Essays on Inflation Expectations, Heterogeneous Agents, and the Use of Approximated Solutions in the Estimation of DSGE models

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A mis padres,
a Christina.

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Abstract

In this thesis I evaluate the departures of three common assumptions in macroeconomic modeling and estimation, namely the Rational Expectations (RE) hypothesis, the representative agent assumption and the use of first-order approximations in the estimation of dynamic stochastic general equilibrium models. In the first chapter I determine how the use of survey data on inflation expectations in the estimation of a model alters the evaluation of the RE assumption in comparison to an alternative assumption, namely *learning*. In chapter two, I use heterogeneous agent models to determine the relationship between income volatility and the demand for durable goods. In the third chapter I analyze if the use of first-order approximations affect the evaluation of the Great Moderation.

Resumen

En esta tesis analizo desvíos de tres supuestos comunes en la elaboración y estimación de modelos macroeconómicos. Estos supuestos son la Hipótesis de Expectativas Racionales (ER), el supuesto del Agente Representativo, y el uso de aproximaciones de primer orden en la estimación de los modelos de equilibrio general. En el primer capítulo determino como el empleo de datos de expectativas de inflación en la estimación de un modelo puede alterar la evaluación del supuesto de ER en comparación a un supuesto alternativo como *learning*. En el segundo capítulo, utilizo modelos de agentes heterogéneos para determinar la relación entre la volatilidad de los ingresos y la demanda de bienes durables. En el tercer capítulo, analizo si el uso de aproximaciones de primer orden afecta la evaluación de la Gran Moderación.

Foreword

The main objective of macroeconomic research is to study fluctuations in economic aggregates such as production, employment and inflation. In doing so, economists rely on models, or theoretical constructs, that represent the underlying relationships among these aggregates. Due to the enormous complexity of the real world, these models are based on various assumptions which allow them to be analytically tractable and to be estimated. The present thesis contains three essays that evaluate the relevance of some of these assumptions.

In the first chapter I compare the performance of the New Keynesian model by Smets and Wouters (2007) in describing a set of macroeconomic indicators when this model is solved using two alternative assumptions of expectations formation: Rational Expectations (RE) and learning. The former one has become the benchmark assumption in macroeconomics since the influential work of Robert Lucas Jr. during the 1970s. It implies that agents have perfect knowledge of the structure of the economy and of the stochastic shocks that affect it, and therefore form their expectations in a model-consistent way. On the contrary, under learning agents do not have a perfect knowledge of the economy. As a consequence, they use historical data to update their perceptions about how the economy works and form their expectations about future variables using forecasting models that are updated whenever new data become available (see Evans and Honkapohja 2001).

The set of macroeconomic variables used in this chapter includes survey data on inflation expectations. This type of information has received a significant attention in monetary economics (e.g. Adam and Padula (2011), Nunes (2010), Roberts (1997, 1998)). Yet, few studies have used such data in the estimation of a dynamic stochastic general equilibrium (DSGE) model. I fill this gap by including survey data on inflation expectations in the estimation of one of the benchmark models for empirical analysis, namely the medium-size New Keynesian DSGE model developed by Smets and Wouters (2007). To be more precise, one goal of this chapter is to determine how the use of survey data in the estimation of this model alters the evaluation of the two alternative assumptions of expectations formation, RE and learning, regarding their relative fit.

Using a model comparison analysis, I find that the RE and learning solutions of the model by Smets and Wouters fit standard macroeconomic series in a similar way when not using survey data. This situation changes, however, once survey data are incorporated in the analysis: the learning solution is now clearly preferred as it is flexible enough to match the increases and decreases of inflation expectations during the late 1970s and the early 1980s.

In the second chapter, which was written in collaboration with Wouter Den Haan, I evaluate the effect of income uncertainty on the demand for durable goods. The setup used in this chapter differs from the representative agent assumption on which most of the dynamic general equilibrium models rely. Under this assumption, the economy behaves as if it is inhabited by a single type of agent. Such an assumption could be justified by the existence of complete insurance markets for the agents' idiosyncratic risk. However, it is hard to belief that this type of market exists. For this reason, I assume that agents can only imperfectly protect against income volatility, and I analyze how the existence of such uncertainty affects their decisions about purchases of durable goods. The relationship between uncertainty and the demand for this type of goods is currently used in consumption theories to explain the lack of a strong response of consumption to persistent income shocks, also known as the excess smoothness puzzle (see Xu (2010) and Luengo-Prado (2006) among others). Carroll and Dunn (1997) also use this relationship to explain some features of the US recession occurred during the period 1990-1991. These models are, however quite complex and difficult to incorporate into a fully-fledged macroeconomic model.

The purpose of the second chapter is thus to illustrate the basic conditions necessary to generate a negative relationship between income uncertainty and the demand for durable goods. Our results show that neither the existence of an alternative saving asset, such as bonds, nor a non-negativity constraint in the purchases of durable goods are sufficient conditions to get a negative relationship. Using two-period and infinite-horizon models we find that the savings aspect of durable goods typically dominates the aversion of buying this type of goods when income volatility increases.

Finally, in the third chapter I evaluate the reliability of the widespread practice to estimate first-order approximated policy functions (or solutions) of DSGE models with Bayesian methods. In particular, I focus on this type of estimation as a tool to determine the key factors behind the substantial decline in the volatility of many macroeconomic indicator in the US (and other developed countries) over the last three decades, a phenomenon known as the Great Moderation. First-order approximations describe well the dynamic of a model around a particular equilibrium. Yet, the presence of significant nonlinearities in the model and the prevalence of large shocks may reduce their accuracy. Since the standard analysis of the Great Moderation compares parameter estimates obtained using first-order approximations for a period of low volatility (usually starting from the 1980s) with those obtained for a period of higher volatility (i.e. before the 1980s) – in the latter case both nonlinearities and shocks are more likely to occur – the appropriateness of first-order approximations is questionable.

Based on a simulation exercise that consists of generating different series of three macroeconomic indicators (output gap, inflation and interest rate) which display similar volatility levels to the ones observed for the US during the period 1953-1984, we find significant biases in the estimations obtained when using first-order approximations. This is the case when the simulated series are obtained using second-order approximations and a monetary policy rule that does not react strongly to changes in inflation. Note, however, that we do not encounter such problems when the volatility level of the period 1953-1984 is due to higher volatility of the shocks affecting the economy. Changes in the volatility of the shocks and in the way how monetary policy was conducted are two of the most popular explanations of the Great Moderation (known as "good luck" and "good policy" hypothesis, respectively). Thus, the results presented in this chapter show that when high volatility data are generated according to the “good policy” hypothesis, omitting second-order terms of the approximated policy functions could lead to an erroneous misinterpretation of the sources of the Great Moderation.

Contents

1. Using Survey Data on Inflation Expectations in the Estimation of Learning and Rational Expectations Models

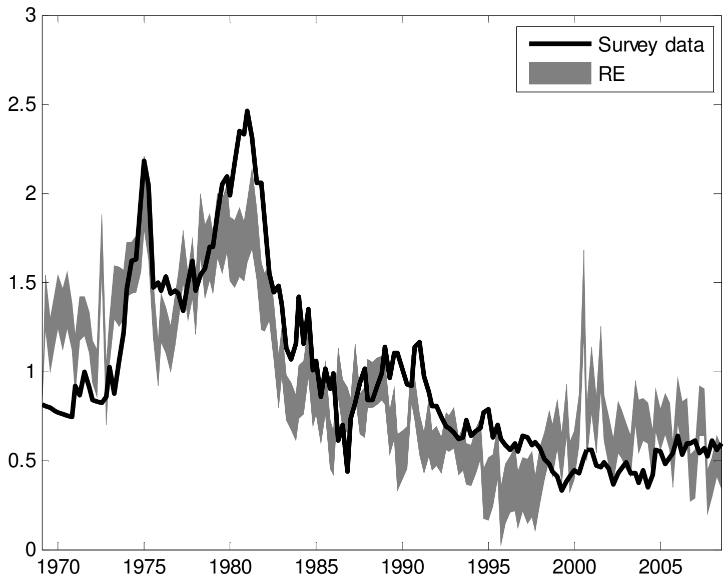
1.1 Introduction

Survey data on inflation expectations have received significant attention in monetary economics. In particular, this information has been used as proxy for expected inflation in estimations of the Phillips curve (Adam and Padula (2011), Nunes (2010)), in calibrations of hybrid models with both backward-looking and forward-looking expectations (Roberts (1997, 1998)), to test rational expectations and informational rigidities (Mankiw et. al. (2003), Coibion and Goridnichenko (2010)), among other applications. However, few studies have used this information in the estimation of a dynamic stochastic general equilibrium (DSGE) model. We contribute to fill this gap by including survey data on inflation expectations in the estimation of one of the benchmark models for empirical analysis, namely the medium-size New Keynesian DSGE model developed by Smets and Wouters (2007).

We estimate this model using two alternative ways to model expectations: on the one hand, we consider the benchmark assumption about expectations formation, namely Rational Expectations (RE), and on the other hand, we consider learning. The reasons for choosing these two alternatives are as follows.

First, even though the model by Smets and Wouters explains well the evolution of inflation and has higher predictive power than Bayesian VARs, it fails to match the evolution of survey data on inflation expectations when solved under the RE assumption and estimated with the standard set of macroeconomic indicators (see Figure 1.1). Therefore, the additional moment restriction implied by the use of survey data on inflation expectation in the estimation of the model might arguably affect the estimations of the parameters.

Figure 1.1
Inflation expectations: survey data and model-implied expectations under RE



Notes: The model-implied inflation expectations are obtained using the Kalman-filtered estimates at each set of parameter values that conforms the posterior distributions. The grey area represents the distance between the 5th and 95th percent confidence bands.

Second, as indicated by Slobodyan and Wouters (2009a,b), when assuming learning the estimation of the Smets and Wouters' model leads to different outcomes depending on the employed forecasting model.[1] This result reflects the main criticism on learning, namely that it relies heavily on the researcher's (arbitrary) selection of the forecasting model used by agents to generate their expectations. In order to address this criticism, we employ survey data to determine the forecasting model that agents most likely use to predict inflation.

Additionally, we use survey data on inflation expectations to pursue a model-comparison analysis between the RE and learning solutions. Similar to Del Negro and Eusepi (2010), we determine how the use of survey data in the estimation alters the evaluation of the two alternative assumptions of expectations formation regarding their relative fit.

Our findings reveal the following. First, according to the model comparison analysis, the RE and learning solutions of the Smets and Wouters´ model fit the standard macroeconomic series in similar way. This situation changes, however, once survey data are incorporated in the analysis: the learning solution is now clearly preferred as it is flexible enough to match the increases and decreases in inflation expectations during the late 1970s and the

[1] Slobodyan and Wouters (2009a,b) point out that the dynamics of the model under learning do not tend to deviate from the RE outcomes when the forecasting models are compatible with the solution under RE, but they deviate significantly when small forecasting models are considered.

early 1980s. Second, the better performance of learning can be mainly explained by the selection of a small forecasting model for inflation with a high speed of learning. As mentioned previously, survey data are employed to determine the specification of this forecasting model.

Third, with respect to the parameter estimates, we find that the additional moment restriction that represents the inclusion of the survey data on inflation expectations results in a higher persistence of exogenous shocks under RE, despite the fact that the model by Smets and Wouters incorporates nominal frictions such as price stickiness and indexation. In contrast, both price indexation and the learning process itself are the main sources of inflation persistence under learning. Additionally, under learning the use of survey data reduces the time-variability of the coefficients of the agents´ forecasting model. As a result, most of the stronger and more persistent responses of inflation to exogenous shocks are concentrated in the 1970s. In particular, in the same vein as Boivin and Giannoni (2008), we observe that unexpected monetary policy shocks have more destabilizing effects on inflation during the 1970s than afterwards.

So far, few studies have incorporated survey data into the estimation of a DSGE model. Del Negro and Eusepi (2010) use survey data on inflation expectations in order to discriminate between a model with imperfect information about a time-varying inflation target as in Erceg and Levin (2003), and a model where agents have perfect information about this target. Additionally, Carboni and Ellison (2009) incorporate the Greenbook unemployment forecast in the estimations implemented by Sargent et al. (2006). In so doing, they remove volatile and unrealistic beliefs of the Federal Reserve about unemployment-inflation dynamics. In contrast to the existing studies, we do not only exploit the additional moment restrictions implied by the use of survey data, but we also employ this information to “discipline” the way how the forecasting model for inflation is selected under learning.

The remainder of this chapter is organized as follows. In the next section we summarize the main features of the model, characterize its solution under RE and learning, and discuss the specific learning setup employed in this study. Section 1.3 presents the series of macroeconomic indicators and the prior distributions used in the Bayesian estimation. Section 1.4 describes the forecasting model used for the learning specification and the results of the model comparison analysis. It also evaluates the changes in the parameter estimates obtained when using survey data in the estimation of the SW model and their effects over the relative importance of the sources of inflation persistence, the composition of inflation expectations, and the Impulse-Response functions analysis. Section 1.5 contains some robustness exercises. Section 1.6 concludes and outlines possible avenues for future research.

