

CHAPTER 6

SICONOS Platform

*“Part of the inhumanity of the computer is that,
once it is competently programmed and working smoothly,
it is completely honest.”*

ISAAC ASIMOV (1920-1992)

In this chapter the SICONOS software, dedicated to simulation of non-smooth dynamical systems (NSDS), is presented. After motivating the development of this tool, we give a overview of the SICONOS software and the way NSDS are modeled and simulated within the platform. Routines for analysis (stability, bifurcations, invariant manifolds, ...) of NSDS implemented in the platform are explained in detail. To conclude, several representative samples are shown in order to illustrate the SICONOS platform capabilities.

6.1 Introduction and motivation

SICONOS software development is part of an European project involving different research teams (more details on <http://siconos.inrialpes.fr>) and is dedicated to modeling, simulation, analysis and control of nonsmooth dynamical systems (NSDS). Basically, the SICONOS platform aims at providing a general and common tool for NSDS present in various scientific fields as applied mathematics, electrical networks, mechanics, robotics, etc. Currently, researchers in different areas of engineering and applied science

often write their own numerical codes for dealing with systems characterised by nonsmooth nonlinearities. These codes are typically specific to the system of interest. The SICONOS platform will try to fill this gap creating a new software for solving nonsmooth problems under a common framework.

The scarcity of theoretical results connected to the characterisation of nonsmooth dynamics has made impossible the development of a general simulation tool for NSDS similar to those available for their smooth counterpart. For smooth systems, software packages such as MATLAB, SCILAB, SIMULINK, DSTOOL, CONTENT and AUTO97 (and its more recent version AUTO2000) allow the time-simulation of smooth systems as well as the continuation of trajectories and their bifurcations. Up to now, only a recently numerical tool called SLIDECONT, an AUTO97 driver for sliding bifurcation analysis of Filippov systems, has been developed. SLIDECONT has the ability to continue equilibria, limit cycles, and their sliding bifurcations but to date it still lacks the capability to perform direct numerical simulations of Filippov systems and automatically switch between sliding and non-sliding motions. Moreover, this tool is only useful for Filippov systems and therefore mechanical systems with impact and other NSDS are not included.

In parallel to numerical analysis studies of NSDS, a few pieces of software incorporating specific nonsmooth tools for complementarity systems have been developed. The existing software developed either for hybrid systems (MODELLICA, MatLab/Simulink/Stateflow, Dymola, etc.) or mechanical systems (Adams, Mechanical, Simpack) do not include specific tools for the simulation of complementarity systems (like LCP or NCP solvers, state re-initialization rules, and/or event detection modules), or propose inadequate models (like contact models in mechanics with hardly identifiable parameters or contact models yielding odd results like contact forces with the wrong sign during the course of the integration), to say nothing of how accumulations of state jumps (Zeno behaviour) are treated numerically. Consequently potential users of nonsmooth mechanical or electrical systems with state reinitializations and unilateral constraints, most often either have to develop their own code, or to adapt existing software not dedicated to their application. Except for very simple cases (one degree of freedom mechanical systems with separated impacts), such tasks rapidly grow cumbersome and unreliable.

None of the codes mentioned above includes routines for continuation of the trajectories and their bifurcations. Issues such as the structural

stability of these systems as well as their robustness to parameter variations and external disturbances are therefore difficult to assess numerically. Consequently there is an urgent need for appropriate numerical simulation packages, both for academic use in control, robotics, civil and mechanical engineering departments, and for industrial use for virtual prototyping and testing of complementarity systems of relevance in applications.

This chapter is devoted to present an overview of the SICONOS software and the capacities developed until now. The chapter is organized as follows. An overview of the software, the different NSDS models and the simulation techniques considered in the platform are described in section 2. Section 3 is devoted to the presentation of the tools for numerical analysis, such as calculations of basins of attraction and bifurcation diagrams, implemented in the software. Finally, three specific examples of nonsmooth problems, solved with the SICONOS platform, are briefly presented in section 4. We study two electrical systems, a buck converter and a parallel resonant converter (PRC), and a mechanical system, an impact oscillator, in order to show the accuracy of the platform.

6.2 NSDS in SICONOS platform

6.2.1 Overview of SICONOS platform

The SICONOS platform is mainly dedicated to modeling and simulation of NSDS. SICONOS is a free software, under GPL GNU license, available on the Gforge web pages (<http://siconos.gforge.inria.fr/>) of the project, where one can also find documentation, support and all that sort of utilities. SICONOS is mainly composed in four parts: Front-End, Numerics, Analysis and Kernel.

Front-End provides interfaces with some specific command-languages such as PHYTON or SCILAB. This supplies more pleasant and easy-access tools for users during pre and post treatment. The Numerics part holds all low-level algorithms to compute basic well-identified problems (ordinary differential equations, LCP, QP solvers, Blas-Lapack linear algebra routines...). The Analysis part supplies a toolbox for analysis of NSDS as stability, bifurcations, domains of attractions, etc. Finally, the Kernel is the core of the software, providing high level description of the studied system and numerical solving strategies. It is fully written in C++, and is composed of several modules. The Utils module contains tools, mainly to handle classical objects such as matrices or vectors. The Input-Output

module is concerned with objects for data management in XML format, thanks to the libxml2 library. Finally, a plugin system is also available, essentially to allow users to provide his own computation methods for some specific functions (vector fields of the dynamical system, mass, viscosity matrix,...) without having to re-compile the whole platform.

6.2.2 NSDS modeling in SICONOS platform

As has been explained during this thesis, a nonsmooth dynamical system is a set of dynamical systems that interact altogether in a nonsmooth way. Therefore, three objects are mainly needed: the *Dynamical System*, the *Relations* between constrained variables and state variables, and the definition of the *Nonsmooth Law* between the constrained variables.

The role of the Kernel modeling part is to provide tools for these systems description. Main objects are the above mentioned `DynamicalSystem`, `Relation` and `NonSmoothLaw`, both embedded in `Iinteraction`. In the following paragraphs, the types of systems, relations and laws implemented in the SICONOS software will be described in more detail.

6.2.2.1 Dynamical Systems

The most general case available in the platform is a first order system of the form

$$\dot{x} = f(x, \dot{x}, t) + T(x)u(x, \dot{x}, t) + r \quad (6.1)$$

where r is the nonsmooth part (typically contact forces for mechanical systems). The terms Tu introduce the control variable into the system.

All other dynamical systems in the software are derivations from the one above. They are:

- **Linear Dynamical Systems:**

$$\dot{x} = A(t)x + Tu(t) + b(t) + r \quad (6.2)$$

- **Lagrangian (second order) systems**, which usually appear in mechanical problems:

$$M(q)\ddot{q} + C(q, \dot{q}) = F_{int}(q, \dot{q}, t) + F_{ext}(t) + r \quad (6.3)$$

where q denotes the generalized coordinates, M the mass matrix, C the nonlinear inertia operator, F_{int} the internal nonlinear forces and F_{ext} the external forces depending only on time.

- **Lagrangian Linear Time Invariant systems:**

$$M\ddot{q} + C\dot{q} + Kq = F_{ext}(t) + r \quad (6.4)$$

where C and K are respectively the classical viscosity and stiffness matrices.

The dimension of the state vector can range from a few degrees of freedom to more than several hundred thousand.

6.2.2.2 Relations

In a general way, the dynamical system is completed by a set of nonsmooth laws. The set of such variables, denoted by y , on which we apply the constraints, depends, in a very general way, of the state vector x , the time t and possibly the force r :

$$y = h(x, r, t) \quad (6.5)$$

In the same way, we have to specify the relation between r , the force due to the constraints, and λ (λ is associated to y through a nonsmooth law):

$$r = g(x, \lambda, t) \quad (6.6)$$

Any other relation is derived from this general one. Possible cases are:

- **Linear Time Invariant Case.** In the linear time invariant case the relations are directly given by matrices defined by:

$$y = Cx + Fu + D\lambda + e, \quad (6.7)$$

$$r = B\lambda + a \quad (6.8)$$

- **Lagrangian system.** In Lagrangian systems, the structure of these relations is very particular and we assume that they can be written as:

$$y = h(q), \quad (6.9)$$

$$\dot{y} = H^T(q)\dot{q}, \quad (6.10)$$

$$r = H(q)\lambda \quad (6.11)$$

- **Lagrangian Linear system.** We can also consider the linear case such as

$$y = H^T q + b, \quad (6.12)$$

$$\dot{y} = H^T \dot{q}, \quad (6.13)$$

$$r = H\lambda \quad (6.14)$$

which can be stated from the beginning or derived by a linearization procedure of (6.9).

6.2.2.3 NonSmooth Laws

Several nonsmooth laws may be formulated in the SICONOS platform. The different laws are:

- Complementarity condition or unilateral contact:

$$0 \leq y \perp \lambda \geq 0 \quad (6.15)$$

- Newton impact law:

$$\text{if } y(t) = 0, \quad 0 \leq \dot{y}(t^+) + e\dot{y}(t^-) \perp \lambda \geq 0 \quad (6.16)$$

- Relay

$$\begin{cases} \dot{y} = 0, |\lambda| \leq 1 \\ \dot{y} \neq 0, \lambda = \text{sign}(y) \end{cases} \quad (6.17)$$

- Unilateral contact and Coulomb's Friction, $y = [y_n, y_t]^T, \lambda = [\lambda_n, \lambda_t]^T,$

$$\text{if } y_n = 0, \begin{cases} 0 \leq \dot{y}_n \perp \lambda_n \geq 0 \\ \dot{y}_t = 0, ||\lambda_t|| \leq \mu\lambda_n \\ \dot{y}_t \neq 0, \lambda_t = -\mu\lambda_n \text{sign}(\dot{y}_t) \end{cases} \quad (6.18)$$

- Piece-wise linear relations associated to saturation, relay with dead zone, etc.

6.2.3 Simulation techniques in SICONOS platform

A first step towards understanding dynamics of nonsmooth transitions is often to perform direct numerical simulations, where it is of great importance that the time and location of any nonsmooth events are resolved as accurately as possible. This idea can be compared with an alternative one for simulating nonsmooth systems, which is to recast the nonsmoothness in terms of a complementarity formulation. Then one can use time stepping methods accompanied with nonsmooth problems (NSP) solvers to simulate the systems without the need for accurate event detection. That is, the solver can only note that one or more events have occurred during a time step without finding the actual event time and location. Such methods have been proven to be effective in simulating mechanical systems with a large number of constraints. However, they suffer from the disadvantage that they are typically only low-order algorithms and nonsmooth events can be lost. Another strategy to simulate nonsmooth systems, that has been adopted in different papers [], is to approach the non-smooth system by a smooth one. The dynamics of the resulting approximate system is then governed by differential equations with sufficient smoothness to be handled through standard numerical techniques. However, a drawback of this method is that an accurate simulation requires the use of very stiff approximate laws. This results a long time simulation because the time-stepping procedures have to resort to a very small step-length. Moreover, the effect of the artificial modifications may blur the simulation results.

