Essays on Monetary Policy and Labor Markets

Author: Sebastian Diz

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THESIS SUPERVISOR
Davide Debortoli i Jordi Galí
Department d'Economia i Empresa

To my parents.

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Abstract

This thesis consists of three self-contained essays. The first chapter investigates the redistributive role minimum wages can play over the cycle and the implied effects on macroeconomic stability. To this end, I develop a two-agent model featuring idiosyncratic uncertainty and limited asset markets participation. I find that the minimum wage has the potential to redistribute against the minimum wage earners during an economic decline due to employment losses it originates. In addition to its detrimental effects on low income individuals' welfare, redistribution can have a quantitatively relevant impact on spending, and hence, on the severity of recessions. The second chapter explores the optimal design of monetary policy in a multisector model where agents' preferences are characterized by sector specific minimum consumption requirements. We find that this specification of preferences alters the optimal measure of inflation that the monetary authority should target. The third chapter studies wage flexibility as a means to absorb adverse shocks. Our focus is on economies with unequal access to financial markets and where the monetary authority is constrained by the zero lower bound. We show that in this particular setting the economy becomes more volatile when wages are less rigid, and hence, the usual recommendation of making labor markets more flexible to restore high output levels is misleading.

Resumen

Esta tesis se compone de tres ensayos independientes. El primer capítulo investiga el papel redistributivo que pueden desempeñar los salarios mínimos durante el ciclo y los consecuentes efectos sobre la estabilidad macroeconómica. Con este fin, desarrollo un modelo de dos agentes que incorpora riesgo idiosincrásico y participación limitada en los mercados financieros. Encuentro que el salario mínimo tiene el potencial de redistribuir en contra de quienes lo perciben durante un declive económico debido a las pérdidas de empleo que ocasiona. Adicionalmente a sus efectos negativos sobre el bienestar de trabajadores de bajos ingresos, la redistribución de ingresos puede tener un impacto cuantitativamente relevante sobre el gasto y, por lo tanto, sobre la severidad de las recesiones. El segundo capítulo explora el diseño óptimo de la política monetaria en un modelo multisectorial donde las preferencias de los agentes se caracterizan por incorporar requisitos mínimos de consumo específicos para cada sector. Encontramos que esta especificación de las preferencias altera la medida óptima de inflación que la autoridad monetaria debe estabilizar. El tercer capítulo estudia la flexibilidad salarial como un medio para absorber shocks adversos. Nos enfocamos en economías con acceso limitado a los mercados financieros y donde la autoridad monetaria está limitada por encontrarse la tasa de política en cero. Mostramos que en este contexto la economía se torna más volátil cuando los salarios son menos rígidos y, por lo tanto, la recomendación habitual de flexibilizar mercados laborales para restaurar los altos niveles de producción resulta inadecuada.

Preface

This thesis consists of three self-contained essays. The first chapter investigates the redistributive role minimum wages can play over the cycle and the implied effects on macroeconomic stability. To this end, I develop a two-agent model featuring idiosyncratic uncertainty and limited asset markets participation. In my framework, the minimum wage constraint binds only for a subset of the workforce, which is thus subject to stronger wage rigidities. The latter implies that the minimum wage constraint alters the dynamics of relative sectoral wages (i.e. the wage gap between the minimum wage earners sector and the rest of the workforce), and thus, that of relative sectoral labor earnings, that is, it constitutes a source of income redistribution over the cycle. Such resource redistribution, in interaction with financial frictions, influence aggregate spending and thus, economic activity. I find that under plausible calibrations of the model, the minimum wage has the potential to redistribute against the minimum wage earners during an economic decline due to employment losses it originates. In addition to its detrimental effects on low income individuals' welfare, redistribution can have a quantitatively relevant impact on spending, and hence, on activity. In particular, economic downturns can deepen, as a consequence of redistribution of current earnings against low income households, characterized by higher propensities to consume, and heightened uncertainty, which further depress aggregate demand by raising desired precautionary savings.

The second chapter, co-authored with Cesar Blanco, explores the optimal design of monetary policy in a multisector model where agents' preferences are non-homothetic. Non-homotheticity derives from the existence of a minimum consumption requirement for agricultural goods that households need to satisfy for subsistence. We find that the introduction of a minimum consumption requirement alters the optimal measure of inflation that the monetary authority should target. More precisely, non-homotheticity results in a reduced weight on agricultural inflation in the optimal index. On the one hand, under this specification of preferences stabilizing inflation in this sector turns more costly, as it requires larger deviations of output from the efficient level. In addition, proximity to the subsistence level implies a low income elasticity for agricultural goods. This translates into a reduced slope on aggregate output in the Phillips curve for agriculture. As a consequence, a more aggressive policy is required to control inflation in this sector, which imposes costs as a stronger response of the central bank can destabilize the rest of the economy. An additional channel relates to the effects of non-homotheticity on the composition of the marginal consumption basket. In particular, this type of preferences imply households spend only a small share of any additional income on agricultural goods. This means prices in this sector have a reduced effect on households' decisions, such as the intertemporal allocation of consumption demand. Given that aggregate demand turns more unresponsive to the evolution of prices in agriculture, responding to it becomes less important.

The third chapter, co-authored with Mario Giardia and Damián Romero, studies wage flexibility as a means to absorb adverse shocks. We focus on economies with unequal access to financial markets and where the monetary authority is constrained by the zero lower bound. We show that in is particular setting the economy becomes more volatile when wages are more flexible. In the model, due to financial frictions, volatility in wages can translate into output instability because of a redistribution channel operating through aggregate demand. When the central bank is constrained by the zero lower bound this feature is exacerbated, making crisis more severe. We conclude that in these kinds of economies, the usual recommendation of making labor markets more flexible to restore high output levels is misleading.

Contents

Chapter 1

MINIMUM WAGES, REDISTRIBUTION AND MACROECONOMIC VOLATILITY

1.1 Introduction

This paper investigates the redistributive role minimum wages can play over the cycle and the implied effects on macroeconomic stability. To this end, I develop a two-agent model featuring idiosyncratic uncertainty and limited asset markets participation. In my framework, idiosyncratic productivity determines earned wage, and hence, whether the minimum wage is binding for the worker. This gives rise to the existence of a subset of low wage, low income workers, for whom the minimum wage is binding, and a subset of high wage, high income individuals, whose wages lie above the established wage limit. Since the minimum wage constraint binds only for a subset of the workforce, such sector is subject to stronger wage rigidities. The latter implies that the minimum wage limit alters the dynamics of relative sectoral wages (i.e. the wage gap between the high and low income households), and thus, that of relative sectoral labor earnings, that is, it constitutes a source of income redistribution over the cycle. Importantly, the key factor shaping the direction of redistribution is the elasticity of substitution across labor from the different sectors, which governs the response of relative employment to relative wages. Such redistribution of resources, in interaction with financial friction, influence aggregate spending and thus, economic activity. Particularly, the model captures two channels through which redistribution operates. First, given limited participation, agents feature differentiated marginal propensities to consume, and hence distribution of resources impacts on aggregate spending. Secondly, the model incorporates idiosyncratic risk, that is, it assumes agents can transition across productivity states, and hence are subject to the risk of transiting to a state in which the minimum wage becomes binding. Uninsurable uncertainty gives rise to precautionary saving motives, and hence, has effects on aggregate consumption. In particular, via redistribution, the minimum wage alters the income gap across sectors, and thus, the magnitude of the drop in consumption workers experience in the event they transit to a state for which the minimum wage binds. Accordingly, it has effects on income uncertainty, and hence, on desired precautionary savings and spending.

I find that under plausible calibrations of the model, the minimum wage has the potential to redistribute against the minimum wage earners during an economic decline due to employment losses it originates. In addition to its detrimental effects on low income individuals' welfare, redistribution can have a quantitatively relevant impact on aggregate spending and hence on the cyclical behavior of the economy. In effect, economic downturns can deepen, as a consequence of redistribution of current income against low income households, characterized by higher propensities to consume, and heightened uncertainty, which further depress spending by raising desired precautionary savings.

Given that minimum wages tend not to bind for large shares of the workforce the current redistribution channel has a limited quantitative impact. Conversely, uncertainty can, under certain circumstances, become relevant. Particularly, whenever the income gap experiences long-lasting variations, precautionary savings motives turn out to be sizable. The model simulations illustrate that such persistence can either be inherited from the shock or arise from a large divergence in the degree of wage rigidities across labor types.

There is an extensive body of research investigating the impact of labor market frictions on macroeconomic volatility, looking at rigidities affecting inflows and outflows to and from employment and rigidities affecting the adjustment of wages. Among others, Campolmi and Faia (2011), Galí (2013), Walsh (2005) and Trigari (2006) perform a theoretical study on the implications of labor markets frictions. On the empirical side, Gnocchi et al. (2015), Bowdler and Nunziata (2007), Abbritti and Weber (2010), Christoffel et al. (2006), Christoffel et al. (2009), among others, investigate this topic. This papers fits within the literature exploring labor market frictions restricting the adjustments of wages. Differently from existing work, by considering a minimum wage constraint, I investigate the effects from rigidities affecting only a subset of the agents in the economy. This gives rise to a redistributive channel not explored in previous work.

My results are in line with the empirical findings of Gnocchi et al. (2015), who uncover a positive correlation across countries between the incidence of minimum wages and macroeconomic volatility. Their estimates are replicated in table 1.1, which reports Spearman partial rank correlations between a set of labor market indicators, including minimum wages, and the volatility of a set of macroeconomic variables. The authors find that minimum wages amplify not only the volatility of the real economy but also of nominal macro-variables. My model replicates such findings.

My work also relates to the literature exploring the effect of minimum wages on unemployment. Conclusions in this regard have been varied. Though empirical studies tend to find a adverse impact on unemployment, meta-regression analysis like Card and Krueger (1995) and Doucouliagos and Stanley (2009), conclude no effect, attributing preceding results to publication bias. Recently, Jardim et al. (2017) find strong effects, attributing the difference in their results with existing literature to data limitations in previous work. This paper aims at exploring the effect of minimum wages on the business cycle properties of the economy via redistribution, through their impact on employment.

The paper is organized as follows. Section 2 introduces a simple version of the model, which in section 3 I use to illustrate the transmission mechanism from frictions in the labor market to macroeconomic volatility. Section 4 presents a more realistic version of the model employed to quantify the redistributive channel. Section 5 reports the results from the quantitative exercise. In section 6 I perform a welfare analysis. Finally, section 7 concludes.

1.2 The model

1.2.1 Households

Households are endowed with an idiosyncratic productivity status $Z \in \{Z^h, Z^l\}$, with $Z^h > Z^l$. Idiosyncratic productivity is stochastic and follows a Markov process with transition probabilities $Pr(Z_t = Z^l|Z_{t-1} = Z^h) = \lambda_{hl}$ and $Pr(Z_t = Z^h|Z_{t-1} = Z^l) = \lambda_{lh}$. Accordingly the (constant) fraction of low productivity (L) households, denoted by ρ, is given by $\rho = (1-\rho)\frac{\lambda_{hl}}{\lambda_{lh}}$. This expression tells us there is a negative link between the probability of leaving a certain state and the steady state share of agents in the corresponding category. Each household is integrated by a continuum of members that offer differentiated labor varieties denoted by $i \in [0,1]$[1]. Wage for each of the labor varieties, corresponding to each of the productivity status, is set by a union operating in a monopolistically competitive market. Due to insurance within the household, members consumption is equalized. In the absence of a minimum wage, high productivity (H) workers' wage is higher than that of L individuals, that is, $W_t^h > \tilde{W}_t^l$, where $\tilde{W}_t^l$ is the wage that would prevail for L agents absent a minimum wage. I assume the government sets a minimum wage W_t^m, such that the following relation holds $W_t^h > W_t^m > \tilde{W}_t^l$ $\forall t$, that is, the constraint never binds for H workers whereas for the L type it is permanently binding. This implies wages for the latter follow the evolution of the minimum wage, i.e., $W_t^l = W_t^m \, \forall t$.

Households' lifetime utility is given by:

$$U_t = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\gamma}}{1-\gamma} - \int_0^1 \frac{\bar{N}_{t+k}^{1+\varphi}(i)}{1+\varphi} di \right)$$

where C_t is a consumption index and $\bar{N}_t(i)$ represents hours worked per member supplying labor variety i. In order to have a tractable model, which does not

[1] All members of a given household have the same productivity status.

require a numerical solution method, I follow Krusell et al. (2011), and assume households can trade bonds but face the following borrowing constraint: $B_t \geq 0$.

From households' first order conditions the Euler equation is obtained:

$$C_t^{-\gamma} \geq \beta R_t \mathbb{E}_t \left[\Pi_{t+1}^{-1} C_{t+1}^{-\gamma} \right]$$

Given the borrowing constraint, no one can borrow in this economy, then, in equilibrium, no one can save. This implies households posses zero wealth. Accordingly, all H households chose the same consumption level, and equivalently, all L agents consume the same. The Euler equations for H and L households are thus given by:

$$\left(C_t^h\right)^{-\gamma} \geq \beta R_t \mathbb{E}_t \left[\Pi_{t+1}^{-1} \left((1-\lambda_{hl}) \left(C_{t+1}^h\right)^{-\gamma} + \lambda_{hl} \left(C_{t+1}^l\right)^{-\gamma} \right) \right]$$

$$\left(C_t^l\right)^{-\gamma} \geq \beta R_t \mathbb{E}_t \left[\Pi_{t+1}^{-1} \left((1-\lambda_{lh}) \left(C_{t+1}^l\right)^{-\gamma} + \lambda_{lh} \left(C_{t+1}^h\right)^{-\gamma} \right) \right]$$

I further assume that shares are evenly distributed and not tradable. The latter implies that perceived dividends are the same for H and L households, and so labor earnings represent the only source of income heterogeneity across agents. Given that dividend income is equalized across households and H agents perceive a higher wage (and a higher labor income), then H-type agents have a higher total income, and hence consumption, relative to L households. Accordingly, at steady state H households expect a fall in consumption, as they might transit to the L state in the future, whereas L families expect a rise in their consumption level, as they might eventually become H. Then the borrowing constraint must be more binding for L households, given that relative to H families, they should have a stronger desire to borrow. A more binding borrowing constraint for L households is a condition that holds not only at steady state, but in its proximity as well. I will consider an equilibrium of this economy in which the Euler equation holds with equality for H agents, in the sense that, given the real rate, zero asset holdings is their optimal choice, whereas the borrowing limit binds for the L-type, and hence, they are hand to mouth.

Unconstrained households' consumption. As stated above, H households are unconstrained and their Euler equation holds with equality. We can express it in deviation from steady state as:

$$\hat{c}_t^h = -\frac{1}{\gamma}\mathbb{E}_t\left(\hat{r}_t - \hat{\pi}_{t+1}\right) + (1-\lambda_{hl})\mathbb{E}_t\hat{c}_{t+1}^h + \lambda_{hl}\left(\frac{C^h}{C^l}\right)^{\gamma}\mathbb{E}_t\hat{c}_{t+1}^l \tag{1.1}$$

Constrained households' consumption. Households in the L state are financially constrained, then consumption equals their current income, composed of labor and dividend earnings:

$$C_t^l = \frac{\int_0^1 W_t^l(i)\bar{N}_t^l(i)di}{P_t} + D_t$$

The above relation can be expressed in deviation from steady state as:

$$\hat{c}_t^l = \frac{W^l\bar{N}^l}{PC^l}\int_0^1 (\hat{\omega}_t^l(i) + \hat{n}_t^l(i))di + \frac{D}{C^l}\hat{d}_t \tag{1.2}$$

Aggregate consumption. Aggregate consumption is given by:

$$C_t = (1-\rho)C_t^h + \rho C_t^l$$

which can be rewritten in deviation from steady state as:

$$\hat{c}_t = (1-\rho)\frac{C^h}{C}\hat{c}_t^h + \rho\frac{C^l}{C}\hat{c}_t^l \tag{1.3}$$

From equations (1.1) and (1.3), the following Euler relation for aggregate consumption is obtained:

$$\begin{aligned}\hat{c}_t = -\frac{1}{\gamma}\mathbb{E}_t\left(\hat{r}_t - \hat{\pi}_{t+1}\right) + \mathbb{E}_t\hat{c}_{t+1} + \frac{\rho C^l}{C}(\mathbb{E}_t(\hat{c}_{t+1}^h - \hat{c}_{t+1}^l) - (\hat{c}_t^h - \hat{c}_t^l)) \\ -\lambda_{hl}\mathbb{E}_t\frac{\gamma(C^h)^{-\gamma}\hat{c}_{t+1}^h - \gamma(C^l)^{-\gamma}\hat{c}_{t+1}^l}{\gamma(C^h)^{-\gamma}}\end{aligned} \tag{1.4}$$

Solving forward the above expression yields:

$$\begin{aligned}\hat{c}_t &= -\frac{1}{\gamma}\mathbb{E}_t\sum_{k=0}^{\infty}(\hat{r}_{t+k}-\hat{\pi}_{t+k+1}) - \frac{\rho C^l}{C}(\hat{c}_t^h - \hat{c}_t^l)\\ &-\lambda_{hl}\mathbb{E}_t\sum_{k=0}^{\infty}\frac{\gamma(C^h)^{-\gamma}\hat{c}_{t+k+1}^h - \gamma(C^l)^{-\gamma}\hat{c}_{t+k+1}^l}{\gamma(C^h)^{-\gamma}}\end{aligned} \tag{1.5}$$

According to equation (1.5) aggregate consumption depends on the path of the real rate, the current consumption gap ($\hat{c}_t^h - \hat{c}_t^l$), and the future path of the gap in consumption (more precisely, the gap in the marginal utility of consumption between unconstrained and constrained agents, given by $\gamma(C^h)^{-\gamma}\hat{c}_{t+k}^h - \gamma(C^l)^{-\gamma}\hat{c}_{t+k}^l$). The current consumption gap measures current income redistribution, and its impact on aggregate consumption is increasing in the steady state share of constrained households' consumption in total consumption ($\frac{\rho C^l}{C}$). The gap in the marginal utility of consumption captures uncertainty. More precisely, it measures the value of consumption in the L state (relative to the H state). Accordingly, it determines desired precautionary savings by unconstrained agents and its effect on aggregate consumption is increasing in the probability they face of becoming constrained (λ_{hl}).

The third term in the RHS of (1.5), capturing idiosyncratic uncertainty, can be decomposed into two components, one related to the future gap in consumption, that is, to future redistribution, and the second related to the state of the cycle. By performing such decomposition equation (1.5) can be rewritten as:

$$\begin{aligned}\hat{c}_t &= -\frac{1}{\gamma}\mathbb{E}_t\sum_{k=0}^{\infty}(\hat{r}_{t+k}-\hat{\pi}_{t+k+1}) - \frac{\rho C^l}{C}\left(\hat{c}_t^h - \hat{c}_t^l\right)\\ &- \lambda_{hl}\left(\frac{C^h}{C^l}\right)^{\gamma}\mathbb{E}_t\sum_{k=0}^{\infty}\left(\hat{c}_{t+k+1}^h - \hat{c}_{t+k+1}^l\right) + \lambda_{hl}\mathbb{E}_t\sum_{k=0}^{\infty}\left(\frac{(C^l)^{-\gamma}-(C^h)^{-\gamma}}{(C^h)^{-\gamma}}\right)\hat{c}_{t+k+1}^h\end{aligned}$$

Interestingly, the above expression illustrates that, conditional on $C^h > C^l$ (i.e. in a setting with steady state inequality) and $\gamma > 0$, uncertainty, and thus desired precautionary savings, fluctuate over the cycle even absent redistribution (i.e., even when the consumption gap is acyclical). The intuition for this result is simple. Precautionary saving motives emerge from the risk unconstrained agents face of experiencing a drop in consumption in the event they switch to the L state. How costly such drop in consumption is will depend on the state of the cycle. In effect, given the curvature of the utility function (i.e. $\gamma > 0$), during recession episodes a drop consumption is particularly costly (i.e. the value of consumption in the event they transit to the L state becomes particularly high), and hence precautionary savings are high relative to normal times. In other words, conditional on $C^h > C^l$ and $\gamma > 0$, uncertainty, and thus precautionary savings, are countercyclical.

Notice that absent redistribution the Euler equation for aggregate consumption becomes:

$$\hat{c}_t = -\tfrac{1}{\gamma}\mathbb{E}_t\left(\hat{r}_t - \hat{\pi}_{t+1}\right) + \left(1 + \lambda_{hl}\left(\left(\tfrac{C^h}{C^l}\right)^{\gamma} - 1\right)\right)\mathbb{E}_t\hat{c}_{t+1}$$

where $1 + \lambda_{hl}\left(\left(\frac{C^h}{C^l}\right)^{\gamma} - 1\right) > 1$ if $C^h > C^l$ and $\gamma > 0$.

The above relation shows that current aggregate consumption reacts more than proportionally to future aggregate consumption, reflecting variations in precautionary saving motives (which add to smoothing motives).

Let us now consider the role of redistribution. For simplicity I assume steady state consumption is equalized across productivity states, which yields:

$$\hat{c}_t = -\frac{1}{\gamma}\mathbb{E}_t\sum_{k=0}^{\infty}\left(\hat{r}_{t+k} - \hat{\pi}_{t+k+1}\right) - \rho\hat{c}^r_t - \lambda_{hl}\mathbb{E}_t\sum_{k=0}^{\infty}\hat{c}^r_{t+k+1} \tag{1.6}$$

where $\hat{c}^r_t \equiv \hat{c}^h_t - \hat{c}^l_t$. It is clear from equation (1.6) that the response of aggregate consumption to a given path of the real rate depends on the cyclicality of the consumption gap. More precisely, if the consumption gap is acyclical (i.e. $\hat{c}^r_t = 0\ \forall t$), then the second and third terms in the RHS vanish and aggregate consumption depends solely on the path of the real rate. If, on the other hand, the consumption gap is procyclical/countercyclical, the second and third terms adopt the opposite/same sign of the first, and thus the response of aggregate consumption to a given path of the real rate gets dampened/amplified.

To get an intuition for the previous results let us consider the effects from a rise in the real rate, for the particular case of a countercyclical consumption gap. Given a drop in aggregate consumption following the real rate increase, a countercyclical gap implies a rise in current relative consumption, i.e. constrained agents' consumption fall relatively more. But consumption by constrained households falling by more relative to that of the unconstrained (which moves according to the path of the real rate) means an amplified contraction of aggregate consumption given the path of the real rate. This effect is captured by the second term in the RHS of (1.6). Additionally, a countercyclical consumption gap implies uncertainty is countercyclical as well. To see this, note that a countercyclical gap means that future relative consumption rises in the face of the economic downturn. Follows from the latter that a larger fall in consumption is experienced by unconstrained agents in the event they switch to the L state, that is, they face greater income risk. Accordingly, spending by unconstrained households and thus aggregate consumption drop further due to precautionary saving motives. This effect is captured by the third term in the RHS of equation (1.6).

1.2.2 Wage setting

***H* workers.** H-type households offer differentiated labor varieties. Wage for each of the varieties is flexible and set by a union operating in a monopolistically competitive market. Unions set the wage in order to maximize households' utility subject to the demand schedule:

$$N_t^h(i) = \left(\frac{W_t^h(i)}{W_t^h}\right)^{-\epsilon_w} N_t^h$$

where $N_t^h(i) = (1-\rho)\bar{N}_t^h(i)^2$, ϵ_w is the elasticity of substitution across labor varieties, and variables N_t^h and W_t^h are a labor index and a wage index, which are defined later.

Optimization implies:

$$\frac{W_t^h(i)}{P_t} = \mathcal{M}_w (C_t^h)^\gamma \bar{N}_t^h(i)^\varphi$$

[2]Equivalently, $N_t^l(i) = \rho\bar{N}_t^l(i)$.

The above expression tells us that wages (in real terms) are set as a markup $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$ over the marginal rate of substitution.

Given symmetry across H workers, wages can be expressed in deviation from steady state as:

$$\hat{\omega}_t^h = \gamma \hat{c}_t^h + \varphi \hat{n}_t^h \tag{1.7}$$

***L* workers.** L-type households offer differentiated labor varieties. Since the minimum wage binds for them, it follows that the wage for all L agents is the same and equal to the minimum wage.

A standard rule followed in practice by governments for setting the minimum wage is indexing to inflation. Indexing takes place periodically, for instance, annually or semiannually. This implies variations in the price level are not immediately incorporated to the minimum wage, instead this occurs with lag, which varies according to the periodicity of indexing. For the simple version of the model I consider the limiting case of a rule of this type, where indexing occurs with no lag, that is, any variation in the price level is immediately incorporated to the nominal minimum wage. Accordingly, the minimum wage in real terms is constant and so we have:

$$\hat{\omega}_t^m = \hat{\omega}_t^l = 0 \tag{1.8}$$

1.2.3 Firms

Final good. Firms producing the final good operate in a perfectly competitive market and combine the different varieties of intermediate goods $Y_t(z)$ into a homogeneous final good Y_t according to the following technology:

$$Y_t = \left(\int_0^1 Y_t(z)^{\frac{\epsilon_p - 1}{\epsilon_p}} dz \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

Where ϵ_p is the elasticity of substitution.

Solving the optimization problem of the firm we obtain the following demand function for intermediate inputs:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon_p} Y_t$$

where $P_t \equiv \left(\int_0^1 P_t(z)^{1-\epsilon_p} dz\right)^{\frac{1}{1-\epsilon_p}}$ is the price of the final good.

Intermediate goods. Each firm z in the intermediate good sector has access to the following production technology:

$$Y_t(z) = A_t N_t(z)^{1-\alpha}$$

where A_t is the productivity level, with $ln(A_t) = a_t = \rho_a a_{t-1} + \varepsilon_t^a$, and total labor $N_t(z)$ is given by:

$$N_t(z) = \left(\varpi_h (Z^h N_t^h(z))^{\frac{\psi-1}{\psi}} + \varpi_l (Z^l N_t^l(z))^{\frac{\psi-1}{\psi}}\right)^{\frac{\psi}{\psi-1}}$$

where ψ is the elasticity of substitution between H and L labor types and $N_t^x(z)$ is a labor index defined as:

$$N_t^x(z) \equiv \left(\int_0^1 N_t^x(i,z)^{\frac{\epsilon_w-1}{\epsilon_w}} di\right)^{\frac{\epsilon_w}{\epsilon_w-1}}$$

for $x \in \{h, l\}$.

Given the demand for labor of type x, $N_t^x(z)$, cost minimization yields the following demand schedule for labor of type x and variety i, $N_t^x(i,z) = \left(\frac{W_t^x(i)}{W_t^x}\right)^{-\epsilon_w} N_t^x(z)$, where $W_t^x \equiv \left(\int_0^1 W_t^x(i)^{1-\epsilon_w} di\right)^{\frac{1}{1-\epsilon_w}}$ is an aggregate wage index.

Additionally, given total labor demand $N_t(z)$, from firms minimization of total expenditure, a demand schedule for each of the labor types is obtained:

$$N_t^x(z) = \left(\frac{W_t^x}{\varpi_x (Z^x)^{\frac{\psi-1}{\psi}} W_t}\right)^{-\psi} N_t(z)$$

where W_t is the cost of one unit of labor $N_t(z)$, and is given by:

$$W_t = \left(\varpi_h^{\psi} (Z^h)^{\psi-1} (W_t^h)^{1-\psi} + \varpi_l^{\psi} (Z^l)^{\psi-1} (W_t^l)^{1-\psi} \right)^{\frac{1}{1-\psi}}$$

The last two equations can be written in deviation from steady state as:

$$\hat{n}_t^x = -\psi(\hat{\omega}_t^x - \hat{\omega}_t) + \hat{n}_t \tag{1.9}$$

and:

$$\hat{\omega}_t = \Omega \hat{\omega}_t^h + (1 - \Omega)\hat{\omega}_t^l \tag{1.10}$$

where $\Omega \equiv \frac{W^h N^h}{WN}$ is the steady state share of H labor expenses in total labor expenditure[3].

I assume that intermediate good producer firms set the price of their product according to Calvo contracts, that is, in each period a portion $1 - \theta_p$ of them can adjust their price.

Intermediate good producers which are able to reoptimize set their price in order to maximize a discounted sum of their future flow of funds subject to the demand for their product from final good producers.

Aggregating price decisions we obtain the following inflation equation in deviation from the steady state:

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \lambda_p \hat{mc}_t \tag{1.11}$$

Where $\hat{mc}_t = \hat{\omega}_t + \alpha \hat{n}_t - a_t$ is the real marginal cost and $\lambda_p \equiv \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$.

1.2.4 Equilibrium

The market of final goods is in equilibrium when households demand equals firms production:

$$C_t = Y_t$$

or:

[3]For the simplified version of the model it can be shown that $\Omega = 1 - \rho$.

$$\hat{c}_t = \hat{y}_t \tag{1.12}$$

The relation between aggregate output and employment of workers offering labor type x can be written as:

$$N_t^x = \Delta_t^x \Delta_t^p \left(\frac{W_t^x}{\varpi_x (Z^x)^{\frac{\psi-1}{\psi}} W_t} \right)^{-\psi} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}}$$

where $\Delta_t^x \equiv \int_0^1 \left(\frac{W_t^x(i)}{W_t^x} \right)^{-\epsilon_w} di$ and $\Delta_t^p \equiv \int_0^1 \left(\frac{P_t(z)}{P_t} \right)^{\frac{-\epsilon_p}{1-\alpha}} dz$, are measures of wage and price dispersion respectively.

This above relation can be expressed in deviation from steady state as follows:

$$\hat{n}_t^x = -\psi(\hat{\omega}_t^x - \hat{\omega}_t) + \frac{1}{1-\alpha}(\hat{y}_t - a_t) \tag{1.13}$$

Finally, to close the model, I assume the central bank determines the path of the real interest rate $\hat{r}_t^r \equiv \hat{r}_t - \mathbb{E}_t \hat{\pi}_{t+1}$.

1.3 Minimum wages and redistribution

1.3.1 Redistribution under demand shocks

In this section I investigate the redistributive effects from minimum wages. For simplicity I assume the existence of transfers that eliminate steady state inequality and shocks to the interest rate as the only drivers of economic fluctuations.

