Model Predictive Control of Complex Systems including Fault Tolerance Capabilities: Application to Sewer Networks

Carlos A. Ocampo Martínez

B.Sc, M.Sc

Advisors

Dr. VICENÇ PUIG
Dr. JOSEBA QUEVEDO

Programa de Doctorat en Control, Visió i Robòtica
Automatic Control Department (ESAII)
Technical University of Catalonia

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08222 - Terrassa (Spain)

Advisors:

Dr. VICENÇ PUIG

Dr. JOSEBA QUEVEDO

To Maria

ABSTRACT

Real time control (RTC) of sewer networks plays a fundamental role in the management of hydrological systems, both in the urban water cycle, as well as in the natural water cycle. An adequate design of control systems for sewer networks can prevent the negative impact on the environment that Combined Sewer Overflow (CSO) as well as preventing flooding within city limits when extreme weather conditions occur. However, sewer networks are large scale systems with many variables, complex dynamics and strong nonlinear behavior. Any control strategy applied should be capable of handling these challenging requirements. Within the field of RTC of sewer networks for global network control, the Model Predictive Control (MPC) strategy stands out due to its ability to handle large scale, nonlinear and multivariable systems. Furthermore, this strategy allows performance optimization, taking into account several control objectives simultaneously.

This thesis is devoted to the design of MPC controllers for sewer networks, as well as the complementary modeling methodologies. Furthermore, scenarios where actuator faults occur are specially considered and strategies to maintain performance or at least minimizing its degradation in presence of faults are proposed. In the first part of this thesis, the basic concepts are introduced: sewer networks, MPC and fault tolerant control. In addition, the modeling methodologies used to describe such systems are presented. Finally the case study of this thesis is described: the sewer network of the city of Barcelona (Spain).

The second part of this thesis is centered on the design of MPC controllers for the proposed case study. Two types of models are considered: (i) a linear model whose corresponding MPC strategy is known for its advantages such as convexity of the optimization problem and existing proofs of stability, and (ii) a hybrid model which allows the inclusion of state dependent hybrid dynamics such as weirs. In the latter case, a new hybrid modeling methodology is introduced and hybrid model predictive control (HMPC) strategies based on these models are designed. Furthermore, strategies to relax the optimization problem are introduced to reduce calculation time required for the HMPC control law.

Finally, the third part of this thesis is devoted to study the fault tolerance capabilities of MPC controllers. Actuator faults in retention and redirection gates are considered. Additionally, hybrid modeling techniques are presented for faults which, in the linear case, can not be treated without loosing convexity of the related optimization problem. Two fault tolerant HMPC strategies are compared: the active strategy, which uses the information from a diagnosis system to maintain control performance, and the passive strategy which only relies on the intrinsic robustness of the MPC control law. As an extension to the study of fault tolerance, the admissibility of faulty actuator configurations is analyzed with regard to the degradation of control objectives. The method, which is based on constraint satisfaction, allows the admissibility evaluation of actuator fault configurations, which avoids the process of solving the optimization problem with its related high computational cost.

Keywords: MPC, sewer networks, hybrid systems, MLD, fault tolerant control, constraints satisfaction.

RESUMEN

El control en tiempo real de redes de alcantarillado (RTC) desempeña un papel fundamental dentro de la gestión de los recursos hídricos relacionados con el ciclo urbano del agua y, en general, con su ciclo natural. Un adecuado diseño de control para de redes de alcantarillado evita impactos medioambientales negativos originados por inundaciones y/o alta polución producto de condiciones meteorológicas extremas. Sin embargo, se debe tener en cuenta que estas redes, además de su gran tamaño y cantidad de variables e instrumentación, son sistemas ricos en dinámicas complejas y altamente no lineales. Este hecho, unido a unas condiciones atmosféricas extremas, hace necesario utilizar una estrategia de control capaz de soportar todas estas condiciones. En este sentido, dentro del campo del RTC de redes de alcantarillado se destacan las estrategias de control predictivo basadas en modelo (MPC), las cuales son alternativas adecuadas para el control de configuraciones multivariable y de gran escala, aplicadas como estrategias de control global del sistema. Además, permiten optimizar el desempeño del sistema teniendo en cuenta diversos índices de rendimiento (control multiobjetivo).

Esta tesis se enfoca en el diseño de controladores MPC para redes de alcantarillado considerando diversas metodologías de modelado. Adicionalmente, analiza las situaciones en las cuales se presentan fallos en los actuadores de la red, proponiendo estrategias para mantener el desempeño del sistema y evitando la degradación de los objetivos de control a pesar de la presencia del fallo. En la primera parte se introducen los conceptos principales de los temas a tratar en la tesis: redes de alcantarillado, MPC y tolerancia a fallos. Además, se presenta la técnica de modelado utilizada para definir el modelo de una red de alcantarillado. Finalmente, se presenta y describe el caso de aplicación considerado en la tesis: la red de alcantarillado de Barcelona (España).

La segunda parte se centra en diseñar controladores MPC para el caso de estudio. Dos tipos de modelo de la red son considerados: (i) un modelo lineal, el cual aproxima los comportamientos no lineales de la red, dando origen a estrategias MPC lineales con sus conocidas ventajas de optimización convexa y escalabilidad; y (ii) un modelo híbrido, el cual incluye las dinámicas de conmutación más representativas de una red de alcantarillado como lo son los rebosaderos. En este último caso se propone una nueva metodología de modelado híbrido para redes de alcantarillado y se diseñan estrategias de control predictivas basadas en estos modelos (HMPC), las cuales calculan leyes de control globalmente óptimas. Adicionalmente se propone una estrategia de relajación del problema de optimización discreto para evitar los grandes tiempos de cálculo que pudieran ser requeridos al obtener la ley de control HMPC.

Finalmente, la tercera parte de la tesis se ocupa de estudiar las capacidades de tolerancia a fallos en actuadores de lazos de control MPC. En el caso de redes de alcantarillado, la tesis considera fallos en las compuertas de derivación y de retención de aguas residuales. De igual manera, se propone un modelado híbrido para los fallos que haga que el problema de optimización asociado no pierda su convexidad. Así, se proponen dos estrategias de HMPC tolerante a fallos (FTMPC): la estrategia activa, la cual utiliza las ventajas de una arquitectura de control tolerante a fallos (FTC), y la estrategia pasiva, la cual sólo depende de la robustez intrínseca de las técnicas de control MPC. Como extensión al estudio de tolerancia a fallos, se propone una evaluación de admisibilidad para configuraciones de actuadores en fallo tomando como referencia la degradación de los objetivos de control. El método, basado en satisfacción de restricciones, permite evaluar la admisibilidad de una configuración de actuadores en fallo y, en caso de no ser admitida, evitaría el proceso de resolver un problema de optimización con un alto coste computacional.

Palabras clave: control predictivo basado en modelo, sistemas de alcantarillado, sistemas híbridos, MLD, control tolerante a fallos, satisfacción de restricciones.

RESUM

El control en temps real de xarxes de clavegueram (RTC) desenvolupa un paper fonamental dins de la gestió dels recursos hídrics relacionats amb el cicle urbà de l'aigua i, en general, amb el seu cicle natural. Un adequat disseny de control per a xarxes de clavegueram evita impactes mediambientals negatius originats per inundacions i/o alta pol·lució producte de condicions meteorològiques extremes. No obstant, s'ha de tenir en compte que aquestes xarxes, a més de la seva grandària i quantitat de variables i instrumentació, són sistemes rics en dinàmiques complexes i altament no lineals. Aquest fet, unit a les condicions atmosfèriques extremes, fan necessari utilitzar una estratègia de control capaç de suportar totes aquestes condicions. En aquest sentit, dins del camp del (RTC) de xarxes de clavegueram es destaquen les estratègies de control predictiu basat en model (MPC), les quals són alternatives adequades per al control de configuracions multivariable i de gran escala, aplicades com estratègies de control global del sistema. A més, permeten optimitzar la resposta del sistema tenint en compte diversos índexs de rendiment (control multiobjectiu).

Aquesta tesi s'enfoca en el disseny de controladors MPC per a xarxes de clavegueram considerant diverses metodologies de modelat. Addicionalment, analitza les situacions en les quals es presenten fallades als actuadors de la xarxa, proposant estratègies per a mantenir la resposta del sistema amb la menor degradació possible dels objectius de control, malgrat la presència de la fallada. En la primera part s'introdueixen els conceptes principals dels temes a tractar en la tesi: xarxes de clavegueram, MPC i tolerància a fallades. Seguidament, es presenta la tècnica de modelat utilitzada per a definir el model d'una xarxa de clavegueram. Finalment, es presenta i descriu el cas d'aplicació en la tesi: la xarxa de clavegueram de Barcelona (Espanya).

La segona part es centra en dissenyar controladors MPC per al cas d'estudi. S'han considerat dos tipus de model de xarxa: (i) un model lineal, el qual aproxima els comportaments no lineals de la xarxa, donant origen a estratègies MPC lineals amb les seves conegudes avantatges de l'optimització convexa i escalabilitat; i (ii) un model híbrid, el qual inclou les dinàmiques de commutació més representatives d'una xarxa de clavegueram com són els sobreeixidors.

En aquest últim cas es proposa una nova metodologia de modelat híbrid per a xarxes de clavegueram i es dissenyen estratègies de control predictives basades en aquests models (HMPC), les quals calculen lleis de control globalment òptimes. Addicionalment, es proposa una estratègia de relaxació del problema d'optimització discreta per a evitar els grans temps de còmput requerits per a calcular la llei de control HMPC.

Finalment, la tercera part de la tesi s'encarrega d'estudiar les capacitats de tolerància a fallades en actuadors de llaços de control MPC. En el cas de xarxes de clavegueram, la tesi considera fallades en les comportes de derivació i de retenció d'aigües residuals. A més, es proposa un modelat híbrid per a fallades que faci que el problema d'optimització associat no perdi la seva convexitat. Així, es proposen dos estratègies de HMPC tolerant a fallades (FTMPC): l'estratègia activa, la qual utilitza les avantatges d'una arquitectura de control tolerant a fallades (FTC), i l'estratègia passiva, la qual només depèn de la robustesa intrínseca de les tècniques de control MPC. Com a extensió a l'estudi de tolerància a fallades, es proposa una avaluació d'admissibilitat per a configuracions d'actuadors en fallada agafant com a referència la degradació dels objectius de control. El mètode, basat en satisfacció de restriccions, permet avaluar l'admissibilitat d'una configuració d'actuadors en fallada i, en cas de no ser admesa, evitaria el procés de resoldre un problema d'optimització amb un alt cost computacional.

Paraules clau: control predictiu basat en model, sistemes de clavegueram, sistemes híbrids, MLD, control tolerant a fallades, satisfacció de restriccions.

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Carlos Ocampo Martínez
Terrassa, April 2007

NOTATION

Throughout the thesis and as a general rule, scalars and vectors are denoted with lower case letters (e.g., a, x, $\ldots$), matrices are denoted with upper case letters (e.g., A, B, $\ldots$) and sets are denoted with upper case double stroke letters (e.g., $\mathbb{F}$, $\mathbb{G}$, $\ldots$). If not otherwise noted, all vectors are column vectors.

$\mathbb{R}$	set of real numbers
$\mathbb{R}_+$	set of non-negative real numbers, defined as $\mathbb{R}_+ \triangleq \mathbb{R} \setminus (-\infty, 0]$
$\mathbb{Z}$	set of integer numbers
$\mathbb{Z}_+$	set of non-negative integer numbers
$\mathbb{Z}_{\geq c}$	set defined as $\mathbb{Z}_{\geq c} \triangleq k \in \mathbb{Z} \mid k \geq c$, for some $c \in \mathbb{Z}$
$\mathbb{Y}^m$	$\underbrace{\mathbb{Y} \times \mathbb{Y} \times \cdots \times \mathbb{Y}}_{m \text{ times}}$
$\lVert q \rVert_p$	arbitrary Hölder vector p-norm with $1 \leq p \leq \infty$
$\mathbb{A} \subseteq \mathbb{B}$	$\mathbb{A}$ is a subset of $\mathbb{B}$
$\mathbb{A} \subset \mathbb{B}$	$\mathbb{A}$ is a proper subset of $\mathbb{B}$
$\rightarrow$	mapping
$\mapsto$	maps to
$\leftrightarrow$	if and only if

H_p	prediction horizon
H_u	control horizon
$\mathcal{U}_{H_p}$	admissible input sequence
$\mathbb{O}$	set of control objectives
$\mathbf{x}_k$	sequence of states ($\mathbf{x}_k$), control inputs ($\mathbf{u}_k$), logical variables (Δ_k) or auxiliary variables ($\mathbf{z}_k$) over a time horizon m, denoted by $\mathbf{x}_k \triangleq (x_1, x_2, \ldots, x_m)$
$J(\cdot)$	MPC cost or objective function (also denoted as V_{MPC})
w_i	i-th cost function weight

u^*	optimal value of u
v	tank volume (state variable)
q_u	manipulated flow (control input)
d	rain inflow (measured disturbance)
φ	ground absorption factor
S	surface area for a sub-catchment
S_w	wetted surface area of a sewer
β	Volume/Flow Conversion coefficient
L	Level gauge (limnimeter)
T_i	i-th tank
P	rain intensity
Δt	sampling time
$\overline{x}$	upper bound of the interval where x is defined
$\underline{x}$	lower bound of the interval where x is defined
$\square\mathbb{A}$	interval hull of set $\mathbb{A}$
diag	diagonal matrix
min	minimum
max	maximum
sat	saturation function
dzn	*dead zone* function

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CHAPTER 1

INTRODUCTION

1.1 Motivation

Water, an essential element for life, has a paramount importance in the future of mankind because it is a scarce resource in a global scale. Water is the most important renewable natural resource and, at the same time, the most endangered one. The pressure arising from decades of human action results in non-sustainable management and control policies. The water problem is particularly severe in the Mediterranean coast, as a consequence of ongoing climate changes: reports from IPCC (Intergovernmental Panel on Climate Change, `http://www.ipcc.ch/`) sponsored by the World Meteorological Organization and United Nations will be presented in Paris in February 2007; such reports indicate that the availability of hydrological resources in the above mentioned region may decrease up to a 30% in the coming decades.

But problems around the water can be associated according to its cycle in the nature and the human influence over this natural cycle. Water management has become an increasingly important environmental and socioeconomic subject worldwide. High costs associated to processes such as pumping, transportation, storage, treatment and distribution, as well as for the collection and treatment of urban drainage, limit the accessibility of water for a large portion of the world. Processes mentioned before, among others, conform the *urban water cycle*, which details the long journey of a drop of water from when it is collected for use in an urban community to when it is returned to the natural water cycle [MMM01].

Knowing the urban water cycle, it is easier to figure out clearly the difficult process of its management and to infer the critical problems in order to propose some ways of solution. Figure 1.1 shows the urban water cycle, which includes different stages from source, transport,

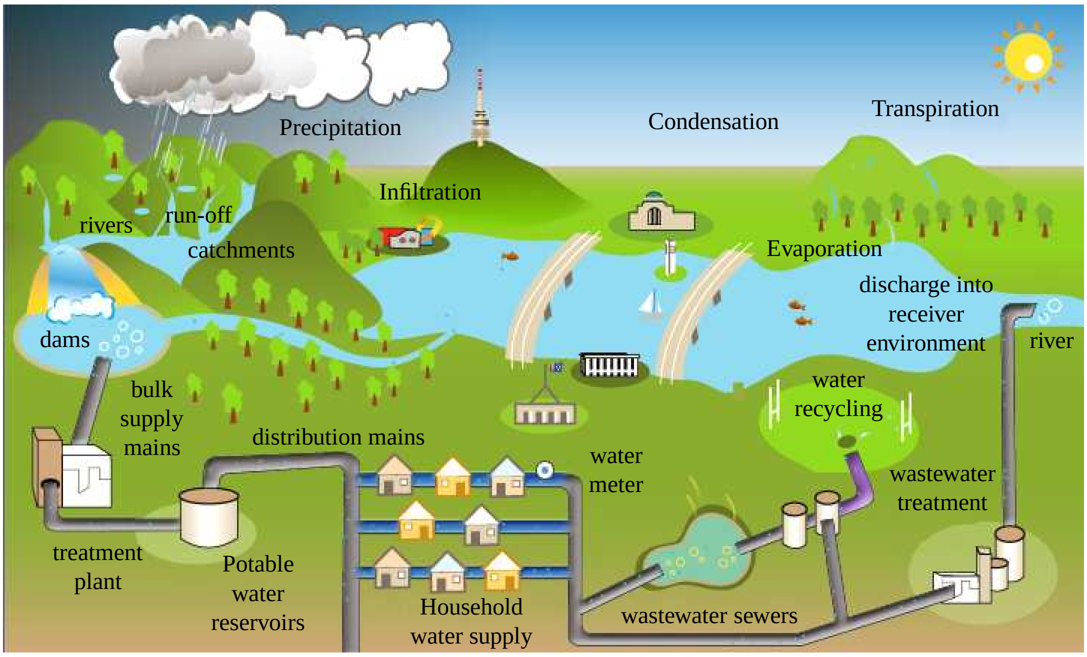

Figure 1.1: Urban water cycle and its main elements and processes.

purification and conditioning for human needs, distribution, consumption, waste water pipelines, depuration and finally reuse or disposal in the natural environment.

This thesis focuses on studying the part related to collecting sewage produced by homes and businesses for being carried to treatment plants in order to avoid pollution in the environment. All the used water from buildings leaves as wasterwater through a set of pipes called *sewer pipes*. Then, the set of linking pipes is called *sewer network*, that is the kind of systems this thesis is focused. Moreover, sewer networks might also integrate a stormwater system, which collects all run-off from rainwater such as road and roof drainage, and a wastewater treatment system, which is used to treat the sewage in order to return it to the natural water cycle free of pollution. The integration of all of these subsystems increases the complexity of the whole system in the sense of its management and the potential risks related to a possible wrong operation.

1.1.1 Sewer Networks as a Complex System

According to the discussion presented before, sewage systems present some specific characteristics which make them especially challenging from the point of view of analysis and control.

They include many complex dynamics and/or behaviors which can be outlined as follows:

- Nonlinear dynamics, which can be seen as structural nonlinearities and changes in the system parameters according to the operating point, e.g. in open-flow channel dynamics and in water quality decay models.

- Compositional subsystems with important delays, e.g. in dynamics related to rivers and open-flow channels.

- Compositional subsystems containing both continuous-variable elements, such as pipe flows and discrete on-off control devices such as fixed-speed pumps.

- Storage and actuator elements with operational constraints, which are operated within a specific physical range.

- Stochastic disturbances, such as rain intensities affecting the urban drainage modeling and operation.

- Partially unknown subsystems and/or behaviors, e.g. networks which have been in operation for many years are partially unknown. Relevant physical characteristics such as diameter, bumpy and slope change in function of time. Similarly, water leakage is an important unknown factor.

- Distributed, large-scale architecture, since water systems may have hundreds or even thousands of sensors, actuators and local controllers.

All the features mentioned before should be taken into account not only in the topology design of a sewer network but also in the definition of an adequate control strategy in order to fulfill a set of given control objectives. In the case of sewer networks, these objectives are related to the environmental protection and the prevention of disasters produced by either the wrong system management or by faulty elements within the network (sensors, actuators or other constitutive elements). For instance, in Figure 1.2, the terrible effects caused by heavy rain episodes occurred in the city of Barcelona can be seen. During these rain episodes, the sewer network capacity could not support the huge water volume fallen, causing flooding and high pollution in the Mediterranean Sea and in the rivers close to the city.

Figure 1.2: Some effects of flooding in Barcelona.

1.1.2 Model Predictive Control

To avoid the rain consequences shown in Figure 1.2, the analysis of sewer networks sets up new challenges in the scientific community, requiring top-level skills in the different control methodologies. Such methodologies have to handle the effect of rain disturbances in a robust way and should be as simple as possible in the sense of complexity and computation time. Since there are many sensors and actuators within a sewer network, this system should be governed using a strategy which can handle multivariable models and can compensate the effect of undesired dynamics such as delays, dead times, as well as consider physical constraints and nonlinear behaviors.

Thus, within the field of control of sewer networks, there exists a suitable strategy, which

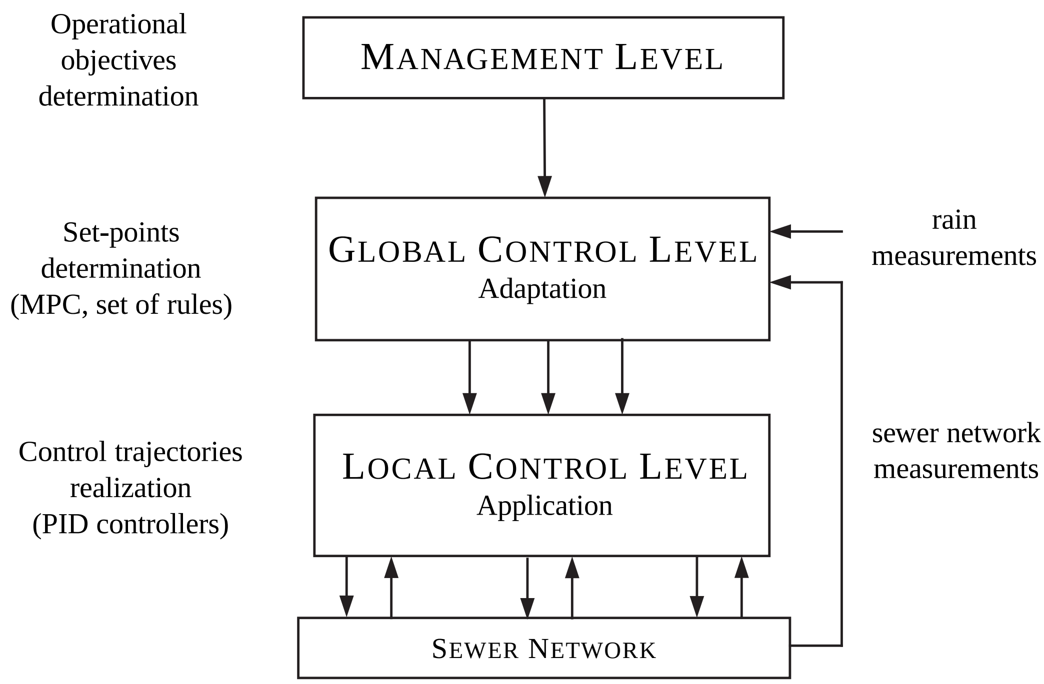

Figure 1.3: Hierarchical structure for RTC system. Adapted from [SCC+04] and [MP05].

fits with the particular issues of such systems. This strategy is known as Model-based Predictive Control (or simply Model Predictive Control - MPC), which more than a control technique, is a set of control methodologies that use a mathematical model of a considered system to obtain a control signal minimizing a cost function related to selected indexes of the system performance. MPC is very flexible regarding its implementation and can be used over almost all systems since it is set according to the model of the plant [CB04]. As will be discussed in Section 2.2.1, MPC has some features to deal with complex systems as sewer networks: big delays compensation, use of physical constraints, relatively simple for people without deep knowledge of control, multivariable systems handling, etc. Hence, according to [SCC+04], such controllers are very suitable to be used in the global control of urban drainage systems within a hierarchical control structure [Pap85, MP05]. Figure 1.3 shows a conceptual scheme for a hierarchical structure considered on the control of sewer networks.

Notice in Figure 1.3 that MPC, as the global control law, determines the references for local controllers located on different elements of the sewer network. These references are computed using measurements taken from sensors distributed along the network and rain sensors. Management level is used to provide to MPC the operational objectives, what is reflected in the controller design as the performance indexes to be minimized. In the case of urban drainage systems, these indexes are usually related to flooding, pollution, control energy, etc.

1.1.3 Fault Tolerant Control

In sewer networks framework, Real-time control (RTC) is a custom-designed management program for a specific urban sewage system that is activated during a wet-weather event. In some cities such as Barcelona, the sewer network uses telemetry (rain gauges and water level sensors in sewers, among other types of sensors) and *telecontrol* associated to water diversion or water detention infrastructure. These elements make possible to implement an active RTC of sewer water flows and levels to achieve flooding control, reducing risks of polluting discharges to receiver waters such as the sea or rivers.

RTC systems are designed for the system in nominal conditions, i.e., with all its elements working correctly. However, if for instance a sensor within the telemetry system fails, then RTC should compensate the miss of information and avoid the collapse of the system. Generally, these latter faults are caused by extreme meteorological conditions, typical of the Mediterranean weather. On the other hand, suppose a fault that restricts the flow through a network control gate. In a heavy rain scenario, this situation could cause that sewage goes out to the city streets causing flooding and/or pollution in the sea or another receiver environment. This situation should be compensated by the RTC in order to avoid problem and disasters, maintaining the system performance.

Therefore, sewer networks not only needs a control strategy designed to improve the system performance but also needs a set of fault tolerance mechanisms that ensure that the control continues working despite the influence of a fault over the system. MPC controllers could guarantee a certain level of implicit tolerance due to their inherent capabilities but the performance would be better if a additional fault tolerant policies were considered into the closed-loop system.

1.2 Thesis Objectives

According to discussions presented beforehand, this thesis focuses on the modeling and control of sewage systems within the framework of the MPC. Therefore, the main objective of the thesis consists in designing MPC strategies to control sewer networks taking into account some of their inherent complex dynamics, the multi-objective nature of their control objectives and the performance of the closed-loop when rain episodes are considered as system disturbances. Complementary, the incorporation of the mentioned closed-loop system within a fault tolerant architecture and the consideration of faults on system actuators is also studied. For this case, only control gates are considered as actuators. The particular sewage system used as case study of the thesis is a representative part of the sewer network of Barcelona. From the whole sewer network, real rain episodes measurements as well as other real data regarding its behavior are available.

To fulfill this global objective, a sequence of specific objectives should be fulfilled as well. They are the following:

1. To develop the formalization of the sewer network modeling and control in the framework of MPC, including the determination of particular aspects related to the control strategy such as costs functions, physical and control problem constraints, tuning methods, etc.

2. To analyze the performance of MPC on sewer networks for controller set-ups different from the reported ones in the literature. It implies the exploration of aspects such as mixing norms in cost functions, proving different tuning methods and constraints managements, etc.

3. To use the hybrid systems theory in order to model a sewer network including its implicit switching dynamics given by overflows in tanks, weirs and sewers in order to design predictive controllers.

4. To explore alternative ways of solution for the problem of high computation times when MPC controllers are used with sewer network hybrid models having many states and logical variables.

5. To analyze the influence of actuator faults in a closed-loop system being governed by a predictive controller based on either linear or hybrid models and to determine the limitation of fault tolerant control schemes (FTC) and strategies. Moreover, to take into to account the hybrid nature of the FTC system by using an hybrid systems modeling, analysis and control methodology. This allows to design the three levels of a FTC system in an integrated manner and verify its global behavior

6. To explore numerical techniques of constraints satisfaction in order to determine off-line the feasibility of fulfilling the control objectives in the presence of actuator faults. This

way of tolerance evaluation avoids solving an optimization problem in order to know whether the control law can deal with actuator fault configuration.

1.3 Outline of the Thesis

This dissertation is organized as follows:

Chapter 2: Literature Review

This chapter aims to bring the main ideas about the different topics considered in this thesis. First part focuses on giving concepts and definitions about the particular treated systems. The chapter also presents a brief state of the art about the RTC on sewer networks and the new research directions in this field. Moreover, concepts and definitions regarding MPC and hybrid systems formalisms are outlined. Finally, concepts and methods on Fault Tolerance Mechanisms are presented and a literature review about such topic is presented.

Related Publications

Section 2.3 is entirely based on

V. Puig, J. Quevedo, T. Escobet, B. Morcego, and C. Ocampo. Control tolerante a fallos (Parte II): Mecanismos de tolerancia y sistema supervisor. Tutorial. *RIAI: Revista Iberoamericana de Automática e Informática Industrial*, 1(2):5-12, 2004.

Chapter 3: Principles for the Mathematical Model of Sewer Networks

Once the structure and operation mode of sewer networks are introduced, a modeling methodology for control design and analysis is required. This chapter introduces the modeling principles for sewer networks by following a *virtual tanks* approach. In this way, a network can be considered as a set of interconnected tanks, which are represented by a first order model relating inflows and outflows with the tank volume. The calibration technique for a whole sewer network model, based on real data of rain inflows and sewer levels, is explained and discussed. Section 3.3 presents and describes in detail the case study of this thesis on which the control techniques and methodologies will be applied. The case study corresponds to a portion of the sewer network under the city of Barcelona. Particular mathematical model is obtained and calibrated using real data from representative rain episodes occurred in Barcelona during the period 1998-2002.

Chapter 4: Model Predictive Control Problem Formulation

Based on the system model determined for the case study in Chapter 3, this chapter considers just the linear representation of the network, i.e., ignores some inherent switching dynamics given by network components such as weirs and overflow elements related to sewers and/or tanks. The idea is to have a optimization problem with linear constraints in order to formalize a Linear Constrained MPC for sewer networks. In this framework, the chapter studies the effect of having different norms in the multiobjective cost function related to the MPC problem and proposes a control tuning approach based on lexicographic programming. This latter approach allows obtaining the global optimal solution without considering the tedious procedure of adjusting the weights in the multiobjective cost function.

Publications

This chapter is entirely based on

C. OCAMPO-MARTÍNEZ, A. INGIMUNDARSON, V. PUIG, AND J. QUEVEDO. Objective prioritization using lexicographic minimizers for MPC of sewer networks. *IEEE Transactions on Control Systems Technology*, 2007. In press.

Chapter 5: Predictive Control Problem Formulation based on Hybrid Models

Limitations regarding the MPC design proposed in Chapter 4 have motivated the search of different modeling techniques in order to have a model that can include the inherent switching dynamics for some of constitutive elements within the sewer network while the global optimal solution of the MPC problem is ensured. Therefore, modeling methodology of hybrid systems is taken into account to reach the desirable features discussed before. Section 5.1 proposes a detailed methodology to obtain an hybrid model considering the whole sewer network as a compositional hybrid system. Hence, the Hybrid MPC for sewer networks is then discussed and the associated MIP problem is presented. Results obtained by using the HMPC application over the case study are given while the main conclusions are discussed.

Publications

Preliminary results of predictive control formulation based on sewer network hybrid models are presented in

C. OCAMPO-MARTÍNEZ, A. INGIMUNDARSON, V. PUIG, AND J. QUEVEDO. Hybrid Model Predictive Control applied on sewer networks: The Barcelona Case Study. F. LAMNABHI-LAGARRIGUE, S. LAGHROUCHE, A. LORIA AND E. PANTELEY (editors): *Taming Heterogeneity and Complexity of Embedded Control (CTS-HYCON Workshop on Nonlinear and Hybrid Control)*. International Scientific & Technical Encyclopedia (ISTE), 2006.

Complementary results and discussions collected in this chapter are reported in

C. OCAMPO-MARTÍNEZ, A. INGIMUNDARSON, A. BEMPORAD AND V. PUIG. On Hybrid Model Predictive Control of Sewer Networks. R. SÁNCHEZ-PEÑA, V. PUIG AND J. QUEVEDO (editors): *Identification & Control: The gap between theory and practice*, Springer-Verlag, 2007.

Chapter 6: Suboptimal Hybrid Model Predictive Control

Results obtained from Chapter 5 show the improvement of the system performance when the HMPC is used on sewer networks. However, the main problem of this control technique is the computation time required to solve the discrete optimization problem associated. From simulations and tests, it could be noticed that the MIP problem behind the HMPC design is very random in the sense of solution times since it depends on the initial condition of the system. Therefore, one possible way of solution to these problems consists in relaxing the MIP in order to reduce the computation time, what lies on possible suboptimal solutions, i.e., improving the solving time by sacrificing the performance. Section 6.2.2 outlines some general strategies to relax the MIP problem associated to the HMPC design.

The chapter presents a MPC strategy for Mixed Logical Dynamical (MLD) systems where the number of differences between the mode sequence of the plant and a reference sequence is limited over the prediction horizon. The aim is to reduce the amount of feasible nodes in the MIP problem and thus reduce the computation time. In Section 6.3, stability of the proposed scheme is proven and practical issues regarding how to find the reference sequence are discussed in Section 6.4. The strategy is then applied over the sewer network model in the case study but applying particular considerations related to its behavior.

Publications

Mode sequence constraints definition and the stability proof of the suboptimal approach are reported in

A. INGIMUNDARSON, C. OCAMPO-MARTINEZ AND A. BEMPORAD. Suboptimal Model Predictive Control of Hybrid Systems based on Mode-Switching Constraints. Submitted to *Conference on Decision and Control (CDC)*, 2007.

Chapter 7: Model Predictive Control and Fault Tolerance

Faults are very undesirable events for all control systems. As was said before, in the case of a sewer network, the fault effect can stop completely the global control loop, what could imply severe flooding and increase of pollution. MPC controllers, as well as all techniques using feedback, have an implicit capability to reject partially the influence of faults. Moreover, if the predictive controller governs the closed-loop within a fault tolerant architecture, faults can be compensated in a better way. This chapter takes the definitions and concepts about fault tolerant mechanisms collected in Section 2.3 and involve them within the predictive control of sewer networks. The fault tolerance capabilities inherent to the MPC strategy are discussed in Section 7.2 where the idea of having a parametrization of the system in function of the faults is explained by means of a simple motivational example.

When the internal model of the predictive controller is obtained considering the plant as an hybrid system, the inclusion of fault tolerance in MPC leads to the Fault Tolerant HMPC (FTHMPC). In this framework, Section 7.3.1 discuses two strategies: the natural robustness of MPC facing faults in the plant (Passive FTHMPC) and the strategy which takes into account fault tolerance mechanisms (Active FTHMPC). Finally, chapter proposes the ways of implementation for a fault tolerant architecture over sewer networks considering faults in the control gates as the actuators of the system.

Publications

Discussions regarding fault tolerance on linear MPC are based on

C. OCAMPO-MARTINEZ, V. PUIG, J. QUEVEDO, AND A. INGIMUNDARSON. Fault tolerant model predictive control applied on the Barcelona sewer network. In *Proceedings of IEEE Conference on Decision and Control (CDC) and European Control Conference (ECC)*, Seville (Spain), 2005.

while the extension to hybrid modeling framework for FTC is preliminary presented in

C. OCAMPO-MARTÍNEZ, A. INGIMUNDARSON, V. PUIG, AND J. QUEVEDO. Fault tolerant hybrid MPC applied on sewer networks. In *Proceedings of IFAC SAFEPROCESS*, Beijing (China), 2006.

Chapter 8: Actuator Fault Tolerance Evaluation

As an extension of the study in fault tolerance, Chapter 8 proposes the fault tolerant evaluation of a certain actuator fault configuration (AFC) considering a linear predictive/optimal control law with constraints. Faults in actuators cause changes in the constraints on the control signals which in turn change the set of feasible solutions. This may derive on the situation where the set of admissible solutions for the control objective was empty. Therefore, the admissibility of the control law regarding actuator faults can be determined knowing the set of feasible solutions. One of the aims of this chapter is to provide methods to compute this set and then to evaluate the admissibility of the control law. In particular, the admissible solutions set for the predictive control problem including the effect of faults (either through reconfiguration or accommodation) can be determined using different approaches as presented in Section 8.3. Finally, the proposed method is tested on a reduced expression of the case study, which is enough to see the advantages of the presented approach.

Publications

Chapter 8 is almost entirely based on

C. OCAMPO-MARTÍNEZ, P. GUERRA, V. PUIG AND J. QUEVEDO. Fault Tolerance Evaluation of Linear Constrained MPC using Zonotope-based Set Computations. Submitted to *Journal of Systems & Control Engineering*, 2007.

P. GUERRA, C. OCAMPO-MARTÍNEZ AND V. PUIG. Actuator Fault Tolerance Evaluation of Linear Constrained Robust Model Predictive Control. Accepted in *ECC*, 2007.

C. OCAMPO-MARTÍNEZ, V. PUIG, AND J. QUEVEDO. Actuator fault tolerance evaluation of Nonlinear Constrained MPC using constraints satisfaction. In *Proceedings of IFAC SAFEPROCESS*, Beijing (China), 2006.

Chapter 9: Concluding Remarks

This chapter summarizes the contributions made in this thesis and discusses the ways for future research.

Other Related Publications

Several of the publications below provide the basis for the manuscripts included in this thesis.

C. OCAMPO-MARTÍNEZ, V. PUIG, J. QUEVEDO, AND A. INGIMUNDARSON. Fault tolerant optimal control of sewer networks: Barcelona case study. *International Journal of Measurement and Control*, Special Issue on Fault tolerant systems, 39(5):151-156, June 2006.

V. PUIG, J. QUEVEDO, T. ESCOBET, B. MORCEGO, AND C. OCAMPO. Control tolerante a fallos (Parte I): Fundamentos y diagnóstico de fallos. Tutorial. *RIAI: Revista Iberoamericana de Automática e Informática Industrial*, 1(1):15-31, 2004.

C. OCAMPO-MARTÍNEZ, S. TORNIL, AND V. PUIG. Robust fault detection using interval constraints satisfaction and set computations. In *Proceedings of IFAC SAFEPROCESS*, Beijing (China), 2006.

V. PUIG, J. QUEVEDO, AND C. OCAMPO. Benchmark for Fault Tolerant Control based on Barcelona sewer network. In *Proceedings of NeCST Workshop*, Ajaccio, Corsica, October 2005.

C. OCAMPO-MARTÍNEZ, P. GUERRA, AND V. PUIG. Actuator fault tolerance evaluation of linear constrained MPC using Zonotope-based set computations. In *VI Jornades en Automàtica, Visió i Robótica*. Universitat Politècnica de Catalunya, 2006.

C. OCAMPO-MARTINEZ. Benchmark definition for fault tolerant control based on Barcelona sewer network. Technical report, Universidad Politécnica de Catalunya (UPC) - ESAII, May 2004.

C. OCAMPO-MARTINEZ. Barcelona sewer network problem: Model based on piecewise functions. Technical report, Technical University of Catalonia (UPC) - ESAII, July 2005.

Part I

Background and Case Study Modeling

CHAPTER 2

BACKGROUND

This chapter collects briefly the basic fundamentals for the main topics treated in this thesis. Three sections gather concepts, definitions and schemes about sewer networks, model predictive control (including hybrid models) and fault tolerance mechanisms. Moreover, bibliographical references to relevant scientific contributions in journals, impact congress and research reports are given for each topic framework and their contents is briefly presented and discussed.

2.1 Sewer Networks: Definitions and Real-time Control

2.1.1 Description and Main Concepts

First of all, this section introduces some important concepts regarding sewer networks and relevant definitions in this framework. The basic concept is in itself what a sewer network is and its objective. In general, *sewers*[1] are pipelines that transport wastewater from city buildings and rain drains to treatment facilities. Sewers connect this staff to horizontal mains. The sewer mains often connect to larger mains and then to the wastewater treatment site. Vertical pipes, called *manholes*, connect the mains to the surface. Sewers are generally gravity powered, though pumps may be used if necessary.

The main type of wastewater collected and transported by a sewer network is in general the sewage, which is defined as the liquid waste produced by humans which typically contains

[1] The word *sewer* comes from the old French *essouier* (to drain), which comes as well from the Latin *exaquaria*: ex- "out" + aquaria, feminine of aquarius "pertaining to water".

washing water, faeces, urine, laundry waste and other liquid or semi-liquid wastes from households and industry. These sewer networks are known as *sanitary sewer network*[2].

Also, there exist the called *storm sewers*, which are large pipes that transport storm water runoff from streets to natural bodies of water in order to avoid street flooding. Otherwise, the kind of network which collects not only sewage from houses and industry but also collects the storm water runoff is called *unitary network* or *Combined Sewer System* (CSS). These sewer networks were built in many older cities because having a mixed system was cheaper but problems came for heavy rains. Hence, these combined systems were designed to handle certain size storms and, when the sewer was overloaded with too much flow, the water would exit the sewer system and into a nearby body of water through a relief sewer to prevent back-up into the street or houses and buildings. This dissertation considers the case of unitary networks so all concepts and descriptions presented in the sequel are related with such networks.

According to the literature, sewer networks can be considered as a collection of elements which are recognized depending its particular function. In general way, a set of few typical elements are going to be described below and Figure 2.1 gives a certain idea of their interrelation for a scheme of a very small and simple sewer network. Some of the presented figures are taken particularly from the Barcelona sewer network, which is described in Chapter 3 as the case study of this dissertation.

Hydrodynamic Links

These elements are used not only as connection between network pieces but also as storage element when the inner capacity of them reaches important values. Regarding their hydrodynamics, this fact also requires the consideration of inherent phenomena in a framework where the sewer network inflow is manipulated using throttle gates. In these cases, the called *backwater effect* may occur, what makes more complex the modeling and simulation of the links behavior. Moreover, due to the network magnitude, transport delays and other nonlinearities can be taken into account in the dynamic description of these elements. Within a sewer network, there exist many kinds of links according to their size. Figure 2.2 shows a picture of a big diameter sewer corresponding to a real sewer network.

[2] also called *foul sewer*, especially in the UK.

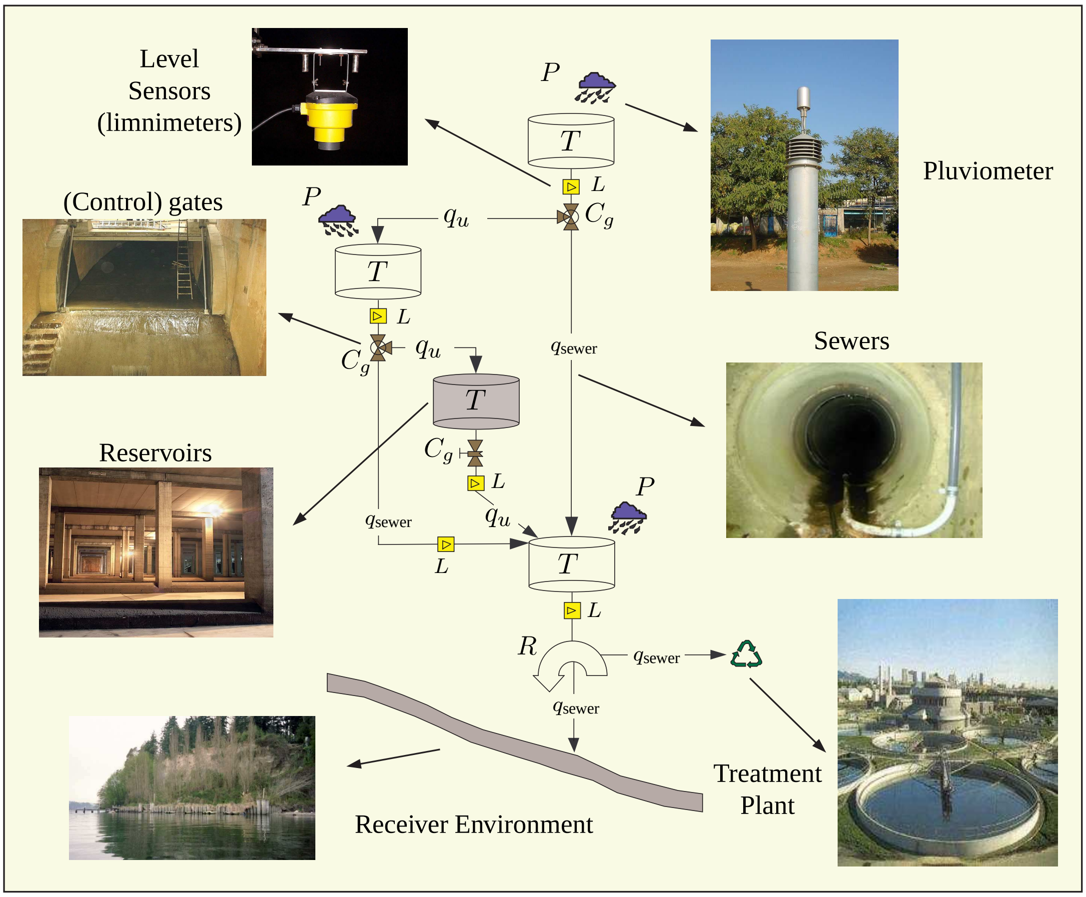

Figure 2.1: Components for a basic scheme of a sewer network.

Tanks or Reservoirs

These elements are used as storage devices with a dual function. First of all, they make their outflow be laminar, what means that the inflow is greater than the outflow. This aspect allows the easier manipulation of the flows in elements located in a low position within the network, mainly in case of heavy rain episodes. In second place, these elements have a environmental function in the sense of retaining highly contaminated sewage. It prevents the spill of this dirty water on beaches, rivers and ports and allows its treatment by the plants. On the other hand, the retained water diminishes its contamination degree due to the sedimentation caused by the retention process.

About their model, these elements can have overflow capability, which means that when the

Figure 2.2: Big diameter sewer. Taken from [CLA05].

water volume reaches the maximum capacity a new flow appears. Such flow is related to the water volume not stored. However, some model proposals consider that a suitable manipulation of a redirection gate located in the tank input can be the strategy which replaces the overflow capability of the reservoirs[3]. The maximum capacity of the tank is a control constraint for the input gate [OMPQI05]. The usefulness of each one of these ways depend in a straightforward manner of the modeling and the control strategy applied to the sewer system. In Figure 2.3 the inner part of a retention tank is shown.

Gates

Within a sewer network, gates are used as control elements because they can change the flow downstream. Depending on the action made, gates can be classified as follows:

Redirection gates: These gates are used to change the direction of the water flow. This group of gates can be located before a reservoir or anywhere a water redirection can be needed.

Retention gates: These gates are used to retain the water flow in a certain point of the network. They are generally located at the reservoir output, what allows to retain the sewage within the tank and benefits the wastewater sedimentation process.

[3]In these cases, the overflow capacity is not a nominal mode of operation and becomes a security mechanism.

Figure 2.3: Retention tank (inner face). Taken from [CLA05].

In sewer network control, the control signals can correspond to the manipulated outflows in control gates. Taking into account the scheme in Figure 1.3, when the global control level computes these outflows, a local controller handles the mechanical actions of the physical gates (actuators) using such computed outflows as set-points. This procedure avoids the consideration of inherent nonlinearities associated to the gate. Figure 2.4 corresponds to a typical retention gate within a sewer network.

Nodes

According to [MP05], these elements correspond with points where water flows are either propagated or merged. Propagation means that the node has one inflow and one outflow so the objective of this point is the connection of sewers with different geometries. On the other hand, merging means that more than one inflow merge to one greater outflow. Therefore, two types of nodes can be considered:

- Nodes with one inflow and multiple outputs (splitting nodes).
- Nodes with multiple inputs and one outputs (merging nodes).

Figure 2.4: Typical retention gate. Taken from [CLA05].

In particular topologies, these elements can have a maximum outflow capacity, what produces an overflow appearance under a given conditions. Hence, the node would have not only one output related to the natural outflow but also a second output corresponding to the considered overflow. Such elements are called *weirs*, which add to the system behavior a switching dynamic, difficult to consider, depending on the used model.

Instrumentation

Many variables have to be measured within a sewer network in order to implement an RTC system. The main devices used to fulfill this objective are, among others:

Rain gauges Rain can be considered the main external input. Hence, it is necessary to measure the rain intensity in order to know the rain inflow. Rain intensity is measured using a *tipping bucket* rain-gauge, whose scheme is presented in Figure 2.5. This gauge technology uses two small *buckets* mounted on a fulcrum (balanced like a see-saw). The tiny buckets are manufactured with tight tolerances to ensure that they hold an exact amount of precipitation. The tipping bucket assembly is located underneath the rain sewer, which funnels the precipitation to the buckets. As rainfall fills the tiny bucket, it becomes overbalanced and tips down, emptying itself as the other bucket pivots into place for the next reading.

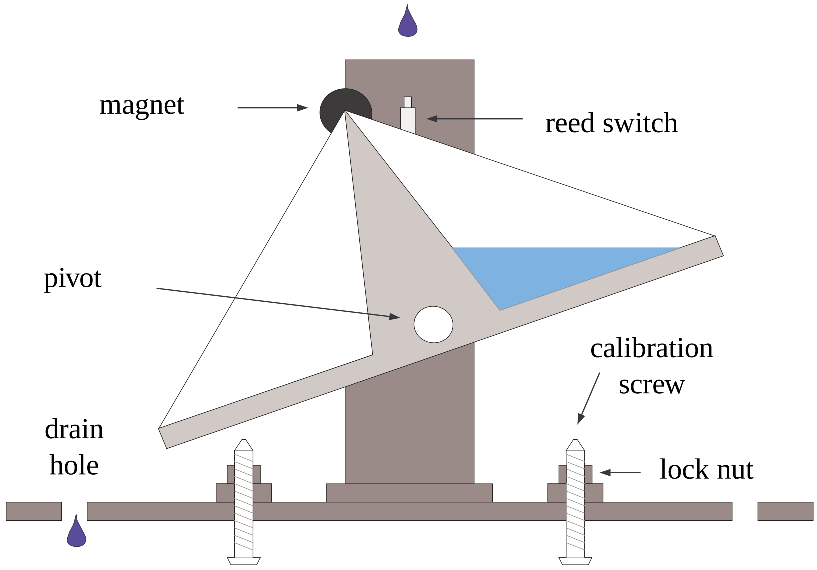

Figure 2.5: Rain measurement principle using a tipping bucket rain gauge.

The action of each tipping event triggers a small switch that activates the electronic circuitry to transmit the count to the indoor console, recording the event as an amount of rainfall. Once the rain intensity is determined, the rain inflow can be computed using the procedure proposed and explained in Chapter 3.

Limnimeters These devices measure the sewage level within the sewers. They are located on strategy points of the network and their given information is related to the water volume and flow by means of Manning formula, see Chapter 3. They are mainly used in points where the sewer slope allows the water flow by gravity.

Velocity sensors According to the geometry and topology of the considered sewer, flow information can be inaccurate due to the level measurements. Then, these sensors are used to measure the sewage velocity in an specific place of the sewer network. Using this information, the sewage flow can be computed in a more accurate manner. This fact for instance avoids situations where the sewer slope is almost null, and despite the water flow exists, the level of the water remains constant.

