

# ***Table of Contents***

<b>Table of Contents</b>	<b>I</b>
<b>List of Variables and Parameters</b>	<b>IX</b>
<b>List of Figures</b>	<b>XIII</b>
<b>List of Tables</b>	<b>XIX</b>
<b>1. Introduction</b>	<b>1</b>
1.1 THE MOTIVATION .....	1
1.2 THE OBJECTIVES .....	2
1.2.1 To Deepen the Knowledge of Hydroelectric System Models .....	2
1.2.2 To Design Controllers from Well Proven Models .....	2
1.3 RESEARCH AIMS.....	2
1.3.1 Hydroelectric Generation Systems .....	2
1.3.2 The Frequency Control.....	5
1.4 ORGANISATION OF THE DISSERTATION .....	8
<b>2. Previous Related Work</b>	<b>11</b>
2.1 MODELS OF HYDROELECTRIC SYSTEMS .....	11
2.1.1 Nonlinear Models.....	12
2.1.2 Linearized Models.....	13
2.1.3 Models for Particular Applications.....	14
2.2 FREQUENCY (SPEED) CONTROLLERS.....	15
2.2.1 Controllers Designed from Linearized Models .....	15

2.2.1.1	Classical Controllers .....	15
2.2.1.2	Controllers with Specific Characteristics .....	17
2.2.2	Controllers Designed from Nonlinear Models .....	18
2.2.3	Controller Implementation .....	18
2.3	CONCLUSIONS .....	18

### **3. Hydroelectric System Models** 21

3.1	PRELIMINARY CONCEPTS .....	21
3.1.1	Definitions.....	23
3.1.2	System Dynamic Equations .....	24
3.1.3	Linearized Equations.....	25
3.1.4	Classification of the Models .....	26
3.2	NONLINEAR MODELS .....	26
3.2.1	Models with Surge Tank Effects.....	28
3.2.1.1	Model with an elastic water column in the penstock and a non-elastic water column in the tunnel (Kundur, 1994) - Model K5, K51, K52 .....	28
3.2.1.2	Model with an elastic water column in the penstock and a non-elastic water column in the tunnel (IEEE Working Group, 1992; Quiroga and Riera, 1999) - Models WG5, QR52, QR51.....	29
3.2.1.3	Model with non-elastic water columns (Kundur, 1994) – Model K4 .....	31
3.2.1.4	Model with non-elastic water columns (IEEE Working Group, 1992) – Model WG4.....	31
3.2.1.5	Comparisons between the models with an elastic water column in the penstock and a non-elastic water column in the tunnel (3.2.1.1 and 3.2.1.2) .....	31
3.2.1.6	Comparison between models with non-elastic water columns (3.2.1.3 and 3.2.1.4) .....	33
3.2.2	Models with no Surge Tank Effects.....	34
3.2.2.1	Model with an elastic water column in the penstock (Kundur, 1994) – Models K3, K32, K31 .....	34
3.2.2.2	Model with an elastic water column in the penstock (IEEE Working Group, 1992; Quiroga and Riera, 1999) – Models WG3, QR33, QR32, QR31 .....	34
3.2.2.3	Model with a non-elastic water column in the penstock (Kundur, 1994) – Model K2 .....	35
3.2.2.4	Model with a non-elastic water column in the penstock (IEEE Working Group, 1992) – Model WG2 .....	35
3.2.2.5	Comparison between the models 3.2.2.1 and 3.2.2.2 .....	35