As a first step, I explore the relationship between the consumption and wage gaps, given by the following expression:

$$\hat{c}_t^r = \frac{(1-\alpha)(1-\psi)}{\mathcal{M}_p} \hat{\omega}_t^r \tag{1.14}$$

where $\hat{\omega}_t^r \equiv \hat{\omega}_t^h - \hat{\omega}_t^l$ and $\mathcal{M}_p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ is the gross firms markup. Equation (1.14) tells us that the consumption gap depends on relative wages and the elasticity of substitution across labor types (ψ), as this parameter determines the response of relative employment to the wage gap. The above relation makes clear the manner minimum wages can redistribute over the cycle. A minimum wage constraint, as a source of rigidities that affect only a subset of the workforce, alters the dynamics of relative wages, and hence, that of the income (and consumption) gap. Equation (1.14) also illustrates that redistribution crucially depends on the degree of substitutability. In particular, under $\psi = 1$, the consumption gap remains invariant regardless of the evolution of relative wages. In this context, variations in relative wages trigger movements in relative employment that perfectly offset the effect of the former on relative income/consumption. Differently, under $\psi \neq 1$, movements in relative wages alter relative income and hence the consumption gap. Thus, if $\psi > 1$ holds, a rise/fall in relative wages is associated to a fall/rise in relative employment of a larger magnitude, then relative income (and consumption) falls/rises, i.e. the wage and consumption gaps move in opposite directions. On the other hand, if $\psi < 1$, a rise/fall in relative wages leads to a fall/rise in relative employment of a smaller magnitude, then relative income (and consumption) rises/falls, i.e. the wage and consumption gaps move in the same direction.

The second piece necessary to understand the role of minimum wages is the evolution of relative wages over the cycle, given by:

$$\hat{\omega}_t^r = \Psi_c \hat{y}_t \tag{1.15}$$

where $\Psi_c \equiv \frac{\gamma + \varphi \frac{1}{1-\alpha}}{1+\varphi\psi\rho + \mathcal{M}_p^{-1}\gamma(1-\alpha)(\psi-1)\rho}$, which is positive under reasonable parameter calibrations. According to (1.15) the wage gap is procyclical, which results from rigidities affecting the adjustment of wages in the market for L workers.

Substituting equation (1.15) into (1.14) yields:

$$\hat{c}_t^r = \tilde{\Psi}_c \hat{y}_t \tag{1.16}$$

Where coefficient $\tilde{\Psi}_c \equiv \frac{(1-\alpha)(1-\psi)\Psi_c}{\mathcal{M}_p}$ determines the cyclicality of the consumption gap.

As is clear from equation (1.16), and in line with the analysis above, the value adopted by parameter ψ is crucial for the dynamics of the gap in consumption. If $\psi = 1$, relative income is unresponsive to relative wages, and thus the consumption gap is acyclical ($\tilde{\Psi}_c = 0$). Differently, when $\psi < 1$ relative income moves in same direction of relative wages, then, given the procyclicality of the latter, the consumption gap is procyclical as well ($\tilde{\Psi}_c > 0$). On the other hand, if $\psi > 1$, relative income moves in the opposite direction of relative wages, then, given the procyclicality of the latter, the gap in consumption is countercyclical ($\tilde{\Psi}_c < 0$).

The discussion above helped us to understand the mechanism underlying the redistributive effects from minimum wages. In the next section, I consider the impact of income redistribution on the cyclical behavior of the economy.

1.3.2 Redistribution and the cycle

Given equations (1.4), (1.16) and the goods market clearing condition, the following Euler relation for aggregate output is obtained:

$$\hat{y}_t = -\frac{1}{\gamma}\chi_1 \mathbb{E}_t \left(\hat{r}_t - \hat{\pi}_{t+1}\right) + \chi_2 \mathbb{E}_t \hat{y}_{t+1} \tag{1.17}$$

where $\chi_1 \equiv 1 - \frac{\rho\tilde{\Psi}_c}{1+\rho\tilde{\Psi}_c}$ and $\chi_2 \equiv 1 - \frac{\lambda_{hl}\tilde{\Psi}_c}{1+\rho\tilde{\Psi}_c}$.

It is easy to verify that $\chi_1 = \chi_2 = 1$ when $\tilde{\Psi}_c = 0$, that is, if the consumption gap is acyclical the Euler equation adopts its usual form.

In addition, the following relations hold:

$$\frac{\partial\chi_1}{\partial\tilde{\Psi}_c} = -\frac{\rho}{(1+\rho\tilde{\Psi}_c)^2} < 0$$

$$\frac{\partial\chi_2}{\partial\tilde{\Psi}_c} = -\frac{\lambda_{hl}}{(1+\rho\tilde{\Psi}_c)^2} < 0$$

Accordingly, if the consumption gap is procyclical (i.e. $\tilde{\Psi}_c > 0$), then $\chi_1 < 1$ and $\chi_2 < 1$. In contrast, under a countercyclical consumption gap (i.e. $\tilde{\Psi}_c < 0$), $\chi_1 > 1$ and $\chi_2 > 1$ hold.

Solving the Euler equation forward and assuming for simplicity a unitary intertemporal elasticity of substitution yields[4]:

[4]The RHS of (1.18) is assumed to be well defined (i.e. I assume the real rate converges to zero faster than the inverse of χ_2^k.)

$$\hat{y}_t = -\mathbb{E}_t \chi_1 \sum_{k=0}^{\infty} \chi_2^k \left(\hat{r}_{t+k} - \hat{\pi}_{t+k+1}\right) \tag{1.18}$$

Equation (1.18) can be used to illustrate the effects from redistribution. Note that aggregate output is a function of the path of the real rate, with coefficient $-\chi_1\chi_2^k$ determining the response to a change in the real rate taking place at time k. As emphasized earlier, when $\psi = 1$ the consumption gap is acyclical, then $\chi_1 = \chi_2 = 1$ and hence $\chi_1\chi_2^k = 1$. In this scenario, redistribution is null despite of the existence of a minimum wage, and the response of aggregate output is identical to that in an economy with no minimum wage constraint. Differently, when $\psi < 1$ the consumption gap is procyclical, then $\chi_1 < 1$, $\chi_2 < 1$ and thus $\chi_1\chi_2^k < 1$, that is, the minimum wage dampens the response of aggregate output to real rate movements. Finally, when $\psi > 1$ the consumption gap turns countercyclical, then $\chi_1 > 1$, $\chi_2 > 1$ and hence $\chi_1\chi_2^k > 1$ holds, that is, the minimum wage amplifies the responsiveness of aggregate output.

Note that coefficient χ_1 captures the impact from current redistribution. Accordingly, the larger the share of constrained agents ρ, the more parameter χ_1 departs from one. Coefficient χ_2 on the other hand captures the effects from uncertainty, and hence, the larger parameter λ_{hl} is, the more this coefficient deviates from unity. Said it differently, the strength of the current redistribution and uncertainty channels is increasing in parameters ρ and λ_{hl}, respectively.

Importantly, the strength of the uncertainty channel is also a function of k, i.e., the timing of variations in the real rate. Such relationship has already been pointed out in previous work, e.g., Werning (2015). To understand this result consider a recession episode triggered by an increase in the real rate taking place in period k, in a scenario characterized by a countercyclical consumption gap. Note that a greater k associates to a more extended period of depressed activity. Given a countercyclical consumption gap, relative consumption rises for a more extended period as well. But a larger gap represents a surge in income risk. Hence, a greater k associates to a longer spell of increased uncertainty, and thus, a stronger cut in spending due to precautionary savings follows.

The intuition for the link between uncertainty persistence and the magnitude of the output response is easy to understand and turns out to result from the interaction between precautionary savings and consumption smoothing. To make this point clear consider the following example. Assume the consumption gap (and thus risk) increases from period t to $t+k$. At $t+k-1$ output will contract due to higher risk arising from a larger consumption gap at $t+k$. The latter implies that at $t+k-2$ consumption (and production) will drop due to the combination of higher risk (deriving from the larger consumption gap in $t+k-1$) and lower expected consumption at $t+k-1$ (due to the before mentioned contraction in that period). Continuing our reasoning this way we can see that, via consumption smoothing, the effects from higher uncertainty from periods t to $t+k$ sum up leading to a large contraction at t.

Also interesting to notice is the link between the share of constrained agents ρ and the strength of the risk channel. Particularly, observe that:

$$\frac{\partial \chi_2}{\partial \rho} = \frac{\lambda_{hl}\left((\tilde{\Psi}_c)^2 - \frac{\partial \tilde{\Psi}_c}{\partial \rho}\right)}{(1+\rho\tilde{\Psi}_c)^2}$$

To interpret the above derivative, let us begin by assuming that the cyclicality of the consumption gap is independent of the share of constrained agents, i.e. $\frac{\partial \tilde{\Psi}_c}{\partial \rho} = 0$, which implies $\frac{\partial \chi_2}{\partial \rho} > 0$. In this scenario the effects from changes in uncertainty are amplified by the share of constrained agents when the consumption gap is countercyclical (i.e., $\chi_2 > 1$[5] rises with ρ, thus departing from 1) whilst under a procyclical gap the impact from variations in uncertainty is dampened as ρ grows (i.e., $\chi_2 < 1$ rises with ρ, thus approaching 1). The explanation for this result is as follows. Uncertainty influences H households' saving and spending decisions. The latter impacts on economic activity, which ultimately has effects on L-type households' income (and spending). In fact, given a countercyclical gap, L households' income/spending vary more than proportionally relative to that of the H-type. Hence, the larger the share of L households ρ, the more changes in uncertainty affect aggregate demand and thus output. Conversely, in the context of a procyclical gap L households' income/spending change less than proportionally relative to that of the H. Accordingly, the larger ρ, the lower the impact of variations in uncertainty on aggregate consumption will be.

[5]Recall $\chi_2 > 1$ holds when the consumption gap is countercyclical while under a procyclical gap we have $\chi_2 < 1$.

Additionally to the mechanism just described, the share of constrained agents can also influence on the strength of the uncertainty channel by changing the cyclicality on the consumption gap, i.e, when $\frac{\partial \tilde{\Psi}_c}{\partial \rho} \neq 0$. In effect, by changing the cyclicality of the gap parameter ρ affects the cyclicality of risk (recall the consumption gap is a measure of uncertainty). Accordingly, whenever the cyclicality of uncertainty increases with ρ (formally, when $\frac{\partial |\tilde{\Psi}_c|}{\partial \rho} > 0$, which implies $\frac{\partial \tilde{\Psi}_c}{\partial \rho} < 0$ in the case of a countercyclical gap and $\frac{\partial \tilde{\Psi}_c}{\partial \rho} > 0$ in the context of a procyclical gap) the strength of the uncertainty channel amplifies. Conversely, whenever the cyclicality of uncertainty goes down as ρ increases (formally, when $\frac{\partial |\tilde{\Psi}_c|}{\partial \rho} < 0$, which implies $\frac{\partial \tilde{\Psi}_c}{\partial \rho} > 0$ in the case of a countercyclical gap and $\frac{\partial \tilde{\Psi}_c}{\partial \rho} < 0$ in the context of a procyclical gap) the strength of the uncertainty channel is dampened.

For the particular case of our model $\frac{\partial \Psi_c}{\partial \rho} < 0$ and hence $\frac{\partial \tilde{\Psi}_c}{\partial \rho} > 0$ hold in the case of a countercyclical gap, i.e., the cyclicality of the consumption gap reduces with ρ, while in the case of a procyclical gap we have $\frac{\partial \Psi_c}{\partial \rho} \gtrless 0$ and thus $\frac{\partial \tilde{\Psi}_c}{\partial \rho} \gtrless 0$. Putting together the two mechanisms we get that the impact of the share of constrained agents on the strength of the uncertainty channel is indeterminate and will depend on the specific model parametrization.

Finally, notice that while the effects from precautionary savings depend on the probability H households face of transitioning to the L status (λ_{hl}), they are unaffected by the expected duration in such state, i.e., they are independent of λ_{lh}. In fact, notice that parameter λ_{lh} only enters coefficient $\chi_2 \equiv 1 - \frac{\lambda_{hl} \tilde{\Psi}_c}{1+\rho \tilde{\Psi}_c}$ through ρ, given the link between the transition probabilities and the steady state mass of households in each of the productivity categories (i.e. $\frac{\rho}{1-\rho} = \frac{\lambda_{hl}}{\lambda_{lh}}$). Particularly, this relation tells us that a higher probability of staying at L, that is, a lower value of λ_{lh}, associates to a larger steady state share of agents in the L status (a larger ρ). Yet, the reason why changes in λ_{lh} can affect χ_2 (through ρ) is unrelated to the expected duration of the L state, as is clear from our preceding discussion on the link between ρ and χ_2. The fact that the magnitude of the effects from precautionary savings is unrelated to the expected permanence in the L status seems counterintuitive, yet it can be easily understood. Note that even if an eventual transition to the L state next period is expected to be short lived, due to smoothing motives H agents seek to move today's consumption one-to-one with expected consumption in the next period, which, given general equilibrium effects, end up provoking a large contraction in today's output.

1.3.3 Introducing productivity shocks

With the introduction of shocks to productivity equation (1.15) is modified as follows:

$$\hat{\omega}_t^r = \Psi_c \hat{y}_t - \Psi_a a_t \tag{1.19}$$

where $\Psi_a \equiv \frac{\frac{\varphi}{1-\alpha}}{1+\varphi\psi\rho+\mathcal{M}_p^{-1}\gamma(1-\alpha)(\psi-1)\rho}$, which is positive under reasonable parameter calibrations.

Now, in addition to output fluctuations, productivity influences the evolution of relative wages as well. More precisely, given aggregate output, variations in productivity require movements in employment, which impact constrained and unconstrained workers wages differently and hence gives rise to movements in relative wages.

Given (1.19), equations (1.16) and (1.17) become:

$$\hat{c}_t^r = \tilde{\Psi}_c \hat{y}_t - \tilde{\Psi}_a a_t$$

where $\tilde{\Psi}_a \equiv \frac{(1-\alpha)(1-\psi)}{\mathcal{M}_p}\Psi_a$.

And:

$$\hat{y}_t = -\frac{1}{\gamma}\chi_1 \mathbb{E}_t\left(\hat{r}_t - \hat{\pi}_{t+1}\right) + \chi_2 \mathbb{E}_t \hat{y}_{t+1} + \chi_3 a_t \tag{1.20}$$

where $\chi_3 \equiv \frac{\rho\tilde{\Psi}_a(1-\rho_a)+\lambda\tilde{\Psi}_a\rho_a}{1+\rho\tilde{\Psi}_c}$.

Notice that absent redistribution (i.e., with $\psi = 1$ and thus $\tilde{\Psi}_a = 0$), and given the real rate, output is insensitive to productivity changes (i.e. $\chi_3 = 0$), that is, productivity shocks only transmit to output through their effect on the real rate. In contrast, when $\psi \neq 1$, and given the real rate, movements in relatives wages following the productivity shock translate into changes in the consumption gap. This implies redistribution of current income and variations in risk, both affecting demand and thus output.

1.4 Quantitative simulations

The present section seeks to evaluate the quantitative relevance of redistribution arising form the existence of the minimum wage. To this end, and given that redistribution crucially hinges on the dynamics of relative wages, a more realistic setup for the labor market is considered. In particular, for H workers I introduce frictions in the form of staggered wage setting, whereas for the minimum wage affecting L workers, a more realistic updating rule is assumed.

1.4.1 Wage setting

***H* workers.** Wage for each of the labor varieties is set by a union operating in a monopolistically competitive market. Unions set wages according to Calvo contracts, that is, in each period a fraction $1-\theta_w$ of them can adjust wages.

Unions set the wage $W_t^h(i)$ in order to maximize:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left(\left(C_{t+k}^h\right)^{-\gamma} \frac{W_t^h(i)}{P_{t+k}} \bar{N}_{t+k}^h(i) - \frac{\bar{N}_{t+k}^h(i)^{1+\varphi}}{1+\varphi} \right)$$

Subject to the demand for their labor variety:

$$N_{t+k}^h(i) = \left(\frac{W_t^h(i)}{W_{t+k}^h}\right)^{-\epsilon_h} N_{t+k}^h$$

Optimization implies:

$$\hat{\pi}_t^h = \beta \mathbb{E}_t \hat{\pi}_{t+1}^h + \lambda_w \left(m\hat{r}s_t^h - \hat{\omega}_t^h \right) \tag{1.21}$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varphi\epsilon_h)}$.

***L* workers.** Two alternative resetting rules are considered, a target on the real minimum wage and a target on the nominal minimum wage.

Target on the real minimum wage. I assume the government has a constant target for the minimum wage in real terms denoted by $\bar{\omega}^m$. The effective real minimum wage nevertheless does not necessarily coincide with its target, since after variations in the price level the government resets the nominal minimum wage W_t^m with a lag s:

$$W_t^m = P_{t-s}\bar{\omega}^m$$

The minimum wage in real terms can then be written as:

$$\omega_t^m = \frac{\bar{\omega}^m}{\Pi_{t-s,t}} \tag{1.22}$$

We can see the real minimum wage equals to its target after discounting inflation taking place from period $t-k$ to t.

The above relation can be expressed in deviation from steady state as:

$$\hat{\omega}_t^m = \hat{\omega}_t^l = -\hat{\pi}_{t,t-s} \tag{1.23}$$

As (1.23) shows, higher inflation implies a lower minimum wage in real terms. This occurs because the nominal minimum wage catches up with variations in prices with a lag, and thus it does not incorporate inflation that took place in the last s periods.

Combining the identity relating real wage variations to price and wage inflation $\hat{\omega}_t^m \equiv \hat{\pi}_t^m - \hat{\pi}_t + \hat{\omega}_{t-1}^m$ with (1.23) an expression for the evolution of the nominal minimum wage inflation is obtained:

$$\hat{\pi}_t^m = \hat{\pi}_t^l = \hat{\pi}_{t-s,t-s-1}$$

Target on the nominal minimum wage. Alternatively I will assume the government has a nominal target, that is, it sets a fixed minimum wage in nominal terms, which is unaffected by variations in the price level.

Hence, the minimum wage is set as:

$$W_t^m = \bar{W}^m$$

where $\bar{W}^m$ is the nominal target.

Such rule implies:

$$\hat{\pi}_t^m = \hat{\pi}_t^l = \hat{w}_t^m - \hat{w}_{t-1}^m = 0 \tag{1.24}$$

Whether a rule like (1.24) or one in the form of (1.23) is a more reasonable assumption depends on the situation considered. Nominal targeting can be a more realistic assumption if we were to analyze an economy experiencing price deflation. In such context an invariant minimum wage seems a more sensible assumption than allowing for a reduction in the nominal minimum wage in line with the fall in prices.

1.4.2 Monetary policy

I consider two alternative policy rules. A Taylor rule of the form:

$$\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + z_t \tag{1.25}$$

And alternatively, an exogenous path for real interest rate:

$$\hat{r}_t^r \equiv \hat{r}_t - \hat{\pi}_{t+1} = z_t \tag{1.26}$$

where: $z_t = \rho_r z_{t-1} + \varepsilon_t^r$.

1.5 Results

1.5.1 Calibration

Parameter α is set to 0.25 and the discount factor β to 0.99. Utility parameters γ and φ are set to 1. Calvo price and wage parameters are set to 0.75, implying an average contract duration of 4 quarters. Parameters ϵ_p and ϵ_w are set to 6, consistent with a steady state markup of 20%. Based on our earlier discussion, calibration of parameter ψ turns crucial, and, at the same time, subject to controversy. The introduction emphasized the contradictory conclusions documented by the empirical literature in what concerns to the impact of minimum wages on employment, with estimates ranging from no effects to sizable associated employment losses. For the model simulation I will follow recent estimates by Jardim et al. (2017), as well as findings by Ciccone and Peri (2005) and Mollick (2011), who provide estimates for the elasticity of substitution amongst labor skill levels. The former reports an elasticity estimate of hours to wages amounting to -3 after a rise in the prevailing minimum wage, whilst the latter two report elasticities of substitution of 1.5 and 2 to 3.2, respectively. Based on such studies I set the elasticity of substitution among the H and L types to 2.5. Additionally, I assume the fraction of minimum wage earners ρ is 0.1[6]. When a real target is assumed, parameter s is set to 2, which implies the minimum wage is actualized with a delay of 2 quarters. This calibration seeks to reflect the fact that minimum wages are usually adjusted at intervals of one year. Resetting it at intervals of one year, say at the end of each year, implies that inflation taking place at the begging of such time period will be incorporated to the minimum wage after 4 quarters, whereas inflation taking place at the end of the year will be immediately incorporated. Then, on average, there will be a lag of 2 quarters for inflation to be incorporated to the new nominal minimum wage. The transition probability from the H to the L state, λ_{hl}, is set to 0.02. Note that the calibration of the transition probability is directly linked to that of ρ. Particularly, I use the results from Floden and Lindé (2001), who estimate the wage process for the US. Based on their results I estimate the probability for a worker whose wage lies above the 10% percentile of the wage distribution (which is the value I assume the minimum wage is set to) to experience a reduction in his salary that brings it below the 10 percentile (i.e., below the assumed minimum wage). Parameters related to the Taylor rule ϕ_π and ϕ_y are set to 1.5 and 0, respectively. The autoregressive coefficients of the productivity and monetary exogenous processes, ρ_a and ρ_r , are set to 0.9, whereas the standard deviation of

[6]Figure 1.1 presents Eurostat statistics reporting the proportion of employees earning the minimum wage across the EU Member States. Such fraction varies from 0.4% in Belgium to 19.1% in Slovenia (without considering Turkey). Based on these statistics parameter ρ is set to 0.1.

the innovations is set respectively to 1% and 0.25%.

1.5.2 IRFs analysis

Monetary shock. Consider first a scenario where the real interest rate follows an exogenous path, reported in figure 1.2. The figure displays the responses for three different economies. One corresponds to the benchmark setting, where wages in both labor markets are freely set (i.e. no minimum wage is in place)[7]. The other two economies incorporate a minimum wage constraint but differ in their demand side. For the first, which I call economy A, idiosyncratic risk is ruled out, and so its response captures uniquely the effects from current redistribution. The second setup introduces uncertainty, then the response of this economy captures both current redistribution and income risk effects. I call this economy B. The different behavior between former and the benchmark provides a measure of the effects from current income redistribution whereas divergence between the latter and the former reflects the isolated effects from idiosyncratic uncertainty. Note that in the benchmark economy wages across labor types move identically and hence redistribution is null. Differently, with the introduction of a minimum wage the gap in wages turns procyclical. Given an assumed elasticity of substitution greater than 1, the latter implies a countercyclical consumption gap and thus redistribution amplifies the drop in activity. Comparison across the three economies illustrates that current redistribution has a modest impact whereas idiosyncratic uncertainty turns out to be more relevant. The former is explained by the assumed small fraction of constrained households (the minimum wage earners), implying that the sizable fall in demand by these agents only slightly impacts on aggregate consumption. The latter is explained by a highly persistent rise in the consumption gap. A lasting rise in the gap implies uncertainty persistently rises, triggering a large cut in consumption due to precautionary savings.

Also worth noting is the response of price inflation. In particular, observe that in economy B inflation is more volatile relative to the benchmark, and this despite of the higher degree of nominal rigidities associated to the minimum wage. The explanation is as follows. On the one hand, rigidities in wages associated to the

[7] In this economy both H and L households set wages a la Calvo.

minimum wage transmit to firms costs and through them to prices, which accordingly become less volatile. On the other hand, the associated redistributive effects turn output and employment more volatile, resulting in a stronger variability in costs and thus prices. In economy B the latter effect dominates, resulting in a more volatile inflation. Note that the dynamics of economy B are in line with empirical findings in Gnocchi et al. (2015), reported in table 1.1, that is, the minimum wage is associated not only to an increase in output volatility but also to greater variability in price inflation.

Let us now consider the scenario where the central bank follows a standard Taylor rule, reported in figure 1.3. Allowing for an endogenous response of the policy rate can either counteract or reinforce the effects from redistribution. On the one hand, an active reaction function implies the monetary authority will partly offset the higher volatility emerging from redistribution. On the other hand, and as stated above, rigidities in wages associated to the minimum wage transmit to prices, which consequently become less responsive to shocks. A reduced responsiveness of inflation in turn implies a weaker response of the policy rate, which amplifies volatility. Such transmission from wage rigidities to the endogenous response of the policy rate has been described in Galí (2013). In economy A the latter effect dominates, and hence, a weaker response of the central bank reinforces the effects from income redistribution. Instead, in economy B the former dominates, and thus, a more aggressive response of the central bank partially offsets the effects from redistribution.

Monetary shock. Target on the nominal minimum wage. Earlier it was emphasized that a long-lasting surge in the consumption gap was required for the effects from uncertainty to be quantitatively relevant. In the context of a real target explored above, such persistence crucially hinged on the assumed persistence of the shock. However, this is not a necessary condition as lasting variations in the consumption gap can also arise from a greater divergence in the degree of wage stickiness across labor types. A scenario characterized by large disparities in the degrees of stickiness is explored next. Such divergence will arise in a context of a nominal target, which imply nominal wages for L agents remain invariant in the face of an adverse shock, i.e., wages in this sector are fully rigid. The corresponding IRFs are reported in Figure 1.4 [8]. Observe that, in line with the discussion above, the consumption gap persistently rises, yet, contrary to what could be expected, comparison between economies A and B appears to indicate that uncertainty only moderately exacerbates the decline in output. Decomposing the response of aggregate consumption according to equation (1.5) can provide an interpretation for this result. Figure 1.5, reporting such decomposition, makes clear that uncertainty has in fact a sizable contractive effect on aggregate consumption, consistent with the persistent rise in income risk. However, the figure also uncovers a strong expansionary effect from the real rate. The latter reflects the reaction of the central bank, which strongly cuts the policy rate in response to the slump and deflation, mostly offsetting the effects from uncertainty.

Productivity shock. Figure 1.6 depicts the response to a negative shock to productivity. As stated in section 1.3.3, productivity affects redistribution through two different channels. Firstly, given the real rate, a drop in productivity is associated to a rise in employment and inflation. The rise in employment puts upward pressure on unconstrained workers wages while increased inflation reduces the real value of the minimum wage. This positively impacts on relative wages. Secondly, the inflationary pressures emerging from the drop in productivity induce a rise in the policy rate. The more contractionary policy stance depresses output, which negatively impacts relative wages. Given the opposing effects, relative wages have a relatively moderate response. The latter adds to a less contractive policy in economy B to result in a negligible impact of the minimum wage on output volatility.

1.6 Welfare analysis

In the present section I conduct an optimal policy analysis. To this end I derive the welfare loss function for the model economy incorporating a minimum wage constraint. For its derivation I assume the central bank seeks to maximize a weighted

[8] For the present simulation parameters ϕ_π and ϕ_y are respectively set to 1.5 and 0.15, since, under a nominal target, the determinancy region of the parameter space shrinks. Additionally, ρ_r is reduced to 0.8.

average of H and L type agents' utility, with the weights coinciding with the mass of households in each category. For simplicity, I further assume the existence of a labor subsidy that corrects for the inefficiencies generated by monopolistic competition and the minimum wage, and, at the same time, equates the steady state consumption of H and L households. By performing a second order approximation of the utility around the efficient steady state with no inequality the following loss function is obtained:

$$L_t = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Xi_y x_t^2 + \Xi_w (\hat{\omega}_t^r)^2 + \Xi_\pi \pi_t^2 \right\}$$

where $\Xi_\pi \equiv \frac{\epsilon_p}{\lambda_p}$, $\Xi_y \equiv \gamma + \frac{\varphi+\alpha}{1-\alpha}$ and $\Xi_w \equiv -\rho(1-\rho)\gamma\left((1-\psi)(1-\alpha)\right)^2 + (1-\alpha)(1+\varphi)\rho(1-\rho)\psi^2$. $x_t \equiv y_t - y_t^e$ is the welfare relevant output gap, where y_t^e denotes the efficient production level. Notice that the efficient output and its natural counterpart, denoted by y_t^n, differ from each other. The former corresponds to the production level that would prevail under flexible prices and absent a minimum wage constraint whereas the latter represents the output level prevailing under flexible prices and a minimum wage in place.

In addition to the usual terms related to the output gap and price inflation, the existence of a minimum wage introduces and additional term related to relative wages. This term captures two types of inefficiencies. First, movements in relative wages lead to the inefficient substitution between H and L labor types by firms. These losses are captured in the term $(1-\alpha)(1+\varphi)\rho(1-\rho)\psi^2$, which is increasing in the degree of substitutability ψ. Second, movements in relative wages give rise to variations in the consumption gap. Given a steady state where consumption across agents is equalized, changes in the consumption gap provoke inequality, and hence, reduce social welfare. These losses are captured by the term $\rho(1-\rho)(\gamma-1)\left((1-\psi)(1-\alpha)\right)^2$, which increases as the degree of substitutability ψ departs from 1. In effect, when $\psi \neq 1$ variations in relative wages translates into movements in the consumption gap and ultimately into welfare losses.

Next, I compute the optimal Ramsey policy for the simple economy introduced in section 1.2. To this end, the welfare loss function is minimized subject to the following constraints:

$$\hat{\omega}_t^r = \Psi_c x_t + \left(\Psi_c - \Psi_a \frac{\gamma(1-\alpha)+\varphi+\alpha}{1+\varphi} \right) \hat{y}_t^e \tag{1.27}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa x_t + \kappa(\hat{y}_t^e - \hat{y}_t^n) \tag{1.28}$$

where $\Psi_c \frac{1+\varphi}{\gamma(1-\alpha)+\varphi+\alpha} - \Psi_a$ and

$\kappa \equiv \lambda_p \left(\frac{\alpha}{1-\alpha} + (\gamma(1-\alpha)(1-\psi)\rho(1-\rho) - \varphi\psi\rho(1-\rho)) \Psi_c + \rho \left(\gamma + \frac{\varphi}{1-\alpha} \right) \right)$ are positive under reasonable parameter calibrations.

The first constraint tells us that in the face of productivity shocks it is not possible to achieve simultaneously a zero wage differential and output gap. This result is easy to understand. Productivity shocks alter the efficient level of output, hence a zero output gap requires effective output to vary accordingly. Given its cyclicality, relative wages need to move as well.