For the time being, time stepping is the only method implemented in the SICONOS platform together with nonsmooth problem solvers to simulate the systems and Moreau time-stepping is the only available strategy. Several strategies as Adams or Lsodar will be available in the future as well as event-driven schemes. The nonsmooth problems solvers available in SICONOS software can be classify as follows:

- Linear Complementarity Problem solvers. The available solvers are Lemke method, Lemke lexicographic method, Latin method, Non-Smooth Newton method, Non Linear Gauss-Seidel method and Conjugated Projected Gradient method.
- Primal relay problems solvers. The only available solver use a Latin method.
- Primal resolution of contact problems with friction. There are several algorithms depending on the dimension of the problem.

- Resolution of problems with impacts. A Newton impact law is applied.

6.3 Routines for analysis in SICONOS

The numerical tools for the analysis of NSDS implemented in the SICONOS software until now are aimed to the calculations of bifurcation diagrams and domains of attraction. The numerical algorithms presented in this section have been partially developed during my stages at the University of Bristol (England) and INRIA Rhone-Alpes (France), with the collaboration of Petri Piiroinen and Franck P erignon.

6.3.1 Domain of attraction routines

In order to get a complete global analysis for non-smooth dynamical systems we need to give the entire phase portrait, with attractors, their basins of attraction, invariant manifolds, unstable limit sets, etc. Basically, there are two different methods to simulate domains of attraction (DOA), namely the *brute force* method and the *cell mapping* method. The method of brute force consists in trying a large number of initial conditions given by the region of interest and watching what comes out. The drawback of this method is the necessity of simulating a large number of initial conditions for a long time in order to obtain interesting features. On the contrary, the brute force method [84], technique of approximating a map using discrete cells, reduces the amount of computational work needed to get a reasonably accurate picture of basins of attraction for dynamical systems. The method implemented in the SICONOS platform is the cell mapping and in the following part will be explained in detail.

6.3.1.1 Cell mapping method

The method of cell mapping for the study of dynamical systems was developed by Hsu [84] who also provided a detailed mathematical foundation for the technique. The dynamic of the system is formulated as a mapping using, for example, a Poincar e map or a stroboscopic map. Then the region of interest is divided into a cell grid and we map cells to cells using the center point of each cell. We map each center point (initial condition) under the point mapping and locate the cell containing the image. This defines a cell mapping. A sketch is given in Figure 6.1. The cells have

been numbered from left to right and from bottom to top, and the cell number 0 has been given to the region outside of the cell grid, the so called sink cell. The sink cell is conventionally mapped to itself. In Table 6.1 and Table 6.2, it is shown part of the outputs for the global analysis of the cell mapping. In the cell table, for each cell its image under the cell mapping is given, which periodic cycle it will be attracted to (i.e. the group it belongs to), the period of this attractor cycle and the number of iterates before landing on the cycle. The group table records for each periodic cycle its period and the cells that form each periodic cycle.

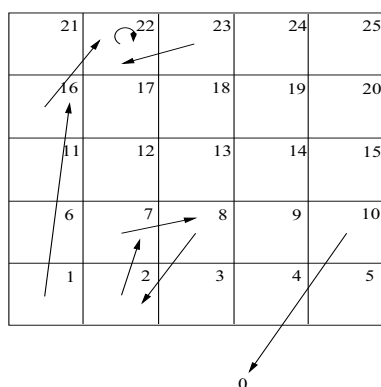


Figure 6.1: Sketch of a cell mapping method.

| Cell | Image | Group | Transient | Period |
|------|-------|-------|-----------|--------|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 16 | 1 | 2 | 1 |
| 2 | 7 | 2 | 0 | 3 |
| 7 | 8 | 2 | 0 | 3 |
| 8 | 2 | 2 | 0 | 3 |
| 10 | 0 | 0 | 1 | 0 |
| 16 | 22 | 1 | 1 | 1 |
| 22 | 22 | 1 | 0 | 1 |
| 23 | 22 | 1 | 1 | 1 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

Table 6.1: The cell table

| Group | Period | Zero transient cells |
|-------|--------|----------------------|
| 0 | 0 | 0 |
| 1 | 1 | 22 |
| 2 | 3 | 2,7,8 |
| ⋮ | ⋮ | ⋮ |

Table 6.2: The group table

In the same way, the user can find an explanation of the cell mapping method following the flowcharts given by figure 6.2.

6.3.1.2 Specifications

In this part we will describe all the tool specification that the user need to provide in order to simulate DOAs with the SICONOS software. The standard cell-to-cell mapping algorithm have been implemented in PHYTON to be used together with the SICONOS platform. Basically, inputs are the same that the inputs required by the simulator except that the user has to specify the following inputs (see flowchart in figure 6.3):

1. Functions defining the Poincaré surface (Poincaré map case) or time T for sampling the flow (Stroboscopic map case).
2. Boundaries of the Poincaré surface (Poincaré map case) or the phase space (Stroboscopic map case) where the initial conditions will be taken.
3. Number of cells and their distribution:
 - Cells uniformly distributed.
 - The user provides the cell distribution.
 - Random cells (Montecarlo method, ...).
4. Criterion to join disjoint cell cycles that represent the same attractor.

6.3.1.3 Output

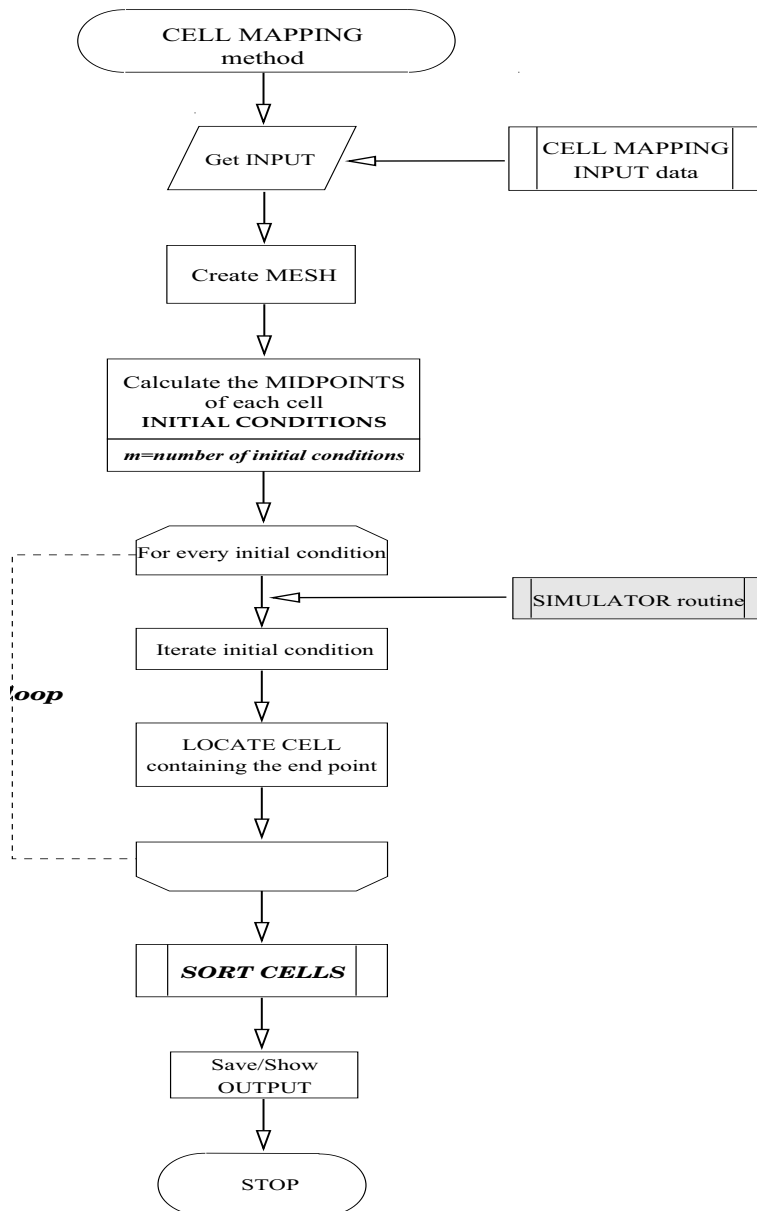


Figure 6.2: The main organization of Cell Mapping method.

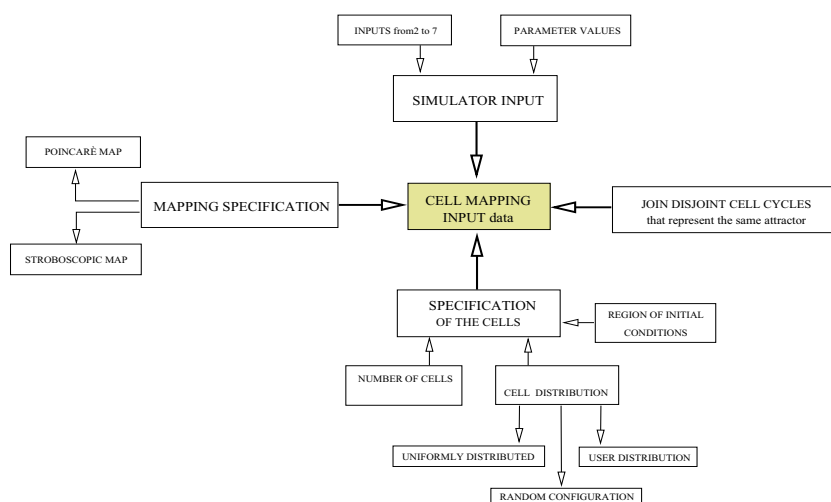


Figure 6.3: The main organization of Cell Mapping input data.

Output is divided in two parts. The first one is composed by two tables, as the Table 6.1 and the Table 6.2. In these tables the user can find the following data:

1. The cell table:

- Cell list. The numbered cells recorded in a list.
- Image list. The image cell of every cell under the cell mapping.
- Group list. The group which every cell belongs to (which periodic cycle it will be attracted to).
- Transient list. The number of iterates of every cell before landing on the cycle.
- Period list. The period of the attractor cycle of every cell.

2. The group table:

- Group list. All the different groups.
- Period list. The period of each group.
- Zero transient cells. The cells that form each periodic cycle.

The other part is given by a graphical output. Each basin of attraction and its periodic cycle is plotted with one different colour in a picture, see example in Figure 6.4.

