# Chapter 6

# *Nonlinear Controllers (II): Based on the Lyapunov Function Technique*

This chapter presents the design of nonlinear controllers by using the Lyapunov function technique. In this second part the controllers are designed from nonlinear models of hydraulic turbines with surge tank effects. Moreover, this chapter proposes comparative studies, where the cost function ($f_{cost(B)}$ defined in Chapter 5) is used to compare these controllers with those designed by using the partial state feedback linearization technique.

This chapter is organised as follows: Section 6.1 presents an introduction. Section 6.2 proposes nonlinear controllers for a hydraulic power plant with surge tank effects. Section 6.3 describes comparative studies. Section 6.4 presents load rejection studies. Finally, Section 6.5 summarises the contents of this chapter.

## 6.1 Introduction

An important step for the design of nonlinear controllers by using the Lyapunov function technique is given in (Batlle, 1998) where there is designed a family of controllers from nonlinear models with no surge tank effects and a non-elastic water column in the penstock. That family of controllers guarantees asymptotic stability in a neighbourhood of the rotor speed ($w_r$=1 [pu]).

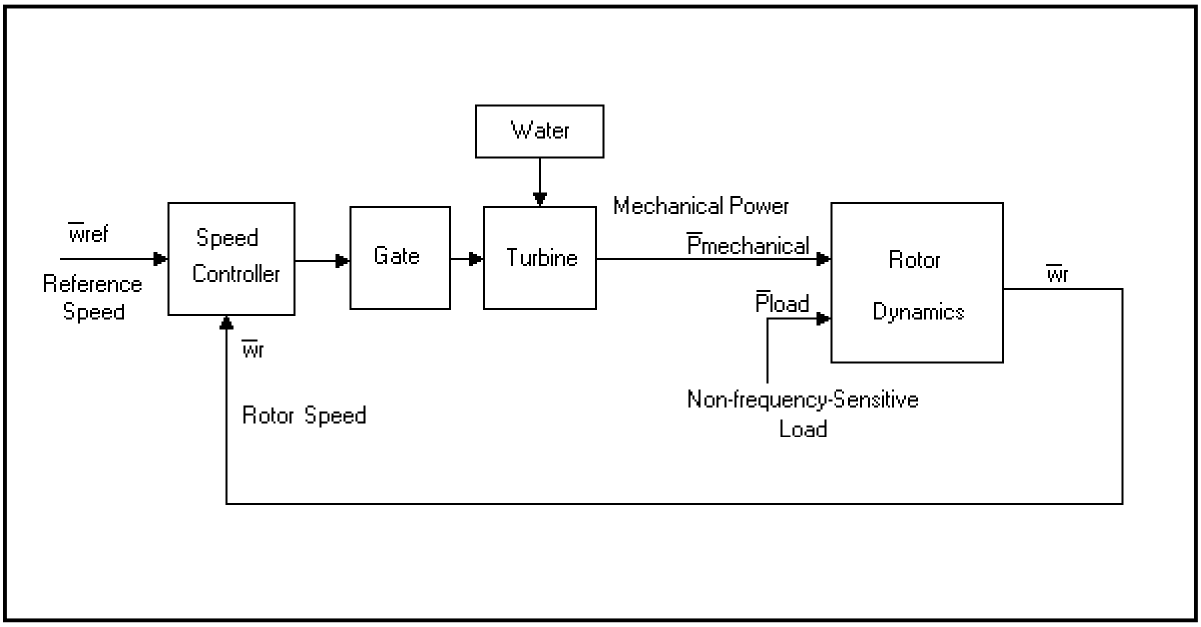


Figure 6.1: General block diagram showing the speed control loop.

Figure 6.1 represents in a functional block diagram the speed control loop. The frequency of a hydroelectric system depends on the active power balance of the system. When a change in the active power demand occurs ($\overline{P}_{load}$ in this case), this affects the power balance and hence the velocity of the turbine and the frequency of the synchronous generator. The speed controller is activated by varying the gate opening in order to change the water flow, then the turbine generates the necessary mechanical power and allows the rotor speed to reach the steady state value, which for a hydraulic plant in isolated conditions coincides with the speed reference value.

## 6.2 Nonlinear Controllers for Hydraulic Plants with Surge Tank Effects

In this section the design of nonlinear controllers based on the Lyapunov function technique from nonlinear models with surge tank effects is presented. The required steps to obtain the control laws and alternative cases are considered.

### 6.2.1 Models for Hydraulic Turbines with Surge Tank Effects

The equations of different dynamics described in Chapter 5 are rewriting in order to clarify the models needed for the design of the nonlinear controllers. The equations are:

- Dynamics of the penstock

$$\overline{H}_l = f_p \cdot \overline{U}_t^{\,2} \tag{6. 1}$$

$$\frac{d\overline{U}_t}{dt} = \frac{\overline{H}_r - \overline{H}_t - \overline{H}_l}{T_{WP}} \tag{6. 2}$$

$$\overline{U}_t = \overline{G} \cdot \sqrt{\overline{H}_t} \tag{6. 3}$$

- Mechanical power

$$\overline{P}_{mechanical} = A_t \cdot \overline{H}_t \cdot \left(\overline{U}_t - \overline{U}_{NL}\right) \tag{6. 4}$$

- Dynamic of the gate servomotor

$$T_g \cdot \frac{d\overline{G}}{dt} + \overline{G} = u \tag{6. 5}$$

- Equation of motion in the turbine

$$\overline{P}_{mechanical} - \overline{P}_{load} = 2 \cdot H \cdot \frac{d\overline{\omega}_r}{dt} + D \cdot \overline{\omega}_r \tag{6. 6}$$

- Dynamics of the tunnel

$$\overline{H}_{l2} = f_{p2} \cdot \overline{U}_c \cdot \left|\overline{U}_c\right| \tag{6. 7}$$

$$\frac{d\overline{U}_c}{dt} = \frac{\overline{H}_0 - \overline{H}_r - \overline{H}_{l2}}{T_{WC}} \tag{6. 8}$$

- Dynamics of the surge tank

$$\frac{d\overline{H}_r}{dt} = \frac{\overline{U}_c - \overline{U}_t}{C_s} \tag{6. 9}$$

Equations (6. 1) to (6. 9 represent the nonlinear model with non-elastic water columns (see the model WG4 in Chapter 3). The nonlinear model with an elastic water column in the penstock and a non-elastic water column in the tunnel (see the model QR51 in Chapter 3) is obtained by replacing equation (6. 2) by

$$\overline{H}_t = \overline{H}_r - \overline{H}_l - z_p \cdot \tanh\left(T_{ep} \cdot s\right) \cdot \overline{U}_t = \overline{H}_r - \overline{H}_l - z_p \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi}\right)^2\right)} \cdot \overline{U}_t \qquad (6.\ 10)$$

In this equation the hyperbolic tangent function has the approximation n=1 and is based on equation 3.12.

