Exploring through prototyping embodied interaction design

Developing systems for mathematics learning for children with visual impairments and without

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“People cannot learn from their experience as long as they are entirely immersed in it. There comes a time when they need to step back, and reconsider what has happened to them from a distance. They take on the role of an external observer, or critic, and they revisit their experience “as if” it was not theirs. They describe it to themselves and others, and in so doing, they make it tangible and shareable.”

-- Ackermann, Edith K

The PhD process has been a deep immersive one, it might be the time to step back, write down the whole story and reflect.

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Abstract

In the last decades new interaction paradigms have emerged: Tangible User Interfaces, ubiquitous computing, wearable devices, mixed-reality among others. Such paradigms extended the user interface beyond the keyboard and mouse, and physical interaction has gained importance. This transformation represents a challenge-opportunity for interaction and experience designers. As a consequence, design frameworks are incorporating embodied cognition theories, getting inspiration from phenomenology and aiming to integrate body, mind and technology. This interaction design perspective is known as embodied interaction. This dissertation aims to understand how to design and implement embodied interactive systems for mathematics learning for children, including sighted children and children with visual impairments (VIs). Thus, we might capitalize technological progress into actual opportunities to better support learning. In this context, the thesis explores the development of three interactive systems for mathematics learning and the evaluation of two of them. Through this prototyping approach we discuss design implications for embodied interaction systems in learning contexts, contributing with the generation of intermediate-level knowledge. Finally, we also confirm and extend previous research in this field.

Resumen

En la últimas décadas han emergido nuevos paradigmas de interacción: Interfaces de Usuario Tangibles, computación ubicua, dispositivos “vestibles”, realidad mixta entre otros. Estos paradigmas han extendido la interfaz de usuario más allá del ratón y el teclado, provocando que la interacción física ganase trascendencia. Para los diseñadores de interacción y experiencia de usuario, esta transformación representa un desafío y oportunidad al mismo tiempo. Consecuentemente, los frameworks de diseño han estado virando hacia la incorporación de teorías inspiradas en fenomenología como la cognición encarnada, buscando la integración de cuerpo, mente y tecnología. A esta perspectiva de diseño de interacción se le ha llamado interacción encarnada (*embodied interaction)*. Esta tesis busca comprender cómo diseñar sistemas de interacción encarnada para el aprendizaje de matemáticas tanto para niños videntes como para niños con discapacidad visual. Entonces, seríamos capaces de capitalizar el avance tecnológico en oportunidades concretas que apoyen el aprendizaje. En este contexto, esta tesis explora el desarrollo de tres sistemas interactivos para el aprendizaje de matemáticas y la evaluación de dos de ellos. A través del desarrollo de estos prototipos, discutimos implicaciones de diseño para sistemas de interacción encarnada en contextos de aprendizaje, contribuyendo a la generación de conocimiento intermedio (*intermediate-level knowledge*). Finalmente también confirmamos y extendemos trabajos previos de investigación en este campo.

Resum

Durant les darreres dècades han aparegut nous paradigmes d'interacció: interfícies d'usuari tangibles, computació ubiqua, "wearable devices" (dispositius vestibles), o la "mixed-reality" (realitat mixta), entre d'altres. Aquests paradigmes han estès la interfície d'usuari més enllà del teclat i el ratolí, i la interacció física ha guanyat importància. Aquesta transformació representa un repte/oportunitat pels dissenyadors d'interacció i d'experiència d'usuari. A conseqüència d'això els "frameworks" de disseny estan incorporant teories d'"embodied cognition" (cognició corporal), prenent inspiració de la fenomenologia amb l'objectiu d'integrar cos, ment i tecnologia. Aquesta perspectiva de disseny d'interacció es coneix com "embodied interaction". Aquesta dissertació té l'objectiu d'entendre com dissenyar i implementar sistemes d'"embodied interaction" per l'aprenentatge de matemàtiques dels infants, incloent tant nens amb capacitats visuals intactes com aquells amb discapacitats visuals. Per tant, podríem capitalitzar el progrés tecnològic convertint-lo en oportunitats reals per millorar el suport a l'aprenentagte. En aquest context, aquesta tesis explora el desenvolupament de tres sistemes interactius d'aprenentatge matemàtic i la evaluaciόde dos d'ells. Mitjançant aquesta aproximació a través del prototipatge discutirem les implicacions dels sistemes d'"embodied interaction" en contextos d'aprenentatge, contribuint amb la generació de coneixement de nivell intermedi. Finalment, també confirmem i estenem coneixement previ en aquest mateix camp.

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1.Introduction

This thesis lies in the field of Human Computer Interaction (HCI); within this field it specifically concerns the exploration of embodied interaction environments for mathematics learning. We have covered two different contexts that vary in the user needs: sighted children and children with visual impairments (VIs). This allowed us to explore a wider domain and discuss the design similarities and differences in terms of experience design.

1.1. Research areas, context and motivation

In the last decades new interaction paradigms have been emerging: Tangible User Interfaces (TUIs), ubiquitous computing, wearable devices, mixed-reality and virtual reality among others. The context where computers are used evolved beyond the desktop personal computer. Technology miniaturization and low production costs gave place to an extensive and rich spectrum of smart devices with promising potential. Nowadays, these devices are being pervasively deployed through society with a tendency to be expanded in the short term future. The user interface has been extended beyond the keyboard and mouse, the limits between computers and users are sometimes fused in the environment, and physical interaction has been gaining importance.

This continuous computer transformation represents an opportunity but also a big challenge for interaction and experience designers. Rules, goals, constraints and materials are constantly evolving in their form and functionality [38]. As a consequence, in order to take advantage and benefits from this challenge-opportunity, experience design frameworks are shifting towards the inclusion of body-mind theories such as embodied cognition aiming to integrate body, mind and technology [3,19, 37, 15].

Embodiment

The relation between body and mind has been under discussion since Plato and Aristotle, and in more recent times through the French philosopher René Descartes (1596-1650), who argued that body and mind are split. His theory is known as cartesian dualism and basically states that the subject is an immaterial mind with a physical body. Unlike Descartes, the phenomenological philosopher Merleau-Ponty (1908-1961) argues that we are not cartesian subjects, i.e., that our body is not detached from our mind. He considers that first, we exist as subjects in the world and that our self-awareness is the result of the interaction with the physical environment and with other subjects [37]. This perspective claims for the importance of the body and the active role in the perceptive process; for Merleau-Ponty we are lived bodies (active bodies) in the world and there is no perception without action [28].

Embodied Interaction

Many HCI researchers have built on top of the Merleau-Ponty theory, stressing the importance of the body in the perceptive process while interacting with systems. Paul Dourish [13] introduces the term Embodied Interaction as a new interaction design perspective. It is focused not only on the (meaningful) physical role while interacting with systems, but also incorporates the social implications that embodiment has:

> "*By embodiment, I don't mean simply physical reality, but rather, the way that physical and social phenomena unfold in real time and real space as a part of the world in which we are situated, right alongside and around us.*"
> Paul Dourish in the seminal book of embodied interaction "Where the action is" [13]

Dag Svanæs [37] extended Dourish applying specific phenomenology concepts to HCI field, for instance the concept of *embodied perception* defined as the active and embodied nature of perception, including the ability to extend the sensory apparatus with external elements. In the HCI context these elements could be digital devices. Antle et al. [3] made extensive research about the different research opportunities related to embodied interaction applied to children. They concisely define embodied interaction as: "*A perspective on interaction that foregrounds embodied cognitive processes is called embodied interaction*" [3]

We are aligned with the aforementioned perspectives and this thesis is framed under an embodied interaction design perspective.

Tangibles for learning

Physical objects have been widely used as learning materials to introduce abstract concepts, for instance, mathematical operations and geometrical relations. These materials have been classically called “manipulatives”. For instance, Cuisenaire rods [12] are a popular mathematical manipulative that consists of wooden rods raging in length and with different colours representing numbers from 1 to 10. Normally, manipulatives serve as tools which enable children to focus on the underlying concepts. In the beginning, children interact directly with objects and later they internalize those relations into metaphors. Then, such metaphors will be applied to understand mathematical concepts [29].

When it comes to the development of this learning methodology, Maria Montessori (1870-1952) is one of the most influential points of reference. Actually, tangibles that allow modelling abstract structures, such as Cuisenaire Rods, have been classified as Montessori inspired Manipulatives (MiMs) [42]. Nowadays many schools all over the world follow the “Montessori method”. Adepts to her method argue that hands-on activities and physical manipulation and experimentation might not be replaced by digital devices. Actually, the potential reduction of physical interaction is one of the main concerns with respect to the inclusion of technology in classrooms [9].

However, Tangible User Interfaces (TUIs), i.e., digitally augmented physical objects, present a valuable opportunity for learning purposes, including the exploration of new ideas and mathematical concepts (among others) through physical actions [23, 26]. When traditional manipulatives are digitally enhanced they are known as “digital manipulatives” [33].

Embodied interaction can contribute to the acquisition of basic mathematical skills both in sighted and VI children by enhancing traditional manipulatives with digital feedback. In particular, for children with VIs, auditory or vibrotactile feedback can be used to represent abstract concepts. For instance, the cardinality of a set could be represented as a group of sounds or vibrations. Indeed, a more advanced approach could allow children with VIs to perceive quantities beyond slow sequential counting strategies [22]. Digital manipulatives that enable perception "at a glance" would mean a significant step towards making the experience of VI children and sighted ones more similar to each other.

Our main motivation is to contribute to the design, development and incorporation of technologically enhanced learning materials in classrooms, looking for a real benefit for children. To this aim, some boundaries like screens, simulated environments, bits and virtuality might be broken. Enhancing user experience through embodiment implies, in the first place, the reduction of cognitive load invested in the interfaces, and in the second place the exploitation of useful metaphors and actions (pragmatic or epistemic) for problem solving. This diverse nature somehow suggests following a multidisciplinary approach, involving different knowledge areas like engineering, design and psychology to work synergistically.

1.2. Approach and research questions

We have followed a research through design [41] methodology in the context of a making/prototyping approach. By the development of three prototypes for mathematics learning, CETA, iCETA and LETSMath, this thesis explores the design of embodied interaction in two main contexts: involving children with full vision (CETA) and children with visual impairments (iCETA and LETSMath).

The intrinsic value of making

The creation of real prototypes carries three main advantages. Firstly, through the materialization of designs into prototypes, we are able to test existing learning and embodiment theories with real apparatus and users. In addition, we incorporated users along the whole design and test process providing valuable feedback. Secondly, the artifact itself, which might become a product and incorporated in classrooms. Also, the theory based design and the user evaluation and system observation will contribute to generating intermediate-level knowledge (ILK) [7]. This kind of knowledge is situated in between general theories and concrete instances (artifacts), and its generation is important for the HCI community since it enables the application of design concepts across different domains and contexts, transcending the specificity of a prototype or technology. It allows designers to capitalize and state design knowledge with a longer life span. The final advantage is the irreplaceable vale of surprise while creating, as Klemmer et al., states:

> "*The epistemic production of concrete prototypes provides the crucial element of surprise, unexpected realizations that the designer could not have arrived at without producing a concrete manifestation of her ideas.*" [21]

In a similar vein, Dag Svanæs introduces the concept of "*kinaesthetic creativity*" [37] as: "*active use of the body through abstract movements to explore possible features*". He also explains that:

> "*The design materials and the physical environment enable the participants to become creative, and much care should therefore be taken during the design of the prototyping materials. This has similarities to the Montessori Method focus on the materiality of toys designed for learning*" [37]

To sum up, prototyping might complement the theory based design enabling the exploration of unexpected possibilities which might lead to solutions not initially considered.

Users involvement and system evaluation

For each prototype, we have followed a User Centered Design (UCD) [30] methodology, involving users actively during the development process. This way we obtained feedback at very early stages and iterated over the prototypes designs.

Regarding the evaluation of the prototypes, we might make the following distinction: on the one hand, we evaluated the embodied interaction itself (CETA and LetsMath), meaning the actions performed on the physical objects, children's strategies and the interaction pace. On the other hand, in the case of CETA, learning outcomes were also evaluated. We designed a long term study with pre and post tests in order to assess the actual impact of the system on learning compared with the mixed-reality solution with a pure virtual one.

Research questions

The aim of this thesis is to understand how to make embodied interactive systems in order to enhance mathematics learning experiences for both children with full vision and children with VIs. To tackle this issue, we need to determine how and which design specificities of (embodied) interactive systems impact on children's perception, abstraction and reflection within the learning experience. To this aim, we address the following specific research questions:

RQ1: To what extent and how embodied interactive systems might benefit mathematics learning?

a) Related to the theoretical background, which are the most relevant underlying cognitive theories that might support the system design?
b) Which are the main requirements that these systems might cover?

RQ2: How might an embodied interactive system be designed in order to enhance the mathematics learning experience?

a) How to shape the level of exposition to abstract representations?
b) How to encourage reflection during the learning activity?
c) How might we incorporate cognitive and learning theories as design features?

d) Which are the similarities and differences when designing for sighted children and children with visual impairments? In terms of perception, actions and feedback.

RQ3: Which is the impact of CETA in terms of learning gain and children's strategies in comparison to pure virtual and traditional approaches?

1.3. Contributions

We performed a literature review of several research areas related to cognitive psychology, embodied interaction and tangibles for learning, for children with full vision (chapter 2) as well as for VI children (chapter 5). We contribute in the understanding and incorporation of the connections between body and cognition to the context of interactive systems design. Combining the knowledge from these research areas with the analysis of already existing prototypes (chapter 2-related work, chapter 5.1-related work, chapter 5.4-related work), we propose the design of three tangible systems (CETA, iCETA and LETSMath) for mathematics learning (RQ2), oriented to first grade children in the process of grasping the number concept and additive composition. We detail how we incorporated background theories as specific features among the three prototypes (RQ2-c).

We explored different technical solutions depending on the context. CETA is a mixed-reality system with passive (non-actuated) tangible objects providing its main feedback through the visual channel (on-screen). iCETA also has passive objects but feedback is mainly provided by sound, exploiting the auditory channel as it was designed for children with visual impairments. Lastly, LETSMath incorporates active tangible objects which provide feedback through sound and vibration, and high contrast graphics although they are not imprescriptible for the activity. This way, attending to user specific needs (RQ2-d), LETSMath foregrounds haptic and auditory channels. In terms of design knowledge, we contribute to the discussion of how physical objects design, actions and feedback might vary depending on users' perception (chapter 6-RQ2). The main outcome of such discussion is a set of considerations when designing inclusive environments for children with and without user impairments.

The construction of CETA, iCETA and LETSMath and evaluation of CETA and LETSMath allowed us to address the questions related to the design (RQ2) and impact (RQ3) of tangibles on the learning experience. We propose two alternatives to gradually incorporate abstract representations within the learning experience (RQ2-a). Aligned with previous research [26, 4, 31], we confirm that the interaction pace is determinant for the learning experience and by modulating it we might trigger reflection (RQ2-b). We concretely proposed two strategies to slow the interaction pace: feedback modulation and physical constraints on the working area (chapter 2.2-discussion, chapter 5.4-discussion, chapter 6-RQ2).

In addition, as a consequence of combining theory based design with evaluations of concrete artifacts, we managed to contribute with the generation of intermediate-level knowledge (ILK). On the one hand, each prototype was carefully designed taking into consideration relevant theoretical multidisciplinary background. On the other hand, prototypes were actually implemented and tested, with real users. Observations and data analysis permitted to state intermediate knowledge between theories and artifacts, a valuable piece of work for future designers. We concretely propose the strong concept "Embodied Interactive Mediated Reflection" (chapter 3) that has generative power towards answering how reflection might be encouraged in this kind of environments (RQ2-b).

Lastly, CETA is open source and open hardware and specifically developed to work with low cost tablets (Appendix A). This is a contribution for the community either for further research and to build knowledge on top of it, or to replicate the system and incorporate it in educational contexts.

Thus, this thesis has approached the possibility of incorporating tangible technologies for the development of basic mathematical skills for children with VIs and without.

1.4. Structure of the dissertation

Chapter 2 is dedicated to discussing the user centered design process carried out to develop CETA. The whole development process took around one year, we formed an interdisciplinary team of psychologists, designers and engineers, which went on to develop iCETA and collaborate with LETSMath. Educators from two schools in Montevideo, Uruguay also took part in the design and evaluation process, and of course the first grade children from these institutions. As a result of this process, two papers were published, a short paper (section 2.1) which was complemented with a demo session at the MobileHCI ’17 conference, and a full paper that obtained an honorable mention at the same conference (section 2.2). In Appendix A we include implementation details, links to repository were software sources, hardware design files (for 3d printers or laser cut) and a full reproducibility guide can be found.

We dedicated chapter 3 to accomplish one of the objectives of this thesis, generating ILK capitalizing the efforts made during CETA development. In section 3.1 we briefly introduce what ILK is and its importance to the HCI community. Next, in section 3.2 we include a position paper for the workshop “ *Intermediate-level knowledge in child-computer interaction*” at the IDC '18 conference, presenting the *strong concept* “Embodied Interactive Mediated Reflection”.

Chapter 4 somehow closes the CETA process conducting a long term study in a public school in Montevideo, Uruguay. The study took three weeks and followed a quasi-experimental structure. Three groups were compared: Tangible Interaction (using CETA), Virtual Interaction (using a virtual version of the game) and Control group (following traditional practices in the curricula). This study derived in the publication of a journal paper included in section 4.1. Later in Chapter 6-RQ3 we complement this article with deeper discussion regarding the impact of virtual and physical materials in children’s strategies.

All the research presented in Chapter 5 is dedicated to understanding, design, development and testing of interactive systems for children with VIs. First, in section 5.1 we include a short paper presented in the conference ASSETS ’19 conveying the design of iCETA, a mixed-reality system that arose as an immediate adaptation of CETA for children with VIs. Such experience gave us valuable insights (explained in section 5.2) to develop LETSMath, a tangible system for mathematics learning for VI children. The design, implementation and testing of this system took around two years. Again, we followed a user centered design within an interdisciplinary team. Educators from institutions from two countries were involved in it: two public schools in Montevideo, Uruguay and the National Organization of Blinds in Spain (ONCE). These efforts were capitalized in the form of a short paper (section 5.3) complemented with a demo session in the conference MobileHCI’18, a journal article to be submitted (section 5.4) describing the design process and evaluation of LETSMath, and the system itself, that to the best of our knowledge is the first tangible system incorporating active feedback specifically designed for number composition training for children with VIs.

Finally, chapter 6 is dedicated to concluding the thesis. In section 6.1 we address the research questions, present the main conclusions and we discuss relevant design implications. In section 6.2 we proposed an extension to the Tangible Learning Design Framework developed by Antle, A. and Wise, A. [4] which we used for the design conceptualization of CETA and LETSMath. Finally, in section 6.3 we discuss limitations and future work perspectives.

2. Mixed-reality interaction for mathematics learning

In this chapter we introduce CETA, a mixed-reality system for mathematics learning. We explain the design and open source and hardware implementation of the system. We also contribute with a literature review of cognition theories and their translation to system design specificities. Furthermore, we analyse the limitations of the system and the exploratory evaluations, leading to a future work research agenda.

Section 2.1 describes the open source and hardware implementation of the system, while Section 2.2 details the system design and user tests. In appendix A we provide complementary material for system replication, including hardware and software sources.

2.1. CETA: open, affordable and portable mixed-reality environment for low-cost tablets

The content of this section was published in the Proceedings of the 19th International Conference on Human-Computer Interaction with Mobile Devices and Services (MobileHCI '17)

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Available
https://repositori.upf.edu/handle/10230/32646

CETA: Open, Affordable and Portable Mixed-reality Environment for Low-cost Tablets

Figure 1: CETA: Mixed-reality system for low cost Android tablets. The tangible blocks represent numbers while joining them represents the addition operation

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Abstract

Mixed-reality environments allow to combine tangible interaction with digital feedback, empowering interaction designers to take benefits from both real and virtual worlds. This interaction paradigm is also being applied in classrooms for learning purposes. However, most of the times the devices supporting mixed-reality interaction are neither portable nor affordable, which could be a limitation in the learning context. In this paper we propose CETA, a mixed-reality environment using low-cost Android tablets which tackles portability and costs issues. In addition, CETA is open-source, reproducible and extensible.

Author Keywords

Mixed-reality; open-source; open-hardware; tangible interaction

ACM Classification Keywords

H.5.2 [Information interfaces and presentation]: User interfaces; H.5.1 [Multimedia Information Systems]: Artificial, augmented, and virtual realities

Introduction

Mixed-reality environments allow the combination of tangible interaction and digital feedback, empowering interaction designers to take benefits from both real and virtual worlds. In the learning context, many alternatives for mixed-reality environments are being explored, for instance tabletops [10, 4]. However, the portability of these devices is quite limited and the production cost is considerably high. OSMO [2], the mixed-reality play system for iPads, tackles the portability issue but it is a commercial product and is not suitable for low profile tablets, such as the ones distributed by the *One Laptop Per Child* (OLPC) program.

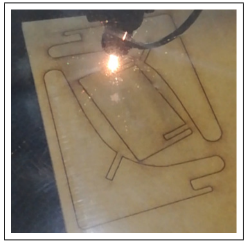

Figure 2: Laser cutter making the holder.

We propose CETA, a mixed-reality environment highly inspired in OSMO [2] that satisfies portability and low-cost requirements. In addition CETA is an open source platform, thus contributing to the digital sovereignty. Thus, the environment (hardware + software) can be adapted for different devices such as smartphones or tablets. This is mainly possible because the full environment is open and reproducible. All the software and hardware (excluding the tablet) are open and available at https://github.com/smarichal/ceta. The software is under GPL-3.0 license [1] and the hardware under CC BY-NC-SA 4.0 license [3].

Software Reproducibility and Extensibility Besides the source code is available in a public github repository, extensive documentation is provided to compile and install the system in Android devices. In addition, later in this paper we explain how to extend CETA to use other digital manipulatives and which is the impact in terms of the system architecture.

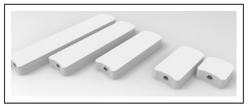

Figure 3: 3D model of the tangible blocks. Magnets are placed in the small holes to encourage users to join the blocks.

Hardware reproducibility The hardware design is open and available for 3D printing, laser cut or any other 3D building technique.

Hardware Extensibility It is easily extensible, giving the possibility to customize the tangible objects with moderate programming skills. This means that instead of only using blocks for mathematics learning as we did in [9], it is possible to create alternative designs according to the objectives of the activity/game (see Figure 12). The steps needed to create new tangibles are explained in the Extending CETA section.

System Description

CETA was developed to promote the use of tangible technologies such as the tablets deployed in public schools as part of the *OLPC* program in Uruguay. The main requirement, and challenge at the same time, was to create an affordable mixed-reality environment for low-cost Android tablets. It was specially designed for a mathematics learning task, thus the tangible elements are inspired in the cuisenaire rods [5], i.e., rectangular blocks ranged in length representing numbers. Cuisenaire rods are widely used for learning basic mathematical skills such as additive composition. In CETA, these blocks become digital manipulatives through augmented reality markers that are detected using the frontal camera of the tablet (see Figure 1).

Hardware

Tablet As explained before, CETA was designed to work in the OLPC tablets. These devices have a Quad Core ARM 1.3GHz CPU, 1 GB of RAM memory and a 0.3 megapixels frontal camera. Their main limitation is the frontal camera, which gives a poor image quality under artificial or low lighting conditions.
The rest of the environment was custom designed and it is composed of:
Holder The holder (see Figure 1) is the wooden structure that maintains the tablet in vertical orientation. It is 10 cm high in order to expand the field of view of the camera enabling a larger interaction zone on the table. The svg file is available for laser cut. It can be easily adapted for tablets with different dimensions although most of the tablets may fit in it.

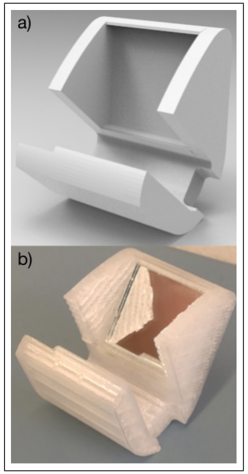

Figure 4: a) 3D model of the mirror b) 3D printed mirror

Mirror The vision of the front camera is redirected to the table using a mirror, and as a result, the tablet can “see” and detect the tangibles on the table. The model of the piece that holds the mirror (see Figure 4) is available for 3D printing and also for laser cut, it might be adapted to the specific dimensions of the tablet (thickness and camera position).
Tangible blocks As mentioned before, the tangible blocks are inspired in cuisenaire [5] rods and have different lengths depending on the represented value, going from 1 to 5 (see Figure 3). We included magnets in the extremities of the blocks in order to provide an affordance to join them, representing the addition concept and the number line representation as discussed in the system’s interaction design [9]. We also provide two versions of the block design, for laser cut and for 3D printers. However, the only requirement for any block to become digital manipulatives is to place the markers on top of it, so we can also build blocks with many other materials and then just paste the markers on top. Thus, the only requirement to create the digital manipulatives is to print the markers and create blocks like the one shown in Figure 8. This makes the system even more affordable and adaptable to different contexts where it may not be possible to access either to 3d printers or laser cutters.

Software
All the software has been developed in Java and it is organized in 3 layers (see figure 7). The third party libraries used are OpenCV [8], libgdx [11] and TopCode [6]. Bellow we discuss each layer and explain the block detection algorithm in layer 2.

Layer 1 - TopCode Computer Vision Module This is the computer vision layer, and in the current design it is implemented by the TopCode library [6]. This is a computer vision library able to identify up to 99 circular markers/tags (see figure 9). It is specially designed for quick identification and tracking of tangible objects on a flat surface. For each identified marker the library provides: id, location, angular orientation and diameter of the tag. This library has been chosen because it is fast and reliable (works in a variety of lighting conditions), is available for Android, is free and open-source, and recognizes small tags (25x25 pixels). This library has been also used in Strewbies, an Osmo based tangible game for programming learning [7].
Finally, to support alternative detection techniques (based on the shape or the color of the objects, for example) this layer might be modified probably also impacting on layer 2.

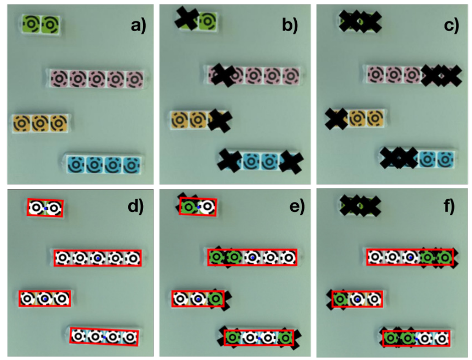

Figure 5: CETA supporting partial occlusions. Frames presented in temporal order from left to right. a,b,c are input frames while d,e,f are the detected blocks. Green markers were inferred.

Layer 2 - Detection Module In this layer we implemented the Augmented Rods detection module, this code is specific for the design of our tangible blocks. The detection module

Block	TopCode Markers
1	31, 61, 103, 179, 227, 271, 283, 355, 391, 453
2	93, 117, 185, 203, 793
3	563, 651, 361, 309
4	171, 555, 421
5	1173, 1189, 677

Table 1: Tangible Blocks-TopCode markers mapping. Specific markers must be used for each block.

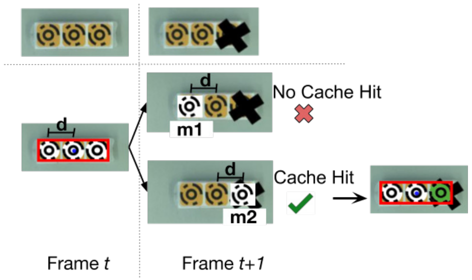

Figure 6: Top images are the input while the bottom images are the output of the detection module. In frame t+1 one of the markers is occluded, so the algorithm computes the candidates m1 and m2, and after querying the cache determine that the missing marker is m2.

and the TopCode library can also be used in desktop platforms, in both cases the input is an image (see Figure 10) that could be loaded from local storage or, as in our case, captured in real time by the camera.

A tangible block is identified as an aligned collection of the same TopCode marker, repeated from 1 to 5 times depending on which block it is. The smallest blocks are those representing the number 1, and they have a single TopCode maker, while the largest contains 5 aligned TopCode markers and they represent the number 5. In addition, we only use a subset of all the available TopCode markers and each marker can only be used within a predefined block, i.e., the marker with id 185 can only be used within blocks of size 2 (see table 1). Next, we use the example in Figure 8 to explain the constraints to be considered when creating augmented rods for CETA:

1. Equal distances between markers within a block, i.e., d1=d2=d3. Let's call this distance d.
2. The distance d must be the same in all the blocks.
3. Just one marker id can be used in each block. This is M1=M2=M3, let's call this marker M.
4. The marker must be mapped with the value of the block, i.e., in this case M has to be mapped with blocks representing number 3 (see table 1).
5. Background color could be changed. However, the TopCode vision algorithm performs better with higher contrast. The best scenario is black rings and white background.

A tangible object that satisfies the previous conditions is an augmented rod and will be detected by our module implemented in layer 2. Having multiple markers per block is redundant but provides robustness to the block interpretation algorithm. When some markers of the block are not visible, we infer their position using the detected markers and a cache system where we store all the detected markers in the last 5 frames. Using this strategy we are able to support partial occlusions in all the blocks except in the block 1 given that it only contains one marker. The figure 5 shows how the system support partial occlusions. The figure shows three frames in temporal order from left to right, where a,b and c are the input image of the detection module and d,e and f show how the system detects the markers (white), infer the non visible markers (green) and computes the middle point of the block (blue point) and the contour (red rectangles). In Figure 6 we explain the cache algorithm for the specific case of the block 3. Basically, when a marker is not visible the algorithm computes the possible candidates using the distance d, i.e., it computes where the occluded markers could be. In this case, m1 and m2 are the candidates. Once the candidates are determined, the

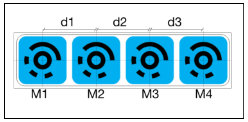

Figure 8: Augmented rod design using TopCode markers

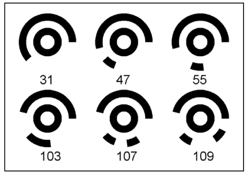

Figure 9: TopCode tags

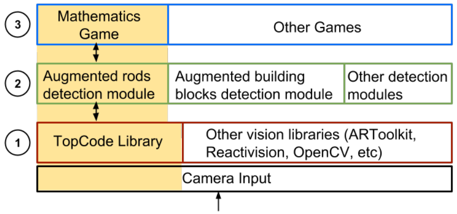

Figure 7: CETA software architecture in three layers

cache is queried. The result is that m1 is discarded and m2 is kept as the missing marker of the block 3.
Lastly, for each augmented rod the detection module provides: center and vertices (4 points) coordinates, rotation angle, value (from 1 to 5) and dimensions (width and height).

Layer 3 - The Game The top layer contains the code of the game, i.e., an Android application that receives information of the tangibles blocks and updates the state of the game while providing feedback as well. The code in this layer does not know how the blocks are detected. However, given that the input of the detection module in layer 2 is an image, the layer 3 is also in charge of capturing the image from the tablet's camera, for example using the Android API to access the hardware, and provide it to the detection module in the second layer (see figure 10).

Extending CETA
In the mathematics game where we initially used CETA [9], we followed the yellow path shown in Figure 7, i.e., android activity + augmented rod detection module + TopCode library. However, it is possible to extend CETA in many ways. In this section we discuss how extending CETA impacts on each layer.
Changing the game/activity: In order to use the augmented rods but with another purpose, we just need to make changes in layer 3. This is typically an Android activity which interprets the position and rotation of the augmented rods to update the digital model and provide feedback. In our mathematics learning game [9] we interpreted the rods as numbers and the action of putting them together as the addition operation. It is possible to design other activities where the augmented rods would have other meaning, for example the input control for a game (see Figure 11) or other learning tasks such as magnitude comparison.

Creating new tangibles using markers: It is also possible to design other tangibles using the TopCode markers. In this case the layer 1 remains unchanged but it is necessary to implement or modify the detection module in layer 2 since the layout of the markers might be interpreted in a different way. For example, we could create tangible geometric bodies (see Figure 12) for a geometry learning activity or a module to detect building blocks as it is suggested in the Figure 7. Writing code in this layer is not so complex and offers many opportunities.

Creating new tangibles without markers: The most difficult but also the most powerful extension is to change the computer vision algorithm in order to detect the tangibles using a different approach. This would enable, for example, to detect objects by their shape or color putting aside the markers. This implies a change in layer one (computer vision algorithm) and is also very likely to impact on layer 2 since the output of layer 1 will probably be different.

Figure 11: Possible use of the augmented rods to control a game.

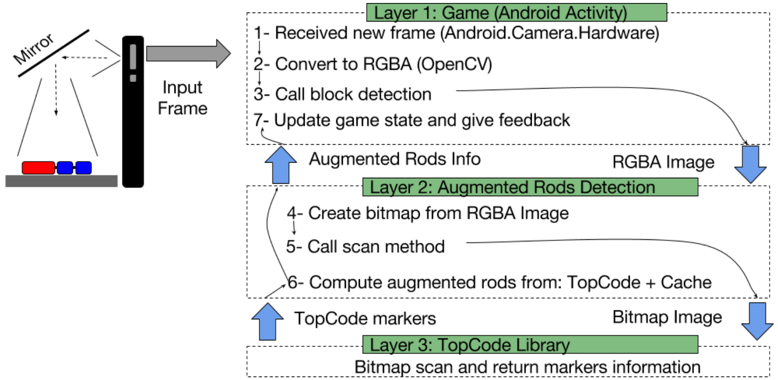

Figure 10: Information flow: The captured frame is scanned by the TopCode library which detects the markers that are interpreted by the augmented rods detection module to compute the augmented rods. Lastly, the game updates its internal state and provides feedback.

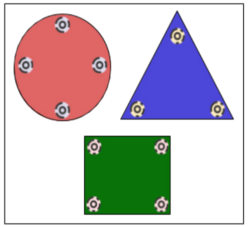

Figure 12: Hypothetical tangibles design for geometric bodies using TopCode markers.

Conclusions and Future work

We proposed a low-cost and portable mixed-reality environment. Both hardware and software are open and reproducible. We expect that these efforts mean a step forward in the inclusion of tangible interaction in classrooms in order to take advantage of the technology deployed worldwide by programs such as *OLPC*.
As future work we expect to build high level tools allowing people without programming skills to design tangibles (such as teachers in many cases) and create customized educational apps. We also encourage to the community to build their own projects using CETA and to improve it.

ACKNOWLEDGMENTS

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2.2. CETA: designing mixed-reality tangible interaction to enhance mathematical learning

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CETA: Designing Mixed-reality Tangible Interaction to Enhance Mathematical Learning

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ABSTRACT
The benefits of applying technology to education have been often questioned. Learning through digital devices might imply reducing the children's physical interaction with the real world, when cognitive theories hold that such interaction is essential to develop abstract concepts in Mathematics or Physics. However, conflicting reports suggest that tangible interaction does not always improve engagement or learning. A central question is how cognitive theories can be successfully applied to the design of interactive systems in order to achieve enhanced learning experiences. In this paper we discuss the interaction design of a mixed-reality system for mathematics learning for school-aged children. Our design approach combines inspiration from previous frameworks with a user-centered design process with early prototype evaluations. As a result of this process we have created a mixed-reality environment for low-cost tablets and an augmented version of the Cuisenaire rods, a milestone of the manipulatives for mathematics learning.

ACM Classification Keywords
H.5.2 Information interfaces and presentation: User interfaces; H.5.1 Multimedia Information Systems: Artificial, augmented, and virtual realities

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Author Keywords
Interaction design; Technology enhanced learning; Embodied interaction, Mixed-reality

INTRODUCTION
Nowadays children have earlier access to digital technology. Specifically, programs such as *One Laptop Per Child* (OLPC) have provided computers and/or tablets to school children worldwide. Educational content is continuously being developed and easily spread through online platforms able to reach the remotest locations. As this technology is already deployed in the classrooms, it is reasonable to devote efforts to create content that encourages the learning process.

However, some question the learning benefits of applying digital technology in education [9]. A potential problem is that some physical interaction with the environment is replaced by mouse-keyboard or multitouch interaction without considering the impact it may have. Several theories such as constructivism, embodied cognition [5, 42] and physically distributed learning [29], support the idea that physical interaction plays a key role in the learning process [6]. The general aim of this paper is to discuss how these theories can ground the design of interactive systems to enhance learning.

Conceptually, Tangible User Interfaces (TUIs) augment everyday physical objects and environments with digital information to become interaction devices [23]. Throughout this paper we refer to this meaning, 'where physical interaction goes beyond touching a mouse, a keyboard or a (touch) screen'. Tangibility might have a different impact depending on the learning task; physics and mathematics are subjects where tangible interaction and real world observation seems to play an important role [46, 44, 28, 27]. However, there is still little formal evidence of tangibles enhancing learning, and how cognitive theories might be translated into the design of interactive systems to support an enhanced learning. To fill this gap, this paper focuses on mathematics learning and discusses the design of CETA (Ceibal Tangible), a mixed-reality system with tangible interaction for school-aged children.