3.3 LINEARIZED MODELS .....	36
3.3.1 Models with Surge Tank Effects .....	37
3.3.1.1 Model with an elastic water column in the penstock and a non-elastic water column in the tunnel (Quiroga, 1998a) – Model $Q_{lin}$ .....	37
3.3.1.2 Model with non-elastic water columns (Quiroga, 1998a) – Model $Q_{lin0}$ .....	37
3.3.2 Models with no Surge Tank Effects .....	37
3.3.2.1 Model with an elastic water column in the penstock (Kundur, 1994) – Model $K_{lin}$ .....	37
3.3.2.2 Models with a non-elastic water column in the penstock (Gaden, 1945) – Model $G_{lin0}$ .....	38
3.4 CONCLUSIONS OF THE PRESENTED MODELS .....	38
3.5 STATIC ANALYSIS.....	39
3.5.1 Determination of the Flow of the Turbine in Steady State.....	40
3.5.1.1 Deduction of $\bar{U}_{tss}$ for a nonlinear model with surge tank effects .....	40
3.5.1.2 Deduction of $\bar{U}_{tss}$ for a nonlinear model with no surge tank effects .....	41
3.5.1.3 Determination of $\bar{U}_0$ used in the models of Kundur (1994).....	41
3.5.1.4 Determination of $\bar{U}_0$ by using the comparison between equations .....	43
3.5.1.5 Verification of supposition made in 3.5.1.4 .....	43
3.5.2 Determination of the Mechanical Power.....	44
3.5.2.1 Calculations for nonlinear models with and with no surge tank effects.....	44
3.5.2.2 Calculations for a linearized model with and with no surge tank effects....	45
3.5.2.3 Calculation of the gate opening ( $\bar{G}$ ) for $\bar{P}_{mecss} = 0$ .....	46
3.6 TIME DOMAIN ANALYSIS OF MODELS .....	48
3.6.1 Nonlinear Models with Surge Tank Effects .....	48
3.6.1.1 Models WG4, QR51, QR52, WG5 .....	48
3.6.1.2 Models K4, K51, K52 .....	52
3.6.2 Linearized Models with Surge Tank Effects .....	54
3.6.2.1 Model $Q_{lin0}$ .....	55
3.6.3 Nonlinear Models with no Surge Tank Effects .....	56
3.6.3.1 Models WG2, QR31, QR32, QR33.....	57
3.6.3.2 Models K2, K31, K32 .....	58
3.6.4 Linearized Model with no Surge Tank Effects.....	59
3.6.4.1 Model $K_{lin}$ .....	59

3.6.4.2	Classic linear model with ideal turbine (Gaden, 1945) - Model $G_{lin0}$	60
3.6.5	Conclusions of Time Domain Analysis	60
3.7	FREQUENCY RESPONSE ANALYSIS OF MODELS	62
3.7.1	Models with no Surge Tank Effects	62
3.7.1.1	Models $K_{lin}$ and $G_{lin0}$	62
3.7.1.2	Stability study	63
3.7.2	Models with Surge Tank Effects	64
3.7.2.1	Models $Q_{lin}$ and $Q_{lin0}$	64
3.7.2.2	Stability Study	66
3.8	SUGGESTIONS FOR MODELLING HYDROELECTRIC POWER PLANTS	69
3.8.1	Models with Surge Tank Effects	69
3.8.1.1	Nonlinear Models	69
3.8.1.2	Linearized Models	70
3.8.2	Models with no Surge Tank Effects	70
3.8.2.1	Nonlinear Models	70
3.8.2.2	Linearized Models	70
3.9	SUMMARY	70
<b>4.</b>	<b>Identification of a Hydroelectric Power Plant</b>	<b>73</b>
4.1	CHARACTERISTICS OF THE HYDROELECTRIC POWER STATION	74
4.2	GENERAL NONLINEAR EQUATIONS	75
4.2.1	Adjustment of the Equations	79
4.2.2	Output Power Adjustment	79
4.2.3	Surge Tank Natural Period	80
4.3	SIMULATION RESULTS	81
4.3.1	Identification Using the Models A	81
4.3.2	Identification Using the Models B	84
4.3.3	Identification Using the Model C	86
4.3.4	Identification Using the Model D	87
4.4	COMPARATIVE STUDY OF THE QUADRATIC ERROR	88
4.5	SUMMARY AND CONCLUSIONS	89