### 6.2.2 Construction of a Lyapunov Function

First, the above equations must be written as a nonlinear system in the state space. Hence, in the case of a model with surge tank effects and non-elastic water columns, the variables of the state system are $x_1 = \overline{U}_t$, $x_2 = \overline{H}_r$, $x_3 = \overline{U}_c$ and $x_4 = \overline{\omega}_r$. Thus, the combination of the above equations yields a nonlinear system in the state space. The first equation is obtained by combining equations (6. 1), (6. 2) and (6. 3). For the second, (6. 9 is used. For the third equation it is necessary to combine (6. 7) and (6. 8). Finally, for the fourth equation, (6. 4) and (6. 6) are combined. Thus, the equations in state space are given by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{T_{WP}} \cdot \left(x_2 - \left(f_p + \frac{1}{\overline{G}^2}\right) \cdot x_1^2\right) \\ \frac{x_3 - x_1}{C_s} \\ \frac{1}{T_{wc}} \cdot \left(\overline{H}_0 - x_2 - f_{p2} \cdot x_3 \cdot \left|x_3\right|\right) \\ -\frac{D}{2 \cdot H} \cdot x_4 - \frac{\overline{P}_{load}}{2 \cdot H} + \frac{A_t}{2 \cdot H} \cdot \left(x_1 - \overline{U}_{NL}\right) \cdot \frac{x_1^2}{\overline{G}^2} \end{pmatrix} \qquad (6.\ 11)$$

The nonlinear controller is found by following the procedure given in Batlle (1998) to obtain a controller designed from a model of a hydroelectric plant with no surge tank. For simplicity $f_p=f_{p2}=0$ are considered. The control effort u can be constructed from $\overline{G}$ by using (6. 5). Hence, $\overline{G}$ may be considered as the control signal.

The equilibrium point is calculated as

$$\begin{pmatrix}0\\0\\0\\0\end{pmatrix}=\begin{pmatrix}\frac{1}{T_{WP}}\cdot\left(x_2-\left(f_p+\frac{1}{\overline{G}^2}\right)\cdot x_1^2\right)\\ \frac{x_3-x_1}{C_s}\\ \frac{1}{T_{WC}}\cdot\left(\overline{H}_0-x_2-f_{p2}\cdot x_3\cdot|x_3|\right)\\ -\frac{D}{2\cdot H}\cdot x_4-\frac{\overline{P}_{load}}{2\cdot H}+\frac{A_t}{2\cdot H}\cdot\left(x_1-\overline{U}_{NL}\right)\cdot\frac{x_1^2}{\overline{G}^2}\end{pmatrix}$$

the solution for this system equation is given by

$$x_1^*=x_3^*=\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t},\ \text{and}\ \ x_2^*=x_4^*=1\quad .$$

It is necessary to change variables in (6. 11), so that the equilibrium point is mapped onto the origin (0,0,0,0):

$$x=x_1-x_1^*=x_1-\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\qquad\qquad y=x_2-x_2^*=x_2-1$$

$$z=x_3-x_3^*=x_3-\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\qquad\qquad w=x_4-x_4^*=x_2-1$$

Replacing these new variables in (6. 11) yields

$$\begin{pmatrix}\dot{x}\\\dot{y}\\\dot{z}\\\dot{w}\end{pmatrix}=\begin{pmatrix}\frac{1}{T_{WP}}\cdot\left[(y+1)-\frac{1}{\overline{G}^2}\cdot\left(x+\overline{U}_{NL}+\left(\frac{D+\overline{P}_{load}}{A_t}\right)\right)^2\right]\\ (z-x)/C_s\\ -y/T_{WC}\\ \frac{1}{2\cdot H}\cdot\left[-D\cdot(w+1)-\overline{P}_{load}+A_t\cdot\left(x+\left(\frac{D+\overline{P}_{load}}{A_t}\right)\right)\cdot\frac{1}{\overline{G}^2}\cdot\left(x+\overline{U}_{NL}+\left(\frac{D+\overline{P}_{load}}{A_t}\right)\right)^2\right]\end{pmatrix}\tag{6. 12}$$

The Lyapunov function is chosen so that the origin point (0,0,0,0) becomes an asymptotically stable fixed point of the system (6. 12). The chosen function is

$$V(x,y,z,w) = a\cdot x^2 + b\cdot y^2 + c\cdot z^2 + d\cdot w^2 + e\cdot x^4 + f\cdot y^4 + g\cdot z^4 + h\cdot w^4 \qquad (6.\ 13)$$

with $a \geq 0, b \geq 0, c \geq 0, d \geq 0, e \geq 0, f \geq 0, g \geq 0, h \geq 0$ and $a+b+c+d+e+f+g+h > 0$.

Lyapunov's theorem for local stability guarantees that if $V > 0$ (positive definite) and $\dot{V} \leq 0$ (negative semi-definite) then the equilibrium point (0,0,0,0) is stable (Slotine and Li, 1991).

Therefore, the first step is to calculate $\dot{V}$ and then multiply it by $\overline{G}^2$

$$\begin{aligned} \overline{G}^2 \cdot \dot{V} = \overline{G}^2 \cdot \big(& 2\cdot a\cdot x\cdot \dot{x} + 2\cdot b\cdot y\cdot \dot{y} + 2\cdot c\cdot z\cdot \dot{z} + 2\cdot d\cdot w\cdot \dot{w} + \\ & + 4\cdot e\cdot x^3\cdot \dot{x} + 4\cdot f\cdot y^3\cdot \dot{y} + 4\cdot c\cdot z^3\cdot \dot{z} + 4\cdot d\cdot w^3\cdot \dot{w}\big) = \\ = & -Q(x,y,z,w)\cdot \overline{G}^2 + P(x,y,z,w) \end{aligned} \qquad (6.\ 14\ )$$