1.2. The model and its solution under the RE assumption and learning

Our estimation is based on a New Keynesian model similar to the model by Smets and Wouters (2007) (hereafter SW) with only one modification that is explained below. The optimization problem of the households, firms and government as well as the equilibrium conditions are described in detail in the Online Appendix. Readers interested in more details of the model are encouraged to refer to SW.

In the remainder of this section we describe the model's participants and its frictions, how participants' decisions depend on forecasts of future variables, the minor modification to the SW model, and the representation of the model solution under the assumption of RE and learning.

1.2.1 Model's participants, main frictions and forward variables

The New Keynesian model by SW is based on a neoclassical growth model augmented with several frictions affecting both nominal as well as real decisions of households and firms. Households maximize a utility function that depends on the consumption of goods and on the amount of labor supplied, over an infinite lifetime horizon. Consumption enters in the utility function relative to a time-varying external habit variable. This feature together with the possibility of consumption (and labor) smoothing, which is possible through the purchasing and selling of a one-period bond, generates that current consumption depends on past and expected future consumption, on current and expected future hours worked and the ex-ante real interest rate of this bond. Households also rent capital services to firms and decide how much capital to accumulate given the capital adjustment costs they face. This friction creates a link between investment, the market value of the capital stock and past and expected future investment. In addition, the arbitrage condition for the value of the capital stock implies that this stock reacts positively to its expected future value and the expected future real rental rate of capital, but negatively to the ex-ante real interest rate. Variations in the rental price of capital affect the level of utilization of the capital stock, which can be adjusted at increasing costs.

Labor is differentiated by a union which determines wages taking into account the existence of nominal rigidities à la Calvo (1983). Thus, given the possibility of not being re-optimized within one period but only partially indexed to past inflation, wages depend on past and expected future wages and inflation. Firms produce differentiated goods, decide on the amount of labor and capital services to hire, and set prices. Prices are also affected by Calvo-type rigidity and when not re-optimized they are partially indexed to past inflation rates. Therefore, prices are set as a function of current and expected future marginal costs, but are also determined by the past inflation rate.

Finally, the model is closed by adding an empirical monetary policy reaction function: the policy controlled interest rate is adjusted in response to inflation and to changes in the level of output from one period to another. Notice that in the original SW specification, the monetary policy rule does not react to the output growth but to the output gap (i.e. the difference between the output obtained under nominal rigidities and under flexible prices). This modification allows us to avoid the estimation of a parallel economy under flexible prices, which reduces the number of forward variables in the model considerably. As reported by Slobodyan and Wouters (2009a), we find that this modification, however, does not affect the results obtained by SW.

The model contains the following 13 endogenous variables: output, y; consumption, c; investment, i; value of the capital stock, Q^k; installed stock of capital, $\bar{k}$; stock of capital, k; inflation, π; capital utilization rate, u; real rental rate on capital, r^k; real marginal cost, mc; real wages, w; hours worked, L; and interest rate, R. In addition, the stochastic part of the model is characterized by seven exogenous autoregressive processes, each of them including an iid-normally distributed error.[2] The model is de-trended with respect to the deterministic growth rate of the labor-augmenting technological progress and linearized around the

[2] These shocks are the risk premium shocks and the investment-specific technology shocks (both of them affecting the intertemporal margin); the wage mark-up shocks and the price mark-up shocks (which impact on the intratemporal margin); two policy shocks, the exogenous spending and the monetary policy shocks; and last the total factor productivity shocks.

steady-state of the de-trended variables. The set of equations that describes the linearized dynamism of this model can be represented as follows:

(1) $$\Theta_0 \tilde{E}_t Y_{t+1} + \Theta_1 Y_t + \Theta_2 Y_{t-1} + \Psi e_t = 0$$

(2) $$e_t = \Gamma_e e_{t-1} + \Gamma_\varepsilon \varepsilon_t$$

Y is a vector that contains the 13 endogenous variables of the model, e is the vector of exogenous shocks and ε is the vector of *iid*-normal innovations. $\tilde{E}_t(\cdot)$ is the expectations operator and indicates that expectations can be either rational (for which we use $E_t(\cdot)$) or that they come from a learning process (represented by $\hat{E}_t(\cdot)$). The matrices Θ_0, Θ_1, Θ_2, and Ψ contain non-linear combinations of the parameters of the model but also zeros. The zero elements in these matrices reflect the fact that the model does not include the expected future values or past values of all the endogenous variables. Γ_e is a diagonal matrix that contains the autoregressive coefficients of the exogenous shocks, and Γ_ε is an identity matrix that additionally incorporates one element that reflects the effect of a productivity innovation over the exogenous spending shocks.[3]

[3] SW include this element motivated by the fact that, in estimation, exogenous spending includes net exports, which may be affected by domestic productivity developments.

1.2.2 The Rational Expectation solution of the DSGE model

When dealing with expectations, researchers have traditionally adopted the RE assumption. This assumption implies that agents have perfect knowledge about the true stochastic process of the economy. There are several algorithms available to solve Equation (1) under the RE assumption. In particular, we use Uhlig (1999), although alternative algorithms include, among others, Blanchard and Kahn (1980), Binder and Pesaran (1997), Christiano (2002) and Sims (2002).

Since we focus on the case of determinacy and restrict the parameter space accordingly, the resulting law of motion takes the following form:

(3) $$Y_t = \Phi_1^{RE} Y_{t-1} + \Phi_2^{RE} e_{t-1} + \Phi_3^{RE} \varepsilon_t$$

Equations (2) and (3) imply a state-space representation of the DSGE model that can be estimated with Kalman filter, where the vector $\begin{bmatrix} Y & e \end{bmatrix}'$ can be viewed as a partially latent state vector.

1.2.3 The Learning solution of the DSGE model

Since the high level of cognitive abilities and computational skills implied under the RE assumption are implausible in practice, researchers have developed models of imperfect knowledge and associated learning processes. One of the most popular learning mechanisms used in macroeconomics is a form of adaptive learning.

Under this approach agents use historical data to update their perceptions about how the economy works and form their expectations about future variables using forecasting models that are updated whenever new data become available (see Evans and Honkapohja 2001).

Before presenting the learning algorithm used in this study, it is important to note that the forward looking nature of Equation (1) leads to a simultaneity problem in case the solution for Y_t depends on reduced form coefficients of forecasting models relying on information up to t. The standard way to overcome this problem is to assume that agents make their forecast of Y_{t+1} based on estimates of the reduced form coefficients from the period t-1.[4] Thus, expectations adopt the following representation:

(4) $$\hat{E}_t Y_{t+1} = \beta'_{t-1} X_t, \quad X \subset [1 \quad Y \quad e]'$$

where X is a vector that could include all endogenous and exogenous variables of the model as well as only a subset of them. It could also include a constant term in case agents use observable non-zero mean time series in their forecasting models. β'_{t-1} is a matrix of linear combinations of the reduced form coefficients that defines the projection of X_{t-2} over Y_{t-1}.[5]

[4] Alternatively, as indicated by Carceles-Poveda and Giannitsarou (2007), one may assume that Y_t is not included in the information set when forming expectations, i.e. expectations are formed using data up to t-1.

[5] Due to the use of observable time series in the forecasting model for inflation, the row of β' related to inflation expectations includes linear combinations of

In practice, in applied studies the forecasting model that agents use to form their expectations has been arbitrarily chosen by the researcher. All state variables of the model can be included to not depart to far from the RE setup; exogenous shocks and variables which are not observed in reality could be excluded under the assumption that agents and the researcher have the same set of information; one could even include a subset of those observed variables arguing that in so doing the overall fit of the model to the data is higher. One of the contributions of this study is to deviate from this arbitrary choice of the forecasting model, but to use survey data on inflation expectations to determine the actual forecasting models for inflation that agents most likely use.[6]

With respect to the estimation of β, the literature on learning commonly assumes that agents update the coefficients of their forecasting models using constant-gain least squares (CG-LS). Under CG-LS, the most recent observations receive higher weights in the least square estimation. More precisely, the weight decreases geometrically depending on the distance in time to the most recent observation. This learning mechanism implies that agents are concerned about structural changes of the economy, which is a realistic feature of any type of econometric estimation in the real world. Additionally, the CG-LS receives empirical support because it outperforms other recursive parameter updating algorithms such

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the coefficients that define the projections among observable series (see derivations in subsection 1.4.1).

[6] Section 1.4 contains the list of variables included in the forecasting model.

as recursive least squares and the Kalman filter in out-of-sample forecasting of inflation and output growth (see Branch and Evans (2006)).

The recursive expression for the estimate of β under CG-LS conditional on information up to t is as follows:

(5a) $$\beta_t = \beta_{t-1} + g\left(R_t\right)^{-1} X_t\left(Y_t - \beta'_{t-1}X_{t-1}\right)'$$

(5b) $$R_t = R_{t-1} + g\left(X_{t-1}X'_{t-1} - R_{t-1}\right)$$

where g represents the *constant-gain parameter* and R_t the variance-covariance matrix of the regressors included in the forecasting model. The gain refers to the relative weight of the most recent observation and $1-g$ is the discount factor over less recent observations (in ordinary least squares, the gain is not a constant value but equals *1/t*, where *t* is the position of the observation since the beginning of the sample). When g =0, β is constant and equal to the value that starts the recursion. Otherwise, β changes with the arrival of new information. Note that g is the only parameter that is added to the set of structural parameters of the model.

Substituting Equation (4) in Equation (1), and using Equation (2), we get the following expression:

(6) $$Y_t = \Phi^L_{0,t-1} + \Phi^L_{1,t-1}Y_{t-1} + \Phi^L_{2,t-1}e_{t-1} + \Phi^L_{3,t-1}\varepsilon_t$$

The matrices $\Phi^L_{0,t-1}$, $\Phi^L_{1,t-1}$, $\Phi^L_{2,t-1}$ and $\Phi^L_{3,t-1}$ are non-linear combinations of the parameters of the model and the reduced form coefficients of β_{t-1}. The presence of the latter type of elements potentially makes these matrices time-varying.

To sum up, the state-space representation of the model estimated under learning consists of Equations (6) and (2). Equations (5a) and (5b) are additionally required to get estimates for β as well as some initial values for β and R necessary for the CG-LS algorithm.[7]

1.2.4 The Learning setting used in this study

Survey data on inflation expectations are used to determine the forecasting model of inflation under learning. However, inflation is not the only variable that is measured in expectations in the model but also six further variables (consumption, investment, hours worked, real wages, real rental rate on capital and value of the capital stock).[8] Thus, in order to restrict the differences in the estimates of the model solved under RE and learning to the use of survey data on inflation expectations, we restrict the learning setting for the other variables appearing in expectations to be as close as possible to the RE setting.[9] The latter implies that the forecasting models for these variables include as regressors the same variables

[7] The criterion that we follow to define these initial values is explained in the following subsection.

[8] This study focuses only on the use of survey data on inflation expectations. Three reasons motivate the selection of this series. First, survey data on inflation expectations have received significant attention in monetary economics (e.g. Roberts (1997,1998), Adam and Padula (2011), Nunes (2010), among others). Second, the quality of this information, jointly with survey data on output growth, has been evaluated by many studies (e.g. Ang.et.al. 2007). Third, they are available for most of the sample of interest in this study.

[9] Notice that it is not possible to have both learning and rational expectations at the same time.

that appear in their solution under the RE assumption (Equation 3). It furthermore implies that the initial values of the elements of β and R related to these variables correspond to the respective rows of the matrices Φ_1^{RE}, Φ_2^{RE} and Φ_3^{RE} and the unconditional second moments resulting from the RE solution, respectively.[10,11] Additionally, the mentioned variables and inflation have separated CG-LS recursion processes (therefore we consider two sets of equations (5a) and (5b)), which implies two gain parameters.

Due to the incompatibility of the forecasting model for inflation with the rational expectation solution for this variable, it is unfeasible to use the coefficients or the implied second moments of this solution as initial conditions of the learning algorithm.[12] For this reason, we use pre-sample estimates of β and R to initialize the learning algorithm for inflation. As explained in Section 1.4, survey data on inflation expectations also play a role in the selection of these values.

Finally, it is important to indicate that learning dynamics in our model is incomplete because it has no chance of converging to the RE equilibrium. This result arises by two reasons. First, the forecasting model for inflation is no compatible with the RE

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[10] In case the gain parameter for these variables is equal to zero, their forecasting model will be completely compatible with the RE solution, not only at the beginning but through the entire sample.

[11] One advantage of choosing the initial conditions of the learning algorithm is that it avoids a significant increase in the number of parameters to be estimated (they more than duplicate the number of structural parameters of the model).