The second constraint illustrates that it is not feasible for the monetary authority to achieve the simultaneous stabilization of the welfare relevant output gap and inflation. Such trade-off emerges because in the presence of a minimum wage limit shocks to productivity alter the gap between the natural output and its efficient level. In effect, in the face of an adverse shock the minimum wage prevents the natural real wage to fall in line with its efficient counterpart. Accordingly, a positive gap between the efficient and natural outputs arise, and hence, conditional on output being at its efficient level, inflationary pressures are present.

By solving the constrained minimization problem the following optimal rule is obtained:

$$\Xi_y x_t + \Psi_c \Xi_w \hat{\omega}_t^r = -\kappa \Xi_\pi \hat{p}_t$$

where $\hat{p}_t \equiv p_t - p_{-1}$.

The optimal rule tells us that the central bank should tolerate a positive gap in output (or a positive inflation) in order to moderate a fall in relative wages (i.e., a rise in the consumption gap) and to tolerate a negative output gap in order to moderate a rise in inflation.

Next, I simulate the response of the economy to a negative productivity shock under the optimal Ramsey policy. Figure 1.7 presents the corresponding IRFs. Observe that in the benchmark economy redistribution is null and thus the efficient allocation is feasible. Consider next the economies with a minimum wage constraint. We know from (1.28) that when the economy is at its efficient level inflationary pressures are present as the minimum wage keeps wages too high. The central bank could curb inflation by inducing a negative gap in output. On the other hand, we know from (1.27) that if output reaches its efficient level, then relative wages become negative, and thus, the consumption gap turns positive, i.e., income redistributes in favor of unconstrained agents. The central bank could mitigate the fall in relative wages by allowing for a positive output gap. Given a higher weight on inflation relative to the wage gap in the loss function the optimal policy prescribes to induce a negative gap in output in order to contain inflationary pressures.

Comparison between economies B and C makes clear that idiosyncratic risk does not alter the optimal response of the economy, it only affects the path of the real interest rate required to implement the optimal allocation. This result is to be expected since the introduction of uncertainty does not modify any of the constraints the monetary authority faces when solving the loss minimization problem. Note that a in the economy featuring idiosyncratic risk a milder rise in the real rate is required. On the one hand, given the real rate, a drop in productivity reduces risk, which boost output. This implies a higher real rate is needed to achieve the negative output gap advised by optimal policy. On the other hand, we know that with a procyclical consumption gap monetary policy is more powerful when agents face idiosyncratic risk. This implies a lower real rate is required to achieve the prescribed negative output gap. Given that the latter effect from idiosyncratic uncertainty dominates, a milder rise in the real rate is required to achieve the prescribed contraction in output.

1.7 Conclusion

In this paper I investigate the redistributive role minimum wages can play over the cycle and the implied effects on macroeconomic stability. To this end, I develop a two-agent model featuring idiosyncratic uncertainty and limited asset markets participation. In my framework, the minimum wage constraint binds only for a subset of the workforce, which is thus subject to stronger wage rigidities. The latter implies that the minimum wage constraint alters the dynamics of relative sectoral wages (i.e. the wage gap between the minimum wage earners sector and the rest of the workforce), and thus, that of relative sectoral labor earnings, that is, it constitutes a source of income redistribution over the cycle.

Importantly, the key factor shaping the direction of redistribution is the elasticity of substitution across labor from the different sectors (ψ), which governs the response of relative employment to relative wages. Particularly, if $\psi = 1$, relative income is unresponsive to relative wages, and thus the income/consumption gap is acyclical. Differently, when $\psi < 1$ relative income moves in same direction of relative wages, then, given the procyclicality of the latter, the consumption gap will be procyclical as well and thus the minimum wage dampens fluctuations. On the other hand, if $\psi > 1$, relative income moves in the opposite direction of relative wages, then, given the procyclicality of the latter, the gap in consumption will be countercyclical, and we obtain amplification.

I find that under plausible calibrations of the model, the minimum wage has the potential to redistribute against the minimum wage earners during an economic decline due to employment losses it originates. In addition to its detrimental effects on low income individuals' welfare, redistribution can have a quantitatively relevant impact on aggregate spending and hence on the cyclical behavior of the economy. In particular, economic downturns can deepen, as a consequence of redistribution of current income against low income households, characterized by higher propensities to consume, and heightened uncertainty, which further depress spending by raising desired precautionary savings.

Given that minimum wages tend not to bind for large shares of the workforce the current redistribution channel has a limited impact. Conversely, uncertainty can, under certain circumstances, become quantitatively relevant. Particularly, whenever the income gap experiences a long-lasting rise, precautionary savings motives turn out to be sizable. The model simulations illustrate that such persistence can either be inherited from the shock or arise from a large divergence in the degree of wage rigidities across labor types.

Figures and tables

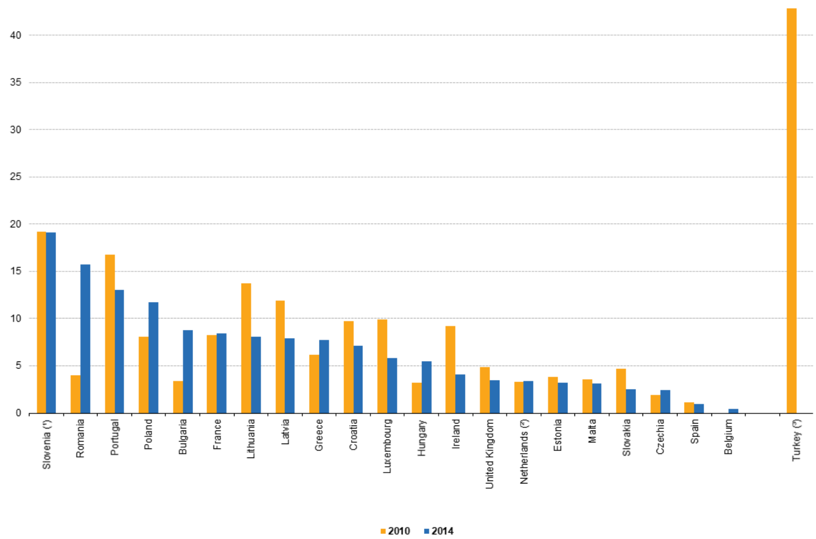
Spearman partial rank correlations: each LMI separately.

LMIs	Business cycle indicators					
	Var(y)	Var(n)	Var(u)	Var(w)	Var (y/n)	Var(π)
RR	−0.04	0.11	0.33***	0.15	−0.10	0.15
UD	0.50***	0.21*	0.10	0.60***	0.59***	0.50***
UC	0.03	0.03	0.15	0.18	0.17	0.19
CONC	0.23**	0.13	0.12	0.37***	0.26**	0.08
CENT	0.24**	0.13	−0.06	0.49***	0.32***	0.24**
WCOORD	0.25**	−0.03	−0.08	0.43***	0.32***	0.16
GOVINT	0.20*	0.29**	0.37***	0.35***	0.29**	0.31***
LEVEL	0.23*	0.14	0.15	0.46***	0.29**	0.21*
EXT	−0.30***	−0.11	0.18	−0.11	−0.20*	−0.14
MIN − WAGE	0.27**	−0.00	−0.08	0.45***	0.46***	0.26**
EPL	−0.09	0.00	0.21*	0.15	0.02	0.16
EPR	−0.08	−0.07	0.07	0.10	0.06	0.07
EPT	−0.08	−0.02	0.14	0.15	0.02	0.16

Note: Business cycle data are detrended by differencing the log series.
* Indicates correlations that are significant respectively at the 10% level.
** Indicates correlations that are significant respectively at the 5% level.
*** Indicates correlations that are significant respectively at the 1% level.

Source: Gnocchi et al. (2015)

Table 1.1: Minimum wages and macroeconomic volatility.



Note. Full-time employees, 21 years or older, working in enterprises with 10 employees or more, NACE Rev. 2 Sections B to S excluding Section O, apprentices excluded. Denmark, Germany, Italy, Cyprus, Austria, Finland and Sweden: no national minimum wage. Germany introduced minimum wage as of 1 January 2015.

(¹) In October 2010 each business entity could pay any amount between EUR 654.69 and EUR 734.15, so the proportion given is only an estimate. In October 2014 the minimum wage was EUR 789.15.
(²) The national minimum wage applies to employees aged 23 years or older, but the scope of this analysis covers employees aged 21 years or older for comparability with other countries.
(³) Data for 2014 not available.

Source: Eurostat (online data code: earn_mw_cur) and Structure of Earnings Survey 2014; special calculation made for the purpose of this publication; the special calculation's result data is not available in Eurostat's online database.

eurostat

Figure 1.1: Proportion of employees earning the minimum wage in EU Member States.

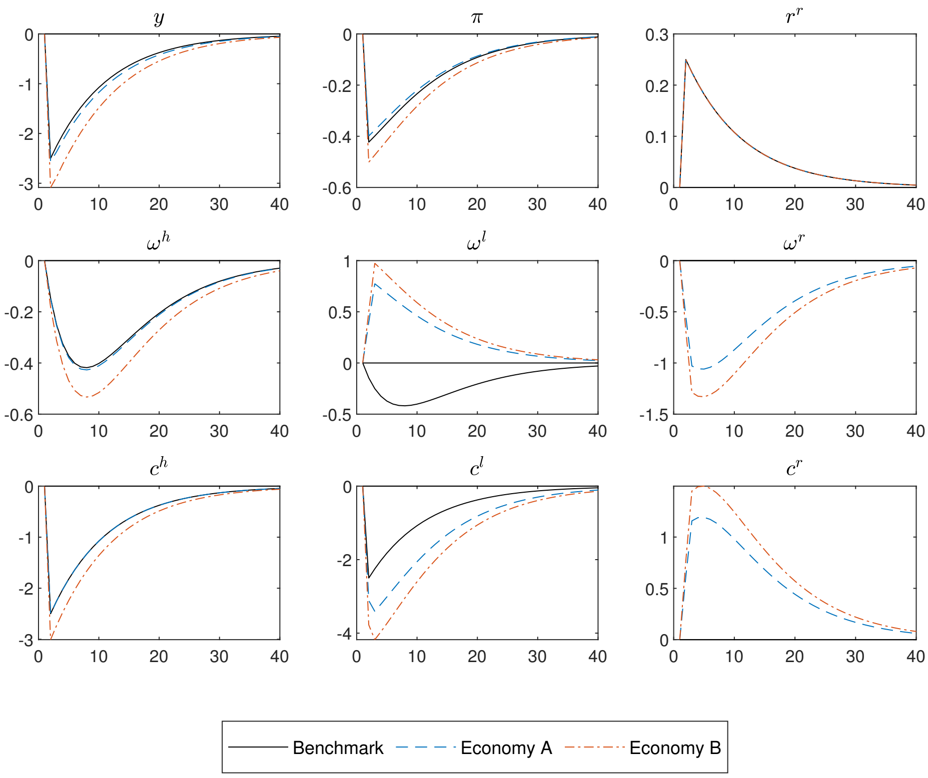


Figure 1.2: Monetary shock. IRFs correspond to an economy where the real interest rate follows and exogenous path.

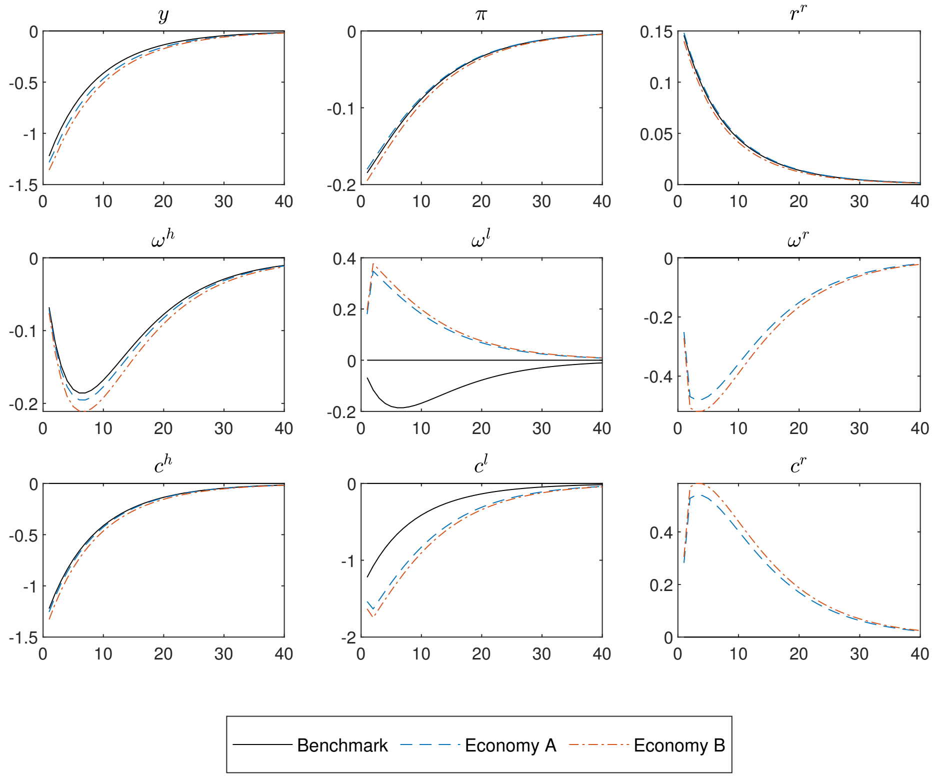


Figure 1.3: Monetary shock. IRFs correspond to an economy where the central bank follows a Taylor rule.

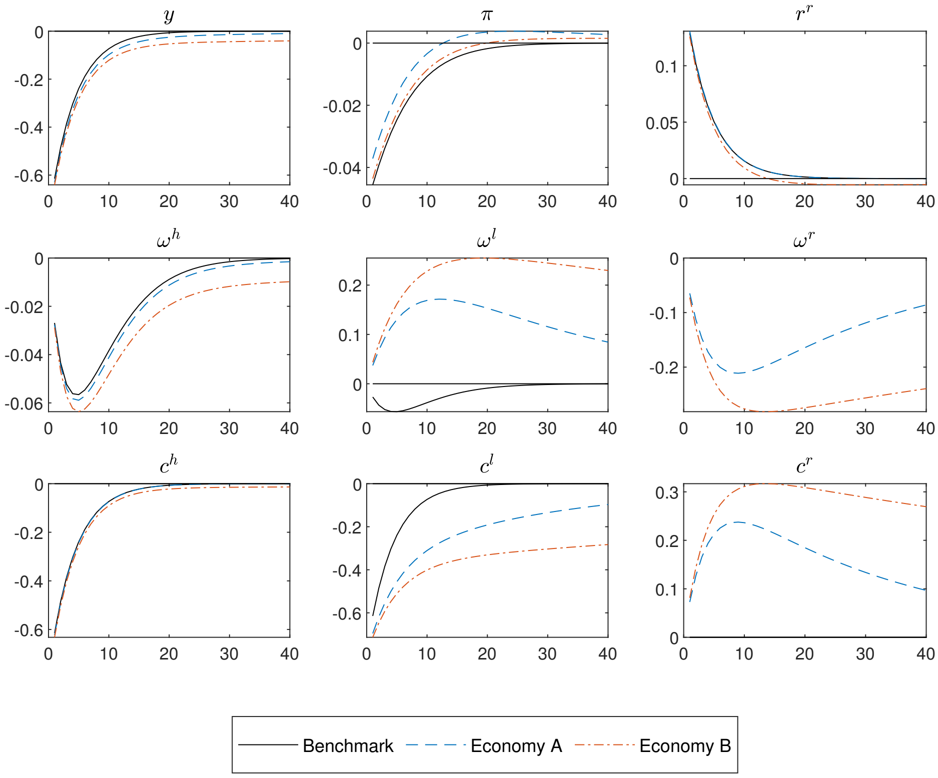


Figure 1.4: Monetary shock. IRFs correspond to an economy where the central bank follows a Taylor rule and the government has a target on the nominal minimum wage.

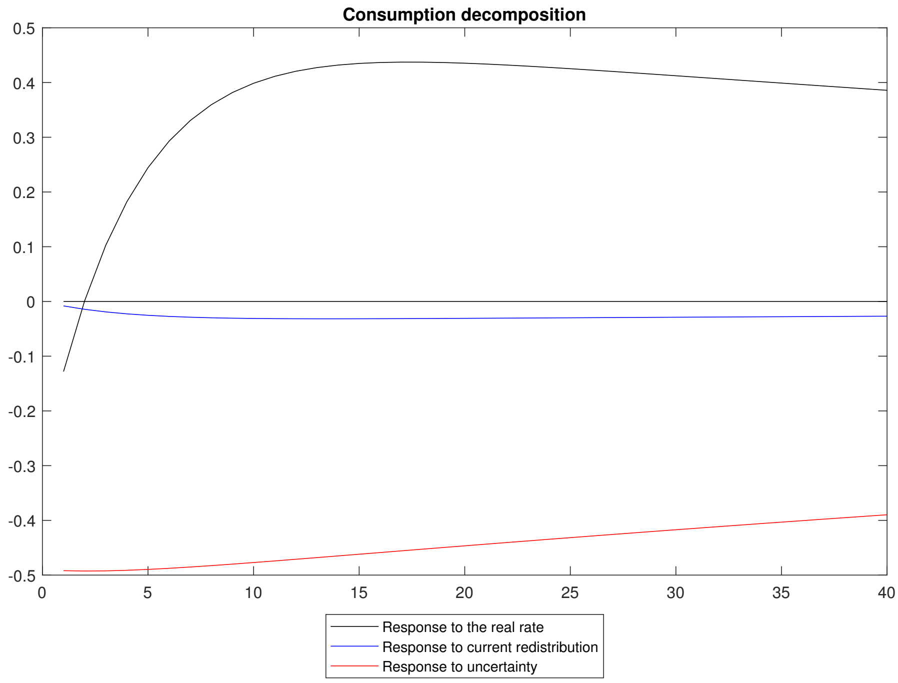


Figure 1.5: Monetary shock. IRFs correspond to an economy where the central bank follows a Taylor rule and the government has a target on the nominal minimum wage.

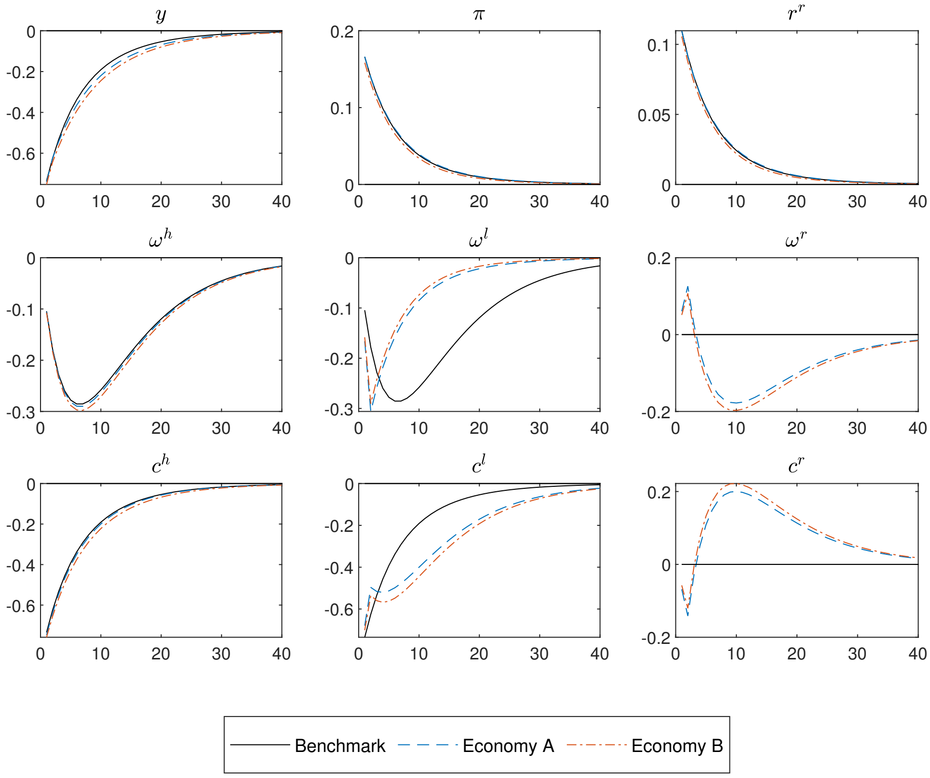


Figure 1.6: Productivity shock.

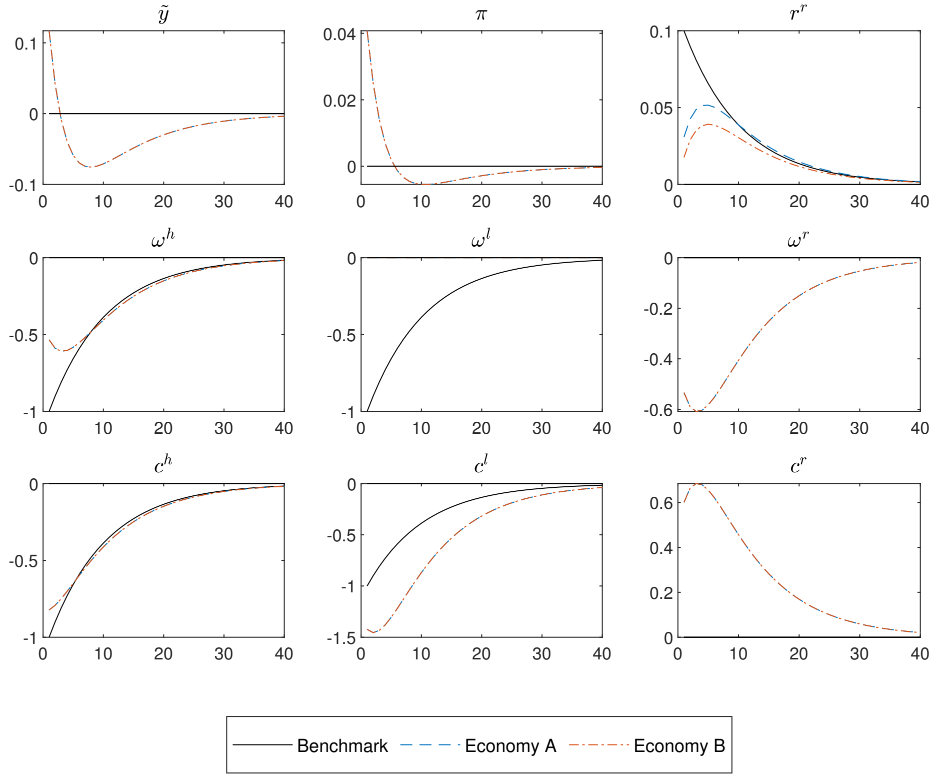


Figure 1.7: Productivity shock. IRFs under the optimal policy.

Chapter 2

OPTIMAL MONETARY POLICY WITH NON-HOMOTHETIC PREFERENCES

with Cesar Blanco*

2.1 Introduction

From the structural change literature, we know that sectoral composition changes as the economy grows, with the share of agricultural output falling as the country develops. Changes in sectoral composition can be explained by agents' preferences featuring non-homotheticity, resulting from the existence of a minimum consumption requirement for agricultural goods, which households need to satisfy for subsistence. In this paper we analyze the implications for the conduct of monetary policy of preferences incorporating sector specific minimum consumption requirements. To this end, we build a multisector model that combines features from the structural change and the New Keynesian literature. More specifically, we consider an economy with two sectors: agriculture and manufacturing. In the model, as a result of the introduction of a minimum consumption requirement for the former, agricultural goods demand has an income elasticity that is lower-than-

*Banco Central del Paraguay. E-mail: zesarb@gmail.com.

one, which implies that households' average and marginal expenditure composition will differ, and price elasticity is non-unitary. Regarding the new Keynesian features of the model, we consider an economy with sticky prices in both agriculture and manufacturing and flexible wages. In addition, we assume there is perfect labor mobility across sectors.

We find that the introduction of a minimum consumption requirement for agricultural goods alters the optimal measure of inflation that the monetary authority should target. More precisely, non-homotheticity results in a reduced weight on agricultural inflation in the optimal index. On the one hand, non-homothetic preferences turn stabilization of agricultural inflation more costly, as it requires larger deviations of output from the efficient level. In addition, proximity to the subsistence level implies a low income elasticity for agricultural goods. This translates into a reduced slope on total output in the Phillips curve for agriculture. As a consequence, a more aggressive policy is required to control inflation in this sector, which imposes costs as a stronger response of the central bank can destabilize the rest of the economy. An additional channel relates to the effects of non-homotheticity on the composition of the marginal consumption basket. We will see that this type of preferences implies households spend only a small share of any additional income on agricultural goods. This means prices in this sector have a reduced impact on households' decisions, such as the intertemporal allocation of consumption demand. Given that aggregate demand turns more unresponsive to the evolution of prices in agriculture, responding to inflation in this sector becomes less important.

Regarding the literature on optimal monetary policy in a multisector economy, Aoki (2001) provides one of the classical results. In a model where one of the sectors has sticky prices while the other is characterized by price flexibility, he finds that stabilizing inflation in the former is sufficient to achieve the efficient allocation. This analysis was expanded in Mankiw and Reis (2003). In that paper, the authors ask what is the measure of inflation that central banks should target in order to stabilize the economy. They establish the difference between the consumption price index and the stabilization price index. The former is weighted by the share of each good in the budget of consumers, and is used to measure the cost of living. The latter has an entirely different purpose. It assigns weights such that central banks can attain the maximum stability of economic activity. Their results indicate that central banks should weight a sector in the stabilization price index given its characteristics, which include not only price stickiness, but size, cyclical sensitivity and magnitude of sectoral shocks. In Benigno (2004), optimal monetary policy in a two region economy is studied. He finds that conditional on the degree of price stickiness being the same across regions, the central bank can replicate the optimal outcome by fully stabilizing a weighted average of regional inflations, with the weights coinciding with the size of the regions. On the other hand, if the degree of rigidities is different, a higher weight should be assigned to the region with a higher degree of stickiness.

More recent studies, and more closely related to ours, are those of Anand et al. (2015) and Portillo et al. (2016), who analyze monetary policy in a multisector economy which incorporates a minimum consumption requirement for agricultural goods. In the former, the authors consider segmented labor and incomplete credit markets. That is, workers cannot move across sectors in the economy, while households in the agricultural sector do not have access to banking services. They find that, under incomplete markets, it is optimal for the central bank to target headline inflation after a negative productivity shock in the agricultural sector. The reason is that such a shock increases real wages of households in this sector (due to a rise in agricultural goods relative prices). This in turn affects aggregate demand positively. To curb demand and price volatility, the central bank must include agricultural prices in their target. Portillo et al. (2016) consider a two sector model, one with sticky prices, and the other characterized by flexible prices. Their findings indicate that the introduction of a minimum consumption requirement does not alter the desirability of sticky price inflation targeting. In our paper, as opposed to Anand et al. (2015) and Portillo et al. (2016), we consider an economy where prices are sticky in both, agriculture and manufacturing. On the one hand, by ruling out differences in terms of nominal rigidities the minimum consumption requirement becomes the only source of heterogeneity across sectors, which makes this a suitable setting for the analysis of the implications of non-homotheticity. On the other hand, our assumption introduces a trade-off for the monetary authority, which turns the existence of sector specific minimum consumption requirements non-trivial. We justify the assumption of price rigidities in agriculture by considering that this category comprises both the processed and unprocessed food goods sectors.

Finally, Galesi and Rachedi (2016) study the effect of structural change on the transmission of monetary policy. They argue that structural change is accompanied by a process of services deepening, that is, both manufacturing and services become more intensive in inputs from the service sector. They also argue that prices in services are more sticky than in manufacturing. Therefore, structural transformation, from manufacturing to services, dampens the response of aggregate and sectoral inflation to monetary policy shocks.

The rest of the paper is organized as follows. The next section introduces the model. Section 3 illustrates the dynamics of the economy absent price rigidities. Section 4 explores the implications of non-homotheticity for the conduct of monetary policy. Section 5 performs a quantitative exercise. Finally, section 6 concludes.

2.2 The model

2.2.1 Firms

The economy consists of two sectors: agriculture and manufacturing, denoted by $s \in \{a, m\}$. In each sector there is a continuum of firms, indexed by $i \in [0, 1]$, each producing a single-differentiated good and with monopoly power to set prices. The production technology is given by:

$$Y_{s,t}(i) = A_{s,t} N_{s,t}(i)^{1-\alpha_s}$$

where $Y_{s,t}(i)$ is output and $N_{s,t}(i)$ represents labor input demanded for firm i in sector s. Productivity levels, denoted by $A_{s,t}$, are common across firms in the same sector.

In every period, firms in sector s reset prices with probability $(1 - \theta_s)$, as in Calvo (1983). A firm in sector s that last reset prices in period t, chooses the price that maximizes the following sum of discounted profits:

$$\sum_{k=o}^{\infty} \theta_s^k \mathbb{E}_t \left\{ Q_{t,t+k} \left(P^*_{s,t} Y_{s,t+k|t} - TC_{t+k}(Y_{s,t+k|t}) \right) \right\}$$

subject to the demand constraint given by:

$$Y_{s,t+k|t} = \left(\frac{P^*_{s,t}}{P_{s,t+k}} \right)^{-\epsilon_p} C_{s,t+k}$$

where $P^*_{s,t}$ is the optimal price of a firm that last reset its price at t, $Y_{s,t+k|t}$ is the output of that firm, $P_{s,t+k}$ is a price index, which we define later, and $C_{s,t+k}$ indicates total demand for goods from sector s. ϵ_p is the elasticity of substitution across goods varieties. The total cost of producing $Y_{s,t+k|t}$ units of output is defined as $TC_{t+k}(Y_{s,t+k|t}) \equiv W_{t+k} N_{s,t+k|t}$.

Maximization implies:

$$\sum_{k=o}^{\infty} (\theta_s)^k \mathbb{E}_t \left\{ Q_{t,t+k} Y_{s,t+k|t} \left(P^*_{s,t} - \mu_p MC_{s,t+k|t} \right) \right\} = 0 \tag{2.1}$$

where $MC_s \equiv \frac{\partial TC(Y_s)}{\partial Y_s}$ is the nominal marginal cost of producing one more unit of output in sector s and $\mu_p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ is the desired markup, common to both sectors.

