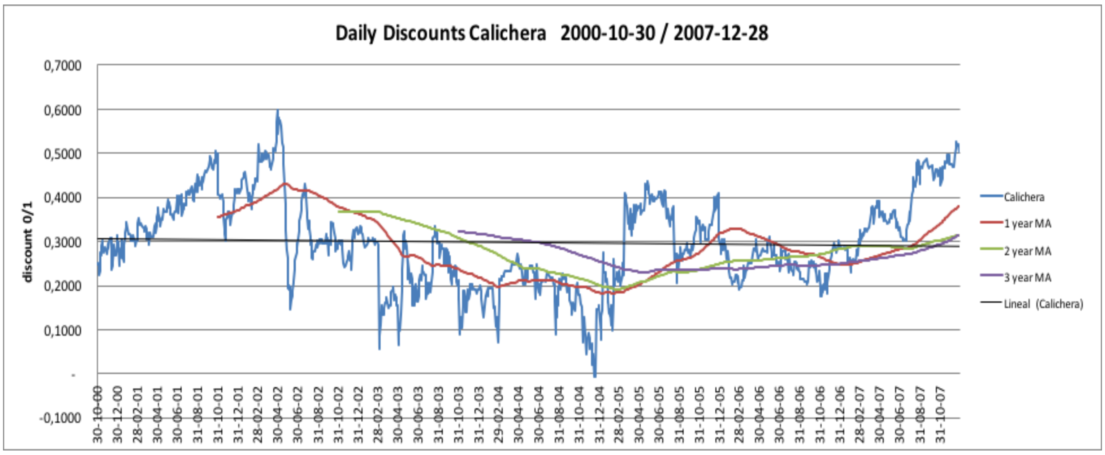
Grupo 5: Oro Blanco



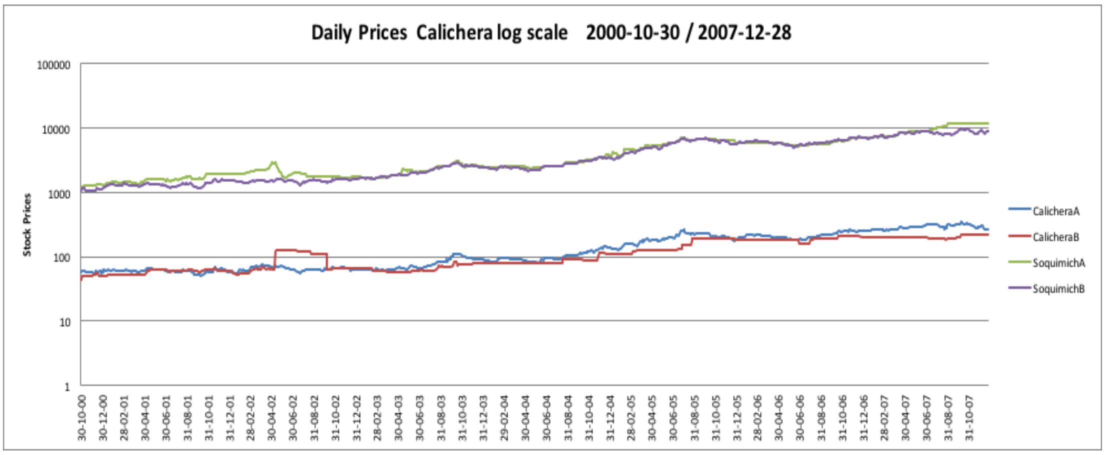
Grupo 5: Oro Blanco



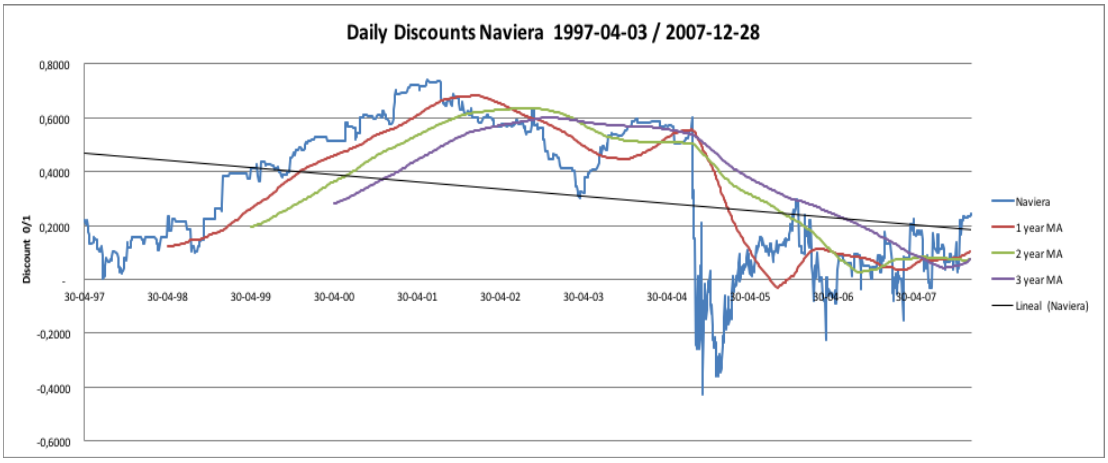
Grupo 6: Calichera



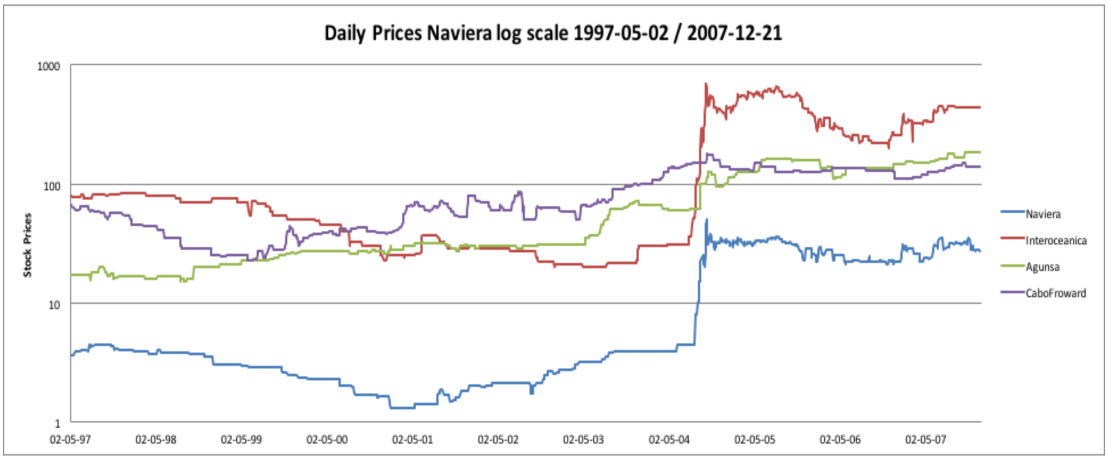
Grupo 6: Calichera



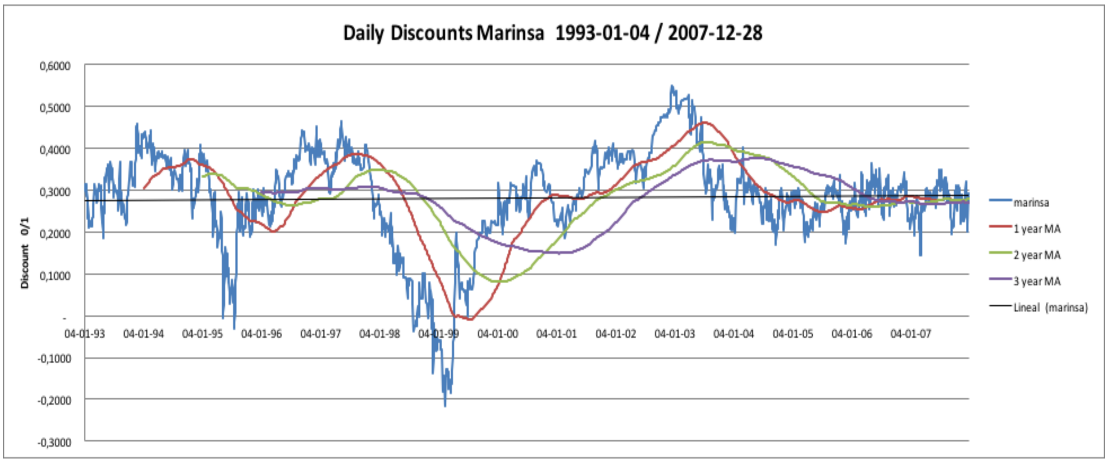
Grupo 7: Naviera



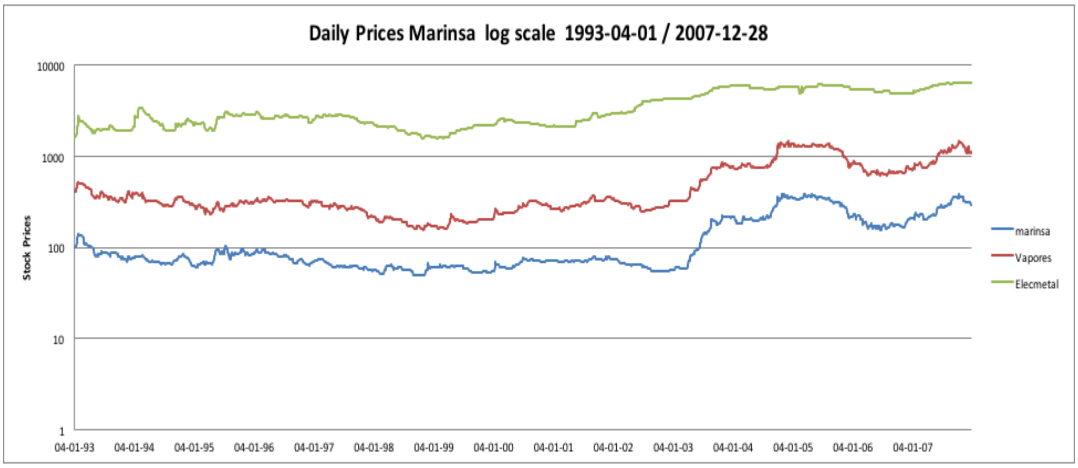
Grupo 7: Naviera



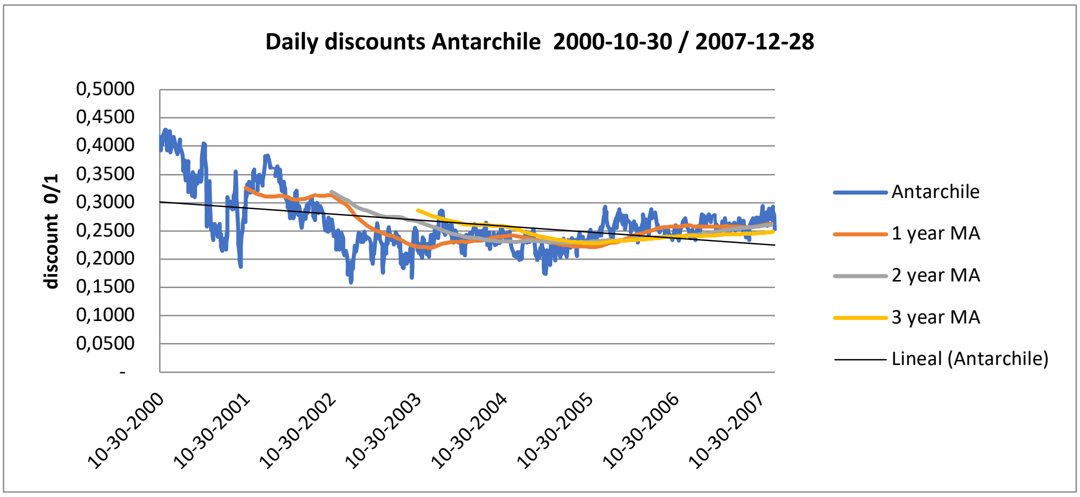
Grupo 8: Marinsa



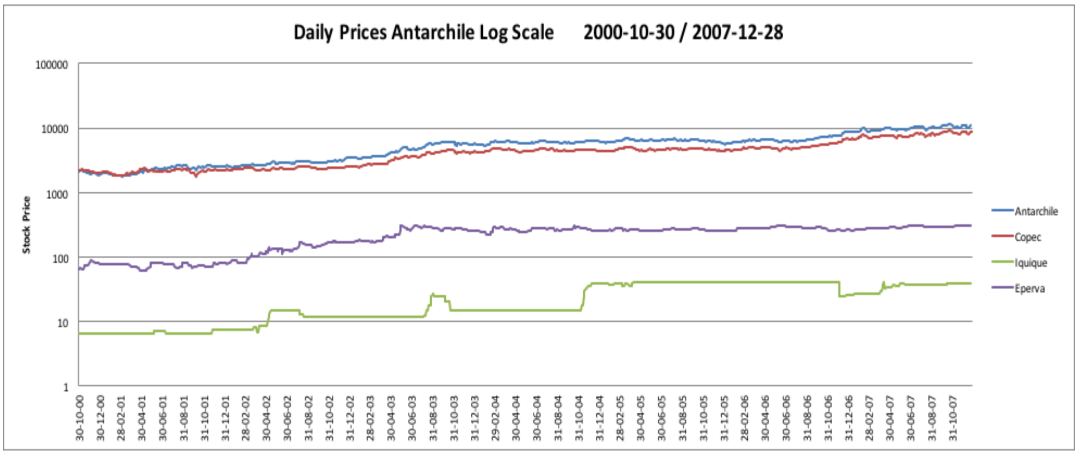
Grupo 8: Marinsa



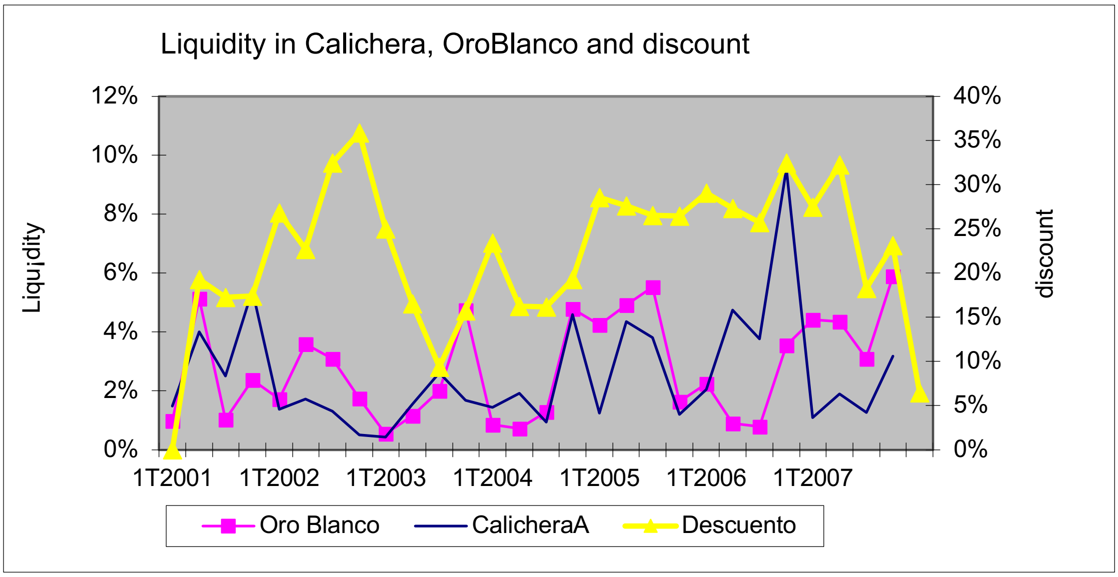
Grupo 9: Antarchile