We need to understand the relation between physical actions and cognitive processes to successfully design a tangible interactive system supporting learning. This means to understand and trace the relation between the physical and digital elements through actions, the system feedback and the impact of these elements on the problem solving processes. Such elements are identified and described as physical objects, digital objects, actions, informational relations and learning activities in the Tangible Learning Design Framework (TLDF) [6]. Along this paper we use this terminology and framework. Then, we can formally specify the role of each interaction element and argue its inclusion in the system, i.e., how we envision that specific design features will help users to achieve specific goals. Besides the framework the research questions proposed in [6] inspire us in the application to a mathematics learning context. We aim to answer the following questions:

Q1: In a mathematics learning activity: *how can we shape the level of abstraction by changing the actions and informational relations between physical and digital objects?*

Q2: Regarding physical actions and objects: *Which actions (such as 'pick up' or 'group') are relevant and desirable in this specific mathematical learning activity? How can we promote these actions through the design of particular affordances? Which complementary epistemic actions might be supported in order to enhance the problem solving process?*

Q3: Considering a mixed-reality system, *which is the most effective and less disruptive way to slow down the interaction pace and encourage reflection?*

We address these questions following a research through design approach. Our design was grounded in previous research results related to tangible interaction design for mathematics learning [28, 27] and informed by literature related to the use of classical manipulatives in mathematics learning [11, 12]. When theories or previous evidence were inconclusive, our design explored different possibilities following a user centered design with user tests. We discuss the CETA system and design decisions, including the application of some of the TLDF [6] guidelines. As a result of the user tests, we were able to validate previous research. Indeed, from the evidence gathered, manipulation itself is not enough to enhance learning: the modulation of the interaction pace is essential to encourage reflection between the children's actions and the system feedback. We addressed this issue through what we call 'action submit' and observed that pacing down the interaction reduced trial and error strategy and encouraged reflection. The main resulting artifact of the whole process is the CETA mixed-reality environment and the digital augmented version of the the Cuisenaire rods, which aims to the inclusion of low-cost tangible and meaningful technology in classrooms.

BACKGROUND

In this section we introduce some cognitive concepts relevant to ground our design: cognition offloading, physically distributed learning, image schemas, conceptual metaphors, and epistemic actions.

Cognitive offloading:

Operations with concepts such as mathematical ones involve the elaboration of mental representations of both abstract and concrete objects. For instance, a group of items should be conceived by assembling different elements in a joint group, being the group itself a mental representation that must be stored during a mathematical operation. Also an abstract concept such as the addition of two new units must be conceptualized demanding increasing cognitive resources (keeping in mind the meaning of this operation). Cognitive offloading refers to the possibility of lightening these cognitive demands by the inclusion of actual objects representing abstract concepts. Since these objects are available to the perceptual system they release working memory load [18, 29].

In the case of operations, actual actions over objects aid in the realization of abstract relationships facilitating mathematical thinking [19]. That is how manipulatives help to decrease cognitive load by giving place to external representations of objects and operations [27, 34].

Physically distributed Learning:

As stated above, manipulatives can aid abstract thinking when objects work as external representations of the learning concepts. For the Physically Distributed Learning theory (PDL) [29], it is crucial for the learners to have a deep understanding of the way in which concrete objects represent abstract entities. A single one to one correspondence between an object and a concept would not be sufficient. Instead, knowledge about how different objects relate to each other and how they can be rearranged would be required to represent the conceptual structure behind mathematical operations. Indeed, for PDL, a richer understanding is achieved when children are allowed to rearrange the environment (i.e., a group of objects) in order to represent the solution to a posed problem (i.e., select the fourth of the group) [29]. Thus, the environment is reinterpreted in order to reflect the abstract structure of the operation to be performed. Therefore, PDL goes beyond simple cognitive offloading, demanding a deeper comprehension of the link between an abstract structure and the structure of an interactive environment. The exploitation of such structures in a stable form has been studied under the labels of image schemas and conceptual metaphors.

Image schemas and conceptual metaphors: Some specific spatial configurations of objects and actions performed over them are typically found when abstract operations are carried out. For instance, the action of taking apart a subgroup of objects within a bigger group will be linked to the operation of subtraction [20]. These spatial arrangements and actions give place to stable external representations, which are stored in memory and can be recovered to aid the accomplishment of symbolic operations as mathematical.

Conceptual metaphors enable the understanding of abstract concepts in terms of more concrete concepts, by providing a cognitive mechanism that enables us to translate inferences made in one domain to another one. For instance, to group and to count small collections of objects can result in neural connections deriving from sensory-motor physical operations (like adding (n+1) or subtracting (n-1)), which, in turn, may result in conceptual metaphors at the neural level: from physical objects to mental operations with numbers [25]. Collections (of objects) with different magnitudes help to learn that numbers also have magnitude; bigger collections of objects represent a metaphor for bigger numbers, the smallest collection represents the number one; taking out a collection from another collection represents subtracting and so forth. These kind of analogies have been proven to be useful for intuitive interaction design [**?**].

Epistemic Actions:

Defined as complementary actions on objects that make problems easier to solve but are not necessarily part of the solution [6]. These actions are performed to exploit the advantages of offloading cognition and conceptual metaphors. Moreover, these actions may reveal information that is hidden or that is hard to compute mentally [24]. For example, rotating a *Tetris* block while we are developing a solution or rotating a map in the mobile phone to follow directions. Research in the use of manipulatives for math learning showed that concrete material fosters the discovery of more strategies to solve mathematical problems [28].

RELATED WORK

In this section we present a selection of studies that are related with the design of CETA regarding the use of tangibles or digital manipulatives and concretely, the use of TUIs in education, and the Cuisenaire rods.

TUIs or digital manipulatives Similar to the concept of TUI [23], digital paradigms and technologies applied to traditional manipulatives are known as *digital manipulatives* [33]. In [33] four computationally-augmented versions of traditional manipulatives are discussed (blocks, beads, balls and badges). Beyond the intrinsic value of the traditional manipulatives, digital manipulatives enable children to familiarize in advance with concepts related with dynamic systems.

Virtual and physical manipulatives were compared in a number partitioning task [28], making efforts to determine which is the role of the physical representation. On the one hand, benefits of virtual manipulatives are: potential to link representations, audiovisual feedback, tracking of the past actions, adaptability and availability. On the other hand, physical manipulatives offers unique benefits such as tactile feedback (size, shape and quantity up to certain limit) and proprioception which allows children to know the position of the block in relation with their body just by touching them [27]. In the case of a mixed-reality system it is possible to exploit benefits from both worlds.

Digital tangibles for education can be distinguished between "Froebel-inspired Manipulatives" (FiMs) and "Montessori-inspired Manipulatives" (MiMs) [47]. The former are building toys that enables children to design real world objects while the later are focused in the modeling of more abstract structures. According to them, TUI are useful for learning abstract concepts in the sense that they provide: sensory engagement (multimodal), accessibility (easier for younger children, novices and people with learning disabilities), and group learning (multi-hand interface enabling natural group interaction).

In TUI there could be sensible, sensable and desirable movements. "Sensible movements are those that users naturally perform; sensable are those that can be measured by a computer; and desirable movements are those that are required by a given application" [8]. In TUI design, this classification is useful in order to detect interaction conflicts and opportunities.

Augmenting the Cuisenaire rods Cuisenaire rods were created in 1952 by educator Georges Cuisenaire [13]. He was inspired in Friedrich Fröbel who had previously designed a set of wooden building blocks [15], but Cuisenaire's design consisted on smaller rods incorporating different colors for each length. However they are considered MiMs, as they allow to model abstract structures related with numbers [47]. He showed that some students who had learned using traditional methods and were rated as 'weak', when they later changed to use the manipulative rods they became 'very good' at traditional arithmetic [16]. Cuisenaire rods supports children's mathematics learning, for example, allowing them to explore and discover the concept of additive composition joining smaller rods to form larger ones.

With respect to cuisenaire rods, most of the digital approaches are *virtual manipulatives* [2, 30, 1], i.e., traditional GUI based programs where the rods are represented with graphics and children manipulate them through mouse-keyboard based interaction or in the best case using multitouch screens. Otherwise, TICLE (Tangible Interfaces for Collaborative Learning) table [35], use a mixed-reality environment that enables tangible interaction with real objects on a table and provide audiovisual feedback on a side monitor. An augmented version of Tangram (an old Chinese geometry puzzle composed by seven pieces) [37] was implemented using this device, where many children can collaborate having equal access to the device at the same time [36]. Also an application to work the concepts of odd and even numbers through Cuisenaire rods was developed [36]. While the tangible interaction proposed by this system it is valuable and allows to explore mathematical concepts in a collaborative way, the main drawback it is the size (a big table and a computer) and probably the cost of producing it and its mobility. To the best of our knowledge, this is the closest approach to develop an augmented cuisenaire rod.

Tangibles for education Other tangible interaction approaches, have been applied in learning contexts using tablets or laptops. They use a mirror in the front camera to redirect the camera vision and computer vision techniques to enable objects detection. Osmo [3] is a mixed-reality play system for iPads. It is used for different learning fields such as mathematics, physics, geometry (also through a Tangram activity) and programming. Strewbies is an Osmo based tangible game for learning programming [22]. They used the topcode vision library [21] to detect real objects, as we did later in CETA. A similar approach had been previously used together with laptops to design tangible educational contents for children with motor impairments [10].

Two previous researches conducted through the design and evaluation of educational interactive systems are especially relevant to CETA. The first one is “Towards Utopia” [7] a tangible environment to enable children to learn concepts related to land use planning and sustainable development, whose design was informed with cognitive load theory and constructivist learning theories. The thorough evaluation of the environment showed that it supports learning; and the paper provides a set of design guidelines that were included and discussed in the TLDF [6].

The second is the mixed-reality system EarthShake designed to support children’s learning of physics principles [45]. It was evaluated [46, 44] through a 2x2 experiment, crossing mixed-reality vs screen-only (pure virtual) with physical or without physical control. It was concluded that the real world physical observation supported learning while the simple hands-on control (pressing a physical button or shaking a tablet) did not add learning, and the authors hypothesized that this could be because these physical controls were not relevant to the learning objectives [46, 44]. They also explain that a key component for the success of the mixed-reality system for learning enhancement is the interactive feedback. This feedback was developed as guides and a self-explanation menu synchronized with the physical world [44].

We use the TLD framework [6] to conceptualize and describe how the tangible interaction supports cognition, going from the design of the learning activity, physical and digital objects, to actions and the relation between them. It provides a taxonomy of system elements: Physical Objects, which are used to interact with the system, and have visual, haptic and optionally auditory attributes. Digital Objects with visual and auditory attributes too, and a temporal property that makes their attributes dynamically change over time. Actions, which are the set of input manipulations that users perform on physical objects or on digital objects in particular cases (e.g. multitouch) whose discoverability by users is important. Informational Relations, the mappings between physical objects, digital objects and actions, which can be perceptual (physical objects representing digital objects) or behavioral (specific actions on physical objects impacts on digital objects), and whose structures, for example the cardinalities (one-to-one or many-to-one), must be considered. And Learning Activities which frame the learner interaction with the system.

SYSTEM OVERVIEW

CETA is a mixed-reality environment inspired in OSMO. It is composed by an Android low cost tablet, a mirror, a holder and a set of wooden blocks, which play the role of manipulatives (see Figure 1). Blocks have rectangular shape and are ranged in length and divided in square sub-elements going from one to five per block. To “see” and detect the blocks on the table, the camera is redirected towards it using the mirror. The blocks become digital manipulatives through markers, which are recognized through the use of the TopCode vision library [21], which works in Android under flexible light conditions. To deal with partial occlusions, which will happen when children

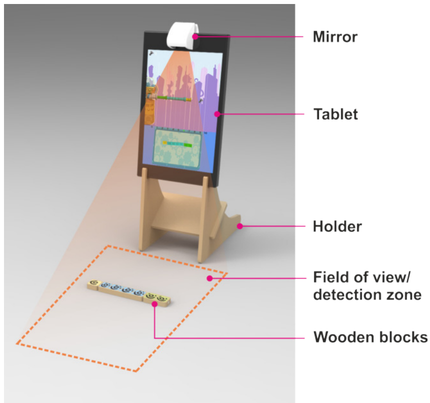

Figure 1. CETA environment setup.

manipulate blocks, we implemented a cache. The cache stores the markers detected in the previous five frames, using the visible markers to infer the position and orientation of the non-visible ones and estimate the position and orientation of the entire block. We included one marker per sub-element within the blocks, e.g., block 1 has just one marker, while block 5 has five. This strategy matches the number of markers with the value of the blocks (see Figure 4-b) and detecting one marker is enough to estimate the position of a block. The software

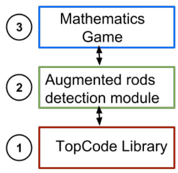

Figure 2. CETA software architecture in three layers: (1) Application, in CETA Game it is an Android Activity (2) Augmented rods detection module (3) Vision library, we used TopCode to detect markers.

architecture is divided in three layers (Figure 2) splitting the game logic from the object detection module. At the same time, the object detection module could use any computer vision library and, for example, detect objects by color or shape instead of using a marker-based approach. All the technical description, software design and implementation are discussed in depth in [**?**].

DESIGN CONCEPTUALIZATION

In this section we discuss the design of CETA in terms of the five element taxonomy proposed in the TLDF [6].

Learning Activity

The goal of the game is to learn the concepts of additive composition and the number line representation. The additive composition implies understanding how numbers can be composed by smaller numbers in different ways (4=2+2, 4=1+1+1+1,

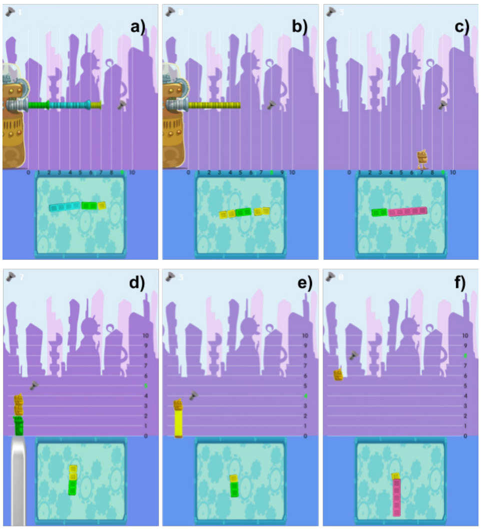

Figure 3. Stages of the CETA game. (a) Bruno composes blocks and creates a long arm to reach the screw (b) Bruno enlarges its arm to reach the screw (c) Bruno moves forward to reach the screw (d) Bruno and friends make a tower to reach the screw (e) Bruno grows to reach the screw (f) Bruno flies to reach the screw

etc.), while the number line representation requires the understanding of the order of numbers represented on a line that in general is vertical or horizontal. Both concepts are taught in the first year of elementary school to 5-6 year old children. The game narrative is about a robot called Bruno that needs to collect some screws appearing at a certain distance from it. Using the blocks, children must compose the number that matches this distance. Once they put the blocks on the table the robot will perform an action to pick the screw (see Figure 3). Horizontal and vertical orientations of the number line are used (see Figure 3 a-c, d-f). Bruno also changes its actions to reach the screws, going from more concrete to more abstract ones. This is discussed in detail in the informational relations section.

Physical Objects

We detail the physical objects design specifying which are the relevant actions in this context and how, through our design, we can promote them.

Blocks: Our block design is inspired in Cuisenaire rods [13] (Figure 4). Each rod represents a different number and has different length and color. In the original set, the smallest cuisenaire rod represents 1 and the largest 10; this mapping is linked with the image schema “shorter is less”. Our design also includes sub-elements representing units, i.e., block N is composed by N sub-elements (see Figure 4-b), this variation is also popular and commonly used. Due to the interaction space constraints given by the field of view of the camera, we only included blocks from 1 to 5. Information is distributed across visual and haptic channels using different (arbitrary) color and (meaningful) size for each block. The multimodality should enhance the learning process by increasing effective working memory capacity, while conceptual metaphors based on image schemas should support learning as it is suggested in the TDLF guideline 7 [6].

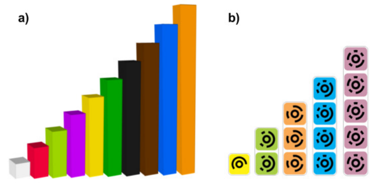

Figure 4. a)Original cuisenaire rods. b)Final design of CETA blocks

Different contemporary approaches to the use of manipulatives tend to highlight the number composition by making explicit the presence of sub units. This trend reflects a modern debate about how number is instantiated in the mind. The theories proposing a general system for magnitude, irrespective of spatial or temporal modality [41] favor analogies linking size and number, and recommend the use of manipulatives based on size. On the other hand, theoretical approaches advocating for the existence of an approximate number system [17] propose that numbers are understood as a group of items, very early in life, and recommend representations that explicitly highlight the composition of units. We followed this second approach, including a counting affordance in the composition task within the game designed.

Blocks contain magnets in their extremities providing an affordance that increases the probability of joining blocks imitating the number line representation. Physical manipulatives not only represent the object itself but also actions are required to be performed with them [32] and their design should be combined with programs to foster certain strategies [26]. Thus, the magnets play the role of facilitators of the representation. They also decrease the probability that children put blocks on top of each other, a sensible movement for some [26] but neither sensable (this action would occlude the markers of the blocks that are not on top of the stack) nor desirable for our learning activity (we encourage children to create linear representation imitating the number line) [8]. We also expected that magnets, as a novelty for children, would increase their enjoyment and engagement.

Digital Objects

The interaction zone (sector of the table) is virtually represented as a colored square. Each physical block is virtually represented through a virtual block with the same color and shape on the screen (Figure 1) below the number line. It is a scaled representation of the reality, included to help children understand how the system is interpreting their actions in a fluent and continuous way [14], not competing for user’s attention and allowing him/her to focus on the consequences of the actions, and also inspired in full body interactive research [38] where it is argued that: “In unmediated full-body interactive experiences, objects should respond continuously and directly to the changing full-body gestures of users, rather than restrict the body to act as a pointer that activates buttons and widgets".

The robot itself is the most relevant digital object, it is the main character of the game and children control his actions and movements combining the blocks. In order to increase the engagement and joy we provided him a name, Bruno, and a friendly and funny appearance. Actions taken on the blocks are mapped to its shape and movements, along the levels of the game it will perform different actions in order to reach the rewards (screws), for example stretch, fly and skate. The details of this mapping are discussed in the Informational Relations section.

Actions

In CETA, children can move the blocks freely, although not all the sensible actions for them are sensable or desirable for the system [8]. Below we present the most relevant actions that may be taken with the blocks, just a subset of them are effectively interpreted as actions in the sense of TLDF and have impact on the digital objects.

The action of joining blocks has two main meanings: Group and Align. Grouping objects is related with the conceptual metaphor that putting objects close somehow adds, composes, creating a new object. Through this action children adapt and reinterpret the environment, supporting Physically Distributed Learning [29]. They also might be offloading cognition by taking action on objects [6] and by making external representation of groups [28]. This is a sensible, sensable and desirable action, and it is the most significant in our system since it represents the addition (group) and number line (align) concepts.

When the blocks are joined it is easy to visualize the result as a new block composed by smaller blocks, while at the same time each block is also composed of units. This might be interesting in order to play with the composition concept, children may visualize the result as the composition of the blocks or as the composition of the units considering the result as a big block without paying attention to the subdivisions given by the union of the blocks (Figure 6). When children align the blocks and then count the sub-units to calculate the addition, the action is considered as an epistemic action since they change the world to make the task easier, i.e., it is easier to count elements aligned than dispersed on the table. This specific action reduces the memory involved and the probability of error in mental computation[24]. This type of interaction enhances children's conceptual learning possibilities [**?**, **?**].

Blocks can be moved individually or in groups, using one or many fingers, or even with the edge of the hand. During the movement, occlusions can occur and therefore the system cannot momentarily detect blocks, but this is overcome when the child moves their hand releasing the block. Rotations are the most meaningful within our game since they enable to interchange the horizontal and vertical representations of the number line. The most obvious and direct impact on the digital objects is given by the virtual blocks since they are a one-to-one mapping of the physical blocks. However, more sophisticated interpretations could be done, for example constrain children to orientate the blocks with the orientation of the number line on the screen.

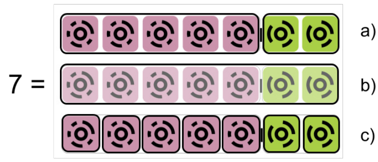

Figure 5. Different perspectives of 5 and 2 making up number 7. a) 7 = 5 + 2, b) 7 as a single bigger block, c) 7 = 1+1+1+1+1+1+1.

Putting blocks away is not sensable and the system does not interpret this action explicitly, but this is the aim of the movement. Children put pieces away to exclude them from the solution [28]. This lack of sensing is an opportunity rather than a problem. Through this action children might offload cognition since it is used to exclude blocks from the solution, and if they want they can put it back in a visible place.

Stacking blocks is a sensible movement for children [28] but given the implementation of the system are not sensable. To learn the concept of number line representation stacking objects is not desirable and, as it was explained before, magnets reduce the probability of this action being taken. However, for the addition concept this might be another way to offload cognition by making external representation of groups [28] and would be a plus if the system was able to interpret it. With the actual computer vision approach, the main drawback is that stacks tend to occlude objects behind them and children may not realize, and as a consequence the natural interaction could be affected since they could be more concerned about the camera vision than about the problem solving.

Another action is telling the system that the actual configuration of blocks on the table has to be processed and interpreted as the solution proposed by the child. In typical interfaces this is commonly carried out through clicking OK, Send, or Submit buttons. In a tangible paradigm, an approach might be to continuously process the current physical objects situation as a solution, i.e., to consider that the child is proposing a solution all the time. However, in a learning context where users have to solve problems, this strategy might not be recommendable because the reflection time is suppressed. Indeed, higher interaction pace may enable the exploration of many different solutions but reduces the reflection time [31]. Manches et al. [28] suggest that adding delays between actions is a good strategy to foster reflection, while Antle et al. [6] recommend to use spatial, physical, temporal or relational properties to slow down the interaction pace and trigger reflection.

Within our design, if the blocks on the table are not moved by the child for one second, a countdown appears on the screen and when it finishes the solution is represented. If the blocks are moved while the countdown is going on, it is automatically canceled. This strategy seems more natural and consistent with the interaction paradigm than having to touch the screen or use a special block as a "send button" as proposed in other systems [39].

Informational relations

As informational relations, we use three mappings between physical and digital objects and actions at different levels of abstraction:

A: *One to one, object to object:* Each block is represented through a single robot or a part of the robot of proportional size. For example, one block of size 2 and one of size 1 are represented through two robots, one on top of the other with size proportional to that of their physical counterparts (Figure 3-a and 3-d).

B: *Many to one, object to object:* Several blocks are mapped to the height or length of a single robot: One block of size 3 and one of size 2 resulting in a robot size 5 without visible subdivisions. This mapping is more abstract since the addition of two numbers is represented (Figure 3-b and 3-e).

C: *Many to one, object to action:* The blocks are mapped to actions of the robot, not objects. Placing one block of size 3 and one of size 1 makes the robot going up or forward 4 units. It is more abstract, as the robot moves in terms of an addition (Figure 3-c and 3-f).

We designed the game narrative through a path from more concrete to more abstract informational relations.

USER TESTS METHODS

We carried out two informal user tests with school age children in their everyday context, to validate the design of the game, as it was evolving and to test different design alternatives, in order to make more informed design decisions.

Participants: Both user tests took place in a public school in Montevideo, Uruguay, with first grade students, aged five to six. The second user test took place 6 weeks after the first one. The One Laptop Per Child program, have provided all children in this school, and all public schools in Uruguay a low end android tablet. Concretely, children from this school have the tablets since 2013. A total, of 19 children, nine girls and 10 boys participated in the study. 10 children participated in the first user test and 18 during the second one, from whom nine participated in the first user test. This difference in numbers is explained by the absence of many children on the day of the first test due to inclement weather. The user tests took place in a classroom using tables and chairs designed for children. Parents were previously informed of the activity and they provided consent for their children to participate.

Levels: During each user test, each child had a turn to play the CETA game individually. In the first user test the game had one level that used the one-to-one object-object mapping and two problems to solve. In the second user test, the game had three levels, and 6 problems to solve, each one using one of the mappings described above, and each level with one with horizontal and one vertical problem. The duration of each user test depended on the child but in average it was around 10 minutes. There was no previous training but in the second user test some of the participants had already participated in the first user test.

During the first user test, as the system was in a very early stage, we used the wizard of oz strategy. In this case, one member of the team performed as a wizard. He was situated behind the child and using a second tablet. He reproduced the same block configuration than the child had in a custom multitouch application and it was communicated to the child's tablet using the OSC protocol [43]. For the second user test we already had the computer vision system working so we used it to detect the tangible blocks.

Data collection and Analysis User test were conducted by a six-person team which organized the logistics , acted as participant observers and recorded the sessions. Personal observations were registered in the observations notes of each researcher, during and / or after the user test, depending on their role. This information was only reachable by the researchers themselves. After each user test, researchers had a debriefing meeting, to make design decisions, based on the annotations from all of them and of video analysis.

OBSERVATIONS

In the following lines we present the more common observations, and the design decisions made based on the two iteration user test. Following the same structure of the game presentation, we introduce the observations under the TLDF categories [6]. Some of these design decisions were implemented for the second user test, and others are to be implemented in a future version of the prototype.

Learning Activity Regarding the learning activity, which is understanding how numbers can be composed, we identified three key points that should be improved.

Identifying the goal: In both user tests children understood that they had to compose a number using blocks. In the first user test the target number was indicated with the position of a screw, and in some cases, it was not easy to identify. Considering that could be a limitation, we decided to highlight the selected number in the number line for the second user test (see Figure 3). As a result, we observed that children somehow simplified the task in two steps, they looked at the highlighted number and composed it with the blocks. We wonder if this could mean a limitation since the screw and/or number line might not be perceived, or at least not actively used when developing the solution.

Selecting vertical or horizontal arrangement: For the first user test we only considered horizontal number line arrangements and the one-to-one, object-to-object mapping. During the second user test the prototype was more advanced and we were able to test both number line orientations (horizontal and vertical) and the three mappings. In the first user test we observed that most children followed the horizontal orientation with the physical blocks. Thus, we hypothesized that this was because they were imitating the orientation of the number line on the screen. However, during the second user test we observed that even when the vertical number line was shown on the screen they still set the blocks in the horizontal orientation. Thus, it is not clear to what extent children's actions can be shaped through on-screen examples.

Closing each independent task: The game presented the problems consecutively, and sometimes children forgot that there were already blocks from the previous solution on the table. For example, in the previous problem they composed a 5 using a block of size 3 and a block of size 2, and in the new problem the system is asking for a 6, sometimes in this case they added a block of size 4 and a block of size 2 considering that just the last two blocks were going to be processed by the system, but in fact they were presenting 11 (3+2+4+2). In this case it might be desirable that the system could detect the situation and show a hint for either clean the table or re use the blocks of the previous problem. To mitigate this drawback the system might test if, the blocks of the previous solution are still in the same place and if the proposed solution is equal to the previous one plus the actual solution. When both conditions are true, the system should display a hint to suggest the removal of the previous blocks.

In accordance with previous research [28, 44] it seems that considering just the working materials is not enough, the context and how the activity is presented and guided through helps and hints play a key role. As a general implication we recommend designing the game/activity to guide and encourage the child to accomplish the goals, this might include providing hints and unlocking children when commit common errors.

Physical Objects Three features of the physical objects were tested during the user tests: Size, magnets and the subdivision in sub-elements.

Block Size: Two different block sizes were tested in the first user test but no differences were observed (unit square side: 1,5cm or 2cm). The smaller size (1.5cm side) was successfully used for the second user test. We conclude that blocks from 1,5cm (block 1) to 7,5cm (block 5) are suitable. Children can manipulate them easily and they are small enough to detect 10 units in-line (horizontal and vertical) within the field of view of the camera.

Magnets: In both user tests we observed that most children took advantage of the magnets to join blocks. Magnets suggest the in-line join of the blocks which is relevant in this context since we are working with the number line. Almost no child put blocks in a stack, which would be sensible but non sensable or desirable. The main drawback observed is that children are disturbed when magnets repel. They keep trying to join them shifting the attention focus. To overcome this drawback we might design asymmetric blocks that can only be joined by the extremities of opposite polarities (see Figure 11). In some cases children did not align the blocks using the magnets. However, it was not considered a major problem for the learning activity. Moreover, requiring a precise alignment of the blocks might reduce the enjoyment and therefore has a negative impact on the user experience.

Blocks sub-elements: Different to the Cuisenaire rods, in the first user test we introduced colored squares within the blocks as sub-elements (see Figure 8-a) and we observed that most of the children used them to count. In the second user test we included in each sub-element a marker of the TopCode computer vision library (see Figure 8-b). We observed that the inclusion of these markers has no negative effect and that all the children used the sub-elements to count. We conclude that the division of the block in sub-elements is very useful for children and that the markers have no negative effects that interfere with the task. Touching blocks is a strategy to offload cognition, and joining blocks is an epistemic action performed to solve the problem. We observed extensive use of both strategies. From this observation we might derive two general implications, the first one is the inclusion of sub-elements as a valuable feature of Cuisenaire rods, and the second one is that vision-based systems must support partial occlusions of the physical blocks since touching is a valuable offloading action while children resolve mathematical problems.

Height: In the first user test we observed that when the tablet is on the table, the children slightly tilted down their heads, causing ergonomic discomfort. To solve this problem, we lifted the tablet, with a box (see Figure 9), and this phe-nomenon was reduced. Using the tablet at a higher position, also expands the field of view of the camera. For the second user test we used a higher tablet holder. We did not observe any inconvenient related to the height of the device. The user test took place in the classroom using children's every day tables and chairs. We realized that ergonomic considerations should be taken into account to adjust the height of the tablet in relation with the height of the children and the furniture used. In this sense, it would be ok to have an adjustable tablet holder that can be adjusted if needed.

Digital Objects Observations of the digital objects might be split in those related to the virtual objects that provide continuous feedback, and the robot as the digital object where actions with blocks are mapped to reach the rewards. They are represented in the bottom and the upper area of the screen respectively.

Virtual Blocks: In both user tests we observed that children do not pay special attention to the virtual representation of the blocks on the screen. A possible explanation is that while they are manipulating the physical blocks and therefore developing the solution of the problem, they do not see the screen and most of the times they do not perceive the continuous feedback given by the virtual blocks.

Interaction Area: The virtual representation of the interaction area is included in the system in order to help children to infer the real interaction space constrained by the field of view of the camera. We tested both conditions, using a sheet to delimit the interaction space (i.e the field of view of the camera) and without the sheet (see Figure 9).

During the first user test we observed that with the sheet the children understood better the interaction area limits. As for this user test we used the wizard of oz technique, therefore, the system feedback was not as continuous and fluid as it should be. For this reason we hypothesized that with real time feedback of the virtual blocks, children would be able to infer the detection area after some tries.

For the second user test, in order to help the users to understand which are the boundaries of the detection area, we designed a fade-off behavior: when the blocks get close to the vision boundary, the virtual representation on screen starts to gradually disappear. However, despite the efforts, we still observed that without the sheet of paper on the table, most of the children do not infer the detection zone on their own, but they needed some help.

We realized that the inclusion of a physical object to delimit the working zone is required. We did it with a paper on the table. However, the main potential drawback is that the sheet could be damaged and that the position is relative to the tablet, but this can be solved by attaching the defined area to the tablet holder.

Possible improvements might be to include on-screen animations to help the children realize that the block is on the boundary (for example arrows pointing to the center) of the detection zone or explicitly explain the existence of these boundaries in a tutorial at the beginning of the game.

Auditive Feedback: We just tested auditive feedback during the second user test and it was just background music and basic sound effects when the robot reached the screws. For this reason, we did not directly observe any conclusive behavior related to the auditive channel. However, we gained some insights related to potential uses of it. For example, we observed that as the setup splits children's attention between the table and the tablet, sometimes they miss events on the screen because they are looking or manipulating the physical blocks. However, as the auditive channel is not affected it could be exploited to provide feedback reducing this negative effect. This seems to point towards a more general implication: in environments where the visual attention is split, the auditive channel might be exploited as a complementary strategy. It is expected that after some trials, children could learn that when they hear a certain sound something is happening on the screen.

Robot: The robot actions were understood by the children. Actually, these actions allowed them to realize if they had achieved the goal or not. In future versions of the game we will design a more active behavior adding animations and hints coming from the robot.

Actions Three main actions are key to achieving the objectives of the game: grouping and aligning the blocks, the third one is submit. The grouping and aligning actions are already described in the physical objects section since they are encouraged by the magnets. In the following lines we discuss the submitting action which is probably the one with the biggest impact on learning.

Action Submit (through countdown): During the first user test this property was not faithfully assessed, since with wizard mediation, countdown could have a great variation. However, we observed that without countdown the child could solve the problem without realizing it, e.g. he puts the random blocks on the table without watching the goal on the screen, if by chance this is a good solution, the system understands this is the children's answer and activate the robot movement.

In the second user test, we controlled the countdown time. Concretely, we used a two-second countdown starting after

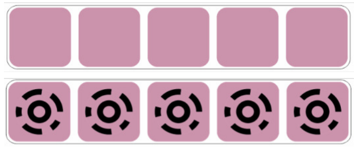

Figure 6. a) Sub-elements as colored squares b) Sub-elements including the TopCode markers.

one second that the blocks are still. We observed that it takes two or three trials before children realizes that after the countdown the robot performs an action. In order to facilitate this understanding, explicit instructions could be shown in a short tutorial at the beginning of the game. In some cases, children pick the blocks before the countdown finishes. To avoid this, we can play a sound and add an animation of the robot getting ready to perform the action in order to attract their attention. This might be very useful to guide children during the first trials. In general the two second countdown is not excessive. However, shorter times might be tested in order to achieve a more fluent interaction without losing the pause for reflection. Playing a sound at the beginning of the countdown might help to highlight the event and also to attract the attention if the child is not looking at the screen.

Thus, without the action submit countdown many times children resolved the problems by chance. The system processes the blocks immediately favoring a trial and error strategy, placing different blocks until the problem is solved. However, there is no reflection in this process. This observation is consistent with previous research [31, 6, 28], although it is important to note that the delay must be introduced between children's actions and the system's response when such response is relevant for the learning goal, i.e., in our case when the system interpret the blocks and performs the addition. The continuous feedback and fluency of the system must no be affected by this delay.

Informational Relations As it was explained before, during the first user test we just had the one-to-one, object-to-object mapping. During the second user test we tested all the mappings between physical objects, actions and digital objects. In general, children understood the different shapes and movements of the robot as it is the key element that allowed them to determine if they had reached the reward or not. As a possible design issue we observed that the action of the robot is not strictly linked with the development of the solution, this means that children can reduce the task to first identifying the number and then composing it with the blocks. We do not know if the action that the robot performs to reach the reward is being perceived by children and if it has a real impact on the level of abstraction of their reasoning, which is in fact our intended purpose. In order to find out further and specific user test might be done.

DISCUSSION

As it was already suggested in previous research [6, 31, 28], the interaction pace has a determinant impact in the reflection during the learning process. In this particular context

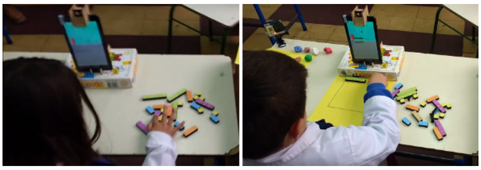

Figure 7. Children using the sub-elements to count while they play with CETA. Left: Without delimiting the interaction area. Right: Using the sheet of paper to delimit the interaction area.

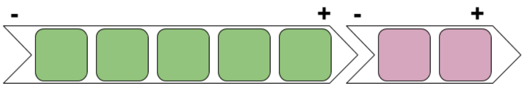

Figure 8. The block's shape constraints the way that it can be joined ensuring that magnets will not repeal.

we observed that when the system does not provide delays between children's proposed solutions (physical blocks configuration) and system evaluation (feedback), the strategy tends to be more like trial and error rather than mediated. Manipulating physical blocks might allow children to 'dive-in' and explore, while the delay gives place to 'step-out' and reflect, ideally leading them through the "ongoing dance" composed by diving-in and stepping-out described by Ackermann [4].

Despite having carefully conducted a theory-based design of our artifact, significant insights were gained during the user test, either validating the initial design or providing valuable feedback to improve it. In this way, we support the idea that prototyping is a fundamental practice that should not be skipped since it enables the active exploration evoking *kinaesthetic creativity* [40] in both users and researchers. It also allows us to observe how theories work when they are put in practice.

Regarding the research question Q1, we managed to design different mappings between physical objects, digital objects and actions in order to modulate the level of abstraction during the game. However, the children's strategy is not affected since the abstraction is given during the feedback phase, i.e the robot actions, and it does not require them to change their actions at anytime, i.e children might limit their strategy just by looking at highlighted number on the screen (see Figure 3) and then represent it with the blocks. As a consequence, it is not clear if children are effectively perceiving it and therefore if it has any impact in the learning experience. Further research needs to be conducted in order to determine the impact of our mappings design modulating the abstract level.

CONCLUSIONS

We have developed the CETA environment, a mixed-reality environment for low-cost tablets and used in the design of the CETA game, a mathematics learning activity for school-aged children. We took advantage of the already deployed tablets in public schools as a part of the OLPC program, aiming to enhance the learning process through the use of augmented manipulatives. The development cost is significantly low and the source and design files are open and accessible at https://github.com/smarichal/ceta. Therefore CETA, the environment and the game, might be a significant step towards bringing tangible technology to classrooms.