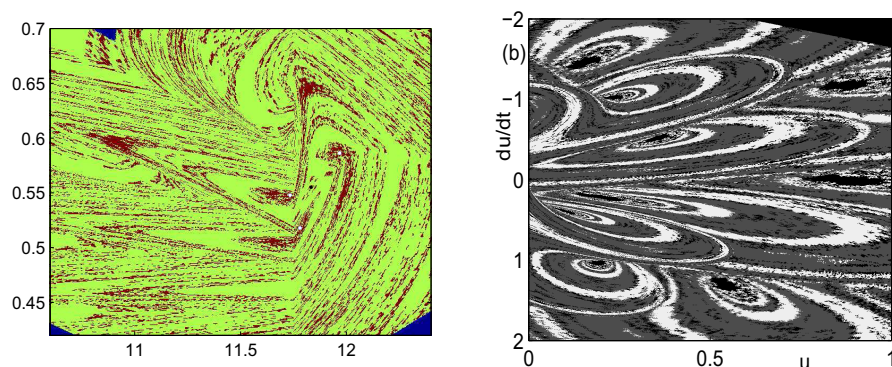


Figure 6.4: Examples of the graphical output.

6.3.2 Bifurcation diagram routines

The changes of the phase portrait as one parameter is varied can be a fundamental reason by which we are interested in studying one specific dynamical system. This can be done by mean of bifurcation diagrams. Basically, there are two different approaches to constructing bifurcation diagrams, using “*brute force*” methods or using *continuation* methods. The brute force method simulates the system for a certain time to watch what comes out for every chosen parameter value. This method has the drawback that long simulation times are generally necessary so that the transient dies out. Another drawback is due to the fact that this method only can find stable solutions, i.e. unstable solutions are not observable directly. However, using a continuation method, one can track unstable and stable solutions varying a parameter and the amount of computational work needed is reduced. Some tools that indicate whether a bifurcation point appear are needed. Therefore, numerical algorithms for the automatic detection of parameter tracing and branch switching at the bifurcations identified during a numerical analysis of solutions of complementarity systems must be implemented. However, the development of this numerical algorithms are still under investigation. For this reason, the only tool implemented in the SICONOS platform for the moment is the “brute force” method. In the following part we will explain the brute force method in detail and how is implemented in the SICONOS Platform.

6.3.2.1 Bifurcation diagram using the brute force method

To simulate bifurcation diagrams using a brute force method the user has to specify the region of interest that he wants to study. Once decided that, a partition of the region must be created. Then, for every parameter value a initial condition is chosen (there are several ways to do this, *e.g.* fixing an initial condition, using random initial conditions, etc.). After that, the system is simulated a certainly time to watch what comes out for every chosen parameter value. An explanation of the program that calculates bifurcation diagrams under one-parameter variation is given by the flowchart in Figure 6.5.

There are some things to take into account. One of them is the time that we simulate every initial condition. If we simulate for a long time from a given initial condition, the motion can be close to an attractor or not depending on the features of the attractor. Then we would need to simulate so long to be sure that transients have died out.

Another thing that the user has to take into account is how to choose the initial condition. Sometimes one can be confused because of the co-existence of attractors. For example, if a fixed initial condition is chosen for every parameter value, that initial condition can belong to the basin of different attractors as the parameter value is varied. One example of that can be seen in Figure 6.6.

The following possibilities for choosing initial conditions are implemented:

- Fix an initial condition for every parameter value.
- Fix the initial condition for the first parameter value. After that, the following initial conditions will be taken using the last value of the previous simulation.
- Take random initial conditions for every parameter value.
- The user gives the initial condition for every parameter value.

6.3.2.2 Specifications

In this part we will describe all the data that the user needs to provide in order to simulate DOAs with the SICONOS software. Basically, the inputs are the same that the inputs required by the simulator except that the user needs to specify the following extra items:

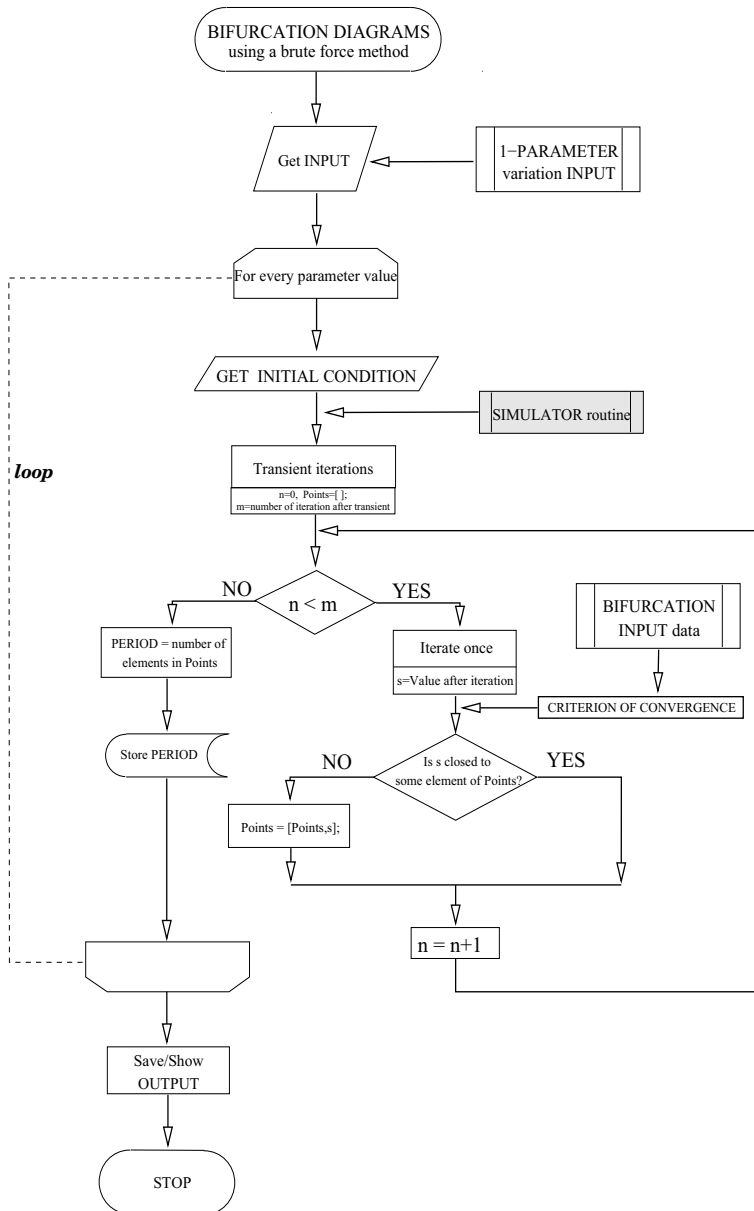


Figure 6.5: Explanation of the bifurcation diagram program.

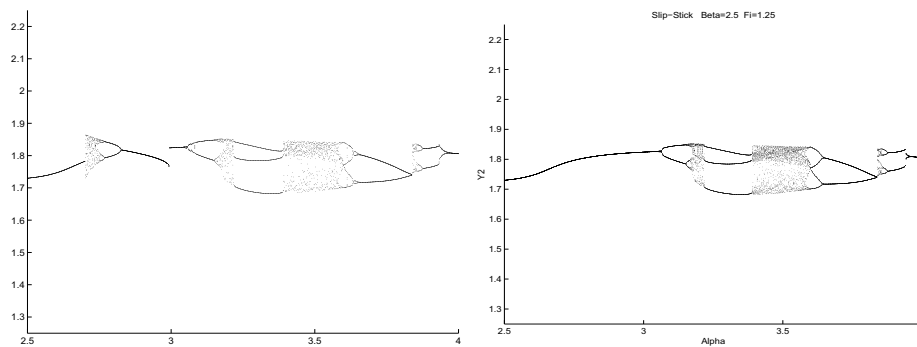


Figure 6.6: Bifurcation diagrams: Difference between to take a fixed initial condition (left picture) or a variable initial condition (right picture).

1. Functions defining the Poincaré surface (Poincaré map case) or time T for sampling the flow (Stroboscopic map case)
2. The parameter or parameters under variation (depending on the choice).
3. The interval of the parameters under variation.
4. Distribution of the parameters values in the interval. There are several ways to do that:
 - Parameters values uniformly distributed.
 - The user provides the distribution.
 - Random values (Montecarlo method, ...).
5. How to take the initial conditions:
 - Fix an initial condition for every parameter value.
 - Fix the initial condition for the first parameter value. After that, the following initial conditions will be taken using the last value of the previous simulation.
 - Take random initial conditions for every parameter value.
 - The user gives the initial condition for every parameter value.
6. Number of transient iterations.
7. Number of iterations after the transient.

8. Criterion of convergence (parameter to identify points very closed).

In the flowchart shown in figure 6.7 it is possible to see an outline of all the inputs.

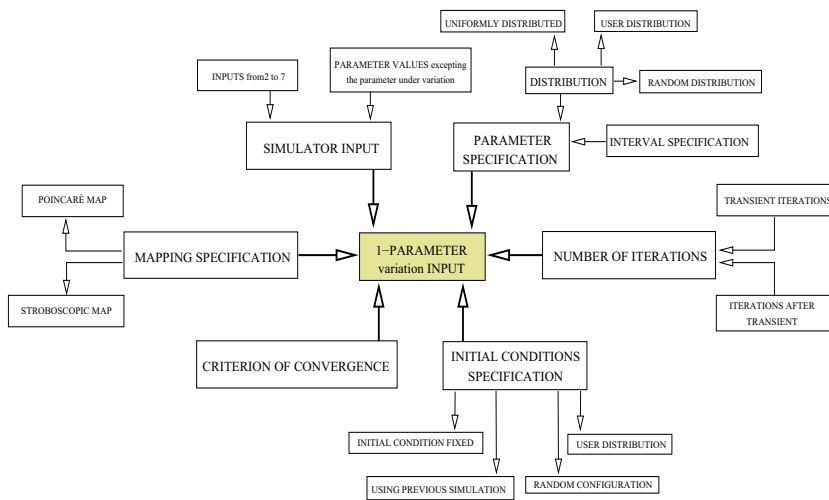


Figure 6.7: The main organization of Bifurcation input data.

6.3.2.3 Output

Output is provided both graphically and by mean of a table. The table shows all the parameter values and the period of the limit cycle (see example in Table 6.3):

| Parameter value α | Period |
|--------------------------|----------|
| 1.00 | 1 |
| 1.01 | 2 |
| 1.02 | 4 |
| \vdots | \vdots |

Table 6.3: Table of bifurcation diagram under one-parameter variation.

