Essays on Information Frictions in Macroeconomics

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To my parents

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Abstract

This dissertation consists of three chapters related to questions in macroeconomics and information frictions. In the first chapter, I relax the complete information assumption in the standard New Keynesian framework to show how the stance of monetary policy can affect the non-fundamental composition of fluctuations, introducing a novel trade-off between stabilizing output and inflation. A strong response to inflation increases the variance of non-fundamental fluctuations. In the second chapter, I study the optimal design of monetary policy in the presence of nominal and informational frictions. Non-fundamental fluctuations are shown to be suboptimal. The Taylor rule is no longer sufficient to rule out indeterminacy. Instead, a more lax response to inflation eliminates non-fundamental fluctuations and hence the output-inflation tradeoff. In the third chapter, I provide evidence that shocks to sentiments and uncertainty as identified in the literature are correlated and may not be truly structural.

Resumen

Esta tesis consta de tres capítulos relacionados con preguntas en macroeconomía y fricciones de información. En el primer capítulo, relajo el supuesto de información completa en el marco del modelo neokeynesiano estándar para mostrar cómo la postura de la política monetaria puede afectar la composición no fundamental de las fluctuaciones, introduciendo un nuevo impedimento para la estabilización simultanea del producto y la inflación. Una respuesta agresiva a la inflación aumenta la varianza de las fluctuaciones no fundamentales. En el segundo capítulo, estudio el diseño óptimo de la política monetaria en presencia de fricciones nominales y de información. Demuestro que las fluctuaciones no fundamentales son subóptimas y que la regla de Taylor ya no es suficiente para descartar la indeterminación. En cambio, una respuesta más laxa a la inflación elimina las fluctuaciones no fundamentales y, por lo tanto, la imposibilidad de estabilizar simultáneamente el producto y la inflación. En el tercer capítulo, aporto pruebas de que los shocks de sentimientos y de incertidumbre identificados en la literatura están correlacionados y, por tanto, podrían no ser verdaderamente estructurales.

Preface

As modern economies are characterized by a network of interconnected agents, many settings feature a coordination motive: an agent's decisions depend not only on their beliefs about the state of the economy, but also on their expectations of how others will respond. For example, the hiring and investment decisions of a firm depend on its expectations of demand for its product, which in turn depend on consumption. Meanwhile, consumers' decisions depend on expectations of income and labor-market conditions, which in turn depend on the decisions of other firms and consumers. However, implicit in workhorse models is the assumption that agents are able to confer with each other, reaching consensus on the current state and future trajectory of the economy. Such models form the basis for our interpretation of data, which inform policy.

In the first chapter, I study the effect of monetary policy in the presence of particular information frictions which impede the ability of agents to coordinate. The first friction is strategic uncertainty, which refers to the uncertainty that agents face about the behavior of others and the resulting macroeconomic outcomes. However, agents do not make decisions in vacuums; they condition their responses on information, which is endogenous in most situations of interest. Endogenous signals capture the role of market research, prices, and macroeconomic indicators in coordinating actions and beliefs. Taken together, these frictions allow for sentiments, or beliefs about aggregate demand, to be self-fulfilling. The perfect information benchmark in the New Keynesian framework turns out to be non-trivial, and relaxing it in this manner allows for an alternate channel in which monetary policy can affect outcomes. Through its effect on aggregate variables, the stance of monetary policy determines the precision of endogenous signals that firms receive, and consequently, the degree of coordination in firms' production (pricing) decision. As a result, the distribution of non-fundamental shocks is no longer independent of policy, introducing a novel tradeoff between stabilizing output and inflation. Strong inflation targeting increases the variance of non-fundamental fluctuations.

The second chapter analyzes welfare properties of the framework introduced in chapter one. I consider an appropriate efficiency benchmark, one which represents the best allocation among those that respect resource feasibility and the decentralization of information. The endogenous information structure in the decentralized equilibrium is shown to result in non-fundamental fluctuations, which characterizes an inefficiency in both the use and aggregation of information. The Taylor rule is no longer sufficient to rule out such indeterminacy. Instead, a simple interest rate rule that mitigates the degree to which it targets inflation implements the efficient allocation, by eliminating non-fundamental fluctuations and precluding the output-inflation trade-off.

Motivated by theories of aggregate fluctuations arising from shocks to agents' expectations in incomplete information settings, the third chapter considers the relationship between sentiment and uncertainty shocks. Both shocks are related to information and the formation of beliefs, and each are typically identified as short-run sources of co-movement in macroeconomic aggregates. Sentiments, defined as a change to expectations about economic activity and orthogonal to news about future technology, can be interpreted as rational optimism or pessimism. As such, they may affect confidence, or uncertainty. A maximum forecast error variance approach is used to identify sentiment and uncertainty shocks in a structural vector autoregressive model. Sentiment shocks are shown to account for more variation than news. However, they explain less of the variation in GDP and hours than previous studies have shown, as uncertainty is also an important source of short-run fluctuations. This chapter also provides evidence that sentiments and uncertainty shocks as identified in the literature are correlated and may not be truly structural.

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Chapter 1

Monetary Policy and Sentiment Driven Fluctuations

1.1. Introduction

The New Keynesian model has become the workhorse for understanding the link between monetary policy and macroeconomic outcomes. The study of monetary policy has typically considered how policymakers should stabilize fluctuations arising from changes in exogenous fundamentals such as technology, markups, or news.[1] Most central banks use the interest rate as the monetary policy instrument. It is well known that an interest rate rule that fails to respond sufficiently strongly to changes in inflation renders the price level indeterminate: shifts in the price level can occur without corresponding changes in fundamentals, but simply as a result of sunspots fluctuations.

However, sunspot fluctuations may also arise in models deviating only slightly from the assumption of perfect information (Benhabib et al. (2015), Angeletos and La'O (2013)). To study the effects of monetary policy in this setting, I introduce nominal rigidities into the model of Benhabib et al. (2015). In this framework, a continuum of firms commit to production (pricing) before shocks are known, conditioning their decision on a dispersed signal of an endogenous variable, aggregate output. Under these assumptions, aggregate fluctuations arise naturally from an information complementarity, as agents make decisions that depend on the actions of others, while responding to an endogenous signal that confounds aggregate demand with idiosyncratic demand. Beliefs about aggregate output, termed *sentiments* can be a self-fulfilling source of fluctuations without movements in underlying economic fundamentals. The set of outcomes are rational ex-

[1] See Galí (2015) and Woodford (2003) for a survey.

pectations equilibria, as firms and households behave optimally, given the information that is available. Most importantly, the distribution of beliefs is disciplined by deep structural parameters and corresponds to the distribution of aggregate output.

When nominal rigidities are introduced, the range of outcomes will depend on the stance of monetary policy. As firms commit to production (prices) based on an endogenous signal of aggregate and idiosyncratic demand, monetary policy, through its effect on aggregate variables, will determine the precision of firms' signals and the degree to which their actions are coordinated. In summary, aggregate fluctuations are the result of coordinated actions, and their volatility is parameterized by the stance of policy. In direct contrast to the Taylor principle, adjusting the nominal interest rate too strongly in response to inflation generates non-fundamental fluctuations. In other words, the region of determinacy in the standard New Keynesian model is not robust to alternative assumptions on the information structure. Instead, to guarantee determinacy, I show that the interest rate rule should be sufficiently lax in responding to inflation.

The main contribution of this paper is to consider an alternate channel through which monetary policy may affect outcomes. Through its effects on aggregate variables, the stance of policy affects the behavior of firms, particularly how they use their signals to respond to aggregate demand. In the aggregate, firms' actions affect the precision of signals they receive. In contrast to the standard view that monetary policy should mitigate the distortionary effects of shocks, policy itself becomes a source of shocks.

While the New Keynesian literature has focused on sunspot driven fluctuations that can arise in models where agents have perfect information, how monetary policy should respond to those that arise in an imperfect information framework is not well-understood. Contrary to the conclusions of the standard New Keynesian model with perfect information, targeting inflation strongly may have a destabilizing role, as the endogeneity of equilibrium outcomes to the stance of monetary policy implies the following. First, the volatility of non-fundamental shocks is no longer independent of policy. Second, the Taylor principle is no longer sufficient to rule out indeterminacy. Third, these fluctuations introduce a novel trade-off for the policymaker whose goal is to stabilize output and inflation.[2] Although these fluctuations induce the same co-movement between output and the price level as a supply shock, replicating the flexible wage or price allocation is suboptimal because it increases the volatility of output. While the Taylor principle is sufficient to rule out nominal indeterminacy in models in which agents have perfect information, it is not sufficient to rule out real indeterminacy in this model. For robustness, I show that these results also hold in the case of price stickiness and when the nominal interest rate targets price inflation.

[2] In the standard New Keynesian model, cost-push shocks are needed in order to produce this trade-off.

1.2. Literature Review

The idea that beliefs can be a cause, and not simply a consequence of macroeconomic outcomes goes back to Pigou (1927) and Keynes (1936). Economists have tried to rationalize shifts in aggregate outcomes without corresponding movements in fundamentals using models that either depart from rational expectations or feature multiple equilibria. Previous attempts have relied on randomization over multiple certainty equilibria, as in Cooper and John (1988), which features strong strategic complementarities in actions. Similar dynamics can be found in models with non-convexities in technology or preferences. In Benhabib and Farmer (1994), Farmer and Guo (1994), Wen (1998), increasing returns in production (from input externalities or monopolistic competition) yield a local indeterminacy, whereby a continuum of deterministic equilibrium paths converge to a unique steady state. Cass and Shell (1983) construct sunspot equilibria that are not necessarily randomizations between certainty equilibria.

I follow a more recent strand of literature by Angeletos and La'O (2013), Benhabib et al. (2015), and Chahrour and Gaballo (2017), which relies on incomplete information as a mechanism for generating sentiment-driven fluctuations in a micro-founded, unique-equilibrium rational expectations macroeconomic model.[3] As the equilibrium conditions impose more structure on equilibrium outcomes, these models facilitate policy analysis. In Angeletos and La'O (2013), the extrinsic source of fluctuations is aggregate noise in information technology used to infer a trading partner's beliefs. Sentiments, as referred to in Benhabib et al. (2015) and Chahrour and Gaballo (2017), correspond to an endogenous variable, aggregate output, and are captured by dispersed signals that can coordinate agents' actions. As a result, the distribution of sentiments is determined by structural parameters and corresponds to the self-fulfilling distribution of aggregate output.[4]

The rest of the paper is organized as follows. Section (1.3) presents a stylized model to illustrate how endogenous signals may lead to indeterminacy in aggregate outcomes, the distribution of which is pinned down by structural parameters. Section (1.4) introduces the benchmark model. It embeds the dynamics of the previous section in a richer, micro-founded business cycle model with Calvo wage rigidity in order to analyze the effect of monetary policy on equilibrium outcomes. Section (1.6) concludes. Appendix (B.2) considers an extension of the model with price rigidity. For reference, the flexible wage and flexible price case can be found in appendices (A) and (B.1).

[3] Information frictions also play an important role in explaining macroeconomic dynamics in Woodford (2003), Adam (2007), Lorenzoni (2009), and Angeletos and Lian (2016).

[4] In this model, multiple equilibria arise from correlated decisions by firms, conditioning on endogenous signals. In this respect, it is similar to Aumann (1987) and Maskin and Tirole (1987), where partially correlated signals lead to correlated equilibria.

1.3. Information Frictions in a Beauty Contest Model

Many environments feature a coordination motive, where an agent's optimal action depends not only on their expectations of exogenous fundamentals (idiosyncratic or aggregate), but also on their expectations of how others will respond.[5]

These dynamics can be captured by a beauty contest (Morris and Shin (2002)), a class of games featuring a linear best response, which agents take under incomplete information. A continuum of agents, indexed by $j \in [0,1]$, choose action $y_{j,t}$ in response to a fundamental (in this case, $\varepsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$), while also minimizing the distance between its action and the actions of others,[6]

$$\min_{y_{j,t}} \mathbb{E}[\beta_0 (y_{j,t} - \varepsilon_{j,t})^2 + \beta_1 (y_{j,t} - y_t)^2 | I_{j,t}].$$

Let $y_t = \int_0^1 y_{j,t}\,\mathrm{d}j$ represent the action profile across agents, and denote the information set of agent j by $I_{j,t}$.[7] The parameters β_0 and β_1 capture the importance that agents place on their action being close to the fundamental and their desire to coordinate, respectively. It follows that the best response of agent j is a linear combination of two terms: the fundamental and the aggregate action,

$$y_{j,t} = \mathbb{E}[\beta_0 \varepsilon_{j,t} + \beta_1 y_t | I_{j,t}].$$

If $\beta_1 < 0$, agents' actions are characterized by strategic substitutability. Otherwise, if $\beta_1 > 0$, we refer to their actions as strategic complements.

1.3.1. Complete Information

In the complete information case,

$$y_{j,t} = \beta_0 \varepsilon_{j,t} + \beta_1 y_t.$$

[5]Macroeconomic applications of beauty contests include the pricing decision of monopolistically competitive firms (Woodford (2003), Hellwig and Veldkamp (2009)) and investment decision of firms Angeletos and Pavan (2007).

[6]The term "fundamental" refers to the fact that the realization of $\varepsilon_{j,t}$ is payoff-relevant to agent j.

[7]The information set may include priors, private signal, or a public signal.

Assuming the law of large numbers applies, the aggregate actions is found by summing across agents,

$$\begin{aligned} y_t &= \int_0^1 y_{j,t} dj, \\ &= \int_0^1 (\beta_0 \varepsilon_{j,t} + \beta_1 y_t) dj, \\ &= \beta_1 y_t. \end{aligned}$$

In the case of $\beta_1 \neq 1$, the only equilibrium is $y_t = 0$. If $\beta_1 = 1$, then multiple equilibria exist and any y_t is a solution.

1.3.2. Incomplete Information

In the case of incomplete information, agents do not observe $\varepsilon_{j,t}$ and y_t. Instead, they condition their response on a unique information set, denoted by $I_{j,t}$. In particular, let $I_{j,t} = s_{j,t}$, a private signal that is endogenous, as it aggregates the idiosyncratic fundamental and the action profile across agents, an endogenous variable,[8]

$$s_{j,t} = \lambda \varepsilon_{j,t} + (1-\lambda) y_t.$$

To consider an equilibrium where y_t may be stochastic, conjecture $y_t \sim N(0, \sigma_y^2)$. Under Bayesian updating, agents will weight their signal according to

$$y_{j,t} = \underbrace{\frac{\beta_0 \lambda \sigma_\varepsilon^2 + \beta_1 (1-\lambda) \sigma_y^2}{\lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_y^2}}_{\mu} \underbrace{[\lambda \varepsilon_{j,t} + (1-\lambda) y_t]}_{s_{j,t}}.$$

The aggregate action across agents is

$$y_t = \int_0^1 y_{j,t} \,\mathrm{d}j = \frac{\beta_0 \lambda \sigma_\varepsilon^2 + \beta_1 (1-\lambda) \sigma_y^2}{\lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_y^2} (1-\lambda) y_t. \tag{1.1}$$

This can be decomposed as follows,

$$y_t = \beta_0 \underbrace{\frac{\lambda \sigma_\varepsilon^2}{\lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_y^2} (1-\lambda) y_t}_{\text{pass-through of } y_t \text{ to } \mathbb{E}[\varepsilon_{j,t} | s_{j,t}]} + \beta_1 \underbrace{\frac{(1-\lambda) \sigma_y^2}{\lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_y^2} (1-\lambda) y_t}_{\text{pass-through } y_t \text{ to } \mathbb{E}[y_t | s_{j,t}]}. \tag{1.2}$$

[8] See appendix (C) for an explanation of why, when firms' actions are strategic substitutes, a sentiment driven equilibrium exists only if the private signal contains $\varepsilon_{j,t}$ and z_t in proportions different from the firms' first order condition; i.e. where $\lambda \neq \beta_0$ and $(1-\lambda) \neq \beta_1$.

Equation (1.1) is satisfied for $y_t = 0$, which is referred to as the *fundamental equilibrium*. In addition to the fundamental equilibrium, any y_t from a distribution with volatility σ_y^2 is also an equilibrium, where σ_y^2 is such that

$$\frac{\beta_0 \lambda \sigma_\varepsilon^2 + \beta_1 (1-\lambda)\sigma_y^2}{\lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_y^2}(1-\lambda) = 1.$$

These additional equilibria are *non-fundamental, or sentiment equilibria*. In this case, the volatility of y_t is determined by the parameters of the model $(\beta_0, \beta_1, \lambda)$,

$$\sigma_y^2 = \frac{\lambda}{1-\lambda}\left(\frac{\beta_0 - \frac{\lambda}{1-\lambda}}{1-\beta_1}\right)\sigma_\varepsilon^2. \tag{1.3}$$

In this framework, multiple equilibria does not rely on non-convexities in technology or preferences, or randomizations over fundamental equilibria, but on information frictions. More specifically, coordination in agents' actions reflect an information externality, the result of conditioning their response on an endogenous signal. This signal conveys information about an endogenous variable, and captures the role that macroeconomic indicators, market research, or prices in coordinating agents' actions and beliefs.

Proposition 1. *In the sentiment equilibrium, the set of outcomes is determined endogenously. The distribution of y_t that satisfies the equilibrium conditions depends on the parameters of the model: β_0, β_1, λ, σ_ϵ^2. Policy, which may affect strategic interactions among agents (parameterized by β_1), will influence σ_y^2.*

$$\frac{\partial \sigma_y^2}{\partial \beta_1} = \frac{\sigma_y^2}{1-\beta_1}.$$

If $\beta_1 < 1$, then $\frac{\partial \sigma_y^2}{\partial \beta_1} > 0$.

To give intuition for this result, suppose policy has an effect on how agents choose to respond to the aggregate component of their signal and for simplicity, consider a decrease in $\beta_1 < 0$. Then for the same y_t as before, agents now increase $y_{j,t}$ by more.[9] The equilibrium condition,

$$y_t = \int_0^1 y_{j,t}\,\mathrm{d}j$$

holds if σ_y^2 increases, so that agents attribute more of their signal to the y_t component. Agents must believe that their signal reflects more of this component so that there is

[9]We say that agents' actions are characterized by less substitutability.

sufficient pass-through of the common component of their signal, which is required for equilibrium under the change in β_1.

An equilibrium with sentiment-driven fluctuations requires agents to misattribute some of y_t to $\varepsilon_{j,t}$. A decomposition of y_t, as in (1.2), shows that as a result of agent j's signal extraction problem, what agents perceive to be the idiosyncratic fundamental actually contains the aggregate, endogenous component of their signal,

$$\mathbb{E}(\varepsilon_{j,t}|s_{j,t}) = \beta_0 \frac{\lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2}[\lambda\varepsilon_{j,t} + (1-\lambda)y_t].$$

Across agents, this coordinated misattribution of components of their signal contributes to aggregate fluctuations.

$$\int_0^1 \mathbb{E}(\varepsilon_{j,t}|s_{j,t})\,\mathrm{d}j = \beta_0 \frac{\lambda\sigma_\varepsilon^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2}(1-\lambda)y_t.$$

Proposition 2. *An equilibrium with sentiment-driven fluctuations does not require a particular type of strategic interaction. From (1.3), note that $\sigma_y^2 > 0$, regardless of the sign of β_1. In order for $\sigma_y^2 > 0$,*

- *If $\beta_1 < 1$, then $\beta_0 > \frac{\lambda}{1-\lambda}$.*
- *If $\beta_1 > 1$, then $\beta_0 < \frac{\lambda}{1-\lambda}$.*

Equilibrium multiplicity requires agents to react differently to the idiosyncratic fundamental than to the aggregate action. If agents' actions are strategic substitutes or relatively minor in strategic complementarity, then the elasticity of their response with respect to the idiosyncratic component must be relatively large. If agents' actions are relatively high in complementarity, then the elasticity of their response with respect to the idiosyncratic component must be relatively small. The elasticities of response (β_0 and β_1) must be large or small, relative to the signal content ($\frac{\lambda}{1-\lambda}$). If agents want to respond differently to idiosyncratic shock and to the aggregate variable, but can not distinguish between the two in their signal, then a coordinated over-response or under-response across agents can lead to sentiment-driven equilibria. Note that if $\beta_1 > 1$, information frictions are not needed for multiplicity of equilibria (see Cooper and John (1988)). In the case of $\beta_1 = 0$, multiple equilibria would still exist, if

$$\sigma_y^2 = \frac{\lambda}{1-\lambda}\left(\beta_0 - \frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2.$$

In this case, y_t can be considered aggregate noise in the signal that agents receive about their idiosyncratic fundamental.

1.4. Monetary Policy with Sticky Wages (Calvo)

1.4.1. Households

Following Erceg et al. (2000), consider a continuum of households, indexed by $i \in [0,1]$, each of which specializes in one type of labor which it supplies monopolistically.[10] The households face Calvo wage rigidity: in each period, only a constant fraction $(1-\theta_w)$ of labor types, drawn randomly, are able to adjust their nominal wage.

Optimal Wage Setting

Consider the wage chosen by a household that is able to re-optimize. Household i, supplying labor $N_{i,t}$, chooses wage $W_{i,t}$ to maximize utility,

$$\max_{W_{i,t}} \mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{{C_{i,t+k|t}}^{1-\gamma}}{1-\gamma} + \Psi(1 - N_{i,t+k|t}) \right) \right] \tag{1.4}$$

Let $C_{i,t+k|t}$ and $N_{i,t+k|t}$ represent the consumption and labor supply in period $t+k$ of a household that last reset its wage in period t. Household i's consumption index is given by

$$C_{i,t} = \left[\int_0^1 \epsilon_{i,j,t}^{\frac{1}{\theta}} C_{i,j,t}^{1-\frac{1}{\theta}} \, \mathrm{d}j \right]^{\frac{\theta}{\theta-1}},$$

where $C_{i,j,t}$ represents household i's consumption of good j and $\theta > 1$ the elasticity of substitution between goods. The idiosyncratic preference shock for good j is log normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\epsilon^2)$). The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is intended to simply expressions.

As the Calvo type wage setting is a constraint on the frequency of wage adjustment, equation (1.4) can be interpreted as the expected discounted sum of utilities generated over the period during which the wage remains unchanged at the level set in the current period. Optimization of (1.4) is subject a sequence of labor demand schedules and flow budget constraints that are effective while $W_{i,t}^*$ is in place. Labor expenditure minimization by firms implies the following demand for labor,[11]

$$N_{i,t+k|t} = \left(\frac{W_{i,t}^*}{W_{t+k}} \right)^{-\epsilon_w} N_{t+k}, \tag{1.5}$$

[10] Alternatively, one can consider a continuum of unions, each of which represents a set of households specialized in a type of labor, and sets the wage on their behalf.

[11] See appendix (E) for details.

where $N_{t+k} = \int_0^1 N_{j,t+k}\,\mathrm{d}j$ denotes aggregate employment in period $t+k$. Households face budget constraint

$$P_{i,t+k}C_{i,t+k|t} + E_{t+k}\{Q_{i,t+k,t+k+1}D_{i,t+k+1|t}\} \leq D_{i,t+k|t} + W^*_{i,t}N_{i,t+k|t} + \Pi_{t+k}, \quad (1.6)$$

where $D_{t+k|t}$ represents the market value of the portfolio of securities held in the beginning of the period by a household that last re-optimized their wage in period t, while $E_{t+k}\{Q_{t+k,t+k+1}D_{t+k+1|t}\}$ is the corresponding market value in period $t+k$ of the portfolio of securities purchased in that period, yielding a random payoff $D_{t+k+1|t}$. Π_t represents dividends from ownership of firms.

The first order condition associated with this problem,

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k\mathbb{E}_t\left[N_{i,t+k|t}U_c(C_{i,t+k|t}, N_{i,t+k|t})\left(\frac{W^*_{i,t}}{P_{t+k}} - \frac{\varepsilon_w}{\varepsilon_w - 1}MRS_{i,t+k|t}\right)\right] = 0$$

where $U(C,N) \equiv \frac{C^{1-\gamma}}{1-\gamma} + \Psi(1-N)$, $U_c \equiv \frac{\partial U}{\partial C}$, and $MRS_{i,t+k|t} \equiv -\frac{U_n(C_{i,t+k|t},N_{i,t+k|t})}{U_c(C_{i,t+k|t},N_{i,t+k|t})}$.

Log-linearizing this expression, an approximate expression for the optimal wage,

$$w^*_{i,t} = \log\left(\frac{\varepsilon_w}{\varepsilon_w - 1}\right) + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k\mathbb{E}_t(mrs_{i,t+k|t} - p_{t+k})$$

Under the assumption of full consumption risk sharing across households (through the assumption of a complete set of securities markets, which equalizes the marginal utility of consumption across households), all households resetting their wage in a given period will choose the same wage, w^*_t, as they face the same problem. An alternative expression for the optimal nominal wage chosen by monopolistically competitive households households who can adjust in time t is given by

$$w^*_t = \beta\theta_w\mathbb{E}_t(w^*_{t+1}) + (1-\beta\theta_w)(w_t - [1-\varepsilon_w\varphi]^{-1}\hat{\mu}^w_t) \quad (1.7)$$

where $\hat{\mu}^w_t \equiv \mu^w_t - \mu^w$ defines the deviations of the economy's log average wage markup ($\mu^w_t \equiv w_t - p_t - mrs_t$) from its steady state level (μ^w).

Defining W_t as the aggregate nominal wage index,

$$W_t \equiv \left[\int_0^1 W_{i,t}^{1-\epsilon_w}di\right]^{\frac{1}{1-\epsilon_w}},$$

the evolution of the aggregate wage index is given by

$$W_t = \left[\theta_w W_{t-1}^{1-\varepsilon_w} + (1-\theta_w)(W^*_t)^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}}.$$

Log-linearized around a zero wage inflation steady state,

$$w_t = \theta_w w_{t-1} + (1 - \theta_w) w_t^*. \tag{1.8}$$

Combining (1.7) and (1.8) yields the wage inflation equation

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w - \lambda_w \hat{\mu}_t^w$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\varphi)}$.

In addition, optimizing consumption inter-temporally for a household that last reset its wage in $t-k$,

$$Q_t = \beta \mathbb{E}_t \left[\frac{U_c(C_{t+1}, N_{t+1,t-k})}{U_c(C_t, N_{t,t-k})} \frac{P_t}{P_{t+1}} \right]. \tag{1.9}$$

At this point, households only form demand schedules for each differentiated good and labor supply schedules, all contingent on shocks to idiosyncratic demand ($\epsilon_{j,t}$) and shocks to aggregate demand (Z_t), which have not been realized.

1.4.2. Intermediate goods firms

A continuum of monopolistic intermediate goods producers indexed by $j \in [0,1]$ decide production level $Y_{j,t}$ before knowing idiosyncratic demand ($\epsilon_{j,t}$) or aggregate demand (Z_t). Instead, they infer these shocks from a signal ($S_{j,t}$) that is endogenous in the sense that it captures aggregate demand, an endogenous variable. This signal may be interpreted as early orders, advance sales, or market research, and captures idiosyncratic preference for good j, as well as households' sentiments about expected aggregate income.

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \tag{1.10}$$

Let $\log \epsilon_{j,t} \sim N(0, \sigma_\epsilon^2)$ and if Z_t is stochastic, conjecture $\log Z_t \sim N(\phi_0, \sigma_z^2)$.

Given the household's labor supply schedule and demand schedule for good j, intermediate foods producers choose $Y_{j,t}$ to maximize nominal profits ($\Pi_{j,t} = P_{j,t}Y_{j,t} - W_t N_{j,t}$) subject to production function $Y_{j,t} = AN_{j,t}$,

$$\max_{Y_{j,t}} \mathbb{E}_t \left[P_t Y_{j,t}^{1-\frac{1}{\theta}} (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A} Y_{j,t} | S_{j,t} \right].$$

The firms' first order condition is

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) A \mathbb{E}_t \left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}} \frac{P_t}{W_t} | S_{j,t} \right) \right]^{\theta}. \tag{1.11}$$

Higher aggregate demand affects firm j's optimal production decision in two ways; while it implies an increase in demand for good j, it also implies that the real wage will be higher. The first effect derives from households' optimal consumption across goods, while the second follows from the labor supply decision of household. Given a nominal wage, the aggregate price level will be lower as aggregate demand increases. This will result in a fall in demand for $C_{j,t}$, which decreases firm j's optimal output level. As $\frac{1}{\theta} - 1 < 0$, the latter effect dominates, with the result that firm j's optimal output decreases with aggregate output. Although firms' actions are strategic substitutes, the rational expectations equilibrium may not be unique if firms condition production on an endogenous signal containing aggregate and idiosyncratic demand. Log-linearizing (1.11) around the steady state,

$$\hat{y}_{j,t} = \mathbb{E}_t[\hat{\varepsilon}_{j,t} + \hat{y}_t - \theta \hat{w}_t^r | s_{j,t}].$$

1.4.3. Central bank

A credible central bank commits to setting the nominal interest rate to target wage inflation and output,

$$i_t = \rho + \phi_\pi^w \pi_t^w + \phi_y \hat{y}_t. \tag{1.12}$$

1.4.4. Timing

Letting Z_t denote aggregate demand and $\epsilon_{j,t}$ represent idiosyncratic preference for good j, the timing of this model is as follows:

1. Households form labor supply schedule ($N_t(Z_t)$) and demand schedules for each good j, ($C_{j,t}(Z_t, \epsilon_{j,t})$), contingent on shocks to be realized. They also hold nominal balances $B_t(Z_t)$.
2. The central bank commits to setting the nominal interest rate on bonds $Q_t(Z_t)$, contingent on shocks to be realized.
3. Z_t, $\epsilon_{j,t}$ realized.
4. Firms receive a private signal, capturing aggregate demand and idiosyncratic preference for their good ($S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}$).
5. Firms commit to production and hence labor demand, based on an endogenous private signal.[12] They produce $Y_{j,t}(S_{j,t})$ and demand labor $N_{j,t}(S_{j,t}) = \frac{Y_{j,t}(S_{j,t})}{A}$.

[12] Firms can not write state contingent schedules for their labor demand, otherwise this would remove the possibility of sentiment-driven fluctuations.

6. The goods market opens and Z_t, $\epsilon_{j,t}$ are observed by all agents. $P_{j,t}$ adjusts so that goods market clears ($C_{j,t} = Y_{j,t}$, $C_t = Y_t$), and state contingent contracts are settled: $\frac{W_t}{P_t} = \frac{\epsilon_w}{\epsilon_w - 1}\Psi Z_t^{\gamma}$ for the $(1-\theta_w)$ households who have reset wages. $\Pi_t(Z_t)$ and $\Pi_t^w(Z_t)$ are consistent with Z_t.

The key friction is that intermediate goods firms commit to labor demand and output, based on an imperfect signal of the aggregate demand and firm level demand, prior to goods being produced and exchanged and before marketing clearing prices are realized. After production decisions are made, the goods market opens, demand is realized, and prices adjust to clear the market.