We have presented the concept of the CETA game, it is an augmentation of one of the most basic manipulatives applied in learning, the Cuisenaire rods. In the conceptualization of interaction, we have considered relevant cognitive theories, including the cognitive offloading, the conceptual metaphors and epistemic actions. In the design of the game, we slightly changed physical design of the Cuisenaire rods, including magnets in the extremities as an affordance to join the rods. The mixed-reality approach enables adding digital representations, and therefore incorporating or changing properties that could not be changed in the real object, for example adding sounds and changing sizes and colors. In addition, system feedback guides and helps children to understand the goals and if they are performing well. Lastly, digital systems and learning through games always mean an extra motivation for children, increasing the engagement and joy.

Initial prototypes of the system were tested with school-aged children in their school context, and their experiences contributed in the design of the system. Through this design we addressed three research questions related to the design of a tangible system for Mathematics learning with school-aged children. This includes the design of the learning activity, physical and digital objects, the actions and the informational relations following the TLDF [6].

Q1: *how can we shape the level of abstraction by changing the actions and informational relations between physical and digital objects?* Our approach was to change the structure of the mappings achieving three levels of abstraction altogether. The most basic and concrete one is the representation of each physical object with a single digital object, i.e., one robot per block or one subdivision of the robotic arm per block. The intermediate level is where many physical objects are represented by the shape of a single digital object, i.e., a composition of blocks determines the arm's length or height of the robot. Lastly, there is the most abstract level in which many physical objects are mapped to actions on the digital objects, i.e., a composition of blocks make the robot skate or fly the same distance as the composed number.

Q2: *Which actions are relevant and desirable in this specific mathematical learning activity?* In our case, given that the learning goal is additive composition and number line, the most relevant actions on physical objects are composing groups and align them imitating the number line. To this aim, we designed objects with magnets in the extremities, a specific affordance to create groups and align the blocks. We observed that some children first join the blocks and then count the sub-elements in order to compute the sum, this is an epistemic action supported by the system and relevant for the problem solving strategy.

Q3: *Which is the most effective and less disruptive way to slow down the interaction pace and encourage reflection?* Adding delays between children's actions and the system evaluation and response is a non disruptive strategy to slow down the interaction pace. Most children understood it and we observed that it encouraged reflection instead of trial and error strategies. These delays should not affect the continuous feedback of the system, children should realize that the system is processing the information and that they must wait.

LIMITATIONS AND FUTURE WORK

Although the user experience would benefit from a virtual representation of the blocks (see Figure 1) that would be displayed in the same space as the physical blocks in order to integrate the input and output space and support exploration [7], we used a different approach. To settle this issue, we provided continuous feedback, however, children do not perceive it until they look at the tablet screen. This requires them to lift their head and somehow "change the context", probably disrupting their active exploration. Otherwise, given the implementation of the system using a computer vision approach, we can not support the total occlusion of the blocks. As a consequence, some positive properties of manipulatives might be affected, including *proprioception* and *haptic subitizing* [27]. What we want to stress here is that the technology is not completely seamless and that it might be constraining some aspects of the natural interaction that children have with physical objects, and in some cases forcing them to adapt to the system. Although there could be better technologies to accomplish these goals, we focused on using the limited features of the low-end tablets already distributed in all public schools in Uruguay and other places where programs such as OLPC have arrived, in order to take advantage of this infrastructure.

Besides we conducted two field studies with prototypes and children, we did not follow a rigorous methodology to formally evaluate the different possibilities of the interactive system. We did a first approximation based on observation of some specific features conducting an exploratory study in real life settings. However, it was useful to make basic design decisions that complemented all the theoretical background behind each design option.

Based on the results of the two user tests, we will improve the system including features such as adding hints to guide children through the activity or when they get stuck and change the physical block design to avoid blocks to repeal (see Figure 11). In addition, we plan to make extensive use of the auditive channel to mitigate the drawback of having separated input and output spaces, and also to reinforce the sense of magnitude mapping sounds with each block following a similar strategy than with the size, but in this case through the image schema "louder is more", i.e., bigger blocks will be mapped with louder sounds.

Once the final prototype will be developed, a multimodal evaluation approach [31] would be useful to analyze the embodied interaction and formally classify and describe the interaction features of the system. In addition, an evaluation of the learning outcome using pre-test and post-test is required to compare our system with non tangible digital approaches, i.e pure virtual, or with traditional methods employed in schools, once the final prototype is developed. Such evaluations, although interesting, were beyond the purpose of this paper, which focuses on the conceptual design of the mixed reality system and the game.

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3. Embodied Interactive Mediated Reflection: A piece of Intermediate-Level Knowledge

In this chapter we introduce some key concepts of intermediate-level knowledge and its relevance for theory informed designed systems (Section 3.1). We contribute with a new strong concept named "Embodied Interactive Mediated Reflection", detailed in Section 3.2 in a short workshop position paper.

3.1. Introduction to Intermediate-Level Knowledge

Intermediate-level knowledge (ILK), is a type of knowledge situated between theories and specific artifacts [7, 16]. That is, knowledge more abstract than particular instances but not as general as a theory [16]. It has been pointed out that design-oriented research, as conducted in this dissertation, has potential to generate this kind of knowledge [16]. There are many forms of ILK, for instance Guidelines, Patterns, Methods and Tools, Bridging Concepts and Strong Concepts [7].

In particular, the so-called strong concepts have generative power and play an active role when it comes to the creation of new designs, they were defined in its seminal work [16] as:

> "*Strong concepts are design elements abstracted beyond particular instances which have the potential to be appropriated by designers and researchers to extend their repertoires and enable new particulars instantiations. We connect the notion of abstraction to scope of applicability. A specific artifact is fully concrete, that is, not abstracted at all, and as such, it is (primarily) applicable only in the situation for which it was designed. Elements of that particular artifact, or instance, can be isolated and abstracted to the level that they are applicable in a whole class of applications, a whole range of use situations, or a whole genre of designs.*"

Strong concepts can be identified through literature reviews of related work [7] as well as from the examination and analysis of our own prototypes (instances), or by combining both approaches. In our specific case, we propose “Embodied Interactive Mediated Reflection” as a strong concept as the result of previous work analysis [31, 5] combined with the design and evaluation of the concrete instance CETA [27]. We observed that promoting reflection during learning activities is, in terms of interaction design, challenging. Tangible systems encourage and facilitate the manipulation of elements from the problem domain easily. This accelerates the interaction pace, resulting in a fast and broad exploration of solutions, which might trigger trial and error strategies lacking reflection.

EIMR is presented as a strong concept in order to help and inspire future embodied interactive systems designs in a learning context. Recently, ILK has been formalized as a concrete contribution to the HCI field. This thesis contributes with a piece of ILK in the form of a strong concept.

3.2. Embodied Interactive Mediated Reflection

The content of this section was presented as a position paper for the workshop:
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Embodied Interactive Mediated Reflection

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Abstract
Technology enhanced learning have been explored during the last decades with some controversy with respect to the real benefits for the learning process. Virtual environments have been pointed to discourage physical interaction and therefore the use of our bodies for learning. Embodied interaction in form of mixed-reality and tangible environments somehow tackle this issue and promise a more balanced combination of the virtual and real world that might enhance learning. One of the main challenges of designing embodied interaction for learning is promoting the reflection required for appropriating new concepts. In this paper we describe why and how we identified Embodied Interactive Mediated Reflection as a strong concept for the design of virtual learning environments.

Author Keywords
Intermediate-level knowledge, Learning, Embodied interaction, strong concepts

ACM Classification Keywords
H.5.2 [Information interfaces and presentation]: User interfaces

Introduction
Embodied interactive learning experiences represent an opportunity for collaborative and group learning, using the

body for active exploration engaging multiple senses in a constructive process [3, 22, 2], and the possibility to translate learning theories such as Physically distributed learning [14] and Constructivism to the domain of digital interactive systems. However, reflective thinking and reflective activities are important in the meaning construction process [7]. In accordance to Ackerman's model of cognitive growth, it is necessary to combine exploration and reflection stages as an "Ongoing dance", diving-in and stepping-out are equally important in the learning process [1]. Moreover, she argues that *"separateness resulting from momentary withdrawal does not necessarily entail disengagement"* [1]. Thus, embodied interaction environments that are highly exploratory should also provide the elements and proper context to encourage reflection. However, the lack of methodologies to understand the relationship between body actions and real-time meaning making [16] as well as the importance of mechanisms to promote reflection into embodied interactive learning experiences has been noticed in previous research [3, 16, 10, 12, 11].

Based on our previous work [12] and other related work [16, 20, 3, 11, 10] we identify reflection as one of the main challenges of embodied interactive learning experiences design. Within this context, we understand that it is a strong concept [9] with generative power. In this paper we describe this concept in terms of horizontal and vertical grounding [4].

Description

Embodied Interactive Mediated Reflection (EIMR) is a strong concept on digital embodied interaction for learning. It builds on learning and cognitive theories and different kinds of interactive systems such as digital manipulatives and tangibles (see figure 1).

Based on cognitive theories related with the role of our body in the learning process, there is an increasing interest in the development of embodied interaction systems for learning purposes [21, 3, 16, 10, 11, 12]. However, contradictory results have been reported about the effective enhancement of these systems in the learning process [5, 23]. We are specially interested in *reflection* as a key element in cognitive growth, and therefore in the strategies to give place and encourage reflection through embodied interaction.

Embodied interaction based systems are highly exploratory and dynamic where the human body plays two roles at the same time. On the one hand, the body is part of the user in the traditional meaning, it is used as a medium to interact with the system. On the other hand, it is also part of the system itself and therefore it is a potential tool for the designer. Being in the world [19] and co-existing in the reality with the system make users active performers, and through this ability to "perform" within the system users might be able to shape their own and unique experience. Our interest and challenge as designers is to create the proper conditions to trigger reflection along this embodied interaction experience.

Horizontal Grounding

Related concepts include technology applied to learning, the role of our body in the learning process (embodied component) as well as the intrinsic role of reflection during learning activities.

Manipulatives are physical learning materials that have been used in education for a long time. Froebel gifts [8], Cuisenaire rods [6] and Montessori materials [15] are some popular instances of manipulatives. They share the objectives of EIMR about exploring through our body and reflect-

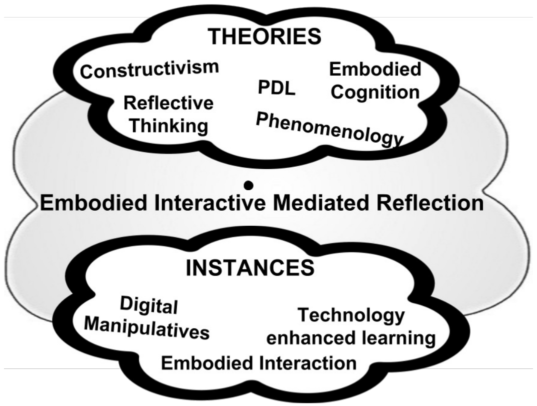

Figure 1: EIMR as intermediate-level knowledge

ing as a learning methodology. However, manipulatives in their original formulation do not involve interactive systems or digital technology.

Virtual manipulatives [10] are pure digital instances of physical manipulatives, i.e., graphically represented on a computer screen. They present interesting opportunities in order to extend traditional manipulatives, e.g. incorporating temporal properties that might change during the activity such as sound, color or size. In addition, they expand the opportunities to trigger reflection by adding pauses or questions and answers on the screen. However, they do not involve the use of the body. Otherwise, digital manipulatives are computationally-augmented versions of traditional manipulatives [17] and somehow they embody good properties from both, physical and digital manipulatives. However, we should not take for granted that just for the fact of interacting with digital manipulatives reflection will emerge.

Reflection is key in the meaning construction process [9], creating relationships between the involved elements and giving meaning to the experience [18]. For instance, in a hands-on mathematics problem solving context, reflection means to be aware of our actions understanding and linking them to abstract concepts such as mathematical operations.

Vertical Grounding

The concept of EIMR builds on the design emerged from at least 3 prototypes of embodied interactive systems for learning, CETA [12], Towards Utopia [3] and a tabletop system [16].

CETA is a mixed-reality system for mathematics learning for children. The interaction is through physical blocks that are automatically detected by the camera of the tablet (see figure2) [13, 12]. One of the main challenges of the design of CETA was, ensuring, that the user does not provide right answers by chance, following a trial and error strategy. Thus, it was relevant to encourage a stage of reflection [12] which is key in the learning process [1]. Our strategy was to add delays between user actions and the system evaluation and response. This slows down the interaction pace and gives place to a momentary withdrawal of the exploration phase, in which children can trace this relationship between their actions, the elements of the environment and the system response.

Price and Jewitt [16] conducted a multimodal analysis of the interaction of groups of 2 children with a tabletop system designed to explore concepts related to the physics of the light. Observing how children interact with the system, authors conclude that a higher interaction pace permitted to explore more configurations of the elements of the system, but also reduced the amount of reflection time. They also argue that reflection time is important to understand the science and also to plan future actions.

Towards Utopia is a TUI learning tabletop environment specifically designed for learning concepts related with sustainable development [3]. It is also a collaborative environment that supports hand-on exploratory activities. Theories such as perspective taking and reflective thinking are applied to the system design. These theories are the main underpinning of EIMR. Authors explain the importance of experiential and reflective learning, agreeing with Ackerman in that both experiences are required for knowledge construction [1, 3]. Their strategy to encourage reflection is also based on pausing actions and slowing down the interaction pace. They explain that spatial, physical, temporal or relational properties can be used to slow down the interaction and trigger reflection.

Thus, in the case studies analyzed, reflection has been encouraged through slowing down interaction pace. However, this could be further analyzed, in terms of how it has been implemented, and which other techniques could be explored.

Conclusion

We have identified Embodied Interactive Mediated Reflection as a strong concept on digital embodied interaction for learning. This strong concept builds on a double challenge; a challenge addressed in the learning theories, and a challenge in the design of embodied interaction to enhance learning.

We expect its generative power to contribute in the design process of embodied interactive systems for learning.

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4.Comparing physical and virtual interaction effects

This chapter presents a long term study in public schools where CETA learning outcomes were evaluated. Actually, CETA was compared with a full virtual version of the game (no tangibility) and with a control group which followed a traditional teaching approach. Results suggest that children who used CETA obtained better results compared to the control group. We also observed that children using CETA developed solutions involving more blocks than children using the virtual version of the game.

Beyond the suggested learning benefit when using CETA compared to traditional teaching practices (control group), we discuss later in the general conclusions (chapter 6-RQ3) how virtual and tangible versions of the game shaped children's strategies in different ways. We also reflect on how we as interaction designers might take advantage of it.

4.1. Building Blocks of Mathematical Learning: Virtual and Tangible Manipulatives Lead to Different Strategies in Number Composition

During this study I mainly collaborated in the interaction design and implementation of the mixed-reality system prototype. Additionally, I also took part in the literature review and discussion from an interaction design perspective analysing how the affordances of the system might shape and constrain users' strategies.

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Building Blocks of Mathematical Learning: Virtual and Tangible Manipulatives Lead to Different Strategies in Number Composition

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Multiple kinds of manipulatives, such as traditional, virtual, or technology-enhanced tangible objects, can be used in primary education to support the acquisition of mathematical concepts. They enable playful experiences and help children understand abstract concepts, but their connection with cognitive development is not totally clear. It is also not clear how virtual and physical materials influence the development of different strategies for solving instructional tasks. To shed light on these issues, we conducted a 13-day intervention with 64 children from first grade, divided into three groups: Virtual Interaction (VI), Tangible Interaction (TI), and Control Group (CO). The VI group played a fully digital version of a mathematics video game and the manipulation of the blocks took place on the tablet screen. The TI group played the same video game with digitally augmented tangible manipulatives. Finally, the CO group continued with their classroom curricular activities while we conducted the training, and only participated in the Pre and Post-Test evaluations. Our results highlighted that the use of tangible manipulatives led to a positive impact in children's mathematical abilities. Of most interest, we recorded children's actions during all the training activities, which allowed us to achieve a refined analysis of participants' operations while solving a number composition task. We explored the differences between the use of virtual and tangible manipulatives and the strategies employed. We observed that the TI group opted for a greater number of blocks in the number composition task, whereas the VI group favored solutions requiring fewer blocks. Interestingly, those children whose improvement in mathematics were greater were the ones employing a greater number of blocks. Our results suggest that tangible interactive material increases action possibilities and may also contribute to a deeper understanding of core mathematical concepts.

Keywords: digital manipulatives, tangible manipulatives, technology-enhanced learning activities, mathematics, additive composition

1. INTRODUCTION

Learning mathematics at an early age is fundamental to ensuring academic success in STEM (science, technology, engineering, and mathematics) disciplines and maximizing future integration into professional life (Wang and Goldschmidt, 2003). Research has been concerned with how to foster this core cognitive ability and enable a deep understanding of mathematical concepts. This research explores how virtual and tangible manipulatives can be used to strengthen math learning at 6 years of age.

In the current study, we used the activity of composing and decomposing sets of manipulatives representing numbers, an exercise that has been traditionally practiced with concrete material in order to foster an understanding of numerosity (Geary et al., 1992; Morin and Franks, 2009). We focused on a set of three properties (additive composition, commutativity, and associativity) and the mastery of the basic number combinations. Additive composition is the knowledge that larger sets are made up of smaller sets; the commutative property implies that changing the order of the operands doesn't affect the result; the associative property allows us to add (or multiply) numbers, no matter how the factors are grouped $[(a + b) + c = a + (b + c)]$; while mastering the basic number combinations leads to understanding how numbers can be composed. These properties are crucial for cardinality and number concept acquisition; and lead to the development of key strategies in arithmetical problem solving, such as addition and subtraction (Fuson, 1992; Verschaffel et al., 2007).

In mathematics curricula, teaching is frequently supported by tangible objects (three-dimensional models of geometrical shapes, etc.) that help young students to better understand abstract concepts, for instance in the acquisition of cardinality (Geary et al., 1992; Morin and Franks, 2009). The pioneer in this tradition was Maria Montessori who developed materials for geometry and mathematics specifically aimed at providing children with autonomy during the learning process (Montessori, 1917). Georges Cuisenaire, in turn, created a special set of tiles for arithmetics learning known as Cuisenaire rods (Cuisenaire, 1968). His proposal was based on the relationship between size and number and exploited the possibility of different spatial arrangements to exemplify mathematical principles like number composition. A new version of these materials can be found in Singapore Math's tiles (Wong, 2009; Wong and Lee, 2009); which is considered one of the more influential methods for teaching basic mathematics nowadays (Deng et al., 2013).

Following this vein, the acquisition of the number concept—one of the building blocks of mathematical learning—would benefit from direct interaction with objects (Dienes, 1961; Chao et al., 2000; Anstrom, 2006; McGuire et al., 2012). Interaction with objects may facilitate the passage from a concrete construal (I can see/manipulate three things in front of me) toward an abstract one (3 = * * *). This transformation begins with a process which is strongly based on perceptual, non verbal operations and turns into a symbolic one supported by an abstract association (Feigenson et al., 2004). The first stage has to do with the understanding that a given group of objects has a certain quantity of components (Gelman and Gallistel, 1978); the second with associating this quantity (of objects) to an exact number and its symbolic expression, and then understanding that any time the number is seen or heard it means that an exact quantity is being referred to (Kilpatrick et al., 2001).

The sensitivity to numerosity is improved gradually as the infant develops (Izard et al., 2009). Infants even just a few hours old are already sensitive to numerosity (e.g., Antell and Keating, 1983; Izard et al., 2009). Allegedly, this is possible due to two innate parallel number systems (see Feigenson et al., 2004; for a review see Piazza, 2010): an object file system (Feigenson and Carey, 2003) which accounts for the immediate identification of a discrete quantity of elements—subitizing (Kaufman and lord, 1949)—and is limited by the capability to attend to different objects at the same time; and an approximate number system (ANS) which accounts for a non-symbolic continuous numerical representation involving large numbers (Gallistel and Gelman, 1992; Dehaene, 2011).

Nevertheless, children are not able to explicitly identify simple quantities involving numbers from 1 to 4 until 4 years old, and up to 5 until 5 years old. To do so, different skills must be developed such as counting and conceptual subitizing; the combination of two "subitizable" numbers, for e.g., recognizing the presence of a 3 (***) and a 4 (****) and implicitly composing a set of 7 (*******) (Steffe and Cobb, 1988; Clements, 1999). Toddlers recognize that sets can be combined in different ways, but this understanding is based on nonverbal, perceptual processes (Sophian and McCorgray, 1994; Canobi et al., 2002). Commutativity is only acquired later between 4 and 5 years old, as also the understanding that commutativity of added groups leads to associativity (Gelman and Gallistel, 1978; Canobi et al., 2002). Thus, associativity reflects conceptual reasoning about how groups can be decomposed and recombined (Sarama and Clements, 2009). Further, as children learn basic number combinations, they can master a broad set of heuristics when faced with addition and subtraction problems.

To foster the conceptualization of unit items children may rely on hand actions such as pointing or grasping (Steffe and Cobb, 1988). For instance, in the case of subtraction, small children often represent the minuend with the fingers (or objects) and fold their fingers (or remove objects) for the value of the subtrahend (Groen and Resnick, 1977; Siegler, 1984). In fact, most children cannot solve complex numerical problems without the support of concrete objects until 5.5 years old (Levine et al., 1992). Later on, children acquire retrieval strategies, accessing results directly from long term memory (Rathmell, 1978; Steinberg, 1985; Kilpatrick et al., 2001). For this to be possible, children need to master basic number combinations (Baroody and Tiilikainen, 2003), but also understand associativity (Sarama and Clements, 2009). Children typically progress throughout three phases to achieve mastery on basic number combinations: (a) Counting strategies—using object counting (e.g., with blocks, fingers) or verbal counting (b) Reasoning strategies—using known information (facts and relationships) to deduce the answer of an unknown combination; (c) Mastery-efficient responses [i.e., fast and accurate (Kilpatrick et al., 2001)].

Children's addition and subtraction strategies also evolve during childhood. For instance, in order to solve 9 + 8, 4 to 5-year-old children would count from 1 to 9 for the first addend and then from 9 to 17 for the total sum ("counting all strategy"; Fuson, 1992; Verschaffel et al., 2007). Later on between 5 and 6 years old children would develop the more refined strategy of "counting on" in which the count starts from the cardinal of the larger addend (i.e., from 9 to 17; Carpenter and Moser, 1982; Siegler and Jenkins, 2014). More sophisticated part-whole strategies are developed with the achievement of associativity and the knowledge of how numbers from 1 to 10 can be composed (6–7 years old; Canobi et al., 2002). To solve 9 + 8 children would be able to retrieve that 9 + 1 is one of the forms to compose 10, and then solve the problem by the easier 10 + 7 (also retrieving that 8−1 equals 7; Carpenter and Moser, 1984; Fuson, 1992; Miura and Okamoto, 2003).

Interaction with objects may supports the development of different strategies by diminishing cognitive load and freeing up working memory, given that the perceived entities are cognitively available through the objects that represent them in space (Manches and O'Malley, 2016). Object manipulation gives rise to operations that can work as analogies of abstract operations. For example, joining 2 elements to a group of another 3 forms a new group of 5. This concrete activity would be a metaphor of act of addition: 2 + 3 = 5. These conceptual metaphors work as scaffolding that allows children to grasp abstract ideas such as commutativity or associativity (Manches and O'Malley, 2016).

With the appearance of digital technologies, researchers have been exploring how the manipulation of digital (Yerushalmy, 2005; Moyer-Packenham and Westenskow, 2013) and/or technology-enhanced concrete material (Tangible User Interfaces or TUIs; Manches, 2011) can benefit learning processes, finding promising results (see Sarama and Clements, 2016). Beyond the encouraging results obtained in several technology-based interventions, it has been claimed that the application of digital technology in the classroom posits the risk of replacing rich physical interactions with the environment by much more constrained interactions such as the use of the mouse–keyboard or multi-tactile interfaces (Bennett et al., 2008). In this vein, theories like constructivism, embodied cognition (Wilson, 2002; Anderson, 2003) and physically distributed learning (Martin and Schwartz, 2005) support the idea that physical interaction plays a key role in the learning process (Antle and Wise, 2013; for a review in this matter see Sarama and Clements, 2016).

In this study, we focus on the kinds of actions virtual and physical manipulatives offer and their impact on numerical learning. On one hand, interaction with virtual manipulatives is limited to dragging objects on the screen, but it still allows children to displace, join and isolate objects as traditional manipulatives allow (Moyer-Packenham and Westenskow, 2013). On the other hand, classic manipulatives offer interactive advantages (to grasp the object, for instance) that could have relevant consequences for educational activity (Martin and Schwartz, 2005; Manches and O'Malley, 2016). Several studies have been dedicated to this comparison, providing results which are slightly favorable to physical manipulatives (Martin and Schwartz, 2005; Schwartz et al., 2005; Klahr et al., 2008).

Technology-enhanced tangible manipulatives offer several advantages when compared with traditional or virtual manipulatives (Moyer-Packenham and Westenskow, 2013). They allow autonomous and active learning by using physical material and enable us to record a child's performance. In addition, they enable us to explore which kind of actions are relevant in specific learning activities. Importantly for the present research, our system permits analyzing and comparing the use of physical and virtual manipulatives to solve a task of additive composition. This comparison is of special theoretical interest given that it makes possible to explore the role of physicality/three-dimensionality in learning mathematics. In other words, the present research aims to investigate if it is indispensable that objects may be grasped, lifted, and explored or would it be enough to interact with virtual manipulatives? And specifically, we ask how the objects' affordances (i.e., the possibility to grasp physical objects or drag virtual ones) will shape and constrain children's composing strategies.

2. MATERIALS AND METHODS

2.1. Participants

We recruited participants from one state school in Montevideo (Uruguay) with a medium-high sociocultural status consisting of 64 children (three classrooms) from first grade. All children had an informed consent form signed by their parents or legal guardians. A research protocol was approved by the Local Research Ethics Committee of the Faculty of Psychology, and is in accordance with the 2008 Helsinki Declaration. We employed a quasi-experimental design and each classroom became one of the following experimental groups: Control (CO), Virtual Interaction (VI), and Tangible Interaction (TI).

Four children (two from the VI group and another two from the TI group) failed to correctly answer 25% of the trials in our training game. Therefore, we performed subsequent analyses with the remaining 60 children (33 girls and 27 boys). Group descriptive information is shown in **Table 1**. We examined the effect of age and sex by conducting separated *t*-tests on assessment scores, but we did not find any effect.

TABLE 1 | Mean and standard deviations at pre- and post-tests by groups.

				TEMA-3	
	n	**Age (years)**	**Sex (*girls)**	**Pre**	**Post**
Passive Group (PA)	20	6.6 (0.3)	13	25.6 (5.7)	28.8 (4.6)
Virtual Interaction Group (VI)	20	6.8 (0.5)	11	31.8 (9.6)	35.1 (9.3)
Tangible Interaction Group (TI)	20	6.8 (0.6)	11	30.2 (10.3)	34.4 (10.5)

2.2. Procedure

To evaluate the impact of both game modalities in the acquisition of mathematical abilities, we planned an intervention with three phases. A first and last phase of evaluations (Pre- and Post-Test), and a training of 13 days in between.

2.2.1. Pre-test

To evaluate children's mathematical abilities before and after training we used the third edition of the standardized Test of Early Mathematics Ability (TEMA-3, Bliss, 2006) for children between 3 and 8 years of age. The test was verbally administered and consisted of 72 items to assess: counting ability, number comparison facility, numeral literacy, mastery of number facts, basic calculation skills, and understanding of mathematical concepts. This test has high content validity (Baroody, 2003) and high reliability ranging from 0.82 to 0.97. Indeed, we found a high test–retest reliability measured by calculating TEMA-3 correlation between Pre-Test and Post-Test measures across children within each training group (TI: 0.94; VI: 0.94; CO: 0.78). We calculated scores by the sum of all the correct answers (taking into account ceiling and floor effects that are part of the test administration). Two trained evaluators conducted the evaluation and it took about 30 min per participant. This phase took one week, with 12 children evaluated per day.

2.2.2. Training/Playing

The three classes selected to participate in the study continued with their regular formal learning activities as part of the school curriculum. Apart from the fact that each class had a different teacher, teachers followed the same program and protocol, and were committed to giving the same math curricula information for the three classes. Both the TI and VI group played over 13 days (3 weeks). Sessions had a duration of 20 min each, from Monday to Friday. Two researchers were present in every session to help with any technical problems that may have arisen. In the first session, we introduced the game dynamics and made explicit the relation between size and value of each tangible and virtual block to facilitate effective use of manipulatives. The CO group continued with their regular curricular activities while the other two groups had 20 min per day of training. The CO group only participated in the Pre- and Post-Tests assessments.

2.2.3. Post-test

The same evaluators assessed the groups again with TEMA-3 and the scores were analyzed in the same manner as in the Pre-Test evaluation.

2.3. Training Game BrUNO

The video game BrUNO was developed to give the learning activity a more attractive and playful format. We took gamification theory into consideration in order to incorporate some gamification elements in BrUNO, such as: microworlds, a main-character, a tutorial, several types of prizes, and funny sounds. During the development of BrUNO, we carried out two informal user tests to inform the game design (Marichal et al., 2017a).

BrUNO is a video game designed to work on additive composition. Children played BrUNO by using five types of blocks whose length and color were associated with their value (see **Figure 1**). The block of 1 represents the number "1"; the block of 2 represents the number "2," and so forth until 5. Each block has a different length which is proportional to the value that it represents).

To facilitate visual recognition of the location of the number required to build, a horizontal or vertical number line (depending on the scenario) is shown on the screen (see **Figure 2**). It is known that as numerosity develops, a hierarchical mental representation of how numbers should be ordered arises in the form of a number line. This line, which is based on a spatial analogy, represents the numbers from lowest to highest and locates them according to their cardinality. Thus, to reinforce this mental representation and to facilitate the additive composition task, we presented a number line to guide the players while they compose the required number. It helps to count the missing/spare units and deduce

Block	Block value	Digital block dimensions	Tangible block dimensions	Color
	1	40px x 40px	16mm x 16mm	yellow
	2	80px x 40px	32mm x 16mm	green
	3	120px x 40px	48mm x 16mm	blue
	4	160px x 40px	64mm x 16mm	orange
	5	200px x 40px	80mm x 16mm	red

FIGURE 1 | Block values, dimensions, and color.

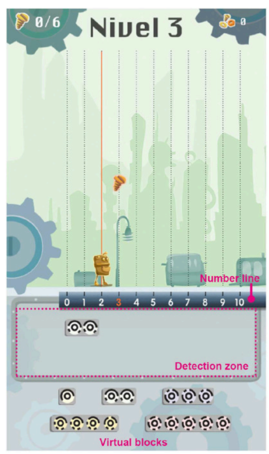

FIGURE 2 | Fully virtual version of BrUNO. Prize placed in number three (as indicated by the orange color). The player has already introduced 1 block of value 2. To reach the prize, he must add one block of value 1. In this example, a horizontal number line is present to help children locating numbers and to help in adding and subtracting operations.

how the target number can be correctly composed. If the child has to build the number 4 and she has already put one block of 3, she can observe that the game character is 1 unit away from the prize and compose the target number by adding the block of 1. This way, the child can learn that 3 + 1 = 4. Additionally, the game helps to demonstrate that, for example, the distance between 1 and 3 is the same as between 21 and 23—a fact that is not so obvious for young children (Siegler and Booth, 2004).

We developed two conditions for the evaluation of manipulatives: the Tangible Interaction Group (TI) and the Virtual Interaction Group (VI). In both cases, children played BrUNO, but the interaction with the blocks differed. In the first case, children manipulated technology-enhanced tangible blocks, and in the second case, virtual blocks.

2.3.1. Tangible Interaction Device

We designed a low cost tangible interaction device named CETA (Marichal et al., 2017a), with three main components (see **Figure 3**): a mirror that changes the webcam's viewing direction, allowing the system to detect objects over the table; a wooden holder that keeps the tablet vertically in portrait orientation; and a set of tangible blocks of different sizes similar to Cuisenaire Rods (representing numbers from 1 to 5; see **Figure 1**).

We used the webcam of the tablet and a mirror to capture the image of the surface in front of the tablet holder in real-time. This image is constantly analyzed to detect blocks in the detection zone (for more details see Marichal et al., 2017b). The limits of the detection zone are determined by the webcam hardware and height of the holder. Blocks outside the detection zone are not visible to the computer vision system.

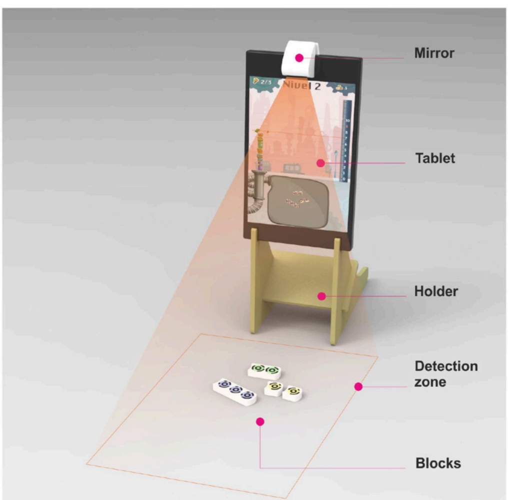

FIGURE 3 | Tangible setting for BrUNO. Figure reproduced with author's permission (Marichal et al., 2017a).

We designed a set of 25 blocks for 3D printing. The handling capabilities of the children at target age, the dimensions of the detection zone of the computer vision system, and the numeric quantities required by the different game challenges determined the dimensions of the blocks. All blocks contain magnets at their extremities, providing an affordance that increases the probability of joining blocks imitating the number line representation. Every block has a positive and a negative extremity. The concave and convex block's terminations constrain the way it can be joined. On the top face of each block we placed a set of colored markers (TopCodes; Horn, 2012) used by the computer vision system. The number of markers on each block corresponds to the block value.

2.3.2. Virtual Interaction Device

The virtual version allows to play BrUNO without CETA device. The blocks are virtual and the child has to place them in the detection zone to submit its answer to the system (**Figure 2**).

2.3.3. Data Collection

We recorded the children's actions to trace the quantity and the type of blocks employed in children's solutions over time. This allowed us to analyze the game strategies developed by each group and follow the performance of every single participant. After each response our system recorded the following data: (1) the number required to form, (2) the number actually formed, and (3) the blocks used to form the number.

We assumed that if the child wanted to respond with two blocks but put the first block in the detection zone while looking for the other, then we should develop a strategy to avoid considering this incomplete answer as a child's final solution. Thus, to avoid recording partial solutions we implemented what we call "action submit," which consists of two steps. The first step is to wait for a stable solution. By stable solutions, we mean invariant responses by children for 1.5 s meaning that the blocks placed in the detection zone were not moved for 1.5 s and no blocks were added or removed. If this condition was completed, then we move to the second step in which the game character prepares itself for 1 s to execute the movement. If, during this time the child changed his or her answer, the time counter resets and "action submit" starts over again. If the answer did not change, the game character moves and the system records the blocks that composed the child's solution. To avoid duplicate responses (e.g., the child leaves the blocks in the detection zone and goes to the bathroom) we only registered the solutions that differed from the last recorded solution.

3. RESULTS

3.1. Differences Between Groups

To test the effect of playing our training game over 13 sessions, we assessed the children's mathematics performance using TEMA-3 before and after training or without training as in the case of the CO group.

While we had a quasi-experimental design in which the groups were non-randomized at baseline, there were no significant differences between groups on Pre-Test, $p = 0.84$. To test for conditional differences, we used an ANCOVA with the Post-Test scores as the dependent variable, the Pre-Test as the covariate, and the Group as the independent variable. ANCOVA is advocated in this type of context because it controls for minor variations in the Pre-Test scores (Oakes and Feldman, 2001; Schneider et al., 2015). The assumptions of the ANCOVA were satisfied (as noted above, the covariate levels did not differ between conditions, and homogeneity of slopes held, as verified by running an ANOVA and customizing the model to include the interaction between the covariate and independent variable, $p = 0.5$). The ANCOVA identified a significant effect of Group, $F_{(2, 54)} = 20.9$, $p < 0.001$, $r = 0.44$. We followed up this analysis with pairwise comparisons between Post-Test scores adjusted by the ANCOVA with the baseline Pre-Test scores. Both experimental groups obtained higher Post-Test scores than the control group (VI_{Mean}: 32.54, $VI_{SD} = 0.77$; TI_{Mean}: 33.27, $TI_{SD} = 0.74$ and CO_{Mean}: 30.93, $CO_{SD} = 0.86$). However, only Post-Tests scores significantly differed when comparing TI vs CO ($p = 0.044$). We found no other significant effects between groups.

3.2. Virtual and Tangible Interaction Groups and the Minimum Blocks Coefficient (MBC)

We focused on the possible problem-solving strategies employed by the children when resolving the number composition task, and how the type of interaction could have affected their actions. To do so, we carried out exploratory analysis using participants' log files. It allowed us to observe which blocks were used to compose each number by all the participants, at every successful trial.