On the other hand, in the graphical output the user can visualize the changes of the attractor when varying the chosen parameter (see example in Figure 6.8):

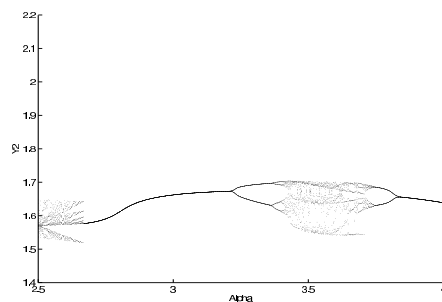


Figure 6.8: Example of the graphical output.

6.4 Benchmarks

6.4.1 Buck converter

As first example we have simulated a DC-DC buck converter whose output voltage is controlled by a PWM with natural sampling and constant frequency using the framework of linear complementarity systems (LCS) in the SICONOS software. This circuit is one of the simplest but most useful power converters, a chopper circuit that converts a dc input to a dc output at a lower voltage (many switched mode power supplies employ circuits closely related to it). An application of current importance is conversion of the standard 5V dc supply used in computers to the 3.3V needed by a Pentium CPU chip. A buck converter for this purpose can achieve a practical efficiency of 92%, whereas a linear regulator would be only 66% efficient, producing six times as much waste heat. Although this example is at a low power level, buck converters are also used at several kilowatts.

The circuit we study is of second order and its block diagram is shown in Fig 6.9. We assume throughout that the components in the circuit are ideal. The comparator A_2 has infinite gain, the switch S has zero on, and infinite off resistance, and can switch instantly. During the interval when switch S is on, the input provides energy to the load as well as to the inductor. During the interval when switch S is off, the inductor current transfers some of its stored energy to the load. One of the methods for controlling the output voltage employs switching at a constant frequency (hence, a constant switching time period $T = t_{on} + t_{off}$), and adjusting the on-duration of the switch to control the average output voltage. In this method, called pulsewidth modulation (PWM) switching, the switch duty

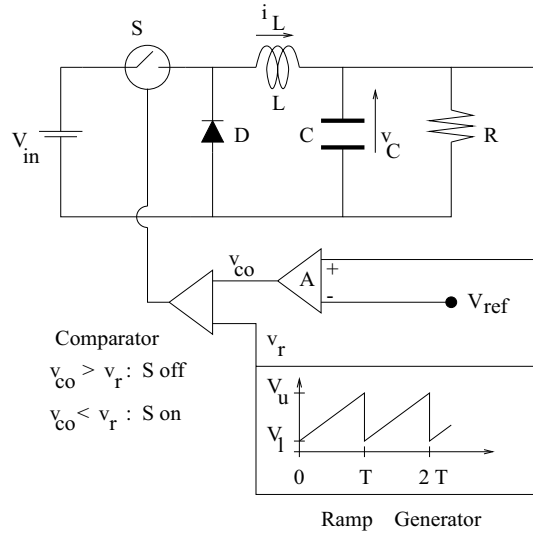


Figure 6.9: Buck converter diagram

ratio d , which is defined as the ratio of the on-duration to the switching time period, is varied. Discontinuous conduction mode is allowed as well.

Considering that the linear amplifier A_1 has gain a , we can write

$$v_{co}(t) = a \cdot (v(t) - V_{ref})$$

Then, both v_{co} and v_{ramp} , the voltage of the ramp, are applied to the comparator, and every time the output difference changes its sign the position of the switch S is commuted in such a way that S is open when the control voltage exceeds the ramp voltage; otherwise S is closed.

6.4.1.1 Modeling

The general form of an LCP is given by

$$\begin{aligned} \dot{x} &= Ax + Bu + E, \\ y &= Cx + Du + F, \end{aligned}$$

with $x \in \mathbb{R}^n$. Here y , u are p pairs of complementarity variables

$$0 \leq y \perp u \geq 0,$$

and A , B , C , D , E and F are (constant) matrices and vector of suitable dimensions.

For each switch position of the buck converter we obtain an LCS given by

$$\boxed{S \text{ closed: } v_{co}(t) < v_{ramp}(t)}$$

$$\begin{aligned} L\dot{x}_1 &= -x_2 + V_{in}, \\ C\dot{x}_2 &= x_1 - \frac{1}{R}x_2, \\ x_1 &= i_D, \\ 0 \leq i_D \perp v_D &\geq 0, \end{aligned}$$

$$\boxed{S \text{ open: } v_{co}(t) > v_{ramp}(t)}$$

$$\begin{aligned} L\dot{x}_1 &= -x_2 - v_D, \\ C\dot{x}_2 &= x_1 - \frac{1}{R}x_2, \\ x_1 &= i_D, \\ 0 \leq i_D \perp v_D &\geq 0. \end{aligned}$$

where $x_1 = i_L$ and $x_2 = v_C$ are the current in the inductance and the voltage in the capacitor and i_D and v_D are the current and the voltage in the diode.

6.4.1.2 Simulations

Here we show simulations of the buck converter using the complementarity framework in the SICONOS Platform. The numerical simulations are performed with the following parameter values: $L = 20mH$, $C = 47\mu F$, $R = 22\Omega$, $a = 8.4$, $V_{ref} = 11.3V$, $V_L = 3.8V$, $V_U = 8.2V$, and $T = 400\mu s$, as in [78],[118]. In Fig. 6.10 we present a one-periodic and a two-periodic orbit for V_{in} equal to $15V$ and $25V$ respectively.

In Fig. 6.11 (a) a bifurcation diagram of this system using a brute force method is shown. We have used the SICONOS platform together with algorithms developed in PHYTON. For each value of V_{in} in a range between $12V$ and $40V$ a fixed initial condition was simulated with the SICONOS Platform. This process was done for 300 periods without plotting and then the dynamics plotted for a further 100 periods. As we have remarked before,

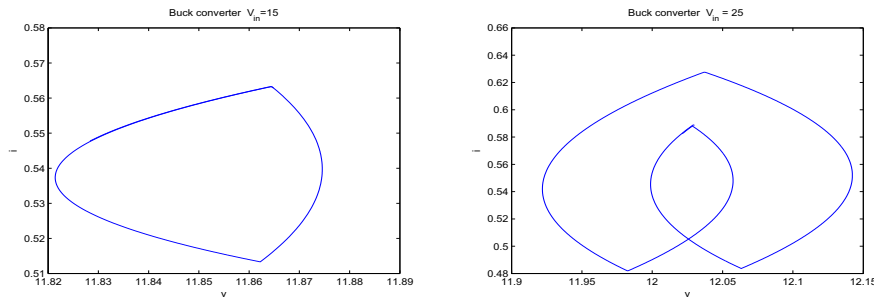


Figure 6.10: Time simulations of a Buck Converter in SICONOS Platform. (a) $V_{in} = 15V$. (b) $V_{in} = 25V$.

this procedure is effective in capture the range of possible stable asymptotic behaviours of the system, though it will miss unstable behaviour.

In the bifurcation diagram we observe that a stable 1T-periodic orbit is initially found and continued until some value near $24.5V$. Then, a period-doubling bifurcation occurs, and the stability of the 1T-periodic orbit is lost in favour of the 2T-periodic orbit which appears at this value. This 2T-periodic orbit also loses stability in a period-doubling bifurcation near $31.5V$. Near the last period-doubling bifurcation, suddenly and at approximately $32.5V$, there is a large chaotic behaviour. This occurs because of a corner-collision bifurcation [118].

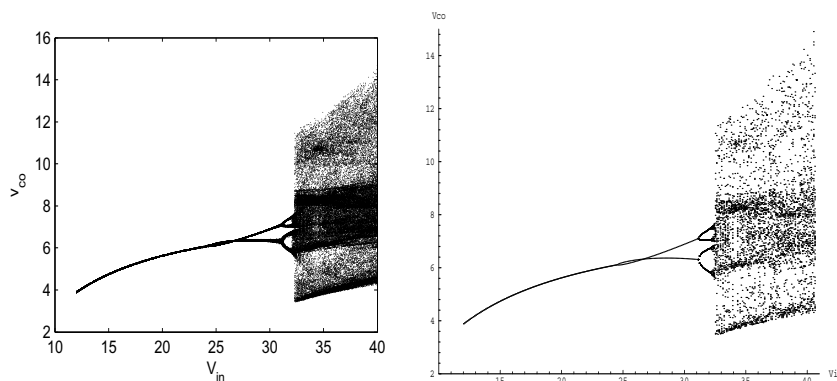


Figure 6.11: Bifurcation simulations (a) Using SICONOS Platform. (b) Using an own MatLab code.

In Fig. 6.11 (b) we have performed the same bifurcation diagram as in Fig. 6.11 (a) with MATLAB. We have employed our own codes developed in MATLAB using event-driven methods. As can be seen the pictures have the same appearance. This provides some evidence that simulations in SICONOS give results comparable to commercial packages.

In Fig. 6.12 (a) we have computed the domains of attraction for the two stable periodic orbits at $V_{in} = 13.8V$ using the SICONOS software. We have used the standard cell-to-cell mapping algorithm explained before. We have considered the domain of interest divided into a number of equally sized squares (1000×1000 cells). The center point of each square has been mapped forward in time one forced period, T , and the location of the trajectory at this point has been recorded. This was done for each square and finally all data was post-processed to build the complete picture. We have obtained the basin of the $1T$ -periodic orbit shown before (green color) and the basin of a $3T$ -periodic orbit (red color).

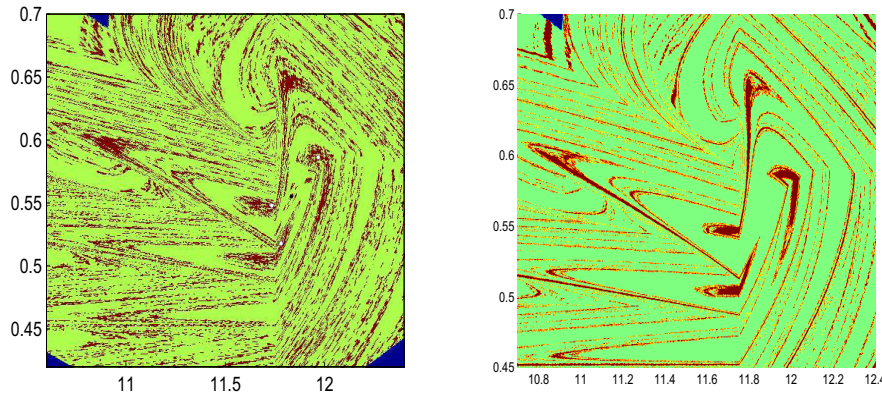


Figure 6.12: Domains of attraction simulations (a) Using SICONOS Platform. (b) Using an own MatLab code.