[12] The use of pre-sample estimations of β and R related to the other variables that appear in expectations does not affect our results.

solution for this variable. Second, the use of CG-LS in the presence of random shocks prevents the resulting dynamics to converge to the RE solution (Evans and Honkapohja (1995)). Honkapohja and Mitra (2003) show that incomplete learning with finite memory can have several attractive properties in standard frameworks. In particular, learning could be asymptotically unbiased in the sense that the mean of the first moment of the forecast is correct. Additionally, dynamics of incomplete learning result in good approximation to actual data, as argued by Marcet and Nicolini (1998) and Sargent (1999).[13]

1.3. Data and priors

The model is estimated using the same quarterly macroeconomic indicators for the US as in SW, but in addition we use survey data on inflation expectations provided by the Survey of Professional Forecasters (SPF). Although the model specifies that many of the important forecasts are made by households, and thus, it would be more appropriated to use survey data that collect directly these expectations, provided for instance by the University of Michigan's Surveys, we opt for the SPF for the following two reasons. First, as pointed out by Del Negro and Eusepi (2010), the Michigan's Surveys ask households about inflation in general, making it

[13] Marcet and Nicolini (1998) employ incomplete learning to explain the existence of hiperinflation processes in some Latin American countries. In the similar way, Sargent (1999) considers incomplete learning as an important element to explain the rises and decreases of inflation in the US.

impossible to relate this measure to a specific measurement of changes in prices, such as the Consumer Price Index or GDP deflator inflations. Second, the Michigan's surveys started collection information about inflation expectations in quantitative form only from 1978 onwards. As a consequence, it does not cover the years of the sample considered by SW, our benchmark reference. The SPF collects expectations on future GDP deflator inflation, a measurement that is compatible with the inflation series used by SW. Moreover, this series starts ten years earlier than the University of Michigan´s Surveys, and thus covers almost completely the sample considered by SW.

Using the SPF we calculate the median value of the quarterly one-period-ahead forecast for the percentage increase of the GDP deflator. The resulting series is referred to as "$dlP^{e}_{t,t+1}$". As this information is only available from 1968:4 onwards, this date marks the starting point of our sample. The sample covers all quarters until 2008:2. Further macroeconomic indicators considered are the first difference of the logarithm of real GDP ("*dlGDP*"), of real consumption ("*dlCons*"), of real investment ("*dlInv*"), the real wage ("*dlWage*") and the GDP deflator ("*dlP*"), as well as the logarithm of hours worked ("*lHours*") and the federal funds rate ("*FedFunds*"). Please refer to the Online Appendix for a detailed description of the data.

The following set of measurement equations relates the mentioned macroeconomic indicators to the variables of the model when survey data on inflation expectations are not included:

$$(7) \qquad \begin{bmatrix} dlGDP_t \\ dlCons_t \\ dlInv_t \\ dlWage_t \\ lHours_t \\ dlP_t \\ FedFunds_t \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{l} \\ \overline{\pi} \\ \overline{r} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} \\ \hat{c}_t - \hat{c}_{t-1} \\ \hat{i}_t - \hat{i}_{t-1} \\ \hat{w}_t - \hat{w}_{t-1} \\ \hat{l}_t \\ \hat{\pi}_t \\ \hat{R}_t \end{bmatrix}$$

where $\overline{\gamma}$ represents the common quarterly trend growth rate, $\overline{l}$ the steady state hours worked, $\overline{\pi}$ the quarterly steady state inflation rate and $\overline{r}$ the quarterly steady state nominal interest rate.

When survey data are incorporated in the estimation of the DSGE model, we need to add an additional measurement equation. Under the RE solution, this equation has the following form:

$$(8a) \qquad dlP^e_{t,t+1} = \overline{\pi} + E_t\hat{\pi}_{t+1} + \zeta_t = \overline{\pi} + \Phi^{RE}_{1,\pi}Y_t + \Phi^{RE}_{2,\pi}e_t + \zeta_t$$

where $\Phi^{RE}_{1,\pi}$ and $\Phi^{RE}_{2,\pi}$ are the row of Φ^{RE}_1 and Φ^{RE}_2 in Equation (3), respectively, that relate inflation to the vectors Y and e. ζ_t represents an *iid* measurement error related to the surveys on inflation expectations. Hence, survey data are viewed as a noisy measure of actual expectations. Under learning, the extra measurement equation has the following form:

(8b) $$dlP^{e}_{t,t+1} = \overline{\pi} + \beta'_{\pi,t-1} X_t + \zeta_t$$

where $\beta'_{\pi,t-1}$ is the corresponding row of β'_{t-1} in Equation (4). In this equation as well as in the previous one, we are implicitly assuming that both, inflation and inflation expectations, have the same steady state.

The structural model contains 38 parameters. 33 of them are estimated while the remaining 5 are fixed at the values used in SW.[14] The learning estimation adds two further parameters (the gains for inflation and for all the other variables appearing in expectations). When estimating the model with survey data, we consider one extra parameter, namely the standard deviation of the measurement error of the surveys (ζ_t). The prior distributions of the structural parameters are as in SW. Additionally, we use uniform distributions over the [0,1] domain for the gains and an inverse gamma distribution with zero mean and standard deviation of 2 for the standard deviation of ζ_t. The prior distributions for all the parameters are presented in Appendix A.

The estimation of the DSGE model is performed using Bayesian estimation methods. Employing the random walk Metropolis-Hastings algorithm, we obtain 250 000 draws from each model's posterior distribution. The first half of these draws is discarded and

[14] These parameters are the depreciation rate (fixed at 0.025), the exogenous spending-GDP ratio (0.18), the steady state mark-up in the labor market (1.5) and the curvature parameters of the Kimball (1995) aggregators in the goods and labor market (both set at 10).

1 out of every 10 draws is selected to estimate the moments of the posterior distributions.

1.4. Results

The first step is to determine the forecasting model that agents most likely use to generate their expectations on future inflation. The resulting forecasting model defines the setup of learning used in this section. In a second step, we implement a model-comparison analysis between the solutions under RE and learning. Then we evaluate the changes in the parameter estimates obtained when using survey data in the estimation of the SW model and their effects over the relative importance of the sources of inflation persistence, the composition of inflation expectations, and the Impulse-Response functions analysis.

1.4.1 Forecasting models for inflation

In order to determine the forecasting model for inflation used under learning, we estimate different linear models for inflation where the regressors consist of (besides an intercept) all possible combinations of the lagged series of dlGDP, dlCons, dlInv, dlWage, dlP, FedFunds and lHours. These are the same macroeconomic series used in the estimation of the DSGE model, and thus, their use implies that the representative agent of the model has the same information as the econometrician.

We rank these models (127 in total) according to the resulting similarities between the one-period-ahead inflation forecast series and the survey data on inflation expectations. For the ranking we employ the Mean Squared Error (MSE). Table 1.1 shows the five best-performing forecasting models for the period 1968:4 – 2008:2 and Figure 1.2 represents the one-period-ahead inflation forecast series of the best three models and the survey data.[15] In general, the one-period-ahead forecasting series yielded by the best performing models are very similar . Moreover, they all track relatively well the increase in survey expectations during the 1970s and the reduction at the beginning of the 1980s. However, during some years of the 1980s and 1990s the forecast series underestimate the survey data, while during the 2000s they overestimate it. In particular, note that the forecasting models under-predict inflation expectations during the year 1983. This result is related to the important reduction of inflation in the previous quarters which was not accompanied by a reduction in inflation expectations of the same magnitude. Thus, as indicated below, the evolution of inflation expectations is difficult to match during this year regardless of the expectation formation assumption.

[15] We elaborate the ranking following these four steps. First, we estimate each model using a recursive CG-LS. Second, we initialize this algorithm using pre-sample estimates for which we employ ordinary least squares. Third, different values of the constant gain are employed to produce forecasts for each of the models (these values are taken from a grid of point between 0 and 1). Then we establish a ranking of these models taking into account the value of the constant gain that results in the lowest MSE for each of the model. Finally, given that the ordering depends on the choice of the pre-sample, we try different pre-samples and select the one with the lowest MSE among the top models.

Table 1.1
Ranking of forecasting models for inflation
Sample 1968:4-2008:2

Rank	Regressors	Gain	MSE
1	dlP	0.125	0.0294
2	dlP lHours	0.113	0.0300
3	dlP dlCons	0.100	0.0302
4	dlP dlCons lHours	0.125	0.0303
5	dlP dlGDP	0.125	0.0315

Note: the models are estimated by recursive CG-LS. The initial conditions are obtained from the period 1950:1-1968:3. Regression: dlP_t = intercept + $regressor_{t-1}$

The benchmark forecasting model for inflation is a model that includes as regressors only lagged inflation and an intercept (the first model in Table 1.1). In this case, the measurement equation for inflation expectations looks as follows:[16,17]

(9) $$dlP^e_{t,t+1} = \beta_{0,\pi,t-1} + \beta_{1,\pi,t-1} dlP_t + \zeta_t$$

$$= \beta_{0,\pi,t-1} + \beta_{1,\pi,t-1}\bar{\pi} + \beta_{1,\pi,t-1}\hat{\pi}_t + \zeta_t$$

Note, that the second equality is obtained using the measurement equation of dlP_t.

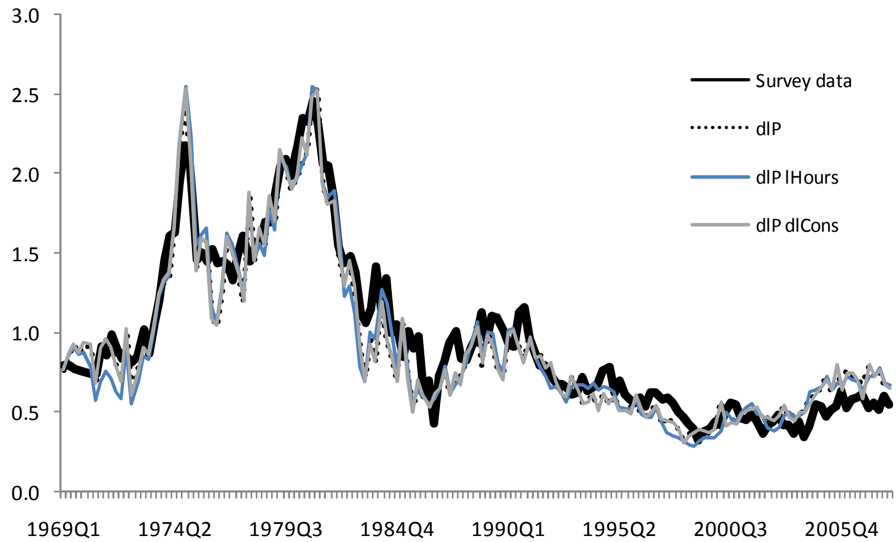
[16] The time-variability of the intercept included in the forecasting models implies that agents do not know the steady state values of the macroeconomic series used in the estimation.

[17] Omitting ζ_t and replacing $dlP^e_{t,t+1}$ by $\hat{E}_t\hat{\pi}_{t+1} + \bar{\pi}$, we get:

$\hat{E}_t\hat{\pi}_{t+1} = \left[-\bar{\pi} + \beta_{0,\pi,t-1} + \beta_{1,\pi,t-1}\bar{\pi} \quad \beta_{1,\pi,t-1}\right]\left[1 \quad \hat{\pi}_t\right]'$, which represents the row of Equation (4) that corresponds to inflation expectations. If we add in both sides $\bar{\pi}$ and the measurement error in the RHS, we get Equation (8b).

The other forecasting models contained in Table 1.1 are considered in the robustness analysis in section 1.5.

Figure 1.2
Inflation forecasts and survey data on inflation expectations



1.4.2 Model comparison

In this subsection we analyze which of the two assumptions of expectations formation (RE and learning) fits the data better. In particular, and similar to Del Negro and Eusepi (2010), we want to determine how the use of survey data on inflation expectations in the estimation of the SW model alters the evaluation of these two alternative assumptions regarding their fit.

Table 1.2 shows the logarithm (log) of the marginal likelihoods of the RE and learning solutions, when survey data are included in the estimation and when they are not. In the latter case, both solutions show similar log marginal likelihoods (see column 1). The log marginal likelihood difference of 3.96 points is not very robust (it goes down to less than 1 when choosing different priors)[18], and thus, there is no clear evidence in favor of learning. However, this result changes significantly when survey data are included in the estimation (see column 2). Now learning clearly outperforms the RE solution with a difference of 64.36 points in the log marginal likelihood, which implies a posterior odd of 8.93E+27 in favor of the former specification.

Table 1.2
Model comparison

Log Marginal Likelihood	Dataset without survey data (1)	Dataset with survey data (2)	(3) =(2)-(1)
RE	-146.78	-19.14	127.64
Learning	-142.82	45.22	188.04

Notes: This table shows the log marginal likelihood for RE and Learning. Survey data on inflation expectations come from the SPF one-quarter-ahead median forecast of the GDP deflator.