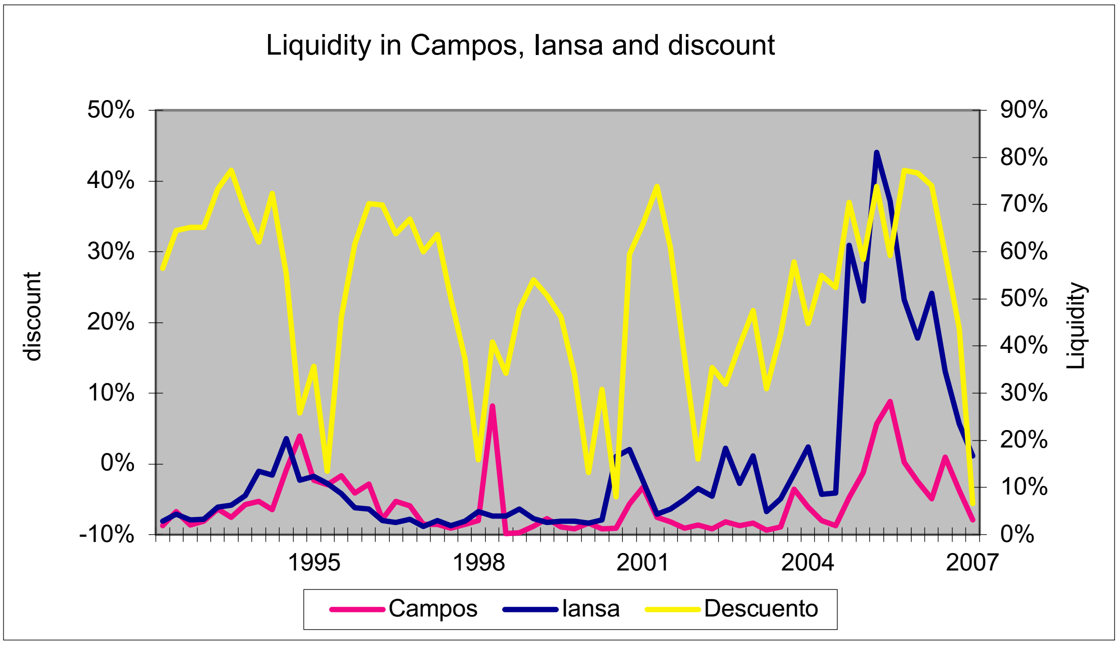


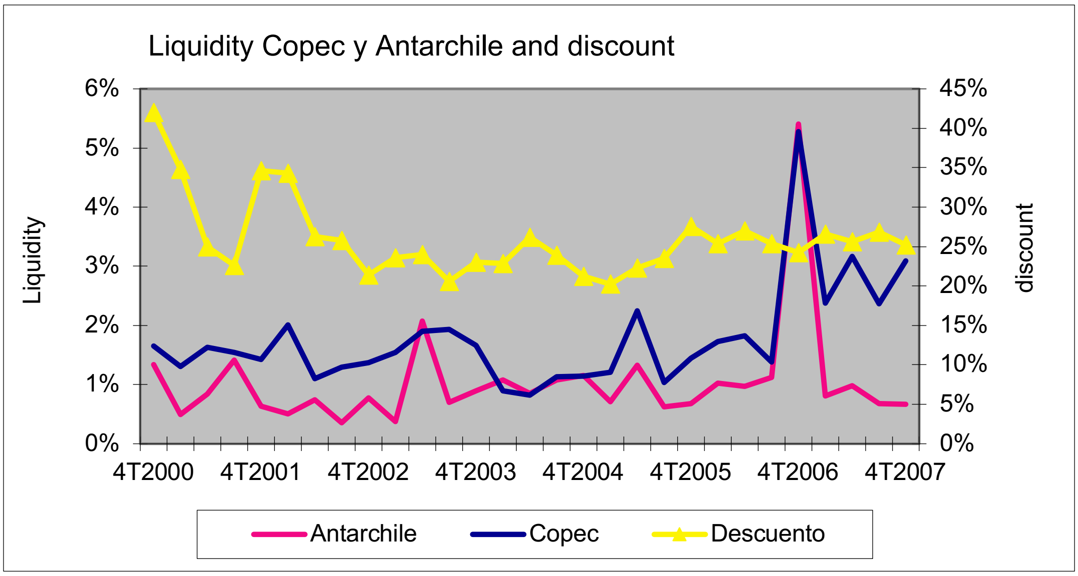
Grupo 9: Antarchile

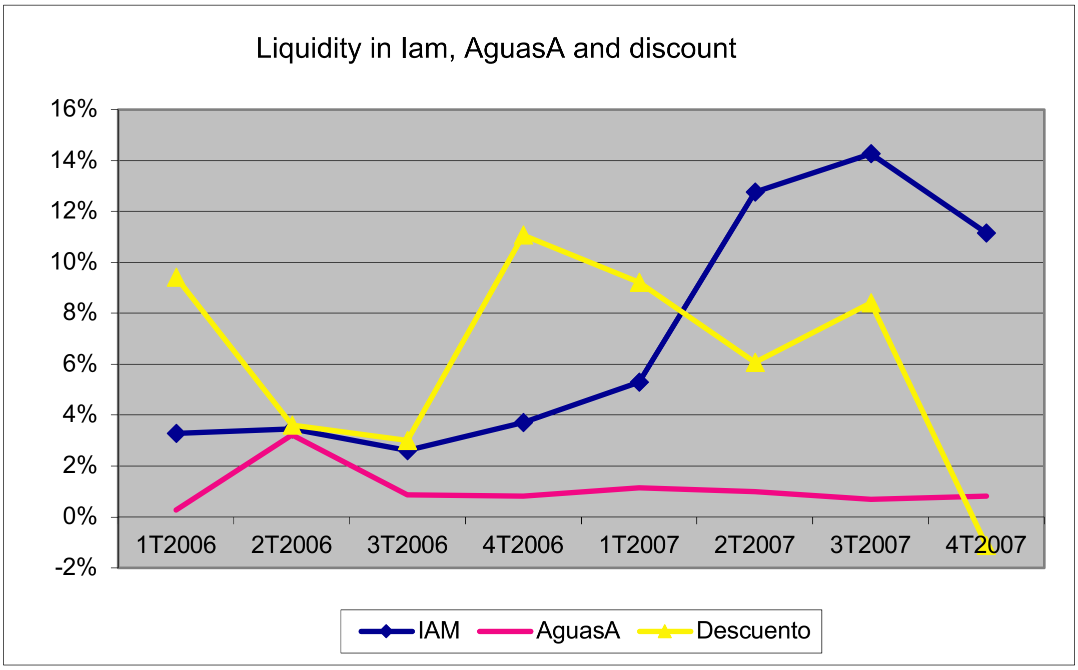


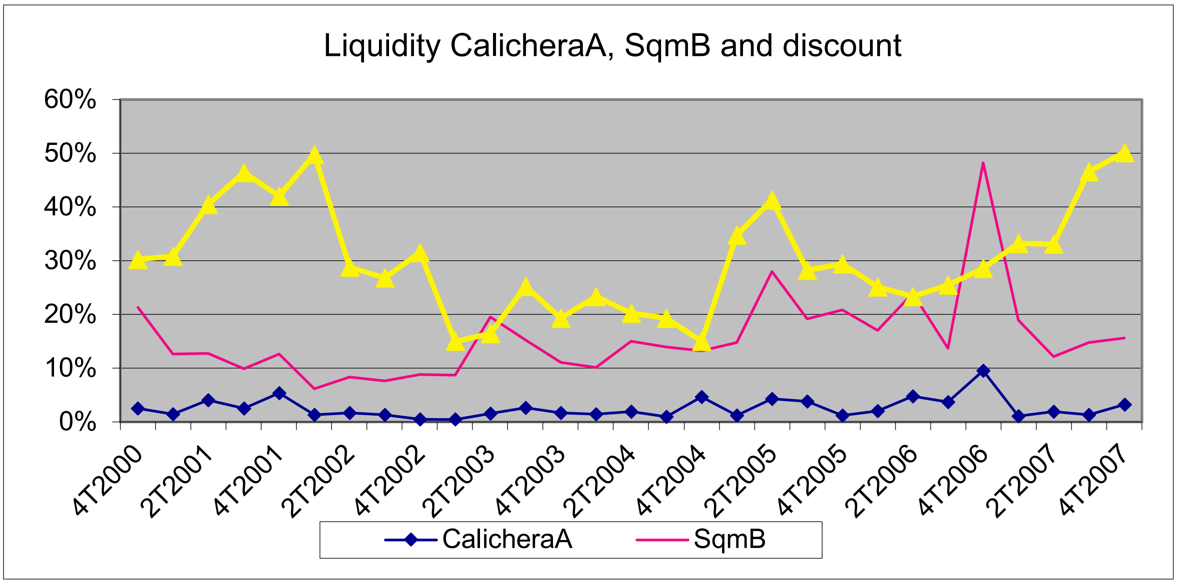
Appendix 6: Liquidity of investment company shares and underlying shares together with discount levels

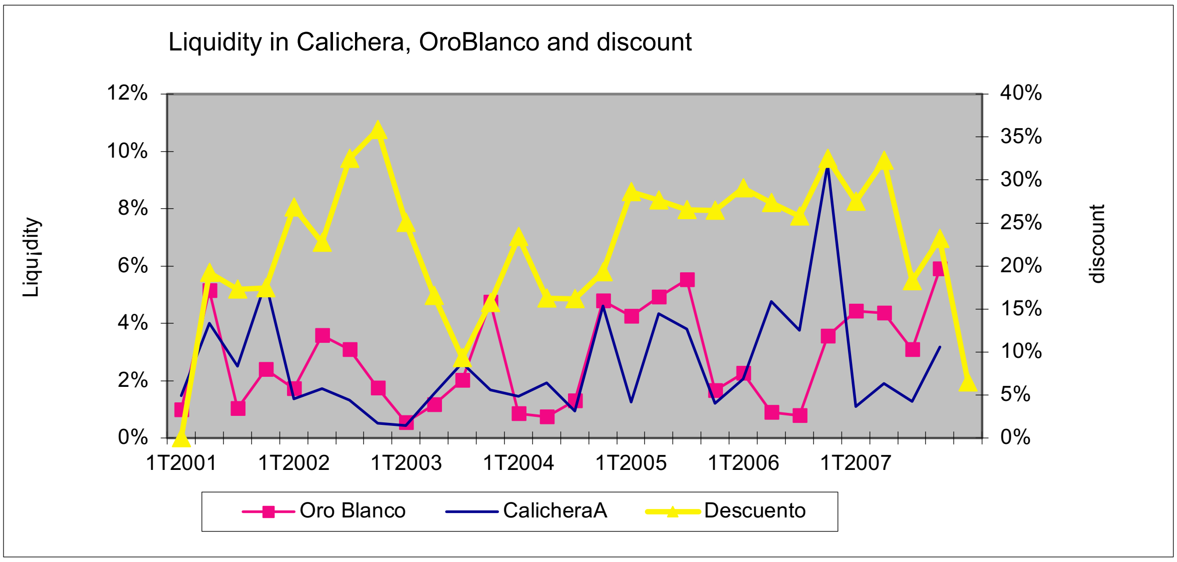




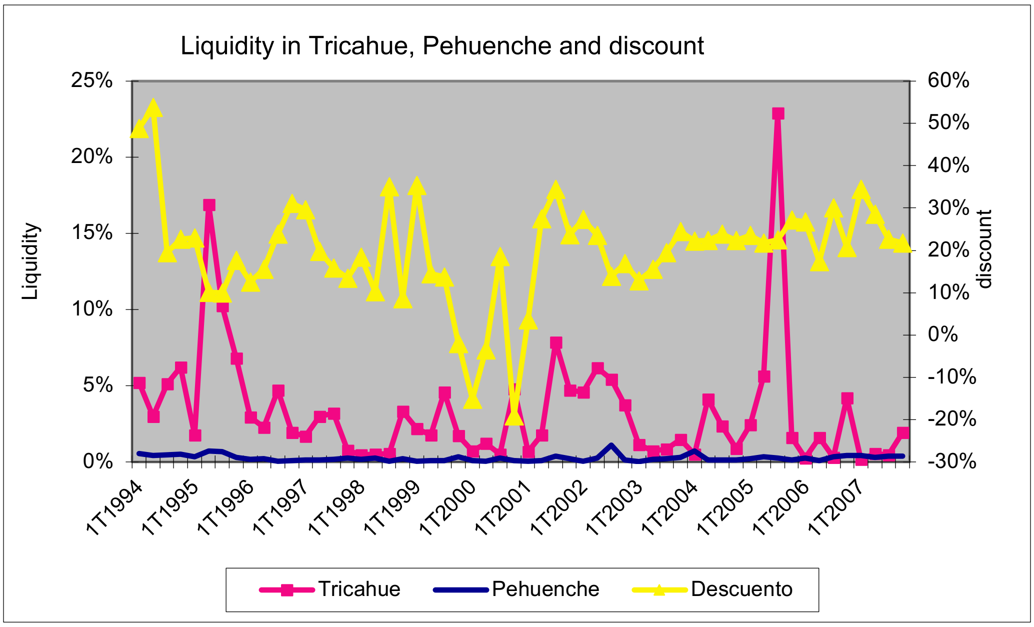

Liquidity Copec y Antarchile and discount
Liquidity
discount
6%
5%
4%
3%
2%
1%
0%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
4T2000
4T2001
4T2002
4T2003
4T2004
4T2005
4T2006
4T2007
Antarchile
Copec
Descuento