1.4.5. Rational Expectations Equilibrium

Definition 1. *A rational expectations equilibrium is a sequence of allocations* $\{C(Z_t), Y(Z_t), C_j(Z_t, \epsilon_{j,t}), Y_j(Z_t, \epsilon_{j,t}), N(Z_t), N_j(Z_t, \epsilon_{j,t}), \Pi(Z_t)\}$*, prices* $\{P_t = 1, P_j(Z_t, \epsilon_{j,t}), W_t = W(Z_t), Q_t = Q(Z_t)\}$*, and a distribution of* Z_t*,* $\mathbf{F}(Z_t)$*, such that for each realization of* Z_t*, (i) equations (1.7), (1.9) maximize household utility given the equilibrium prices* $P_t = P(Z_t), P_{j,t} = P_j(Z_t, \epsilon_{j,t})$*,* $W_t = W(Z_t)$*, and* $Q_t = Q(Z_t)$ *(ii) equation (1.11) maximizes intermediate goods firm's expected profits for all* j *given the equilibrium prices* $P_t = P(Z_t), W_t = W(Z_t)$*, and the signal (1.10) (iii) a credible central bank commits to setting the nominal interest rate in response to wage inflation and output (1.12),* $Q_t = Q(Z_t)$ *(iv) all markets clear:* $C_{j,t} = Y_{j,t}, N(Z_t) = \int N_{j,t}\,\mathrm{d}j$*, and (v) expectations are rational: household's beliefs about* W_t*,* P_t *and* Π_t^w, Π_t *are consistent with its belief about aggregate demand* Z_t*, and* $Y_t = Z_t$*, so that actual aggregate output follows a distribution consistent with* $\mathbf{F}$.

There exist two rational expectations equilibria among the class of linear Gaussian random variables. The first is a *fundamental equilibrium*, where aggregate output and prices are all constant and where sentiments play no role in determining the level of aggregate output, while the second is a *stochastic equilibrium* where sentiments matter and the volatility of beliefs about aggregate demand is endogenously determined and equal to the variance of aggregate output.

We can now consider a rational expectations equilibrium in the context of the model introduced in the previous section, which can be summarized by the following system of equations. The wage inflation inflation equation,

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w - \lambda_w \hat{\mu}_t^w, \tag{1.13}$$

where $\hat{\mu}_t^w \equiv \mu_t^w - \mu^w = \hat{w}_t^r - \gamma \hat{c}_t$ denotes deviations of the wage markup from its steady state level and $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$ is a measure of wage flexibility. Optimal inter-temporal consumption is given by the Euler equation (let $i_t \equiv -\ln Q_t, \rho \equiv -\ln \beta$)

$$\hat{c}_t = \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\gamma}(i_t - \rho - \mathbb{E}_t \hat{\pi}_{t+1}). \tag{1.14}$$

Firm production, conditional on signal $s_{j,t}$ is

$$\hat{y}_{j,t} = \mathbb{E}_t[\hat{\varepsilon}_{j,t} + \hat{y}_t - \theta \hat{w}_t^r | s_{j,t}], \tag{1.15}$$

where

$$s_{j,t} = \lambda \hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t.$$

The central bank follows the policy rule

$$i_t = \rho + \phi_\pi^w \hat{\pi}_t^w + \phi_y \hat{y}_t.$$

As there are no savings in this model, market clearing implies

$$\hat{y}_t = \hat{c}_t.$$

The real wage identity can be used to determine price inflation in equilibrium,

$$\hat{w}_{t+1}^r = \hat{w}_t^r + \mathbb{E}_t \hat{\pi}_{t+1}^w - \mathbb{E}_t \hat{\pi}_{t+1}.$$

Lastly, beliefs about aggregate demand are correct,

$$\hat{z}_t = \hat{y}_t.$$

1.4.6. Sentiment Equilibrium

Effect of an iid shock to sentiments

When firms condition on endogenous signals, there exists a sentiment driven equilibrium where aggregate output, $\hat{y}_t$, is stochastic and equal to the sentiment $\hat{z}_t$. To analyze the effect of an iid shock to sentiments on the volatility of output in a equilibrium where sentiments are self-fulfilling, conjecture $\hat{z}_t \sim N(0, \sigma_z^2)$ and policy functions for $\hat{c}_t$, $\hat{w}_t^r$, $\hat{\pi}_t$, and $\hat{\pi}_t^w$ where the state variables are $\hat{z}_t$, $\hat{w}_{t-1}^r$. The following policy functions verify the conjecture

$$\hat{c}_t = \hat{z}_t, \tag{1.16}$$

$$\hat{w}_t^r = \frac{\gamma(1+\lambda_w \phi_\pi^w) + \phi_y}{1+\lambda_w \phi_\pi^w} \hat{z}_t, \tag{1.17}$$

$$\pi_t^w = -\frac{\lambda_w \phi_y}{1+\lambda_w \phi_\pi^w} \hat{z}_t, \tag{1.18}$$

$$\pi_t = -\left[\frac{\gamma(1+\lambda_w \phi_\pi^w) + \phi_y(1+\lambda_w)}{1+\lambda_w \phi_\pi^w}\right] \hat{z}_t + \hat{w}_{t-1}^r. \tag{1.19}$$

Note that for a reasonable parameterization of the CRRA parameter ($\gamma > 1$) and Taylor rule coefficient for output ($\phi_y > 0$), the real wage increases in response to a positive sentiment shock. This occurs through a decrease in price inflation that is greater in magnitude than the decrease in wage inflation.

Firm j's optimal production decision (1.15), incorporating the relationship between the real wage and sentiments (1.17):

$$\hat{y}_{j,t} = \mathbb{E}_t \left[\hat{\varepsilon}_{j,t} + \left(1 - \theta \underbrace{\left[\frac{\gamma(1 + \lambda_w \phi_\pi^w) + \phi_y}{1 + \lambda_w \phi_\pi^w} \right]}_{D} \right) \hat{z}_t | s_{j,t} \right]. \tag{1.20}$$

Through its effects on aggregate variables, the stance of monetary policy (ϕ_π^w relative to ϕ_y) and the degree of wage flexibility (λ_w) affect the strategic interaction among firms, parameterized by coefficient $1 - \theta D(\phi_\pi^w, \phi_y, \lambda_w)$. Conditional on signal $s_{j,t} = \lambda \hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t$, the firms' best response is

$$\hat{y}_{j,t} = \frac{\lambda \sigma_\varepsilon^2 + (1-\lambda)(1 - \theta D)\sigma_z^2}{\lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_z^2} (\lambda \hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t). \tag{1.21}$$

Summing across firms, aggregate supply is

$$\hat{y}_t = \frac{\lambda \sigma_\varepsilon^2 + (1-\lambda)(1 - \theta D)\sigma_z^2}{\lambda^2 \sigma_\varepsilon^2 + (1-\lambda)^2 \sigma_z^2} (1-\lambda)\hat{z}_t.$$

In equilibrium, beliefs about aggregate demand are correct ($\hat{y}_t = \hat{z}_t$), which implies

$$\sigma_y^2 = \sigma_z^2 = \frac{1}{D} \frac{\lambda(1 - 2\lambda)}{(1-\lambda)^2 \theta} \sigma_\varepsilon^2. \tag{1.22}$$

The volatility of sentiments and hence output is determined by structural parameters. In a rational expectations equilibrium, monetary policy affects the optimal response of firm production to aggregate output, which has implications for the precision of the endogenous signals firms receive.

Proposition 3. *Let $\lambda \in (0, \frac{1}{2})$. There exists a sentiment-driven rational expectations equilibrium where aggregate output is stochastic, with variance increasing in ϕ_π^w and λ_w, and decreasing in ϕ_y,*

$$\sigma_z^2 = \frac{1 + \lambda_w \phi_\pi^w}{\gamma(1 + \lambda_w \phi_\pi^w) + \phi_y} \frac{\lambda(1 - 2\lambda)}{(1-\lambda)^2 \theta} \sigma_\varepsilon^2. \tag{1.23}$$

As $\phi_\pi^w \to \infty$, $\sigma_z^2 \to \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2$, its value under the flexible wages (A.20).

In a model with sticky wages and a central bank that targets wage inflation, the mechanism through which a sentiment shock is realized is inter-temporal. For a positive sentiment to be self-fulfilling, the real interest rate must fall in order for households to shift consumption to the current period. In this framework, the real interest rate decreases in one of two ways, either through a decrease in the nominal interest rate (which occurs if there is a decrease in wage inflation), or an increase in expected price inflation. Which combination of these changes will take place for a given sentiment shock to be self-fulfilling depends on the parameters ϕ_π^w, λ_w, γ, which respectively determine the extent to which the central bank targets wage inflation, the degree of wage flexibility, and the risk aversion of households. As these parameters affect how the real wage changes in equilibrium, they determine how much of their endogenous signal firms attribute to aggregate demand.[13]

Consider the process by which a belief about an increase in aggregate demand is self-fulfilling in this model. A belief about increased aggregate demand is self-fulfilling through a decrease in the real interest rate, not solely through an increase in the real wage. Instead, what happens to the real wage is a *consequence* of how the real interest rate changes in order for a sentiment shock to be fulfilled.

On the *demand side*, by the IS relation (1.14), in order for households to increase consumption, the real interest rate must fall. In this model, the real interest rate,

$$r_t = i_t - \mathbb{E}_t \pi_{t+1},$$

falls in one of two ways: either the nominal interest rate falls and/or expected price inflation increases (current price level falls), as

$$\mathbb{E}_t \pi_{t+1} \equiv \mathbb{E}_t p_{t+1} - p_t.$$

Expected price inflation is no longer zero in response to an iid sentiment shock if the central bank targets wage inflation, but is equal to the real wage (1.19). In this model, for expected price inflation to increase, either the real wage increases or the current price level falls. Next, for a central bank that targets wage inflation, the nominal interest rate decreases when wage inflation falls. By the New Keynesian Philips Curve for wage inflation, for wage inflation to fall when aggregate demand increases, the real wage must increase.

[13] In a model with flexible wages (see section A), a positive sentiment shock is self-fulfilling solely through an increase in the real wage (which implies that the price level falls, given a nominal wage). As the price level falls, households increase consumption and supply more labor. As the real wage increases, and all else equal, firms decrease production. However, if firms condition production on an endogenous signal of aggregate demand, there is an equilibrium level of output volatility such that firms misattribute enough of their signal to idiosyncratic demand, that aggregate supply equals beliefs about aggregate demand that households hold.

These effects can be verified by the policy functions (1.17-1.19). Following a positive sentiment shock and for reasonable parameterizations ($\gamma > 0, \lambda_w > 0, \phi_y \geq 0, \phi_\pi^w \geq 0$), the real wage increases through a decrease in price inflation that exceeds the fall in wage inflation ($\frac{\partial \pi_t}{\partial z_t} < \frac{\partial \pi_t^w}{\partial z_t}$),

$$\frac{\partial \pi_t}{\partial z_t} = \frac{\partial \pi_t^w}{\partial z_t} - \underbrace{\left(\frac{\gamma(1 + \lambda_w \phi_\pi^w) + \phi_y}{1 + \lambda_w \phi_\pi^w}\right)}_{>0}.$$

On the *supply side*, the real wage increases ($\frac{\partial w_t^r}{\partial z_t} > 0$) with a positive sentiment shock, raising marginal cost. However, an increase in aggregate demand also increases demand for good j. As the first effect dominates, ($\theta D > 1$) the optimal response of a firm to a sentiment shock will be to reduce production (see (1.20)). In other words, firm production is characterized by strategic substitutability. By Proposition 3, there can be a rational expectations equilibrium where Z_t is stochastic, and any realization from a distribution parameterized by σ_z^2 clears markets.

Next, consider how equilibrium outcomes are affected by wage flexibility and the response of monetary policy. The parameters ϕ_π^w, λ_w, and γ affect the degree to which a fall in the nominal interest rate substitutes for an increase in the real wage, required for a positive sentiment shock to be self-fulfilling. In summary, an increase in wage flexibility and a stronger response to wage inflation both have the same effect of mitigating the degree to which the real wage rises when beliefs about aggregate output increase.

A strong response to wage inflation (ϕ_π^w) caps the amount by which wage inflation needs to decrease in order to trigger a fall in the nominal interest rate required for households to consume what they believe will be aggregate output.[14] By the wage inflation equation (1.13), in order for wage inflation to fall when aggregate demand rises, the real wage must increase. However, if the nominal interest rates are very sensitive to changes in wage inflation, or if wages are flexible, this mitigates the extent to which the real wage must increase to reach a given level of wage deflation. See appendix (I.1) and (G) for details.[15]

[14] A strong response to wage inflation also implies that an increase in expected inflation (fall in the current price level) is not required for the real interest rate to decrease to a level such that a positive sentiment shock is fulfilled.

[15] Another way to see this is to replace w_t^r in (1.13) with the real wage identity, and rearranging terms,

$$\pi_t^w = -\frac{\lambda_w}{1 + \lambda_w}(\pi_t + c_t - w_{t-1}^r)$$

As $\frac{\lambda_w}{1+\lambda_w}$ is increasing in λ_w, the less price inflation needs to fall to reach a given level of wage inflation. The net effect is that the real wage increases by less when wages are more flexible.

Both wage flexibility and a strong response to wage inflation mitigate the degree to which the real wage increases in equilibrium. Firm production with respect to aggregate demand is characterized by less substitutability. All else equal, aggregate supply will exceed aggregate demand. In order for markets to clear, firms must attribute more of their signal to aggregate demand (σ_z^2 must increase), which will induce them to reduce output in response. The result is that sentiment volatility must be higher in equilibrium.

Therefore, although the co-movement of variables caused by z_t is similar to a technology shock, implementing the flexible wage allocation through a strong response to wage inflation increases volatility in beliefs about aggregate output. In an equilibrium where these beliefs can be self-fulfilling, targeting inflation strongly increases the volatility of realized output. By (1.23),

$$\frac{\partial \sigma_z^2}{\partial \phi_\pi^w} = \frac{\lambda_w \phi_y}{[\gamma(1+\lambda_w\phi_\pi^w)+\phi_y]^2}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 > 0,$$
$$\frac{\partial \sigma_z^2}{\partial \lambda_w} = \frac{\phi_\pi^w \phi_y}{[\gamma(1+\lambda_w\phi_\pi^w)+\phi_y]^2}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 > 0.$$

Proposition 4. *In an equilibrium with sentiment driven fluctuations, the central bank faces a tradeoff in stabilizing output and inflation. Equation (1.18) can be used to derive a relationship between the volatility of inflation and the volatility of output:*

$$\sigma_{\pi^w}^2 = \left(\frac{\lambda_w\phi_y}{1+\lambda_w\phi_\pi^w}\right)^2\sigma_y^2.$$

Expressing σ_y^2 and $\sigma_{\pi^w}^2$ in terms of model parameters,

$$\sigma_y^2 = \frac{1+\lambda_w\phi_\pi^w}{\gamma(1+\lambda_w\phi_\pi^w)+\phi_y}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2,$$

$$\sigma_{\pi^w}^2 = \frac{(\lambda_w\phi_y)^2}{(1+\lambda_w\phi_\pi^w)[\gamma(1+\lambda_w\phi_\pi^w)+\phi_y]}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2.$$

As the central bank increases its response to wage inflation (ϕ_π^w), the volatility of wage inflation declines, but this comes at the expense of higher volatility of output. Assuming $\gamma+\phi_y > 1$,

$$\frac{\partial \sigma_z^2}{\partial \phi_\pi^w} = \frac{\lambda_w \phi_y}{[\gamma(1+\lambda_w\phi_\pi^w)+\phi_y]^2}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 > 0.$$

Conversely, the more the central bank responds to output, the less volatile output becomes, but the more volatile wage inflation is in equilibrium.

$$\frac{\partial \sigma_{\pi^w}^2}{\partial \phi_y} = \frac{\lambda_w^2\phi_y[\phi_y+2\gamma(1+\lambda_w\phi_\pi^w)]}{[\gamma(1+\lambda_w\phi_\pi^w)+\phi_y]^2}\frac{1}{1+\lambda_w\phi_\pi^w}\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2 > 0.$$

1.5. Productivity shock

The previous section has shown how monetary policy that targets inflation strongly can increase the volatility of sentiment-driven fluctuations, which arise under a minor deviation from the perfect information benchmark of a standard New Keynesian model. In this extension, I consider the robustness of these results to the case where aggregate output is composed of both a non-fundamental and fundamental component, in the form of a technology shock that is unobservable (A_t). I show that when both types of shocks co-exist, the equilibrium features fluctuations from both sources.

1.5.1. Flexible wages

When aggregate technology is exogenous, the firms' first order condition is

$$Y_{j,t} = \left(\mathbb{E} \left[\frac{\theta - 1}{\theta} \frac{1}{\Psi} \epsilon_{j,t}^{\frac{1}{\theta}} Z_t^{\frac{1}{\theta} - \gamma} A_t | S_{j,t} \right] \right)^{\theta}.$$

This expression incorporates the household's optimal labor supply, demand for good j, and its production function

$$\begin{aligned} \frac{W_t}{P_t} &= \frac{1}{\Psi} Z_t^{\gamma}, \\ Y_{j,t} &= \left(\frac{P_t}{P_{j,t}} \right)^{\theta} \epsilon_{j,t} Y_t, \\ Y_{j,t} &= A_t N_{j,t}. \end{aligned}$$

Let $a_t \equiv \log A_t \sim N(\bar{a}, \sigma_a^2)$. The signal and aggregate production are

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Y_t^{1-\lambda}, \tag{1.24}$$

$$Y_t = \left[\int Y_{j,t}^{\frac{\theta-1}{\theta}} \epsilon_{j,t}^{\frac{1}{\theta}} \, \mathrm{d}j \right]^{\frac{\theta}{\theta-1}}. \tag{1.25}$$

Certainty equilibrium

Under complete information, firm j produces

$$Y_{j,t} = \left(\frac{\theta - 1}{\theta} \frac{1}{\Psi} \epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta} - \gamma} A_t \right)^{\theta}.$$

Proposition 5. *When firms perfectly observe shocks $\epsilon_{j,t}$ and A_t, there is a certainty equilibrium in which Y_t responds only to fluctuations in technology.*

$$Y_t = \left(\frac{\theta - 1}{\theta} \frac{1}{\Psi} A_t \left[\int \epsilon_{j,t} \, \mathrm{d}j \right]^{\frac{1}{\theta - 1}} \right)^{\frac{1}{\gamma}}$$

$y_t \equiv \log Y_t$ *has mean and variance*

$$\begin{aligned} \phi_0^{A*} &= \frac{1}{\gamma} \left[\log \left(\frac{\theta - 1}{\theta} \frac{1}{\Psi} \right) + \bar{a} + \frac{\sigma_\epsilon^2}{2(\theta - 1)} \right], \\ \sigma_y^2 &= \frac{1}{\gamma^2} \sigma_a^2. \end{aligned}$$

Sentiment equilibrium

Proposition 6. *When firms condition output on an endogenous signal, Y_t features fluctuations from both fundamental and non-fundamental sources, A_t and ζ_t. Aggregate output, $y_t \equiv \log Y_t \sim N(\phi_0, \sigma_y^2)$, is stochastic, with mean and variance*

$$\begin{aligned} \phi_0^{A} &= \frac{1}{\gamma} \left[\log \left(\frac{\theta - 1}{\theta} \frac{1}{\psi} \right) + \bar{a} + \frac{\Omega_s}{2} \right] + \frac{1}{\theta} \log \kappa_1, \\ \sigma_y^2 &= \frac{1}{\gamma\theta} \tilde{\sigma}_z^2 + \frac{1}{\gamma^2} \sigma_a^2, \end{aligned}$$

where $\kappa_1 \equiv \left[\int \epsilon_{j,t}^{\frac{1}{\theta} + \frac{\theta-1}{\theta} \lambda B} \, \mathrm{d}j \right]^{\frac{\theta}{\theta - 1}}$, $B = \frac{1}{1-\lambda}$, *and* $\tilde{\sigma}_z^2 \equiv \frac{\lambda}{1-\lambda} \left(1 - \frac{\lambda}{1-\lambda} \right) \sigma_\epsilon^2$. *The volatility of non-fundamental fluctuations is*

$$\sigma_\zeta^2 = \frac{1}{\gamma\theta} \tilde{\sigma}_z^2.$$

See Appendix (J).

As long as endogenous signals capture aggregate demand and firms are unable to distinguish between its fundamental and non-fundamental components, their signal extraction problem will entail misattributing a portion of aggregate demand, y_t, to idiosyncratic demand, $\epsilon_{j,t}$, which leads to sentiment driven fluctuations as in the baseline model. The source of the output-inflation trade-off can not be eliminated, and policymakers can not respond optimally to both technology shocks and sentiments.

1.5.2. Sticky wages

Proposition 7. *When firms condition output on an endogenous signal, Y_t features fluctuations from both fundamental and non-fundamental sources of fluctuations, A_t and ζ_t. Aggregate output, $y_t \equiv \log Y_t \sim N(\phi_0, \sigma_y^2)$, is stochastic, with variance increasing in ϕ_π^w and λ_w,*

$$\sigma_y^2 = \frac{1}{D}\left(\frac{1}{\theta}\tilde{\sigma}_z^2 + \psi_{ya}^2\sigma_a^2\right),$$

where $D \equiv \frac{\partial \hat{w}_t^r}{\partial \zeta_t} = \gamma + \frac{\phi_y}{1+\lambda_w \phi_\pi^w}$. The volatility of non-fundamental fluctuations is

$$\sigma_\zeta^2 = \frac{1}{D\theta}\tilde{\sigma}_z^2 + \frac{\psi_{ya}}{D}\sigma_a^2 - \psi_{ya}^2\sigma_a^2.$$

See Appendix (K).

1.6. Conclusion

This chapter has shown how conventional optimal monetary policy is not robust to a deviation from perfect information. In a model that departs slightly from the standard New Keynesian framework in assuming that firms condition on endogenous signals to decide production (pricing) before shocks are known, beliefs about aggregate demand can be self-fulfilling. Although these fluctuations have a non-fundamental source, the range of outcomes is disciplined by rational expectations and pinned down by deep structural parameters.

Through its effect on aggregate variables, the stance of monetary policy determines the precision of endogenous signals that firms receive, and consequently, the degree of coordination in firms' production (price setting). As a result, the distribution of non-fundamental shocks is no longer independent of policy, introducing a novel tradeoff between stabilizing output and inflation. Strong inflation targeting increases the variance of non-fundamental fluctuations.

The broader implication of the paper is to highlight the importance of models that deviate from full information when considering the effects of policy. In particular, modeling dispersed information provides a role for strategic uncertainty, which complements the literature that has considered uncertainty about fundamentals. This paper studies a model where sentiment driven fluctuations can arise when households and firms make decisions simultaneously, while endogenous signals allow the sentiments of one class of agents to feed into another. Monetary policy, through its effects on payoff relevant aggregate variables, can influence firms' motives to coordinate, while firms' actions affect the precision of the endogenous signals they receive. In providing an alternative channel through which policy may affect outcomes, this framework offers a different perspective on what constitutes stabilization policy.

Appendix

A. Flexible Wages

Consider a representative household and a continuum of monopolistic intermediate goods producers indexed by $j \in [0, 1]$. Households supply labor and form *demand schedules* for differentiated goods conditional on shocks that have not yet been realized. The key friction is that intermediate goods firms commit to labor demand and output, based on an imperfect signal of the aggregate demand and firm level demand, prior to goods being produced and exchanged and before marketing clearing prices are realized.

After production decisions are made, the goods market opens, demand is realized, and prices adjust to clear the market. The firms' signal extraction problem can lead to multiple equilibria and endogenous fluctuations in aggregate output.

A.1. Households

The representative household chooses labor N_t to maximize utility

$$\max_{N_t} \log C_t + \Psi(1 - N_t)$$

subject to budget constraint

$$C_t \leq \frac{W_t}{P_t} N_t + \frac{\Pi_t}{P_t}$$

where C_t is aggregate an consumption index, $\frac{W_t}{P_t}$ is the real wage, $\frac{\Pi_t}{P_t}$ is real profit income from all firms, Ψ is disutility of labor. Their first order condition is

$$C_t = \frac{1}{\Psi} \frac{W_t}{P_t} \tag{A.1}$$

where

$$C_t = \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{A.2}$$

C_t represents an aggregate consumption index, $\theta > 1$ is the elasticity of substitution between goods, $C_{j,t}$ denotes the quantity of good j consumed by the household in period t. The idiosyncratic preference shock for good j is log normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$). The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is solely intended to simplify expressions.

The household allocates consumption among j goods to maximize C_t for any given level of expenditures $\int_0^1 P_{j,t} C_{j,t}\,\mathrm{d}j$, where $P_{j,t}$ is the price of intermediate good j.

Optimizing its consumption allocation, household's demand for good j is given by

$$C_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} C_t \epsilon_{j,t}. \tag{A.3}$$

The resulting aggregate price level is obtained by substituting (A.3) into (A.2),

$$P_t = \left(\int_0^1 \epsilon_{j,t} P_{j,t}\,\mathrm{d}j\right)^{\frac{1}{1-\theta}}.$$

In this model, households form demand schedules for each differentiated good and supply labor, all contingent on shocks to idiosyncratic demand and shocks to aggregate income/consumption are be realized. Let Z_t represent the household's beliefs about aggregate income/consumption at the beginning of period t. Households form consumption *plans* using (A.3)

$$C_{j,t}(Z_t, \epsilon_{j,t}) = \left(\frac{P_t(Z_t)}{P_{j,t}(Z_t, \epsilon_{j,t})}\right)^{\theta} C_t(Z_t)\epsilon_{j,t}, \tag{A.4}$$

and decide labor supply, using (A.1) to obtain an implicit function of labor supply as a function of sentiments, $N_t = N(Z_t)$, given a nominal wage W_t,

$$P_t(Z_t) = \frac{W_t}{\Psi\left[\frac{1}{P_t(Z_t)} N_t + \frac{\Pi_t(Z_t)}{P_t(Z_t)}\right]}. \tag{A.5}$$

Note that $\Pi_t(Z_t) = P_t(Z_t)Z_t - W_t N_t$.

A.2. Intermediate goods firms

The intermediate goods firms decide production level $Y_{j,t}$ without perfect knowledge of idiosyncratic demand ($\epsilon_{j,t}$) or aggregate demand (Y_t). Instead, they infer these quantities from a signal $S_{j,t}$ that may be interpreted as early orders, advance sales, or market research,

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda},$$

where $\log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$ and $\log Z_t \sim N(\phi_0, \sigma_z^2)$.

Given the nominal wage, intermediate goods producers choose $Y_{j,t}$ to maximize nominal profits ($\Pi_{j,t} = P_{j,t}Y_{j,t} - W_tN_{j,t}$) subject to production function ($Y_{j,t} = AN_{j,t}$) and demand for its good (A.3). Substituting out labor demand of firm j, ($N_{j,t} = \frac{Y_{j,t}}{A}$) and the price of its good ($P_{j,t}$) using (A.3), firm j's problem is

$$\max_{Y_{j,t}} \mathbb{E}_t \left[P_t Y_{j,t}^{1-\frac{1}{\theta}} (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} - \frac{W_t}{A} Y_{j,t} | S_{j,t} \right],$$

The first order condition of intermediate goods firm j is given by,

$$\left(1 - \frac{1}{\theta}\right) Y_{j,t}^{-\frac{1}{\theta}} \mathbb{E}_t \left[P_t (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} | S_{j,t} \right] = \frac{W_t}{A}.$$

Rearranging terms,

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) A \mathbb{E}_t \left[(\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} \frac{P_t}{W_t} | S_{j,t} \right] \right]^{\theta}, \tag{A.6}$$

Substitute P_t with the household's first order condition, $P_t = \frac{1}{\Psi} \frac{W_t}{Y_t}$, where $Y_t = C_t$ due to the absence of savings in this model. As nominal variables are indeterminate in the flexible wage extension, the nominal wage can be normalized to 1,

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) \frac{A}{\Psi} \mathbb{E}_t [\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}] \right]^{\theta}$$

Higher aggregate demand affects firm j's optimal production decision in two ways; while it implies an increase in demand for good j, it also implies that the real wage will be higher. The first effect derives from households' optimal consumption across goods, while the second follows from the labor supply decision of household. Given a nominal wage, the aggregate price level will be lower as aggregate demand increases. This will result in a fall in demand for $C_{j,t}$, which decreases firm j's optimal output level. As $\frac{1}{\theta} - 1 < 0$, the latter effect dominates, with the result that firm j's optimal output decreases with aggregate output. Although firms' actions are strategic substitutes, the rational expectations equilibrium may not be unique if firms condition production on an endogenous signal containing aggregate and idiosyncratic demand.

A.3. Timing

With Z_t as aggregate demand and $\epsilon_{j,t}$ as idiosyncratic preference for good j, the timing of this model is as follows,

1. Households form labor supply schedule ($N_t(Z_t)$) and demand schedules for each good j, ($C_{j,t}(Z_t, \epsilon_{j,t})$), contingent on shocks to be realized.

2. Z_t, $\epsilon_{j,t}$ realized.
3. Firms receive a private signal of aggregate demand and idiosyncratic preference for their good ($S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}$).
4. Firms can not write contingent schedules for their labor demand, otherwise this would remove the possibility of sentiment-driven fluctuations. Instead, firms must commit to production and hence labor demand, based on an imperfect private signal. They produce $Y_{j,t}(S_{j,t})$ and demand labor $N_{j,t}(S_{j,t}) = \frac{Y_{j,t}(S_{j,t})}{A}$.
5. Goods market opens. Z_t, $\epsilon_{j,t}$ observed by everyone. $P_{j,t}$ adjusts so that goods market clears ($C_{j,t} = Y_{j,t}$, $C_t = Y_t$), and $P_t = \frac{1}{\Psi Z_t}$.

A.4. Equilibrium

In equilibrium, aggregate output, intermediate goods supply, and the private signal are given by

$$Y_t = \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} \,\mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \tag{A.7}$$

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right)\frac{A}{\Psi}\mathbb{E}[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}]\right]^{\theta}, \tag{A.8}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \tag{A.9}$$

The first equation indicates that in equilibrium, goods markets clear: $Y_t = C_t$, $C_{j,t} = Y_{j,t}$. In the sentiment driven equilibrium, an additional condition stipulates that beliefs about aggregate demand are correct in equilibrium,

$$Z_t = Y_t. \tag{A.10}$$

After the realization of Y_t, and after goods markets clear, the aggregate price index, market clearing prices for each good, aggregate labor, and aggregate profits are given by

$$P_t = \frac{1}{\Psi Y_t}, \tag{A.11}$$

$$P_{j,t} = (\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} Y_{j,t}^{-\frac{1}{\theta}} P_t, \tag{A.12}$$

$$N_t = \int_0^1 N_{j,t} \,\mathrm{d}j = \int_0^1 \frac{Y_{j,t}}{A} \,\mathrm{d}j, \tag{A.13}$$

$$\Pi_t = P_t Y_t - N_t = \frac{1}{\Psi} - N_t. \tag{A.14}$$

In the first equation, the actual aggregate price level in equilibrium is determined by realized aggregate output. The second equation indicates that in equilibrium, the market clearing price for good j is determined by realized aggregate output, production of good j, and the realized aggregate price level. In the third equation, labor supply equals aggregate labor demand. In the fourth equation, aggregate profits equal aggregate revenue minus aggregate production costs.