Radars An alternative way to measure rain intensity is using *weather radars*. The weather radar is an instrument used to obtain a detailed description of the spatial and temporal rainfall field. This information is needed to model in the hydrologically sense a certain

Figure 2.6: Typical pumping station for reservoir. Taken from [CLA05].

region with sufficient resolution. However, such devices are complex instruments. They measure a property of the rainfall drops. This property is related to the portion of the power of the beam put out by the radar and that returns to it once the beam has hit its target. This property, known as the rainfall reflectivity, is indirectly related to the rainfall intensity (through the raindrop sizes distribution). It is also indirectly related to the intensity of the rainfall that reaches the ground [GRA07].

Pumping Stations

Once a rain episode has finished, the tanks are drained towards the treatment plant. For this procedure two elements can be needed: a retention gate and a pumping station. About first element, some ideas have been presented beforehand. Pumping stations are needed to take out the water that can not get out by gravity. Hence, these pumping stations are also manipulated, allowing the flow control downstream. Figure 2.6 shows a typical pumping station for a sewer network.

Figure 2.7: View of the wastewater treatment plan of Columbia, Missouri (USA). Taken from `http://www.gocolumbiamo.com/`.

Treatment Elements

This element consists in plants where, through physical-chemical and biological processes, organic matter, bacteria, viruses and solids are removed from wastewaters before they are discharged in rivers, lakes and seas. It receives all the water that has got into the sewer network and has not got out through the overflows. Nowadays the inclusion of such elements within the sewer networks is of great significance in order to preserve the ecosystem and maintain the environmental balance inside the water cycle. In this sense, the separation of the storm sewers from waste sewers would be a great strategy because the huge water inflow during a rainstorm can overwhelm the treatment plant, resulting in untreated sewage being discharged into the environment. In this sense, some cities have dealt with this aspect by adding large storage tanks or ponds to hold the water until it can be treated. Another way to deal with this aspect consists in design a suitable control strategy which prevents all type of pollution and Combined Sewage Overflow (CSO) in the sewer network and then the damage to the environment. Figure 2.7 presents a picture of an important treatment plant located in Columbia, Missouri (USA).

2.1.2 RTC of Sewage Systems

This section explores the contributions reported in the literature about the real-time control of sewer networks. However, this literature exploration also takes into account modeling aspects of sewage systems due to the close relation between modeling and control for thee particular systems. Real-time sewer network control systems play an important role in meeting increasingly restrictive environmental regulations to reduce release of untreated waste or CSO to the environment. Reduction of CSO often requires major investments in infrastructure within city limits and thus any improvement in efficient use of existing infrastructure, for example by improved control, is of interest. The advantage of sewer network control has been demonstrated by a number of researchers in the last decades, see [GR94], [PMLC96], [Mar99], [PCL$^+$05], [MP05]. A common control strategy to deal with urban drainage systems is Model Predictive Control (MPC), see [GR94], [PPC$^+$01] [MP05]. This fact is because the urban drainage control problem is often multi-input, multi-output and the goal consists in using existing infrastructures to their limits, characteristics that make MPC specially suitable with its inherent capacity to deal with constraints.

A very important aspect on sewer networks is their modeling. Several modeling approaches have been presented in the literature about sewer networks [Erm99], [Mar99], [DMDV01], [MP05]. Specifically and due to its complex nature, several hydrological models have been proposed [PMLC96], [ZHM01]. Sewer networks are systems with complex dynamics since water flows through sewer in open channels. As will be discussed later, flow in open canals are described by Saint-Vennant's partial differential equations that can be used to perform simulation studies but are highly complex to solve in real-time. For the purpose of control, modeling techniques have been presented that deal with sewer networks, see [DMDV01], [OMIPQ06]. However, when an implementation of a real-time control (RTC) strategy is implemented, the complexity of the models could be an important problem because it implies higher computation times and difficulties when a control sequence for a desired performance is computed [ZHM01], [MP05]. This problem is also consequence of the high model dimension, proper characteristic of the large-scale systems. Often the purpose of the model is to perform simulation studies and they range from highly complex partial differential equations to simpler conceptual models.

In an early reference on MPC ([GR94]), a linear model of a sewer network was used for prediction. Good performance of identified linear models in simulation of flow in urban drainage networks with rain measurements as input has also been reported in [PLM99]. The use of nonlinear models for predictive control of urban drainage systems has also been reported, see

[RL95], [MP98].

Improvements in prediction achieved by using nonlinear models need to be compared to the uncertainty introduced due to the error in predicting the rain over the horizon. Short term rain prediction or nowcasting is an active field of research, see [SA00]. With a combination of radar, rain gauge measurements and advanced data processing, prediction of rain has improved greatly lately and the potential for the use in predictive control of urban drainage systems has been pointed out in [YTJC99].

Then, an operational model of an urban drainage system would be a set of equations which provide a fast approximate evaluation of the hydrological variables of the network and their response to control actions on the gates. In [RL95], nonlinear model predictive control (NMPC) was implemented over a large-scale system with 26 states and 10 manipulated inputs. It was shown that a complex nonlinear model is always better but differences with linear MPC may be too small to justify the NMPC effort. [MP97] justifies the use of simpler models for optimization-based control of sewer networks due to

- the model inaccuracies impact is reduced solving the control problem iteratively and updating inflow predictions and initial conditions, and
- the details of local elements and catchments are considered in local control loops.

About control strategies, extensive research has been carried out on RTC of urban drainage systems. Comprehensive reviews that include a discussion of some existing implementations are given by [SAN$^+$96] and its cited references, while practical issues are discussed by [SBB02], among others. The common idea is the use of optimization techniques to improve the system performance trying to avoid the street flooding, prevent the CSO discharges, minimize the pollution, get uniform the utilization of sewer system storage capacity and, in most of cases, minimize the functioning costs, among another objectives [Erm99], [Wey02], [STJB02], [SCC$^+$04]. In this way, [GR94] proposed the implementation of model MPC over the Seattle urban drainage system. In this work, authors organized the fundamental ideas about the use of these techniques in sewer networks: definition of appropriate cost functions, creation and maintenance of models and use of the prediction for minimizing the uncertainty effect of the rain estimation, aspect that, in these systems is crucial for the right operation in closed-loop. Their results confirm the effectiveness of the control law over large-scale systems relative to the used automatic controls in that moment, which were based on heuristics.

Years later, [MP97] proposed the application of optimal control retaking the hierarchical control philosophy [Pap85]. This philosophy suggests that a RTC structure that combines high efficiency and low implementation cost would have three layers:

- a adaptation layer, where the inflow prediction (rain) and state estimation in real-time is done,
- a optimization layer, which is responsible of the global control and the reference trajectories computation, and
- a decentralized control layer, which is responsible of the control trajectories realization.

A similar idea of hierarchical control for RTC can be found in [SCC+04]. In [MP01], the authors combine the work presented in [MP97] with the receding horizon philosophy, that is, optimal control in finite horizon and prediction in a sliding time window. [DMV04] implements the global control level introduced in Figure 1.3 within the framework of predictive control for minimizing the overflow volumes from combined sewers during rainfalls on the urban area drained by the Marigot interceptor in Laval, Canada. The results have shown that allowing surcharged flows in the interceptor during rainfalls leads to important decreases in overflow volumes.

Although the application of optimization methods, and, more generally, the development of control procedures, usually aims to determining the optimal (best possible) control action under the given conditions, a suboptimal control decision is sometimes often enough for RTC (as long as it can be ensured that this decision does not lead to a performance of the system worse than the no-control scenario). However, under specific model conditions and for MPC strategies, it could be ensured that the best possible solution is obtained.

2.2 MPC and Hybrid Systems

2.2.1 MPC Strategy Description

Model predictive control (MPC), also *referred as model based predictive control*, *receding horizon control* or *moving horizon optimal control*, is one of the few advanced methodologies which has significant impact on industrial control engineering. MPC is being applied in process industry because it can handle multivariable control problems in a natural form, it can take into account actuator limitations and it allows constraints consideration. Predictive Control methods are developed around certain common ideas, which are basically [Mac02], [GSdD05]:

- The explicit use of a model in order to predict the process output in a time horizon.
- The obtaining of a control sequence which minimizes a cost (objective) function.
- The application of the first control signal from the computed sequence and the displacement of the horizon towards the future.

MPC as a wide field of control methods is developed around a set of basic elements in common. Its parameters can be modified giving rise to different algorithms. These main elements can be outlined as:

- Prediction model, which should capture all process dynamics and allows to predict the future behavior of the system.
- Objective (cost) function, which is, in general form, the element that penalizes derivations of the predicted controller outputs from a reference trajectory. It represents a performance index of the system studied.
- Control signal computation.

This control strategy presents important advantages over other control methods. Some of these advantages are outlined below [Bor00].

- It is very easy to use for people without deep knowledge in control. Its concepts are very intuitive and the tuning is relatively simple.
- Can be used to control a wide type of processes, including simple dynamics towards systems with big delays, unstable and nonminimum phase systems.
- It is very useful for multivariable systems.
- It has inherently the delay compensation.
- It allows the use of constraints, which can be added during the design process.

However, it also has some disadvantages such as the high computational cost in the control law obtaining process. But the main problem of this strategy lies on the dependence of a accurate system model. The design algorithm is based on the previous knowledge of the system behavior so the performance is related to the quality of the plant representation.

MPC Formulation

In most of the cases presented in the research literature, the MPC formulation is expressed in state space. However, in order to present a generic and simple representation of the strategy, let

$$x_{k+1} = g(x_k, u_k) \tag{2.1}$$

be the mapping of states $x_k \in \mathbb{X} \subseteq \mathbb{R}^n$ and control signals $u_k \in \mathbb{U} \subseteq \mathbb{R}^m$ for a given system, where $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is the arbitrary system state function and $k \in \mathbb{Z}_+$. Let

$$\mathbf{u}_k(x_k) \triangleq \left(u_{0|k}, u_{1|k}, \ldots, u_{H_p-1|k}\right) \in \mathbb{U}^{H_p} \tag{2.2}$$

be the input sequence over a fixed time horizon H_p. Moreover, the *admissible input sequence* with respect to the state $x_k \in \mathbb{X}$ is defined by

$$\mathcal{U}_{H_p}(x_k) \triangleq \left\{\mathbf{u}_k \in \mathbb{U}^{H_p} | \mathbf{x}_k \in \mathbb{X}^{H_p}\right\}, \tag{2.3}$$

where

$$\mathbf{x}_k(x_k, \mathbf{u}_k) \triangleq \left(x_{1|k}, x_{2|k}, \ldots, x_{H_p|k}\right) \in \mathbb{X}^{H_p} \tag{2.4}$$

corresponds to the state sequence generated applying the input sequence (2.2) to the system (2.1) from initial state $x_{0|k} \triangleq x_k$, where x_k is the measurement or the estimation of the current state. Hence, the receding horizon approach is based on the solution of the open-loop optimization problem (OOP) [BM99b]:

$$\min_{\{\mathbf{u}_k \in \mathcal{U}_{H_p}\}} J\left(\mathbf{u}_k, x_k, H_p\right) \tag{2.5a}$$

subject to

$$H_1 \mathbf{u}_k \leq b_1 \tag{2.5b}$$

$$G_2 \mathbf{x}_k + H_2 \mathbf{u}_k \leq b_2 \tag{2.5c}$$

where $J(\cdot) : \mathbb{X}_f(H_p) \mapsto \mathbb{R}_+$ is the cost function with domain in the *set of feasible states* $\mathbb{X}_f(H_p) \subseteq \mathbb{X}$ [LHWB06], H_p denotes the *prediction horizon* or *output horizon* and G_2, H_i and

b_i are matrices of suitable dimensions. In sequence (2.4), $x_{k+i|k}$ denotes the prediction of the state at time $k+i$ done in k , starting from $x_{0|k} = x_k$. When $H_p = \infty$, the OOP is called *infinite horizon problem*, while with $H_p \neq \infty$, the OOP is called *finite horizon problem*. Constrains stated to guarantee system stability in closed-loop would be added in (2.5b)-(2.5c).

Assuming that the OOP (2.5) is feasible for $x \in \mathbb{X}$, i.e., $\mathcal{U}_{H_p}(x) \neq \emptyset$, there exists an optimal solution given by the sequence

$$\mathbf{u}_k^* \triangleq \left(u_{0|k}^*, u_{1|k}^*, \ldots, u_{H_p-1|k}^*\right) \in \mathcal{U}_{H_p} \tag{2.6}$$

and then the receding horizon philosophy sets [Mac02], [CB04]

$$u_{\text{MPC}}(x_k) \triangleq u_{0|k}^* \tag{2.7}$$

and disregards the computed inputs from $k = 1$ to $k = H_p - 1$, repeating the whole process at the following time step. Equation (2.7) is known in the MPC literature as *the MPC law*. Summarizing, Algorithm 2.1 briefly describes the basic MPC law computing process.

Algorithm 2.1 Basic MPC law computation.

$k = 0$
loop
    $x_{k+0|k} = x_k$
    $\mathbf{u}_k^*(x_k) \Leftarrow$ solve OOP (2.5)
    Apply only $u_k = u_{k+0|k}^*$
    $k = k + 1$
end loop

2.2.2 Hybrid Systems

In the dynamical systems behavior there exist several phenomena produced by the interaction of signals of different nature. In general, systems are composed of both continuous and discrete components, the former are typically associated with physical first principles, the latter with logic devices, such as switches, digital circuitry, software code. This mixture of logical conditions and continuous dynamics gives rise to a *hybrid system*.

For instance, in the case of sewer networks there exist several phenomena (overflows in sewers and tanks) and elements in the system (redirection gates and weirs) that present a different behavior depending on the flow/volume in the network. This leads naturally to the use of hybrid models in order to describe such behaviors. The hybrid models considered here belong to the class of discrete-time linear hybrid systems. The condition of discrete-time avoids certain mathematical problems (like Zeno behavior, see [HLMR02], [AS05]) and allows to derive models for which analysis and optimal/predictive control problems can be posed.

Mixed Logical Dynamical Systems

The *Mixed Logical Dynamical* (MLD) modeling framework, introduced in [BM99a], is a way, among others, that allows one to represent hybrid systems, which can be described by interdependent physical laws, logical rules and operating constraints. MLD models have recently been shown to be equivalent to representations of hybrid systems such as *Linear Complementarity* (LC) systems, *Min-Max-Plus Scaling* (MMPS) systems and *Piecewise Affine* (PWA) systems, among others, under mild conditions, see [HDB01]. MLD systems are described by linear dynamic equations subject to linear mixed-integer inequalities, i.e., inequalities involving both continuous and binary (or logical, or 0-1) variables. These include physical/discrete states, continuous/integer inputs, and continuous/binary auxiliary variables. The ability to include constraints, constraint prioritization and heuristics are powerful features of the MLD modeling framework. The general MLD form is [BM99a]:

$$x_{k+1} = Ax_k + B_1 u_k + B_2 \delta_k + B_3 z_k \tag{2.8a}$$

$$y_k = Cx_k + D_1 u_k + D_2 \delta_k + D_3 z_k \tag{2.8b}$$

$$E_2 \delta_k + E_3 z_k \leq E_1 u_k + E_4 x_k + E_5 \tag{2.8c}$$

where the meaning of the variables is the following:

- x are the continuous and binary states:

$$x = \begin{bmatrix} x_c \\ x_\ell \end{bmatrix}, \quad x_c \in \mathbb{X} \subseteq \mathbb{R}^{n_c}, \quad x_\ell \in \{0,1\}^{n_\ell} \tag{2.9}$$

- y are the continuous and binary outputs:

$$y = \begin{bmatrix} y_c \\ y_\ell \end{bmatrix}, \quad y_c \in \mathbb{Y} \subseteq \mathbb{R}^{p_c}, \quad y_\ell \in \{0,1\}^{p_\ell} \tag{2.10}$$

- u are the continuous and binary inputs:

$$u = \begin{bmatrix} u_c \\ u_\ell \end{bmatrix}, \quad u_c \in \mathbb{U} \subseteq \mathbb{R}^{m_c}, \quad u_\ell \in \{0,1\}^{m_\ell} \tag{2.11}$$

- Auxiliary binary variables: $\delta \in \{0,1\}^{r_\ell}$
- Auxiliary continuous variables: $z \in \mathbb{R}^{r_c}$.

Notice that by removing (2.8c) and by setting δ and z to zero, (2.8a) and (2.8b) reduce to an unconstrained linear discrete time system in state space. The variables δ and z are introduced when translating logic propositions into linear inequalities. All constraints are summarized in the inequality (2.8c).

The transformation of certain hybrid system descriptions into the MLD form requires the application of a set of given rules. To avoid the tedious procedure of deriving the MLD form by hand, a compiler was developed in [TB04] to generate matrices A, B_i, C, D_i and E_i in (2.8) through the specification language HYSDEL (HYbrid System DEscription Language).

2.2.3 MPC Problem on Hybrid Systems

Different methods for the analysis and design of hybrid systems have been proposed in the literature during the last few years [BM99a], [LTS99], [BZ00]. The implementation of these methods are related in a straightforward manner to the hybrid system representation. One of the most studied methods involves the class of optimal controllers, which may use the MLD form in order to compute the corresponding control law according to the system performance objectives. The formulation of the optimization problem in Hybrid MPC (HMPC) follows the approach in standard MPC design, see [Mac02]. The desired performance indexes are expressed as affine functions of the control variables, initial states and known disturbances. However, due to the presence of logical variables, the resulting optimization problem is a *mixed integer quadratic* or *linear program* (MIQP or MILP, respectively). The control law obtained in this way is referred to as *mixed integer predictive control* (MIPC).

In general, the MIPC structure is defined by the OOP (2.5) described as beforehand but including the logical dynamics part. Hence, the OOP considering the hybrid systems framework is presented as follows. Assume that the hybrid system output should track a reference signal y_r and x_r, u_r, z_r are desired references for the states, inputs and auxiliary variables, respectively.

For a fixed prediction horizon $H_p \in \mathbb{Z}_{\geq 1}$, the input sequence (2.2) is applied to the system (2.8), resulting the sequences (2.4) and

$$\Delta_k(x_k, \mathbf{u}_k) \triangleq \left(\delta_{0|k}, \delta_{1|k}, \ldots, \delta_{H_p-1|k}\right) \in \{0,1\}^{r_\ell \times H_p} \tag{2.12}$$

$$\mathbf{z}_k(x_k, \mathbf{u}_k) \triangleq \left(z_{0|k}, z_{1|k}, \ldots, zH_p - 1|k\right) \in \mathbb{R}^{r_c \times H_p} \tag{2.13}$$

under the same conditions as in problem (2.5).

Hence, the OOP for hybrid systems is now defined as:

$$\min_{\{\mathbf{u}_k \in \mathcal{U}_{H_p}\}, \Delta_k, \mathbf{z}_k} J\left(\mathbf{u}_k(x_k), \Delta_k, \mathbf{z}_k, x_k\right) \triangleq \left\|Q_{x_f}\left(x_{H_p|k} - x_f\right)\right\|_p$$
$$+ \sum_{i=1}^{H_p-1} \left\|Q_x\left(x_{k+i|k} - x_r\right)\right\|_p + \sum_{i=0}^{H_p-1} \left\|Q_u\left(u_{k+i|k} - u_r\right)\right\|_p$$
$$+ \sum_{i=0}^{H_p-1} \left(\left\|Q_z\left(z_{k+i|k} - z_r\right)\right\|_p + \left\|Q_y\left(y_{k+i|k} - y_r\right)\right\|_p\right) \tag{2.14a}$$

$$\text{subject to} \begin{cases} x_{k+i+1|k} &= Ax_{k+i|k} + B_1 u_{k+i|k} + B_2 \delta_{k+i|k} + B_3 z_{k+i|k} \\ y_{k+i|k} &= Cx_{k+i|k} + D_1 u_{k+i|k} + D_2 \delta_{k+i|k} + D_3 z_{k+i|k} \\ E_2 \delta_{k+i|k} &+ E_3 z_{k+i|k} \leq E_1 u_{k+i|k} + E_4 x_{k+i|k} + E_5 \\ x_f &= x_{r\,H_p|k} \end{cases} \tag{2.14b}$$

for $i = 0, \ldots, H_p - 1$, where x_f corresponds to the final desired value for the state variable over H_p and p is related to the selected norm (1-norm, quadratic norm or infinity norm). Q_{x_f}, Q_x, Q_u, Q_δ, Q_z and Q_y are the weight matrices of suitable dimensions, which fulfill the following conditions:

$$\begin{array}{lll} Q_{x_f,x,u} = Q^T_{x_f,x,u} \succ 0, \quad Q_{\delta,z,y} = Q^T_{\delta,z,y} \succeq 0 & & (p = 2) \\ Q_{x_f}, Q_x, Q_u, Q_\delta, Q_z, Q_y \quad \text{nonsingular} & & (p = 1, p = \infty) \end{array} \tag{2.15}$$

Assuming that the MIPC problem related to the OOP (2.14) is feasible for $x \in \mathbb{X}$, there exists an optimal solution given now by the sequence

$$\left(u^*_{0|k}, u^*_{1|k}, \cdots, u^*_{H_p-1|k}, \delta^*_{0|k}, \delta^*_{1_k}, \cdots, \delta^*_{H_p-1|k}, z^*_{0|k}, z^*_{1|k}, \cdots, z^*_{H_p-1|k}\right)$$

which, applying the receding horizon strategy, yields the MPC law in (2.7). Notice that the described procedure corresponds to the extension of MPC formulation in Section 2.2.1 over hybrid systems but the solution is obtained solving the OOP (2.14).

Some theoretical aspects about control of hybrid systems can be discussed and they have been a research topic during the last few years. For instance, notice that H_p should be finite. Infinite horizon formulations are not pragmatic neither theoretically nor in practical implementation. The approximation of H_p as large as possible implies a great amount of logical variables within the MIPC problem, what yields an almost impossible computation treatment [BM99a]. The assumption $H_p \to \infty$ is already worse in the case of large scale systems. On the other hand, the constraint $x_f = x_{r\,H_p|k}$, related to the final state within the MIPC problem (2.14), can be relaxed as $x_{H_p|k} \in \mathbb{X}_T \subseteq \mathbb{X}$, where $\mathbb{X}_T$ is defined as the *target state set* [LHWB06]. According to this assumption, the sequence $\mathcal{U}_{H_p}(x(k))$ in (2.3) is redefined with respect to $\mathbb{X}_T$ as

$$\mathcal{U}_{H_p}(x_k) \triangleq \{\mathbf{u}_k \in \mathbb{U}^{H_p} \,|\, x_k \in \mathbb{X}^{H_p},\, x_{H_p|k} \in \mathbb{X}_T\}. \tag{2.16}$$

All concepts, formulations and definitions presented so far are used in next chapters to present the MPC formulation not only for linear but also for hybrid systems. Chapters 4 and 8 considers the definition of a OOP where the model is purely linear while Chapters 5, 6 and 7 consider the OOP for an hybrid system.

2.3 Fault Tolerance Mechanisms

The aim of RTC on sewer networks is to improve its performance in extreme meteorological conditions. Under these conditions, it is very likely that a fault occurs in any constitutive element of the network, what leads to losing the control effectiveness, degrading the performance and even causing dangerous situations such as severe flooding or pollution. Then, it is extremely needed to have fault tolerance mechanisms that reduce the faults effect, ensuring at least the partial fulfilling of the control objectives.

Fault Tolerant Control (FTC) is a new idea recently introduced in the research literature [Pat97] which allows to have a control loop that fulfills its objectives (maybe with a possible degradation) when faults in components of the system (instrumentation, actuators and/or plant) appear. A control loop could be considered fault tolerant if there exist:

- Adaptation strategies of the control law included in the closed-loop.
- Mechanisms that introduce redundancy in sensors and/or actuators.

Figure 2.8 presents a possible classification of the fault tolerance mechanisms considered in

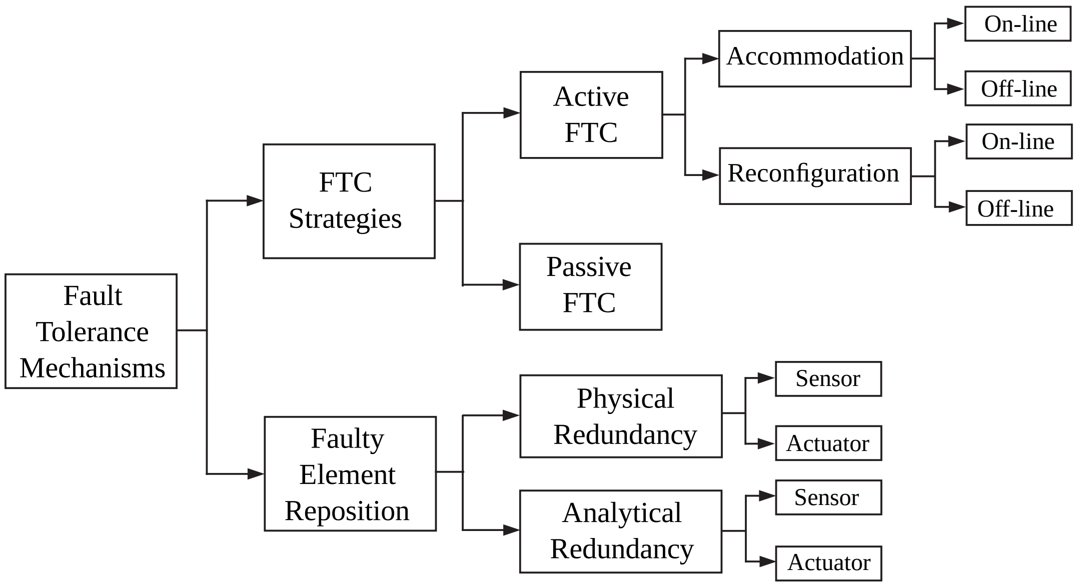

Figure 2.8: Classification of Fault Tolerance Mechanisms

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this section.

2.3.1 Fault Tolerance by Adaptation of the Control Strategy

From the point of view of the control strategies, the literature considers two main groups: the *active* and the *passive* techniques. The passive techniques are control laws that take into account the fault appearance as a system perturbation. Thus, within certain margins, the control law has inherent fault tolerant capabilities, allowing the system to cope with the fault presence. In [CPC98], [LLL00], [QIYS01], [LWY02] and [QIYS03], among many others, complete descriptions of passive FTC techniques can be found.

On the other hand, the active fault tolerant control techniques consist in adapting the control law using the information given by the FDI block. With this information, some automatic adjustments are done trying to reach the control objectives.

Scheme of Figure 2.9 proposes a particular architecture of an active FTC loop introduced by [Bla99], which contains three design levels: the *control loop* (level 1), the *Fault Diagnosis and Isolation* (FDI) system (level 2) and the *supervisor system* (level 3), which closes the outer loop and adds the fault tolerance capabilities.

The feedback control loop shown in Figure 2.9 is composed by a *control law*, an *actuator* or *set of actuators*, the *plant* and a *sensor* or *set/array of sensors*. In parallel with sensor and actuator blocks, there exist other hardware or software blocks used to provide *redundancy* in the signal measurement as in the application of the control action. This redundancy could be introduced in physical form (redundant sensors or actuators) or in analytical form (through mathematical models). From the input and output signals of sensors, actuators and the plant, FDI system detects and isolates the faults, quantifies their magnitude and identifies the specific faulty elements, if possible. Next, FDI system sends this information to the *Automatic Supervisor* (AS), which takes the corresponding decisions in order to maintain the control loop operative in spite of the fault.

Notice that AS block is a discrete event system while the supervised system is defined in continuous time. The information exchange between both systems is done through the FDI block. Due to the whole system has a mixed nature, its corresponding analysis and design could be done using the *hybrid systems theory* (see [CLO95], [BM99a], [AHS01], [MBB03], among many others), this being an open area that is been currently explored and developed in the research literature. In this way, this idea is further developed and discussed in Chapter 7.

Once the AS block receives the information from the FDI module, it evaluates the admissibility of the system performance taking into account the fault presence. To do this, AS considers whether the control objectives: (i) are fulfilled accepting a certain degradation level (region of degraded performance), or (ii) are not fulfilled but there is still the possibility of activating corrective actions (region of unacceptable performance). Otherwise, the process should be stopped (region of danger). Figure 2.10 shows the above mentioned regions of operation for a two-states system. Chapter 8 of this thesis presents a methodology that helps on doing the the admissibility task.

Accommodation and Reconfiguration Strategies

In order to understand the operation of the different strategies within the active FTC philosophy, the standard feedback control problem is defined by [BKLS03]:

$$\langle \mathbb{O}, \mathbb{C}(\theta), \mathbb{U} \rangle, \tag{2.17}$$

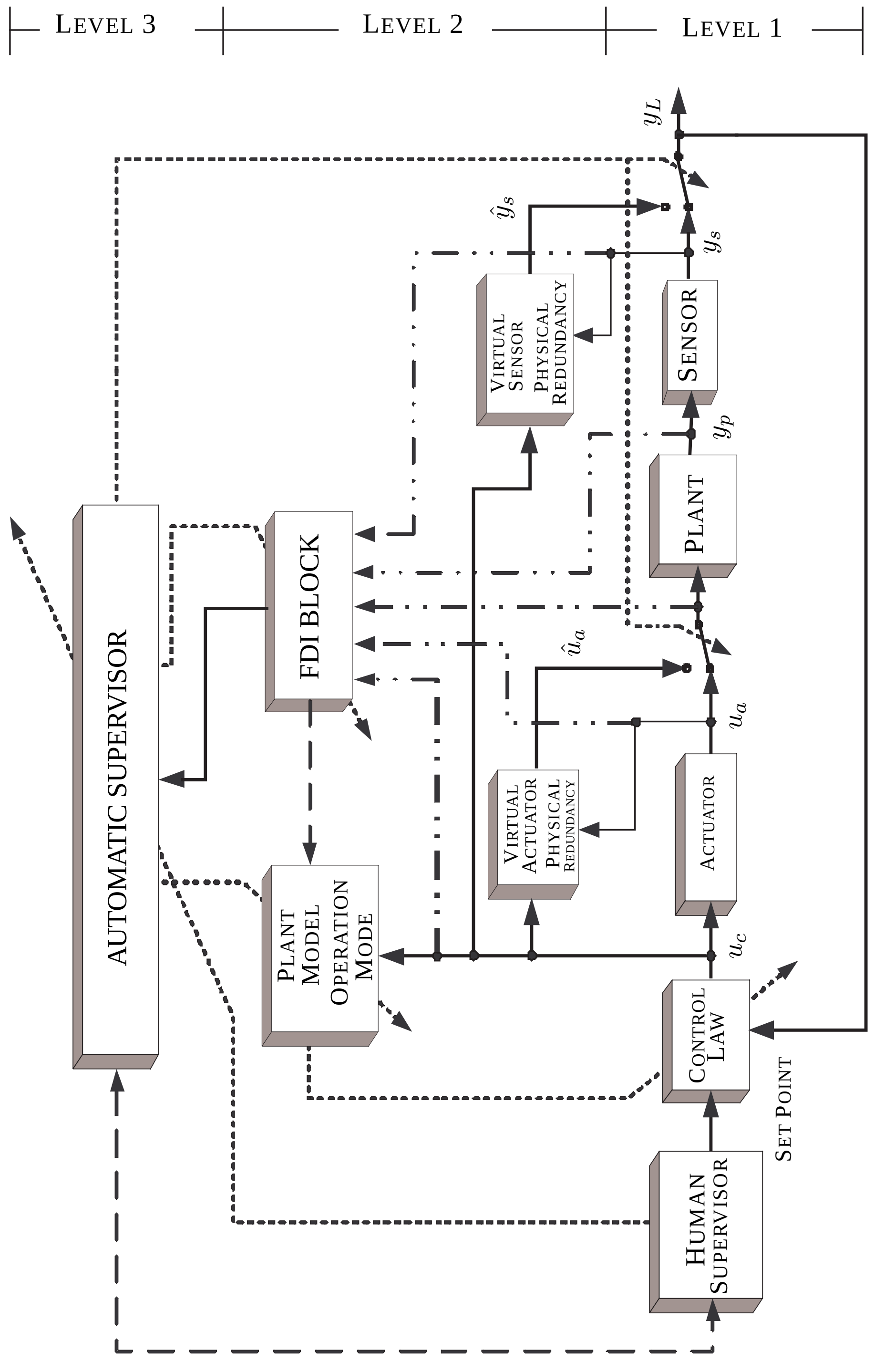

Figure 2.9: Proposed architecture for a fault tolerant control system.

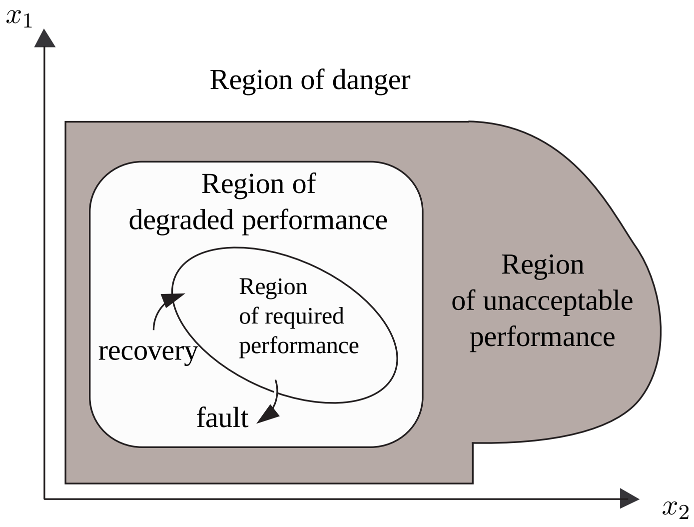

Figure 2.10: Regions of operation according to system performance.

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where $\mathbb{O}$ is the set of *control objectives*, $\mathbb{C}$ is the set of *system constraints*, θ is the vector of *system parameters* and $\mathbb{U}$ is the *control law*. Hence, the faults impact is considered over the problem expressed in (2.17), where $\mathbb{C}(\theta)$ indicates how the systems constraints depend on the parameters that, in turn, depend on the faults. The FDI block provides the detection and isolation of the fault with or without estimating its magnitude.

Depending on the information provided by the FDI module about the fault magnitude, two main strategies to adapt the control loop in order to introduce fault tolerance are possible. The first strategy consists in modifying the control law without changing the rest of the elements within the control loop. This is known as *system accommodation to the fault effect* and it could be done in the case of all changes in system structure and parameters due to the fault could have been accurately estimated. More formally, the following definition is introduced.

Definition 2.1 (*Fault Accommodation*)**.** Fault accommodation consists in solving the control problem $\langle \mathbb{O}, \hat{\mathbb{C}}_f(\hat{\theta}_f), \hat{\mathbb{U}}_f \rangle$, being $\hat{\mathbb{C}}_f(\hat{\theta}_f)$ an estimation of actual system constraints provided by the FDI algorithm.

Second strategy to adapt the control loop is based on changing the control law and another elements of the closed-loop as needed. This is known as *system reconfiguration due to the fault presence* and it could be applied when there is not available information about the fault estimation. In this case, faulty components will be unplugged by FDI block and the control objective will try to be reached using the non faulty components. Then, a formal definition is given as follows.

Definition 2.2 (*System Reconfiguration*)**.** System reconfiguration due to the fault presence consists in finding a new set of constraints $\mathbb{C}_f(\theta_f)$ such that the control problem $\langle \mathbb{O}, \mathbb{C}_f(\theta_f), \mathbb{U}_f \rangle$ can be solved. Then, this solution is found and applied.

On-line/Off-line Control Law Adaptation

Once the adaptation approach of the control law is selected, there are two main ways of implementation within the control loop. The basic difference between them is that in one case (off-line adaptation), the control law parameterized with respect to the faults is pre-computed off-line while in the other case (on-line adaptation), the control law is recomputed on-line taking into account the faults. These ways are:

Off-line Adaptation (Also known as adaptation using precalculated controller). In this case, the control law could be written as $\mathbb{U}_f = U(f)$, where f corresponds to the determined fault. Thus, within the FTC architecture there exists a block used to determine the operation mode of the system once the fault occurs, what allows to compute $\mathbb{U}_f$ (see Figure 2.11(a)). A possible characterization of the control laws on this framework according to the plant nature (mathematical model) is given in [The03] as follows:

- *Control Laws for LTI Models*: techniques based on LTI system models, such as *Model Matching* [Kun92], *Model Following* [Jia94], *LQR* and *Eigenstructure assigment* [Jia94] [ZJ02], among others.
- *Control Laws for a LTI Models family*: techniques based on LTI models obtained by linearization around a set of equilibrium points, covering a certain portion of the whole state space. Some examples are *Multi-models*, *Gain-Scheduling* and *LPV*, among others.
- *Control Laws for Nonlinear Models*: techniques based on nonlinear mathematical model of the plant. In this case, *soft-computing* techniques to design the controller are usually implemented. Examples of these laws are Fuzzy Control, Neural Control, Neuro-Fuzzy Control, among others [DP01].

On-line Adaptation (Also known as adaptation by using an on-line computed controller). In this case, the control law $\mathbb{U}$ is obtained on-line from an estimation of the actual system restrictions $\hat{\mathbb{C}}_f(\hat{\theta}_f)$ once the fault occurs. Figure 2.11(b) shows the basic operation scheme for this case. Also, for the estimation of the fault effect on the system constraints, two alternatives exist:

- *Off-line estimation*: The fault effect over the system constraints has been considered off-line. This fact allows to express such constraints in function of the fault and to change the control law according to the fault information provided by the FDI module. In this way, the controller is always recomputed taking into account the fault effect in the system constraints. Examples of control techniques of this group are the *Model Predictive Control* (MPC) [MR93], [ML01] and *Static Feedback Linearization* [ZJ03].
- *On-line estimation*: The fault effect over the system constraints is computed on-line so the controller will change on-line as well. Examples of control techniques of this group are *Adaptive Control* [IS95], [DP02], *Dynamic Feedback Linearization* [ZJ03] and *Dual Predictive Control* [VX98].

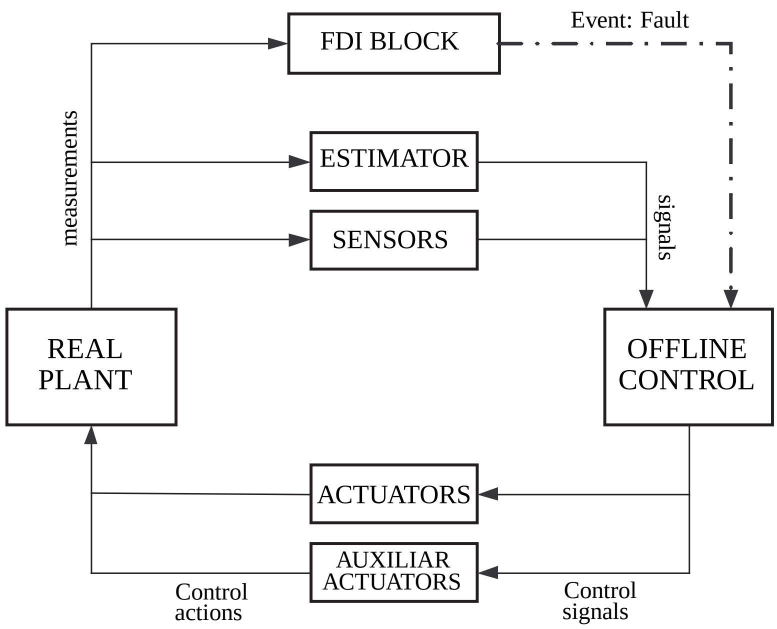
2.3.2 Fault Tolerance by Reposition of Sensors and/or Actuators

Serious faults in sensors or actuators break the control loop. In order to maintain the system in operation, some redundancy should be present in such a way that a new set of sensors (plant inputs) or actuators (plant outputs) is used. To do this, an accommodation block is implemented to work together with the plant and the other non-faulty elements. The main objective consists in having a closed-loop with almost the same performance of the non faulty closed-loop trying to maintain the desired control objectives.

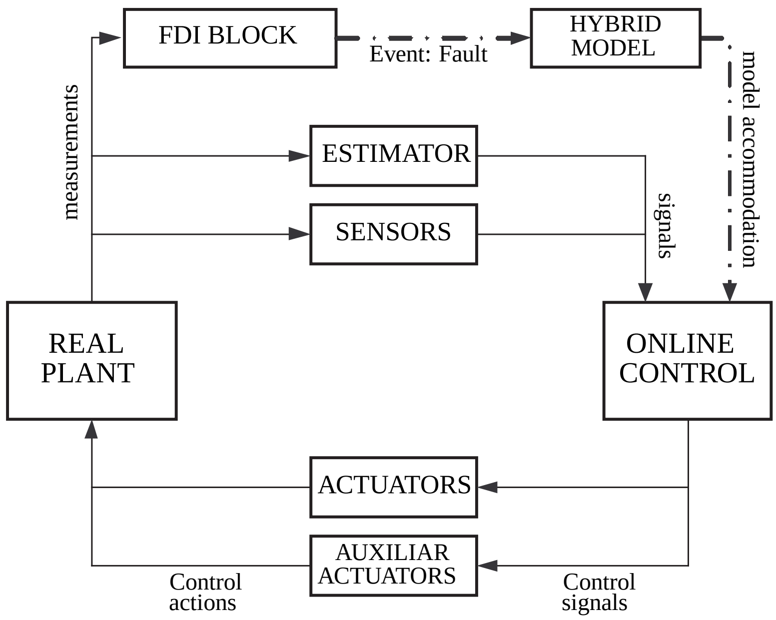
The required redundancy for sensors/actuators fault tolerance can be achieved either using physical redundancy (also called *hardware redundancy*) or using analytical redundancy (also known as *software redundancy* or *redundancy by virtual element*).

Fault Tolerance in Sensors

In the case of sensors, the physical redundancy consists in having an odd number of measurement elements which outputs are multiplexed in a decision block. Such block gives the correct measurement from the determination of the more common signal value of all multiplexed signals. On the other hand, tolerance mechanism by using analytical redundancy consists in using an observer in order to rebuild the system measurements from other existing sensors. For this



(a) Off-line



(b) On-line

Figure 2.11: Conceptual schemes for FTC law adaptation.

reason, this technique is also known *virtual or software sensor*. The design of a sensors network considering the criteria of fault tolerance, system observability, costs and robustness is nowadays an important subject of study in the literature [HSA00], [AHS01]. In [SHA04], estimations of fault tolerance associated to the design of sensor networks is proposed. In this work, aspects like reliability of a set of sensors, its fault tolerance capabilities and minimum number of redundant sensors are evaluated. Applications of these mechanisms can be found in aeronautics [LDC99], [HIM01], in AC systems [BPD99], among many others.

Fault Tolerance in Actuators

As in the case of sensors, the physical redundancy in actuators consists in having additional units that can be multiplexed in a decision block by unplugging the faulty actuator and connecting an alternative non faulty.

On the other hand, in the case of a over-actuated system, some kind of physical redundancy exists. This fact allows to adapt the control law (either through accommodation or reconfiguration strategies)in order to find a suitable control actions for non-faulty actuators. In this way, the control objectives can be fulfilled with an acceptable degradation level [DMHN99]. Thus, the incorporation of new hardware to the closed-loop is avoided, which makes cheaper the implementation. For instance, in the case of large scale water systems where there are thousands of actuators, this approach is suitable for achieving actuator fault tolerance (see Chapters 7 and 8).

From the theoretical point of view, analytical redundancy used to achieve actuator fault tolerance has been recently proposed a dual strategy of the virtual sensor known as *virtual actuator* [LS03] but the proposal is currently limited to be treated and discussed only in the field of the FTC analysis and design.

2.4 Summary

This chapter has presented the main aspects for each one of the constitutive topics related to the thesis. First section has collected definitions, concepts and discussions from literature about sewer networks and their constitutive elements. Moreover, a brief state of the art regarding real-time control of such systems has been also outlined. In this part, references to actual research state have been outlined and described.

In the second section, MPC strategy, hybrid systems and the MPC formulation as RTC control strategy on sewage systems have been presented and discussed.

Finally, the third section collects the main ideas about the existing fault tolerance mechanisms. In Chapter 3, sewer network elements presented in Section 2.1 are described using a given mathematical modeling principles in order to obtain a model of the case study considered in this thesis. MPC formulations and concepts for linear systems are applied in Chapter 4 while for hybrid systems are applied in Chapters 5, 6 and 7. Moreover, also in Chapter 7 and in Chapter 8, descriptions and definitions about fault tolerance and FTC introduced in Section 2.3 are considered and their application is discussed.

CHAPTER 3

PRINCIPLES OF MATHEMATICAL MODELING ON SEWER NETWORKS

One of the most important stages on the RTC of sewer networks, and in general in the control of dynamical systems, lies on the definition of the model for the considered system. Some control techniques such as MPC are very dependent of this issue in order to obtain acceptable performance and satisfactory results due to the accuracy of the open-loop model. This chapter is focused on the determination of a control oriented sewer network model taking into account the trade-off between model accuracy and model complexity [GR94]. Moreover, this chapter proposes and describes a case study based on a real sewer network based on the Barcelona urban drainage system. Using such case study, in subsequent chapters control strategies and their associated advantages and problems are discussed not only in nominal mode but also in faulty mode.

3.1 Fundamentals of the Mathematical Model

The water flow in sewers is open-channel, i.e., the flow shares a free surface with an empty space above. The Saint-Venant equations[1], based on physical principles of mass conservation and energy, allow to describe accurately the flow in open-flow channels as, for instance, the

[1]Adhémar Jean Claude Barré de Saint-Venant (1797 - 1886) was a mechanician who developed the one-dimensional unsteady open channel flow shallow water equations or Saint-Venant equations that are a fundamental set of equations used in modern hydrological engineering.

sewer within a network [May04]. These equation in general form are expressed as:

$$\frac{\partial q_{x,t}}{\partial x} + \frac{\partial A_{x,t}}{\partial t} = 0 \quad (3.1a)$$

$$\frac{\partial q_{x,t}}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q_{x,t}^2}{A_{x,t}}\right) + gA_{x,t}\frac{\partial L_{x,t}}{\partial x} - gA_{x,t}\left(I_0 - I_f\right) = 0 \quad (3.1b)$$

where $q_{x,t}$ is the flow (m^3/s), $A_{x,t}$ is the cross-sectional area of the sewage flow (m^2), t is the time variable (s), x is the spatial variable measured in the direction and the sense of the water flow (m), g is the gravity (m/s^2), I_0 is the sewer slope (dimensionless), I_f is the friction slope (dimensionless) and $L_{x,t}$ is the water level inside the sewer (m). This pair of partial-differential equations constitutes a nonlinear hyperbolic system, that for an arbitrary geometry lacks on analytical solution. Notice that these equations get high detail level in the description of the system behavior. However, such detail level is not useful for real-time control implementation due to the complexity of obtaining the solution of (3.1) and the high computational cost associated [Cro05].

Alternatively, several modeling techniques have been presented in the literature that deal with real-time control of sewer networks, see [MP98], [Erm99], [DMDV01], [MP05], among many others. The modeling approach used in this chapter is based on the proposal presented in [GR94]. There, the sewer system was divided into connected subgroups of sewers and treated as interconnected *virtual tanks* (see Figure 3.1). At any given time, the stored volumes represent the amount of water inside the mains associated with the tank and are calculated on the basis of area rainfall and flow exchanges between the interconnected virtual tanks. The volume is calculated through the mass balance of the stored volume, the inflows and the outflow of the tank and the input rain intensity.

Using the virtual tanks approach and the sewer network elements presented in Section 2.1, the following elementary models are introduced:

Tanks (both virtual and real) The mass balance of the stored volume, the inflows and the outflow of the tank and the input rain intensity mentioned before can be written as the difference equation

$$v_{ik+1} = v_{ik} + \Delta t \varphi_i S_i P_{ik} + \Delta t\left(q_{ik}^{\text{in}} - q_{ik}^{\text{out}}\right), \quad (3.2)$$

where φ_i is the *ground absorption coefficient* of the ***i-th*** catchment, S is the surface area, P is the *rain intensity* in each sample, with a sampling time Δt. q_{ik}^{in} and q_{ik}^{out} are the

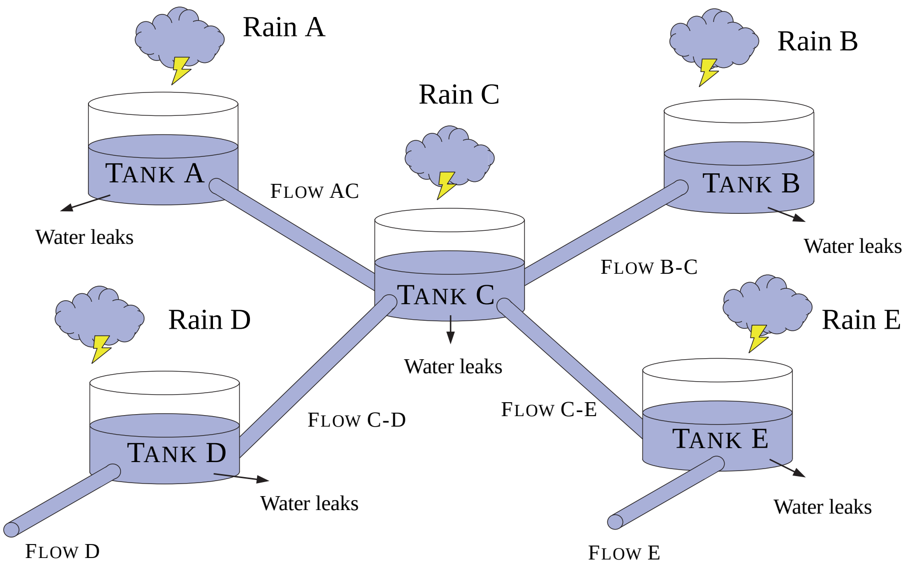


Figure 3.1: Sewer network modeling by means of *virtual tanks*.

sum of inflows and outflows, respectively. *Real retention tanks*, which corresponds to the sewer network reservoirs, are modeled in the same way but without the precipitation term. The tanks are connected with flow paths or links which represents the main sewage pipes between the tanks. The manipulated variables of the system, denoted as q_{u_i}, are related to the outflows from the control gates. The tank outflows are assumed to be proportional to the tank volume (linear tank model approach), that is,

$$q_{ik}^{\text{out}} = \beta_i v_{ik}, \tag{3.3}$$

where β_i (given in s^{-1}) is defined as the *volume/flow conversion* (VFC) coefficient as suggested in [Sin88]. Notice that this relation can be made more accurate (but more complex) if 3.3 is considered to be non-linear (non-linear tank model approach).