Firstly, we analyzed whether the number of blocks used to build the correct solution was different across groups. For example, to build the number 3, it is possible to use three blocks of 1 ("1-1-1"), one block of 1 and one block of 2 ("1-2"), or directly use one block of 3 ("3"). To evaluate how close the child was to using the minimum number of blocks that were necessary to build a number (one block in the case of numbers from 1 to 5, two blocks in case of numbers from 6 to 10, or three blocks if the number is greater than 10), we developed a score called the "Minimum Blocks Coefficient" (MBC). MBC is a metric that allows us to observe the different solutions in composing numbers while training additive composition. We aim to explore how children compose numbers using different types of manipulatives. For each correct solution it takes the minimum number of blocks necessary to build the number requested, and divides it by the number of blocks actually used. For example, in the case of number 3 the variant "1-1-1" becomes the score 1/3 = 0.33, because just one block is necessary to build the number (block of 3), and in reality, three blocks were used. The combination "1-2," becomes 1/2 = 0.5, and "3," becomes the score of 1.0. To calculate the MBC for one particular number and one particular group (TI or VI), we take all the correct solutions of the number formed by the participants of the group and calculate the mean value. Error rates were not analyzed because we observed that the tangible system required more time for the physical manipulation and during that time some partial solutions were recorded as errors before the child's final answer. For example, if the child wanted to respond with two blocks, but he or she put the first block in the detection zone while looking for the other and no changes occur in the detection zone for 2.5 s, the system registered the child's uncompleted solution as a response (error in this case). The algorithm is explained with more detail in the section "2.3.3." For the aforementioned reasons we decided to only analyze the correct answers, so we were confident that we analyzed explicitly correct answers rather than random solutions.

3.2.1. Minimum Blocks Coefficient by Numbers (1–13)

We applied a two-way ANOVA considering the MBC as the dependent variable and Group and Numbers as the independent variables. Numbers is the variable that represents the number the child is asked to build. We divided all the Numbers that appear in the game (1–13) into three ranges based on the theoretical MBC that could be used for those numbers. Specifically, the theoretical MBC for numbers ranged from 1 to 5 is one block (i.e., they have the possibility to respond with a minimum of one block); for the numbers ranged 6–10 is two (i.e., they have the possibility to respond with a minimum of two blocks) and for the numbers ranged from 11 to 13 is three blocks (i.e., they have the possibility to respond with a minimum of three blocks).

The results showed that the type of manipulatives (TI or VI group) [$F_{(1, 126)} = 6.21$, $p = 0.014$, $r = 0.076$] and the Number [$F_{(2, 126)} = 10.8$, $p < 0.001$, $r = 0.060$] (see **Figure 4**) significantly influenced the MBC. We found no further interaction. The TI group used significantly more pieces (lower MBC) comparing with the VI group ($TI_{Mean} = 0.65$, $TI_{SD} = 0.19$, $VI_{Mean} = 0.72$, $VI_{SD} = 0.15$). These differences between TI and VI may be a result of the diverse composing strategies used when solving the number composition task.

Considering the variable Number, the number of blocks used were significantly fewer for the numbers ranging from 1 to 5 compared to the numbers ranging from 6 to 10 ($p = 0.0002$) and also compared to the numbers ranging from 11 to 13 ($p = 0.0003$).

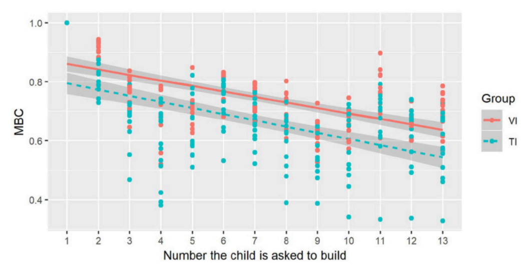

FIGURE 4 | The Minimum Blocks Coefficient (MBC) for each number the child was asked to build. We applied a linear model to data points with a 95% confidence level for each Experimental Group: Virtual Interaction (VI) and Tangible Interaction (TI).

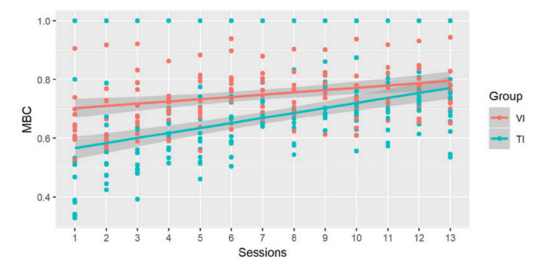

FIGURE 5 | Minimum Blocks Coefficient (MBC) for each session and experimental group. We applied a linear model to data points with a 95% confidence level.

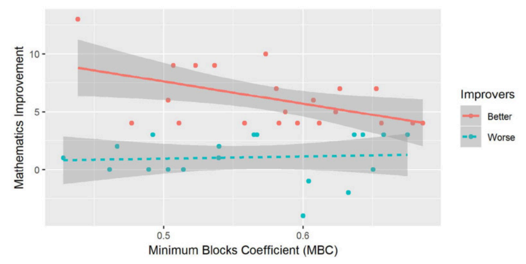

FIGURE 6 | Minimum Blocks Coefficient by mathematics improvement for better and worse improvers. We applied a linear model to data points with a 95% confidence level.

3.2.2. Minimum Blocks Coefficient Over Time

Participants reduced the number of blocks used during the 13 sessions that our intervention lasted (see **Figure 5**). We found a significant positive correlation ($ps < 0.0001$) between the MBC and sessions for VI (0.84) and for TI (0.87) groups. We also explored whether the number of blocks employed was significantly different at different moments of our intervention by analysing the MBC Mean for the first and last three sessions for both groups. Interestingly, in the first three sessions, the MBC was greater for the VI group, i.e., children used fewer blocks ($p < 0.0001$). In contrast, when analysing the last three sessions, the MBC did not differ between either group.

3.2.3. Minimum Blocks Coefficient and Mathematics Improvement

We explored the relationship between the number of blocks employed during the intervention (measured by MBC) and the amount of mathematical improvement (dScores: Post-Test scores − Pre-Test Scores) and found no correlation ($p > 0.05$). Neither TI nor VI groups showed a significant correlation between MBC and dScore when analyzed separately ($p > 0.05$).

Further, we decided to analyze the differences in the number of blocks employed comparing the performance of the Better and Worse Improvers. Thus, we divided all participants by the median of the dScore comprising two groups. The Better Improvers were the children with a dScore above the median, while the Worse Improvers were the ones whose dScore was below the median (see **Figure 6**). We found a significant negative correlation between MBC and dScores for the Better Improvers (cor = −0.50, $p = 0.021$), but not for the Worse Improvers. In conclusion, the children that had a greater improvement were the children using more blocks than the minimum blocks necessary to build the numbers required by the game. In contrast, we did not observe any change in the number of blocks used by the children who did not improve in mathematics.

3.2.4. Minimum Blocks Coefficient and Mathematics Performance

We were also interested in the relationship between the Minimum Blocks Coefficient (MBC) and mathematical performance (Pre-Test scores). Analysis indicated that Pre-Test scores were positively correlated with the MBC (cor = 0.41, $p = 0.009$; see **Figure 7**). Children who had greater Pre-Test scores at the beginning of this study had the tendency to use less number of blocks during the game.

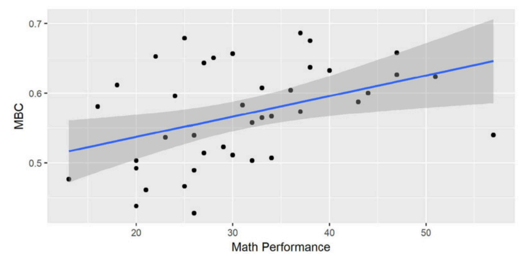

FIGURE 7 | Minimum Blocks Coefficient (MBC) by math performance (pre-test scores). We applied a linear model to data points with a 95% confidence level.

4. DISCUSSION

4.1. Impact of Manipulatives on Mathematical Learning

Our results indicate that the tangible manipulative group showed an advantage in mathematics scores after training compared to the control group. Our findings highlight the possibility of improving mathematical ability by practicing implicit number composition tasks assisted by tangible manipulatives.

We did not find significant differences either between the two types of manipulatives (virtual and tangible), or between virtual manipulatives and the control group when considering mathematical improvement tested by TEMA-3. It may be the case that virtual tangibles also have an impact in Post-Test scores, which was not observed due to the lack of statistical power of the present study.

4.2. Virtual and Tangible Manipulatives Led to Different Strategies in Number Composition

We analyzed children's behavior during our intervention to look for possible differential profiles in their evolution during training. Our tablet-based intervention allowed us to record the children's responses every time they submitted a block to compose a number. Our results enabled us to reflect on the role of specific actions performed by children affecting the learning process, and how learning could be influenced by the interactive properties of the blocks rendered as a representational assistance (Manches and O'Malley, 2016).

It was observed that the TI and VI groups significantly differed in the numbers of blocks used to compose a number. VI employed significantly fewer blocks compared with TI, showing that the different type of manipulatives could have led to different problem solving strategies. TI children opted to compose numbers using more varied combination of blocks, i.e., they used more number composition strategies. This suggests that the affordances of physical objects do trigger more diverse solutions (Manches and O'Malley, 2016), which have been advocated to prompt better learning experiences in numerosity knowledge (Alibali and Goldinmeadow, 1993; Chi et al., 1994; Siegler and Shipley, 1995) and specifically foster mastery of basic number combinations (Baroody and Tiilikainen, 2003; Sarama and Clements, 2009).

Our results are in accordance with Manches et al. (2010) results that found that children employed a significantly greater number of solutions when they used plastic blocks as manipulatives, comparing with a condition in which children were aided with a visual representation drawn on paper. For instance, it is easier to detect the "reversion" strategy (5-2, 2-5) when you can hold and displace objects representing these quantities (2 and 5). This finding supports the view that objects affordances implicitly carry information that could be relevant to reflect on abstract concepts, through conceptual metaphors. In our study, we compared tangible blocks (TI group) against virtual blocks (VI group). The use of virtual blocks allowed the children to drag, transform, and move blocks which allows a richer interaction compared to blocks drawn on paper. However, when compared to virtual blocks, tangible blocks enabled a more diverse combination of blocks to compose numbers as also observed elsewhere (Manches et al., 2010).

4.2.1. Strategies Evolution in Number Composition

When we analyzed strategies during training sessions we found that at the beginning of the training both groups employed more blocks to compose numbers with a tendency to diminish in the last sessions. This tendency to diminish may represent an approach to optimal performance (when the number is composed by the minimal quantity of possible blocks), probably reflecting learning toward increasing efficient and fastest strategies in number composition (Baroody and Dowker, 2003).

This is in line with the fact that composing and decomposing strategies becomes semiautomatic or automatic with effective and faster answers to basic number combinations. Children may automatize some combinations of a number through practice, resulting in an association with their counting knowledge. This association encourages efficiency, preventing children from repeatedly practicing all the possible combinations (Baroody, 2006). In our study, children at the beginning started by practicing various combinations of numbers. For instance, in the first sessions to form the number 5 children might use several combinations as 1+1+1+1+1, 2+2+1, 2+1+1+1, reflected by low MBC scores. Nevertheless, at the end of the training sessions children were able to answer more effectively, reflected by high MBC scores. For instance, to form the number 5 they answered with the block 5 or by adding just two blocks as 2+3 or 4+1, which is quicker and more direct.

Analyses showed that the mean of blocks used in the first three sessions was significantly smaller for the VI group, whereas both groups employed the same number of blocks in the last three sessions. This suggests that besides the tendency of both groups to optimize responses, they presented a different profile in their evolution during training. Children who used tangible manipulatives had the tendency to use more blocks and showed a more pronounced decrease in the number of blocks used during the intervention compared to children who used virtual manipulatives. This finding may be connected to the observed improvement in maths scores (measured by TEMA-3) for the TI group. The number of combinations used in the TI may have contributed to achieving mastery in mathematical knowledge, since mastery in basic number composition is enriched by experiencing more varied possibilities (Markman, 1978; Bowerman, 1982; Karmiloff-Smith, 1992). In this study, physical object affordances offered the user a richer set of action possibilities, and most probably also a more comprehensive understanding of the phenomenon explored.

4.2.2. Strategies in Additive Composition Task and Mathematical Improvement

We did not find a correlation between the number of blocks employed by children and mathematical improvement in general (all children analyzed together). Nevertheless, when children were divided according to their improvement in mathematics (Post-Test − Pre-Test) after the intervention, it was observed that the greater improvement group showed a positive correlation between number of blocks employed and gain in mathematical knowledge, which was not found for the Worse Improvers.

Therefore, children who showed a greater improvement tended to use more blocks. This outcome may suggest that an optimal performance in number composition (understood as fewer pieces used to form a number equals better performance) would not necessarily lead to a better learning experience. Another hypothesis would be that children who do not already have this mastery in number combinations, i.e., efficient, fast and accurate responses, would benefit more from employing manipulatives to solve additive composition and this might be the case for the "Better Improvers." Children who improved at maths during training were the ones using more varied block combinations. This is connected to the fact that the use of a greater variety of strategies can result in a better learning outcome (Markman, 1978; Bowerman, 1982; Karmiloff-Smith, 1992).

4.2.3. Strategies in Additive Composition Task and Mathematics

Interestingly, a negative correlation was found between mathematical scores at the Pre-Test (how good the children were at the beginning of the study) and the number of blocks employed. That is, being better at mathematics at Pre-Test implied the use of fewer manipulative blocks, probably due to a better knowledge of retrieval strategies while composing numbers (Rathmell, 1978; Steinberg, 1985; Kilpatrick et al., 2001). Children who were good at maths at the beginning of the training will not necessarily use more strategies because they already have a deeper knowledge in number concept and composition. That is to say, children who have already learned basic combinations of numbers have the ability to use such knowledge to answer quickly and efficiently in a familiar and unfamiliar learning context (Baroody, 2006).

It may seem contradictory that children who obtained the best scores at TEMA-3 (better at mathematics at baseline) used fewer blocks whereas the Better Improvers tended to employ more. However, according to Sarama and Clements (2009), despite seeming paradoxical, those who are better at solving problems with objects, fingers or counting are less likely to persist in these strategies in the future—as already reported by Siegler (1993)—but this is because they trust their answers and therefore move toward more precise strategies based on the retrieval of number combinations, leaving behind what once served as a scaffolding.

These results also suggest that children who will benefit more from the use of manipulative blocks are the children who do not have already mastery in number combinations. The use of enhanced manipulatives may be more suitable for younger children who need to practice and automatize simple number combinations.

4.3. Limitations

The present study has several limitations that should be considered when interpreting the results. It may lack statistical power since the number of participants in each group is small and for such reason, a larger confirmatory study is needed to strengthen the conclusions of the present study. The quasi-experimental design of the current study has more ecological validity (children were kept in their school groups), but it is susceptible to threats on internal validity compared to controlled experimental designs and for that reason we consider our results as exploratory and conclusions are drawn carefully.

4.4. Conclusions

Current findings indicate that the use of tangible manipulatives had a positive impact on mathematical learning. We were able to observe interesting relationships between the level of mathematics and the kind of manipulative strategies chosen by the children when solving number composition tasks. Our results suggest that tangible manipulatives increase action possibilities and may also contribute to a deeper understanding of core mathematical concepts. Playing the game BrUNO with tangible manipulatives promotes meaningful practice of more varied number combinations by encouraging children to focus on patterns and relationships in basic number combinations. In addition, we were able to observe how their responses pattern changed throughout the training leading to the use of less but efficient strategies in the last sessions which may reflect that they achieved mastery in doing such combinations. Thus, training in this basic combinations led to an improvement in mathematics and hopefully may lead children to effectively apply this knowledge in new and unfamiliar number combinations.

From an interaction design perspective (for more details regarding this research and perspective, see Marichal et al., 2017a), the most relevant observation is how the objects' affordances (i.e., the possibility to grasp physical objects or drag virtual ones) somehow shape and constrain users' strategies. In our study, tangible blocks meant a richer interaction, providing the opportunity to explore more number composition possibilities. This possibly led to an improvement in mathematical performance. Thus, depending on the learning task objective (context), we might take advantage of this phenomena, by choosing either tangible, virtual or mixed learning environments. The current study invites researchers to delve deeper in the exploration of the potential for designing interactive activities aimed at fostering learning of specific target content.

ETHICS STATEMENT

All children that participated in this research had the informed consent form signed by their parents or legal guardians. The intervention current protocol was approved by the Local Research Ethical Committee of the Faculty of Psychology, and is in accordance with the 2008 Declaration of Helsinki.

AUTHOR CONTRIBUTIONS

AP: substantial contributions to the conception or design of the work, analysis and interpretation of data for the work, drafting and revising it critically for important intellectual content. FG: drafting the work or revising it critically for important intellectual content, interpretation of data for the work. EB: substantial contributions to the conception and design of the work and data acquisition. BF: drafting the work or revising it critically for important intellectual content, analysis, and interpretation of data for the work. GS: substantial contributions to the design of the work. SM: substantial contributions to the design of the work, drafting and revising it critically for important intellectual content.

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5.Designing TUIs for children with VIs

Along this chapter we describe the design, development and evaluation of two embodied interactive systems for children with VIs in a mathematics learning context. We provide design knowledge and opportunities for further research in the context of VIs.

Section 5.1 corresponds to a short conference demo paper describing iCETA, and adaptation of CETA for children with VI. The paper provides a set of design adaptations for children with VIs in mathematics learning context. As a result of several participatory design sessions with children with VI and educators, iCETA explores a mixed-reality solution with passive blocks, like CETA, but incorporating a richer auditory channel exploitation.

In Section 5.2 we describe the main design drawbacks of iCETA and argue the value that a tangible system with active feedback in the blocks might signify.

Lastly, Section 5.3 and 5.4 corresponds to a short conference demo paper and journal article respectively describing the design, development and evaluation of LETSMath.

5.1.A tangible Math Game for Visually Impaired Children

The content of this section was published in the proceedings of the 21st International ACM SIGACCESS Conference on Computers and Accessibility (ASSETS ’19).

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A Tangible Math Game for Visually Impaired Children

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ABSTRACT
We present iCETA, an inclusive interactive system for math learning, that enables children to autonomously engage and solve additive composition tasks. It was designed through a set of participatory sessions with visually impaired children and their educators, and supports math learning through the combination of tangible interaction with haptic and auditory feedback. Tangible blocks representing numbers 1 to 5 were used to add or subtract and correctly solve the task embedded in a computerized game. Our approach aims to provide better scaffolding for understanding the abstract concept of a number by working with different representations of that number, as size of a block, Braille, color and audio feedback.

ACM Classification Keywords
H.1 Information Systems: Models and principles: User/ Machine Systems: Software psychology

Author Keywords
Tangibles; Multimodal; Visually impaired; Cognitive training.

INTRODUCTION
One common way to introduce mathematical concepts is the use of manipulatives as external representations, as an additional resource of information to focus on the underlying concepts. Embodied, constructivist and constructionism theories shed light on the importance to manipulate and operate concrete material, using the body to deepen abstract conceptualizations [10, 8, 7]. The use of external representations as manipulatives decreases cognitive load, allowing the children to focus on the understanding of the abstract concept, reinforces its understanding, and increases the effective capacity of working memory by the stimulation of several sensory modalities simultaneously [1]. Also, distributing pieces of mental operations into actions on physical or digital objects may simplify and help gain deeper knowledge [1].

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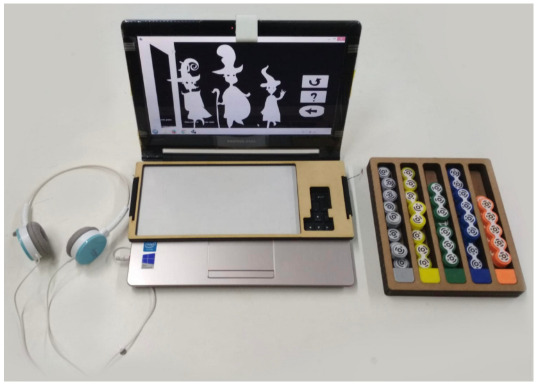

Figure 1. iCETA: headphones, computer, mirror in the camera, tangible blocks and working area on top of the keyboard.

There has been limited research dedicated to help visually impaired (VI) users to incorporate mathematical concepts taking advantage of tangible manipulatives and multimodal systems. Jafri et al. propose several educational activities based on the distribution of three-dimensional geometric figures that are analyzed by the computer that provides immediate feedback through sound codes [4]. Manshad et al. propose another system based on interactive multimodal cubes for object orientation, and on an interactive table that provides auditory feedback about the current state of the system, which helps guiding a blind user in the spatial task at hand [5].

Building on these ideas, and on an identified need of tools to promote math learning, we set out to develop an auditory computerized game to be played with tangible blocks. These efforts build on our previous work in developing a visual multimodal tangible system, CETA.[6]. In that system, an audio-visual game was displayed in the tablet and children had to solve the additive composition tasks by manipulating the tangible blocks. We observed that the use of tangible blocks had a positive impact in their mathematical ability after two weeks of training [9]. Hence, the adaptation of CETA could be very suitable for fostering mathematical abilities of VI children while playing with tangible objects that represent numbers, if adequate adaptations were made. In this paper, we present iCETA and detail the main design changes performed to make the approach to be inclusive.

DESIGN PROCESS

The iterate design of iCETA involved the participation of 11 VI children (6 legally blind) aged between 5 and 10 years old from special education schools in Montevideo (Uruguay), as well as other relevant stakeholders (2 school directors, 3 elementary teachers, 1 music teacher and 1 IT teacher). In participatory design sessions, children and stakeholders helped to define and create proper training tasks, interface elements and interaction modalities, exploiting the possibility of the interaction with objects, multisensory experiences, narrative and the potential of digital tools. Our design process began with semi-structured interviews to stakeholders in order to identify the objectives and needs, materials and tasks for math learning in the school. Findings informed the design of early prototypes that were iteratively discussed and improved. Each session with prototypes was the input for a new iteration in the design of the interface, blocks, working area, music and sound, and narrative of the game, following an iterative process of development-feedback-development.

ICETA

Inclusive CETA (iCETA) is the adaptation of CETA, a mixed-reality, open source, low-cost and portable system created to be used in school settings [6]. iCETA is a system that consists of a set of blocks detected by the camera (see Figure 1) and the camera is redirected towards the working area using a mirror. The blocks are recognized through TopCode markers [3]) and the computer provides auditory feedback.

Blocks

We were inspired by the cuisenaire rods [2] so largely used with young children in schools. Blocks represent values from 1 to 5. They vary in size (e.g., "2" is twice the size of "1"), texture, Braille and colors. Blocks have tactile division marks to split the units, and each unit has the shape of a circle. The TopCode markers were also used inside each circle to reinforce the recognition of each unit. In our final design sessions, children easily identified that the blocks were sums of units.

Working area and storage box

Besides a working area, we created a box where each of the blocks can be organized by its number (see Figure 2). This allowed the children to have the blocks organized ready to be used, which is likely to reduce cognitive load and enable more efficient manipulation of the blocks.

Learning TUI Objectives

We identified two levels of learning objectives: a) multimodal reinforcement - auditory and haptic feedback, and b) math training tasks of additive composition and decomposition of numbers from 1 to 10. All the tasks were designed to first display a specific sound that repeats *n* times and the children have to put the blocks that together compose such number.

Figure 2. Blocks representing values from 1 to 5, similar to cuisenaire rods. Blocks varies in size, texture, braille and colors.

Music and sounds

Three sound parameters were tested: tempo (temporal sequence between 1 and 2, for instance), pitch and timbre. Tempo is personalized because children exhibit differences to distinguish between the sound of the end of the block and the start of the sound of a new block. We created different timbres and pitches related to the action required by the character of the game (steps, knocking door, stir the magic potion). Binaural sound was used to differentiate the sound of the recognition of the blocks from the one of the number required by the game.

Narrative and gamification

We created a new game, "Logarin", named after the main character: a young magician that does everything wrong and needs help to achieve the objective of becoming a great magician. To do that, children must help him by performing spells, or organizing a music band, etc. The creation of levels according to the narrative allowed children with high and low performance in mathematics to have fun and be challenged. We also took into account gamification elements as the main character, microworlds, obstacles and levels.

OUTLOOK

iCETA has been piloted in schools and was welcomed by both children and educators. It provides a playful and rich multi-sensorial environment for children with different visual abilities to learn math. One of our future goals is to expand the amount of math concepts to be conveyed by the game. We are also exploring the usage of intelligent objects with built-in electronics that can provide more diverse feedback (speech, sound, vibration and force), to support novel ways of interaction and learning. In addition, we intend to apply this approach to other learning domains as computational thinking.

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5.2. Channels matters: From mixed-reality to tangibles

In iCETA we included two external number representations associated with each individual block: The shape of the block and the braille sign. Both require physical scanning and were implemented through passive non-augmented objects. However, it has been pointed out that processing information distributed across different perceptual channels (modalities) might be cognitively more efficient [5]. Thus, for LETSMath, we decided to incorporate active objects which provide individual vibrotactile haptic feedback as well as auditory feedback. This makes a more extensive exploitation of sensory channels in a VI context providing interaction alternatives.

In CETA we have modulated the level of abstraction by changing the mapping between the physical blocks and the digital objects on the screen, starting from simple one to one, object to object mapping until a many to one, object to action mapping (see section CETA). However, this strategy mainly relies on the visual channel since it is given by graphic representations and movements. Thus, in LETSMath, we explore different vibro tactile haptic feedback covering different abstraction levels. In addition, also the translation between sensory channels might lead to abstract conceptualization [2].

Lastly, the incorporation of sensors in the blocks gives us also the opportunity to change the detection system. CETA and iCETA employed computer vision strategies in order to detect the blocks. However, in iCETA, we observed that the extensive use of physical scanning causes occlusion problems, i.e., VI children cover the blocks more often causing occlusions. As a consequence, the camera does not detect the blocks provoking malfunctioning. This drawback was also noticed by Ducasse [14], who discourages the use of tabletop with the camera above for VI users. Thus, in LETSMath, we designed a TUI embedding sensors inside the blocks and enabling wireless communication between them and also with the computer where the game is running.

5.3. LETSMath: Learning Environment for Tangible Smart Mathematics

The content of this section was published in the Proceedings of the 20th International Conference on Human-Computer Interaction with Mobile Devices and Services Adjunct(MobileHCI '18).

Sebastián Marichal, Andrea Rosales, Gustavo Sansone, Ana Cristina Pires, Ewelina Bakala, Fernando Gonzalez Perilli, Bruno Fleischer, and Josep Blat. 2018. LETSmath. In Proceedings of the 20th International Conference on Human-Computer Interaction with Mobile Devices and Services Adjunct(MobileHCI '18). Association for Computing Machinery, New York, NY, USA, 313–320. DOI:https://doi.org/10.1145/3236112.3236157

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Abstract
Visual information can be decoded very fast, letting us perceive and process a large amount of data in parallel. There is a lot of knowledge organized as guidelines and recommendations for GUI design. However, for blind people that perceive the world through auditory and haptic channels, GUIs might not fit their needs. In this paper we present a prototype of LETSMath (Learning Environment for Tangible Smart Mathematics), a tangible system for mathematics learning for blind children. LETSMath consists of tangibles blocks with tactile and auditory feedback, a working space, and a tablet-mediated audio game.

Author Keywords
Interaction design; Technology enhanced learning; Embodied interaction; Tangible Interaction; Visual Impairments

ACM Classification Keywords
H.5.2 [Information interfaces and presentation]: User interfaces

Introduction
When interacting with a system through a Graphical User Interface (GUI) users see the screen. Icons, windows, colors, characters, pictures and other digital elements represent the information mediated by the system. With a quick view users can perceive the state of the system and which

(inter)actions are possible. Visual information can be decoded very fast, letting us perceive and process a large amount of data in parallel. There is a lot of knowledge organized as guidelines and recommendations for GUI design. However, for blind people that perceive the world through auditory and haptic channels, classical GUIs might not fit their needs. The aim of the present research is to exploit opportunities provided by smart objects to create richer experiences in learning, and more specific, in the number acquisition in/of blind children/in the context of blindness. This requires a design that considers knowledge about user experience, cognitive development, and multimodal integration for blind people.

We present the initial prototype of LETSMath, a tangible system designed to exploit the haptic and auditive channels in a mathematics learning context. In this paper we discuss the initial design and prototype as well as we explain the next steps of this ongoing project.

Related Work

Many authors investigate the benefits of physical interactive learning systems. Price [14, 15] states that physical world augmented through digital information can lead to more awareness, exploration, collaboration and reflection in learning activities, and so, offers better support for active and playful learning. Indeed, Rosales et al. [19] showed that school-aged children could effectively incorporate a movement-to-sound interaction accessory into their free-play, which encouraged creative and diverse free-play. Rogers [17, 18] investigates the relationships that can be established between physical and digital actions. She states that combining familiar physical actions with unfamiliar digital effects promotes reflection and creativity.

Some authors highlight the potential of tangible interaction to support numerical development of mathematic skills. Actually, manipulatives have been used in the context of mathematics learning for a long time, for instance the cuisenaire rods [3] consisting of wooden rods of different colors for each length, representing numbers from 1 to 10. There also exists some virtual (traditional GUIs) [13, 1, 2] and tangible [20, 12] systems augmenting the cuisenaire rods. However, non of these approaches is adapted for blind children.

Manches et al. [7, 8] argue that digitally augmented physical objects present unique opportunities for learning purposes for children in the early years. Zuckerman et al. [23] present technology enhanced building blocks that enable children to physically explore abstract mathematical concepts like counting and probability. They promote hands-on modeling as an engaging strategy for learning. Graphmaster [21] aims at helping children to develop intuitions about graph theory before they get to know the underlying mathematical concepts and formal notation. This tangible graph construction kit poses connectors, illuminated edges, and capacitive sensing that allow children to create physical graphs and interact with graph theory concepts in a tangible way. Smart Blocks [4] are three-dimensional cubes to explore the relation between different configurations of the units and their surface areas and volumes.

The feedback provided by many tangible systems is visual, such as values rendered on the GUI in Smart Blocks or light messages in graph edges in Graphmaster, which is unsuitable for blind children. But there are also research projects focused on helping blind users to incorporate mathematical concepts that take into account advantages of tangible manipulatives and interactive systems. Jafri [5] proposes various learning activities based on spatial distribution of three-dimensional geometrical figures that are further analyzed by the computer to provide real-time audio feed-

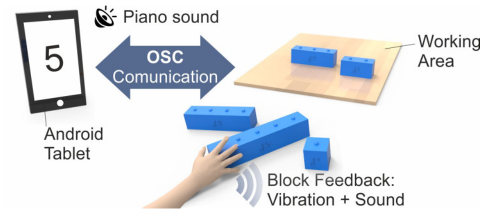

Figure 1: LETSMath environment.

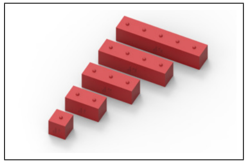

Figure 2: Blocks ranged in length including braille representation of the number and medium sized dots on top representing units.

back regarding discovered shapes and their spatial relationships. Manshad et al. [9] propose a system based on MICOO (multimodal interactive cubes for object orientation) and an interactive table, where diagrams and graphs can be created and modified. It also provides audio feedback on the current state of the system which guides the user. In subsequent work Manshed [10] extends the proposed system with new hardware components to provide more diverse feedback (speech, sound/music, vibration and force feedback), supporting new types of actions of the units like stack, roll, or connect and enabling collaborative and distance learning.

System Description

The system consists of a set of blocks technologically augmented, a low cost Android tablet and a surface used to delimit the working area (see figure 1). Both the blocks and the tablet give feedback to the user. Children have to put the blocks on the working area composing numbers. When the blocks are placed on the working area they communicate with the tablet, which provides feedback according to the composition. Next, we detail the design of LETSMath including physical blocks, system feedback, working area as well as a preliminary game design.

Blocks

The tangible blocks represent the numbers 1 to 5 according to their length (see figure 2), following the same strategy of [12] based on cuisenaire rods [3]. The block 1 is actually a cube whose side is 4.2 cm, and the block 5 is five times longer, reaching 21 cm. Each block has two additional representations of the number: as many 3D circular marker points as units - to allow children to touch units as physical elements and count, an epistemic action that seems to offload cognition [12] -, and the braille sign of the number - which is an abstract representation of the quantities and let children perceive the number simultaneously rather than following the serial counting strategy of touching unit by unit, that perhaps could limit the development of more complex computation strategies [6].

Feedback There are two kind of feedback, the one provided by each individual block and the one provided by the tablet.

The blocks provide feedback only when they are being touched. Synchronized beep sounds and vibration match the cardinality of the block, i.e., when the user touches the block 2 vibrates and beeps twice, this feedback is repeated in a loop until the block is released (see figure 3).

The tablet provides auditive feedback as notes matching the composition of the blocks placed on the working area. For instance, if the user places blocks 3 and 1 on it, the tablet plays four consecutive piano notes followed by a silence. Each block has a different musical note associated (see figure 4). This provides an identity to the number represented by the block with the aim to let users perceive numbers beyond the serial counting strategy, that in long term could be an limitation to think in abstract concepts like quantities and operations [6].

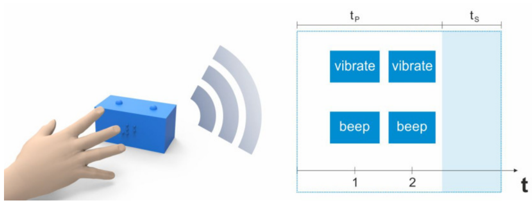

Figure 3: Block 2 giving feedback in the form of sound + vibration

Figure 4: The tablet gives auditive feedback according to the blocks composition

Both feedbacks from tablet and block are provided as rhythmic cycles playing consecutive notes during t_p time followed by a silence lasting t_s (see figures 3 and 4). When a block is on the working area and being touched at the same time, both feedbacks will be played simultaneously, from the block itself and from the tablet.

Working Area It is a special surface where the children perform the compositions by placing blocks on it. When the block is on the working area, this gets activated and the tablet receives a notification of the event.

Implementation

Interactive blocks Blocks are simple 3D printed blocks with a small circuit in them. Each circuit contains an ESP-07 module, a small board that works as a standalone micro-controller with Wifi connectivity and 9 input/ouput pins that can be used to add other electronic components.

Block feedback We included a speaker able to produce different sounds depending on the supplied current and a mini vibrator motor to produce the haptic feedback of the block.

Sensing capabilities We included a reed switch and a touch sensor. The reed switch is an on/off switch that is activated through a magnetic field. Placing one of these switches at the bottom of each block makes detecting presence/absence of a magnetic field possible. By putting a magnetic surface as the working area, when the block is placed on it, the reed switch is immediately activated.
We used a touch sensor module connected to a conductive textile wrapping the block in order to detect user's touch all over the block(see Figure 5). This way, when the user touches the block the system can detect it and give feedback.

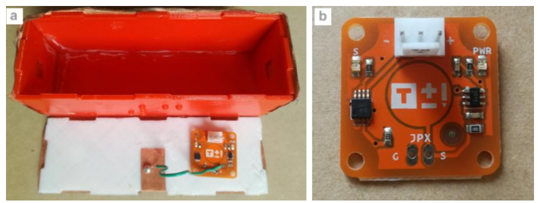

Figure 5: a) Touch sensor connected to the conductive textile on the top side of the block (disassembled in this picture) b) Tinkerkit touch sensor module.

Communication The tablet and the blocks are connected to the same wireless network and we use the Open Sound

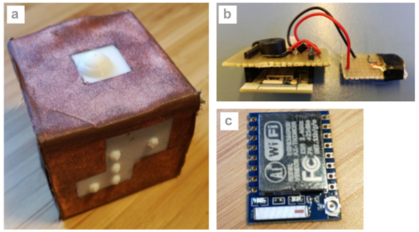

Figure 6: a) Conductive textile wrapping block 1. b) Block circuit. c) ESP-07 microcontroller and wifi module

Control (OSC) protocol [22] to exchange messages. In this first prototype the only communication is when blocks are placed on the working area: each sends a message to the tablet which computes their composition and gives feedback. The next version of the system will incorporate a 3-way communication: among blocks, from blocks to tablet and vice versa (see future work section).

Composition Game The system offer a simple composition game where additive composition is the main trained skill. In this game there are two modalities, exploration and challenge. In the exploration mode the tablet only provides the auditive feedback according to the blocks placed on the working area. This mode is useful as an introduction to the system, letting users explore and understand the interaction, there are no right or wrong answers. In the challenge mode the tablet says aloud a random number and the child has to compose it by placing blocks on the working area. If the solution is correct, a positive feedback is played and another number is said, continuing the challenge. Figure 7 shows an user playing this game during an early prototype evaluation.

Figure 7: User playing the composition game during an early evaluation.

Future work

This is an ongoing work and we are planning to include the improvements next described for the demo session:

Join detection Cubes will detect when they are joined each other. This will make available the joining action in the system, giving place to design richer interaction. For instance, we could constraint compositions to happen only when the blocks are joined rather than when the blocks are just placed on the working area. Using magnets to attract blocks as we previously did in [12, 11] we also provide a physical affordance for the joining action. In order to detect this action we could use conductive contacts that close a circuit when blocks are joined [16] or infrared communication. This functionality requires a communication between blocks.

Synchronized feedback During the first preliminary user test we observed that it is quite confusing when the tablet and the blocks provide feedback at the same time. Thus, this behavior needs to be changed avoiding the simultaneous feedback. Furthermore, blocks' feedback will be configurable being able to choose individual or group feedback. The individual feedback was already explained, when the user touch the block it plays a cycle of sound and vibration matching its cardinality. The new modality will enable group feedback for the composed blocks. For instance, if the block 1 is joined with block 3 and the user touches the composition, then both block will give feedback simultaneously according to the composed number, in this case both blocks will play a cycle of 4 beeps and vibrations at the same time, behaving as a block 4. The tablet will be in charge of synchronizing blocks sending OSC messages indicating the starting time of the feedback and which number has to be represented (amount of consecutive beeps and vibrations).