<b>5. Nonlinear Controllers (I): Based on the Partial State Feedback Linearization Technique</b>	<b>91</b>
5.1 INTRODUCTION .....	92
5.2 MODELS FOR HYDRAULIC TURBINES.....	94
5.3 NONLINEAR CONTROLLERS FOR HYDRAULIC TURBINES WITH NO SURGE TANK EFFECTS	95
5.3.1 The Cost Functions.....	95
5.3.1.1 The Cost Function A .....	96
5.3.1.2 The Cost Function B .....	96
5.3.2 PID Controllers.....	96
5.3.2.1 Fixed PID controllers .....	96
5.3.2.2 PI-PD controller .....	97
5.3.2.3 Gain Scheduling Controllers .....	97
5.3.3 Nonlinear Controllers .....	98
5.3.4 The Zero Dynamics of Nonlinear System with no Surge Tank Effects .....	101
5.3.5 Comparative Studies Using $f_{cost(A)}$ .....	102
5.3.5.1 Comparison of Rotor Speed Behaviour for Different Load Changes .....	104
5.3.5.2 Comparison of Cost Function Values ( $f_{cost(A)}$ ) .....	105
5.3.6 Controller Adjustment Surfaces Using $f_{cost(A)}$ .....	106
5.3.6.1 Adjustment for the PI-PD Controller .....	107
5.3.6.2 Adjustment for the Nonlinear Controller NL B .....	108
5.3.7 Comparative Studies Using $f_{cost(B)}$ .....	109
5.3.7.1 Comparison of Cost Function Values ( $f_{cost(B)}$ ).....	110
5.3.8 Controllers Adjustment Surfaces Using $f_{cost(B)}$ .....	111
5.3.8.1 Adjustment for the PID Controller .....	111
5.3.8.2 Adjustment for the Nonlinear Controller NL B .....	113
5.4 NONLINEAR CONTROLLERS FOR HYDRAULIC TURBINES WITH SURGE TANK EFFECTS.....	114
5.4.1 Nonlinear Controllers .....	114
5.4.2 The Zero Dynamics of Nonlinear System with Surge Tank Effects .....	117
5.4.3 Comparative Studies Using $f_{cost(A)}$ .....	119
5.4.3.1 Comparison of Rotor Speed Behaviour for Different Load Changes .....	119
5.4.3.2 Comparison of Cost Function Values $f_{cost(A)}$ .....	120

5.4.4 Controller Adjustment Surfaces Using $f_{cost(A)}$ .....	121
5.4.4.1 Adjustment for the PI-PD Controller .....	121
5.4.4.2 Adjustment for the Nonlinear Controller NL D .....	122
5.4.5 Comparative Studies Using $f_{cost(B)}$ .....	123
5.4.5.1 Comparison of Cost Function Values $f_{cost(B)}$ .....	123
5.4.6 Controller Adjustment Surfaces Using $f_{cost(B)}$ .....	124
5.4.6.1 Adjustment for the PI-PD Controller .....	124
5.4.6.2 Adjustment for the Nonlinear Controller NL D .....	126
5.5 LOAD REJECTION STUDIES .....	127
5.5.1 Study for the Controller NL B.....	127
5.5.2 Study for the Controller NL D .....	128
5.6 SUMMARY AND CONCLUSIONS.....	128
<b>6. Nonlinear Controllers (II): Based on the Lyapunov Function Technique</b>	<b>131</b>
6.1 INTRODUCTION.....	132
6.2 NONLINEAR CONTROLLERS FOR HYDRAULIC PLANTS WITH SURGE TANK EFFECTS ..	133
6.2.1 Models for Hydraulic Turbines with Surge Tank Effects .....	133
6.2.2 Construction of a Lyapunov Function.....	134
6.2.3 Consideration of ' $a_2$ ': Alternative Cases.....	139
6.3 COMPARATIVE STUDIES .....	140
6.3.1 Comparisons of Hydro Plants with Surge Tank Effects .....	140
6.3.1.1 Comparison of Cost Function Values Using $f_{cost(A)}$ .....	140
6.3.1.2 Comparison of Rotor Speed Behaviour .....	142
6.3.1.3 Comparison of Cost Function Values Using $f_{cost(B)}$ .....	142
6.3.2 Comparison of Hydro Plants with no Surge Tank Effects .....	143
6.3.2.1 Comparison of Cost Function Values Using $f_{cost(A)}$ .....	142
6.3.2.2 Comparison of Cost Function Values Using $f_{cost(B)}$ .....	145
6.4 LOAD REJECTION STUDIES .....	146
6.4.1 Study for the Lyapunov 2 Controller .....	146
6.4.2 Study for the Lyapunov 4 and Lyapunov 51 Controllers .....	146
6.4 SUMMARY AND CONCLUSIONS.....	148
<b>7. Conclusions and Contributions</b>	<b>149</b>
7.1 CONCLUSIONS .....	149