The limit on the range of real tanks is expressed as:

$$0 \leq v_{ik} \leq \overline{v}_i \tag{3.4}$$

where $\overline{v}_i$ denotes the maximum volume capacity given in m^3. As this constraint is physical, it is impossible to send more water to a real tank than it can store. Notice that reservoirs without overflow capability have been considered. The virtual tanks do not have a physical limit on their capacity. When they rise above a decided level an overflow situation occurs. This represents the case when the level in the sewers has reached a limit so that an overflow situation can occur in the streets.

Gates In the case of a real tank, a *retention gate* is present to control the outflow. Virtual tank outflows can not be closed but can be redirected using *redirection gates*. The redirection gates divert the flow from a nominal flow path which the flow follows if the redirection gate is closed. This nominal flow is denoted as Q_i in the equation below, which expresses mass conservation at the redirection gate:

$$q_{ik}^{\text{in}} = Q_{ik} + \sum_j q_{u_i k}^j, \tag{3.5}$$

where j is an index over all manipulated flows $q_{u_i}^j$ coming from the gate and q_i^{in} is the flow coming to the gate. The flow path which Q_i represents is assumed to have a certain capacity and when this capacity reaches its limit, an overflow situation occurs. This flow limit will be denoted $\overline{Q}_i$. When Q_i reaches its maximum capacity, two cases are considered:

1. The water starts to flow on the streets, causing a flooding situation.
2. The water exits the sewer network and is considered lost to the environment.

In the first case, the overflow water either follows the nominal flow path and ends up in the same tank as Q_i or it is diverted to another virtual tank. Flow to the environment physically represents the situation when the sewage ends up in a river, in the sea or in another receiver environment. When using this modeling approach where the inherent nonlinearities of the sewer network are simplified by assuming that only flow rates are manipulated, physical restrictions need to be included as constraints on system variables. For example, variables $q_{u_i}^j$ that determinate outflow from a tank can never be larger than the outflow from the tank. This is expressed with the inequality

$$\sum_j q_{u_i k}^j \leq q_{ik}^{out} = \beta_i v_{ik}. \tag{3.6}$$

Usually the range of actuation is also limited so that the manipulated variable has to fulfill $\underline{q}_{u_i}^j \leq q_{u_i k}^j \leq \overline{q}_{u_i}^j$, where $\underline{q}_{u_i}$ denotes the lower limit of manipulated flow and $\overline{q}_{u_i}$ denotes its upper limit. When $\underline{q}_{u_i}^j$ equals zero, this constraint is convex but if the lower bound is larger than zero, constraint (3.6) has to be included in the range limitation. This leads to the following non-convex inequality:

$$\min(\underline{q}^j_{u_i}, q_{ik}^{out} - \sum_{t \neq j} q^t_{u_i k}) \leq q^j_{u_i k} \leq \overline{q}^j_{u_i} \tag{3.7}$$

The sum in the expression is calculated for all outflows related to tank i except j. A further complexity is that whether the control signal is a inflow to a real tank that has hard constraints on its capacity, then the situation can occur that this lower limit is also limited by this maximum capacity and the outflow from the real tank.

Weirs (Nodes) This elements includes in the model of a sewer network the switching behavior since describes the situation when sewage flow has a restriction due to sewer capacity and "jumps" trying to find another path. According to discussion in Section 2.1, these elements can be classified as *splitting nodes* and *merging nodes*. The first can be treated considering a constant partition of the flow in predefined portions according to the topological design characteristics. Merging nodes exhibits a switching behavior. In the case of a set of n inflows q_i, with $i = 1, 2, \ldots, n$, an outflow q_{out} and a maximum outflow capacity $\overline{q}_{out}$, the expressions for this element under the aforementioned conditions are:

$$q_{in} = \sum_{i=0}^{n} q_i \tag{3.8a}$$

$$q_{out} = \min\{q_{in}, \overline{q}_{out}\} \tag{3.8b}$$

$$q_{over} = \max\{0, q_{in} - \overline{q}_{out}\} \tag{3.8c}$$

where q_{over} corresponds to the node overflow. Notice that these expressions define a nonlinear model of the element with all their possible implications.

Remark 3.1. Overflows in sewers (links) follow almost the same description as the nodes since the overflow phenomenon in these elements can be considered as the case of a merging node having a maximum capacity in the nominal outflow path related to the flow capacity of the sewer.

3.2 Calibration of the Model Parameters

In order to estimate the parameters of virtual tanks from real data, measurements coming from sensors are available. Water level measurements in sewers are taken using ultrasonic limnimeters. Notice that the sewer level is measured instead of the flow. This is because the level sensors do not have contact with the water flow, what prevents problems such as wrong measurements caused by sensor faults. From these level measurements, the flow entering and exiting each virtual tank can be estimated assuming steady-uniform flow and using Manning formula[2] [May04]

$$q = vS_w, \tag{3.9}$$

where S_w is the wetted surface that depends on the cross-sectional sewer area A and water level L within the sewer. The dependence of A and L on x and t are omitted for compactness. Moreover, v is the water velocity computed according to the parameters relation

$$v = \frac{K_n}{n} R_h^{2/3} I_0^{1/2}, \tag{3.10}$$

where K_n is a constant whose value depends on the measurement units used in the equation, n is the Manning coefficient of roughness which depends on flow resistance offered by the sewer material, R_h is the hydrological radius defined as the relation of the cross sectional area of flow and the wetted perimeter p as $R_h = A/P$, and I_0 is the sewer slope. For a given geometry of the sewer cross-section, wetted perimeter and hydrological radius can be expressed in function of the sewer level L. For instance, given a rectangular cross-section of width b, the wetted surface S_w is bL, p is $b + 2L$ and the hydrological radius is given by $R_h = \frac{bL}{b+2L}$.

Using the rain intensities P_i and the stated input/output flows, by combining (3.2) and (3.3), the following input/ouput equation in function of the flow in sewers and rain intensity in catchments can be obtained and expressed as:

$$q_{ik+1}^{out} = a\, q_{ik}^{out} + b_1\, P_{ik} + b_2\, q_{ik}^{in}, \tag{3.11}$$

where $a = (1 - \beta_i \Delta t)$, $b_1 = \beta_i \Delta t \varphi_i S_i$ and $b_2 = \beta_i \Delta t$. Figure 3.2 represents this equation and the interaction of all described parameters and measurements.

Equation (3.11) is linear in the parameters, what allows to estimate them using classical parameter estimation methods based on *least-squares algorithm* [Lju99]. Hence, the parameter associated to the ground absorption coefficient is estimated as:

$$\varphi_i = \frac{b_1}{b_2 S_i} \tag{3.12}$$

[2]The Manning formula is an empirical formula for open channel flow, or flow driven by gravity. It was developed by the French engineer *Robert Manning* and proposed on 1891 in the Transactions of the Institution of Civil Engineers (Ireland).

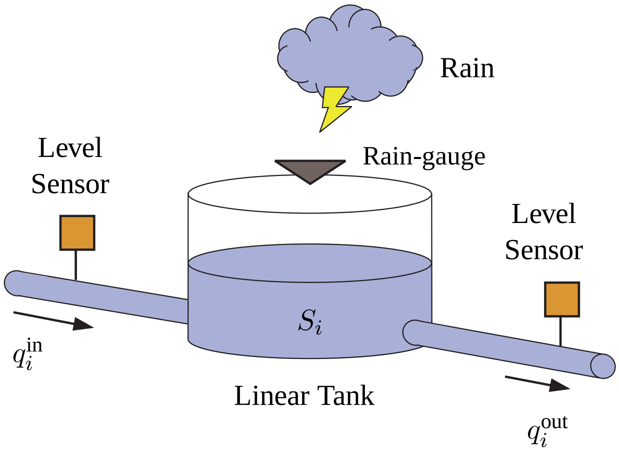


Figure 3.2: Scheme of an individual virtual tank and its parameters and measurements.

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and the VFC coefficient is estimated as:

$$\beta_i = \frac{b_2}{\Delta t} \tag{3.13}$$

for the *i-th* catchment.

Ground absorption and volume/flow conversion coefficients could be estimated on-line at each sampling time using (3.11) and the recursive least-squares (RLS) algorithm [Lju99]. Once estimated, these parameters are supplied to the MPC controller in order to take into account their time variation and neglected nonlinearities.

3.3 Case Study Description

3.3.1 The Barcelona Sewer Network

The city of Barcelona has a CSS of approximately 1697 Km length in the municipal area plus 335 Km in the metropolitan area, but only 514.43 Km are considered as the main sewer network. Its storage capacity is of 3038622 m^3, which implies a dimension three times greater than other cities comparable to Barcelona. The network gathers waters of about 160000 points between the connections with buildings (more than 81500 houses and factories) and the grates of rain entrance, denominated *scuppers*, which can be found in the sidewalks and roads. The entry points for visits to the network, denominated wells, are throughout the network and each 50 ms in average, being altogether 30000.

The main problems of the Barcelona sewer network are caused by three factors: the city topology and its environment, the population and the weather.

The City Topology and Environment

The topological profile of Barcelona has a strong slope in the zone near to the mountain (around 4%), which decreases in direction towards the Mediterranean sea (less than 1%). This aspect causes the fast water concentration in zones in the middle of the city and close to the beach when heavy rainstorms occur in just a short time. Furthermore, the coast dynamic phenomena plus the coincidence of short-time heavy rainstorms with bad marine weather make the drainage difficult, taking into account that the occurrence of heavy rainstorms can suppose an increase on the sea level in almost 50 cm [CLA05]. Similar phenomenon occurs with the water drainage to the rivers Llobregat and Besòs.

The Population

An important characteristic of Barcelona is its population. Practically the totality of its 98 Km^2 approximately of urban territory is urbanized. Over this surface live around 1593000 inhabitants[3], what means a very high density of population (almost 16000 habitants per Km^2). The fast growth of the city during the XX century has left some parts of the sewer network obsolete so the sewage overflow from these areas tends to search its natural way, which implies the occurrence of flooding in certain zones downstream.

The Weather

The Mediterranean weather of the city and its surroundings can represent another problem of vital importance. The yearly rainfall is approximately of 600 mm (600 l/m^2/year), including heavy storms, i.e., rains with an intensity greater than 90 mm/h during half a hour and decennial frequency. These particular episodes can correspond with the half yearly precipitation or,

[3]According to the official report from Spanish Institute of Statistics on January 1th, 2005.

in other words, an episode can concentrate in 30 minutes the fourth part of the yearly rain. These rainstorms are typical of the Mediterranean weather and represents a headache for the management of the sewer network.

Moreover, it is proved that the urban environment affects the local climatology, which implies a correlation between the second and the third problem factors. The thermal difference between Barcelona and its surroundings can reach 3 or 4 degrees Celsius. This phenomenon can benefit rainstorm process not only causing them but also augmenting their intensity.

Sewer Network Managment

Clavegueram de Barcelona, S.A. (CLABSA) is the company in charge of the sewer system management in Barcelona. There is a remote control system in operation since 1994 which includes, sensors, regulators, remote stations, communications and a Control Center in CLABSA. Nowadays, as regulators, the urban drainage system contains 21 pumping stations, 36 gates, 10 valves and 8 detention tanks which are regulated in order to prevent flooding and CSO. The remote control system is equipped with 56 remote stations including 23 rain-gauges and 136 water-level sensors which provide real-time information about rainfall and water levels into the sewer system. All this information is centralized at the CLABSA Control Center through a supervisory control and data acquisition (SCADA) system (see Figure 3.3). The regulated elements (pumps, gates and detention tanks) are currently controlled locally, i.e., they are handled from the remote control center according to the measurements of sensors connected only to the local station.

3.3.2 Barcelona Test Catchment

From the whole sewer network of Barcelona, which was described beforehand, this dissertation considers a portion that represents the main phenomena of the most common characteristics appeared in the entire network. This representative portion is selected to be the case study where a calibrated and validated model of the system following the methodology explained in Section 3.2 is available as well as rain gauge data for an interval of several years.

The considered Barcelona Test Catchment (BTC) has a surface of 22,6 Km^2 and includes typical elements of the larger network. Due to its size, there is a spacial difference in the rain intensity between rain gauges. Figure 3.4 shows the catchment over a real map of Barcelona.

Figure 3.3: CLABSA Control Center.

The expressions V_i in the figure show in different colors the different sub-catchments considered within this thesis using the notation T_i. Notice that the case study corresponds an important piece of the network and it is completely representative of the whole sewage system. On the other hand, the equivalent system is presented in Figure 3.5 using the virtual reservoir methodology described in Section 3.1.

The BTC has 1 retention gate associated with 1 real tank, 3 redirection gates and 1 retention gate, 11 sub-catchments defining equal number virtual tanks, several level gauges (limnimeters) and a pair of links connected to equal number of treatment plants. Also, there are 5 rain-gauges in the BTC but some virtual tanks share the same rain sensor. These sensors count the amount of tipping events in 5 minutes (sampling time) and such values is multiplied by 1.2 mm/h in order to obtain the rain intensity P in m/s at each sampling time, after the appropriate units conversion. The difference between the rain inflows for virtual tanks that share sensor lies in the surface area S_i and the ground absorbtion coefficient φ_i in (3.12) of the i-th sub-catcment, what yields in different amount of the rain inflows. The real tank corresponds to the *Escola Industrial* reservoir, which is located under a soccer field of the Industrial School of Barcelona (see Figure 3.6). It has a rectangle geometry of 94×54 m with a medium depth of 7 m and a maximum water capacity of 35000 m^3 [CLA05].

The related system model has 12 state variables corresponding to the volumes in the 12 tanks

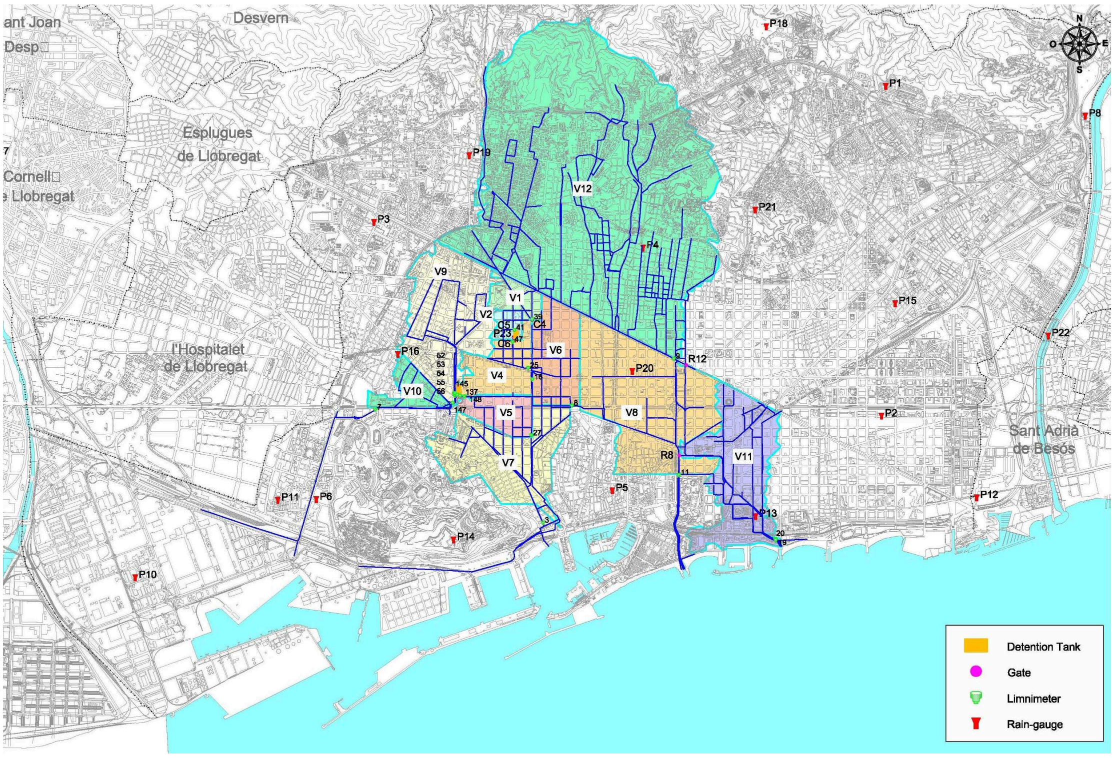


Figure 3.4: Test Catchment located over the Barcelona map. Courtesy of CLABSA.

(1 real, 11 virtual), 4 control inputs corresponding to the manipulated links and 5 measured disturbances corresponding to the measurements of rain precipitation over the virtual tanks. Two water treatments plants can be used to treat the sewer water before it is released to the environment. It is supposed that all states (virtual tank volumes) are estimated by using the limnimeters shown with capital letter L in Figure 3.5. The free flows to the environment as pollution ($q_{10\mathrm{M}}$, $q_{7\mathrm{M}}$, $q_{8\mathrm{M}}$ and $q_{11\mathrm{M}}$ to the Mediterranean sea and q_{12s} to the other catchment) and the flows to the treatment plants ($Q_{7\mathrm{L}}$ and $Q_{11\mathrm{B}}$) are shown in the figure as well. Variable d_i for $i \in [1, 12], i \in \mathbb{Z},\ i \neq 3$ is related to the rain inflow in function of one of the rain intensities P_{13}, P_{14}, P_{16}, P_{19} and P_{20} according to the case. The 4 manipulated links, denoted as q_{u_i} have a maximum flow capacity of 9.14, 25, 7 and 29.3 $\mathrm{m}^3/\,\mathrm{s}$, respectively, and these amounts can not be relaxed, being physical constraints of the system.

Figures 3.7(a) and 3.7(b) present the comparison between real level (from real data) and predicted level (using model described in Section 3.1) corresponding to the output sewers of virtual tank T_1 and T_2, respectively. It can be noticed the fit obtained with this modeling approach.

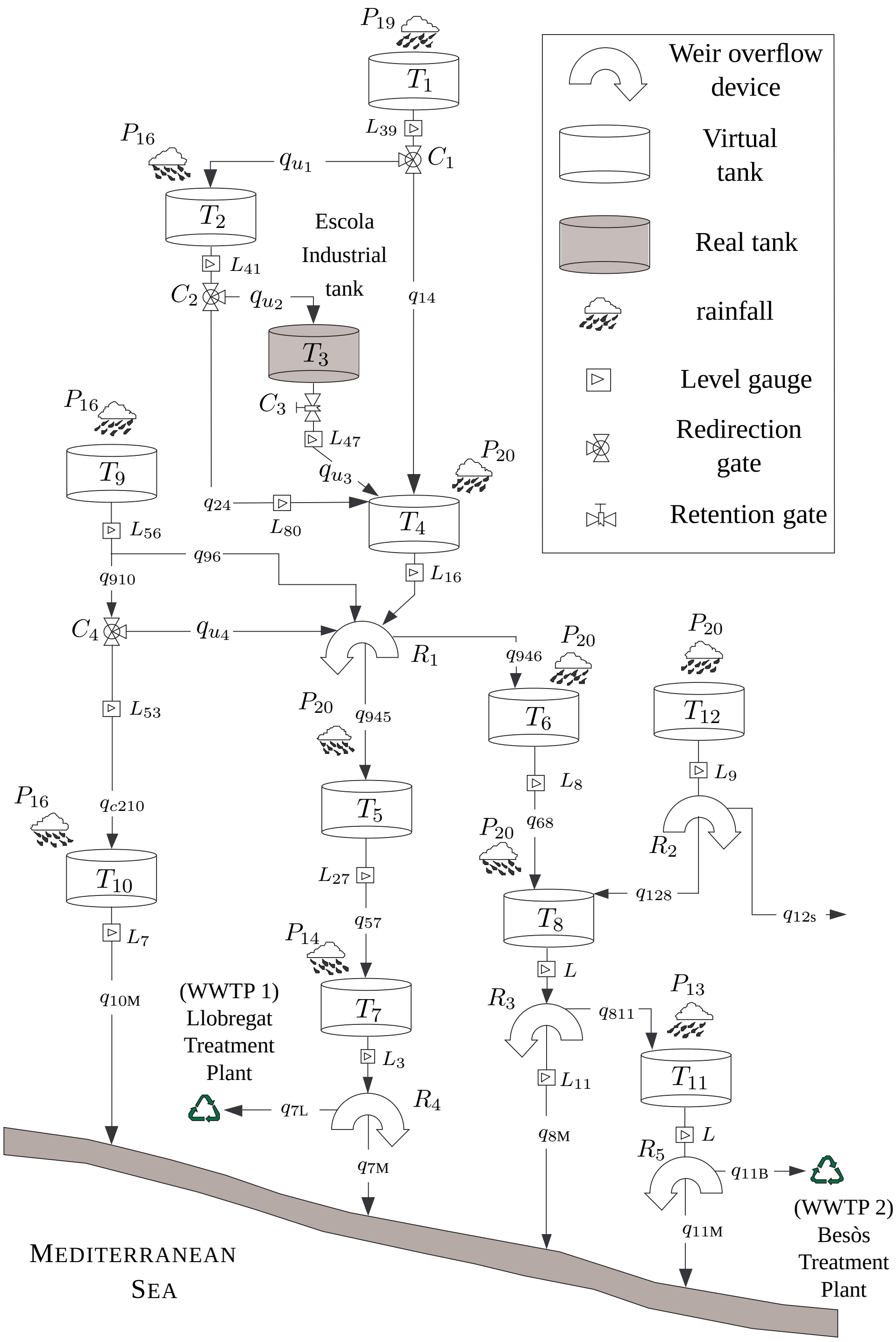


Figure 3.5: Barcelona Test Catchment.

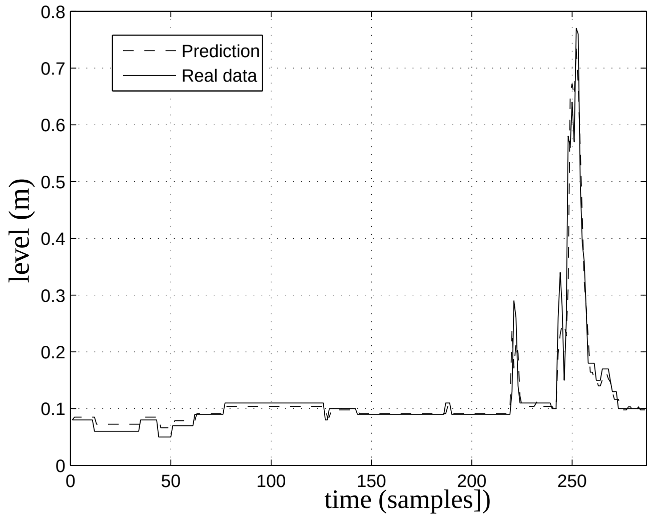
Figure 3.6: Retention tank located at Escola Industrial de Barcelona.

Table 3.1: Parameter values related to the sub-catchments within the BTC.

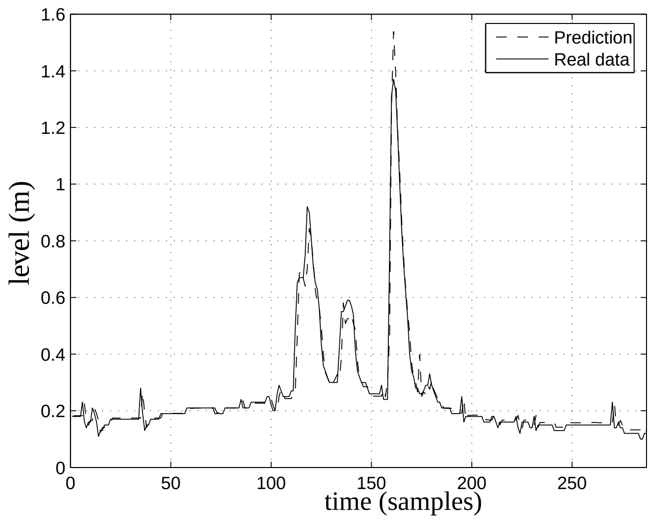
Tank	S (m^2)	φ_i	β_i (s^{-1})	$\overline{v}_i$ (m^3)
T_1	323576	1.03	7.1×10^{-4}	16901
T_2	164869	10.4	5.8×10^{-4}	43000
T_3	5076	–	2.0×10^{-4}	35000
T_4	754131	0.48	1.0×10^{-3}	26659
T_5	489892	1.93	1.2×10^{-4}	27854
T_6	925437	0.51	5.4×10^{-4}	26659
T_7	1570753	1.30	3.5×10^{-4}	79229
T_8	2943140	0.16	5.4×10^{-4}	87407
T_9	1823194	0.49	1.3×10^{-4}	91988
T_{10}	385274	5.40	4.1×10^{-4}	175220
T_{12}	1913067	1.00	5.0×10^{-4}	91442
T_{12}	11345595	1.00	5.0×10^{-4}	293248

Tables 3.1 and 3.2 summarize the description of the case study variables as well as the value of the parameters obtained by calibrating the system model following the procedures described in Section 3.2. In Table 3.1 (and also in Figure 3.5), T_i for $i \in [1, 12], i \in \mathbb{Z}$, $i \neq 3$ denotes the i-th sub-catchment associated to a virtual tank and T_3 denotes the real tank. In Table 3.2, $\overline{q}$ denotes the maximum flow capacity related to the corresponding sewer.

Weirs R_i can be seen as nodes where sewage takes different paths according to the flow



(a) Output level in T_1.



(b) Output level in T_2.

Figure 3.7: Results of model calibration using the approach given in Section 3.2.

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capacity of the sewer located immediately downstream. The presence of these elements within the network causes the addition of nonlinear expressions in the system model due to their nature and dynamics. This fact motivates the use of modeling methodologies which include such switching dynamics and, if possible, allows the use of linear preditive/optimal control, taking

Table 3.2: Maximum flow values of the main sewers for the BTC.

Sewer	$\overline{q}$ (m^3/s)	Sewer	$\overline{q}$ (m^3/s)
q_{14}	9.14	q_{128}	63.40
q_{24}	3.40	q_{57}	14.96
q_{96}	10.00	q_{68}	7.70
$q_{\mathrm{c}210}$	32.80	$q_{12\mathrm{s}}$	60.00
q_{945}	13.36	q_{811}	30.00
q_{910}	24.00	$q_{7\mathrm{L}}$	7.30
q_{946}	24.60	$q_{11\mathrm{B}}$	9.00

advantage of the linear or quadratic programming algorithms in order to obtain the control laws.

3.3.3 Rain Episodes

The rain episodes used for the simulation of the BTC and the design of control strategies are based on real rain gauge data obtained within the city of Barcelona on the given dates (year-month-day) as presented in Table 3.3. These episodes were selected to represent the meteorological behavior of Barcelona, i.e., they contain representative meteorologic phenomena in the city. The table also shows the maximum *return rate*[4] among all five rain gauges for each episode. In the third column of the table, the return rate for the whole of Barcelona is shown. The number is lower because it includes in total 20 rain gauges. Notice that one of the rain storms had a return rate of 4.3 years related to whole of Barcelona while for one of the rain gauges the return rate was 16.3 years.

In Figure 3.8, the reading of the rain gauges for two of these episodes is shown. The rain storm presented in Figure 3.8(a) caused severe flooding in the city area under study.

3.3.4 Modeling Approaches

The description of the dynamical behavior of the case study depends on the desired accuracy level and the tradeoff between complexity and time computation. As was discussed in Chapter 2, each constitutive element of the network can be considered having complementary dynamics that describe the real behavior of the sewage during its flow through the sewer network.

[4]The return rate or return period is defined as the average interval of time within which a hydrological event of given magnitude is expected to be equaled or exceeded exactly once. In general, this amount is given in years.

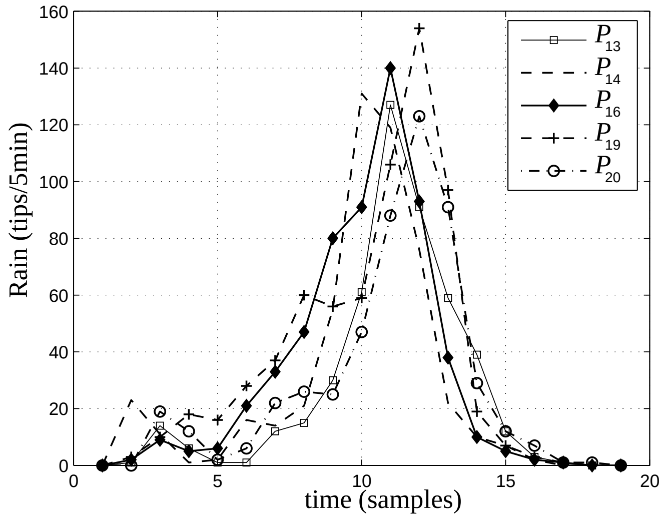
Table 3.3: Description of rain episodes using with the BTC.

Rain Episode	Maximum Return Rate (years)	Return Rate average (years)
99-09-14	16.3	4.3
02-07-31	8.3	1.0
02-10-09	2.8	0.6
99-09-03	1.8	0.6
99-10-17	1.2	0.7
00-09-28	1.1	0.4
98-10-05	1.4	0.2
98-09-25	0.6	0.3
98-10-18	0.4	0.1
00-09-19	0.3	0.2
01-09-22	0.3	0.1
02-08-01	0.2	0.2
00-09-27	0.2	0.2
01-04-20	0.2	0.2
98-09-23	0.1	0.1

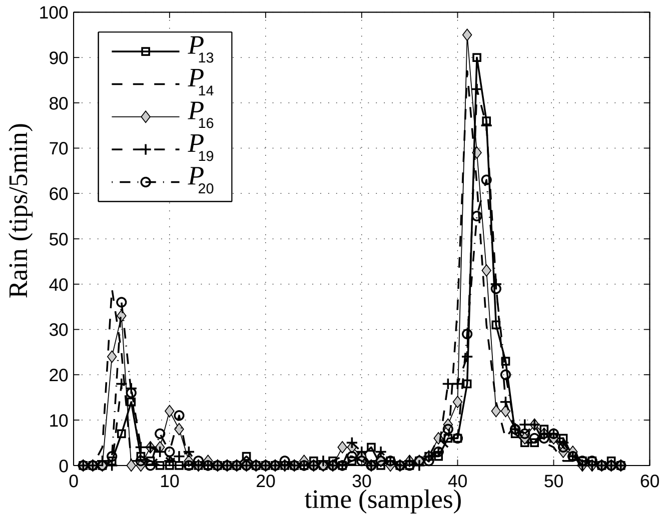
In fact, a collection of expressions of the type (3.2) related to the scheme in Figure 3.5, complemented with dynamics of the form (3.8), make that mathematical expressions related to the sewer network model is collected in a nonlinear state space representation. In order to design a optimal/predictive controller for the case study, such system representations can difficult the computation of a suitable control not only by the suboptimal nature of the solution (non-convex model) but also by the computational effort and time required.

This type of modeling has been implemented in order to design an optimal control law for the BTC [CQS+04]. Software tools such as CORAL [FCP+02] can generate the set of equations that represent the corresponding behavior and dynamics of the constitutive elements considered within a sewer network. Once this set of equations has been obtained, CORAL computes a optimal control sequence which minimizes a given performance indexes. Using this strategy, suboptimal solutions of the control problem are found with a pre-established trade-off between complexity and computation time.

Another approach consists in using functions that are continuous and monotonic in order



(a) September 14, 1999.



(b) October 17, 1999.

Figure 3.8: Examples of rain episodes occurred in Barcelona. Each curve represents a rain gauge P_i.

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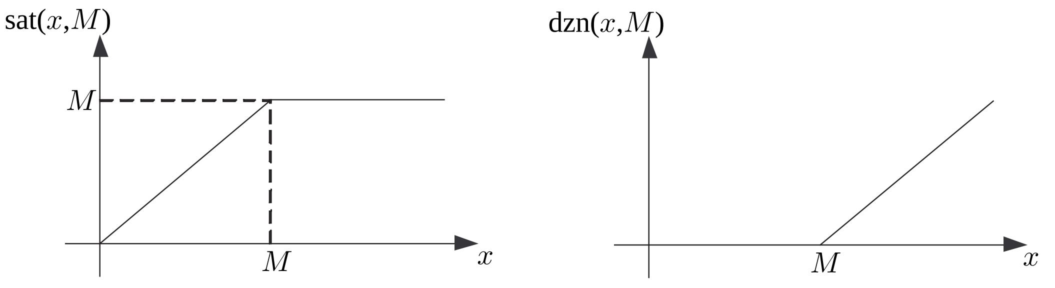
to approximate the expressions of the form (3.8) and/or expressions that describe the weirs behavior and overflow capability of reservoirs. These properties are very useful to obtain a quasi-convex system and can ensure that a global optimal solution in the optimization process can be obtained [BV04]. Despite this approach is been described within the modeling description of



(a) Saturation function. (b) Dead zone function.

Figure 3.9: Continuous and monotonic piecewise functions for sewer network modeling.

the BTC, it is valid as a modeling approach for any sewer network.

The continuous and monotonic functions, in the case of sewer networks, can have the form:

- Saturation (Figure 3.9(a)), given by

$$\text{sat}(x, M) = \begin{cases} x & \text{if} \quad 0 \leq x \leq M \\ M & \text{if} \quad x > M \\ 0 & \text{if} \quad x < 0 \end{cases} \tag{3.14}$$

- Dead Zone (Figure 3.9(b)), given by

$$\text{dzn}(x, M) = \begin{cases} x - M & \text{if} \quad x \geq M \\ 0 & \text{if} \quad x < M \end{cases} \tag{3.15}$$

Hence, element expressions such as (3.8b)-(3.8c) can be now expressed as:

$$q_{out} \quad = \quad \text{sat}(q_{in}, \overline{q}_{out}) \tag{3.16a}$$

$$q_{over} \quad = \quad \text{dzn}(q_{in}, \overline{q}_{out}) \tag{3.16b}$$

and in general all elements and their associated dynamics can be expressed using this methodology. The main objective of this method consists in converting a non-convex problem to a (quasi) convex problem an then trying to find a global optimal solution. Despite such approach is theoretically valid, closed-loop simulations have shown that the computation times are high and the real-time restriction leads to suboptimal performances that do not justify the use of the proposed methodology [OM05].

3.4 Summary

This chapter deals with modeling aspects related to sewer networks. The system model is obtained from the assumption that the network is composed by virtual tanks, which correspond to the storage capacity of a set of sewers in a given catchment. From this moment and in the sequel, the network is considered as a collection of tanks, connective links (main sewers) and gates (among other elements), which constitute a representation of functional elements that can model any sewer network. However, this set of elements as a entire system has a mathematical model which includes nonlinear expressions. This fact adds more modeling considerations when a constrained optimal/predictive control law is designed. Moreover, once the system model is determined, a parameter calibration method using real data is described.

Finally, this chapter presents a brief description of a particular sewage system: the Barcelona sewer network. Its main problems are outlined and a case study is proposed taking a representative portion from the entire network. The case study is named Barcelona Test Catchment (BTC). The BTC model is calibrated using real data of rain episodes occurred in the city of Barcelona between 1998 and 2002. The rain episodes used to simulate this system and design the control strategies in next chapters are also presented. Some modeling methodologies and approaches are also discussed for this particular case study.

An exhaustive mathematical description of the BTC as well as the description of a MATLAB® SIMULINK® tool for simulation the case study behavior and preliminary optimal control design can be found in [OM04].

Part II

Model Predictive Control on Sewer Networks

CHAPTER 4

MODEL PREDICTIVE CONTROL PROBLEM FORMULATION

4.1 General Considerations

One of the most effective and accepted control strategy for the sewage control problem is Model Predictive Control (MPC). An early reference where this approach was suggested is [GR94]. There, an implementation of linear model predictive control over the Seattle urban drainage system was presented. Their results confirmed the effectiveness of the global predictive control law relative to the conventional local automatic controls and heuristic rules that were used to control and coordinate the overall system. Other articles where predictive control ideas were developed further are [MP97, MP01, DMV04, Wah04].

The predictive controller is usually thought to occupy the middle level of a hierarchical control structure where on the top, the states are estimated and the rain is predicted over the control horizon. This information is the input into the MPC problem. The outputs of the MPC controller are reference values for decentralized local controllers that implement the calculated set-points. See [MP97, SCC$^+$04] for references where this hierarchical structure is followed.

As there are many control objectives associated with the sewer network control problem, the optimization problem associated with the MPC controller has multiple objectives as well. The most common approach to solve multi-objective optimization problems is to form a scalar cost function, composed of a linearly weighted sum of cost functions associated with each objective. When objectives have a priority, that is when certain objectives are considered more important than others, then the aim is to reflect this importance with the selection of weights. However, finding appropriate weights is not a trivial problem, especially for large scale control problems with multiple objectives as in the case of sewer networks. Due to different numerical values of cost functions during scenarios, weights that are appropriate for one situation might not be appropriate for another. The weights therefore serve to normalize the cost functions as well as organize their priority. Furthermore, in the case of sewer network control, some objectives are only relevant under specific circumstances. For example, when there is little precipitation, there is no risk of flooding while release of untreated sewage maintains its importance as a control objective. A selection of weights chosen with regard to flooding might not be appropriate when this phenomenon is not present.

Generally, the selection of weights is done by heuristic trial and error procedures involving a lot of numerical simulations, see [TM99]. This complicates and increases the cost of the implementation of predictive controllers for sewer networks. Furthermore, maintaining the controllers and adapting to changes in the system is complicated as weights need to be revised in these cases.

As an alternative to weight based method, the lexicographic approach is based on assigning “a priori” different priorities to the different objectives and then focus on optimizing the objectives in their order of priority. Establishing priorities between objectives by using lexicographic minimization is conceptually very simple, especially in sewer network control, and requires a marginal implementation effort compared to the weight based approach. The main contribution of this chapter is to present the application of lexicographic minimization to eliminate the weight selection process when designing model predictive controllers for sewer networks. Lexicographic minimization has been mentioned in the context of MPC, see [TM99, VSJF01, KM02] but few real applications, specially regarding large scale systems, have been presented.

Keep in mind that only linear models of the sewer network are considered. The main reason is to maintain the optimization problem convex. Convexity is an important property to guarantee applicability of the MPC methodology to large scale problems. The use of nonlinear models for predictive control of urban drainage systems has also been reported, see [MP98]. However, improvements in prediction achieved by using nonlinear models should be compared to the uncertainty present due to the error in predicting the rain over the horizon. If the improvement due to the use of nonlinear models is marginal compared to the uncertainty related to rain prediction, nonlinear models are difficult to justify as often the related predictive control optimization problem is non-convex with the related difficulties due to convergence to local minima and numerical efficiency when large scale problems are considered. In [PLM99], identification of higher order linear models for sewer systems was presented with good results.

It should be pointed out that short term rain prediction or nowcasting is an active field of research, see [SA00]. With a combination of radar, rain gauge measurements and advanced data processing, prediction of rain has improved greatly lately and the potential for the use in predictive control of urban drainage systems has been pointed out in [YTJC99]. But it is out of the scope of the current chapter to explore the tradeoffs between the use of linear and nonlinear models in the context of modern rain prediction methods.

4.2 Control Problem Formulation

As the model and constraints are linear, the MPC controller presented in this chapter is designed using text book formalisms, see for example [Mac02, GSdD05]. Using the modeling formalism presented in Chapter 3, the model of the sewer network can be written as:

$$x_{k+1} = A\,x_k + B\,u_k + B_p\,d_k \tag{4.1}$$

where x_k is the state vector collecting the tank volumes v_i (including virtual and real ones), u_k represents the vector of manipulated flows $q_{u_i k}$, vector d_k corresponds to rain perturbations and constant matrices A, B and B_p are the system matrices of suitable dimensions. Equation (4.1) is created by using (3.2), (3.3), (3.5) and the topology of the sewer network. When the lower limit on $q_{u_i k}$ is zero, the model constraints can be written as:

$$Ex_k + Hu_k \leq b \tag{4.2}$$

where E, H and b are matrices of suitable dimensions created by using (3.6) and (3.4) as well as range limitations to the manipulated flows $q_{u_i k}$.

The model presented in (3.2) is a first order model relating inflows and outflows with a tank volume. In [PLM99] higher order linear models were identified as a function of inflows and outflows. Good results were obtained, even when Output Error models were used for simulation. The control methodology presented can be applied virtually unchanged if a more general linear filter, for example obtained from online calibration procedures, would replace the model in (3.2). In the software implementation, the states are expressed as affine functions of the changes in the control signal, i.e., $\Delta u_k = u_k - u_{k-1}$ for a prediction horizon H_p. The control signal is, on the other hand, only allowed to change over the control horizon H_u.

4.2.1 Control Objectives

The sewer system control problem has multiple objectives with varying priority, see [MP05]. There exist many types of objectives according to the system design. In general, the most common objectives are related to the manipulation of the sewage in order to avoid undesired sewage flows outside of the main sewers. Another type of control objectives are related for instance to the control energy, i.e., the energy cost of the regulation gates movements. According to the literature of sewer networks, the main objectives for the case study of this thesis are listed below in order of decreasing priority:

- *Objective 1:* minimize flooding in streets (virtual tank overflow) (f_1).
- *Objective 2:* minimize flooding in links between virtual tanks (f_2).
- *Objective 3:* maximize sewage treatment (f_3).
- *Objective 4:* minimize control action (f_4).

A secondary purpose of the third objective is to reduce the volume in the tanks to anticipate future rainstorms. This objective also indirectly reduces pollution to the environment. This is because if the treatment plants are used optimally with the storage capacity of the network, pollution lost to the environment should be at a minimum. Moreover, this objective can be complemented by conditioning minimum volume in real tanks at the end of the prediction horizon. It could be seen as a fifth objective. It should be noted that in practice the difference between the first two objectives is small.

The variables related to the first two objectives are overflow variables, that depend on the state. These variables can be treated as slack variable to the overflow constraints, see [GR94] for a similar approach. In the case of virtual tank overflow, these variables are expressed as:

$$(\hat{v}_{ik+j|k} - \overline{v}_i)/S_i \leq \epsilon^v_{i\,k+j|k} \tag{4.3a}$$

$$0 \leq \epsilon^v_{i\,k+j} \tag{4.3b}$$

for all tanks $i = 1 \ldots n$ and for $j = 1 \ldots H_p$. $\hat{v}_{ik+j|k}$ denotes the prediction of the state at time $k \in \mathbb{Z}_+$, j samples into the future. For the first two objectives, the vectors of slack variables are defined as:

$$\Psi_v = [\epsilon^v_{k+1|k}, \ \dots \ , \epsilon^v_{k+H_p|k}] \tag{4.4a}$$

$$\Psi_{q_s} = [\epsilon^{q_s}_{k|k}, \ \dots \ , \epsilon^{q_s}_{k+H_p-1|k}]. \tag{4.4b}$$

Vector Ψ_{q_s} has $N_{q_s} \cdot H_p$ elements where N_{q_s} is the amount of overflow links. Notice that slack variable $\epsilon^v_{k|k}$ is not defined as it depends on $\hat{v}_{k|k}$ or the measured state at time k which can not be affected by control action. For the same reason, Ψ_{q_s} does not include variables for time $k + H_p$. The third and fourth objectives are expressed with vectors:

$$\Psi_{\text{TP}} = [\overline{q}^{\text{TP}} - q^{\text{TP}}_{k|k}, \ \dots \ , \overline{q}^{\text{TP}} - q^{\text{TP}}_{k+H_p-1|k}] \tag{4.5a}$$

$$\Psi_{\Delta u} = [\Delta u_{k|k}, \ \dots \ , \Delta u_{k+H_p-1|k}] \tag{4.5b}$$

The variable $q^{\text{TP}}_{k|k+i}$ is a vector containing the flows to the treatment plants located in the network, $\overline{q}^{\text{TP}}$ is its maximum, and finally, Δu is a vector containing the changes of control action between samples and is defined within this framework as $\Delta u = q_{u_i k} - q_{u_i k-1}$. From variables (4.4a)-(4.4b) and (4.5a)-(4.5b), the control objectives described above can be formulated mathematically as the minimization of the following cost functions:

$$f_1 = \|\Psi_v\|_\infty, \quad f_2 = \|\Psi_{q_s}\|_\infty, \quad f_3 = \|\Psi_{TP}\|_1 \quad \text{and} \quad f_4 = \|\Psi_{\Delta u}\|_1. \tag{4.6}$$

The ∞-norm is used because it is desirable to minimize the maximum flooding over the horizon. For waste water treatment on the other hand, the total volume treated is the important quantity and not specific peaks.

4.2.2 Constraints Included in the Optimization Problem

The physical constraints of the system that were presented in (3.6) and, in the case of real tanks, (3.4), are added as constraints in the optimization problem. For each variable ϵ, restrictions in (4.3) are included as well.

4.2.3 Multi-objective Optimization

The optimization problem associated with the MPC controller is multi-objective. A recent survey of multi-objective optimization can be found in [Mie99]. In general, such a problem can be formulated in the following way:

$$\min_{z \in \mathcal{Z}} \quad [f_1(z), f_2(z), \cdots, f_r(z)] \tag{4.7}$$

where $z \in \mathcal{Z}$ is a vector containing the optimization variables, $\mathcal{Z} \subseteq \mathbb{R}^p$ is the admissible set of optimization variables, and f_i are scalar valued functions of z. A solution z^* is said to be *Pareto optimal* if and only if there does not exist another $z \in \mathcal{Z}$ such that $f_i(z) \leq f_i(z^*)$ for all $i = 1, \cdots, r$ and $f_j(z) < f_j(z^*)$ for at least one index j. In other words, a solution is Pareto optimal if an objective f_i can be reduced at the expense of increasing at least one the other objectives. In general, there may be many Pareto optimal solutions to an optimization problem.

A common approach to solving multi-objective optimization problems is by scalarization, see [Mie99]. This means converting the problem into a single-objective optimization problem with a scalar-valued objective function. A common way to obtain a scalar objective function is to form a linearly weighted sum of the functions f_i:

$$\sum_{i=1}^{r} w_i f_i(z) \tag{4.8}$$

The priority of the objectives are reflected by the weights w_i. Although this type of scalarization is widely used, it has serious drawbacks associated with it, see [Mie99]. Practical drawbacks to this approach are detailed in [TM99].

If a priority exists between the objectives, a unique solution exists on the Pareto surface where this order is respected (see [KM02] and the references therein). Let the objective functions be arranged according to their priority from the most important f_1 to the least important f_r.

Definition 4.1 (Lexicographic Minimizer)**.** A given $z^* \in \mathcal{Z}$ is a lexicographic minimizer of (4.7) if and only if there does not exist a $z \in \mathcal{Z}$ and an i^* satisfying $f_{i^*}(z) < f_{i^*}(z^*)$ and $f_i(z) = f_i(z^*), i = 1, \ldots, i^* - 1$. The corresponding solution $f(z^*)$ is the lexicographic minima.

An interpretation of the above definition is that a solution is a lexicographic minima if and only if an objective f_i can be reduced only at the expense of increasing at least one of the higher-prioritized objectives $\{f_1, ..., f_{(i-1)}\}$. Hence, a lexicographic solution is a special type of Pareto-optimal solution that takes into account the order of the objectives. This hierarchy defines an order on the objective function establishing that a more important objective is infinitely more important that a less important objective.

A standard method for finding a lexicographic solution is to solve a sequential order of single objective constrained optimization problems. After ordering, the most important objective function is minimized subject to the original set of constraints. If this problem has a unique solution, it is the solution of the whole multi-objective optimization problem. Otherwise, the second most important objective function is minimized. Now, in addition to the original constraints, a new constraint is added. This new constraint is there to guarantee that the most important objective function preserves its optimal value. If this problem has a unique solution, it is the solution of the original problem. Otherwise, the process goes on as above. Algorithm 4.1 states formally the sequential solution method to find the lexicographic minimum of (4.7).

Algorithm 4.1 Lexicographic multi-objective optimization using the sequential solution method

$f_1^* = \min_{z \in \mathcal{Z}} f_1(z)$
for $i = 2$ to r **do**
  $f_i^* = \min \left\{ f_i(z) | f_j(z) \leq f_j^*, j = 1, ..., i-1 \right\}$
end for
Determine the lexicographic minimizer set as: $z^* \in \left\{ z \in \mathcal{Z} | f_j(z) \leq f_j^* \,, j = 1, \cdots, r \right\}$

Other approaches to finding the lexicographic minima beside the sequential solution approach have been presented. In [TM99] and [KBM$^+$00] it was shown how the sequential solution approach could be replaced by solving a single Mixed Integer Program (MIP). In [VSJF01] it was shown how the weights for scalar objective function (4.8) could be found so that the solution of the scalar problem would be a lexicographic minima. The weights were found by solving a multi-parametric LP (mpLP). The parameters were the components of the measured state $\hat{x}_{k|k}$ of the system to be controlled. In the current case, where large scale systems are considered and where the disturbances (rain) are predicted over the prediction horizon and included in the optimization problem, the amount of parameters for which the mpLP would be solved off-line would be not only related to the amount of states but also related to the amount of disturbances multiplied by the length of the prediction horizon. This would lead to a very large multi-parametric problem and the advantages over using the sequential solution approach would be lost.

The sampling time in sewer network control is generally large (in the order of several minutes). This gives plenty of time for modern LP solvers to solve many large scale problems, enabling the sequential solution implementation of lexicographic minimization to obtain the control signal. The lexicographic minima was found in this thesis by using the sequential solution method described in Algorithm 4.1 [Mie99].

For a thorough comparison of the lexicographic minimization approach with the weight based approach for control objective prioritization, the performance of the closed-loop system was compared for 15 episodes of different rain intensities representative of Barcelona weather. The strategies were only compared in simulation as it is impossible to repeat experiments on the real process for obvious reasons.

4.3 Closed-Loop Configuration

4.3.1 Model Definition

Notice that the approach discussed in the current chapter considers a linear model of the system. Despite tanks (real and virtual) are modeled using first order linear models, weirs R_i in Figure 3.5 can not be modeled adequately with a linear expression. Hence, the assumption done during this chapter consists in considering these elements as redirection gates with the propose of showing the application of the methodology presented. In Figure 4.1, the reconfigured system is then shown and the modified elements are denoted as *manipulated overflow elements*. Keep in mind that all particular descriptions, concepts and parameter values for the BTC defined in Chapter 3 remain the same for this modified case study.