where

$$\begin{aligned} P(x,y,z,w) = & -2\cdot \frac{a\cdot x\cdot \left(x + \overline{U}_{NL} + \dfrac{D+\overline{P}_{load}}{A_t}\right)}{T_{WP}} - 4\cdot \frac{e\cdot x^3\cdot \left(x + \overline{U}_{NL} + \dfrac{D+\overline{P}_{load}}{A_t}\right)}{T_{WP}} + \\ & + \left(d\cdot w + 2\cdot h\cdot w^3\right)\cdot \frac{A_t\cdot \left(x + \dfrac{D+\overline{P}_{load}}{A_t}\right)\cdot \left(x + \overline{U}_{NL} + \dfrac{D+\overline{P}_{load}}{A_t}\right)^2}{H} \end{aligned}$$

and

$$\begin{aligned} -Q(x,y,z,w) = & -2\cdot b\cdot y\cdot \left(\frac{z-x}{C_s}\right) + \frac{2\cdot c\cdot z\cdot y}{T_{WC}} - \frac{(2\cdot a\cdot x + 4\cdot e\cdot x^3)\cdot (y+1)}{T_{WP}} + \\ & + \frac{4\cdot g\cdot z^3\cdot y}{T_{WC}} + (2\cdot d\cdot w + 4\cdot h\cdot w^3)\cdot \left(\frac{\left(D\cdot (w+1) + \overline{P}_{load}\right)}{2\cdot H}\right) - \\ & - 4\cdot f\cdot y^3\cdot \left(\frac{z-x}{C_s}\right) \end{aligned} \qquad (6.\ 15)$$

The second step is to choose $\overline{G}$ so that $\dot{V} \leq 0$ and $\dot{V} = 0$ only at the origin. A positive definite function $\alpha(x, y, z, w)$ is introduced

$$-\alpha(x, y, z, w) = -Q(x, y, z, w) \cdot \overline{G}^2 + P(x, y, z, w) \tag{6. 16}$$

Hence, the feedback control, which results in a Lyapunov function, is given by

$$\overline{G}^2(x, y, z, w) = \frac{P(x, y, z, w) + \alpha(x, y, z, w)}{Q(x, y, z, w)} \tag{6. 17}$$

Moreover, it is necessary to choose $\alpha(x, y, z, w)$ in a way that $\overline{G}(x, y, z, w)$ is well defined in a neighbourhood of the origin. Therefore, the zeros of $Q(x, y, z, w)$ have to be cancelled out by choosing $\alpha(x, y, z, w)$ as

$$\alpha(x, y, z, w) = -P(x, y, z, w) + a_1 \cdot Q(x, y, z, w) + a_2 \cdot Q^3(x, y, z, w) + ........ \tag{6. 18}$$

By replacing (6. 18) in (6. 17), then:

$$\overline{G}^2(x, y, z, w) = \frac{P(x, y, z, w) + (-P(x, y, z, w) + a_1 \cdot Q(x, y, z, w) + a_2 \cdot Q^3(x, y, z, w) + .......)}{Q(x, y, z, w)} =$$

$$\overline{G}^2(x, y, z, w) = a_1 + a_2 \cdot Q^2(x, y, z, w) + ....... \tag{6. 19}$$

At the same time $\alpha(x, y, z, w)$ must be positive definite. This can be satisfied by choosing the parameters available so that the linear terms in (6. 18) are cancelled out, and the second order terms form a positive definite quadratic form.

The linear terms are

$$\text{linear part of } (\alpha(x, y, z, w)) = \text{linear part of } (-P(x, y, z, w) + a_1 \cdot Q(x, y, z, w)) \tag{6. 20}$$

where

$$\text{linear part of } (a_1 \cdot Q(x, y, z, w)) = a_1 \cdot \left[ -2 \cdot \frac{a \cdot x}{T_{WP}} - 2 \cdot d \cdot w \cdot \left( \frac{-D - \overline{G}^2 \cdot \overline{P}_{load}}{2 \cdot H} \right) \right]$$

$$\text{linear part of } (-P(x,y,z,w)) = -2\cdot\frac{a\cdot x\cdot\left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)^2}{T_{WP}} - \frac{d\cdot w\cdot\left(D+\overline{P}_{load}\right)\cdot\left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)^2}{H}$$

$$\text{linear part of } (\alpha(x,y,z,w)) = -2\cdot\frac{a\cdot x\cdot\left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)^2}{T_{WP}} - \frac{d\cdot w\cdot\left(D+\overline{P}_{load}\right)\cdot\left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)^2}{H} +$$
$$+a_1\cdot\left[-2\cdot\frac{a\cdot x}{T_{WP}}-2\cdot d\cdot w\cdot\left(\frac{-D-\overline{G}^2\cdot\overline{P}_{load}}{2\cdot H}\right)\right].$$

Since the linear part of $(\alpha(x,y,z,w))$ must be equal to zero, then:

$$a_1 = \left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)^2 \tag{6. 21}$$

The second order terms are:

$$\text{second order terms of } (\alpha(x,y,z,w)) = \frac{d\cdot w^2\cdot D\cdot a_1}{H} + \left(\frac{4\cdot a\cdot D}{T_{WP}\cdot A_t}+\frac{4\cdot a\cdot\overline{P}_{load}}{T_{WP}\cdot A_t}+\frac{4\cdot\overline{U}_{NL}}{T_{WP}}\right)\cdot x^2 -$$
$$-\left(\frac{6\cdot d\cdot D\cdot\overline{P}_{load}}{A_t\cdot H}+\frac{3\cdot d\cdot D^2}{A_t\cdot H}+\frac{d\cdot A_t\cdot\overline{U}_{NL}^{\,2}}{H}+\frac{4\cdot d\cdot\overline{U}_{NL}\cdot\overline{P}_{load}}{H}+\frac{3\cdot d\cdot\overline{P}_{load}^2}{A_t\cdot H}\right)\cdot w\cdot x$$

(6. 22)

Replacing $a_1$ in (6. 22); the following expression is obtained:

$$\text{second order terms of } (\alpha(x,y,z,w)) = 2\cdot\left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)\cdot$$
$$\cdot\left[\frac{2\cdot a}{T_{WP}}\cdot x^2+\frac{d\cdot D}{2\cdot H}\cdot\left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)\cdot w^2-\frac{d}{2\cdot H}\cdot\left(A_t\cdot\overline{U}_{NL}+3\cdot(D+\overline{P}_{load})\right)\cdot w\cdot x\right].$$