Does learning provide a better description of the survey data on inflation expectations than rational expectations? To answer this question we follow Del Negro and Eusepi (2010) and calculate how well the model fits the series of inflation expectations conditional

[18] Using uniform prior distributions, we find log marginal likelihood values for the RE and learning specifications of -120 and -119.2, respectively. Moreover, Del Negro and Schorfheide (2008) shows that even 5 points in the log marginal likelihood can be overturned by choosing a slightly different prior.

on the parameter distribution delivering the best possible fit for the rest of macroeconomic indicators. The object of interest has the following representation:

$$p(dlP_{1,T}^{e} \left| Y_{1,T}, M_i \right.) = \int p(dlP_{1,T}^{e} \left| \theta, Y_{1,T}, M_i \right.)\, p(\theta \left| Y_{1,T}, M_i \right.)\, d\theta$$

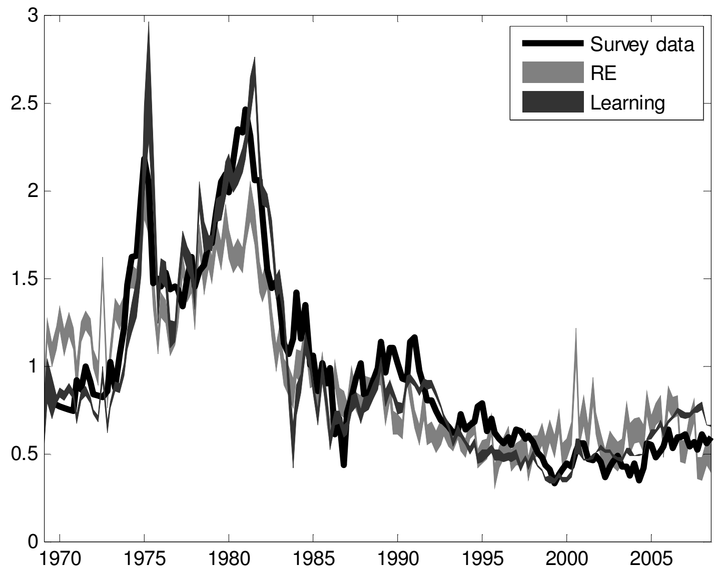
where $Y_{1,T}$ and $dlP_{1,T}^{e}$ represent the series of macroeconomic indicators of Equation (1.7) and the survey data on inflation expectations, respectively, with observations going from 1 to T. $p(\theta \left| Y_{1,T}, M_i \right.)$ represents the posterior distribution of the parameters of the model, θ, which are obtained from the estimation of the model ignoring survey data. Finally, M_i corresponds to the solution of the model that could be obtained either under RE or learning.

Column (3) of Table 1.2 shows the logarithm of $p(dlP_{1,T}^{e} \left| Y_{1,T}, M_i \right.)$, which is determined by the difference between column (2), logarithm of $p(dlP_{1,T}^{e}, Y_{1,T} \left| M_i \right.)$, and column (1), logarithm of $p(Y_{1,T} \left| M_i \right.)$. According to this measure, learning clearly outperforms RE in describing the evolution of the survey data. This result implies that the predictive power of the SW model can be improved by resorting to available survey data and to an admissible learning rule for the formation of expectations.

A graphical evaluation of the model-implied series of inflation expectations also shows that learning improves the description of the evolution of survey expectations (see Figure 1.3). In particular, the RE solution under-predicts survey expectations during the late 1970s and the early 1980s. It also over-predicts surveys at the beginning and end of the sample. On the contrary, the solution under learning is flexible enough to match more closely the fluctuations in the survey data with a couple of exceptions.[19] First, the model-implied inflation forecast over-predicts survey expectations in 1974:4. This result can be explained by the significant and sharp increase in inflation observed after the first oil crisis. And second, the model-implied inflation forecast obtained under learning, but also under RE, under-predicts survey data in 1983. During this year and the previous one, the important reduction in inflation was not accompanied with a similar-sized reduction in inflation expectations of the SPF. The closely related dynamics of inflation and inflation expectations in the RE solution, on the one side, and the highly perceived inflation persistence in the learning specification estimated for this time, on the other side, constitute the reasons why both specifications fail to track the evolution of survey data for this period.

[19] The better performance of learning in matching the survey data on inflation expectations can also be measured by the correlation between surveys and the model-implied series of inflation expectations. When measured in levels, the correlation coefficients equal 0.870 and 0.938 for RE and learning, respectively. In first differences, the correlation coefficients for both cases are 0.201 and 0.276, respectively. These differences are statistically significant.

Figure 1.3
Inflation expectations: survey data and model-implied expectations
Database includes survey data



Notes: The model-implied inflation expectations are obtained using the Kalman-filtered estimates at each set of parameter values that conforms the posterior distributions. The grey and black areas represent the distance between the 5th and 95th percent confidence bands.

1.4.3 Posterior estimates

The next step is to compare the posterior estimates obtained under RE and learning when survey data on inflation expectations are not employed in the estimation of the DSGE model (see Table 1.3). Taking the estimates of the RE solution as the benchmark case (column 1), the estimation under learning (column 2) results in a lower autocorrelation coefficient of the price mark-up shock, lower price stickiness, and higher price indexation. These results are compatible with Slobodyan and Wouters (2009b), but not, however, with the ones of Milani (2007). Milani (2007) finds that the introduction of learning forces the degree of habits in consumption and inflation indexation almost down to zero, while the autocorrelation coefficient of the supply shocks increases significantly (from a posterior mean of 0.02 in his rational expectations estimation (Table 1.3) to 0.854 in his benchmark learning estimation (Table 1.2)). As in Slobodyan and Wouters (2009b), we use small forecasting models for inflation while Milani uses forecasting models that are compatible with the RE solution of his model.[20] Additionally, we employ external habits in consumption, and not internal type as Milani does. These differences may explain our discrepancies.

When estimating both the RE and learning solutions using survey data on inflation expectations, we find that the most important changes in the parameter estimates are observed in the RE solution. In particular, we find that the price indexation significantly decreases (from a posterior median of 0.327 to 0.052), the autocorrelation coefficient of the price mark-up shocks increases (from a posterior median of 0.448 to 0.726), and the wage stickiness is slightly lower (from 0.554 to 0.468) (see Table 1.3, column 3). In the learning estimation, the only significant change in the parameter

[20] We refer as "small" forecasting model to those models that use fewer regressors than the implied RE solution of the DSGE model.

estimates is observed for the gain parameter for inflation, which decreases from 0.188 to 0.141 (see Table 1.3, column 4).[21] As a result, the additional moment restriction that represents the inclusion of the survey data on inflation expectations highlights the differences in the sources of inflation persistence. Under the RE solution, despite the fact that the model incorporates nominal frictions such as price stickiness and indexation, inflation persistence depends on the persistence of the price mark-up shock. In contrast, under learning, both price indexation and the learning process itself are the main sources of inflation persistence.

[21] The use of more data in the estimation of a DSGE model could eliminate flat areas of the likelihood function related to some parameters or combinations of them, and thus, may help to solve problems of weak identification (as discussed by Canova and Sala (2009)). However, in this study, we basically do not observe such improvements when incorporating survey data on inflation expectations in the estimation of the DSGE model (with the exceptions of the gain parameter for inflation in the learning specification and the standard deviation of the price mark-up shock under rational expectations).

Table 1.3
Posterior distribution statistics

		(1)		(2)		(3)		(4)	
		WITHOUT survey data				WITH survey data			
		RE		Learning		RE		Learning	
	Symbol	Median	Std	Median	Std	Median	Std	Median	Std
Wage stickiness	ξ_w	0.554	0.045	0.547	0.049	0.468	0.043	0.563	0.049
Price stickiness	ξ_p	0.648	0.044	0.481	0.035	0.629	0.058	0.480	0.035
Wage indexation	ι_w	0.482	0.131	0.314	0.107	0.442	0.124	0.319	0.107
Price indexation	ι_p	0.327	0.155	0.544	0.108	0.052	0.025	0.515	0.119
TR: inflation	r_π	1.666	0.130	1.396	0.116	1.711	0.114	1.398	0.104
TR: lag interest rate	ρ_R	0.760	0.028	0.763	0.028	0.706	0.030	0.777	0.029
TR: change in output	$r_{\Delta y}$	0.199	0.046	0.203	0.046	0.187	0.044	0.210	0.047
aut. Price Mk up shock	ρ_p	0.448	0.195	0.140	0.070	0.726	0.078	0.173	0.087
std. Price mkup shock	σ_p	0.145	0.026	0.213	0.017	0.112	0.013	0.204	0.014
gain - inflation	g^{π}			0.188	0.014			0.141	0.009
gain - others	$g^{non\pi}$			0.031	0.042			0.019	0.031
Measurement exp error	σ_{exp}					0.265	0.016	0.176	0.010
Log. Mg. Likelihood		-146.8		-142.8		-19.1		45.2	

Notes: this table shows the median and standard deviation of the posterior distributions of those parameters most closely related to the dynamics of inflation. The Online Appendix contains the same statistics for the complete list of parameters of the model, their prior and posterior distributions and a convergence check of the random walk Metropolist-Hasting.

Additionally, I would like to comment particularly on the posterior median estimate obtained for the gain parameter for inflation as it is higher than the estimates reported by previous studies.[22] For instance, Orphanides and Williams (2005a) consider a baseline calibrated value of the gain parameter of 0.02 and Milani (2007) and Slobodyan and Wouters (2009a) find posterior mean estimates that range between 0.0161 and 0.0247, and between 0.002 and 0.02, respectively. The high values for this parameter obtained in our study are related to the specification of the forecasting model. As the econometric exercise implemented in the subsection 1.4.1 shows, the forecasting models for inflation that best fit the survey data require significant time variation of their coefficients (the gain parameters are equal or higher than 0.10). Moreover, the fewer variables are included in the forecasting model, the smaller the impact of the time-variation of their coefficients on the stability of the DSGE model. Thus, not only does the forecasting model for inflation require high levels of time-variability in its coefficients, but its specification actually allows us to estimate the DSGE model for these levels of time-variability. Finally, it is important to mention that Slobodyan and Wouters (2009b) also find a high

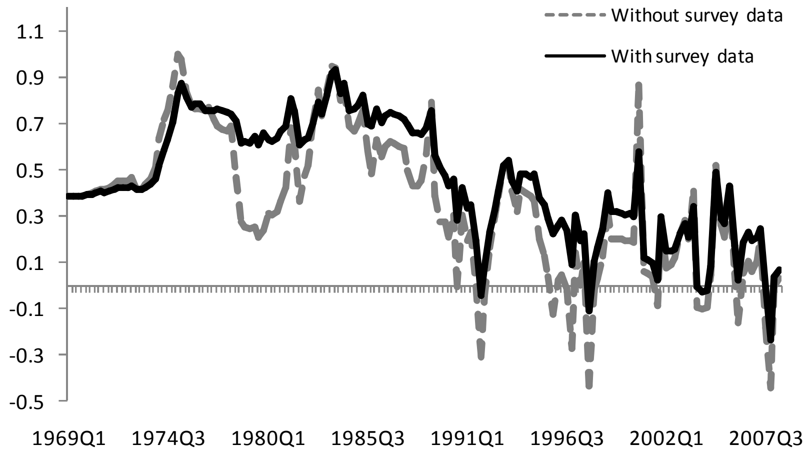
[22] Constant-gain parameter values of 0.188 and 0.141 imply that 75 percent of the information that people employ to generate their inflation expectations is contained in the 6.7 and 9.1 most recent quarterly data observations, respectively. Because the relative weight of the 'j' most recent observations in the estimation of the forecasting model is $g(1-g)^{j-1}$, the number of observations required to accumulate the 'p' percent of information used in this estimation is given by log(1-p)/log(1-g).

degree of time-variability in the coefficients of the small forecasting models employed in their learning estimation. However, this result is not reflected in a high gain parameter because they do not use constant-gain but Kalman-filter learning.

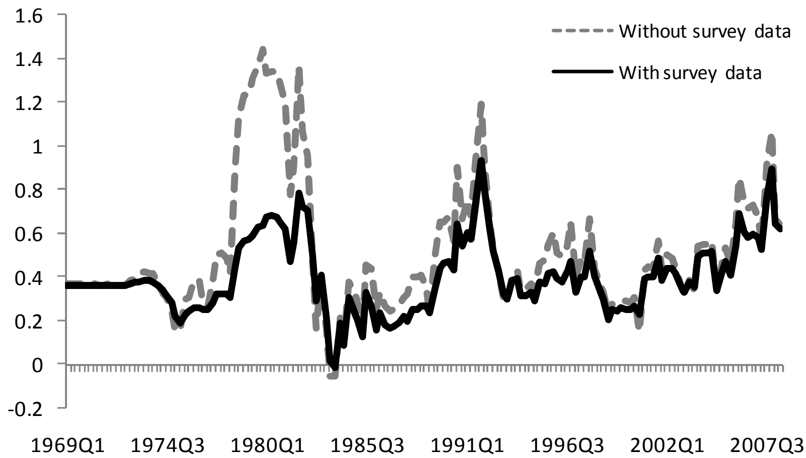
The reduction in the posterior mean of the gain parameter for inflation obtained once survey data are employed in the estimation of the DSGE model has some interesting effects on the evolution of the coefficients of the forecasting model of inflation, the composition of inflation expectations, and the evolution of the inflation target implied by the model.

As shown by Figure 1.4, when survey data are not included in the estimation, the perceived inflation persistence $(\beta_{1,\pi})$ shows a sharp decline at the end of the 1970s with a subsequent increase. When survey data are included, the evolution of this coefficient does not exhibit this decline but remains high through all the second half of the 1970s and all the 1980s. Additionally, the increases in the intercept of the forecasting model $(\beta_{0,\pi})$ observed during the late 1970s and the early 1980s are less important.