7.2 GENERAL CONTRIBUTIONS .....	152
7.3 FUTURE WORK.....	153
<b>Appendix A</b>	<b>157</b>
<b>Appendix B</b>	<b>165</b>
<b>References</b>	<b>169</b>



# *List of Variables and Parameters*

- a: wave velocity [m/s].  
A: cross section [ $m^2$ ].  
 $A_c$ : tunnel cross section [ $m^2$ ].  
 $A_p$ : penstock cross section [ $m^2$ ].  
 $A_s$ : surge tank cross section [ $m^2$ ].  
 $A_t$ : turbine gain [pu].  
 $c_{1,2,3}$ : weight coefficients of the cost function [pu].  
D: load-damping constant [pu].  
 $D_1$ : turbine damping [pu/pu].  
 $D \cdot \bar{\omega}_r$ : frequency-sensitive load [pu].  
E: Young's modulus of pipe wall material.  
f: wall thickness of penstock [m].  
 $f_0$ : surge chamber orifice head loss coefficient [pu].  
 $f_p, f_{p1}$ : penstock head loss coefficient [pu].  
 $f_{p2}$ : tunnel head loss coefficient [pu].  
g: acceleration due to gravity [ $m^2/s$ ].  
G : gate opening [%]  
 $\bar{G}$  : gate opening [pu].  
 $\bar{G}_0$  : initial value of the gate opening [pu].

$\bar{G}_d$ : desired value of the gate opening [pu].

$H$ : inertia constant [MW·s/MVA].

$\bar{H}$ : hydraulic head at gate [pu].

$H_0$ : total head [m].

$\bar{H}_0$ : initial steady state value of  $\bar{H}$  [pu].

$H_{base}$ : base head [m].

$H_1 + H_{12}$ : head losses [m].

$\bar{H}_1$ : head loss in the penstock [pu].

$\bar{H}_{12}$ : head loss in the tunnel [pu].

$\bar{H}_r$ : surge tank head [pu].

$\bar{H}_{rss}$ : steady state value of the surge tank head [pu].

$\bar{H}_t$ : turbine head [pu].

$k_f$ : Head loss constant due to friction [pu].

$K_p$ : proportional gain of a PID.

$K_{p1}$ : proportional gain of a PID.

$K_i$ : integral gain of a PID.

$K_d$ : derivative gain of a PID.

$L_c$ : length of the tunnel [m].

$L_p$ : length of the penstock [m].

$P_{elec}$ : electric power [MW].

$\bar{P}_{electric}$ : electric power [pu], where:  $\bar{P}_{electric} = \bar{P}_{load} + D \cdot \bar{\omega}_t$ .

$\bar{P}_{mechanical}$ : turbine mechanical power [pu].

$\bar{P}_{mecss}$ : steady state value of the turbine mechanical power [pu].

$\bar{P}_{load} \equiv P_l$ : non-frequency-sensitive load [pu].

$Q$ : flow [ $m^3/s$ ].

$Q_{base}$ : base flow [ $m^3/s$ ].

$Q_{max}$ : maximum low [ $m^3/s$ ].

$Q_{rated}$ : rated flow [ $m^3/s$ ].