2.2.2 Households

Lifetime utility of the representative household is given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^+)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

where C_t^+ is a consumption index and N_t represents labor supply. The consumption index is an aggregate of agricultural and manufacturing goods. It is defined as:

$$C_t^+ \equiv \Xi (C_{a,t} - \tilde{C}_a)^{\omega} C_{m,t}^{(1-\omega)}$$

where $\Xi \equiv (\omega^{\omega}(1-\omega)^{1-\omega})^{-1}$, while $C_{a,t}$ and $C_{m,t}$ are consumption indexes comprising the different varieties of goods available in each sector, and are defined as:

$$C_{s,t} \equiv \left(\int_0^1 C_{s,t}(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

where $C_{s,t}(i)$ denotes household's consumption of good variety i available in sector s.

Parameters ω and $1-\omega$ are the utility weights of agriculture and manufacturing and $\tilde{C}_a \geqslant 0$ is the agricultural minimum consumption requirement. When $\tilde{C}_a > 0$, preferences are non-homothetic.

Households' budget constraint is given by:

$$\int_0^1 P_{a,t}(i) C_{a,t}(i) di + \int_0^1 P_{m,t}(i) C_{m,t}(i) di + Q_t B_t = W_t N_t + B_{t-1} + \Pi_t$$

They receive labor income, $W_t N_t$, and profits, Π_t, from equal ownership of firms. They spend income on consumption and to accumulate the asset B_t, valued at price Q_t.

Intratemporal optimization

In each period, households determine the optimal consumption of good i from sector s given total expenditure in that sector. Optimization implies the following demand function:

$$C_{s,t}(i) = \left(\frac{P_{s,t}(i)}{P_{s,t}} \right)^{-\epsilon_p} C_{s,t} \tag{2.2}$$

where the price index in sector s is defined as $P_{s,t} \equiv \left(\int_0^1 P_{s,t}(i)^{1-\epsilon_p} di \right)^{\frac{1}{1-\epsilon_p}}$.

In turn, households' total demand of good s is given by:

$$C_{a,t} = \tilde{C}_a + \omega \left(\frac{P_{a,t}}{P_t^+} \right)^{-1} C_t^+ \tag{2.3}$$

and

$$C_{m,t} = (1-\omega)\left(\frac{P_{m,t}}{P_t^+}\right)^{-1} C_t^+ \tag{2.4}$$

where the aggregate price index is defined as:

$$P_t^+ \equiv P_{a,t}^{\omega} P_{m,t}^{(1-\omega)} \tag{2.5}$$

Notice from (2.3) that the existence of the agricultural minimum consumption requirement $\tilde{C}_a$ implies price and income elasticities of demand for the agricultural goods bundle that are lower than 1.

Using equation (2.2) we can derive the aggregate expenditure as:

$$E_t \equiv \int_0^1 P_{a,t}(i)C_{a,t}(i)di + \int_0^1 P_{m,t}(i)C_{m,t}(i)di = P_{a,t}C_{a,t} + P_{m,t}C_{m,t}$$

Besides, from households' optimal allocation problem we obtain the following relation $P_t^+C_t^+ = E_t - P_{a,t}\tilde{C}_a$. Using this expression we can rewrite the budget constraint as:

$$P_t^+C_t^+ + Q_tB_t = W_tN_t + B_{t-1} + \Pi_t - P_{a,t}\tilde{C}_a \tag{2.6}$$

where $P_t^+C_t^+$ is households' total expenditure excluding the value of the minimum consumption requirement, $P_{a,t}\tilde{C}_a$.

Average and marginal expenditure composition

From equations (2.3) and (2.4) we obtain the following expressions relating expenditure on agricultural and manufactured goods with total expenditure:

$$P_{a,t}\left(C_{a,t} - \tilde{C}_a\right) = \omega\left(E_t - P_{a,t}\tilde{C}_a\right)$$

$$P_{m,t}C_{m,t} = (1-\omega)\left(E_t - P_{a,t}\tilde{C}_a\right)$$

Let $\tilde{E}_t \equiv E_t - P_{a,t}\tilde{C}_a$ denote income remaining after the agricultural minimum consumption requirement has been covered. By differentiating the two previous expressions with respect to $\tilde{E}_t$ we get:

$$\frac{\partial\big(P_{a,t}\big(C_{a,t}-\tilde{C}_a\big)\big)}{\partial\tilde{E}_t} = \omega$$

$$\frac{\partial(P_{m,t}C_{m,t})}{\partial\tilde{E}_t} = 1 - \omega$$

The above derivatives tell us that once the agricultural minimum requirement has been covered, households will spend a fraction ω of any additional income on agricultural goods and a fraction $1 - \omega$ on manufactured products. We call these the *marginal* expenditure shares.

Average expenditure composition on the other hand is given by:

$$\eta_t = \frac{P_{a,t}\tilde{C}_a + \omega\tilde{E}_t}{E_t} = \omega + (1-\omega)\frac{P_{a,t}\tilde{C}_a}{E_t}$$

$$1 - \eta_t = \frac{(1-\omega)\tilde{E}_t}{E_t} = (1-\omega) - (1-\omega)\frac{P_{a,t}\tilde{C}_a}{E_t}$$

where $\eta_t \equiv \frac{P_{a,t}C_{a,t}}{E_t}$ is the average expenditure share in agriculture.

If $\tilde{C}_a > 0$ (and therefore $\tilde{E}_t < E_t$), the marginal expenditure share of agricultural goods is smaller than its average expenditure share, i.e., $\frac{\partial\big(P_{a,t}\big(C_{a,t}-\tilde{C}_a\big)\big)}{\partial\tilde{E}_t} = \omega < \eta_t$. This occurs because households spend all of their income on agricultural goods up to the point where their subsistence needs are covered and only then they will spend a fraction ω of any additional income on agricultural products.

The marginal expenditure share of manufactured goods on the other hand is larger than its average share, i.e., $\frac{\partial(P_{m,t}C_{m,t})}{\partial\tilde{E}_t} = 1 - \omega > 1 - \eta_t$. This occurs because households only begin to spend a fraction $1-\omega$ of any additional income on manufactured goods once they have covered their subsistence needs.

Notice that with homothetic preferences we have $\tilde{C}_a = 0$ and therefore $\tilde{E}_t = E_t$. Then the marginal and average expenditure shares are the same:

$$\frac{\partial(P_{a,t}C_{a,t})}{\partial\tilde{E}_t} = \omega = \eta_t$$

$$\frac{\partial(P_{m,t}C_{m,t})}{\partial\tilde{E}_t} = 1 - \omega = 1 - \eta_t$$

Intertemporal problem

Maximization of lifetime utility subject to (2.6) implies the following Euler equation:

$$Q_t = \beta \mathbb{E}_t \left\{ \left(\frac{C^+_{t+1}}{C^+_t} \right)^{-\sigma} \frac{P^+_t}{P^+_{t+1}} \right\} \tag{2.7}$$

Notice that the relevant price index for households' intertemporal allocation is given by (2.5), that is, a price index that weights agricultural and manufactured goods prices according to the composition of households' marginal rather than average consumption basket. Since for intertemporal allocation decisions agents care about the marginal utility of consumption over time, the relevant price index is that of the marginal consumption basket, given by P^+_t, which correctly weighs agricultural and manufactured goods by their shares in marginal expenditure.

Labor supply

Intratemporal optimization implies:

$$\frac{W_t}{P^+_t} = (C^+_t)^{\sigma} N^{\varphi}_t \tag{2.8}$$

Note that for labor supply the relevant price index is also given by (2.5). Again, for labor supply decisions agents care about the marginal utility of consumption and therefore about the price of the marginal consumption basket, given by P^+_t.

2.2.3 Aggregate output and inflation

Due to the existence of a minimum consumption requirement for agricultural goods, the consumption index C^+_t will not be a good measure of aggregate consumption (and output).

We introduce therefore a measure of aggregate output, which we define as the index of sectoral production weighed by their steady state relative prices:

$$Y_t \equiv \tfrac{P_a}{P} Y_{a,t} + \tfrac{P_m}{P} Y_{m,t}$$

Similarly, we introduce a measure of the aggregate price level, which we call the measured price index, defined as the index of sectoral prices weighed by the corresponding steady state sectoral production levels:

$$P_t \equiv \tfrac{Y_a}{Y} P_{a,t} + \tfrac{Y_m}{Y} P_{m,t}$$

Differently from P_t^+, the measured price index weighs sectoral prices according to the steady state (or average) rather that the marginal expenditure shares.

2.2.4 The central bank

The central bank sets the nominal rate following a simple interest rate rule, given by:

$$R_t = \frac{1}{\beta}(\frac{\Pi_t^*}{\Pi^*})^{\phi_\pi} \tag{2.9}$$

where $R_t = Q_t^{-1}$ is the nominal interest rate.

The measure of inflation that the central bank targets is defined as $\Pi_t^* \equiv \Pi_{a,t}^{\Omega} \Pi_{m,t}^{1-\Omega}$, where Ω is the weight assigned to agricultural inflation.

2.2.5 Shocks

The model includes temporary shocks to agricultural, manufacturing and aggregate productivity. The exogenous process for sector s is given by:

$$A_{s,t} = A_s e^{a_{s,t}}$$

The shock $a_{s,t}$ evolves according to:

$$a_{s,t} = \rho_s a_{s,t-1} + \nu_{s,t} + \nu_{am,t}$$

where $\nu_{s,t}$ and $\nu_{am,t}$ are respectively the sectoral and aggregate IID innovations with zero mean and standard deviation σ_{vs} and σ_{vam}. Parameter ρ_s determines shock persistence and A_s is the sectoral steady state productivity level.

2.2.6 The linearized system

In this section we present the log-linear approximation (around the zero inflation steady state) of the system of equations that describe our economy.

The sectoral Phillips curves and production functions are given by:

$$\pi_{s,t} = \lambda_s \hat{mc}_{s,t} + \beta \mathbb{E}_t \pi_{s,t+1}$$

and

$$\hat{y}_{s,t} = a_{s,t} + (1 - \alpha_s)\hat{n}_{s,t}$$

Labor supply is:

$$\hat{\omega}_t^+ = \sigma \hat{c}_t^+ + \varphi \hat{n}_t$$

where $\omega_t^+ = log\left(\frac{W_t}{P_t^+}\right)$. Aggregate employment and output are:

$$\hat{n}_t = \frac{N_a}{N}\hat{n}_{a,t} + \frac{N_m}{N}\hat{n}_{m,t}$$

and

$$\hat{y}_t = \eta \hat{y}_{a,t} + (1 - \eta)\hat{y}_{m,t}$$

where $\eta \equiv \frac{P_a Y_a}{E}$ is the steady state share of agriculture in total expenditure. The Taylor rule is given by:

$$\hat{r}_t = \phi_\pi \hat{\pi}_t^* = \phi_\pi(\Omega \hat{\pi}_{a,t} + (1 - \Omega)\hat{\pi}_{m,t})$$

The relation between aggregate output and the consumption index is (see Appendix):

$$\hat{c}_t^+ = \frac{1-\omega}{1-\eta}\hat{y}_t$$

where $\frac{1-\omega}{1-\eta} > 1$.

Inflation associated with price indexes P_t and P_t^+ is given by:

$$\pi_t = \eta \pi_{a,t} + (1 - \eta)\pi_{m,t}$$

and

$$\pi_t^+ = \omega \pi_{a,t} + (1 - \omega)\pi_{m,t}$$

we will refer to π_t as the measured inflation, as this index resembles the manner total inflation is measured in practice, i.e., by aggregating sectoral inflation in accordance to the average expenditure shares.

The Euler equation is given by:

$$\hat{c}_t^+ = -\frac{1}{\sigma}\mathbb{E}_t\left(\hat{r}_t - \hat{\pi}_{t+1}^+\right) + \mathbb{E}_t\hat{c}_{t+1}^+$$

and the sectoral demands are the following:

$$\hat{c}_{a,t} = \frac{\omega(1-\eta)}{(1-\omega)\eta}(-(1-\omega)\hat{p}_{r,t} + \hat{c}_t^+)$$

and

$$\hat{c}_{m,t} = \omega\hat{p}_{r,t} + \hat{c}_t^+$$

where $\frac{\omega(1-\eta)}{(1-\omega)\eta} < 1$ and $p_{r,t} \equiv p_{a,t} - p_{m,t}$ represents relative prices.

2.3 The flexible price economy

We begin by studying the implications of the minimum agricultural consumption requirement in the flexible price economy. For the present analysis we assume $\alpha_a = \alpha_m$. One can show that (see Appendix) absent nominal rigidities the response of the economy to the sectoral productivity shocks is given by:

$$\hat{y}_{a,t}^n = \Upsilon_a a_{a,t}$$

$$\hat{y}_{m,t}^n = \Upsilon_m a_{a,t} + a_{m,t}$$

$$\hat{y}_{a,t}^n - \hat{y}_{m,t}^n = \Upsilon_{ry} a_{a,t} - a_{m,t}$$

$$\hat{p}_{r,t}^n = -\frac{1-\alpha\Upsilon_{ry}}{1-\alpha} a_{a,t} + a_{m,t}$$

$$\hat{y}_t^n = \left(\eta\Upsilon_a + (1-\eta)\Upsilon_m\right) a_{a,t} + (1-\eta) a_{m,t}$$

$$(\hat{\omega}_t^+)^n = \Upsilon_\omega a_{a,t} + (1-\omega) a_{m,t}$$

where the superscript n denotes natural levels, i.e., variables level under flexible prices,

$\Upsilon_a \equiv \frac{1+\varphi}{\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)\varphi(1-\eta)+\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)+\varphi\eta+\alpha\varphi(1-\eta)+\alpha}$,

$\Upsilon_m \equiv \frac{\left(\frac{(1-\omega)\eta}{\omega(1-\eta)}-1\right)(1-\alpha)\varphi\eta}{\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)\varphi(1-\eta)+\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)+\varphi\eta+\alpha\varphi(1-\eta)+\alpha}$,

$\Upsilon_\omega \equiv \frac{\omega}{1-\alpha}\left(1-\alpha(\Upsilon_a+\frac{1-\omega}{\omega}\Upsilon_m)\right)$ and

$\Upsilon_{ry} \equiv \frac{1+\varphi-\left(\frac{(1-\omega)\eta}{\omega(1-\eta)}-1\right)(1-\alpha)\varphi\eta}{\left(\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)+\alpha\right)+\left(\left(\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)+\alpha\right)(1-\eta)+\eta\right)\varphi}$.

With homothetic preferences we have that:

$$\Upsilon_a = 1,\ \Upsilon_m = 0,\ \Upsilon_{ry} = 1,\ \Upsilon_\omega = \eta,\ \eta\Upsilon_a + (1-\eta)\Upsilon_m = \eta \text{ and } \frac{1-\alpha\Upsilon_{ry}}{1-\alpha} = 1$$

Non-homothetic preferences on the other hand imply:

$$\Upsilon_a < 1,\ \Upsilon_m > 0,\ \Upsilon_{ry} < 1,\ \Upsilon_\omega < \eta,\ \eta\Upsilon_a + (1-\eta)\Upsilon_m < \eta \text{ and } \frac{1-\alpha\Upsilon_{ry}}{1-\alpha} > 1$$

Let us first consider the manner the minimum agricultural consumption requirement alters the response of the flexible price economy to a (negative) agricultural productivity shock. In the economy with non-homothetic preferences the proximity to the minimum consumption requirement implies it is costly for households to reduce the consumption of agricultural goods. As a consequence, agents offset the effect of lower productivity on agricultural output by moving labor from manufacturing towards this sector. This results in a contained fall in agricultural production at the cost of a larger contraction in manufactured output relative to the economy characterized by homothetic preferences.

The behavior of sectoral output explains the response of the sectoral output differential $\hat{y}^n_{a,t} - \hat{y}^n_{m,t}$. With homothetic preferences manufacturing output remains invariant while agricultural production moves one to one with changes in agricultural productivity. As stated above, when preferences are non-homothetic agricultural output falls by less while output in manufacturing reduces by more after the negative shock in agriculture. As a consequence, the sectoral output differential becomes less responsive to shocks in agriculture with non-homothetic preferences.

The weaker response of the output differential implies a larger volatility in relative prices with non-homothetic preferences. The explanation is as follows. Variations in agricultural sector productivity have an effect on relative prices through two different channels. One is the direct effect of productivity on marginal costs. This channel implies agricultural relative prices increase after the negative shock in this sector. The second channel relates to the effect of productivity on relative marginal costs through its impact on relative sectoral output. With homothetic preferences, given that relative agricultural output moves in the same direction as agricultural productivity, this channel implies a reduction in relative agricultural marginal costs after the adverse shock in this sector that offsets the direct effect of productivity on relative prices. With non-homothetic preferences the response of the output differential is more moderate and hence the indirect channel weakens. Note that the higher variability of relative prices under non-homothetic preferences requires $\alpha > 0$, since the impact of relative sectoral output on relative sectoral marginal costs depends on labor returns being decreasing. In the particular case when $\alpha = 0$, the response of relative prices is independent of the type of preferences.

Finally, total output and the real wage are less volatile when preferences are non-homothetic. We postpone the explanation of this result to a later section.

Regarding the behavior under flexible prices when shocks are to manufacturing productivity, notice that the response for the economies featuring homothetic and non-homothetic preferences is identical, except for the behavior of the real wage. More precisely, the real wage becomes more volatile in the non-homothetic case. An explanation for this result is presented in a following section as well.

2.4 Monetary policy with non-homothetic preferences

Next we evaluate the implications of non-homotheticity for the conduct of monetary policy. To this end we compare our economy with preferences incorporating the minimum agricultural consumption requirement to a benchmark economy featuring homothetic preferences. To isolate the effects from non-homotheticity we will start by considering economies where agricultural and manufacturing sectors are identical, except for the existence of the minimum consumption requirement in the former. Later, when performing a quantitative exercise, we adopt a more realistic calibration.

2.4.1 Calibration

For the baseline calibration we assume $\sigma = 1$ and $\varphi = 1$, which are common values in the literature. The discount factor β is set to 0.99, which implies an annual interest rate of 4%. The elasticity of substitution across goods varieties ϵ_p is set to 6, implying a markup of 1.2 in steady state. The same production technology is assumed for both sectors, i.e., $\alpha_a = \alpha_m = 0.25$. We assume $\theta_a = \theta_m = 3/4$, implying an average price duration of four quarters.

For the economy featuring non-homothetic preferences we set parameter ω, related to the share of agricultural goods in marginal expenditures, to 0.05, that is, households will spend only 5% of any additional income on agricultural goods. The minimum consumption requirement for agricultural goods $\tilde{C}_a$ and the steady state sectoral productivity levels A_a and A_n are set targeting a steady state share of agriculture in total expenditure of 50%. Our purpose is to compare this economy to another having the same steady state share of agriculture in total expenditure but featuring homothetic preferences. Consequently, for the economy characterized by homothetic preferences we set $\omega = \eta = 0.5$, implying that both, the share of agriculture in total expenditure in steady state (or on average) and in the margin is 50%.

We set the response to inflation in the Taylor rule ϕ_π to 1.5. The productivity shock parameters are set to $\rho_a = \rho_m = 0.9$ and $\sigma_{va} = \sigma_{vm} = \sigma_{vam} = 0.02$.

2.4.2 Results

IRFs

Figure 2.1 displays the IRFs to a negative agricultural productivity shock for the economies featuring homothetic and non-homothetic preferences. The central bank is assumed to follow a Taylor rule with the weights on sectoral inflation coinciding with sectoral sizes, i.e. $\Omega = 0.5$. Note that in the economy with homothetic preferences there is a large contraction in agricultural output while manufacturing production falls moderately. This results from a rise in the policy rate, that responds to agricultural goods inflation, and an increase in relative prices, which switches consumption from agricultural to manufactured goods. Differently, in the economy with non-homothetic preferences agricultural production falls slightly while output in manufacturing experiences a large contraction. This results from the low income and price elasticities of agricultural goods demand. Such behavior is in line with the preceding analysis of the flexible price economy. Given the proximity to the minimum consumption requirement, labor moves from manufacturing to agriculture to prevent a fall in agricultural goods consumption, but at the cost of a large contraction in manufacturing. Note also the higher volatility of sectoral inflation in the economy featuring non-homothetic preferences. We postpone the explanation of this outcome to a later section.

In the remainder of the paper we evaluate the implications of the minimum consumption requirement for the optimal design of monetary policy. We will consider three scenarios, the optimal policy under commitment, the optimal design when we restrict to fully optimized simple rules and, finally, the optimal choice of Ω under a standard Taylor rule.

Optimal policy under commitment

In this section we explore the optimal policy under commitment. To this end we derive the welfare loss function for the model economy incorporating a minimum consumption requirement. For its derivation we assume $\alpha = 0$, as this is required to obtain an analytical expression for welfare losses. In addition, we assume the existence of a labor subsidy that corrects for the inefficiencies generated by monopolistic competition in the goods market. By performing a second order approximation of households' utility around the efficient steady state welfare losses can be expressed as:

$$L_t = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left\{ \varXi_y \tilde{y}_t^2 + \varXi_{rp} (\tilde{p}_{r,t})^2 + \varXi_a \pi_{a,t}^2 + \varXi_m \pi_{m,t}^2 \right\}$$

where $\tilde{y}_t$ and $\tilde{p}_{r,t}$ represent output and relative prices in deviation from their natural levels, $\varXi_y \equiv \frac{1-\omega}{1-\eta} + \varphi$, $\varXi_{rp} \equiv \omega(1-\eta)$, $\varXi_a \equiv \eta \frac{\epsilon_p}{\lambda_a}$ and $\varXi_m \equiv (1-\eta)\frac{\epsilon_p}{\lambda_m}$.

Welfare losses result from deviations of output and relative prices from their natural counterparts, as well as from sectoral inflation. We have seen that homothetic and non-homothetic preferences imply $\omega = \eta$ and $\omega < \eta$, respectively. Accordingly, the weight on the output gap is higher when preferences are non-homothetic, reflecting higher costs associated to output variations, which result from agricultural consumption being close to its subsistence requirement. Given the agricultural expenditure share η, the weight associated to the gap in relative prices is smaller with non-homothetic preferences. This results from a smaller degree of substitutability associated to this preferences. Also, given the agricultural expenditure share η, weights on sectoral inflation are not affected by the type of preferences. These weights are directly related to the degree of sectoral rigidities, reflected in λ_s, and the sectoral expenditure shares.

The loss function is minimized subject to the following constraints:

$$\pi_{a,t} = \lambda_a \left(\frac{1-\omega}{1-\eta} + \varphi \right) \tilde{y}_t - \lambda_a (1-\omega) \tilde{p}_{r,t} + \beta \mathbb{E}_t \pi_{a,t+1} \tag{2.10}$$

$$\pi_{m,t} = \lambda_m \left(\frac{1-\omega}{1-\eta} + \varphi \right) \tilde{y}_t + \lambda_m \omega \tilde{p}_{r,t} + \beta \mathbb{E}_t \pi_{m,t+1} \tag{2.11}$$

$$\tilde{p}_{r,t} = \tilde{p}_{r,t-1} - \Delta p^n_{r,t} + \pi_{a,t} - \pi_{m,t} \tag{2.12}$$

Equations (2.10) and (2.11) are the sectoral Phillips curves, while (2.12) reflects the evolution of the relative price gap. According to (2.12) the monetary authority faces a trade-off whenever a shock impacts on natural relative prices $p^n_{r,t}$. In effect, if $\Delta p^n_{r,t} \neq 0$ it will not be possible to simultaneously stabilize sectoral inflation and the gap in relative prices. Note that given the implications of non-homotheticity for the relation between ω and η, the nature of preferences affects both, the weights in the welfare loss function and the constraints.

The FOCs of the minimization problem are the following:

$$-\varXi_y \tilde{y}_t - \vartheta_{1,t} \lambda_a \left(\tfrac{1-\omega}{1-\eta} + \varphi \right) - \vartheta_{2,t} \lambda_m \left(\tfrac{1-\omega}{1-\eta} + \varphi \right) = 0$$

$$-\varXi_a \pi_{a,t} + \vartheta_{1,t} - \vartheta_{1,t-1} - \vartheta_{3,t} = 0$$

$$-\Xi_m \pi_{m,t} + \vartheta_{2,t} - \vartheta_{2,t-1} + \vartheta_{3,t} = 0$$

$$-\Xi_{rp}(\tilde{p}_{a,t} - \tilde{p}_{m,t}) + \vartheta_{1,t}\lambda_a(1-\omega) - \vartheta_{2,t}\lambda_m\omega + \vartheta_{3,t} - \beta\mathbb{E}_t\vartheta_{3,t+1} = 0$$

where $\vartheta_{1,t}$, $\vartheta_{2,t}$ and $\vartheta_{3,t}$ are the Lagrange multipliers associated to the constraints.

Next we impose equal degree of price stickiness across sectors, implying $\lambda_a = \lambda_m = \lambda$. Under this assumption the minimum consumption requirement in agriculture becomes the only source of sectoral heterogeneity. For this particular scenario the following optimality condition is derived from the FOCs:

$$\pi_t = \eta\pi_{a,t} + (1-\eta)\pi_{m,t} = -\frac{1}{\epsilon_p}\Delta\tilde{y}_t \qquad (2.13)$$

Now let us focus on the optimal policy solution for the case of homothetic preferences. From the sectoral Phillips curves we derive the following Phillips equation for aggregate inflation:

$$\pi_t = \lambda\left(\tfrac{1-\omega}{1-\eta} + \varphi\right)\tilde{y}_t - \lambda(\eta(1-\omega) - (1-\eta)\omega)\tilde{p}_{r,t} + \beta\mathbb{E}_t\pi_{t+1}$$

With homothetic preferences $\omega = \eta$ holds, then the above expression reduces to:

$$\pi_t = \lambda\left(\frac{1-\omega}{1-\eta} + \varphi\right)\tilde{y}_t + \beta\mathbb{E}_t\pi_{t+1} \qquad (2.14)$$

Equation (2.14) tells us that it is feasible for the monetary authority to simultaneously achieve the full stabilization of measured inflation and the output gap.

By solving the system of equations composed by (2.13) and (2.14) we get that the optimal policy under commitment effectively prescribes to set $\pi_t = \tilde{y}_t = 0\ \forall t$.

Notice that $\pi_t = 0$ implies $\eta\pi_{a,t} + (1-\eta)\pi_{m,t} = 0$, that is, the optimal policy prescribes to fully stabilize an inflation index with the weights coinciding with the size of the sectors. This result accords the findings in Benigno (2004).

Next we compare the IRFs under the Ramsey policy for the homothetic and non-homothetic cases. For this exercise we follow our baseline calibration. Figure 2.2 presents the IRFs corresponding to a negative productivity shock in agriculture. The negative shock results in inflationary pressures for agricultural goods. To contain the rise in prices the central bank implements a contractive policy, which, by containing wage inflation, offsets the effects of productivity on agricultural prices but at the cost of provoking deflation in manufacturing. The type of preferences will determine to what extent the central bank finds it optimal to counter the effects of the fall in productivity on agricultural inflation. In line with our previous results, measured inflation and the output gap are fully stabilized when preferences are homothetic. Given $\eta = 0.5$ (i.e. equal sectoral sizes), setting $\pi_t = 0$ implies $\pi_{a,t} = -\pi_{m,t}$, that is, equal inflation volatility across sectors is advised by the optimal policy. Differently, in the economy featuring non-homothetic preferences the optimal policy prescription is biased towards the stabilization of manufacturing inflation. In the remainder of this section we seek to clarify this result.

Note that with homothetic preferences the slopes on relative prices in the sectoral Phillips curves are given respectively by $\lambda(1 - \omega^H) = \lambda(1 - \eta)$ and $\lambda\omega^H = \lambda\eta$ (here we use the superscripts H and NH to denote homothetic and non-homothetic preferences, respectively). Then, conditional on $\tilde{y} = 0$, the sectoral Phillips curves imply $\frac{\pi_{a,t}}{\pi_{m,t}} = \frac{1-\omega^H}{\omega^H} = \frac{1-\eta}{\eta}$. Since the sectoral size is directly related to the weight on sectoral inflation in the loss function, a policy that implies an inverse relation between sectoral sizes and sectoral inflation variability (i.e. $\frac{\pi_{a,t}}{\pi_{m,t}} = \frac{1-\eta}{\eta}$) will minimize inflation related losses. As such policy is compatible with $\tilde{y} = 0$, setting $\frac{\pi_{a,t}}{\pi_{m,t}} = \frac{1-\eta}{\eta}$ is prescribed by the optimal policy. In the particular case presented in figure 2.2, the latter result implies that setting $\pi_{a,t} = -\pi_{m,t}$, which minimizes inflation related losses, is compatible with $\tilde{y} = 0$, which minimizes losses related to the gap in output. Consequently, equal inflation volatility across sectors is advised by the optimal policy.

Differently, with non-homothetic preferences the slopes on relative prices are given respectively by $\lambda(1-\omega^{NH}) > \lambda(1-\eta)$ and $\lambda\omega^{NH} < \lambda\eta$. Then, conditional on $\tilde{y} = 0$, we get from the sectoral Phillips curves that $\frac{\pi_{a,t}}{\pi_{m,t}} = \frac{1-\omega^{NH}}{\omega^{NH}} > \frac{1-\eta}{\eta}$. Accordingly, given the relative sectoral sizes, setting $\tilde{y} = 0$ implies a too volatile inflation in agriculture relative to manufacture. The previous result also tells us that further stabilizing agricultural inflation would be costly, as it requires to induce a gap in output. Going back to our simulation in figure 2.2, this implies that the central bank finds it optimal to allow for a higher volatility in agricultural relative to manufacturing inflation, as further containing inflation in the former would require an increase in the volatility of the output gap.

We have seen that, conditional on output being at its natural level, non homothetic preferences imply inflationary pressures in agriculture are too high. Behind this result is the different behavior of wages according to the nature of preferences. To make this point clear we need to look at the behavior of real wages in the flexible price economy, which, as stated in section 3, become less responsive to shocks in agriculture when preferences are non-homothetic. Next we explore in more detail the link between such result and the optimal policy design.

Notice that absent nominal rigidities sectoral labor demand is given by:

$$(\hat{\omega}_t^{+})^n - (\hat{p}_{s,t}^{+})^n = a_{s,t} = \widehat{mrt}_{s,t}^{\,n}$$

where $p_{s,t}^{+} = log\left(\frac{P_{s,t}}{P_t^{+}}\right)$ and $\widehat{mrt}_{s,t}^{\,n}$ denotes the sectoral marginal rate of transformation.