4.3.2 Simulation of Scenarios

For simplicity, control objective 2 was omitted in the simulation of the case study for this chapter. This is advantageous to the weighted approach as extra control objectives only mean one more optimization in the lexicographic case while in the weight based approach, more objectives make the selection of weights more difficult.

The lexicographic control law was calculated by using Algorithm 4.1 with cost functions given by (4.6). Notice that f_i have different norms, namely the ∞-norm and the 1-norm. Both of these norms result in a linear program to solve the MPC problem, which in turn means that when passing the result of an optimization to a constraint in the subsequent optimization in Algorithm 4.1, it can be done using linear constraints.

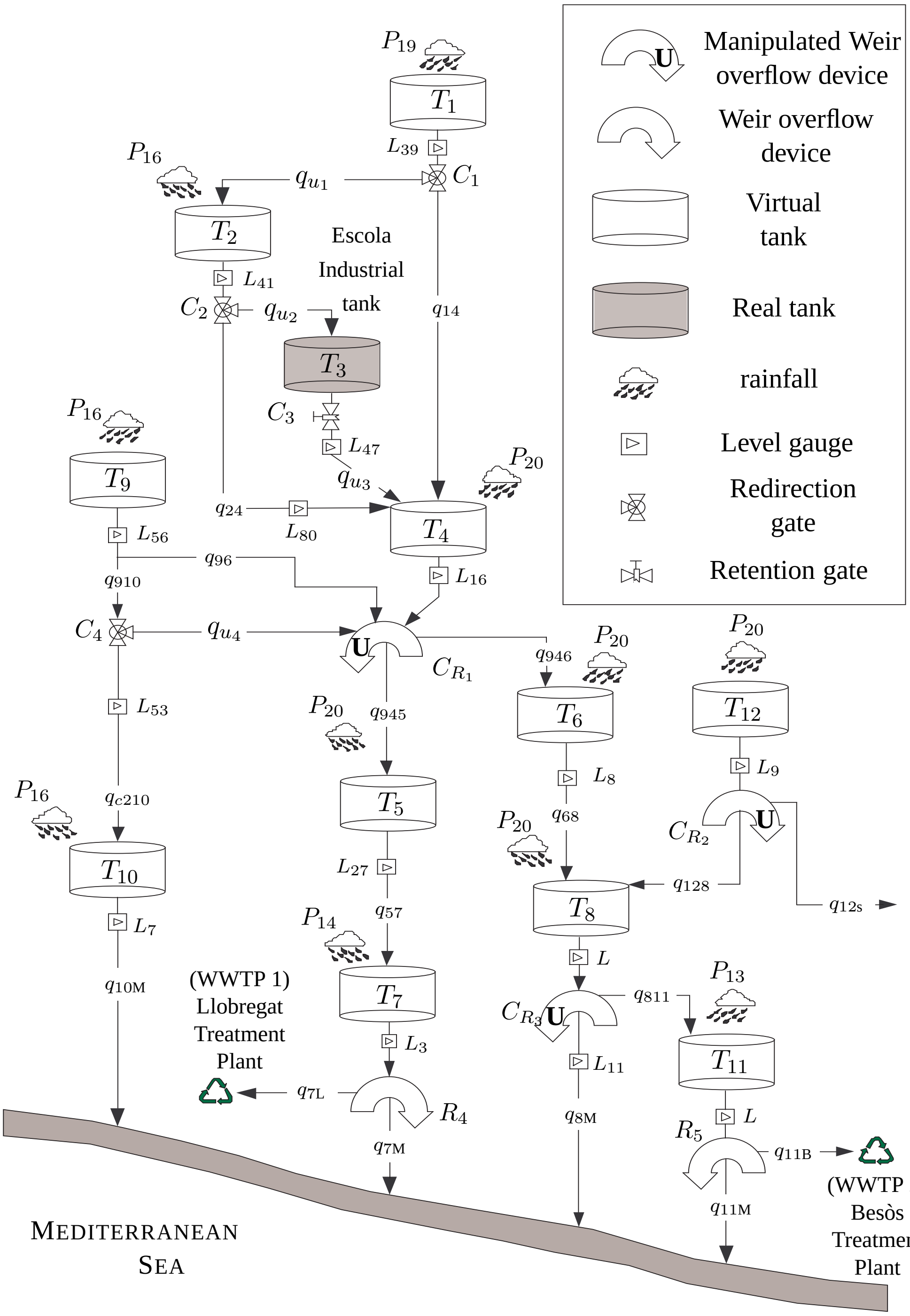


Figure 4.1: Barcelona Test Catchment considering some weirs as redirection gates.

The weight based approach used the same cost functions f_i to express the control objectives but the control signals of the MPC controller were found as the solution when cost function (4.8) was minimized. For an exhaustive comparison, a range of ratios between the first two weights, w_1 and w_3 were considered. In one extreme of this range ($w_1/w_3 = 200$), objective 1 obtained the same value as if the other terms of the cost functions were not present. On the other extreme of this range ($w_1/w_3 = 0.4$), the first objective started to suffer and worse performance could be observed, even causing a reversal of priorities between objectives. Notice that the numerical values of f_i are quite different over the scenarios. The value of the other weights were carefully selected so that the numerical values of the terms $w_i f_i$ would be considerably smaller in the scenarios considered.

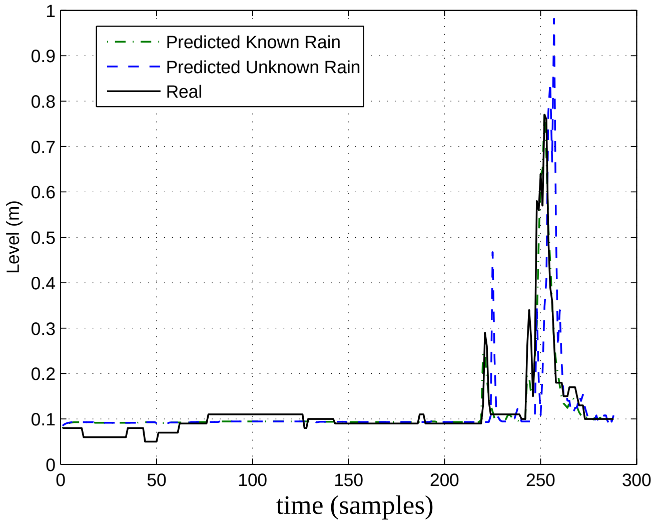
For comparison of strategies for one rain episode, the best performance from the range of w_1/w_3 ratios was compared to the performance of the lexicographic control strategy. The values shown in Table 4.1 correspond to this selection. Thus, no one ratio was considered for all scenarios but the optimal weight was selected after the simulation of each scenario.

The control strategies/tunings were compared by simulating the closed-loop system for each rain episode. The model used for simulation (open-loop model) was the same as was used for the model predictive controller.

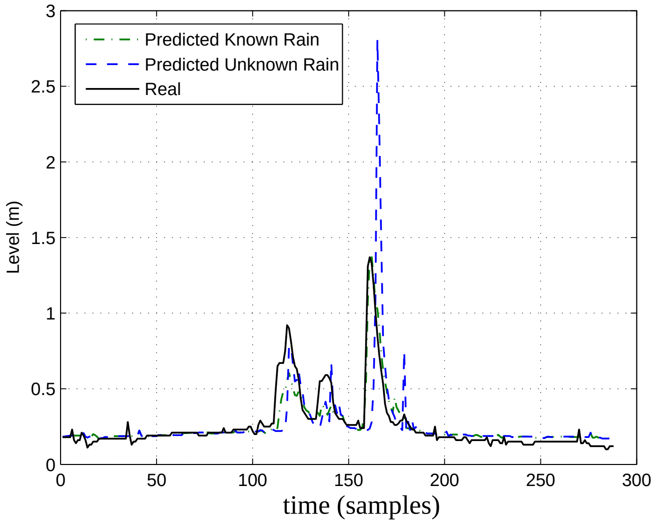
The duration of the simulations was selected as 80 samples or 6.5 hours approximately as the rain storm generally had peaks of duration around 10 samples or 50 minutes. The tanks were empty in the beginning of the scenarios. The rain storm peaks generally occurred around the first 25 samples. Some of the considered rain episodes could have more duration of significant rain.

The prediction horizon and control horizon were selected as 6 samples or 30 minutes which corresponds to the time of concentration[1] for the Barcelona sewer network. This selection has been done according the heuristic knowledge of the CLABSA engineers and field tests made in the sewer network. Another reason for the selection of these prediction and control horizon values is that prediction provided by the used sewer network model becomes less reliable for larger time horizons. In this sense, Figures 4.2(a) and 4.2(b) present the comparison between real sewage level (from real data) and predicted sewage level (using model described in Section 3.1) corresponding to the output sewers of virtual tanks T_1 and T_2, respectively, when the model is used to predict 6 steps ahead. It can be noticed that the fit obtained with the proposed modeling

[1]The time of concentration of a sewer network is determined as the time required for water to travel from the most remote catchment to its outlet to the environment [May04].



(a) Output level in T_1.



(b) Output level in T_2.

Figure 4.2: Results of model calibration using the approach given in Section 3.2 for a prediction of 6 steps.

approach is not as accurate as in the case of Figures 3.7, where the prediction is made for 1 step ahead. Moreover, it can also be noticed that the fact of considering constant rain or known rain during the prediction also affects the quality of the prediction.

Table 4.1: Results obtained for the considered control objectives.

RAIN EPISODES	WEIGHTED APPROACH Flooding (cm)	Pollution 10^3 (m^3)	Treated water 10^3 (m^3)	LEXICOGRAPHIC PROGRAMMING Flooding (cm)	Pollution 10^3 (m^3)	Treated water 10^3 (m^3)
99-09-14	11.5	200	292	11.5	197	295 (1.0%)
02-07-31	6	236	310	6	226	312 (0.6%)
02-10-09	3.6	384	504	3.2	380	508 (0.8%)
99-09-03	0	52	222	0	48	225 (1.3%)
99-10-17	0.3	71	274	1	70	275 (0.4%)
00-09-28	1.2	108	271	1.4	109	271 (0%)
98-10-05	0	3	85	0	0	89 (4.5%)
98-09-25	0	7	299	0	4	304 (1.6%)
98-10-18	0	5	125	0	0	130 (3.8%)
00-09-19	0	3	64	0	0	67 (4.5%)
01-09-22	0	30	181	0	28	183 (1.1%)
02-08-01	0	7	259	0	.4	266 (2.6%)
00-09-27	0	8	101	0	0	109 (7.3%)
01-04-20	0	42	224	0	39	228 (1.7%)
98-09-23	0	2	70	0	0	72 (2.7%)

4.3.3 Criteria of Comparison

To compare the strategies over the simulation scenarios, values related to the control objectives were calculated for each scenario. For the first objective, the maximum flooding over the whole scenario, that is the maximum value of $\|\Psi_v\|_\infty$ for the whole scenario was compared for each control strategy. For the second objective, the total volume of water treated was added up over the scenario. The water released to the environment was added in the same way. These values, obtained for each control strategy were compared for each rain episode. The results are summarized in Table 4.1. Finally, to determine which control strategy was better, the values related to the control objectives were compared in a lexicographic manner, i.e., in an order related to the priorities. If the first values were equal, the second values were compared and so on.

4.4 Results Discussion

The main obtained results, derived from numerical results summarized in Table 4.1, are now discussed.

1. Important performance improvements were obtained in secondary objectives when lexicographic minimization was used. No performance improvements with regard to the first objective were observed. Maximum flooding remained the same for the two control strategies with the exception of two cases (episodes 99-10-17 and 00-09-28). The average percent increase in treated sewage was 2.3 %. For rain episodes where return rate was lower than 0.3, the increase was 3.4 %. The percentage increase is shown in parenthesis in the last column of Table 4.1.

   The reasons for the increase in sewage treatment was that the virtual tanks were used more efficiently to keep the levels in tanks 7 and 11 higher. This in turn enabled the inflow into the waste water treatment plants to be higher, as the outflow of these tanks could be redirected to the treatment plants.

   The improvement in sewage treatment lead to an important decrease in pollution released to the environment. It can be seen in the table that for six rain episodes, sewage released to the environment was reduced to zero when lexicographic minimization was used. Pollution released (flow and total volume) to the environment is demonstrated in Figure 4.3 for episode 00-09-27. The reason why no performance improvement was observed for the first objective was that the same performance could always be achieved by selecting the weight ratio w_1/w_3 large enough.

2. In episode 99-10-17, the maximum flooding in the virtual tanks over the whole scenario was higher when lexicographic minimization was used compared to when the weight based approach was used. The reason for this is explained in Figure 4.4. The rain episode has two peaks, a smaller one peaking at time 5 while the larger one at time 43. Looking at the levels in the tanks it can be seen that the in the lexicographic case, the level is higher at time 40 when the second peak starts. This causes the flooding to become considerably larger around time 46. The other scenario where lexicographic minimization performs worse (episode 00-09-28) has double peaks as well.

   The double peak episodes are complicated because poor performance in these cases is really related to the quality of prediction of rain during the scenarios. The controller based on lexicographic minimization is operating correctly until the second peak arrives. It accumulates sewage with the purpose of maximizing sewage treatment. On the other hand, when the second peak arrives, this behavior is counter productive. Notice that if the objective related to sewage treatment were dropped from the objective functions, both controllers could reduce flooding to zero for episode 99-10-17 and for the other episode, substantially.

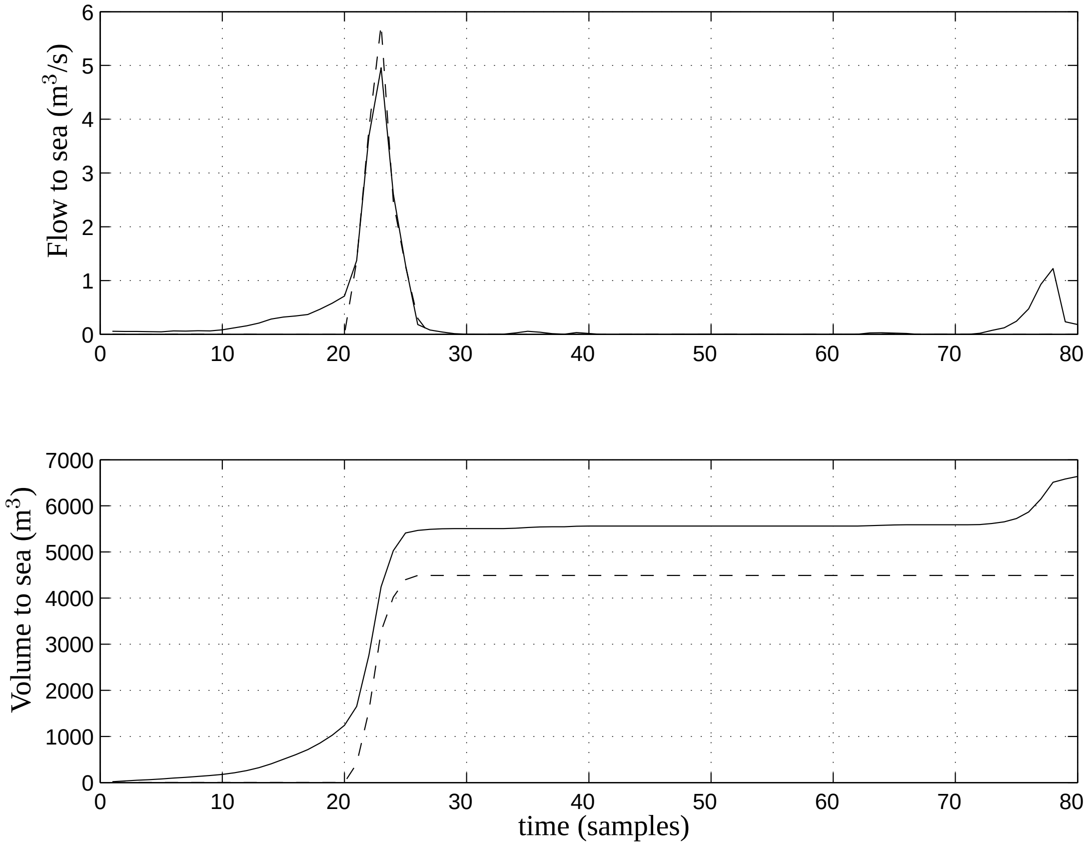


Figure 4.3: Flow and total volume to environment for episode 00-09-27. Solid curve (−), weight based prioritizing, dashed curve (−−) lexicographic prioritizing.

The basic problem is that the two first objectives encourage opposite behaviors in the controllers. The optimal behavior with regard to the first objective is to have the virtual tanks as empty as possible to have capacity to be able to receive new peaks of rain. The second objective strives to store sewage in the system to maximize the sewage treatment. In the double peak episodes, the lexicographic controller suffers due to its superior ability to achieve the second objective in the absence of flooding.

If prediction of rain could be improved to the point of being able to recognize multi-peak episodes, a remedy to the problem described would be to simply drop the secondary objectives until it is sure that flooding danger is not present.

3. Initially, two cases were considered with regard to the prediction of rain $d(k)$ over the control horizon (30 minutes). In the first case, $d(k)$ was assumed to be equal to the

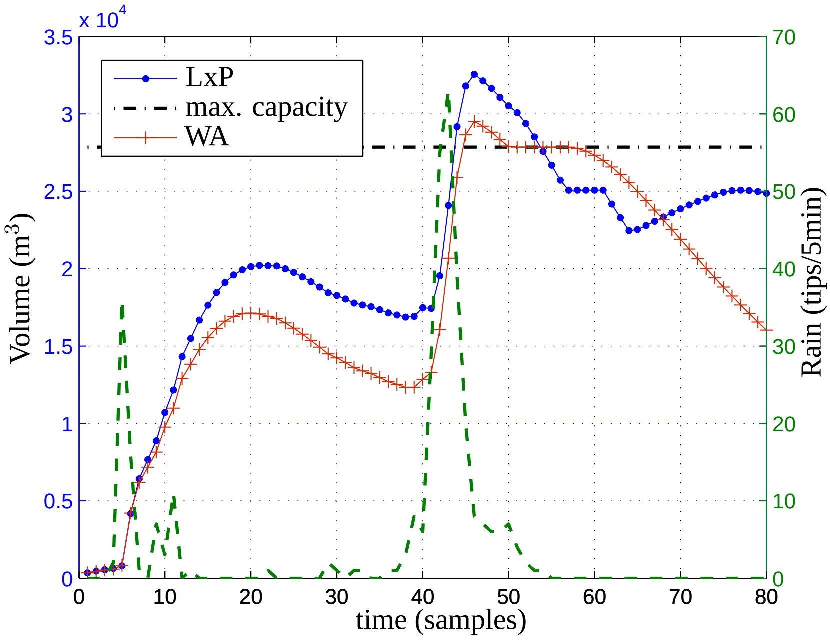


Figure 4.4: Case when lexicographic minimization exhibited poor performance. The dashed curve is the rain intensity (right axis).

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last measurement over the whole horizon. In the other case, the rain was assumed to be accurately predicted over the horizon. Real rain prediction within an urban area of the type considered would be somewhere between these two cases, better than assuming rain constant over the horizon but worse than accurate prediction. The MPC strategy allows updating the control actions taking into account the real state of the system and the precipitation intensity at each sampling time. This state feedback reduces the effect of the rain prediction error on the control performances. In fact, the difference between the two cases was found to be small.

4.5 Summary

This chapter presents the lexicographic approach as a solution technique for the multi-objective optimization that arises in the application of MPC on sewer networks. The possible benefits of using lexicographic minimization have been demonstrated using a modified version of the BTC where the most important rain episodes that occurred over an interval of 4 years were studied and the control performance compared with the traditional weight based approach to express priorities between control objectives. It was shown that performance is similar with regard to the objective with the highest priority while performance is improved with regard to objectives of lower priority. For light rain episodes, sewage treatment could be improved 3.5%. The average improvement for all scenarios was 2.4%.

The numerical value of the performance improvements should be viewed in the context of the high cost of infrastructures for urban waste water systems. MPC based on lexicographic minimization is conceptually very simple and requires a marginal implementation effort compared to the weight based approach once the model is available. The future increase of complexity in the sewer networks of big cities, with multiple reservoirs and actuators (gates) will cause an increment of criteria to take into account in the global optimization function and this imply an important and difficult off-line effort to find a compromise to tune all the parameters in the weight-based approach. The lexicographic approach is therefore a very interesting tool for efficient solution of these kind of problems. The only disadvantage is the increase in the amount of optimization problems required to obtain the lexicographic minima. However, if a linear model is used, modern convex optimization routines can easily handle the large scale systems that occur in the predictive control of urban drainage systems.

Notice that the assumption of a linear system model limits the accuracy in the description of the real behavior of the sewer network. This motivates to find a different model methodology that covers all dynamics without loosing the advantages of the MPC for linear systems. Next chapter deals with this problem and proposes a way to model a sewer network in order to design a optimization-based controller.

CHAPTER 5

PREDICTIVE CONTROL PROBLEM FORMULATION BASED ON HYBRID MODELS

Chapter 4 introduced the application of the predictive control on sewer networks showing important improvements in the closed-loop system performance and the advantages of this control strategy over this kind of systems. However, the linear system model considered did not reflect accurately some particular dynamics and behaviors for some of its components (e.g., weirs, tank overflows). Therefore, a model methodology that allows to reflect the dynamics of those components without leaving the linear framework and without losing all the advantages of the MPC on the linear system is then needed. This fact has motivated to propose a model methodology where the nonlinear dynamics of the form (3.8) are taken into account for some of the considered sewer network elements. These dynamics have been figured out as mode commutations, where a logic variable determines the continuous behavior of the particular elements and then a new global behavior of the whole sewer network. Hence, this mixture of continuous dynamics and logical events define the well known hybrid systems.

This chapter deals with the modeling of a generic sewer network using the hybrid systems framework. The system is in itself decomposed in functional subsystems in order to understand clearly the "hybridity" related to each component. Using the MLD form, the entire model is expressed by means of a discrete linear state space representation that is used to design a MPC law computed by solving a discrete optimization. Both the modeling methodology and the control design are applied over the BTC, showing the advantages of the proposal and the improvement of the closed-loop system performance respect to the open-loop performance.

5.1 Hybrid Modeling Methodology

The presence of intense precipitation causes some sewers and virtual tanks to surpass their limits. When it happens, the volume above the maximum volume flows to another tank. In this way, flow paths appear that are not always present and depend of the system state and inputs. According to this observed behavior in most of the sewer network parts, a model methodology in order to consider and incorporate overflows and other logical dynamics is needed. Using hybrid MLD modeling methodology, the global sewer system as well as each constitutive element can be modeled. Here the most common elements are considered. Other elements such as pumping stations can be easily modeled and added into a sewer network design using the proposed hybrid modeling methodology. Notice that the hybrid model for the constitutive sewer network elements discussed in this chapter can be easily modified, keeping their hybrid nature and becoming new elements.

The hybrid behavior in the considered model of sewer network is present in the flow links between tanks, in the tanks themselves (either virtual or real tanks), in the redirection gates and in the weirs. Network sensors (rain and level gauges) can be also represented as hybrid systems due to their internal dynamics but they have not been taken into account in this dissertation. According to this, the network model has been divided into functional parts in order to make easier the definition of the logical variables and their relation with corresponding flows. These parts are *Virtual Tanks* (VT), *Real Tanks with Input Gate* (RTIG), *Flow Links* (FL) and *Redirection Gates* (RG). In this section, each element is described and its equations within the MLD framework are expressed. In order to obtain the MLD forms, the following equivalences are used

$$[f(x) \leq 0] \longleftrightarrow [\delta_k = 1] \quad \text{is true iff} \quad \begin{cases} f(x) \leq M(1-\delta_k) \\ f(x) \geq \epsilon + (m-\epsilon)\delta_k \end{cases} \tag{5.1}$$

and

$$z_k = \delta_k f(x) \quad \text{is true iff} \quad \begin{cases} z_k \leq M\delta_k \\ z_k \geq m\delta_k \\ z_k \leq f(x) - m(1-\delta_k) \\ z_k \geq f(x) - M(1-\delta_k) \end{cases} \tag{5.2}$$

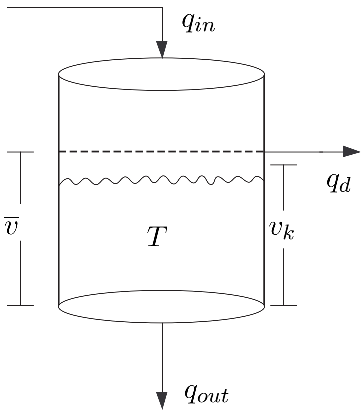

Figure 5.1: Scheme of virtual tank.

where $M, m \in \mathbb{R}$ are the upper and lower bounds on the linear function $f(x)$ for $x \in \mathbb{X}$. $\epsilon > 0$ is the computer numeric tolerance, see [BM99a], [TB04].

5.1.1 Virtual Tanks (VT)

This element is modeled as a hybrid system considering the behavior described as follows. When the maximum volume in virtual tanks is reached, the volume above this maximum amount is redirected to another tank within the network. This phenomenon can be expressed mathematically as:

$$q_{dk} = \begin{cases} \frac{(v_k - \overline{v})}{\Delta t} & \text{if} \quad v_k \geq \overline{v} \\ 0 & \text{otherwise} \end{cases} \tag{5.3a}$$

$$q_{out k} = \begin{cases} \beta\overline{v} & \text{if} \quad v_k \geq \overline{v} \\ \beta v_k & \text{otherwise} \end{cases} \tag{5.3b}$$

where v_k corresponds to the tank volume (system state) and $\overline{v}$ is its maximum volume capacity. Flow q_{dk} is the overflow from the tank, see Figure 5.1. For a feasible solution of the control problem, the virtual tank volume can not be limited with hard constraints.

Hence, in order to obtain the MLD model, the condition of overflow existence is considered defining the logical variable

$$[\delta_k = 1] \longleftrightarrow [v_k \geq \overline{v}], \tag{5.4}$$

which implies that flows q_{dk} and $q_{out\,k}$ are defined through this logical variable as:

$$\begin{aligned} z_{1k} &= q_{dk} \\ &= \delta_k \left[\frac{(v_k - \overline{v})}{\Delta t} \right] \qquad (5.5a) \\ z_{2k} &= q_{out\,k} \\ &= \delta_k \beta \overline{v} + (1 - \delta_k) \beta v_k. \qquad (5.5b) \end{aligned}$$

The corresponding difference equation for the tank in function of the auxiliary variables is written as:

$$v_{k+1} = v_k + \Delta t [q_{in\,k} - z_{1k} - z_{2k}], \tag{5.6}$$

where $q_{in\,k}$ is the tank inflow, z_{1k} is related to the tank overflow and z_{2k} is related to the tank output. Notice that $q_{in\,k}$ collects all inflows to the tank, which could be outflows from tanks located at a more elevated position within the network, link flows, overflows from other tanks and/or sewers and rain inflows.

Hence, the MLD expression (2.8) for this element, taking the tank volume as the system output, would be written as follows:

$$\begin{aligned} v_{k+1} &= v_k + [\Delta t]\, q_{in\,k} + \begin{bmatrix} -\ \Delta t & -\Delta t \end{bmatrix} \begin{bmatrix} z_{1k} \\ z_{2k} \end{bmatrix} \\ y_k &= v_k \end{aligned}$$
$$\begin{bmatrix} -M_v + \overline{v} \\ \overline{v} \\ (M_v - \overline{v})/\Delta t \\ \overline{v}/\Delta t \\ -\overline{v}/\Delta t \\ -(M_v - \overline{v})/\Delta t \\ \beta\overline{v} \\ \beta(M_v - \overline{v}) \\ -\beta(M_v - \overline{v}) \\ \beta\overline{v} \\ 0 \\ 0 \end{bmatrix} \delta_k + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_{1k} \\ z_{2k} \end{bmatrix} \leq \begin{bmatrix} -1 \\ 1 \\ -1/\Delta t \\ 1/\Delta t \\ 0 \\ 0 \\ 0 \\ 0 \\ -\beta \\ \beta \\ 1 \\ -1 \end{bmatrix} v_k + \begin{bmatrix} \overline{v} \\ 0 \\ M_v/\Delta t \\ 0 \\ 0 \\ 0 \\ 0 \\ \beta M_v \\ 0 \\ 0 \\ 0 \\ M_v \end{bmatrix} \tag{5.7}$$

where M_v is related to the maximum value of the state variable which in the case of virtual tanks

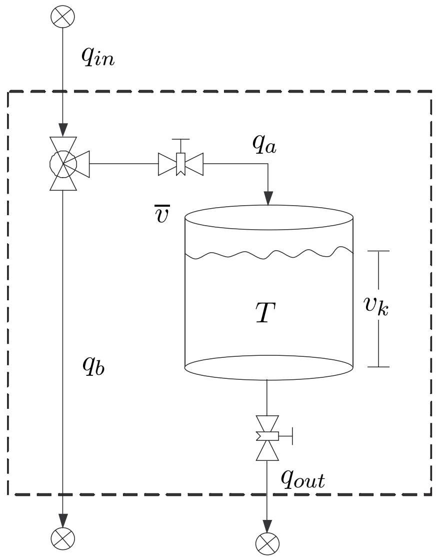

Figure 5.2: Scheme of a real tank with input gate.

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would be unbounded, i.e., $M_v \rightarrow \infty$.

5.1.2 Real Tanks with Input Gate (RTIG)

As was said before, the real tanks are elements designed to retain water in case of severe weather. For this reason, both the tank inflow and outflow could be controlled. On the other hand, the tank inflow is constrained by the actual volume present in the real tank, by its maximum capacity and by the tank outflow. Since this category of tanks are considered without overflow capabilities, the inflow is pre-manipulated using a redirection gate, reason that causes the consideration of this component within the modeling of this element. Figure 5.2 shows a scheme of this element.

When the model for open-loop is considered (no manipulated links), the flow through q_a due to its maximum capacity is defined by

$$q_{a1_k} = \begin{cases} \overline{q}_a & \text{if} \quad q_{in} \geq \overline{q}_a \\ q_{in_k} & \text{otherwise} \end{cases} \tag{5.8}$$

where q_{in_k} is the inflow to the element and $\overline{q}_a$ represents the maximum flow through sewer q_a.

However, if the inflow q_{a1k} causes the real tank to overflow, such inflow have to be reduced to a flow equivalent to the volume that fills the tank. That is:

$$q_{a2k} = \begin{cases} v_{rk} & \text{if} \quad q_{a1k} - q_{out\,k} \geq v_{rk} \\ q_{a1k} & \text{otherwise} \end{cases} \tag{5.9}$$

where $v_{rk} = \left(\frac{(\overline{v}-v_k)}{\Delta t} + q_{out\,k}\right)$ and $q_{out\,k}$ denotes the tank outflow. The MLD form under the initial assumptions is obtained as follows. Defining

$$[\delta_{1k} = 1] \longleftrightarrow [q_{in\,k} \geq \overline{q}_a], \tag{5.10}$$

it is possible to obtain an auxiliary continuous variable

$$\begin{aligned} z_{1k} &= q_{ak} \\ &= \delta_{1k}\,\overline{q}_a + (1-\delta_{1k})\,q_{in\,k}. \end{aligned} \tag{5.11}$$

For q_{a2k}, the following logical variable is defined as:

$$[\delta_{2k} = 1] \longleftrightarrow \left[z_{1k} - q_{out\,k} \geq \frac{(\overline{v}-v_k)}{\Delta t}\right] \tag{5.12}$$

and hence the auxiliary variable equivalent to the new real tank inflow is expressed as:

$$\begin{aligned} z_{2k} &= q_{a2k} \\ &= \delta_{2k}\left[\frac{(\overline{v}-v_k)}{\Delta t} + q_{out\,k}\right] + (1-\delta_{2k})\,z_{1k}. \end{aligned} \tag{5.13}$$

Finally, according to the mass conservation in the included input gate, q_{bk} is defined as $q_{bk} = q_{in\,k} - z_{2k}$ and the corresponding difference equation for the tank in function of the auxiliary variables is:

$$v_{k+1} = (1-\beta\Delta t)v_k + \Delta t z_{2k} \tag{5.14}$$

The MLD expression (2.8) taking the tank volume as the system output is written as:

$$\begin{aligned} v_{k+1} &= [1-\beta\Delta t]\,v_k + \begin{bmatrix} 0 & \Delta t \end{bmatrix} \begin{bmatrix} z_{1k} \\ z_{2k} \end{bmatrix} \\ y_k &= v_k \end{aligned} \tag{5.15}$$

and the Inequality (2.8c) collecting 12 linear inequalities with:

$$\begin{aligned}
E_1 &= \begin{bmatrix} 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
E_{21} &= \begin{bmatrix} -\overline{q}_a & (\overline{q}_{in} - \overline{q}_a) & 0 & 0 & (\overline{q}_{in} - \overline{q}_a) & \overline{q}_a \\ 0 & 0 & -\frac{\overline{v}}{\Delta t} & (\overline{q}_{in} - \beta\overline{v}) & 0 & 0 \end{bmatrix} \\
E_{22} &= \begin{bmatrix} -\overline{q}_a & -(\overline{q}_{in} - \overline{q}_a) & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\overline{v}}{\Delta t} & (\overline{q}_{in} - \beta\overline{v}) & 0 & 0 \end{bmatrix} \\
E_2 &= \begin{bmatrix} E_{21} & E_{22} \end{bmatrix}^T \\
E_3 &= \begin{bmatrix} 0 & 0 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{bmatrix}^T \\
E_4 &= \begin{bmatrix} 0 & 0 & \frac{1}{\Delta t} - \beta & -\left(\frac{1}{\Delta t} - \beta\right) & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\Delta t} - \beta & -\left(\frac{1}{\Delta t} - \beta\right) \end{bmatrix}^T \\
E_{51} &= \begin{bmatrix} -\overline{q}_a & \overline{q}_{in} & -\frac{\overline{v}}{\Delta t} & \left(\overline{v}\left(\frac{1}{\Delta t} - \beta\right) + \overline{q}_{in}\right) & (\overline{q}_{in} - \overline{q}_a) & \overline{q}_a \end{bmatrix} \\
E_{52} &= \begin{bmatrix} -\overline{q}_a & \overline{q}_a & (\overline{q}_{in} - \beta\overline{v}) & \frac{\overline{v}}{\Delta t} & -\frac{\overline{v}}{\Delta t} & \frac{\overline{v}}{\Delta t} \end{bmatrix} \\
E_5 &= \begin{bmatrix} E_{51} & E_{51} \end{bmatrix}^T
\end{aligned} \tag{5.16}$$

According to the use of this element regarding either the simulation or the prediction model, both the tank inflow and output could be manipulated. Hence, two possibilities are considered and outlined below.

1. The RTIG element is used to build a global hybrid model for simulation within a control loop. In this case, both flows q_{a_k} and q_{out_k} have to fulfill the respective physical constraints

$$\underline{q}_a \leq \quad q_{a_k} \quad \leq \overline{q}_a \tag{5.17}$$

$$\underline{q}_{out} \leq \quad q_{out_k} \quad \leq \overline{q}_{out} \tag{5.18}$$

where $\underline{q}_a$ and $\underline{q}_{out}$ are the minimum flows through sewers q_a and q_{out}, respectively. The values of these minimum flows are supposed to be zero in nominal configuration (no faults). For this case, the MLD model of the element includes the constraints (5.17)-(5.18) and validates the possible inputs out of the range.

The expressions of the dynamics in MLD form according to this assumption are obtained as follows. The given input value $q_a^\star$ have to be smaller than the inflow to the element. Otherwise, $q_a^\star$ does not have sense because the difference $q_a^\star - q_{in_k}$ corresponds to a nonexistent water entering to the element. This idea is expressed mathematically as:

$$q_{a1_k} = \begin{cases} q_a^\star & \text{if} \quad q_a^\star \geq q_{in_k} \\ q_{in_k} & \text{otherwise}. \end{cases} \tag{5.19}$$

Now, the upper bound of the link flow adds another condition for $q_a^\star$, given by the upper physical bound as:

$$q_{a2_k} = \begin{cases} q_{a1_k} & \text{if} \quad q_{a1_k} \leq \overline{q}_a \\ \overline{q}_a & \text{otherwise}. \end{cases} \tag{5.20}$$

Finally, the maximum tank capacity also constrains the value of $q_a^\star$ to:

$$q_{a3_k} = \begin{cases} v_r & \text{if} \quad q_{a2_k} - q_{out_k} \geq v_r \\ q_{a2_k} & \text{otherwise}. \end{cases} \tag{5.21}$$

Moreover, the given value $q_{out}^\star$ is also restricted by its physical constraint in (5.18). The corresponding expression that restricts this flow due to the upper physical bound is:

$$q_{out1_k} = \begin{cases} q_{out}^\star & \text{if} \quad q_{out}^\star \leq \overline{q}_{out} \\ \overline{q}_{out} & \text{otherwise} \end{cases} \tag{5.22}$$

and then the expression that restricts this flow due to the actual tank volume is written as:

$$q_{out2_k} = \begin{cases} q_{out1_k} & \text{if} \quad q_{out1_k} \geq \beta v_k \\ \beta v_k & \text{otherwise}. \end{cases} \tag{5.23}$$

Again, the flow q_{b_k} is defined by mass conservation condition in the input gate as:

$$q_{b_k} = q_{in_k} - q_{a3_k}. \tag{5.24}$$

The MLD model for this case is then obtained from the expressions presented in Table 5.1, which define the corresponding δ and z variables. For this case, $q_{b_k} = q_{in_k} - z_{5_k}$.

The difference equation related to the tank is:

$$v_{k+1} = v_k + \Delta t \left(z_{5_k} - z_{4_k} \right). \tag{5.25}$$

Finally, the MLD expression (2.8) taking the volume as the system output can be written

Table 5.1: Expressions for δ and z variables in RTIG element considering manipulated links in closed-loop simulation.

Logical variable δ	Auxiliary variable z
$[\delta_{1k} = 1] \longleftrightarrow [q_a^\star \geq q_{ink}]$	$z_{1k} = \delta_{1k} q_a^\star + (1 - \delta_{1k}) q_{ink}$
$[\delta_{2k} = 1] \longleftrightarrow [z_{1k} \leq \overline{q}_a]$	$z_{2k} = \delta_{2k} z_{1k} + (1 - \delta_{2k}) \overline{q}_a$
$[\delta_{3k} = 1] \longleftrightarrow [q_{out}^\star \leq \overline{q}_{out}]$	$z_{3k} = \delta_{3k} q_{out}^\star + (1 - \delta_{3k}) \overline{q}_{out}$
$[\delta_{4k} = 1] \longleftrightarrow [z_{3k} \geq \beta v_k]$	$z_{4k} = \delta_{4k} z_{3k} + (1 - \delta_{4k}) \beta v_k$
$[\delta_{5k} = 1] \longleftrightarrow [z_{2k} - z_{4k} \geq v_{rk}]$	$z_{5k} = \delta_{5k} v_{rk} + (1 - \delta_{5k}) z_{2k}$

as:

$$\begin{aligned} v_{k+1} &= v_k + \begin{bmatrix} 0 & 0 & 0 & -\Delta t & \Delta t \end{bmatrix} \begin{bmatrix} z_{1k} \\ z_{2k} \\ z_{3k} \\ z_{4k} \\ z_{5k} \end{bmatrix} \\ y_k &= v_k \end{aligned} \tag{5.26}$$

and the inequality (2.8c) collecting 30 linear inequalities, which is automatically generated by HYSDEL.

2. The RTIG element is used to build a global hybrid model for prediction within a control loop. In this case, constraints (5.17) and (5.18) are included in the control law design so they are not taken into account when the MLD model of the element is done. Thus, the expressions of the element dynamics can be obtained as follows. In order to restrict the value of the given input $q_a^\star$ in order to fulfill the mass conservation condition in the input gate, the flow through link q_a is expressed as:

$$q_{a1k} = \begin{cases} q_a^\star & \text{if} \quad q_a^\star \leq q_{ink} \\ q_{ink} & \text{otherwise} \end{cases} \tag{5.27}$$

However, the tank capacity also restricts the tank inflow according to the expression:

$$q_{ak} = \begin{cases} q_{a1k} & \text{if} \quad q_{a1k} - q_{outk} \leq \frac{\overline{v} - v_k}{\Delta t} \\ \frac{\overline{v} - v_k}{\Delta t} & \text{otherwise} \end{cases} \tag{5.28}$$

Table 5.2: Expressions for δ and z variables in RTIG element considering manipulated links in closed-loop prediction.

Logical variable δ	Auxiliary variable z
$[\delta_{1k} = 1] \longleftrightarrow [q_a^\star \leq q_{in\,k}]$	$z_{1k} = \delta_{1k} q_a^\star + (1 - \delta_{1k}) q_{in\,k}$
$[\delta_{2k} = 1] \longleftrightarrow \left[z_{1k} - z_{3k} \leq \frac{\overline{v} - v_k}{\Delta t}\right]$	$z_{2k} = \delta_{2k} q_{a1\,k} + (1 - \delta_{2k}) \frac{\overline{v} - v_k}{\Delta t}$
$[\delta_{3k} = 1] \longleftrightarrow [q_{out}^\star \leq \beta v_k]$	$z_{3k} = \delta_{3k} q_{out}^\star + (1 - \delta_{3k}) \beta v_k$

About the tank outflow, the given input $q_{out}^\star$ is restricted according to the outflow related to the actual volume. That is:

$$q_{out\,k} = \begin{cases} q_{out}^\star & \text{if} \quad q_{out}^\star \leq \beta v_k \\ \beta v_k & \text{otherwise} \end{cases} \tag{5.29}$$

The expressions for δ and z variables in order to obtain the corresponding MLD model are collected in Table 5.2.

The MLD expression (2.8) for this element in this case, taking the tank volume as the system output, is written as follows:

$$\begin{aligned} v_{k+1} &= v_k + \begin{bmatrix} 0 & -\Delta t & \Delta t \end{bmatrix} \begin{bmatrix} z_{1k} \\ z_{2k} \\ z_{3k} \end{bmatrix} \\ y_k &= v_k \end{aligned} \tag{5.30}$$

and the inequality (2.8c) collecting 18 linear inequalities, which is automatically generated by HYSDEL.

Notice that a MLD model with fewer logical variables can be used for instance in the design of a model predictive controller, what reduces the complexity and the computation time of the control problem solution.

5.1.3 Redirection Gates (RG)

These type of elements within a sewer network are used to redirect flow at a certain point in the network. Assuming that $q_{a\,k}$ is manipulated, outflow $q_{b\,k}$ in Figure 5.3 has to fulfill the mass

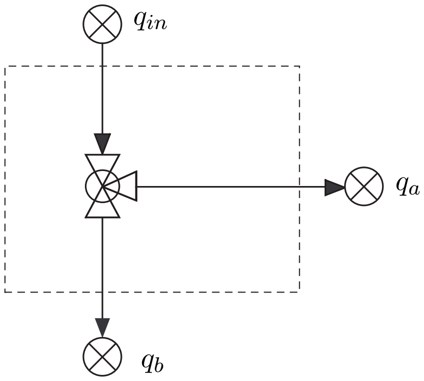


Figure 5.3: Scheme of redirection gate element.

conservation law in this point. Generally, q_{a_k} is assumed to be limited, in which case q_{b_k} is unlimited. The reason for this assumption is that if both outflows are limited, the situation could occur that q_{in_k} would be larger than the sum of these limits, causing the total network to have no feasible solution. The expressions that describe the element dynamics are:

$$q_{a_k} = \begin{cases} q_{in_k} & \text{if} \quad q_{in_k} \leq \overline{q}_a \\ \overline{q}_a & \text{otherwise} \end{cases} \tag{5.31a}$$

$$q_{b_k} = q_{in_k} - q_{a_k}. \tag{5.31b}$$

If the flow through sewer q_a is imposed (for instance computed by means of a control law), its new expression is now written as:

$$q_{a_k} = \begin{cases} \overline{q}_a & \text{if} \quad q_a^\star > \overline{q}_a \\ q_a^\star & \text{otherwise} \end{cases} \tag{5.32}$$

where $q_a^\star$ corresponds to the imposed/computed value for the flow q_{a_k}, while q_{b_k} follows the expression (5.31b).

This element may have a kind of MLD model depending if q_a is considered as a manipulated link or not. In any case, this is an static element, i.e., there is not any state variable in the MLD model. This element only adds more δ and z variables to the global MLD model of the sewer network.

1. q_a *as free link:* Basically this model is used for open-loop simulation. The hybrid dynamics are defined by the maximum flow through sewer q_a, what causes the definition of the auxiliary logic variable

$$[\delta_k = 1] \longleftrightarrow [q_{in\,k} \geq \overline{q}_a] \tag{5.33}$$

and redefine the flow $q_{a\,k}$ as:

$$\begin{aligned} z_k &= q_{a\,k} \\ &= \delta_k\, \overline{q}_{a\,k} + (1 - \delta_k)\, q_{in\,k} \end{aligned} \tag{5.34}$$

Thus, the flow through sewer q_b is immediately defined by the mass conservation as:

$$q_{b\,k} = q_{in\,k} - z_k \tag{5.35}$$

In this case, 6 linear inequalities are defined as in (2.8c) as:

$$\begin{bmatrix} -\overline{q}_a \\ \overline{q}_{in} - \overline{q}_a \\ \overline{q}_{in} - \overline{q}_a \\ \overline{q}_a \\ -\overline{q}_a \\ -\overline{q}_{in} + \overline{q}_a \end{bmatrix} \delta_k + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} z_k \leq \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} q_{in\,k} + \begin{bmatrix} \overline{q}_a \\ \overline{q}_{in} \\ \overline{q}_{in} - \overline{q}_a \\ \overline{q}_a \\ -\overline{q}_a \\ \overline{q}_a \end{bmatrix} \tag{5.36}$$

Notice that sewer q_a is considered as the main path for the flow.

2. q_a *as manipulated link:* In this case, a closed-loop is considered so the element could be used within either a simulation or a prediction model. In the first case (simulation model), the hybrid dynamics are defined taking into account the system constraint $\underline{q}_a \leq q_a \leq \overline{q}_a$. This constraint is supposed to be added to the global model when the control law is designed. Thus, a couple of δ and z variables are defined in order to validate that the given value of q_a fulfills to the mentioned constraint. The definitions of these δ variables depending on the hybrid condition are explained as follows. For the condition of water sufficiency, the logical variable δ_1 is determined as:

$$[\delta_{1\,k} = 1] \longleftrightarrow [q_{a\,k} \geq q_{in\,k}] \tag{5.37}$$

and for the condition related to the upper limit of sewer q_a the logical variable δ_2 is determined as:

$$[\delta_{2k} = 1] \longleftrightarrow [q_{ak} \leq \overline{q}_a] \,. \tag{5.38}$$

Then, the corresponding auxiliary continuous variables are:

$$z_{1k} = \delta_{1k} q_{ak} + (1 - \delta_{1k}) q_{in k} \tag{5.39a}$$

$$z_{2k} = \delta_{2k} z_{1k} + (1 - \delta_{2k}) \overline{q}_{ak}. \tag{5.39b}$$

Again, the flow through sewer q_b is immediately defined by the mass conservation as:

$$q_{bk} = q_{in k} - z_{2k}. \tag{5.40}$$

Here, 12 linear inequalities are defined according to Inequality (2.8c) with:

$$\begin{aligned}
E_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{bmatrix}^T \\
E_{21} &= \begin{bmatrix} -\overline{q}_a & \overline{q}_{in} & 0 & 0 & \overline{q}_a & -\overline{q}_{in} \\ 0 & 0 & -\overline{q}_a & 0 & 0 & 0 \end{bmatrix} \\
E_{22} &= \begin{bmatrix} -\overline{q}_{in} & \overline{q}_a & 0 & 0 & 0 & 0 \\ 0 & 0 & (\overline{q}_{in} - \overline{q}_a) & \overline{q}_a & -\overline{q}_a & -(\overline{q}_{in} - \overline{q}_a) \end{bmatrix} \\
E_2 &= \begin{bmatrix} E_{21} & E_{22} \end{bmatrix}^T \\
E_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{bmatrix}^T \\
E_5 &= \begin{bmatrix} 0 & \overline{q}_{in} & -\overline{q}_a & \overline{q}_a & \overline{q}_a & \overline{q}_{in} & 0 & 0 & (\overline{q}_{in} - \overline{q}_a) & \overline{q}_a & -\overline{q}_a & \overline{q}_a \end{bmatrix}^T
\end{aligned} \tag{5.41}$$

On the other hand, when a prediction model is considered, q_a theoretically fulfills the design constraint $\underline{q}_a \leq q_a \leq \overline{q}_a$, but the values of $\underline{q}_a$ and $\overline{q}_a$ are information data of the control algorithm, so they could be different from the physical bounds (this could be caused, for instance, by a fault effect). Hence, the definitions

$$[\delta_k = 1] \longleftrightarrow [q_{ak} \geq q_{in k}] \tag{5.42}$$

and

$$z_k = \delta_{1k} q_{ak} + (1 - \delta_{1k}) q_{in k} \tag{5.43}$$

are given in order to fulfill the mass conservation condition. Then, the element adds to the global model the following set of 6 inequalities according to Inequality (2.8c):

$$\begin{bmatrix} -\overline{q}_a \\ \overline{q}_{in} \\ \overline{q}_a \\ \overline{q}_{in} \\ -\overline{q}_{in} \\ -\overline{q}_a \end{bmatrix} \delta_k + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} z_k \leq \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 0 & -1 \\ 0 & 1 \\ -1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} q_{in\,k} \\ q_{a\,k} \end{bmatrix} + \begin{bmatrix} 0 \\ \overline{q}_{in} \\ \overline{q}_a \\ \overline{q}_{in} \\ 0 \\ 0 \end{bmatrix} \tag{5.44}$$

In conclusion, the selection of whatever of these two models is directly related to the use of the RG element within the global model and the use of the global model in itself.