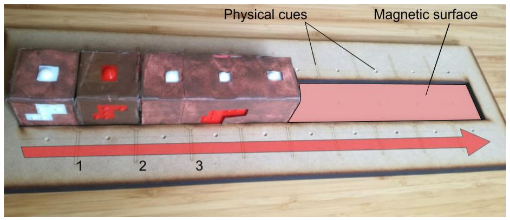

Figure 8: Prototype of a wooden number line working area.

Number line Figure 8 shows the prototype of a wooden number line where blocks can be placed representing number composition and training the number line concept. The existence of a physical line is a constraint of the physical space from which blind children might benefit at the time of searching the blocks. It is also a strategy to encourage the composition of blocks along the line, matching the result with the number of the line. In short, the wooden number line is a physically constrained working area.

Technology miniaturization All the sensors and the main board will be integrated in a customized printed circuit board. This will allow as to reduce the size of the blocks gaining portability and enabling children to pick up and hold multiple blocks.

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5.4. How auditory and haptic feedback contribute to developing basic mathematical skills for children with visual impairments

The content of this section contains a journal article to be published:

Marichal, S., González Perilli, F., Rosales, A., Pires, A. C., & Blat, J. (2020). How auditory and haptic feedback contribute to developing basic mathematical skills for children with visual impairments. To be submitted

González Perilli, F., Rosales, A. — Equal contribution.

How auditory and haptic feedback contribute to developing basic mathematical skills for children with visual impairments

1. Introduction

Traditional manipulatives such as (physical) counting rods or tiles exploit the haptic channel allowing users to scan, grasp and count elements (dots for instance). These physical objects support embodied cognition [51,52], allow the execution of epistemic actions [53] - which we later discuss in 2 - and the creation of conceptual metaphors [50] during the processes that are common in the acquisition of mathematical skills, e.g., with the famous Cuisenaire Rods [68]. Users typically complement haptics with vision, with a quick view users can perceive the state of the system and which (inter)actions are possible. Visual information can be decoded very fast, letting us perceive and process a large amount of data in parallel [12, 39]. However, for visually impaired (VI) children, classical manipulatives are limited. Missing information should be compensated to reduce the intrinsic handicap of their visual condition and provide opportunities of embodied cognition similar to children without visual impairment (VI). Indeed, complementing haptic feedback with auditory feedback provides an alternative strategy to scan, grasp or count different elements in one shot, rather than having to go through sequential physical scanning one at a time.

Following this vein, the research we present has a double objective. First, building on cognitive theories, we propose a concrete model [18] of augmented manipulatives for mathematical learning for children with VIs. Second, we validate the model through an iterative prototyping and testing approach that included three phases, to understand how to design tangible material with auditory and haptic feedback in order to enhance mathematics concepts acquisition for children with VI. Particularly, we explore different feedback modalities to provide simultaneous number perception to children with VI.

We conducted three user tests during different stages of the design, which followed a user centered design approach. In total, 19 children from 6 to 12 years old and their teachers, participated in the studies. All children were VI and in the process of developing basic mathematical skills. Different participants were involved in the three user studies with the exception of one that took part in the first and second ones. We recruited them through special educational programs for children with VI in their own cities. Their degrees of VI could go from *low vision* to *blind*. Many of them also presented cognitive disabilities, generically labeled as *Pervasive developmental disorder* (PDD). Most of them showed delays in the accomplishment of mathematical skills compared to sighted children, mainly as a side effect of their VI condition. For instance, counting skills (or cardinality skills) were not fully established in most of the children. All the tests were conducted in the facilities used for their own programs.

The first test was a preliminary exploration on an early prototype, and followed a wizard of oz approach, involving six children and interviews with their teachers. After re-designing the prototype taking into consideration the outcomes of the first study, we conducted a second test with six children including one that had participated in the previous one, and interviewed the school director. During this study children completed three tasks (find a block, composition and broken blocks game) using the tangible blocks with three different feedback setups: a) *no feedback*, b) *fast vibration* and c) *sound + vibration*. Finally, in a third instance, we incorporated Logarin, a narrative video game that challenges children to solve problems by composing numbers on a wooden number line using the tangible blocks. Eight children took part of this final evaluation and educators from the center. At least two observers followed each user test, and took note of the observations following a semi structured questionnaire.

We state three research questions aiming to evaluate the comprehension, incorporation and impact of LETSMath in the problem solving process:

RQ1: Comprehension, do children understand the blocks' concrete model? This includes the physical static properties of the blocks such as shape and size, the digital feedback they provide and the informational relations (mapping).

RQ2: Incorporation, is the proposed auditory and haptic feedback incorporated during: **a)** the blocks recognition process? **b)** the composition task?

RQ3: Impact, Which is the impact of the number line working area in terms of children's strategies and error rate?

With respect to (Q1), our results show that all the participants understood the concrete model, they were able to train number composition skills in a potentially unsupervised composition task. Children understood the mapping between the physical form of the block and the digital feedback associated. As for the incorporation of digital feedback for recognition tasks (Q2), the results suggest that both low vision and blind children incorporated the digital feedback. However, only blind ones incorporated the digital feedback into the composition task. Users complemented their different levels of limited vision with physical scanning, spatial memory and digital feedback when needed. Lastly, regarding the number line working area (Q3), the results suggest that composing directly on the number line might provoke more errors, probably due to a lack of reflection in the problem solving process.

This is a substantial contribution for the design knowledge for the digital/computationally augmented [44] manipulatives for children with VI. The insights gained and reported come from combining a theory grounded design with user studies and validations. Our prototype is, to the best of our knowledge, the first interactive system with tangible objects incorporating active feedback specifically designed for number composition training for children with VIs. The results for these very special users should be useful to the research community in terms of providing a more precise understanding of the integration of different modalities, and their design affordances, submitted to the challenging task of learning early mathematical skills.

The rest of this article is organized as follows: Section 2 presents some background concepts about interaction and cognitive processes, and on special needs as users like children with VIs. Section 3 discusses an overview of related work done. In Section 4 we present our design rationale. In Section 5,6 and 7 we explain the methodology used to validate the model and the results of each user study. In Section 8 we discuss findings and implications for the improvement of the system. In Section 9 we discuss future directions and Section 10 concludes the article.

2. Background

Several concepts play a key role in the paper and are presented in this section. Most of them relate to understanding better the special needs as users of VI people, i.e., how interaction takes place in case of VI, while other ones are concepts related to interaction and cognitive processes, which play a key role in the paper.

Visual Impairments (VI) classification

The *International Classification of Diseases* discriminates between moderate and severe VI and blindness [105]. A person is considered blind when her visual acuity is worse than 3/60 or has a visual field no greater than 10% [105]. When the visual acuity is worse than 6/18 visual impairment is classified as moderate, and when it is worse than 3/18 and greater or equal than 3/60, it is severe [105] . Both children that qualify as blind and children with low-vision that have moderate or severe VI took part in our study.

Objects as interfaces for people with VI

Everyday objects can be understood as interfaces that allow the communication between the user and a concept [80]. The interface can afford this dialog through different senses. For instance, a simple glimpse allow us to evaluate if we can grasp, push or throw an object [3].

When interacting with a system through a Graphical User Interface (GUI) users rely on vision. Virtual objects as icons, windows, characters, pictures and other digital elements represent the information mediated by the system. Visual information can be decoded very fast, letting users perceive and process a large amount of data in parallel. Consequently, after a quick view users can understand the system state and which (inter)actions are available. There is a significant amount of HCI knowledge organized as guidelines and recommendations for GUI design. However, classical GUIs are not suitable for people with VI, as they perceive the world through auditory and haptic channels.

Audition is the main source of information for people with VI when interacting with information technology (IT) devices, such as mobile phones and computers, typically provided by translating GUI elements into words.

However, auditory information is less effective when spatial information has to be conveyed. Haptic interaction is a natural strategy to compensate for this issue, and has been exploited in *Tangible User Interfaces* (TUIs), which are everyday physical objects and environments augmented with digital information which become interaction devices [25]. The term is often used for other objects beyond the mouse, the keyboard or the (touch) screen. Some examples which are relevant for this paper include MapSense [89] or Torino [88].

Sound can also be incorporated as non-speech audio [21] in order to externally represent abstract concepts. For instance, the cardinality of a set can be represented as a group of sounds, in an analogy to a group of dots. Indeed, Leuders [12] argues that designing materials that use rapid sequences of sounds as representations of quantities, could be an alternative for VI people, beyond counting slower sequences of beats. Thus, auditory feedback can be much more complex than just translating a GUI into words.

Development of counting skills and number concept acquisition

Number concept acquisition benefits from perceptual processes, in the sense that it can be regarded as the passage of a perceptual construal (I can see three things in front of me) towards an abstract one (3 = * * *) [8].

In this process, it is key to perceive the whole group and its components at the same time. In fact, prior to the acquisition of the number concept (cardinality), children can distinguish among groups of different quantities, despite not knowing the exact amount of components. This ability is supported by the *approximate number system* (ANS) [8], an innate analogical system that is present in newborns [7].

Children start understanding that sets have different magnitudes. Later on they learn that each magnitude relates to an exact quantity, and finally they learn that these quantities can be represented with symbolic expressions (numbers) [9].

In the case of children with VIs, the simultaneous perception of the whole and its parts is very difficult, delaying mastery of cardinality [90]. In order to assist children with VIs in this process it is important to convey the quantity of a set instantly. Theories linking physical interaction and abstract cognitive processes are of special interest for this purpose and inspire our design approach.

Physical interaction and cognitive processes

Abstract information, like number, is usually represented by spatial structures which are typically perceived through vision. In the case of children with VI these structures are usually detected by touch [97]. Children with VI develop strategies for active touch to successfully perform counting [55]. They usually start by a preliminary scanning (analog to a glimpse); next, they search for perceptual keys for counting (e.g. detecting dots), and, finally, they usually partitionate space by setting aside already checked elements [55].
That is, haptics can be a source of information that can contribute to the instantiation of abstracts models. Indeed, there is a substantial amount of research about the role of physical interaction and body movements in high level cognitive processes (embodiment theories) [26,27,28]. These views support the development of tangibles for learning [1,29,30,31,32,33,34] and more specifically tangibles for mathematics learning [35,36,37,38].

Physically distributed learning theory

The application of embodiment theories in the field of interaction design for educational purposes has been summarized by Martin and Schwartz [31] through their *physically distributed learning* (PDL) theory.

PDL [31] stresses the importance of allowing children to rearrange the environment in order to represent the solution of a certain problem. For instance, dividing by two might be performed as splitting a single group of objects in two parts of the same size, understanding that two different subgroups created by the proximity of their elements do appear, and that the result of the division is exposed by the cardinality of each subgroup. Reinterpreting the environment allows children to reveal the abstract structure of the underlying operation.

It has been claimed that these spatial rearrangement operations give place to *conceptual metaphors* [50], i.e., analogies that enable the understanding of abstract concepts in terms of more familiar and well known concrete concepts. For instance, a typical spatial representation reflecting number knowledge is the *number line* standing for ordinality and making explicit relationships among cardinals [91], which we discuss later in the context of the experiments.

Epistemic and Pragmatic Actions

When solving a problem, a set of actions is performed in order to take the agent closer to her physical goal, which are called *pragmatic actions* [53]. However, there exists another kind of actions - *epistemic actions* - which are performed to reveal information that might be partially hidden or hard to detect [53], but are not necessarily part of the solution [29]. For instance, a tetris player might move a piece to the left of the screen and then back to the right, leaving the system in the same state: she might have spent some time doing so, but during this process the player might have learnt or compute something that makes it worth [53]. Kirsh and Maglio explain that the primary function of epistemic actions is to improve cognition by reducing the memory, number of steps and probability of error in mental computation [53]. Thus, in the context of manipulatives for mathematics learning, we are interested in detecting/observing/encouraging this kind of actions, which might allow children to save cognitive resources and discover more or better strategies to solve a problem [35].

Concrete Models

Mix refers to manipulatives as concrete models [18]. Her definition grasps the idea that, besides material features (as shape and size), it is important for manipulatives to be included within a model involving interconnected knowledge of physical objects, actions performed on them and symbolic representations [31]. Children's direct experience with objects is key here, and some authors have argued that this is what is lacking when children first face symbols [18].

Designing a learning experience might imply the elaboration of a concrete model, by proposing a certain placement of objects and some rules about how they interact and respond. The aim of the model is to reflect in a directly perceivable way the abstract relationships to be learned. For instance, when the user joins several blocks, the system interprets this action as a composition. Thus, concrete models provide relevant opportunities to reveal abstract relationships.

How children with VI count with manipulatives

Children with VIs exploit spatial structures for the development of **active touch counting strategies** [55]. For example, it is important for them to know which elements they are able to count and where they are, so that they do not count the same element twice. These strategies should be taken into account for systems to support the development of mathematical knowledge for children with VIs. More specifically, they should develop their strategies in the following dimensions [55],:

- *Preliminary scanning* – Children learn to deliberately scan the structure before counting.
- *Count organizing* – Children learn to follow given structures like dot lines or circles.
- *Partitioning* – Children develop strategies to keep track of elements already counted by moving them aside or by using one hand to indicate a partition.

Thus, educational materials should foster the possibility of arranging the space in order to facilitate preliminary scanning, of giving salient keys for counting, and provide a spatial structure which favors counting activity. Moreover, these concepts are especially relevant to frame the presentation of physical models for abstract ideas, and this is crucial for children with VIs, for whom abstracts concepts as cardinality are hard to conceptualize. Haptic interaction is a very valuable approach to overtake this restriction, but auditory feedback is also needed to design a richer informational model.

Auditory assistance in manipulatives when dealing with VI

When acquiring cardinality, the passage from a concrete conception to an abstract one is more difficult for blind children, since they do not generally perceive objects in a simultaneous manner but in a serial one (when scanning the environment they have to touch them one by one) [12]. In addition, this sensory system is slower than vision, requiring more working memory and cognitive load [18].

Thus, when thinking in a way to foster blind children capabilities to acquire the number concept, we may focus on how to enable the simultaneous perception of a group of objects, e.g., giving keys to immediately understand the cardinality of a group even if visual perception of all components is not possible. Specifically, the task of additive composition has been claimed to favour cardinality acquisition [92, 93]. It implies number recognition, grouping and awareness of results. Manipulatives have been proven to be a valuable assistance for this activity [98]. However, it is much harder to perform for children with VIs.

Auditory information may be crucial [12] as a complement, although auditory feedback cannot convey as much information as does the visual channel and it is more sequential [39,58]. This is one of the reasons why we propose an interactive system that combines tactile and auditory feedback, reinforcing the complementary perceptual channels when vision is not available. Moreover, fast auditory patterns can be perceived as a unique event but at the same time allowing rapid counting of components [87]. Thus, leading to an experience that can be closer to the simultaneous sight of a group of objects.

Tangible Learning Design Framework

Physical objects, digital objects, actions, informational relations and learning activities are the elements of the taxonomy proposed within the Tangible Learning Design Framework (TLDF) [29]. This framework provides a structure to design TUI for learning purposes aiming. Aiming to generate compatible knowledge with previous research [4, 29, 38], we apply the TLDF along this article to describe the design of our system.

Reflection in tangible interaction environments

Cognitive growth as an ongoing dance

Tangible interaction environments represent an opportunity to use the body for active exploration, and to engage multiple senses in a constructive process [29,30,34]. They also enable the possibility to materialise learning theories such as PDL [31] into the domain of interactive systems. However, exploration and engagement of different sense does not always shape a reflective activity or ensure reflective thinking, which is important in the meaning construction process [106,107].

Ackerman [100] proposes a model of cognitive growth combining exploration and reflection stages in an "ongoing dance". She argues the necessity to dive-in and step-out, which are important factors in the learning process. Moreover, she explains that separateness as a consequence of momentary withdrawal does not necessarily represents disengagement [100]. Thus, TUIs and embodied interaction environments in general afford easy and fast exploration, but they should also encourage reflection to balance the "ongoing dance" in the learning context.

However, previous research has pointed to the lack of methodologies to understand the relationship between body actions and real-time meaning making [101], as well as the importance of mechanisms to promote reflection into embodied interactive learning experiences [1,30,35, 38].

Feedback and interaction pace modulation

In interactive systems, feedback can be an important behaviour regulator. Two kinds of feedback are considered in TUI. On one hand, *process feedback* refers to the real time coupling between user actions and system augmentations, i.e., users get a continuous feedback about their actions [102,108]. On the other hand, *task feedback* is about the correctness of the solution of a proposed task, e.g., the evaluation of a number composition.

Previous research suggests that providing continuous feedback at the process level encouraged students to dive into action and proceed with trial and error strategies [102, 38]. Thus, it might result into more intuitive interaction or that solutions become easier to find, but this does not mean that reflective thinking is taking place. Actually, students who did not have immediate process level feedback reflected more and reached a higher learning gain [102]. This remark is not intended to generate a debate about feedback, but to stress the fact that different feedbacks and delays might modulate the interaction pace so that reflection is more or less stimulated.

Several studies suggest that slowing down the interaction pace might trigger reflection [29, 35, 38, 101]. In a similar vein, also physical disposition of inputs and outputs might cause delays favouring reflection or not [30].

3. Related Work

In this section we introduce and discuss some relevant previous work related with manipulatives for mathematics learning, both traditional and augmented ones. Then, we present related work in TUIs specifically for children with VI. Finally, we discuss research on TUIs for mathematics learning for children with VI.

Manipulatives for Mathematics Learning

For many years mathematical manipulatives have been included in schools, both for sighted as well as for children with VI. Building blocks and tiles, such as the cuisenaire rods or the abacus have been used to instruct mathematics in early stages [94]. This kind of learning materials permits the physical exploration but rely on visual abilities for an optimal comprehension of the metaphors behind them, e.g., those coming from placing cuisenaire rods one next to the other and visually comparing the sizes.

In schools, children with VIs also currently learn mathematics through traditional manipulatives, or other ones specifically designed for them, such as the Taylor Frame for arithmetic calculations [63] or braille math blocks [66] (see Figure 1). However, none of these examples exploit the auditory channel neither as an external representation nor to provide guidance during the learning task. Particularly, the braille math blocks [66] are not ranked by length as traditional manipulatives i.e., there is no conceptual metaphors supporting their design. They also force children with VIs to perform serial scanning without any aid to identify objects with one quick gesture like size estimation which would be equivalent to what sighted children do at a glance.

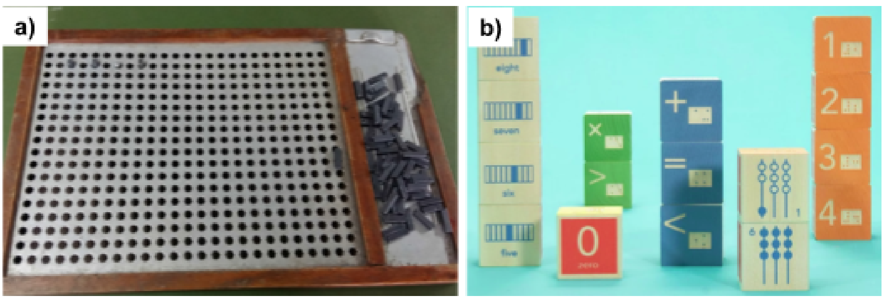

Figure 1. a) Taylor frame (extracted from [63]), b) Braille blocks (extracted from [66])

Virtual/Digital manipulatives in learning

Virtual manipulatives are on-screen representations of physical manipulatives that can be used in education (e.g. [1]). Virtual manipulatives might also offer concrete representations which could be virtually manipulated (for instance on a tablet screen) supporting children thinking [64]. However, for children with VI, using virtual representations would limit them to an audio-based interface. Thus, while sighted users can benefit from parallel processing, VI users have to sequentially process the information through audio feedback [58].

Traditional manipulatives enhanced with digital technologies are known as *digital manipulatives* [23]. The subset of these manipulatives focused on modeling abstract structures (such as the number composition) is known as "Montessori-inspired Manipulatives" (MiMs) [34]. Digital manipulatives and MiMs in particular, are key educational technologies for children with VI, as they can use haptic and audio feedback to compensate for the lack of visual information.

In relation to mathematics learning, there exist some virtual representations of the aforementioned Cuisenaire rods [75,76,77], as well as digital manipulatives [38,78]. We have explored this line with CETA, a system that enable the use of tangibles, similar to Cuisenaire rods, to solve additive tasks embedded in a digital game [38]. This approach is promising for younger children: they improved at mathematics after playing 13 sessions of the game that required the use of manipulatives to solve addition and subtraction tasks [98]. Zuckerman et al. [34] developed two MiMs (FlowBlocks and SystemBlocks) to encourage children to physically explore abstract mathematical concepts like counting and probability. In a similar vein, Smart Blocks [74] are 3D physical cubes designed to explore the relation between different configurations of the units and their surface areas and volumes. Regarding graphs learning, Graphmaster [79] is a tangible graph construction kit composed of connectors, illuminated edges and capacitive sensors that allow children to create physical graphs representations and interact with graph theory concepts in a tangible way. However, none of these approaches adapt the haptic and audio feedback for children with VI, for instance the digital feedback provided in SmartBlocks takes place through a GUI.

TUI for Learning for children with VI

In this section we review those systems that at least combine tangible interaction with auditory feedback.

The development of interactive tangible maps for VI has attracted increasing interest in the last years. MapSense [89] is a multi-sensory interactive map especially designed for children with VI. It incorporates passive haptic feedback as a tactile raised-line map overlay over a touch-screen, auditory, olfactory and gustatory feedback. After conducting several user studies, the authors propose valuable design guidelines stressing the importance of an inclusive and collaborative design using multi-sensory interactions, and the efficacy of storytelling to stimulate engagement and reflection. In a similar vein, Ducasse investigated the design of tangible maps and diagrams for VI [39] . During her research she developed three prototypes (see figure 2) combining tangible and auditory feedback. In all cases the solution is based on tabletop TUI, which is appropriate for map exploration. The materials are mainly paper-based, 3D printed objects, usb keyboards and cameras. Furthermore, these systems allow VI and sighted users to collaborate through an inclusive design.

Figure 2. Left: Pupils interacting with the map of France using the Tangible Reels system. Middle: The tangible box setup. Right: A blind user zooming a map with a slider using BotMap. Figures extracted from [39].

Introducing children with VI to computational thinking is also getting increasing interest. Torino is a collaborative and inclusive tangible environment for learning computer programming [88]. The system is composed by physical 'instruction beads' (three types: play, pause and loop). Children can connect them in order to program stories or digital music. Thus, the system output is highly based on audio, but also provides passive haptic feedback when beads are manipulated (see Figure 3).

The authors draw insights after they conducted three sessions of collaborative work in groups of two children combining low vision, blind and sighted. Among other observations, they discuss the crucial role of audio feedback to understand the state of the program. They highlight that children paid special attention to the sound that marked when the beads were added/removed, i.e., the immediate feedback that indicates a change of state in the system. However, when playing in groups, sometimes this sound led to confusion and children could not distinguish between the audio feedback produced by one's actions or the partner's ones, losing some plug/unplug events.

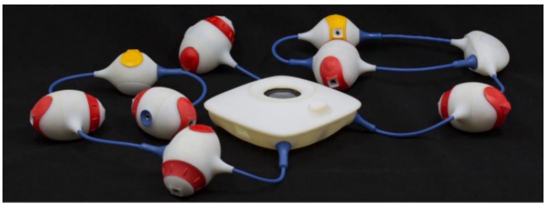

Figure 3. Torino: Different instruction beads connected to the main hub in order to create an audio-based computer program. Figure extracted from [88].

Jafri et al. [70] developed a tangible tabletop, as a low-cost (based on 3d printed objects, a lamp and a USB camera) system (see Figure 4) for teaching the tactual shape perception and spatial awareness for children with VI. Manshad et al. [71] also developed a tangible tabletop system based on MICOO (multimodal interactive cubes for object orientation) and an interactive table, for learning diagrams and graphs related concepts. The system provides audio feedback on the current state of the system guiding the user. In subsequent work Manshad [72] extends the system by incorporating new hardware components providing more diverse feedback (speech, sound/music, vibration and force feedback), supporting new actions of the units (stack, roll, or connect) and enabling collaborative and distance learning. Despite Jafri et al.'s work [70] is low-cost and objects can be 3d-printed, the systems are based on tabletop TUIs, which always require a non-trivial setup and a dedicated space in the classroom, i.e., they are not portable.

Figure 4. Interactive systems for children with VI learning. a) Low-cost tangible system for spatial awareness and tactual shape perception learning. Figure extracted from [70]. b) Appcessory, 3d printed object for geometry learning. Figure extracted from [73]. c)

Multitouch tabletop and tangible objects within a system for interacting with graphical representations such as graphs and diagrams. Figure extracted from [47].

Rühmann et al. [73] present a tangible system for children with VI for geometry learning. The system is made of an Android application and its physical counterpart formed by *appcessories*. An *Appcessory* is a 3d printed object of rods and nodes that can be combined to create different physical representations of geometric figures (see Figure 4-c), whose shapes are sensed by a tablet that provides auditory feedback. This is a promising portable solution which still needs user studies and further investigation to assess its usability and learning impact.

iCETA [99] is an adaptation of CETA [38] for children with VI, which used tangibles inspired in Cuisenaire rods. Children can compose blocks to solve basic addition and subtraction operations and the experience is shaped within a narrative game. However, the digital feedback is only provided in the computer, i.e., the physical blocks are passive. Thus, the system could be further explored and tangibles could be enhanced to provide more multisensorial information to help in the abstract representation of numbers.

To the best of our knowledge, there is no tangible and portable system providing active haptic and acoustic feedback to train additive composition and number line representation in VI context. To address this gap we designed LetsMATH (Learning Environment for Tangible Smart Mathematics) consisting of a set of tangibles, electronic blocks with tactile and auditory feedback, a working area, and a computer-mediated narrative game for mathematics learning.

4. Design rationale

General Design Requirements

Based on the related work and background theories, we identified the following design requirements (DR) that have driven our design: **DR1) *portability and size*:** design suitable technology in terms of portability and size that could potentially be included in classrooms [39], **DR2) *inclusivity*:** design inclusive and collaborative environments where VI and sighted children can work together having shared experiences [60,46], **DR3) storytelling**: as a powerful tool to stimulate engagement and reflection [89], and favouring to train mathematical skills autonomously through the computer game.

In addition to these wider principles, we also identified the following specific requirements for children with VI in a mathematics learning context: **DR4) *suitable for active touch strategies:*** design systems where children can easily understand spatial structures and are able to organize the space in order to perceive informational relationships, **DR5) *continuity:*** build on previous tangible manipulatives, **DR6) *digital enhancement for number recognition:*** provide digital feedback as vibration and or sound [39, 47], i.e., incorporate digital manipulatives to provide abstract number representations [12].

Next we provide a system overview and then we detail the final version of the design rationale of LetsMATH in terms of the *Tangible Learning Design Framework* (TLDF) [29]. This framework describes the system in terms of: physical objects, digital objects, actions, informational relations and learning activities.

System overview

The system consists of a set of blocks technologically augmented, a laptop running a narrative based game and a wooden number line which is the working area (see figure 5). Both the blocks and the laptop give feedback to the user. The working area is designed to add blocks. When the blocks are on the working area they communicate with the computer which evaluates the composition as a composed block and provides feedback.

The goal of the system is to help children in the number concept acquisition process, and learn the concept of additive composition as well as number line representation, two key concepts in cardinality acquisition [96]. The former implies to understand how numbers can be composed by smaller numbers, and the latter refers to the numbers represented on a line that could be either horizontal or vertical.

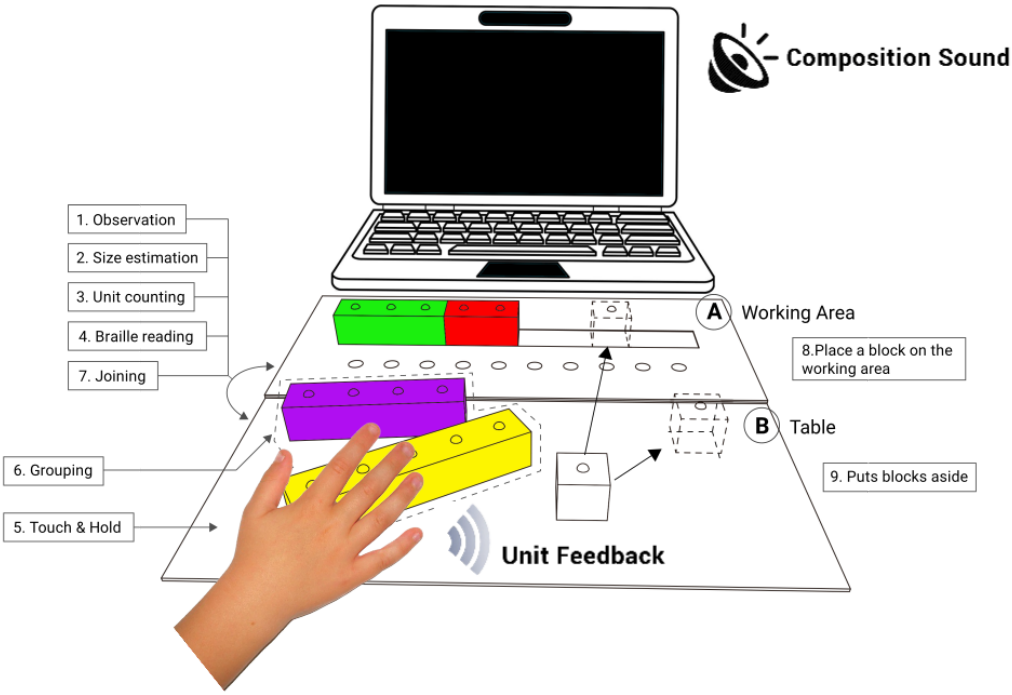

Figure 5. LETSMath prototype environment

Learning Activity

The learning activity is implemented through a computer game called *Logarin*: It is based on an audio interface complemented with a high contrast GUI where a representative drawing of each level is displayed (see figure 6). The GUI is suitable for children with both full and low vision, but is not essential for the game, so that the activity can be conducted by blind children.

Figure 6. Logarin screenshot. In this level the wizard has to stir N times and the user has to help him by composing the number N with the blocks.

The game narrative is about a ‘recently graduated wizard’ called Logarin who needs help to make different spells. In the different levels Logarin has to perform different actions (stir, knock on a door, put ingredients, etc) N times in order to prepare the spells. The child has to compose the number N by combining blocks. Once they do it the wizard finishes the spell and advances to the next level. The activity of additive composition fosters mastery of number combinations, an important milestone in cardinality acquisition [96].

The game is appropriate either for blind, low vision and sighted children fulfilling the ***inclusivity*** **(DR2)** requirement. The learning activity also fulfills the ***storytelling*** **(DR3)** requirement through a narrative game.

Physical Objects

In this section we detail the physical objects design and discuss the actions that we intend to encourage through this design. These actions might be related to the learning goal (additive composition and number line representation) as well as to specific needs of children with VI, for example object identification through physical scanning in one gesture.

Blocks: The design of the blocks is inspired in Cuisenaire rods [68] following the same strategy of [38] to be consistent with the ***continuity*** **(DR5)** requirement. The length of a block is proportional to the number it represents, which is also identifiable by a different color (see Figure 7). The actual dimensions of block 1, represents one unit, are 5 x 5 x 5 cm, and those of block 2, which represents two units, are 10 x 5 x 5 cm and so on. In order to adapt the Cuisenaire rods for children with VI, we included some extra physical features (see figure 7), but they are still appropriate for children with full vision too. Each block has two additional physical representations of the number: equally spaced circular spots as a 3D relief as *unit markers* - to allow children to touch units as physical elements and count, an epistemic action that seems to offload cognition [38] -, and the braille sign of the number - which is a symbolic representation of the quantities. Through the unit markers, the blocks expose the whole-part relationship with the aim to be discovered, comprehended and trained. We also included magnets inside the left and right sides of the blocks in a way such that they always attract one another, providing an affordance to encourage the joining action, and also mimicking the number line representation and creating (composing) a longer block. Moreover, magnets have a novelty effect and increase children's enjoyment and engagement [38]. A guiding line connects the unit markers to support haptic exploration.

Compared with usual Cuisenaire rods the size of the blocks is bigger to facilitate their appropriation by children with low vision.

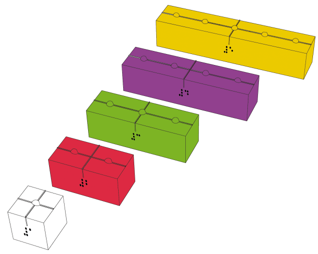

Figure 7. Physical blocks ranged in length with unit markers, braille signs and guiding lines to support exploration

Working Area: It is a board representing the number line (see figure 8) with a guide to place and fit the blocks, so that children can create their compositions. It includes braille signs of the numbers and magnets that the blocks detect when placed on and send a notification to the laptop. Thus, the system only evaluates the compositions performed on the working area. This physically constraints children to submit the solution using the number line representation.

Similar to other learning materials often used in the classrooms, the design of the system is aligned with the ***portability and size*** **(DR1)** requirement. It can be used and stored in classrooms without any special demand.

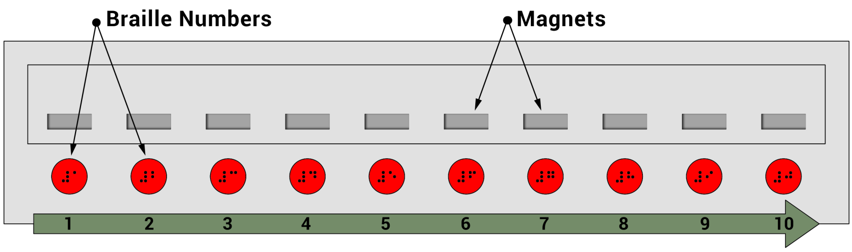

Figure 8. Working area.

The system is ***suitable for active touch strategies*** **(DR4)** which are developed by blind children when counting through tactile patterns [12]. As touch does not provide a preliminary overview as vision does, children have to organize materials and develop their own abilities to count [55]: Preliminary physical scanning of the structure to be counted, count organizing (following lines and/or dots) and partitioning (for instance using one hand to indicate a partition which was already counted and thus keep track and avoid repetition).

The number line has two roles, it is the input space and, at the same time, a stable structure. While blocks allow physical rearrangement of the environment and active exploration as PDL theory suggests [31], the number line is the stable structure which might scaffold the construction of the composition [30].

Digital Objects

There are two different kinds of digital objects. First, the sound and vibration feedback. And second, Logarin as the main character of the story that is mainly represented through its voice.

Logarin, whose voice guides the user through the game, asking for help, giving hints and congratulating the child when the answer is correct. Some high contrast pictures are displayed on the screen (see figure 6).

Composition sound: A sound representation of the number is provided by the game indicating the target number to be composed (N). This *composition sound* is thematic as sounds change depending on the context of the story, e.g. the sound of drops when Logarin is preparing a spell or of knocks when it is trying to open a magic door. The children have to listen to how many times the sound is repeated (N) and compose a block of the same size as the number on the working area.

The *composition sound* provides rhythmic cycles of consecutive sounds (see figure 9-a). This is followed by a hint, provided by Logarin, that depends on the current solution on the working area, for instance if N = 4 and only block number 2 is on the working area, the hint might say “come on, try adding more blocks to the solution”. This cycle, thematic sounds followed by a hint, is looped until the correct solution is composed on the working area. When this happens a special sound (called *blocks sound*) is played along with the thematic sound instruction just to indicate that the solution is correct (see figure 9-b). After this, Logarin congratulates the child and the next level starts.

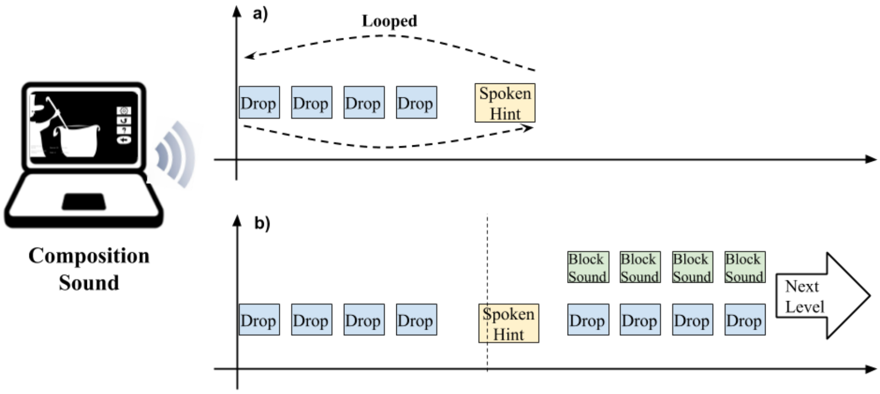

Figure 9. Composition sound. a) Looped composition sound b) After composing the correct solution

Unit feedback: While a block is being touched it plays the *unit feedback* which represents the cardinality of the block, i.e., a sound is repeated as many times as the units of the block. In the final version of the system, two patterns are combined with different speeds (COMB, see table 2 and Figure 11).