This can be made positive definite, a perfect square, if ‘d’ and ‘a’ satisfy the relation

$$\frac{d}{2\cdot H}\cdot\left(A_t\cdot\overline{U}_{NL}+3\cdot(D+\overline{P}_{load})\right)^2 = \frac{8\cdot a\cdot D}{T_{WP}}\cdot\left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)^2 \tag{6. 23}$$

Therefore the value of ‘a’ is

$$a = \frac{d \cdot \left(A_t \cdot \overline{U}_{NL} + 3 \cdot (D + \overline{P}_{load})\right)^2 \cdot T_{WP}}{16 \cdot H \cdot D \cdot \left(\overline{U}_{NL} + \dfrac{D + \overline{P}_{load}}{A_t}\right)^2} \tag{6. 24 }$$

Condition (6. 23) does not determine the coefficients $a_2$, $a_3$.... Therefore, those values could be chosen trying to make the neighbourhood of the origin where $\alpha(x, y, z, w)$ is positive as large as possible. The simplest choice is $a_2 = a_3 = .... = 0$, and replacing in (6. 19), the resulting control is

$$\overline{G}(x, y, z, w) = \left(\overline{U}_{NL} + \frac{D + \overline{P}_{load}}{A_t}\right) \tag{6. 25}$$

This equation is the controller law called Lyapunov 4 since it is related to the nonlinear model WG4, i.e. equations (6. 1) to (6. 9).

The equations (6. 1), (6. 3) to (6. 10) represent the nonlinear model QR51. These equations can be written as a nonlinear system in the state space (eight equations and eight state variables) and by utilising the same study applied in this Section it may be shown that equation (6. 25) is also the control law for that model. In this case the controller is called Lyapunov 51, since it is related to the nonlinear model QR51.

### 6.2.3 Consideration of ‘$a_2$’: Alternative Cases

Since the expression of the control is given by $\overline{G}^2(x, y, z, w) = a_1 + a_2 \cdot Q^2(x, y, z, w) + ..$, there are many options to consider in the design of the nonlinear controllers for hydraulic plants. This subsection proposes to add the term ‘$a_2$’ to the expression of the control.

In the first place, it is necessary to replace (6. 24 ), the value of ‘a’, in (6. 15); then,

$$\begin{aligned} -Q(x, y, z, w) = & -\frac{d \cdot \left(\overline{U}_{NL} \cdot A_t + 3 \cdot D + 3 \cdot \overline{P}_{load}\right)^2 \cdot x \cdot (y+1)}{8 \cdot H \cdot D \cdot \left(\overline{U}_{NL} + \dfrac{D + \overline{P}_{load}}{A_t}\right)^2} - 2 \cdot b \cdot y \cdot \left(\frac{z - x}{C_s}\right) + \frac{2 \cdot c \cdot z \cdot y}{T_{WC}} + \\ & + (2 \cdot d \cdot w + 4 \cdot h \cdot w^3) \cdot \left(\frac{\left(D \cdot (w+1) + \overline{P}_{load}\right)}{2 \cdot H}\right) - \frac{4 \cdot e \cdot x^3 \cdot (y+1)}{T_{WP}} - \\ & - 4 \cdot f \cdot y^3 \cdot \left(\frac{z - x}{C_s}\right) + \frac{4 \cdot g \cdot z^3 \cdot y}{T_{WC}} \end{aligned}$$

Two control laws are presented bellow

<u>Considerations for the *Case 1*:</u> b = c = e = f = g = 0

$$\overline{G}=\sqrt{\left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)^2+a_2\left[\frac{d\cdot\left(\overline{U}_{NL}\cdot A_t+3\cdot D+3\cdot\overline{P}_{load}\right)^2\cdot x\cdot(y+1)}{8\cdot H\cdot D\cdot\left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)^2}-(2\cdot d\cdot w+4\cdot h\cdot w^3)\cdot\left(\frac{\left(D\cdot(w+1)+\overline{P}_{load}\right)}{2\cdot H}\right)\right]^2} \quad (6.\ 26)$$

In this case the control law uses the state variables x (turbine flow), y (surge tank head) and w (rotor speed). The turbine flow and the surge tank head are rather difficult to measure; hence, to implement this control law, state observers are required.

<u>Considerations for the *Case 2*:</u> b = c = d = e = f = g = 0

$$\overline{G}=\sqrt{\left(\overline{U}_{NL}+\frac{D+\overline{P}_{load}}{A_t}\right)^2+a_2\left[4\cdot h\cdot w^3\cdot\left(\frac{\left(D\cdot(w+1)+\overline{P}_{load}\right)}{2\cdot H}\right)\right]^2} \quad (6.\ 27)$$

The responses of the rotor speed of these two control laws (equations (6. 26) and (6. 27)) are compared with the control law given by equation (6. 25) (Lyapunov 4). The results are presented in the next section.

## 6.3 Comparative Studies

This section presents comparisons, by means of the cost functions $f_{cost(A)}$ and $f_{cost(B)}$, of the controllers designed in this chapter and some controllers designed in Chapter 5.

### 6.3.1 Comparisons of Hydro Plants with Surge Tank Effects

This subsection proposes the comparison studies between the Lyapunov 4 and Lyapunov 51 nonlinear controllers by using the cost functions $f_{cost(A)}$ and $f_{cost(B)}$. In these studies the parameters from the IEEE Working Group (1992) are utilised (i.e. the Parameters 1 of Table 3.5, Chapter 3). Furthermore, these studies verify the constraints of maximum gate opening rate and maximum gate closing rate for all controllers. Typical values for these constraints are 0.16 [pu/s] (Kundur, 1994).

#### 6.3.1.1 Comparison of Cost Function Values Using $f_{cost(A)}$

Figure 6.2 shows the comparison of the cost function values obtained for the Lyapunov51 and Lyapunov 4 controllers.