In Fig. 6.12 (b) a basin of attraction calculation of the same region is shown using a MATLAB code of our own. The similar appearance between the domain of attraction pictures confirms the good results obtained by the platform.

6.4.2 Forced Harmonic Oscillator

As second example we have simulated in collaboration with Petri Piironen, an unforced harmonically oscillating particle u satisfying the differential equation

$$\frac{d^2u}{dt^2} + u = 0, \quad u > z \quad (6.19)$$

impacting with a smoothly oscillating wall at $z(t) = \sin(\omega t)$. We take $r = 0.8$ to be the restitution coefficient, so that

$$\frac{du^+}{dt} = -r \frac{du^-}{dt}, \quad u = z \quad (6.20)$$

leading to energy loss at each impact and consequently to a dissipative system. This example has been widely studied, see for instance [] and it is known that u can exhibit periodic or chaotic motion depending upon the value of ω .

6.4.2.1 Modeling

Our system belongs to the abstract class of Lagrangian Linear Time Invariant systems described in section 3. The general equation of this class is governed by the equation

$$M\ddot{q} + C\dot{q} + Kq = F_{ext}(t) + r$$

where C and K are respectively the classical viscosity and stiffness matrices. Then, in our case, $q = u$, $M = 1$, $C = 0$ and $K = 1$. The external force is given by $F_{ext} = \sin(\omega t)$.

The unilateral constraint requires that $u \geq 0$, so we identify the terms of the equation (6.12):

$$\begin{aligned} y &= H^T q + b \\ H^T &= 1, b = 0 \end{aligned}$$

In the same way, the reaction due to the constraint is written as follows:

$$r = H\lambda$$

with $H = 1$.

In this case, there is just one unilateral constraint such that

$$0 \leq y \perp \geq 0$$

Newton's impact law is given by :

$$\text{if } y = 0, \quad \dot{y}(t^+) = -e\dot{y}(t^-)$$

6.4.2.2 Simulations

In Fig. 6.13 (a) a bifurcation diagram of this system using a brute force method is shown. We have used the SICONOS platform together with algorithms developed in PHYTON. For each value of ω a fixed set of initial conditions $(u, \frac{du}{dt})$ was chosen at $t = 0$. The trajectories resulting from each such condition were then computed with the SICONOS Platform. This process was continued for 50 periods, where one period is $\frac{2\pi}{\omega}$ time units long, without plotting and then the dynamics was plotted for a further 20 periods. This was then repeated for other values of ω . As we have remarked before, this procedure is effective in capturing the range of possible stable asymptotic behaviours of the system, though it will miss unstable behaviour.

In the bifurcation diagram we observe simple (resonant) periodic motion, when $\omega = 2$ and $\omega = 4$, surrounded by more complex dynamics, including chaotic behaviour for ω close to 3. Such complex dynamics can arise either through smooth bifurcations, in particular period-doubling bifurcations, or through *grazing* bifurcations which appear when u has a grazing impact with z with $\frac{du}{dt} = 0$.

In Fig. 6.13 (b) we have performed the same bifurcation diagram as in Fig. 6.13 (a) with MATLAB. We have used own codes developed in MATLAB using event-driven methods. As can be seen the pictures have the same appearance. This implies again that simulations in SICONOS give good results.

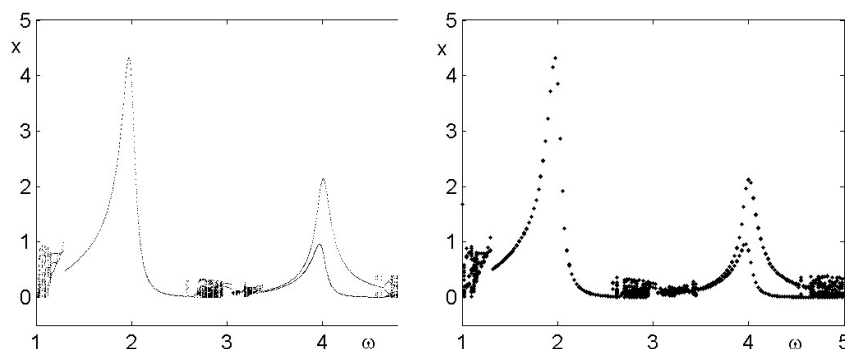


Figure 6.13: Bifurcation simulations (a) Using SICONOS Platform. (b) Using an own MatLab code.

In Fig. 6.14 (a) we have computed the domains of attraction for the

two stable periodic orbits at $\omega = 2.6$ using the SICONOS software. We have used the standard cell-to-cell mapping algorithm explained before. We have considered the domain of interest divided into a number of equally sized squares (1000×1000 cells). The center point of each square has been mapped forward in time one forced period ($\frac{2\pi}{\omega}$), and the location of the trajectory at this point has been recorded. This was done for each square and finally all data was post-processed to build the complete picture. Obviously, the smaller each cell square is, the more refined the final picture will be.

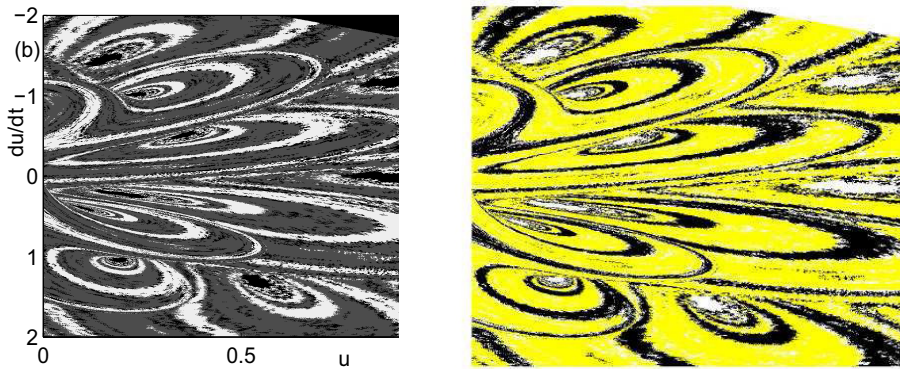


Figure 6.14: Domains of attraction simulations (a) Using SICONOS Platform. (b) Using an own MatLab code.

Figure 6.14 compares again the results of the SICONOS platform and the same MATLAB code of the first example. The similar appearance between the domain of attraction pictures confirms the good results obtained by the platform.

6.4.3 Parallel Resonant Converter

In this last example a Parallel Resonant Converter (PRC) has been simulated using the SICONOS software. Basically, a Parallel Resonant Converter is a dc-dc power converter. The schematic of the PRC is shown in Figure 6.15. As can be seen, it consists of four parts: an inverter block, a resonant tank in series, a rectifier block and an output filter. In our case, the inverter block is a full-bridge inverter. It is called parallel resonant converter because the load is in parallel with the resonant capacitor. More accurately, this converter should be called series resonant converter with

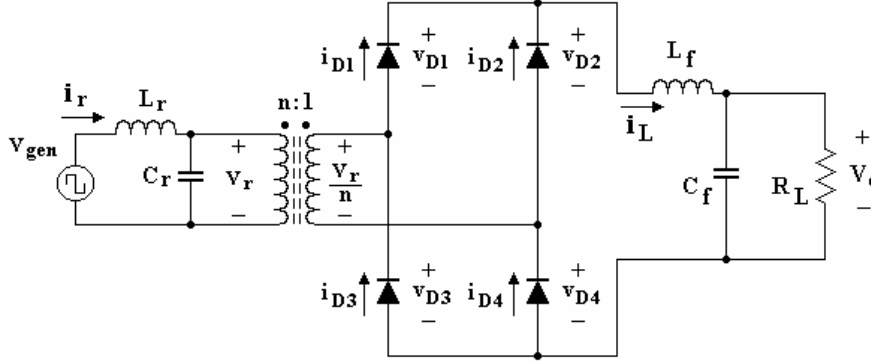


Figure 6.15: A Parallel Resonant Convert diagram

parallel load. Since the transformer primary side is a capacitor, an inductor is added on the secondary side to match the impedance.

6.4.3.1 Modeling

The general form of an LCS is given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ew(t), \\ y(t) &= Cx(t) + Du(t) + Fw(t), \\ 0 &\leq y \perp u \geq 0 \end{aligned} \quad (6.21)$$

with $x \in \mathbb{R}^n$, $w \in \mathbb{R}^k$, and y, u are p pairs of complementarity variables. Here x denotes the state (the voltage across the capacitors and the currents through the inductors), w denotes the external source, (u_i, y_i) denotes either the voltage-current of the current-voltage pairs of the i th port, and A, B, C, D, E, F are (constant) matrices and vectors of suitable dimensions.

According to the general form of an LCS we can model our parallel resonant converter (PRC) as follows.

We take as state variables $x_1 = i_r, x_2 = v_r, x_3 = i_L$ and $x_4 = v_0$, and $u_1 = i_{D1}, u_2 = i_{D3}, u_3 = v_{D2}, u_4 = v_{D4}, y_1 = v_{D1}, y_2 = v_{D3}, y_3 = i_{D2}$ and $y_4 = i_{D4}$ as complementarity variables. Then, in matrix notation, we have

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + E\text{Sign}(\sin(\omega t)), \\ y(t) = Cx(t) + Du(t) + F\text{Sign}(\sin(\omega t)), \\ 0 \leq y \perp u \geq 0 \end{cases} \quad (6.22)$$

with

$$A = \begin{pmatrix} 0 & -\frac{1}{L_r} & 0 & 0 \\ \frac{1}{C_r} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{L_f} \\ 0 & 0 & \frac{1}{C_f} & -\frac{1}{R_L C_f} \end{pmatrix}, B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\frac{1}{nC_r} & \frac{1}{nC_r} & 0 & 0 \\ 0 & 0 & \frac{1}{L_f} & \frac{1}{L_f} \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 & -\frac{1}{n} & 0 & 0 \\ 0 & \frac{1}{n} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, D = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix},$$

$$E = \begin{pmatrix} \frac{1}{L_r} \\ 0 \\ 0 \\ 0 \end{pmatrix}, F = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

6.4.3.2 Simulations

We present different dynamical behaviours when the load (R_l) and the frequency (F_0) of the input voltage are changed. The numerical simulations are performed with the following parameter values: $L_r = 150 \mu H.$, $L_f = 0.4 mH.$, $C_r = 68 nF.$, $C_f = 2.2 \mu F.$

Figure 6.16 shows the different behaviours of the PRC with a frequency $F_0 = 55 kHz.$ and different loads. Figure 6.16 (a),(b) are displayed with a load 3Ω . In particular, it is shown in Figure 6.16 (a) that the orbit has a sliding region in $x_2 = v_r = 0$ due to a generalised discontinuous conduction mode (GDCM). For a load value of 33Ω such GDCM disappears as can be observed in Figure 6.16 (c).