$R_{1,2,3,4}$ : feedback gains [pu].

$R_p$ : temporary droop [pu].

$T$ : surge tank natural period [s].

$T_e$ : elastic time [s].

$T_{ec}$ : elastic time of the tunnel [s].

$T_{ep}$ : elastic time of the penstock [s].

$T_g$ : main servo time constant [s].

$T_p$ : pilot valve and servomotor time constant [s].

$T_{\bar{w}}$ : water starting time at any load [s].

$T_w$ : water starting time at rated or base load [s].

$T_{wc}$ : water starting time of the tunnel at rated or base load [s].

$T_{wp}$ : water starting time of the penstock at rated or base load [s].

$u$ : control effort [pu].

$U$ : water velocity [m/s]

$\bar{U}$ : water velocity [pu].

$\bar{U}_0$ : initial steady state value of water velocity or steady-state flow [pu].

$\bar{U}_c$ : water velocity in the tunnel or tunnel flow [pu].

$\bar{U}_{css}$ : steady state value of the tunnel flow [pu].

$\bar{U}_p$ : water velocity in the penstock or penstock flow [pu].

$U_{rated}$ : rated water velocity [m/s].

$\bar{U}_t$ : water velocity in the turbine or turbine flow [pu].

$\bar{U}_{tss}$ : steady state value of the turbine flow [pu].

$\bar{U}_s$ : water velocity in the surge tank or surge tank flow [pu].

$\bar{U}_{NL}$ : no-load flow [pu].

$x_1, x_2, x_3, x_4$ : state variables [pu].

$x, y, z, w$ : state variables [pu].

$z_1, z_2, z_3, z_4$ : state variables [pu].

$z_0$ : hydraulic surge impedance of the conduit.

$z_c$ : hydraulic surge impedance of the tunnel.

$z_n$ : normalised hydraulic surge impedance of the conduit.

$z_p$ : hydraulic surge impedance of the penstock.

$\Delta\bar{P}_m$ : deviation of the mechanical power [pu].

$\Delta\bar{\omega}$ : deviation of the rotor speed [pu].

$\phi$ : internal penstock diameter [m].

$\phi_i$ : initial diameter of the penstock [m].

$\phi_f$ : final diameter of the penstock [m].

$\Phi_c$ : friction coefficient of the tunnel [pu].

$\Phi_p$ : friction coefficient of the penstock [pu].

$\kappa$ : bulk modulus of water [ $\text{kg}/\text{m} \cdot \text{s}^2$ ].

$\eta(\bar{G})$ : nonlinear function [pu].

$\rho$ : density of water [ $\text{kg}/\text{m}^3$ ].

$\omega$ : angular frequency [ $\text{rad}/\text{s}$ ] ( $s=j\omega$ ).

$\bar{\omega}_r$ : rotor speed [pu].

$\bar{\omega}_{ref}$ : reference speed [pu].

# *List of Figures*

Figure 1.1: Representation of a hydroelectric power plant .....	4
Figure 1.2: Functional block diagram that shows the relation between the hydroelectric system and the controls for a complete system.....	6
Figure 3. 1: Plot of the distribution of parameters in a hydraulic power plant.....	23
Figure 3. 2: Plot of the distribution of heads and flows in a hydraulic power plant .....	23
Figure 3. 1: Block Diagram of the hydroelectric system used in the models of Kundur (1994) .....	26
Figure 3.4: Functional diagram of model WG5 from the IEEE Working Group (1992) including associated dynamics.....	30
Figure 3. 5: Diagram of heads and flows distribution in models WG3, QR33, QR32, QR31 and WG2 .....	36
Figure 3.6: Plot of $\bar{U}_0$ in function of the variable $\bar{G}$ for two hydroelectric power stations ...	42
Figure 3.7: Mechanical power generated by the turbine in hydroelectric plants with surge tank effects as function of $\bar{G}$ .....	46
Figure 3.8: Representation of the block used for the calculation of the hyperbolic tangent (model WG5) .....	48
Figure 3.9: Representation of the block that can be used for the calculation of the hyperbolic tangent function in the models WG4 (n=0), QR51 (n=1) and QR52 (n=2).....	49
Figure 3.10: Comparison among the models WG4, QR51, QR52 and WG5, detail .....	51

