Appendix B

Linearized Models

B.1 Models with Surge Tank Effects

B.1.1 Model Q_{lin}

- Equation of the flow in the penstock:

$$\Delta\overline{U} = \frac{\partial\overline{U}}{\partial\overline{H}} \cdot \Delta\overline{H} + \frac{\partial\overline{U}}{\partial\overline{G}} \cdot \Delta\overline{G} = a_{11} \cdot \Delta\overline{H} + a_{13} \cdot \Delta\overline{G}$$

- Equation of the mechanical power:

$$\Delta\overline{P}_m = \frac{\partial\overline{P}_m}{\partial\overline{H}} \cdot \Delta\overline{H} + \frac{\partial\overline{P}_m}{\partial\overline{U}} \cdot \Delta\overline{U} = a_{21} \cdot \Delta\overline{H} + a_{23} \cdot \Delta\overline{U}$$

- Transfer functions F(s) and G(s):

$$F(s) = \frac{\overline{U}_t - \overline{U}_0}{\overline{H}_t - \overline{H}_0} = -\frac{1 + \frac{G(s)}{z_p} \cdot \tanh(T_{ep} \cdot s)}{\Phi_p + G(s) + z_p \cdot \tanh(T_{ep} \cdot s)}$$

$$G(s) = \frac{\overline{H}_s - \overline{H}_0}{\overline{U}_p - \overline{U}_0} = \frac{\Phi_c + s \cdot T_{WC}}{1 + s \cdot C_s \cdot \Phi_c + s^2 \cdot T_{WC} \cdot C_s}$$

Ideal values for the Francis turbine are a_{11}= 0.5, a_{13}= 1, a_{21}= 1.5 and a_{23}= 1

$$\frac{\Delta\overline{P}_m}{\Delta\overline{G}} = \frac{1-\Phi_p - z_p \cdot \tanh(T_{ep}\cdot s) + \dfrac{G(s)}{z_p}\cdot \tanh(T_{ep}\cdot s) - G(s)}{1+0.5\cdot\Phi_p + 0.5\cdot z_p\cdot T_{ep}\cdot s + 0.5\cdot G(s) + \dfrac{G(s)}{z_p}\cdot\tanh(T_{ep}\cdot s)}$$

B.1.2 Model Q_{lin0}

- Equation of the flow in the penstock:

$$\Delta\overline{U} = \frac{\partial\overline{U}}{\partial\overline{H}}\cdot\Delta\overline{H} + \frac{\partial\overline{U}}{\partial\overline{G}}\cdot\Delta\overline{G} = a_{11}\cdot\Delta\overline{H} + a_{13}\cdot\Delta\overline{G}$$

- Equation of the mechanical power:

$$\Delta\overline{P}_m = \frac{\partial\overline{P}_m}{\partial\overline{H}}\cdot\Delta\overline{H} + \frac{\partial\overline{P}_m}{\partial\overline{U}}\cdot\Delta\overline{U} = a_{21}\cdot\Delta\overline{H} + a_{23}\cdot\Delta\overline{U}$$

- Transfer functions F(s) and G(s):

$$F(s) = \frac{\overline{U}_t - \overline{U}_0}{\overline{H}_t - \overline{H}_0} = -\frac{1+\dfrac{G(s)}{z_p}\cdot(T_{ep}\cdot s)}{\Phi_p + G(s) + z_p\cdot(T_{ep}\cdot s)}$$

$$G(s) = \frac{\overline{H}_s - \overline{H}_0}{\overline{U}_p - \overline{U}_0} = \frac{\Phi_c + s\cdot T_{WC}}{1+s\cdot C_s\cdot\Phi_c + s^2\cdot T_{WC}\cdot C_s}$$