Definition A.1. *A rational expectations equilibrium is a sequence of allocations* $\{C(Z_t), Y(Z_t), C_j(Z_t, \epsilon_{j,t}), Y_j(Z_t, \epsilon_{j,t}), N(Z_t), N_j(Z_t, \epsilon_{j,t}), \Pi(Z_t)\}$*, prices* $\{P(Z_t), P_j(Z_t, \epsilon_{j,t}), W_t = 1\}$*, and a distribution of* Z_t*,* $\mathbf{F}(Z_t)$ *such that for each realization of* Z_t*, (i) equations (A.4) and (A.5) maximize household utility given the equilibrium prices* $P_t = P(Z_t), P_{j,t} = P_j(Z_t, \epsilon_{j,t})$*, and* $W_t = 1$ *(ii) equation (A.8) maximizes intermediate goods firm's expected profits for all* j *given the equilibrium prices* $P(Z_t), W_t = 1$*, and the signal (A.9) (iii) all markets clear:* $C_{j,t} = Y_{j,t}, N(Z_t) = \int N_{j,t}\,\mathrm{d}j$*, and (iv) expectations are rational such that the household's beliefs about* P_t *and* Π_t *are consistent with its belief about aggregate demand* Z_t *(according to its optimal labor supply condition) and* $Y_t = Z_t$*: actual aggregate output follows a distribution consistent with* $\mathbf{F}$.

There exist two rational expectations equilibria: (1) a fundamental equilibrium with a degenerate distribution of sentiments, where aggregate output and prices are all constant and where sentiments play no role in determining the level of aggregate output and (2) a stochastic equilibrium where sentiments matter and the volatility of beliefs about aggregate demand is endogenously determined and equal to the variance of aggregate output. As firms make their production decisions based on the correctly anticipated distribution of aggregate demand and their own idiosyncratic demand shocks, these self-fulfilling stochastic equilibria are consistent with rational expectations.

Fundamental equilibrium

Under perfect information, firms receive signals that reveal their idiosyncratic demand shocks, and we will show that there is a unique rational expectations equilibrium in which output, aggregate demand, and the aggregate price level are constant. Using the equilibrium conditions in (A.8), (A.7), (A.12), and (A.11), $Y_t, P_t, Y_{j,t}$ and $P_{j,t}$ in the fundamental equilibrium are as follows: From (A.8),

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) \frac{A}{\Psi} \epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1}\right]^{\theta}. \tag{A.15}$$

Using (A.7), and substituting $Y_{j,t}$ with (A.15),

$$\begin{aligned}
Y_t &= \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{1-\frac{1}{\theta}}\,\mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \\
&= \left[\int_0^1 \epsilon_{j,t}^{\frac{1}{\theta}} \left[\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1}\right]^{\theta-1}\mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \\
&= \left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\left[\int_0^1 \epsilon_{j,t}\,\mathrm{d}j\right]^{\frac{1}{\theta-1}}.
\end{aligned}$$

Let variables with * denote their counterparts in the fundamental equilibrium. As $C_t = Y_t$ in equilibrium,

$$C^* = Y^* = \left(1-\frac{1}{\theta}\right)\frac{A}{\Psi}\left[\int_0^1 \epsilon_{j,t}\,\mathrm{d}j\right]^{\frac{1}{\theta-1}}. \tag{A.16}$$

Using (A.11), the equilibrium aggregate price level is

$$P^* = \frac{1}{\Psi Y^*} = \frac{\theta}{\theta-1}\frac{1}{A}\left[\int_0^1 \epsilon_{j,t}\,\mathrm{d}j\right]^{\frac{1}{1-\theta}}.$$

In the fundamental equilibrium, as Y_t is known, $S_{j,t}$ reveals $\epsilon_{j,t}$ perfectly. Any shift in $\epsilon_{j,t}$ results in a corresponding change in $Y_{j,t}$ without affecting $P_{j,t}$. Substituting the previous expressions for Y_t, P_t, and $Y_{j,t}$ into (A.12),

$$P_{j,t} = \frac{\theta}{\theta-1}\frac{1}{A}.$$

Let $y^* \equiv \log(Y^*)$. Without loss of generality, let $\left(1-\frac{1}{\theta}\right)\frac{A}{\Psi} = 1$. Then another way of expressing (A.16)

$$y^* = \frac{1}{2(\theta-1)}\sigma_\varepsilon^2. \tag{A.17}$$

$$\tag{A.18}$$

Sentiment-driven equilibrium

When firms face information frictions, there exists a sentiment driven equilibrium, in addition to the fundamental equilibrium. The sentiment driven equilibrium is a rational expectations equilibrium where aggregate output is not constant but equal to a sentiment (Z_t). Let $\hat{z}_t$ and $\hat{y}_t$ denote Z_t and Y_t in log deviation from the steady state of this equilibrium, respectively.[16] $\hat{z}_t \sim N(0, \sigma_z^2)$, where σ_z^2 is a constant to be determined below.

Equation (A.8) gives firm j's optimal output conditional on its signal. As it is derived using equations (A.1) and (A.3), it already incorporates market clearing for labor and consumption.

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right)\frac{A}{\Psi}\mathbb{E}[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}]\right]^{\theta}. \tag{A.19}$$

Firm j's private signal is

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}.$$

Log-linearizing around the steady state,

$$\hat{y}_{j,t} = \mathbb{E}_t[\hat{\varepsilon}_{j,t} + (1-\theta)\hat{y}_t | s_{j,t}].$$

Conditional on its signal, firm j's best response is

$$\begin{aligned}
\hat{y}_{j,t} &= \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2} s_{j,t}, \\
&= \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\hat{\varepsilon}_{j,t} + (1-\lambda)\hat{z}_t).
\end{aligned}$$

Aggregate supply is then

$$\begin{aligned}
\hat{y}_t &= \int_0^1 \hat{y}_{j,t}\,\mathrm{d}j, \\
&= \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda)\hat{z}_t.
\end{aligned}$$

In this equilibrium, household's beliefs about aggregate demand are correct ($\hat{y}_t = \hat{z}_t$). This implies

$$1 = \frac{\lambda\sigma_\varepsilon^2 + (1-\theta)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda).$$

[16] See the next section (appendix A.4) for a calculation of the steady state in this equilibrium.

Then, the volatility of actual aggregate output and beliefs about aggregate demand are determined by the parameters of the model. If $\lambda \in (0, \frac{1}{2})$ and $\sigma_\varepsilon^2 > 0$, then there exists a sentiment driven rational expectations equilibrium with $\hat{y}_t = \hat{z}_t$ where[17]

$$\sigma_y^2 = \sigma_z^2 = \underbrace{\frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2}_{B}. \tag{A.20}$$

Let B denote the volatility of sentiments under the baseline model. The volatility of the sentiment shock must be commensurate with the degree of complementarity/substitutability in actions across firms (θ), information content of the private signal (λ), and the volatility of idiosyncratic demand (σ_ε^2), all of which affect the firm's response to a sentiment shock.

Note that if $\lambda = 1$, the signal contains only the idiosyncratic preference shock, the result is that an equilibrium with constant output is the unique equilibrium. If $\lambda = 0$ or $\sigma_\varepsilon^2 = 0$, then the private signal conveys only aggregate components. The result is also that the unique equilibrium is the fundamental equilibrium, due to substitutability of firms' outputs.

The intuition for why the sentiment-driven equilibrium is a rational expectations equilibrium is as follows: Given the parameters of the model, σ_z^2 is determined such that for any aggregate demand sentiment, all firms misattribute enough of the sentiment component of their signal to an idiosyncratic preference shock such that aggregate output will be equal to the sentiment in equilibrium. The volatility of the sentiment process (σ_z^2) determines how much firms attribute their signal to $\hat{z}_t$. In particular, when firms' actions are strategic substitutes, the optimal output of a firm is declining in σ_z^2 as this leads the firms to attribute more of the signal to an aggregate demand shock. Since firms' optimal output depends negatively on the level of $\hat{z}_t$ and positively on the idiosyncratic preference shock $\hat{\varepsilon}_{j,t}$, if they are unable to distinguish between the two components in their signal, then there can be a coordinated over-production (under-production) in response to a positive (negative) aggregate sentiment shock, such that $\hat{y}_t$ equals $\hat{z}_t$ in equilibrium if σ_z^2 is as in (A.20). The rational expectations equilibrium pins down the variance of the sentiment distribution, although sentiments are extrinsic. The result is an additional rational expectations equilibrium that is characterized by aggregate fluctuations in output and employment despite the lack of fundamental aggregate shocks.

[17] Alternatively, $\sigma_y^2 = \sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{1-\frac{\lambda}{1-\lambda}}{\theta}\sigma_\varepsilon^2$, where the elasticities of firm j's production with respect to $\epsilon_{j,t}$ and y_t are $\beta_0 = 1$ and $1 - \beta_1 = \theta$, as in section (1.3).

Steady state of the Sentiment-driven equilibrium

The firm's optimal production, incorporating households' optimal labor supply decision (A.1), and contingent on signal $s_{j,t}$ is

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right)\frac{A}{\Psi}\mathbb{E}_t[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | S_{j,t}]\right]^{\theta}.$$

Let $\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$ and $z_t \equiv (\log Z_t) - \phi_0 \sim N(0, \sigma_z^2)$, firm j's signal is

$$S_{j,t} = \varepsilon_{j,t}^{\lambda} Z_t^{1-\lambda}.$$

Without loss of generality, normalize $\left(1 - \frac{1}{\theta}\right)\frac{A}{\Psi}$ to 1. Firm production is then

$$Y_{j,t} = \left(\mathbb{E}_t[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-1} | s_{j,t}]\right)^{\theta}.$$

Define $y_t \equiv (\log Y_t) - \phi_0$. Unless specified otherwise, let lower-case letters represent the variable in logs. In this equilibrium, as aggregate demand is sentiment driven, we can replace y_t in the firm's response with z_t

$$y_{j,t} = (1-\theta)\phi_0 + \theta \log \mathbb{E}_t\left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right) | s_{j,t}\right].$$

To compute the conditional expectation, note that $\mathbb{E}_t\left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right) | s_{j,t}\right]$ is the moment generating function of normal random variable $\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right) | s_{j,t}$. Then

$$\begin{aligned}
&\mathbb{E}_t\left[\exp\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t\right) | s_{j,t}\right] \\
&= \exp\left[\mathbb{E}_t\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t | s_{j,t}\right) + \frac{1}{2}\text{Var}\left(\frac{1}{\theta}, \varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t | s_{j,t}\right)\right],
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{E}_t\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t | s_{j,t}\right) &= \frac{\text{cov}(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta} z_t, s_{j,t})}{\text{var}(s_{j,t})} s_{j,t}, && \text{(A.21)}\\
&= \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\varepsilon_{j,t} + (1-\lambda)z_t). && \text{(A.22)}
\end{aligned}$$

For now, let $\Omega_s \equiv \text{Var}\left(\frac{1}{\theta}\varepsilon_{j,t} + \frac{1-\theta}{\theta}z_t | s_{j,t}\right)$. As $\frac{1}{\theta}\varepsilon_{j,t}, \frac{1-\theta}{\theta}z_t$ are Gaussian, Ω_s does not depend on $s_{j,t}$.

$$y_{j,t} = (1-\theta)\phi_0 + \theta\frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}(\lambda\varepsilon_{j,t} + (1-\lambda)z_t) + \frac{\theta}{2}\Omega_s, \tag{A.23}$$

$$= \varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t). \tag{A.24}$$

where

$$\mu \equiv \frac{\frac{1}{\theta}\lambda\sigma_\varepsilon^2 + \frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_z^2}, \tag{A.25}$$

$$\varphi_0 \equiv (1-\theta)\phi_0 + \frac{\theta}{2}\Omega_s. \tag{A.26}$$

Using equilibrium condition (A.7) which equates aggregate demand and aggregate supply, get an expression for y_t in terms of $y_{j,t}$

$$\left(1 - \frac{1}{\theta}\right)\log Y_t = \log\left(\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} dj\right),$$

$$\left(1 - \frac{1}{\theta}\right)(\phi_0 + z_t) = \log \mathbb{E}_t\left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}}\right),$$

$$= \log \mathbb{E}_t\left(\exp\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}y_{j,t}\right]\right).$$

Replacing $y_{j,t}$ with (A.24) and using the properties of a moment generating function for normal random variable $\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}\left[\varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t)\right]\right]$,

$$\left(1 - \frac{1}{\theta}\right)(\phi_0 + z_t) = \log \mathbb{E}_t\left(\exp\left[\frac{1}{\theta}\varepsilon_{j,t} + \frac{\theta-1}{\theta}\left[\varphi_0 + \theta\mu(\lambda\varepsilon_{j,t} + (1-\lambda)z_t)\right]\right]\right), \tag{A.27}$$

$$= \left(1 - \frac{1}{\theta}\right)\varphi_0 + \left[\frac{\theta-1}{\theta}\theta\mu(1-\lambda)\right]z_t + \frac{1}{2}\left[\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right]^2\sigma_\varepsilon^2, \tag{A.28}$$

$$\left(\frac{\theta-1}{\theta}\right)(\phi_0 + z_t) = \frac{\theta-1}{\theta}\varphi_0 + \frac{\theta-1}{\theta}\theta\mu(1-\lambda)z_t + \frac{1}{2}\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right)^2\sigma_\varepsilon^2. \tag{A.29}$$

Match the coefficients in (A.29) to get two constraints for the parameters to be determined (ϕ_0, σ_z^2)

$$\theta\mu = \frac{1}{1-\lambda}, \tag{A.30}$$

$$\frac{\theta-1}{\theta}\phi_0 = \frac{\theta-1}{\theta}\varphi_0 + \frac{1}{2}\left(\frac{1}{\theta} + \frac{\theta-1}{\theta}\theta\mu\lambda\right)^2\sigma_\varepsilon^2. \tag{A.31}$$

Next, σ_z^2 can be solved for in terms of the structural parameters using (A.30) and (A.25)

$$\sigma_z^2 = \frac{\lambda(1-2\lambda)}{(1-\lambda)^2\theta}\sigma_\varepsilon^2. \tag{A.32}$$

Rearranging terms for a more intuitive expression,

$$\sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{1-\frac{\lambda}{1-\lambda}}{\theta}\sigma_\epsilon^2.$$

Next, solve for the steady state (ϕ_0), using (A.29),

$$\phi_0 = \varphi_0 + \frac{1}{2}\frac{\theta-1}{\theta}\left[\frac{1}{\theta-1}+\frac{\lambda}{1-\lambda}\right]^2\sigma_\epsilon^2.$$

Substituting for φ_0 and simplifying,

$$\phi_0 = \frac{\Omega_s}{2} - \log\psi + \frac{1}{2\theta}\frac{\theta-1}{\theta}\left[\frac{1}{\theta-1}+\frac{\lambda}{1-\lambda}\right]^2\sigma_\epsilon^2.$$

As $\Omega_s \equiv \text{var}\left(\frac{1}{\theta}\varepsilon_{j,t}+\frac{1-\theta}{\theta}z_t|s_{j,t}\right)$,

$$\begin{aligned}
\Omega_s &= \text{var}(\frac{1}{\theta}\varepsilon_{j,t}+\frac{1-\theta}{\theta}z_t) - \frac{[\text{cov}(\frac{1}{\theta}\varepsilon_{j,t}+\frac{1-\theta}{\theta}z_t, s_{j,t})]^2}{\text{var}(s_{j,t})},\\
&= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)^2\sigma_z^2 - \mu\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2+\frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right],\\
&= \left(\frac{1}{\theta}\right)^2\sigma_\varepsilon^2 + \left(\frac{1-\theta}{\theta}\right)^2\sigma_z^2 - \left(\frac{1}{\theta}\frac{1}{1-\lambda}\right)\left[\frac{1}{\theta}\lambda\sigma_\varepsilon^2+\frac{1-\theta}{\theta}(1-\lambda)\sigma_z^2\right],\\
&= \frac{1}{\theta^2}\left(1-\frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2 + \frac{1-\theta}{\theta^2}(-\theta\sigma_z^2),
\end{aligned}$$

where the third equality uses (A.21) and (A.25). Incorporating (A.32),

$$\Omega_s = \frac{1}{\theta^2}\left(1-\frac{\lambda}{1-\lambda}\right)\left(1+(1-\theta)\left(-\frac{\lambda}{1-\lambda}\right)\right)\sigma_\epsilon^2.$$

Simplifying,

$$\Omega_s = \frac{(1-\lambda)(1-2\lambda)+(\theta-1)\lambda(1-2\lambda)}{\theta^2(1-\lambda)^2}\sigma_\varepsilon^2.$$

Then by (A.26) and (A.31),

$$\phi_0 = \frac{(1-\lambda)(\theta-1)\lambda}{\theta(1-\lambda)} \underbrace{\frac{1}{2(\theta-1)}\sigma_\varepsilon^2}_{\phi_0^*},$$

where ϕ_0^* denotes the steady state of the fundamental equilibrium (A.17).

B. Price Setting Firms

The first section will consider the decisions of households, intermediate goods firms, and equilibrium conditions of the flexible price model. The mechanism behind a self-fulfilling equilibrium with sentiments will be described. The second section will consider the impact of monetary policy on the volatility of output in the sentiment-driven equilibrium.

B.1. Flexible Prices

There is a representative household and a continuum of monopolistic intermediate goods producers indexed by $j \in [0,1]$. Households supply labor and form *demand schedules* for differentiated goods conditional on shocks that have not yet been realized. The key friction is that intermediate goods firms must set prices first and commit to meeting demand at the announced price, based on an imperfect signal of the aggregate demand and firm level demand.

After prices are set, the goods market opens, demand is realized, and production adjust to meet demand at the announced price. The firms' signal extraction problem can lead to multiple equilibria and endogenous fluctuations in aggregate output.

Households

The representative household's problem is[18]

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \Psi(1-N_t) \right),$$

[18]For non-linear disutility of labor, see Appendix (I.2). Specifying the utility function in this way ($\gamma \neq 1$) will allow sentiments to affect the real wage, by γ, the CRRA parameter. This will affect the firms' marginal cost and their optimal response to sentiments.

subject to

$$C_t \equiv \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

$$\int P_{j,t} C_{j,t} dj + Q_t B_t \leq B_{t-1} + W_t N_t + \Pi_t.$$

where C_t is an aggregate consumption index and $C_{j,t}$ denotes the quantity of good j consumed by the household in period t. The idiosyncratic preference shock for good j is log normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$. Ψ is disutility of labor, while $\theta > 1$ is the elasticity of substitution between goods. The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is solely intended to simplify calculations. Π_t is profit income from all firms, while W_t is the wage.

The household allocates consumption among j goods to maximize C_t for any given level of expenditures. Optimizing its consumption allocation, household's demand for good j is given by

$$C_{j,t} = \left(\frac{P_t}{P_{j,t}} \right)^\theta C_t \epsilon_{j,t}. \tag{B.33}$$

The resulting aggregate price level is obtained by substituting (B.33) into the aggregate consumption index,

$$P_t = \left(\int_0^1 \epsilon_{j,t} P_{j,t} dj \right)^{\frac{1}{1-\theta}},$$

and implies $\int P_{j,t} C_{j,t} dj = P_t C_t$.

Choosing labor (N_t) optimally, the households' labor supply condition is

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}, \tag{B.34}$$

$$\Psi C_t^\gamma = \frac{W_t}{P_t}, \tag{B.35}$$

where $\frac{W_t}{P_t}$ is the real wage. Taking the log of this expression,

$$w_t - p_t = \gamma c_t + \log \Psi.$$

Intertemporal consumption is

$$Q_t = \beta \mathbb{E}_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right).$$

In logs,

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\gamma}[i_t - \mathbb{E}_t \pi_{t+1} - \rho].$$

In this model, households form demand schedules for each differentiated good and supply labor, all contingent on shocks to idiosyncratic demand and shocks to aggregate income/consumption to be realized. Let Z_t represent the household's beliefs about aggregate income/consumption at the beginning of period t. Households form consumption *plans* using (B.33)

$$C_{j,t}(Z_t, \epsilon_{j,t}) = \left(\frac{P_t(Z_t)}{P_{j,t}(Z_t, \epsilon_{j,t})}\right)^{\theta} C_t(Z_t)\epsilon_{j,t}, \tag{B.36}$$

and decide labor supply, using (B.35) to obtain an implicit function of labor supply as a function of sentiments, $N_t = N(Z_t)$, given a nominal wage W_t

$$P_t(Z_t) = \frac{W_t}{\Psi\left[\frac{W_t}{P_t(Z_t)}N_t + \frac{\Pi_t(Z_t)}{P_t(Z_t)}\right]^{\gamma}}. \tag{B.37}$$

Note that $\Pi_t(Z_t) = P_t(Z_t)Z_t - W_t N_t$.

Intermediate goods firms

Sentiment driven equilibria requires a signal extraction problem with two shocks, to each of which the optimal response of the firm's price setting decision is different. The Dixit-Stiglitz structure of the model implies that the optimal price for intermediate goods firm j under perfect information does not depend on the idiosyncratic preference shock for good j. To circumvent this, assume that a firm's marginal cost is positively correlated with its demand.

The intermediate goods firms decide price $P_{j,t}$ without perfect knowledge of idiosyncratic demand or aggregate demand. Instead, they infer $\epsilon_{j,t}$ and $Y_{j,t}$ from a signal $S_{j,t}$ that may be interpreted as early orders, advance sales, or market research,

$$S_{j,t} = \varepsilon_{j,t}^{\lambda} Y_t^{1-\lambda}.$$

Let $\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$ and $y_t \equiv (\log Y_t) - \phi_0 \sim N(0, \sigma_y^2)$.

Given an aggregate price index (P_t), intermediate goods producers choose $P_{j,t}$ to maximize nominal profits

$$\max_{P_{j,t}} \mathbb{E}_t \left[P_{j,t} Y_{j,t} - W_t N_{j,t}\right]$$

subject to production function

$$Y_{j,t} = \epsilon_{j,t}^{\tau} N_{j,t}.$$

Note that idiosyncratic demand $\epsilon_{j,t}$ will also need to affect production technology for the sentiment equilibrium to exist (for example, if demand affects marketing costs). Under this assumption, the two components of the signal, $\epsilon_{j,t}$ and Z_t will affect marginal cost differently, and fluctuations are possible when agents misattribute the latter to the former.

Demand schedule for good j (imposing the market clearing condition, $C_t = Y_t$ and $C_{j,t} = Y_{j,t}$)

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} \epsilon_{j,t} Y_t,$$

Substituting $N_{j,t}$ using firm j's production function and $Y_{j,t}$ from its demand schedule, the firms' problem is

$$\max_{P_{j,t}} \mathbb{E}_t \left[P_{j,t}^{1-\theta} P_t^{\theta} \epsilon_{j,t} Y_t - W_t P_t^{\theta} P_{j,t}^{-\theta} \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t} \right]. \tag{B.38}$$

The first order condition is given by

$$(1-\theta) P_{j,t}^{-\theta} P_t^{\theta} \mathbb{E}_t(\epsilon_{j,t} Y_t | S_{j,t}) + \theta P_t^{\theta} P_{j,t}^{-\theta-1} \mathbb{E}_t(W_t \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t}) = 0.$$

As nominal variables are indeterminate in the flexible price case, the nominal aggregate consumption price index (P_t) can be normalized to 1. Rearranging terms,

$$P_{j,t} = \left(\frac{\theta}{\theta-1}\right) \frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | S_{j,t}]}.$$

Replacing W_t with the household's labor supply decision, firm j's optimal price is

$$P_{j,t} = \left(\frac{\theta}{\theta-1}\right) \Psi \frac{\mathbb{E}[\epsilon_{j,t}^{1-\tau} Y_t^{\gamma+1} | S_{j,t}]}{\mathbb{E}[\epsilon_{j,t} Y_t | S_{j,t}]}.$$

Timing

Letting Z_t denote aggregate demand and $\epsilon_{j,t}$ represent idiosyncratic preference for good j, the timing of this model is as follows:

1. Households form labor supply schedule $(N_t(Z_t))$ and demand schedules for each good j, $(C_{j,t}(Z_t, \epsilon_{j,t}))$, contingent on shocks to be realized.

2. Z_t, $\epsilon_{j,t}$ realized.

3. Firms receive a private signal of aggregate demand and idiosyncratic preference for their good $(S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda})$.

4. Firms can not write contingent schedules for their labor demand, otherwise this would remove the possibility of sentiment-driven fluctuations. Instead, firms must commit to a price $(P_{j,t}(s_{j,t}))$, based on an imperfect private signal.

5. Goods market opens. Z_t, $\epsilon_{j,t}$ observed by everyone. Firms meet supply at posted price $Y_{j,t}(P_{j,t})$, so that goods market clears $(C_{j,t} = Y_{j,t}$, $C_t = Y_t)$, and $W_t = \Psi Z_t^{\gamma}$.[19]

Equilibrium

In equilibrium, the aggregate price index, intermediate goods price, and the private signal are given by

$$P_t = \left[\int \epsilon_{j,t} P_{j,t}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \tag{B.39}$$

$$P_{j,t} = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | s_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | S_{j,t}]}, \tag{B.40}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \tag{B.41}$$

Note that the firm's price setting decision already incorporates the household's optimal labor supply decision, $\frac{W_t}{P_t} = \Psi Y_t^{\gamma}$. In the sentiment driven equilibrium, one additional condition applies: that beliefs about aggregate demand are correct in equilibrium.

$$Z_t = Y_t \tag{B.42}$$

After the realization of Z_t, and after goods markets clear, market clearing quantities for each good, aggregate output, aggregate labor, nominal wage, and aggregate profits are

[19]Thus, wages are realized at the end of the period.

given by

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} \epsilon_{j,t} Y_t, \tag{B.43}$$

$$Y_t = \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{1-\frac{1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{B.44}$$

$$N_t = \int_0^1 N_{j,t} dj = \int_0^1 Y_{j,t} \epsilon_{j,t}^{-\tau} dj, \tag{B.45}$$

$$\frac{W_t}{P_t} = \Psi Y_t^{\gamma}, \tag{B.46}$$

$$\Pi_t = P_t Y_t - W_t N_t = Y_t - W_t N_t. \tag{B.47}$$

The first equality, which follows from the household's demand equation, indicates that in equilibrium, the market clearing quantity of good j is determined by aggregate price index, price of good j, and realized aggregate output. The second follows from optimal aggregate consumption by households in conjunction with market clearing, the third from the firm's production function, and the fourth from the household's optimal labor supply condition. Finally, in the fifth equality, aggregate profits equal aggregate revenue minus aggregate production costs.

Definition B.2. *A rational expectations equilibrium is a sequence of allocations* $\{C(Z_t), Y(Z_t), C_j(Z_t, \epsilon_{j,t}), Y_j(Z_t, \epsilon_{j,t}), N(Z_t), N_j(Z_t, \epsilon_{j,t}), \Pi(Z_t)\}$*, prices* $\{P_t = 1, P_j(Z_t, \epsilon_{j,t}), W_t = W(Z_t)\}$*, and a distribution of* Z_t*,* $\mathbf{F}(Z_t)$ *such that for each realization of* Z_t*, (i) equations (B.36) and (B.37) maximize household utility given the equilibrium prices* $P_t = 1, P_{j,t} = P_j(Z_t, \epsilon_{j,t})$*, and* $W_t = W(Z_t)$ *(ii) equation (B.40) maximizes intermediate goods firm's expected profits for all* j *given the equilibrium prices* $P_t = 1, W_t = W(Z_t)$*, and the signal (B.41) (iii) all markets clear:* $C_{j,t} = Y_{j,t}, N(Z_t) = \int N_{j,t} dj$*, and (iv) expectations are rational such that the household's beliefs about* W_t *and* Π_t *are consistent with its belief about aggregate demand* Z_t *(according to its optimal labor supply condition), and* $Y_t = Z_t$*, so that actual aggregate output follows a distribution consistent with* $\mathbf{F}$.

There exist two rational expectations equilibria: (1) a fundamental equilibrium with a degenerate distribution of sentiments, where aggregate output and prices are all constant and where sentiments play no role in determining the level of aggregate output (2) a stochastic equilibrium where sentiments matter and the volatility of beliefs about aggregate demand is endogenously determined and equal to the variance of aggregate output.

Fundamental equilibrium

Under perfect information, there is a unique rational expectations equilibrium in which the price of good j, aggregate price level, and aggregate demand are constant. aggregate output is constant and known. Then, the private signal that firms receive reveals their idiosyncratic demand shocks. Using the equilibrium conditions in (B.40), (B.44), (B.43), and (B.46), $Y_t, P_t, Y_{j,t}$ and $P_{j,t}$ in the fundamental equilibrium are as follows.

Under perfect information, the price of good j (B.40) is

$$P_{j,t} = \frac{\theta}{\theta - 1} \frac{W_t \epsilon_{j,t}^{1-\tau} Y_t}{\epsilon_{j,t} Y_t}.$$

Replacing W_t with (B.46),

$$P_{j,t} = \frac{\theta}{\theta - 1} \Psi P_t Y_t^{\gamma} \epsilon_{j,t}^{-\tau}.$$

Without loss of generality, normalizing $\frac{\theta}{\theta-1}\Psi$ to 1,

$$P_{j,t} = P_t Y_t^{\gamma} \epsilon_{j,t}^{-\tau}. \tag{B.48}$$

Substituting (B.48) into (B.39), the aggregate price index with flexible prices is indeterminate:

$$\begin{aligned} P_t &= \left[\int \epsilon_{j,t} [P_t Y_t^{\gamma} \epsilon_{j,t}^{-\tau}]^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \\ &= \left[\int \epsilon_{j,t}^{1-\tau(1-\theta)} dj \right]^{\frac{1}{1-\theta}} P_t Y_t^{\gamma}. \end{aligned}$$

Without loss of generality, normalize P_t to 1.

Next, the normalization of $P_t = 1$ can be used to find Y_t,

$$Y_t = \left[\int \epsilon_{j,t}^{1-\tau(1-\theta)} dj \right]^{\frac{1}{\gamma(\theta-1)}}. \tag{B.49}$$

Taking the log of this expression (let $y_t \equiv (\log Y_t) - \phi_0$),

$$y_t + \phi_0 = \frac{1}{\gamma(\theta - 1)} \log \mathbb{E}_t \left[\epsilon_{j,t}^{1-\tau(1-\theta)} \right].$$

As $\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$, by the properties of a moment generating function for a normally distributed random variable,

$$y_t + \phi_0 = \frac{1}{\theta - 1} \frac{1}{2} \text{Var}_t([1 - \tau(1-\theta)]\varepsilon_{j,t}), \tag{B.50}$$

$$= \frac{1}{\gamma(\theta - 1)} \frac{[1 - \tau(1-\theta)]^2}{2} \sigma_\varepsilon^2. \tag{B.51}$$

Equating coefficients implies $y_t = 0$ and

$$\phi_0^* = \frac{1}{2(\theta - 1)} \frac{(1 + \tau[\theta - 1])^2}{\gamma} \sigma_\varepsilon^2.$$

As expected, output in the fundamental equilibrium when firms choose quantity (A.17), $(\gamma = 1, \tau = 0)$ is equivalent to its counterpart when firms choose prices,

$$\phi_0^* = \frac{1}{2(\theta - 1)} \frac{(1 + \tau[\theta - 1])^2}{\gamma} \sigma_\varepsilon^2. \tag{B.52}$$

Finally, an expression for $Y_{j,t}$ can be found by using the demand curve (B.43), and substituting $P_{j,t}$ with (B.48)

$$\begin{aligned} Y_{j,t} &= \left(\frac{P_t}{P_{j,t}}\right)^\theta \epsilon_{j,t} Y_t, \\ &= [Y_t^\gamma \epsilon_{j,t}^{-\tau}]^{-\theta} \epsilon_{j,t} Y_t, \\ &= \epsilon_{j,t}^{1+\tau\theta} Y_t^{1-\gamma\theta}. \end{aligned}$$

Replacing Y_t with (B.49),

$$Y_{j,t} = \epsilon_{j,t}^{1+\tau\theta} \left[\int \epsilon_{j,t}^{1-\tau(1-\theta)} dj\right]^{\frac{1-\gamma\theta}{\gamma(\theta-1)}}.$$

Sentiment-driven equilibrium

When firms set prices conditional on an endogenous signal of aggregate demand, there exists a sentiment driven equilibrium, in addition to the fundamental equilibrium. The sentiment driven equilibrium is a rational expectations equilibrium where aggregate output is not constant but equal to a sentiment (Z_t). Let $\hat{z}_t$ and $\hat{y}_t$ denote Z_t and Y_t in log deviation from the steady state of this equilibrium, respectively.[20] To solve for this equilibrium, conjecture $\hat{z}_t \sim N(0, \sigma_z^2)$, where σ_z^2 is a constant to be determined below.