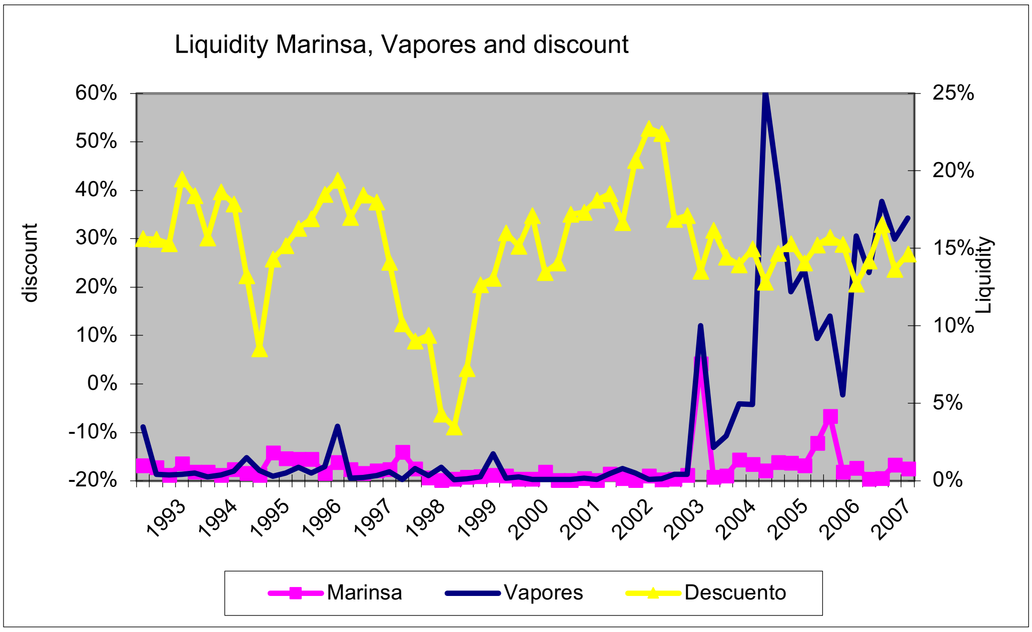

Liquidity in Iam, AguasA and discount
16%
14%
12%
10%
8%
6%
4%
2%
0%
-2%
1T2006
2T2006
3T2006
4T2006
1T2007
2T2007
3T2007
4T2007
IAM
AguasA
Descuento

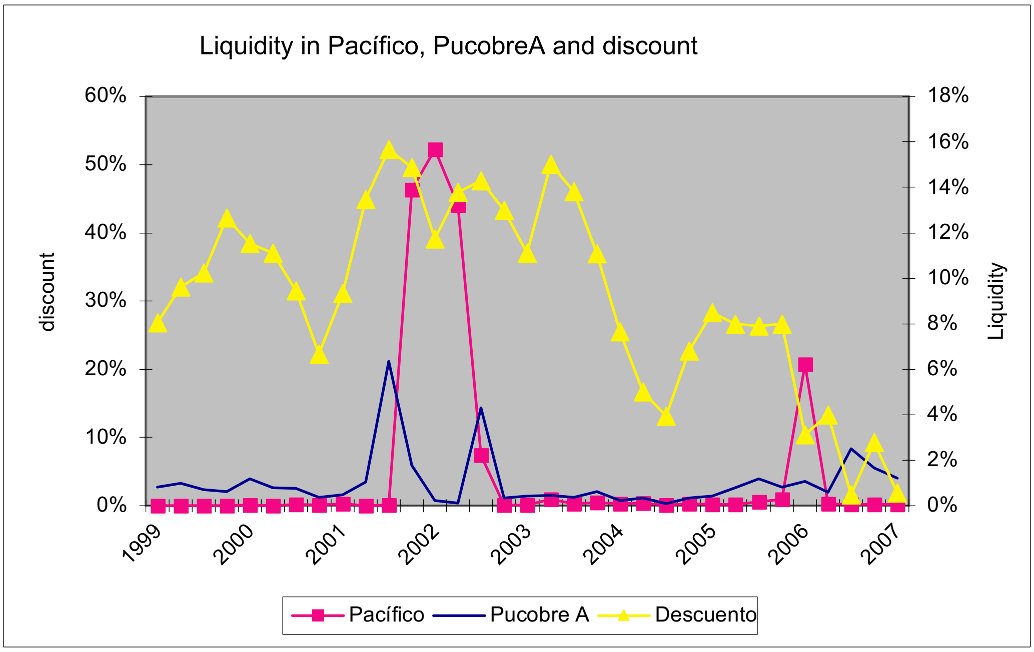




In the following case, in the Tricahue investment company, I include two figures, the first is the same as the previous but the second shows the amounts traded in millions of pesos, since the liquidity of the Pehuenche electricity company is far lower than that of Tricahue.


Liquidity in Tricahue, Pehuenche and discount
25%
20%
15%
10%
5%
0%
Liquidity
60%
50%
40%
30%
20%
10%
0%
-10%
-20%
-30%
discount
1T1994
1T1995
1T1996
1T1997
1T1998
1T1999
1T2000
1T2001
1T2002
1T2003
1T2004
1T2005
1T2006
1T2007
Tricahue
Pehuenche
Descuento


Liquidity Marinsa, Vapores and discount
60%
50%
40%
30%
20%
10%
0%
-10%
-20%
discount
25%
20%
15%
10%
5%
0%
Liquidity
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
Marinsa
Vapores
Descuento


Liquidity in Pacífico, PucobreA and discount
discount
Liquidity
60%
50%
40%
30%
20%
10%
0%
18%
16%
14%
12%
10%
8%
6%
4%
2%
0%
1999
2000
2001
2002
2003
2004
2005
2006
2007
Pacífico
Pucobre A
Descuento

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Chapter 4. Holding company discount and Business Groups Optimal Bailout of Subsidiaries[48,49]

Abstract

We develop a simple two-times one period model of the holding company-subsidiary relationship that analyzes the option faced by controlling shareholders to optimally bailout subsidiary companies using the holding company's financial resources. This behavior is rational under several incentive structures originated in the presence of group external benefits, private benefits of control and synergies. The results of our model and the pervasive presence of holding company discounts suggest that controlling shareholders of business groups are prone to share the financial costs of keeping pyramids in place but are less willing to share the conglomeration benefits with their minority partners in the holding company. Our empirical analysis states a hypothesis and, although partially because lack of data, the analysis seems to reflect that our model correctly captures some empirical findings.

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49 Early versions of this paper (without empirical section) are published as submitted to Documentos de Trabajo #34, Facultad de Economía y Empresa Universidad Diego Portales (2011) and published in ResearchGate Network; it is listed but was not submitted to FMA Congress in Orlando 2015, and submitted but not presented to World Finance Conference Río de Janeiro (2012). This doctoral candidate keeps absolute responsibility of the empirical section of this chapter, and keeps exclusive liability for it.

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Section 1. Introduction

A business group is a collection of separate firms tied by ownership relations and which, in spite of their legal autonomy, tend to respond to a common "group" strategy. As it turns out, most firms in emerging economies and several developed economies are linked in some way or another to a business group that exercises tight control over the firms and owns a large fraction of their shares (La Porta et al. (1999), Colpan et al. (2010) and Lefort (2010).

As a consequence, a growing body of literature in corporate governance and corporate strategy has shifted its focus away from the standard agency problem between managers and dispersed shareholders, and has, instead, looked closely into the relationship between minority and majority shareholders. In particular, it has been argued that business groups are prone to carrying inefficient investment and generating minority shareholder expropriation, especially when control is exercised through complex mechanisms such as pyramid schemes, crossholdings, and dual-class shares. In those cases, the agency problem is exacerbated because, on the one hand, ownership concentration insulates the controller from the market for corporate control, and on the other, control is executed by a shareholder that holds a relatively small fraction of the cash flow rights.

Interestingly, many of these studies recognize that one of the most salient characteristics of business groups, especially in emerging economies, is that they are persistent in time and able to adapt to most changing situations. The large amount of evidence compiled in Colpan et al. (2010) shows that business groups have been able to grow and increase their scope and self-intermediation practices even during times of fierce economic reform and deregulation. This kind of evidence has supported a more favorable view of business groups, arguing that business groups are a natural and efficient way for firms to deal with imperfect capital markets, poor institutions, corruption, and other imperfections that frequently plague emerging economies. In this context, business groups arise to fill (or to take advantage of) the voids left by poor institutions. Hence, they could represent an advantage in dealing with a corrupt government, a highly regulated economy, and a poor judiciary system (Khanna and Palepu, 1997).

The combined control of several companies may also create operational synergies, which might be related to economies of scale and scope in product and factor markets, due to poor basic services like electricity, postal service, or others. In particular, internal capital markets, that is, the headquarters' collection and allocation of funds to the different companies in a group could create value in a credit constrained world (Stein, 1997). Other financial synergies may arise because of the possibility for business groups to liquidate assets of specific firms in response to a general downturn (Shleifer and Vishny, 1997), and because of risk diversification that might be valuable to investors in economies with underdeveloped capital markets.

An interesting consequence of the existence of synergies and private benefits of controlling different companies through a business group is the creation of incentives that can trigger an opportunistic behavior consisting in bailing out distressed companies controlled by the group with financial resources belonging to other companies of the same business group. In the academic literature, this behavior has been labeled as propping or inverse tunneling (Riyanto and Toolsema, 2008). Naturally, in order for this behavior to be optimal, the initial costs of bailing out should be expected to be recovered in excess by the business group in the future. Future benefits may arise either from the possibility of keeping tunneling benefits (Friedman et al., 2003), or from the possibility of benefiting from synergies obtained by the group that otherwise would be lost.

A common way to organize a business group is through a pyramid (Almeida and Wolfenzon, 2006; Bertrand et al., 2002; Colpan et al., 2010). Usually, at the top of such structures, a holding company, controlled by the business group's controlling shareholder, owns shares of the group's different subsidiary companies, which, in turn, own shares of other firms in the group. Complicated pyramidal structures may have several of these layers of companies in order to allow business groups to achieve control of large assets with small amounts of capital. A possible rationale for such a behavior is the possibility of tunneling private benefits through the pyramid.

Almeida and Wolfenzon (2006) have noticed that many business groups organized as pyramids have very minor separation between cash flow rights and voting rights. Hence, their explanation for the existence of pyramids is that business groups choose "…the pyramidal structure because of the payoff and financing advantages it provides when new firms are expected to yield low-security benefits relative to the required investments." Almeida and Wolfenzon's explanation is interesting, among other things, because it seemingly turns around the standard vision regarding the existence of pyramids. Common wisdom associates pyramids to a tool implemented to extract private benefits. However, the results of Almeida and Wolfenzon's model indicate that pyramids are the best way to finance the creation of new companies that yield relatively low-security benefits.

Both views are, in fact, consistent, since the pyramid provides a financial advantage to fund new acquisitions in which security benefits are low relative to private benefits. Hence, the controlling shareholder is willing to invest other people's money to gain access to private benefits.

In this paper, we want to contribute to the literature on business groups by adding an overlooked empirical regularity to the complex issues that theoretical models on pyramid structures attempt to explain. Recently, several authors have drawn attention to the existence of a holding company discount in many business groups. Such a discount means that the market capitalization is less than the market capitalization or economic value of the investments it holds minus the holding company's debt (Net Asset Value or NAV). Rommens, Deloof and Jegers (2004) show that holding companies, which play an important role in corporate finance in Belgium and in other Continental European countries, often trade at a discount to their estimated net asset value (NAV). Gálvez (2009) finds that in the Chilean case, even holding companies in which all assets are shares in traded companies and thus net asset values are calculated straightforwardly, trade at an average 30% gross discount over NAV.

Similarly to close-end fund discounts, holding company discounts can be attributed to limited free float of the holding company, tax inefficiencies associated with the holding company, and the additional administrative costs any holding company incurs. Rommens et al (2004) hypothesize that, in addition, holding company discounts could be due to the extraction of private benefits by controlling shareholder at the expense of minority shareholders[50],[51], and to the fact that the estimated NAV may be an overestimation of the actual value of the holding company's investments. However, as Lefort et al. (2010) show, discounts are still present when net asset values are obtained directly from market values.

The relationship between tunneling and holding company discounts is not straightforward. To illustrate this, consider the simple case where a holding company sole asset are shares of a subsidiary. Private benefits tunneled from the subsidiary to the holding company, could increase the holding company's security value to all shareholders of the holding company. Instead, private benefits extracted from the subsidiary directly to the controlling shareholder's pockets, will not be reflected in the holding company security value. It is easy to show that the former case would imply a holding company premium, while the later would imply a holding discount.

Unless the subsidiary is wholly owned by the holding company, the controlling shareholder fraction of cash flow rights will be higher in the holding company than in the subsidiary, and incentives will be structured so as to eventually tunnel private benefits from the subsidiary to the holding company, and not the other way around. Consistent with this intuitive result, Almeida and Wolfenzon's (2006) model predicts that business groups will find it optimal to structure pyramids in order to profit from their financing advantage only in situations where the model parameters predict the presence of holding company premiums.

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50 Around Rommens et al. (2004) an ample literature has developed studying the effects of the extraction of private benefits of control at the expense of minority shareholders in business groups, as examples only, I have Bertrand et al. (2002), Friedman et al. (2003) and Riyanto and Toolsema (2009).
51 The holding discount has also been of interest in business practice. Some recent articles can be found: TSI whealth Daily Advice, "How to realize big benefits from a holding company discount" October 21, 2019; TSI wealth Daily Advice 2019.

In this paper, we develop a simple two period model of the holding company-subsidiary relationship that analyzes the option faced by the controlling shareholder to optimally bailout the subsidiary using the holding company's financial resources. This behavior is rational under several incentive structures originated in the presence of group external effects, synergies and private benefits of control[52]. The general intuition is that when the bankruptcy of a financially distressed subsidiary may cause a loss of benefits to the controlling shareholder, it may be optimal for him or her to pay a cost in order to bailout the distressed company. This cost will be shared with minority shareholders at the holding company and will induce a holding company discount.

Similarly, to our model, Almeida and Wolfenson's (2006) model predicts that an unexpected acquisition of a new company into the pyramidal structure will cause a loss of value in the holding company, because this implies additional private benefits to the controlling shareholder, at the expense of the holding company's minority shareholders, which finance part of the acquisition.

The results of our model are consistent with empirical evidence of holding company discounts and suggest that holding companies bailout subsidiaries in order to maintain a flow of conglomeration benefits to the controlling shareholder. In other words, the picture that emerges from this model, the holding company discount evidence, and casual evidence of holding companies effectively funding subsidiary companies when they face financial distress, is one of diffuse limits between the subsidiary companies of a group. This representation of a business group is consistent with the numerous descriptions of specific cases occurring in a variety of countries depicted in Colpan et al. (2010).

In particular, a conclusion of this paper is that legal boundaries to firms in the context of a business group are less meaningful than expected. Business groups not only may eventually tunnel private benefits from the firms they control or allocate strategic

52 I can interpret the bailout of a distressed subsidiary as the intention to maintain group benefits such as reputation and synergies, in the same line of reasoning as in Morck (2010).

resources among them, but they appear to be willing to forfeit the benefits associated with limited liability rights in the firms they control. The result is a benefit accrued by minority shareholders on downstream companies of the group, partially financed by minority shareholders of upstream companies. When the expected resources at stake in this intercompany resource allocation surpass the expected value of tunneling or other benefits from subsidiaries, the result is the appearance of holding company discounts.