The *combined unit feedback* mode provides two types of digital feedback, first the fast vibration followed by a slower sound pattern. Actually, the vibration also generates an intrinsic collateral sound, i.e, vibration might be perceived by both the haptic and auditory channels. Thus, there are two patterns, whose key difference is the stimulus frequency, high and low.

The sequence of vibrations has a periodicity of 100 ms, the beeps of 850 ms. Despite the low frequency beeps pattern is easier to count, there is a risk of children getting stuck in counting strategies, preventing them to acquire abstract number conceptualization [12]. High frequency patterns, might allow a kind of simultaneous perception of the whole and the part in a similar way as visual perception does [12] (sounds played at less than 200 ms start being perceived as a continuum [87]).

Finally, both feedbacks are concrete-iconic number representations since there is a direct perceptual relation to the represented object [24]. Within the proposed concrete model, this kind of feedback has the role of perceptual scaffolding [18] in the process of acquiring cardinality.

The design of acoustic and vibrational material favours number units perception contributing to the ***digital enhancement for number recognition*** **(DR6)** requirement. Specifically, fast vibration (and collateral plastic sound) allows to be perceived simultaneously and then stands as an approximation to a more symbolic representation but still providing a direct link to the number. Also, the different representations proposed through different sensory channels: vibration-haptic, sound-auditory, might lead to better learning since it is cognitively more efficient to process information across modalities [30] and also might facilitate the translation between sensory channels prompting the emergence of abstract conceptualization [24].

Children will learn better because it is more cognitively efficient to process information distributed across modalities including haptic (form), visual (images, text), and auditory (voice, sound).

Actions

In this section we discuss the actions that could be taken with the blocks. Some of them are not strictly under the TLDF classification since they do not have an impact on the digital objects. However, they might still be taken during the development of the solution as epistemic actions (see Figure 5 and table 1 for a summary).

We describe how the different system design features might help/encourage children to discover and perform these actions, and which their role is during the problem solving process.

Unit counting is one of the most significant actions since it is present in the block recognition process as well as in the composition task. The physical unit markers expose the whole-part relationship which might be interpreted within a single block, i.e., understand that the block 3 is composed by 3 units, this would help children to recognize the number represented in each block. This action can also be performed when composing a number with several blocks, i.e., grouping or joining blocks and then counting the unit markers to calculate the result.

Braille reading is an action that might allow children, who have already basic cardinality and braille skills, to identify each block. A physical guiding line leads the children to the braille sign. On one hand, children should have previous skills to use the braille sign, and it could demand more cognition since they are facing a symbol, i.e., an abstract number representation. On the other hand, it is more precise, less ambiguous than the size estimation, and would allow children to continue practicing mathematical skills, beyond cardinality.

The ***size estimation*** action is mainly performed as a first or quick alternative to compare-recognize blocks. It is typically performed when children are searching for a block. For instance, if one is looking for block 3, but finds block 1 instead, it might be quite easy to discard it. This is possible when the blocks' size is big enough to perceive double and triple ratios quickly; this size avoids the blocks to "get lost" on the table too.

Grouping relates to the conceptual metaphor that bringing objects together somehow adds, compose and create a new object [38]. This grouping action supports the PDL [31] since children are adapting and reinterpreting the environment. Children might also offload cognition by taking actions on objects [29] and by the creation of external representation of groups [35].

To join blocks implies creating an aligned group of blocks, making use of the magnets and imitating the number line representation. Thus, for the sake of clarity, *joining* is a specific case of *grouping*. However, within this design, *joining* also implies to align the blocks exposing the whole-part relation between blocks and unit markers, leading to different interpretations (see Figure 10). Although this is more powerful when vision is available, still might help children with VIs to organize the blocks, facilitate the computation of the result and avoid errors such as counting a unit marker or a block twice. We might say that *joining* is an epistemic action, i.e., makes the problem easier to solve but is not necessarily part of the solution [53].

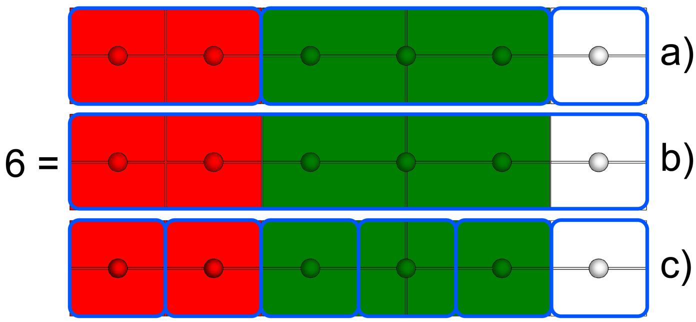

Figure 10. Different perspectives of blocks 2, 3 and 1 making up number 6. a) 6 = 2 + 3 + 1, b) 6 as a single bigger block, c) 6 = 1+1+1+1+1+1.

To place blocks on the working area is the only pragmatic action involved in the development of the solution. The opposite action is to ***put blocks aside*** in order to exclude them from the solution and reduce the interference when composing, an action that have been previously observed with sighted children [30].

Touch or Hold provides the *unit feedback,* that includes sound and haptic feedback. It is an epistemic action that might aid children to recognize the number of a block without a full physical scanning.

	Action	Role	Design facilitator/affordance
Visual	Observation(size estimation/comparison/unit counting)	Might be used for blocks recognition. Could be part of the composition if children performs mental computation	● Big size ● Blocks ranged in length ● 3D unit markers
Physical	Size estimation/comparison	Might be used for blocks recognition	● Big size, two hands grasping ● Blocks ranged in length
	Unit counting (part-whole identification)	Might be used for recognition and composition	● 3D unit markers ● 3D guiding lines
	Braille reading	Might be only used for blocks recognition	● Standard braille sign size ● 3D guiding lines
	Touch & Hold (digital feedback perception)	Used for blocks recognition. Could be part of the composition if children performs mental computation	● Sensors ● Unit feedback (sound/vibration)
	Grouping	Epistemic action that might be used for composition	● Freedom to move blocks and rearrange the environment
	Joining	Epistemic action that might be used for composition	● Magnets at the extremities of the blocks
	Place a block on the working area	Pragmatic action, might be used for composition	● Matching number line and blocks width
	Put blocks aside	Epistemic action that might be used during composition or recognition	● Freedom to move blocks and rearrange the environment

Table 1. Possible visual or physical actions with the blocks and their respective design affordances and facilitators

Visual observation As part of the target users present low vision we considered it as one of the available actions. Blocks are big enough to permit some of the children to extract information through the visual channel. Children might estimate block's size, compare among them and even count the unit markers depending on their degree of vision.

All the actions except braille reading and visual observation fulfill the ***suitable for active touch strategies*** **(DR4)** requirement.

These actions are fostered by specific physical design features detailed in table 1. When it comes to the composition task specifically, **joining** and **grouping** are relevant actions that allow children to modify the environment to reduce cognitive load. In addition, these actions have been observed as relevant epistemic actions in previous work [38] and originally emerge from traditional manipulatives such as Cuisenaire rods, which contributes to fulfill the **continuity (DR5)** requirement.
The combination of all these alternative actions might contribute to the ***inclusivity*** **(DR2*)*** objective, for instance allowing low-vision and sighted children to recognize the number represented in each block visually.

Informational Relations

Informational relations make reference to the mapping between physical objects, digital objects and actions. Within this system, it is represented through the feedback provided to the actions. On one hand, the unit feedback, designed for children to recognize the number. It provides alternative external representations of the number and uses multiple representations through different sensory channels, that might help to gain abstraction [24]. The mapping that triggers the unit feedback when a block is being touched is especially aligned with the ***digital enhancement for number recognition*** **(DR6)** requirement.

On the other hand, when the blocks are placed on the working area, the system evaluates their composition and provides feedback. In case that the solution is correct, it also plays the blocks sounds and then congratulates the child.

5. Three iterative studies

In section 4 we detailed the final design rationale of the system. As the system evolved through a user-centered design iterative process, in this section, we detail each user study (see Tables 3 and 4) and provide a description of the system at the point of each study (see Table 5).

All tests were individuals and in case there were video recordings (study 1 and 2), they were only accessible to the researchers participating in the study were used for video analysis. In all the cases, parents or tutors were previously informed of the activity and they provided consent for children's participation.

Code	Name	Description
P_OFF	*Powered off*	No digital feedback
FAST_V	*Fast Vibration*	The block only vibrates. The delay between each vibration is quite short (100 ms)
S_AND_V	*Parallel sound and vibration*	The block vibrate and play a beep at the same time*. The delay between each vibration-beep is longer than in Mode B (850 ms)
COMB	*Combined*	In this case the block first provides the fast vibration and then plays only the beeps. It is like a combination of FAST_V and S_AND_V

Table 2. Unit feedback modes.

* Actually the vibration is first lasting 100ms and immediately after the sound lasting 50ms. This simulates the effect of a unique stimulus.

Study	Participants		Average Duration	Data Collection & Analysis
	Low vision Blind	Age		
1	N=6 (2 with low vision, 4 legally blind) Named S1.1 to S1.6	7-9	18	Conducted by five researchers who acted as participant observers and performed post video analysis
2	N=6 (2 with low vision, 4 legally blind) Named S2.1 to S2.6	7-12	20	Conducted by three researchers. They acted as participant observers and performed post video analysis
3	N=8 (4 with low vision, 4 legally blind) Named S3.1 to S3.8	6-8	18	Conducted by two researchers and one educator. They acted as participant observers and took structured notes

Table 3. Methods of the User studies

Learning Activities	
Name	**Description**
Exploration	The tablet provides the composition sound according to the blocks placed on the working area. It is an introduction where there are no right or wrong answers. The objective is to provide a warming up and observe the understanding of the concrete model.
Composition (tablet or experimenter guided)	The tablet or experimenter says aloud a number and the child has to compose it (on the working area or table depending on the study). If the solution is correct, the child receives positive feedback (from the tablet or experimenter) and another number is said, continuing the challenge. The objective is to evaluate the usability and usefulness of the blocks when training additive composition skills is assessed.
Order	Children are instructed to order three blocks of different values (value one, two and three) in increasing or decreasing order, as they preferred. The objective of this activity is evaluating the concrete model comprehension in terms of physical representation (Q1).
Broken Blocks	The *unit feedback* mapping is misconfigured making some blocks to be “broken”, for instance, block 3 might play only two sounds. The experimenter explains that some blocks are broken without specifying that the feedback is the dysfunctional element. There are four different feedbacks mappings for each of the three blocks, making twelve combinations. Children are asked to identify the broken blocks and they also have to explain their answer: “Is this block right or wrong? Why?”. The objective of this activity is evaluating the concrete model comprehension in terms of informational relations (Q1).
Find a Block	In this game the participants are asked to find and give to the experimenter a specific block. The four blocks are on the table and, for example, the experimenter asks “could give me the block number 3?”. This is repeated three times, one for each block number. The underlying intention with this game was to observe the strategies used to recognize the blocks (Q2).
Logarin (composition video game)	Composition activity shaped by a narrative video game. Children have to perform compositions on the wooden number line working area (see section 4 for a detailed description). This is the main activity where the final version of LETSMath with all features incorporated is tested.

Table 4. Description of the learning activities

Study	Unit Feedback	Learning Activities	Working Area
1	P_OFF S_AND_V (in loop)	Exploration, Composition, Order	Rectangular wooden working area (no detection capabilities, wizard of oz)
2	P_OFF, FAST_V and S_AND_V (not looped, counterbalanced)	Find a Block, Composition, Broken Blocks	Wooden number line (only 2 participants)
3	COMB (not looped)	Logarin (composition video game) Broken Blocks	Wooden Number line with magnets

Table 5. Description of the environments and activities used in each study

Study 1

The aim of this study was to validate the initial design and to gain insights for the next design iterations. Thus, the intention was to start addressing Q1, i.e., if the concrete model is understood and if the materials are appropriate for children with VI. Physical (size) and digital (feedback) mappings were tested regarding their understanding and appropriation.

Methods

Participants: The experiment took place in a school for children with special needs in Uruguay and some of the participants had additionally cognitive deficits (see table 3).

Procedure: The individual tests were carried out in a classroom followed by an interview. The interview took the form of a radio program in order to make it more engaging for the children. During this interview an educator from the school was present.

In four cases the blocks were on from the beginning, thus giving the *unit feedback*. In the other two cases the blocks were off to focus on the identification of physical aspects: size, unit markers and braille signs.
First, we presented blocks 1 and 2 to the pupils and we guided them to discover its size, unit markers, braille signs and unit feedback if available. We also guided them to identify the relationship between the value of the block and its size. After this brief introduction children are guided through the learning activities.

System:

Physical objects: For this study we used a first prototype of the blocks which included a conductive textile wrapping them implementing a touch sensor (see Figure 12).

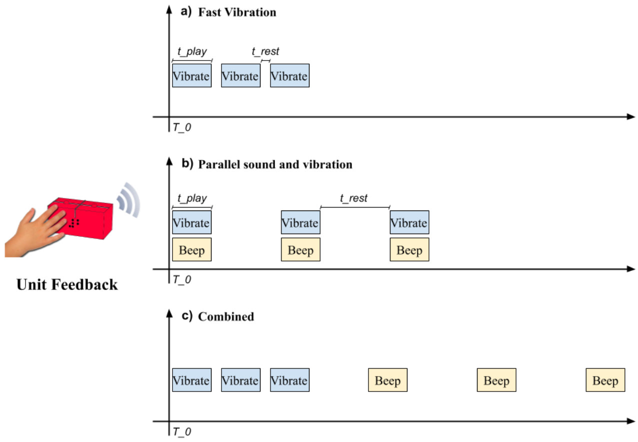

Figure 11. Unit Feedback modes

When the block is on the working area, it gets activated and the tablet receives a notification of the event (see Figure 12, table 5). At this point the prototype was not finished, so we resorted to a Wizard-of-Oz strategy. A team member used a laptop to send OSC messages to the tablet when a block was touched or placed on the working area.

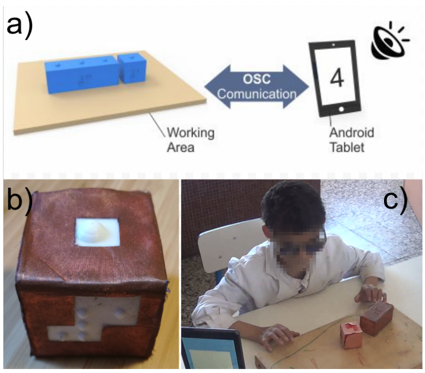

Figure 12. a) System environment for study 1 b) Block 1 wrapped by conductive textile c) Blind participant testing the prototype

Learning activities
We implemented three games to support the learning objectives: Exploration, Composition (guided by tablet) and Order (see table 4).

Results

This study allowed us to validate that children partially understood the concrete model (Q1), and how to improve the design. We analyze the data of the study through each of the components of the design rationale:

Learning Activity: In this opportunity the games that conformed the whole learning activity were quite straightforward and all the participants were able to understand the goals.

Physical Objects and Actions:
Children explored the objects in a similar vein at the three different games. Only two participants, S1.1 and S1.5, used the **unit markers for counting** for discovering the block value. The distance between the unit markers make it difficult to discover all of them, only a few children were able to find all of them during the exploration game.

All participants had problems to identify the **braille** signs, which are made of 3D points. Due to technical 3D printing limitations we decided to increase the braille sign size, but this led children to misunderstand it and they often described these braille sign as "small points" or even confuse them with the unit markers. In addition, one of the educators also stressed the importance of using the standard braille sign size [81] because each sign might fit under their fingertips. The same educator also suggested to add the guiding lines between unit markers to lead the exploration to the next point, so in both cases the expert opinion matched with the field observations.

Digital Objects and Informational Relations:
We observed that many children did not use the *unit feedback* to recognize the number represented in each block, although all of them showed enthusiasm with this feature.This led us to wonder if they understood the mapping and as a consequence we designed an activity specifically addressed to check the mapping (broken blocks game, see table 4) for the following evaluations in order to find it out.
In addition, each block reproduced the unit e feedback in a loop while being touched by the user. In such configuration we observed that the children did not realize that they should "reset" their counting to obtain the correct value of the block between the loops. Thus, for the second and third study we decided to provide one time unit feedback until blocks are released and touched again.

Many feedbacks at the same time can hinder the understanding of the current situation during the game. We observed this problem while the children were touching the blocks on the working area. In this version the tablet provided auditory feedback to reveal the presence of the blocks on the working area; at the same time each block provided *unit feedback when touched* making the situation quite hard to interpret. Thus, for study 3 we decided that blocks only give unit feedback when the working area is empty.

In sum, this study allowed us to partially answer Q1, i.e., children understood the physical model but it showed some drawbacks which were improved in the design and tested in the second or third study. Digital feedback emerged with an engagement effect but in most of the cases was not used neither to recognize the blocks nor to compose numbers. Actually, some children were confused by the auditory feedback loop and they mostly relied on the physical factor to accomplish the goals. Therefore a special activity was designed in order to explore this issue in the following study.

Study 2

Methods

Study 1 provided a valuable validation of the model, however many children seemed not to rely on digital feedback to recognize the blocks. Moreover, despite the fact that they understood the physical design (*unit markers* and size relation) some drawbacks were noted as the lack of guiding lines and the braille size. Thus, for the second study, we aimed to verify that the informational relations were understandable for children and validate the improvements done on the physical design (Q1). At the same time we aimed to observe whether the different representations available support the number/block recognition (Q2-a). Finally, we also intended to observe which elements of the model were used during the composition task, specifically if digital feedback was incorporated (Q2-b). The composition task involves not just recognizing the number depicted by the blocks but also the sum of the values of the composed set, requiring larger working memory resources. To this aim, participants played three games in a within subject design: *find a block, composition game and broken blocks game* (see table 4); under three different feedback conditions.

Participants: The study took place in two schools for children with special needs in Uruguay. We also had an interview with the director of one of the schools (see Table3).

Procedure: At the beginning of the session the experimenters explained the relationship between the size, unit markers and braille sign. Additionally, under the feedback FAST_V and S_AND_V, the experimenters explained the mapping, e.g., “this block represents a number 3 and vibrates three times”. After a brief introduction to the blocks, participants played the games always in the same order: find a block, composition and broken blocks game.

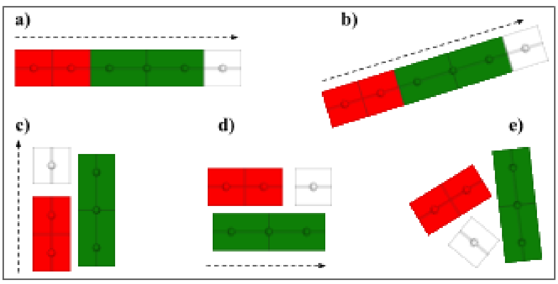

Figure 13. Observed composition arrangements

Data Collection and Analysis: Two researchers analyzed the videos and coded the so called **recognition actions** (see table 1) performed on the blocks during a) number/block recognition and b) composition task. During a first exploration of the video recorded sessions six action categories were identified: Visual recognition, Physical Dot Counting, Braille reading, Physical Size estimation, Sound counting, Vibration perception/counting. For the composition game, also the blocks arrangement was observed and classified under four categories: a) Horizontal line, b) Diagonal line, c) Vertical 2d, d) Horizontal 2d, e) No order (see figure 13). The two researchers who analysed the videos discussed each time they had different criteria in the video coding and interpretation until agreement. In some cases the participation of a third observer was required in order to reach agreement.

In some cases, children received significant help from experimenters who explicitly asked them to perform an action. For instance, participant S2.1 was not able to perceive and count sounds because he was always counting the physical unit markers, in this case the experimenter held his hand constraining movement in order to avoid physical dot counting and asked him to count the sounds. In addition, participant S2.1 and S2.4 also used the number line working area to place the blocks over, so the analysis of the composition arrangement has no sense and as a consequence they were excluded for this analysis but these cases are discussed separately. Also, participant S2.4 did not perform under the fast vibration condition and did not understand the Broken Blocks game.

System

Blocks: For this instance we incorporated the guiding lines to the blocks and reduced the braille sign size. There were four blocks: one 1, two 2 and one 3. We tested three different *unit feedback* modes: P_OFF, FAST_V and S_AND_V (see tables 2 and 3, and Figure 11).

Learning activities: We designed three games to support the learning goals: Find a Block, Composition (guided by experimenter) and Broken blocks (see table 4).

Working area: The two blind children in this study used a working area representing a number line (see figure 8). Although for the aforementioned games there is no real need to use the working area we still tested it in order to gain insights about its affordances to guide them during the composition game. For the rest of the games and participants the problems were solved on the basic table.

Results

Game 1. Find a Block

This task is connected to the block's recognition process (Q2-a). In general, children were able to *find a block* without difficulties. Children understood the proposed model including the availability of different external representations to recognize the blocks through multiple perceptual channels.

We observed that children varied their interaction strategies with the blocks depending on their visual impairment. For this reason and besides we only had two blind children in this study, we still consider that the results should be analysed grouped by type of visual impairment.

LOW VISION CHILDREN

Children with low vision privileged the vision to recognize the blocks, they also used their hands to count the number of dots when no digital feedback was available (see Figure 14-b). However, when other sensorial representations become available, we observed a change in the pattern of recognizing a block. They still greatly relied on visual cues but partially replaced unit counting with *fast vibration* (see Figure 14-e) or *sound and vibration* (see Figure 14-h).

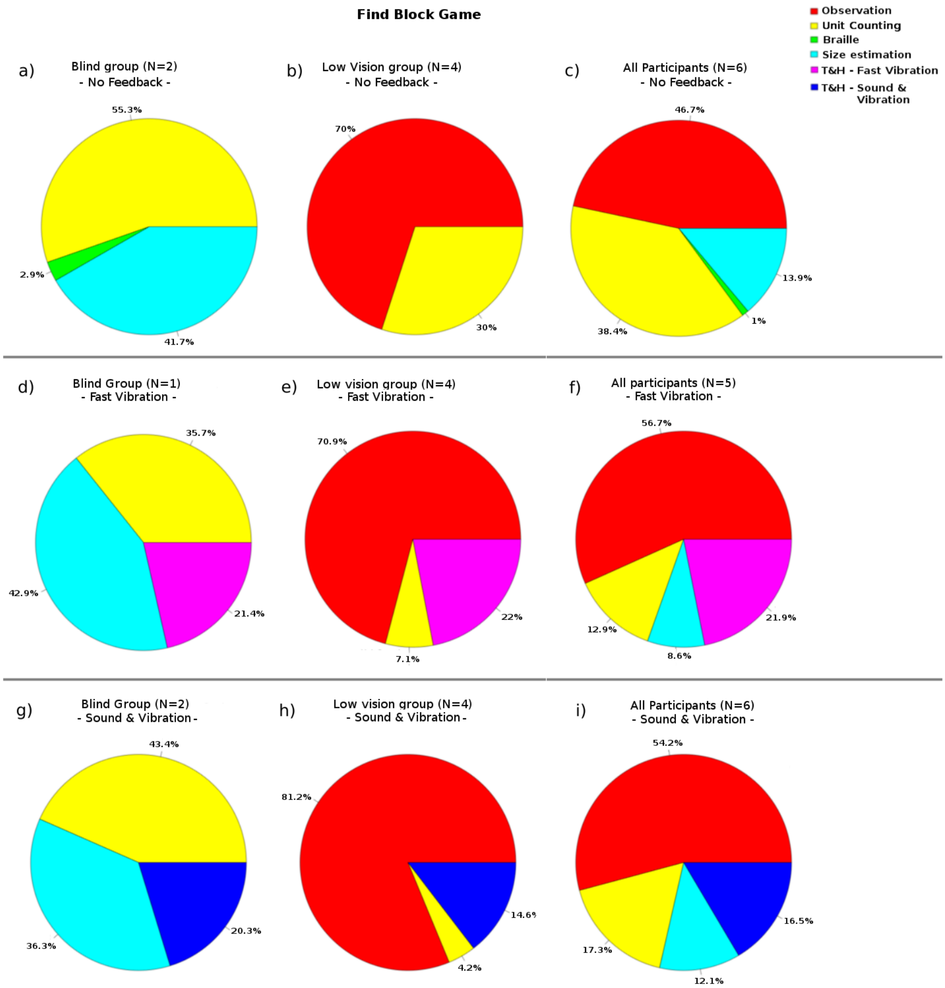

Figure 14. Actions performed on Blocks during the Find Block game through the three conditions: No feedback, Fast vibration and Sound and Vibration. Data is grouped by visual condition group blind (a,d,g) low vision (b,e,h) as well as all together (c,f,i).

That is to say, when children with low vision received haptic or sound representations, they showed opened to use such representations to solve this task. This could be due to the novelty effect or a real usefulness in block's recognition, or both.

BLIND CHILDREN

For blind children, unit counting was the most employed strategy to recognize the blocks, followed by size estimation using both hands. When sound and vibration representations came available they tended to exploit such new representations to recognize the block: feeling vibrations or listening to the sounds of each block.
Thus, for those who were blind, we observed a more distributed type of sensorial representation used whereas for children with low vision we observed a strong tendency to use vision to accomplish the task in detriment of other sensorial representations (see Figure 14-a,d,g and Figure 14-b,e,h). In addition, dot counting was used less when other sensorial representations become available.

Therefore, children tended to adopt digitally enhanced strategies when looking for a block. Interestingly, vibration emerged as a valid strategy for number recognition, even when children had little time to habituate.

In relation to braille reading, we observed that they only did it when the blocks did not present other sensorial representation as sound or vibrations, and even in such cases it represents a very small portion of the performed actions.

Game 2. Composition

All children accomplished this task with no problems showing a good comprehension of the interaction model. It is worth considering that in this task children had to detect block number but also had to make a composition.

Interestingly, the differences in frequency between unit counting and digital strategies were modulated by the task and visual condition.

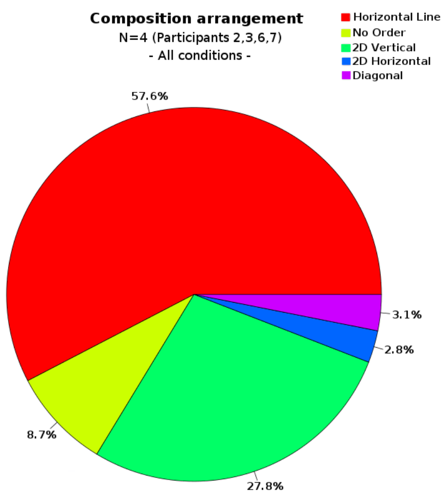

Figure 15. Blocks arrangement observed during the composition game for low vision group

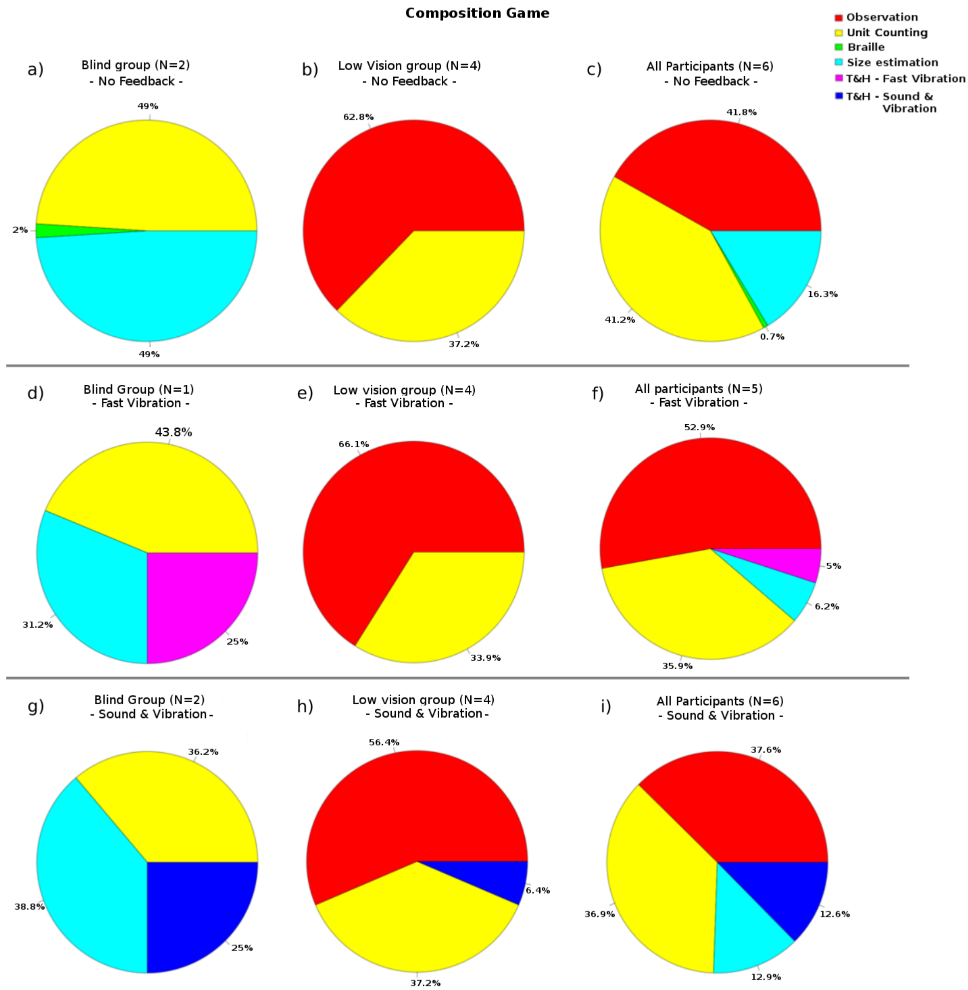

Figure 16. Actions performed on Blocks during the Composition game through the three conditions: No feedback, Fast vibration and Sound and Vibration. Data is grouped by visual condition group: blind (a,d, g) low vision (b,e,h) as well as all together (c,f,i).

LOW VISION CHILDREN

Compared to the recognition task, during the composition task, children with low vision tended to decrease observation and make more use of dot counting (see Figure 14-b,e,h and 16-b,e,h). Interestingly, low vision children showed a higher utilization of the observation strategy for the three feedback conditions during the find block game than in the composition game.

Additionally, they abandoned strategies involving *unit feedback*. They did not make use at all of *fast vibration* and barely used *sound and vibration* (see Figure 16-b). Our hypothesis is that for low vision children, when it comes to a composition task, *unit feedback* does not involve any benefit or cognition offloading. That is to say, *unit feedback* does not help to

compute the additive composition result since it only provides feedback representing each individual block. Contrary, physical unit markers might allow children to touch and count them computing the composition result.
It is worth noting that unit counting allows a larger cognitive offloading than the transient auditory/vibratory stimulation. For this more demanding task it seems reasonable that children relied in a slower but more reliable resource as haptic counting.

BLIND CHILDREN

For the blind children, on the one hand we observed that they relied on *fast vibration* and *sound and vibration* (see Figure 16-d,g) in a similar proportion during both games. On the other hand, when digital feedback was available during the composition game, they reduced the size estimation action on the blocks in a higher proportion than dot counting (see Figure 16-d,g), a phenomena that was not clearly observed during the find a block game.

These results suggest that, in contradiction with children with low vision, digital feedback is still useful in the context of the composition task. Actually, they replaced in a higher proportion the size estimation action than the dot counting action. Thus, dot counting might be an action more useful for the composition task than size estimation, which makes sense since it allows to count the result of a composition, and even more when blocks are joined/aligned as it was the case for blind children since they used the number line working area.

COMPOSITION ARRANGEMENT
When the composition arrangement involves blocks' alignment (see Figure 13-a,b) the benefit is even higher, children might count going from one extremity to another ensuring to count all the unit markers and avoid accidentally repetition exploiting the physical rearrangement to unload cognition as the PDL theory explains [31]. We analysed composition arrangement for children with low vision and the horizontal line is the most used arrangement (see Figure 15), this might facilitate counting and it is also a signal of the incorporation of number line concept. Interestingly, this strategy supports the count organizing and partitioning skills proposed by Sicilian [55], and at the same time is something that we have observed before with sighted children [38].

Game 3. Broken Blocks

From the results presented below we had to exclude participant S2.4 since he did not finish the session and had problems understanding the broken blocks game.

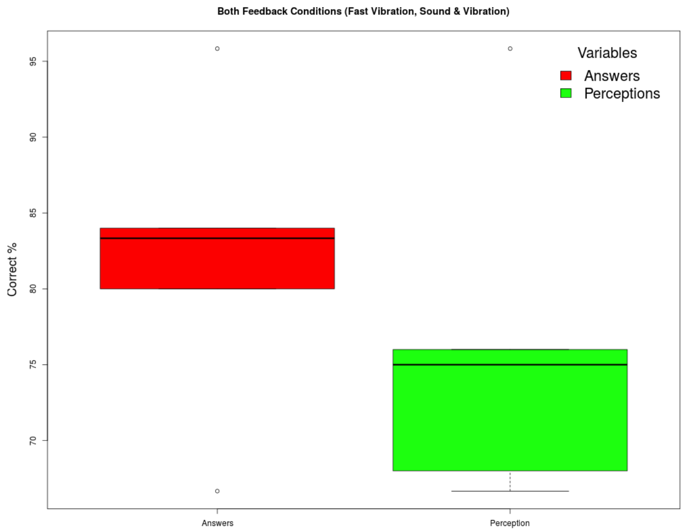

Figure 17. Broken blocks game results, FAST_V and S_AND_V unit feedback conditions (Study 2)

This game was specifically designed to check informational relations understanding (Q1). It has been observed that children were highly efficient assessing if the digital mapping matched with the physical block (84%), and when asked to report why, their responses were at a similar level (75%); a little decay reflected that sometimes children had trouble explaining why they choose an answer (see figure 17). Interestingly, children were not equally accurate in all conditions.

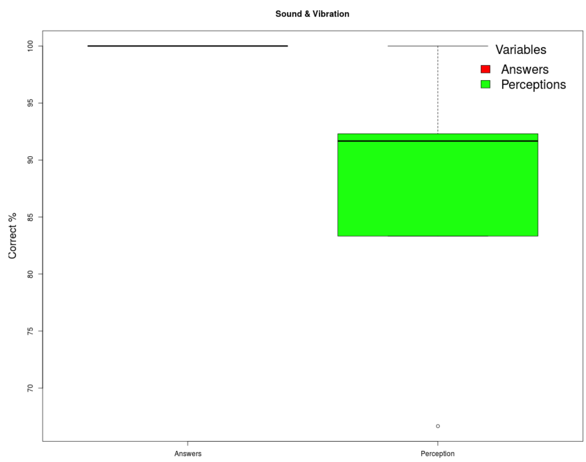

Figure 18. Broken blocks game results, S_AND_V unit feedback condition (Study 2)

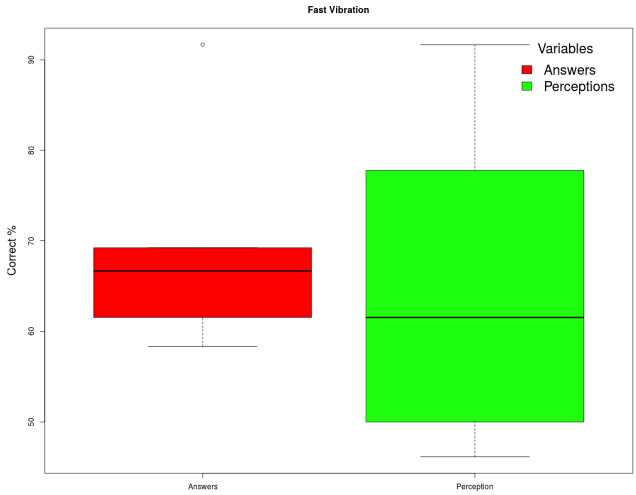

Figure 19. Boxplot of average of correct answers and perceptions of participants in the broken block games, FAST_V unit feedback condition (Study 2)

At the S_AND_V unit feedback condition, responses achieved almost 100% accuracy (correctly justified in 95% of the cases, see Figure 18). Whereas under the FAST_V unit feedback condition a 65% was reached (perception 59%, see Figure 19). It seems that the slower and parsimonious strategy was more successful when children were faced to avoid deception.

Study 3

Methods

Through the second study we were able to provide reliable evidence in order to address Q1 and gain insights for Q2. The aim of the third study was to test the concrete model in the context of a narrative game, Logarin, shaping an engaging learning activity. The inclusion of a narrative video game chase multiple objectives. On the one hand, we aimed to incorporate and validate the concrete model as part of a system which might be used to train mathematics autonomously (Q1). On the other hand, we aimed to formally test the number line working area and its impact on the strategies and performance (Q3). Additionally, as we already probe that children understand both unit feedback modalities (S_AND_V and FAST_V), we incorporated the COMB unit feedback.

Participants: The experiment took place in the resource center of the National Organization of Blind people of Spain (ONCE). We also had interviews with the pedagogical technical director of the center and with two educators.

Procedure: For the unit feedback we always used mode COMB (see tables 2 and 5). The test included four stages: 1) Warm up, 2) Logarin (composition game), 3) Broken blocks game 4) Optional Interview. In the first stage, the warm up, the experimenters explained that the goal of the game was to help a wizard composing numbers on the wooden number line. Additionally, they explained the relationship between the size, unit markers and braille sign, as well as the unit feedback mapping. After this, participants had a moment to try the blocks. In the second stage, we introduced the Logarin game, with a tutorial. The game included five levels in which children were challenged to compose three (random) numbers from 1 to 7 except in level one that it was from 1 to 5 in order to make it easier with the blocks provided. In the third stage, participants played the broken blocks game. In the fourth stage, if the child did not look like tired, we had the chance to ask how did they recognize the blocks and it they would have liked to continue playing.