Consider the case of a positive sentiment shock in the flexible wage and flexible price model. A self-fulfilling equilibrium is possible when σ_z^2 is sufficiently low such that firms attribute just enough of z_t to $\epsilon_{j,t}$ and so that the increase in sentiment leads firms to lower $p_{j,t}$. When goods markets open, the quantity of firm j's product, $(y_{j,t}(p_{j,t}))$, demanded at price $p_{j,t}$ is higher than that under perfect information. Thus, there is a σ_z^2 such that aggregate supply across firms exactly fulfills the positive sentiment formed by households.

Proposition B.1. *Let $\lambda \in (0,1)$. There exists a sentiment-driven rational expectations equilibrium where aggregate output is stochastic with variance*

$$\sigma_z^2 = \frac{\lambda}{1-\lambda} \frac{\tau + B\frac{\lambda}{1-\lambda}}{\gamma} \sigma_\epsilon^2,$$

where $B = \frac{\partial p_t}{\partial z_t}$.

Proof. Equation (B.40) gives firm j's optimal price conditional on its signal. As it is derived using equations (B.46) and (B.43), it already incorporates market clearing for labor and consumption.

$$\begin{aligned} P_{j,t} &= \frac{\theta}{\theta-1} \frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | S_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | S_{j,t}]}, \\ &= \frac{\theta}{\theta-1} \Psi \frac{\mathbb{E}_t[P_t \epsilon_{j,t}^{1-\tau} Z_t^{\gamma+1} | S_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Z_t | S_{j,t}]}, \end{aligned}$$

where the second equality results from substituting W_t with the household's optimal labor supply (B.46). Taking logs,

$$p_{j,t} = \log\left(\frac{\theta}{\theta-1}\Psi\right) + \log \mathbb{E}_t[P_t \epsilon_{j,t}^{1-\tau} Z_t^{\gamma+1} | s_{j,t}] - \log \mathbb{E}_t[\epsilon_{j,t} Z_t | s_{j,t}].$$

Conjecture a solution of the form $p_{j,t} = D + B s_{j,t}$. According to this guess, $p_t = A + B(1-\lambda) z_t$ where A incorporates $\mathbb{E}(\epsilon_{j,t})$, which affects the steady state. Substituting our guess for p_t,

[20] See appendix (A.4) for a calculation of the steady state in this equilibrium.

$$p_{j,t} = \log\left(\frac{\theta}{\theta-1}\Psi\right) + \log \mathbb{E}_t[\exp(p_t + (1-\tau)\varepsilon_{j,t} + (\gamma+1)(z_t+\phi_0))|s_{j,t}] \quad \text{(B.53)}$$

$$- \log \mathbb{E}_t[\exp(\varepsilon_{j,t} + z_t + \phi_0)|s_{j,t}] \quad \text{(B.54)}$$

$$= \log\left(\frac{\theta}{\theta-1}\Psi\right) + \gamma\phi_0 + A \quad \text{(B.55)}$$

$$+ \log \mathbb{E}[\exp(B(1-\lambda) + \gamma + 1)z_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}] \quad \text{(B.56)}$$

$$- \log \mathbb{E}_t[\exp(\varepsilon_{j,t} + z_t)] \quad \text{(B.57)}$$

$$= \log\left(\frac{\theta}{\theta-1}\Psi\right) + \gamma\phi_0 + A + \frac{\Omega_1 - \Omega_2}{2} + (\mu_1 - \mu_2)s_{j,t} \quad \text{(B.58)}$$

$$= \varphi_0 + \bar{\mu}s_{j,t} \quad \text{(B.59)}$$

where

$$\varphi_0 \equiv \log\left(\frac{\theta}{\theta-1}\Psi\right) + \gamma\phi_0 + A + \frac{\Omega_1 - \Omega_2}{2}, \quad \text{(B.60)}$$

$$\bar{\mu} \equiv \mu_1 - \mu_2, \quad \text{(B.61)}$$

$$\mu_1 \equiv \mathbb{E}_t[B(1-\lambda) + \gamma + 1)z_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}], \quad \text{(B.62)}$$

$$\Omega_1 \equiv \frac{1}{2}\text{Var}_t[B(1-\lambda) + \gamma + 1)z_t + (1-\tau)\varepsilon_{j,t}|s_{j,t}], \quad \text{(B.63)}$$

$$\mu_2 \equiv \mathbb{E}_t[\varepsilon_{j,t} + z_t|s_{j,t}], \quad \text{(B.64)}$$

$$\Omega_2 \equiv \frac{1}{2}\text{Var}[\varepsilon_{j,t} + z_t|s_{j,t}]. \quad \text{(B.65)}$$

Variables in lowercase denote the log of their counterparts, with the exception of $z_t = \log Z_t - \phi_0$. Note that the firm's price is a constant projection of $s_{j,t}$. Hence, in a sentiment-driven equilibrium, all firms set prices in the same proportion to their signal.

Taking the log of the aggregate price index (B.39) and substituting for $p_{j,t}$ with (B.59),

$$\begin{aligned}
(1-\theta)p_t &= \log \mathbb{E}_t[P_{j,t}^{1-\theta}\epsilon_{j,t}], \\
&= \log \mathbb{E}_t[\exp([1-\theta]p_{j,t} + \varepsilon_{j,t})], \\
&= (1-\theta)\varphi_0 + (1-\theta)\bar{\mu}(1-\lambda)z_t + \log \mathbb{E}_t[e^{([1-\theta]\bar{\mu}\lambda+1)\varepsilon_{j,t}}], \\
A + Bz_t &= \varphi_0 + \bar{\mu}(1-\lambda)z_t + \frac{[(1-\theta)\bar{\mu}\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2.
\end{aligned}$$

Equating coefficients on z_t,

$$B = \bar{\mu}(1-\lambda). \quad \text{(B.66)}$$

Evaluating (B.62) and (B.64), we have

$$B = \frac{(\gamma + B)(1-\lambda)\sigma_z^2 - \tau\lambda(1-\lambda)\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + (1-\lambda)^2\sigma_z^2}(1-\lambda),$$

which implies[21]

$$\sigma_z^2 = \frac{\lambda}{1-\lambda}\frac{\tau + B\frac{\lambda}{1-\lambda}}{\gamma}\sigma_\epsilon^2. \tag{B.67}$$

From equating the constant terms, we have

$$A = \varphi_0 + \frac{[(1-\theta)\bar{\mu}\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2$$

Applying (B.66) and (B.60),

$$\phi_0 = \frac{1}{\gamma}\left(\frac{[(1-\theta)\frac{\lambda}{1-\lambda}B + 1]^2}{2(\theta-1)}\sigma_\epsilon^2 - \log\left(\frac{\theta}{\theta-1}\Psi\right) - \frac{\Omega_1 - \Omega_2}{2}\right)$$

Note that A is the steady state for the price level, which is indeterminate, while ϕ_0 is the steady state for aggregate output. The conditional variances are constants, and functions of σ_ϵ^2, σ_z^2, and other parameters of the model,

$$\Omega_1 - \Omega_2 = [(\gamma + B)^2 + (2-\mu_1)(\gamma + B) - B]\sigma_z^2 + \left[\tau^2 + (\mu_1 - 2)\tau - B\frac{\lambda}{1-\lambda}\right]\sigma_\epsilon^2.$$

□

Thus, the volatility of actual aggregate output and beliefs about aggregate demand are determined by the parameters of the model. If $\lambda \in (0,1)$, $\tau > 0$, and $\sigma_\varepsilon^2 > 0$, then there exists a sentiment driven rational expectations equilibrium with $\hat{y}_t = \hat{z}_t$ where

$$\sigma_y^2 = \sigma_z^2. \tag{B.68}$$

Expression B.67 implies that the following factors affect sentiment volatility (1) structural parameters such as the degree of complementarity/substitutability in actions across firms (τ, γ), information content of the private signal (λ), and the volatility of idiosyncratic demand (σ_ε^2), all of which affect the firm's response to a sentiment shock. (2)

[21] The relationship between the price level and sentiments is indeterminate in the flexible price case.

Note that if $\tau = 0, \lambda = 0$ or $\sigma_\varepsilon^2 = 0$, then the private signal conveys only aggregate demand or price depends only on aggregate demand. The result is also that the unique equilibrium is the fundamental equilibrium, due to substitutability of firms' outputs. (3) Sentiment volatility is decreasing in $1 - \lambda$; as the private signal becomes more informative about aggregate demand ($1 - \lambda$ increases), we approach the certainty equilibrium of the previous section. (4) Sentiment volatility is increasing in $\sigma_\varepsilon^2 > 0$, which implies that a sentiment driven equilibrium needs sufficient coordination. All firms set the same price regardless of their individual signal, but depending on the (known) distribution of signals. The more volatile the idiosyncratic component of the signal, the more difficult it is to attain coordination. In this case, sentiment volatility must be commensurately larger.

The intuition for why the sentiment-driven equilibrium is a rational expectations equilibrium is as follows: Given the parameters of the model, σ_z^2 is determined such that for any aggregate demand sentiment, all firms misattribute enough of the sentiment component of their signal to an idiosyncratic preference shock such that price-setting decisions lead to aggregate output equaling the sentiment in equilibrium. The volatility of the sentiment process (σ_z^2) determines how much firms attribute their signal to $\hat{z}_t$. Firms increase their price in response to aggregate demand, and decrease their price in response to idiosyncratic demand. Through prices, firms' output decision are strategic substitutes. When firms actions are strategic substitutes, the optimal output of a firm is declining in σ_z^2 as this leads the firms to attribute more of the signal to an aggregate demand shock. Since firms' optimal price depends negatively on the idiosyncratic preference shock $\hat{\varepsilon}_{j,t}$ and positively on the level of aggregate demand, $\hat{z}_t$, if they are unable to distinguish between the two components in their signal, then there can be a coordinated over-production (under-production) in response to a positive (negative) aggregate sentiment shock, such that $\hat{y}_t$ equals $\hat{z}_t$ in equilibrium if σ_z^2 is as in (B.67). The rational expectations equilibrium pins down the variance of the sentiment distribution, although sentiments are extrinsic. The result is an additional rational expectations equilibrium that is characterized by aggregate fluctuations in output and employment despite the lack of fundamental aggregate shocks.

B.2. Monetary Policy with Sticky Prices (Calvo)

Under Calvo price setting, a fraction θ_p of firms can not adjust their price in period t. Instead, $(1 - \theta_p)$ of firms choose their optimal price taking into account the probability of not being able to adjust for $\frac{1}{\theta_p}$ periods. The representative households sets wages flexibly. As multiple equilibria arises from coordinated actions, when signals are correlated, sticky prices will hinder coordination, reducing the set of equilibria. As a result, sentiment driven fluctuations are less volatile. Due to the endogeneity of sentiment volatility, when the central bank targets inflation strongly or prices are more flexible, this leads to higher volatility of output. Note that although sentiment shocks are iid (and thus price setting with sticky prices is equivalent to price setting under flexible prices), the Calvo parameter affects inflation through the proportion of firms who can reset prices.

The following section will consider the micro-foundations of the baseline model (decisions of households, firms, equilibrium condition). The quantity of output in the fundamental equilibrium is derived, followed by the mean level of output in the sentiment driven equilibrium. In addition, the mechanism behind a self-fulfilling equilibrium with sentiments will be described.

Households

The representative household's problem is[22]

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} + \Psi(1-N_t) \right),$$

subject to

$$C_t \equiv \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} C_{j,t}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}},$$

$$\int P_{j,t} C_{j,t} dj + Q_t B_t \leq B_{t-1} + W_t N_t + \Pi_t,$$

where C_t represents an aggregate consumption index and $C_{j,t}$ denotes the quantity of good j consumed by the household in period t. The idiosyncratic preference shock for good j is log normally distributed ($\varepsilon_{j,t} \equiv \log \epsilon_{j,t} \sim N(0, \sigma_\varepsilon^2)$). $\theta > 1$ is the elasticity of substitution between goods, while Ψ represents disutility of labor. The exponent $\frac{1}{\theta}$ on $\epsilon_{j,t}$ is solely intended to simplify calculations.

From the households problem, we obtain optimal conditions for demand, labor supply, and intertemporal consumption. The household allocates consumption among j goods to maximize C_t for any given level of expenditures $\int_0^1 P_{j,t} C_{j,t} dj$, where $P_{j,t}$ is the price of intermediate good j. Optimizing its consumption allocation, household's demand for good j is given by

$$C_{j,t} = \left(\frac{P_t}{P_{j,t}} \right)^{\theta} C_t \epsilon_{j,t} \tag{B.69}$$

[22]For non-linear disutility, see Appendix (I.2). Specifying the utility function in this way ($\gamma \neq 1$) will allow sentiments to affect the real wage, by γ, the CRRA parameter. This will affect the firms' marginal cost and their optimal response to sentiments.

The resulting aggregate price level,

$$P_t = \left(\int_0^1 \epsilon_{j,t} P_{j,t} dj \right)^{\frac{1}{1-\theta}},$$

implies $\int P_{j,t} C_{j,t} dj = P_t C_t$.

Choosing labor supply (N_t) optimally,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}, \tag{B.70}$$

$$\Psi C_t^{\gamma} = \frac{W_t}{P_t}. \tag{B.71}$$

In logs,

$$w_t - p_t = \gamma c_t + \log \Psi.$$

Choosing intertemporal consumption optimally,

$$Q_t = \beta \mathbb{E}_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right).$$

In logs,

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\gamma}[i_t - \mathbb{E}_t \pi_{t+1} - \rho].$$

In this model, households form demand schedules for each differentiated good and supply labor, all contingent on shocks to idiosyncratic demand and shocks to aggregate income/consumption to be realized. Let Z_t represent the household's beliefs about aggregate income/consumption at the beginning of period t. Households form consumption *plans* using (B.69)

$$C_{j,t}(Z_t, \epsilon_{j,t}) = \left(\frac{P_t(Z_t)}{P_{j,t}(Z_t, \epsilon_{j,t})} \right)^{\theta} C_t(Z_t) \epsilon_{j,t} \tag{B.72}$$

and decide labor supply, using (B.71) to obtain an implicit function of labor supply as a function of sentiments, $N_t = N(Z_t)$, given a nominal wage W_t

$$P_t(Z_t) = \frac{W_t}{\Psi \left[\frac{W_t}{P_t(Z_t)} N_t + \frac{\Pi_t(Z_t)}{P_t(Z_t)} \right]^{\gamma}} \tag{B.73}$$

Note that $\Pi_t(Z_t) = P_t(Z_t) Z_t - W_t N_t$.

Intermediate Goods Firms

Marginal cost. Derive the firms' marginal cost from the following minimization problem,

$$\min_{N_{j,t}} W_t N_{j,t}$$

subject to

$$Y_{j,t} \leq \epsilon_{j,t}^{\tau} N_{j,t}.$$

The Lagrangian is

$$L = W_t N_{j,t} - \Phi_t(\epsilon_{j,t}^{\tau} N_{j,t} - Y_{j,t}).$$

Substituting for W_t using (B.71), nominal marginal cost is

$$\Phi_t = \Psi \epsilon_{j,t}^{-\tau} Z_t^{\gamma} P_t.$$

In logs,

$$\phi_t = \log(\Psi) - \tau\epsilon_{j,t} + \gamma z_t + p_t.$$

NKPC. With Calvo price setting, the aggregate price index is as follows,

$$P_t^{1-\theta} = \int_{\times_t^c} P_{j,t}^{1-\theta} \epsilon_{j,t} dj + \int_{\times_t} P_{j,t}^{*(1-\theta)} \epsilon_{j,t} dj,$$

where $\times_t^c$ denotes the set of firms who can not re-adjust prices in period t and $\times_t$ as the complement of this set. Let

$$P_{t-1}^{1-\theta} \equiv \frac{1}{\theta_p} \int_{\times_t^c} P_{j,t}^{1-\theta} \epsilon_{j,t} dj, \tag{B.74}$$

$$P_t^{*(1-\theta)} \equiv \frac{1}{1-\theta_p} \int_{\times_t} P_{j,t}^{*(1-\theta)} \epsilon_{j,t} dj. \tag{B.75}$$

Then

$$P_t^{1-\theta} = \theta_p P_{t-1}^{1-\theta} + (1-\theta_p) P_t^{*(1-\theta)}, \tag{B.76}$$

$$\Pi_t^{1-\theta} = \theta_p + (1-\theta_p)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\theta}. \tag{B.77}$$

A first order approximation to (B.77) around a zero inflation steady state yields

$$\pi_t = (1-\theta_p)(p_t^* - p_{t-1}). \tag{B.78}$$

The firm's profit-maximizing price is

$$p_{j,t}^* - p_{t-1} = (1-\beta\theta_p)\mathbb{E}_t[\gamma z_t - \tau\varepsilon_{j,t}|s_{j,t}] + \mathbb{E}_t[\pi_t|s_{j,t}].$$

Substituting π_t with (B.78),

$$p_{j,t}^* - p_{t-1} = (1-\beta\theta_p)\mathbb{E}_t[\gamma z_t - \tau\varepsilon_{j,t}|s_{j,t}] + \mathbb{E}_t[\pi_t|s_{j,t}]. \tag{B.79}$$

To solve for p_t^*,

1. Conjecture $p_t^* = \tilde{D} + \mu(1-\lambda)z_t$.
2. Use conjecture and (B.79) to find $p_{j,t}^*$

$$\begin{aligned} p_{j,t}^* &= (1-\beta\theta_p)\mathbb{E}_t[\gamma z_t - \tau\varepsilon_{j,t}|s_{j,t}] + (1-\theta_p)\mathbb{E}_t[\tilde{D} + \mu(1-\lambda)z_t|s_{j,t}] + \theta_p p_{t-1}, \\ &= (1-\theta_p)\tilde{D} + \theta_p p_{t-1} \\ &\quad + \mathbb{E}_t([(1-\beta\theta_p)\gamma + (1-\theta_p)\mu(1-\lambda)]z_t - (1-\beta\theta_p)\tau\varepsilon_{j,t}|s_{j,t}). \end{aligned}$$

   Let $p_{j,t}^* = D + \mu s_{j,t}$ where

$$\begin{aligned} D &\equiv (1-\theta_p)\tilde{D} + \theta_p p_{t-1}, \\ \mu &\equiv \frac{\text{cov}([(1-\beta\theta_p)\gamma + (1-\theta_p)\mu(1-\lambda)]z_t - (1-\beta\theta_p)\tau\varepsilon_{j,t}, s_{j,t})}{\text{var}(s_{j,t})}. \end{aligned}$$

3. Substitute $p_{j,t}^*$ into (B.75) and equate coefficients to find the steady state for $p_{j,t}^*$ and p_t^*, as well as their responses to z_t. Taking the log of (B.75) and defining $\mathbb{E}_{\times t}$ as $\frac{1}{1-\theta_p}\int_{\times t}$,

$$\begin{aligned} (1-\theta)p_t^* &= \ln \mathbb{E}_{\times t} e^{(1-\theta_p)p_{j,t}^* + \varepsilon_{j,t}}, \\ p_t^* &= D + \mu(1-\lambda)z_t + \frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2. \end{aligned}$$

   Equating coefficients,

$$\begin{aligned} \tilde{D} &= p_{t-1} + \frac{1}{\theta_p}\frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2, \\ D &= p_{t-1} + \frac{1-\theta_p}{\theta_p}\frac{[(1-\theta)\mu\lambda + 1]^2}{2(1-\theta)}\sigma_\epsilon^2, \\ \mu &= (1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma_z^2 - \tau\lambda\sigma_\epsilon^2}{\lambda^2\sigma_\epsilon^2 + \theta_p(1-\lambda)^2\sigma_z^2}. \end{aligned}$$

Note that μ is close to $\mathbb{E}_t[\gamma z_t - \tau\varepsilon_{j,t}|s_{j,t}]$ if $\theta_p \to 1$. The more flexible prices are ($\theta_p \to 0$), the larger is μ, and the more pass through of z_t to $p^*_{j,t}$ and thus to p^*_t. When prices are sticky, coordination is more difficult to achieve. The θ_p in the denominator is from the effect of z_t on p^*_t. The more p^*_t is composed of the non-fundamental component, the more μ will weight $s_{j,t}$ to pass through more of z_t. As a result, $p^*_{j,t}$ contains more of the non-fundamental component. The implied processes are

$$p^*_{j,t} = p_{t-1} + \frac{1-\theta_p}{\theta_p}\frac{[(1-\theta)\mu\lambda+1]^2}{2(1-\theta)}\sigma^2_\epsilon + (1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma^2_z - \tau\lambda\sigma^2_\epsilon}{\lambda^2\sigma^2_\epsilon + \theta_p(1-\lambda)^2\sigma^2_z}s_{j,t}, \tag{B.80}$$

$$p^*_t = p_{t-1} + \frac{1}{\theta_p}\frac{[(1-\theta)\mu\lambda+1]^2}{2(1-\theta)}\sigma^2_\epsilon + (1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma^2_z - \tau\lambda\sigma^2_\epsilon}{\lambda^2\sigma^2_\epsilon + \theta_p(1-\lambda)^2\sigma^2_z}(1-\lambda)z_t. \tag{B.81}$$

Substituting for p^*_t in (B.78) with (B.81), we get a form of the NKPC, which results from the price setting behavior of firms with imperfect information,

$$\pi_t = \frac{1-\theta_p}{\theta_p}\frac{[(1-\theta)\mu\lambda+1]^2}{2(1-\theta)}\sigma^2_\epsilon + (1-\theta_p)(1-\beta\theta_p)\frac{\gamma(1-\lambda)\sigma^2_z - \tau\lambda\sigma^2_\epsilon}{\lambda^2\sigma^2_\epsilon + \theta_p(1-\lambda)^2\sigma^2_z}(1-\lambda)z_t. \tag{B.82}$$

Note that the degree of pass through of z_t to π_t is increasing in the degree of price flexibility (θ_p) decreases.

Central bank

The central bank sets the nominal interest rate as a function of price inflation and output

$$Q_t^{-1} = \beta^{-1}\Pi_t^{\phi_\pi} + Y_t^{\phi_y}.$$

In logs,

$$i_t = \rho + \phi_\pi\pi_t + \phi_y y_t.$$

Equilibrium

In equilibrium, aggregate price index, intermediate goods price, and the private signal are given by:

$$P_t = \left[\int \epsilon_{j,t} P_{j,t}^{1-\theta} dj\right]^{\frac{1}{1-\theta}}, \tag{B.83}$$

$$0 = \sum_{k=0}^{\infty} \theta_p^k \mathbb{E}_t[Q_{t,t+k} Y_{t+k|t}(P_{j,t}^* - M\psi_{t+k|t})], \tag{B.84}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}. \tag{B.85}$$

With iid sentiments, (B.84) simplies to

$$P_{j,t}^* = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | s_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | s_{j,t}]}.$$

In the sentiment driven equilibrium, one additional condition applies: that beliefs about aggregate demand are correct in equilibrium,

$$Z_t = Y_t. \tag{B.86}$$

After the realization of Z_t, and after goods markets clear, market clearing quantities for each good, aggregate output, aggregate labor, nominal wage, and aggregate profits are given by

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}}\right)^{\theta} \epsilon_{j,t} Y_t, \tag{B.87}$$

$$Y_t = \left[\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{1-\frac{1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{B.88}$$

$$N_t = \int_0^1 N_{j,t} dj = \int_0^1 Y_{j,t} \epsilon_{j,t}^{-\tau} dj, \tag{B.89}$$

$$\frac{W_t}{P_t} = \Psi Y_t^{\gamma}, \tag{B.90}$$

$$\Pi_t = P_t Y_t - W_t N_t = Y_t - W_t N_t. \tag{B.91}$$

The first equality follows from the household's demand equation and indicates that in equilibrium, the market clearing quantity of good j is determined by aggregate price index, price of good j, and realized aggregate output. The second follows from optimal aggregate consumption by households in conjunction with market clearing, the third from the firm's production function, and the fourth from the household's optimal labor supply condition. Finally, in the fifth equality, aggregate profits equal aggregate revenue minus aggregate production costs.

Effect of an iid shock to sentiments

Proposition B.2. *Let $\lambda \in (0,1)$. Under Calvo price setting, there exists a sentiment-driven rational expectations equilibrium where aggregate output is stochastic, with variance increasing in ϕ_π and decreasing in ϕ_y.*

$$\sigma_z^2 = \frac{\lambda}{1-\lambda} \frac{\tau - \frac{\lambda}{1-\lambda} \frac{1}{(1-\beta\theta_p)(1-\theta_p)} \frac{\gamma+\phi_y}{\phi_\pi}}{\gamma + \frac{\theta_p}{(1-\beta\theta_p)(1-\theta_p)} \frac{\gamma+\phi_y}{\phi_\pi}} \sigma_\epsilon^2 \tag{B.92}$$

The Euler equation, Taylor rule imply the following relationship between inflation and sentiments in partial equilibrium

$$\pi_t = -\frac{\gamma+\phi_y}{\phi_\pi} z_t \tag{B.93}$$

while the NKPC (B.82) describes another relation. In a sentiment driven equilibrium, the σ_z^2 that satisfies both relationships is

$$\sigma_z^2 = \frac{\lambda}{1-\lambda} \frac{\tau - \frac{\lambda}{1-\lambda} \frac{1}{(1-\beta\theta_p)(1-\theta_p)} \frac{\gamma+\phi_y}{\phi_\pi}}{\gamma + \frac{\theta_p}{(1-\beta\theta_p)(1-\theta_p)} \frac{\gamma+\phi_y}{\phi_\pi}} \sigma_\epsilon^2 \tag{B.94}$$

Under sticky prices, the self-fulfilling equilibrium has a different mechanism than in the case where firms set prices and households set wages flexibly. Here, a positive sentiment shock is realized when the nominal interest rate falls, which follows from a decrease in price inflation. For price inflation to fall when sentiment increases, σ_z^2 must be sufficiently low such that firms must misattribute enough of the increase in z_t to $\epsilon_{j,t}$ instead, leading them to lower prices. When goods markets open, households demand $y_{j,t}(p_{j,t})$, which is higher than the quantity that would have been demanded if firms had set prices under perfect information. There is a σ_z^2 such that aggregate supply is equal to the sentiment that households have formed.

Note that as price flexibility facilitates the pass through of z_t, sentiment volatility is increasing in the degree to which firms are able to adjust prices.

The central bank can suppress these non-fundamental fluctuations with sufficiently lax monetary policy,

$$\phi_\pi < \frac{\lambda}{1-\lambda}\frac{1}{\theta_p\lambda_p}\frac{\gamma+\phi_y}{\tau}$$

Optimal Policy

How do information frictions affect the optimal design of monetary policy? In this model, imperfect information is both a source of real (as firms choose inputs) and nominal frictions (as firms set prices) based on an imperfect signal of underlying shocks. The framework introduces a new role for policy in coordinating production decisions among firms.

The interest rate rule that implements constrained efficient allocation ($\sigma_z^2 = 0$) is

$$i_t = -\gamma y_t.$$

By insulating prices from changes in beliefs, this policy allow firms to place less weight on others' choices, and to rely more on their own signal as information about the fundamental. The endogeneity of the signal implies that the signal also becomes more informative. For the IS relation, this interest rate rule implies that any z_t is attained without a fall in the price level, solely through a change in the interest rate.

C. Private signal correct up to iid noise

When agents actions are strategic substitutes, a private signal that conveys perfectly information needed for the agents' first order condition, but with iid noise, results in only the fundamental equilibrium. Consider the first order condition of a general beauty contest model, where a continuum of agents indexed by $j \in [0,1]$ take action conditional on a private signal $s_{j,t}$

$$y_{j,t} = \mathbb{E}[\underbrace{\beta_0\varepsilon_{j,t} + \beta_1 y_t}_{x_{j,t}} | s_{j,t}],$$
$$s_{j,t} = \beta_0\varepsilon_{j,t} + \beta_1 y_t + \nu_{j,t}.$$

Note that $s_{j,t} = x_{j,t} + \nu_{j,t}$. Agent j's optimal response depends on an idiosyncratic iid shock $\varepsilon_{j,t} \sim N(0, \sigma^2_{\varepsilon_{j,t}})$, as well as on the aggregate response of other agents ($y_t =$

$\int_0^1 y_{j,t}dj)$, where $y_t \sim N(0,\sigma_y^2)$. The parameters β_0 and β_1 capture the elasticity of actions to the idiosyncratic shock and the aggregate variable. If $\beta_1 > 0$, agents face strategic complementarities. If $\beta_1 < 0$, agents face strategic substitutabilities.

Agent j's optimal response is

$$y_{j,t} = \frac{\beta_0^2\sigma_\varepsilon^2 + \beta_1^2\sigma_y^2}{\beta_0^2\sigma_\varepsilon^2 + \beta_1^2\sigma_y^2 + \sigma_\nu^2}(\beta_0\varepsilon_{j,t} + \beta_1 y_t + \nu_{j,t}).$$

As $\frac{\beta_0^2\sigma_\varepsilon^2+\beta_1^2\sigma_y^2}{\beta_0^2\sigma_\varepsilon^2+\beta_1^2\sigma_y^2+\sigma_\nu^2} \in (0,1)$, we can only have sentiment driven equilibrium with this private signal if $\beta_1 > 1$.

However, if the private signal is instead $s_{j,t} = \lambda\varepsilon_{j,t} + (1-\lambda)y_t + \nu_{j,t}$, where $\lambda \neq \beta_0$ and $(1-\lambda) \neq \beta_1$, then

$$y_{j,t} = \frac{\beta_0\lambda\sigma_\varepsilon^2 + \beta_1(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2 + \sigma_\nu^2}(\lambda\varepsilon_{j,t} + (1-\lambda)y_t + \nu_{j,t}),$$

$$y_t = \int_0^1 y_{j,t}dj = \frac{\beta_0\lambda\sigma_\varepsilon^2 + \beta_1(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2 + \sigma_\nu^2}(1-\lambda)y_t.$$

In this case, any y_t is an equilibrium if

$$\frac{\beta_0\lambda\sigma_\varepsilon^2 + \beta_1(1-\lambda)\sigma_y^2}{\lambda^2\sigma_\varepsilon^2 + (1-\lambda)^2\sigma_y^2 + \sigma_\nu^2}(1-\lambda) = 1.$$

The volatility of y_t is determined by parameters of the model,

$$\sigma_y^2 = \frac{\beta_0\lambda(1-\lambda) - \lambda^2}{(1-\lambda)^2(1-\beta_1)}\sigma_\varepsilon^2 - \frac{1}{(1-\lambda)^2(1-\beta_1)}\sigma_\nu^2.$$

The private signal that is correct up to iid noise allows firms to respond to the two shocks in the correct proportions. In order for sentiment driven equilibria to exist when firms' actions are strategic substitutes, information frictions must be such that firms misattribute some of the sentiment component in their signal to idiosyncratic preference for their good.