Figure 1.4
Evolution of the coefficients of the forecasting model for inflation
(a) Perceived persistence of inflation



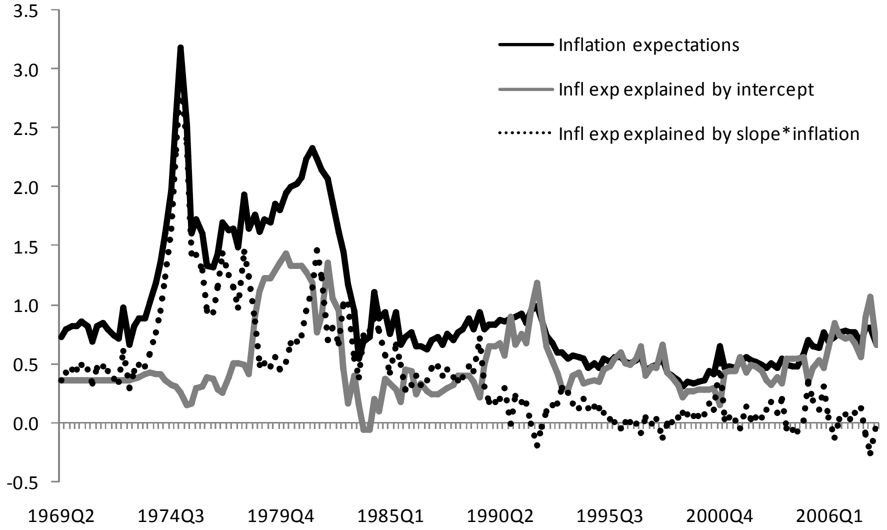
(b) Intercept



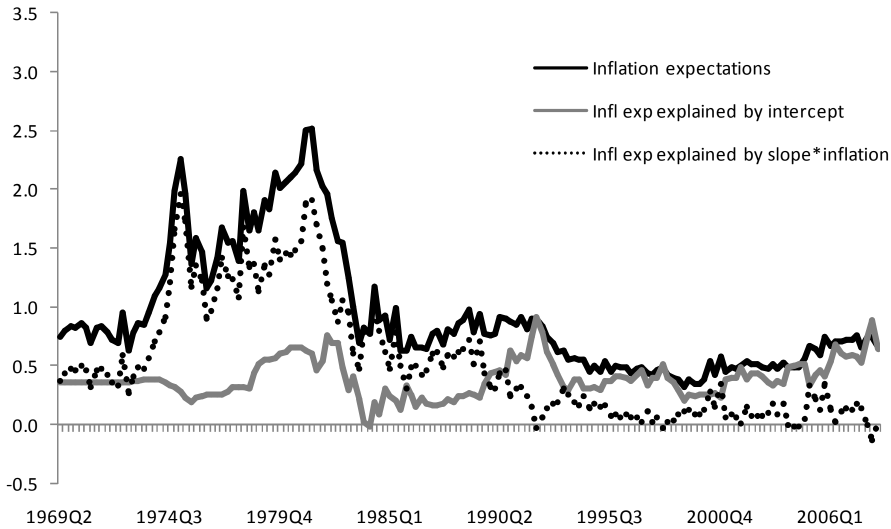
These differences in the evolution of the coefficients of the forecasting model affect the composition of inflation expectations. Considering the form of the forecasting model, conditional inflation expectations can be represented as $E_t(dlP_{t+1}) = \beta_{0,\pi,t} + \beta_{1,\pi,t} dlP_t$. When survey data are included in the estimation, these expectations are closely related to the evolution of the perceived persistence of inflation, $\beta_{1,\pi,t} dlP_t$, up to the beginning of the 1990s (see Figure 1.5b). However, this is not the case when survey data are absent (Figure 1.5a). Notice that since the 1990s, both estimations indicate that inflation expectations are no longer related to the perceived persistence of inflation but to the perceived inflation mean.[23]

[23] Given the structure of the forecasting model for inflation, if the perceived persistence coefficient is close to zero, the intercept can be interpreted as the perceived mean of inflation.

Figure 1.5
Composition of inflation expectations under learning
(a) Not using survey data



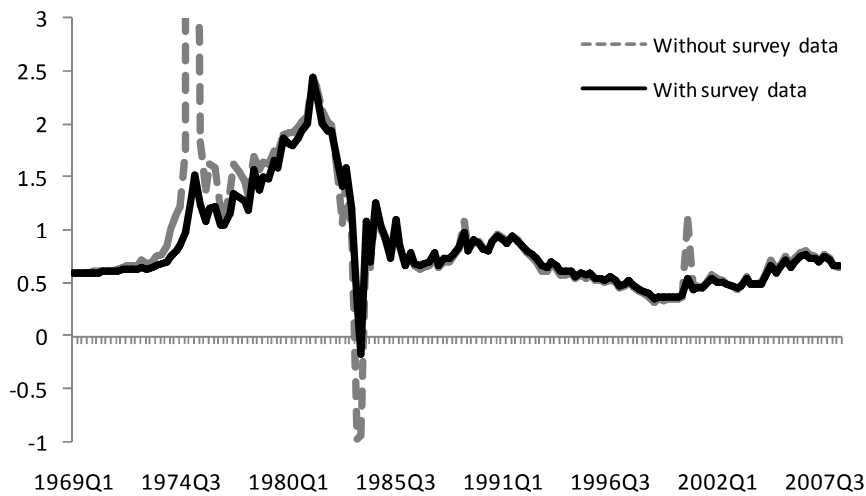
(b) Using survey data



Finally, Figure 1.6 shows the evolution of the perceived long-run inflation target at each point in time expressed by $\beta_{0,\pi} / (1-\beta_{1,\pi})$.

According to this figure, at the beginning of the 1970s the expected quarterly inflation target was 0.59 percent, but it kept increasing until it reached the level of 2.44 percent in 1981:2. Afterwards, we observe an important reduction down to a level between 0.6 and 1 during the 1980s. The timing and the magnitude of the reduction in expected inflation target are consistent with the belief that the Volcker recession at the beginning of the 1980s reduced inflation expectations. During the 1990s, the target steadily decreases until the early 2000s (in 2000:1 the target is situated at 0.37). After this point, the expected inflation target follows a positive path that is interrupted by the outbreak of the financial crisis in 2007. The use of survey data in the estimation avoids the presence of some outliers observed in the evolution of the inflation target.

Figure 1.6
Evolution of the inflation target under learning



1.4.4 Impulse-Response analysis

Finally, we analyze how the use of survey data affects the Impulse-Response functions (IRFs) analysis for inflation. We only focus on this variable as the introduction of survey data does not affect the IRFs for any further variables of the model.[24]

Under the RE solution, there are no significant differences in the IRFs obtained with and without the use of survey data, except in the response of inflation to the wage mark-up shock. The reduction of the wage stickiness results in a less persistent inflation response to this type of shocks (see Figure 1.7a). Interestingly, despite the large reduction in the degree of inflation indexation, the impact of a price mark-up shock on inflation does not significantly alter with the use of survey data (see Figure 1.7b). The underlying reason is the compensation of declining price indexation by the increasing autocorrelation coefficient of the price mark-up shock.

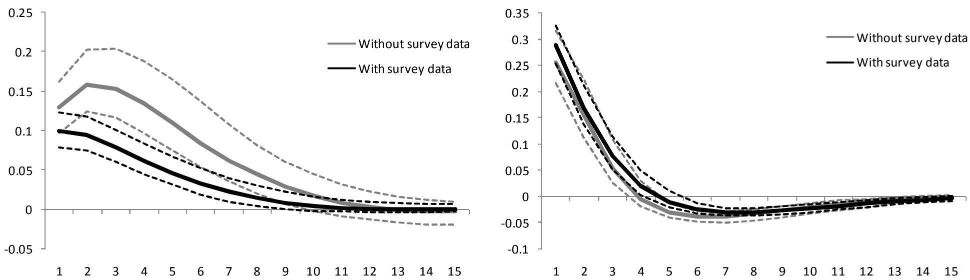
Under learning, adding survey data leads to a reduction in the time-variability of the coefficients of the forecasting model for inflation, and thus to a reduction in the time-variability of the IRFs. As a result, most of the stronger and more persistent responses of inflation are concentrated in the 1970s. For instance, it is observed that unexpected monetary policy shocks have more destabilizing

[24] The Online Appendix contains the variance-covariance analysis for inflation. We find that the relative importance of the shocks depends on the assumption about how expectations are formed rather than on the use of survey data.

effects on inflation during the 1970s than afterwards (see Figure 1.8).

Figure 1.7
IRFs under RE: response of inflation to price and wage mark-up shocks

(a) Wage mark-up shock (b) Price mark-up shock



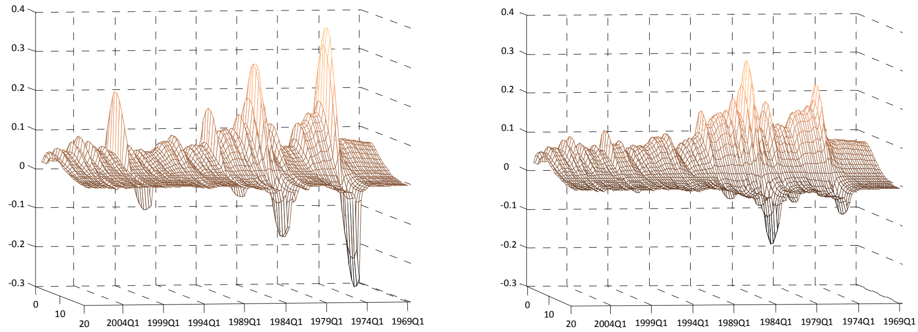
Notes: This figure shows the responses of inflation to a price and a wage mark-up shocks. Dotted lines are the 90% confidence intervals.

This result is compatible with the study by Boivin and Giannoni (2008), who find weaker responses of inflation to unexpected changes in the interest rate for the post-1979 period in comparison to the previous period. When survey data are excluded from the estimation of the DSGE model, the higher time-variability of the coefficients of the forecasting model for inflation, generated by a higher gain parameter for inflation, produces some important responses of inflation to structural shocks during the 1990s and 2000s, that otherwise would not be observed.

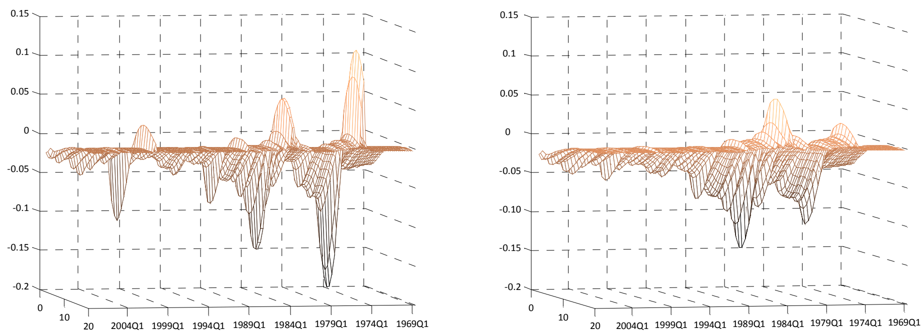
Figure 1.8
IRFs under learning: Response of inflation to structural shocks

Without using survey data   Using survey data

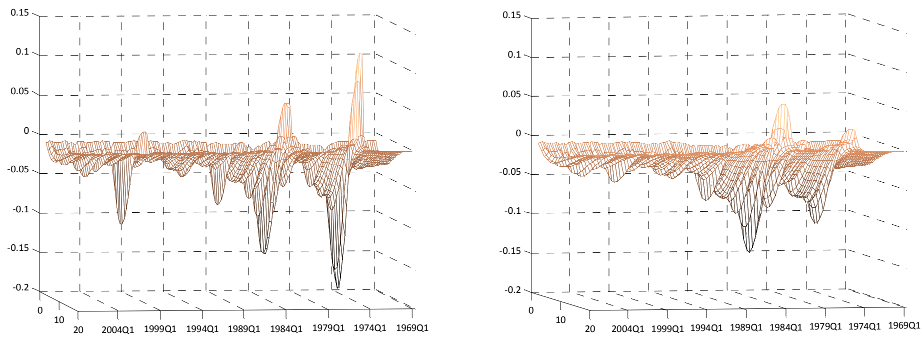
(a) Wage mark-up shock



(b) Productivity shock



(c) Monetary policy shock



Notes: This figure shows the responses of inflation to a wage mark-up, productivity and monetary policy shocks using the structure of the economy at every point in time.

1.5. Robustness exercises

In order to test for robustness of our findings, we evaluate the following three variations of our benchmark specification. First, we use alternative specifications of the forecasting model for inflation under learning. Second, we analyze how our results change when loose uniform priors are used.[25] Finally, we replace the CG-LS algorithm employed under learning by ordinary least squares (OLS).

When using alternative forecasting models for inflation, the posterior statistics obtained under learning barely alter (see Table 1.4, columns 2 to 5).[26] In particular, the median values of the posterior distribution of price indexation and the autocorrelation coefficient of the price mark-up shock are close to 0.60 and 0.161, respectively and the gain parameter for inflation is close to 0.135. These numbers are very similar to those obtained under the benchmark specification of the forecasting model for inflation (column 1).