Figures 6.17 (a)-(f) show different behaviours with a frequency $F_0 = 75 kHz.$ and different loads.

6. SICONOS Platform

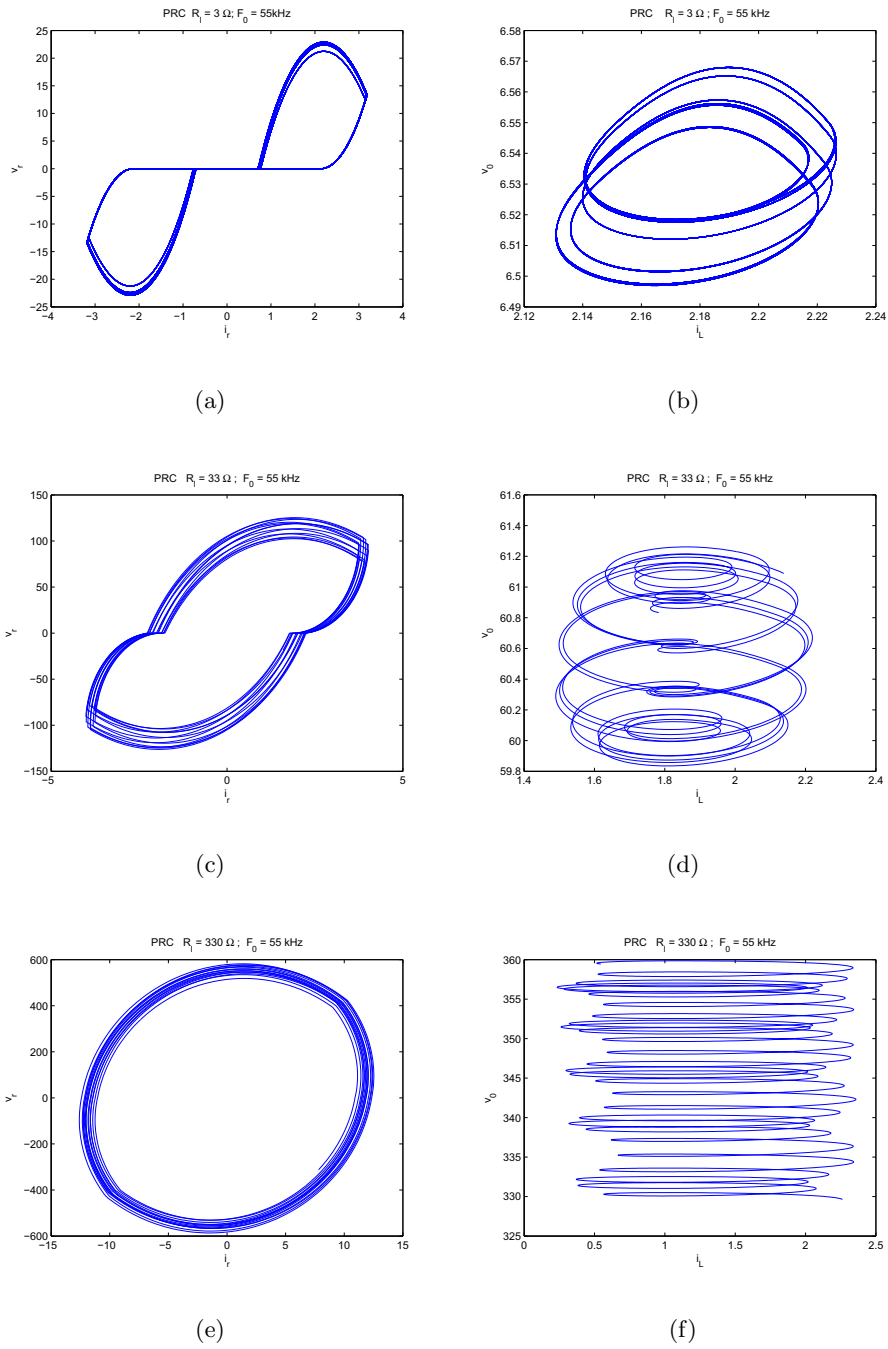
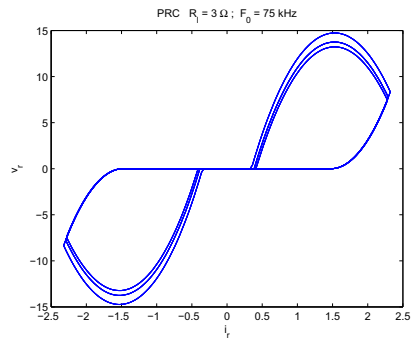
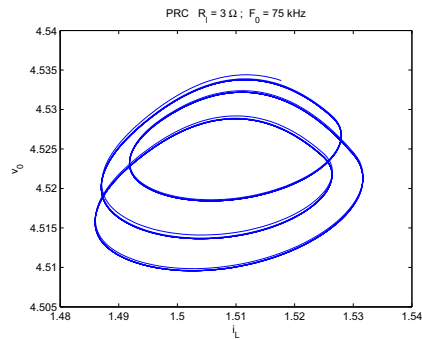


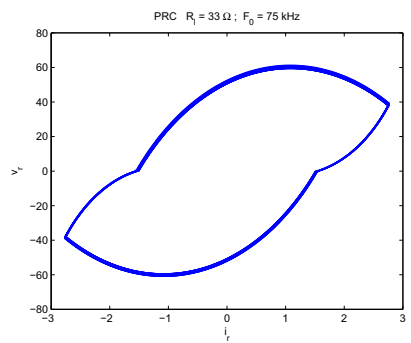
Figure 6.16: Simulations in SICONOS Platform.



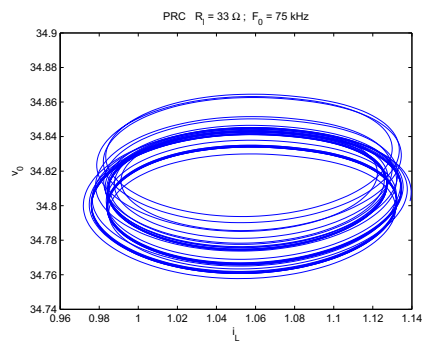
(a)



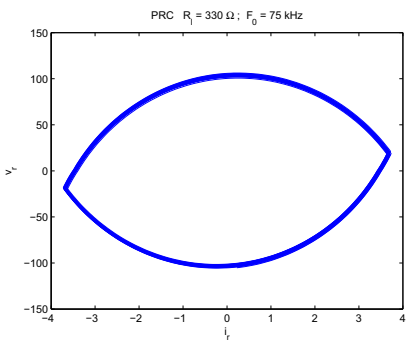
(b)



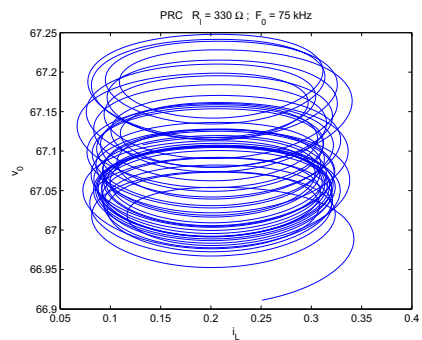
(c)



(d)



(e)



(f)

Figure 6.17: Simulations in SICONOS Platform 2.

6. SICONOS Platform

CHAPTER 7

Conclusions and future research

“It is better to know some of the questions than all of the answers.”

JAMES THURBER (1894-1961)

This chapter gives an overview of the thesis indicating the contributions. The thesis is closed with recommendations for further research.

7.1 Overview and Summary of Contributions.

The work developed for this Ph.D. thesis contributes to the study of non-smooth dynamical systems with emphasis on the numerical computing side. This thesis can be divided into two parts, one concerned with the modeling of power converters using the complementarity formalism and the other one dedicated to the numerical study of mechanical systems with impact and dry friction. Also, from an engineering point of view, this thesis contributes to answering some questions about the behaviour observed in experiments, and to generating new questions to be answered by the engineering and applied math community. Among these, the experimental search for some discontinuity-induced bifurcations detected numerically and the development of a general control theory for complementarity systems.

Chapter 1 gives some background information on nonsmooth dynamical systems after motivating this work. We have defined the objective and scope of the thesis and finally, we have outlined the structure and contents.

Chapter 2 contains some background of the theory for modeling non-smooth dynamical systems using the complementarity formalism. Then, we have modeled some basic dc-dc power converters with a single diode (buck, boost, buck-boost and Čuk) as linear cone complementarity systems. It is shown that, for each position of the switches, the dynamics is given by a linear complementarity problem to which standard techniques can be applied. For systems with a single diode, an analytical condition for the presence of generalised discontinuous conduction modes (GDCM), characterised by a reduction of the dimension of the effective dynamics, have been stated. We have presented analytical state-space conditions for the presence of a GDCM in each example and simulation results, showing a variety of behaviours, such as persistent or re-entering GDCM. A Parallel Resonant Converter, which has four diodes, has been also modeled as a linear complementarity system. In this converter the diodes are in parallel, therefore, known results about existence and uniqueness cannot be applied because some assumptions do not hold. However, we have proved that the state-space solution is unique although the solutions of the complementarity variables are not. Finally, we have presented the simulation of a boost converter with a sliding mode control in the complementarity formalism, even though control theory for complementarity systems is not still developed.

The main contributions of this Chapter are:

- Modeling of some basic dc-dc power converters with a single diode (buck, boost, buck-boost and Čuk) in the complementarity formalism.
- Analytical state-space conditions for GDCM in systems with a single diode. We have applied such results for each example and we have presented simulation results, showing a variety of behaviours, such as persistent or re-entering GDCM.
- Modeling, analysis and simulation of a Parallel Resonant Converter (PRC) which has four diodes. We have also proved that the state-space solution is unique although the solutions of the complementarity variables are not unique.
- Simulation of a boost converter with a sliding mode control in the complementarity formalism.