Almeida and Wolfenson's view, and this chapter's view are parallel in the sense that pyramids are useful devises to finance new ventures, as well as for protecting existing ones, creating an economic network between the companies of the group. Consistent with the above, strong incentives may exist to preserve the structure of a group in the presence of financial distress in one of the subsidiaries, in spite of the costs that controlling and minority shareholders must bear.

In our model, holding discounts and premiums arise because the net asset values computed for holding companies do not consider the amount of conglomeration benefits shared with the minority shareholders of the holding company and the expected financial cost incurred by the same shareholders when the group decides to bailout a distressed subsidiary. Hence, if conglomeration benefits to the controlling shareholder are not high enough, holding companies will let subsidiaries go bankrupt and, in the presence of conglomeration benefits perceived by the minority shareholders of the holding company, a holding premium will be observed. On the contrary, if conglomeration benefits to the controlling shareholder are high enough, holding companies will rescue financially distressed subsidiaries and, if the financial cost of rescue borne by the holding company minority shareholders exceeds their share of conglomeration benefits, a holding discount will be observed.

The rest of the paper is structured as follows: Section II discusses empirical evidence of holding discounts and relates it to the close-end fund literature. Section III presents a model of optimal bailout of subsidiaries. We model holding company discounts for a number of cases, depending on the nature of the benefits that the controlling shareholder can optimally derive from the pyramid, ranging from external benefits, to tunneling, to synergies. The decision to bail-out the distressed subsidiary and the welfare implications to minority shareholders given the existence of potential conflicts of interest are also analyzed in this section. Section 4 summarizes the main results and conclusions of Sections 1 to 3, Section 5 is the empirical section of this chapter and develops and tests (at least partially) a hypothesis, of the basic determinant of the holding discount: hypothesis H3 Null: The holding discount is positively related to the debt riskiness of the business group as reflected risk spread of bonds issued by the holding group. The debt riskiness is a proxy to the probability of default or bad times in the economy.

Section 2. Holding vs. Close End Funds discount

Several authors have drawn attention to the existence of a holding company discount in many business groups. Such a discount means that the market capitalization of a business group holding company is less than the market capitalization or economic value of the investments it holds once the holding company's debt is subtracted (Net Asset Value or NAV). Rommens et al. (2004) show that holding companies, which play an important role in corporate finance in Belgium and in other Continental European countries, often trade at a discount to their estimated net asset value (NAV). Gálvez (2009) finds that in the Chilean case, even holding companies that only hold shares of traded companies, trade at an average 30% discount over NAV.

holding discounts remind of a similar phenomenon documented in the closed-end funds literature. Interestingly, close-end funds generally trade in the U.S. at a discount on their net asset value. Moreover, they are rarely leveraged. Though they generally trade at a discount, they can also show premiums over their NAV. Possible explanations for the close-end fund discount are transaction, administrative and agency costs, effects arising from portfolio diversification and liquidity, tax effects and poor estimation of NAV values. Many of these explanations could also apply to the holding discount puzzle (Malkiel 1977, Boudreaux 1973; Rommens et al. 2004).

Lee et al. (1991) and Brickley et al. (1991) analyze most of these explanations. Administration and agency costs appear as far too small to explain observed close-end fund discounts while the theory of lack of liquidity is rejected in the case of closed-end funds that mainly invest in liquid shares. However, Chan et al. (2008) provide evidence suggesting that market illiquidity plays a significant role in explaining both time series and cross-sectional variation in closed-end country fund discounts.

The tax effect argument deserves some attention. Malkiel (1995) argues that funds with high unrealized capital appreciation should sell at discounts from NAV because the holder of such a fund would be assuming a potential tax liability that depends on the holding period of the investor. He finds a positive relationship between discounts and unrealized capital appreciation. Brickley et al. (1991) suggest that discounts are partly driven by the fact that, as a result of holding shares in a closed-end fund, investors lose valuable tax-trading opportunities associated with the movements of the individual portfolio constituents (tax-timing option). They find that discounts are positively correlated with the average variance of the constituent assets in the fund. These studies suggest that tax effects would explain a 5% to 10% discount. Finally, the fund discount could also be the result of a bad estimate of its true net asset value. However, Brickley, et al. (1991) analyze the changes in the value of 16 closed-end funds that went through liquidation or merger concluding that discounts are not the result of calculation errors.

Since holding companies tend to be investing vehicles through which business groups hold shares of companies affiliated to the group, the parallelism between close-end fund discounts and holding discounts is obvious. The main difference between both structures is that, in general, holding companies are used by the business groups to control and influence investing, operating and corporate decisions made by the affiliate companies, while close-end funds are passive investors in their companies.

In spite of this important distinction, most of the reasons that could explain holding company discounts are similar to the ones explored by the close-end fund discount literature. holding company discounts are normally attributed to limited free float of the holding company, tax inefficiencies associated with the holding company, and the additional administrative costs any holding company incurs. In particular, since a holding company constantly monitors the subsidiaries that controls, large administrative costs of holding companies may explain observed holding discounts. Rommens et al. hypothesize that, in addition, holding company discounts could be due to the extraction of private benefits by controlling shareholder at the expense of minority shareholders, and to the fact that the estimated NAV may be an overestimation of the actual value of the holding company's investments.

However, Galvez (2009) shows that discounts are still present when net asset values can be directly obtained from market values. These authors compute daily holding discounts for the period from January 1993 to December 2007 for nine business groups which comply the following conditions. First, the holding company must be a public company, traded in the Chilean stock market, with no activities other than holding shares of other publicly traded companies. Secondly, the book value of the investments in subsidiaries must add up to an important percentage of the holding company's total book value of assets. These conditions imply that holding discounts are calculated mainly based on market values and, hence, it is less likely that NAV calculations are overestimated.[53] In addition, the selected holding companies tend to be undiversified.[54] Table 28 reproduces some of Lefort et al. (2010) calculations.

53 For these groups, subsidiaries represent 89% to 99% of the holding company's investment in companies.

54 Campos in the agriculture industry; Aguas Andinas in urban water and sewage; Pacífico and Oro Blanco in mining; Pampa Calichera in fertilizers; Naviera and Marinsa in maritime transportation and port agency services; and finally, Antarchile, the most diversified group in the sample, in fuel import and distribution, forestry, cellulose, fishing and mining.

Table 28: Monthly holding company discounts for selected Chilean Business Groups

holding company	Industrial Sector	Listed subsidiaries % of total assets	holding company discounts (%) Average	Standard Deviation	Max/ Min	Time Period
Tricahue	Electric Generation	89-99	29	21.4	93/-16	1994-2009
Campos Chilenos	Agriculture Industry	90-99	20	12.5	65/-29	1993-2009
Aguas Metropolitanas	Water and Sewage Urban	95-98	21	8	36/5	2005-2009
Pacífico	Mining	43-96	9	6	26/-1	1999-2009
Oro Blanco	Mining	97-99	25	20	54/-47	2001-2009
Pampa Calichera	Fertilizers	96-99	28	12	52/-17	2000-2009
Empresas Navieras	Maritime Operations/Port	93-97	29	21	69/-27	1997-2009
Marinsa	Maritime Freight /services, Metallurgy	94-99	31	10	55/-7	1993-2009
Antarchile	Fuel distribution, Forestry, Fishing	94-99	26	5	42/18	2000-2009

Monthly average discounts reported by these authors fluctuate in the 20% to 31% range, with the exception of one group presenting an average discount of 9%. As in the case of close-end funds, holding discounts show a high time series volatility including brief periods where holding shares are traded at a premium above NAV values.[55] Like close-end funds, the evidence provided by Lefort et al. shows that holding discounts present a mean reversion behavior over a twelve-month period.

Galvez (2009) explores some possible explanations to the holding discount evidence. On November 7th, 2001, Law 19.768 modified Chilean Income Tax Law introducing a tax exemption to capital gains made on stock market listed shares. If holding discounts were explained by an implicit tax liability, there should have been a significant reduction in holding discount levels after such a Law was passed. Gálvez (2009) did not find any significant effect in their data. Furthermore, they did not find

55 holding discounts were calculated as one minus the holding company's market value of equity divided into Net Asset Value of the holding Company (NAV). NAV in turn, is calculated as the market value of the holding company's assets minus the market value of the holding company's liabilities, using face values of liabilities as proxies of their respective market value. Finally, the holding company market value of assets has two components: the market value of investment in subsidiaries, and short term assets using their face value as a proxy. Raw market data is the daily prices obtained from Economatica Data Base, financial data is released quarterly by public companies and published by the regulating entity (SVS), giving 68.880 group-day and 1.148 group-quarter observations, respectively.

any significant relationship between the degree of liquidity of holding company shares and the level of holding discounts observed.

The key difference between a closed-end fund and a holding company is that the later are used by business groups to control and influence investing, operating and corporate decisions made by the affiliate companies. Holding companies are at the top of pyramidal structures used by business groups to achieve control over a large collection of assets and, among other things, tunnel private benefits of control through the pyramid. Hence, as suggested by Rommens et al. holding company discounts could be due to the extraction of private benefits of control by controlling shareholders at the expense of minority shareholders.

However, it is simple to see that tunneling private benefits through a pyramid structure may induce either a holding company premium or discount. To illustrate this, consider the simple case where a holding company sole asset are shares of a subsidiary. If private benefits are tunneled from the subsidiary to the holding company and shared with minority shareholders at the holding company, they will increase the holding company's security value producing a holding premium. If instead, private benefits are extracted from the subsidiary directly to the controlling shareholder's pockets, they will tend to generate a holding company discount.

In this paper, we postulate that controlling shareholders of business groups have a variety of incentives to bail-out financially distressed subsidiary companies. In such a context, observed holding company discounts arise because the market anticipates the financial cost that minority shareholders of holding companies will pay when the controlling shareholders of the business group will decide to bail out distressed subsidiaries.

Section 3. A model of optimal bailout of subsidiaries

3.1. A simple Model

We model a business group as two related but legally separate entities: an unlevered holding company (HC), controlled by a controlling shareholder and a partially owned levered subsidiary, controlled by the holding company. The HC's assets are its investments in the subsidiary's shares plus cash (retained earnings). The controlling shareholder holds a fraction h of the holding company's shares, and the holding company holds a fraction f of the subsidiary. We assume $0.5 < h, f < 1$, and that h and f are enough to exercise complete control over the holding company and the subsidiary.

In our model, agents are rational, so they maximize expected cash flows in an educated manner and use all available information. There is a competitive securities market to which shareholders and debt holders have free access; there are neither taxes, information asymmetries nor bankruptcy costs, and discount rates are assumed to be zero.

The subsidiary is a risky productive asset financed with debt and equity. In a two period time frame *(t=0, 1)*, the state of nature is revealed in t=1 as asset value is achieved by taking one of two possible values: V_u and V_d with probabilities q and $(1-q)$ respectively, where $V_u > V_d$. We denote equity market values as S, we use superscripts H and Aff to denote holding company and the subsidiary company respectively. When it is not obvious, we use subscripts *0* and *1* to denote t=0 and t=1. Face value of debt in t=1 is D. We assume $V_u > D > V_d$, so that in the "down or bad" state of nature the subsidiary becomes financially distressed.

Limited Liability (LL) is the legal rule by which, shareholders of a distressed company, have the right but not the obligation to go bankrupt by transferring ownership and control of assets to creditors. For simplicity, we assume that in the case of bankruptcy no negotiations will take place with creditors and no further equity costs will be incurred. Hence, the subsidiary's equity value is equivalent to a call option on the company assets with a strike price equal to the face value of debt. Conversely, we refer to "rescue" or "bailout", the case in which shareholders, while having the legal right to go bankrupt, avoid bankruptcy and keep ownership and control of assets by fully repaying debt.

In *t=1* the state of nature is revealed, and the subsidiary's asset value is known. If the state is "u" assets are kept or cashed-out at a profit. If the state is "d", controlling shareholder must decide whether to go bankrupt or to rescue the subsidiary. We assume that cash held initially at the holding company (i.e. retained earnings in the form of liquid assets) equals the amount shareholders must pay to avoid bankruptcy (*Cash=D*).[56] If the controlling shareholder decides to bailout the subsidiary, he will force an equity call at the subsidiary and, assuming minority shareholders do not concur, all new equity will be paid by the holding company from existing cash.[57]

Table 29 summarizes holding equity valuations for both types of shareholders and in both states of nature under these assumptions.

Table 29: Holding company shareholders net wealth, in t=1, in $

		State of nature "u"	State of nature "d"
HC lets distressed subsidiary go bankrupt	Controlling	$h \cdot \left(f \cdot \left(V_u - D \right) + Cash \right)$	$h \cdot Cash$
	Minority	$(1-h) \cdot \left(f \cdot \left(V_u - D \right) + Cash \right)$	$\left(1-h\right) \cdot Cash$
HC bails out distressed subsidiary	Controlling	$h \cdot \left(f \cdot \left(V_u - D \right) + Cash \right)$	$h \cdot V_d$
	Minority	$(1-h) \cdot \left(f \cdot \left(V_u - D \right) + Cash \right)$	$\left(1-h\right) \cdot V_d$

Note: Base Case: No Benefits

In this simple setup, there is no economic rationale for the holding company to bailout the subsidiary and give up limited liability. Hence, the expected present value of the holding company assets is $E(s^H) = q \cdot \left(f \cdot \left(V_u - D \right) \right) + Cash$.

56 For simplicity, our model considers an exogenous probability (*q*) of facing financial distress. In general, however, the probability of financial distress will not be independent from other parameters of the model such as the level of debt (*D*).

57 By assumption, controlling shareholders have enough voting power to unilaterally approve the equity call. Minority shareholders in the subsidiary will find it optimal not to concur.

For simplicity we compute holding discounts as the difference between expected present equity value for minority shareholders in the holding company (Security Value in Almeida and Wolfenson (2006) terminology) and their share in the net asset value (NAV). Similar to market analysts, we calculate NAV as the market value of the holding company's assets, i.e. its share in the subsidiary's equity market value, plus excess cash, minus the holding company's financial debt. We use minority shareholders' valuations as proxies to market value since we assume their shares are the only shares traded in the securities market.[58] For this reason, holding discounts computed in this way implicitly include relative liquidity issues.