5.1.4 Flow Links (FL)

Flow links between tanks have limited capacity. As the flow from virtual tanks can not be controlled, when this limit is exceeded, the resulting overflow might be redirected to a tank to which the original link was not connected. When RG are used to redirect the virtual tank outflow, the unmanipulated link associated to the RG could surpass the maximum sewer capacity. Hence, the sewer overflow is sent to a tank located in a lower level of the sewer network or flows to the environment as water losses. The behavior explained can be represented with the following equations (see Figure 5.4):

$$q_{b\,k} = \begin{cases} \overline{q}_b & \text{if} \quad q_{in} > \overline{q}_b \\ q_{in\,k} & \text{otherwise} \end{cases} \tag{5.45a}$$

$$q_{c\,k} = \begin{cases} q_{in\,k} - \overline{q}_b & \text{if} \quad q_{in} > \overline{q}_b \\ 0 & \text{otherwise} \end{cases} \tag{5.45b}$$

where $\overline{q}_b$ is the maximum flow through q_b, $q_{in\,k}$ is the element inflow and $q_{c\,k}$ corresponds to the outflow.

For the MLD model of this element, only one logical variable is needed. It is defined from the hybrid overflow condition as:

$$[\delta_k = 1] \longleftrightarrow [q_{in} \geq \overline{q}_b] \tag{5.46}$$

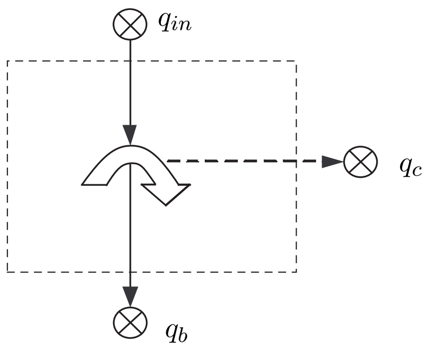


Figure 5.4: Scheme for flow links.

and the auxiliary continuous variables that define the flows q_{bk} and q_{ck} are respectively:

$$\begin{aligned} z_{1k} &= q_{bk} \\ &= \delta_k \, \overline{q}_b + (1-\delta_k) \, q_{in\,k} \end{aligned} \tag{5.47a}$$

$$\begin{aligned} z_{2k} &= q_{ck} \\ &= \delta_k \, (q_{in\,k} - \overline{q}_b). \end{aligned} \tag{5.47b}$$

Notice that, as in the case of the element presented before, this is also a static element, so it only adds more δ and z variables to a global MLD model of the sewer network as well as more constraints in the corresponding Inequality (2.8c) as follows:

$$\begin{bmatrix} -\overline{q}_{in}+\overline{q}_b \\ \overline{q}_b \\ \overline{q}_b \\ \overline{q}_{in}-\overline{q}_b \\ -\overline{q}_{in}+\overline{q}_b \\ -\overline{q}_b \\ \overline{q}_{in}-\overline{q}_b \\ \overline{q}_b \\ -\overline{q}_b \\ -\overline{q}_{in}+\overline{q}_b \end{bmatrix} \delta_k + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} z_k \leq \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ -1 \\ 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} q_{in\,k} + \begin{bmatrix} \overline{q}_b \\ 0 \\ 0 \\ \overline{q}_{in} \\ 0 \\ 0 \\ \overline{q}_{in} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5.48}$$

where $\overline{q}_{in}$ corresponds to the maximum inflow to the element.

Both overflow in VT and overflow in FL do not necessarily go to the immediately next element in the network. Generally, FL overflows do not only go to VT but also to the environment. In these cases, these flows represent losses to the environment (flows to the sea or rivers without previous treatment).

5.1.5 The Whole MLD Catchment Model

The total sewer network is constructed by connecting the network inflows (rain) and outflows (sewer treatment plants or pollution) with the inflows and outflows of the tanks as well as connecting the tanks themselves. Notice that it is possible to obtain the global MLD model by connecting all elements in order to build the equivalent model scheme as shown in Figure 5.5 for the BTC. However, a software tool that allows to interpret the corresponding inputs and outputs of each element in its MLD form and translates them into a global MLD is now pending of implementation. The final tool would have the advantages of a tool for sewer networks known as CORAL [FCP+02], but including the hybrid model framework into its kernel, what allows to improve the computation of optimal control laws since the considered lineal model of the system despite of the global dimension of the network.

The manipulated variables of the system, denoted as q_u, are the manipulated variables of each component as described before. The whole sewer network expressed in MLD form can be written as:

$$v_{k+1} = Av_k + B_1 q_{uk} + B_2\delta_k + B_3 z_k + B_4 d_k \quad (5.49a)$$

$$y_k = Cv_k + D_1 q_{uk} + D_2\delta_k + D_3 z_k + D_4 d_k \quad (5.49b)$$

$$E_2\delta_k + E_3 z_k \leq E_1 q_{uk} + E_4 v_k + E_5 + E_6 d_k \quad (5.49c)$$

where $v \in \mathbb{V} \subseteq \mathbb{R}_+^{n_c}$ corresponds to the vector of tank volumes (states), $q_u \in \mathbb{U} \subseteq \mathbb{R}_+^{mi_c}$ is the vector of manipulated sewer flows (inputs), $d \in \mathbb{R}_+^{md}$ is the vector of rain measurements (disturbance), logic vector $\delta \in \{0,1\}^{r_\ell}$ collects the Boolean overflow conditions and vector $z \in \mathbb{R}_+^{r_c}$ is associated with variables that appear depending on system states and inputs. Variables δ and z are auxiliary variables associated with the MLD form. Equation (5.49c) collects the set of system constraints as well as translations from logic propositions. Notice that this model is a more general MLD than was presented in [BM99a] due to the addition of the measured disturbances.

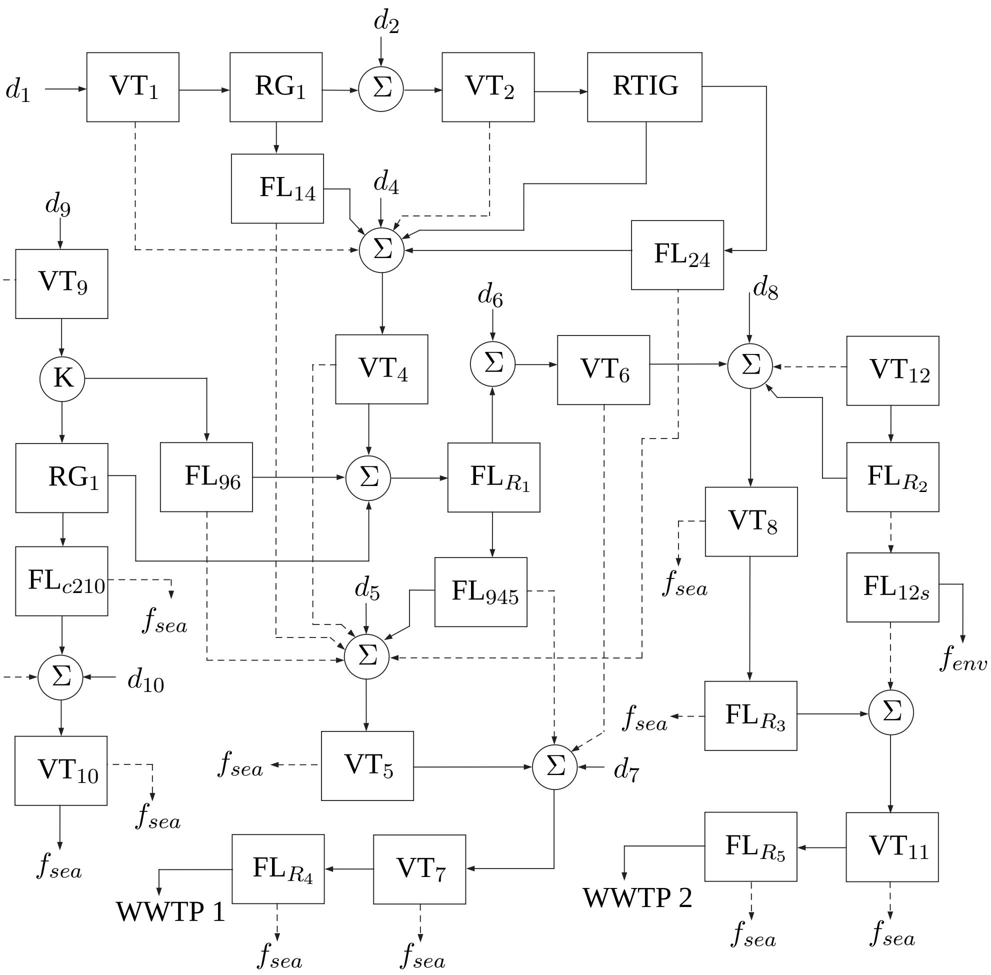


Figure 5.5: BTC scheme using hybrid network elements.

Likewise, measured disturbances may be considered even as inputs or system states. According to the case, q_u and d could be collected in a single vector of system inputs depending on the control law algorithm used. Hence, the MLD form could be rewritten as:

$$v_{k+1} = Av_k + \tilde{B}\,\tilde{q}_{uk} + \begin{bmatrix} B_2 & B_3 \end{bmatrix} \begin{bmatrix} \delta_k \\ z_k \end{bmatrix} \tag{5.50a}$$

$$y_k = Cv_k + \tilde{D}\,\tilde{q}_{uk} + \begin{bmatrix} D_2 & D_3 \end{bmatrix} \begin{bmatrix} \delta_k \\ z_k \end{bmatrix} \tag{5.50b}$$

$$\begin{bmatrix} E_2 & E_3 \end{bmatrix} \begin{bmatrix} \delta_k \\ z_k \end{bmatrix} \leq \tilde{E}\,\tilde{q}_{uk} + \begin{bmatrix} E_4 & E_5 \end{bmatrix} \begin{bmatrix} v_k \\ 1 \end{bmatrix} \tag{5.50c}$$

where vector $\tilde{q}_{uk}$ collects the control inputs and measured disturbances. Moreover, $\tilde{B} = [B_1 \quad B_4]$, $\tilde{D} = [D_1 \quad D_4]$ and $\tilde{E} = [E_1 \quad E_6]$.

On the other hand, assuming that the rain prediction over H_p obeys to a preestablished linear dynamic described as $d_{k+1} = A_d d_k$, the MDL (5.49) form could be rewritten in this case as:

$$\begin{bmatrix} v_{k+1} \\ d_{k+1} \end{bmatrix} = \begin{bmatrix} A & B_4 \\ 0 & A_d \end{bmatrix} \begin{bmatrix} v_k \\ d_k \end{bmatrix} + \begin{bmatrix} B_1 & B_2 & B_3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_{uk} \\ \delta_k \\ z_k \end{bmatrix} \quad (5.51a)$$

$$y_k = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} v_k \\ d_k \end{bmatrix} + \begin{bmatrix} D_1 & D_2 & D_3 \end{bmatrix} \begin{bmatrix} q_{uk} \\ \delta_k \\ z_k \end{bmatrix} \quad (5.51b)$$

$$\begin{bmatrix} E_2 & E_3 \end{bmatrix} \begin{bmatrix} \delta_k \\ z_k \end{bmatrix} \leq \begin{bmatrix} E_4 & E_6 \end{bmatrix} \begin{bmatrix} v_k \\ d_k \end{bmatrix} + \begin{bmatrix} E_1 & E_5 \end{bmatrix} \begin{bmatrix} q_{uk} \\ 1 \end{bmatrix} \quad (5.51c)$$

In general, this assumption is an open research topic. Different types of rain prediction can be considered since this procedure can be development in a either theoretical way (statistical, AR models, etc) [SA00] or practical way (radars, meteorological satellites, etc) [YTJC99]. According to [CQ99], different assumptions can be done for the rain prediction when an optimal control law is used in the RTC of sewer networks. Results show that the assumption of constant rain over a short prediction horizon gives results that can be compared with the assumption of known rain over the considered horizon, confirming similar results reported in [GR94] and [OMPQI05]. According to this, matrix A_d can be set as a identity matrix of suitable dimensions.

5.2 Predictive Control Strategy

This section presents the detailed description of the control strategy applied on sewer networks considering an hybrid system model. The different aspects discussed here are presented for the particular case study of this thesis but can be easily extrapolated to other sewage system topologies. The concepts and definitions of Section 2.2 are applied in this section in a straightforward manner but taking into account the particular notation used for sewer networks. Hence, the sequences for manipulated flow and volumes are defined as:

$$\begin{aligned} \mathbf{v}_k &= \{v_{1|k}, v_{2|k}, \ldots, v_{H_p|k}\} \\ \mathbf{q_u}_k &= \{q_{u0|k}, q_{u1|k}, \ldots, q_{uH_p-1|k}\} \end{aligned} \quad (5.52)$$

and the expression for the admissible input sequence respect to the initial volume $v_{0|k} \triangleq v_k \in \mathbb{V}$ is now written as:

$$\mathcal{Q}_{\mathcal{U}}(v_k) \triangleq \{\mathbf{q_u}_k \in \mathbb{U}^{H_p} | \mathbf{v}_k \in \mathbb{V}^{H_p}\}. \tag{5.53}$$

5.2.1 Control Objectives

In the design process of the MPC controller based on a hybrid model of the system, the control objectives are the same as in the controller design done in Chapter 4 (Section 4.2.1). According to the hybrid model for sewer networks proposed in Section 5.1, all overflows and flows to treatment plants are defined by auxiliary variables z. However, they can be also defined as system outputs, these being either individual (over)flows or sums of (over)flows according to the case.

5.2.2 The Cost Function

Each control objective defines or can define one term in the cost function. Hence, the expression of that function depends on its constitutive variables (auxiliary or output type). In general form, the structure for the cost function in (2.14a) has the form:

$$J(\mathbf{q_u}_k, \Delta_k, \mathbf{z}_k, v_k) \triangleq \sum_{i=0}^{H_p-1} \left\| Q_z(z_{k+i|k} - z_r) \right\|_p + \sum_{i=0}^{H_p-1} \left\| Q_y(y_{k+i|k} - y_r) \right\|_p \tag{5.54}$$

where Q_z and Q_y correspond to weight matrices of suitable dimensions fulfilling the conditions in (2.15) and z_r, y_r are reference trajectories related to auxiliary and output variables, respectively. For the objectives 1 and 2, the references are zero flow. For the third objective, the references are the maximum capacity of the associated sewage treatment plants. Priorities are set by selecting matrices Q_z and Q_y. The norm p can be selected as $p = 1, 2$ or $p = \infty$. Notice that since all performance variables are positive, the case when $p = 1$ is actually a simple sum of the performance variables.

According to the definition of the control objectives done in Section 5.1 and in the case of having the complementary fifth objective, notice that the cost function adopts the form:

$$J(\mathbf{q_u}_k, \Delta_k, \mathbf{z}_k, v_k) \triangleq \| Q_{v_f} v_{rt(H_p|k)} \|_p + \sum_{i=0}^{H_p-1} \left\| Q(z_{k+i|k} - z_r) \right\|_p \tag{5.55}$$

where Q_{v_f} is the corresponding weight matrix of suitable dimensions.

5.2.3 Problem Constraints

The modeling approach is based on mass conservation. Due to this, physical restrictions have to be included as constraints in the optimization problem. The sum of inflows into nodes that connect links have to be equal the outflow. The control variables are limited to a range given in (3.7). The constraints associated to the MIPC problem are in general the constraints associated with the hybrid behavior as well as the system physical constraints for manipulated links and real tanks and the initial condition corresponding to the measurements of the tanks volumes at time instant $k \in \mathbb{Z}_+$.

All the constraints can be expressed on the form given by $(5.49c)$. The physical constraints are considered as *hard constraints* into the control problem. On the other hand, the overflows in sewers and virtual tanks are considered as *soft constraints* and a constraint manager could be designed and implemented to solve the control problem with constraints prioritization [KBM+00].

5.2.4 MIPC Problem

According to the aspects described before, the predictive control problem for a sewer network considering its hybrid model is defined as the OOP

$$\min_{\mathbf{q_u}_k \in \, \mathcal{Q}_\mathcal{U}(v_k), \Delta_k, \mathbf{z}_k} J\left(\mathbf{q_u}_k, \Delta_k, \mathbf{z}_k, v_k\right) \tag{5.56a}$$

$$\text{subject to} \quad \begin{cases} v_{k+i+1|k} &= A\, v_{k+i|k} + B_1\, q_{u k+i|k} + B_2\, \delta_{k+i|k} + B_3\, z_{k+i|k} + B_4\, d_{k+i|k} \\ y_{k+i|k} &= C\, v_{k+i|k} + D_1\, q_{u k+i|k} + D_2\, \delta_{k+i|k} + D_3\, z_{k+i|k} + D_4\, d_{k+i|k} \\ E_2\, \delta_{k+i|k} &+ \; E_3\, z_{k+i|k} \leq E_1\, q_{u k+i|k} + E_4\, v_{k+i|k} + E_5 + E_6\, d_{k+i|k} \\ d_{k+i+1|k} &= A_d\, d_{k+i|k} \end{cases} \tag{5.56b}$$

for $i = 0, \ldots, H_p - 1$. Assuming that the problem is feasible for $v \in \mathbb{V}$, i.e., $\mathcal{Q}_\mathcal{U}(v) \neq \emptyset$, the receding horizon philosophy is then used considering as the MPC law

$$q_{u_{\text{MPC}}}(v_k) \triangleq q_{u_{0|k}}^{*} \tag{5.57}$$

and the entire optimization process is repeated for time $k + 1$.

5.3 Simulation and Results

5.3.1 Preliminaries

The purpose of this section is to show the performance of HMPC for realistic episodes of rain storms. The assumptions made for the implementation will be presented and their validity discussed before the results are given.

The transformation of the hybrid system equations into the MLD form requires the application of the set of given rules in (5.1) and (5.2). The higher level language and associated compiler HYSDEL is used here to avoid the tedious procedure of deriving the MLD form by hand. Given the MLD model, the controllers were designed and the scenarios simulated using the HYBRID TOOLBOX for MATLAB® (see [Bem06]). Moreover, ILOG CPLEX 9.1 has been used for solving MIP problems.

The considered system is shown in Figure 5.6. The dashed lines represent the overflow from tanks and sewers. These lines therefore represent the hybrid behavior of the network. The catchment hybrid model has 12 state variables corresponding to the volumes in the tanks (11 virtual and 1 real), 4 control signals related to the manipulated flows in gates (3 redirection gates and 1 retention gate) and 11 perturbation signals related to the inflow rain of each virtual tank.

The nominal operating ranges of the control signals, the description of the variables in Figure 5.6 and all needed parameters are given in Tables 3.1 and 3.2. Table B in Appendix B collects the auxiliary variables z defined for the BTC and relate them with the control objectives discussed in Section 4.2.1. In order to define the outputs in the global MLD of the network, the following outputs are defined:

$$y_1 = \sum_i z_{\text{str}_v}, \quad y_2 = \sum_i z_{\text{str}_q}, \quad y_3 = z_{41} \quad \text{and} \quad y_4 = z_{43}.$$

5.3.2 MLD Model Descriptions and Controller Set-up

Two different MLD models can be needed to simulate the scenarios, one for the HMPC controller, MLD_C, and one to simulate the plant, MLD_P. Notice that physical constraints are included into the model MLD_C and the solution to the optimization problem respects these constraints in the nominal case when there is no mismatch between the model and the plant.

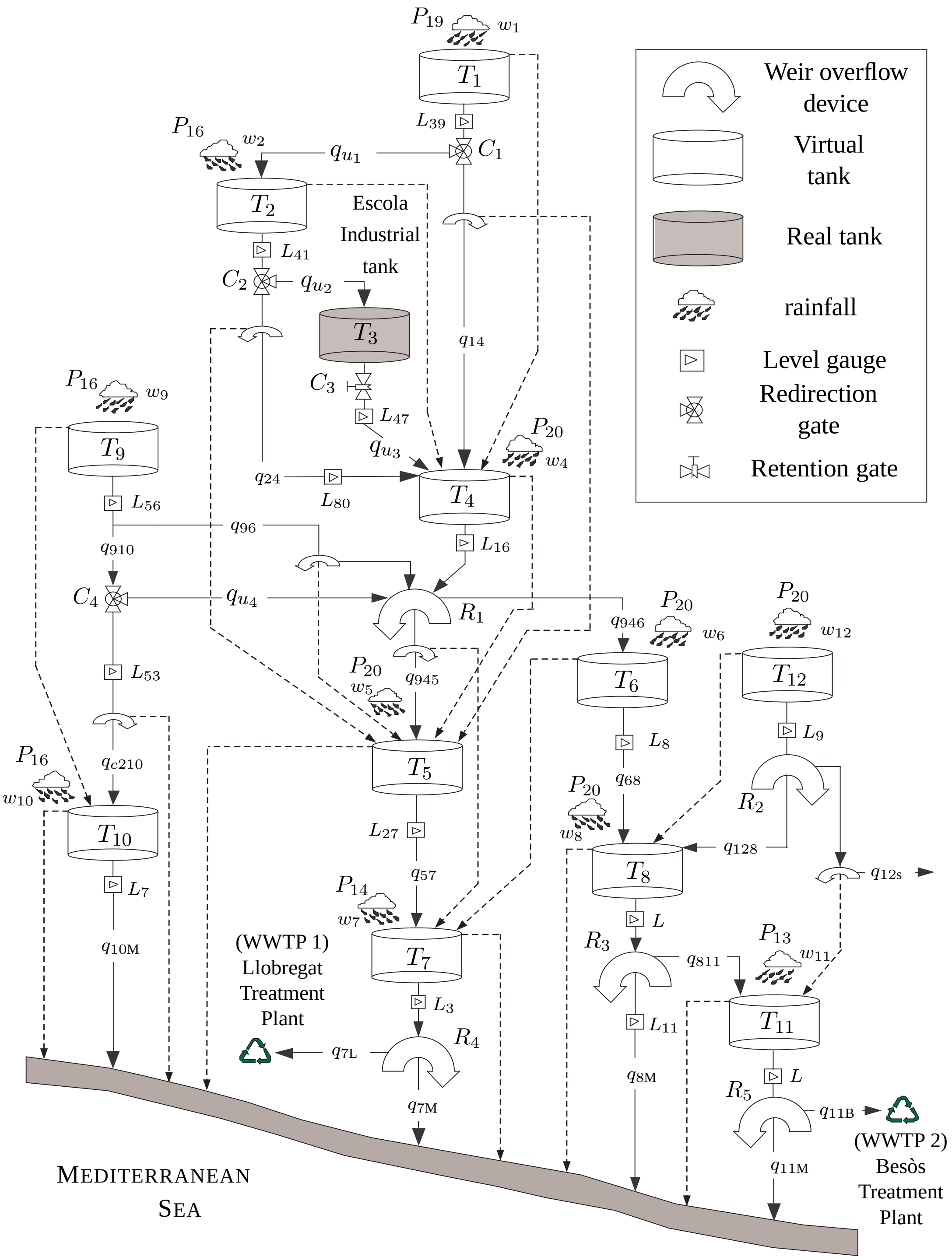


Figure 5.6: BTC diagram for hybrid design.

Otherwise, the solution to the optimization problem might not respect the physical restrictions of the network. The model MLD_P is therefore augmented so that the control signals from the controller are adjusted to respect the physical restrictions of the whole network. MLD_P contained more auxiliary variables for this reason.

Another alternative consists in using the set of equations given by the virtual tank modeling of the network as the plant, which implies that the MLD_P is not needed. For the simulations and results presented next, this second alternative is used. The MLD_C model implemented has 22 logical variables and 44 auxiliary variables, which implies that for each time instant $k \in Z_+$ and considering the prediction horizon $H_p = 6$, $2^{22\times 6} = 5.4\times 10^{39}$ LP problems (for $p = 1, \infty$) or QP problems (for $p = 2$) could be solved in the worst case.

In this case of control design where a hybrid model of the system is taken into account, tuning techniques based on weighted approach are implemented. Tuning proposes such as the lexicographic approach presented in Chapter 4 for linear MPC is not suggested to be applied in predictive control of hybrid systems because the complexity of the optimization problem when the system is considered of large scale. Notice that if the lexicographic approach is implemented, for each time instant k over the scenario, a number of discrete optimization problems equals to the number of control objectives have to be solved. This fact can cause high computation costs and important complexity in the solution algorithms. Chapter 6 deals with these problems and proposes some possible solutions.

Hence, for this case where the tuning by weighted approach is done, the weight matrices used for the cost function (5.54) are given by

$$Q_y = \begin{cases} \text{diag}(w_{\text{str}_v} I_n \quad w_{\text{str}_q} I_n \quad w_{\text{WWTP}} I_n) & \text{if} \quad \text{only } y \text{ are used in } J \\ w_{\text{WWTP}} I_n & \text{if} \quad z \text{ and } y \text{ are used in } J \end{cases} \tag{5.58a}$$

and

$$Q_z = \begin{cases} \text{diag}(w_{\text{str}_v} I_n \quad w_{\text{str}_q} I_n) & \text{if} \quad z \text{ and } y \text{ are used in } J \\ 0 & \text{otherwise} \end{cases} \tag{5.58b}$$

where the description of the weight parameters w_i are collected in Table 5.3 and I_n corresponds to a identity matrix of suitable dimensions. Moreover, the vector of references z_r is always zero

Table 5.3: Description of parameters related to weight matrices in HMPC.

Parameter	Description
w_{str_v}	Weight for the flow to the street due to tanks
w_{str_q}	Weight for the flow to the street due to link
w_{WWTP}	Weight for the flow to treatment plants

and y_r is set as:

$$y_r = \begin{cases} [\mathbb{0}_n \quad \mathbb{0}_n \quad \mathbb{0}_n \quad \mathbb{1}_n\,\overline{q}_{7\text{L}} \quad \mathbb{1}_n\,\overline{q}_{11\text{B}}]^T & \text{if} \quad \text{only } y \text{ are used in } J \\ [\mathbb{0}_n \quad \mathbb{1}_n\,\overline{q}_{7\text{L}} \quad \mathbb{1}_n\,\overline{q}_{11\text{B}}]^T & \text{if} \quad z \text{ and } y \text{ are used in } J \end{cases} \tag{5.59}$$

where $\mathbb{0}_n$ is a vector of zeros and $\mathbb{1}_n$ is a vector of ones, both with suitable dimensions for each set of variables related to each control objective.

As was said before, the prediction horizon H_p is set as 6, what is equivalent to 30 minutes (with the sampling time $\Delta t = 300$ s). The optimal solutions are computed for a bounded time interval $k \in [0, 100]$, which implies around 8 hours. The computational times refer to a MATLAB® implementation running on a INTEL® PENTIUM® M 1.73 GHz machine.

5.3.3 Performance Improvement

The performance of the control scheme is compared with the simulation of the sewer network without control when the manipulated links have been used as passive elements, i.e., the amount of flows $q_{u_1\,k}$, $q_{u_2\,k}$ and $q_{u_4\,k}$ only depend on the inflow to the corresponding gate and they are not manipulated (see Section 5.1) while $q_{u_3\,k}$ is the natural outflow of the real tank given by (3.3). The control tuning is done taking into account the prioritization of the control objectives.

In a preliminary study, different norms, cost function structures and cost function weights w_i have been used. In order to give a hierarchical priority to the control objectives, the relation of w_i between objectives is an order of magnitude. Table 5.4 summarizes the obtained results for a heavy rain episode occurred on September 14, 1999 (see Figure 3.8(a)).

The use of infinity norm for the first and second objectives implies the minimization of the greatest flow to the street caused by one of the 11 virtual tanks and/or the 6 considered sewers.

Table 5.4: Obtained results of closed-loop performance using rain episode occurred on September 14, 1999.

Norm	Variables in J	Tuning w_{str_v}	Flooding $\times 10^3$ (m^3)	Pollution $\times 10^3$ (m^3)	Treated W. $\times 10^3$ (m^3)
∞	only y	10	84.1	225.3	279.4
∞	only y	0.1	84.2	225.3	279.4
∞	y and z	100	100.9	225.9	278.7
∞	y and z	0.1	103.2	225.6	279.1
2	y and z	1	94.3	228.3	276.1
2	only y	0.01	92.8	223.5	280.8

However, in the case of two or more virtual tanks or sewers have overflow, only the worst case will be minimized.

Taking into account that the system performance in open-loop for the considered rain episode has a flooding volume around 108000, a pollution volume of 225900 and a volume treated water of 278300 (all in m^3), the improvement reached is between 4.5$\%$ and 22.1$\%$ for the first objective and the other objectives keep almost in the same values for most of the cases, fulfilling the desired prioritization principle implemented making the control tuning by weighted approach. However, it was observed that some simulations did not run with some combination of selected parameters because numerical problems or the parameters setting made that no improvement was reached in the system performance.

Table 5.5 summarizes the results for ten of the more representative rain episodes in Barcelona between 1998 and 2002. The results were obtained considering $p = 2$ and a cost function containing only output variables with $w_{\text{str}_v} = 10^{-2}$. The system performance in general is improved when the hybrid model based predictive control strategy is applied (see percentages for some values).

5.4 Summary

The possibility of having a linear model of a sewer network taking into account the logical dynamics given by some constitutive elements is discussed in this chapter. A new modeling methodology for sewer networks using a MLD form is proposed and widely explained. This fact makes possible to take advantage of the MPC capabilities in order to design a control strategy

Table 5.5: Obtained results of closed-loop performance using ten representative rain episodes.

Rain Episodes	Open-Loop Flooding $\times 10^3$ (m^3)	Open-Loop Pollution $\times 10^3$ (m^3)	Open-Loop Treated W. $\times 10^3$ (m^3)	Closed-Loop Flooding $\times 10^3$ (m^3)	Closed-Loop Pollution $\times 10^3$ (m^3)	Closed-Loop Treated W. $\times 10^3$ (m^3)
99-09-14	108	225.8	278.4	92.9 (14%)	223.5	280.7
02-10-09	116.1	409.8	533.8	97.1 (16%)	398.8	544.9
99-09-03	1	42.3	234.3	0 (100%)	44.3	232.3
02-07-31	160.3	378	324.4	139.7 (13%)	374.6	327.8
99-10-17	0	65.1	288.4	0	58.1 (11%)	295.3
00-09-28	1	104.5	285.3	1	98 (6%)	291.9
98-09-25	0	4.8	399.3	0	4.8	398.8
01-09-22	0	25.5	192.3	0	25	192.4
02-08-01	0	1.2	285.8	0	1.2	285.8
01-04-20	0	35.4	239.5	0	32.3 (9%)	242.5

for the entire system.

The control design is then proposed and the discrete optimization problem is described and discussed. Both the main improvements of the technique and the possible problems of implementation are pointed out. The modeling methodology as well as the controller design are implemented in simulation over the BTC and the main obtained results are presented and also discussed.

The main issue of this chapter lies on the real implementation of the proposed control design. Some of the simulations presented in Tables 5.4 and 5.5 need high computation times, which implies that the use on-line of the controller could not be possible in some cases. This fact makes that the idea of having a predictive controller when the system model is hybrid and has an important size should be thought only for off-line proposes. In this sense, Chapter 6 deals with the time computation problem, proposing and discussing some ways of solution.

CHAPTER 6

SUBOPTIMAL HYBRID MODEL PREDICTIVE CONTROL

6.1 Motivation

The results obtained in the simulation study of the previous chapter show that important performance improvements can be accomplished when HMPC is applied to sewer networks. Furthermore, the hybrid modeling methodology is very rich and allows the straightforward treatment of hybrid phenomena such as overflow and flooding. HMPC has been applied successfully to a variety of control problems the last years using several approaches, see [BBM98], [Sch99], [TLS00], [LR01], [BBM02], among others.

However, the underlying optimization problem of HMPC is combinatorial and $\mathcal{NP}$-hard [Pap94]. The worst-case computation time is exponential in the sense of the amount of combinatorial variables. In Figure 6.1 it is shown how this problem manifests itself for the application under investigation in this thesis.

In the top graph, rain intensity is shown related to the five rain gauges in the BTC for the critical portion (second rain peak) of the rain episode occurred on October 17, 1999. This episode was relatively intensive with a return rate of 0.7 years within the city of Barcelona.

In the second graph, the computation time to solve the MIP is shown as a function of sample for the same scenario. Recalling that the desired sampling time for this system is 300 seconds, it can be seen that the MIP solver is incapable of finding the optimum within the desired sampling time. Furthermore, it is seen that computation time varies greatly. Before sample 16 the

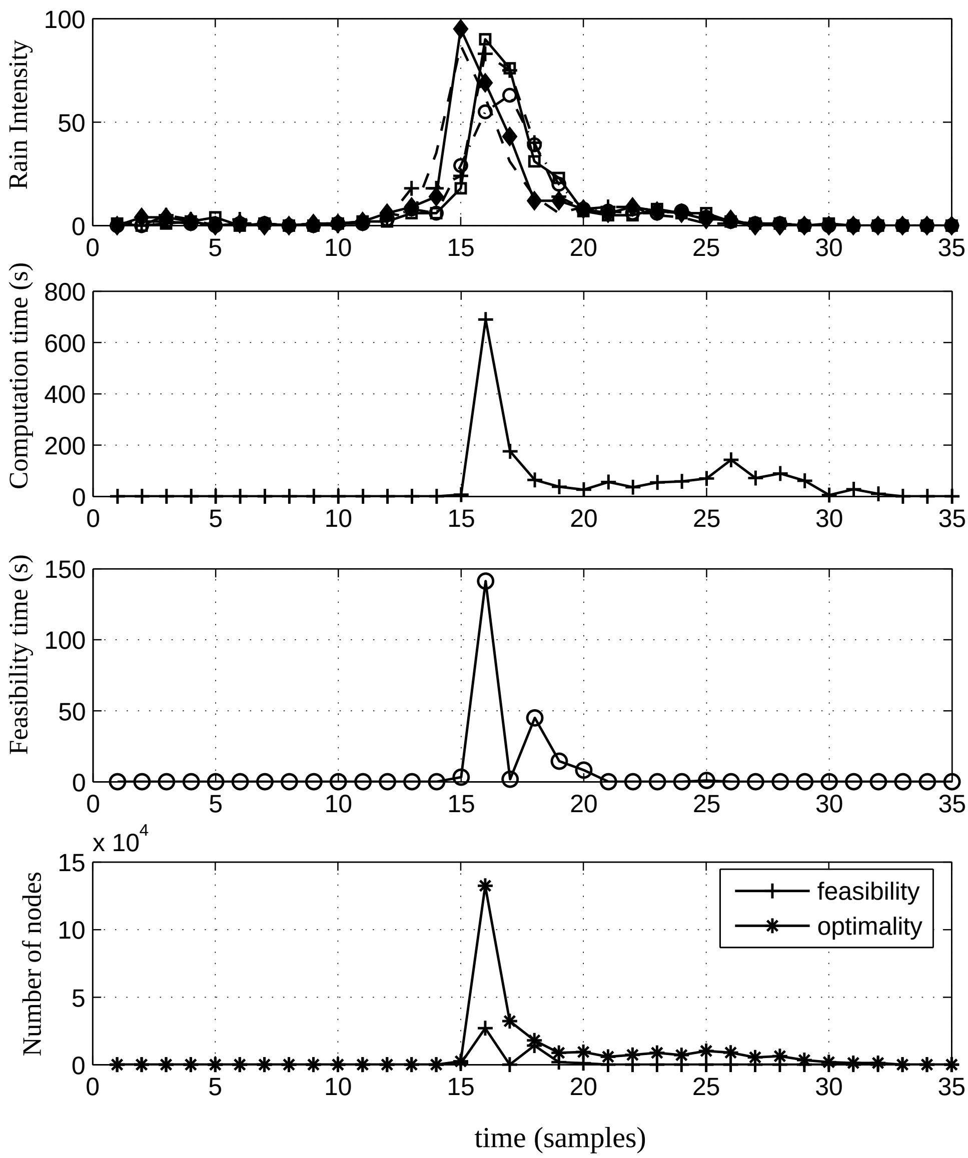


Figure 6.1: MIP problem characteristics for the rain episode occurred on October 17, 1999.

computation time is very small.

If optimality is not achieved within a desired sampling time, feasibility is at least required. Often feasibility is sufficient for proving of stability of MPC schemes, see [SMR99]. The ILOG CPLEX solver used in the current application can be configured to put special emphasis on finding a feasible solution before an optimal one [A.03b]. It is also possible to limit the time the solver has to solve the problem at hand. In the second graph of Figure 6.1, the time required to find a feasible solution is shown. It was found by iteratively increasing the maximum solution time allowed for the solver until a feasible solution was found. The feature of CPLEX to put emphasis on finding feasible solution was activated. Again it can be seen that the time required to find a feasible solution varies considerably. Furthermore, it should be stated that often the feasible solutions found had a poor quality as the solutions found by running the system in open-loop.

In the current application (BTC), the MIQP problem given in (5.56) is solved in each sample having the following generical form:

$$\min_{\rho} \quad \rho^T H \rho + f^T \rho \tag{6.1}$$

$$\text{s.t} \quad A\rho \leq b + Cx_0 \tag{6.2}$$

where vector x_0 collects the system initial conditions and predicted disturbances (rain), which is the only thing that changes from sample to sample.

The ability of the MIP solver to reduce computation time from the worst-case depends on its ability to exclude from consideration as many nodes as possible when branching and bounding. This is done either by proving them to be infeasible or that their solution is suboptimal to other solutions. The increase in computation time is thus linked to an increase in the amount of feasible nodes. In the bottom graph of Figure 6.1, the number of nodes the CPLEX solver explored during branching is shown. It is seen that there was a huge increase in number of explored nodes between samples. There is thus a dramatic change in the complexity of the optimization problem for certain values of x_0.

Physical insight into the process can explain the increase in complexity at time 11. At that time, due to the rain, many of the virtual tanks are very close to their overflow limit. This in turn means that more trajectories for distinct switching sequences Δ_k are feasible. Similar behavior was encountered in other rain episodes and when other cost functions were used.

As was said before, simulations done in Chapter 5 have shown the improvement of the system performance when a HMPC controller is used. However, for some rain episodes the obtained computation times were too high respect to the sampling time of the system. This fact shows the extreme randomness of the computation time and the dependence of the initial conditions of the corresponding MIP for each sample. In Table 6.1, the computation time results are collected. These results correspond to a particular simulation of the closed-loop system for

Table 6.1: Obtained results of computation time using 10 representative rain episodes.

Rain episodes	Total CPU time (s)	Maximum CPU time in a sample (s)
99-09-14	1445.9	1058.3
02-10-09	1328.8	188.4
99-09-03	135.6	60.9
02-07-31	1806.8	391.9
99-10-17	815.7	689.9
00-09-28	254.7	65.6
98-09-25	45.6	40.3
01-09-22	63.3	14.4
02-08-01	47.1	5.6
01-04-20	129.1	18.9

each rain episode but they can vary from one simulation to another.

The same problem regarding computation time occurs in nonlinear MPC, see [Mac02]. When the optimization problem is no longer convex, a fundamental question is how long will the optimization take and will the quality of the solution be sufficient to justify the application of the MPC control approach.

Sewer networks can be considered large sale systems. Research literature have shown that the computation time of such systems is very difficult to predict when the associated MIP problem is solved. As the HMPC is based on solving a MIP problem (MILP or MIQP), it is well known that general MIP problems belong to the class of $\mathcal{NP}$-hard [Pap94] and solution algorithms of polynomial complexity do not exist [TEPS04].

Notice that the whole large scale system is not only the hybrid model of the sewer network but the entire MIP problem associated. Each logic variable induces a particular *mode* in the continuous part [GTM03]. Therefore, the complexity in large scale systems is related to the amount of logical variables related to the system model, what yields a great amount of possible modes. In a MIP problem, the number of *possible modes* Γ is given by

$$\Gamma = 2^{r_\ell H_p}. \tag{6.3}$$

Hence, in the sequel, a system is said to be of large scale in the sense of large amount of logical variables and then a high value of Γ.

MIP solvers such as ILOG CPLEX [A.03b] include modern Branch&Bound search algorithms which construct successively a decision tree. The tree complexity is given by the total number of decision/logical variables of the system r_ℓ and the time horizon H_p associated to the optimization problem. In each node, a feasibility probing is done and the cost function amount is computed and compared against the lowest upper bound found so far. If the obtained value is greater than that upper bound, the corresponding branch is ignored and it is not explored anymore. The upper bound is taken from the best integer solution found prior to the actual node [FP01]. Notice that the total number of tree nodes corresponds to the value of Γ. As greater is Γ, greater is the computation time of the MIP problem solution.

However, in the worst case whether a MIP solver finds the solution of a large scale problem taking into account all its possible modes, the computation time would tend to infinity [A.03b]. Moreover, there exist some modes that could not be reached due to the system constraints and the initial conditions of the states. This fact implies the determination of a subset of Γ which collects the *feasible modes* defined in function of the hybrid model equations (MLD form, PWA form, etc.), the prediction horizon and the initial conditions of the system states. Thus, the problem computation time depends in a straightforward manner on the number of feasible modes (see bottom graph in Figure 6.1).

6.2 General Aspects

6.2.1 Phase Transitions in MIP Problems

Performance of MIP solvers has improved greatly the last years [BFG$^+$00]. The size limit of problems considered to be practically solvable has increased steadily. Part of the reason lies in the many order of magnitudes improvement of desktop computing power over the years. But there has also been tremendous improvement in solution algorithms for LP's and QP's, which are a cornerstone of MIP solvers [Bix02]. Furthermore, modern solvers have incorporated many performance improving features that have existed in the literature such as cutting plane capabilities. Generally the solvers apply a barrage of techniques on each problem. A recent improvement in solving the optimal control problem of HMPC by using symbolic techniques to solve constraint satisfactions problems (CSP), was presented in [BG06].

The MIP problem is equivalent to the archetypal $\mathcal{NP}$-complete K-satisfiability problem

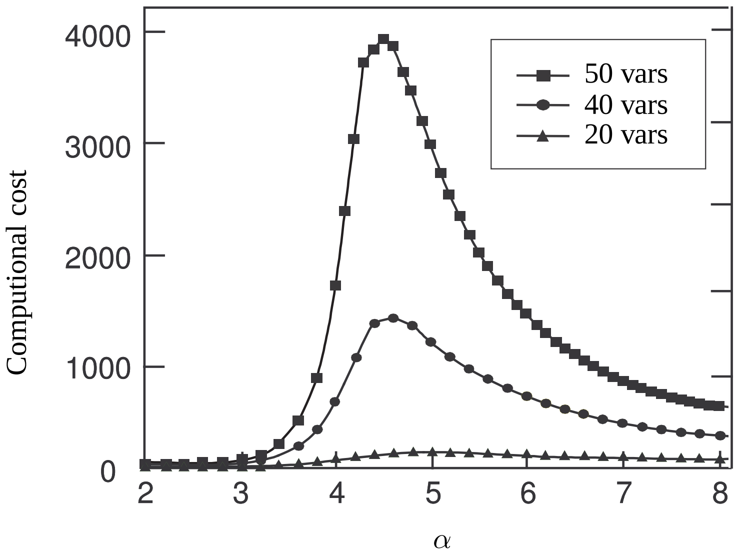


Figure 6.2: Solution complexity pattern for a $\mathcal{NP}$-complete typical problem for a different number of problem variables.

[MZK+99] (or the *Zeroone Integer programming* problem (ZIOP), see [BT00]). It has recently been shown that K-satisfiability problems exhibit phase transitions in terms of computational difficulty and solution character when these aspects are considered as a function of parameters such as the ratio of number of constraints to number of variables. According to [GW94], the phase transition between satisfiability (feasibility) and unsatisfiability (infeasibility) of the discrete (optimization) problem appears as α, defined as the ratio of its constraints to its variables, is varied [1].

Figure 6.2 shows a typical "easy/hard/less-hard" pattern in computation cost (difficulty) for a given MIP problem in function of α. Notice that at low values of α, there are relatively few constraints and many variables, which means that the problem is relatively "easy" to solve due to it is under-constrained. On the other hand, at high values of α, the problem is over-constrained and is almost always unfeasible ("less-hard" region). Besides these two regions, it can be noted a third region corresponding to the edge between the regions aforementioned, where the problems are hardest to solve ("hard" region) [EP04].

Depending on the problem structure, i.e., the number of constraints and variables for a MIP

[1] The experimental work with this $\mathcal{NP}$-complete problems has been done using the random k-SAT model as it has several features which makes it useful for benchmarks. SAT (propositional satisfiability) is the problem of deciding if there is an assignment for a variables in a propositional formula that makes the formula true [GW94].

problem for a given sewer network, the value of α can be located in any place on the x-axis of Figure 6.2. To change the complexity level of the MIP problem might not be enough to suppress a small number of logical variables or, alternatively, to add some additional constraints. Therefore, this approach needs to be applied by considering a strong reduction on hybrid dynamics either by suppressing many logical variables or by adding an important number of constraints in order to change considerably the value of α. This procedure lies on the reduction of computation time due to the straightforward reduction of Γ and therefore the reduction of feasible system modes. Experimental results have shown explicitly the clear relation between $\mathcal{NP}$-complete problems runtime and α [NLBH$^+$04]. Phase transition behavior has been reported for example in multi-vehicle task assignment problems, see [ED05].

6.2.2 Strategies to Deal with the Complexity in HMPC

Given the problem of the computation time, it is needed to explore some ways to relax and/or simplify the discrete optimization problem and find methodologies that make the HMPC methodology practically applicable to large problems such as the MIP one on sewer networks. The majority of the hybrid control approaches presented in the literature have been applied to rather small examples. In the large scale systems framework, there does not exist a standard strategy to relax the problem in order to find a tradeoff between optimality and acceptable amount of computation time.

Control strategies have been proposed where the HMPC problem is relaxed to make it computationally tractable. In [BKB$^+$05] a *decentralized control* approach to HMPC was presented. The class of systems considered were those made up of dynamically uncoupled subsystems but where global control objectives were formulated with a global cost function.

A number of authors have also presented methods where the intent is to reduce complexity off-line. In [BBBM05] an explicit solution to the constrained finite-time optimal control problem was presented for discrete-time linear hybrid systems. *Mode enumeration* (ME) [GTM03] is an off-line technique to compute and enumerate explicitly the feasible modes of piecewise affine PWA models. The technique allows the designer to understand the real complexity of the system and moreover to take advantage of its topology. Thus, once the feasible modes are recognized, the model can be efficiently translated to a specific hybrid system framework such as MLD, MMPS, LC, etc, see [HDB01]. The difference of this technique and the similar problem solved in [Bem04] is the computation of the cells in the hyperplane formed by the input-state space yielding in the PWA model. The approximation in [Bem04] is based on multi-parametric programming and mixed integer programming and deals directly in the MLD model form.

In order to apply a MPC in the closed-loop, ME allows to prune unnecessary modes of the resulting system and to reduce the combinatorial explosion of the algorithms. This allows to introduce cuts on the modes of the complete prediction model over a given horizon. Even though the technique has been reported to be very efficient [Gey05], its application over large scale systems has a huge computational complexity due to the dense space state explicit partition. This off-line procedure can lie in the determination of few regions to prune and the result could remain a hybrid model with many logical variables, which implies even a large scale MIP problem.

In this chapter, a HMPC strategy is proposed which limits on-line the number of feasible nodes in the MIP problem. This is done by adding constraints to the MIP based on insight into the system dynamics. The idea consists in helping the MIP solver by adding cuts in the search space. In this way, the main source of complexity, namely the combinatorial explosion related to the binary search tree, is reduced at the expense of a suboptimal solution. Despite this suboptimal nature of the solution, stability is proved using recent results for HMPC [LHWB06]. It has been recognized in the MPC literature that even though the solution applied is only suboptimal, stability can often be proven [MRRS00]. Infeasibility is avoided by restricting the number of combinations around a nominal feasible trajectory.

6.3 Model Predictive Control for Hybrid Systems including Mode Sequence Constraints

This section explains the details of the proposed suboptimal approach which consists in limiting the system commutation between its dynamical models. This limitation is done considering a given mode sequence reference. The section also presents the conditions for feasibility an closed-loop stability within the framework of the proposed suboptimal approach. Some definitions and results of this section follow closely Sections II. and III. in [LHWB06].

Assume that there are no disturbances and that polyhedra $\mathbb{X}$ and $\mathbb{U}$, containing the origin in their interior, represent state and input constraints respectively. The mapping of state x_k and control signal u_k defined by the MLD (2.8) is denoted as in (2.1):

$$x_{k+1} = g(x_k, u_k) \tag{6.4}$$

where g is a discontinuous function in the case of MLD forms. It is assumed that the origin is an equilibrium state with $u_k = 0$, i.e., $g(0, 0) = 0$.

Consider the sequences (2.2), (2.4), (2.12) and (2.13), which have been presented and explained in Section 2.2.1. In the sequel, sequence $\Delta_k(x_k, \mathbf{u}_k)$ in (2.12) will be called the *mode sequence*. Let $\mathbb{X}_T$ (target state set) contain the origin in its interior. Let

$$\bar{\Delta}_k = \left(\bar{\delta}_{0|k}, \ldots, \bar{\delta}_{H_p-1|k}\right) \in \{0, 1\}^{r_l \times H_p} \tag{6.5}$$

be a *reference sequence* of binary variables $\bar{\delta}_k$ of the same dimension as Δ_k and define the related sets $\mathcal{D}_{M_i}(\bar{\Delta}_k) \subseteq \{0, 1\}^{r_l \times H_p}$ and $\mathcal{D}_M(\bar{\Delta}_k) \subseteq \{0, 1\}^{r_l \times H_p}$ in the following manner:

$$\mathcal{D}_{M_i}(\bar{\Delta}_k) = \left\{\Delta_k \in \{0, 1\}^{r_l \times H_p} | \sum_{k=0}^{H_p-1} |\bar{\delta}_k^i - \delta_k^i| \leq M_i\right\} \tag{6.6a}$$

$$\mathcal{D}_M(\bar{\Delta}_k) = \left\{\Delta_k \in \{0, 1\}^{r_l \times H_p} | \sum_{i=1}^{r_l} \sum_{k=0}^{H_p-1} |\bar{\delta}_k^i - \delta_k^i| \leq M\right\} \tag{6.6b}$$

where $M, M_i \in \mathbb{Z}_+$ and $i = 1 \ldots r_l$. The dependence of Δ_k on x_k and $\mathbf{u}_k$ is omitted for compactness. These sets contain the sequences Δ_k with a limited number of differences from a reference sequence $\bar{\Delta}_k$. Thinking of $\Delta_k, \delta_k, \bar{\Delta}_k$ and $\bar{\delta}_k$ as binary strings, the inequalities that define these sets limit the Hamming distance[2] between the strings involved. In what follows, the discussion will be limited to the set $\mathcal{D}_M(\bar{\Delta}_k)$ for compactness reasons. The proof of stability follows through in the exact way if M is replaces with M_i.