In Chapter 3 the analysis of a cam-follower system has been presented. Specifically, we have studied a simplified model of an automotive camshaft system. This kind of system can be considered as a forced impact oscillator. Therefore, several nonsmooth phenomena as first detachment, transition from complete to uncomplete chattering, and discontinuity-induced bifurcations of periodic orbits can be exhibited. We have analysed these complex behaviours under variations of the rotational speed of the cam. In order to have a better understanding of the dynamical behaviour we have constructed bifurcation diagrams. Once we have observed the different behaviours occurring in our system, we have stated analytical explanations of some phenomena. We have calculated the regions with possible detachment points and, particularly, the rotational speed for the first detachment. After the first detachment occurs, we have a sequence of chattering in the system. We have study analytically the accumulation points of such impacts explaining some phenomena that happen in our system. Another phenomena is the nonsmooth transition from complete to uncomplete chattering. We have observed the destruction of the period one chattering sequence for a certain value of the rotational speed parameter due to the crossing of the accumulation point to the next forcing period. A detailed study of this discontinuity-induced bifurcation is a subject of further research. We have also given necessary conditions for periodic orbits with a single impact. Using these necessary conditions we have continued a periodic orbit of period one and one impact. Such periodic orbit has a suddenly jump to chaos due to a corner-impact bifurcation, and we have been able to explain this bifurcation in an analytical way. Another corner-impact bifurcation of a period 2 orbit is also explained. Finally, coexistence of attractors is shown using domains of attraction calculated with a standard cell-to-cell mapping method.

The main contributions of this Chapter are:

- Simulation of a cam-follower system, which is a class of forced impact oscillator.
- Detection of several nonsmooth phenomena such as first detachment, transition from complete to uncomplete chattering and discontinuity-induced bifurcation of periodic orbits.
- Calculation of the regions with detachment points and in particular, the rotational speed value for the first detachment.

- Calculation of the accumulation points in the sequence of chattering occurring after the first detachment.
- Description of a nonsmooth behaviour due to the transition from complete to uncomplete chattering.
- Necessary conditions for periodic orbits with a single impact.
- Explanation of the suddenly jump from a periodic orbit to chaos (corner-impact bifurcation) using analytical calculations.
- Calculations of basins of attraction using a standard cell-to cell mapping in order to show coexisting solutions.

Chapter 4 deals with a dry friction oscillator introduced by Popp [?]. This kind of systems can be studied using Filippov theory and is affected by discontinuity-induced bifurcations (DIBs) due to “stick-slip” motions. Such bifurcations have been recently classified as *sliding bifurcations*. Basically, four distinct cases of such bifurcations can be identified: *crossing-sliding*, *grazing-sliding*, *switching-sliding* and *adding-sliding*. We have presented detailed examples of all these different bifurcation scenarios. Furthermore, a degenerate switching-sliding bifurcation has been shown. In that case of degenerate switching-sliding bifurcation two curves of codimension-one sliding bifurcation, crossing-sliding and adding-sliding, branch out from the codimension-two point. Also, a smooth codimension-two cusp bifurcation has been presented. Coexistence of periodic orbits in the region between both fold codimension-one curves have been shown to exist by means of domain of attraction diagrams computed using a cell-to-cell mapping method.

The main contributions of this Chapter are:

- Simulation of a dry friction oscillator introduced by Popp [?], which can be studied using Filippov theory.
- Detection of the four distinct cases of sliding bifurcations: adding-sliding, grazing-sliding, crossing-sliding and switching-sliding.
- A degenerate codimension-two switching-sliding bifurcation have been explained.
- Detection of a smooth codimension-two cusp bifurcation.

- Calculations of basins of attraction using a standard cell-to cell mapping in order to show coexisting solutions.

In Chapter 5 the dynamic behaviour of the two-block Burridge-Knopoff model for earthquake simulations has been investigated. This model can be studied as a Filippov system with two discontinuity surfaces. Previous numerical studies investigated in [?] verified that, with a friction force of Coulomb type (that is the dynamic friction coefficient being constant), the system presents only periodic behaviour. We have shown that chaotic regions can be observed in a symmetric configuration even if a Coulomb friction is considered with the relaxation of the assumption that the driving block does not move during the slipping events. Furthermore, we have studied the behaviour of the system with asymmetric configuration. Different periodic solutions and regions of chaos can be observed varying the asymmetry of the system. With respect to the bifurcation point of view, this system can exhibit smooth and discontinuity-induced bifurcations as we have presented in this chapter.

The main results are

- Simulation of a two-block Burridge-Knopoff model for earthquake simulations.
- We have proven that chaotic regions can be observed considering a symmetric configuration and Coulomb friction with the relaxation of the assumption that the driving block does not move during the slipping events.
- Simulation of codimension-two bifurcation diagram in order to show the complex behaviours in an asymmetric configuration.
- Explanation of some smooth and discontinuity-induced bifurcations.

Chapter 6 presents the SICONOS software, dedicated to simulation of nonsmooth dynamical systems (NSDS). After motivating the development of this tool, we have given a overview of the SICONOS software and the way NSDS are modeled and simulated within the platform. Routines for analysis (stability, bifurcations, invariant manifolds, ...) of NSDS implemented in the platform have been explained in detail. To conclude, several representative samples have been shown in order to illustrate the Siconos platform abilities.

Results are milestones of this chapter include

- Participation of this Ph.D. candidate in the development of the SICONOS Platform as an expert user.
- Development of some routines for analysis such as calculations of bifurcation diagrams and domains of attraction.
- Implementation of several examples in the SICONOS Platform such as a Buck converter and a Parallel Resonant Converter (PRC).

7.2 Publications

The main contributions of this thesis have been published in the following journals and congresses:

- **Generalized Discontinuous Conduction Modes in the Complementarity Formalism**, *C. Battle, E. Fossas, I. Merillas and A. Miralles*, IEEE Transactions on Circuits and Systems II, 52 (8), pp. 447-451, 2005.
- **Complex Dynamics of Cam Follower Systems**, *I. Merillas, G. Osorio, P.T. Piiroinen, M. di Bernardo and E. Fossas*. To be submitted to International Journal Bifurcation and Chaos.
- **Sliding Bifurcations in a stick-slip system**, *I. Merillas, U. Galvanetto and C. Battle*. To be submitted to International Journal Bifurcation and Chaos.
- **Bifurcations in a two-block stick-slip system**, *I. Merillas and G. Olivar*, In preparation.
- **Simulation of a boost converter with sliding mode control using the complementarity formalism**, *I. Merillas, E. Fossas and C. Battle*, SICONOS Internal report, 2004.
- **Modeling and Numerical Study of Nonsmooth Dynamical Systems**, *I. Merillas*, DDays workshop, Sevilla-Islantilla, 18-21 October 2006.
- **Sliding Bifurcations in a Stick-slip System**, *I. Merillas, U. Galvanetto and C. Battle*, SICONOS General Meeting, Capri, 14-15 September 2006.

- **Some Non-smooth Bifurcations in a stick-slip system**, *I. Merillas and G. Olivar*, Proceedings of Workshop on Bifurcations in Non-smooth and Hybrid Dynamical Systems, Milano, 21-22 October 2004.
- **Bifurcaciones No Suaves en Sistemas Stick-slip**, *I. Merillas and G. Olivar*, Proceedings of NO LINEAL 2004, Toledo, 1-4 Juny 2004.
- **Non-Smooth Continuation of Periodic Orbits in a Chaotic Buck Converter**, *I. Merillas and G. Olivar*, Proceedings of Workshop on Analysis and Continuation of Bifurcations, Sevilla, 19-21 May 2004.
- **Bifurcations in a Two-Block Stick-Slip Model**, *I. Merillas and G. Olivar*, Proceedings of SCICADE 2003, Trondheim, 30 Juny 2003 - 04 July 2003.

7.3 Future research

Although the work presented in this Ph.D. thesis has achieved some interesting results for the modeling, simulation and analysis of nonsmooth dynamical systems, many problems remain still open and will be the subject of future investigations.

As has been seen in Chapter 2, two main problems are still open to investigate in this part. First, known results can not assure existence and uniqueness of solutions for power electronic switches with diodes in parallel. This is due the fact that the results in the literature only consider the existence and uniqueness of solutions with P -matrices. We believe that having a P_0 -matrix in the Rational Complementarity Problem is a necessary and sufficient condition to assure the existence and uniqueness.

Another open problem in this part is concerned with control theory for complementarity systems. Design of robust and efficient controllers for complementarity systems is now being investigated but still there are no general results. In order to get some ideas for the development of a general control theory we have considered the particular case of controlling a boost converter with coupled inductors.

For Chapter 3 we propose different open problems. As we have outlined before, the theoretical study of discontinuity-induced bifurcations due to the transition from complete to uncomplete chattering is still an open question. In this thesis only a first numerical approximation to this problem has been considered.

Another discontinuity-induced bifurcation explained in this chapter is due to an impact in the discontinuity point of the acceleration of cam profile. It is also interesting to study cam-follower systems with other cam profiles which can present discontinuities in the position, velocity, etc. This study can contribute to the classification of corner-impact bifurcations in these systems. In relation with this problem we have begun to study the behaviour of a grain down a rough inclined staircase.

We can also consider the study of invariant manifolds in systems with impacts as subject for a future research. Up to our knowledge there is nothing written about this field and it is an interesting problem from a mathematical point of view.

The study of invariant manifolds for Filippov systems is an open problem which arises from Chapter 4. These systems are characterised by not being integrable backward in time and therefore, the techniques available for smooth systems to calculate stable manifolds cannot be used.

Detection of other nonsmooth codimension-two bifurcation points in dry-friction oscillators or power converters is an interesting subject in order to complete the first tentative classification given in [92]. In this thesis, we have presented a degenerate codimension-two switching-sliding bifurcation that has been never reported before, but other codimension-two bifurcation points has not been found yet.

A two-block Burridge-Knoppoff is an example of Filippov system with two discontinuity surfaces if back-slip is allowed. This example can be used as a workbench to find discontinuity-induced bifurcations only possible due to the two discontinuity surfaces.

Concerning to the SICONOS Platform, several things need to be implemented. We need to add routines for continuation of periodic orbits and detection of bifurcation points. Also, it is necessary the implementation of more examples in order to test the Platform.

Finally, a detailed study of the consequences about approximating non-smooth systems by smooth ones is needed, in order to get a better understanding of this technique.

Appendices

Appendix A: Moreau's Time-stepping

This appendix is concerned with the simulation of complementarity systems using a Moreau's Time-stepping scheme.

Dynamical system and Boundary conditions Consider the Lagrangian (second order) system:

$$M(q)\ddot{q} + C(q, \dot{q}) = F_{int}(q, \dot{q}, t) + F_{ext}(t) + r \quad (1)$$

where q denotes the generalized coordinates, M the mass matrix, C the nonlinear inertia operator, F_{int} the internal nonlinear forces, F_{ext} the external forces depending only on time and r is the possible force due to the constraints.

A particular case is the Lagrangian Time Invariant system which is defined by

$$M\ddot{q} + C\dot{q} + Kq = F_{ext}(t) + r \quad (2)$$

where C and K are respectively the classical viscosity and stiffness matrices.