From the sectoral demand schedules the following expression for aggregate demand is obtained:

$$(\hat{\omega}_t^{+})^n = \omega a_{a,t} + (1-\omega) a_{m,t} = \widehat{mrt}_t^{\,n} \tag{2.15}$$

with $\widehat{mrt}_t^{\,n} \equiv \omega \widehat{mrt}_{a,t}^{\,n} + (1-\omega)\widehat{mrt}_{m,t}^{\,n}$ denoting the aggregate marginal rate of transformation.

According to equation (2.15) the nature of preferences affects the response of labor demand to shocks. Particularly, given $\omega^{NH} < \omega^{H}$, demand turns less sensitive to variations in agricultural productivity under non-homothetic preferences. In the particular case of a fall in agricultural productivity, labor demand and therefore the real wage fall by less. Then, conditional on output (and the real wage[1])

[1]The following relation between the output and real wage gaps $\tilde{\omega}_t = \left(\frac{1-\omega}{1-\eta} + \frac{\varphi}{1-\alpha}\right)\tilde{y}_t$ imply that when output is at its natural level, so is the real wage.

being at their natural levels, wages and therefore inflationary pressures are higher in the economy with non-homothetic preferences. This implies the central bank will need to contract output below the efficient level to further reduce wages, and therefore, inflationary pressures.

Intuitively, given that in the margin the share of agricultural goods in the consumption basket is small in the context of non-homothetic preferences, the aggregate marginal rate of transformation is reduced only slightly after the fall in agricultural productivity. As a result, the wage firms are willing to pay only moderately reduces as well. But this means wages are high when output is at its natural level, thus, inflationary pressures in agriculture are high as well.

Now let's consider the response to a shock in manufacturing productivity, reported in figure 2.3. As before, when preferences are non-homothetic the optimal policy prescription is biased towards the stabilization of manufacturing inflation. This result can also be explained by the slopes on the relative price gap. Conditional on output being at its natural level, inflationary pressures are higher in agriculture. Accordingly, further containing inflation in this sector would require to induce a gap in output.

The analysis of the flexible price economy can also provide an intuition for this result. With shocks to manufacturing productivity, the marginal rate of transformation, and therefore, the natural real wage, become more responsive when preferences are non-homothetic. The explanation for this result can also be found in the composition of the marginal consumption basket. Since the marginal share of manufactured goods is higher with this type of preferences, productivity variations in this sector alter the aggregate marginal rate of transformation by more. Accordingly, after a negative shock in manufacturing the natural real wage falls by more. Then, deflationary pressures in agriculture are higher.

The optimal simple rule

In this section we restrict ourselves to the analysis of fully optimized simple rules. We compute the optimal simple rule allowing for the central bank to choose both, the weight on agricultural inflation, Ω, in its targeted index, and the strength of the policy rate response to inflation, determined by ϕ_π. We find that for the homothetic and non-homothetic cases welfare losses are minimized when the central bank fully stabilizes (at zero) the targeted inflation index, i.e. when $\phi_\pi \to \infty$, with Ω set to 0.5 and 0.46, respectively.

With the purpose to better understand our results, next we compute losses associated to alternative choices of Ω, under the assumption that the central bank fully stabilizes the targeted index.

Figures 2.4 and 2.5 show the results. Shocks are to both, agricultural and manufacturing productivity. The horizontal axis represents the weight on agricultural inflation Ω while the vertical axis represents consumption equivalent welfare losses. Losses are decomposed according to their source. The blue and red lines represent losses resulting from inflation in agriculture and manufacturing, respectively, while the green line represents the sum of sectoral inflation related losses. The purple and light blue lines correspond to losses associated to the gaps in output and relative prices and the black line represents total losses. First let us examine the results for the economy with homothetic preferences, illustrated in figure 2.4. In line with findings in the previous section, setting $\Omega = 0.5$ minimizes losses related to both the output gap and inflation. Consider now the economy characterized by non-homothetic preferences, presented in figure 2.5. If the central bank only cared about minimizing losses related to inflation, setting $\Omega = 0.5$ would be optimal, just as in the homothetic case. Yet, if the central bank were uniquely concerned about losses related to the output gap it would be optimal to set $\Omega = 0.05$. Note that under this choice of Ω the output gap if fully stabilized. Here we can observe the same trade-off noticed in the previous section. Conditional on output being at its natural level, inflationary pressures in agriculture are too high, then the central bank needs to induce a gap in output in order to further stabilize it. Taking into account all sources of losses we get $\Omega^* = 0.46$[2], which lies very close to the optimal Ω under homothetic preferences. A much higher weight in the loss function on sectoral inflation relative to the output gap explains this result. Notice that even when non-homotheticity does not considerable alter the policy prescription, it does imply that controlling inflation becomes more costly. In fact, as figure 2.6 illustrates, setting $\Omega = 0.46$ implies the monetary authority will have to tolerate a large volatility in the output gap. Finally, observe that un-

[2] We use the superscript $*$ to denote the optimal choice of parameter Ω.

der the optimized reaction function, the Ramsey optimal allocation is replicated under homothetic preferences, whilst in the non-homothetic case welfare losses are essentially the same as those corresponding to the Ramsey outcome (whose corresponding welfare losses are indicated by an horizontal dashed line).

Also important to mention is that when sectoral shocks are considered separately the same parametrization of the Taylor rule, i.e. $\Omega^* = 0.46$, delivers the optimal outcome (not shown).

The optimal Ω when the central bank follows a standard Taylor rule

Now we limit our attention to the optimal policy design when the central bank follows a standard Taylor rule. In particular, the rules considered will be of the form of (2.9), where we will allow for the central bank to choose the weight on agricultural inflation Ω in the targeted index, while keeping ϕ_π fixed to its baseline calibration. As before, shocks are to both, agricultural and manufacturing productivity. Interestingly, as figure 2.8 shows, in this setting a lower weight on agriculture is advised (in particular, $\Omega^* = 0.37$) when preferences are non-homothetic, while $\Omega = 0.5$ remains optimal under homotheticity (shown in figure 2.7).

The reduction in the optimal Ω relates to the role monetary policy plays in the control of inflation expectations. To see this, recall the Euler equation is given by:

$$\hat{c}_t^+ = -\mathbb{E}_t \left(\hat{r}_t - \hat{\pi}_{t+1}^+ \right) + \mathbb{E}_t \hat{c}_{t+1}^+$$

where $\hat{\pi}_{t+1}^+ = \omega \hat{\pi}_{a,t+1} + (1-\omega)\hat{\pi}_{m,t+1}$, that is, the inflation index that is relevant for households' intertemporal allocation is one that weights sectoral inflation according to the marginal expenditure composition.

Since the dynamics of $\hat{\pi}_t^+$ influence aggregate demand, it is desirable for the central bank to respond to this index. In the economy characterized by homothetic preferences $\omega = \eta$ holds, and therefore $\hat{\pi}_t^+ = \hat{\pi}_t$. Accordingly, by targeting an index with the weights coinciding with sectoral sizes the central bank is, at the same time, responding to inflation expectations. Differently, with non-homothetic preferences we have $\omega < \eta$. Then responding to inflation expectations requires to target an index that assigns agriculture a weight that falls below its share in total expenditure. Hence, given the sectoral composition of the economy, a lower Ω is desirable if preferences are non-homothetic.

To provide support to the claim that expectations underlie our results in this section we perform the following exercise. Consider a setup where the central bank neutralizes any effect from inflation expectations on households' demand by moving the nominal rate one-to-one with the latter. In particular, let us assume a reaction function of the form $\hat{r}_t = \mathbb{E}_t\hat{\pi}^+_{t+1} + \phi_\pi(\Omega\hat{\pi}_{a,t} + (1-\Omega)\hat{\pi}_{m,t})$. By computing the optimal weight on agricultural inflation in this setting we obtain $\Omega^* = 0.57$, i.e., the bias towards the stabilization of manufacturing inflation vanishes. The fact that the optimal Ω goes beyond the value prescribed under the optimal Ramsey policy indicates the existence of an additional force associated with non-homotheticity whose effect opposes that of expectations.

Our exercise in the present section is also useful to illustrate an additional implication of the minimum consumption requirement. Comparison between figures 2.7 and 2.8 makes clear that welfare costs resulting from departures from the optimal Ramsey policy are particularly high under non-homotheticity. The direct conclusion is that the lack of an adequate policy response to inflation dynamics will result particularly costly if preferences adopt this form (this would be the case of developing economies which are close to the minimum consumption requirement and thus are characterized by highly non-homothetic preferences).

The optimal policy when $\alpha > 0$

In this section we abandon the assumption of a linear production technology. Such assumption is key as setting $\alpha = 0$ breaks any link between relative sectoral output and relative sectoral marginal costs. Since the nature of preferences significantly alters the dynamics of relative sectoral output, by disregarding such link we could be missing relevant aspects of non-homotheticity.

Figure 2.9 presents the response to a negative shock in agriculture under the Ramsey policy with parameter α set to its baseline calibration[3]. Comparison with figure 2.2 illustrates that the assumed production technology is in fact relevant,

[3]The optimal Ramsey policy is computed by maximizing households' lifetime utility subject to the non-linear system describing the private sector optimality conditions.

since in the present setup the bias towards the stabilization of manufacturing inflation exacerbates.

To understand this result, note that under $\alpha > 0$, the sectoral Phillips curves, given by equations (2.10) and (2.11), become:

$$\begin{aligned}\pi_{a,t} = \lambda \left(\frac{1-\omega}{1-\eta} + \varphi \frac{1}{1-\alpha} + \frac{\alpha}{1-\alpha} \frac{1-\omega}{1-\eta} \frac{\omega(1-\eta)}{(1-\omega)\eta} \right) \tilde{y}_t \\ -\lambda \left(1 + \frac{\alpha}{1-\alpha} \frac{\omega(1-\eta)}{(1-\omega)\eta} \right) (1-\omega) \tilde{p}_{r,t} + \beta \mathbb{E}_t \pi_{a,t+1}\end{aligned} \quad (2.16)$$

$$\begin{aligned}\pi_{m,t} = \lambda \left(\frac{1-\omega}{1-\eta} + \varphi \frac{1}{1-\alpha} + \frac{\alpha}{1-\alpha} \frac{1-\omega}{1-\eta} \right) \tilde{y}_t \\ +\lambda \left(1 + \frac{\alpha}{1-\alpha} \right) \omega \tilde{p}_{r,t} + \beta \mathbb{E}_t \pi_{m,t+1}\end{aligned} \quad (2.17)$$

Given $\frac{\omega(1-\eta)}{(1-\omega)\eta} < 1$, the slope on the output gap reduces for agricultural relative to manufactured goods inflation. The explanation for this result is as follows. Conditional on $\alpha > 0$, changes in sectoral output affect sectoral marginal costs, and consequently, sectoral inflation. With non-homothetic preferences demand for agricultural goods becomes more income inelastic relative to demand for manufactured goods. Given that agricultural goods demand is relatively less sensitive to income (i.e. to aggregate output), so is the marginal cost, and hence inflation, in this sector. Going back to figure 2.9, this means that inducing a negative gap in output will translate into only small gains in terms of containing inflation in agriculture relative to losses associated to deflation in manufacturing that output contraction would provoke. As a consequence, stabilizing agricultural inflation becomes less desirable.

Let us now compute the optimal Ω in the present setting (i.e. with $\alpha > 0$), assuming the central bank follows a standard Taylor rule. For this exercise we compute households' welfare under the different regimes relative to the Ramsey optimal policy. We denote by Λ to utility losses associated to the adoption of a particular Taylor rule relative to the optimal Ramsey policy. More precisely, as in Schmitt-Grohé and Uribe (2007), Λ is defined as the fraction of consumption under the Ramsey policy that household should renounce for welfare under the optimal policy and the alternative regime to be equated, i.e.:

$$V_0^T = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(log((1-\Lambda)C_t^R) - \frac{(N_t^R)^{1+\varphi}}{1+\varphi} \right)$$

where T and R denote variables under the Taylor rule and the Ramsey policy regimes respectively.

Λ can by computed according to the following formula[4]:

$$\Lambda = 1 - e^{(1-\beta)\left(V_0^T - V_0^R\right)}$$

Figure 2.10 reports the results for the economy with the minimum consumption requirement and both, agricultural and manufacturing productivity shocks as source of fluctuations. Note that the optimal policy assigns a weight of only 0.28 to agricultural inflation, significantly below for the 0.5 prescribed under homothetic preferences.

Importantly, when the central bank follows a Taylor rule, decreasing returns play an additional role, which relates to the response of natural relative prices to shocks. In section 2.3 we stated that natural relative prices are more responsive to variations in agricultural productivity when preferences are non-homothetic. There we explained that the stronger response results from the effect of non-homotheticity on the the behavior of relative sectoral output ($\hat{y}_{a,t}^n - \hat{y}_{m,t}^n$). Since the nature of preferences does not alter the response of relative sectoral output when shocks are to manufacturing productivity, it does not affect the response of relative prices either. Then, conditional on sectoral productivities having the same volatility, shocks in agriculture will result in higher volatility in natural relative prices as compared to shocks in manufacturing when preferences are non-homothetic.

It is the differentiated impact of sectoral shocks on natural relative prices (together with a reduced slope for the agricultural goods Phillips curve) what lays behind the effects of non-homotheticity on the optimal Ω. In effect, in a multisector economy where the central bank follows a Taylor rule it is desirable to target an inflation index where the sectoral weights are in inverse relation to the volatility of the corresponding sectoral shocks. But natural relative prices being

[4] V_0^T and V_0^R are approximated by computing the second order accurate solution of the model.

more responsive to agricultural relative to manufacturing sector shocks is equivalent to shocks in agriculture being relatively more volatile, since it is through their impact on natural relative prices that shocks to productivity have an effect on the economy. Equations (2.16), (2.17) and (2.12), describing the supply side of the economy, make this point clear. Productivity shocks impact on natural relative prices, which affect the gap in relatives prices, which in turn determines sectoral inflation dynamics.

Optimal policy with aggregate shocks

At last, we explore the implications of the minimum consumption requirement when aggregate productivity shocks hit the economy. With aggregate shocks, non-homotheticity will alter the behavior of the economy due to its effects on the dynamics of natural relative prices. More precisely, assuming a common shock the evolution of natural relative prices is given by:

$$\hat{p}_{r,t}^{n} = \frac{\alpha(1-\Upsilon_{ry})}{1-\alpha} a_t$$

When preferences are homothetic we have $\Upsilon_{ry} = 1$, and therefore $\hat{p}_{r,t}^{n} = 0$. Differently, with non-homothetic preferences $\Upsilon_{ry} < 1$ holds, and thus $\hat{p}_{r,t}^{n} \neq 0$.

Intuitively, non-homotheticity results in variations of natural relative prices because it entails a differentiated sensitivity of sectoral outputs to the shock. Since the economy is close to its subsistence level, agricultural output is less responsive than production in manufacturing. Under the assumption of decreasing returns to labor, this implies marginal costs and therefore prices in agriculture will be relatively less sensitive, then, natural relative prices will vary.

Given this result, equation (2.12) tells us that the optimal allocation is achievable when preferences are homothetic. Differently, if the economy is characterized by non-homothetic preferences aggregate shocks will alter natural relative prices, giving rise to a trade-off for the monetary authority.

Next, let us consider the response of the economy to a negative aggregate productivity shock under the Ramsey policy, presented in figure 2.11. In the economy with homothetic preferences the central bank can fully stabilize the gaps in output and relative prices as well as sectoral inflation. In the economy with non-homothetic preferences, the rise in natural relative prices imply inflationary pressures are higher in agriculture. Accordingly, stabilizing inflation in this sector requires to provoke deflation in manufacturing. More interestingly, notice that, analogous to the case of sectoral shocks, the Ramsey policy prescribes a bias towards the stabilization of manufacturing inflation. The same explanation from the previous sections applies here. Conditional on output being at its natural level, inflationary pressures are higher in agriculture when preferences are non-homothetic, which implies that further stabilization can only be achieved at the cost of depressing output below its efficient level. Additionally, the slope on the output gap in the Phillips curve is lower for agricultural relative to manufacturing inflation, implying that further stabilizing inflation in the former can only be achieved by provoking a large deflation in the latter.

2.5 Quantitative simulations

2.5.1 Calibration

In previous sections symmetry across sectors was imposed to isolate the effects from the minimum consumption requirement. Next, with the purpose to quantitatively assess the relevance of policy prescriptions under non-homotheticity, a more realistic calibration is adopted. Particularly, we assume prices in agriculture are more flexible relative to manufacturing. Based on results in Alvarez et al. (2006), who find that food prices (comprising processed and unprocessed products) are revised roughly twice as frequently as non-food goods prices, we set $\theta_a = 0.5$ and $\theta_m = 0.75$. In addition, to capture the fixed nature of the land input in agriculture we set $\alpha_a > 0$, while $\alpha_m = 0$ is assumed to reflect constant returns to scale in manufacturing. More precisely, we follow Gollin et al. (2007) and Gollin and Rogerson (2014), who employ shares of land ranging from 0.3 to 0.4. Since in our model agriculture encompasses the processed and unprocessed food sectors, the share of land, α_a, is set to 0.2. We will consider alternative calibrations for the steady state sectoral technology A_s. More precisely, we examine what occurs as A_s grows and the economy moves away from the subsistence consumption level. As we will see, as A_s increases, agricultural consumption will grow relative to its minimum requirement, that is, $\frac{\tilde{C}_a}{C_a} \to 0$, and the steady state expenditure composition will converge to the marginal shares, i.e., $\eta \to \omega$. In other words, as sectoral productivity grows the economy converges to one with homothetic preferences.

2.5.2 The optimal Ω

In this section we compute the optimal Ω when the central bank follows a standard Taylor rule. Figure 2.12 depicts the results, with agricultural and manufacturing productivity shocks as source of fluctuations. The horizontal axis represents the steady state share of agricultural goods in total expenditure η while the vertical axis represents the optimal weight on agricultural inflation in the Taylor rule. The share of agriculture in total expenditure will be related to the relevance of the non-homothetic component. More precisely, a larger η is associated with a higher ratio of the minimum requirement to total agricultural consumption, given by $\frac{\tilde{C}_a}{C_a}$, i.e., the larger η is, the closer the economy will be to its minimum consumption requirement. Table 2.1 shows the ratio $\frac{\tilde{C}_a}{C_a}$ as a function of η. To better assess the effects of the minimum consumption requirement figure 2.12 also depicts the optimal Ω as a function of η for the case of homothetic preferences. We can observe that the optimal choice of Ω differs for the scenarios with homothetic and non-homothetic preferences, with the difference increasing in η (and $\frac{\tilde{C}_a}{C_a}$). For the particular case of $\eta = 0.5$, figure 2.13 plots welfare losses (Λ) as a function of Ω under non-homothetic preferences. Note that the optimal outcome with homothetic and non-homothetic preferences is achieved by setting respectively $\Omega = 0.45$ and $\Omega = 0.30$.

At last, we compare the response of the economy under a Taylor rule to that under the Ramsey policy. First, we consider a scenario where the central bank ignores the nature of preferences, and thus, chooses always the Ω that is optimal under homotheticity. Figures 2.14 and 2.15 report the simulations for the homothetic and non-homothetic cases, respectively. When the central bank follows a Taylor rule, as can be expected, sectoral inflation becomes more volatile than under the Ramsey policy, and, importantly, the more so if preferences are non-homothetic. Recall the same pattern of increased sectoral inflation volatility under non-homotheticity was observed in figure 2.1. The present exercise, and that in section 2.4.2, illustrate the relevance of controlling inflation expectations. In the economy with the minimum consumption requirement, manufacturing prices disproportionally influence inflation expectations, and accordingly, by choosing the same Ω as in the homothetic case, the central bank fails to respond to expectations, and thus, to stabilize the economy. Figure 2.15 also reports the response with the optimal Ω under non-homotheticity[5]. Observe that such policy, which takes into account the nature of preferences, visibly improves stability.

2.6 Conclusion

In this paper we study how monetary policy should be conducted in a multi-sector model where agents' preferences are non-homothetic. Non-homotheticity derives from the existence of a minimum consumption requirement for agricultural goods, which households need to satisfy for subsistence. The minimum consumption requirement results in a lower-than-one income elasticity, which implies households' average and marginal expenditure composition will differ, and a non-unitary price elasticity.

We find that the introduction of the minimum consumption requirement alters the measure of inflation that the monetary authority should target. More precisely, non-homotheticity results in a reduced weight for agricultural inflation in the optimal index. On the one hand, this type preferences turns stabilization of agricultural sector inflation more costly, as this requires larger deviations of output from the efficient level. In addition, proximity to the subsistence level imply a low income elasticity for agricultural goods. This translates into a reduced slope on total output in the Phillips curve for agricultural inflation. As a consequence, a more aggressive policy is required to control inflation in this sector, which imposes costs as a stronger response of the central bank can destabilize the rest of the economy. An additional channel relates to the control of inflation expectations. We have seen that inflation expectations will be relatively less responsive to the evolution of prices in agriculture. Then responding to inflation in this sector becomes less important.

[5] The optimal Ω under homothetic and non-homothetic preferences amount respectively to 0.325 and 0.075.

Figures and tables

η	$\frac{\tilde{C}_a}{C_a}$
0.1	0.53
0.2	0.79
0.3	0.88
0.4	0.92
0.5	0.95

Table 2.1: Relative size of the minimum consumption requirement and steady state expenditure composition.

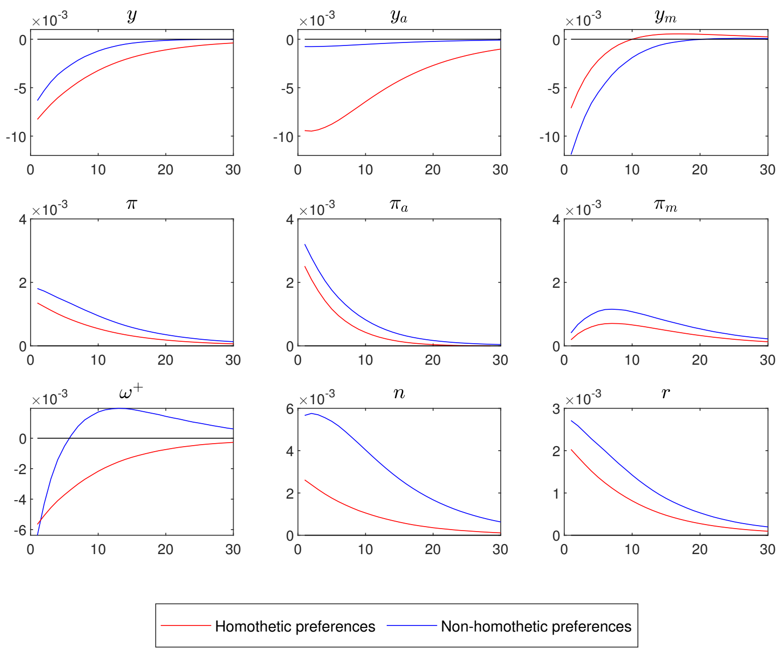


Figure 2.1: Shock to agricultural productivity. The central bank follows a Taylor rule.

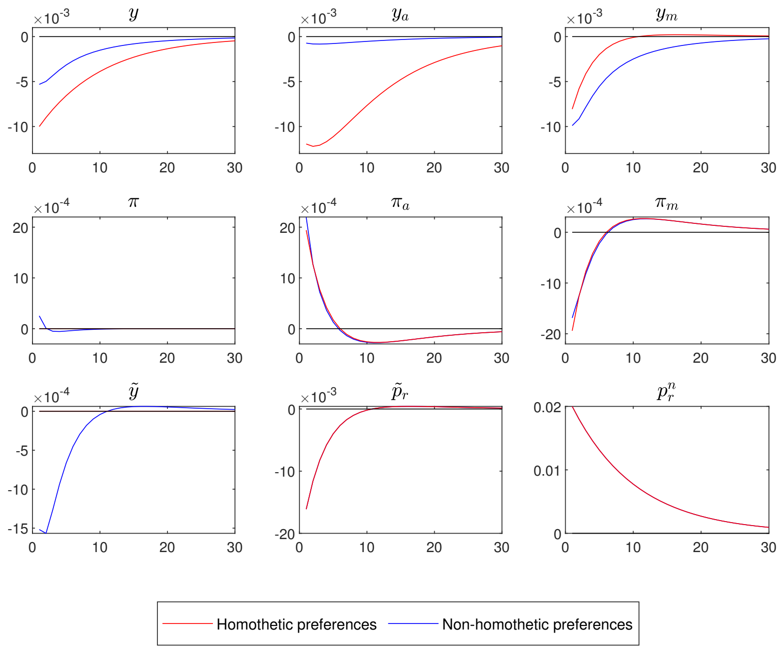


Figure 2.2: Shock to agricultural productivity. Optimal policy under commitment.

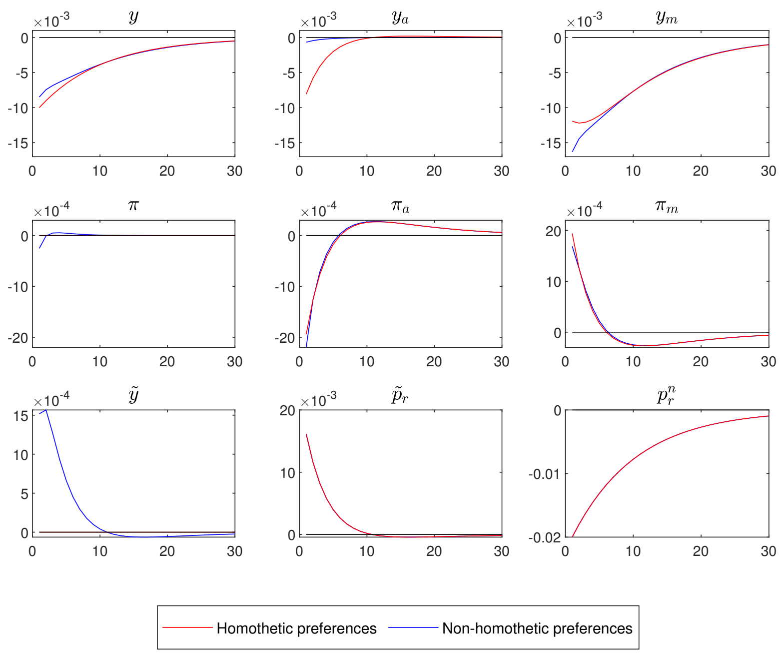


Figure 2.3: Shock to manufacturing productivity. Optimal policy under commitment.

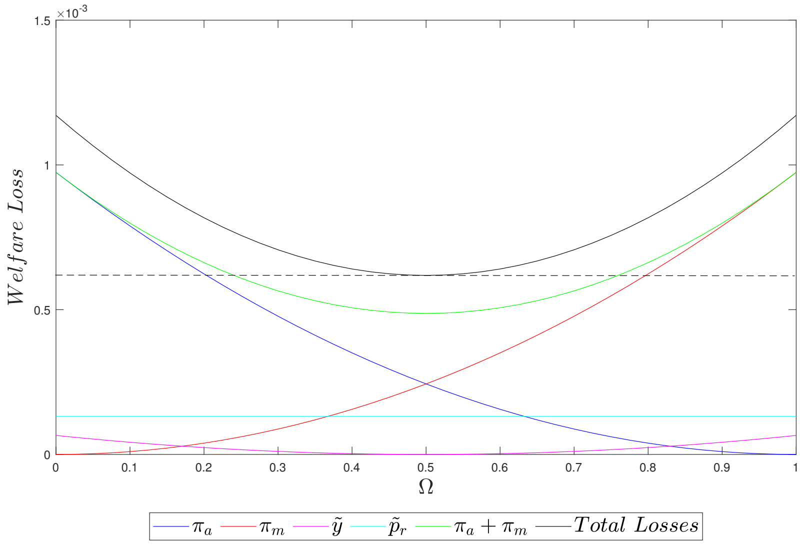


Figure 2.4: Optimal simple rule. Homothetic preferences. The horizontal dashed line represents losses under the Ramsey policy.

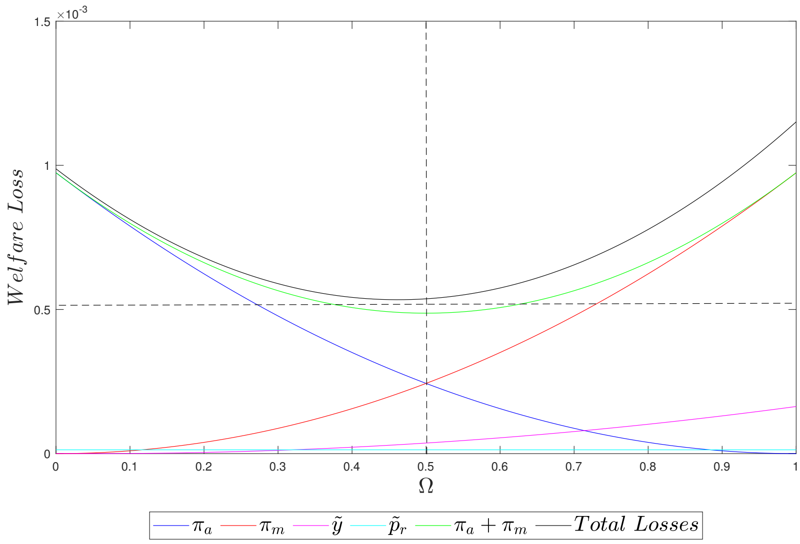


Figure 2.5: Optimal simple rule. Non-homothetic preferences. The vertical dashed line indicates the optimal Ω under homothetic preferences while the horizontal dashed line represents losses under the Ramsey policy.