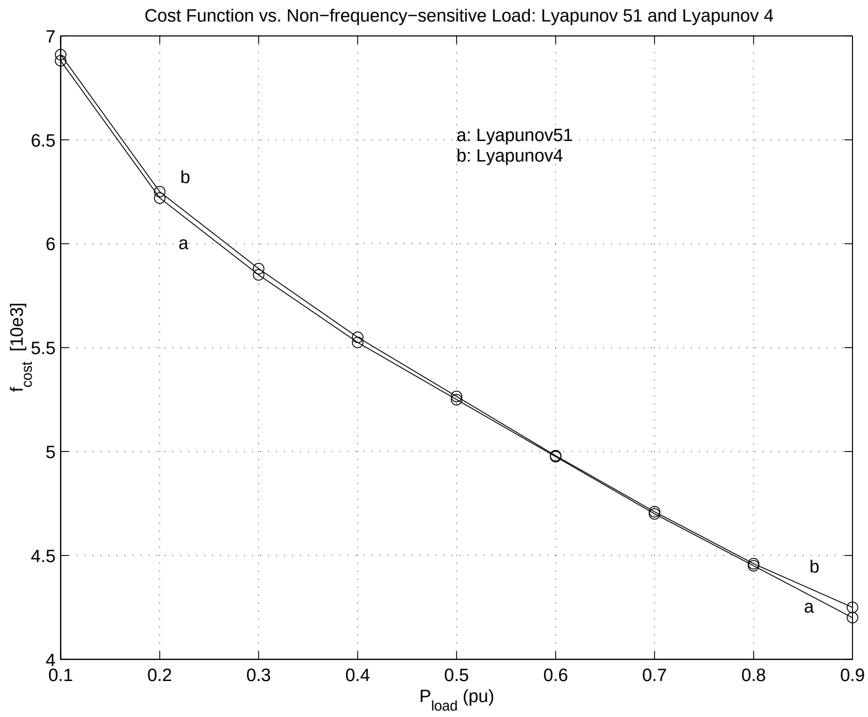


Figure 6.2: Comparison of the cost function ($f_{cost(A)}$) for the Lyapunov 4 and Lyapunov51 controllers.

This figure shows interesting results. The first one is that the differences between the values of the cost function for the Lyapunov 4 and Lyapunov 51 controllers are around one per cent.

From the point of view of the cost function ($f_{cost(A)}$), the second result points out that these controllers do not improve plant behaviour. The cost function values of both controllers are greater than the value obtained with the NL C controller (Chapter 5).

The third result is in relation to the model complexity of the hydraulic plant. When the model complexity is increased, this means considering an elastic water column in the penstock, and by using the models WG4 and QR 51 of the hydraulic turbine, the results from the cost function point of view are equivalent, differing only in a small value of one per cent. This is an important result since it shows that increasing the hydraulic model complexity does not mean better results, i.e. a low cost function.

### 6.3.1.2 Comparison of Rotor Speed Behaviour

Figures 6.3 and 6.4 show the rotor speed response for the disturbance $\overline{P}_{load}$ from 0.8 [pu] to 0.9 [pu] for the Lyapunov 4 and Lyapunov 51 controllers.

Figure 6.4 shows in detail the little difference in the rotor speed value between both controllers. Moreover, it can be observed that to reach the steady-state value of the rotor speed a considerable period of time is needed, and this effect is reflected in large values of the cost function ($f_{cost(A)}$) for these controllers.

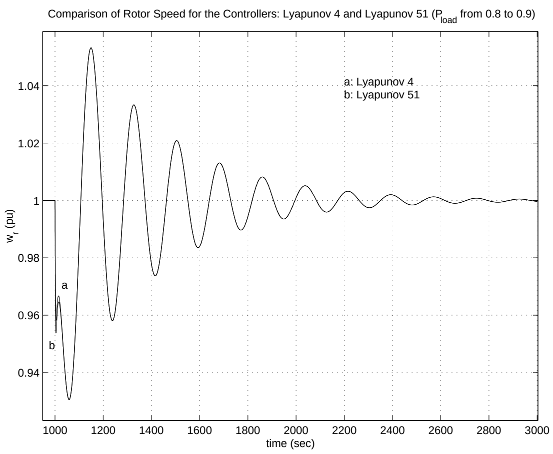


Figure 6.3: Comparison of rotor speed.

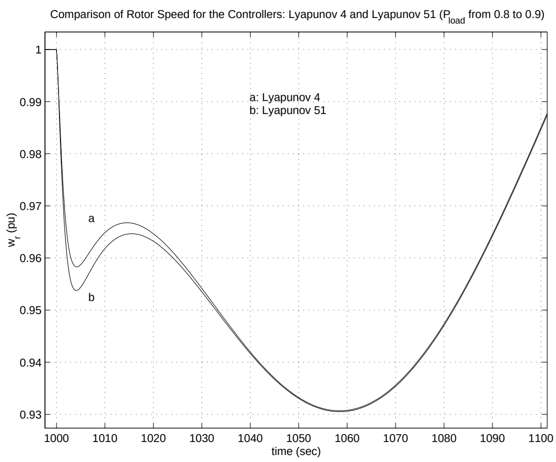


Figure 6.4: Comparison of rotor speed, detail.

The control law of Lyapunov 4 given by (6. 25) and the alternative cases given by (6. 26) and (6. 27) are compared; however, the speed rotor has the same responses for theses three control laws. This can be observed in a graphic that is similar to the representation of Lyapunov 4, which is depicted in Figure 6.4. This is due to the fact that the terms that are multiplied by ‘$a_2$’ are four orders of magnitude smaller than the term given by ‘$a_1$’.

### 6.3.1.3 Comparison of Cost Function Values Using $f_{cost(B)}$

Figure 6.5 shows the comparison of the cost function ($f_{cost(B)}$) for the Lyapunov51, Lyapunov 4, NL C, PID, PI-PD and NL D controllers.