In a general way, the dynamical system is completed by a set of non-smooth laws. The set of such variables, denoted by y , on which we apply the constraints, depends, in a very general way, of the state vector x , the time t and possibly the force r :

$$y = h(x, r, t) \quad (3)$$

In the same way, we have to specify the relation between r , the force due

to the constraints, and λ (λ is associated to y through a nonsmooth law):

$$r = g(x, \lambda, t) \quad (4)$$

In this appendix we only consider a complementarity nonsmooth law:

$$0 \leq y \perp \lambda \geq 0 \quad (5)$$

However, the same method can be used for other nonsmooth laws such as a Newton impact law.

Description of the numerical strategy: the Moreau's Time-stepping scheme.

We provide in this section a time discretization method of the Lagrange dynamical system (1), consistent with the non smooth character of the solution. Let us consider here only the linear time invariant case. The equation may be reformulated equivalently in terms of an integral over a time step $[t_i, t_{i+1}]$ of length h such that :

$$\int_{[t_i, t_{i+1}]} M\ddot{q} + C\dot{q} + Kq dt = \int_{[t_i, t_{i+1}]} F_{ext}(t) dt + \int_{[t_i, t_{i+1}]} r d\nu \quad (6)$$

Due to the non smooth character of the motion, the first term is integrated by an one order scheme(backward Euler-like) such that :

$$\int_{[t_i, t_{i+1}]} M\ddot{q} \approx M(\dot{q}(t_{i+1}) - \dot{q}(t_i)) \quad (7)$$

For the other terms, a θ -method is used :

$$\int_{[t_i, t_{i+1}]} C\dot{q} + Kq dt \approx h[\theta(C\dot{q}(t_{i+1}) + K\dot{q}(t_{i+1})) + (1 - \theta)(C\dot{q}(t_i) + K\dot{q}(t_i))] \quad (8)$$

$$\int_{[t_i, t_{i+1}]} F_{ext}(t) dt \approx h[\theta F_{ext}(t_{i+1}) + (1 + \theta)F_{ext}(t_i)] \quad (9)$$

Discretizing the Nonsmooth laws and considering $\theta = 1$ we obtain the Backward-Euler Time-stepping method used in Chapter 2.

Backward-Euler Time-stepping method

This method comes down to the computation of u_{k+1}^h, y_{k+1}^h , and x_{k+1}^h given x_k^h through the linear complementarity problem given by

$$\frac{x_{k+1}^h - x_k^h}{h} = Ax_{k+1}^h + Bu_{k+1}^h \quad (10)$$

$$y_{k+1}^h = Cx_{k+1}^h + Du_{k+1}^h \quad (11)$$

$$0 \leq y_{k+1}^h \perp u_{k+1}^h \geq 0 \quad (12)$$

Here \bullet_k^h denotes the value at the k th step of the corresponding variable for the step size $h > 0$. Based on this scheme, one can construct approximations of the transient response of a LCS by applying the algorithm below.

Algorithm 1 : $(\{u_{k+1}^h\}, \{x_{k+1}^h\}, \{y_{k+1}^h\}) = \text{LCPsimulator}(A, B, C, D, T_{end}, h, x_0)$.

1. $N_h = \lfloor \frac{T_{end}}{h} \rfloor$.
2. $x_{-1}^h := x_0$.
3. $k := 1$.
4. *solve the one-step problem*

$$y_{k+1}^h = C(I - hA)^{-1}x_k^h + [D + hC(I - hA)^{-1}B]u_{k+1}^h$$

$$0 \leq u_{k+1}^h \perp y_{k+1}^h \geq 0$$

5. $x_{k+1}^h := (I - hA)^{-1}x_k^h + h(I - hA)^{-1}Bu_{k+1}^h$
6. $k := k + 1$.
7. *if* $k < N_h$ *goto* 4.
8. *stop*.

The one-step problem is given by a linear complementarity problem in step 4. In general a linear complementarity problem may have multiple solutions or have no solutions at all. We shall proceed by assuming unique solvability of the problem. The assumption is introduced here for reasons of generality, but this assumption is implied by passivity. In [?], [?] is described the consistency of the algorithm when we have passivity.

Appendix B: Sliding Mode Control

In this appendix the method of sliding mode control for controlling power electronics systems is reviewed.

Dynamical systems with sliding mode control. Consider the non-linear dynamical system:

$$\dot{x} = f(x) + g(x)u \quad (13)$$

where $x \in \mathbf{X}$, an open set of \mathbb{R}^n ; the control input function $u : \mathbb{R}^n \rightarrow \mathbb{R}$ is a discontinuous function; and f, g are smooth vector fields defined on \mathbf{X} with $g(x) \neq 0, \forall x \in \mathbf{X}$. Let s denote a smooth function $s : \mathbf{X} \rightarrow \mathbb{R}$, with non-zero gradient on \mathbf{X} . The set

$$\mathbf{S} = \{x \in \mathbb{R}^n : s(x) = 0\} \quad (14)$$

defines a locally regular (n-1)-dimensional sub-manifold in \mathbf{X} (the *sliding manifold* or *switching surface*). The scalar function s will often be addressed as the *sliding surface coordinate function*.

A *variable structure control law* is obtained by letting the control function u take one of two values according to the sign of $s(x)$ ¹, as defined by

$$u = \begin{cases} u^+(x) & \text{for } s(x) > 0 \\ u^-(x) & \text{for } s(x) < 0 \end{cases} \quad (15)$$

¹Actually, the control function values depend on the sign of $(\nabla s) \cdot g$ which can locally be assumed positive, without loss of generality.

The control laws $u^+(x), u^-(x)$ are assumed to be smooth functions of x . Let $L_h\sigma$ denote the directional derivative of the scalar function σ with respect to the vector field h . Suppose that as a result of the control policy (3) the state trajectories of (1) locally reach the sliding surface \mathbf{S} and, from there on, their motion is constrained to the immediate vicinity of \mathbf{S} . We say that the sliding regime exists on \mathbf{S} whenever

$$\lim_{s \rightarrow 0^+} L_{f+gu^+} s < 0, \quad \lim_{s \rightarrow 0^-} L_{f+gu^-} s > 0 \quad (16)$$

i.e. the rate of the change of the scalar surface coordinate function $s(x)$, measured in the direction of the controlled field, is such that a crossing of the surface is guaranteed, from each side of the surface, by use of the switching policy (15).

Let ds denote the one-form corresponding to the gradient of $s(x)$ and let \langle, \rangle denote the standard scalar product of vectors and co-vectors in their functional relationship. Conditions (16) are equivalent to

$$\lim_{s \rightarrow 0^+} \langle ds, f + gu^+ \rangle < 0, \quad \lim_{s \rightarrow 0^-} \langle ds, f + gu^- \rangle > 0 \quad (17)$$

which alternatively explains that, on \mathbf{S} , the projections of the controlled vector fields $f + gu^+$ and $f + gu^-$ on the gradient vector to s are opposite in sign and hence the controlled fields locally point towards the surface \mathbf{S} .

These definitions are equivalent to:

S is said to be a sliding surface for the dynamical system defined by (13), (14) and (15) if there exists θ , an open set in U containing S , in such a way that $\forall x \in \theta \setminus S$, one of the following conditions holds.

1. there exists a finite time $t_s > 0$ such that

$$s(\phi(x, t)) \neq 0 \quad 0 \leq t < t_s \quad \text{and} \quad s(\phi(x, t)) = 0 \quad t \geq t_s$$

2. there exist t_s and \hat{t}_s , $0 < t_s < \hat{t}_s < \infty$ such that

$$s(\phi(x, t)) \neq 0 \quad 0 \leq t < t_s \quad \text{and} \quad s(\phi(x, t)) = 0 \quad t_s \leq t < \hat{t}_s$$

$$\text{and } \phi(x, \hat{t}_s) \in \partial(S \cap U)$$

Roughly speaking, the trajectories starting in a neighbourhood of S must fall down to S and remain there or, should one escape, it must go through $\partial(S \cap U)$.

As a first consequence of the definition, two questions arise, namely

1. **Existence.** Which conditions on f , g , u , σ and S , if any, guarantee that S is a sliding surface?
2. **Ideal sliding dynamics.** On the one hand the dynamics defined by (13), (14) and (15) does not consider S ; on the other hand, if S is a sliding surface for this dynamics, which vector field governs the system on S ?

Manifold invariance conditions

A definition of the *ideal sliding dynamics* was given by Utkin. This definition is based on the *method of equivalent control*. In this approach, ideal sliding motions are described by using the manifold invariance conditions $s = 0$ and $\dot{s} = 0$. Namely,

$$s = 0, \quad L_{f+gu_{eq}}s = \langle ds, f + gu_{eq}(x) \rangle = 0 \quad (18)$$

where $u_{eq}(x)$ is a smooth control law for which \mathbf{S} is a local integral manifold or a local invariant manifold of (1). The control function $u_{eq}(x)$ is called the *equivalent control*. From the definition of the directional derivative and (6), the equivalent control is explicitly given by

$$u_{eq}(x) = -\frac{L_f s}{L_g s} = -\frac{\langle ds, f \rangle}{\langle ds, g \rangle} = -\left(\frac{\partial s}{\partial x} g\right)^{-1} \frac{\partial s}{\partial x} f \quad (19)$$

The dynamical system $s = 0$ and $\dot{x} = f(x) + g(x)u_{eq}(x)$ is said to describe the *ideal sliding dynamics*.

Existence conditions

Now, some results on existence for the equivalent control and sliding motion will be given.

The equivalent control is said to be well defined whenever it exist and it is uniquely determined from the invariance conditions.

Lemma 1

A necessary and sufficient condition for the equivalent control to be well defined is that the transversality condition

$$\langle ds, g \rangle \neq 0 \quad (20)$$

be locally satisfied on S

Lemma 2

Let us assume there are no restrictions on the control action. Then, a necessary condition for the existence of a local sliding motion on S is that the equivalent control be well defined on S

Theorem 1

A necessary and sufficient condition for the local existence of a sliding regime on S is that locally in \mathbf{X} , for $x \in S$,

$$\min\{u^-(x), u^+(x)\} < u_{eq}(x) < \max\{u^-(x), u^+(x)\} \quad (21)$$

Corollary 1

Suppose a sliding regime locally exists on S and $\frac{\partial s}{\partial x} \cdot g > 0$, then a switching logic which achieves the sliding regime is given by

$$u = (-k + u_{eq}(x)) \text{sign}(s(x)) \quad (22)$$

with $k > 1$.

The proofs of these results can be found in [64], [133].

In practice, sliding motion is not attainable; imperfections such as hysteresis, delays, sampling and unmodelled dynamics will result in a chattering motion in a neighbourhood of the sliding surface.

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