Data Collection and Analysis: During Logarin game play we annotated the number of errors in each trial and the composition strategy. The composition strategy was categorized as: a) Clear the working area, compose the number on the table and then put the blocks on the working area, b) Clear the working area and compose the numbers directly on the working area block by block, c) Do not clear the working area and perform the whole composition directly on it. General observations were also taken by both researchers and one educator of the center (see Figure 20). Some of these observations aimed to assess the video game understanding as a control measure in order to isolate the concrete model validation, i.e., mitigate the possibility of the game acting as a confounding variable.

System

Blocks: For this instance, we used exactly the same blocks than in the previous study but configured with unit feedback COMB (see table 2 and Figure 11).

Learning activities: This time we incorporated the narrative video game Logarin and kept the Broken blocks game (see table 4) with the same aim than in Study 2, validate the understanding of the informational relations (Q1).

Working area: In this opportunity, all the participants used the number line working area (see figure 8 and table 5). As it was explained in Section 4, the video game only evaluate those compositions on the working area. In order to help children in their first approach to the system, the game asks them to clear the working area during the tutorial and the first two levels, i.e., they are forced to follow the composition strategy a) or b) (See section Study 3, data collection & analysis). For levels 3,4 and 5 they are free to work without clearing the working area, so they can follow the strategy c).

Results

Game 1. Logarin Gameplay

The observations of researchers and educators indicate that all the participants were able to understand the introductory tutorial, to successfully use the prototype, and play the five levels of the game (see Figure 20-O1,O2).
Importantly, children showed a “learning effect” diminishing in their error rate as they advanced through the game (see Figure 21).

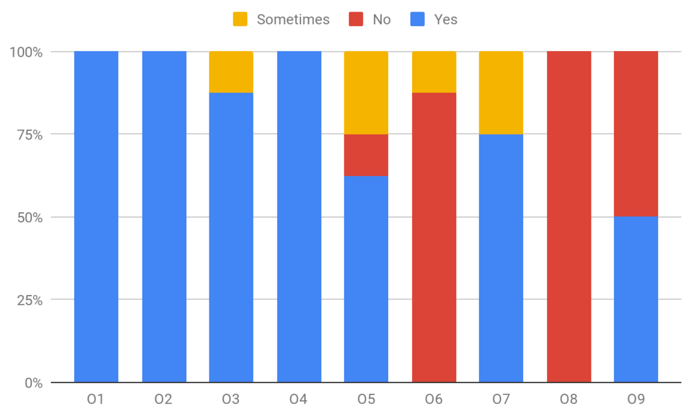

Figure 20. Researcher and educator observations: O1 Understand the tutorial, O2 Understand the task, O3 Identify/recognize the proposed number to be composed , O4 Understand the game’s rules, O5 Match the requested number with the blocks placed on the working area, O6 Try blocks randomly, O7 Understand when the blocks are detected by the system, O8 Ask to play one more level, O9 Would you like to continue playing? (Study 3)

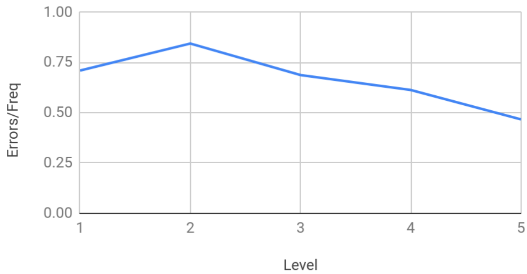

Figure 21. Error rate per level of the Logarin game (Study 3)

Also, the representation of the number through the *composition sound* was clear in most of the cases (see Figure 20-O3). Only participant S3.2 had problems to interpret the *composition sound* having to wait until the game gave him an explicit hint like "You have to compose the number N". The game rules were understood (see Figure 20-O4). We observed that some participants confused the concept of units with blocks (see Figure 20-O5). One of the reasons might be the oral hints provided by the game like: "try with more/less blocks" which is not always accurate, it might say "try with smaller/bigger blocks" instead, this issue was also stressed by one of the teachers.

In some cases, children did not realize that the blocks were detected by the system when they put them on the working area (see Figure 20-07). As a consequence, in some cases as they did not perceive an immediate feedback or reward, they removed the blocks before the system evaluates the composition. This issue might be solved introducing a stood out synchronic feedback when a block is placed on the working area. However, it is important to delay the system evaluation in order to avoid trial and error strategies by putting on and off random blocks, a phenomena that was observed in previous studies, and was almost not observed during this study (see Figure 20-6).

Observations O1 to O7 confirm the comprehension of the concrete model (Q1) applied in the context of a narrative game. However, no one asked to continue playing (Figure 20-O8) and when they were explicitly asked if they wanted to continue playing, 50% of the participants said no (Figure 20-O9).

	Composition strategy			
	a)	**b)**	**c)**	Missed observations
Must clear number line	39.58%	54.17%	-	6.25%
Free to work without clearing	25.42%	33.51%	30.45%	10.63%

Table 4. Percentage of Observed composition strategies (see Study 3, Methods, Data collection & analysis) by clearing line conditions (Study 3)

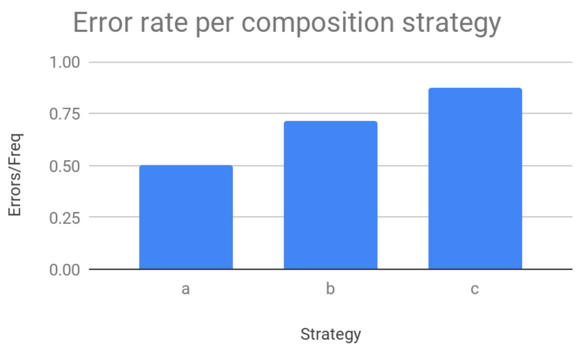

Figure 22. Error rate per composition strategy (Study 3)

Regarding the incorporation of the number line working area and its impact on the strategies and error rates (Q3), the results suggest a preference to compose the solution directly on the number line, i.e., strategies b) and c) (see Table 4). At the same time, results also suggest that children make less mistakes when they clear the number line before composing the solution (a and b). Working directly on the number line without clearing it (c) produces the highest error rate (see Figure 22).

Game 2. Broken Blocks

Regarding Q1, we might say that through this second instance of the broken blocks game we probed that that children understood the informational relations in the *COMB* mode (see Table 2), i.e., the mapping between physical blocks, touch action and digital feedback (see Table 5 and Figure 23).

	Right	**Wrong**	**Missed observations**
Answer	75	10	-
Perception	54	9	22

Table 5. Total right and wrong answers and perceptions among all participants of the Broken Blocks game (Study 3)

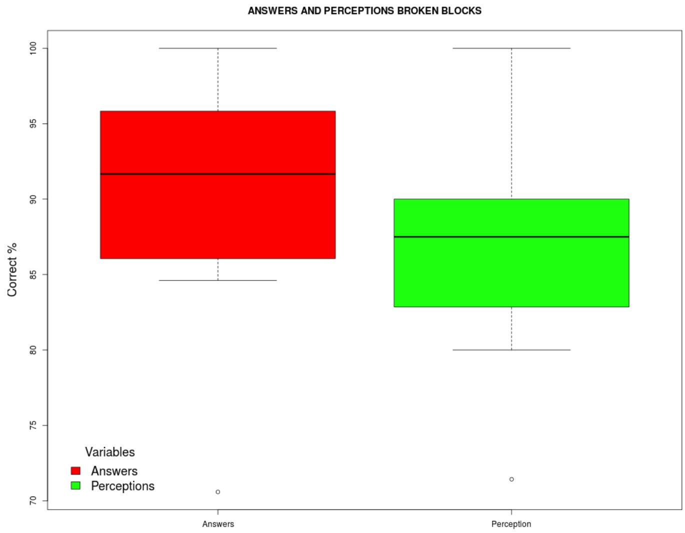

Figure 23. Broken blocks game results, Fast Vibration and Sound & Vibration unit feedback conditions (Study 2)

In addition, this test validates the usefulness of the *COMB* unit feedback as a number representation. One participant (S3.P6) relied only in vibration when perceiving the digital feedback during the broken blocks game achieving and good performance, showing that fast vibration can be understood. However, another participant (S3.P5) during the broken blocks game did not wait until the sounds were finished and this led to confusion and provoking errors. A possible solutions might be to always play the full unit cycle feedback even when the child already raised the hand.

6. Discussion

We conducted three user studies as part of a user centered design process around the development of LETSMath. These studies allowed us to observe and evaluate the (embodied) interaction of the children with different features of the system. As a result, key insights were gained in order to answer the research questions that were initially stated in this article.

This section address each research question making use of the TLDF and linking the results with the general design requirements stated in section 4.

Concrete model comprehension (Q1)

a) Physical Objects

During the first two studies children were able to accomplish the goals under the no feedback condition. Thus, we might conclude that the blocks' physical design is understood by children, i.e., they comprehend the mapping between the physical design and the number representation. Our design follows the same line than cuisenaire rods [68], representing the number through size and unit markers, elements over which we observed extensive utilization.
Moreover, these elements were used when scanning and recognizing blocks as well as during the composition task.

Even when children were not constrained to use the number line (low vision participants from the study 2), during the composition task they showed a tendency to align blocks horizontally, imitating a number line and taking advantage of this arrangement in order to partitionate and count. It is interesting to note that the magnets in the extremities of the blocks suggest this action and shape user's strategies. We might confirm that the designed system is **suitable for *active touch strategies* (DR4)** supporting the strategies proposed by Sicilian [55]: Preliminary scanning, count organizing and partitioning.

b) Informational relations (blocks)

The understanding of the informational relations (mapping between physical blocks, touch action and digital feedback) has been proved through the broken blocks games in studies 2 and 3.

We provided two digital representations of concrete numbers which were understood by children with VI. On the one hand, sound and vibration (S_AND_V) is a slower feedback which trigger a counting strategy. On the other hand, fast vibration (FAST_V) might be closer to an abstract representation providing a faster access to the number. In particular, we understand this "abstract" representation as a step forward Leuders [12] direction, providing vibrotactile material closer for simultaneous perception rather than sequential counting.

Lastly, combining both representations (COMB) children have two instances to perceive the number *through digital enhancement (f)* at different abstraction levels.

Children understood these digital representation, however they did not made use of them neither immediately nor for every task.

Digital feedback incorporation (Q2)

Once we proved the comprehension of digital feedback, we might discuss the effective incorporation of such number representations as active strategies to solve specific tasks.

a) Actions performed during block recognition (Q2-a)

Low vision group took advantage of their limited vision to perform the task, probably relying on the size or performing visual dot counting. Indeed, they only incorporated unit feedback during the Find block game. This suggests that despite the novelty of the auditory and haptic feedback, it was appropriated by children mostly as part of their strategies used for recognition.

Blind children mostly relied on dot counting to recognize the number of each block followed by size estimation. They also showed a higher utilization of the unit feedback (fast vibration as well as sound and vibration) in both games (Find Block and Composition) than low vision children.

b) Actions performed during number composition (Q2-b)

For the composition task, counting strategies are commonly exploited. In particular, dot counting was shown as an efficient strategy when it comes to count the composed number. All the participants performed dot counting while composing numbers, which is inline with the horizontal block arrangement (57,6%) which was observed as the most popular arrangement during the composition task (see figure 15). The model itself facilitate this action by the inclusion of magnets that suggest joining blocks. It is interesting to stress how once again the model affordances shape users' strategies as we have noticed in previous research [38, 98].

Low vision group did not make use of the *fast vibration* feedback at all and barely used the *sound and vibration* for the composition task. This might have several interpretations. Unit feedback does not represent combined blocks, it only plays feedback representing individual blocks, which is not useful to count the composition, only to recognize blocks. According to Sarama & Clements children who are learning to sum need to count to check the result of the operation [96]. Later on, when they start mastering number combinations, they rely on the knowledge of addends' cardinals to solve the operation without checking by counting [96]. In this case, children might take advantage of individual block recognition, but for low vision children it seems that observation is the fastest and easiest strategy to do so. As a consequence, unit feedback is ignored again.

Specifically regarding *fast vibration* feedback, in spite of the fact that does not provide feedback representing the composition, we hypothesize that it was even more ignored by low vision children because it might be perceived as a more abstract and symbolic representation, even closer to symbolic representation such as braille or written numbers are and, as a substantial difference with sound and vibration, it does not even trigger counting strategies. An abstract representation as *fast vibration* would not be helpful to accomplish the task.

To sum up, the fact that low vision children do not incorporate unit feedback for number composition seems to be aligned with number composition developmental theories [96], and the possibility that fast vibration is perceived as more abstract representation not triggering counting strategies make such representations specially interesting in the search of non visual number representations [12].

Blind children strategies at the find a block and composition task were similar They relied on *fast vibration* and *sound and vibration* in a similar proportion in both tasks. Our interpretation is that in their case it is harder to disentangle the recognition process from the composition task. This means that for blind children, the composition is a demanding task but previously they have to find the blocks that will take part of the solution. In this retrieval process they might also count the unit markers, perceive the digital feedback and compose in a more homogenous vein.

The number line working area influence (Q3)

a) Children appropriation and preferences

Results show a preference to compose solutions directly on the number line, i.e., strategies b) when they were forced to clear the working area, and b) + c) when they were not constrained to do so (see Table 4).

Place blocks on the working area is the only pragmatic action involved in the development of the solution. The opposite action is to *put blocks aside* in order to exclude them from the solution and reduce the interference when composing, an action that have been previously observed with sighted children in a similar context [30]. However, in a VI context, putting aside could mean to temporarily lose the location of the block, and as a consequence a re-scanning of the environment to retrieve it again, this could explain children's preference to work directly on the number line. In sum, this component fulfilled the ***suitable for active touch strategies*** **(DR4)** requirement.

b) A constraining role on the interaction pace

While clearing the number line might represent a risk of losing the blocks and force a re-scanning, results shows that children make less mistakes when: 1) They clear the number line before starting the composition (strategies a and b), and 2) when they compose on the table and then put the solution on the working area (only strategy a). To make it clearer, working directly on the number line without clearing it has the highest error rate (strategy c, see Figure 22).

Our interpretation is that this spatial restriction (working area - table) somehow split the working space in two and make them to slow down the interaction pace as it was previously suggested [29,30]. This extra time, when clearing the number line and rearranging the blocks on the table, might also discourage the trial and error strategy as well as force momentary withdrawal causing reflection [100].

c) Informational relations (Blocks, working area, feedback)

When a block is placed on the working area, a sound is played immediately in order to indicate that the system has detected it. This is an immediate feedback coupling the actions on physical objects with a digital response. In our case, such immediate feedback is not encouraging trial and error because does not provide any assessment on the task, i.e., the composition is not evaluated immediately but only when a new loop starts (see Figure 9-a).To make it clearer, putting a new block on the working area does not interrupt the current feedback loop. However, we observed (see Figure 20-O7) some cases where children do not understand that they have to wait and sometimes they showed anxious and removed the block from the working area before the system evaluation.

Many studies also reveal that slowing down the interaction pace is key in order trigger reflection [29,35,101]. Tangible environments allow fast and seamless exploration but it might not lead to reflection. Thus, delaying the system evaluation of the composition seems to be a good strategy to slow down the interaction pace giving place for monetary withdrawal and reflection [100]. In previous work with sighted children we named this delay as "action submit" [38]. The main difference with the current system is that in [38] an animation was shown on the screen with an explicit countdown. So, in VI context and specifically in our development, we might have further considerations and improve this waiting state communication to the user.

7. Future work

The system was tested through three user studies in different contexts. However, all these were “one session” user studies where generally users have little time to get used to the system. The novelty effect was also present and first experiences might be cognitive demanding since everything is new, and interaction rules have to be incorporated at the same time that math problems are solved. In this sense, intuitiveness is a key. However, tangible interaction is not only about intuitiveness, it might be helpful during the first approaches to the system, but in the long term it can also neglect user’s skills [102]. Lastly, sometimes also teachers have shown anxiety helping children instead of letting them to struggle with the system.

Therefore, considering the reasons explained above, we wonder how deep we were able to test and to which extent shall we complexify the semantics of use. Once the children have incorporated the rules and master the system, more complex and abstract mappings might be incorporated in order to explore and exploit the possibilities of the tangible environment.

Future work should include a longer term user study where children make use of the system in classrooms during a longer period of time. To this aim, the system could be incorporated in the curricula and contextualized as a classroom activity. This would also increase the ecological validity but maintain the experimental approach, which is a challenge when it comes to educational materials [69]. This might allow us to assess the learning gain and the efficiency of the system, for instance using pre and post tests as it was done before [98].

Additionally, we plan to incorporate and explore other representations that might add complexity but also expressivity and advanced interaction techniques. The increase of the complexity should be done as the children's skills and understanding of mathematical concepts also increase [103], giving them time to get used to the material in order to handle it more naturally and reduce the processing load [12]. Thus, we plan the gradual incorporation and testing of the following features:

- **Binaural sound**: Making use of binaural sound would be possible to provide *composition sound* (the number to be composed) and *composition feedback* (the current composition on the working area) at the same time. This requires certain mastery of the system and might be harder at the beginning. However, with stereo mode users can perceive the state of the current partial solution by the auditory feedback of the game. Thus, they would not need to scan the physical environment in order to compute the current composition on the number line. This could mean that somehow children with VI might have access to the whole system state in a continuous and faster approach, more similar to the sighted users’ experience.This hypotheses have to be tested with children with VI.

- **Symbolic number representation:** During the whole process we have tested different unit feedback modalities, including a slower pattern of sound & vibration and a faster vibration. However, more advanced designs might still be explored. As Leuders [12] suggests, it might be possible to represent numbers through prosody, i.e., to exploit auditory properties such as timber and mainly rhythmic patterns in order to represent numbers in a more symbolic manner, similar to braille but through sounds. In a similar vein, an analogous effect might be achieved by using haptic technology that enables to precisely control parameters as frequency and power, or even using electric vibrations [104].

- **Synchronized unit feedback**: One of the reasons that we attribute to the abandonment of the digital unit feedback by low vision children during the composition game (study 2), is that this kind of feedback only represents blocks individually. Thus, a future design consideration could be to provide synchronized unit feedback representing the composition done when blocks are joined and touched outside the working are. This might let children to test and perceive the composition through digital feedback before submitting the result on the working area.

To sum up, while the aforementioned features might increase the complexity of the interaction and semantics, we understand that in a long term use and potential mastering of the system, it might also provide children with richer and more abstract number representations beyond the individual blocks identification and sequential elements (dots, sounds) counting.

8. Conclusions

Nowadays children with VI learn mathematics in the school mainly using traditional manipulatives (building blocks and tiles) or in some cases specially adapted materials. This current approach present tree main problems. First, while active haptic exploration might be supported by these materials, auditory channel is not. Second, in general all these materials demand an active supervision and guidance from educators. Lastly, none of these materials permit the number identification through one gesture, even braille codes demands and active exploration and physical scanning. Thus, there is a lack of educational material that provide children with VI similar opportunities than sighted children to learn mathematics.

Building on cognitive theories and design requirements identified from previous related work, we have proposed a concrete model that exploits auditory and haptic feedback for mathematics learning especially but not exclusively for children with VI. Based on the model we designed and implemented a tangible system that provides multiple representation exploiting auditory and haptic sensory channels.

We validated this model through an iterative user centered design process in which 19 children with VI were involved as well as some educators from their institutions. This process let us to gain insights, validate interaction techniques and contribute with design knowledge in this context.

We probed that almost all the children understood the proposed concrete model as well as the narrative game, and they were able to train mathematical skills with this system showing an improvement in the use of the system during a single session. So far, however, we can not claim a learning effect as a consequence of the system, this has to be done through a longer term study as it was mentioned in the future work section.

We understand that the inclusion of this kind of tangible systems enabling the exploitation of auditory and haptic channels in schools is a step forward to the equality of opportunities for children with VI.

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6.Conclusions

In this chapter the main conclusions are presented with the following structure: First, section 6.1 is dedicated to address each research question. By summarizing the main cognitive background theories and the requirements that an embodied interactive system for learning might fulfill, we conclude that by following a responsible and properly informed design, such systems can benefit mathematics learning in several ways (RQ1). Embracing inclusivity, autonomy, abstraction and reflection within the learning domain. Next, in order to address RQ2, we detail how we actually incorporated such theories as design features within the three prototypes CETA, iCETA and LETSMath. We also discuss embodied interaction design implications for learning environments and reflect on similarities and differences of designing for children with and without visual impairments. Finally, in order to answer RQ3, we discuss some implications of the study presented in section 4 and confirm previous research in the same field.

In section 6.2 we present an extension of the Tangible Learning Design Framework (TLDF) [4] by proposing a new category called "*Offline Actions*". We understand that such category is missing in the original categorization and that it is relevant, thus it somehow complement and completes the framework.

Finally, in sections 6.3 we discuss the limitations of this thesis and future work, covering a research agenda that includes concrete steps to continue with this research.

6.1. Addressing research questions and thesis contributions

RQ1: To what extent and how embodied interactive systems might benefit mathematics learning?

According to the previous literature, while some studies report no benefit from physical interaction in a learning task [40, 11], there are others reporting the contrary [26, 5]. Thus, physical interaction is not enough to enhance the learning experience. And specifying which is the role of each interaction element for achieving a meaningful design taking real advantage of manipulation is required. From a theoretical point of view, this might imply the translation of cognitive theories into interaction design features.

We have developed three prototypes, CETA, iCETA and LETSMath augmenting the Cuisenaire rods [12], a traditional manipulative applied for mathematics learning. In the cases of CETA and LETSMath, we conducted user studies validating the proposed concrete models. Most of the children understood the meaning of digital and physical objects, and how they relate to each other through actions. In general, children had no problems playing the video games Bruno and Logarin and trained the number composition skill satisfactorily.

Regarding the development of interfaces for VI children, instead of translating GUIs into voice interfaces, this thesis proposes the exploitation of auditory and haptic channels, aiming to represent elements and actions from the learning domain making use of conceptual metaphors and schemas. Specifically, in LETSMath, given that children are in the process of acquiring the number cardinality, to represent a group of *n* objects without visual information that could be achieved with a simple gesture, we use *n* repetitions of a sound or a vibration. Results suggest that children actually understood this representation and were able to train mathematics while playing Logarin. So far, for this system, we are missing a long term formal evaluation in order to assess the impact on learning, which is part of the future work agenda.

Next, the most relevant background concepts and theories that motivated the design of our prototypes are summarised. We finally conclude the answer to RQ1 by summarizing a set of requirements identified from these theories and related work.

Main background concepts and cognitive theories (RQ1-a)

Cognitive offloading is a broad concept that refers to the possibility of lightning cognitive demands while solving a problem. In an embodied interaction context, this might be applied as the inclusion of actual physical objects and actual actions representing abstract concepts and operations. The creation of stable structures and external representations where information is “stored” in the environment and can be later retrieved to operate, decreases cognitive load and saves working memory. Thus, designers might incorporate these elements aiming at lightning cognition. When it comes to learning, manipulatives (traditional and augmented) enable not only physical manipulation but in general, also enable freedom for movement and rearrangement. Physically Distributed Learning (**PDL)** theory stresses the importance of the environment rearrangement in order to represent solutions, in our case within a mathematical context. From the interaction design point of view, this theory demands a deeper comprehension between abstract structures and the structures of the interactive environment. **Image schemas** (e.g., louder is more, shorter is less) might foster this link to emerge. In addition, the spatial rearrangement operations might give place to **conceptual metaphors** (e.g., aligning blocks imitating a number line), complementing the image schemas in the translation of abstract concepts into terms of more familiar domains.

When designing an interactive system, the inclusion of pragmatic actions is mandatory, otherwise, it would not be possible to solve the problems. One of the design challenges resides in the inclusion of complementary **epistemic actions**, those taken to make the problem solving process easier. Many of these epistemic actions might not be detected by the system, but they are still sensible for users. As interaction designers, we might incorporate affordances encouraging these actions to be taken. This is how embodied interaction actually aid users to reveal useful information by changing the environment, and as a consequence, we might say that this is where “real benefits” from physical interaction starts.

Narrowing the design domain specifically to mathematics learning, our designs are inspired in the **approximate number system theory**, which states that numbers are understood as a group of items and, therefore, the incorporation of representation that highlights this composition of units is key.

All the aforementioned concepts and theories composed our theoretical cognitive framework. We pretend neither to say that these are the only theories to be considered nor that we are the first ones taking inspiration from them. However, we find them suggestive and with generative power at the time of answering if embodied interactive systems can enhance mathematics learning. Next, in order to conclude addressing RQ1, we summarize some of the main requirements that embodied interaction systems for learning should be covered.

Main requirements that embodied interactive systems for learning might cover (RQ1-b)

One of the main limitations of some current embodied interactive systems is their size. For instance, tabletop or projector based interactive systems demand too much space. As a consequence, this issue also prevents them from being included in classrooms, a fact previously noted [14, 32]. Thus, as far as possible if circumstances permit it, we might consider designing suitable technology in terms of ***portability and size*** that could potentially be included in classrooms. Additionally, ***continuity*** as the act of building on previous manipulatives might encourage the acceptance and incorporation from teachers, and therefore the inclusion in classrooms.

Guiding children in a smart way has been pointed out as critical drawbacks of tangibles of VI children [18] as well as a key design guideline for tangibles interactive systems for children with full vision specifically for mathematics learning [26]. While it might not be the only valid approach, we identified ***storytelling*** as a powerful resource to provide guidance and autonomy as well as to stimulate engagement and reflection [10].

The ***inclusivity*** requirement encourages the design of inclusive and collaborative systems where VI children and children with full vision can work together [39, 36]. At this point, we would like to remark on some important issues and concepts to have in mind when designing inclusive learning tangible systems. First, the information presented only as audio output might be simple to develop but it would not work. While sighted users can perceive a lot of information in parallel from the visual channel, VI users will have to process the information sequentially through the auditory channel [6], placing them in a handicapped position. Second, it would be really difficult to design conceptual metaphors, foster image schemas, enable epistemic actions and apply PDL just through an audio based interface. Lastly, sighted children would be deprived of exploiting their vision placing them in a disadvantaged position and therefore not contributing to satisfy the inclusivity requirement. The complexity of designing inclusive systems goes far beyond the translation and serialization of GUIs into voice based interfaces or the ad-hoc incorporation of braille labels to already tangible systems. In particular, in a mathematics learning context, the ***suitability for active touch strategies*** is key. In particular, supporting preliminary scanning, count organizing and partitioning as Sicilian explains [35]. Finally, ***digital enhancement for number recognition*** implies providing digital feedback in the form of sounds and/or vibrations complementing physical structures. If we are able to design feedback that children perceive as abstract numbers by non sequential strategies, then we might be a step closer to Leuders' [22] objective, i.e., provide tactile and acoustic material equivalent to visuospatial representations, letting VI children perceive "at a glance" and process bigger amounts of information in parallel, and finally having a similar experience to sighted children. Then, we might summarise these requirements as ***inclusivity and perceptual parity opportunities***.

RQ2: How might an embodied interactive system be designed in order to enhance the mathematics learning experience?

This dissertation aims to go beyond the design contributions in terms of novelty, which could be relative considering that many of the design features from our prototypes were at least already suggested by previous research and/or background theories. However, we contribute to the translation and interpretation of theories into design features and to the application of already suggested embodied interaction guidelines. We also contribute with the implementation of concrete instances (artifacts) which were user tested validating our designs and confirming previous related work.

Next, we present the two main implications to design of embodied interactive systems for enhanced mathematics learning (RQ2): Abstraction shaping (RQ2-a) and Embodied Interactive Mediated Reflection (RQ2-b). Then, we summarize the actual implementation of theories and background concepts within our prototypes CETA, iCETA and LETSMath (RQ2-c, see Table 1). Finally, we conclude reflecting on the similarities and differences when designing for sighted children and children with visual impairments (RQ2-d).

Embodied interaction design implications for learning environments

Two main implications for designing embodied interactive learning environments have emerged from this thesis. First one refers to the need to shape the level of abstract representations exposition, and second refers to the strategies required for support embodied interactive mediated reflection. In the following section we describe its rationale and application.

Abstraction Shaping (RQ2-a):
Children are intended to learn abstract concepts such as symbolic number representations or mathematical operations. Manipulatives might help in this process by fostering links between physical experiences and those abstract concepts [25]. In an embodied interactive environment, we can expose children to abstract representations and encourage them to reflect on how entities relate to each other, for instance how physical and virtual blocks are connected through the composition operation. Gradually incorporating more abstract digital representations we expect to scaffold children acquisition of numerical properties; that can be understood as the passage from concrete to abstract conceptions.

As it is suggested in [4], from an embodied perspective, this could be achieved by using conceptual metaphors through the design of appropriate informational relations. Depending on the perceptual channel, such metaphors could be graphically represented, through the auditory channel or even haptically [32]. Shaping gradually such exposition might be key to scaffolding abstract thinking, i.e., presenting too abstract representations from scratch could generate confusion or misunderstanding. We provide different solutions in order to shape the level of abstraction for children with visual impairments (LETSMath) and without (CETA).

In CETA we changed the structure of the mappings between the physical (rods) and digital objects (on-screen robot called BrUNO). In other words, we manipulate digital representation concreteness in order to present children the kind of transformation that occurs when a number becomes another as the result of an arithmetic operation. We achieved three levels of abstraction, the most basic where each physical object is represented as a digital object; the intermediate where many physical objects are represented as a single digital object and lastly the most abstract where many physical objects were represented as actions of the robot. This means, going from schemas more similars or closer to those relationships (one physical element represented with one virtual element), i.e., more concretes, to more abstract associations, tracing a gradual path. In all these cases we mainly exploited visual feedback by changing digital objects' design and behavior on the screen. However, with this approach, the abstraction is provided during the feedback phase, i.e, the robot aspect or actions. As a consequence, it does not necessarily affect children's strategies.

When working with VI children in LETSMath, we decided to change the approach in order to shape the abstraction level for two reasons: 1) we could no longer rely on visual digital objects, 2) we aimed for children to perceive the abstraction and therefore generate an actual impact on their strategies. Thus, we designed different unit feedback played on the physical objects: S_AND_V, FAST_V, and COMB. These modalities were perceived by children provoking changes in their strategies depending on their visual condition and the task (find a block or composition). While S_AND_V provided slower feedback and triggered counting strategies, we hypothesize that FAST_V was perceived as a more abstract, closer to a symbolic representation. Lastly, the COMB unit feedback provided children with two instances to perceive the number at different abstraction levels through digital enhancement.

Embodied interactive mediated reflection (RQ2-b)

Aligned with previous research [26, 4, 31] we confirmed the key importance of modulating interaction pace along the learning activity. We confirmed that **direct physical manipulation enables fast and easy exploration**, which is not necessarily negative but it has to be modulated. In both cases, CETA and LETSMath, we observed that when the interaction pace is not lowered, children tend to perform trial and error strategies and make more mistakes.

The exploration phase has to be balanced with reflection time in order to give place to cognitive growth shaped as the Ackerman's "ongoing dance" [1]: Dive-in (explore) and step-out (reflect).

We proposed the strong concept *Embodied Interactive Mediated Reflection* (chapter 4) aiming to encourage the design of interactive elements within highly exploratory systems that allow reflection. During this thesis, we have explored two techniques to slow down the interaction pace and give place for reflection: feedback modulation and physical constraints.

Feedback modulation: In CETA we successfully implemented the feedback modulation through what we called *Action Submit*. Adding a delay between children's actions and system evaluation presented good results avoiding trial and error strategies. At the same time, by showing an animation on the screen, the system clearly communicated that the evaluation was in progress, so this idle time was completed with the animation (sound and graphics) and naturally incorporated by children.

In LETSMath the scenario was slightly different, when children place a new block on the working area the system does not evaluate the composition until the current sound loop is finished (see chapter LETSMath-Figure 9). During this time, the children have to wait, and in spite the system playing a single sound indicating that a new block was placed on the working area, some children showed themselves anxious and removed the block from the working area changing the solution before getting the system evaluation. In other words, the way we communicated that the system was evaluating the solution was clearer in CETA than in LETSMath, and it was probably due to the fact that we used two channels (visual and auditory) and because the animation covered the whole waiting time.

We might conclude that feedback modulation is an effective strategy to lower the interaction pace, specifically adding delays between actions and system evaluation. However, it must be carefully implemented in order to avoid extraneous cognitive loads or misunderstandings.

Physical Constraints: The interaction space in LETSMath is divided in two areas: table and the number line working area. When children cleared the number line before starting a new composition, they performed better. Clearing the number line takes some time lowering the interaction pace giving place to reflection and avoiding trial and error strategies. Thus, aligned with previous research [4, 5], we confirm that physical constraints are an effective strategy when it comes to modulating the interaction pace in tangible environments.

Translating theories to embodied interaction design (RQ2-newC)

In Table 1 we detail how elements related to embodied cognition and embodied interaction theories, that were described in this thesis, were actually incorporated as system features in the three prototypes CETA, iCETA and LETSMath.

Embodied theories	**System features**		
	CETA	iCETA	LETSMath

<table>
<tr><td rowspan="5">Image schemas and conceptual metaphors</td><td colspan="3">Blocks ranging in length represent the image-schema “shorter is less, longer is more”</td></tr>
<tr><td colspan="3">Joining blocks means addition (magnets affords this action)</td></tr>
<tr><td colspan="3">Joining make blocks to be aligned, simulating the number line representation (conceptual metaphor)</td></tr>
<tr><td colspan="3">Abstraction Shaping: change informational relations between digital and physical objects</td></tr>
<tr><td>Physical blocks, virtual blocks and actions were mapped providing 3 levels of abstraction (one-to-one, many-to-one and objects to actions)</td><td>Using different thematic sounds</td><td>Using different thematic sounds

Unit feedback modalities (auditory and haptic channels) and representations (faster, slower or combined patterns)</td></tr>
<tr><td rowspan="3">Epistemic Actions</td><td colspan="3">Blocks can be grouped and can be set aside</td></tr>
<tr><td colspan="3">Block can be joined and are provided with count units</td></tr>
<tr><td>Using visual markers</td><td>Using the tactile divisions</td><td>Using unit dots</td></tr>
<tr><td rowspan="2">Enable active exploration and environment modification and reinterpretation supporting PDL</td><td colspan="3">Blocks can be freely moved</td></tr>
<tr><td>Individually or in groups</td><td>Individually or in groups</td><td>Mainly individually, the size does not afford group moving</td></tr>
<tr><td rowspan="2">Provide stable structures (scaffold composition,</td><td colspan="3">The working space is delimited</td></tr>
<tr><td>Working area delimitation on a sheet of paper</td><td>Physical delimited working area and storage box</td><td>Wooden number line working area</td></tr>
</table>

<table>
<tr><td>afford and suggest actions)</td><td></td><td></td><td></td></tr>
<tr><td>Distribute information encouraging translation between sensory channels. Multimodality increases effective working memory</td><td>Haptic (passive): size and shape;andvisual: colors, unit subdivisions</td><td>Haptic (passive), auditory and visual</td><td>Haptic (passive and active), auditory and visual</td></tr>
<tr><td>Approximate number system theory (numbers as a group of items)</td><td>Blocks visually subdivided in units</td><td>Blocks haptically subdivided in units

Sounds matching number cardinality (on the tablet)</td><td>Physical unit dots
Sounds matching number cardinality (on the laptop)

Sounds and vibrations matching number cardinality (unit feedback on the blocks)</td></tr>
<tr><td>Modulate Interaction Pace to encourage reflection, prevent trial and error strategies</td><td>Delayed system evaluation with an on-screen animation</td><td>Delayed system evaluation</td><td>Delayed system evaluation

Physical constraints imposed by the wooden number line</td></tr>
<tr><td rowspan="3">Real-time immediate feedback and system guidance</td><td colspan="3">Hints and guidance when children get stuck</td></tr>
<tr><td rowspan="2">Virtual one to one on-screen representation of physical blocks</td><td colspan="2">Sound indicating that a block is detected on the working area</td></tr>
<tr><td>Hints and guidance when children get stuck</td><td>Hints and guidance when children get stuck

Immediate unit feedback when blocks are touched</td></tr>
</table>

Storytelling	BrUNO game	iCETA Logarin game	LETSMath Logarin game

Table 1. Actual implementation of theories and background concepts in prototypes CETA, iCETA and LETSMath

Similarities and differences when designing for sighted children and children with visual impairments (RQ2-d)

Perception and feedback: In the case of sighted children, the visual channel is predominant, the most important information is conveyed graphically. In CETA, we used graphical representations to state the challenge (number to be composed) represented as a distance between the robot BrUNO and the spare parts to be collected. We also implemented images-schemas and conceptual metaphors (horizontal and vertical number line) and progressive abstraction shaping (changing virtual objects and actions on the screen) based on graphical representations. Also the story of BrUNO is mainly graphical. However, in environments like CETA where the visual attention is split (table and screen), the auditory channel could be exploited to integrate them. For instance, when a child is diving-in the solution by physical manipulation of the blocks, her visual attention is fully on the table, an alternative to get her attention is by playing a sound in order to notify that an event is taking place on the screen. After conducting the user studies described in chapter 2.2, we incorporated a sound during the Action Submit countdown. This strategy was successfully tested along the study of section 4.1.