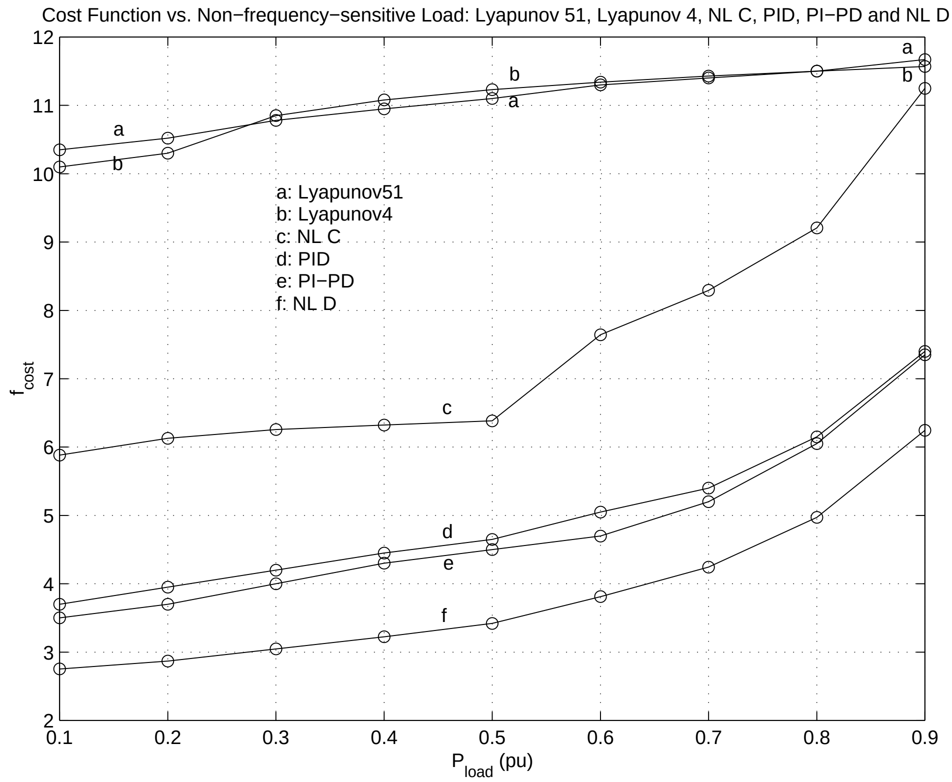


Figure 6.5: Comparison of the cost function for the controllers: Lyapunov 4, Lyapunov51, NL C, PID, PI-PD and NL D ($f_{cost(B)}$).

Moreover, Figure 6.5 points out that for the 0.9 [pu] load the cost function value of the Lyapunov 4 and Lyapunov51 controllers are respectively 2.7 and 3.5 per cent greater than the cost function value of the controller NL C. Those differences for the remaining points are increased due to the slow damping.

When $f_{cost(B)}$ is considered, the values of the cost function of the Lyapunov 4 and Lyapunov51 controllers decrease when the values of $\overline{P}_{load}$ decrease. On the other hand when $f_{cost(A)}$ is taken into account, the cost function values for the Lyapunov 4 and Lyapunov51 controllers increase when the values of $\overline{P}_{load}$ decrease. This is due to the penalisation of large values of time considered in the cost function $f_{cost(A)}$.

### 6.3.2 Comparison of Hydro Plants with no Surge Tank Effects

Batlle (1998) presents the design of a controller from a nonlinear model of a hydroelectric plant without a surge tank and a non-elastic water column in the penstock. This means that the hydroelectric model used in that paper is WG2 (equations (6. 1) to (6. 6)), so the controller could be called Lyapunov 2. In these studies the parameters from St. Lawrence power plant are used (i.e. the Parameters 5, Table 3.3 of Chapter 3).

#### 6.3.2.1 Comparison of Cost Function Values Using $f_{cost(A)}$

The analyses for different operating points defined by the non-frequency-sensitive load $\overline{P}_{load}$ are performed. The parameters of the controllers are adjusted according to the minimal value of the cost function after applying a step function on the disturbance $\overline{P}_{load}$ from 0.8 [pu] to 0.9 [pu]. This operating point corresponds to the worst case for a fixed parameter controller of a hydroelectric power plant.

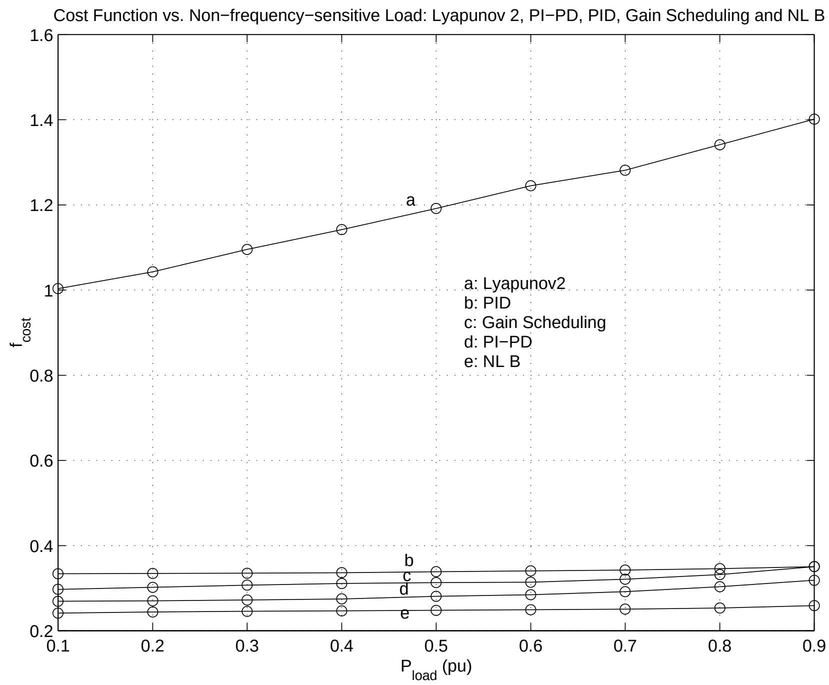


Figure 6.6: Comparison of the cost function for the Lyapunov 2, Gain Scheduling PID, PI-PD and NL B controllers ($f_{cost(A)}$).

Figure 6.6 shows the differences in the cost function values between the Lyapunov 2 controller, and the Gain Scheduling PID, PI-PD and NL B controllers when the $f_{cost(A)}$ is considered. Once more, these differences are due to the penalisation of large values of time considered in the cost function $f_{cost(A)}$.

#### 6.3.2.2 Comparison of Cost Function Values Using $f_{cost(B)}$

This point presents the comparison of cost function values taking into account $f_{cost(B)}$, which does not penalise large values of time duration.