In this simple model, both the security value of HC's minority shareholders and their share in NAV is $E(S^H) = (1-h)\cdot\left(q\cdot f\cdot(V_u - D) + Cash\right)$. Therefore, in this case the holding discount is simply zero. However, it is interesting to see that, if against basic economic sense, the controlling shareholder of HC decided to rescue the distressed subsidiary, HC's minority shareholders security value would be: $(1-h)\cdot\left(q\cdot\left(f\cdot(V_u - D) + Cash\right) + (1-q)\cdot f\cdot V_d\right)$[59], while their share in NAV would be $(1-h)\cdot\left(q\cdot f\cdot(V_u - D) + Cash\right)$, resulting in a holding discount given by $(1-q)\cdot(1-h)\cdot\left(V_d - D\right) < 0$.[60] Note that this corresponds to the HC minority shareholder's share in the expected net cost of rescuing the distressed subsidiary.

Intuitively, the controlling shareholder of a business group will only consider rescuing a distressed subsidiary if the benefits of bailing it out are greater than the associated costs. Hence, in the following section we consider variations on our simple model based on including different types of conglomeration benefits associated to maintaining the pyramidal structure in place by rescuing the distressed subsidiary. The inclusion of these benefits changes the incentive structure of the model, allowing for situations where optimal bailouts of the distressed subsidiary arise. The benefits to be

58 The international evidence shows that controlling shareholders of holding companies trade blocks of shares privately or engage in public tender offers.

59 In the case of rescue, as minority shareholders in the subsidiary would not concur to the equity call, the share of the holding company in the subsidiary increases to 1.

60 As *Cash* = *D* by assumption.

considered are external benefits (characterized by reputation effects) in section B1, private benefits of control (characterized by tunneling) in section B2, and synergies in section B3.

3.2. A Rationale for bailing out subsidiaries.

3.2.1. External Benefits: Reputation

The model is the same as in section A, but we assume the existence of external benefits X derived from outside the economic boundaries of the business group and perceived directly by the shareholders of the holding company. These benefits may arise from reputation in the business community that produces more favorable customer, financing or supplier relations and new business opportunities (Walker and Kent, 2010). External benefits are lost if the subsidiary goes bankrupt.

We assume that X is received by the HC´s shareholders in addition to the security value of the subsidiary assets. We divide the external benefit X in two parts: X_1 received by all shareholders of the holding company[61] and, X_2 received exclusively by the controlling shareholder. X_1, X_2 are positive and not state dependent. X_2 does not affect cash flows, financial statements or public information of the holding company and the subsidiary, and therefore is not added to security benefits in our calculations. Table 30 summarizes wealth values in t=1, which include security values and external benefits, for both types of shareholders of the holding company, in both states of nature.

Table 30: Holding company shareholders net wealth in t=1

		State of nature "u"	**State of nature "d"**
HC lets distressed subsidiary go bankrupt	Controlling	$h \cdot \left(X_1 + f \cdot \left(V_u - D\right) + Cash\right) + X_2$	$h \cdot Cash$
	Minority	$(1-h) \cdot \left(X_1 + f \cdot \left(V_u - D\right) + Cash\right)$	$\left(1-h\right) \cdot Cash$
HC bails out distressed subsidiary	Controlling	$h \cdot \left(X_1 + f \cdot \left(V_u - D\right) + Cash\right) + X_2$	$h \cdot \left(X_1 + V_d\right) + X_2$
	Minority	$(1-h) \cdot \left(X_1 + f \cdot \left(V_u - D\right) + Cash\right)$	$\left(1-h\right) \cdot \left(X_1 + V_d\right)$

Note: External Benefits

61 For simplicity I assume benefit X_1 is received in the same proportion they hold shares in the holding company.

Under the previous assumptions and as shown in table 30, if the state of nature is "u" external benefits will remain in place. Alternatively, if the state of nature is "d" and the controlling shareholder lets the subsidiary go bankrupt, all shareholders in the HC will lose their share of the external benefits and keep their share of the cash. On the contrary, if the controlling shareholder decides to bail out the distressed subsidiary, shareholders in the HC will keep their share of the external benefits by paying the face value of debt using the retained earnings of the holding company.

Since, by assumption, the controlling shareholder at the holding company has enough voting power to unilaterally decide the subsidiary's fate, the decision will be based on the relative wealth of the controlling shareholder under the bad state of nature. Hence, the bailout decision will be made if and only if the following condition holds.

$$(1) \left(X_1 + \frac{X_2}{h} \right) \geq \left(D - V_d \right)$$

Therefore, in the presence of external benefits, the controlling shareholder will decide to bailout a distressed subsidiary when the external benefits he receives from it are larger than the direct cost of rescue. It is interesting to note, that the relative value of the external benefits to the controlling shareholder is higher when he/she controls the HC with a low *h*.

Assume that the direct cost of rescue exceeds the amount of external benefits received by the controlling shareholder, so that condition (1) does not hold. Then the controlling shareholder will let the subsidiary fall in the bad scenario. Under perfect information, a holding premium would be observed if the holding company minority shareholders receive some external benefits generated by the group, X_1.

$$(2) \qquad HP(\text{external benefits, bankrupcy}) = (1-h) \cdot q \cdot X_1 \geq 0$$

Of course, if minority shareholders in the holding company do not receive any of the external benefits, there would be no holding premium or discount.

If condition (1) holds and the controlling shareholder decided to rescue the distressed subsidiary, a holding discount or premium would be observed in t=0 depending on the relative amount of external benefits received by the holding company minority shareholders.

$$(3) \qquad HD(\text{external benefits, bailout}) = (1-h)\left[X_1 - (1-q)(D-V_d)\right] \geq\leq 0$$

If X_1 is zero or small enough, a holding discount will be observed reflecting the expected net cost incurred by the minority shareholders of the holding company in case the subsidiary is bailed out by the holding company. Under no information asymmetries, however, the market anticipates the bailout of the subsidiary in the down estate of nature and minority shareholders will incur in no loss of wealth unless unexpected changes in the parameters occur. Despite this, if a holding discount is observed, when the time comes to rescue the subsidiary in t=1, minority shareholders will oppose the corporate action. Alternatively, if a holding premium is observed ($\left[X_1 - (1-q)(cash - V_d)\right] > 0$), both types of shareholders will obtain a value gain by bailing out the subsidiary and no horizontal agency conflict of interests will arise.

3.2.2. Internal benefits: Private benefits of control

Based on the model in section A, we now turn to the existence of private benefits of control, characterized as wealth transfers from the subsidiary´s shareholders to the controlling shareholder of the holding company. This activity is customarily referred to as tunneling and it is described elsewhere in the literature, including Friedman et al. 2003. A comprehensive description of different types of tunneling activities can be found in Atanasov et al. 2008.

In our model, a total of T resources are extracted directly from the subsidiary, lowering its security value. Analogous to the previous cases, we assume that T is decomposed into two parts: T_1 which is extracted through the holding company, thereby increasing its security value, thus benefiting all its shareholders, and T_2 which is extracted directly from the subsidiary to the controlling shareholder´s pockets. This is not reflected in the HC´s security value, as it is received only by its controlling shareholder. T_1 and T_2 take positive values and are not state dependent. Hence, the subsidiary's asset values are $V_u - T_1 - T_2$ and $V_d - T_1 - T_2$ in states *"u"* and *"d"* respectively.

We assume that if the financially distressed subsidiary went bankrupt, then the private benefits of control T would be lost. We also assume that cash held at the holding company is just enough to pay debt holders to avoid the subsidiary's bankruptcy, and $V_u - T_1 - T_2 > D > V_d - T_1 - T_2 > 0$. Table 31 summarizes wealth values in t=1, which include security values as well as private benefits for both types of shareholders and states of nature.

Table 31: Holding company shareholder net wealth in t=1

		State of nature "u"	**State of nature "d"**
HC lets distressed subsidiary go bankrupt	Controlling	$h \cdot \left(T_1 + f \cdot \left(V_u - T_1 - T_2 - D\right) + Cash\right) + T_2$	$h \cdot (Cash)$
	Minority	$(1-h) \cdot \left(T_1 + f \cdot \left(V_u - T_1 - T_2 - D\right) + Cash\right)$	$(1-h) \cdot Cash$
HC bails out distressed subsidiary	Controlling	$h \cdot \left(T_1 + f \cdot \left(V_u - T_1 - T_2 - D\right) + Cash\right) + T_2$	$h \cdot \left(T_1 + (V_d - T_1 - T_2)\right) + T_2$
	Minority	$(1-h) \cdot \left(T_1 + f \cdot \left(V_u - T_1 - T_2 - D\right) + Cash\right)$	$(1-h) \cdot (T_1 + (V_d - T_1 - T_2))$

Note: Private Benefits of Control

As shown in table 31 when state of nature is "u" internal benefits of control will remain in place. Alternatively, if state of nature is "d" and the controlling shareholder lets the subsidiary go bankrupt, all shareholders will lose their share of the internal benefits and keep their share of the cash. On the contrary, if the controlling shareholder of the HC decides to bail out the distressed subsidiary, all shareholders in the HC will keep their share of the internal benefits by paying the face value of debt using the retained earnings of the holding company.

Since, by assumption, the controlling shareholder at HC has enough voting power to unilaterally decide the subsidiary's fate, the decision will be based on the relative wealth of the controlling shareholder under the bad state of nature. Accordingly, the controlling shareholder will decide to bail out the subsidiary depending on the relative importance of his share of the private benefits and the direct cost of rescue. The subsidiary will be rescued if condition (4) holds.

$$(4) \qquad (1-h)\cdot T_2 > h\cdot\left(D-V_d\right).$$

Therefore, in the presence of private benefits of control, the controlling shareholder will decide to bailout a distressed subsidiary when the private benefits directly tunneled to his/her pockets are larger than his/her share of the cost of rescue. Intuitively, condition (4) shows that it is more likely to observe a bailout, when the controlling shareholder controls the holding company with a low *h*.

Assume that condition (4) does not hold, and the controlling shareholder will decide to let the subsidiary fall in the bad estate of nature. In such a case, the minority shareholders in the holding company will receive tunneling benefits only in the good estate but would never have to give away their share of the existing cash. Hence, their holding company shares would trade at a premium given by equation (5).[62]

$$(5) \qquad H\text{Premium}(Tunneling, going\ bankrupt) = q\cdot(1-h)\cdot T_1 \geq 0\cdot$$

This premium is equivalent to the expected present value of private benefits of control to be extracted from the subsidiary through the holding company.

Assume now that condition (4) holds and, hence, the controlling shareholder would rescue the distressed subsidiary in the bad scenario. In such a situation, the shares of the minority shareholders at the holding company would be traded either at a premium or discount depending on whether the portion of tunneling benefits common to all shareholders of the holding company is greater than the expected net cost of rescue.[63]

$$(6) \qquad H\text{P}\ (tunneling, bailout) = (1-h)\left\{T_1-(1-q)\left[D-\left(V_d-T_1-T_2\right)\right]\right\} \gtrless 0\,.$$

62 The minority shareholders' share in total security value of the HC would be $(1-h)\cdot\left\{q\cdot\left(T_1+f\cdot\left(V_u-T_1-T_2-D\right)\right)+Cash\right\}$ while their share of the NAV would be $(1-h)\cdot\left\{q\cdot f\cdot\left(V_u-T_1-T_2-D\right)+Cash\right\}$.

63The minority shareholders' share in the HC´s value is $q\cdot\left[(1-h)\cdot\left(T_1+f\cdot\left(V_u-T_1-T_2-D\right)+Cash\right)\right]+(1-q)\cdot\left[(1-h)\cdot\left(T_1+\left(V_d-T_1-T_2\right)\right)\right]$, while their share in NAV is $(1-h)\cdot\left(q\cdot\left[\left(f\cdot\left(V_u-T_1-T_2-D\right)\right)\right]+\left[Cash\right]\right)$.

The existence of private benefits of control may induce the controlling shareholder of a holding company to bailout a financially distressed subsidiary depending on the relative sizes of benefits and costs of rescue. When the perceived private benefits are low in comparison to the rescue costs, the controlling shareholder will let the subsidiary fail in the bad states of nature. In that scenario, the holding company's shares will be traded at a premium with respect to the NAV, which is proportional to the expected common private benefits accrued by minority shareholders. However, when private benefits tunneled directly to the business group controlling shareholder are large enough, he/she will rescue the distressed subsidiaries in order to maintain the tunneling of such benefits. Minority shareholders will trade their shares considering the expected impact of the bailout costs. Intuitively, when these costs are large compared to their share of the private benefits extracted from the subsidiary, the minority shareholders shares will trade at a discounted value with respect to the NAV. Notwithstanding, a holding premium can be observed when the amount of private benefits obtained by the minority shareholders of the holding company is large enough.

Assume that bailing out the subsidiary is the optimal course of action for the controlling shareholder of the holding company. Minority shareholders of the holding company may eventually obtain also a gain if $(1-h)\left[T_1-\left(D-\left(V_d-T_1-T_2\right)\right)\right]>0$, on the contrary, a horizontal agency conflict of interest will arise.

When most of the private benefits are tunneled directly to the controlling shareholders (large T_2), minority shareholders at the holding company will more likely be harmed by the bailout decision and a holding discount will be observed. All else equal, the decision to bailout the distressed subsidiary will more likely harm the minority shareholders at the holding company, when the stake of the controlling shareholder in the holding company is relatively small (h is close to 0.5 in our model) because minority shareholders will bear a larger fraction of the financing cost of the bailout.

3.3.3. Synergies

Finally, we allow for the presence of synergies, *Y*. Synergies may arise from many sources vastly treated elsewhere in the economics and management literature, like economies of scope, shared resources, shared markets, shared costs and distribution channels, network effects, economies of scale in joint operations, and financial synergies. Unexploited synergies are a clear incentive for conglomeration of a business group.

In the context of our model, synergies, unlike external and private benefits increase the security values of both the subsidiary and the holding company. We label synergies that increase security value Y_1 and divide the total value between the value increase of the subsidiary, Y_1^{Aff}, and the value increase of the holding company, Y_1^{hc}. For consistency, we also consider the existence of synergy value realized by the controlling shareholder in other non-related companies, Y_2.

We assume that all synergies arise from the consolidation with the subsidiary. If the subsidiary went bankrupt all synergies would be lost. As in cases B1 and B2, cash held at the holding company is assumed to be the amount that must be paid to debt holders to avoid the subsidiary's bankruptcy, D. We also assume $V_u + Y_1^{Aff} > D > V_d + Y_1^{Aff}$, $Y_1^{Aff}, Y_1^{hc}, Y_2 \geq 0$, and that they are not estate dependent.

Table 32 summarizes wealth values in t=1, including security and synergy values for both types of shareholders and for both states of nature.