Figure 3.11: Comparison among the models WG4, QR51, QR52 and WG5 .....	51
Figure 3.12: Block diagram for the models of Kundur (1994) and his derived models .....	52
Figure 3.13: Comparison among the models K4, K51 and K52, detail .....	53
Figure 3.14: Comparison among the models K4, K51 and K52 .....	53
Figure 3.15: Simulation of the linearized model $Q_{lin0}$ , for a variation of 0.01 [pu] in the gate opening, detail .....	55
Figure 3.16: Simulation of the linearized model $Q_{lin0}$ , for a variation of 0.01 [pu] in the gate opening .....	55
Figure 3.17: Functional scheme of a model WG3 from (IEEE Working Group, 1992) .....	56
Figure 3.18: Comparison among the models WG2, QR31, QR32, QR33 and WG3.....	57
Figure 3.19: Comparison among the models K2, K31 and K32 .....	58
Figure 3.20: Simulation of the model $K_{lin}$ ( $n=0$ ) by using the parameters of G-Gilboa 3, St. Lawrence 32 and Niagara 1 .....	59
Figure 3.21: Simulation of the classic linear model $G_{lin0}.....$	60
Figure 3.22: Bode plot for models of $K_{lin}$ , approximations $n = 0, 1$ . Parameters B Gilboa3 ..	62
Figure 3.23: Nyquist diagram for the model of $K_{lin}$ , approximation $n = 0$ . Parameters from B Gilboa3 .....	63
Figure 3.24: Nyquist diagram for the model of $K_{lin}$ , approximation $n = 1$ . Parameters from B Gilboa3 .....	63
Figure 3.25: Bode plot for the model of $Q_{lin0}$ , approximation $n = 0$ . Parameters from (IEEE Working Group, 1992) .....	64
Figure 3.26: Bode plot for the model of $Q_{lin}$ , approximations $n = 1, 2$ . Parameters from (IEEE Working Group, 1992) .....	65
Figure 3.27: Bode plot for the model of $Q_{lin0}$ and $Q_{lin}$ , approximations $n = 0, 1$ . Parameters from Appalachia power plant .....	65
Figure 3.28: Nyquist diagram for the model $Q_{lin}$ , approximation $n = 0$ . Parameters from (IEEE Working Group, 1992).....	66
Figure 3.29: Nyquist diagram for the model of $Q_{lin}$ , approximation $n = 1$ . Parameters from (IEEE Working Group, 1992).....	67
Figure 3.30: Detail of Figure 3.29.....	67

Figure 3.31: Nyquist diagram for the model $Q_{lin}$ , approximation $n = 2$ . Parameters from (IEEE Working Group, 1992).....	68
Figure 3.32: Detail of Figure 3.31 .....	68
Figure 4.1: Plot of heads and flows distribution in a hydroelectric plant with surge tank effects .....	74
Figure 4.2: Functional diagram for a general nonlinear model with surge tank effects .....	77
Figure 4.3: Schematic representation of the “In-Out” block used to calculate the exact hyperbolic tangent given by the mathematical expression $z_p \cdot \tanh(T_{ep} \cdot s)$ .....	78
Figure 4.4: Representation of the block used to calculate the approximations $n=0$ , $n=1$ and $n=2$ of the hyperbolic tangent function.....	78
Figure 4.5: Plot of the nonlinear function $\eta(\bar{G})$ calculated by the quadratic and cubic polynomials.....	80
Figure 4.6: Identification of Susqueda using the Models A .....	82
Figure 4.7: Identification of Susqueda using the Models A .....	83
Figure 4.8: Detail of Figure 4.7 .....	83
Figure 4.9: Identification of Susqueda using the Models A .....	83
Figure 4.10: Detail of Figure 4.9 .....	83
Figure 4.11: Identification of Susqueda using the Models A .....	83
Figure 4.12: Detail of Figure 4.11 .....	83
Figure 4.13: Identification of Susqueda using the Models A .....	83
Figure 4.14: Detail of Figure 4.13 .....	83
Figure 4.15: Identification of Susqueda using the Models B .....	85
Figure 4.16: Identification of Susqueda using the Model C.....	86
Figure 4.17: Identification of Susqueda using the Model C.....	86
Figure 4.18: Identification of Susqueda using the Model D .....	87
Figure 4.19: Identification of Susqueda using the Model A .....	88
Figure 4.20: Detail of Figure 4.19 .....	88