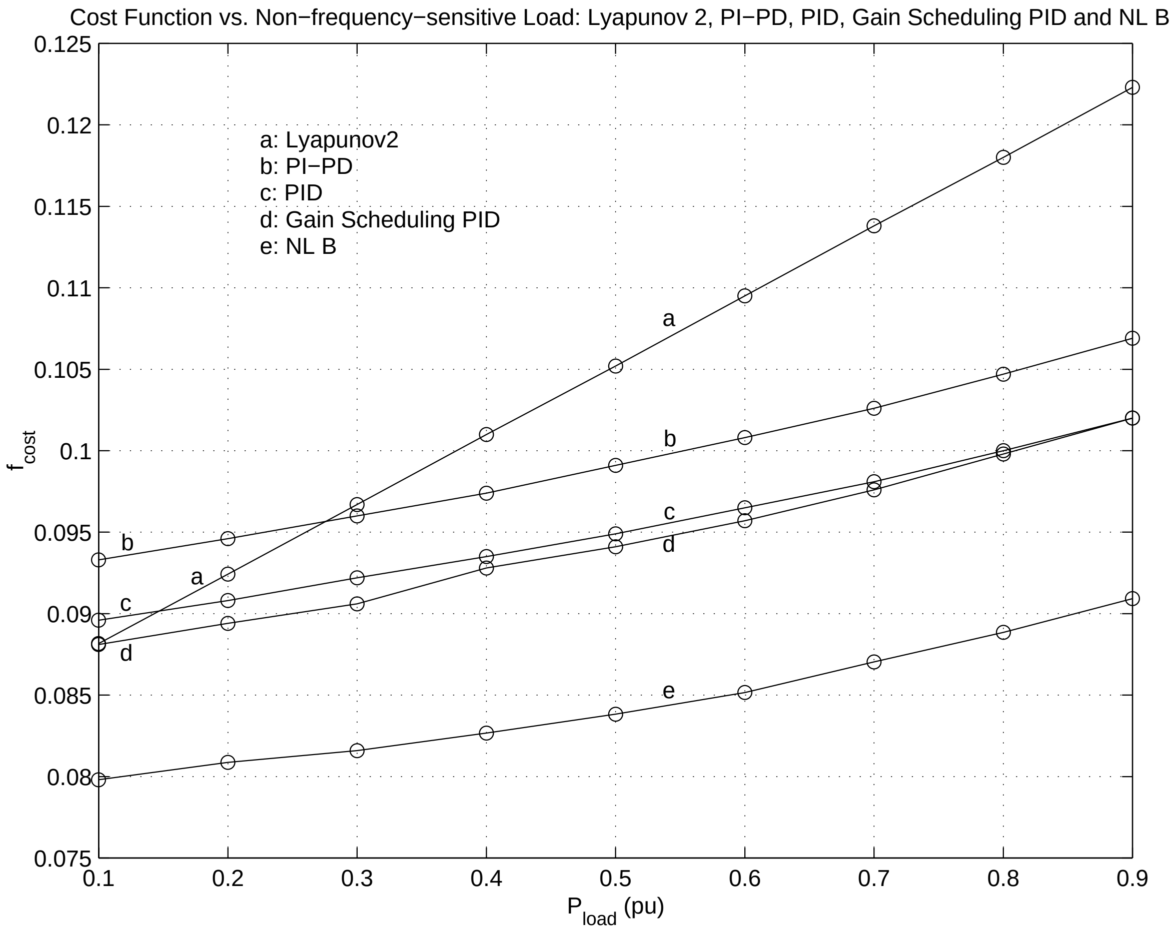


Figure 6.7: Comparison of the cost function for the Lyapunov 2, PI-PD, PID, Gain Scheduling PID and NL B controllers ($f_{cost(B)}$).

Figure 6.7 points out the comparison of the $f_{cost(B)}$ for Lyapunov 2, PI-PD, PID, Gain Scheduling PID and NL B controllers.

For the 0.9 [pu] load the cost function value for the Lyapunov 2 controller is greater than the value for the PI-PD controller (16.5 per cent). For decreasing loads, the cost function values of the Lyapunov 2 controller decrease more rapidly than the cost function values of the PI-PD, PID and Gain Scheduling controllers. For the 0.1 [pu] load the cost function value for the Lyapunov 2 controller is almost coincident with the cost function value for the Gain Scheduling PID, which is 10 per cent greater than the cost value for the NL B controller.

## 6.4 Load Rejection Studies

This study is performed for the nonlinear controllers Lyapunov 2, 4 and 51. The comparison of the rotor speed for different loads is presented for the Lyapunov 4 controller. Moreover, three figures where the cost function $f_{cost(B)}$ versus discrete increments of non-frequency-sensitive load ( $\Delta\overline{P}_{load}$ ) for these three controllers are represented.

### 6.4.1 Study for the Lyapunov 2 Controller

Figure 5.48 shows a “linear” relation between $f_{cost(B)}$ and $\Delta\overline{P}_{load}$ . This relation is due to in this case there are not surge tank effects and the controller reaches the steady state rapidly.

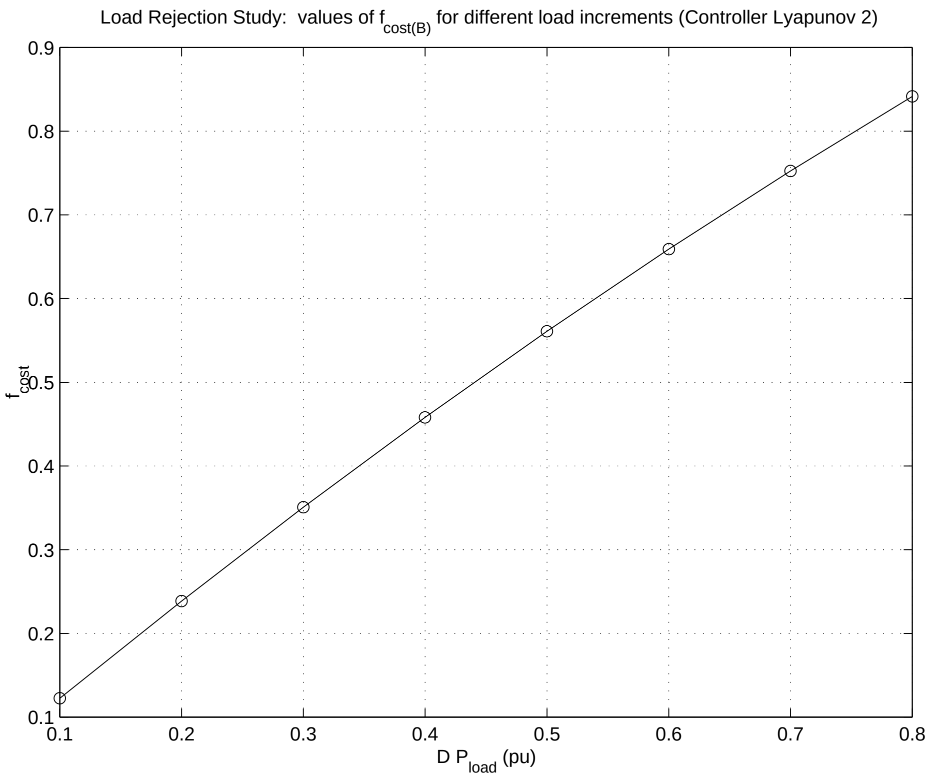


Figure 6.8: Representation of the relation between $f_{cost(B)}$ and $\Delta\overline{P}_{load}$ .