---

[25] The Online Appendix contains the list of prior distributions used for these estimations.

[26] The forecasting models for inflation that are considered are those presented in Table 1.1.

Table 1.4

Posterior distribution statistics: different specifications of the forecasting model for inflation

Estimations include survey data on inflation expectations

		(1)		(2)		(3)		(4)		(5)	
		dlP (benchmark)		dlP lHours		dlP dlCons		dlP dlCons lHours		dlP dlGDP	
	Symbol	Median	Std	Median	Std	Median	Std	Median	Std	Median	Std
Wage stickiness	ξ_w	0.563	0.049	0.555	0.048	0.548	0.046	0.548	0.046	0.551	0.048
Price stickiness	ξ_p	0.480	0.035	0.446	0.033	0.467	0.037	0.453	0.035	0.462	0.037
Wage indexation	ι_w	0.319	0.107	0.314	0.101	0.342	0.110	0.326	0.106	0.319	0.109
Price indexation	ι_p	0.515	0.119	0.584	0.118	0.553	0.118	0.618	0.112	0.674	0.101
TR: inflation	r_π	1.398	0.104	1.432	0.111	1.404	0.121	1.468	0.111	1.423	0.115
TR: lag interest rate	ρ_R	0.777	0.029	0.775	0.029	0.767	0.029	0.778	0.023	0.776	0.031
TR: change in output	$r_{\Delta y}$	0.210	0.047	0.210	0.045	0.203	0.046	0.214	0.044	0.206	0.046
aut. Price Mk up shock	ρ_p	0.173	0.087	0.180	0.083	0.168	0.083	0.175	0.080	0.124	0.062
std. Price mkup shock	σ_p	0.204	0.014	0.221	0.014	0.206	0.015	0.223	0.013	0.221	0.014
gain - inflation	g^π	0.141	0.009	0.132	0.006	0.137	0.007	0.130	0.005	0.140	0.008
gain - others	$g^{non\pi}$	0.019	0.031	0.023	0.022	0.028	0.049	0.019	0.021	0.019	0.032
Measurement exp error	σ_{exp}	0.176	0.010	0.177	0.011	0.175	0.009	0.173	0.009	0.184	0.011
Log. Mg. Likelihood		45.2		38.6		43.3		40.4		24.1	

Notes: this table shows the median and standard deviation of the posterior distributions of those parameters most closely related to the dynamics of inflation. Each of the columns indicates the use of different specifications of forecasting models for inflation. These specifications are the ones that generate the series of one-period-ahead inflation forecasts closer to the series of survey data on inflation expectations.

With respect to the introduction of loose uniform prior distributions, we find that learning does not display a major change in the parameter estimates with the exception of the increases in wage stickiness and the coefficient of output growth in the Taylor rule (for details please refer to the Online Appendix). The latter result is also observed under RE.

Table 1.5
Model comparison: estimation using OLS learning

Log Marginal Likelihood	Dataset without survey data (1)	Dataset with survey data (2)	(3)=(2)-(1)
RE	-146.78	-19.14	127.64
Learning	-148.35	-77.20	71.14

Notes: This table shows the log marginal likelihood for RE and Learning. Survey data on inflation expectations come from the SPF one-quarter-ahead median forecast of the GDP deflator.

Finally, the use of the OLS algorithm instead of CG-LS decreases significantly the log marginal likelihood of learning when survey data are included (Table 1.5, column 2). This result can be explained by the inability of this specification to match the evolution of the survey data. Using the OLS algorithm keeps the coefficients of the forecasting model very close to the initial conditions. Given that the initial conditions are obtained during a period of low and not persistent inflation (period 1950:1-1968:3), the model fails to replicate the increases in the expectations during the 1970s and beginning of the 1980s (see Online Appendix). Yet, the RE solution also fails to match the evolution of the survey data as well as learning when the CG-LS is employed.

1.6 Conclusions

In this chapter, we provide evidence that the predictive power of DSGE models is improved when resorting to available survey data and to an admissible learning rule for the formation of expectations. In particular, we find that the solution under learning of the New Keynesian model developed by SW fits the data better than the RE solution once survey data on inflation expectations are included in the analysis.

Moreover, we employ survey data on inflation expectations in the selection of the forecasting model for inflation under learning and thus, reduce to some extent the degree of freedom the researcher faces at the time of choosing the forecasting models. The resulting small forecasting model for inflation and the high speed of learning allows the SW model, when solved under learning, to match the increases and decreases in inflation expectations observed during the late 1970s and the early 1980s.

Finally, the additional moment restriction that represents the inclusion of the survey data on inflation expectations leads to parameter estimates that highlight the differences in the sources of inflation persistence between RE and learning. Under RE, a highly persistent price mark-up shock is observed, despite the fact that this model incorporates nominal frictions such as price stickiness and indexation. In contrast, both price indexation and the learning process itself are the main sources of inflation persistence under learning.

There are several important issues that are not addressed in this study. First, we only use the median value of inflation expectations reported by the forecasters included in the SPF at each point in time. However, it is possible to exploit information about other moments – such as the dispersion –to evaluate issues like the credibility of the central bank or the effect of periods of high disagreement in expectations on the conduct of monetary policy. Second, survey data on inflation expectations may be employed to evaluate models particularly designed to better explain the low frequency movements of inflation observed during the late the 1970s and the early 1980s in many developed countries. In light of our results, it is interesting to ask whether other perfect information setups (such as Sbordone (2007) or Ireland (2007)) can provide better descriptions of the survey data than learning. Finally, survey data are also available for a variety of other macroeconomic indicators besides inflation expectations. For instance, survey data on expectations of future output and investment growth might contain useful information for the identification of the mechanisms underlying the business cycle.

To conclude, this study is one of the first to show that survey data contain useful information when estimating DSGE models. Yet, so far, the information collected by surveys such as the Survey of Professional Forecasters (SPF), the Livingstone and Michigan surveys or the Greenbook, have been largely neglected by empirical macroeconomic studies. The use of this information could improve our understanding of how expectations are formed and their impact on the economy.

2. Macroeconomic Models with Durables and Uncertainty*

2.1. Introduction

Investment in durable consumption changes a lot more over the cycle than nondurable consumption. One reason is that the stock of durables is large relative to the investment in durables, which means that a certain drop in the investment in durables leads to much smaller drop in the level of durables. In addition, one would think that the increased probability of unemployment induces agents to be careful in making large purchases. Especially, if there is little risk sharing then an increase in idiosyncratic risk would imply a reduction in the demand for durables. This relationship between uncertainty and the demand for durables can be found in the model of Carroll and Dunn (1997) and Xu (2010). These models are quite complex and would be difficult to incorporate into a full-fledged macroeconomic model.

The objective of this chapter is to investigate whether an increase in uncertainty also leads to a reduction in models that incorporate a demand for durables using the standard (simple) setup. The short answer is no. The savings aspect of durables typically dominates and an increase in uncertainty tends to lead to an increase in the demand for durables. There are a few exceptions, for example if

* This chapter is written in collaboration with Wouter den Haan.

there is an elastic supply of bonds available to the economy as would be the case in a small open economy.

2.2. Two-period models

In this section, we use a two-period version of the models considered later in this chapter to derive some analytical results and to build intuition.

2.2.1 Two-period model; no bonds and no non-negativity constraint

We start with a model in which the agent can spend his income only on purchases of nondurable and durable consumption. This means that the only way he can save is through purchases of durables. The agent's maximization problem is the following: [1]

$$(1) \quad \max_{c,c_{+1},i,i_{+1},d,d_{+1}} \frac{c^{1-\gamma_c}-1}{1-\gamma_c}+\psi_1\frac{d^{1-\gamma_d}-1}{1-\gamma_d}+E\left[\beta\left(\frac{c_{+1}^{1-\gamma_c}-1}{1-\gamma_c}+\psi_2\frac{d_{+1}^{1-\gamma_d}-1}{1-\gamma_d}\right)\right]$$

s.t.

$c+pi=y$

$d=i+(1-\delta)d_{-1}$

$c_{+1}+p_{+1}i_{+1}=y_{+1}$

[1] The scaling of the coefficients ψ is different in the two periods. We choose the value of ψ_2 relative to ψ_1 such that the steady state values for c and d are the same in each period, a property that is also true in the infinite-horizon version of the model.

$$d_{+1} = i_{+1} + (1-\delta)d$$

Throughout this section, the distribution of y_{+1} is described by the following assumption.

Assumption 1. *The value of* y_{+1} *is equal to* $y+\sigma$ *with probability 1/2 and equal to* $y-\sigma$ *with probability 1/2.*

Assumption 1 implies that

(2) $$E\left[\left(y_{+1} - E\left[y_{+1}\right]\right)^2\right] = \sigma^2.$$

The solution is determined by the following set of equations:

(3) $$pc^{-\gamma_c} = \psi_1 d^{-\gamma_d} + E\left[\beta(1-\delta)\psi_2 d_{+1}^{-\gamma_d}\right],$$

(4) $$p_{+1}c_{+1}^{-\gamma_c} = \psi_2 d_{+1}^{-\gamma_d},$$

(5) $$c + pi = y,$$

(6) $$c_{+1} + p_{+1}i_{+1} = y_{+1},$$

(7) $$d = i + (1-\delta)d_{-1},$$

(8) $$d_{+1} = i_{+1} + (1-\delta)d.$$

Partial equilibrium. The main purpose of this chapter is to understand the effect of uncertainty on the demand for durables. Consequently, we take prices of durables as given. In particular, we assume that

(9) $$p = p_{+1} = 1.$$

The following proposition describes the effect of arbitrarily small increases in uncertainty on investment.

Theoretical results.

Proposition 1 *If assumption 1 holds and* c, c_{+1}, d, d_{+1}, i, *and* i_{+1} *are determined by equations 3 through 8, then*

$$\frac{\mathrm{d}i(\sigma)}{\mathrm{d}\sigma} = 0 \textit{ if } \sigma = 0,$$
$$\frac{\mathrm{d}i(\sigma)}{\mathrm{d}\sigma} > 0 \textit{ if } \sigma > 0.$$

Proof *From the second-period first order condition, we get*

(10) $$\gamma_c d_{+1}\mathrm{d}c_{+1} = \gamma_d c_{+1}\mathrm{d}d_{+1} = \gamma_d c_{+1}(\mathrm{d}i_{+1} + (1-\delta)\mathrm{d}i)$$

From the budget constraint, we get

(11) $$\mathrm{d}c_{+1} = \pm\mathrm{d}\sigma - \mathrm{d}i_{+1}.$$

Combining these last two equations gives

$$\mathrm{d}d_{+1} = \frac{\gamma_c d_{+1}}{\gamma_c d_{+1} + \gamma_d c_{+1}}(\pm\mathrm{d}\sigma + (1-\delta)\mathrm{d}i)$$

From the budget constraint in the first period, we get

(12) $$\mathrm{d}c = -\mathrm{d}i.$$

Using the last two results and the Euler equation for the first period, we get

(13) $$\left(\gamma_c c_\sigma^{-\gamma_c-1} + \psi_1\gamma_d d_\sigma^{-\gamma_d-1}\right)\mathrm{d}i$$

$$= -\frac{\beta(1-\delta)}{2}\left(\begin{array}{c} d_{hi,\sigma,+1}^{-\gamma_d-1}\dfrac{\psi_2\gamma_d\gamma_c d_{hi,\sigma,+1}}{\gamma_c d_{hi,\sigma,+1} + \gamma_d c_{hi,\sigma,+1}}\left(+\mathrm{d}\sigma + (1-\delta)\mathrm{d}i\right) + \\ d_{lo,\sigma,+1}^{-\gamma_d-1}\dfrac{\psi_2\gamma_d\gamma_c d_{lo,\sigma,+1}}{\gamma_c d_{lo,\sigma,+1} + \gamma_d c_{lo,\sigma,+1}}\left(\text{-}\mathrm{d}\sigma + (1-\delta)\mathrm{d}i\right) \end{array}\right).$$

Here the subscript σ indicates the value of the variable at the value for σ at which the increase in σ is considered. If the increase in σ is considered starting at σ = 0, then

$$c_{hi,\sigma,+1} = c_{lo,\sigma,+1} \textit{ and } d_{hi,\sigma,+1} = d_{lo,\sigma,+1},$$

which means that

(14) $$\frac{\mathrm{d}i}{\mathrm{d}\sigma} = 0.$$

Now consider an increase in σ starting at a positive value for σ. Then

(15) $$\mathrm{d}i = -\frac{\beta(1-\delta)}{2\left(\gamma_c c_\sigma^{-\gamma_c-1} + \psi_1 \gamma_d d_\sigma^{-\gamma_d-1} + \beta(1-\delta)^2 X\right)} \ldots$$

(16)

$$\ldots \times \left(\frac{\psi_2 \gamma_d \gamma_c d_{hi,\sigma,+1}^{-\gamma_d}}{\gamma_c d_{hi,\sigma,+1} + \gamma_d c_{hi,\sigma,+1}} - \frac{\psi_2 \gamma_d \gamma_c d_{lo,\sigma,+1}^{-\gamma_d}}{\gamma_c d_{lo,\sigma,+1} + \gamma_d c_{lo,\sigma,+1}} \right) \mathrm{d}\sigma,$$

where

(17) $$X = \left(\frac{\psi_2 \gamma_d \gamma_c d_{hi,\sigma,+1}^{-\gamma_d}}{\gamma_c d_{hi,\sigma,+1} + \gamma_d c_{hi,\sigma,+1}} + \frac{\psi_2 \gamma_d \gamma_c d_{lo,\sigma,+1}^{-\gamma_d}}{\gamma_c d_{lo,\sigma,+1} + \gamma_d c_{lo,\sigma,+1}} \right) /2.$$

It follows that

(18) $$\frac{\mathrm{d}i}{\mathrm{d}\sigma} > 0$$

since

(19) $c_{hi,\sigma,+1} > c_{lo,\sigma,+1}$ *and* $d_{hi,\sigma,+1} > d_{lo,\sigma,+1}$ *for* σ>0.