The class of admissible input sequences (2.3) now defined with respect to $\mathbb{X}_T$ and set $\mathcal{D}_M$ is:

$$\mathcal{U}_{H_p}(x_k, \bar{\Delta}_k) \triangleq \left\{\mathbf{u}_k \in \mathbb{U}^{H_p} | \mathbf{x}_k(x_k, \mathbf{u}_k) \in \mathbb{X}^{H_p}, x_{H_p|k} \in \mathbb{X}_T, \Delta_k(x_k, \mathbf{u}_k) \in \mathcal{D}_M(\bar{\Delta}_k)\right\} \tag{6.7}$$

Remark 6.1. Notice that this set can be characterized exactly with a mixed integer linear inequality (2.8c).

The MPC problem described in Section 2.2.1 is now stated in a similar way as in [LHWB06]. Some sequences and definitions given in such section are rewritten here for the completeness of this formulation.

[2]In information theory, the Hamming distance between two strings of equal length is the number of positions for which the corresponding symbols are different. Put another way, it measures the number of substitutions required to change one into the other, or the number of errors that transformed one string into the other.

Problem 6.1 (MPC problem with Mode Sequence Constraints)**.** *Let the target set* $\mathbb{X}_T \subset \mathbb{X}$ *and* $H_p \in \mathbb{Z}_{\geq 1}$ *be given. Minimize the cost function*

$$J(x_k, \mathbf{u}_k) = F(x_{H_p|k}) + \sum_{i=0}^{H_p-1} L(x_{i|k}, u_{i|k}) \tag{6.8}$$

over $\mathbf{u}_k \in \mathcal{U}_{H_p}(x_k, \bar{\Delta}_k)$*, where* $F : \mathbb{R}^n \to \mathbb{R}_+$ *and* $L : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_+$ *are functions that fulfill* $F(0) = 0$ *and* $L(0, 0) = 0$*.*

An initial state $x_0 \in \mathbb{X}$ is feasible if there exist a reference sequence $\bar{\Delta}_k$ such that $\mathcal{U}_{H_p}(x_0, \bar{\Delta}_k) \neq \emptyset$. Hence, Problem 6.1 is feasible if there exist a feasible $x \in \mathbb{X}$ such that $\mathcal{U}_{H_p}(x, \bar{\Delta}_k) \neq \emptyset$. Let the set $\mathbb{X}_f(H_p) \subseteq \mathbb{X}$ denote the *set of feasible states*. The function

$$V_{\text{MPC}}(x_k) = \min_{\mathbf{u}_k \in \mathcal{U}_{H_p}(x_k, \bar{\Delta}_k)} J(x_k, \mathbf{u}_k) \tag{6.9}$$

related to Problem 6.1 is called the *MPC value function*. It is assumed that there exists an optimal control sequence (2.6)

$$\mathbf{u}_k^* = \left(u_{0|k}^*, u_{1|k-1}^*, \ldots, u_{H_p-1|k}^*\right)$$

for the above problem and any state $x_k \in \mathbb{X}_f(H_p)$. Using the receding horizon philosophy, the MPC control law is defined as in (2.7):

$$u_{\text{MPC}}(x_k) \triangleq u_{0|k}^* \tag{6.10}$$

where $u_{0|k}^*$ is the first element of $\mathbf{u}_k^*$.

Remark 6.2. The selection of the reference sequence between samples k and $k+1$ can be done considering the sequence $\Delta_k^* = (\delta_{0|k}^*, \delta_{1|k}^*, \ldots, \delta_{H_p-1|k}^*)$ obtained from the solution of Problem 6.1 at time k. Given a $\vartheta \in \mathbb{U}$ that fulfills $x_{H_p+1|k} = g(x_{H_p|k}^*, \vartheta) \in \mathbb{X}_T$, the reference mode sequence in time $k+1$ is set as:

$$\bar{\Delta}_{k+1} = \left(\delta_{1|k}^*, \ldots, \delta_{H_p-1|k}^*, \delta_+(x_{H_p|k}^*, \vartheta)\right) \tag{6.11}$$

where $\delta_+(x_{H_p|k}^*, \vartheta)$ is found by using the system equations in (6.4).

However, to determine the reference sequence (6.11), input ϑ have to fulfill certain specific conditions [MRRS00]. In this sense and according to [LHWB06], both feasibility and stability can be ensured by using a terminal cost and constraint set method as in [MRRS00] but with the conditions and assumptions adapted to hybrid systems. Therefore, the following assumption is now presented to prove stability of the closed-loop system (6.4) and (6.10). This assumption is taken unchanged from [LHWB06].

Assumption 6.1 (see [LHWB06]). *Assume there exist strictly increasing, continues functions* $\alpha_1, \alpha_2 : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ *that fulfill* $\alpha_1(0) = \alpha_2(0) = 0$*, a neighborhood of the origin* $\mathcal{N} \subset \mathbb{X}_f(H_p)$ *and a nonlinear, possibly discontinues function* $h : \mathbb{R}^n \rightarrow \mathbb{R}^m$*, such that* $\mathbb{X}_T \subset \mathbb{X}_\mathbb{U}$*, with* $0 \in \mathrm{int}(\mathbb{X}_T)$*, is a positively invariant set for system* (6.4) *in closed-loop with* $u_k = h(x_k)$*.* $\mathbb{X}_\mathbb{U}$ *denotes the safe set with respect to state and input constraints for* $h(\cdot)$*. Furthermore,*

$$L(x,u) \leq \alpha_1(\|x\|), \quad \forall x \in \mathbb{X}_f(H_p), \forall u \in \mathbb{U}, \tag{6.12a}$$

$$F(x) \geq \alpha_2(\|x\|), \quad \forall x \in \mathcal{N} \qquad \textit{and} \tag{6.12b}$$

$$F(g(x,h(x))) - F(x) + L(x,h(x)) \leq 0, \quad \forall x \in \mathbb{X}_T. \tag{6.12c}$$

The following theorem is now presented for stability of MPC controllers with mode sequence constraints. Its proof follows closely the proof presented in [LHWB06] but for completeness of this chapter it is repeated here considering the concepts and sequences defined for the proposed approach. The proof rests on Lyapunov stability results for systems with discontinuous system dynamics developed in [Laz06] but it also takes into account the mode sequence constraints.

Theorem 6.1. *For fixed* H_p*, suppose that Assumption 6.1 holds. Then it holds that:*

1. *If Problem 6.1 is feasible at time* k *for state* $x_k \in \mathbb{X}$*, then Problem 6.1 is feasible at time* $k+1$ *for state* $x_{k+1} = g(x_k, u_{\text{MPC}}(x_k))$ *and* $\mathbb{X}_T \subseteq \mathbb{X}_f(H_p)$*.*

2. *It holds that* $\mathbb{X}_T \subset \mathbb{X}_f(H_p)$*;*

3. *The origin of the MPC closed-loop system formed by applying control law* (6.10) *to plant* (6.4) *is asymptotically stable in the Lyapunov sense for initial conditions in* $\mathbb{X}_f(H_p)$*.*

Proof. See Appendix A. □

Therefore, according to the Assumption 6.1 and results given by Theorem 6.1, $\delta_+(x^*_{H_p|k}, \vartheta)$ in Remark 6.2 can be computed using the local control law $\vartheta = h(x_k)$ ensuring feasibility and stability.

Assuming the cost function (6.8) defined by $F(x_k) = \|Px_k\|_p$ and $L(x_k, u_k) = \|Qx_k\|_p + \|Ru_k\|_p$, and the local control law set as $h(x_k) = Kx_k$, the computation of the weigh matrix P and the piecewise linear state-feedback gain K fulfilling Assumption 6.1 and Remark 6.2 are reported in [Laz06] for norms $p = 1, \infty$ and $p = 2$. These computations are done off-line based on different methods and algorithms discussed in the mentioned reference and the references therein.

6.4 Practical Issues

An important practical problem in the proposed method is to find $\bar{\Delta}_k$ so that $\mathcal{U}_{H_p}(x_k, \bar{\Delta}_k)$ is non-empty and the MIP with mode sequence constraints has a solution. When states are measured and disturbances are present, the assumption that $x_{0|k} = x_{1|k-1}$ will not hold and the shifted sequence from the previous sample will not necessary be feasible. Finding $\bar{\Delta}_k$ using (6.11) is then not an option.

The problem of finding $\bar{\Delta}_k$ for distinct cases of state and input constraints will now be analyzed. The main tool to find this sequence is solving a constraints satisfaction problem (CSP)[3]. A natural candidate solution, which might be close to the optimum is the shifted sequence from the last sample given by (see proof of Theorem 6.1)

$$\mathbf{u}_{k+1}^1 \triangleq \left(u_{1|k}^*, \ldots, u_{H_p-1|k}^*, h(x_{H_p-1|k+1}) \right). \qquad (6.13)$$

This control sequence can be used to simulate the system in open-loop. If all constraints are respected, $\mathbf{u}_k^1$ is a feasible solution. If the measured state is close to the predicted state, it is reasonable to believe that this sequence provides at least with a good initial guess, close the the optimum.

6.4.1 No State Constraints

If $\mathbb{X} = \mathbb{R}^n$ (no state constraints) and system (6.4) is stable, then using $\mathbf{u}_k^1$ defined in (6.13) in open-loop simulation from the new initial state $x_{0|k}$ results in a sequence $\bar{\Delta}_k$ that can be used to form $\mathcal{U}_{H_p}(x_k, \bar{\Delta}_k)$. If $\mathbf{u}_k^1$ is not available or the system (6.4) is unstable, the way for finding the

[3]Depending on the case, the CSP is equivalent to a simulation of the open-loop model. However, when input, state and/or output constraint are present, only CSP has sense. CSP concepts will be presented and discussed in Chapter 8.

sequence $\bar{\Delta}_k$ is not clear and it is needed to use the heuristic knowledge of the system.

6.4.2 State Constraints

State constraints are generally related to either physical constraints of the model such as conservation equations and physical limitations of the process, or to control objectives.

If the open-loop simulation (CSP) fails and some constraints are violated, in the worst case, the problem of finding $\bar{\Delta}_k$ is to find a feasible trajectory for the problem without mode sequence constraints from the new initial state. This is in turn a MIP feasibility problem. The reduction in time that can be achieved with the presented methodology then depends on the complexity of feasibility problem compared to the optimization problem, something that is difficult to analyze a priori. This is a restriction to the presented method but if constraints related to safety or high risk are present in $\mathbb{X}$, and feasibility can not be assured within a pre specified time-frame, neither the presented method nor other HMPC strategies that depend on a MIP to find a feasible solution would be applicable in practice.

6.4.3 Constraints Management

Constraints management is an important issue in constrained predictive control, see [Mac02]. A common approach to deal with infeasibilities is to change constraints from "hard" to "soft", that is, add terms containing slack variables of the constraints to the cost function. If the constraints thus changed represent physical characteristics, the resulting control signal might be of little use as the model from which the control signal is obtained might not fulfill basic physical laws. If the constraints are related to safety considerations, the resulting control signal might not be applicable either.

Constraints management is equally important in the presented scheme as a straight forward way to obtain an initial feasible solution is to change any unfulfilled constraints in $\mathbb{X}$, when $\mathbf{u}_k^1$ is used in open-loop simulation, into soft constraints. As mentioned previously, this approach is only appropriate if the relaxed constraints do not represent physical or safety characteristics of the system.

When forming the cost function containing the slack variables relates to the soft constraints, frequently, some constraints have higher priority than others. The common way to deal with distinct priorities is to assign weights to each slack variable that reflects their importance. Finding these weights is generally done with trial and error procedures involving simulations of typical disturbance and reference value scenarios. If the relative importance of the relaxed constraints is known, objective prioritization schemes implemented with propositional logic, see [TM99] represent an interesting option as these schemes are implemented with MIP solvers.

6.4.4 Finding a Feasible Solution with Physical Knowledge and Heuristics

Physics or heuristical knowledge of the system can often be used to find a feasible solution that fulfills the physical constraints of the system. For example, in steady state, all integer variables have fixed values which could be used in the sequence $\bar{\Delta}_k$.

State constraints representing physical limitations can often be incorporated into the hybrid model by using propositional logic. As an example consider a tank with an upper limit on its level and with its inflow controlled with a valve. The upper limit on the tank could be modeled by adding a constraint to the optimization problem so that any controlled signal to the valve causing the level to surpass the physical limit, would be infeasible in the optimization problem. Within the hybrid modeling framework, a logical statement could be incorporated guaranteeing that the inflow to the tank would never cause the level to surpass the physical level, irrespective of the control signal to the valve.

This hybrid modeling approach actually represents the physical behavior better and would enable the removal of a state constraint where infeasibility could occur during the open-loop simulation. On the other hand, it would increase the amount of binary variables in the system.

6.4.5 Suboptimal Approach and Disturbances

Consider now the system (6.4) including disturbances and then being rewritten as follows:

$$x_{k+1} = g(x_k, u_k, d_k) \tag{6.14}$$

where $d_k \in \mathbb{R}_+^{md}$ denotes the vector of bounded disturbances. In the presence of uncertainty and disturbances, a reference sequence $\bar{\Delta}_k$ can not be obtained in the manner proposed in Section 6.3 as the measured state at time $k \in \mathbb{Z}_{\geq 2}$ will not correspond to the predicted from the previous sample ($x_{0|k} \neq x_{1|k-1}$). That means that the sequence $\overline{\delta}_k$ would not be necessarily feasible and at each time instant this problem will appear. In the worst-case, this problem reduces to obtaining a feasible solution to a MIP. This is also the case where measured disturbances or reference signals are taken into account over the prediction horizon. These can be transformed into an equivalent MIP feasibility problem given an extended initial state.

6.5 Suboptimal HMPC Strategy on Sewer Networks

Once the proposed suboptimal strategy has been presented and discussed, it is applied in the HMPC design of sewer networks. In fact, the strategy has been inspired by this type of large scale systems where computation time is an important problem for online implementation proposes. As was discussed before and since the rain is the main influence to be considered in the sewer network control design process, this section will use some of the ideas presented in Section 6.4 in order to explain how the suboptimal strategy was applied. Finally, simulations made in order to obtain the results reported in Chapter 5 were made again using the suboptimal controller. Then, the main obtained results are given and the corresponding conclusions are outlined.

6.5.1 Suboptimal Strategy Setup

First of all, two facts are taken into account regarding the case study:

1. Sewer networks are in general stable systems according to the modeling framework and the hierarchical control philosophy [Pap85]. Computed control signals just modify the value of the performance indexes. For this reason, target state set is not considered.

2. The associated optimization has always a feasible solution since the state constraints are soft. Only states related to real tanks within the virtual tank modeling methodology are hard constrained but these restrictions can be assumed by their related inflows (manipulated link in the control gate upstream).

Let

$$\mathbf{d}_k = \left(d_{0|k}, d_{1|k}, \ldots, d_{H_p-1|k}\right) \in \mathbb{R}_+^{md} \tag{6.15}$$

is the sequence containing the measured disturbances. Generally only $d_{0|k}$ is measured while the other values are predicted over the prediction horizon (see Section 5.1.5).

To obtain a reference mode sequence $\bar{\Delta}_{k+1}$ from the optimal sequence Δ_k^*, the optimal state value $v^*_{H_p|k} \in \mathbf{v}_k$, the value $d_{H_p-1|k} \in \mathbf{d}_k$ and the last value of the sequence $\mathbf{q_u}_k^* \triangleq \left(q_{u}{}^*_{0|k}, q_{u}{}^*_{1|k}, \ldots, q_{u}{}^*_{H_p-1|k}\right)$ are taken for computing the last value of $\bar{\Delta}_{k+1}$, $\delta_+(v^*_{H_p|k}, \vartheta, d_{H_p-1|k})$. However, it could be possible that $\vartheta = q_{u}{}^*_{H_p-1|k}$ makes the optimization problem unfeasible since the value of $q_{u}{}^*_{H_p-1|k}$ can cause that the physical constraints of the system in (5.49c) are not fulfilled. Hence, considering the discussion done in Section 6.4.4 and having a MLD$_P$ explained in Section 5.3.2, $q_{u}{}^*_{H_p-1|k}$ is validated by simulating the system using the mode detailed model MLD$_P$. Then, validated control signal $\tilde{q}_u$ is obtained, which is used to set $\delta_+(v^*_{H_p|k}, \tilde{q}_u, d_{H_p-1|k})$ within the reference sequence for $k+1$.

Remark 6.3. Computation of $\tilde{q}_u$ from $q_{u}{}^*_{H_p-1|k}$ using the system model MLD$_P$ can be seen as a Constraints Satisfaction Problem (CSP) [JKBW01]. This technique is applied to refine a system set according to the problem constraints.

Finally, consider the constraints which define sets $\mathcal{D}_{M_i}(\bar{\Delta}_k)$ and $\mathcal{D}_M(\bar{\Delta}_k)$ in (6.6) given by

$$\sum_{k=0}^{H_p-1} |\bar{\delta}_k^i - \delta_k^i| \leq M_i \tag{6.16a}$$

$$\sum_{i=1}^{r_l} \sum_{k=0}^{H_p-1} |\bar{\delta}_k^i - \delta_k^i| \leq M \tag{6.16b}$$

where the first limits the number of changes of δ_k^i from the sequence $\bar{\delta}^i$ over the control horizon and (6.16b) limits the total number of changes, counting all binary variables δ_k^i and for the whole prediction horizon.

Therefore, the control strategy applied is the following. At time k, do

1. Obtain the reference sequence $\bar{\Delta}_k$ by using Δ^*_{k-1} and the MLD$_P$ model.
2. Add to the MIP problem related to the HMPC problem the corresponding set of constraints in (6.16).
3. Solve the MIP problem and obtain a new sequence $\mathbf{q_u}_k^*$.
4. Apply the control law (5.57) to the process.

Notice that adding restrictions of type (6.16) to the MIP problem will not cause infeasibility as the trivial solution $\delta_k^i = \bar{\delta}_k^i$ always fulfills the problem constraints.

6.5.2 Simulation of Scenarios

The suboptimal strategy were applied by simulating the closed-loop system for rain episodes listed in Table 3.3. The structure of the closed-loop is the same as the one performed for simulations in Chapter 5. The duration of the simulated scenarios was determined by the duration of the rain peak and the system reaction time to that rain since the sample with maximum CPU time was generally after the rain peak. The approach efficiency was measured not only regarding CPU time but also in system suboptimality for different values of M and M_i in each case.

Previous tests were done where CPLEX parameters were modified to convenience [A.03a]. In this case, the default value (10^{75} s) of the parameter that sets the maximum time for a call to an optimizer (`CPX_PARAM_TILIM`) was modified to a number smaller than Δt in order to fulfill time requirements. However, in some scenarios this time was not enough to find at least a feasible solution. Therefore, another parameter which sets the balance between the feasibility and optimality of the solver solutions (`CPX_PARAM_MIPEMPHASIS`) was also modified in order to generate feasible solutions in less time for be used as suboptimal solution (the default value balances the feasibility and optimality). This change reduced the CPU time for each sample but the system performance was reduced as well so the option was ruled out.

6.5.3 Main Obtained Results

Simulations have been done adding both type of constraints (6.16) and results were obtained using the HYBRID TOOLBOX for MATLAB® [Bem06] and ILOG CPLEX 9.1 as MIP solver [A.03b]. CPLEX parameters discussed before were set to their default values.

The suboptimality level of the strategy was measured using the relation between the value of the cost function for the system with the suboptimal controller $J_k^{\mathrm{s}}(\mathbf{q_u}_k, \Delta_k, \bar{\Delta}_k, \mathbf{z}_k, v_k)$, and the value of the cost function for the closed-loop system without suboptimal approaches $J_k^{\mathrm{n}}(\mathbf{q_u}_k, \Delta_k, \mathbf{z}_k, v_k)$. This relation is expressed as:

$$\mathcal{S}_k = \frac{J_k^{\mathrm{s}}(\mathbf{q_u}_k, \Delta_k, \bar{\Delta}_k, \mathbf{z}_k, v_k)}{J_k^{\mathrm{n}}(\mathbf{q_u}_k, \Delta_k, \mathbf{z}_k, v_k)} \tag{6.17}$$

According to Table 6.1, the rain episode occurred on September 14, 1999 had the highest computational load. Using the suboptimal approach proposed, the computation time was reduced for small values of M or M_i without important reduction of the system performance. In

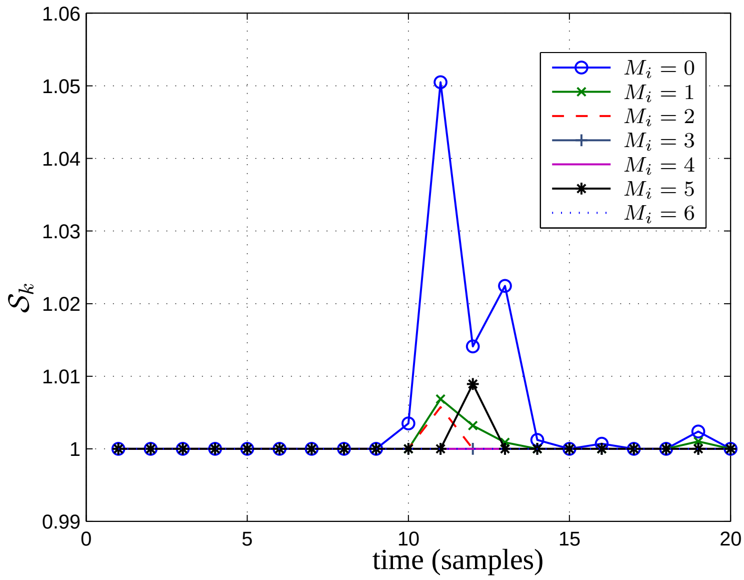
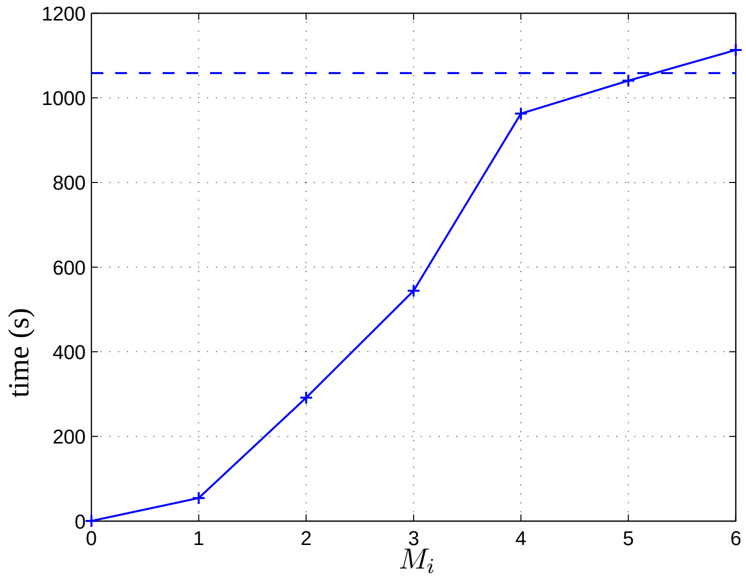


Figure 6.3: Suboptimality level in rain episode 99-09-14 for different values of M_i in (6.16a).

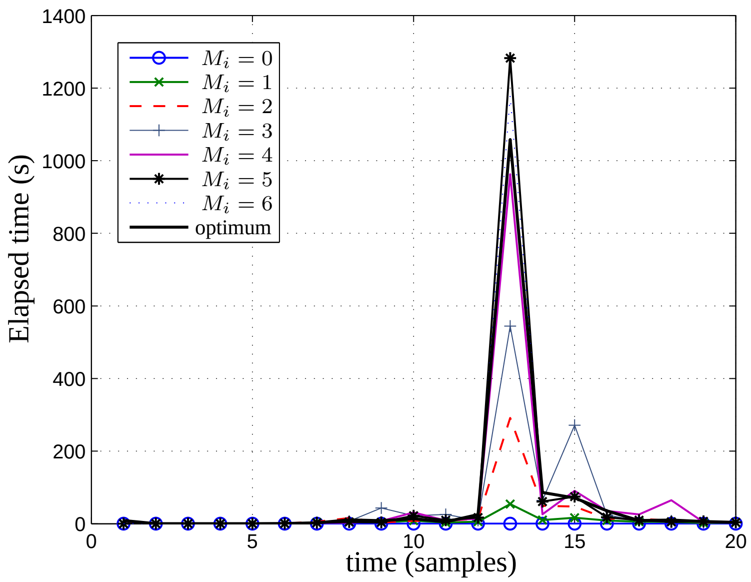
fact, the suboptimality was just around 5% for the critical sample in the considered portion of the scenario. In this way, Figure 6.3 shows the behavior of $\mathcal{S}_k$ for each sample during the heavy part of the rain episode.

About the CPU time reduction for this rain episode, Figure 6.4(a) shows the value of M_i versus the maximum CPU time over the scenario for the system with the mode sequence constraints. Moreover, Figure 6.4(b) shows the evolution of the CPU time for each sample considering some values of M_i. It can be noticed that decreasing M_i makes easier the MIP problem and then the solver takes less time to find the suboptimal solution. When M_i (or M, according to the case) is zero, the MIP problem is just a QP or LP problem. Keep also in mind that when $M \geq H_p$, the mode sequence constraints do not have any influence over the optimality of the compute solution since logical variables sequences can take any value. For this rain episode, in order to fulfill system time requirements, M_i should be strictly less than 2.

Another critical rain episode in the sense of CPU time was the one occurred on October 17, 1999. For this case, constraints in (6.16b) were taken into account in the suboptimal approach. Figure 6.5 presents the maximum CPU time over the scenarios with different values of M. It can be noticed that, once $M \geq H_p$, CPU time varied around the value obtained when the mode



(a) Maximum CPU time over scenario.



(b) Evolution of the CPU time for each sample.

Figure 6.4: CPU time considerations for different values of M_i in (6.16a) in the rain episode 99-09-14. In (a), dashed curve ($--$), optimal simulation time.

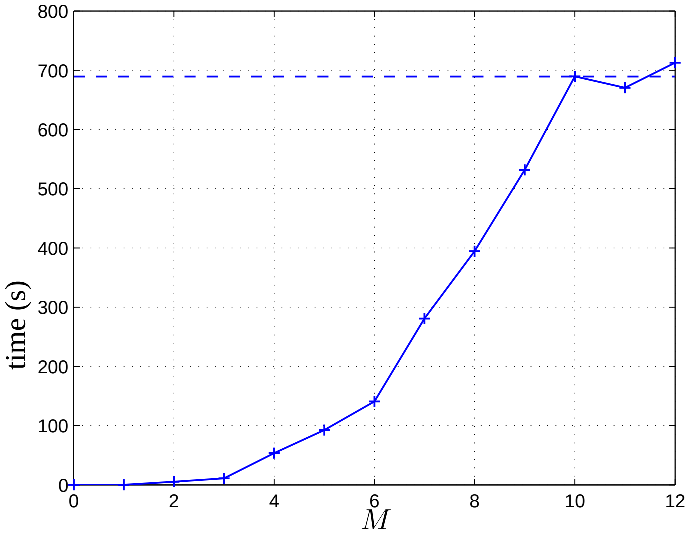


Figure 6.5: Maximum CPU time in rain episode 99-10-17 for different values of M in (6.16b). Dashed curve ($--$), optimal simulation time.

sequence constraints are not considered. For small values of M, the CPU time is reduced and the optimality of the solutions in general was not critically affected. Only when $M = 0$ and $M = 1$, the suboptimality level reached 30% for the critical samples within the scenarios. For this rain episode, in order to fulfill system time requirements, M should be less than 7.

6.6 Summary

Motivated by the high CPU times obtained when the BTC was simulated in closed-loop using an HMPC controller (Chapter 5), it is easy to conclude that the computation time spent by computing the HMPC control law is sometimes very high. The computational cost increases drastically with the value of the initial conditions vector at each time instant $k \in \mathbb{Z}_+$. Furthermore, results obtained have shown that the computation time for obtaining the HMPC law is very difficult to predict when an associated MIP problem has to be solved. As the HMPC is based on solving a MIP problem (MILP or MIQP), it is well known that general MIP problems belong the class $\mathcal{NP}$-complete [Pap94] and solution algorithms of polynomial complexity do not exist [TEPS04].

Given the problem of the computation time, it is needed to explore some ways to relax and/or simplify the discrete optimization problem and find methodologies that make the HMPC methodology practically applicable to large problems such as the MIP one on sewer networks. The majority of the hybrid control approaches presented in the literature have been applied to rather small examples. In the large scale systems framework, there does not exist a standard strategy to relax the problem in order to find a trade-off between optimality and acceptable amount of computation time. Some existing strategies could be considered to deal with the MIP problem before trying to obtain its solution. Such techniques could simplify the initial hybrid model of the system, split the MIP problem in small subproblems, add more constraints to the discrete optimization problem in order to reduce the amount of feasible modes, among other approaches.

This chapter proposes a suboptimal model predictive control scheme for discrete time hybrid system where optimality is sacrificed for a reduction of computation time. The approach is based on limiting the system commutation between its dynamical modes and takes advantage of the optimal solution computed in the previous sample within the receding horizon strategy. Stability of the proposed scheme has been proven when states are measured or accurately estimated and no disturbances are present. The proof is done using results reported in [LHWB06] but adding the limitations of switching between dynamical modes.

Once the proposed approach is explained and discussed under the consideration of no disturbances, it is included within the HPMC on sewer networks. Some important practical issues have been outlined and ways of solution are then discussed. It has been shown that by using the suboptimal scheme, computation time is reduced consistently as a function of the parameter M and, in the case of the BTC, the suboptimality level is not critical.

Another proposed approach in order to relax/symplify the HMPC problem on sewer networks and in general on MIP problems, consists in modeling the hybrid dynamics using piecewise functions such as *max* or *min* (see discussion in Section 3.3.4). This approach avoids the logical variables handling (discrete optimization) but includes nonlinear functions, what yields in non-convex problems. However, the problem of finding a feasible solution is not avoided and the Non-linear Programming Algorithms applied reach only a local optima which implies a suboptimal solution [BSS06]. Generally heuristical methods are used to find an initial feasible solution. It is a common engineering practice to use NLP algorithms in the optimal control of sewer networks [Mar99], [MP05], or water distribution networks [BU94], being accepted the possible suboptimality introduced that is compensated with the capability of dealing with very huge networks, see [BU94], [QGP$^+$05], among others.

Part III

Fault Tolerance Capabilities of Model Predictive Control

CHAPTER 7

MODEL PREDICTIVE CONTROL AND FAULT TOLERANCE

Once the MPC technique has been presented and discussed in the framework of sewer networks, the fact of considering a fault event at some actuator of the sewage system is treated in this chapter. Hence, the tolerance strategies within MPC theory are considered and different aspects of the application of a Fault Tolerant Scheme using the MPC as the control law of the closed-loop are outlined, discussed and some problems are solved using the methodology proposed. Moreover, the hybrid modeling methodology developed before is used to deal with the faults and their modeling in order to have a global solution of the sewer network control problem despite the fault presence. The expression of faults in the hybrid systems framework complements the plant model and allows to take advantage of MPC capabilities within a FTC architecture.

7.1 General Aspects

As discussed in Chapter 2, FTC is concerned with the control of faulty systems. In general, the control algorithms have just been designed to achieve control objectives only in the case of non-faulty situation. Hence, the presence of a fault would imply changing the control law or even the whole control loop configuration. This way to achieve fault tolerance relies on employing a fault diagnosis scheme on-line and on reacting to the results of diagnosis. Another possible way to achieve fault tolerance is to make use of the robustness of feedback control systems that gives rise to an implicit fault tolerance. In this case, the control algorithm has been designed to achieve control objectives either in healthy or in faulty situations.

A fault is a discrete event that acts on the system, changing some of the properties of the system (either the structure or the parameters, or both). In turn, fault-tolerant control responds to the occurrence of the fault by accommodation or by reconfiguration. Due to these discrete event nature of fault occurrence and the reconfiguration/accommodation, a FTC system is hybrid system by nature. Therefore, the analysis and design of FTC systems is not trivial. For design purposes of these systems, traditionally the hybrid nature has been neglected in order to facilitate a simple design, reliable implementation, and systematic testing. In particular, the whole FTC scheme can be expressed using the three-level architecture for FTC systems proposed by Blanke (see Figure 2.9), where [BKLS03]:

- Level 1 (*Control Loop*). This level comprises a traditional control loop with sensor and actuator interfaces, signal conditioning and filtering and the controller.
- Level 2 (*Fault Diagnosis and Accommodation*). The second level comprises a given amount of detectors, usually one per each fault effect which will be detected, and effectors that implement the desired reconfiguration or other remedial actions given by the autonomous supervisor. The functions of this module are: detection based on hardware or analytic redundancy based on fault detection and isolation methods, detection of faults in control algorithms and application software and effector modules to execute fault accommodation.
- Level 3 (*Supervision*). The supervisor is a discrete-event dynamical system (DEDS) comprises state-event logic to describe the logical state of the controlled object. Transition between states is by events. The supervisor functionality includes an interface to detectors for fault detection and demands remedial actions to accommodate a fault.

The reasons for separating a FTC systems in three layers are that it provides a clear development structure, independent specification and development of each layer, and last but not least, testability of detector and supervisor functions. However, there is no guarantee that all the whole FTC system works when all subsystems are integrated.

One of the main objectives of this chapter consists in taking into to account the hybrid nature of the FTC system by using an hybrid systems modeling, analysis and control methodology. This allows to design the three levels of a FTC system in an integrated manner and verify its global behavior.

In fact, comparing the three-level structure with a conceptual scheme of an hybrid system according to Figure 7.1, there is a quite precise correspondence since the Control Loop Level

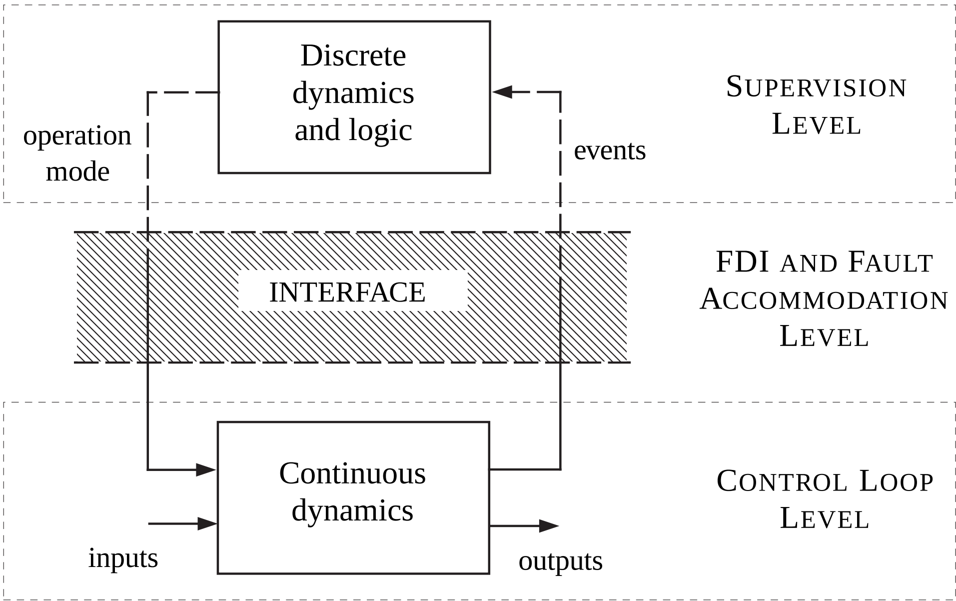


Figure 7.1: Parallel between the basic scheme of an hybrid system and the three levels FTC architecture proposed on [Bla01].

matches the continuous part of the hybrid system, the FDI and Fault Accommodation Level matches the interface between continuous and discrete system dynamics in both ways. Finally, the Supervision Level matches the discrete dynamics part of the hybrid system. Moreover, the events might be associated to faults within the FTC architecture and accommodation actions are related to changes in the operation mode of the continuous part.

7.2 Fault Tolerance Capabilities of MPC

7.2.1 Implicit Fault Tolerance Capabilities

As was said before, the robustness of feedback control systems gives rise to an implicit fault tolerance. Faults that occur under closed-loop control are often compensated by the control action. The same applies when MPC is used at control level. It has been furthermore demonstrated that even when knowledge of the fault is not available, when the estimation of external disturbances affecting the loop is performed in a special way and the input levels have hard constraints, the MPC controller automatically takes advantage of actuator redundancy when available, see

[Mac98]. However, when the states are assumed measurable or estimable, this fault tolerance property does not apply.

On the other hand, it is possible, when using the MPC formalism, to increase fault tolerance, if knowledge of faults is available by modifying parameters of the optimization problem which is solved in each sample. Faults that affect the internal model or system constraints can in this way be incorporated into a MPC controller in a natural way. Furthermore, due to the flexibility that control objectives can be expressed within the MPC formalism, when faults cause control objectives to become unattainable, they can be dropped from the optimization problem or degraded in priority, for example, by changing hard constraints to soft ones. The information of a fault occurrence can be included in a MPC law in the following ways:

- Changing the constraints in order to represent certain kinds of fault, being specially "easy" to adapt the algorithms for faults in actuators.
- Modifying the internal plant model used by the MPC in order to reflect the fault influence over the plant.
- Relaxing the initial control objectives in order to reflect the system limitations under fault conditions.

However, these ways relies on several assumptions [Mac99]:

- The nature of the fault can be located and its effects modeled.
- The internal model of the plant can be updated, essentially in an automatic manner.
- The set of control objectives $\mathbb{O}$ defined in the MPC design process can be left unaltered once the fault has occurred.

These strong assumptions can be treated by using a reliable FDI an taking advantage of the emerging technologies not only for system management but also for the friendly interaction between the designer/user and the complex systems.

The idea of using FTC considering MPC as the control law of the tolerant architecture has been reported in the literature during the last few years. The first steps on this field were discussed in [Mac97] and the theory was implemented over an aircraft system in [MJ03]. The main results reported allow to conclude, among other ideas, that MPC has a good degree of fault tolerance to some faults, especially actuator faults, under a certain conditions, even if the faults are not detected. Subsequently, MPC and its advantages related to fault tolerance have inspired other contributions to the FTC field, see [PMNS06], [AGBG06], [MVSdC06], among others. In [PNP05], the authors propose an scheme where the fault tolerance and MPC work together in chemical applications since MPC controllers are typically used to control key operations in chemical plants, so this can have an impact on safety an productivity of the entire system. Based the model predictive control, [Zho04] proposes a FTC design considering faults on actuator elements. There, the simple fault detection and fault complement approach are presented and discussed.

7.2.2 Explicit Fault Tolerance Capabilities

Linear constrained MPC is based on the solution of an optimization problem using either linear or quadratic programming, which determines the optimal control action. As the coefficients of the linear term in the cost function and the right hand side of the problem constraints depend linearly on the current state, in particular the quadratic programming can be viewed as a multiparametric quadratic programming (mpQP). In [BMDP02], the authors analyze the properties of mpQP, showing that the optimal solution is a piecewise affine function of the vector of parameters. As a consequence, the MPC controller is a piecewise affine control law which not only ensures feasibility and stability, but also is optimal with respect to LQR performance. An algorithm based on a geometric approach for solving mpQP problems in order to obtain explicit receding horizon controllers was proposed in [BMDP02].

The explicit form of the MPC controller provides also additional insight for better understanding the control policy. Moreover, this methodology allows introducing faults as additional parameters into the parametric programming algorithms thanks to the information given by a FDI module. For instance, in the case of faults affecting actuator bounds, since the maximum control input from an actuator is often constrained in the optimization formulation, this constraint can be considered as a parameter. Thus, if an actuator has failed, the situation can be handled by constraining the corresponding control input to be null (reconfiguration strategy) or, using the fault information available, by constraining the control related input to have the new (faulty) operating ranges (accommodation). Example 7.1 allows to understand how a MPC controller handles a fault situation.

Example 7.1. Consider a first-order continuous system described by the transfer function

$$G(s) = \frac{0.8}{2s+1},$$

whose equivalent discrete-time state-space description, using a sampling time $\Delta t = 0.1$ s, is given by

$$\begin{aligned} x_{k+1} &= 0.9512 x_k + 0.0975 u_k \\ y_k &= 0.8 x_k \end{aligned} \tag{7.1}$$

where $|u_k| \leq \mu$ and $\mu = 1$. For notation proposes, $u_k \in [\underline{\mu}, \overline{\mu}] = [-1, 1]$. It is clear that in this case, $-\underline{\mu} = \overline{\mu}$. An MPC controller is used in the closed-loop system satisfying the associated control constraints and considering the cost function

$$J(x_k, u_k) = P x_{H_p}^2 + \sum_{i=0}^{H_p-1} \left(Q x_i^2 + R u_i^2 \right) \tag{7.2}$$

where $H_p = 2$ and the terminal weight matrix P is determined using the Ricatti equation with $Q = 1$ and $R = 0.1$. According to Theorem 6.2.1 in [GSdD05], since in this particular case the prediction horizon is 2, the explicit form of the optimal control law $u_k^* = \mathcal{K}_2(x)$, which depends on the current system state $x_0 = x$, is given by[1]

$$\mathcal{K}_2(x) = \begin{cases} -\mathrm{sat}_\mu(Gx + h) & \text{if} \quad x \in \mathbb{Z}^- \\ -\mathrm{sat}_\mu(Kx) & \text{if} \quad x \in \mathbb{Z} \\ -\mathrm{sat}_\mu(Gx - h) & \text{if} \quad x \in \mathbb{Z}^+ \end{cases} \tag{7.3}$$

where the saturation function $\mathrm{sat}_\mu(\cdot)$ is defined, for the saturation level μ, as:

$$\mathrm{sat}_\mu(u_k) = \begin{cases} \mu & \text{if} \quad u_k > \mu \\ u_k & \text{if} \quad |u_k| \leq \mu \\ -\mu & \text{if} \quad u_k < -\mu \end{cases} \tag{7.4}$$

K and P are obtained by the algebraic Ricatti equation

$$P = A^T P A + Q - K^T (R + B^T P B) K, \qquad K = (R + B^T P B)^{-1} B^T A,$$

[1] Results used to show analytically the MPC fault tolerance capabilities are limited to be applied considering $H_p = 2$. Results related to mp-programming are more general.

which gives $P = 3.2419$ and $K = 2.2989$ for the current example. Also from Theorem 6.2.1 in [GSdD05], the gain $G \in \mathbb{R}^{1\times n}$ and the constant $h \in \mathbb{R}$ are given by

$$G = \frac{K + KBKA}{1 + (KB)^2} \quad \text{and} \quad h = \frac{KB}{1 + (KB)^2}\mu, \tag{7.5}$$

which gives $G = 2.6557$ and $h = 0.2135$. The state space partitions for control law (7.3) are defined by

$$\begin{aligned} \mathbb{Z}^- &= \{x : K(A - BK)x < -\mu\} \\ \mathbb{Z} &= \{x : |K(A - BK)x| \leq \mu\} \\ \mathbb{Z}^+ &= \{x : K(A - BK)x > \mu\} \end{aligned} \tag{7.6}$$

which determines the following sets for the particular case:

$$\begin{aligned} \mathbb{Z}^- &= \{x : 1.6713x < -\mu\} \\ \mathbb{Z} &= \{x : 1.6713x \leq \mu\} \\ \mathbb{Z}^+ &= \{x : 1.6713x > \mu\} \end{aligned}$$

In can be noticed that control law (7.3) depends indirectly of the actuator limits given by μ, through expressions in (7.5) and (7.6). Therefore, it is clear how the effect of a fault over the actuator operating range can modify the expression of control law. This suggests that (7.3) can be parameterized in function of the actuator faults (limits). This parametrization is possible using results given in [BMDP02], where state-feedback explicit control law for the MPC controller, piece-wise affine with respect to the states, can be derived using multiparametric quadratic programming (mpQP). Using this approach in the current example, the expression of $\mathcal{K}_2(\cdot)$ is given in function of the parameters $\theta = [\ x \quad \underline{\mu} \quad \overline{\mu}\ ]^T$, which constitutes an extended system composed by the system state and the control input bounds of its operating range. Thus, expression

$$\mathcal{K}_2(\theta) = \begin{cases} [\,-2.299\ 0\ 0\,]\,\theta & \text{if} \quad \begin{bmatrix} 1.6713 & 1 & 0 \\ -1.6713 & 0 & -1 \end{bmatrix} \theta \leq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ & \quad \text{(Region \#1)} \\ [\,-2.656\ 0\ -0.2135\,]\,\theta & \text{if} \quad \begin{bmatrix} 0 & 1 & -1 \\ 1.272 & 0 & 0.7611 \end{bmatrix} \theta \leq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ & \quad \text{(Region \#2)} \\ [\,-2.656\ -0.2135\ 0\,]\,\theta & \text{if} \quad \begin{bmatrix} 0 & 1 & -1 \\ -1.272 & -0.7611 & 0 \end{bmatrix} \theta \leq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ & \quad \text{(Region \#3)} \end{cases} \tag{7.7}$$

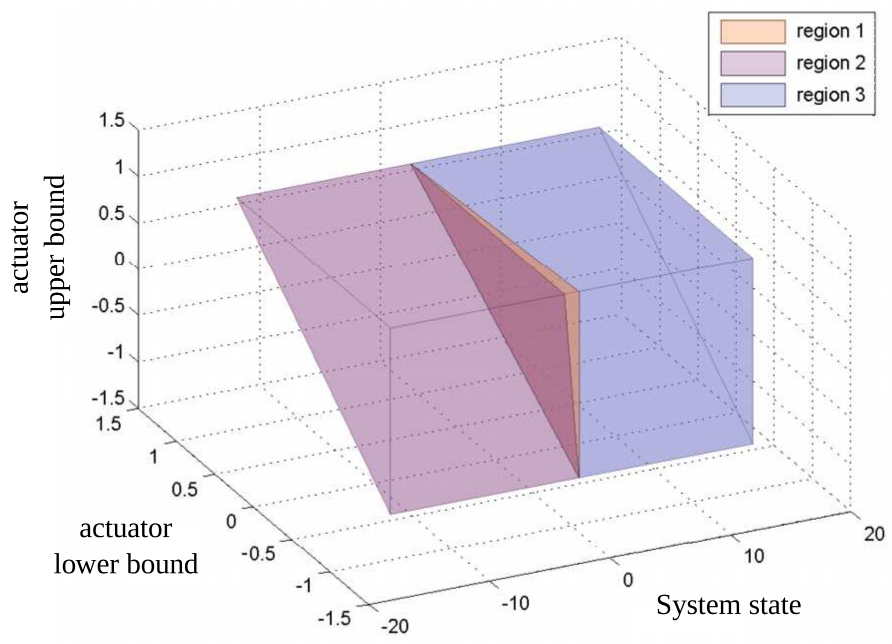


Figure 7.2: Polyhedra partitions related to control law (7.7) for Example 7.1.

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corresponds to the explicit PWA control law, which has been obtained and represented graphically (see Figure 7.2) using the mp-programming tools included in the HYBRID TOOLBOX for MATLAB® [Bem06]. Comparing expressions (7.3) and (7.7), it can be seen that both control laws are equivalent.

7.3 Fault Tolerant Hybrid MPC

As discussed in Section 7.1, a FTC system matches an hybrid system. If an MPC controller is used at Control Loop Level within the three-level architecture discussed before, in order to take advantage of all fault tolerant capabilities described in this chapter, the use of Hybrid MPC methodologies follows naturally in this framework. This section deals with the interaction of blocks within a FTC scheme taking into account the hybrid modeling of the plant for the MPC control proposes.

Looking at the schemes shown in Figures 7.3 and 7.4, which will be used in later sections to explain the fault tolerant hybrid MPC strategies, it can be noticed that the plant is treated

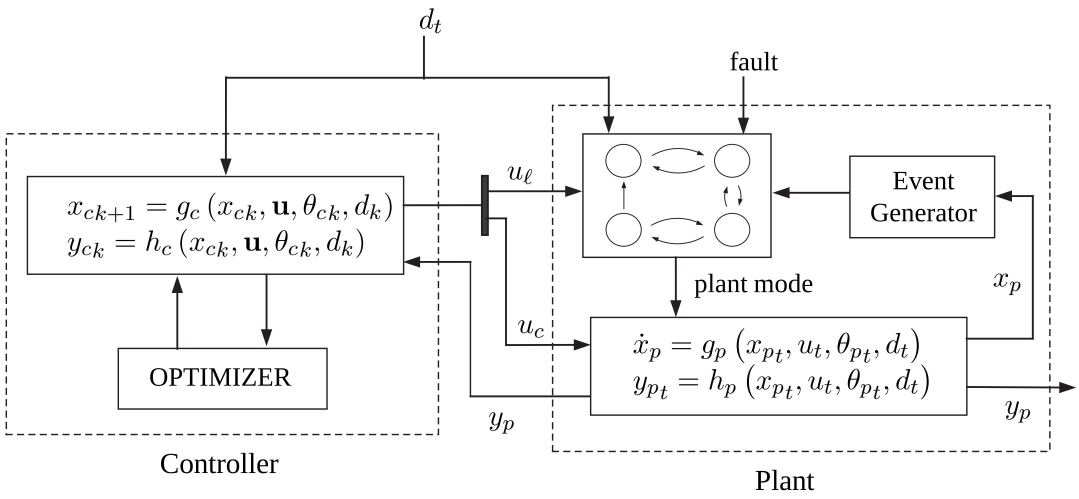

Figure 7.3: Scheme of the PFTHMPC architecture.

as an hybrid system, where the continuous and discrete parts interact using the event generator interface. External effects such as disturbances and/or noise affect selectively parts of the plant. Despite the faults can occur in any constitutive element within the system, they can be seen as external events that affect the nominal behavior altering the system dynamics. Furthermore, the closed-loop controller may apply either continuous or discrete inputs over the plant according to its nature and to the control design. The following sections present the description of the fault tolerant hybrid MPC strategies according to the available information related to the influence of faults in an hybrid system model-based plant.