Ideal values for the Francis turbine are a_{11}= 0.5, a_{13}= 1, a_{21}= 1.5 and a_{23}= 1

$$\frac{\Delta\overline{P}_m}{\Delta\overline{G}} = \frac{1-\Phi_p - z_p\cdot T_{ep}\cdot s + \dfrac{G(s)}{z_p}\cdot T_{ep}\cdot s - G(s)}{1+0.5\cdot\Phi_p + 0.5\cdot z_p\cdot T_{ep}\cdot s + 0.5\cdot G(s) + \dfrac{G(s)}{z_p}\cdot T_{ep}\cdot s}$$

B.2 Models with no Surge Tank Effects

B.2.1 Model K_{lin}

- Equation of the flow in the penstock:

$$\Delta\overline{U} = \frac{\partial\overline{U}}{\partial\overline{H}} \cdot \Delta\overline{H} + \frac{\partial\overline{U}}{\partial\overline{G}} \cdot \Delta\overline{G} = a_{11} \cdot \Delta\overline{H} + a_{13} \cdot \Delta\overline{G}$$

- Equation of the mechanical power:

$$\Delta\overline{P}_m = \frac{\partial\overline{P}_m}{\partial\overline{H}} \cdot \Delta\overline{H} + \frac{\partial\overline{P}_m}{\partial\overline{U}} \cdot \Delta\overline{U} = a_{21} \cdot \Delta\overline{H} + a_{23} \cdot \Delta\overline{U}$$

- Transfer function F(s):

$$F(s) = \frac{\overline{U}_t - \overline{U}_0}{\overline{H}_t - \overline{H}_0} = -\frac{1}{\Phi_p + z_p \cdot \tanh(T_{ep} \cdot s)}$$

Ideal values for the Francis turbine are a_{11}= 0.5, a_{13}= 1, a_{21}= 1.5 and a_{23}= 1

$$\frac{\Delta\overline{P}_m}{\Delta\overline{G}} = \frac{1 - \Phi_p - z_p \cdot \tanh\left(T_{ep} \cdot s\right)}{1 + 0.5 \cdot \Phi_p + 0.5 \cdot z_p \cdot \tanh\left(T_{ep} \cdot s\right)}$$

B.2.2 Model G_{lin0}

- Equation of the flow in the penstock:

$$\Delta\overline{U} = \frac{\partial\overline{U}}{\partial\overline{H}} \cdot \Delta\overline{H} + \frac{\partial\overline{U}}{\partial\overline{G}} \cdot \Delta\overline{G} = a_{11} \cdot \Delta\overline{H} + a_{13} \cdot \Delta\overline{G}$$

- Equation of the mechanical power:

$$\Delta\overline{P}_m = \frac{\partial\overline{P}_m}{\partial\overline{H}} \cdot \Delta\overline{H} + \frac{\partial\overline{P}_m}{\partial\overline{U}} \cdot \Delta\overline{U} = a_{21} \cdot \Delta\overline{H} + a_{23} \cdot \Delta\overline{U}$$

- Transfer function F(s):

$$F(s) = \frac{\overline{U}_t - \overline{U}_0}{\overline{H}_t - \overline{H}_0} = -\frac{1}{\Phi_p + z_p \cdot \tanh(T_{ep} \cdot s)}$$

Ideal turbine: a_{11}= 0.5, a_{13}= 1, a_{21}= 1.5 and a_{23}= 1

$$\frac{\Delta\overline{P}_m}{\Delta\overline{G}} = \frac{1 - z_p \cdot T_{ep} \cdot s}{1 + 0.5 \cdot z_p \cdot T_{ep} \cdot s} = \frac{1 - T_{WP} \cdot s}{1 + 0.5 \cdot T_{WP} \cdot s}$$

Non ideal turbine:

$$\frac{\Delta\overline{P}_m}{\Delta\overline{G}} = a_{23} \cdot \frac{1 + \left(a_{11} - a_{13} \cdot a_{21} / a_{23}\right) \cdot T_{\varpi} \cdot s}{1 + a_{11} \cdot T_{\varpi} \cdot s}$$