When it comes to the design of environments for children with VIs, the auditory and haptic channel gain importance. They mainly perceive the world through these channels, so the information must be available through such modalities. In iCETA and LETSMath the composition challenge was represented through consecutive sounds. This approach serializes the information and triggers counting strategies. Thus, in LETSMath we combined auditory and haptic (vibrations) feedback incorporated in the physical blocks (unit feedback). As it was previously noted by Leuders [22], in VI context a material able to represent numbers beyond counting strategies would be valuable. In tangible systems, children with VIs use auditory and haptic channels to recognize and solve the problems, while sighted children use their vision to recognize the elements and then they manipulate to solve the problem. This does not mean that haptic and auditory channels are not important for children with full vision, but we are trying to stress that for VI children those channels are fulfilling a double objective: recognize the blocks and compose the solution. Therefore, auditory and haptic feedback might be clear and conveniently combined avoiding saturation. For instance, playing the composition sound in the laptop and the unit feedback sound in each individual block was confusing for many children during study 1 presented in chapter 5.4.

In a more general approach, we might recommend the distribution of information across modalities but preventing it from overlapping important information or generating noise. Moreover, children with VIs, especially those who have limited vision, use their limited vision when possible, thus materials for children with VIs, should reinforce the visual channels, with features appropriate for limited vision, including high contrast colors and size adaptations.

Actions: Phenomenology theory states that there is no perception without action [28], therefore as feedback and perception varies depending on the visual condition, actions taken might also do it consistently.

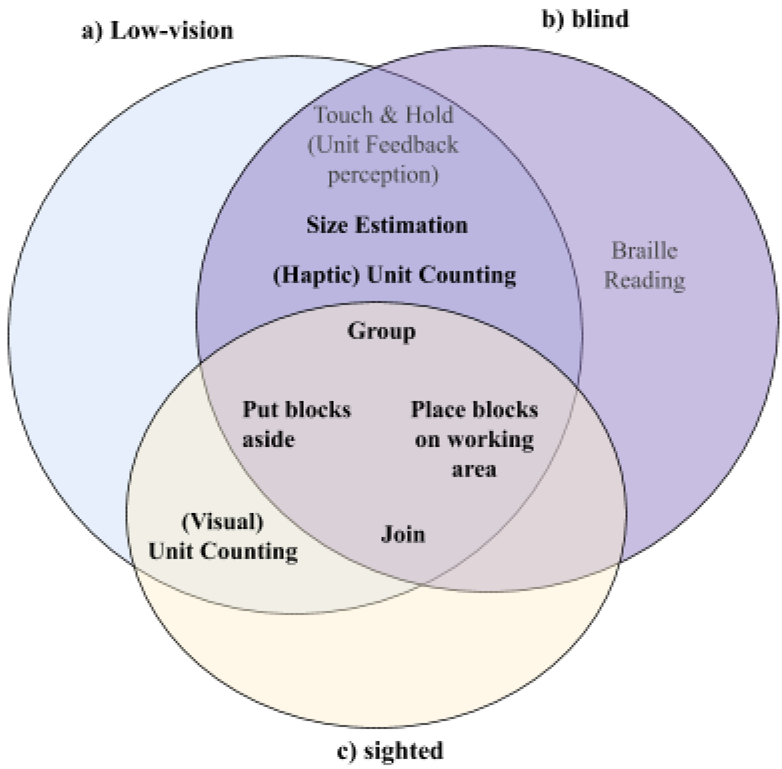

Figure 1. Actions observed in CETA and LETSMath classified by visual condition

It might be the case that epistemic actions like Group, Join and Put blocks aside are taken by the three groups of children (low-vision, blind and sighted, see Figure 1). From the interaction design point of view we conclude that: **a) Environment rearrangement is key**: all these actions implicate the free movement of blocks, **b) *Offline actions* have to be though as part of the system and be afforded by it**: None of these actions can be sensed by the system and they do not have a direct impact on it. However, children under any visual condition used them, **c) Actions strongly linked with the learning domain/task might afford across modalities and independently of the visual condition**: Joining, grouping and ungrouping or placing aside are actions strongly linked to number composition. Probably boosted by internal image-schemas and conceptual metaphors, all the children executed these actions.

In relation to c) we might also note that Unit Counting is an action involved in the composition strategy across modalities, i.e., sighted children and part of children with low-vision performed visual unit counting while blind children and part of children performed unit counting haptically. Thus, as a general implication **d) we should afford intrinsic solving problem actions across modalities within inclusive systems.**

Lastly, unit counting might have a double function. On the one hand, it takes part of the problem solving strategy, probably offloading cognition when blocks are aligned and units are individually counted. On the other hand, they are involved in a recognition strategy to identify the value of a block.

RQ3: Which is the impact of CETA in terms of learning gain and children's strategies in comparison to pure virtual and traditional approaches?

In chapter 4 we presented the long term study comparing CETA (TI group) with a pure virtual version of the game (VI group) and a control group following traditional teaching practices (CO group). Results show that children using CETA, i.e., tangible manipulatives, significatively improved their post-test scores compared to the control group. Children using virtual manipulatives also improved their post-test scores but not significatively. We highlight the effective usefulness of technologically augmented tangible manipulatives to practice mathematical skills through an assisted number composition task.

In both cases blocks could be manipulated, in the virtual scenario dragging them on the screen and in CETA physically. However, results show that children using CETA used significatively more blocks than children using the virtual version of the game within the same number composition task. In this particular scenario, it might be that grasping, dragging and joining blocks (with magnets in the extremities) was easier and faster than on the screen. That is to say, physical objects' affordances do shape children's strategies.
Such phenomena confirms previous research where the high explorative power of physical manipulatives had already been

highlighted [24, 4, 31]. In addition, our results show that children with a higher improvement were those who used more blocks at the beginning. Thus, it might be the case that tangible manipulatives encourage exploration and trigger a more extensive exploration of the solutions' domain, and finally a deeper understanding of the underlying phenomena, which has also been pointed out as positive when it comes to mastering basic number combinations [34, 8].

To sum up, during this study we were able to confirm and expose embodied interaction properties that shape children's learning experiences. Once again, depending on the context we might incorporate them in embodied interactive systems in one way or another.

6.2. Going beyond the scope of the thesis

Extending Tangible Learning Design Framework

Along this dissertation we have been using the TLDF as our main design framework, applying some of the suggested embodied interaction design guidelines and conceptualizing our designs (see chapter 2-Design conceptualization and chapter 5.4-Design rationale) under the five element's taxonomy: Physical Objects, Digital Objects, Actions, Informational Relations and Learning Activity [4].

This framework has shown to be a useful tool with generative power. However, when it comes to the classification of actions we found a limitation that we would like to address contributing to extend the framework. Antle et al. [4], provide the following definition:
"Actions on objects are the set of input manipulations that learners can take on the physical (and in some cases digital) objects that are sensed by the system; for example, tracking the speed with which a learner changes an object's position or orientation."

There exist other actions that might be taken on the physical objects but are not sensed by the system. For instance, in our prototypes, some of these actions are: unit counting, braille reading, size estimation, grouping and joining. Those actions might also be taken into account for the design and analysis of the system. What is more, some of these actions are complementary epistemic actions [20], that make problems easier to solve and offload cognition onto the environment, therefore they are relevant in this context. To address this gap, we propose to add a new element to the taxonomy defined as:

Offline Actions: Actions on system's objects that are not sensed but still have a (probably complementary) role in the problem solution, for instance some epistemic actions. Even when offline actions are not able to be sensed, they might be taken into account when designing the system, making use of affordances, narratives or other system properties to encourage or discourage them depending on the context.

Our intention by adding this category is to encourage and facilitate interaction designers to think about those elements that even when they can not be detected or sensed by the system, they have an effect on users' cognition, and in our specific case on the learning experience.

6.3. Limitations and future work

The observations and conclusions made in this thesis about how children interact, perceive and incorporate the proposed concrete models might not be definitive. The prototypes were informed with cognitive theories and observations were focused on children's actions and perception. We consider that in order to generalize, confirm and dig deeper in the relation of materials and children's strategies, the prototypes should have a higher finishing level closer to a final product. For instance, in CETA, sometimes the computer vision detection system failed. In LETSMath the sensors' accuracy, resolution and the body finish should be improved in order to provide a richer perceptual experience isolating channels, for instance eliminating the collateral plastic sound provoked by the vibration. This kind of technical improvements might allow us to design with more precision and at the same time to assess the interaction with higher reliability, eliminating extraneous load provoked by technical failures. Nevertheless, we achieved high level functional prototypes augmenting the Cuisenaire Rods either for VI children as well as children with full vision, where they were able to train mathematics skills.

In regards to LETSMath we are missing a longitudinal study in order to assess the learning gain and also the interaction and usability after several sessions. This might mitigate the novelty effect and extra cognitive loads during the first approach to the system enabling us to validate our results in more ecological environment and also generalize them to other domains. Nevertheless, for this prototype, we conducted three user studies in two countries under different contexts. Children, teachers and school directors collaborated with the design process giving precious feedback which was taken into consideration when iterating over the system design. Many researchers participated during the experiments and data analysis, collecting formal evidence that makes our contribution relevant.

Despite the aforementioned limitations, this dissertation contributed to the research community with formal theory informed designs and showed that there is place to enhance tangible learning materials with technologies beyond virtuality and screens.

We plan to continue towards the development of abstract representations through auditory and haptic channels. Regarding LETSMath this means the development of abstract number representations through auditory and haptic feedback, beyond braille and serial sounds counting. It is not clear to what extent this kind of abstract representations could be incorporated and mastered by children. We hypothesise that through one session evaluations we are just testing the basic and first interaction, i.e., the tip of the iceberg. Everyday we use skills that were not developed in "one session", for instance, walking or driving cars, it takes long to master such abilities. Thus, we can not expect children to understand and incorporate new abstract representations on the fly, however, it does not mean that they are not able to master and make use of such alternatives over time. To address this issue, long term studies might be carried out allowing for system mastery and hopefully a deeper comprehension and exploitation of the proposed technology.

In a similar vein, we also plan to incorporate the binaural sound proposed in iCETA for LETSMath. Such design might allow VI children to perceive the composition sound (N) and the current composition on the working area. This approach was barely explored and not formally evaluated in iCETA, but again, it might take some time until children get used to it and take real advantage of it. The aim of such design is to enable VI children to compute the difference between N and the current composition on the fly. In other terms, we wonder if such multichannel auditory representation would enable children to perceive the state of the system in a similar way that children with full vision do at a glance.

To this aim, we plan to run experiments with LETSMath using two experimental conditions: 1) The game only provides the composition sound, i.e., as we have done until now; 2) The game provides stereo feedback, i.e., the composition sound and the current composition on the number line. With the stereo mode, users can perceive the state of the current partial solution by the auditory feedback of the game. Thus, they would not need to scan the physical environment in order to compute the current composition on the number line. This could mean that somehow VI users might have access to the whole system state in a continuous and faster approach, more similar to the sighted users' experience. However, this hypothesis has to be tested in a real world environment.

Lastly, we plan to incorporate accelerometers in the prototypes in order to measure the movements of each individual block. This way we might obtain statistical data related to the manipulation of the blocks. This data might be explored in post experimental sessions but can also be processed in real time in order to measure and modulate the interaction pace.

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Appendix A: CETA complementary material

The mixed-reality environment CETA was developed as an open and free educational system. All the software sources are published under GPL 3.0 license and can be accessed in the following repository: https://github.com/smarichal/ceta/. This software can be deployed in low resources tablets running Android.

The system is implemented in two main modules:

1. BrUNO game, an android application running the core of the game. The GUI is implemented here.
2. CETA vision core library. This module has access to the tablet's camera and is in charge of the marker detection and blocks' recognition. This module detects aligned sets of TopCode markers [17] and provides the coordinates of each detected block and the rotation angle. It also has a robust system supporting partial occlusions of the blocks.

Regarding the hardware, all the designs can be also found in the same repository. This includes the ready-to-print A4 formatted sheet with the makers (see Figure A1).

Figure A1. CETA markers ready to print

The 3D models of the blocks, the mirror gadget (see Figure A2) and tablet holder can also be downloaded from the repository. The model of the holder is in svg format ready to be laser cut (see Figure A3).

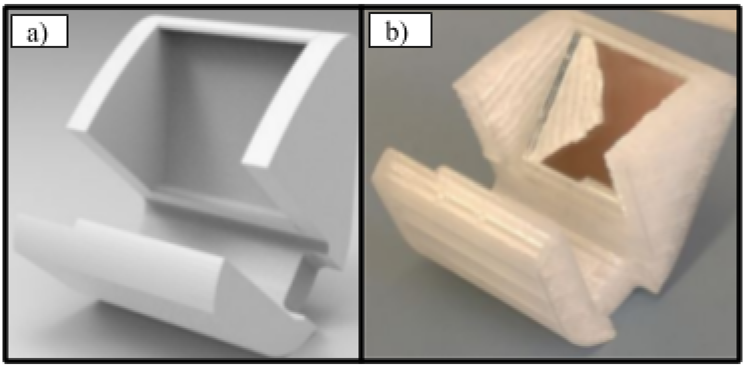

Figure A2. a) 3D model of the mirror gadget b) Mirror gadget 3d printed (with the mirror installed)

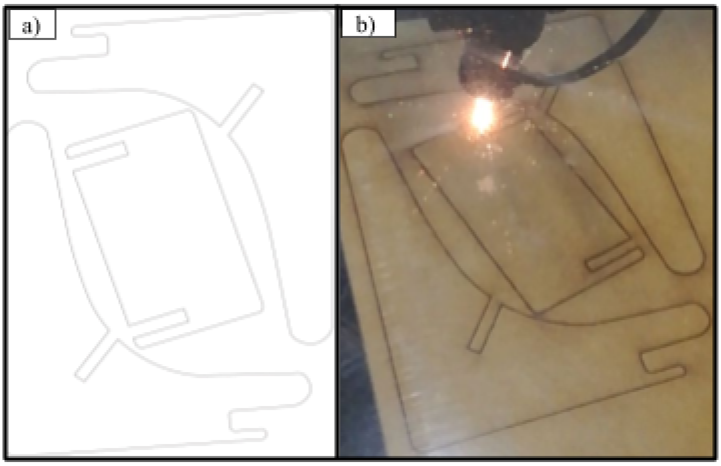

Figure A3. a) Tablet holder model b) Tablet holder being laser cut

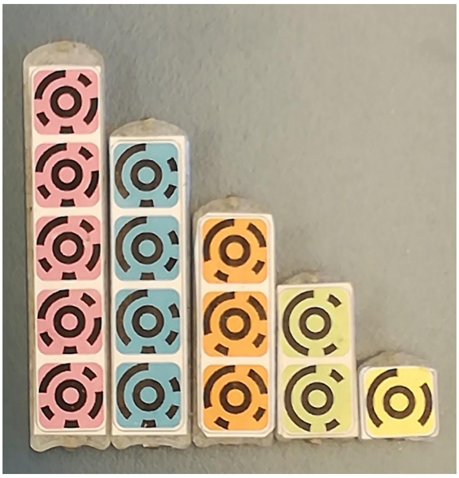

Figure A4. 3D printed blocks with markers on top

Once the blocks and markers are printed, we just have to put the markers on top of the blocks (see Figure A4). Actually, if there is no possibility to 3D print the blocks, the makers can be pasted over any other object representing a block.

We encourage researchers and educational centers to replicate the system and freely use it and improve it.

Appendix B: Additional research on users interfaces for robotics

During the first five months of this dissertation the author did an internship at INRIA in the city of Nancy, France. During this stay he took part in a Human Robot Interaction project whose aim was to design an intuitive user interface for a robotic arm intended to be manipulated by non-experts users. A user study was conducted at LARSEN laboratory giving place to conference paper published on the International Conference on Social Robotics.

In spite of the research field is not directly linked with this thesis, it belongs to the HCI field and during the internship he was able to explore and learn valuable techniques such as designing and conducting focus groups and user studies. Some of these tools were later applied during the dissertation.

One-Shot Evaluation of the Control Interface of a Robotic Arm by Non-experts

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Available:
https://hal.inria.fr/hal-01353809/document

One-Shot Evaluation of the Control Interface of a Robotic Arm by Non-experts

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Abstract. In this paper we study the relation between the performance of use and user preferences for a robotic arm control interface. We are interested in the user preference of non-experts after a one-shot evaluation of the interfaces on a test task. We also probe into the possible relation between user performance and individual factors. After a focus group study, we choose to compare the robotic arm joystick and a graphical user interface. Then, we studied the user performance and subjective evaluation of the interfaces during an experiment with the robot arm Jaco and N = 23 healthy adults. Our preliminary results show that the user preference for a particular interface does not seem to depend on their performance in using it: for example, many users expressed their preference for the joystick while they were better performing with the graphical interface. Contrary to our expectations, this result does not seem to relate to the user's individual factors that we evaluated, namely desire for control and negative attitude towards robots.

Keywords: Human-robot interfaces · User evaluation · Individual factors · Non-experts

1 Introduction

In this paper, we address the question of the preference for a robotic interface by non-experts (or naive users without training in robotics), after one single evaluation of such an interface on a simple task. This refers to situations when non-experts face the decision of adopting a robot for episodic use (i.e., not a regular continuous use as workers in factories): the ease of use of an interface is crucial for the robot acceptance. We do not target users that could have or

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will have the time to receive a proper training on how to use a robot. While in manufacturing, robots are used by skilled workers that receive a proper training for operating the robots, this training is not likely to happen for many assistance and service scenarios: for example, inside an healthcare facility it is likely that the nurses or the patients will never receive a proper training for operating and interacting with the robots. The question arises on how to make the robot easily controllable by such users and facilitate their interaction with the robot. As the interface for controlling the robot is an essential part of the robotics system, this question impacts not only the interaction performance, but also the user acceptance and final adoption of the technology.

In this study, we focus on the Kinova Jaco (see Fig. 1), a lightweight robotic arm which can be controlled with a built-in joystick. It was designed for a daily and regular use for ordinary people after some training: the joystick is easy to manipulate but it has several buttons and control modes that require practice to achieve a fluent interaction. Here, we target a different use and a one-shot evaluation: if the control interface is an obstacle to the use of the robot, the users will not likely adopt the robot even for sporadic use. Several interfaces for robot control have been investigated in HRI. For example [17] investigated touch, speech and gestures for teaching a robot a nursery rhyme, finding that users do not prefer a particular modality but enjoy less touching the robot. In [16] the authors compared haptic interfaces with buttons, finding that users preferred buttons for simple tasks and physical command for complex tasks requiring high precision. Here, We compare the joystick with a ad-hoc graphical user interface (GUI) with buttons.

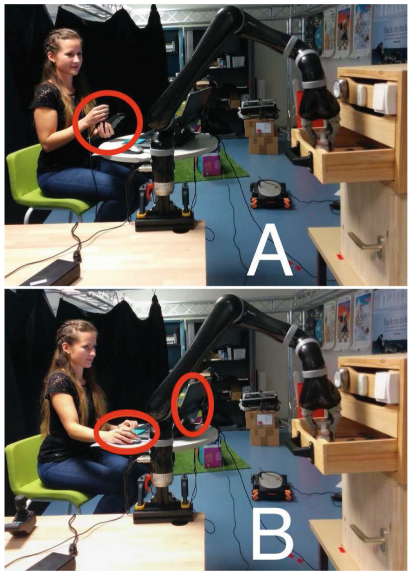

Fig. 1. The experimental setup with the Kinova Jaco arm. The participant moves the arm using (A) the joystick and (B) the graphical interface on the laptop.

We are here interested in *(i)* probing the relation between individual factors and user performances for robot interfaces, and *(ii)* studying the relation between the performances that the user achieve with such interfaces and their preference.

Our main hypothesis is that the preference of an interface is related to the performance of using it. This premise is evident from other studies focused on interfaces evaluation. Guo & Sharlin noted that preferences for a tangible interface was related to a stronger performance in using it [15]. Many studies on control interfaces for robots focused on graphical user interfaces for their better acceptance by non-experts, for example [6] for teaching objects to a robot, [4] for applications in rehabilitation and medicine. In [5] the authors proposed an Android interface for moving the Jaco arm, but unfortunately it was not thoroughly evaluated by final users.

Our second hypothesis is that individual factors, such as traits and attitudes, may influence the user performances with the robot interfaces. There is indeed prior evidence that some personality traits have significant effects on the perceived ease of use of new technologies, such as smartphones [7]. There is also evidence that personality traits and attitudes have some influence in HRI in the context of social robotics [1]. It seems therefore rational to explore the relation between individual factors and the user perception and performance in controlling a robot. Two attitudes seems particularly relevant for our study: the Negative Attitude towards robots (NARS) [3], which captures the anxiety of an imagined interaction with a robot, and the Desire For Control (DFC) [8], which captures the attitude to be in control or control situations. The first could influence for example the time spent on using the robot, while the second could influence the preference for an interface that provides a stronger sensation of controlling the robot.

Our study was split in two phases. In the first, we carried out a focus group study to identify the main concerns of people interacting with a robotic arm, the key elements underlying their imagined interaction and the imagined interfaces to control the robot movement. This set enabled us to formulate the first hypothesis and choose a graphical user interface (GUI) as an intuitive interface alternative to the Kinova joystick. The second phase concerned the experiments with the Jaco robot and the two interfaces. We first performed a pilot study with University students to test the experimental setup and gain preliminary insights for the later final experiments with ordinary adults. The analysis of the pilot study and the outcome of the focus group enabled us to refine the evaluation questionnaires to be used for the final experiments and formulate new hypothesis.

We studied the user performance and subjective evaluation of the interfaces during an experiment with the robot arm Jaco and $N = 23$ healthy adults. We provide quantitative evidence of the different performances obtained by non-experts, using both interfaces for the first time to realize some tasks. We also report on the user feedback in using the two interfaces, which provides us useful information to inform future interface designers.

Our preliminary results show that the user preference for a particular interface does not seem to depend on their performance in using it: for example, many users expressed their preference for the joystick whereas they were better performing with the graphical interface. Also, contrary to our expectations, this result does not seem to relate to the user's aforementioned individual factors.

Research Hypothesis - Given the previous results in the literature, we expect that the GUI will be easier to use than the joystick, for non-trained users. The GUI has the advantage to not require too much training, and it provides some graphical shortcuts to the main robot configurations. To provide a quantitative

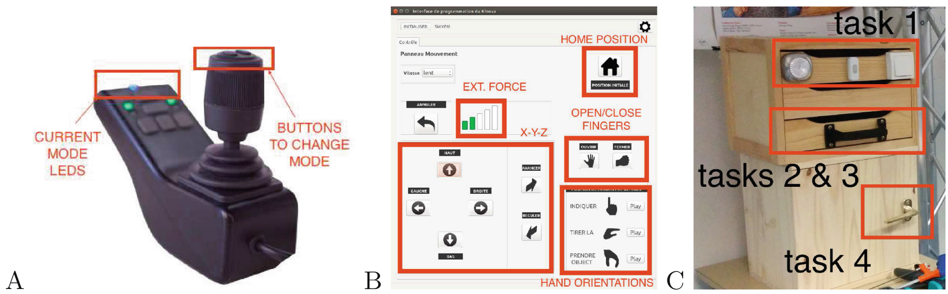

Fig. 2. The two interfaces used for the evaluation: (A) the Kinova joystick and (B) our ad-hoc graphical interface on the laptop. (C) The Activities of Daily Living setup: click the three buttons (*task 1*), open a drawer (*task 2*), take an object inside the drawer (*task 3*), open the door (*task 4*).

measure of the ease of use, we use the duration of execution of tasks performed with an interface, and the number of errors done while using it. We formulate the hypothesis as:

(H1) *The time necessary to complete the tasks with the GUI is shorter than with the joystick.*
(H2) *The number of precision errors with the GUI is lower than with the joystick.*
(H3) *The number of mapping errors with the GUI is lower than with the joystick.*

We also hypothesize that the user personality, attitudes and their prior experience with related technologies may influence the user acceptance of the proposed technologies and the performance in using it. The desire for control could play a crucial role in the preference for the joystick to the GUI, as the users could have the impression to be more in control of the robot while moving it. The negative attitude towards robots could influence the user perception of the interaction and the perceived ease of use. We formulate therefore the following hypothesis:

(H4) *Participants with high score of DFC will prefer the joystick to the GUI.*
(H5) *Participants with a high negative attitude towards robots score will make more errors and have a lower perceived ease of use and user satisfaction.*

2 Methods

Participants. The participants were all French, healthy adults that volunteered to take part in the study. The focus group study was carried out with 6 adults (age: 39.16 ± 15.71, 3 males, 3 females) without or with little robotics experience (1 participant). The pilot study was carried out with 7 University students in cognitive sciences (age: 23.14 ± 1.46, 2 males, 5 females). The final experiments with the robot were carried out with 23 adults (age: 35.13 ± 11.98, 12 males, 11 females) without robotics experience.

Experimental setup. The experiments were carried out at the LARSEN laboratory of INRIA (Nancy, France). The experimental setup was organized as shown in Fig. 1. A desk with a laptop was placed in front of the Kinova Jaco arm, fixed on a table. The arm was positioned in such a way to be able to perform some manipulations on the ADL setup (Fig. 2C), made of two boxes: one with a door handle, one with three buttons and a drawer containing a small object. A video camera, placed behind the participants, was used to record the experiments. Two interfaces (see Fig. 2A and B) for controlling the robot were used: the native joystick by Kinova and our own ad-hoc graphical user interface (GUI). The joystick can move the hand in the Cartesian space (position and rotation), open and close the fingers. Two buttons are used to select whether to move the hand position (mode 1), its orientation (mode 2) or the fingers (mode 3). The GUI was developed with Qt and is open-source[1]. Both interfaces use the same Kinova API for robot control and inverse kinematics solving.

Questionnaires. To probe into the influence of individual factors, we asked the participants to the robot experiment to fill out some questionnaires before the experiments: the Negative Attitude Towards Robots Scale (NARS) [3] and the Desire For Control scale (DFC) [8]. Our French adaptation was used [1]. The participants also filled two post-experimental questionnaires consisting of questions/affirmations adapted from usability and technology acceptance models to a robotic context as it was done in previous works [9,12]. The post-block questionnaire, at the end of each experimental condition (block when one interface is used), was based on the USE questionnaire [13] (typical questions were *"How good will you rate the movement you achieved in the 'open the drawer' task?"*). The post-experimental questionnaire consisted of a set of affirmations to be rated on a 7-points Likert scale, targeting constructs typical of the UTAUT [11] and TAM 3 models [14] (typical questions were *"Controlling the robot with the GUI is easy"*).

Experimental protocol. The study consist of a focus group and two robot experiments: a pilot study with University students, then experiments with ordinary adults. All the data were recorded in anonymous form through a random numerical id attributed to each participant. All participants were equally informed by the experimenter about the purpose of the study and their rights, according to the ethics guidelines of our institute. An informed consent form was signed by each participant. The protocol received the positive approbation of the local Ethics Committee.

Focus group study - We asked a group of 6 adults without or with little experience in robotics to imagine how they would interact with the robot and control it to do some tasks. The group gathered in a closed room around a table. One moderator led the group, while two recorders took notes and annotated

[1] https://github.com/serena-ivaldi/kinova-modules.

sentences and body language. The session lasted about 2 h and was recorded for analysis purposes. The experimenter asked to the group six warm-up questions, such as "*Tell us about your overall experience with robots*", "*In which situation(s) do you imagine that a robotic arm such as the Kinova would be useful?*". In a work in pairs, participants had to present their ideas about interfaces for controlling a robot arm.

Pilot study with the robot - We carried out a pilot study with the Jaco robot and 7 University students. Each participant had to perform the 4 tasks (see Fig. 2C) with the robot, using the joystick and the GUI. The order of the interfaces was randomized across the participants. After the experiment with the robot, we asked the participant to express their preference for one of the two interfaces and provide their feedback and personal evaluations.

Experimental study with the robot - The experiments with the Jaco robot were carried out with 23 adults without expertise in robotics. Each participant filled in the questionnaires NARS and DFC one week before the experiment. The day of the experiment, the participant was welcomed to the laboratory room by the experimenter and seated on a table with a laptop (see Fig. 1) in front of the robot. There were two blocks corresponding to the two experimental conditions: one with the joystick and one with the GUI. In each block, the participant had to perform the 4 tasks with the robot (see Fig.2C). The order of use of the interface was randomized and balanced across the participants. To ensure that all the participants received an equal set of instructions, we provided them with the same instructions, either in paper format and in video format (tutorial). The participant started by reading some paper instructions explaining the 4 tasks to be performed with the robot. After reading the instructions, they had to rate some statements on a 7-items Likert scale, such as "*The required tasks are difficult*" and "*The instructions were difficult to read*". We also added two trick questions to check if they were attentive and had carefully read the instructions. Before each block, the participant watched a 2/3 min video tutorial explaining how to use each interface, then he/she could familiarize and try it for about 1 min. We instructed the participants to follow a think-aloud protocol. When the participant was ready to start, he/she began performing the 4 tasks in sequence. Two experimenters monitored and annotated the experiment. After completion, the participant filled in a questionnaire evaluating the ease of use of the interface. The sequence tutorial-test-tasks-evaluation was repeated for the second interface. After the experiment with the robot, the participant filled in the post-experimental evaluation questionnaire, then answered to some semi-directed questions during an interview with the two experimenters.

Measures and data analysis. During the focus group, two recorders annotated the discussion. Video recordings were used to complete the annotation offline. In the pilot study, we measured the duration of each task and the user preference for each interface. In the robot experiments, we employed both objective and subjective measures. Two experimenters annotated: the *duration of each task*; the *numbers of precision errors*, represented by the number of times the robot hit the ADL board; the *number of mapping errors*, represented by the number of times the robot was moved in the opposite direction with respect to the desired (we could identify this by the explicit verbalization of the participant, or by two consecutive movements in opposite directions where the first was clearly in the wrong direction with respect to the goal of the movement). The questionnaires' score for NARS and DFC were computed according to the authors' recommendations. The subjective measures retrieved from the post-experimental questionnaires are the *perceived ease of use* (PEOU, typical question: *"Controlling the robot with the GUI is easy"*), the *user satisfaction* (US, *"How good will you rate the movement you achieved in the 'open the drawer' task?"*) and the *facilitating condition* (FC, *"The time to test the Joystick before the experiment was enough"*) related to each interface, computed by the sum of the score of the questionnaire items for each construct. The expertise in using joysticks was a self-reported score on a 10-item scale.

Unless otherwise stated, we computed median and standard deviation of all the measured variables; we used Spearman's correlation and verified the statistical significance of the different conditions with a Wilcoxon signed ranked test with continuity correction in R.

3 Results

Focus group - The focus group participants did not have a particular affinity with robotics, and were generally worried about the possibility of robots replacing humans. When asked about the possible use for the Jaco arm, they indicated grabbing objects on very high shelves, assisting people with impairments or arm troubles, doing manual tasks like laundry, ironing and painting walls. Almost all the participants agreed that the robot should not be completely autonomous: they need to be in control of the situation when the robot is acting. They said that they should "*teach the robot to do the things the way we want*" and "*be able to stop the robot anytime*". When we asked how to control the robot, the participants mostly indicated panels with buttons (3/6). In particular, one participant explained that there should be a button for each possible robot gesture.

Pilot study - The only significant difference in terms of task duration with the two interfaces is on the second task (*opening the drawer*, $V = 0$ $p = 0.0156 < 0.05$). We did not find any significant correlation between the task duration and the participants' self-report expertise with joysticks.

Concerning the joystick, the negative points were: the difficulty in controlling the hand orientation and the way to change the modes with the buttons. Positive points were that it was more intuitive to move in the x-y-z space, especially for the students used to play video-games, and that it felt like an "extension of their arm". Concerning the GUI, the negative point was that it required to switch continuously the attention from the laptop to the robot. The positive points were its clearer design that made the actions explicit and the ease of use when choosing pre-determined orientations of the hand for manipulation.

We asked the 7 participants to choose the interface that was easier to use and more intuitive for them: 2 preferred the joystick and 5 the GUI ("*it can be mastered, one makes more errors with the joystick*").

Robot experiments - After reading the instructions, the participants evaluated the tasks to be not difficult (on a 7-item Likert scale, median = 2, stdev = 1.67) and the instructions easy to read (median = 1, stdev = 2.03). We found a significant difference in the overall duration of the tasks ($V = 25$ $p = 0.0006 < 0.001$) for the two conditions, in particular for Task 2 (*opening the drawer*, $V = 10$ $p = 0.0002 < 0.001$) and Task 3 (*grabbing the object*, $V = 28.5$ $p = 0.0009 < 0.001$), a fair difference for Task 4 (*opening the door*, $V = 51.5$ $p = 0.0089 < 0.01$). We also compared the duration of the tasks executed with each interface when the latter is first or second in order of execution: we did not find difference in the execution for the GUI (Mann-Whitney, $W = 82$ $p = 0.347$ (N.S.)), whereas there is a weak evidence for a difference in the execution time of the joystick if it is used as first or second (Mann-Whitney, $W = 27.5$ $p = 0.0193 < 0.05$). In terms of use of the interface, there is a marginal difference in terms of precision ($V = 53$ $p = 0.0531$ (N.S.)), while there is a strong difference in terms of mapping errors ($V = 0$ $p = 2.85\text{e-}05 < 0.001$) - the median number of mapping errors with the joystick is also quite elevated (10). Regarding the subjective measures retrieved by the questions, we found a significant difference in the ratings in terms of ease of use ($V = 251.5$ $p = 5.23\text{e-}05 < 0.001$), satisfaction ($V = 239$ $p = 0.0022 < 0.005$) and facilitating conditions ($V = 159$ $p = 0.0013 < 0.005$): the GUI has higher ratings than the joystick on all the three items. We did not find a significant correlation between the users' performance and their prior expertise in using joysticks nor between the user performance and their NARS.

Among the 23 participants, 11 expressed preference for the joystick and 12 for the GUI. However, in terms of usability, the joystick was favored by 6 participants, while the GUI by 16 (one participant said they were equal). We tested if the interface preference was related to the DFC score of the participants but we did not find any significant difference (Mann-Whitney, $W = 48$, $p = 0.279$ (N.S.)).

We asked the participants to provide their feedback in the post-experimental interview. Many participants highlighted that the joystick made them feel more "in control" when moving in the main Cartesian directions (x,y,z - the first mode of the joystick) and that they could achieve more precise movements with it. Almost all the participants reported that switching the mode with the joystick was very difficult. However, some thought that they could become good users with a dedicated training. One participant, for example, said "*my son is very good with the video-games pad, he will learn in 10 min; for me, I will need some hours*". Many participants appreciated the GUI because of the intuitive buttons where each command/action was explicit.

4 Discussion

In this study we focused on non-expert users controlling a robot for their first time: if the robot-user does not have a proper training, or if he is using the robot only once in a while, which interface could be easier to use and facilitating the robot adoption? From the focus group study, we learned that people imagine to interact with the robot in a structured way (e.g., buttons) that allows them to be in control of the robot decisions (e.g., when to start, when to stop).

To make the robot controllable by non-experts, our conclusion is that we need a very reliable control interface that they can understand and use easily/intuitively, that is robust and that gives them the impression to be in control. From the participants suggestions, a panel with buttons seems appropriate as a control interface: it gives the user the impression that the robot can act upon their orders. For the purpose of this study, we decided that the most appropriate control interface to test against the joystick of the Jaco arm was a GUI with buttons.

Is a GUI really better than a joystick? - From the pilot study with students, we could not strongly conclude that the GUI brings notable improvements over the joystick. In the experiments with ordinary adults, the GUI is better than the joystick in terms of objective performance measures and subjective user evaluation. We found significant difference in the duration of tasks and mapping errors, but not in the precision errors: therefore we accept **H1** and **H3** but reject **H2**. Almost all participants found the GUI easier to use, more understandable and straightforward. Many participants appreciated moving the robot with the joystick as they felt it an "extension of their hand". Interestingly, while most participants appreciated the pre-programmed orientations/configurations of the hand, that were quite difficult to obtain with the joystick, some participants reported them as a constraint that was limiting their freedom to choose different orientations of the hand to realize the tasks. These participants suggested that the two interfaces should be combined to give the user more freedom. It is however important to notice that the GUI performs better than the joystick in our particular experimental conditions, where the participants have a very limited training for using the interfaces (a video tutorial and 1 min to familiarize with the interface and try it). The results could be very different in a case where the participant uses the robot on a regular basis or receives a proper training. We will address this case in future experiments.

Do individual factors play a role in the user performance with an interface? - Our preliminary results show that the user preference for a particular interface does not seem to relate to their performance in using it: for example, many users expressed their preference for the joystick whereas they were better with the GUI. Contrary to our expectations, this result does not seem to relate to the user's individual factors, as we did not find a strong evidence to support our hypothesis. We did not find significant correlations between the user preferences or performances with both NARS and DFC. We therefore reject **H4** and **H5**. Nevertheless, in the post-experimental interviews many participants reported to feel more comfortable with the joystick despite being better with the GUI: this may seem counter-intuitive, but in fact suggests that there may be other individual criteria that drive their choice.

5 Conclusions

Two main questions emerge for future work: Which are the key factors that determine user preference for a robot control interface and if the preference and performance in using an interface would change in a long term scenario (i.e., a scenario where users receive a training for operating the robot with the interface and use such an interface more frequently or on a daily basis). We plan more experiments to investigate more thoroughly all these questions.

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