7.3.1 Fault Tolerant Hybrid MPC Strategies

The are two strategies in order to implement FTC using Hybrid MPC depending on the available information about the existence of faults at the Control Loop Level. The definition of these strategies follows the concepts and features described in Section 2.3.

In case of lacking of knowledge about the presence of the fault, the hybrid controller should deal with a plant that has changed its mode because the fault effect. In this case, fault tolerance relies on the implicit tolerance capabilities of the feedback control loop. This strategy is called in the sequel as Passive FTHMPC (PFTHMPC). Figure 7.3 shows a conceptual scheme of this strategy.

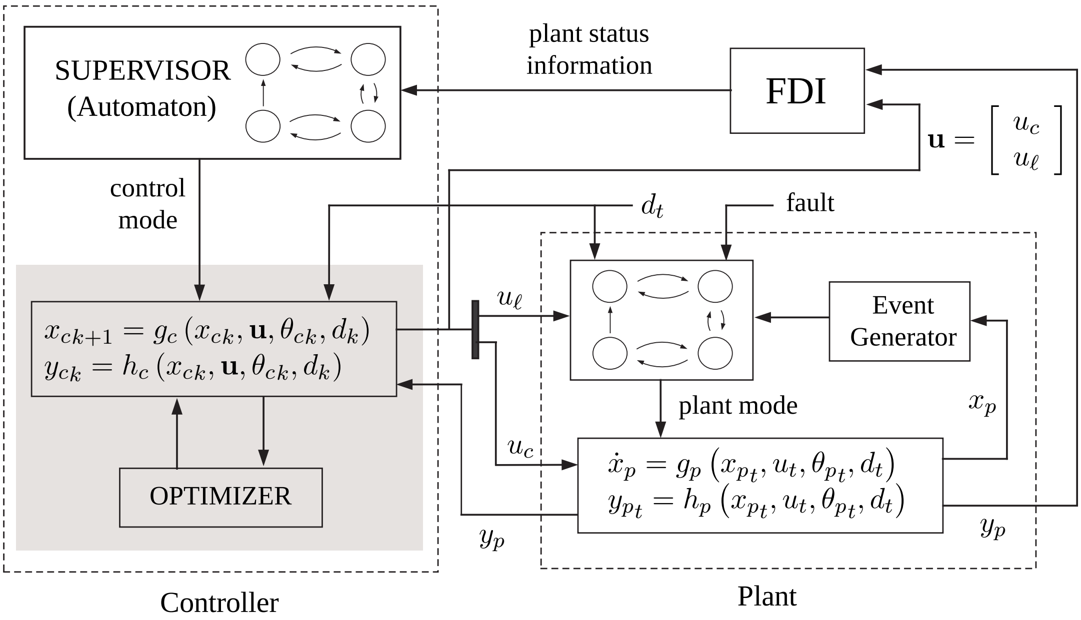

Figure 7.4: Scheme of the AFTHMPC architecture.

On the other hand, in the case of having knowledge about the presence of the fault thanks to the existence of an FDI module, the hybrid controller can adapt its operation mode in order to handle the faulty plant operating mode. In this case fault tolerance take advantage of the the implicit fault tolerant capabilities of MPC as described before. This strategy is called in the sequel as Active Fault Tolerant Hybrid Model Predictive Control (AFTHMPC). Figure 7.4 shows a conceptual scheme of this strategy. Notice that FDI module functionality is assumed to work correctly. Ideally, it detects and isolates the faulty actuator and returns the corresponding information complemented with the fault magnitude. The fault information is assumed readily available and is used to modify the corresponding constraints in the optimization problem.

7.3.2 Active Fault Tolerant Hybrid MPC

Once the “fault tolerant loop” is closed with the FDI module and the Supervisor, the whole FTC system starts an exchange of signals of different nature and objective, following the operating philosophy of a generic hybrid system.

From the plant, continuous and discrete signals are received in the FDI module which are complemented with signals of both natures from the controller. This mixture of information is processed by the algorithms of diagnosis obtaining the plant status information: nominal or faulty; if faulty, where the fault is, its magnitude, etc. The status information is then sent to the Supervisor, which takes the corrective actions related to the fault occurrence. The supervisor block within a FTC architecture is considered in itself a state machine, which implies again an hybrid behavior within the fault tolerance loop. The information given by FDI algorithms is then processed by the supervisor, which determines several aspects of the closed-loop system status and modifies the control law in order to respond facing the fault event. The joint design of the supervisor and control strategy blocks (what is usually done) using hybrid modeling techniques gets high compatibility due to the same nature of their structures.

However, each fault type induces different dynamics in the plant. In the set of the operating plant modes, a new subset of “faulty modes” would be added. Try to define all these faulty dynamics and/or plant operating modes taking into account the fault influence could be a hard work and sometimes an impossible mission. In this sense, an plant hybrid modeling would have to include an complementary model which incorporates the hybrid model representation of the fault effects for a given set of fault scenarios.

In the same way, a compositional hybrid system [Joh00] would have to consider the hybrid model for each constitutive element where the inherent continuous and discrete dynamics and the fault influences are taken into account simultaneously. The main disadvantage of this approach could lie on the high dimension of the logical dynamics, what produces a combinatorial effect in the continuous dynamics. Consider a system represented with a MLD form. The inclusion of the hybrid modeling of the considered faults can increase the dimension of δ variables, what yields a more complex MIP in the design process of the HMPC closed-loop (see Chapter 5). Considering this phenomena, the suboptimal strategies discussed and proposed in Chapter 6 are now an alternative way to deal with these limitations.

7.4 Fault Tolerant Hybrid MPC on Sewer Networks

In this chapter, Fault Tolerant Hybrid MPC strategy (FTHMPC) strategy on sewer networks just considers actuator faults as the change of the bounds of the operating ranges related to input signals, $\underline{q}_{u_i}$ and $\overline{q}_{u_i}$, in the discrete optimization problem solved online. This information would be available once the FDI has detected, isolated and estimated the actuator fault occurred in the sewage system. According to the ideas discussed in previous sections, the FTHMPC is now implemented and tested over the BTC. This section presents the typical fault scenarios occurred on control gates of sewer networks and discusses the modeling of these phenomena in the hybrid systems framework. The use of the hybrid approach is motivated by the results obtained in a previous study where faults were considered within a linear MPC design. To motivate the fault tolerant approach, results are presented making a comparison between the FTHMPC strategies. As was said before, the functionality of the associated FDI module in the AFTHMPC strategy is assumed to work correctly. It detects and isolates the faulty actuator and returns the corresponding information complemented with the fault magnitude. The fault information is assumed readily available and is used to modify the corresponding constraints in the discrete optimization problem.

7.4.1 Considered Fault Scenarios

There may exists many types of fault scenarios related to the control gates within a sewer network. During this chapter, three fault scenarios are considered since they might represent the typical phenomena occurred with these elements under common faulty conditions for these systems and taking into account that the control signal is the outflow rate from the control gates. The flow range can be limited from below due to the inability to close a gate and it can be limited from above due to the inability to open a gate sufficiently (or reduction in pump capacity if pumping elements were considered). A stuck gate means the range is limited to a point or very narrow interval. Hence, the fault scenarios consist in limiting the range of the gates in three ways:

1. Limit range from below (range is 50-100%), denoted as $f\underline{q}_{u_i}$.

2. Limit range from above (range is 0-50%), denoted as $f\overline{q}_{u_i}$.

3. Limit from below and above, simulating stuck gate (50-51%) and denoted as $f\underline{\overline{q}}_{u_i}$.

In scenarios 1 and 3, $\underline{q}_{u_i} = 0.5\,\overline{q}_{u_i}$. For the BTC in Figure 5.6 and particularly in scenario $f\underline{q}_{u_2}$, the lower limit of gate C_2, $\underline{q}_{u_2}$, was set equal to the upper limit of actuator C_3, $\overline{q}_{u_3}$. The reason for this arrangement is that the optimization problem is infeasible if tank T_2 is full and $\underline{q}_{u_2} > \overline{q}_{u_3}$.

7.4.2 Linear Plant Models and Actuator Faults

When the internal model of the predictive controller is considered as in Chapter 4, i.e., a linear model, the AFTMPC strategy deals with a optimization problem whose convexity is given by the type of fault considered. Assuming that any fault model modifies the upper limit for any actuator range, the optimization problem can be solved using fast algorithms of linear programming (LP) in the case of cost function with linear norms or quadratic programming (QP) in the case of quadratic norms. This issue makes that the approach can deal with big systems having many state variables, what yields relative big optimization problems that can be solve very fast obtaining the global optimal solution. Notice that under these conditions, the problem can be easily scalable in the sense of the sewer network size.

However, when the fault effect makes that at least one of the ranges for any system actuator has a nonzero lower bound, the constraint related to this actuator is now non-convex and therefore the optimization problem is non-convex as well. In this case, it is not possible to take advantage of the LP or QP algorithms to solve the optimization problem and its solution is not global (suboptimal). This problem is reported in [OMPQI05], where a small system inspired on the BTC was used. The proposed system contains representative elements of the whole sewer network of Barcelona and considers components enough for testing the FTC strategies based on MPC. The results obtained shown the usefulness of the fault tolerant approach when certain models of faults are considered, including models where $\underline{q}_{u_i}$ is nonzero. Nevertheless, with fault models modifying $\underline{q}_{u_i}$, the obtained solution corresponded to a local minimum of the cost function, which implies that the approach performance is just suboptimal.

7.4.3 Hybrid Modeling and Actuator Faults

The ideas proposed in [OMPQI05] and discussion in Section 7.3 motivates to explore other alternatives of modeling for the considered fault scenarios taking advantage of techniques having a desirable level of accuracy in the expression of the complex dynamics of the system. Hence, this chapter uses the hybrid systems modeling presented in Chapter 5 to take into account actuator faults in sewer networks considering the three fault scenarios as modes related to the behavior of the element. Figure 7.5 shows a conceptual behavior of the system once the fault has occurred. Notice that the system changes between different modes depending on the fault scenario. In this case, it is considered that single fault affects the system. Two or more faults at same time causes here a explosion in the amount of system modes.

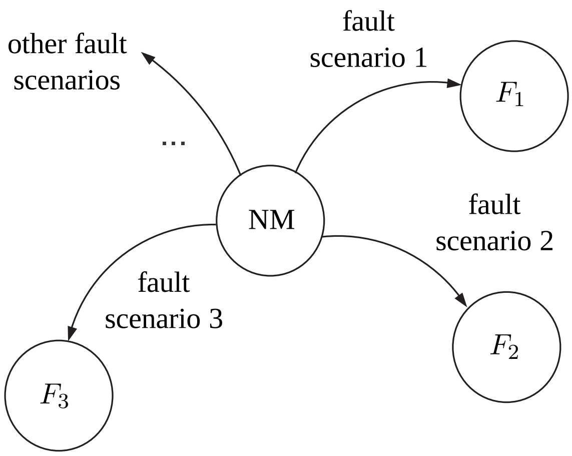

Figure 7.5: Conceptual scheme of actuator mode changing considering fault scenarios. NM denotes the nominal (non-faulty) actuator mode.

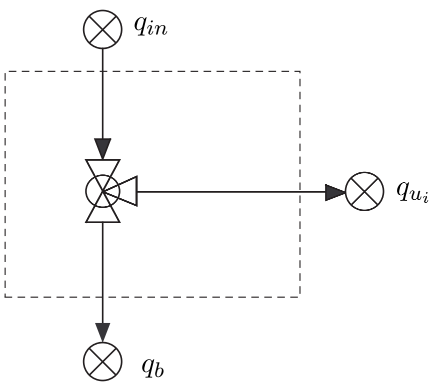

Figure 7.6: Control gate scheme used to explain the fault hybrid modeling.

For instance, consider the fault actuator mode $f\underline{q}_{u_i}$ for the redirection gate presented in Figure 5.3, which is repeated here for simplicity. In this case, it is shown how such mode can be expressed using the proposed hybrid modeling approach[2].

This fault limits explicitly the range of the manipulated outflow q_{u_i}. Then, the following

[2]Similarly, the rest of fault modes described in Section 7.4.1 can be modeled using this approach.

expressions are obtained using the principle of mass conservation:

$$q_{u_i} \leq \min(\overline{q}_{u_i}, q_{in}) \tag{7.8a}$$
$$q_{u_i} \geq \min(\underline{q}_{u_i}, q_{in}). \tag{7.8b}$$

which derives in non-convex constraints on the optimization problem. It can be noticed that this behavior is similar to the switching behavior associated to weirs and suggests the appearance of new system modes. This fact further motivates the use of the proposed hybrid modeling methodology.

Inequality (7.8a) related to the upper bound can be expressed with two linear inequalities as:

$$q_{u_i} \leq \overline{q}_{u_i}, \qquad q_{u_i} \leq q_{in}.$$

Notice that in the case of fault scenarios $f\overline{q}_{u_i}$ and $f\underline{q}_{u_i}$, the fault affects the system when $q_{in} > f\overline{q}_{u_i}$. Otherwise, the fault does not have any influence over the behavior of the network.

The non-convex constraint (7.8b) can be easily treated in the hybrid framework by introducing auxiliary variables

$$[\delta_i = 1] \rightarrow [q_{u_i} \leq q_{in}] \tag{7.9a}$$
$$z_{q_{u_i}} = \begin{cases} \underline{q}_{u_i} & \text{if} \quad \delta_i = 1 \\ q_{in} & \text{otherwise} \end{cases} \tag{7.9b}$$

and replacing (7.8b) by $q_{u_i} \geq z_{q_{u_i}}$. In this way, a non-convex constraint can be expressed with a finite number of linear constraints in the optimization problem (using the equivalences (5.1) and (5.2)), avoiding possible problems due to convergence of optimization routines to local minima. Figure 7.7 shows the set of valid values for q_{u_i} where the actuator constraints are fulfilled. Notice the change of the area when the bound $\underline{q}_{u_i}$ is modified due to the fault effect. Also notice that when $\underline{q}_{u_i}$ is nonzero, points over the line defined by $q_{u_i} = q_{in}$ and (7.8a) belong to that set of valid values.

This fault modeling associated with the hybrid approach for sewer network elements suggest that each element can include typical fault models according to its nature within the network. Hence, the fault tolerance is included in the modeling process when the MPC plant model is

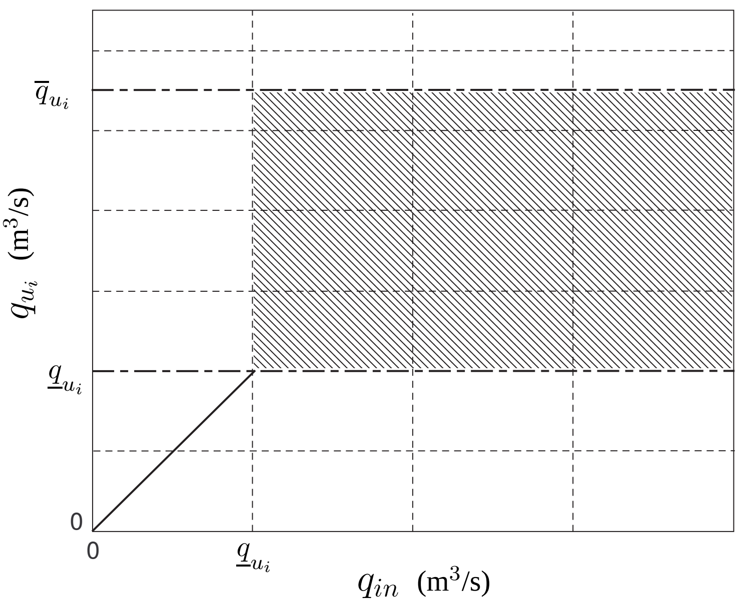

Figure 7.7: Values for q_{u_i} where actuator constraints are fulfilled.

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obtained, assuming that the fault information is provide by a FDI module. This feature allows to have a more accurate representation of the real plant when a given fault effect is considered and makes easier the reconfiguration/accommodation of the control law within the FTC architecture. Another advantage of the fault hybrid modeling lies on the possibility of having continuity in the fault magnitude for a given particular fault model. This fact also increases the accuracy of the element model facing fault occurrence.

7.4.4 Implementation and Results

The purpose of this section lies on comparing AFTHMPC and PFTHMPC for realistic episodes of rain storms (Section 3.3.3) and actuator faults (Sections 7.4.1) for the BTC in Figure 5.6. The assumptions made for the comparison will be presented and their validity discussed before the results are given. In all cases, fault accommodation has been done. For this particular case study, the reconfiguration strategy would consist in considering the control gate totally open when a actuator fault occurs. Thus, the sewage flows downstream by the gravity action fulfilling the mass conservation principle and respecting the main paths. However, notice that this behavior occurs when fault scenarios with nonzero lower bound on the operational ranges are considered.

This observation is valid while the gate inflow is lower than the faulty lower bound.

Simulation of Scenarios

The control strategies were compared by simulating the closed-loop system for all fault scenarios presented in Section 7.4.1 and for the rain episodes listed in Table 3.3. The structure of the closed-loop is the same as the one performed for simulations in Chapter 5. In the PFTHMPC case, the actuator ranges were limited only in the plant model.

The duration of the simulations scenarios was selected to have 8 hours approximately ($k \in [0, 100]$) as the rain storm generally had peaks of duration around 10 samples or 50 minutes. The tanks were empty in the beginning of the scenarios. To compare strategies, total flooding, pollution and treated water released was added over the whole scenario.

The prediction horizon H_p and control horizon H_u were selected as 6 samples or 30 minutes for reasons given in previous chapters. The cost function structure, norm and control tuning used were the same as in the simulations done for nominal HMPC shown in Chapter 5 and the software tools for simulation as well as the solver package were the same as well.

Main Results

Generally, CSO flooding in streets was reduced when AFTMPC was used compared to PFTMPC. The biggest improvements were obtained when precipitation was large enough so that actuators needed to operate close to the upper limit of their range, that is when the precipitation brought the sewer network close to its capacity. Even though results are shown for specific rain episodes, the conclusions presented were based on simulation of various scenarios.

AFTMPC did not yield great improvements when heavy rain episodes as the one occurred on September 14, 1999 (see Figure 3.8(a)) was considered in the simulation. The reduction in CSO that could be achieved was about 0-5%. The reason for this is that the BTC does not have the capacity to handle rain storms with that intensity even in the fault free case. Therefore, it did not matter if actuation limits were known to the HMPC controller or not. This behavior can be seen in Table 7.1 where the main performance indices for the BTC were compared for the considered fault scenarios using AFTHMPC and PFTHMPC. Notice that the largest flooding reduction was obtained for scenarios $f\overline{q}_{u_2}$ and $f\underline{\overline{q}}_{u_2}$. There the flooding was reduced from roughly 135700 m^3 to around 121000 m^3, which corresponds to an improvement about 11%. The other performance

Table 7.1: Results for FTHMPC with rain episode occurred on September 14, 1999.

Fault Scenario		PFTHMPC			AFTHMPC		
Actuator	Type	Flooding $\times 10^3$ (m^3)	Pollution $\times 10^3$ (m^3)	Treated W. $\times 10^3$ (m^3)	Flooding $\times 10^3$ (m^3)	Pollution $\times 10^3$ (m^3)	Treated W. $\times 10^3$ (m^3)
q_{u_1}	$f\underline{q}_{u_1}$	99.5	223.5	280.6	99.5	223.5	280.6
	$f\overline{q}_{u_1}$	93.6	223.9	280.3	93.4	223.8	280.3
	$f\underline{\overline{q}}_{u_1}$	99.9	223.9	280.3	99.9	223.9	281.6
q_{u_2}	$f\underline{q}_{u_2}$	92.9	222.7	281.5	92.9	222.7	281.5
	$f\overline{q}_{u_2}$	135.7	230.2	274.1	121.0	228.8	275.5
	$f\underline{\overline{q}}_{u_2}$	125.5	226.6	277.6	118.3	227.5	276.7
q_{u_3}	$f\underline{q}_{u_3}$	94.3	226.0	278.2	94.2	225.3	278.9
	$f\overline{q}_{u_3}$	97.7	221.1	283.2	95.1	223.1	281.0
	$f\underline{\overline{q}}_{u_3}$	97.6	223.1	281.2	96.0	224.7	279.5
q_{u_4}	$f\underline{q}_{u_4}$	102.1	222.4	281.8	102.1	222.3	281.9
	$f\overline{q}_{u_4}$	92.8	223.5	280.7	92.8	223.5	280.7
	$f\underline{\overline{q}}_{u_4}$	102.1	222.4	281.8	102.1	222.3	281.9

indices were also improved simultaneously. It can occur that flooding was reduced but pollution and/or treated water were not. This fact is caused by the objective prioritization reflected in the tuning of the cost function. However, pollution and flooding indices were improved as well as the flooding was also improved.

When very common rain episodes with little precipitation were studied the same thing occurred, that is, AFTMPC did not give a great improvement. The reason for this is that in those scenarios the constraints are usually not reached and thus faults in actuators rarely affect performance.

Results are shown in Table 7.2 for a rain storm occurred on October 17, 1999. This rain episode has a 0.7-year return period with regard to total amount and 10-year return period with regard to maximum intensity. The particular feature of this episode lies on its behavior during the time window considered. As was seen in Figure 3.8(b), this rain presents a double peak of intensity, what yields that the sewer network behavior is more complex and the nominal HMPC and the FTHMPC designs have a lot of work trying to control the system and avoiding the fault influence. The network is almost not sensitive to faults in gates C_1 and C_4. The modified upper bounds for q_{u_1} and q_{u_4} were always greater than the inflow of the corresponding control

Table 7.2: Results for FTHMPC with rain episode occurred on October 17, 1999.

Fault Scenario		PFTHMPC			AFTHMPC		
Actuator	Type	Flooding $\times 10^3$ (m^3)	Pollution $\times 10^3$ (m^3)	Treated W. $\times 10^3$ (m^3)	Flooding $\times 10^3$ (m^3)	Pollution $\times 10^3$ (m^3)	Treated W. $\times 10^3$ (m^3)
q_{u_2}	$f\underline{q}_{u_2}$	0.0	61.9	291.4	0.0	61.9	291.4
	$f\overline{q}_{u_2}$	15.9	62.8	290.5	10.4	63.3	290.0
	$f\underline{\overline{q}}_{u_2}$	14.8	63.3	290.0	9.6	63.8	289.5
q_{u_3}	$f\underline{q}_{u_3}$	0.0	59.1	294.2	0.0	58.8	294.5
	$f\overline{q}_{u_3}$	3.5	57.7	295.6	0.2	58.7	294.7
	$f\underline{\overline{q}}_{u_3}$	0.8	58.8	294.6	0.0	58.9	294.4

gates. Due to restrictions in lower bounds caused by the faults on these gates, the values for manipulated flows q_{u_1} and q_{u_4} took the same values of the respective inflows in most of the cases. Those values were not big enough to cause overflows downstream.

In this case, the most representative flooding reduction occurred in the fault scenario $f\overline{q}_{u_2}$, with about 35% of improvement caused by the use of the AFTHMPC strategy respect to the PFTHMPC. This improvement is reached by means of a set of procedures caused by the computed control signals. Figure 7.8 shows this set of actions after the second rain peak for different parts of the BTC for both active and passive approaches. In the active case and due to the manipulated flow q_{u_2} has lost capacity, the controller can not take advantage of the the real tank T_3 in a short/medium term[3]. This fact induces that sewage coming from T_1 is conveniently derived through sewer q_{14} (see Figure 7.8, top graph), what produces that sewers close to T_3 do not have as much overflow as in the case of applying the passive strategy (see Figure 7.8, medium graph). The slow filling of T_3 plus its convenient outflow manipulation (control signal related to q_{u_3}, see bottom graph in Figure 7.8) make that a bit of buffer capability benefits the overflow avoidance in T_5. All these actions produce the mentioned improvement of flooding reduction for this fault scenario in this rain episode.

Finally, an intermediate type of rain episode in the sense of rain intensity is for instance the one occurred on September 3, 1999. This episode is well supported by the network topology design, i.e., implementing an adequate control law, the sewer network would not have CSO. Results obtained for this rain episode have shown that the network is almost not sensitive to

[3]Keep in mind that real tanks (reservoirs) are generally used as a buffer within the network. Using this capability, they can store enough water to avoid flooding and/or CSO downstream.

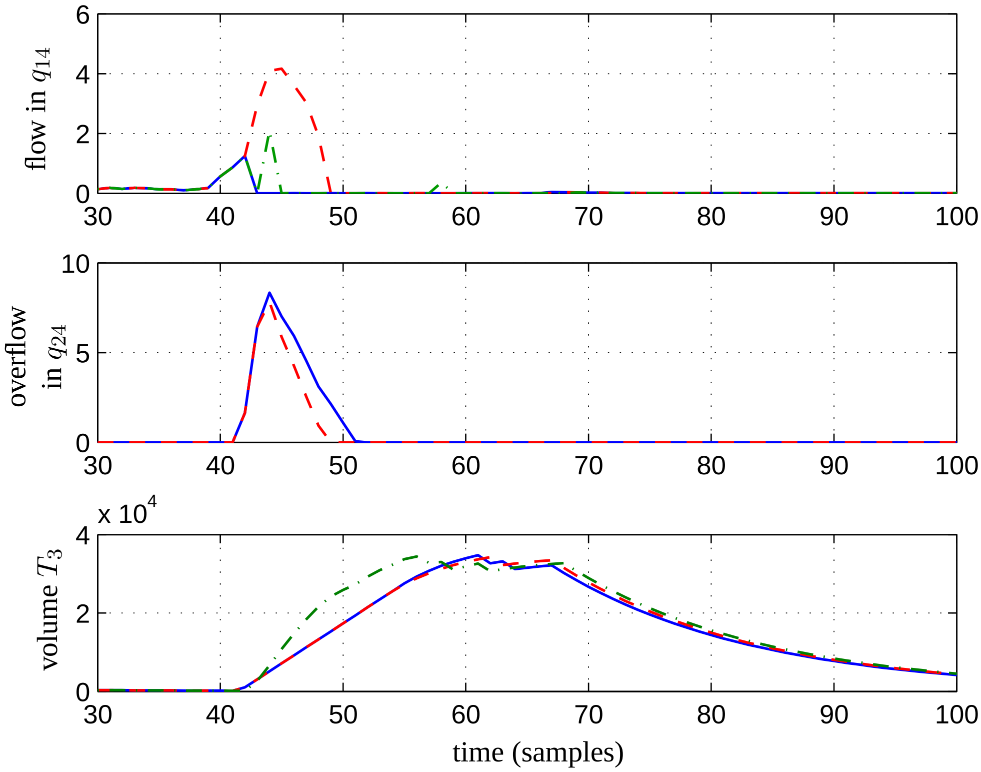

Figure 7.8: Set of actions in different parts of the BTC for both active and passive FTHMPC approaches for fault scenario $f\overline{q}_{u_2}$. Solid curve (—): PFTHMPC; dashed curve (− −): AFTHMPC; dotted-dashed curve (− · −): no-fault HMPC (rain episode: 99-10-17).

all the considered fault scenarios in control gates C_1 and C_4 due to the same reasons discussed for the previous rain episode. Table 7.3 collects the results obtained for the other two actuators where the AFTHMPC yielded improvements respect to the PFTHMPC strategy. In the scenarios $f\overline{q}_{u_3}$ and $f\underline{q}_{u_3}$, the FTC strategy achieved around 100% of flooding reduction. The reason of this improvement is because the AFTHMPC takes advantage of the sewage accumulation in the real tank imposed by the emptying restriction due to the fault (see medium graph in Figure 7.9) and computes adequately the set of control signals in order to redirect the sewage avoiding big quantities around the faulty elements within the sewer network.

When the PFTHMPC strategy is used, the controller computes a control signal $q_{u_3}(k)$ without knowing the fault in the actuator, which implies that the computed control signal and the

Table 7.3: Results for FTHMPC with rain episode occurred on September 3, 1999.

Fault Scenario		PFTHMPC			AFTHMPC		
Actuator	Type	Flooding $\times 10^3$ (m^3)	Pollution $\times 10^3$ (m^3)	Treated W. $\times 10^3$ (m^3)	Flooding $\times 10^3$ (m^3)	Pollution $\times 10^3$ (m^3)	Treated W. $\times 10^3$ (m^3)
q_{u_2}	$f\underline{q}_{u_2}$	0.0	44.3	232.3	0.0	44.3	232.3
	$f\overline{q}_{u_2}$	15.2	44.5	232.1	12.2	44.7	231.9
	$f\overline{\underline{q}}_{u_2}$	14.7	44.3	232.3	11.8	44.4	232.2
q_{u_3}	$f\underline{q}_{u_3}$	0.0	45.2	231.4	0.0	45.2	231.4
	$f\overline{q}_{u_3}$	4.1	44.1	232.5	0.2	44.3	232.3
	$f\overline{\underline{q}}_{u_3}$	1.5	44.3	232.3	0.0	44.3	232.3

applied signal related to the control action will be different (notice that the applied signal in this case corresponds to the computed signal but saturated according to the faulty upper limit). Hence, due to the physical constraints impose a limit on the real tank inflow (manipulated link q_{u_2}) in function of its actual volume, sewage that enters in C_2 is derived through sewer q_{24} causing overflow in this element and then flooding increase.

On the other hand, the AFTHMPC strategy computes the control signal $q_{u_1}(k)$ in such a way that the sewage from T_1 is redirected through q_{14} and then less water goes towards the real tank and its faulty output actuator. Thus, despite the slow emptying of T_3, the C_2 water inflow is conveniently distributed between sewers q_{u_2} and q_{24}, avoiding the overflow in this latter sewer and therefore preventing the flooding increase. Figure 7.9 shows the obtained signals related to the computed control signal $q_{u_3}(k)$ (top graph), the volume in T_3 (medium graph) and overflow in q_{24} (bottom graph) using both active and passive FTC strategies.

7.5 Summary

This chapter introduces concepts and methods to incorporate fault tolerance in a closed-loop governed by an MPC control law. The both implicit and explicit fault tolerance capabilities of this control technique has been outlined and particular features in this sense have been discussed. Moreover, MPC designs considering hybrid system models are included in the framework of the FTC. In fact, it has been proposed a parallelism between the conceptual structure of an hybrid system and the three-levels FTC architecture. This proposal states that both conceptual schemes

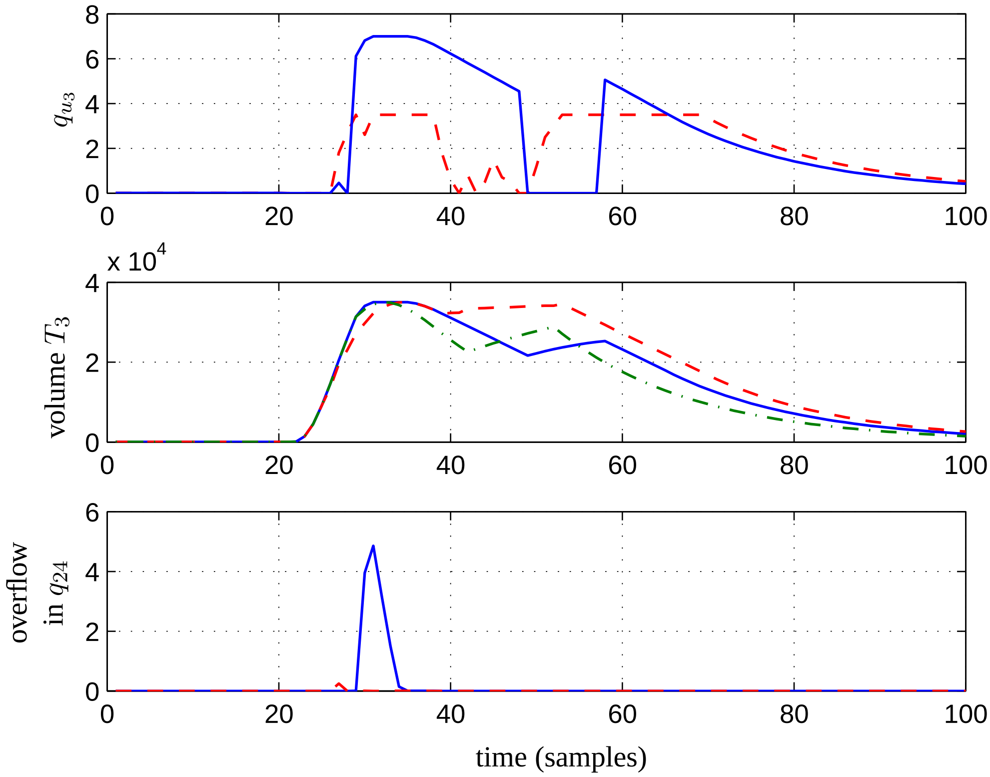

Figure 7.9: Stored volumes in real tank T_3 for fault scenario $f\overline{q}_{u_3}$. Solid curve (–), PFTHMPC, dashed curve (– –), AFTHMPC, dotted-dashed curve (– · –), no-fault HMPC (rain episode: 03-09-1999).

match in the sense of signal natures and exchange between modules status information.

Within the hybrid MPC theory, two FTC strategies have been proposed to deal with faults. The difference between them lies on the available information in the controller related to the fault effects over the plant. The strategies proposed follow the philosophy discussed in Chapter 2 for control loop strategies in a FTC architecture.

Moreover, this chapter has presented a comparison between AFTHMPC and PFTHMPC applied to sewer networks under realistic rain and fault scenarios. The result showed that AFTHMPC reduces CSO flooding in almost all cases. Furthermore, using AFTHMPC could prevent flooding or reduce it considerably when rain episodes considered are supported by the sewer network maintaining inside of its design limits. However, the performance is poorly improved for heavy rains due to sewer network topological limitations. In the other extreme cases, i.e., having light rains, the considered fault scenarios do not have an important influence on the sewer network behavior due to the small internal flows handled.

The study presented motivates the use of FDI algorithms to diagnose actuator faults in sewer networks. The diagnosis algorithm would provide the limits on the actuator range to be useful for FTC.

CHAPTER 8

ACTUATOR FAULT TOLERANCE EVALUATION

8.1 Introduction

In this chapter, fault tolerant evaluation of a certain actuator fault configuration (AFC) considering a linear predictive/optimal control law with constraints is studied. This problem has been already treated in the literature for the case of LQR problem without constraints [Sta03], due to the existence of analytical solution. However, constraints (on states and control signals) are always present in real industrial control problems and could be easily handled using Linear Constrained Model Predictive Control (LCMPC) [Mac02]. But in general, an analytical solution for obtaining these control laws does not exist, which makes difficult the application of this approach.

The method proposed in this chapter is not of analytical but of computational nature. It follows the idea proposed by [LP04] in which the computation of the control law for a predictive/optimal controller with constraints can be divided in two steps: first, the computation of solutions set that satisfies the constraints (feasible solutions) and second the optimal solution determination.

Faults in actuators would cause important changes in the set of feasible solutions since constraints on the control signals have varied. This causes that the set of admissible solutions for the given control objectives could be empty. Therefore, the admissibility of the control law facing actuator faults can be determined knowing the feasible solutions set. One of the aims of this chapter is to provide methods to compute this set and then evaluate the admissibility of the control law.

To find the feasible solutions set for the LCMPC problem, a constraints satisfaction problem could be formulated [LP04]. However, such problems are computationally demanding, what induces to find a approximate solution bounded by interval hulls and in a iterative manner on time. Proceeding in this way, an interval simulation problem is implicitly solved appearing typical difficulties associated with it (such as wrapping effect, among others) [PSQ03]. In order to avoid such problems, the region of possible states could be approximated using more complex domains than intervals, such as subpavings [KJW02], ellipsoids [EC01, PSN+04], zonotopes [Küh98, ABC05], among others.

Therefore, this chapter presents a preliminary study about the mentioned admissibility evaluation of AFC considering linear plant models. This study can be extended to the cases where nonlinear or linear hybrid models are considered. First approximations by the way of nonlinear systems are reported in [OMPQ06] while in the hybrid systems framework are reported in [Tor03].

8.2 Preliminary Definitions

Considering the MPC problem defined in Section 2.2.1, and especially the sequences $\mathbf{u}_k$ and $\mathbf{x}_k$ in (2.2) and (2.4), respectively, the definitions below are given.

Definition 8.1 (Feasible solutions set)**.** The feasible solutions set is given by

$$\Omega = \{\mathbf{x}_k, \mathbf{u}_k \mid \mathbf{x}_{k+1} = g(\mathbf{x}_k, \mathbf{u}_k)\}$$

and corresponds to the input and state sets that satisfy the system constraints.

Definition 8.2 (Feasible control objectives set)**.** The feasible control objectives set is given by

$$\mathbb{J}_\Omega = \{J(\mathbf{x}_k, \mathbf{u}_k) \in \mathbb{R} \mid (\mathbf{x}_k, \mathbf{u}_k) \in \Omega\}$$

and corresponds to the set of all values of J obtained from feasible solutions set.

In the case of a fault, Ω changes to Ω_f and $\mathbb{J}_\Omega$ changes to $\mathbb{J}_{\Omega_f}$.

Definition 8.3 (Admissible solutions set)**.** Given the following subsets:

- Ω_f, defined as the feasible solutions set of a AFC and
- $\mathbb{J}_{\mathbb{A}}$, defined as the admissible control objectives set,

the admissible solutions set is given by

$$\mathbb{A} = \left\{ \mathbf{x}_k, \mathbf{u}_k \in \Omega_f \mid J(\mathbf{x}_k, \mathbf{u}_k) \in \mathbb{J}_{\mathbb{A}} \cap \mathbb{J}_{\Omega_f} \right\}$$

and corresponds to the feasible solutions subset that produces control objectives in $\mathbb{J}_{\mathbb{A}}$. If $\mathbb{A}$ is the empty set, then the considered AFC is not fault tolerant.

Definition 8.4 (Predicted states set)**.** Given the set of states at time *k-1*, the set of predicted states at time *k* is defined as:

$$\mathbb{X}_k^p = \{x_k = g(x_{k-1}, u_{k-1}) \,|\, x_{k-1} \in \mathbb{X}_{k-1}, u_{k-1} \in \mathbb{U}\}$$

and corresponds to the set of states at time *k* originated by the system evolution starting from the set of states at time *k-1*.

Definition 8.5 (Feasible states set)**.** The set of feasible states at time *k* is given by

$$\mathbb{X}_k^c = \left\{ x_k \,|\, x_k \in \mathbb{X}_k^p \cap \mathbb{X} \right\}$$

and corresponds to the set of predicted states that satisfies the system state constraints.

Definition 8.6 (Admissible inputs set)**.** The set of admissible inputs at time *k-1* is given by

$$\mathbb{U}_{k-1}^c = \left\{ u_{k-1} \in \mathbb{U} \,|\, (x_k = g(x_{k-1}, u_{k-1})) \in \mathbb{X}_k^c, x_{k-1} \in \mathbb{X}_{k-1}^c \right\}$$

and corresponds to the set of inputs that produces the set of feasible states.

Remark 8.1. Notice that $\mathbb{U}_{k-1}^c$ in Definition 8.6 is an alternative form for expressing the admissible input sequence in (2.3).

8.3 Admissibility Evaluation Approaches

8.3.1 Admissibility Evaluation using Constraints Satisfaction

This section deals with the methodology proposed in order to evaluate the admissibility of a given AFC by means of the constraint satisfaction approach. First of all, the definition of the constraint satisfaction problem is presented and some particular details related to this approach are presented and discussed. Then, this approach is explained in the framework of the AFC admissibility evaluation.

Constraints Satisfaction Problem

A *Constraints Satisfaction Problem* (CSP) on sets can be formulated as a 3-tuple $\mathcal{H} = (\Upsilon, \Lambda, \mathbb{C})$ [JKBW01], where:

- $\Upsilon = \{\upsilon_1, \cdots, \upsilon_n\}$ is a finite set of variables,
- $\Lambda = \{\Lambda_1, \cdots, \Lambda_n\}$ is the set of their domains represented by closed sets and
- $\mathbb{C} = \{c_1, \cdots, c_n\}$ is a finite set of constraints relating variables of Υ.

A point solution of $\mathcal{H}$ is a n-tuple $(\tilde{\upsilon}_1, \cdots, \tilde{\upsilon}_n) \in \Lambda$ such that all constraints $\mathbb{C}$ are satisfied. The set of all point solutions of $\mathcal{H}$ is denoted by $\mathbb{S}(\mathcal{H})$. This set is called the *global solution set*. The variable $\upsilon_i \in \Upsilon_i$ is *consistent* in $\mathcal{H}$ if and only if

$$\forall \upsilon_i \in \Upsilon_i \;\; \exists\, (\tilde{\upsilon}_1 \in \Lambda_1 \cdots, \tilde{\upsilon}_n \in \Lambda_n) \,|(\tilde{\upsilon}_1, \cdots, \tilde{\upsilon}_n) \in \mathbb{S}(\mathcal{H})$$

with $i = 1, \ldots, n$. The solution of a CSP is said to be *globally consistent*, if and only if every variable is consistent. A variable is *locally consistent* if and only if it is consistent with respect to all directly connected constraints. Thus, the solution of an CSP is said to be locally consistent if all variables are locally consistent.

The principle of algorithms for solving CSP using local consistency techniques consists essentially in iterating two main operations: *domain contraction* and *propagation*, until reaching a stable state. Roughly speaking, if the domain of a variable υ_i is locally contracted with respect to a constraint c_j, then this domain modification is propagated to all the constraints in which υ_i occurs, leading to the contraction of other variable domains and so on. Then, the final goal of such strategy is to contract as much as possible the domains of the variables without loosing any solution by removing inconsistent values through the *projection* of all constraints. To project a constraint with respect to some of its variables consists in computing the smallest set that contains only consistent values applying a contraction operator.

Being incomplete by nature, these methods have to be combined with enumeration techniques, for example bisection, to separate the solutions when it is possible. Domain contraction relies on the notion of *contraction operators* computing over approximate domains over the real numbers [JKDW01].

Admissibility Evaluation Approach

The admissibility evaluation requires the computation of the admissible solutions set introduced in Definition 8.3. It can be noticed that this corresponds naturally to a CSP on sets. Algorithm 8.1 allows the admissibility evaluation of a given AFC by solving the associated CSP defined by the system equations, the operative limits on inputs and states over H_p and the initial state.

Algorithm 8.1 Computation of $\mathbb{A}$ for a horizon H_p and some given $\mathbb{J}_{\mathbb{A}}$

$$\Upsilon \Leftarrow \{\overbrace{x_{0|k}, x_{1|}, \cdots, x_{H_p|k}}^{\mathbf{x}_k}, \overbrace{u_{0|k}, u_{1|k}, \cdots, u_{H_p-1|k}}^{\mathbf{u}_k}, J\}$$
$$\Lambda \Leftarrow \{\mathbb{X}_1, \mathbb{X}_2, \cdots, \mathbb{X}_N, \mathbb{U}_1, \mathbb{U}_2, \cdots, \mathbb{U}_{N-1}, \mathbb{J}_{\mathbb{A}}\}$$
$$\mathbb{C} \Leftarrow \{\mathbf{x}_{k+1} = g(\mathbf{x}_k, \mathbf{u}_k), J(\mathbf{x}_k, \mathbf{u}_k)\}$$
$$\mathcal{H}_{\mathbb{A}} = (\Upsilon, \Lambda, \mathbb{C})$$
$$\mathbb{A} = \mathbf{solve}(\mathcal{H}_{\mathbb{A}})$$

It is well known that the solution of these problems has a high complexity [JKBW01]. This causes that, in practice, the sets that define the variable domains in Algorithm 8.1 are approximated by intervals. Thus, the new domains set for $\mathcal{H}$ are expressed as:

$$\Lambda = \left\{ [x]_{1|k}, [x]_{2|k}, , \cdots, [x]_{H_p|k}, [u]_{0|k}, [u]_{1|k}, \cdots, [u]_{H_p-1|k}, \Box\mathbb{J}_{\mathbb{A}} \right\}.$$

Therefore, a first relaxation consists in approximating the variable domains by means of intervals and finding the solution solving an *Interval Constraints Satisfaction Problem* (ICSP) [Hyv92]. The determination of the intervals that approximate in a more fitted form the sets that define the variable domains requires global consistency, what demands a high computational cost [Hyv92]. A second relaxation consists in solving the ICSP by means of local consistency techniques, deriving on conservative intervals and, of course, on imprecise solutions.

An alternative approach to solve the CSP proposed in Algorithm 8.1 consists in admitting the rupture of the existing relations between variables of consecutive time instants, which makes possible a determination of the interval hull of the feasible solutions set step by step. However, the problem of uncertainty propagation (wrapping effect) could appear when the CSP is solved in this way, since an interval simulation problem is being implicitly solved as well. This problem does not appear in the *isotone systems* [CPSE02] (see also *monotone systems* [AS03]), which are the systems whose state function g is isotone[1].

In this case, it is only necessary to propagate the interval hull of the admissible solution set from the actual iteration to the next. This allows to rewrite the Algorithm 8.1 as is presented in Algorithm 8.2 for the interval hull of some given admissible control objective set $\mathbb{J}_{\mathbb{A}}$. As notation, the expression $\square\mathbb{A}$ (square box) means the interval hull of the set $\mathbb{A}$.

Algorithm 8.2 Computation of $\square\mathbb{A}$ for some given $\square\mathbb{J}_{\mathbb{A}}$

for $k = 1$ to H_p **do**
 $\Upsilon \Leftarrow \{x_k, x_{k-1}, u_k, J_k, J_{k-1}\}$
 $\Lambda \Leftarrow \left\{[x]_k, [x]_{k-1}, [u]_{k-1}, \square\mathbb{J}_{\mathbb{A}_k}, [J]_{k-1}\right\}$
 $\mathbb{C} \Leftarrow \{x_k = g(x_{k-1}, u_{k-1}), J_k(x_{k-1}, u_{k-1})\}$
 $\mathcal{H}_{\square\mathbb{A}_k} = (\Upsilon, \Lambda, \mathbb{C})$
 $\square\mathbb{A}_k = \mathbf{solve}(\mathcal{H}_{\square\mathbb{A}_k})$
end for
$\square\mathbb{A} = \bigcup_{k=0}^{H_p} \square\mathbb{A}_k$

Remark 8.2. If the interval hull of the admissible solution set $\mathbb{A}$ returned by Algorithm 8.2 is empty, then $\mathbb{A}$ is empty as well and the AFC produces a non admissible solution. Otherwise, nothing can be stated since $\square\mathbb{A} \neq \emptyset$ does not imply that $\mathbb{A} = \emptyset$.

For non isotone systems, the iterative algorithm Algorithm 8.2 could not be applied. As possible alternatives to extend the applicability of this algorithm to non isotone systems could be considered:

- Approximate the feasible solution domains through more complex domain forms than interval hull, i.e., zonotopes [Küh98, Bra04], ellipses [Neu93], etc and using set propagation and/or set constraints satisfaction [JKDW01].

- Convert the system in an isotone system by means of feedback state techniques [Mac02].

- Formulate a CSP propagating the initial state and using global consistency techniques.

- Formulate the problem in analytical way (linear systems) and using the corresponding tools to find the solution [BMDP02].

[1] A generic function $g = (g_1, g_2, \ldots, g_n)$ is isotone about x if g_i are non-decreasing with respect to all x_j: $j = 1, \ldots, n$.

Solving the ICSP

Several algorithms can be used to solve the ICSP enunciated in Algorithm 8.2, including Waltz's local filtering algorithm [Wal75] and Hyvönen's tolerance propagation algorithm [Hyv92]. The first only ensures locally consistent solutions while the second can guarantee global consistent solutions.

In this chapter, the ICSP is solved using a tool based on interval constraint propagation, known as *Interval Peeler*. This tool has been designed and developed by research team of the Professor Luc Jaulin [Bag05]. The goal of this software consists in determining the solution of ICSP defined in Section 8.3.1 in the case that the domains are represented by closed real intervals. The solution provides refined interval domains consistent with the set of ICSP constraints. The admissibility evaluation of a AFC using Interval Peeler is based on the procedure described in Figure 8.1.

8.3.2 Admissibility Evaluation using Set Computation

The admissibility evaluation using a set computation approach starts obtaining the feasible solutions set Ω given a set of initial states $\mathbb{X}_0 \subseteq \mathbb{X}$, the system equations and the system operating constraints over H_p. This procedure is described in the Algorithm 8.3.

Algorithm 8.3 Computation of Ω

1: $\mathbb{X}_k \Leftarrow \mathbb{X}_0$
2: $\Omega_0 \Leftarrow \mathbb{X}_0$
3: **for** $k = 1$ to H_p **do**
4: $\quad \mathbb{U}_{k-1} \Leftarrow \mathbb{U}$
5: $\quad$ Compute $\mathbb{X}_k^p$ from $\mathbb{X}_{k-1}$ and $\mathbb{U}_{k-1}$
6: $\quad$ Compute $\mathbb{X}_k^c = \mathbb{X} \cap \mathbb{X}_k^p$
7: $\quad$ Compute $\mathbb{U}_{k-1}^c$ from $\mathbb{X}_k^c$
8: $\quad \Omega_k = \mathbb{X}_k^c \times \mathbb{U}_{k-1}^c$
9: $\quad \mathbb{X}_k \Leftarrow \mathbb{X}_k^c$
10: **end for**
11: $\Omega = \bigcup_{k=0}^{H_p} \Omega_k$

At the same time that Ω is computed, the feasible control objectives set (Definition 8.2) can be obtained. Thus, in time $k = H_p$, $\mathbb{J}_{\Omega_k}$ is computed according to Algorithm 8.4.