■

The result that $\mathrm{d}i/\mathrm{d}\sigma = 0$ at $\sigma = 0$ is not surprising. When σ is equal to 0, then the derivatives when $y_{+1} = y + \sigma$ are equal to the derivatives when $y_{+1} = y - \sigma$. Consequently, starting at the case of no uncertainty, the impact of an unexpected positive shock exactly offsets the impact of an unexpected negative shock. As the gap between $y + \sigma$ and $y - \sigma$ increases, the gap between $d_{hi,\sigma,+1}$ *and* $d_{lo,\sigma,+1}$ increases, which in turn increases the gap between the decrease in marginal utility (for the positive shock) and the increase in marginal utility (for the negative shock). This result described by this proposition is the *opposite* of the perceived wisdom described in the introduction. At best, investment does not respond, which only happens if the starting point is the unlikely situation of having no uncertainty at all and the increase in uncertainty is arbitrarily small. Starting at a positive amount of uncertainty, investment in durables increases when uncertainty rises even for marginal increases.

Another way to understand the results is the following intuitive reasoning. Consider a (discrete) change in σ from 0 to a positive number. Suppose to the contrary that i decreases. If i decreases, then c increases which means that $c^{-\gamma_c} - \psi_1 d^{-\gamma_d}$ decreases which means that $E\left[c_{+1}^{-\gamma_c}\right]$ decreases. This expectation can only decrease if either $E\left[c_{+1}\right]$ increases or the variance of c_{+1} decreases. The latter is not possible if we start at $\sigma = 0$. Suppose that $E\left[c_{+1}\right]$ increases. This means that $E\left[d_{+1}\right]$ decreases. The decrease in

$E\left[c_{+1}^{-\gamma_c}\right]$ implies that $E\left[d_{+1}^{-\gamma_d}\right]$ also decreases. But this is impossible if $E\left[d_{+1}\right]$ decreases. Consequently, it cannot be the case that i decreases.

Parameter values. The value of y is set equal to 2. This is just a normalization. The value of β is equal to 0.99 and the value of δ is equal to 0.025, which are standard values in the literature. As mentioned above, $p = p_{+1} = 1$. We choose the values of ψ_1, ψ_2 such that the values of $c,\ c_{+1},\ i,\ and\ i_{+1}$ are all equal to 1 if σ is equal to 0. The benchmark value for σ is equal to 0.5, but we also consider other values. This is a huge amount of uncertainty. When σ is equal to 0.5, then agents face the possibility of a 25 percent drop and a 25 percent increase in their income. The benchmark value for both γ_c and γ_d is equal to 2, but we do consider other values as well.

Quantitative results. The results of our numerical experiment are reported in top panel of Table 2.1. If we increase σ from 0 to 0.15, then the value of i increases from 1 to 1.000019, an increase of only 0.0019 percent. If we set σ equal to 1, then the increases is still only 0.084 percent. A value of σ equal to 1 means that the agent faces a 50 percent probability that income is equal to 50 percent below expected income and a 50 percent probability that income is 50 percent above expected income. If we set γ_c and γ_d equal to 10 and keep σ at 1, then the increase is 0.31 percent. Given the high values of risk aversion, this is still a low number. Why does an agent with this much curvature not save more? The reason is that an increase in the curvature would increase the demand for buffer stock savings, but also makes it more costly to do so, since building up the buffer stock requires a reduction in current period nondurable consumption. This is more costly with more curvature in the utility function. The importance of this argument can be highlighted with the following experiment. Suppose that we keep the values of γ_c and γ_d equal to 10 in the second period, but set these values equal to 0.5 in the first period.[2] Thus, substituting out of nondurable consumption is now less costly. Now, investment increases with 4.22 percent, a nontrivial number.

Table 2.1
Investment in durables and uncertainty: two-period model without bonds (% change from zero uncertainty)

A. Without non-negativity constraint

	i		c	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
$\sigma = 0$	0	0	0	0
$\sigma = 0.15$	0.002	0.007	-0.002	-0.007
$\sigma = 0.50$	0.021	0.077	-0.021	-0.077
$\sigma = 1$	0.084	0.306	-0.084	-0.306
$\sigma = 1.25$	0.132	0.475	-0.132	-0.475

B. With non-negativity constraint

	i		c	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
$\sigma = 0$	0	0	0	1
$\sigma = 0.15$	0.002	0.007	-0.002	-0.007
$\sigma = 0.50$	0.021	0.077	-0.021	-0.077
$\sigma = 1$	0.084	0.306	-0.084	-0.306
$\sigma = 1.25$	-0.423	-0.092	0.423	0.092

Notes: The table reports the percentage difference between the chosen level at the indicated level of σ relative to the chosen level when σ is equal to 0.

[2] These values of ψ_1 and ψ_2 are such that the steady state remains the same.

2.2.2 Two-period model; no bonds and non-negativity constraint

Next, we consider the version of the model in which agents cannot sell their durables in the second period.

$$\text{(20)} \quad \max_{c,c_{+1},i,i_{+1},d,d_{+1}} \frac{c^{1-\gamma_c}-1}{1-\gamma_c}+\psi_1\frac{d^{1-\gamma_d}-1}{1-\gamma_d}+E\left[\beta\left(\frac{c_{+1}^{1-\gamma_c}-1}{1-\gamma_c}+\psi_2\frac{d_{+1}^{1-\gamma_d}-1}{1-\gamma_d}\right)\right]$$

s.t.

$$c+i=y$$

$$c_{+1}+i_{+1}=y_{+1}$$

$$i=d-(1-\delta)d_{-1}$$

$$i_{+1}=d_{+1}-(1-\delta)d$$

$$i_{+1}\geq 0$$

Prices are again set equal to 1 in both periods. The solution is determined by the following set of equations:

$$\text{(21)} \quad c^{-\gamma_c}=\psi_1 d^{-\gamma_d}+E\left[\beta(1-\delta)\psi_2 d_{+1}^{-\gamma_d}\right],$$

$$\text{(22)} \quad c_{+1}^{-\gamma_c}-\eta_{+1}=\psi_2 d_{+1}^{-\gamma_d},$$

$$\text{(23)} \quad c+i=y,$$

$$\text{(24)} \quad c_{+1}+i_{+1}=y_{+1},$$

$$\text{(25)} \quad d=i+(1-\delta)d_{-1},$$

$$\text{(26)} \quad d_{+1}=i_{+1}+(1-\delta)d,$$

$$\text{(27)} \quad i_{+1}\geq 0,$$

where η_{+1} is the Lagrange multiplier associated with the non-negativity constraint.

Theoretical results. If the value of σ is such that the non-negativity constraint is binding if the agent receives the low income realization in the second period, then the impact of uncertainty on this period´s demand for durables is the opposite of the one found above. As formally stated in the following proposition, a marginal increase in uncertainty reduces the demand for durables when the constraint is binding.

Proposition 2 *If assumption 1 holds and* c, c_{+1}, d, d_{+1}, i, *and* i_{+1} *are determined by equations 21 through 27 and the non-negativity constraint is binding, then*

$$\frac{\mathrm{d}i(\sigma)}{\mathrm{d}\sigma} < 0.$$

Proof *If the non-negativity constraint is binding in case the agent receives the low income realization, then*

(28) $\mathrm{d}d_{lo,\sigma,+1} = (1\text{-}\delta)\mathrm{d}i.$

If the agent has the high income realization, then we have (as before)

$$\mathrm{d}d_{hi,\sigma,+1} = \frac{\gamma_c d_{hi,\sigma,+1}}{\gamma_c d_{hi,\sigma,+1} + \gamma_d c_{hi,\sigma,+1}}(+\mathrm{d}\sigma + (1-\delta)\mathrm{d}i)$$

From the budget constraint in the first period, we get

(29) $\mathrm{d}c = -\mathrm{d}i.$

Using the last two results and the Euler equation for the first period, we get

(30) $$\left(\gamma_c c_\sigma^{-\gamma_c-1}+\psi_1\gamma_d d_\sigma^{-\gamma_d-1}\right)\mathrm{d}i$$

$$=-\frac{\beta(1-\delta)}{2}\left(\begin{array}{c} d_{hi,\sigma,+1}^{-\gamma_d-1}\dfrac{\psi_2\gamma_d\gamma_c d_{hi,\sigma,+1}}{\gamma_c d_{hi,\sigma,+1}+\gamma_d c_{hi,\sigma,+1}}\left(+\mathrm{d}\sigma+(1-\delta)\mathrm{d}i\right)+ \\ d_{lo,\sigma,+1}^{-\gamma_d-1}\psi_2\gamma_d(1-\delta)\mathrm{d}i \end{array}\right).$$

From this we get

(31)

$$\mathrm{d}i=-\frac{\beta(1-\delta)}{2\left(\gamma_c c_\sigma^{-\gamma_c-1}+\psi_1\gamma_d d_\sigma^{-\gamma_d-1}+\beta(1-\delta)^2X\right)}\left(\frac{\psi_2\gamma_d\gamma_c d_{hi,\sigma,+1}^{-\gamma_d}}{\gamma_c d_{hi,\sigma,+1}+\gamma_d c_{hi,\sigma,+1}}\right)\mathrm{d}\sigma,$$

where

(32) $$X=\left(\frac{\psi_2\gamma_d\gamma_c d_{hi,\sigma,+1}^{-\gamma_d}}{\gamma_c d_{hi,\sigma,+1}+\gamma_d c_{hi,\sigma,+1}}+\psi_2\gamma_d d_{lo,\sigma,+1}^{-\gamma_d-1}\right)/2,$$

which implies that

(33) $$\frac{\mathrm{d}i}{\mathrm{d}\sigma}<0.$$

The reason for this result is fairly intuitive. An increase in σ has no effect on $d_{lo,\sigma,+1}$ at all but it does increase $d_{hi,\sigma,+1}$, which means that the expected value of investing in durables is decreased. Consequently the demand for durables drops. ■

The reason for this result is relatively simple. If the non-negativity constraint is not binding, then the increase in uncertainty increases the value of $E\left[d_{+1}^{-\gamma_d}\right]$ and, thus, increases the expected return on the investment in durables. The reason is that the negative shock increase $d_{lo,\sigma,+1}^{-\gamma_d}$ by more than the positive shock decreases $d_{hi,\sigma,+1}^{-\gamma_d}$. If the non-negativity constraint is binding, then durables in the second period are given by

(34) $$d_{lo,\sigma,+1} = (1-\delta)d = (1-\delta)^2 + (1-\delta)i,$$

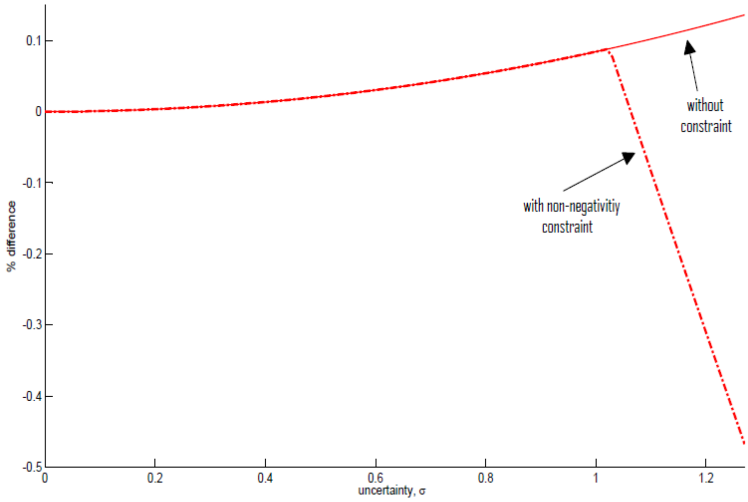
which means that a marginal change in the negative shock has no effect on $d_{lo,\sigma,+1}^{-\gamma_d}$. But the positive shock still decreases $d_{hi,\sigma,+1}^{-\gamma_d}$. Consequently, the value of $E\left[d_{+1}^{-\gamma_d}\right]$ decreases and the investment in durables should decrease as well.