Table 32: Holding company shareholder net wealth in t=1

		State of nature "u"	State of nature "d"
HC lets distressed subsidiary go bankrupt	Controlling	$h\cdot\left(f\cdot\left(V_u+Y_1^{Aff}-D\right)+Y_1^{hc}+Cash\right)+Y_2$	$h\cdot\left(Cash\right)$
	Minority	$\left(1-h\right)\cdot\left(f\cdot\left(V_u+Y_1^{Aff}-D\right)+Y_1^{hc}+Cash\right)$	$\left(1-h\right)\cdot Cash$
HC bails out distressed subsidiary	Controlling	$h\cdot\left(f\cdot\left(V_u+Y_1^{Aff}-D\right)+Y_1^{hc}+Cash\right)+Y_2$	$h\cdot\left(V_d+Y_1^{Aff}+Y_1^{hc}\right)+Y_2$
	Minority	$\left(1-h\right)\cdot\left(f\cdot\left(V_u+Y_1^{Aff}-D\right)+Y_1^{hc}+Cash\right)$	$\left(1-h\right)\cdot\left(V_d+Y_1^{Aff}+Y_1^{hc}\right)$

Note: Synergies.

If the estate of nature is "d" and the controlling shareholder lets the subsidiary go bankrupt, all shareholders will lose their share of the synergies and keep their share of the cash. Alternatively, if the controlling shareholder decides to bail out the distressed subsidiary, shareholders will keep their share of the synergies by paying the face value of debt using the retained earnings of the holding company.

In this case, the controlling shareholder will decide to bail out the subsidiary if the following condition holds.

$$(7) \qquad \left(Y_1^{hc}+\frac{Y_2}{h}\right)>\left[D-\left(V_d+Y_1^{Aff}\right)\right]$$

As before, it is more likely that the controlling shareholder will decide to bail the subsidiary in the case of financial distress, when the benefits of the synergies obtained by him are larger than the financial cost of bailing out the subsidiary.

Assume that condition (7) does not hold and, hence, the controlling shareholder will not bail out the subsidiary in the down estate. Then, provided that part of the synergies increases the holding company security value, the minority shareholders' shares in the holding company will be traded at a premium above net assets value.[64] This premium is equivalent to the expected present value of synergy benefits to be received only by the holding company.

64 In this case the holding company would receive synergy benefits only in the good state and would never use the existing cash; Hence, minority shareholders share in security value of the HC would be $(1-h)\cdot\left\{q\cdot\left[\left(f\cdot\left(V_u+Y_1^{Aff}-D\right)+Y_1^{hc}\right)\right]+Cash\right\}$, their share in NAV would be $(1-h)\cdot\left\{q\cdot\left[\left(f\cdot\left(V_u+Y_1^{Aff}-D\right)\right)\right]+Cash\right\}$.

$$(8) \qquad H\mathrm{P}(Synergies, going\ bankrupt) = q \cdot (1-h) \cdot Y_1^{hc}$$

On the other hand, if condition (7) holds, we would observe either a holding premium or a discount depending on whether the synergy benefits directly received by the shareholders of the holding company are greater than the expected net cost of rescue.[65]

- $$H\mathrm{P}\ (synergies, bailout) = (1-h) \cdot \left\{ Y_1^{hc} - (1-q) \cdot \left[D - \left(V_d + Y_1^{Aff} \right) \right] \right\} \geq\leq 0$$

As in the cases of external benefits and private benefits of control, the existence of synergies may induce the controlling shareholder of a holding company to bail out financially distressed subsidiaries, depending on the relative sizes of his benefits and his costs of rescue. When the perceived synergy benefits are low in comparison to the rescue costs, the controlling shareholder will let the subsidiary fail in the bad estates of nature. In that scenario, the holding company shares will be traded at a premium with respect to the NAV. However, when synergy benefits are large enough, the controlling shareholder will rescue distressed subsidiaries in order to maintain such benefits. Minority shareholders will trade their shares considering the impact of the bailout costs. Intuitively when these costs are large compared to their share of the synergies obtained, the minority shareholders' shares will trade at a discounted value with respect to the NAV. Even in this case, a holding premium can be observed when the number of synergies received by minority shareholders in the holding company is large enough.

Assume that bailing out the subsidiary is the optimal course of action for the controlling shareholder of the holding company. Minority shareholders of the holding company may eventually obtain also a gain if $\left(1-h\right) \cdot \left\{ Y_1^{hc} - \left[D - \left(V_d + Y_1^{Aff} \right) \right] \right\} > 0$, on

65 The minority shareholders' share in the security value of the holding company would be $q \cdot \left[(1-h) \cdot \left(f \cdot \left(V_u + Y_1^{Aff} - D \right) + Y_1^{other} + Cash \right) \right] + (1-q) \cdot \left[(1-h) \cdot \left(V_d + Y_1^{Aff} + Y_1^{other} \right) \right]$, while their share of NAV would be $q \cdot \left[(1-h) \cdot \left(f \cdot \left(V_u + Y_1^{Aff} - D \right) + Cash \right) \right] + (1-q) \cdot \left[(1-h) \cdot Cash \right]$.

the contrary, a horizontal agency conflict of interest will arise at the holding company.[66]

Section 4. Summary and main conclusions of Sections 1 to 3

A business group is a collection of separate firms tied by ownership relations and which, in spite of their legal autonomy, tend to respond to a common "group" strategy. In most cases, business groups exercise control over group affiliated companies through investment vehicles commonly referred as holding companies. A large academic literature indicates that the combined control of several firms using holding companies may allow a business group to benefit from conglomeration through reputational effects, operational and financial synergies, and the appropriation of private benefits extracted from the companies it controls.

On the other hand, the international empirical evidence shows that the market capitalization of a business group holding company is usually less than the market capitalization or economic value of the investments it holds once the holding company's debt is subtracted (Net Asset Value or NAV). This empirical regularity, the holding company discount, is attributed to limited free float of the holding company, tax inefficiencies associated with the holding company, additional administrative costs, and the extraction of private benefits by controlling shareholders at the expense of minority shareholders.

In this paper, we have developed a simple two period model of the holding company-subsidiary relationship that analyzes the option faced by the controlling shareholder to optimally bailout the subsidiary using the holding company's financial resources. In a simple model without taxes, illiquid markets, bankruptcy costs and informational asymmetries, we have shown that in the absence of reputational effects, synergies from conglomeration, or private benefits of control, business groups will not have

66 The controlling shareholder's decision to bailout the distressed subsidiary will more likely harm the minority shareholders at the holding company, all else being equal, when the stake of the controlling shareholder in the holding company is low (h is close to 0.5), which produces incentives to eventually keep the smallest stake possible in order to share the rescue costs if a rescue is going to be carried out. Also note that the wealth change caused to minority shareholders by the bailout can be measured directly by the change in holding premium or discount level.

incentives to forfeit limited liability and rescue financially distressed subsidiaries. In such a setting, holding discounts will not be observed.

However, under several incentive structures originated in the presence of group external effects, synergies and private benefits of control, business groups may find optimal to bailout financially distressed subsidiaries. The general intuition is that when the bankruptcy of a financially distressed subsidiary may cause a loss of conglomeration benefits to the controlling shareholder, it may be optimal for him/her to pay a cost in order to bailout the distressed company. This cost will be shared with minority shareholders at the holding company. In this sense, the conclusions of our model complement those of Almeyda and Wolfenzon because in both situations the controlling shareholder is willing to use other people's money to gain or retain access to his/her share of group related benefits.

In addition, our model sheds some light on the holding discount puzzle. In our model, holding discounts and premiums arise because the net asset values computed for holding companies do not consider the amount of conglomeration benefits shared with the minority shareholders of the holding company and the expected financial cost incurred by the same shareholders when the group decides to bailout a distressed subsidiary. Hence, if conglomeration benefits to the controlling shareholder are not high enough, holding companies will let subsidiaries go bankrupt and, in the presence of conglomeration benefits perceived by the minority shareholders of the holding company, a holding premium will be observed. On the contrary, if conglomeration benefits to the controlling shareholder are high enough, holding companies will rescue financially distressed subsidiaries and, if the financial cost of rescue borne by the holding company minority shareholders exceeds their share of conglomeration benefits, a holding discount will be observed.

In particular, the existence of private benefits of control may induce the controlling shareholder of a holding company to bailout a financially distressed subsidiary depending on the relative sizes of benefits and costs of rescue. When the perceived private benefits are low in comparison to the rescue costs, the controlling shareholder will let the subsidiary fail in the bad states of nature. In that scenario, the holding company's shares will be traded at a premium with respect to the NAV, which is proportional to the expected common private benefits accrued by minority shareholders. However, when private benefits tunneled directly to the business group controlling shareholder are large enough, he/she will rescue the distressed subsidiaries in order to maintain the tunneling of such benefits. Minority shareholders will trade their shares considering the expected impact of the bailout costs. Intuitively, when these costs are large compared to their share of the private benefits extracted from the subsidiary, the minority shareholders shares will trade at a discounted value with respect to the NAV. Notwithstanding, a holding premium can be observed when the amount of private benefits obtained by the minority shareholders of the holding company is large enough.

In the context of our model, the pervasive presence of holding company discounts is consistent with controlling shareholders of business groups that are prone to share the financial costs of keeping the pyramidal structure in place, but are less willing to share the conglomeration benefits with their minority partners in the holding company.

In such a world, we should expect holding discounts to increase as the probability of bankruptcy of business groups subsidiaries raises. Hence, all else equal, there should be a positive correlation between the subsidiary's debt risk premium and the holding discount observed. Also, holding companies that present a higher separation between cash flow and control rights to the controlling shareholder should present, all else equal, higher discounts.

Section 5. Some Empirical Findings and Hypothesis development

In section 3.1 in this chapter, we demonstrated, in a two times-one period (t=0 y t=1) our simplest model, and two states of nature in t=1(up-down), that holding discount may be written, in its simplest form as:

$$(1-q)\cdot(1-h)\cdot\left(V_d - D\right) < 0,$$

which is less than zero indicating a discount, and where:

> Vd: is total asset value of the subsidiary in the bad(down) state of nature.
>
> D: is the value of the debt in the subsidiary, which in case the subsidiary is in financial distress in paid by the holding company bailing the subsidiary out: $V_d < D$.
>
> (1-q): is the probability of a down state of nature in t=1, in which subsidiary enters in financial distress.
>
> h: in the controlling shareholders in the holding company share in equity.

The discount as indicated in the previous formula, may be understood as the share, that the non- controlling shareholders in the holding company face of the total cost of bailout, when the controlling shareholders in the holding company decide to bail it out. This decision in turn may be taken with or without consent of minority shareholders in the holding company as well as in the Subsidiary.

The basis of this section lies in recognizing that the discount amount is, according to the model, is directly related to the probability of financial distress, i.e.:

$$\frac{\partial |Discount|}{\partial (1-q)} > 0.$$

As shown in the previous equation, the holding discount is positively related with the level of risk (probability) of default of the holding company. This probability of default is captured by the market in the spread (Yield minus Risk Free rate) of bonds issued by the company. Therefore, we develop the following hypothesis:

Hypothesis H3 Null: The holding discount is positively related to the debt riskiness of the business group as reflected in the risk spread of bonds issued by the business group[67].

As indicated in Chapter 3: "holding company discount in the Chilean market, 1993-2007: Some facts and Statistical Analysis", several tests, such as linear regressions,

67 See chapter 1, 3.1.2, section for a complete listing of the thesis's hypotheses

(not shown), and cointegration were made to find systematic relations between calculated holding discounts of nine selected Chilean business groups and several macroeconomic independent variables such as interest rates, economic growth, economic business expectations. No statistically significant results were obtained. This indicated that the search of idiosyncratic factors was necessary; a straightforward proxy variable to the probability of insolvency (1-q), is the corporate bonds spread (actual traded bonds yield minus treasury yield considered as risk free). Given that the greater the risk of financial distress, the greater the bonds spread, we can then formulate the following null hypothesis H3 Null: The holding discount is positively related to the debt riskiness o the business group as reflected in the risk spread of bonds issued by the business group. Per our model in section 3.1A, the greater the overall risk of financial distress of the business group, should result in a greater discount.

The spread calculation procedure, see appendix, as well as the spread data were submitted by RiskAmerica®. It basically consists in a parallel upwards shift of the yield curve of risk free of government bonds, with which the problem of comparing spreads of bonds with different remaining life or different duration is avoided.

On the other hand, the intersection of the available information from Chapter 3: "holding company discount in the Chilean market, 1993-2007: Some facts and Statistical Analysis", consisting in the calculated discount for nine selected Chilean business groups, and the information available from RiskAmerica®, permits us to analyze the discount versus spread relation for one group only, IAM Group, for which we have daily discounts and spreads from different corporate bonds traded , for the period November 2005-April 2009.

From "holding company discount in the Chilean market, 1993-2007: Some Facts and Statistical Analysis", Inversiones Aguas Metropolitanas, "Inversiones Aguas Metropolitanas is the holding controlling company of Aguas Andinas through a shareholding of 50.1% in the company, at 31 of December 2008. Aguas Andinas together with its subsidiaries make up the largest water utilities group of Chile and one of the largest in Latin America. In July 2005, IAM became a public joint-stock company and in November of the same year it went public through an IPO. The company limited its business line to investment in shares of Aguas Andinas and the provision of all kinds of consulting services related to the transfer of technology and know-how, technical support, businesses, and project management, specially related to the management and operation of water management businesses. Compañía General de Aguas Barcelona, Agbar, owns 56.6% of IAM´s stocks, and is the only controlling company shareholder. Agbar is the parent company of a holding company that owns more than 150 companies and has 19,000 employees.

Aguas Andinas provides water management services to the city of Santiago and nearby areas. It serves a population of around six million inhabitants. It produces and distributes drinking water, captures, treats and disposes of the wastewater together with other related services.[68] At December 31 2008, its main shareholders were Inversiones Aguas Metropolitanas (50.1%), Corporación de Fomento de la Producción, CORFO, a Chilean State Company (35.0%) and Pension System Fund Managers AFPs (10.1%). The company is part of group Aguas, a set companies related to the sanitary industry. The services of production, distribution of drinking water, capture, treatment, and disposal of wastewater are carried out by Aguas Andinas, Aguas Cordillera, Aguas Manquehue and Aguas Los Dominicos. The services related to the treatment of industrial waste, sale of equipment and laboratory analysis are carried out by unregulated subsidiaries EcoRiles, Gestión y Servicios and Anam.

IAM, Inversiones Aguas Metropolitanas, is a good case for analyzing holding discount. On 31 December 2007, its main asset was 50,1% shares of Aguas Andinas (Water and Sewage Facilities for Santiago de Chile). In this case, the investment in shares is practically 100% of IAM's assets.

68 See web page of the Bolsa de Comercio de Santiago, www.bolsadesantiago.com

Following figure 22 shows the evolution of IAM holding discount for the period 2005 2009.