Figure 5.1: Functional block diagram showing the relation between the hydroelectric system and the controls for a complete system.....	93
Figure 5.2: General speed control scheme for a generic controller.....	94
Figure 5.3: PID Controller .....	97
Figure 5.4: PI-PD Controller.....	97
Figure 5.5: Nonlinear Controller A (NL A).....	100
Figure 5.6: Nonlinear Controller B (NL B).....	100
Figure 5.7: General speed control scheme for the controllers NL A or NL B .....	101
Figure 5.8: Comparison of rotor speed .....	104
Figure 5.9: Comparison of rotor speed .....	104
Figure 5.10: Comparison of rotor speed .....	105
Figure 5.11: Comparison of rotor speed .....	105
Figure 5.12: Comparison of rotor speed .....	105
Figure 5.13: Comparison of rotor speed .....	105
Figure 5.14: Comparison of cost function values ( $f_{cost(A)}$ ) .....	106
Figure 5.15: Adjustment surface for the PI-PD controller ( $f_{cost(A)}$ ) .....	107
Figure 5.16: Plane $K_i = 0.7$ .....	107
Figure 5.17: Plane $K_p = 0.5$ .....	107
Figure 5.18: Adjustment surface for the controller NL B ( $f_{cost(A)}$ ) .....	108
Figure 5.19: Plane $K_i = 0.3$ .....	108
Figure 5.20: Plane $K_p = 1.5$ .....	108
Figure 5.21: Comparison of cost function ( $f_{cost(B)}$ ).....	111
Figure 5.22: Adjustment surface for the PID controller ( $f_{cost(B)}$ ).....	112
Figure 5.23: Plane $K_d = 1.75$ .....	112
Figure 5.24: Plane $K_p = 2.75$ .....	112
Figure 5.25: Adjustment surface for the controller NL B ( $f_{cost(B)}$ ) .....	113

Figure 5.26: Plane $K_i = 0.3$ .....	113
Figure 5.27: Plane $K_p = 1.5$ .....	113
Figure 5.28: Nonlinear Controller C (NL C).....	116
Figure 5.29: Nonlinear Controller D (NL D) .....	117
Figure 5.30: General speed control scheme for the controllers NL C or NL D .....	117
Figure 5.31: Comparison of rotor speed.....	120
Figure 5.32: Comparison of cost function for the controllers PID, PI-PD, Gain Scheduling PI-PD and NL D ( $f_{cost(A)}$ ).....	120
Figure 5.33: Adjustment surface for the PI-PD controller ( $f_{cost(A)}$ ) .....	121
Figure 5.34: Plane $K_i = 0.04$ .....	121
Figure 5.35: Plane $K_p = 1$ .....	121
Figure 5.36: Adjustment surface for the controller NL D ( $f_{cost(A)}$ ) .....	122
Figure 5.37: Plane $K_i = 0.75$ .....	122
Figure 5.38: Plane $K_p = 1$ .....	122
Figure 5.39: Comparison of cost function for the controllers: PID, PI-PD and NL D ( $f_{cost(B)}$ ) .....	124
Figure 5.40: Adjustment surface for the PI-PD controller ( $f_{cost(B)}$ ) .....	125
Figure 5.41: Plane $K_i = 0.04$ .....	125
Figure 5.42: Plane $K_p = 1$ .....	125
Figure 5.43: Adjustment surface for the controller NL D ( $f_{cost(B)}$ ) .....	126
Figure 5.44: Plane $K_i = 0.75$ .....	126
Figure 5.45: Plane $K_p = 0.25$ .....	126
Figure 5.46: Load rejection study of the controller NL B for three different loads .....	127
Figure 5.47: Representation of the relation between $f_{cost(A)}$ and $\Delta\bar{P}_{load}$ .....	127
Figure 5.48: Load rejection study of the controller NL D for two different loads .....	128
Figure 5.49: Graphic of the relation between $f_{cost(A)}$ and $\Delta\bar{P}_{load}$ .....	128
Figure 6.1: General block diagram showing the speed control loop.....	134