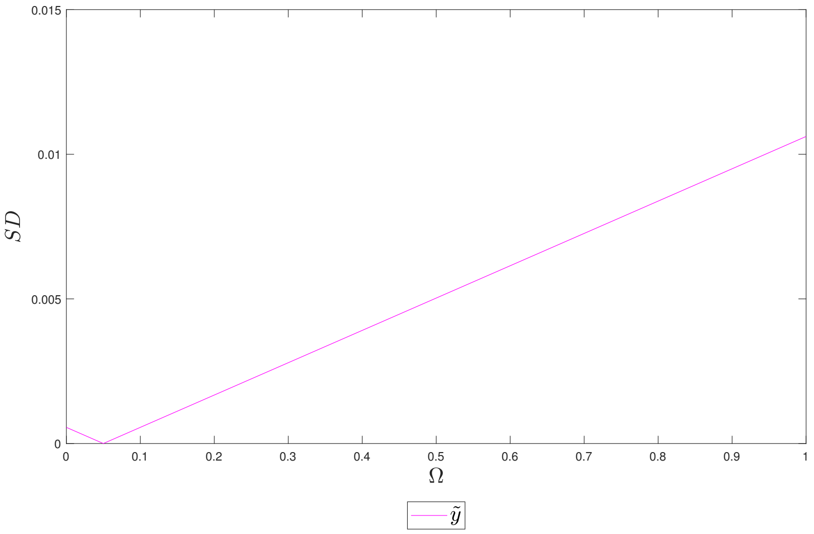


Figure 2.6: Optimal simple rule. Non-homothetic preferences. Output gap volatility.

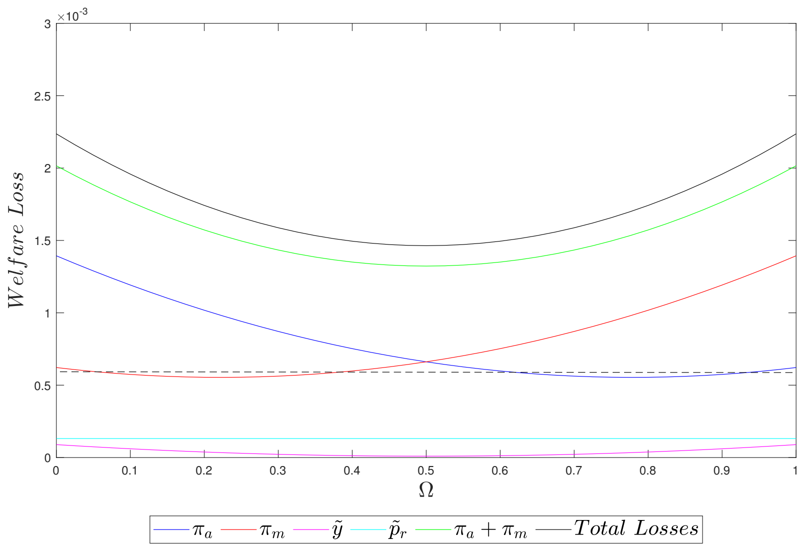


Figure 2.7: Optimal Ω under a standard Taylor rule. Homothetic preferences. The horizontal dashed line represents losses under the Ramsey policy.

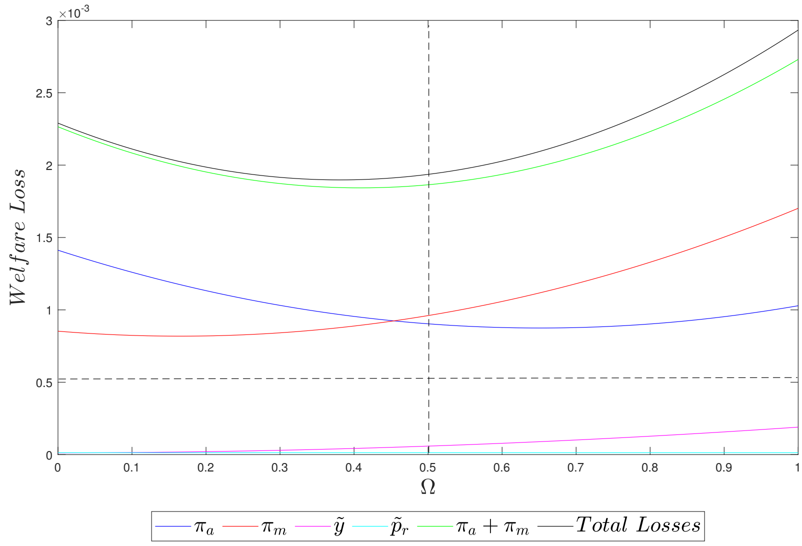


Figure 2.8: Optimal Ω under a standard Taylor rule. Non-homothetic preferences. The vertical dashed line indicates the optimal Ω under homothetic preferences while the horizontal dashed line represents losses under the Ramsey policy.

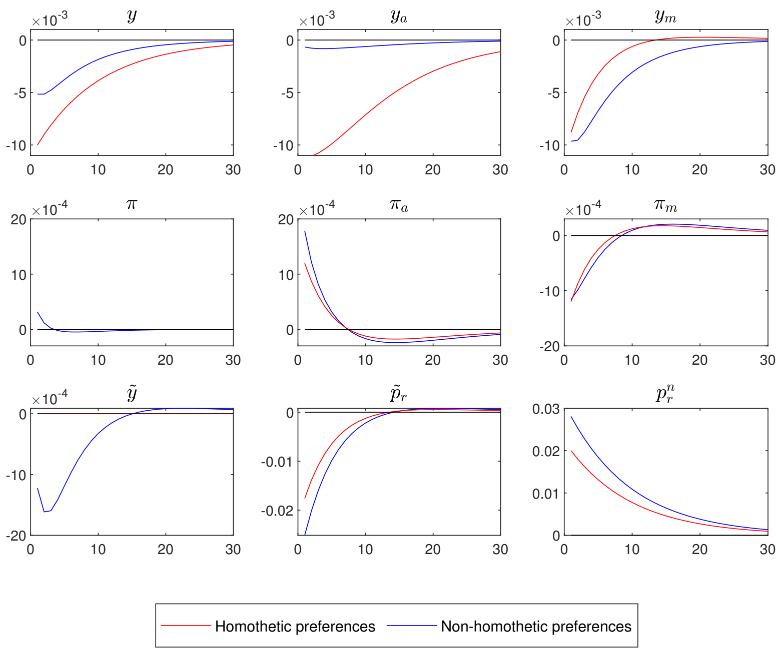


Figure 2.9: Shock to agricultural productivity. Optimal policy under commitment when $\alpha > 0$.

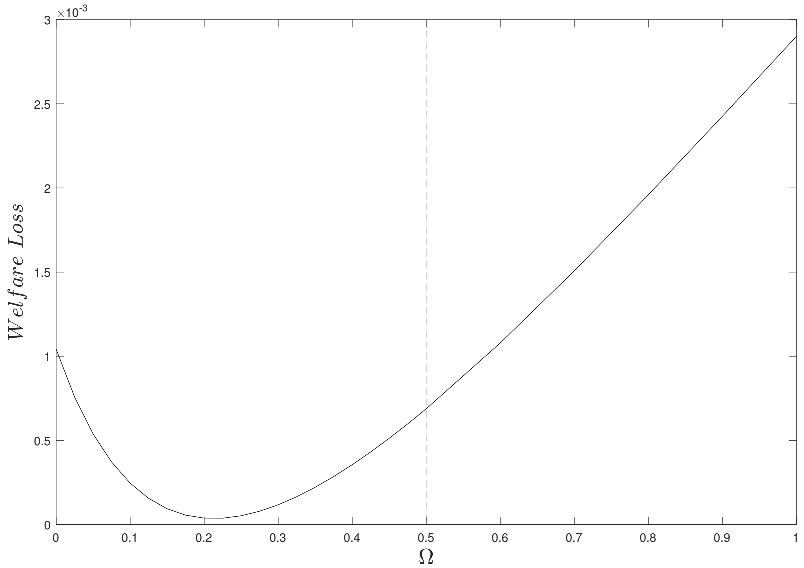


Figure 2.10: The optimal Ω ($\eta = 0.5$). The vertical dashed line indicates the optimal Ω under homothetic preferences.

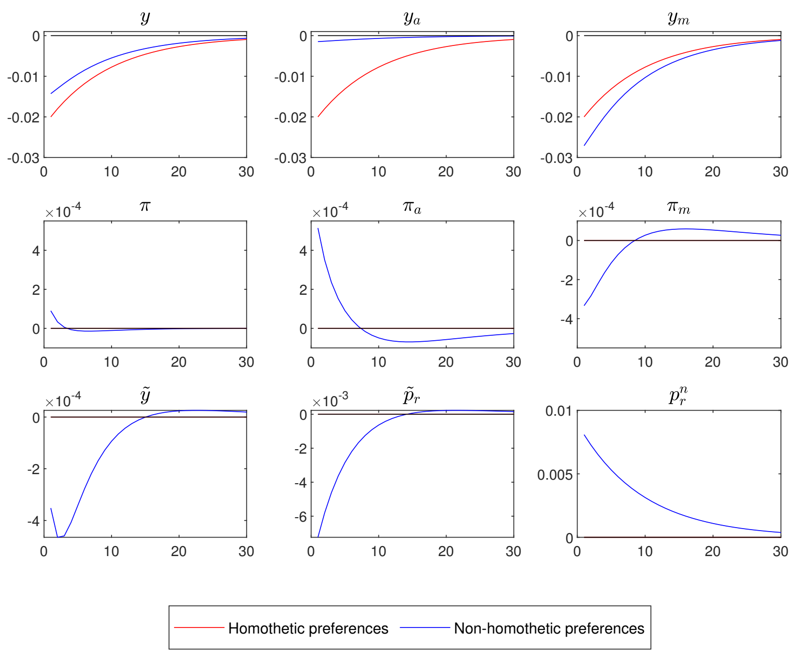


Figure 2.11: Shock to aggregate productivity. Optimal policy under commitment.

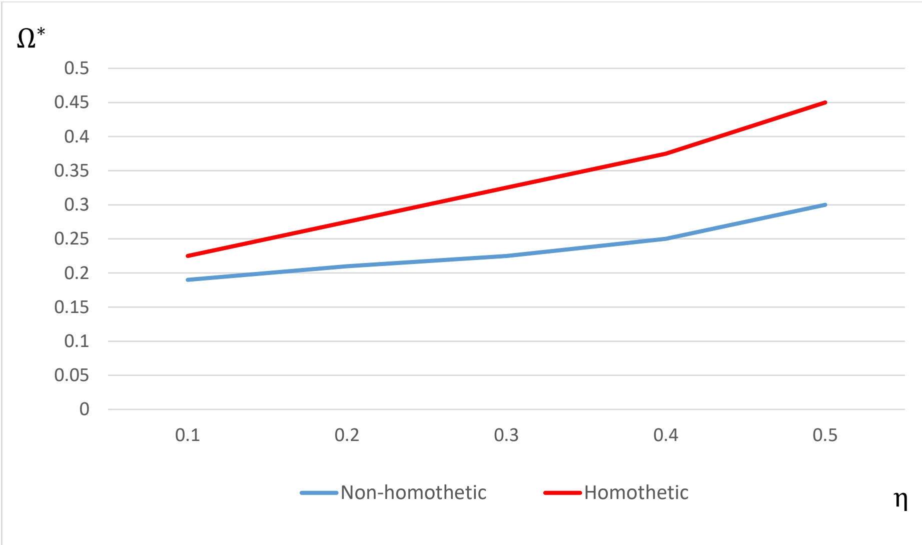


Figure 2.12: The optimal Ω as function of η.

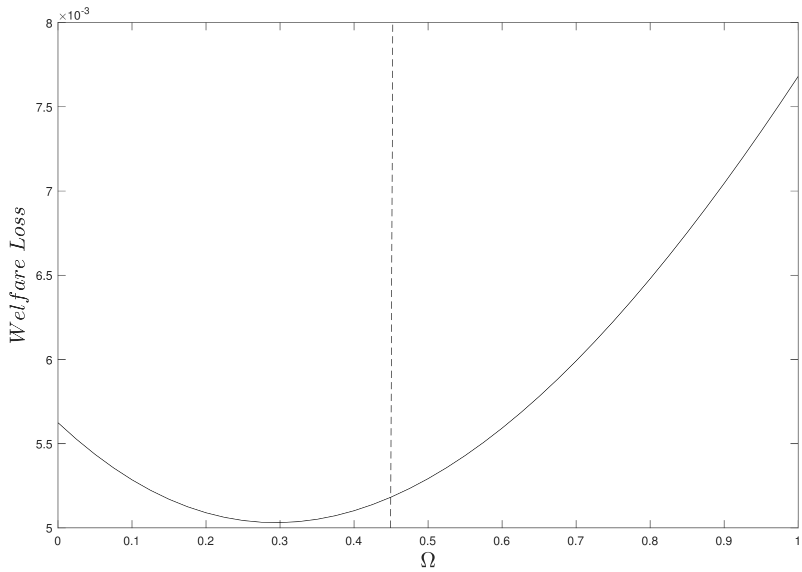


Figure 2.13: The optimal Ω ($\eta = 0.5$). The vertical dashed line indicates the optimal Ω under homothetic preferences.

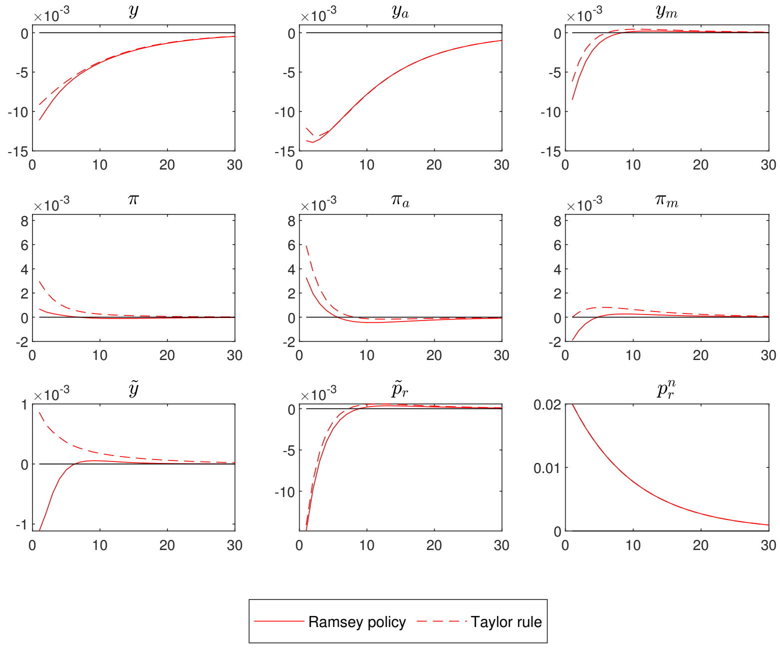


Figure 2.14: Shock to agricultural productivity. Homothetic preferences. Ramsey policy vs. Taylor rule.

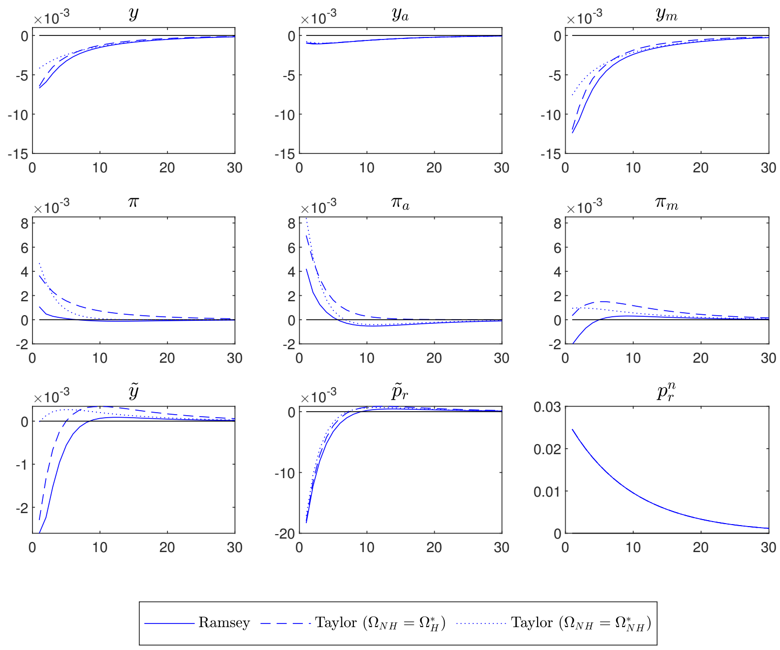


Figure 2.15: Shock to agricultural productivity. Non-homothetic preferences. Ramsey policy vs. Taylor rule.

Appendix

Households' optimal consumption allocation.

Households maximize C_t^+ conditional on the expenditure level E_t:

$$\max_{\{C_{a,t},C_{m,t}\}} \Xi(C_{a,t}-\tilde{C}_a)^{\omega}C_{m,t}^{(1-\omega)} - \aleph(P_{a,t}C_{a,t}+P_{m,t}C_{m,t}-E_t)$$

From the FOCs we obtain:

$$\frac{P_{a,t}(C_{a,t}-\tilde{C}_a)}{P_{m,t}C_{m,t}} = \frac{\omega}{1-\omega}$$

Plugging into the budget constraint:

$$C_{a,t} = \tilde{C}_a + \omega\frac{E_t-P_{a,t}\tilde{C}_a}{P_{a,t}}$$

and

$$C_{m,t} = (1-\omega)\frac{E_t-P_{a,t}\tilde{C}_a}{P_{m,t}}$$

Plugging the above expressions into the consumption index we get:

$$P_t^+C_t^+ = E_t - P_{a,t}\tilde{C}_a$$

where $P_t^+ \equiv P_{a,t}^{\omega}P_{m,t}^{(1-\omega)}$.

Then the demand for sectoral goods can be expressed as:

$$C_{a,t} = \tilde{C}_a + \omega\left(\frac{P_{a,t}}{P_t^+}\right)^{-1}C_t^+$$

and

$$C_{m,t} = \omega\left(\frac{P_{m,t}}{P_t^+}\right)^{-1}C_t^+$$

Aggregate output.

Aggregate output is defined as follows:

$$Y_t \equiv \frac{P_a}{P}Y_{a,t} + \frac{P_m}{P}Y_{m,t}$$

which can be expressed in log-deviation from steady state as:

$$\hat{y}_t = \frac{P_aY_a}{PY}\hat{y}_{a,t} + \frac{P_mY_m}{PY}\hat{y}_{m,t}$$

We know that in steady state $Y = \frac{P_a}{P}Y_a + \frac{P_m}{P}Y_m$, or $PY = P_aY_a + P_mY_m = E$. Then:

$$\hat{y}_t = \frac{P_aY_a}{E}\hat{y}_{a,t} + \frac{P_mY_m}{E}\hat{y}_{m,t}$$

By defining $\eta \equiv \frac{P_aY_a}{E}$ and $1-\eta \equiv \frac{P_mY_m}{E}$ we obtain:

$$\hat{y}_t = \eta\hat{y}_{a,t} + (1-\eta)\hat{y}_{m,t}$$

The aggregate price index.

The aggregate price index is defined as follows:

$$P_t \equiv \frac{Y_a}{Y}P_{a,t} + \frac{Y_m}{Y}P_{m,t}$$

which can be expressed in log-deviation from steady state as:

$$1 = \frac{P_aY_a}{PY}(\hat{\pi}_{a,t} - \hat{\pi}_t) + \frac{P_mY_m}{PY}(\hat{\pi}_{m,t} - \hat{\pi}_t)$$

Since $PY = E$ we obtain:

$$\hat{\pi}_t = \eta\hat{\pi}_{a,t} + (1-\eta)\hat{\pi}_{m,t}$$

The relation between $\hat{y}_t$ and c_t^+.

We know that:

$$P_t^+C_t^+ + P_{a,t}\tilde{C}_a = P_{a,t}C_{a,t} + P_{m,t}C_{m,t}$$

which can be expressed in log-deviation from steady state as:

$$\frac{P^+C^+}{E}\hat{c}_t^+ = \frac{P_a(C_a-\tilde{C}_a)}{E}\hat{p}_{a,t}^+ + \frac{P_mC_m}{E}\hat{p}_{m,t}^+ + \frac{P_aC_a}{E}\hat{c}_{a,t} + \frac{P_mC_m}{E}\hat{c}_{m,t}$$

where $\hat{p}_{s,t}^+ \equiv log\left(\frac{P_{s,t}}{P_t^+}\right) - log\left(\frac{P_s}{P^+}\right)$.
the above expression can be rewritten as:

$$\frac{P^+C^+}{E}\hat{c}_t^+ = \frac{\tilde{E}}{E}\left(\frac{P_a(C_a-\tilde{C}_a)}{\tilde{E}}(1-\omega)\hat{p}_{r,t} - \frac{P_mC_m}{\tilde{E}}\omega\hat{p}_{r,t}\right) + \hat{y}_t$$

but $\frac{P_a(C_a-\tilde{C}_a)}{\tilde{E}} = \omega$ and $\frac{P_mC_m}{\tilde{E}} = 1-\omega$. Then:

$$\hat{c}_t^+ = \frac{E}{P^+C^+}\hat{y}_t$$

but $\frac{E}{P^+C^+} = \frac{E}{\tilde{E}} = \frac{\frac{P_mY_m}{E}}{\frac{P_mY_m}{\tilde{E}}} = \frac{1-\omega}{1-\eta}$. Then:

$$\hat{c}_t^+ = \frac{1-\omega}{1-\eta}\hat{y}_t$$

The flexible price economy.

Absent nominal rigidities and assuming $\alpha_a = \alpha_m = \alpha$ our economy is described by the following system:

The firms optimal pricing condition:

$$(\hat{\omega}_t^+)^n = (\hat{y}_{a,t}^n - \hat{n}_{a,t}^n) + (1-\omega)\hat{p}_{r,t}^n$$

$$(\hat{\omega}_t^+)^n = (\hat{y}_{m,t}^n - \hat{n}_{m,t}^n) - \omega\hat{p}_{r,t}^n$$

The labor supply schedule:

$$(\hat{\omega}_t^+)^n = \sigma(\hat{c}_t^+)^n + \varphi\hat{n}_t^n$$

Aggregate employment:

$$\hat{n}_t^n = \eta\hat{n}_{a,t}^n + (1-\eta)\hat{n}_{m,t}^n$$

The production functions:

$$\hat{y}_{a,t}^n = a_{a,t} + (1-\alpha)\hat{n}_{a,t}^n$$

$$\hat{y}_{m,t}^n = a_{m,t} + (1-\alpha)\hat{n}_{m,t}^n$$

The sectoral demand functions:

$$\hat{c}_{a,t}^n = \tfrac{\omega(1-\eta)}{(1-\omega)\eta}(-(1-\omega)\hat{p}_{r,t}^n + (\hat{c}_t^+)^n)$$

$$\hat{c}_{m,t}^n = \omega\hat{p}_{r,t}^n + (\hat{c}_t^+)^n$$

Final good markets clearing conditions:

$$\hat{y}_{a,t}^n = \hat{c}_{a,t}^n$$

$$\hat{y}_{m,t}^n = \hat{c}_{m,t}^n$$

where the superscript n denotes natural levels.

By solving the previous system of equations we get:

$$\hat{y}_{a,t}^n = \Upsilon_a a_{a,t}$$

$$\hat{y}_{m,t}^{n} = \Upsilon_m a_{a,t} + a_{m,t}$$

$$\hat{y}_{a,t}^{n} - \hat{y}_{m,t}^{n} = \Upsilon_{ry} a_{a,t} - a_{m,t}$$

$$\hat{p}_{r,t}^{n} = -\tfrac{1-\alpha\Upsilon_{ry}}{1-\alpha} a_{a,t} + a_{m,t}$$

$$\hat{y}_{t}^{n} = \left(\eta\Upsilon_a + (1-\eta)\Upsilon_m\right) a_{a,t} + (1-\eta) a_{m,t}$$

$$(\hat{\omega}_t^{+})^{n} = \Upsilon_\omega a_{a,t} + (1-\omega) a_{m,t}$$

where

$\Upsilon_a \equiv \frac{1+\varphi}{\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)\varphi(1-\eta)+\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)+\varphi\eta+\alpha\varphi(1-\eta)+\alpha}$,

$\Upsilon_m \equiv \frac{\left(\frac{(1-\omega)\eta}{\omega(1-\eta)}-1\right)(1-\alpha)\varphi\eta}{\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)\varphi(1-\eta)+\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)+\varphi\eta+\alpha\varphi(1-\eta)+\alpha}$,

$\Upsilon_\omega \equiv \frac{\omega}{1-\alpha}\left(1-\alpha(\Upsilon_a + \frac{1-\omega}{\omega}\Upsilon_m)\right)$ and

$\Upsilon_{ry} \equiv \frac{1+\varphi-\left(\frac{(1-\omega)\eta}{\omega(1-\eta)}-1\right)(1-\alpha)\varphi\eta}{\left(\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)+\alpha\right)+\left(\left(\frac{(1-\omega)\eta}{\omega(1-\eta)}(1-\alpha)+\alpha\right)(1-\eta)+\eta\right)\varphi}$.

Chapter 3

INEQUALITY, THE ZERO LOWER BOUND AND THE GAINS FROM WAGE FLEXIBILITY

with Mario Giarda* and Damián Romero†.

3.1 Introduction

Letting wages adjust freely after an economic downturn is one of the main elements of the classical economists' toolkit. Under this argument, if wages fall, demand for labor increases, and then output returns to its natural level. This logic is behind the usual recommendation in policy debates, that after an economic downturn, to restore competitiveness and potential output levels, wage deflation is a desirable outcome. However, in economies where this prescription has been applied, the result was not restoring pre-crisis employment levels, but increasing social unrest, as in the case of the recent European crisis.

Recent literature challenges this conventional recommendation. On the one

*Universitat Pompeu Fabra and Barcelona GSE. E-mail: mario.giarda@upf.edu.

†Universitat Pompeu Fabra and Barcelona GSE. E-mail: damian.romero@upf.edu.

hand, there is work analyzing the role of monetary policy and its interaction with wage flexibility. Galí (2013) shows that in models where GDP is demand determined the role of monetary policy is central in the transmission from flexibilized wages to macroeconomic stability. In particular, in the face of adverse shocks less rigid wages bring inflation down, and hence, a more accommodative policy stance counteracts activity contraction. The policy stance is also crucial for the dynamics of nominal variables as reducing rigidities in an environment of a passive monetary policy can result in excessively volatile price and wage inflation. So even when higher flexibility can stabilize employment, the final welfare effect depends on the monetary policy stance. On related work, Galí and Monacelli (2016) and Billi and Galí (2019) study the gains from wage flexibility when the central bank is either restricted by a currency union or by the zero lower bound. They show that when these restrictions are in place, the transmission from flexibilized wages to output stability via the policy rate response breaks, and hence, reducing rigidities might not be a desirable outcome. In fact, higher flexibility is associated with more severe deflationary expectations during recessions, which, given the inability of the monetary authority to cut the nominal rate, translate into higher real rates that ultimately deepen economic downturns. On the other hand, the gains from wage flexibility also relate to the capacity of different agents to smooth consumption. In economies with a significant share of agents that are hand-to-mouth, wage fluctuations generate redistribution, which influences aggregate demand and thus output variability.

In this paper, we study the gains from wage flexibility when incomplete markets and the zero lower bound interact. We show that in this environment the undesirable effects from reducing rigidities can worsen. We start by showing that when markets are incomplete the slope of the IS curve depends on the relative degrees of price and wage rigidities, for them determine income redistribution between constrained and unconstrained households. As Bilbiie (2008), we show that inequality amplifies the effect from movements in the interest rate, yet, as shown in Ascari et al. (2017), the presence of wage rigidities generates dampening. The latter is a consequence of the response of wage markups, which offset variations in firms markups, hence diminishing redistribution of income from workers to firms owners during recessions (which is the source of amplification in those papers). In other words, higher rigidities, by containing the cut in wages during episodes of economic downturns, reduce redistribution from workers, characterized by relatively higher marginal propensities to consume, to companies owners, thus dampening activity contraction.

We conclude from our previous discussion that in periods when monetary policy is not constrained by the ZLB, the effects from increased wage flexibility via the policy rate response and through redistribution oppose to each other. We will see that the former channel tends to dominate and thus greater flexibility results in lower volatility in output and employment. In effect, we show that this outcome can only be overturned when the economy moves towards extreme degrees of flexibility. Importantly, in spite of its stabilizing effects, flexibility is associated with more severe drops in workers income during economic recessions due to redistribution. We show that the space of parameters that bring gains from greater flexibility shrinks when we mix incomplete markets and wage rigidities. When the zero lower bound is at play, the parameter space further shrinks. In this scenario the contractive effects via deflationary expectations and redistribution reinforce each other due to a feedback loop between the two channels. As a result, the contractionary effects of enhanced flexibility significantly amplify relative to a scenario without financial frictions.

The remaining of the paper is organized as follows. Section 2 describes the model. Section 3 derives an aggregate Euler equation that captures the effects from wage rigidities and limited asset markets participation. Section 4 conducts a quantitative exercise. Finally, section 5 concludes.

3.2 The model

We build a New Keynesian model with incomplete markets, wage rigidities and the zero lower bound. In particular, we assume there is a share of agents that has limited access to financial markets. Workers supply labor in a monopolistically competitive environment and are subject to staggered wage setting. Firms are also subject to price rigidities. Additionally, we assume that monetary policy follows a Taylor rule which is bounded from below by the zero.

Households. The economy is populated by a continuum of households of mass 1, where a fraction λ cannot borrow or lend while the remainder $1-\lambda$ has full access to financial markets and own the firms in the economy. We refer to the former as constrained agents, denoted by c, and to the latter as unconstrained, denoted by u. Each household is composed by a continuum of members that supply differentiated labor varieties denoted by $j \in [0,1]$. Due to insurance within the household, members consumption is equalized.

Households' lifetime utility is given by:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \chi_{t+k} \left(\frac{(C_{t+k}^K)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{N_{t+k}^K(j)^{1+\varphi}}{1+\varphi} dj \right)$$

for $K \in \{u, c\}$, where χ_t represents a shock to preferences, C_t^K is final good consumption and $N_t^K(j)$ denotes hours worked for member supplying labor variety j.

Households face the following period resource constraint:

$$P_t C_t^K + Q_t B_t^K = B_{t-1}^K + \int_0^1 W_t(j) N_t^K(j) dj + D_t^K + T_t^K$$

Income is composed of total labor earnings $\int_0^1 W_t(j) N_t^K(j) dj$, fiscal transfers T_t^K and profits proceeding from the ownership of firms in the intermediate goods sector D_t^K. $P_t C_t^K$ represents total expenditure in the final good and $Q_t B_t^K$ bond purchases, where P_t and Q_t are the respective prices.