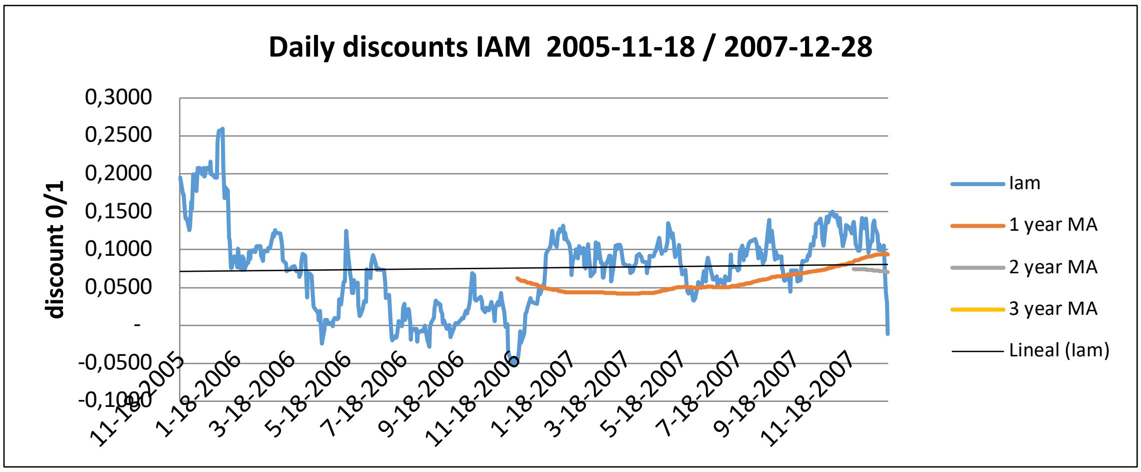

Figure 22. Daily discounts IAM 2005-11-18 / 2007-12-28

The next graph (figure 23) shows the evolution o the pread calculated for the six different corporate bonds, series C1,C2, D1, D2, E, F, with different durations, for the same period, this time measured in working days. The average of the spreads of the different bonds traded in a particular day has been included. This last variable is of particular interest because it reflects the wheigted average of bonds of different durationt in a common measure, because of the calculation procedure. It can be observed that bonds weren´t traded every day, so graph is discontinuous. The variable with the most number of observations is average spread (sprmedio), which, given the spread calculation procedure is a "legitimate" average, representing the spread for corporate debt of IAM in a given particular date.

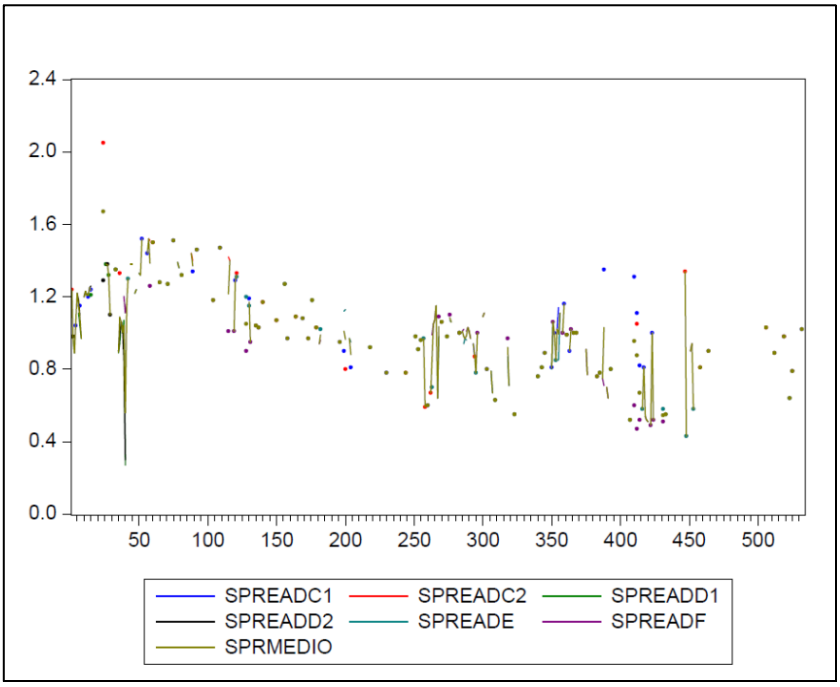

Figure 23. Spreads in % for the various Bonds analyzed

Finally, in the next graph (figure 24) we present both, discount and average spread in a rescaled graph. Spread has been multiplied by ten (x10). At a first glance, some coevolution is observed.

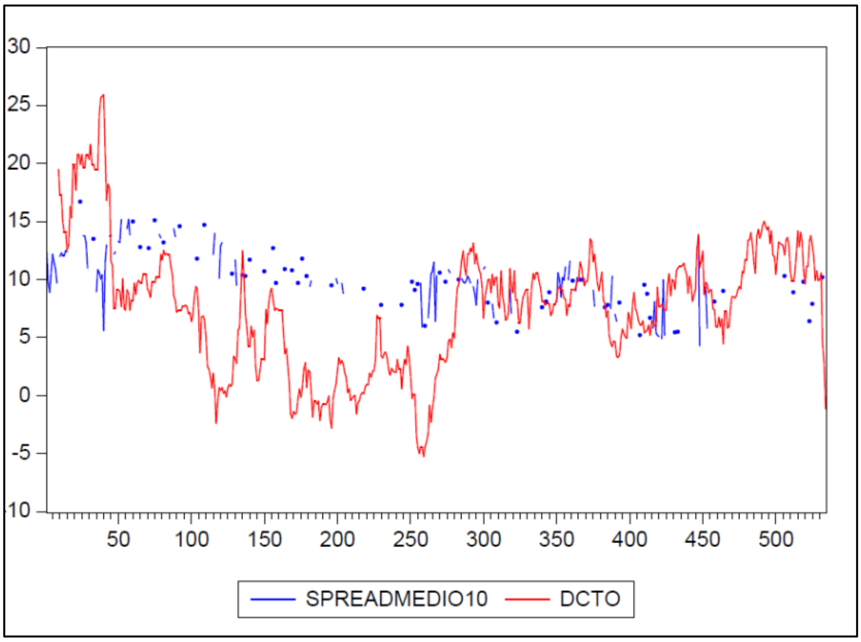

Figure 24. Scaled Spread (medio) versus discount

5.1. Results of the Empirical Section

Next, we provide main results for the different regressions run in order to check whether there is an association between spread and discount, as hypothesized before. Results obtained are interesting. We run linear OLS regressions with intercept (regression) and without intercept (projection) of discount as dependent variable and spreads of the six bonds traded, individually, plus the average spread, as independent variable.

Although it is true that correlation coefficient is low in all regressions (even negative in some cases, a common result in regressions without intercept), what we are looking for if for statistically significant association between variables, and that is observed through students t statistic and this is what we found in the majority of cases. High student´s t statistic implies a very low probability that the true coefficient for the explanatory variable is null. In all projections run, t statistic is very high, and the sign of the association is correct. Nevertheless, not in all the regressions run, the statistic is also very high, showing in those cases a statistically significant association between bond daily spreads and daily discount, but in other cases, the statistic is low and the sign of the association is reversed. Notwithstanding, when we consider average spread, that according to our analysis is representative as a given spread for a given day, the association of average spread and discount is very strong, and the sign is correct. We conclude stating that at least using this kind of methodology, and for one group, IAM, we cannot reject the null hypothesis H3 that riskiness (as reflected in spreads) and discount are significantly related. Table 33 summarizes the results for Projections (OLS without intercept).

Table 33: (OLS) Projections Spread% versus holding discount.

	C1	C2	D1	D2	E	f	Average
#observat.	37	24	11	14	61	59	162
R2 adj.	0,11	0,246	-2,43	-2,63	-0,06	0,007	0,018
Coefficient	6,64	5,75	17,33	16,82	6,95	8,29	7,92
t student	9,35	6,29	6,95	8,24	10,33	10,58	17,04
Prob	0,000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Note: Independent Variable: Spread % bonds different series; Dependent Variable: holding discount %; Frequency: daily

In previous table 33, Projections, it can be seen the coefficient for the independent variable y positive as hypothesized, and t student statistics are very high in all seven cases, showing a high confidence level, greater than 99%, in every case.

Table 34 summarizes the results for Regressions (OLS with intercept).

Table 34: OLS regressions between Spread% and holding discount%

	C1	C2	D1	D2	E	F	average
#observat.	37	24	11	14	61	59	162
R2 adj.	0,086	0,28	0,18	0,38	-0,07	0,01	0,032
Coefficient	7,04	10,4	-8,25	-10,6	1,83	4,52	4,69
t student	2,098	3,17	-1,80	-3,02	0,729	1,265	2,52
Prob	0,0432	0,0044	0,1057	0,0106	0,4684	0,2108	0,0127
Coeff.intercept	-0,45	-5,88	28,9	31,19	5,33	3,61	3,45
T student.int.	-0,12	-1,47	5,78	8,04	2,11	1,08	1,79
Prob.intercept	0,9045	0,1552	0,0003	0,0000	0,0385	0,2836	0,0751

Note: Independent Variable: Spread % bonds different series; Dependent Variable: holding discount %; Frequency: daily.

In previous table 34, OLS with intercept, it can be seen that existence of intercept attenuates and even destroy the strong associations found in the projections; in some cases, the sign of the association is reversed, contrary to the prediction of the simple model tested.

This may be due to many factors, as omitted independent variables, or the existence of tunneling (extraction of private benefits to the controlling shareholder), included in the more complete model where we can predict discounts or premium depending on the size of the effects.

Interesting to note, of the seven regressions presented, four present a statistically insignificant intercept (significance level greater than 5%). In both types of OLS (with and without intercept) the association between average spread and discount is statistically significant, although the direction of causation is to be explored in the future.

Therefore, we cannot reject H3 Null, given that holding discount is positively related to the debt riskiness of the business group as reflected in the spread of bonds issued by the holding group.

Section 6. Summary and conclusions

In this chapter, we develope a simple one period model based on the complete model 4 in chapter 2, thet contrary to that model, specifies the kind of internal and external benefits that a controlling shareholder may have controlling a simple business group.

Some benefits are exclusive to the controlling shareholder, as private benefits of control may be, and some of the benefits are shared with the rest of the shareholders, such as synergies and external benefits like reputation.

We examine the incentives of the controlling shareholder to rescue a distressed affiliate (un-limited liability), instead of letting it go bankrupt. These incentives can be understood as the relative benefit and costs of rescue benefits in place instead of loosing them. We also examine the conditions under which the rescue decision is against minority shareholders interests.

As in model 3 and four in chapter 2, a holding discount arises, which is positively related to the probability of distress. This probability is idiosyncratic to the subsidiary and can be proxied with the riskiness ob the public debt issued by the subsidiary or the group.

To empirically analyze this fact, we formulate Hypothesis H3 Null: The holding discount is positively related to the debt riskiness of the business group as reflected in the risk spread of bonds issued by the business group.

In spite the lach of informstion we have for eight out of nine of the selected business groups from chapter 3, we have information for IAM, permiting it to contrast empirically hypothesis H3, and conclude that H3 cannot be rejected. This result reinforces the theoretical model developed.

Section 7. Avenues for further research

In a further potential research, the model developed in this chapter, could eventually be extended to a second period, for example to two stages, and include corporate taxes, and some asymmetric information. In my opinion, according with the results shown, this would enrich even more the analysis of this chapter, helping future researchers to develop new models that forecast better the holding discount.

Additionally, If I counted on the appropriate data to construct a data base for the recent years, I should be able to extend the analysis done of one group in this chapter, to the rest of the selected groups analyzed in Chapter 3.

Finally, if there existed the information needed to measure internal and external benefits, we could be able to obtain a more quantitative relationship between size of the holding discount and probability of distress. This exercise has proven to be very difficult to do, existing some very interesting excercises in the literature. (Bertrand et al 2002).

Appendix

Spread Calculation ©RiskAmerica Procedure®

Starting from a temporal structure of interest rates, the spread of any transaction with respect to the reference structure, can be calculated according to the following steps.

1.- Zero Coupon Reference Structure Calculation: RiskAmerica calculates, on a daily basis, the temporal interest rate structure implicit in the transaction prices of the different financial instruments of the Chilean market. The spot or zero coupon rates are estimated using a dynamic stochastic model Vasicek type, estimated using a Kalman´s Filter; this model has as main characteristic, its stability and the adjustment of the volatility structure of the observations. In the case of IAM, the reference model for prices is Zero real rates (inflation adjusted), which is utilized as the discount rate temporal structure, which is used as the discount structure for each "instruments family", in this case Corporate Bonds.

2-. Calculation of the present value of the Bond by discounting the remaining coupons of a given Bond on a given day, using the reference interest rate structure of point 1.-

3-. Calculation of the Base IRR (internal rate of return): with the present value calculated in 2-., and the schedule of remaining coupons of the bond, an iterative process is done, so as to find the IRR of the Bond which gives the same present value.
4-. Spread Calculation: Spread is the difference between the effective transaction IRR of the bond in that day (Bond IRR) and the Base IRR calculated in 3.

5-. Example: The following example illustrates the case in which a bond has three remaining coupons in periods T_1, T_2, T_3. Zero Coupon Rates from the Reference Structure are: r_{T1}, r_{T2}, r_{T3} .

Then Present Value of the Bond is:

$$\text{Base Present Value} = \frac{C_{T1}}{(1+r_{T1})^{T1}} + \frac{C_{T2}}{(1+r_{T2})^{T2}} + \frac{C_{T3}}{(1+r_{T3})^{T3}}.$$

With the Base Present Value Base IRR is calculated as

$$\text{Base Present Value} = \frac{C_{T1}}{(1+BaseIRR)^{T1}} + \frac{C_{T2}}{(1+BaseIRR)^{T2}} + \frac{C_{T3}}{(1+BaseIRR)^{T3}}$$

Finally, for the specific bond at a given date is:

Spread = Bond IRR – Base IRR.

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Chapter 5. Summary, Conclusions and Avenues for further research

Index

Section 1. Introduction

At the beginning of my studies in the Ph.D. program, a question troubles my mind: What is the importance of a holding discount?

If a holding company raises capital in the securities markets and has a penalty in form of a holding discount, it has a direct implication: its shares are undervalued, and less capital than expected is raised. In other words, its capital is issued at a high cost. On the other hand, the holding company's shares should be better valued it subsidiaries shares increase their prices in the markets.

For the holding company on its own, an improvement in (lower) discount is highly important because it signals to the market an efficient corporate governance, thus improving its reputation, generating more business opportunities, and consequently, improving its economic value.

In the five chapters of this thesis, I address the problem of studying the holding discount, mainly from the corporate finance perspective. Although several aspects of businesses influence the holding discount, for example business strategy, structure of the group, and corporate governance, they are not analyzed in this thesis.

After reviewing the theoretical contributions and the main results of the selected studies, I conclude two main things:

First: business group discount is well documented in the literature for several countries, including Chile, utilizing an indirect method to calculate the discount, which is the imputation of values to the nonpublic subsidiaries of the business groups. Also, this discount appears to be stable along time on average.