D. Expected future inflation with iid shock to sentiments

Let lower-case variables with a hat symbol represent variables in log-deviation from steady state. If z_t is iid and with mean equal to z, and if we conjecture $\hat{y}_t = \hat{c}_t = \hat{z}_t$, then $\forall\ k \geq 1$,

$$\mathbb{E}_t\hat{c}_{t+k} = 0, \tag{D.95}$$
$$\mathbb{E}_t\hat{y}_{t+k} = 0. \tag{D.96}$$

Following (D.95), it can be shown that

$$\mathbb{E}_t\hat{\pi}_{t+1} = 0,$$
$$\mathbb{E}_t p_{t+1} = p_t.$$

Real interest rate path as a function of iid **shock** z_t**.** The Euler equation in period $t+k$ is

$$\begin{aligned}
\hat{c}_{t+k} &= \mathbb{E}_{t+k}\hat{c}_{t+k+1} - \frac{1}{\gamma}[i_{t+k} - \mathbb{E}_{t+k}\hat{\pi}_{t+k+1} - \rho], \\
&= \mathbb{E}_{t+k}\hat{c}_{t+k+1} - \frac{1}{\gamma}[r_{t+k} - \rho], \\
&= \mathbb{E}_{t+k}\hat{c}_{t+k+1} - \frac{1}{\gamma}\hat{r}_{t+k}.
\end{aligned}$$

where $\rho \equiv log(\frac{1}{\beta})$ and the real interest rate $r_t \equiv i_t - \mathbb{E}_t\pi_{t+1}$. Note that under the assumption of zero inflation in steady state, ρ is both the steady state nominal interest rate and steady state real interest rate. Taking the expectation at time t of both sides and applying the law of iterated expectations,

$$\mathbb{E}_t\hat{c}_{t+k} = \mathbb{E}_t\hat{c}_{t+k+1} - \frac{1}{\gamma}\mathbb{E}_t\hat{r}_{t+k}.$$

Using (D.95), $\forall\, k \geq 1$

$$\mathbb{E}_t\hat{r}_{t+k} = 0. \tag{D.97}$$

Inflation in terms of real interest rate path. Next, use Fisher equation $r_t = i_t - \mathbb{E}_t\pi_{t+1}$ to show that $\mathbb{E}_t\hat{\pi}_{t+1} = 0$. Combining these two expressions gives inflation (and hence the price level) as a function of the path of the real interest rate. Again, under the assumption of zero inflation in the steady state, the Fisher equation is

$$r_t = i_t - \mathbb{E}_t\hat{\pi}_{t+1}.$$

Assume the central bank follows the Taylor rule given by

$$i_t = \rho + \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t,$$

Substituting this expression into the Fisher equation and rearranging terms,

$$
\begin{aligned}
r_t &= i_t - \mathbb{E}_t\hat{\pi}_{t+1}, \\
&= \rho + \phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t - \mathbb{E}_t\hat{\pi}_{t+1}, \\
\hat{\pi}_t &= \frac{1}{\phi_\pi}[\hat{r}_t - \phi_y\hat{y}_t + \mathbb{E}_t\hat{\pi}_{t+1}].
\end{aligned}
$$

Iterating forwards and using (D.96),

$$
\hat{\pi}_t = \sum_{k=0}^{\infty}\frac{1}{\phi_\pi^{k+1}}\mathbb{E}_t\hat{r}_{t+k} - \sum_{k=0}^{\infty}\left(\frac{\phi_y}{\phi_\pi}\right)^{k+1}\mathbb{E}_t\hat{y}_{t+k}.
$$

Then at $t+1$, we will have

$$
\hat{\pi}_{t+1} = \sum_{k=0}^{\infty}\frac{1}{\phi_\pi^{k+1}}\mathbb{E}_{t+1}\hat{r}_{t+k+1} - \sum_{k=0}^{\infty}\left(\frac{\phi_y}{\phi_\pi}\right)^{k+1}\mathbb{E}_{t+1}\hat{y}_{t+k+1}.
$$

Taking the expectation at time t of both sides, and applying the law of iterated expectations,

$$
\mathbb{E}_t\hat{\pi}_{t+1} = \sum_{k=0}^{\infty}\frac{1}{\phi_\pi^{k+1}}\mathbb{E}_t\hat{r}_{t+k+1} - \sum_{k=0}^{\infty}\left(\frac{\phi_y}{\phi_\pi}\right)^{k+1}\mathbb{E}_t\hat{y}_{t+k+1}.
$$

Using (D.97) and (D.96),

$$
\mathbb{E}_t\hat{\pi}_{t+1} = 0.
$$

E. Calvo wage setting

Firm j produces output $Y_{j,t}$ according to the production function

$$
Y_{j,t} = AN_{j,t},
$$

where $N_{j,t}$ is an index of labor input used by firm j and is defined as

$$
N_{j,t} = \left[\int_0^1 N_{i,j,t}^{1-\frac{1}{\epsilon_w}}\,di\right]^{\frac{\epsilon_w}{\epsilon_w - 1}},
$$

capturing the use of a continuum of differentiated labor services. $N_{i,j,t}$ is the quantity of type i labor employed by firm j in period t. The parameter ϵ_w represents the elasticity of substitution among labor varieties. From firm minimization of labor expenditure, the following labor demand schedules are obtained,

$$N_{i,j,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon_w} N_{j,t}.$$

W_t is the aggregate nominal wage index, defined as

$$W_t \equiv \left[\int_0^1 W_{i,t}^{1-\epsilon_w} di\right]^{\frac{1}{1-\epsilon_w}}.$$

Aggregating across firms, the demand for type i labor is

$$N_{i,t} = \int_0^1 N_{i,j,t}\,\mathrm{d}j = \left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon_w} \int_0^1 N_{j,t}\,\mathrm{d}j = \left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon_w} N_t.$$

F. Sentiment equilibrium with Calvo wage setting

The equilibrium is characterized by the Euler equation, the New Keynesian Phillips curve for wage inflation, the firms' optimal production function, the central bank's policy rule, market clearing, and the real wage identity,

$$\begin{aligned}
\hat{c}_t &= \mathbb{E}_t\hat{c}_{t+1} - \frac{1}{\gamma}(i_t - \rho - \mathbb{E}_t\hat{\pi}_{t+1}),\\
\pi^w_t &= \beta\mathbb{E}_t\pi^w_{t+1} - \lambda_w\hat{\mu}^w_t,\\
\hat{y}_{j,t} &= \mathbb{E}_t(\hat{\varepsilon}_{j,t} + \hat{y}_t - \theta\hat{w}^r_t|s_{j,t}),\\
i_t &= \rho + \phi^w_\pi\hat{\pi}^w_t + \phi_y\hat{y}_t,\\
\hat{y}_t &= \hat{c}_t,\\
\hat{w}^r_{t+1} &= \hat{w}^r_t + \mathbb{E}_t\hat{\pi}^w_{t+1} - \mathbb{E}_t\hat{\pi}_{t+1}.
\end{aligned}$$

Also, households' beliefs about aggregate output are correct

$$\hat{y}_t = \hat{z}_t.$$

To find $\hat{w}^r_t, \hat{\pi}_t, \hat{\pi}^w_t$ in terms of $\hat{z}_t$, guess and verify the following policy functions:

$$\begin{aligned}
\hat{c}_t &= \hat{z}_t,\\
\hat{w}^r_t &= a_r\hat{z}_t + b_r\hat{w}^r_{t-1},\\
\pi^w_t &= a_w\hat{z}_t + b_w\hat{w}^r_{t-1},\\
\pi_t &= a_p\hat{z}_t + b_p\hat{w}^r_{t-1}.
\end{aligned}$$

Replacing $\mathbb{E}_t\hat{c}_{t+1}$ and $\mathbb{E}_t\pi_{t+1}$,

$$\hat{c}_t = -\frac{1}{\gamma}(i_t - \rho - \hat{w}_t^r) \tag{F.98}$$

Substituting i_t in (F.98) with the Taylor rule, and imposing $\hat{y}_t = \hat{c}_t$

$$\hat{c}_t = -\frac{1}{\gamma}(\phi_\pi^w \pi_t^w + \phi_y \hat{c}_t - \hat{w}_t^r),$$

to get an expression for π_t^w,

$$\pi_t^w = -\frac{\gamma + \phi_y}{\phi_\pi^w}\hat{c}_t + \frac{1}{\phi_\pi^w}\hat{w}_t^r. \tag{F.99}$$

Similarly, in the New Keynesian Phillips curve, substitute our guess for $\mathbb{E}_t\pi_{t+1}^w$, and set $\hat{y}_t = \hat{c}_t$. Households that can reset their wage will set a markup $\hat{\mu}_t^w = \hat{w}_t^r - \hat{c}_t$,

$$\pi_t^w = \beta\mathbb{E}_t\pi_{t+1}^w - \lambda_w\hat{\mu}_t^w, \tag{F.100}$$

$$= -\lambda_w(\hat{w}_t^r - \hat{c}_t). \tag{F.101}$$

Equating (F.99) and (F.101) gives an expression for the real wage in terms of consumption,

$$\hat{w}_t^r = \frac{\lambda_w + \frac{\gamma+\phi_y}{\phi_\pi^w}}{\lambda_w + \frac{1}{\phi_\pi^w}}\hat{c}_t. \tag{F.102}$$

An alternative expression for the real wage,

$$\hat{w}_t^r = \left(1 - \frac{1 - (\gamma + \phi_y)}{\lambda_w\phi_\pi^w + 1}\right)\hat{c}_t, \tag{F.103}$$

which can be substituted into (F.101) to get an expression for wage inflation in terms of consumption,

$$\pi_t^w = \frac{1 - (\gamma + \phi_y)}{\phi_\pi^w + \frac{1}{\lambda_w}}\hat{c}_t. \tag{F.104}$$

Next, we verify that price inflation takes the conjectured form using the real wage identity

$$\hat{w}_t^r = \hat{w}_{t-1}^r + \pi_t + \pi_t^w.$$

Substituting with (F.103) and (F.104),

$$\pi_t = \left(\frac{(\lambda_w + 1)(1 - [\gamma + \phi_y])}{\lambda_w\phi_\pi^w + 1} - 1\right)\hat{c}_t + \hat{w}_{t-1}^r. \tag{F.105}$$

G. Effect of increasing central bank's response to wage inflation (ϕ_π^w)

- Consider a Taylor rule that does not weight wage inflation ($\phi_\pi^w = 0$). This implies

$$\begin{aligned} \hat{w}_t^r &= (\gamma + \phi_y)\hat{z}_t, \\ \pi_t^w &= \lambda_w[1 - (\gamma + \phi_y)]\hat{z}_t, \\ \pi_t &= [(\lambda_w + 1)(1 - [\gamma + \phi_y]) - 1]\hat{z}_t + \hat{w}_{t-1}^r. \end{aligned}$$

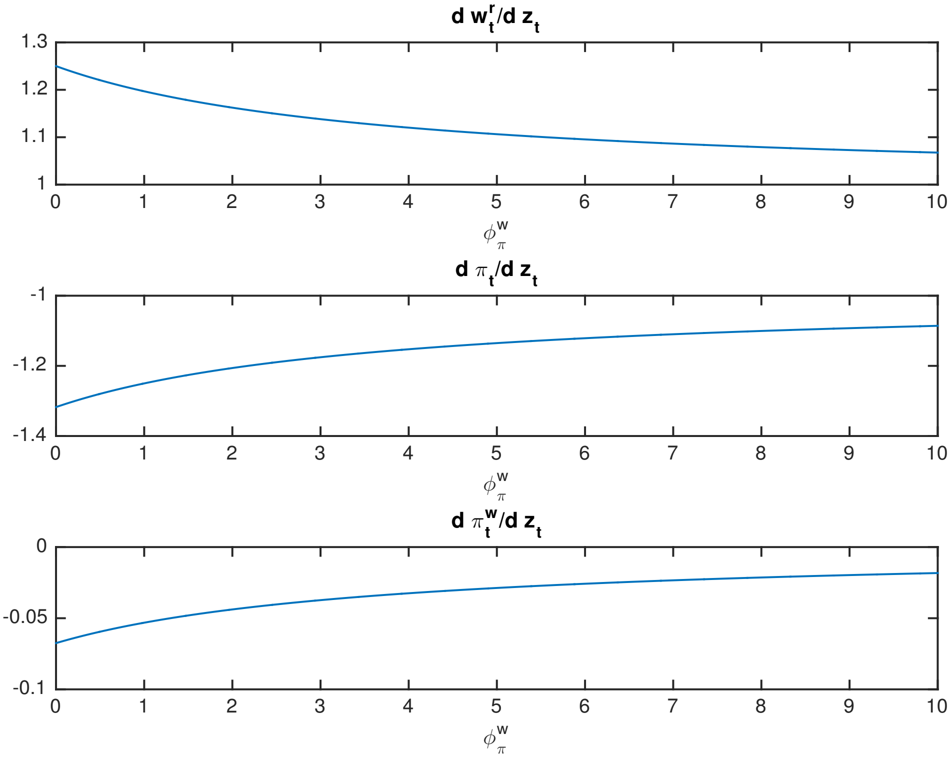
- Consider a Taylor rule that weights wage inflation very strongly ($\phi_\pi^w \to \infty$). This implies

$$\begin{aligned} \hat{w}_t^r &\to \hat{z}_t, \\ \pi_t^w &\to 0, \\ \pi_t &\to -\hat{z}_t + \hat{w}_{t-1}^r. \end{aligned}$$

- **Plots**:

$$\begin{aligned} \frac{\partial}{\partial \phi_\pi^w}\frac{\partial \hat{w}_t^r}{\partial \hat{z}_t} &= \frac{\lambda_w[1 - (\gamma + \phi_y)]}{(1 + \phi_\pi^w \lambda_w)^2} < 0 \\ \frac{\partial}{\partial \phi_\pi^w}\frac{\partial \pi_t^w}{\partial \hat{z}_t} &= \frac{-\lambda_w^2[1 - (\gamma + \phi_y)]}{(1 + \phi_\pi^w \lambda_w)^2} > 0 \\ \frac{\partial}{\partial \phi_\pi^w}\frac{\partial \pi_t}{\partial \hat{z}_t} &= \frac{-\lambda_w(\lambda_w + 1)[1 - (\gamma + \phi_y)]}{(1 + \phi_\pi^w \lambda_w)^2} > 0 \end{aligned}$$

As ϕ_π^w increases, π_t (and thus p_t) decreases by less, π_t^w (and thus w_t) decreases by less, and w_t^r increases by less.



H. Effect of increasing wage flexibility

- Consider $\lambda_w = 0$ **(completely sticky wages)**. When wages are unadjustable, wage inflation is equal to zero, and the nominal interest rate does not change. Then, the real interest rate falls solely through an increase in expected price inflation (fall in p_t).

$$\hat{w}^r_t = (\gamma + \phi_y)\hat{z}_t,$$
$$\pi^w_t = 0,$$
$$\pi_t = -(\gamma + \phi_y)\hat{z}_t + \hat{w}^r_{t-1}.$$

- Consider $\lambda_w \to \infty$ **(completely flexible wages)**. When wages are flexible, wage inflation decreases (w_t falls) in order for the nominal interest rate to fall. Then, the real interest rate falls through a combination of an increase in expected price inflation (fall in p_t) and a decrease in the nominal interest rate. Therefore, expected price inflation does not need to increase by as much, relative to the case where wages are completely sticky, and so p_t falls by less. Since w_t falls and p_t

falls by less, w_t^r increases by less. As $\lambda_w \to \infty$,

$$\hat{w}_t^r = \frac{\phi_\pi^w + \frac{\gamma+\phi_y}{\lambda_w}}{\frac{1}{\lambda_w} + \phi_\pi^w} \hat{z}_t \to \hat{z}_t, \tag{H.106}$$

$$\pi_t^w = \frac{1-(\gamma+\phi_y)}{\frac{1}{\lambda_w} + \phi_\pi^w} \hat{z}_t \to \frac{1-(\gamma+\phi_y)}{\phi_\pi^w} \hat{z}_t, \tag{H.107}$$

$$\pi_t = \left[\frac{\left(1+\frac{1}{\lambda_w}\right)[1-(\gamma+\phi_y)]}{\phi_\pi^w + \frac{1}{\lambda_w}} - 1 \right] \hat{z}_t + \hat{w}_{t-1}^r \to \left[\frac{1-(\gamma+\phi_y)}{\phi_\pi^w} - 1 \right] \hat{z}_t + \hat{w}_{t-1}^r. \tag{H.108}$$

Note that under perfectly flexible wages, the central bank's response to wage inflation (ϕ_π^w) has no effect on the real wage.

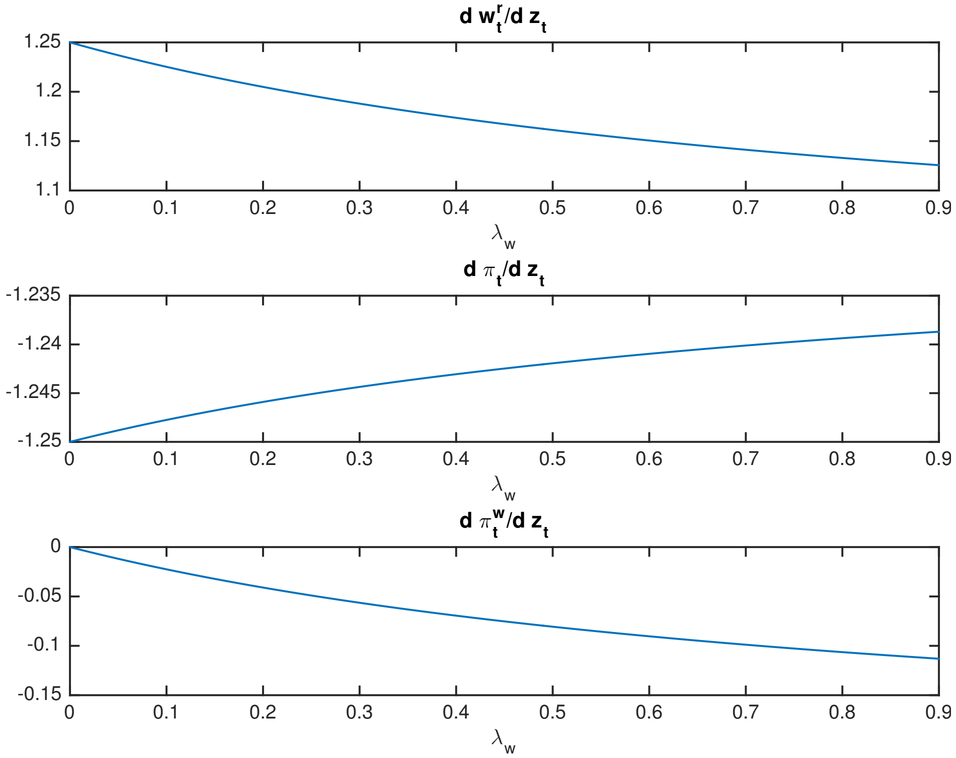
- **Plots**:

$$\frac{\partial}{\partial \lambda_w} \frac{\partial \hat{w}_t^r}{\partial \hat{z}_t} = \frac{\phi_\pi^w [1-(\gamma+\phi_y)]}{(1+\phi_\pi^w \lambda_w)^2} < 0$$

$$\frac{\partial}{\partial \lambda_w} \frac{\partial \pi_t^w}{\partial \hat{z}_t} = \frac{1-(\gamma+\phi_y)}{(1+\phi_\pi^w \lambda_w)^2} < 0$$

$$\frac{\partial}{\partial \lambda_w} \frac{\partial \pi_t}{\partial \hat{z}_t} = \frac{(1-\phi_\pi^w)[1-(\gamma+\phi_y)]}{(1+\phi_\pi^w \lambda_w)^2} > 0$$

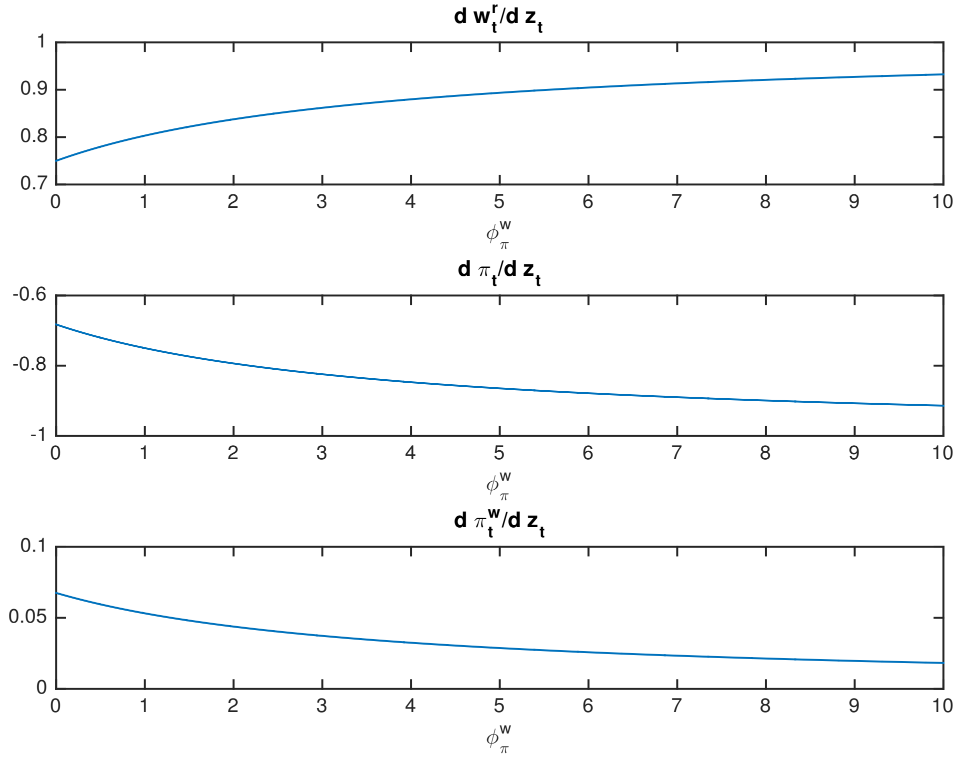
As λ_w increases, π_t (and thus p_t) decreases by less, π_t^w (and thus w_t) decreases by more, and w_t^r increases by less.



The x-axis corresponds to values of λ_w consistent with $\theta_w = 0.4$ to 0.8.

I. Effect of risk-aversion

Note that the result $\frac{\partial \sigma_z^2}{\partial \phi_\pi^w} > 0$ depends on a sufficient level of risk-aversion. Consider $\gamma = 0.5$,

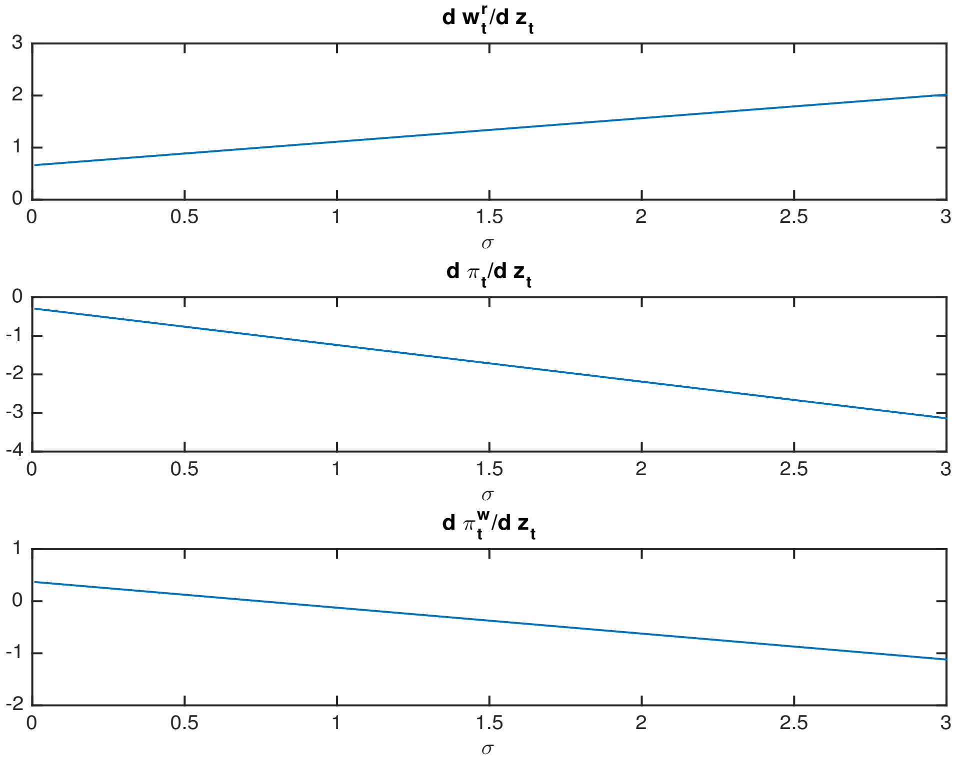


This points to a primary effect and a secondary effect of a response to z_t. The first way in which a self-fulfilling positive z_t is fulfilled is through a decrease in the price level, which results in an increased real wage. As a result, expected price inflation increases without a change in the nominal interest rate. However, if the resulting increase in consumption is not sufficient (if γ is high), wage inflation may need to fall as well so that the real interest rate decreases by more when the nominal interest rate falls. The result is that real interest rate falls through both an increase in expected price inflation and a decrease in the nominal interest rate.

I.1. Role of substitution versus wealth effect (γ)

- A decrease in the real interest rate has two opposing effects on consumption. The *substitution effect*: as the real interest rate falls, consumption increases as the return from savings offers lower utility than additional consumption. Consumption and savings are substitutes, and as the return from savings decreases, consumption increases. The *wealth effect* refers to a less known dynamic: as the real interest rate falls, the reduced return on savings decreases. As a result of this fall in the return to savings, households consume less.

- When γ is sufficiently small, the wealth effect dominates. From the households' optimal inter-temporal consumption decision (1.14), a decrease in γ renders the real interest rate more effective in changing consumption



For γ low, a smaller fall in the real interest rate is required to increase consumption on the household side. Thus, in a self-fulfilling equilibrium, wage inflation does not need to fall by as much. In equilibrium, the real wage increases when by more when γ is low.

I.2. Robustness of results to alternative preferences

Non-linear disutility of labor, firm sets quantity

In the quantity setting case, a non-linear disutility of labor implies that the real wage must increase by more in a sentiment-driven equilibrium (relative to the case of linear disutility of labor).[23] As a result, firm level output is characterized by more substitutability with respect to aggregate output, and sentiments are less volatile.

[23] With a linear disutility of labor, labor supply responds strongly to a change in the real wage.

Consider a more general utility function for households that is non-linear in labor supply. Households choose labor supply (N_t) to maximize utility

$$\max_{N_t} \frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

subject to budget constraint

$$P_t C_t \leq W_t N_t + \Pi_t.$$

The resulting first order condition is

$$\frac{-U_n}{U_c} = \frac{W_t}{P_t},$$
$$C_t^{\gamma} N_t^{\varphi} = \frac{W_t}{P_t}.$$

This implies that the price level is as follows,

$$P_t = \frac{W_t}{C_t^{\gamma} N_t^{\varphi}}.$$

Substituting N_t with the production function $Y_t = AN_t$ and applying the market clearing condition, $Y_t = C_t$,

$$P_t = \frac{W_t}{C_t^{\gamma+\varphi}} A^{\varphi}. \tag{I.109}$$

From (1.11) The firms' first order condition is

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) A \mathbb{E}_t \left[(\epsilon_{j,t} Y_t)^{\frac{1}{\theta}} \frac{P_t}{W_t} | s_{j,t}\right]\right]^{\theta}.$$

Substituting P_t with (I.109),

$$Y_{j,t} = \left[\left(1 - \frac{1}{\theta}\right) A^{1+\varphi} \mathbb{E}_t \left[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-\gamma+\varphi} | s_{j,t}\right]\right]^{\theta}.$$

Alternatively, substituting the real wage with the household's optimal labor supply condition,

$$Y_{j,t}^{\frac{1}{\theta}} = \left[\left(1 - \frac{1}{\theta}\right) A \mathbb{E}_t \left[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}-\gamma} N_t^{-\varphi} | s_{j,t}\right]\right].$$

Replacing $N_t = \int N_{j,t} dj = \int \frac{Y_{j,t}}{A} dj$,

$$Y_{j,t}^{\frac{1}{\theta}} = \left[\left(1 - \frac{1}{\theta}\right) A \mathbb{E}_t \left[\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta} - \gamma} \left(\int \frac{Y_{j,t}}{A} dj\right)^{-\varphi} | s_{j,t}\right]\right].$$

First, conjecture $y_{j,t} = D + B s_{j,t}$. Equating coefficients,

$$D = \frac{1}{1 + \varphi\theta}\left((1 - \gamma\theta)\phi_0 - \varphi\theta\left[\log\frac{1}{A} + \frac{(B\lambda)^2}{2}\sigma_\epsilon^2\right] + \frac{\theta}{2}\Omega_s\right),$$
$$B = \frac{(1 - \gamma\theta)(1 - \lambda)\sigma_z^2 + \lambda\sigma_\epsilon^2}{(1 - \lambda)^2(1 + \theta\varphi)\sigma_z^2 + \lambda^2\sigma_\epsilon^2}.$$

Note that the pass through of z_t to $y_{j,t}$ is mitigated by φ (effect of higher wages with linear disutility of labor). Next, substitute $y_{j,t}$ in aggregate price index (A.7), and equate coefficients,

$$\phi_0 = \frac{1}{\varphi + \gamma}\left[\frac{\Omega_s}{2} - \varphi\log\frac{1}{A} + \frac{1}{\theta}\left(\frac{(1 + \varphi\theta)(1 + [\theta - 1]\frac{\lambda}{1-\lambda})^2}{1\theta(\theta - 1)} - \frac{\varphi\theta(\frac{\lambda}{1-\lambda})^2}{2}\right)\sigma_\epsilon^2\right],$$
$$\sigma_z^2 = \frac{\lambda}{1 - \lambda}\frac{1 - \frac{\lambda}{1-\lambda}}{\theta(\varphi + \gamma)}\sigma_\epsilon^2.$$

Non-linear disutility of labor, firm sets price

Firms set price $P_{j,t}$ optimally according to

$$P_{j,t} = \frac{\theta}{\theta - 1}\frac{\mathbb{E}_t[W_t \epsilon_{j,t}^{1-\tau} Y_t | s_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | s_{j,t}]}.$$

Replacing N_t with $\int \frac{Y_{j,t}}{\epsilon_{j,t}^{\tau}} dj = P_t^{\theta} Y_t \int P_{j,t}^{-\theta} \epsilon_{j,t}^{1-\tau} dj$,

$$P_{j,t} = \frac{\theta}{\theta - 1}\frac{\mathbb{E}_t[P_t^{1+\theta\varphi}\epsilon_{j,t}^{1-\tau} Z_t^{1+\gamma+\varphi}\left(\int P_{j,t}^{-\theta}\epsilon_{j,t}^{1-\tau} dj\right)^{\varphi} | s_{j,t}]}{\mathbb{E}_t[\epsilon_{j,t} Y_t | s_{j,t}]}. \tag{I.110}$$

First, substitute the conjecture for $p_{j,t}$ on the right hand side of (I.110) and simplify. Equating coefficients,

$$\bar{\mu} = \frac{-\tau\lambda\sigma_\epsilon^2 + (\gamma + \varphi + B)(1 - \lambda)\sigma_z^2}{\lambda^2\sigma_\epsilon^2 + (1 - \lambda)^2\sigma_z^2}.$$

In equilibrium, $B = \bar{\mu}(1-\lambda)$, which implies

$$\sigma_z^2 = \frac{\lambda}{1-\lambda} \frac{\tau + B\frac{\lambda}{1-\lambda}}{\gamma + \varphi} \sigma_\epsilon^2.$$

B is indeterminate, and when we introduce monetary policy, it will be equal to $\frac{\partial p_t}{\partial z_t} = -\frac{\gamma + \phi_y}{\phi_\pi}$.