Figure 6.2: Comparison of the cost function ( $f_{cost(A)}$ ) for the Lyapunov 4 and Lyapunov51 controllers .....	141
Figure 6.3: Comparison of rotor speed .....	142
Figure 6.4: Comparison of rotor speed, detail.....	142
Figure 6.5: Comparison of the cost function for the controllers: Lyapunov 4, Lyapunov51, NL C, PID, PI-PD and NL D ( $f_{cost(B)}$ ) .....	143
Figure 6.6: Comparison of the cost function for the controllers: Lyapunov 2, Gain Scheduling PID, PI-PD and NL B ( $f_{cost(A)}$ ) .....	144
Figure 6.7: Comparison of the cost function for the controllers: Lyapunov 2, PI-PD, PID, Gain Scheduling PID and NL B ( $f_{cost(B)}$ ).....	145
Figure 6.8: Representation of the relation between $f_{cost(B)}$ and $\Delta\bar{P}_{load}$ .....	146
Figure 6.9: Load rejection study of the controller Lyapunov 4 for two different loads .....	147
Figure 6.10: Graphic of the relation between $f_{cost(B)}$ and $\Delta\bar{P}_{load}$ , Lyapunov 4 .....	147
Figure 6.11: Graphic of the relation between $f_{cost(B)}$ and $\Delta\bar{P}_{load}$ , Lyapunov51 .....	147

# *List of Tables*

Table 3.1: List of parameters.....	22
Table 3.2: List of variables.....	23
Table 3. 3: Table of nonlinear models.....	27
Table 3. 4: Table of linearized models.....	36
Table 3.5: Parameters for different power plants .....	47
Table 4.1: Plant Characteristics.....	75
Table 4.2: Conduits Characteristics .....	75
Table 4.3: Parameters of Susqueda power station.....	76
Table 4.4: Values of the nonlinear function $\eta(\bar{G})$ for different gate positions deduced from experimental tests .....	79
Table 4.5: Polynomials obtained from the first and third columns of Table 4.4 using the method of the least squares .....	80
Table 4.6: Description of the Models A and their references.....	81
Table 4.7: Description of the Models B and their references.....	84
Table 4.8: Description of the Model C and its references.....	86
Table 4.9: Description of the Model D and its references.....	87

Table 4.10: Quadratic error found using the models A, where the nonlinear function $\eta(\bar{G})$ is approximated by two polynomials with degree 2 and 10 .....	89
Table 5. 1: Parameters of PID, PI-PD, NL A and NL B controllers .....	103
Table 5. 2: Meanings of the parameters of the controllers.....	103
Table 5. 3: Values of the parameters for the Gain Scheduling PID and the Gain Scheduling PI-PD ( $f_{cost(A)}$ ) .....	104
Table 5.4: Parameters of PID, PI-PD, NL A and NL B controllers .....	109
Table 5.5: Parameter values of the Gain Scheduling PID and Gain Scheduling PI-PD ( $f_{cost(B)}$ ) .....	110
Table 5.6: Parameters of the PI-PD, PID, NL C and NL D controllers .....	119
Table 5.7: Parameters of the PI-PD, PID, NL C and NL D controllers .....	123