The preference shock is assumed to follow an exogenous AR(1) process given by:

$$\log \chi_t = (1-\rho_\chi) \log \overline{\chi} + \rho_\chi \log \chi_{t-1} + \sigma_\chi \eta_t$$

Intertemporal optimization implies the following Euler equation for unconstrained households:

$$1 = R_t \mathbb{E}_t \left\{ \beta \frac{\chi_{t+1}}{\chi_t} \left(\frac{C_t^u}{C_{t+1}^u} \right)^{\sigma} \frac{1}{\Pi_{t+1}^p} \right\} \tag{3.1}$$

Constrained households have no access to financial markets, hence their consumption equals current income, from labor earnings and transfers:

$$C_t^c = \int_0^1 \frac{W_t(j)N_t(j)}{P_t} dj + \frac{T_t^c}{P_t} \tag{3.2}$$

Finally, aggregate consumption is given by:

$$C_t = (1-\lambda)C_t^u + \lambda C_t^c \tag{3.3}$$

Wage setting. The wage for each labor variety is set by a union operating in a monopolistically competitive market. Unions choose the wage rate that maximizes a weighted average of unconstrained and constrained lifetime utility, given by:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \chi_{t+k} \Bigg((1-\lambda) \left(\frac{(C_{t+k}^u)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{(N_{t+k|t}^u)^{1+\varphi}}{1+\varphi} dj \right) + \lambda \left(\frac{(C_{t+k}^c)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{(N_{t+k|t}^c)^{1+\varphi}}{1+\varphi} dj \right) \Bigg)$$

subject to households' resource constraint and the sequence of demands for the labor variety they represent:

$$N_{t+k|t}^K = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\epsilon_w} N_{t+k}^K$$

where W_t^* is the optimal wage chosen by a union that last resets its wage at t, $N_{t+k|t}^K$ is labor supply for the household members whose wage was last reoptimized in period t and ϵ_w is the elasticity of substitution among labor varieties.

Assuming firms demand for constrained and unconstrained workers labor is the same, i.e. $N_t(j)^u = N_t(j)^c = N_t(j)$, maximization implies:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \chi_{t+k} N_{t+k|t}^{1+\varphi} \Bigg[\Bigg((1-\lambda) \frac{1}{(C_{t+k}^u)^\sigma N_{t+k|t}^\varphi} + \lambda \frac{1}{(C_{t+k}^c)^\sigma N_{t+k|t}^\varphi} \Bigg) \frac{W_t^*}{P_{t+k}} - \mathcal{M}^w \Bigg] = 0$$

where $\mathcal{M}^w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$ is the desired markup.

Final good producers. Firms producing the final good operate in a perfectly competitive environment and combine a continuum of measure one of intermediate goods $Y_t(i)$ to produce a homogeneous final good Y_t according to the following technology:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

where ϵ_p is the elasticity of substitution among good varieties.

Solving the optimization problem of the firm we obtain the following demand function for intermediate inputs:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_p} Y_t$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon_p} di \right)^{\frac{1}{1-\epsilon_p}}$ is the price of the final good.

Intermediate good producers. There is a continuum of intermediate firms, indexed by $i \in [0, 1]$. These firms operate in a monopolistically competitive environment. Hence, each firm produces a single-differentiated good and operates as a monopoly in its own market. Intermediates production technology is given by:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where $Y_t(i)$ is firm i output and $N_t(i)$ is labor input.

Firms face price stickiness a la Calvo, hence, in every period, they reset prices with probability $(1 - \theta_p)$. A firm that is able to reset prices in period t, chooses the price P_t^* that maximizes the following sum of discounted profits:

$$\mathbb{E}_t \sum_{k=o}^{\infty} \theta_p^k \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - TC_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to the demand constraint given by:

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p} Y_{t+k}$$

Total cost of producing $Y_{s,t+k|t}$ units is defined as $TC_{t+k}(Y_{t+k|t}) \equiv W_{t+k} \left(\frac{Y_{t+k|t}}{A_{t+k}} \right)^{\frac{1}{1-\alpha}}$. Profit maximization implies:

$$\mathbb{E}_t \sum_{k=o}^{\infty} (\theta_p)^k \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M}^p MC_{t+k|t} \right) \right\} = 0$$

where $\mathcal{M}^p \equiv \frac{\varepsilon_p}{\varepsilon_p - 1}$ is the desired markup and $MC_{t+k|t}$ the nominal marginal cost.

Monetary policy. We assume that the monetary authority follows a Taylor rule which is subject to the zero lower bound, given by:

$$R_t = \max \left\{ \overline{R} \left(\frac{\Pi_{p,t}}{\overline{\Pi}_p} \right)^{\phi_\pi} \left(\frac{Y_t}{\overline{Y}} \right)^{\phi_y}, 1 \right\}$$

where $R_t = \frac{1}{Q_t}$, and parameters ϕ_π and ϕ_y measure the response of the central bank to deviations of inflation and output from their steady state level, respectively.

Equilibrium. In this economy all production is consumed:

$$Y_t = C_t$$

The relation between aggregate output and employment can be written as:

$$N_t = \Delta_{w,t} \Delta_{p,t} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}}$$

where $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} di$.

Finally, we assume bonds are in zero net supply. Hence, equilibrium in the bonds market requires:

$$(1-\lambda)B_t^u + \lambda B_t^c = 0$$

Since constrained agents have no access to financial markets, the last expression implies $B_t^u = B_t^c = 0$.

3.3 A simplified economy

The present section seeks to illustrate how the interaction between price and wage rigidities shapes redistribution. In order to clarify the economic mechanisms we are proposing, we simplify the wage and price setting process. As before, workers supply labor in a monopolistically competitive environment. We assume that a fraction of the them face no frictions in the wage setting process while the remainder must set the wage before shocks are realized. Firms also operate in a monopolistically competitive environment, and a share of them must decide their price before the realization of shocks.

Price setting. There is a mass $1-\theta_p$ of optimizing firms (denoted by superscript o) while the remaining fraction θ_p set prices before the shock is realized (denoted by superscript m).

We start by describing the price setting problem optimizers face. Given the (log-linearized) demand function for a given variety i, $y_t(i) = y_t - \epsilon_p(p_t^o(i) - p_t)$, firms maximize profits setting their price as a markup μ^p over the marginal cost. Optimal pricing implies:

$$p_t^o(i) = \mu^p + w_t - \log(1-\alpha) + \frac{\alpha}{1-\alpha} y_t^o(i) - \frac{1}{1-\alpha} a_t$$

where $\mu^p = log\left(\mathcal{M}^p\right)$. Substituting the labor demand function into the firm's optimality condition and rearranging yields:

$$p_t^o(i) = \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\left(\mu^p + \omega_t - log(1-\alpha) + \frac{\alpha}{1-\alpha}y_t - \frac{1}{1-\alpha}a_t\right) + p_t$$

Consider next the price setting problem non-optimizers face. These firms set prices at the end of period $t-1$, and hence, they make their pricing decisions for period t based on the information set available at $t-1$. Optimization implies:

$$p_t^m(i) = \mathbb{E}_{t-1}\left(\mu^p + w_t - a_t - log(1-\alpha) + \alpha n_t^m(i)\right)$$

On the other hand, the aggregate price index is given by:

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon_p} di\right)^{\frac{1}{1-\epsilon_p}}$$

which implies:

$$P_t = \left((1-\theta_p)(P_t^o)^{1-\epsilon_p} + \theta_p(P_t^m)^{1-\epsilon_p}\right)^{\frac{1}{1-\epsilon_p}}$$

Accordingly, the following relation holds around steady state:

$$p_t = (1-\theta_p)p_t^o + \theta_p p_t^m$$

Substituting the pricing rules from optimizers and non-optimizers into the aggregate price index yields the following price inflation equation:

$$\hat{\pi}_t^p = \frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\left(\hat{\omega}_t + \frac{\alpha}{1-\alpha}\hat{y}_t - \frac{1}{1-\alpha}a_t\right) + \mathbb{E}_{t-1}\hat{x}_t^p \qquad (3.4)$$

where $\hat{x}_t^p \equiv \hat{\omega}_t - a_t + \alpha\hat{n}_t^m(i) + \hat{\pi}_t^p$.

Wage setting. Wages are set by unions representing the different labor varieties. There is a mass $1-\theta_w$ of optimizers (denoted by superscript o) and a mass θ_w of unions who set wages before the shock is realized (denoted by superscript m).

Consider first the optimizers' problem. Given the demand function $n_t^o(j) =$

$n_t - \epsilon_w(\omega_t^o(j) - \omega_t)$, unions maximize households' utility by setting the wage (in real terms) as a markup μ^w over the marginal rate of substitution:

$$\omega_t^o(j) = \mu^w + \sigma c_t + \varphi n_t^o(j)$$

where $\mu^w = log(\mathcal{M}^w)$. Substituting the demand function into the optimal wage setting condition and rearranging yields:

$$\hat{\omega}_t^o(j) = \frac{1}{1 + \varphi\epsilon_w}(\sigma\hat{c}_t + \varphi\hat{n}_t + \varphi\epsilon_w\hat{\omega}_t)$$

Consider next the wage setting problem non-optimizers face. Identical to the firms' problem, non-optimizing unions decide wages for period t based on the information set available at $t-1$, implying:

$$w_t^m(j) = \mathbb{E}_{t-1}(\mu^w + \sigma c_t + \varphi n_t^m(j) + p_t)$$

which can be rewritten as:

$$\hat{\omega}_t^m(j) = -\hat{\pi}_t^p + \mathbb{E}_{t-1}(\sigma\hat{c}_t + \varphi\hat{n}_t^m(j) + \hat{\pi}_t^p)$$

On the other hand, the aggregate wage is given by:

$$W_t \equiv \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj\right)^{\frac{1}{1-\epsilon_w}}$$

which implies:

$$W_t = \left(\theta_w (W_t^m)^{1-\epsilon_w} + (1-\theta_w)(W_t^o)^{1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}}$$

The above expression can be written in log-deviation from the steady state as:

$$\hat{\omega}_t = \theta_w\hat{\omega}_t^m + (1-\theta_w)\hat{\omega}_t^o$$

Finally, substituting optimizers and non-optimizers' wage setting rules into the aggregate wage equation and imposing the market clearing condition yields:

$$\hat{\omega}_t = \frac{1-\theta_w}{1+\theta_w\varphi\epsilon_w}(\sigma\hat{y}_t + \varphi\hat{n}_t) - \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w}\hat{\pi}_t^p + \mathbb{E}_{t-1}\hat{x}_t^w \tag{3.5}$$

where $\hat{x}_t^w \equiv \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w}(\sigma\hat{y}_t + \varphi\hat{n}_t^m(j) + \hat{\pi}_t^p)$.

The evolution of the real wage. From equations (3.4) and (3.5) and assuming no shocks to productivity we get:

$$\hat{\omega}_t = \Xi\hat{y}_t + \mathbb{E}_{t-1}\hat{x}_t \tag{3.6}$$

where

$$\hat{x}_t \equiv \left(1 + \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w}\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\right)^{-1}\left(\hat{x}_t^w - \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w}\hat{x}_t^p\right)$$

and

$$\Xi \equiv \frac{\frac{1-\theta_w}{1+\theta_w\varphi\epsilon_w}\left(\sigma + \frac{\varphi}{1-\alpha}\right) - \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w}\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\frac{\alpha}{1-\alpha}}{1 + \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w}\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}}.$$

Note that coefficient Ξ governs the cyclicality of the real wage and hence, as we show next, will be crucial for the the dynamics of income redistribution.

The consumption gap. To obtain the Euler equation in our setup with limited asset markets participation, we first need to derive an expression describing the dynamics of the consumption gap (between unconstrained and constrained agents) over the cycle. Particularly, we define the consumption gap as:

$$\gamma_t = \frac{C_t^u}{C_t^c}$$

Recall unconstrained agents own the firms and hence their income is given by the sum of labor and profit income, i.e. $C_t^u = \frac{W_t}{P_t}N_t + \frac{1}{1-\lambda}\frac{D_t}{P_t}$, whereas constrained households only receive labor income, i.e. $C_t^c = \frac{W_t}{P_t}N_t$. The consumption gap can then be rewritten as:

$$\gamma_t = \frac{W_tN_t + \frac{1}{1-\lambda}D_t}{W_tN_t}$$

As Debortoli and Galí (2017) we express the consumption gap in terms of the economy average price markup:

$$\gamma_t = \frac{1 - \alpha + \frac{1}{1-\lambda}\left(\mathcal{M}_t^p - (1-\alpha)\right)}{1-\alpha}$$

where we have made use of $\frac{W_t N_t}{P_t Y_t} = (1-\alpha)\frac{Y_t}{\mathcal{M}_t^p N_t}\frac{N_t}{Y_t} = \frac{1-\alpha}{\mathcal{M}_t^p}$ and $\frac{D_t}{P_t Y_t} = \frac{P_t Y_t - W_t N_t}{P_t Y_t} = 1 - \frac{1-\alpha}{\mathcal{M}_t^p}$ and $\mathcal{M}_t^p$ is the average price markup.

We can write the above equation in log-deviation from steady state as:

$$\hat{\gamma}_t = \Psi \hat{\mu}_t^p \tag{3.7}$$

where $\Psi \equiv \frac{\mathcal{M}^p}{(1-\lambda)\left(1-\alpha+\frac{1}{1-\lambda}(\mathcal{M}^p-(1-\alpha))\right)}$.

We have determined the relationship between the consumption gap and the average price markup. Next we need to understand how the price markup evolves over the cycle.

Firms' average markup. Firms' average markup is given by:

$$\mathcal{M}_t^p = \frac{P_t}{W_t}(1-\alpha)\frac{Y_t}{N_t}$$

Log-linearizing the above expression around steady state and assuming no shocks to productivity yields:

$$\hat{\mu}_t^p = -\frac{\alpha}{1-\alpha}\hat{y}_t - \hat{\omega}_t \tag{3.8}$$

Finally, combining equations (3.6) and (3.8) we arrive to an expression relating the price markup with output:

$$\hat{\mu}_t^p = -\left(\Xi + \frac{\alpha}{1-\alpha}\right)\hat{y}_t - \mathbb{E}_{t-1}\hat{x}_t \tag{3.9}$$

The cyclicality of the consumption gap. Now we can put equations (3.7) and (3.9) together to obtain an expression describing the dynamics of the consumption gap over the cycle:

$$\hat{\gamma}_t = -\Theta\hat{y}_t - \Psi\mathbb{E}_{t-1}\hat{x}_t \tag{3.10}$$

where coefficient $\Theta \equiv \Psi\left(\Xi + \frac{\alpha}{1-\alpha}\right)$ determines the cyclicality of the consumption gap. Equation (3.10) will be useful for deriving an aggregate Euler equation that considers limited asset markets participation and frictions in the labor market. We do this next.

Deriving the aggregate Euler equation. From (3.1) we obtain the following Euler equation for unconstrained agents in deviation from steady state:

$$\hat{c}_t^u = \mathbb{E}_t(\hat{c}_{t+1}^u) - \frac{1}{\sigma}\mathbb{E}_t(\hat{r}_t - \hat{\pi}_{p,t+1} - (1-\rho_\chi)\hat{\chi}_t) \tag{3.11}$$

Besides, from (3.3) we can get:

$$C_t = C_t^u\left((1-\lambda) + \lambda\frac{1}{\gamma_t}\right)$$

which can be rewritten in log-deviation from steady state as:

$$\hat{c}_t = \hat{c}_t^u - \frac{\lambda}{(1-\lambda)\gamma+\lambda}\hat{\gamma}_t \tag{3.12}$$

Substituting (3.12) into (3.11) and assuming $\rho_\chi = 0$ yields the following aggregate Euler relation involving the consumption gap:

$$\hat{c}_t + \frac{\lambda}{(1-\lambda)\gamma+\lambda}\hat{\gamma}_t = \mathbb{E}_t\left\{\hat{c}_{t+1} + \frac{\lambda}{(1-\lambda)\gamma+\lambda}\hat{\gamma}_{t+1}\right\} - \frac{1}{\sigma}\mathbb{E}_t\left\{\hat{r}_t - \hat{\pi}_{t+1}^p - \hat{\chi}_t\right\}$$

Finally, using (3.10) and imposing market clearing we obtain:

$$\begin{aligned}\hat{y}_t + &\frac{\lambda}{(1-\lambda)\gamma+\lambda}\left(-\Theta\hat{y}_t - \Psi\mathbb{E}_{t-1}\hat{x}_t\right)\\ &= \mathbb{E}_t\left\{\hat{y}_{t+1} + \frac{\lambda}{(1-\lambda)\gamma+\lambda}\left(-\Theta\hat{y}_{t+1} - \Psi\mathbb{E}_t\hat{X}_{t+1}\right)\right\} - \frac{1}{\sigma}\mathbb{E}_t\left\{\hat{r}_t - \hat{\pi}_{t+1}^p - \hat{\chi}_t\right\}\end{aligned}$$

Assume next that the economy starts at steady state in period $t-1$ and an iid demand shock hits at t. Since the shock is unexpected, then at $t-1$ agents forecast that the economy at t will remain at steady state, i.e. $\mathbb{E}_{t-1}\hat{x}_t = 0$. Additionally, since the shock has no persistence all real variables return to steady state at $t+1$.[1]

[1] At t agents correctly anticipate no shocks at $t+1$, hence it is as if the economy was not

The nominal variables at $t+1$ on the other hand will be determined by monetary policy. We assume the central bank implements a policy such that $\hat{\pi}_{p,t+1} = 0$. Accordingly, we have $\mathbb{E}_t\hat{y}_{t+1} = \mathbb{E}_t\hat{\pi}^p_{t+1} = \mathbb{E}_t\hat{X}_{t+1} = 0$, and hence the aggregate Euler equation reduces to:

$$\hat{y}_t = -\frac{1}{\sigma}\frac{1}{1-\frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta}\mathbb{E}_t(\hat{r}_t - \hat{\chi}_t) \tag{3.13}$$

Expression (3.13) is the Euler equation under our simplifying assumptions. Notice that the response of output to the interest rate not only depends on the IES but also on another term, which involves the market incompleteness parameter λ. If $\lambda = 0$ the economy is a RANK and the slope of the Euler equation is given by $1/\sigma$. However, when market incompleteness is present (with $\lambda > 0$) the elasticity to the real rate depends on another parameter, Θ, which governs the cyclicality of the consumption gap.

Recall Θ is given by:

$$\Theta \equiv \Psi\left(\Xi + \frac{\alpha}{1-\alpha}\right)$$

with

$$\Xi \equiv \frac{\frac{1-\theta_w}{1+\theta_w\varphi\epsilon_w}\left(\sigma + \frac{\varphi}{1-\alpha}\right) - \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w}\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\frac{\alpha}{1-\alpha}}{1+\frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w}\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}}.$$

The key object of our analysis will be parameter Ξ. The latter governs the cyclicality of the real wage and hence that of markups and the consumption gap, which ultimately determines the elasticity of output to the real rate. Hence, the slope of our Euler equation depends on good and labor markets features, such as the degrees of price and wage rigidities. Particularly, the slope of the Euler equation depends positively on the degree of price stickiness, determined by θ_p, and negatively on the degree of wage rigidities, governed by θ_w. Intuitively, more sticky prices imply larger price markups during an economic downturn, and thus, redistribution from workers to firm owners, which, given the higher propensity

affected by any friction at $t+1$ and thus the allocation must coincide with that of the flexible price economy. The latter ensures all real variables return to steady state at $t+1$.

to consume of the former, amplifies output contraction. On the other hand, more rigid wages entail lower markups and hence redistribution of resources in the opposite direction, thus dampening the drop in activity.

Interestingly, when wages are fully flexible, the slope of the Euler equation is unaffected by the degree of price rigidities. Such outcome seems counterintuitive given our previous discussion, yet it can be easily understood. Under sticky prices and wages it is the interaction between these two forms of rigidities what determines the cyclicality of the real wage, which ultimately shapes the cyclicality of the consumption gap. However, under flexible wages the real wage is fully determined by the labor supply schedule, given by:

$$\hat{\omega}_t = \sigma \hat{y}_t + \varphi \hat{n}_t$$

which can be rewritten as:

$$\hat{\omega}_t = \left(\sigma + \frac{\varphi}{1-\alpha}\right) \hat{y}_t - \frac{\varphi}{1-\alpha} a_t$$

The above expression makes clear that the cyclicality of the real wage is independent of the degree of price stickiness.

Combining the above expression with (3.7) and (3.8) yields:

$$\hat{\gamma}_t = -\left(\sigma + \frac{\varphi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right) \Psi \hat{y}_t + \frac{\varphi}{1-\alpha} \Psi a_t$$

It follows that the cyclicality of the consumption gap, and hence redistribution, are unaltered by price rigidities either. Note that this result is independent of our assumptions concerning the price and wage setting process and thus carry over to an environment with Calvo style rigidities.

Also important to notice is that under fully flexible prices we have $\Xi = -\frac{\alpha}{1-\alpha}$, and hence $\Theta = 0$. It follows that, independently of the degree wage stickiness, redistribution is zero and the standard Euler equation is recovered. Such outcome has been already described by Ascari et al. (2017). This result is not surprising given (3.7), which under flexible prices yields:

$$\gamma = \Psi \mu^p$$

The above relation illustrates that with $\theta_p = 0$ the price markup remains constant at its desired level and hence the consumption gap becomes constant as well. Observe that also this result is independent of the price and wage setting process. In fact, it only hinges on the assumed production technology, used in the derivation of (3.7).

An interesting question arising from the discussion above is whether the cyclicality of the consumption gap can be reversed under sufficiently large values of parameter θ_w. In such situation the real wage would remain high during economic downturns and thus redistribution in favor of workers could be expected, that is, the consumption gap could be expected to turn procyclical.

To answer this question recall that the cyclicality of the consumption gap is determined by coefficient $\Theta \equiv \Psi \left(\Xi + \frac{\alpha}{1-\alpha} \right)$, which, under sticky prices and flexible wages, has been shown to be positive, implying a countercyclical gap.

Since $\Psi \equiv \frac{\mathcal{M}^p}{(1-\lambda)\left(1-\alpha+\frac{1}{1-\lambda}(\mathcal{M}^p-(1-\alpha))\right)} > 0$, it follows that overturning the cyclicality of $\hat{\gamma}_t$ requires $\Xi < -\frac{\alpha}{1-\alpha}$ to be satisfied. As shown in the appendix, it turns out that for any parameter calibration $\Xi \geqslant -\frac{1}{1-\alpha}$ holds, i.e., the consumption gap can never be procyclical.

To provide an intuition for this result let us consider the response of our economy to a negative iid preference shock. Equation (3.7) makes clear that a procyclical consumption gap requires the price markup to turn negative in face of the adverse shock, i.e., markups need to reduce below their desired level. But such circumstance is not possible in our setting. In fact, at t, when the contractive shock hits, employment must fall along with output while wages experience downward pressure due to reduced employment and consumption. Accordingly, nominal marginal costs reduce. The latter imply a rise in the markup and so firms cut prices. Yet, such reduction in prices can never lead to a negative markup for this would imply markups fall below the desired level, which is sub optimal. Accordingly, redistribution in favor of workers is ruled out.

In our previous discussion we have stressed the negative relation between the degree of wage rigidities and the slope of the Euler equation. Such result seems to indicate that, under limited financial markets participation and in the face of demand shocks, more flexible wages would necessarily lead to an increase in output volatility. Note however that our earlier analysis abstracted from any endogenous response of the policy rate to shocks. Yet, as we will see next, the rate response turns out to be critical for the final outcome from flexibilized wages.

Introducing monetary policy. Let us now close the model by introducing the following rule for the nominal policy rate:

$$\hat{r}_t = \phi_\pi \hat{\pi}_t^p + \phi_y \hat{y}_t$$

Substituting the Taylor rule into the Euler equation delivers:

$$\hat{y}_t = -\frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta} \mathbb{E}_t \left\{ \phi_\pi \hat{\pi}_t^p + \phi_y \hat{y}_t - \hat{\chi}_t \right\}$$

Besides, from equations (3.4) and (3.5) and assuming no shocks to productivity we obtain:

$$\hat{\pi}_t^p = \Upsilon \hat{y}_t$$

where $\Upsilon \equiv \left(1 + \frac{1-\theta_p}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p} \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w}\right)^{-1} \frac{1-\theta_p}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p} \left(\frac{1-\theta_w}{1+\theta_w\varphi\epsilon_w}\left(\sigma + \frac{\varphi}{1-\alpha}\right) + \frac{\alpha}{1-\alpha}\right)$.

Finally, substituting the above relation into the Euler equation and rearranging yields:

$$\hat{y}_t = \left(1 + \frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta} (\phi_\pi \Upsilon + \phi_y)\right)^{-1} \frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta} \hat{\chi}_t$$

Considering the endogenous response of the policy rate introduces a new mechanism through which wage rigidities affect the response of output to the preferences shock. In particular, note that coefficient Υ, determining the sensitivity of price inflation to output fluctuations, is decreasing in θ_w. The reason is that more sticky wages translate into more rigid marginal costs and thus into less responsive prices. But a reduced response of prices imply a weaker response of the policy rate, which amplifies output volatility. Such transmission mechanism from wage rigidities to output volatility has been described in Galí (2013).

We can conclude from our discussion in this section that the effects from higher wage flexibility via the endogenous response of the policy rate and through redistribution oppose to each other. The next section explores the effects from an increase in wage flexibility when these two channels interact and, due to the central role played by monetary policy, considers two different environments: one where the monetary authority has the ability to react to shocks and, alternatively, a setting where the central bank finds itself constrained by the ZLB constraint.

3.4 The gains from wage flexibility

Next we simulate the model economy introduced in section 3.2 to study the gains from wage flexibility in a setup characterized by limited asset markets participation and a monetary authority constrained by the ZLB.

3.4.1 The welfare loss function

In the forthcoming analysis we focus on the link between wage rigidities, macroeconomic stability and the ensuing welfare implications. To this end we derive the welfare loss function for our model featuring limited asset market participation. For its derivation we assume the central bank seeks to maximize a weighted average of unconstrained and constrained agents' utility, with the weights coinciding with the mass of households in each category. For simplicity, we further assume the existence of a labor subsidy that corrects for the inefficiencies generated by monopolistic competition, and transfers that equate the steady state consumption of constrained and unconstrained households. By performing a second order approximation of the utility around the efficient steady state with no inequality, average welfare losses can be expressed as:

$$L = \frac{1}{2}\Bigg[\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) var(\tilde{y}_t) + \sigma\lambda(1-\lambda)var(\hat{\gamma}_t) + \frac{\epsilon_p}{\lambda_p} var(\pi_t^p) + \frac{(1-\alpha)\epsilon_w}{\lambda_w} var(\pi_t^w)\Bigg]$$

where $\lambda_p \equiv \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$ and $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\varphi\epsilon_w)}$.

Welfare losses are a function of the output gap, price and wage inflation volatility, and the consumption gap. The latter term captures inequality and arises from the existence of limited asset markets participation.

3.4.2 Calibration

For the baseline calibration parameter α is set to 0.25 and the discount factor β to 0.994. Calvo price and wage parameters are set to 0.75, implying an average contract duration of 4 quarters. Parameters ϵ_p and ϵ_w are set to 6, which imply a steady state markup of 20%. Households' preferences parameters σ and φ are set to 1. Coefficients on the Taylor rule ϕ_π and ϕ_y are set to 1.5 and 0.125, respectively, while the smoothing parameter ρ_r is set to 0.8. The autoregressive coefficient for the preferences exogenous process ρ_c is set to 0.8. Regarding the fraction of constrained agents two scenarios are considered, a Ricardian economy where all households are unconstrained (i.e. $\lambda = 0$) and an economy with a positive fraction of constrained agents, for which $\lambda = 0.3$ is assumed.

3.4.3 Simulations

The following simulations are based on Billi and Galí (2019), who evaluate the welfare implications of a change in the degree of wage flexibility in a Ricardian economy where monetary authority is restricted by the ZLB. In particular, we simulate the response of the economy for two alternative scenarios, characterized by different degrees of wage rigidities. One corresponds to the baseline calibration, for the other, the Calvo parameter for wages is set to 0.3. Our focus in this exercise will be on the effects of greater wage flexibility on output volatility.

We begin by performing an IRF analysis for a scenario with no ZLB constraint, reported in figure (3.1). In the Ricardian economy, more flexible wages translate into more responsive prices which in turn trigger a stronger response of the policy rate. The more accommodative policy stance ultimately leads to a contained fall in output. Such transmission mechanism from greater wage flexibility to output volatility has been described in Galí (2013).

Consider next the scenario with financial frictions. Now an additional mechanism operates in the transmission from wage flexibility to output volatility. In our setting, given the assumed distribution of firms profits, income redistributes over the cycle and such redistribution is directly affected by the degree of wage adjustments in response to shocks. In particular, during recession episodes higher flexibility results in a larger drop in wages. Accordingly, labor income contracts by more relative to profit earnings, and hence, constrained agents (who only receive labor income) see their income fall by more relative to unconstrained households. Such redistribution against constrained agents, characterized by a higher propensity to consume, depresses demand and hence output. As the figure illustrates the more accommodative police stance dominates the contractionary effects of redistribution, and hence the net effect of wage flexibility is a contained drop in activity.

To discern whether this is a general result or simply a feature of our specific calibration we plot in figures (3.2) and (3.3) the impact and cumulative responses of output for alternative calibrations of parameters θ_w and λ. As the figure makes clear, the policy channel in fact tends to dominate as more flexibility can only result in increased output volatility when parameter θ_w approaches zero, i.e., under extreme degrees of flexibility.

At last, note from figure (3.1) that in spite of the milder contraction, constrained agents' income end up dropping by more in the scenario characterized by higher wage flexibility. In other words, stability is achieved at the cost worsening conditions for the low income, constrained individuals, during recession episodes.