Alternative preferences in the model with wages set one period in advance

Consider a more general utility function for households that is non-linear in labor supply. Households choose the nominal wage for their labor type ($W_{i,t}$) to maximize utility

$$\max_{W_{i,t}} \mathbb{E}_{t-1} \left[\frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \frac{N_{i,t}^{1+\varphi}}{1+\varphi} \right]$$

subject to budget constraint

$$P_t C_{i,t} + Q_t B_{i,t} \leq W_{i,t} N_{i,t} + \Pi_{i,t} + B_{i,t-1},$$

and firms' labor demand

$$N_{i,t} = \left(\frac{W_{i,t}}{W_t} \right)^{-\varepsilon_w} N_t.$$

Maximizing utility with respect to $W_{i,t}$,

$$\frac{\partial U}{\partial W_{i,t}} = \mathbb{E}_{t-1} \left[C_{i,t}^{-\gamma} \frac{\partial C_{i,t}}{\partial W_{i,t}} - N_{i,t}^{\varphi} \frac{\partial N_{i,t}}{\partial W_{i,t}} \right],$$

where

$$\frac{\partial N_{i,t}}{\partial W_{i,t}} = -\varepsilon_w \left(\frac{W_{i,t}}{W_t} \right)^{-\varepsilon_w - 1} \frac{N_t}{W_t} = -\varepsilon_w \frac{N_{i,t}}{W_{i,t}}$$

follows from the labor demand schedule of firms and

$$\frac{\partial C_{i,t}}{\partial W_{i,t}} = \frac{1}{P_t} \left(W_{i,t} \frac{N_{i,t}}{W_{i,t}} + N_{i,t} \right) = \frac{N_{i,t}}{P_t} (1 - \varepsilon_w)$$

follows from the household's budget constraint and the labor demand schedule of firms. Substituting $\frac{\partial C_{i,t}}{\partial W_{i,t}}$ and $\frac{\partial N_{i,t}}{\partial W_{i,t}}$ in the first order condition yields the optimal wage chosen by household i at $t-1$,

$$\mathbb{E}_{t-1}\left[C_{i,t}^{-\gamma}\frac{N_{i,t}}{P_t}(1-\varepsilon_w) - N_{i,t}^{\varphi}(-\varepsilon_w\frac{N_{i,t}}{W_{i,t}})\right] = 0.$$

Rearranging terms,

$$W_{i,t} = \frac{\varepsilon_w}{\varepsilon_w - 1}\frac{\mathbb{E}_{t-1}(N_{i.t}^{\varphi+1})}{\mathbb{E}_{t-1}\left(\frac{N_{i,t}}{C_{i,t}^{\gamma}P_t}\right)}.$$

Since consumption will be the same for all households, the wage set will be the same, and we can remove the i index,

$$W_t = \frac{\varepsilon_w}{\varepsilon_w - 1}\frac{\mathbb{E}_{t-1}(N_t^{\varphi+1})}{\mathbb{E}_{t-1}\left(\frac{N_t}{C_t^{\gamma}P_t}\right)}.$$

The optimal nominal wage chosen by monopolistically competitive households at $t-1$ will be a markup ($\frac{\varepsilon_w}{\varepsilon_w-1}$) over the marginal rate of substitution multiplied by the aggregate price level.

Letting $\mu \equiv \frac{\varepsilon_w}{\varepsilon_w-1}$ and rearranging terms, the households' wage setting equation is

$$1 = \mu\frac{\mathbb{E}_{t-1}(N_t^{\varphi+1})}{\mathbb{E}_{t-1}\left(N_t C_t^{-\gamma}\frac{W_t}{P_t}\right)}. \tag{I.111}$$

Log-linearizing (I.111) around the steady state,

$$\mathbb{E}_{t-1}\hat{w}_t^r = \mathbb{E}_{t-1}(\varphi\hat{n}_t + \gamma\hat{c}_t).$$

J. Sentiment Equilibrium with Flexible Wages and Technology Shocks

To solve for equilibrium output, conjecture $Y_t = MA_t^{\psi_{ya}}\zeta_t$ and $y_t \equiv \log Y_t \sim N(\phi_0, \sigma_y^2)$. In expectation,

$$e^{\phi_0+\frac{\sigma_y^2}{2}} = e^{m+\psi_{ya}\bar{a}+\frac{\psi_{ya}^2\sigma_a^2+\sigma_\zeta^2}{2}}.$$

This implies

$$\phi_0 = m + \psi_{ya}\bar{a},$$
$$\sigma_y^2 = \psi_{ya}^2\sigma_a^2 + \sigma_\zeta^2.$$

Firm level production, in logs,

$$y_{j,t} = \theta \log\left(\frac{\theta-1}{\theta}\frac{1}{\psi}\right) + (1-\gamma\theta)\phi_0 + \theta\bar{a} + \theta\,\underbrace{\mathbb{E}\left[\frac{1}{\theta}\varepsilon_{j,t} + (\frac{1}{\theta}-\gamma)\bar{y}_t + \bar{a}_t|\tilde{s}_{j,t}\right]}_{\mu} + \frac{\theta}{2}\Omega_s,$$

where $\tilde{s}_{j,t} = \lambda\epsilon_{j,t} + (1-\lambda)(\psi_{ya}\bar{a}_t + \bar{\zeta}_t)$, $\bar{a}_t \equiv \log \bar{A}_t \sim N(0,\sigma_a^2)$, $\bar{\zeta}_t \equiv \zeta_t \sim N(0,\sigma_\zeta^2)$, $\bar{y}_t \equiv \log \bar{Y}_t \equiv \log[\bar{A}_t^{\psi_{ya}}\bar{\zeta}_t] \sim N(0,\sigma_y^2)$ and $\Omega_s \equiv \text{Var}[\frac{1}{\theta}\varepsilon_{j,t} + (\frac{1}{\theta}-\gamma)\bar{y}_t + \bar{a}_t|\tilde{s}_{j,t}]$ Let firm production be represented by

$$Y_{j,t} = e^{\varphi_0}\tilde{S}_{j,t}^B,$$

where $\tilde{S}_{j,t} = \epsilon_{j,t}^{\lambda}[\bar{A}_t^{\psi_{ya}}\bar{\zeta}_t]^{1-\lambda}$, $\varphi_0 \equiv \theta\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + (1-\gamma\theta)\phi_0 + \theta\bar{a} + \frac{\theta}{2}\Omega_s$, $\log \bar{Y}_t \sim N(0,\sigma_y^2)$, and $B \equiv \theta\mu$. By (1.25), aggregate output is

$$Y_t = e^{\varphi_0}[\bar{A}_t^{\psi_{ya}}\bar{\zeta}_t]^{B(1-\lambda)}\underbrace{\left[\int \epsilon_{j,t}^{\frac{1}{\theta}+\frac{\theta-1}{\theta}\lambda B}\,\mathrm{d}j\right]^{\frac{\theta}{\theta-1}}}_{\kappa_1}.$$

In logs,

$$y_t = \varphi_0 + B(1-\lambda)[\psi_{ya}\bar{a}_t + \bar{\zeta}_t] + \log\kappa_1.$$

This implies

$$e^{\phi_0+\frac{\sigma_y^2}{2}} = e^{\varphi_0+\log\kappa_1+\frac{1}{2}[B(1-\lambda)]^2[\psi_{ya}^2\sigma_a^2+\sigma_\zeta^2]}.$$

Equating with the conjecture,

$$B = \frac{1}{1-\lambda}, \tag{J.112}$$
$$\phi_0 = \varphi_0 + \log\kappa_1, \tag{J.113}$$
$$= \theta\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + (1-\gamma\theta)\phi_0 + \theta\bar{a} + \frac{\theta}{2}\Omega_s + \log\kappa_1, \tag{J.114}$$
$$\psi_{ya} = \frac{1}{\gamma}, \tag{J.115}$$
$$m = \frac{1}{\gamma}\left[\log\left(\frac{\theta-1}{\theta}\frac{1}{\Psi}\right) + \frac{\Omega_s}{2}\right] + \frac{\log\kappa_1}{\theta}. \tag{J.116}$$

In equilibrium, (J.112) implies

$$\sigma_y^2 = \tilde{\sigma}_z^2 + \frac{1}{\gamma^2}\sigma_a^2 + (1-\gamma\theta)\sigma_\zeta^2,$$

where $\tilde{\sigma}_z^2 \equiv \frac{\lambda}{1-\lambda}\left(1-\frac{\lambda}{1-\lambda}\right)\sigma_\epsilon^2$. Equating with the results from our conjecture,

$$\begin{aligned}\sigma_y^2 &= \frac{1}{\gamma\theta}\tilde{\sigma}_z^2 + \frac{1}{\gamma^2}\sigma_a^2,\\ \sigma_\zeta^2 &= \frac{1}{\gamma\theta}\tilde{\sigma}_z^2.\end{aligned}$$

When firms condition production on an endogenous signal of aggregate demand, there must be an extrinsic component to aggregate output.

K. Sentiment Equilibrium with Sticky Wages and Technology Shocks

Incorporating the household's labor supply condition and its own production function, firm j conditions production ($Y_{j,t}$) on its signal $S_{j,t}$,

$$Y_{j,t} = \left[\left(1-\frac{1}{\theta}\right)\mathbb{E}_t\left(\epsilon_{j,t}^{\frac{1}{\theta}} Y_t^{\frac{1}{\theta}} \frac{1}{W_t/P_t} A_t^\tau | S_{j,t}\right)\right]^\theta.$$

In logs,

$$\begin{aligned} y_{j,t} = {} & \theta \ln\left(1-\frac{1}{\theta}\right) + \mathbb{E}[\varepsilon_{j,t} + y_t - \theta w_t^r + \tau\theta a_t | s_{j,t}] \\ & + \frac{\theta}{2} Var\left[\frac{1}{\theta}(\varepsilon_{j,t} + y_t) - \theta w_t^r + \tau a_t | s_{j,t}\right]. \end{aligned}$$

The other equilibrium conditions include the Euler equation, Taylor rule, New Keynesian Phillips curve for wage inflation, the signal firms receive, labor supply of households, market clearing, and technology process,

$$
\begin{aligned}
\hat{c}_t &= \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\gamma}(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}), \\
\hat{i}_t &= \phi_\pi^w \hat{\pi}_t^w + \phi_y \hat{y}_t, \\
\hat{\pi}_t^w &= \beta \mathbb{E}_t \hat{\pi}_{t+1}^w - \lambda_w \hat{\mu}_t^w, \\
s_{j,t} &= \lambda \varepsilon_{j,t} + (1-\lambda) y_t, \\
\hat{\mu}_t^w &= \hat{w}_t^r - \gamma \hat{c}_t, \\
\hat{y}_t &= \hat{c}_t, \\
\hat{y}_t &= \int_0^1 \hat{y}_{j,t} dj, \\
\hat{a}_{t+1} &= \rho \hat{a}_t + \hat{\varepsilon}_{t+1}^a.
\end{aligned}
$$

Conjecture the following policy functions for output, price inflation, wage inflation, and the real wage,

$$
\begin{aligned}
\hat{c}_t &= \hat{\zeta}_t + b_c \hat{w}_{t-1}^r + \psi_{ya} \hat{a}_t, \\
\hat{\pi}_t &= a_\pi \hat{\zeta}_t + b_\pi \hat{w}_{t-1}^r + c_\pi \hat{a}_t, \\
\hat{\pi}_t^w &= a_{\pi^w} \hat{\zeta}_t + b_{\pi^w} \hat{w}_{t-1}^r + c_{\pi^w} \hat{a}_t, \\
\hat{w}_t^r &= a_w \hat{\zeta}_t + b_w \hat{w}_{t-1}^r + c_w \hat{a}_t.
\end{aligned}
$$

Coefficients that verify the conjecture are

$$
\begin{aligned}
a_w &= \frac{\gamma(1+\phi_\pi^w \lambda_w) + \phi_y}{1+\phi_\pi^w \lambda_w}, \\
b_\pi &= 1, \\
a_\pi^w &= \frac{-\lambda_w \phi_y}{1+\lambda_w \phi_\pi^w}, \\
a_\pi &= \frac{-\gamma(1+\phi_\pi^w \lambda_w) + \phi_y(1+\lambda_w)}{1+\lambda_w}.
\end{aligned}
$$

Assuming technology shocks are iid ($\rho = 0$),

$$
\begin{aligned}
c_w &= \frac{\gamma(1+\phi_\pi^w) + \phi_y}{1+\phi_\pi^w} \psi_{ya}, \\
c_\pi &= \frac{-\gamma(1+\phi_\pi^w \lambda_w) + \phi_y(1+\lambda_w)}{1+\lambda_w} \psi_{ya}, \\
b_\pi^w &= \frac{-\lambda_w \phi_y}{1+\lambda_w \phi_\pi^w} \psi_{ya}.
\end{aligned}
$$

From the wage inflation equation, $b_\pi^w(1-\beta c_w) = \lambda_w \gamma b_c$, which implies $b_\pi^w = b_c = 0$. Note that as $\phi_\pi^w \to \infty$, $c_w \to \gamma\psi_{ya}$ and $a_w \to \gamma$.

Under the assumption that technology shocks are persistent ($\rho > 0$),

$$\begin{aligned}
c_w &= \frac{[\gamma(1-\rho)+\phi_y](1-\beta\rho)+\gamma\lambda_w(\phi_\pi^w-\rho)}{\lambda_w(\phi_\pi^w-\rho)+(1-\beta\rho)(1-\rho)}\psi_{ya}, \\
c_{\pi^w} &= \frac{1}{1-\beta\rho}[-\lambda_w(c_w-\gamma\psi_{ya})], \\
c_\pi &= c_{\pi^w} - c_w.
\end{aligned}$$

Chapter 2

Optimal Policy and Sentiment Driven Fluctuations

2.1. Introduction

A growing literature has explored the macroeconomic implications of information frictions. Models in which agents hold incomplete information about the current state of the economy or heterogeneous beliefs about its future yield important insights for business cycle dynamics. For instance, aggregate variables that adjust slowly to shocks, myopic expectations, and self-fulfilling fluctuations can be rationalized in models featuring dispersed information (Lucas (1972), Morris and Shin (2002), Angeletos and La'O (2013), and Benhabib et al. (2015)), sticky information (Mankiw and Reis (2002)), rational inattention (Sims (2003), Woodford (2001), Maćkowiak and Wiederholt (2009)), or higher-order uncertainty (Morris and Shin (1998), Angeletos and Lian (2016))

However, the normative implications of informational and nominal frictions for the design of monetary policy are less clear. Ball et al. (2003), Lorenzoni (2009), Adam (2007) consider how monetary policy should respond in settings characterized by slow adjustment to shocks, noise, or persistent shocks and strategic complementarity in actions. In related work, Angeletos and La'O (2019) consider optimal policy in a model in which firms' pricing and production decisions are subject to informational frictions. Due to the exogenous information structure that they consider, optimal policy leans against the wind and does not target price stability. By contrast, I will show endogenous signals provide an alternate channel through which monetary policy can affect outcomes.

I study optimal monetary policy in a standard New Keynesian model, with the exception that firms condition their production decisions on heterogenous, private signals about the state of the economy. In addition, households face Calvo wage rigidity. The key friction is that firms make decisions based on a signal that confounds idiosyncratic demand and an endogenous variable, aggregate demand. In this setting, fluctuations may be non-fundamental in nature, introducing a trade-off between stabilizing output and inflation. Through its effect on aggregate variables, the stance of monetary policy affects the volatility of these fluctuations.

To understand the inefficiencies that the dispersion of information may cause, I compare the decentralized equilibrium to an appropriate efficiency benchmark (Angeletos et al. (2007)). Abstracting from policy instruments, the constrained efficient allocation represents the best allocation among those that respect resource feasibility and decentralized information (Angeletos et al. (2007)). I show that this allocation can be implemented with a simple interest rate rule.

In treating information as endogenous, this framework is able to consider questions related to both the allocative and informational efficiency of monetary policy. The key insight is that in the presence of endogenous signals, there is an alternate channel through which monetary policy affects equilibrium outcomes. As the stance of monetary policy affects aggregate variables, it influences how firms use their signals and the degree of coordination in production. Thus, there is an inefficiency in the decentralized equilibrium which stems from an interaction between the use of information and the aggregation of information. As the dispersed signals capture an endogenous variable (aggregate demand), firms' actions affect the signals they receive. In this setting, non-fundamental fluctuations can arise, and the stance of monetary policy can affect the size of these fluctuations. This implies that a policymaker can always reach the constrained efficient allocation, a fundamental equilibrium that features no fluctuations. As simple interest rate rule that does not respond strongly to inflation can eliminate non-fundamental fluctuations, thereby precluding the output-inflation trade-off.

2.2. Literature Review

The literature on optimal monetary policy is extensive. Simple interest rate rules to offset the macroeconomic effects from a fundamental shock have been well-studied (Woodford (2003), Clarida et al. (2000), and Rotemberg and Woodford (1998)). A portion of this literature addresses the optimality of flexible price allocations and its relationship to price stability. This paper considers the same question in a conventional New-Keynesian setting, but by relaxing the complete information assumption.

The topic of optimal monetary policy in the presence of information frictions has been explored in Ball et al. (2003), Adam (2007), Lorenzoni (2009)). Lorenzoni (2009) considers the allocative efficiency of policy, in showing how optimal policy may not eliminate the impact of noise on aggregate output. Similar to this paper, Paciello and Wiederholt (2014) consider the informational efficiency aspect of monetary policy by studying how it can affect the incentives firms face in allocating attention to fundamental shocks. In their setting, price stability is optimal, as it incentivizes price setters to pay less attention to shocks that cause inefficient fluctuations. They also consider signals that are endogenous, as agents choose the precision of idiosyncratic noise about aggregate fundamentals. As a result, price stickiness is variant to policy, depending on how the cost of attention is modeled. By contrast, in the information structure that I consider, fluctuations may be non-fundamental in nature. Policy, through its influence on firms' actions, affect the precision of signals firms receive about an idiosyncratic fundamental. In related work, Angeletos and La'O (2019) consider a model in which firms' pricing and production decisions are subject to informational frictions. The efficient allocation can be obtained with a subsidy that removes the monopoly distortion and a monetary policy that replicates flexible-price allocations. However, optimal policy targets a negative correlation between the price level and real economic activity, not price stability. Unlike Lorenzoni (2009) and Angeletos and La'O (2019), optimal policy in my framework can eliminate the impact of noise on aggregate output, as the endogeneity of outcomes to policy implies an alternate effect of the nominal interest rate and a new trade-off in stabilizing output and inflation.

Angeletos and Pavan (2007) characterize the equilibrium use of information and welfare in an abstract framework featuring strategic interaction, externalities, and heterogeneous information. I follow their example in considering an appropriate efficiency benchmark to assess welfare. However, the signals they consider are not endogenous, which is crucial for the results that I obtain. In the information structure I consider, monetary policy affects the decisions of firms, which determines the precision of their signals.

The fluctuations that arise in this model are the result of a correlated, common component of the endogenous signals, which can be considered noise. However, there are important differences with respect to the literature on news versus noise. The noise-driven fluctuations in models featuring news arise from representative-agent models, not from the dispersion of information.[1] Moreover, noise disappears once once uncertainty about fundamentals diminishes. By contrast, the fluctuations that arise in this model depend on the heterogeneity of information in conjunction with strategic interactions among agents, not on the level of uncertainty about underlying fundamentals. Furthermore, in this model, the common component in signals can generate aggregate fluctuations that are orthogonal to productivity shocks and that resemble the impact of

[1] For models featuring news, noise, and studies analyzing their empirical significance, see Barsky and Sims (2011), Beaudry and Portier (2004), Beaudry and Portier (2006), Christiano et al. (2014), Gilchrist and Zakrajšek (2012), Jaimovich and Rebelo (2009), Lorenzoni (2009)

demand shocks (Lorenzoni (2009)), without exotic preferences (Jaimovich and Rebelo (2009)), or significant departures from the standard DSGE framework (Beaudry and Portier (2004)).

Section 2.3 considers an appropriate efficiency benchmark for the model introduced in Section 1.4, while Section 2.4 discusses the implementation using a simple interest rate rule. Section 2.5 concludes.

2.3. Constrained Efficiency

The framework introduced in Chapter 1 demonstrated the following: despite strategic substitutability in actions and dispersed signals, the decentralized equilibrium of this model features aggregate fluctuations that arise from endogenous signals. This section considers the welfare properties of the decentralized equilibrium, using an appropriate efficiency benchmark. As in Angeletos and Pavan (2007), the benchmark that serves this goal is the strategy mapping from primitive information to actions that maximizes ex-ante utility. This strategy identifies the best allocation subject to the constraint that information cannot be centralized or transferred among the agents. This exercise extends the analysis of Angeletos and Pavan (2007) to an endogenous information structure. Comparing the competitive equilibrium to this benchmark allows us to isolate the discrepancy between the private and social incentives in the use of information.

Consider the social planner's problem in an equilibrium with sentiment-driven fluctuations. Restricting the set of solutions to $Y_t \sim N(\phi_0, \sigma_z^2)$, it chooses the mean and variance of output to maximize expected household utility.

$$\max_{B,\phi_0,\sigma_z^2} \mathbb{E}_t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

subject to the following constraints,

$$Y_{j,t} = F S_{j,t}^{B}, \tag{2.1}$$

$$S_{j,t} = \epsilon_{j,t}^{\lambda} Z_t^{1-\lambda}, \tag{2.2}$$

$$Y_t = \left(\int \epsilon_{j,t}^{\frac{1}{\theta}} Y_{j,t}^{\frac{\theta-1}{\theta}} \, \mathrm{d}j \right)^{\frac{\theta}{\theta-1}}, \tag{2.3}$$

$$Y_{j,t} = A N_{j,t}, \tag{2.4}$$

$$N_t = \int N_{j,t} \, \mathrm{d}j, \tag{2.5}$$

$$Y_{j,t} = C_{j,t}, \tag{2.6}$$

$$Y_t = C_t. \tag{2.7}$$

By (2.1) and (2.2), the planner directs firms' actions depend solely on their own information set. Aggregate output and labor are (2.3) and (2.5), while production and market clearing are given by (2.4), (2.6), and (2.7), respectively.

Proposition 3. *If agents are sufficiently risk averse ($\gamma \geq 1$), the constrained efficient allocation features no non-fundamental fluctuations*

$$\sigma_z^{2*} = 0$$

while the mean of log aggregate output is

$$\phi_0^* = \frac{1+\varphi}{\gamma+\varphi} \log\left(\frac{\kappa_2}{\kappa_1}\right)$$

where $\log\left(\frac{\kappa_2}{\kappa_1}\right) \equiv \frac{\sigma_\epsilon^2}{2}\left(\left[\frac{\lambda}{1-\lambda}\right]^2 - \frac{\theta}{\theta-1}\left[\frac{1}{\theta} + \frac{\lambda}{1-\lambda}\frac{\theta-1}{\theta}\right]^2\right) < 0$*. Assuming a subsidy for monopolistic competition that aligns the steady state of the decentralized equilibrium with that of its counterpart in the constrained efficient allocation, by (1.23), a simple interest rule that targets inflation sufficiently weakly,* $\phi_\pi^w \in (-\frac{1}{\lambda_w}, \frac{\gamma+\phi_y}{\lambda_w})$*, can implement the constrained efficient allocation. If* $\gamma < 1$*,*

$$\begin{aligned}
\sigma_z^{2*} &= \log\left(\frac{1-\gamma}{1+\varphi}\right) - 2(1+\varphi)\log\left(\frac{\kappa_2}{\kappa_1}\right) \\
\phi_0^* &= \frac{2+\varphi-\gamma}{2}\log\left(\frac{1-\gamma}{1+\varphi}\right) + (1+\varphi)\left(\frac{1}{\gamma+\varphi} - [2+\varphi-\gamma]\right)\log\left(\frac{\kappa_2}{\kappa_1}\right)
\end{aligned}$$

In the case of $\gamma < 1$, fluctuations may be desirable ($\sigma_z^{2*} > 0$) if φ (disutility of labor) is sufficiently larger than γ,

$$\gamma < 1 - (1+\varphi)e^{(1+\varphi)\sigma_\epsilon^2\left(\left[\frac{\lambda}{1-\lambda}\right]^2 - \frac{\theta}{\theta-1}\left[\frac{1}{\theta}+\frac{\lambda}{1-\lambda}\frac{\theta-1}{\theta}\right]^2\right)}$$

In the case of $\gamma \geq 1$, the equilibrium allocation is constrained inefficient: there a mapping from signals to actions that improves upon the decentralized equilibrium, which features no sentiment driven fluctuations. Unlike firms in the decentralized equilibrium, the planner takes into account how its actions affect the precision of its signal.

Comparing efficient versus equilibrium allocations allows us to isolate the inefficiency that originates in the way equilibrium processes available information. In the decentralized equilibrium with sentiments, firms respond to their signal with the following weight:

$$B = \frac{\lambda\sigma_\epsilon^2 + (1-\theta D)(1-\lambda)\sigma_z^2}{\lambda^2\sigma_\epsilon^2 + (1-\lambda)^2\sigma_z^2 + \sigma_\nu^2},$$

The decentralized equilibrium features an interaction between the use of information and the aggregation of information that is inefficient. As long as $\sigma_z^2 > 0$, then fluctuations in Z_t are important for firms, since they affect marginal cost through the real wage. Firms' actions will reflect this component of their signal. In addition, due to the endogeneity of the signal, σ_z^2 affects the precision of the signal. As a result of correlated signals, correlated actions by firms leads to aggregate fluctuations in output. In the aggregate, the actions of firms conditioning on an endogenous signal, affects the precision of the signals that they receive, an externality that the social planner internalizes.

As the stance of monetary policy affects aggregate variables, it influences how firms use their signals and the degree of coordination in firm production, thereby determining the degree to which the business cycle is driven by non-fundamental forces. By the same reasoning, the nominal interest rate can be used to eliminate non-fundamental fluctuations.

2.4. Optimal Monetary Policy

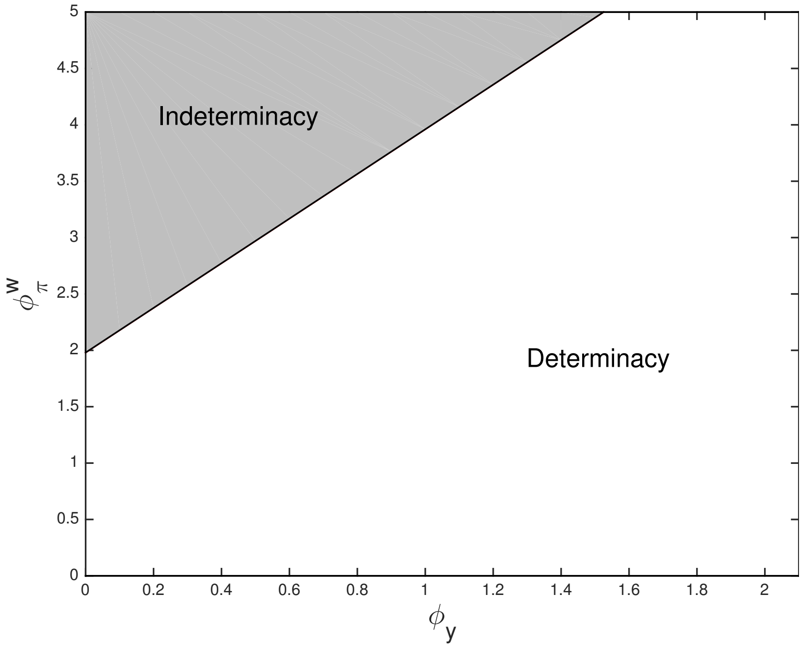
Next, I characterize optimal monetary policy in the presence of non-fundamental fluctuations. The previous section abstracted from policy instruments to show that a social planner choosing among allocations that respect resource feasibility and the decentralization of information can improve upon the competitive equilibrium. The lower welfare in the latter reflects an inefficiency in the use of information, coupled with an inefficiency in the aggregation of information. As the range of fluctuations is endogenously determined by policy parameters, this implies that a policymaker can always reach a fundamental equilibrium that features no fluctuations. Indeed, by mitigating the degree to which it targets inflation, the policymaker eliminates non-fundamental fluctuations, thereby precluding the output-inflation trade-off.

Proposition 4. *There is (real) indeterminacy even when Taylor principle is satisfied. In contrast, by (1.23), the policymaker can eliminate non-fundamental fluctuations with a sufficiently lax response to wage inflation,*

$$\sigma_z^2 \leq 0 \iff \phi_\pi^w \in \left(-\frac{1}{\lambda_w}, \frac{\gamma + \phi_y}{\lambda_w}\right). \tag{2.8}$$

Figure (2.1) shows the indeterminacy region for a model with $\beta = 0.99$ (which implies a steady state real return on bonds of about 4 percent), $\gamma = 1$ (log utility), and $\theta_w = 0.5$ (an average wage duration of 2 years). Finally, assume that the idiosyncratic component of the signal is $\lambda = 0.2$.

Figure 2.1: Indeterminacy and determinacy regions with information frictions



In the absence of non-fundamental fluctuations, the condition for indeterminacy is given by (see Blasselle and Poissonnier (2016))

$$\phi_\pi^w > 1 - \frac{1-\beta}{(1-\nu)\kappa_p + \nu\kappa_w}\phi_y,$$

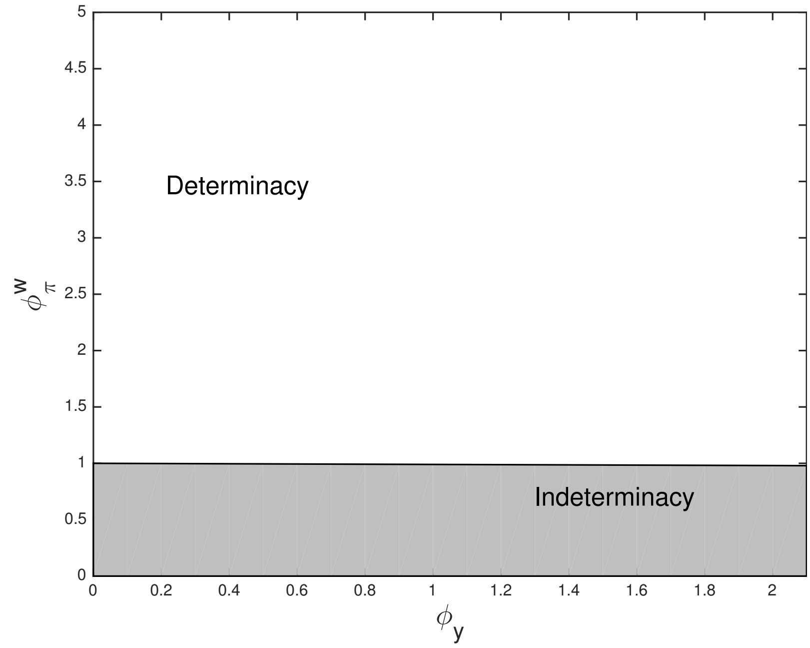
where $\nu = \frac{\lambda_p}{\lambda_p+\lambda_w}$.