### 6.4.2 Study for the Lyapunov 4 and Lyapunov 51 Controllers

Figure 6.9 depicts the load rejection study of the nonlinear Lyapunov 4 controller for two different loads.

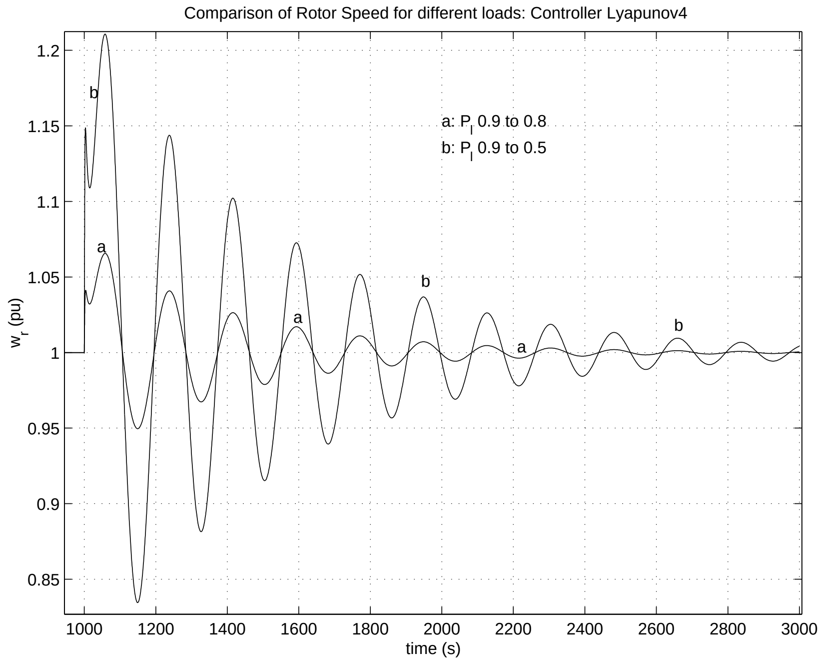


Figure 6.9: Load rejection study of the Lyapunov 4 controller for two different loads.

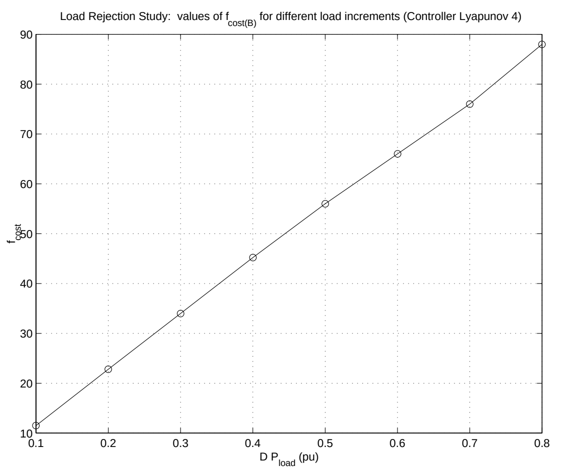


Figure 6.10: Graphic of the relation between $f_{cost(B)}$ and $\Delta\overline{P}_{load}$ , Lyapunov 4.

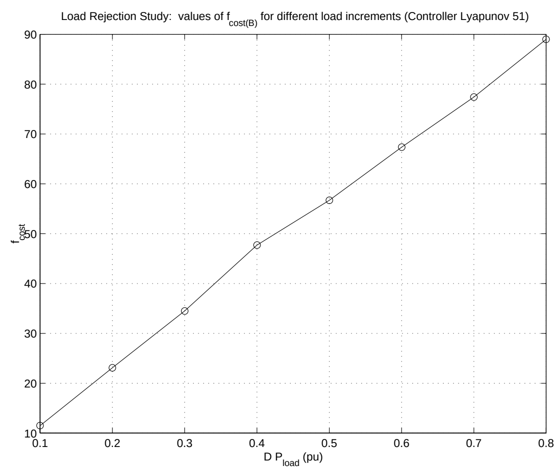


Figure 6.11: Graphic of the relation between $f_{cost(B)}$ and $\Delta\overline{P}_{load}$ , Lyapunov51.

Figure 6.10 and 6.11 show "linear" relations between $f_{cost(B)}$ and $\Delta\overline{P}_{load}$ for the Lyapunov 4 and 51 controllers, respectively. These are due to the effects of the surge tank, and that the cost function $f_{cost(B)}$ penalises the speed errors, the actions of the controller that can produce damage (i.e. the amplitude of the manoeuvres), and the duration of both.

## 6.5 Summary and Conclusions

Though the Lyapunov 4 and Lyapunov 51 controllers from the cost function ($f_{cost(A)}$) point of view have poor responses, they show that by increasing the complexity of the hydroelectric turbine model, the response of the system does not improve and the difference between their cost function values is around one per cent.

When the cost function $f_{cost(B)}$ is considered, the Lyapunov 4 and Lyapunov 51 controllers have cost function values of the same order of magnitude as the NL C, PID, PI-PD and NL D controllers, although the cost function values of the Lyapunov 4 and Lyapunov 51 controllers are greater than the values of NLC, PID, PI-PD and NLD controllers.

On the other hand, when the comparison study using $f_{cost(B)}$ is considered, an interesting result from the cost function point of view is obtained for the Lyapunov2 controller, since at some operating points the cost function is equal to the values of the cost function when using the PID or PI-PD controllers.

The terms that are multiplied by ‘$a_2$’ are smaller than the term ‘$a_1$’ and do not improve the rotor speed responses of the Lyapunov 4 controller, and as a consequence the cost function values are the same.

The load rejection studies show that the relation between $f_{cost(B)}$ versus $\Delta\overline{P}_{load}$ is “linear” when using the Lyapunov 2, 4 and 51 controllers. This is due to the fact that the cost function $f_{cost(B)}$ penalises the speed errors and its duration, and the actions of the controller, as well as the amplitude of the manoeuvres that can produce damage, and its duration.