Second: many causes have been invoked to explain the business discount, for example too much diversification, too much investment, excess od indebtness, double taxation between holding company and subsidiaries, imperfections (such as taxes, poor corporate governance, agency problems (including conflicts of interests between board of directors and shareholders, or within boards of directors), information asymmetries, and extraction of private benefits of control, in a legal system of limited liability of corporations.

In this thesis I study, from a theoretical and from an empirical point of view, the discount of business groups in Chile, attempting to find some of the main determinants in a context of efficient and competitive capital market, without the imperfections already mentioned, to see if discount can be modeled or predicted, consequently studying its predictability over time. Some interesting questions are raised below to carry out this research and the corresponding hypotheses will be formulated.

I now review summaries, conclusions, and avenues for further research, for chapters 2,3 and 4.

Section 2. From Chapter 2

2.1 Summary and conclusions

Chapter 2 was conceived to show a very common practice in holding's financial management that is the use of subsidiaries to address several objectives of the parent company of the group.

I developed a series of recursive models for a business group in which the levered holding company repeatedly faces the decision to bailout a distressed levered subsidiary in bad times. I have stated the conditions for this decision to be made, named exercising the "unlimited liability option" (in the literature this is a propping or bailing out behavior), in several contexts of increasing complexity. This increasing complexity is reflected in models 1 to 4, the former stating the structural basis of the rest of the models, the latter is an extension of model 3.

Model 1 corresponds to a Stand-Alone Firm. Model 2 analyzes an unlevered holding company, that when decided by the controlling shareholder, rescues the distressed affiliate making an equity call. Model 3 analyzes a levered holding company, that when decided by the controlling shareholder rescues the distressed affiliate by issuing debt. Model 4 is a special case of model 3, in which holding company rescues the distressed affiliate using existing cash in the holding company. Specific characteristics of the models developed, can be found in section 5.5, Table 4.

Contrary to models developed in the literature, designed to explain the holding discount under asymmetrical information and other market imperfections, my models rely in a context of full and symmetrical information and rational investors, in the absence of taxes only, and explain the existence of a holding discount under certain plausible conditions; this is a main contribution of this chapter.

The analysis starts by formulating Hypothesis H0: holding discount exists, only when asymmetric information, agency costs or other imperfections, are considered regardless of the legal system. This hypothesis is contrasted against models 2,3 and 4, sequentially.

Additionally, eventual conflicts of interest with minority shareholders in the holding company may arise. Moreover, I can also show that in certain conditions, majority shareholders and minority shareholder's interests will be aligned when the bailout decision is faced, and depending on the assumptions of the models, they can be in conflict, as existing literature prescribes.

Finally, I derive a set of empirical implications from the models.

2.2 Avenues for further research

In practice, what I can observe in the marketplace is a time series of observable variables for several given business groups already conformed, and their stock traded since the IPO. I do not observe all the variables that affect the continuation conditions, the losses or price discounts. What I can infer from the model, are variations in losses (discounts) caused by variations in observed variables that alter the continuation conditions.

The chain of causality is as follows: $\partial(Obs) \Rightarrow \partial(NotObs) \Rightarrow \partial CC \Rightarrow \partial \Pr ice$, where $"\partial"$ means change in a variable while keeping all the rest unchanged, "Obs" are exogenous observed variables firm specific or market conditions, "NotObs" are variables that are in my models, and are functions of observed variables, CC means continuation conditions and price would be holding premium or holding discount.

The questions I can propose can be derived from the following facts:

1. When Controlling shareholder follows the LL rules, I will not observe premium or discount in the context of no market imperfections, no information asymmetries.
2. It follows that a holding discount may arise when the distressed affiliate is rescued, which implies a cost to minority shareholders, with some probability of rescue or occurrence. The concurrence of a favorable continuation conditions for the Controlling Shareholder (*CCcs*), as well as the probability of distress (1-q), and the cost of rescue are crucial. The holding discount can be regarded as the expected loss to minority shareholders, which has embedded the probability of default and the cost of rescue.

Then, the most important variables that I can monitor and include in a model are:

a) Continuation Conditions, mainly indebtness of the subsidiary, and private benefits of control which are not observed in the market.
b) Probability of default. This can be proxied by some variable that reflects that probability in the market, for example the risk classification, or the observed spread of loans existing in the holding company and if possible in the affiliate company. Refer to Chapter 4, "Optimal Bailout of subsidiaries" as a good example of this.
c) Cost of rescue that will be borne by minority shareholders.

As it can be seen in the rest of Section 7, if I had the necessary public information to work with (which unfortunately is not the case), may empirical tests can be done, involving time series of the discount for a specific holding, and a cross section (panel) of several holdings, to make conclusions more robust. The main counter force is the availability of public information.

Let us see several models that can be advanced in the form of panel:

a. Continuation Conditions

   Controlling Shareholder forces rescue when this condition is positive:

$$CC_{cs}: \frac{q\cdot(V_u - D_1)}{R_e} - c\cdot f\cdot m + M = \frac{q\cdot(V_u - D_1)}{R_e} - (D_1 - V_d)\cdot f\cdot m + M\cdot$$

   Stand-alone Company representative shareholder benefits from rescue when positive are:

$$CC: \frac{q\cdot(V_u - D_1)}{R_e} - (D_1 - V_d)\cdot$$

An answer that can be advanced is, for example: if CC gets more positive, the greater the probability of rescue, given a state of nature, and the greater the discount.

Other hypothesis can be: the worst the state of the economy (1-q increases) the greater the probability of rescue.

Then, how do I answer those questions? Starting from an initial operating point, were $CS_{cs}\ can\ be >, =, or > 0$, but $CC < 0$, I can evaluate the changes of CCcs and CC due to a change in the relevant variables. In this way I can write:

$$CC_{cs} = CC_{cs}(f^-, m^-, D_1^-, R_e^-, M^+, q^+, V_u^+, V_d^+)$$
$$CC = CC(D_1^-, R_e^-, q^+, V_u^+, V_d^+)$$

This expression can be read as follows: the likelihood of observing a price reduction on the affiliate as well as the holding companies, increases as variables labeled with "+" increase, and decreases when variables labeled with "-" increase.

b. Shareholder´s Losses (CS forcing vs. CS not forcing)

Majority, affiliate (MajA):

$$LMajA = f \cdot \frac{1-q}{r_e+q}\left(-(D_1 - V_d) + \frac{q\cdot(V_u - D_1)}{R_e}\right) = f \cdot \frac{1-q}{r_e+q} \cdot CC(D_1^-, R_e^-, q^+, V_u^+, V_d^+)$$
$$= LMajA\left(f^+, q^-, CC^+(D_1^-, R_e^-, q^+, V_u^+, V_d^+)\right)$$

Minority, affiliate (MinA):

$$LMinA = (1-f)\cdot \frac{1-q}{r_e+q} \cdot CC(D_1^-, R_e^-, q^+, V_u^+, V_d^+)$$
$$= LMinA\left(f^-, q^-, CC^+(D_1^-, R_e^-, q^+, V_u^+, V_d^+)\right)$$

Majority, holding (MajH):

$$LMajH = m \cdot f \cdot \frac{1-q}{r_e+q} \cdot CC(D_1^-, R_e^-, q^+, V_u^+, V_d^+)$$
$$= LMajH\left(m^+, f^+, q^-, CC^+(D_1^-, R_e^-, q^+, V_u^+, V_d^+)\right)$$

Minority, holding (MinH):

$$LMinH = (1-m)\cdot f \cdot \frac{1-q}{r_e+q} \cdot CC(D_1^-, R_e^-, q^+, V_u^+, V_d^+)$$
$$= \left(m^-, f^+, q^-, CC^+(D_1^-, R_e^-, q^+, V_u^+, V_d^+)\right)$$

c. Price discount (minority shareholders valuation (CS forcing vs. CS not forcing)

Affiliate company (PdA):

$$1 - \frac{\left(E(V_1) - D_1\right)}{q\cdot(V_u - D_1)} \cdot \frac{R_e}{r_e+q}$$

holding company (PdH):

$$1 - \left(\sum_i f_i \cdot S_i - D_0^H + \frac{(1-m)\cdot\left(E(V_1) - D_1\right)}{r_e+q}\right) / \left(\sum_i f_i \cdot S_i - D_0^H + \frac{(1-m)\cdot f \cdot q \cdot (V_u - D_1)}{R_e}\right)$$

Section 3. From Chapter 3

3.1 Summary and Conclusions

This chapter was conceived after observing the real prices in the Chilean stock market. It was very common to see that quoted business groups shown a difference in the price of the holding company and in the stake in the price of its subsidiaries.

To analyze this situation, it is necessary to see the problem from several dimensions: In first place, I have analyzed several Chilean business groups using simple criteria for selection, that is the group must be as simple as possible, in order to eliminate two factors, I did not intend to analyze: the diversification factor, and the non-public company's valuation issue.

With this I select a set of 9 business groups, which are fully described.

In second place, I have used a unique and proprietary data base for the nine selected Chilean groups constructed under my direction and permitted to calculate very reliable groups discounts on three frequencies: daily, weekly and monthly. In the appendixes I document the standard statistics for the discounts. The discounts show high averages and high volatility.

In third place, I tested for stock market inefficiency. My null hypothesis H2 is that the market is efficient, hence the discount is not predictable. To test my hypothesis, I use four sub hypotheses and four complementary methods, whose main objective is to check whether the discounts can be predicted in some way. If that were the case, stock market would present inefficiency, and main actors in the stock market would be correct in arbitraging on the discounts.

The four methods are: Unit Root Tests (test for stationarity -for my purposes predictability- of a time series); Variance Ratio Tests (test for the presence of a mean reversion of a time series, or martingale behavior (hence predictability again); Cointegration Analysis (tests for a long run stationary relationship between the two components of the discount: Market Capitalization of the holding company (Market Value or MV) and its Net Asset Value (NAV); Comovement Analysis (that tests for another aspect of the Efficient Market Hypothesis, that is the presence of investor sentiment, which makes that the stocks, or in this case the discount, of structurally and fundamentally unrelated businesses move together, departing from the random walk hypothesis.

The results of these tests show that there is some degree of inefficiency in the Chilean stock market during the period 1993-2007.

The Unit root tests show that six out of nine groups present random walk behavior regardless of the frequency of data. This must be considered as a not concluding evidence against non-predictability, according to my null hypothesis H2a.

Variance ratio tests go more strongly against my hypothesis H2b. The VR results suggest that only the minority of the groups present martingale behavior, for my purposes unpredictability of the discounts, showing a higher degree of market inefficiency than that showed with the Unit root tests. The results of the VR test permit me to reject the null hypothesis H2b using high frequency data (daily and weekly) but are more in favor -although still against- of the null hypothesis H2b if I use monthly data.

Cointegration analysis results suggest that with a at least 90% confidence level, I have at least one long run stationary relationship between the lnMarket Value and the lnNav, for almost all the groups, which represent the discount, showing reversion periods of several months. I cannot conclude that this kind of information is enough to enter in arbitrage operations, but I can conclude that there is at least weak evidence in favor of market inefficiency, this means not been able to reject hypothesis H2c.

Finally, the comovement analysis is less conclusive against my null hypothesis H2d; there is some evidence of comovement, and if I accept the underlying theory, of investor sentiment, but is far from been a general phenomenon in the time span of analysis.

3.2 Avenues for further research

From an economic perspective, not high-developed countries, like Chile, had not databases containing enough information to contrast several models constructed in high-developed countries. In this thesis I had the opportunity to organize information about the Chilean stock market and accounting information about the firms traded in it, in a robust data base. Using this organized information, I have worked to contrast the hypothesis shown in this chapter. If I had a new opportunity, or if the Chilean financial authorities develop a new database with the relevant information for the nine selected groups, analyzed in this chapter, for the recent years, I have the will, and the tools to expand the conclusions, and if it possible to extend this work to other groups. All the statistical analysis could be done in a new temporal horizon, confirming, or contradicting the conclusions of this chapter along the time.

Section 4. From Chapter 4

4.1 Summary and conclusions

Using the knowledge gained in chapter 2 and chapter 3, it was possible to study another common practice in holding companies. Normally, in this companies there is a set of shareholders that control the whole business group co-habitating with a little group of shareholders that only receive the protection of the law. This situation allows an asymmetrical management about the benefits.

In this chapter, we developed a simple one period model based on the complete model 4 in chapter 2, the contrary to that model, specifies the kind of internal and external benefits that a controlling shareholder may have controlling a simple business group.

Some benefits are exclusive to the controlling shareholder, as private benefits of control may be, and some of the benefits are shared with the rest of the shareholders, such as synergies and external benefits like reputation.

We examine the incentives of the controlling shareholder to rescue a distressed affiliate (un-limited liability), instead of letting it go bankrupt. These incentives can be understood as the relative benefit and costs of rescue benefits in place instead of losing them. We also examine the conditions under which the rescue decision is against minority shareholders' interests.

As in model 3 and four in chapter 2, a holding discount arises, which is positively related to the probability of distress. This probability is idiosyncratic to the subsidiary and can be proxied with the riskiness of the public debt issued by the subsidiary or the group.

To empirically analyze this fact, we formulate Hypothesis H3 Null: The holding discount is positively related to the debt riskiness of the business group as reflected in the risk spread of bonds issued by the business group.

In spite the lag of information we have for eight out of nine of the selected business groups from chapter 3, we have information for IAM, permitting it to contrast empirically hypothesis H3, and conclude that H3 cannot be rejected. This result reinforces the theoretical model developed.

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Finally, if there existed the information needed to measure internal and external benefits, we could be able to obtain a more quantitative relationship between size of the holding discount and probability of distress. This exercise has proven to be very difficult to do, existing some very interesting exercises in the literature. (Bertrand et al. 2002)

Section 5. Final Remarks

With this thesis, I expect to contribute, a little more, to develop the theory of the holding discount phenomenon. After the reading of my work, I would open new avenues for future research.

By following a rigourous hypothetical-deductive method I have designed several hypotheses confronted along the chapter of the thesis, and I have been able to prove several interesting facts:

- Holding discount can be modeled without recourse to several market imperfections, and under several feasible conditions; holding discount exists in Chile, as measured in a direct way, and can be predicted to follow a long-term value or trend, as various statistical tests show.
- Holding discount appears to be linked to idiosyncratic business group risks rather than macroeconomices risks, which is proved in some way with our models,
- Holding discount increases as the extraction of private benefits of control, internal and external synergies increase.

With this basis, it is possible to recognize the most relevant variables to construct new databases that allow to extend the models shown in previous pages. With more appropriate data and extended models it will be possible to produce more robust forecasts about holding discounts and to prevent financial distress and bailout situations.

This thesis and the future research have the aim to help holding companies' managers to achieve a better way to satisfy stakeholders needs, not only to serve the immediate interest of the present shareholders.

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