However, this fundamental equilibrium is not robust to the introduction of fundamental shocks. When a technology shock is introduced, the equilibrium may feature fluctuations from non-fundamental and fundamental sources, re-introducing the trade-off between stabilizing output and inflation (see Section (1.5)). The stance of monetary policy will determine the fundamental and non-fundamental composition of aggregate output.

2.5. Conclusion

This chapter has considered how information frictions and nominal rigidities affect the design of optimal monetary policy. To this goal, I consider a minor deviation form the complete information benchmark of the standard New Keynesian model. Firms condition production on an endogenous signal, one which confounds idiosyncratic and aggre-

Figure 2.2: Indeterminacy and determinacy regions under perfect information



gate demand. Dispersed, endogenous signals, in conjunction with strategic interactions among firms can lead to fluctuations that are non-fundamental.

To address whether the fluctuations in the decentralized equilibrium are socially desirable, I consider an appropriate efficiency benchmark. From the perspective of a social planner that has neither an informational advantage relative to firms, nor the ability to centralize information that is dispersed among agents, non-fundamental fluctuations are not efficient. Assuming subsidies are in place to correct for the distortions due to monopolistic competition in the decentralized equilibrium, there is a mapping from signals to actions that results in higher welfare, without sentiment-driven fluctuations. The source of inefficiency in the decentralized equilibrium is that firms do not internalize how their response to an endogenous signal affects its precision. The policymaker can implement the constrained efficient allocation with a simple rule that targets inflation less strongly. An extension of this model will consider the positive and normative effects of monetary policy when output is driven by both fundamental and non-fundamental sources of fluctuations, and assess whether the policymaker can still implement the constrained efficient allocation. If the policymaker can not distinguish between the two types of shocks, it may not be able to eliminate non-fundamental fluctuations and hence the source of the output-inflation tradeoff.

Chapter 3

Sentiments and Uncertainty

3.1. Introduction

The real business cycle literature has emphasized the role of surprise technology shocks in driving fluctuations. However, a series of empirical contributions shows that the bulk of business cycles can be attributed to non-technology shocks, typically referred to as "demand" shocks.[1]

News shocks potentially fill this role (Beaudry and Portier (2004), Jaimovich and Rebelo (2009)). As anticipated shocks about future total factor productivity that affect the economy in the current period, they provide a compelling narrative: consumption and investment rise in anticipation of future good times while recessions take place when agents realize they have been overly optimistic and revise their expectations about future fundamentals downward.

While a news shock can be considered a fundamental shock, one which affects agents' expectations of payoffs, a long literature has also considered fluctuations driven by shocks that are extrinsic to economic fundamentals (Cooper and John (1988), Benhabib and Farmer (1994), Angeletos and La'O (2013)and Benhabib et al. (2015)). As independent changes in market expectations, sentiments provide another plausible narrative: shifts in market sentiment and aggregate demand seemingly occur without innovations to preferences, technology, or other payoff-relevant fundamentals.

Finally, uncertainty shocks form an alternate source of belief-driven business cycle fluctuations (Bloom (2009)). As agents take consumption and investment decisions based on beliefs about the current and future path of the economy, unexpected changes to their

[1] Blanchard and Quah (1989), Galí (1999), Canova and Nicoló (2002), Basu et al. (2006).

beliefs can lead to fluctuations in aggregate variables.

Shocks to news, sentiments, and uncertainty are all related to information and the formation of beliefs. Motivated by recent theories of sunspot fluctuations which rely crucially on information frictions, whether common uncertainty about fundamentals Lorenzoni (2009) or strategic uncertainty with dispersed information Angeletos and La'O (2013), Benhabib et al. (2015)), this paper provides quantitative evidence for the relationship between sentiments and uncertainty.

Although recent theoretical work has made progress in incorporating non-fundamental fluctuations in dynamic micro-founded rational expectation models by incorporating information frictions, the empirical research on the size of their effect is inconclusive (Fève and Guay (2019), Angeletos et al. (2014)). One plausible explanation for these differences is that the impact of sentiments on macroeconomic variables depend on the degree of uncertainty agents face. In this paper, I provide evidence that sentiments and uncertainty shocks as identified in the literature are correlated and may not be truly structural.

Sentiments, identified here as a change to expectations about economic activity, unrelated to news about future technology, can be interpreted as rational optimism or pessimism. As such, it may affect confidence, or uncertainty.

News, sentiment, and uncertainty shocks are each identified by maximizing the respective forecasting error variances of their measurement proxies using the same reduced form vector autoregression. This identification strategy, first used to identify news shocks in Barsky and Sims (2011), is a variance decomposition extension of the penalty function approach by Faust (1998) and Uhlig (2005).

News shocks are identified as the linear combination of reduced-form innovations that maximizes the productivity variance in the long run (40 quarters), orthogonal to a surprise technological shock. As in much of the literature for news shocks, productivity is measured by the quarterly, utilization-adjusted TFP series from Fernald (2014).

Following Levchenko and Pandalai-Nayar (2018), I identify a non-technology business cycle shock using a structural vector auto-regression that includes an expectational variable, alternatively a GDP forecast from the Philadelphia Fed's Survey of Professional Forecasters or the University of Michigan Consumer Confidence index. The non-technology shock is identified as the shock orthogonal to surprise TFP and news TFP shocks that explains the maximum of the residual forecast error variance of this expectational variable at a short horizon (2 quarters). Because the shock is identified from data on expectations after controlling for shocks to current and future TFP, Levchenko and Pandalai-Nayar (2018) consider this shock to be sentiment.[2]

[2]Although this shock can be driven by anything that makes agents expect higher or lower eco-

Uncertainty shocks are identified as the linear combination of reduced form shocks that maximizes the short-run forecast error variance of a proxy for uncertainty (over 2 quarters), with no restrictions on impact, as in Caldara et al. (2016). For robustness, I analyze the correlation between sentiment and uncertainty shocks using alternative proxies for observed uncertainty. As in Ludvigson et al. (2015) and Cascaldi-Garcia and Galvao (2018), I consider two categories for uncertainty: financial and macroeconomic. Macroeconomic uncertainty measures include professional forecasters' disagreement, which is associated with changes to ambiguity (Ilut and Schneider (2014)), while financial uncertainty measures are related to quantifiable risk as in Christiano et al. (2014).

Compared to news or uncertainty, the sentiment shock accounts for more of fluctuations at business cycle frequencies (10-15% of the forecast error variance in GDP and hours) and generates positive co-movement of GDP and hours. These properties are consistent with the sentiment shock being a transitory demand shock (Galí (1999); Canova and Nicoló (2002)). However, uncertainty is also an important source of fluctuations in the short-term, accounting for 3-7% of GDP and hours. News contributes 2% to GDP and 13-15% of hours, but accounts for a larger share of fluctuations in the long-term.

If sentiment and uncertainty shocks are truly structural, they should be orthogonal even when separately identified. I find that these two shocks are positively correlated, indicating that sentiment, as identified in Levchenko and Pandalai-Nayar (2018) and uncertainty, as identified in Caldara et al. (2016), may not be truly structural. I find evidence of a positive correlation between sentiment shocks and uncertainty, extracted from both macroeconomic and financial uncertainty proxies.

3.2. Literature Review

This paper contributes to the literature quantifying the contribution of sentiment, news, and uncertainty shocks to business cycle variation.

News, or information about the future total factor productivity, may affect the economy in the current period. Although innovations take time to have an impact in the economy, part of this technological impact is foreseen by the economic agents, who react to it in the present. These anticipated shocks provide a plausible narrative for business cycle fluctuations: good news about the future leads to an expansion, which is later reversed by less favorable outcomes. Theoretical foundations for the news-driven business cycles have been explored in Beaudry and Portier (2004) and Jaimovich and Rebelo (2009).

nomic activity, conditional on available information about current and future productivity, Levchenko and Pandalai-Nayar (2018) provide evidence to support this identification as a sentiment shock, and not a monetary policy, fiscal policy, or oil price shock.

Departures from the standard real business cycle framework, such as strong complementarities across sectors, preferences with low wealth effect on labor, varying capacity utilization or high labor supply elasticity, and high adjustments costs to capital are crucial for generating expected co-movements in macroeconomic aggregates.

The evidence on the empirical front has been mixed, as researchers disagree on the significance of news shocks as well as the direction in which it moves macroeconomic aggregates. As beliefs are unobservable, the quantification of news shocks relies on debatable identification strategies. While Cochrane (1994), Beaudry and Portier (2004), Beaudry and Portier (2006), Beaudry and Portier (2014), Kurmann and Otrok (2013), and Fève and Guay (2019) show that a significant component of business cycle fluctuations is comprised of news , Barsky and Sims (2011), Forni et al. (2014) find contradictory results. Blanchard et al. (2013), Levchenko and Pandalai-Nayar (2018), Schmitt-Grohe and Uribe (2012), and Christiano et al. (2014) provide evidence favoring noise, sentiment shocks, or alternative anticipated shocks, respectively, in explaining business cycle fluctuations. The difference in findings can also be attributed to the amount of forward looking data used in the VAR, which represents the information set that agents are assumed to hold.

Unlike news, which can be considered a fundamental shock, an exogenous change in the information technology of agents, shocks to sentiment are purely extrinsic source of fluctuations. While earlier theoretical work on sunspot equilibria emphasized the role of strategic complementarity or increasing returns to scale in production (Cooper and John (1988), Benhabib and Farmer (1994)), recent contributions highlight the importance of information frictions (Angeletos and La'O (2013) and Benhabib et al. (2015)).

Recent papers identify shocks that are interpreted as sentiments, in both VAR settings and fully specified DSGE models (Angeletos et al. (2014); Milani (2017); Nam and Wang (2018)). The procedure used to identify sentiments in this paper follows Levchenko and Pandalai-Nayar (2018), which complements previous contributions by separating a strictly non-technology expectations shock from the TFP news shock and by explaining variation only in an expectational variable. This strategy does not extract a shock that has a particular impact on the key macro aggregates by construction. Using a structural VAR, Levchenko and Pandalai-Nayar (2018) find a significant proportion of business cycle fluctuations to be driven by sentiment, a shock that best explains the short run forecast error variance of expected GDP, net of present and future TFP.

Finally, another strand of the literature on belief-driven business cycles considers the effect of uncertainty shocks (Bloom (2009)). Uncertainty, typically proxied by the volatility of economic variables, captures the unexpected changes to beliefs that agents form about the current and future state of the economy. As agents condition consumption and investment on beliefs, uncertainty can be a source of fluctuations. Theoretical contributions considering the role of uncertainty as a source of fluctuations include 'wait-and-see' effects (Bachmann et al. (2013)), confidence effects (Ilut and Schneider (2014)), growth options effects (Bloom (2014)), and the possibility of uncertainty traps (Fajgelbaum et al. (2017)).

The empirical evidence on the short-run negative effects of uncertainty shocks on economic activity using vector autoregressive models includes Bachmann et al. (2013), Jurado et al. (2015), Baker et al. (2016), Caldara et al. (2016), and Rossi et al. (2016). Recent developments in this literature focus on the effects of financial versus macroeconomic uncertainty (Carriero et al. (2018), Jurado et al. (2015)), and whether these changes arise exogenously or are endogenous responses to other economic movements (Ludvigson et al. (2015)).

The link between news shocks and uncertainty has been explored in Cascaldi-Garcia and Galvao (2018), while Forni et al. (2017), Berger et al. (2019) considers uncertainty as second order news shocks. Benhabib et al. (2015) and Angeletos and La'O (2013) provide a theoretical foundation for information frictions as a source of sentiment driven fluctuations. Proxies for uncertainty can represent aggregate noise, reducing the precision of signals that firms receive. As firms condition pricing or production on noisy signals, fluctuations driven by non-fundamental sources are possible.

In summary, the findings in the empirical literature regarding the contribution of news, sentiments, and surprise technology shocks rest on the following key issues (1) the amount of forward looking data used in the VAR (2) invertibility, or the ability of structural VARs to adequately recover shocks from economic models and (3) methodology for identifying shocks.

The identification of news depends crucially on the information set that agents are assumed to hold. A central idea in Beaudry and Portier (2006) is that financial variables are the type of variables most likely to reflect news. Stock prices, in particular, are clearly forward looking and responsive to expectations. Thus, to measure the contribution of news, financial variables are needed to assess the extent to which their innovations contain information about future technological growth.

Recent work has questioned the ability of structural VARs to recover shocks from economic models (Blanchard et al. (2013)). The problem of non-fundamentalness arises if news shocks cannot be expressed as linear combinations of the current and past observables that are in the agent's information set. Beaudry and Portier (2014) discuss how it may still be possible to approximatively recover news shocks, provided that the non-fundmentalness is not too serious. They illustrate how the addition of more variables to a VAR can mitigate the issue. Forni et al. (2014) address the invertibility issue in a structural VAR with factors extracted from a large data set.

The literature has considered several approaches to identifying uncertainty. Many econometric studies estimate the effects of uncertainty on economic variables by using structural VARs with a recursive identification scheme. This assumes that there is a causal relationship between uncertainty and economic variables and that uncertainty is exogenous.[3] Recursive schemes have the advantage of being simple to implement and interpret. However, the identification strategy is debatable, particularly when the direction of causality between uncertainty and economic activity is unclear. Ludvigson et al. (2015) show that these results can be biased by an endogeneity problem. Using an identification procedure based on external variable constraints, they conclude that macro uncertainty is mostly endogenous, while financial uncertainty is mostly an exogenous source of business cycle fluctuations.

Ludvigson et al. (2015) and Antolín-Díaz and Rubio-Ramírez (2018) consider event based sign restrictions which constrain the structural shocks around key historical events, to ensure that they agree with the established narratives about these episodes. However, this requires assumptions that some shocks play larger roles while others play smaller or negligible roles.

As in from Levchenko and Pandalai-Nayar (2018), Cascaldi-Garcia, Caldara et al. (2016), Uhlig (2005), Barsky and Sims (2011), and Kurmann and Otrok (2013), I identify structural shocks by maximizing the shock's contribution to the forecast error variance of a given proxy variable over a pre-specified forecast horizon. The common assumption in the news shock literature is that a small number of shocks lead to movements in aggregate technology. Levchenko and Pandalai-Nayar (2018) adapt this method for identifying sentiment shocks by assuming that a small number of shocks explain fluctuations in expectations of future economic activity. Thus, the identification strategy for news and sentiments represents a more agnostic approach, by not extracting a shock that has a particular impact on key macroeconomic variables. As a partial identification strategy, this approach can be used with a VAR containing any number of variables without additional assumptions.

In Section 3.3, I describe the identification strategy used to disentangle news, sentiment, and uncertainty and the estimation method. Section 3.4 provides a description of the data set (more details on the variables can be found in the Appendix). Section 3.5 presents empirical evidence on the correlation between sentiment and uncertainty shocks, and an analysis of the responses to sentiment, news, and uncertainty shocks. Section 3.6 concludes.

[3] It is assumed that uncertainty does not react contemporaneously to economic variables, while economic variables react contemporaneously to uncertainty.

3.3. Empirical Strategy

In this section, I describe how news, sentiment, and uncertainty shocks can be estimated using a VAR model and an identification strategy based on maximizing the forecast error variance decomposition of a target variable over a defined number of horizons. This identification is used by Barsky and Sims (2011) and Levchenko and Pandalai-Nayar (2018), and is related to the maximum forecast error variance approach in Uhlig (2005). The same reduced form VAR is used to identify news, sentiment, and uncertainty shocks. However, the matrices required for the identification of these shocks is estimated separately, as if we are interested in either news (Barsky and Sims (2011)), sentiment (Levchenko and Pandalai-Nayar (2018)), or uncertainty shocks (Caldara et al. (2016)).

Let Y_t denote a $k \times 1$ vector of observables, as listed in Table 3.6. The moving average representation of this VAR is

$$Y_t = B(L)u_t,$$

where u_t are reduced form residuals, L denotes the lag operator and $B(L)$ is the matrix of lag order polynomials. Assume there is a linear relationship between innovations (u_t) and structural shocks (ϵ_t)

$$u_t = A\epsilon_t,$$

where A is referred to as the impact matrix. This implies that the structural representation of the VAR is

$$Y_t = C(L)\epsilon_t,$$

where $C(L) = B(L)A$. If the structural shocks have unit variance, then the impact matrix satisfies

$$AA' = \Sigma,$$

where Σ is the variance-covariance matrix of the reduced form innovations ($\Sigma = VCV(u_t)$). The impact matrix A is not unique, as the entire space of permissible impact matrices can be written as $\tilde{A}D$, where D is orthonormal. The Choleski decomposition of Σ provides one such candidate for $\tilde{A}$.

Define the h-step ahead forecast error as

$$Y_{t+h} - \mathbb{E}_{t-1}Y_{t+h} = \sum_{\tau=0}^{h} B_\tau \tilde{A} D \epsilon_{t+h-\tau},$$

where B_τ is the reduced form matrix of lag τ moving average coefficients. The forecast error variance of variable i at horizon h is the sum of the contributions of the k structural shocks. Define $\Omega_{i,j}(h)$ as the contribution of shock j to the forecast error variance of variable i at horizon h,

$$\begin{aligned}\Omega_{i,j}(h) &= \frac{e_i'(\sum_{\tau=0}^{h} B_\tau \tilde{A} D e_j e_j' D' \tilde{A}' B_\tau')e_i}{e_i'(\sum_{\tau=0}^{h} B_\tau \Sigma B_\tau')e_i}, \\ &= \frac{\sum_{\tau=0}^{h} B_{i,\tau} \tilde{A} \gamma \gamma' \tilde{A}' B_{i,\tau}'}{\sum_{\tau=0}^{h} B_{i,\tau} \Sigma B_{i,\tau}'},\end{aligned}$$

where e_i represents selection vectors with one in the ith place and zeroes elsewhere. $A = \tilde{A}D$ is known as the impact matrix. If γ represents the jth column of D, then $\tilde{A}\gamma$ can be interpreted as an impulse vector, a $k \times 1$ vector corresponding to the jth column of a possible orthogonalization. The ith row of the matrix of moving average coefficients are denoted by $B_{i,\tau}$.

3.3.1. News shock

To identify the news shock, assume productivity is driven by two structural shocks, unanticipated TFP (ϵ_t^{sur}) and news about TFP (ϵ_{t-s}^{news}), which agents receive $s > 0$ periods in advance,

$$TFP_t = \lambda_1 \epsilon_t^{sur} + \lambda_2 \epsilon_{t-s}^{news}. \tag{3.1}$$

As only surprise TFP and news affect TFP, the forecast error variance of our proxy for TFP can be decomposed as follows (without loss of generality, assume TFP is ordered first in the VAR so that $i = 1$)

$$\Omega_{1,1}(h) + \Omega_{1,news}(h) = 1 \ \forall h.$$

In a multivariate VAR, this restriction is unlikely to hold for all horizons. Instead, identify the news shock by choosing parts of the impact matrix so that this expression holds over a finite subset of horizons. Define the news shock as the shock orthogonal to surprise technology shocks that best explains future unpredictable movements of utilization-adjusted TFP. More precisely, the news shock is a linear combination of the remaining reduced form innovations $k - 1$ that maximize the residual forecast error variance of TFP $(1 - \Omega_{1,1}(h))$ over a horizon H_{news}[4]. By identifying the reduced form

[4] H_{news} is typically set to 40Q (Barsky and Sims (2011), Kurmann and Otrok (2013), Levchenko and Pandalai-Nayar (2018), Forni et al. (2014)).

innovation in TFP as the first structural shock, $\Omega_{1,1}$ is fixed for all horizons h. As the contribution to the forecast error variance depends only on a single column of the impact matrix, this problem amounts to finding γ_{news} to solve the following optimization problem

$$\gamma_{news} = \arg\max_{\gamma} \sum_{h=0}^{H_{news}} \frac{\sum_{\tau=0}^{h} B_{1,\tau}\tilde{A}\gamma\gamma'\tilde{A}'B'_{1,\tau}}{\sum_{\tau=0}^{h} B_{1,\tau}\Sigma B'_{1,\tau}},$$

subject to

$$D(1,i) = 0 \ \forall i \neq 1, \tag{3.2}$$

$$D \text{ is orthonormal.} \tag{3.3}$$

The first condition specifies that none of the $k-1$ structural shocks has a contemporaneous effect on TFP, while the second condition ensures that the structural shocks are orthogonal to each other. As the lower triangular matrix $\tilde{A}$ is the Choleski decomposition, $\tilde{A}(1,m) = 0$ for all $m > 1$.

Uhlig (2005) rewrites the maximization problem in quadratic form in which the non-zero portion of γ corresponds to the eigenvector associated with the maximum eigenvalue of a weighted sum of the $(k-1) \times (k-1)$ submatrix of $(B_{1,\tau}\tilde{A})'(B_{1,\tau}\tilde{A})$ over τ. In other words, the impact vector for the news shock is the first principal component of observable TFP orthogonal to its own innovation.

As identification is based on the forecast error variance, the signs of the identified shocks might switch with a change in the VAR parameters. To ensure the identification of a positive news shock, I check that the response of utilization-adjusted total factor productivity is positive after 40 quarters. If the response is negative, all computed responses are multiplied by (-1). Similarly, in the case of sentiment (uncertainty) shocks, I check whether the shock has a positive contemporaneous impact on the sentiment (uncertainty) measure and multiply the responses by (-1) if they are negative.

3.3.2. Sentiment shock

To identify the sentiment shock, assume that expectations of future economic activity are driven by surprise innovation in TFP, news about TFP, as well as sentiment (ϵ_t^{sent}). Let F_t represent expectations about GDP, which can be proxied by professional forecasts of GDP or consumer confidence,

$$F_t = \lambda_1^F \epsilon_t^{surp} + \lambda_2^F \epsilon_{t-s}^{news} + \lambda_3^F \epsilon_t^{sent} + \zeta_t. \tag{3.4}$$

By equation (3.4), expectations of better future economic conditions are attributable to either news of high future TFP, or to positive confidence. In order to extract a non-technology shock from data on expectations, it is important to control for news of future productivity. A VAR with utilization-adjusted TFP and expectations must be augmented by other forward-looking macroeconomic aggregates (for example, stock prices, as in Beaudry and Portier (2006)) in order to identify the three shocks of interest. Although rational, forward-looking agents may also respond to other changes in the economy that affect GDP, assume that these three shocks account for the bulk of the variation in expectations of future activity.

Let the subscript $i = F_t$ denote the position of variable F_t in the $k \times 1$ vector of observables Y_t. Holding fixed the identification of news and surprise TFP ($\Omega_{F_t,surp}(h)$ and $\Omega_{F_t,news}(h)$), the share of forecast error variance of sentiment is computed as

$$\Omega_{F_t,sent}(h) = \frac{\sum_{\tau=0}^{H^{sent}} B_{F_t,\tau} \tilde{A} \gamma_{sent} \gamma_{sent}' \tilde{A}' B_{F_t,\tau}'}{\sum_{\tau=0}^{H^{sent}} B_{F_t,\tau} \Sigma B_{F_t,\tau}'},$$

where $H_{sent} = 2Q$, reflecting the short run effects of a sentiment shock. The sentiment shock is the linear combination of the remaining $k - 2$ innovations that maximizes the forecast error variance of F_t,

$$\gamma_{sent} = \arg\max_{\gamma} \sum_{\tau=0}^{H^{sent}} \Omega_{F_t,sent}(h)$$

subject to

$$D(1, i) = 0 \ \forall i \neq 1, \tag{3.5}$$

$$D \text{ is orthonormal}, \tag{3.6}$$

$$D(:, 2) = \gamma_{news}. \tag{3.7}$$

As before, the first restriction indicates that none of the $k-1$ structural shocks has a contemporaneous impact on TFP and the second restriction specifies the orthogonality of the structural shocks. The final restriction conditions the identification of the sentiment shock on the fixed identification of the news shock. Thus, the sentiment shock captures residual variance of the forecast of GDP, once surprise TFP and news are accounted for.

3.3.3. Uncertainty shock

Define the uncertainty shock as the shock that best explains future unpredictable movements of an observable proxy for uncertainty (U_t). Let the subscript $i = U_t$ denote the position of variable U_t in the $k \times 1$ vector of observables Y_t. As in the previous sections, the uncertainty shock is identified by maximizing the forecast error variance decomposition of uncertainty over the short term ($H^{unc} = 2Q$, as in Caldara et al. (2016)),

$$\Omega_{U_t,unc}(h) = \frac{\sum_{\tau=0}^{H^{unc}} B_{U_t,\tau} \tilde{A} \gamma_{unc} \gamma'_{unc} \tilde{A}' B'_{U_t,\tau}}{\sum_{\tau=0}^{H^{unc}} B_{U_t,\tau} \Sigma B'_{U_t,\tau}}.$$

Uncertainty shocks are the linear combination γ_{unc} of the reduced form innovations u_t that maximizes the short-term unexpected variation of uncertainty. In contrast with to the identification of news and sentiment shocks, no additional restrictions are imposed.

$$\gamma_{unc} = \arg\max \sum_{\tau=0}^{H^{unc}} \Omega_{U_t,unc}(h)$$

3.4. Data

The dataset contains 13 quarterly variables measured in log levels to allow for the possibility of co-integration. For variables which are available at a higher frequency, I take the quarterly average. Included are macroeconomic variables that are usually considered in the news shock literature: utilization-adjusted TFP, personal consumption expenditures per capita, GDP per capita, private domestic investment per capita, hours worked, and price deflator. All of the macroeconomic series are obtained in quarterly frequency from the FRED database of the Federal Reserve Bank of St. Louis. The time period for the core, macroeconomic variables spans 1973Q1 to 2018Q4. Alternate measures of macroeconomic and financial uncertainty are incorporated and include the measures computed by Ludvigson et al. (2015), S&P100 futures index implied volatility, policy uncertainty as computed by Baker et al. (2016), forecast dispersion as in Bachmann et al. (2013) and the Survey of Professional Forecasters. Not all uncertainty measures are available for the same duration as the core variables (see Tables 3.8 and 3.9).

The identification of news, sentiment, and technology shocks in a VAR relies on an accurate measure of productivity. Following Beaudry and Portier (2006) and Barsky and Sims (2011), this is measured by the quarterly, utilization-adjusted TFP series from Fernald (2014). The series is the quarterly version of the annual series developed by Basu et al. (2006), which exploits first-order conditions from a firm optimization problem to correct for unobserved factor utilization and is thus preferable to a simple Solow residual measure for exogenous TFP. Accounting for measurement issues arising from changes in capital and labor utilization is crucial. Basu et al. (2006) find that the detrended utilization-adjusted TFP is both less correlated with output, and less volatile than the standard Solow residual. Although the annual series in Basu et al. (2006) allows for non-constant returns to scale, the industry-level data required for controlling for non-constant returns to scale are not available quarterly, so the Fernald (2014) series corrects only for variable capital and labor utilization.

For consumption, I use personal consumption expenditures in non-durable goods and services. Output is measured by non-farm, business GDP. Investment is given by real gross private domestic investment. These series are in real terms, chain-weighted by 2012 dollars. Hours are computed with total hours of wage and salary workers on non-farm payrolls. Prices are measured by the implicit price deflator in the non-farm business sector. All variables are converted to per-capita terms using the series for civilian non-institutional population, aged 16 and older.

Financial variables include the S&P500 stock price index, excess bond premium, federal funds rate, spread between the 10-year Treasury rate and the Federal funds rate. Non-quarterly data are converted to quarterly frequency by computing arithmetic averages over the appropriate time intervals. The S&P 500 composite index, deflated by the consumer price index and in per-capita terms is a forward looking variable required for the identification of a news shock (Beaudry and Portier (2006)). The 10-year Treasury spread and the excess bond premium are measures of credit conditions. The excess bond premium is a component of corporate bond credit spreads that is not directly attributable to expected default risk. It provides an effective measure of investor sentiment or risk appetite in the corporate bond market (Gilchrist and Zakrajšek (2012)).

Sentiment is proxied by the University of Michigan Survey of Consumers, series E12Y (bexp_r_all), constructed from the response to the question: *And how about a year from now, do you expect that in the country as a whole, business conditions will be better, or worse than they are at present, or just about the same?*. One quarter ahead growth rate forecasts of US GDP from the Survey of Professional Forecasters (SPF) serve as an alternate measure of sentiment. The short time horizon of forecasts is chosen to reflect the transitory nature of shocks to sentiment.

Finally, various alternative measures of macroeconomic and financial uncertainty are considered. Macroeconomic uncertainty measures are typically related to the forecasting uncertainty of macroeconomic variables (Bloom (2014) and Ilut and Schneider (2014)), such as real GDP. Financial uncertainty variables are measures of equity markets volatility. The macroeconomic and financial uncertainty indices proposed by Jurado et al. (2015) are based on implied forecast errors for real economic activity derived from a factor model that using hundreds of economic and financial series. Although policy uncertainty and business uncertainty are not typical macroeconomic uncertainty measures, they represent uncertainty surrounding macroeconomic developments. The index of economic policy uncertainty developed by Baker et al. (2016) captures the frequency of words in major U.S. newspapers associated with uncertainty regarding economic policy. As in Bachmann et al. (2013), I consider a measure of forecast dispersion constructed using the Philadelphia Fed's Business Outlook Survey.

Estimation is taken over four lags of each variable, an intercept term, but no time trend. Due to the large number of coefficients, Bayesian methods are used, along with Minnesota prior on the estimation and take draws from the posterior to compute error bands. Results are robust to estimating a VAR with OLS instead and computing error bands by bootstrapping from the estimated VAR.

3.5. Results

In this section, I discuss the impulse response functions for news, sentiment, and uncertainty shocks identified and estimated one at a time by maximizing the forecast error variance decomposition of the corresponding proxy variable. In the case of a news shock, the maximization is over the long term (10 years). As in Levchenko and Pandalai-Nayar (2018) and reflecting the transitory nature of news shocks and uncertainty shocks, maximization is over 2 quarters.

Figure 3.2 shows the responses of macroeconomic variables to a news shock. The shaded gray areas are one standard error confidence bands computed with 1,000 posterior draws. Following the identified news shock, output and investment decline on impact, while consumption increases. The results on impact are not significantly different from zero, as indicated by the confidence bands. All variables continue to increase to a peak 10 - 12 quarters later. Hours increase on impact, and continue this trajectory until 10 quarters later, which is consistent with a VAR that includes forward-looking variables, such as stock prices (Beaudry and Portier (2014)). The dynamic paths of these variables largely track the estimated path of TFP, which increases slowly before declining at a peak of 20 quarters. Consistent with the results in Beaudry and Portier (2006) and Barsky and Sims (2011), stock prices and consumer confidence rise significantly on impact. Macroeconomic uncertainty decreases, but confidence bands cover zero.

In response to a surprise technology shock (Figure 3.1), utilization adjusted TFP increases on impact and decays slowly. By construction, the TFP shock does not affect the other variables on impact. Investment and output fall in the short term but increase again after 2 and 5 quarters, respectively. Consumption is constant on impact, with the response peaking 15 quarters ahead. Consumer sentiment, hours and stock prices increase steadily to a peak at 5 quarters, 12 quarters, and 15 quarters, respectively. Uncertainty exhibits the opposite trend, by falling to a low at 8 quarters, then increasing steadily.