Let us now turn to the scenario with a ZLB constraint, reported in figure (3.4). As emphasized in Billi and Galí (2019), during episodes when the nominal rate reaches the zero limit the before mentioned transmission mechanism from flexibilized wages to output volatility via the policy rate breaks. The figure makes clear that in such events more flexible wages are associated with a more severe deflation. Since the policy rate cannot be further reduced, deflationary expectations induce a drop in demand (due to the associated rise in the real rate), which exacerbates output contraction.

Finally, we explore the effects from the interaction between financial frictions and the ZLB. In this scenario the effects from greater wage flexibility via deflationary expectations and redistribution reinforce each other due to a feedback loop between the two channels. In fact, redistribution puts downward pressure on output and thus prices, which translate into a rise in the real rate that depresses activity. Given a countercyclical income gap, the latter implies redistribution against constrained individuals, which further depresses demand and output. A stronger cut in demand by unconstrained agents, who respond to more severe deflationary expectations, in conjunction with the contractive effects of redistribution, explain the more severe drop in activity as compared to the Ricardian economy.

To further explore the effects from greater wage flexibility, in the next exercise we generate artificial time series under demand shocks for our two alternative parametrizations of the Calvo wage parameter [2]. For this simulation the volatility of the innovation is set so that the ZLB binds 5% of the time. Figure (3.5) reports the results for the Ricardian economy. In line with our previous analysis, a reduction in rigidities affecting wages is associated with lower output variability in periods when the central bank reaction function is active, whilst volatility increases in periods when the ZLB binds. Yet, note that the reduction in rigidities has only a modest effect on output dynamics. Conversely, the volatility of price and wage inflation is largely affected by the reduction in rigidities.

Let us next turn to the scenario with financial frictions, reported in figures (3.6) and (3.7). Note that in this setting more wage flexibility can considerably exacerbate output contraction in periods when the monetary authority is constrained by the ZLB. In accordance with our earlier discussion, the more severe contraction is explained by a larger drop in unconstrained agents' consumption, who respond to increased deflationary expectations, and, to a larger extent, by redistribution. Importantly, notice from figure (3.7) that higher flexibility particularly affects constrained households, whose income is severely reduced due to a large cut in wages, requiring a sizable cut in spending.

Next we compute the volatility of the welfare relevant macro variables for our alternative parametrizations of θ_w. As table (3.1) makes clear, in the Ricardian economy higher flexibility associates with a more stable output while price and wage inflation volatility largely increase. In what concerns to the non-Ricardian economy, two outcomes are worth of mention. Firstly, in this setting more flexible wages destabilize output. Secondly, price and wage inflation volatility are more affected by flexibilized wages. The latter results from the negative impact of redis-

[2] Simulations are performed using the extended path method.

tribution on output, which leads to more severe deflationary episodes in periods when the ZLB binds. Finally, note the sizable impact on the consumption gap, whose standard deviation rises from 1.1% to 5.5% in the scenario characterized by lower rigidities.

At last, we compute welfare losses associated to our alternative scenarios. As table (3.2) illustrates, and in line with the previous discussion, greater wage flexibility increases losses related to all welfare relevant variables in the non-Ricardian economy.

3.5 Conclusion

This paper analyzes the consequences of wage flexibility in the context of incomplete markets and the zero lower bound. In general, wage flexibility is associated with higher inflation volatility. In the context of demand shocks being the driving force of the economy, we show that when the central bank cannot properly offset fluctuations, wage flexibility also reduces welfare due to increased output volatility. Moreover, the combination with market incompleteness reduces even further the ability of the authority to face a crisis, because of a redistribution channel that directly affects aggregate demand, making the zero lower bound a more active problem. In line with other results in the literature, we conclude that the classical policy prescription of flexibilizing wages during a downturn does not improve welfare, an outcome that aggravates in the presence of market incompleteness.

Figures and tables

	$\tilde{y}$	π^p	π^w
$\theta_w = 0.75$	0.030	0.002	0.002
$\theta_w = 0.30$	0.029	0.004	0.015
Ratio	0.96	2.7	6.4

(a) $\lambda = 0$

	$\tilde{y}$	γ	π^p	π^w
$\theta_w = 0.75$	0.027	0.011	0.001	0.002
$\theta_w = 0.30$	0.036	0.055	0.006	0.019
Ratio	1.3	5	3.8	8.9

(b) $\lambda = 0.3$

Table 3.1: Standard deviation of welfare relevant macro-variables

	$\tilde{y}$	π^p	π^w	Total Loss
$\theta_w = 0.75$	0.0012	0.0003	0.0010	0.0025
$\theta_w = 0.30$	0.0011	0.0019	0.0022	0.0052
Ratio	0.9	7.1	2.1	2.1

(a) $\lambda = 0$

	$\tilde{y}$	γ	π^p	π^w	Total Loss
$\theta_w = 0.75$	0.0010	0.0000	0.0002	0.0008	0.0021
$\theta_w = 0.30$	0.0017	0.0003	0.0032	0.0034	0.0087
Ratio	1.7	24.9	14.7	4.1	4.2

(b) $\lambda = 0.3$

Table 3.2: Consumption equivalent welfare losses

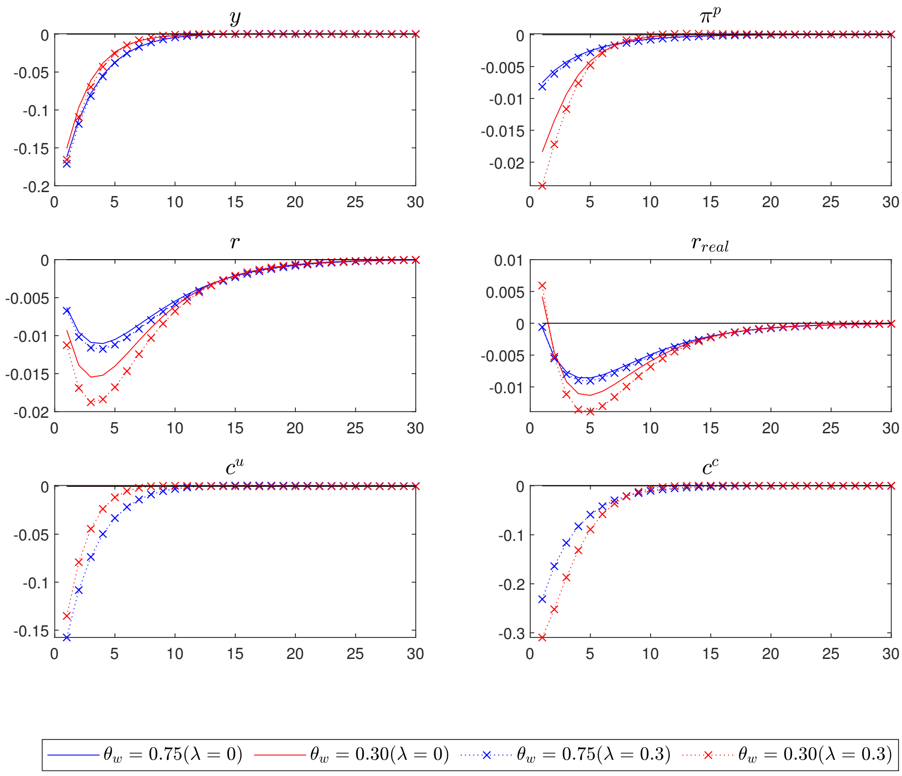


Figure 3.1: IRFs under preference shocks.

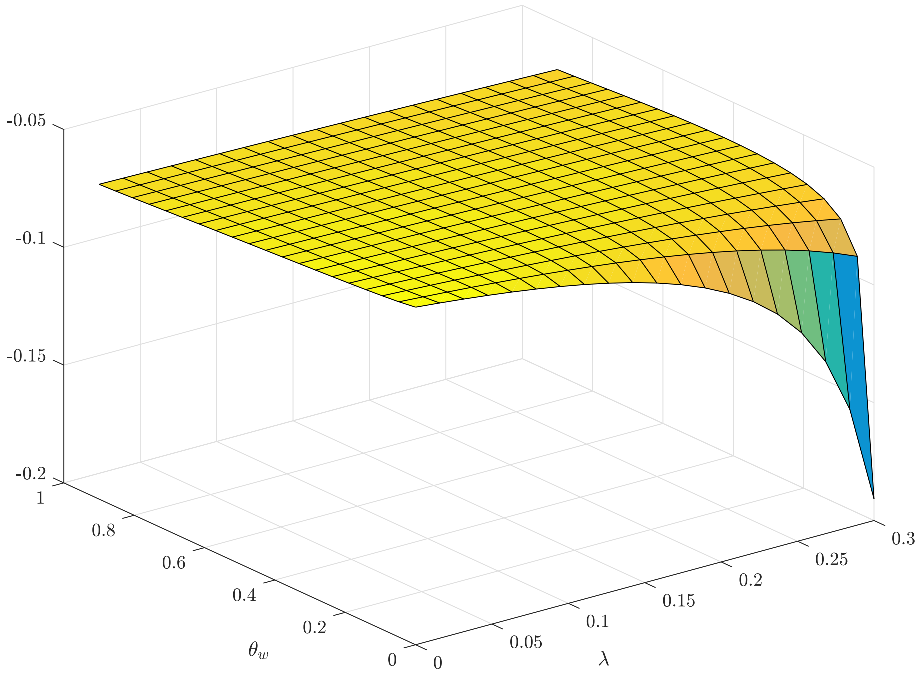


Figure 3.2: Output impact response under preference shocks.

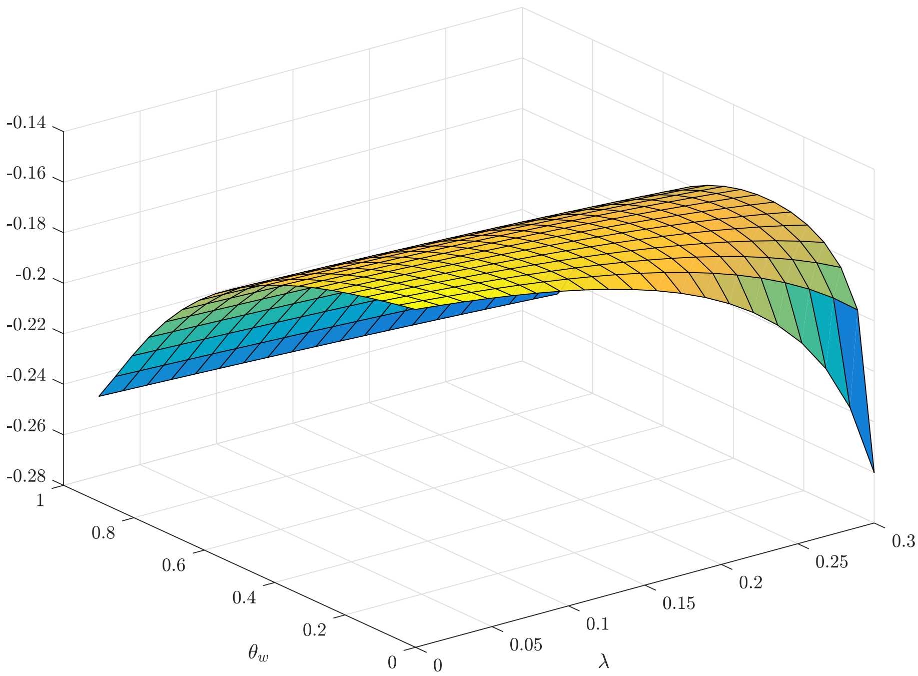


Figure 3.3: Output cumulative response under preference shocks.

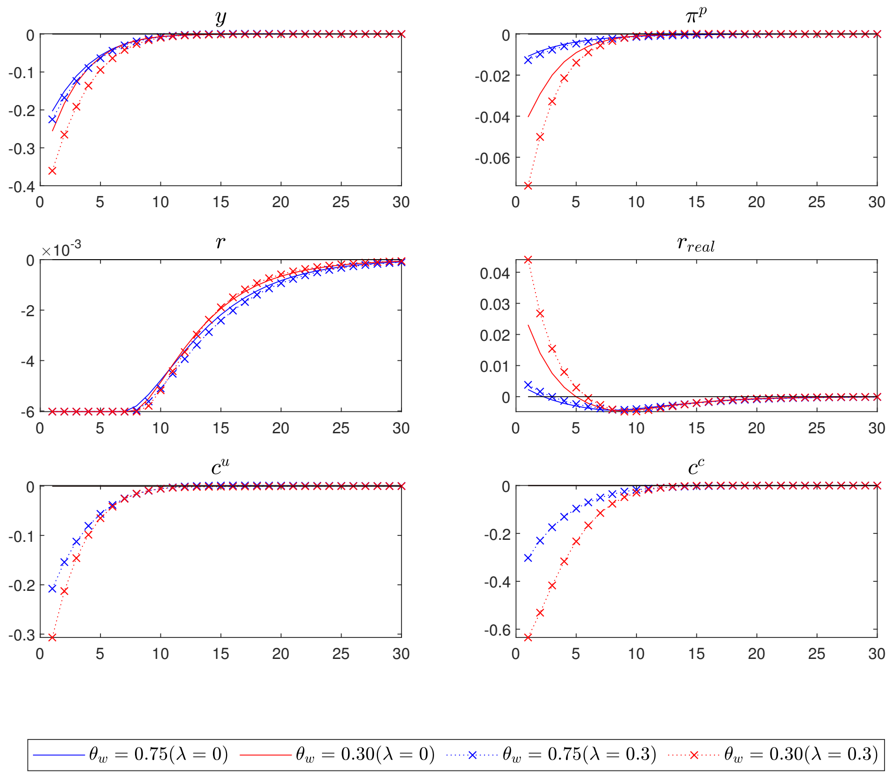


Figure 3.4: IRFs under preference shocks and a ZLB.

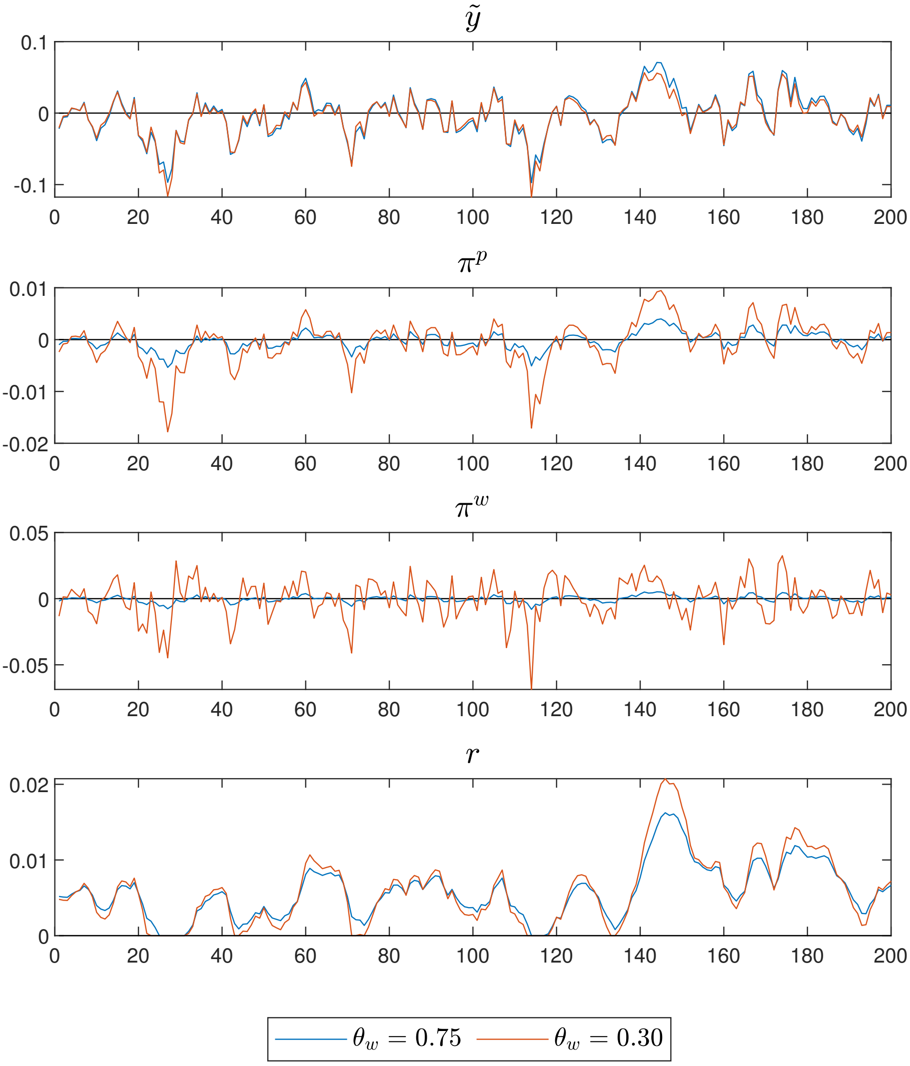


Figure 3.5: Fluctuations under preference shocks ($\lambda = 0$).

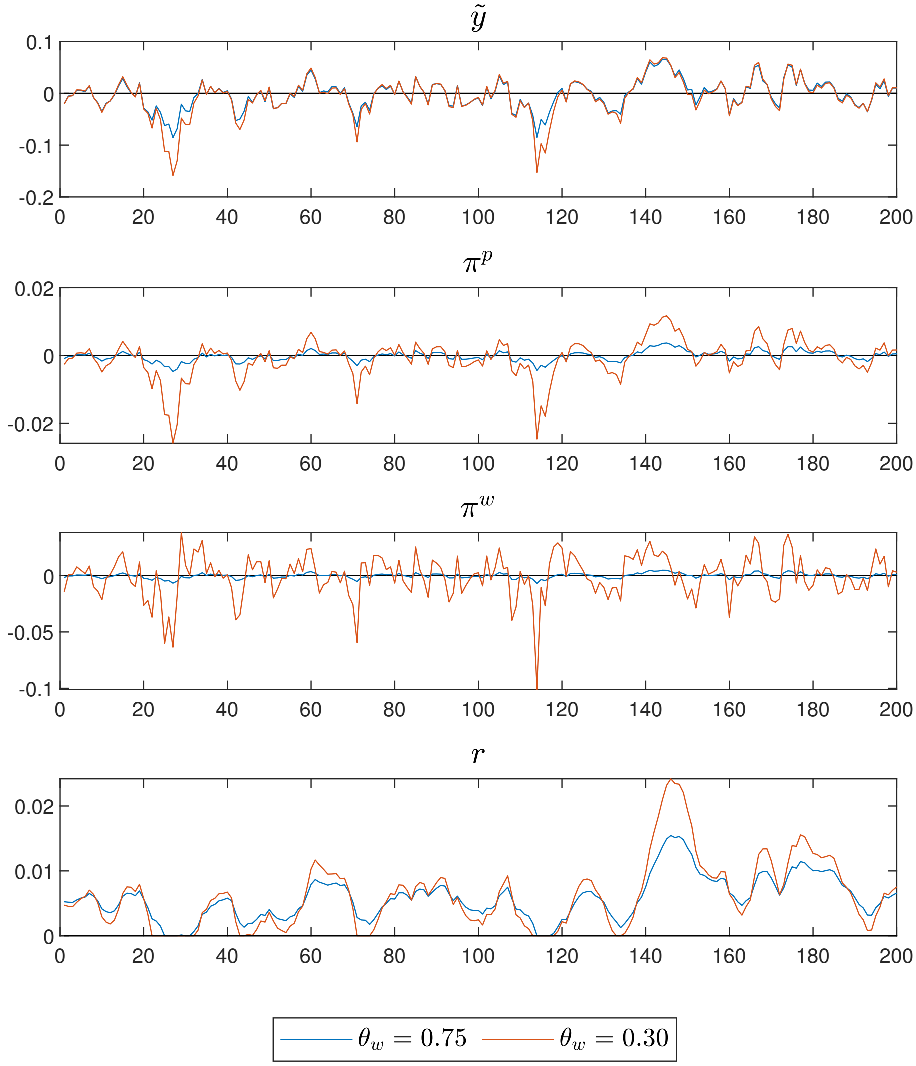


Figure 3.6: Fluctuations under preference shocks $(\lambda = 0.3)$.

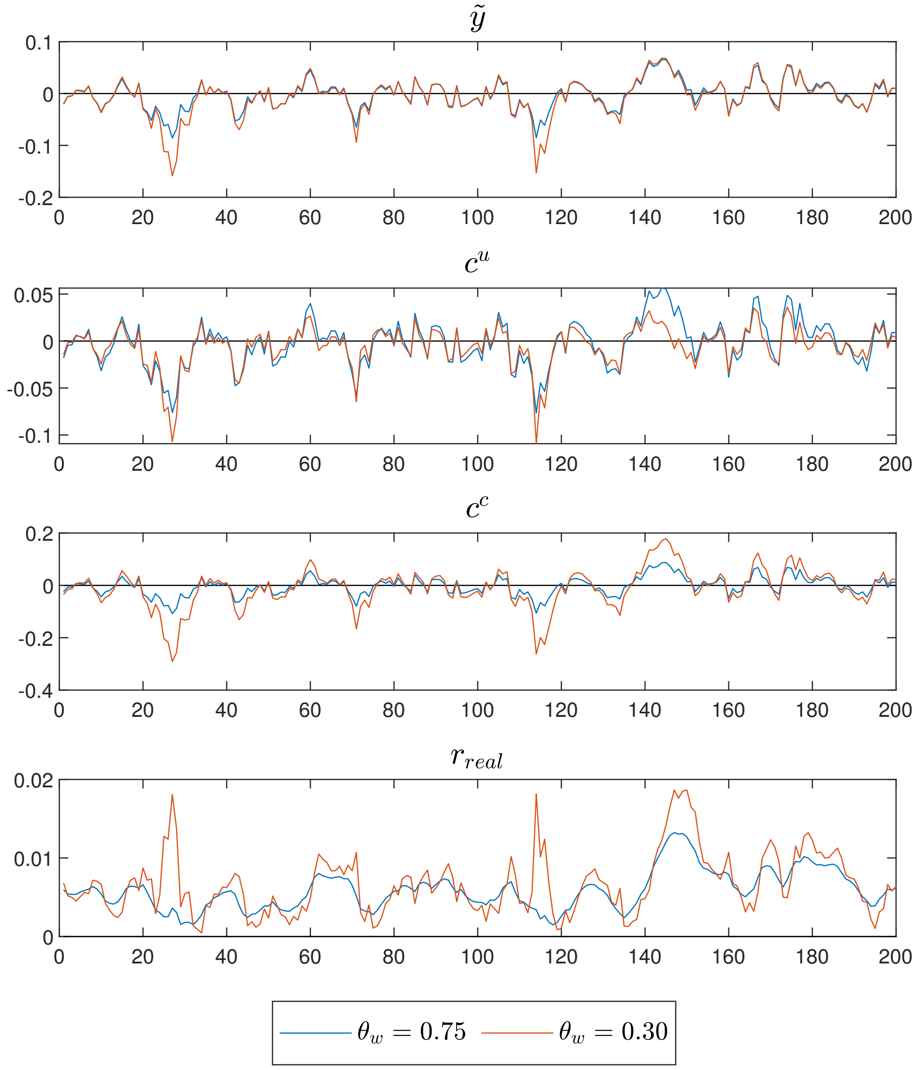


Figure 3.7: Fluctuations under preference shocks ($\lambda = 0.3$).

Appendix

The cyclicality of the real wage

Next we seek to determine the values coefficient Ξ can adopt. Note that:

$$\lim_{\theta_p \to 0} \Xi = \frac{\frac{1-\theta_w}{1+\theta_w \varphi \epsilon_w}\left(\sigma + \frac{\varphi}{1-\alpha}\right) - \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w \varphi \epsilon_w}\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\frac{\alpha}{1-\alpha}}{1 + \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w \varphi \epsilon_w}\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}} = \frac{\infty}{\infty}$$

Applying L'Hospital's rule:

$$\lim_{\theta_p \to 0} \frac{\frac{\partial N}{\partial \theta_p}}{\frac{\partial D}{\partial \theta_p}} = \frac{-\frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w \varphi \epsilon_w}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\frac{\alpha}{1-\alpha}\frac{-\theta_p-(1-\theta_p)}{\theta_p^2}}{\frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w \varphi \epsilon_w}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\frac{-\theta_p-(1-\theta_p)}{\theta_p^2}} = -\frac{\alpha}{1-\alpha}$$

where N and D denote respectively the numerator and denominator of Ξ.

Also note that:

$$\frac{\partial \Xi}{\partial \theta_p} = \frac{\left(\frac{\alpha}{1-\alpha} + \frac{1-\theta_w}{1+\theta_w \varphi \epsilon_w}\left(\sigma + \frac{\varphi}{1-\alpha}\right)\right)\frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w \varphi \epsilon_w}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\frac{1}{\theta_p^2}}{\left(1 + \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w \varphi \epsilon_w}\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\right)^2} > 0$$

We have shown that for any calibration of parameters σ, φ, ϵ_p, θ_w and ϵ_w, coefficient Ξ is an increasing function of θ_p with lower limit $-\frac{\alpha}{1-\alpha}$. Accordingly, for any parameter calibration $\Xi \geqslant -\frac{1}{1-\alpha}$ holds.

Bibliography

Abbritti, M. and Weber, S. (2010). Labor market institutions and the business cycle - unemployment rigidities vs. real wage rigidities. Working Paper Series 1183, European Central Bank.

Alvarez, L. J., Dhyne, E., Hoeberichts, M., Kwapil, C., Le Bihan, H., Lünnemann, P., Martins, F., Sabbatini, R., Stahl, H., Vermeulen, P., et al. (2006). Sticky prices in the euro area: a summary of new micro-evidence. *Journal of the European Economic association*, 4(2-3):575–584.

Anand, R., Prasad, E. S., and Zhang, B. (2015). What measure of inflation should a developing country central bank target? *Journal of Monetary Economics*, 74:102–116.

Aoki, K. (2001). Optimal monetary policy responses to relative-price changes. *Journal of monetary economics*, 48(1):55–80.

Ascari, G., Colciago, A., and Rossi, L. (2017). Limited asset market participation, sticky wages, and monetary policy. *Economic Inquiry*, 55(2):878–897.

Benigno, P. (2004). Optimal monetary policy in a currency area. *Journal of international economics*, 63(2):293–320.

Bilbiie, F. O. (2008). Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory*, 140(1):162–196.

Billi, R. and Galí, J. (2019). Gains from Wage Flexibility and the Zero Lower Bound. Working Paper Series 367, Sveriges Riksbank (Central Bank of Sweden).

Bowdler, C. and Nunziata, L. (2007). Inflation adjustment and labour market structures: Evidence from a multi-country study. *Scandinavian Journal of Economics*, 109(3):619–642.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398.

Campolmi, A. and Faia, E. (2011). Labor market institutions and inflation volatility in the euro area. *Journal of Economic Dynamics and Control*, 35(5):793–812.

Card, D. and Krueger, A. B. (1995). Time-series minimum-wage studies: a meta-analysis. *The American Economic Review*, 85(2):238–243.

Christoffel, K., Kuester, K., and Linzert, T. (2009). The role of labor markets for euro area monetary policy. *European Economic Review*, 53(8):908–936.

Christoffel, K. P., Kuester, K., and Linzert, T. (2006). Identifying the role of labor markets for monetary policy in an estimated DSGE model. Discussion Paper Series 1: Economic Studies 2006,17, Deutsche Bundesbank.

Ciccone, A. and Peri, G. (2005). Long-run substitutability between more and less educated workers: evidence from us states, 1950–1990. *Review of Economics and statistics*, 87(4):652–663.

Debortoli, D. and Galí, J. (2017). Monetary policy with heterogeneous agents: Insights from tank models. Working paper, CREI.

Doucouliagos, H. and Stanley, T. D. (2009). Publication selection bias in minimum-wage research? a meta-regression analysis. *British Journal of Industrial Relations*, 47(2):406–428.

Floden, M. and Lindé, J. (2001). Idiosyncratic risk in the united states and sweden: Is there a role for government insurance? *Review of Economic dynamics*, 4(2):406–437.

Galesi, A. and Rachedi, O. (2016). Structural transformation, services deepening, and the transmission of monetary policy. Working Papers 1615, Banco de España.

Galí, J. (2013). Notes for a new guide to keynes (i): wages, aggregate demand, and employment. *Journal of the European Economic Association*, 11(5):973–1003.

Galí, J. and Monacelli, T. (2016). Understanding the gains from wage flexibility: the exchange rate connection. *American Economic Review*, 106(12):3829–68.

Gnocchi, S., Lagerborg, A., and Pappa, E. (2015). Do labor market institutions matter for business cycles? *Journal of Economic Dynamics and Control*, 51:299–317.

Gollin, D., Parente, S. L., and Rogerson, R. (2007). The food problem and the evolution of international income levels. *Journal of Monetary Economics*, 54(4):1230–1255.

Gollin, D. and Rogerson, R. (2014). Productivity, transport costs and subsistence agriculture. *Journal of Development Economics*, 107:38–48.

Jardim, E., Long, M. C., Plotnick, R., Van Inwegen, E., Vigdor, J., and Wething, H. (2017). Minimum wage increases, wages, and low-wage employment: Evidence from seattle. Technical report, National Bureau of Economic Research.

Krusell, P., Mukoyama, T., and Smith Jr, A. A. (2011). Asset prices in a huggett economy. *Journal of Economic Theory*, 146(3):812–844.

Mankiw, N. G. and Reis, R. (2003). What measure of inflation should a central bank target? *Journal of the European Economic Association*, 1(5):1058–1086.

Mollick, A. V. (2011). The world elasticity of labor substitution across education levels. *Empirical Economics*, 41(3):769–785.

Portillo, R., Zanna, L. F., O'Connell, S., and Peck, R. (2016). Implications of food subsistence for monetary policy and inflation. *Oxford Economic Papers*, 68(3):782–810.

Schmitt-Grohé, S. and Uribe, M. (2007). Optimal simple and implementable monetary and fiscal rules. *Journal of monetary Economics*, 54(6):1702–1725.

Trigari, A. (2006). The Role of Search Frictions and Bargaining for Inflation Dynamics. Working Papers 304, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University.

Walsh, C. E. (2005). Labor market search, sticky prices, and interest rate policies. *Review of economic Dynamics*, 8(4):829–849.

Werning, I. (2015). Incomplete Markets and Aggregate Demand. NBER Working Papers 21448, National Bureau of Economic Research.