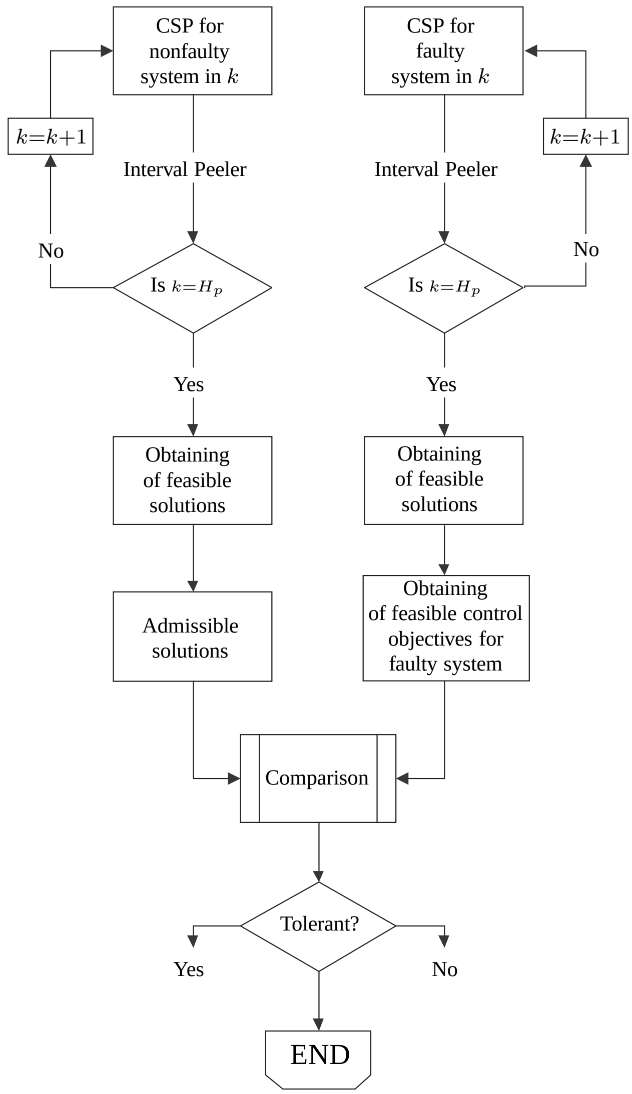

Figure 8.1: Process followed to evaluate admissibility of solutions using ICSP.

Given a fault actuator configuration, $\mathbb{J}_{\Omega_f}$ is obtained from the Algorithm 8.4 from the corresponding sets. Therefore, if some admissible control objective set $\mathbb{J}_{\mathbb{A}}$ is given, the admissibility of that AFC could be determined computing the solution admissible set $\mathbb{A}$, which is given in Definition 8.3.

If the set $\mathbb{A}$ is the empty set, the AFC is not admissible. Otherwise, that configuration would have a certain admissibility degree according to the system designer.

Algorithm 8.4 Computation of $\mathbb{J}_\Omega$ using Ω_k

1: $\mathbb{X}_k \Leftarrow \mathbb{X}_0$
2: $\Omega_0 \Leftarrow \mathbb{X}_0$
3: **for** $k = 1$ to H_p **do**
4: Compute Ω_k *(See Algorithm 8.3)*
5: Compute $\mathbb{J}_{\Omega_k}$ using $\Omega_k = \mathbb{X}_k^c \times \mathbb{U}_{k-1}^c$
6: **end for**
7: $\mathbb{J}_\Omega = \bigcup_{k=0}^{H_p} \mathbb{J}_{\Omega_k}$

Except in very particular cases, it is not possible to evaluate exactly the three sets $\mathbb{X}^p$, $\mathbb{X}^c$ and $\mathbb{U}^c$ required in Algorithm 8.3. Instead guaranteed outer approximations of these sets, as accurate as possible, have been proposed and used in the literature. On this way, some contributions have been done in [OMGVQ] and [OMGPW06], where the problem proposed in the current chapter is solved using zonotope-based set computations. Another approach to be used in this way is based on the proposal reported in [OMTP06] and [POMTI06]. There, the set computations are based on subpavings [JKDW01]. Despite the particular particular proposal reported in the latter mentioned papers is used for state estimation, the computational principle can be applied in the straightforward manner on the admissibility evaluation.

8.3.3 Motivational Example

An example to motivate the usefulness and interest of the proposed method is presented. The presence of constraints in MPC makes very difficult to proceed with the fault-tolerance analysis as proposed by [Sta03]. There, the analysis is possible because the expression of how fault affects the objective function is available using LQR theory. However, in constrained MPC this expression is not available, although an explicit expression for the controller could be derived [BMDP02]. This motivates the usefulness of the proposed approach.

Example 8.1. The double integrator system proposed by [BMDP02] is considered here, whose equivalent discrete-time state-space description using the Euler discretization rule is:

$$x_{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_k \tag{8.1}$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k, \tag{8.2}$$

with the following constraints for states and control signals: $x_1 \in [-15, 15]$, $x_2 \in [-6, 6]$ and $u \in [-1, 1]$. A MPC controller is used to control this system satisfying the associated state and

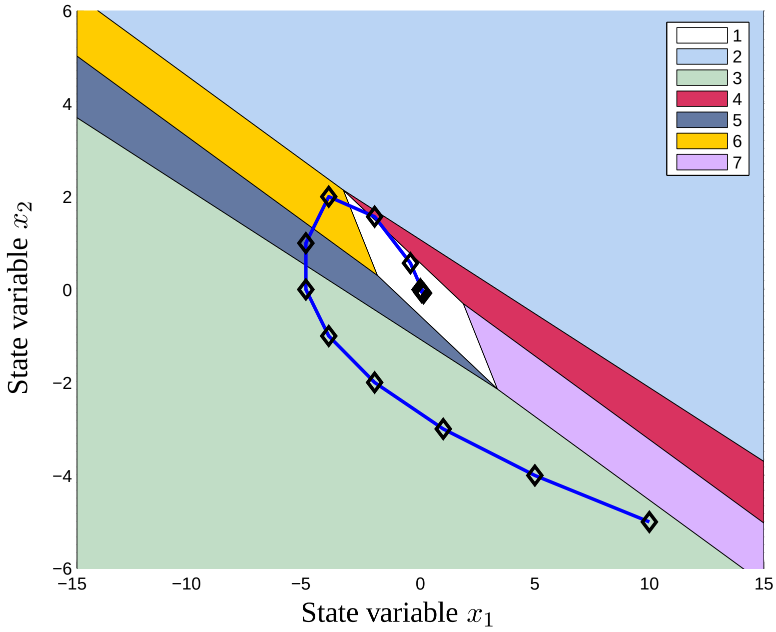

Figure 8.2: Explicit state space polyhedral partition and MPC law for Example 8.1.

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control constraints using the following objective function:

$$J = x_{H_p}^T P x_{H_p} + \sum_{i=0}^{H_p-1} \left(x_i^T Q x_i + R u_i^2 \right) \tag{8.3}$$

where $H_p = 2$, terminal weight matrix P is determined using the Ricatti equation with $Q = \text{diag}([1 \ \ 0])$ and $R = 0.01$. According to [BMDP02], a state-feedback explicit control law for the MPC controller, piece-wise affine with respect to the states, can be derived: $u_k = K_{\text{PWA}}(x_k)\, x_k$. Using the HYBRID TOOLBOX for MATLAB® [Bem06], the expression of K_{PWA} for the proposed example can be determined and represented graphically (see Figure 8.2). Using this law, the closed-loop state trajectory can be computed and represented. Notice that depending on the region of the state space, a different gain for the state feedback is applied.

Using the CSP method proposed in this chapter, the feasible sets for states x_1 and x_2 are computed and represented in Figure 8.3(a) and 8.3(b), respectively. It can be noticed that the closed-loop state trajectories applying the MPC controller are inside the corresponding feasible sets as expected. Now, a fault in the actuator is introduced. This fault corresponds to a reduction of the operating range of the actuator such that $u \in [-0.75, 0.75]$. Computing in this situation the feasible set for states x_1 and x_2 using again the method proposed, it can be noticed that the closed-loop state trajectories applying the MPC controller in the non-faulty situation are outside the corresponding feasible sets for the faulty situation. This means that the performance of the MPC controller will be worse that in the case of the non-faulty actuator since MPC trajectories for the faulty situation are not reachable. This means that if MPC trajectories in the non-faulty situation were inside the corresponding feasible sets, the performance of the MPC controller would not be affected by the fault, i.e., would be fault-tolerant. This example shows how easily can be evaluated the tolerance of a control law with respect to a fault using the method proposed. Moreover, the degradation in the performance can also be evaluated with this method, as it will be shown in the following application example.

8.4 Actuator Fault Tolerance Evaluation on Sewer Networks

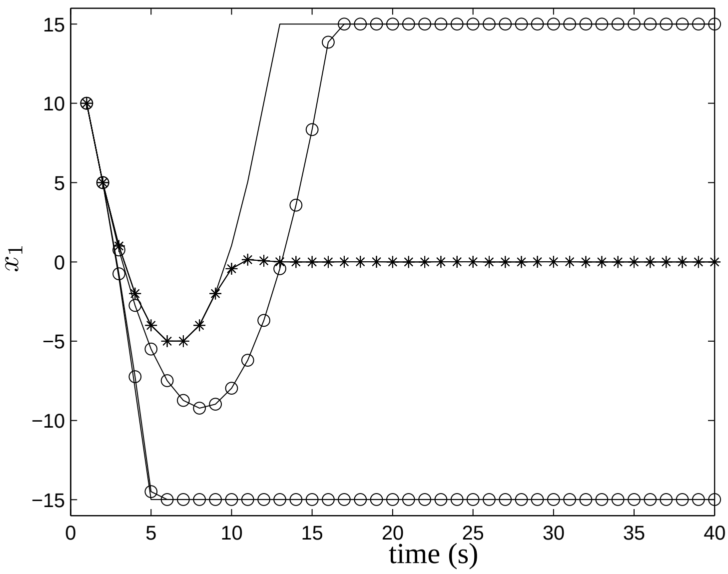
8.4.1 System Description

In order to apply the approach proposed on this chapter for the evaluation of a given AFC within a sewer network, a small system inspired on the BTC is used. As was said before, this 3-tank catchment (3-TC) contains representative elements of the entire sewer network and considers three of the four control gates appeared in the BTC. For these reasons, the 3-TC is enough for showing the effectiveness of the approach proposed. Hence, the 3-TC, presented in Figure 8.4, is described by the discrete-time equation in (4.1), where:

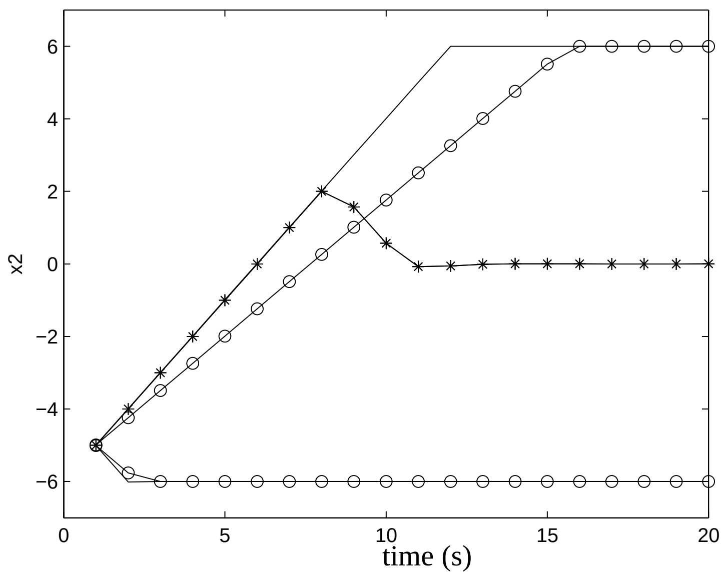
$$A = \begin{bmatrix} 1-\Delta t\beta_1 & 0 & 0 \\ 0 & 1 & 0 \\ \Delta t\beta_1 & 0 & 1-\Delta t\beta_3 \end{bmatrix} \quad B = \Delta t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \quad B_p = \Delta t \begin{bmatrix} 0 & \alpha_2 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & \alpha_3 \end{bmatrix}$$

In Figure 8.4, d_1 is directly a rain inflow because the virtual tank T_1 is not considered due any gate has influence over its dynamic. However $d_{2k} = \alpha_2 P_{16k}$ and $d_{4k} = \alpha_4 P_{20k}$ are the product of the measurements from the rain-gauges (P_i) and the conversion coefficients $\alpha_2 = \varphi_2 S_2 = 0.5951$ and $\alpha_4 = \varphi_4 S_4 = 0.1530$. Parameters β_i, φ_i and S_i are taken from Table 3.1 in Chapter 3.

The system constraints expressed in the notation adopted for sewer networks are:



(a) Feasible state trajectories for x_1.



(b) Feasible state trajectories for x_2.

Figure 8.3: Feasible set corresponding to the state variables in non-faulty (−) and faulty scenarios (−o−) for Example 8.1. The MPC solution in a non-faulty situation is also presented (−*−).

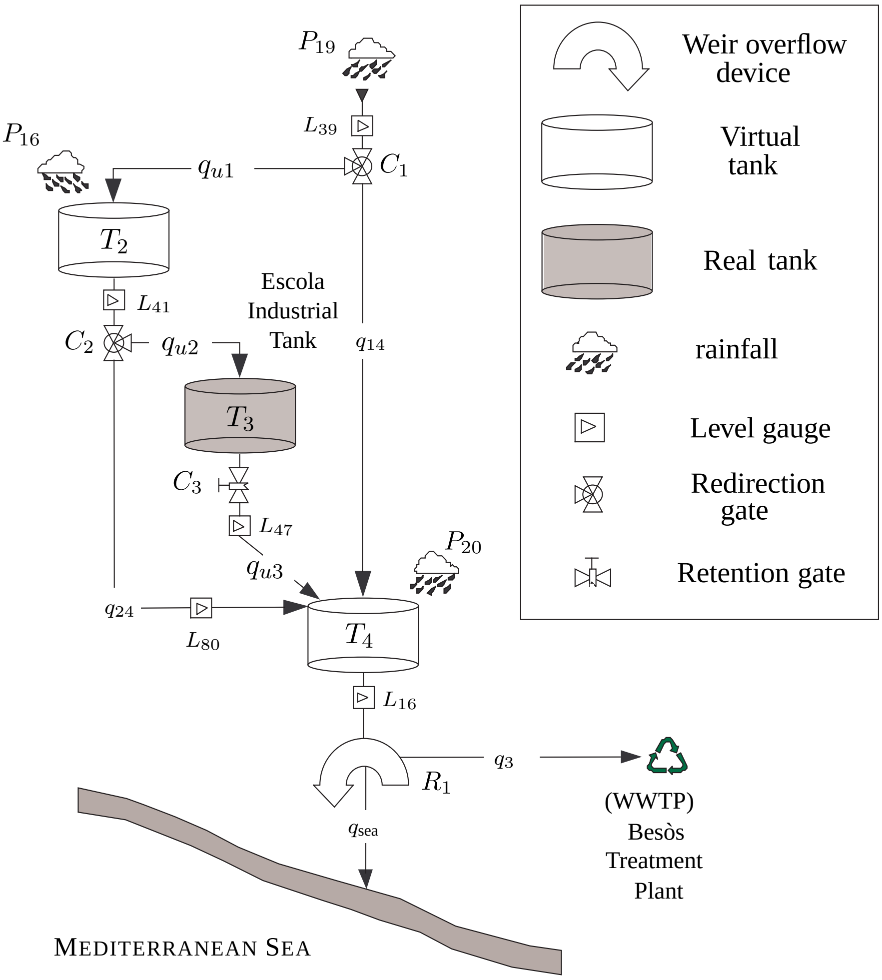


Figure 8.4: Application case: Three Tanks Catchment.

- Bounding constraints: refers to physical restrictions.

$$
\begin{aligned}
v_{2k} &\in [0, +\infty] \qquad & q_{u_1 k} &\in [0, 11] \\
v_{3k} &\in [0, 35000] \qquad & q_{u_2 k} &\in [0, 25] \\
v_{4k} &\in [0, +\infty] \qquad & q_{u_3 k} &\in [0, 7]
\end{aligned}
\tag{8.4}
$$

- Mass conservation constraints

$$\begin{aligned} d_{1k} &= q_{u_1 k} + q_{14k} \\ q_{x_1 k} &= q_{u_2 k} + q_{24k} \\ q_{x_2 k} &\geq q_{u_3 k} \end{aligned} \tag{8.5}$$

For this application, it is supposed that vector d_k (rain) is known at each time instant $k \in \mathbb{Z}_+$, what means known perturbation. This causes that the obtained results have only an interest of design for the tolerant control system.

It is desired to evaluate the admissibility of different AFC not only in reconfiguration but also in accommodation. Configuration admissibility is defined from a control objectives degradation with respect to nominal (without fault) configuration for a given rain episode. The selected rain corresponds to the one occurred on September 14, 1999, see Figure 3.8(a). This day, severe flooding occurred as a consequence of the rain storm. The actuator faults are no simultaneous and they are present from the beginning of the scenario. Their models are described as change of operating limits (accommodation) or operative annulation (reconfiguration).

Setting-up the Algorithm 8.2

The ICSP $\mathcal{H} = (\Upsilon, \Lambda, \mathbb{C})$ associated to the system has, at each time instant k, the set of variables with 9 components:

$$\Upsilon = \{v_{2k}, v_{3k}, v_{4k}, v_{2k+1}, v_{3k+1}, v_{4k+1}, q_{u_1 k}, q_{u_2 k}, q_{u_3 k}\},$$

the domains set

$$\Lambda = \{[v_2]_k, [v_3]_k, [v_4]_k, [0, +\infty], [0, 35000], [0, +\infty], [0, 11], [0, 25], [0, 7]\}$$

and the set of constraints given by the corresponding system model 4.1 and expressions in (8.4) and (8.5).

8.4.2 Control Objective and Admissibility Criterion

The main control objectives are defined as the minimization of the pollution (water volume that goes to the environment) and the minimization of the CSO to streets (flooding caused by the insufficient capacity of sewers q_{14} and q_{24}). Notice that overflows in virtual tanks are not considered in this case. From system variables, the constraints that defines the control objectives are given by

- For pollution

$$J = V_{\text{env}} = \Delta t \sum_{k=0}^{H_p} \max(0, q_{v_4 k} - \overline{q}_4) \tag{8.6}$$

- For flooding

$$J = V_{\text{str}} = \Delta t \sum_{k=0}^{H_p} \max(0, q_{14k} - \overline{q}_{14}) + \max(0, q_{24k} - \overline{q}_{24}) \tag{8.7}$$

Notice that the expression for pollution is expressed in function of a isotone state variable that has an exact interval hull. Therefore, since pollution only depends of this variable, its interval is also exact as well. In this case, $\square\mathbb{J}_{\square\Omega} \supseteq \square\mathbb{J}_{\mathbb{A}}$ holds with equality, what allows an always correct admissibility evaluation.

On the other hand, when the objective related to flooding is taking into account, its expression depends of relations between isotone and no isotone variables and therefore, according to Remark 8.2, the assessment of the non admissible configuration is possible but nothing can be said about admissibility of the configuration.

The admissibility criterion is based on a direct comparison between the obtained minimum final value of volume given by the related envelope $\underline{V}_{obj}(H_p)$ and the same value for the degraded nominal system configuration $V_{obj}^{nom}(H_p)$. Notice that it can be done due to the pollution and flooding indexes correspond to the accumulated masses over a given scenario). The expression for the aforementioned comparison is given by

$$\underline{V}_{obj}(H_p) = \psi \left(V_{obj}^{nom}(H_p) \right)$$

where ψ is the relation of degradation and subscript *obj* denotes the control objective. n order to illustrate the proposed method, in this application example, it is assumed that $\psi = 8$. In the reality, this relation is provided by the network operator according to the directives given by the city authorities based on the heuristic knowledge of the sewage system designers.

Table 8.1: Admissibility of AFC for pollution: Reconfiguration.

Fault Location	Min. Volume (m^3)	Admissibility Status
No fault	1050	—
Fault in q_{u_1}	8800	No Admissible
Fault in q_{u_2}	52200	No Admissible
Fault in q_{u_3}	1050	Admissible

8.4.3 Obtained Results

Reconfiguration Case

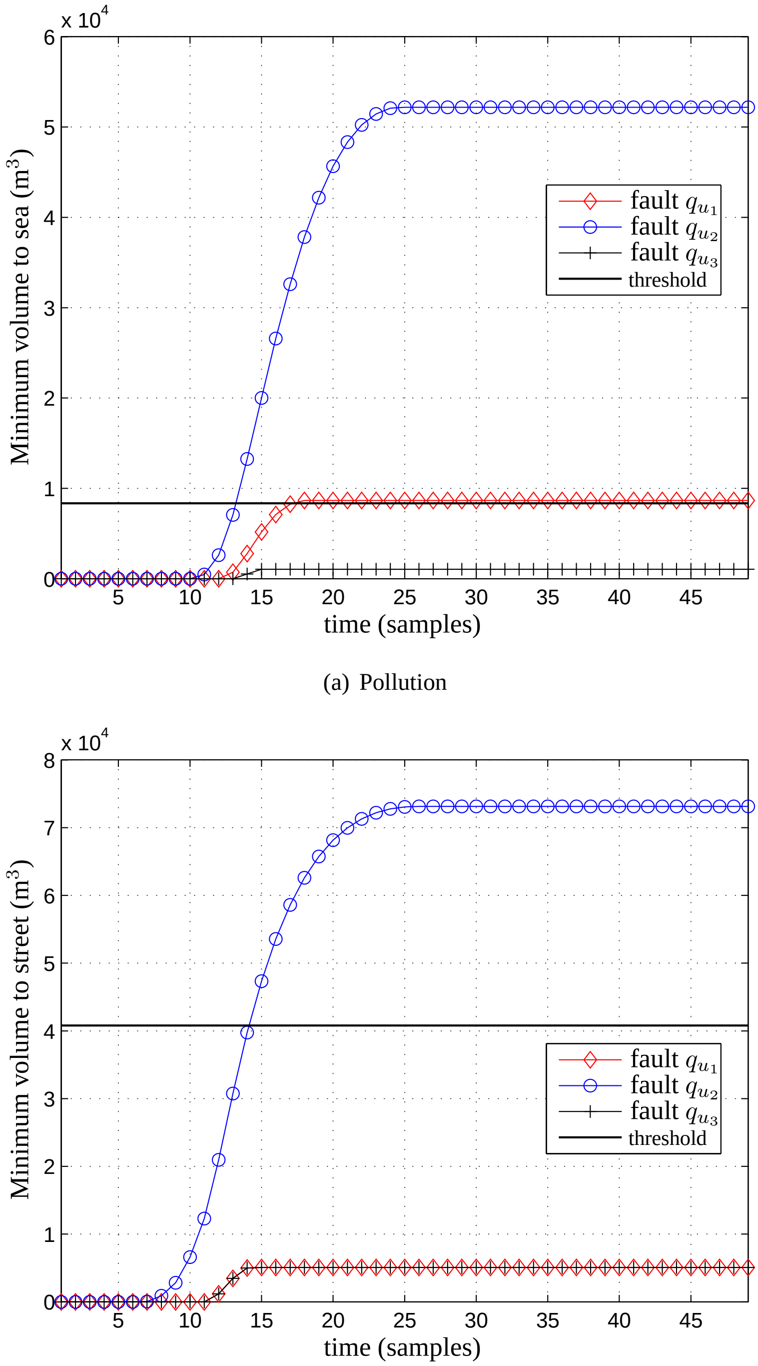
This case considers actuators completely closed or completely open due to the fault. This fact would change the admissibility of the obtained AFC. Table 8.1 resumes the possible fault cases and their admissibility when pollution is considered. Only faults described by actuators completely closed are simulated, that is $q_{u_i} \in [0, 0]$ and $q_i \in [0, +\infty]$. In the contrary case, ICSP can not be solved due to constraints in ranges of q_{u_i} are violated.

On the other hand, Table 8.2 presents the results obtained when flooding is considered. Notice that some configurations are uncertain due to definition of the cost function for this objective (see Section 8.4.2).

Table 8.2: Admissibility of AFC for flooding: Reconfiguration.

Fault Location	Min. Volume (m^3)	Admissibility Status
No fault	5100	—
Fault in q_{u_1}	5100	Uncertain
Fault in q_{u_2}	73200	No Admissible
Fault in q_{u_3}	5100	Uncertain

Figure 8.5 shows the minimum envelopes for pollution (Figure 8.5(a)) and for flooding (Figure 8.5(b)) when the admissibility criterion is considered (threshold).



(a) Pollution

(b) Flooding

Figure 8.5: Minimum envelopes for Reconfiguration.

Accommodation Case

This case considers faults manifested as the reduction of the actuators operative rank (for example from 0-100% to 0-50%). Thus, the amount of admitted AFC varies as it is shown in Tables 8.3 and 8.4, where if the admissibility criterion is maintained as in the reconfiguration case, more configurations could be admitted. Different accommodation ranges are presented. These tables do not consider accommodation for q_{u_3} due to the system insensibility shown for this actuator what is seen in the results collected in Tables 8.1 and 8.2. Figure 8.6 presents the minimum envelopes for pollution and flooding in the case of accommodation.

Table 8.3: Admissibility of fault configurations for pollution: Accommodation.

Fault Location	Operation range	Min. Volume (m^3)	Admissibility Status
No fault	—	1050	—
Fault in q_{u_1}	0-20%	5200	Admissible
Fault in q_{u_1}	0-50%	2300	Admissible
Fault in q_{u_2}	0-20%	34000	No Admissible
Fault in q_{u_2}	0-50%	15700	No Admissible

8.5 Summary

This chapter proposes a method for admissibility evaluation of fault configurations based on the solution of a Interval Constraints Satisfaction Problem (ICSP). This procedure implies the propagation of the feasible solution set at each time instant solving implicitly an interval simulation. The results provide the limits of system performance considering all the feasible solutions and how they are degraded after fault occurrence. This allows to evaluate the admissibility of a given AFC using a degradation criterion established beforehand.

Another techniques could be used for solving the proposed problem of actuator fault tolerance evaluation. These techniques are based on set computation using zonotopes, subpavings and other approximations. However, these techniques are not explained here despite of their applications have been reported, see [OMGVQ], [OMGPW06], [OMTP06] and [POMTI06].

The proposed method has been successfully applied on a linear predictive control system

Table 8.4: Admissibility of fault configurations for flooding: Accommodation.

Fault Location	Operation range	Min. Volume (m^3)	Admissibility Status
No fault	—	5100	—
Fault in q_{u_1}	0-20%	5100	Uncertain
Fault in q_{u_1}	0-50%	5100	Uncertain
Fault in q_{u_2}	0-20%	50000	Uncertain
Fault in q_{u_2}	0-50%	26100	No Admissible

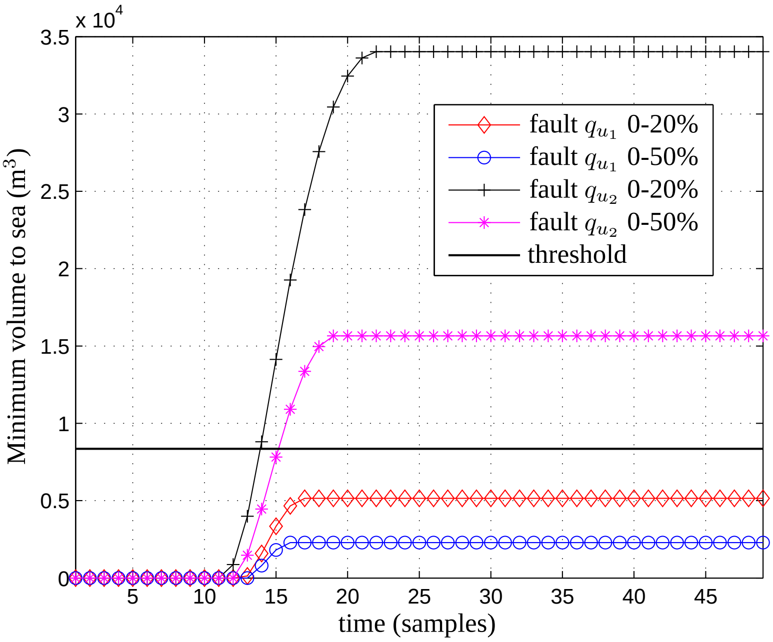
inspired on the BTC. This 3-tank catchment (3-TC) contains representative elements of the entire sewer network and considers three of the four control gates appeared in the BTC. For these reasons, the 3-TC is enough for showing the effectiveness of the approach proposed. The proposed technique on this chapter has been also proved considering a nonlinear model of the 3-TC [OMPQ06]:

$$\begin{aligned} v_{2k+1} &= v_{2k} + \Delta t\,[q_{u_1 k} + d_{2k} - q_{v_{2out} k}] \\ v_{3k+1} &= v_{3k} + \Delta t\,[q_{u_2 k} - q_{u_3 k}] \\ v_{4k+1} &= v_{4k} + \Delta t\,[q_{u_3 k} + d_{3k} + q_{14k} + q_{24k} - q_{v_{4out} k}] \end{aligned} \tag{8.8}$$

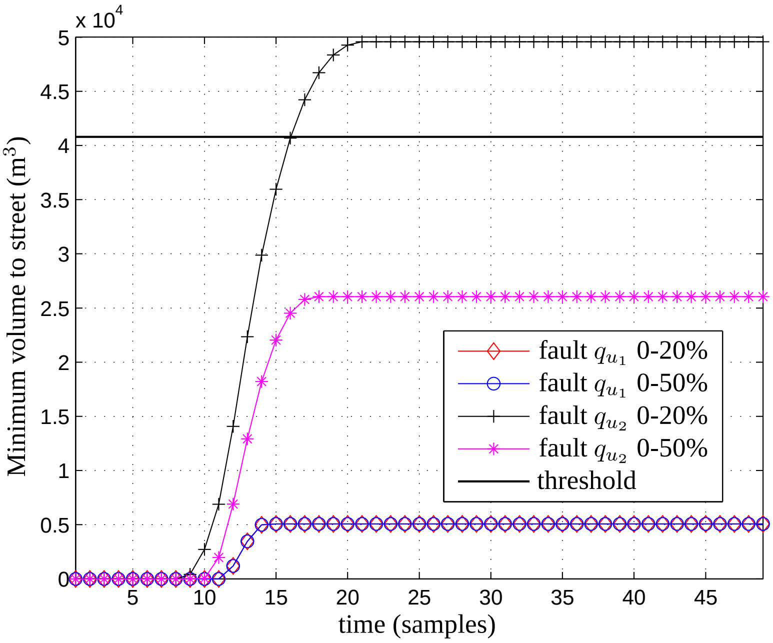
where the nonlinear relation between the i-th tank volume v_i and the tank outflow $q_{v_{iout}} = \beta_i \sqrt{v_{ik}}$ is considered. Obtained results reported in this reference shown the effectiveness of the approach considering the nonlinear model predictive control framework.

On the other hand, in [GOMP07] is proposed the actuator fault tolerance evaluation when Linear Constrained Robust Model Predictive Control (LRMPC) is used taking into account model uncertainty. In this case, the problem considers the parameters as new variables to be refined when the ICSP is stated. Hence, the set of constraints C for the LRMPC problem is now defined as:

$$\mathbb{C} : \begin{cases} x_{k+1} = A(\theta_k)x_k + B(\theta_k)u_k \\ x_k \in \mathbb{X} \qquad k \in [0, H_p] \subset \mathbb{Z}_+ \\ u_k \in \mathbb{U} \qquad k \in [0, H_p - 1] \subset \mathbb{Z}_+ \\ \theta_k \in \Theta \qquad k \in [0, H_p] \subset \mathbb{Z}_+ \end{cases} \tag{8.9}$$



(a) Pollution



(b) Flooding

Figure 8.6: Minimum envelopes for Accommodation.

where

$$\mathbb{X} \triangleq \{x \in \mathbb{R}^n \,|\, \underline{x} \leq x \leq \overline{x}\} \tag{8.10a}$$

$$\mathbb{U} \triangleq \{u \in \mathbb{R}^m \,|\, \underline{u} \leq u \leq \overline{u}\} \tag{8.10b}$$

$$\Theta \triangleq \left\{\theta \in \mathbb{R}^p \,|\, \underline{\theta} \leq \theta \leq \overline{\theta}\right\} \tag{8.10c}$$

and $A(\theta)$ and $B(\theta)$ are the system matrices of suitable dimensions including their associated uncertain parameters. Also the effectiveness of the evaluation approach is proved despite of the presence of parameter uncertainty.

Part IV

Concluding Remarks

CHAPTER 9

CONCLUDING REMARKS

In conclusion, the thesis objectives has been fulfilled. Furthermore, during the thesis process new objectives and tasks were appearing. They have enhanced the proposed approaches and/or have complemented the obtained results. Therefore, this chapter summarizes the main contributions done and proposes future ways to continue the line of the thesis.

9.1 Contributions

The central idea behind this thesis was the design of MPC strategies for sewer networks including considerations about fault tolerance. The main contributions of this thesis are summarized below.

- Lexicographic approach was used as an automatic tuning for the MPC controller of a sewer network. The application of this technique over such complex systems was motivated by the difficulty of determining the suitable weights for a cost function of a traditional MPC controller due to the continuous change of the rain intensity (system disturbances).

- An hybrid modeling methodology was developed for modeling sewer networks. The proposed methodology allows to represent each constitutive element of the network as an hybrid system. To obtain the model of the entire system within this framework, all preestablished hybrid submodels are joined suitably (taking into account their connections within the network), avoiding the tedious and complex process of modeling directly the whole sewer network as an unique hybrid system.

- Predictive control of sewer networks has been proposed by considering their models as hybrid systems. This fact allows to compute the global optimal solution of the associated optimization problem despite the sewage system model includes nonlinear dynamics related to overflows and flood, which implies switching on operating modes.
- A suboptimal HMPC design was derived for reducing the computation time taking by the solution of the MIP associated to the discrete optimization problem. The obtained approach guarantees feasibility in the the optimization process and closed-loop stability. Such suboptimal HMPC design was proved on the sewer network case study, obtaining satisfactory results related not only with computation time but also with the system suboptimality level.
- The hybrid modeling methodology was used to represent actuator faults consisting in the change of operational ranges of such elements. Also, it was taking into to account the hybrid nature of the FTC system by using an hybrid systems modeling, analysis and control methodology. This Fact has allowed to design the three levels of a FTC system in an integrated manner and verify its global behavior.
- A method was described for evaluating the fault tolerance of a linear MPC closed-loop under the effect of faults in actuators. Such method uses constraints satisfaction to know whether a certain configuration of faulty actuators fulfills the problem constraints, or whether the corresponding control objectives are fulfilled despite the presence of actuator faults. This way of tolerance evaluation avoids solving an optimization problem in order to know whether the control law can deal with actuator fault configuration.

9.2 Directions for Future Research

To continue the research proposed in this thesis, some ideas are outlined below.

- It would be very useful to derive sewer network models with adaptive parameters which could be auto-calibrated according to the available information from sensors, statistics and accurate predictions for rain intensities and their effects over the sewage system.
- The inclusion of inherent switching dynamics in the sewer network model for the MPC design of Chapter 4 should be further investigated. There exist some inaccurate approaches for dealing with these nonlinear dynamics[1] which can be tested in order to take advantage

[1] for instance, approximations of min or max function by sets of linear inequalities [A.03b].

of the scalability and self-tuning capabilities without losing the convexity of the optimization problem.

- The implementation of a suitable software tool, inspired on the philosophy of CORAL [FCP+02], should be done. Such tool would make the automatic integration of the different hybrid elements when a given sewer network has been considered as a compositional hybrid system. The software tool would calculate the global MLD model related to the given network.

- The application of tuning methods for MPC controllers based on Lexicographic programming should be extended to HMPC controllers once suboptimal strategies have been included in the design of such controllers. According to Section 4.2.3, it can be noticed that if the mentioned tuning methods are considered without taking into account a suboptimal strategy, the computation time for solving the discrete optimization problem increases excessively due to the implementation features of the tuning method.

- Within the active approach of the FTHMPC, this thesis assumed that FDI module always operates correctly. However it can not be ensured for all situations. Even though a false alarm of fault occurrence (situation when FDI faults) only implies a very conservative controller[2], when FDI does not inform about a fault situation, the active fault tolerant topology become passive. However, these facts should be investigated with more depth.

- Another important situation that should be investigated is related with the existence of delays in the FDI module and their effects in the performance of the closed-loop, mainly by considering the presence of faults. Theoretically, MPC techniques deal with such problems, however this fact would have to be confirmed when assuming the AFTHMCP strategy.

- Rain prediction is an active research area which is currently under development and combines different disciplines. This topic is undoubtedly important in the study of sewer networks and all aspects associated with: modeling, RTC, environmental management, etc.

Finally, constant changes on the Barcelona sewer network topology in order to improve its management have produced. Figure 9.1 shows the actual BTC. Considering the new system topology, it is possible that some of the methodologies, approaches and strategies proposed on this thesis reach better results than with the older BTC version.

[2]under these conditions, control gates are under-used and this situation is as if the actuators had a self-limiter.

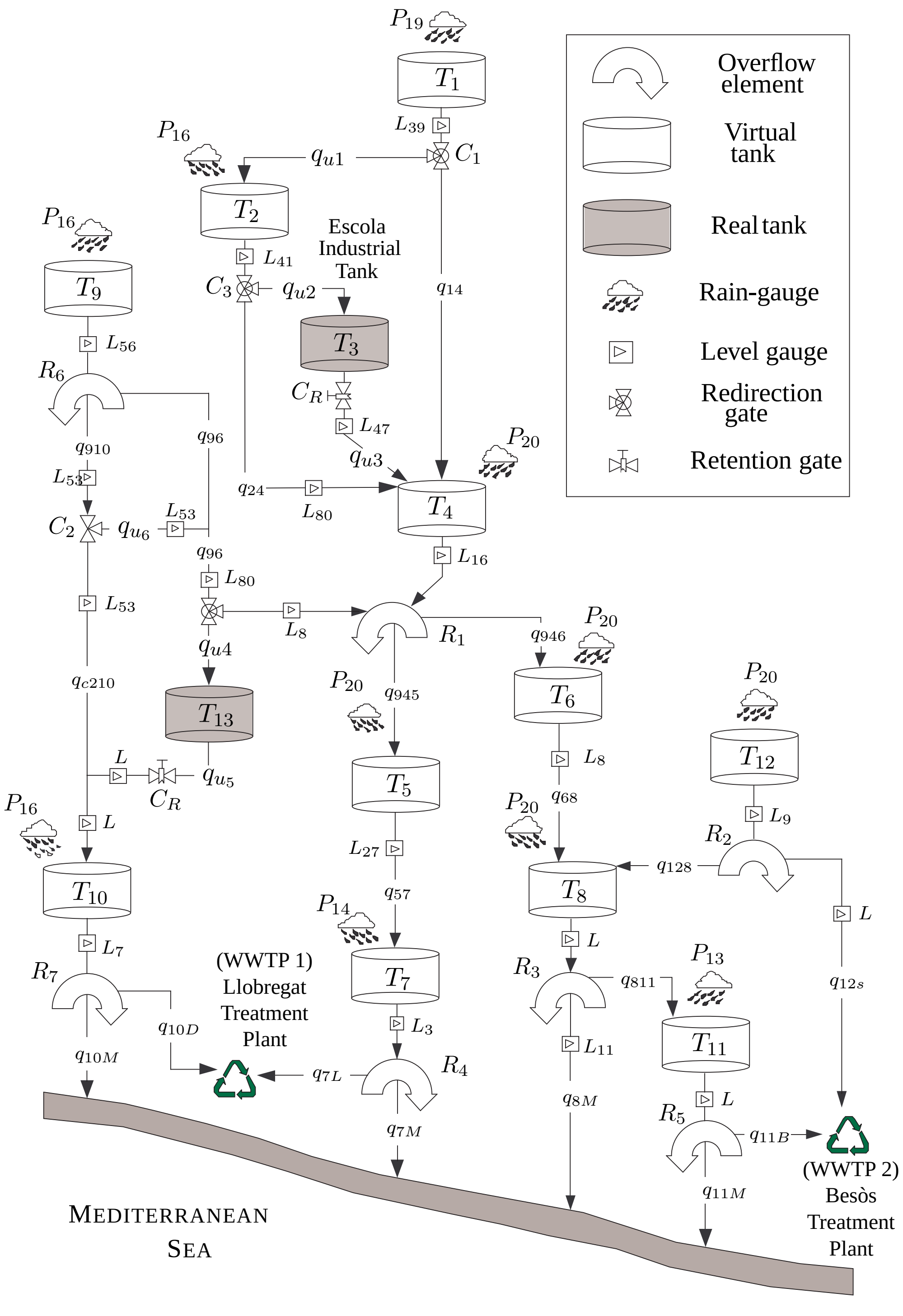


Figure 9.1: New topology for the BTC.

Part V

Appendices

APPENDIX A

PROOF OF THEOREM 6.1

Consider the shifted sequence

$$\mathbf{u}_{k+1}^{1} \triangleq \Big(u_{1|k}^{*}, \ldots, u_{H_p-1|k}^{*}, h(x_{H_p-1|k+1})\Big) \tag{A.1}$$

where $x_{H_p-1|k+1}$ is the state at prediction time $H_p - 1$, obtained at time $k+1$ by applying the input sequence $u_{1|k}^{*}, \ldots, u_{H_p-1|k}^{*}$ to system (6.4) with initial condition $x_{0|k+1} \triangleq x_{1|k}^{*} = x_{k+1} = g(x_k, u_{\text{MPC}}(x_k))$. Note that $x_{H_p-1|k+1} = x_{H_p|k}^{*}$.

1. If Problem 6.1 is feasible at time $k \in \mathbb{Z}_+$ for state $x_k \in \mathbb{X}$ then there exists a mode sequence reference $\bar{\Delta}_k$ such that $\mathcal{U}(x_k, \bar{\Delta}_k)$ is nonempty. The optimal solution to Problem 6.1 is denoted $\mathbf{u}_k^{*}$. Then it follows that $x_{H_p-1|k+1} \in \mathbb{X}_T$. Due to Remark 6.2 and the positive invariance of $\mathbb{X}_T \in \mathbb{X}_{\mathbb{U}}$, it holds that $x_{H_p|k+1} \in \mathbb{X}_T$ and $\mathbf{u}_{k+1}^{1} \in \mathcal{U}(x_{k+1}, \bar{\Delta}_{k+1})$. This implies that Problem 6.1 is feasible at time $k+1$ for x_{k+1} and mode sequence reference $\bar{\Delta}_{k+1}$.

2. Let $\tilde{\mathbf{x}}(x_k) = (\tilde{x}_{1|k}, \tilde{x}_{2|k} \ldots \tilde{x}_{H_p|k})$ denote the state sequence generated by the system $x_{k+1} = g(x_k, h(x_k))$ from initial state $\tilde{x}_{0|k} \triangleq x_k \in \mathbb{X}_T$. Let $\tilde{\mathbf{u}}_k$ denote the corresponding control signal. Since $\tilde{\mathbf{x}}_k \in \mathbb{X}_T^{H_p}$, then $\tilde{\mathbf{u}}_k \in \mathbb{U}$ according to Assumption 6.1. A candidate reference sequence $\bar{\Delta}_k$ so that $\mathcal{U}_k(x_k, \bar{\Delta}_k)$ is nonempty is the one related to $\tilde{\mathbf{u}}_k$ and $\tilde{\mathbf{x}}_k$.

3. Consider again the state sequence $\tilde{\mathbf{x}}_k(x_k)$. Since $\tilde{\mathbf{x}}_k(x_k) \in X_T^{H_p}$, inequality (6.12c) from

Remark 6.2 holds for all elements in the sequence $\tilde{\mathbf{x}}_k$, yielding

$$\begin{aligned} F(\tilde{x}_{1|k}) - F(\tilde{x}_{0|k}) + L(\tilde{x}_{0|k}, h(\tilde{x}_{0|k})) &\leq 0 \\ F(\tilde{x}_{2|k}) - F(\tilde{x}_{1|k}) + L(\tilde{x}_{1|k}, h(\tilde{x}_{1|k})) &\leq 0 \\ &\vdots \\ F(\tilde{x}_{H_p|k}) - F(\tilde{x}_{H_p-1|k}) + L(\tilde{x}_{H_p-1|k}, h(\tilde{x}_{H_p-1|k})) &\leq 0. \end{aligned}$$

From these inequalities, by optimality and by Remark 6.2 it follows that

$$V_{\text{MPC}}(x_k) \leq J(x_k, \tilde{\mathbf{u}}_k) \leq F(x_k) \leq \alpha_2(\|x_k\|), \quad \forall x_k \in \widetilde{\mathcal{N}}$$

where $\widetilde{\mathcal{N}} = \mathcal{N} \cap \mathbb{X}_T$. Again, using optimality, one has:

$$\begin{aligned} V_{\text{MPC}}(x_{k+1}) - V_{\text{MPC}}(x_k) &= J(x_{k+1}, \mathbf{u}^*_{k+1}) - J(x_k, \mathbf{u}^*_k) \\ &\leq J(x_{k+1}, \mathbf{u}^1_{k+1}) - J(x_k, \mathbf{u}^*_k) \\ &= -L(x_k, u_{\text{MPC}}(x_k)) + F(x_{H_p|k+1}) - F(x^*_{H_p|k}) \\ &\quad + L(x^*_{H_p|k}, h(x^*_{H_p|k})). \end{aligned}$$

Then, as $x^*_{H_p|k} \in \mathbb{X}_T$ and by condition (6.12c) in Assumption 6.1, it holds that

$$V_{\text{MPC}}(g(x_k, u_{\text{MPC}}(x_k))) - V_{\text{MPC}}(x_k) \leq -L(x_k, u_{\text{MPC}}(x_k)) \leq -\alpha_1(\|x_k\|) \quad \forall x_k \in \mathbb{X}_f(H_p).$$

Since $\mathbb{X}$ is compact and $\mathbb{X}_f \subset \mathbb{X}$, then according to item 1., $\mathbb{X}_f(H_p)$ is positively invariant. Let x_k be a state reached with the closed-loop system (6.4) and (6.10) from initial state x_0. Chose any $\eta > 0$ such that the ball $\mathcal{B}_\eta \triangleq \{x \in \mathbb{R}^n | \|x\| \leq \eta\}$ satisfies $\mathcal{B}_\eta \subset \widetilde{\mathcal{N}}$. It is possible to chose any $0 < \epsilon \leq \eta$ a $\sigma \in (0, \epsilon)$ such that $\alpha(\sigma) < \alpha(\epsilon)$. For any $x_0 \in \mathcal{B}_\sigma \subset \mathbb{X}_f(H_p)$, due to positive invariance of $\mathbb{X}_f(H_p)$, it follows that

$$... \leq V_{\text{MPC}}(x_{k+1}) \leq V_{\text{MPC}}(x_{k+1}) \leq \cdots \leq V_{\text{MPC}}(x_0) \leq \alpha_2(\|x_0\|) \leq \alpha_2(\sigma) \leq \alpha_1(\epsilon).$$

Since we have $V_{\text{MPC}}(x) \geq \alpha_1(\epsilon)$ for all $x \in \mathbb{X}_f(H_p) \backslash \mathcal{B}_\epsilon$ it follows that $x_k \in \mathcal{B}_\epsilon$ for all $k \in \mathbb{Z}_+$.

APPENDIX B

AUXILIARY DATA FOR CHAPTER 5

Table B.1: Relation between z variables and control objectives.

Objective	*z* vector	*z* variable	Description
1	z_{str_v}	z_2	overflow in T_1
		z_6	overflow in T_2
		z_{10}	overflow in T_4
		z_{12}	overflow in T_9
		z_{20}	overflow in T_5
		z_{22}	overflow in T_6
		z_{24}	overflow in T_7
		z_{26}	overflow in T_{12}
		z_{32}	overflow in T_8
		z_{36}	overflow in T_{10}
		z_{40}	overflow in T_{11}
2	z_{str_q}	z_4	overflow in q_{14}
		z_8	overflow in q_{24}
		z_{14}	overflow in q_{96}
		z_{18}	overflow in q_{945}
		z_{30}	overflow in $q_{12\text{s}}$
		z_{34}	overflow in q_{c210}
3	z_{sea}	z_{29}	flow to environment ($q_{12\text{s}}$)
		z_{35}	flow to sea ($q_{10\text{M}}$)
		z_{38}	flow to sea ($q_{8\text{M}}$)
		z_{42}	flow to sea ($q_{11\text{M}}$)
		z_{44}	flow to sea ($q_{7\text{M}}$)
4	—	z_{43}	flow to Llobregat WWTP
	—	z_{41}	flow to Besòs WWTP

APPENDIX C

ACRONYMS

AFC	Actuator Fault Configuration
AFTHMPC	Active Fault Tolerant Hybrid Model Predictive Control
AFTMPC	Active Fault Tolerant Model Predictive Control
AKF	Adaptive Kalman Filter
AS	Automatic Supervisor
BTC	Barcelona Test Catchment
CLABSA	Clavegueram de Barcelona, S.A.
CSO	Combined Sewage Overflow
CSP	Constraint Satisfactions Problem
CSS	Combined Sewage System
DEDS	Discrete-Event Dynamical System
FDI	Fault Diagnosis and Isolation
FTC	Fault Tolerant Control
FTHMPC	Fault Tolerant Hybrid Model Predictive Control
GPC	Global Predictive Control
HMPC	Hybrid Model Predictive Control
HYSDEL	HYbrid System DEscription Language
ICSP	Interval Constraints Satisfaction Problem
LC	Linear Complementarity
LCMPC	Linear Constraint Model Predictive Control
LP	Linear Program(ming)
LPV	Linear Parameter Variant
LQR	Linear Quadratic Regulator

LCRMPC	Linear Constrained Robust Model Predictive Control
LTI	Linear Time Invariant
MBPC	Model-Based Predictive Control
MILP	Mixed Integer Linear Program(ming)
MIMO	Multi-Input Multi-Output
MIP	Mixed Integer Program(ming)
MIQP	Mixed Integer Quadratic Program(ming)
MLD	Mixed Logical Dynamics
MMPS	Min-Max-Plus Scaling
MPC	Model Predictive Control
NMPC	Nonlinear Model Predictive Control
mpLP	Multi-Parametric Linear Program(ming)
mpQP	Multi-parametric Quadratic Program(ming)
NLP	Nonlinear Programming Algorithms
OOP	Open-loop Optimization Problem
PFTHMPC	Passive Fault Tolerant Hybrid Model Predictive Control
PFTMPC	Passive Fault Tolerant Model Predictive Control
PWA	Piecewise Affine
QP	Quadratic Program(ming)
RLS	Recursive Least-Squares
RTC	Real-Time Control
SCADA	Supervisory Control and Data Acquisition
VFC	Volume/Flow Conversion

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