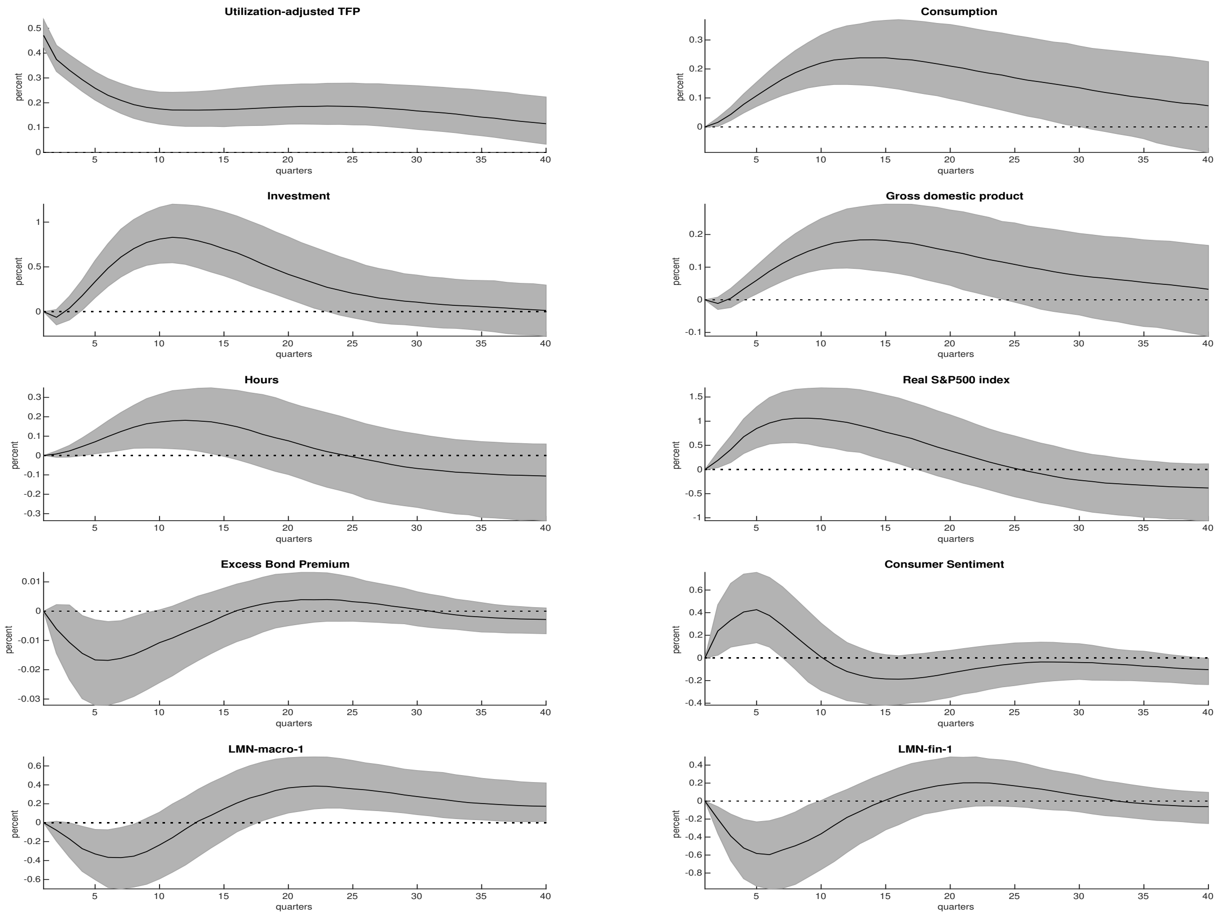
The impulse responses to a sentiment shock (Figure 3.3) are noticeably different from the news shock. Following a positive shock to sentiment, output, investment, and uncertainty increase on impact and decay rapidly, falling below zero between 2 and 7 quarters, consistent with the interpretation of sentiment as a transitory demand shock. Stock prices, hours, and sentiment increase on impact in response to sentiment, similar to the response to a news shock. However, the response declines shortly following the impact of the sentiment shock.

Tables 3.1-3.3 report the share of the forecast error variances of the macroeconomic aggregates accounted for by TFP, news, and sentiment shocks, respectively. At short frequencies (horizons 1 year or less), the sentiment shock accounts for 6-13% of the variation in GDP and 7-14% of the variation in hours. By contrast, at these frequencies, surprise TFP shocks explain almost none of the variation in GDP, consumption, and in hours. The news shock does a little bit better for GDP (2-6%), consumption (2-12%), and hours (13-15%). At longer horizons the news shock increases in importance. Barsky and Sims (2011) reach a qualitatively similar conclusion about the effect of news and surprise TFP shocks, while attributing most of the short-run variation in aggregate variables to unexplained shocks. Macroeconomic uncertainty accounts for 3-15% of the variation in GDP, 6-14% in consumption, and 3-15% in hours in the short run.

Note that the identification strategy only imposes that the sentiment shock has no effect on true TFP on impact. By selecting the news TFP shock that best explains variation in the TFP series, the procedure minimizes the impact of sentiment shock (as well as of the other structural shocks). However, if the surprise and news TFP shocks do not sufficiently account for the forecast error variance of the TFP series, there is room for other shocks to drive TFP (see Table 3.3).

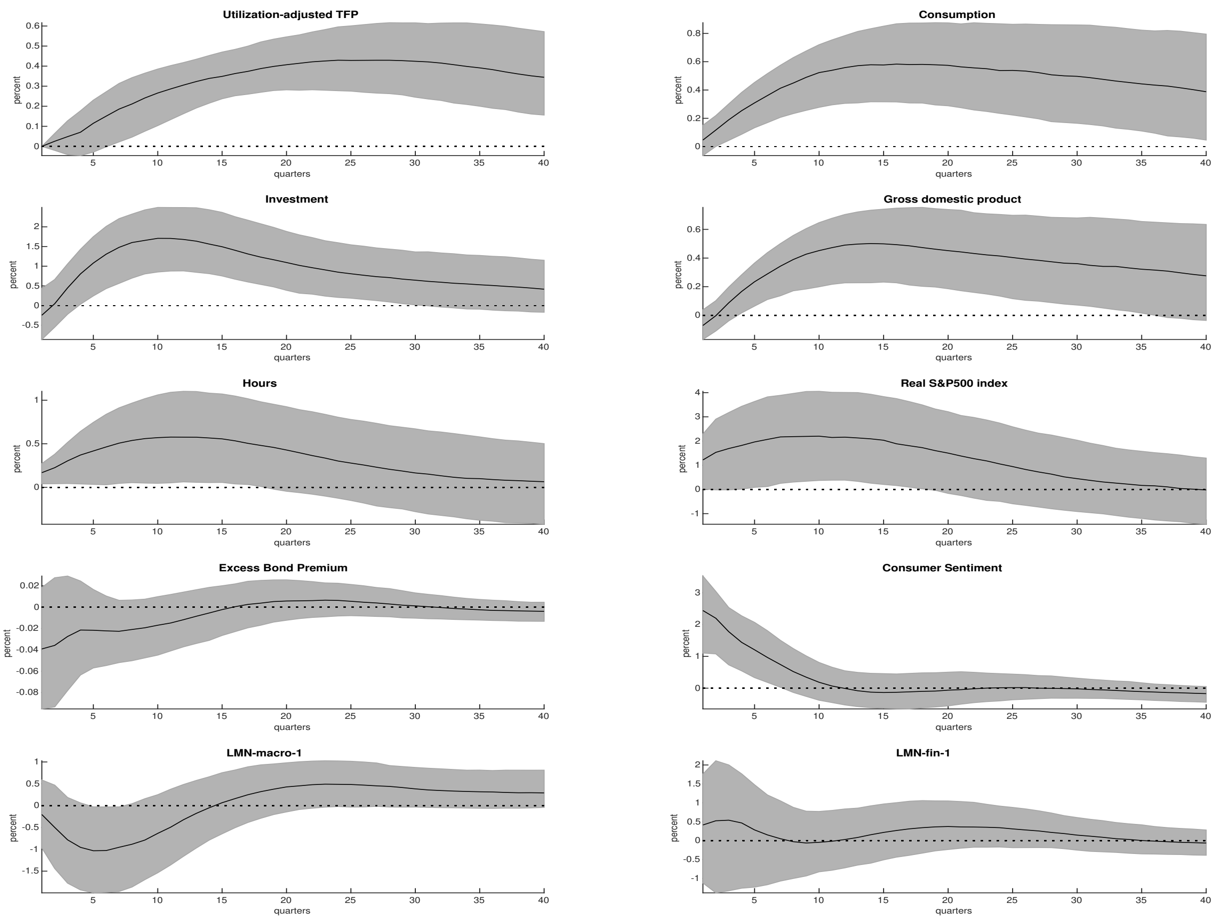
The main result from Table 3.5 is that there is a positive and significant correlation between sentiment and various measures of macroeconomic and financial uncertainty shocks. This finding indicates that these are not truly structural shocks, implying that when estimated separately as in Levchenko and Pandalai-Nayar (2018) and Caldara et al. (2016), their contribution to business cycle variation may be biased.

Figure 3.1: IRFs to Surprise TFP Shock



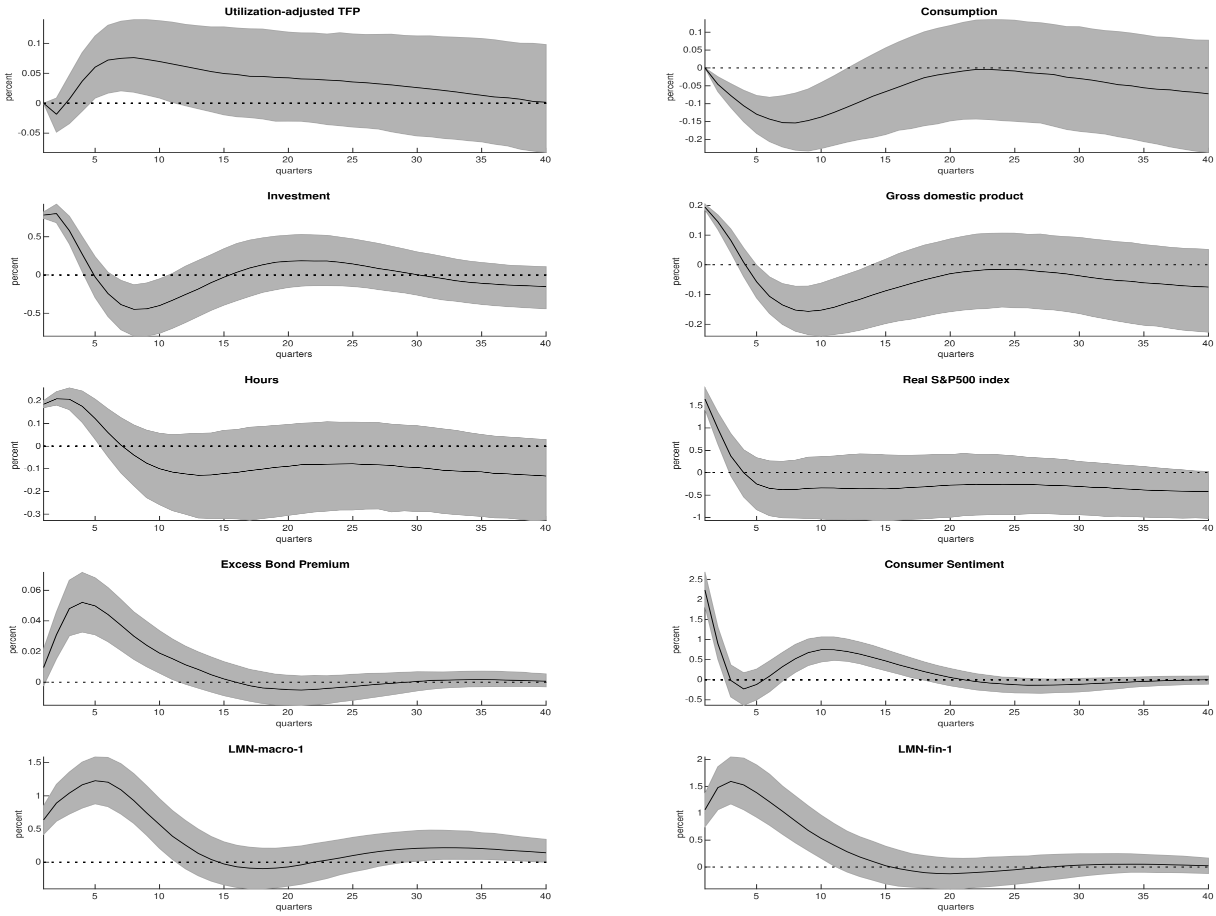
Note: Shaded areas describe 68% confidence intervals computed with 1,000 posterior draws. The baseline identification scheme for news shocks is described in Section 3.3. The VAR model includes all variables in the first panel of Table 3.8, a proxy for macroeconomic uncertainty (LMN-macro-1), and a proxy for financial uncertainty (LMN-fin-1). Sample period: 1973Q1 to 2018Q4.

Figure 3.2: IRFs to News Shock



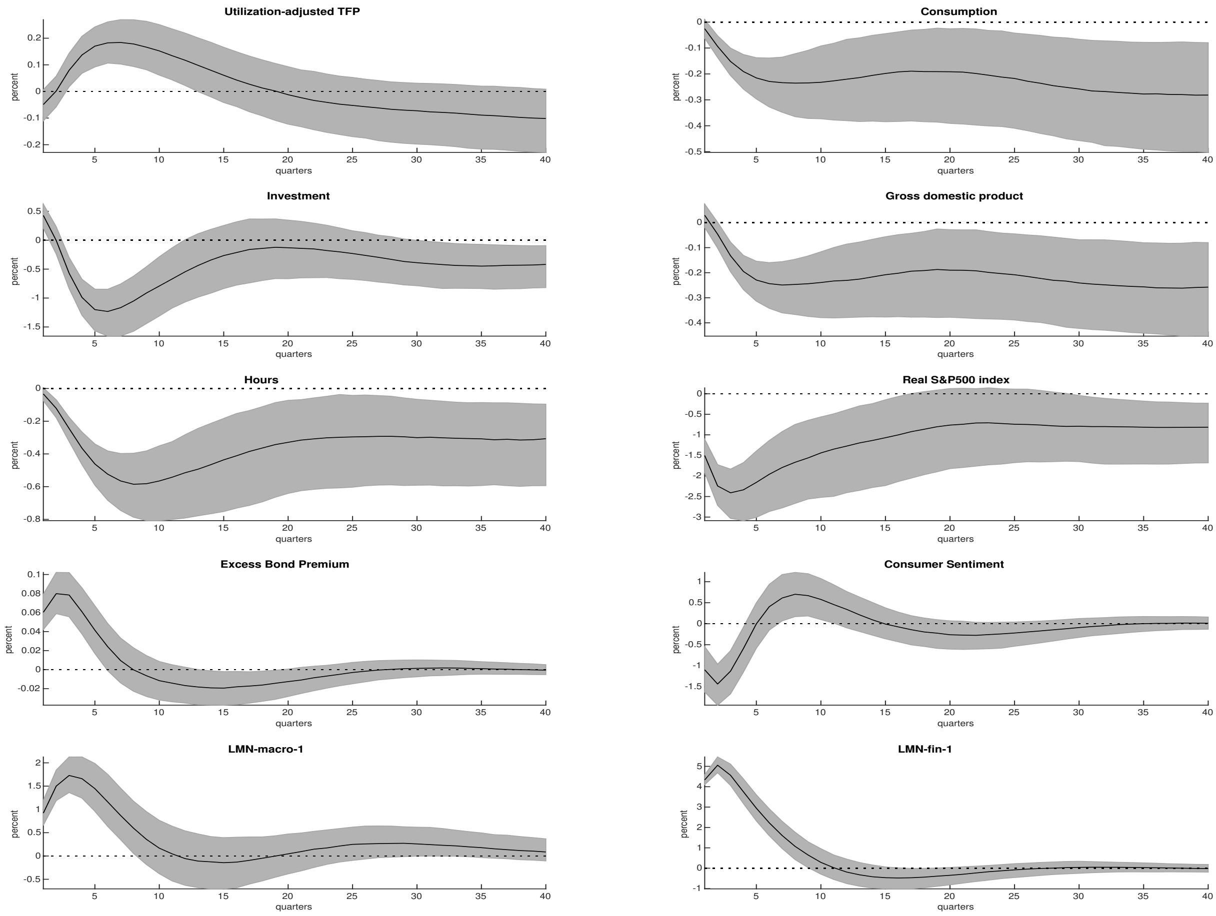
Note: Shaded areas describe 68% confidence intervals computed with 1,000 posterior draws. The baseline identification scheme for news shocks is described in Section 3.3. The VAR model includes all variables in the first panel of Table 3.8 and a proxy for macroeconomic uncertainty (LMN-macro-1), and a proxy for financial uncertainty (LMN-fin-1). Sample period: 1973Q1 to 2018Q4.

Figure 3.3: IRFs to Sentiment Shock



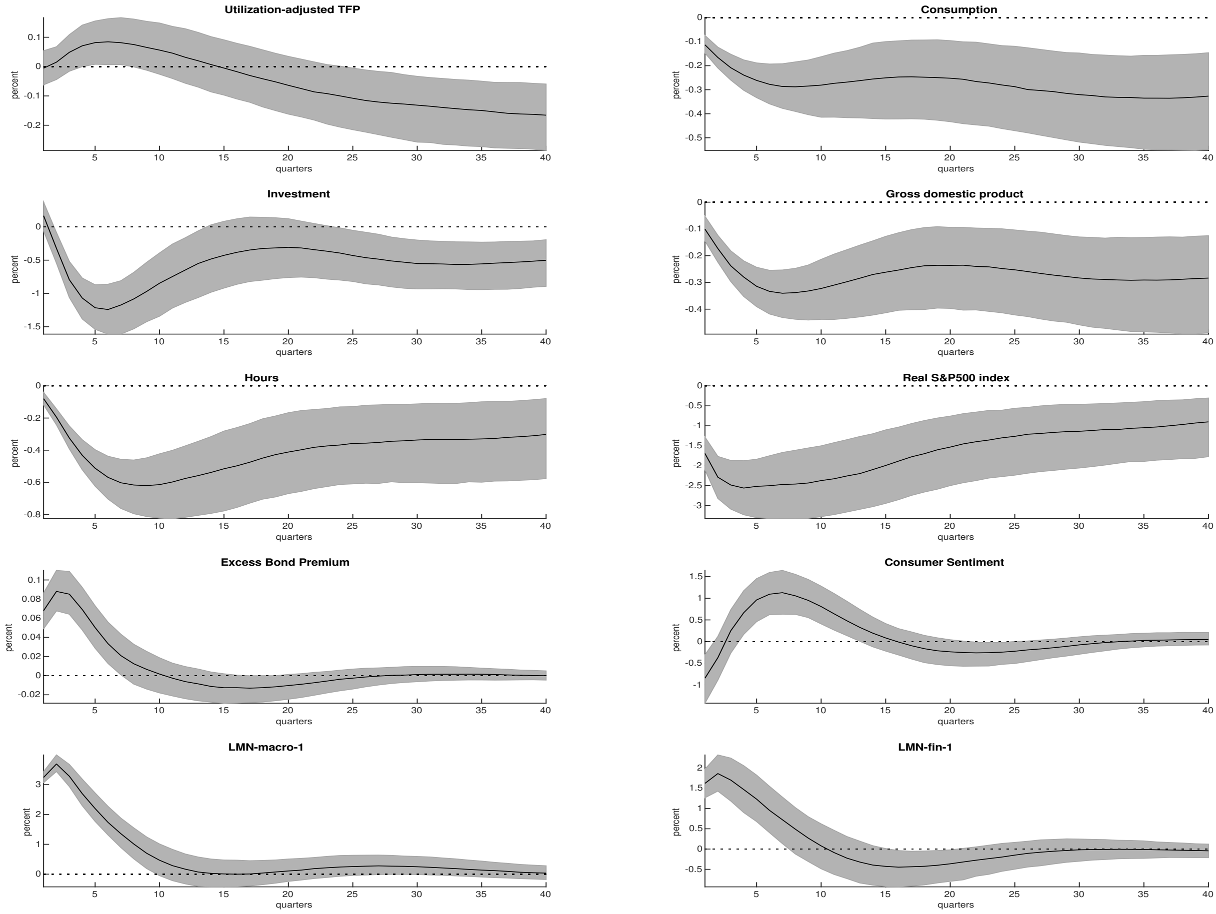
Note: Shaded areas describe 68% confidence intervals computed with 1,000 posterior draws. The baseline identification scheme for sentiment shocks is described in Section 3.3. The VAR model includes all variables in the first panel of Table 3.8 and a proxy for macroeconomic uncertainty (LMN-macro-1), and a proxy for financial uncertainty (LMN-fin-1). Sample period: 1973Q1 to 2018Q4.

Figure 3.4: IRFs to Financial Uncertainty Shock



Note: Shaded areas describe 68% confidence intervals computed with 1,000 posterior draws. The baseline identification scheme for financial uncertainty shocks is described in Section 3.3. The VAR model includes all variables in the first panel of Table 3.8 and a proxy for macroeconomic uncertainty (LMN-macro-1), and a proxy for financial uncertainty (LMN-fin-1). Sample period: 1973Q1 to 2018Q4.

Figure 3.5: IRFs to Macro Uncertainty Shock



Note: Shaded areas describe 68% confidence intervals computed with 1,000 posterior draws. The baseline identification scheme for macroeconomic uncertainty shocks is described in Section 3.3. The VAR model includes all variables in the first panel of Table 3.8 and a proxy for macroeconomic uncertainty (LMN-macro-1), and a proxy for financial uncertainty (LMN-fin-1). Sample period: 1973Q1 to 2018Q4.

Table 3.1: Surprise TFP Shock: Variance Decomposition

Horizon	TFP	GDP	Consumption	Hours	Forecast
1 Q	1.00	0.00	0.00	0.00	0.00
2 Q	0.98	0.00	0.00	0.00	0.00
4 Q	0.90	0.00	0.02	0.00	0.01
8 Q	0.71	0.03	0.08	0.01	0.02
20 Q	0.50	0.09	0.15	0.03	0.03
40 Q	0.36	0.07	0.10	0.04	0.04

Note: The baseline identification scheme is described in section 3.3. The VAR model includes all variables in the first panel of Table 3.8 and a proxy for macroeconomic uncertainty (LMN-macro-1), and a proxy for financial uncertainty (LMN-fin-1). Sample period: 1973Q1 to 2018Q4.

Table 3.2: News Shock: Variance Decomposition

Horizon	TFP	GDP	Consumption	Hours	Forecast
1 Q	0.00	0.02	0.02	0.13	0.14
2 Q	0.00	0.02	0.05	0.14	0.16
4 Q	0.01	0.06	0.12	0.15	0.19
8 Q	0.07	0.18	0.27	0.18	0.20
20 Q	0.33	0.34	0.44	0.22	0.18
40 Q	0.48	0.31	0.40	0.19	0.18

Note: The baseline identification scheme is described in section 3.3. The VAR model includes all variables in the first panel of Table 3.8 and a proxy for macroeconomic uncertainty (LMN-macro-1), and a proxy for financial uncertainty (LMN-fin-1). Sample period: 1973Q1 to 2018Q4.

Table 3.3: Sentiment Shock: Variance Decomposition

Horizon	TFP	GDP	Consumption	Hours	Forecast
1 Q	0.00	0.13	0.00	0.14	0.12
2 Q	0.00	0.10	0.00	0.12	0.09
4 Q	0.00	0.06	0.02	0.07	0.07
8 Q	0.01	0.04	0.03	0.03	0.06
20 Q	0.02	0.03	0.02	0.02	0.08
40 Q	0.02	0.02	0.02	0.02	0.08

Note: The baseline identification scheme is described in section 3.3. The VAR model includes all variables in the first panel of Table 3.8 and a proxy for macroeconomic uncertainty (LMN-macro-1), and a proxy for financial uncertainty (LMN-fin-1). Sample period: 1973Q1 to 2018Q4.

Table 3.4: Macro Uncertainty Shock: Variance Decomposition

Horizon	TFP	GDP	Consumption	Hours	Forecast
1 Q	0.00	0.03	0.06	0.03	0.02
2 Q	0.00	0.07	0.10	0.07	0.01
4 Q	0.01	0.15	0.14	0.15	0.02
8 Q	0.02	0.22	0.16	0.21	0.06
20 Q	0.03	0.16	0.12	0.20	0.07
40 Q	0.06	0.15	0.13	0.16	0.08

Note: The baseline identification scheme is described in section 3.3. The VAR model includes all variables in the first panel of Table 3.8 and a proxy for macroeconomic uncertainty (LMN-macro-1), and a proxy for financial uncertainty (LMN-fin-1). Sample period: 1973Q1 to 2018Q4.

Table 3.5: Financial Uncertainty Shock: Variance Decomposition

Horizon	**TFP**	**GDP**	**Consumption**	**Hours**	**Forecast**
1 Q	0.01	0.00	0.01	0.01	0.03
2 Q	0.01	0.01	0.03	0.03	0.05
4 Q	0.03	0.06	0.07	0.10	0.06
8 Q	0.07	0.11	0.10	0.18	0.06
20 Q	0.06	0.09	0.07	0.15	0.07
40 Q	0.06	0.09	0.08	0.12	0.07

Note: The baseline identification scheme is described in section 3.3. The VAR model includes all variables in the first panel of Table 3.8 and a proxy for macroeconomic uncertainty (LMN-macro-1), and a proxy for financial uncertainty (LMN-fin-1). Sample period: 1973Q1 to 2018Q4.

Table 3.6: Correlation between News and Uncertainty Shocks for Different Uncertainty Proxies

	Correlation	
Financial uncertainty		
Realized volatility		
LMN-fin-1	0.05	[0.000]
LMN-fin-3	0.05	[0.000]
LMN-fin-12	0.06	[0.000]
VXO	0.00	[0.270]
Macro uncertainty		
Policy uncertainty	-0.00	[0.746]
Business uncertainty	-0.21	[0.000]
SPF disagreement	-0.08	[0.000]
LMN-macro-1	-0.11	[0.000]
LMN-macro-3	-0.12	[0.000]
LMN-macro-12	-0.15	[0.000]

Note: Values in brackets are p-values for the test of zero correlation under the null hypothesis ($H_0 : \rho = 0$). These results are computed for a reduced-form VAR model with all variables in the first panel of Table 3.8 and one proxy for uncertainty at a time. Identification schemes are described in section 3.3. The sample period is from 1973Q1 to 2018Q4, unless indicated (see Table 3.9 for details).

Table 3.7: Correlation between Sentiment and Uncertainty Shocks for Different Uncertainty Proxies

	Correlation	
Financial uncertainty		
Realized volatility		
LMN-fin-1	0.14	[0.000]
LMN-fin-3	0.20	[0.000]
LMN-fin-12	0.05	[0.000]
VXO	-0.01	[0.000]
Macro uncertainty		
Policy uncertainty	-0.07	[0.000]
Business uncertainty	0.21	[0.000]
SPF disagreement	0.20	[0.000]
LMN-macro-1	0.21	[0.000]
LMN-macro-3	0.23	[0.000]
LMN-macro-12	0.28	[0.000]

Note: Values in brackets are p-values for the test of zero correlation under the null hypothesis ($H_0 : \rho = 0$). These results are computed for a reduced-form VAR model with all variables in the first panel of Table 3.8 and one proxy for uncertainty at a time. Identification schemes are described in section 3.3. The sample period is from 1973Q1 to 2018Q4, unless indicated (see Table 3.9 for details).

3.6. Conclusion

Sentiments and uncertainty have both been considered as potential sources of short-run fluctuations. Separately identified, they have been shown to contribute significantly to co-movement and fluctuations in GDP and hours (Levchenko and Pandalai-Nayar (2018), Angeletos et al. (2014), Caldara et al. (2016)). As shocks to expectations about economic activity, orthogonal to news, sentiment may affect measures of confidence, or uncertainty. Motivated by recent theories of aggregate fluctuations arising from s to agents' expectations in incomplete information settings, I consider the link between the two types of shocks.

Using a vector autoregression with forward looking data, I extract the shocks that best explain the forecast error variance of proxies for sentiment and uncertainty, respectively. Sentiment shocks are shown to explain more variation in macroeconomic aggregates than news. However, sentiments account for less variation than previous studies have shown, as uncertainty is also an important source of short-run fluctuations. There is also evidence that sentiment and uncertainty shocks are correlated.

Data Appendix

Table 3.8: Description of Core Variables

	Name	Description	Source
1	Utilization adjusted TFP	Utilization adjusted TFP in log levels.	Fernald's website (June 2019)
2	Consumption	Real personal consumption expenditures per capita in log levels. Computed with real PCE (non-durable goods and services) and population.	FRED
3	Output	Real per capita GDP in log levels. Computed using the real GDP (non-farm business) and population.	FRED
4	Investment	Real per capita investment in log levels. Computed using real gross private domestic investment and population.	FRED
5	Hours	Per capita hours in log levels. Computed with total hours in non-farm business sector and population values.	FRED
6	Prices	Price deflator, computed with the implicit price deflator for non-farm business sector.	FRED
7	S&P stock price index	Shiller's real S&P composite stock price index, CPI deflated and per capita	Shiller's website (July 2019)
8	EBP	Excess bond premium as computed by Gilchrist and Zakrajšek (2012).	Gilchrist's website (June 2019)
9	FFR	Effective federal funds rate.	FRED
10	Spread	Difference between the 10-year Treasury rate and the FFR.	FRED
11	Sentiment	University of Michigan Consumer Sentiment Index	

Per-capita variables are adjusted by the Bureau of Labor Statistics' Civilian Non-institutional Population (CNP16OV), Source: FRED. The range for all variables is from 1973Q1 through 2018Q4 except when noted. Monthly series converted to quarterly by averaging.

Table 3.9: Description of Uncertainty Proxies

		Financial Uncertainty Measures	
1	VXO	S&P100 futures index option-implied volatility. Available from 1986Q1.	Chicago Board Options Exchange (CBOE)
2	LMN-fin-1	Financial forecasting uncertainty computed by Jurado et al. (2015), 1 month ahead	Ludvigson's website
3	LMN-fin-3	Financial forecasting uncertainty computed by Jurado et al. (2015), 3 months head	Ludvigson's website
4	LMN-fin-12	Financial forecasting uncertainty computed by Jurado et al. (2015), 1 year ahead	Ludvigson's website
		Macroeconomic Uncertainty Measures	
1	Policy uncertainty	Economic Policy Uncertainty Index in logs computed by Baker et al. (2016). Available from 1985Q1.	Bloom's website
2	Business uncertainty	Business forecasters dispersion computed by Bachmann et al. (2013). Available up to 2011Q4.	AER website
3	SPF disagreement	SPF forecasters dispersion on one-quarter-ahead real GDP quarterly growth forecasts, interdecile range.	Federal Reserve Bank of Philadelphia
4	LMN-macro-1	Macro forecasting uncertainty computed by Jurado et al. (2015), 1 month ahead	Ludvigson's website
5	LMN-macro-3	Macro forecasting uncertainty computed by Jurado et al. (2015), 3 months ahead	Ludvigson's website
6	LMN-macro-12	Macro forecasting uncertainty computed by Jurado et al. (2015), 1 year ahead	Ludvigson's website

The range for all variables is from 1973Q1 through 2018Q4 except when noted. Monthly series converted to quarterly by averaging.

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