Quantitative results. Exact outcomes for some particular values of σ, γ_c and γ_d are given in the bottom panel of Table 2.1. The main result of this section is displayed in Figure 2.1, which plots the value of i as a function of σ for the case when there is and when there is not a non-negativity constraint. For low values of σ, the possible drop in income is not high enough to generate a binding non-negativity constraint. An increase in uncertainty then increases the demand for durables, i, as documented in the previous section. The value of σ has be quite high before the constraint becomes binding, namely 1.025, which is 51.2 percent of the agent´s expected income. If the constraint is binding, a further increase in σ would lead to lower values for i. Given the rise in i up to this point, the value of σ has to increase sufficiently above 1.025 before i is below the level corresponding to the case of no uncertainty. However, this will happen quickly. The figure seems to suggest that the demand for durables drops rapidly with σ when the constraint becomes binding. But this is not true. The demand for durables when the constraint is binding drops relative to the increase when the constraint is not binding, but both are changes are quantitatively small. When σ = 1.2, then the investment in durables is still only 0.31 percent below the level without uncertainty. With such a minor drop in the investment in durables, the level of durables itself changes even less.

The results so far can be summarized as follows. In this model, in which the consumer can respond to changes in uncertainty only by changing the mix of durable and nondurable consumption, the quantitatively effects of uncertainty on durable investment are small, both when the constraint is and when the constraint is not binding.[3]

[3] As mentioned above, the exception occurs when the curvature parameters change over time.

Figure 2.1
Impact of uncertainty on durables investment:
two-period model without bonds



Notes: This graph plots the difference between the investment in durables chosen by an (unconstrained) agent at the indicated level of uncertainty relative to the chosen level when there is no uncertainty.

2.2.3 Two-period model with bonds: with and without a non-negativity constraint

One would expect that an increase in uncertainty would induce agents to consume less. In the model without bonds, the budget constraint implies that nondurable and durable consumption move in the opposite direction. That is,

$$\mathrm{d}c = -\mathrm{d}i.$$

Consequently, the agent cannot reduce both types of consumption commodities. The most logical outcome is that an increase in uncertainty induces the agent to increase i, since durables not only provide utility, but also are a savings vehicle. The results that a decrease in c necessarily implies an increase in d no longer holds if the agent can also invest in bonds. Would an increase in uncertainty lead to a reduction in both types of consumption if the agent can invest in bonds?

We analyze this question using the following model.

(35) $$\max_{c,c_{+1},b,i,i_{+1},d,d_{+1}} u(c)+U(d)+E\left[\beta\left(u(c_{+1})+U(d_{+1})\right)\right]$$

s.t.

$$c+qb+pi=y$$

$$c_{+1}+p_{+1}i_{+1}=y_{+1}+b$$

$$d=i+(1-\delta)d_{-1}$$

$$d_{+1}=i_{+1}+(1-\delta)d$$

The solution to this problem satisfies the following set of equations.

(36) $$pc^{-\gamma_c}=\psi_1 d^{-\gamma_d}+E\left[\beta(1-\delta)\psi_2 d_{+1}^{-\gamma_d}\right],$$

(37) $$qc^{-\gamma_c}=\beta E\left[c_{+1}^{-\gamma_c}\right],$$

(38) $$p_{+1}c_{+1}^{-\gamma_c}=\psi_2 d_{+1}^{-\gamma_d},$$

(39) $$c+qb+pi=y,$$

(40) $$c_{+1}+p_{+1}i_{+1}=y_{+1}+b,$$

(41) $$d=i+(1-\delta)d_{-1},$$

(42) $$d_{+1}=i_{+1}+(1-\delta)d.$$

Theoretical results. Investments in both bonds and durables are both savings vehicles. The gross return on bonds equals $1/q$ and the gross return on durables excluding the utility flow is equal to $p_{+1}(1-\delta)/p$. With constant prices the latter reduces to $(1-\delta)$. The following assumption states that excluding the utility flow, bonds are a better savings vehicle than durables.

Assumption 2

(43) $$\frac{1}{q} > (1-\delta).$$

This is a weak condition. If it wouldn't hold then there could be no positive demand for bonds. If this condition holds, then it is easy to show that the demand for durables decreases with the amount of uncertainty. This is formally stated in the following proposition.

Proposition 3 *If assumptions 1 and 2 hold and* c*,* c_{+1}*,* b*,* d*,* d_{+1}*,* i*, and* i_{+1} *are determined by equations 36 through 42, then*

$$\frac{\mathrm{d}i(\sigma)}{\mathrm{d}\sigma} < 0.$$

Parameter values. We use the same parameters as in the case without bonds. The value of q is set equal to the discount factor.

Quantitative results. Figure 2.2 plots the value of i as a function of σ for the model with bonds. First consider the case when there is no non-negativity constraint (or when the value of σ is low enough so that it would not be binding). In contrast to the case without bonds, investment in durables decreases with σ. Not only the sign changes when bonds are introduced. Quantitatively, the change in σ has a larger impact on the investment in durables. For example, suppose that $\gamma_c = \gamma_d = 2$. If we increase σ from 0 to 0.15, then the value of i decreases with 0.076 percent, still small, but much larger than the 0.002 percent increase observed for the case without bonds.

Nonlinearities are important and the results increase more than proportionally with σ. If we increase σ to 1, then the reduction in i is equal to 3.4 percent (see Table 2.2).

Whereas the impact of changes in σ on the investment in durables depends a lot on whether the agent can invest in bonds or not, the impact on nondurable consumption does not show this dependence. In both the model with and the model without bonds, increases in σ have only a minor impact on nondurable consumption in the first period. In both models, the drop is less than 0.085 percent when σ increases from 0 to 1.

Table 2.2
Investment in durables and uncertainty:
two-period model with bonds (% change from zero uncertainty)

A. Without non-negativity constraint

	i		c		b	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
$\sigma = 0$	0	0	0	0	0	0
$\sigma = 0.15$	-0.076	-0.278	-0.002	-0.007	0.001	0.003
$\sigma = 0.50$	-0.843	-3.082	-0.021	-0.077	0.009	0.032
$\sigma = 1$	-3.374	-12.233	-0.084	-0.306	0.035	0.127
$\sigma = 1.25$	-5.264	-19.005	-0.132	-0.475	0.054	0.197

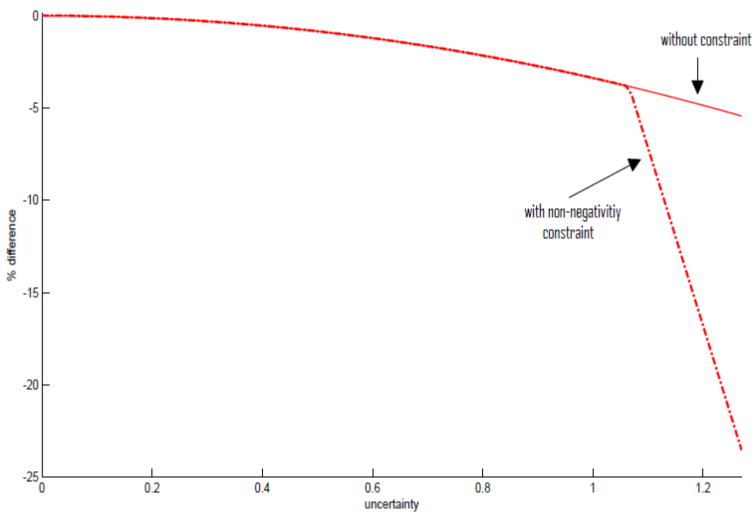
B. With non-negativity constraint

	i		c		b	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
$\sigma = 0$	0	0	0	0	0	0
$\sigma = 0.15$	-0.076	-0.278	-0.002	-0.007	0.001	0.003
$\sigma = 0.50$	-0.843	-3.082	-0.021	-0.077	0.009	0.032
$\sigma = 1$	-3.374	-12.233	-0.084	-0.306	0.035	0.127
$\sigma = 1.25$	-21.608	-21.285	-0.132	-0.475	0.220	0.220

Notes: The table reports the percentage difference between the chosen level at the indicated level of σ relative to the chosen level when σ is equal to 0. For bonds (which are zero when σ = 0), the table reports the actual difference.

Now consider the case with a non-negativity constraint on investment. The constraint becomes binding at a somewhat higher value for σ, namely at σ = 1.066. If the constraint is binding and the value of σ increases, then investment in durables plummets.[4] For example, suppose that $\gamma_c = \gamma_d$ =2. As σ increases from 0 to 1 the non-negativity constraint remains not binding and investment drops by 3.4 percent. As σ increases further to 1.27 then investment falls to a level that is 23.5 percent below the level with no uncertainty. Most of the drop occurs when σ exceeds 1.066, the value at which the non-negativity constraint becomes binding.

Figure 2.2
Impact of uncertainty on durables investment:
two-period model with bonds



Notes: This graph plots the difference between the investment in durables chosen by an (unconstrained) agent at the indicated level of uncertainty relative to the chosen level when there is no uncertainty.

[4] Note that the scale of the y-axis in Figure 2.2 is very different from the scale in Figure 2.1.

The results so far can be summarized as follows. If building up a buffer stock means reducing nondurable consumption to increase durable consumption then the effects of changes in uncertainty are quantitatively small. If building up a buffer stock means reducing the (large stock) of durable consumption to increase bond holdings, then the effects of changes in uncertainty are much larger.

Caveat and general equilibrium. Proposition 3 shows that a very simple model can generate the desired result, namely that an increase in uncertainty leads to a decrease in the demand for durables. Moreover, the numerical results show that the results are quantitatively nontrivial. There is one caveat. The counterpart of the reduction in both types of durables is that the demand for bonds increases. In general equilibrium this can only happen if the supply increases as well. In a small open economy the increase in the supply could be provided by the international financial market. In a closed economy the government may be willing to supply these bonds at fixed bond prices.[5] If there is no such elastic supply of bonds, then the bond price would increase. Suppose that the bond price increases until the demand for bonds is equal to zero. In that case, the demand for durables and nondurable consumption is equal to the outcome of the model without bonds.[6] Consequently, the demand for durables again increases with uncertainty unless there is

[5] Of course, (perceived) effects of this increase in bonds by the government on the agents future budget constraint would have to be considered.

[6] If the supply of bonds is equal to zero, then the bond price does not affect the equilibrium demand for the two consumption commodities. If the supply of bonds is not equal to zero, then the bond price does matter and the "equilibrium" version of the model with bonds would not be equal to the model without bonds.

a non-negativity constraint and the increase in uncertainty is sufficiently large.

2.3. Infinite-horizon models and unconditional means

In this section, we investigate whether the results for the two-period models carry over to models with an infinite number of periods. Here we focus on unconditional means. In particular, we investigate whether an increase in uncertainty increases or decreases the average amount of investment in durables. That is, we are interested in the sign of

$$\frac{\partial E[i]}{\partial \sigma}.$$

There are several reasons why the analysis of the two-period model only provides a limited insight into the answer of this question. In the analysis above, the change in uncertainty only affects the situation in the next period. That is, whereas, y_{+1} could take on a high and a low value, we kept the resources the agent started with fixed. Suppose that an increase in uncertainty induces an agent to hold more bonds to insure himself against unforeseen shocks. To build up this higher wealth level the agent would have to give up consumption. But if the agent has built up the desired buffer stock, the extra interest income will increase the average level of the sum of the two consumption expenditures. The effect of this extra income on i is a channel that is not present in the two-period model.

There is another reason why the infinite-horizon model is not simply a multi-period extension of the two-period model. In the two-period model we kept first-period income fixed when we increased the amount of uncertainty. Moreover, we considered a level of income at which the agent was not at the constraint. An increase in the amount of uncertainty in the infinite-horizon also means considering a wider range of initial conditions, including the possibility that the agent is constrained. A binding constraint means that investment in durables would be less if there were no constraint. As uncertainty increases, then it is more likely that an agent is at the constraint. That is, it is more likely that investment in durables is not allowed to fall as much as the agent would like.

2.3.1 Infinite-horizon without bonds

Let e be an indicator that is equal to 1 if the agent is employed and equal to 0 if the agent is not employed. If there is no non-negativity constraint, then the agent's optimization problem is as follows:

$$(44) \qquad \upsilon(d_{-1}, e) = \max_{c,i,d} \; u(c) + U(d) + \beta E\left[\upsilon(d, e_{+1})\right]$$

s.t.

$$c + pi = ey_e + (1-e)y_u$$

$$d = i + (1-\delta)d_{-1}$$

The stochastic process for income y is characterized in the following assumption.[7]

[7] At the end of this subsection we show the results of a simulation exercise that includes a persistent income process.

Assumption 3 *The process for* y *is determined by a symmetric first-order Markov process. The switching probability is equal to 1/2 and the two possible realizations are* $y_u = y - \sigma$ *and* $y_e = y + \sigma$. *Thus, the unconditional mean is equal to* y *and the unconditional standard deviation is given by* σ.

The solution satisfies the following set of equations:

(45) $$pc^{-\gamma_c} = \psi d^{-\gamma_d} + \beta(1-\delta)E\left[p_{+1}c_{+1}^{-\gamma_c}\right],$$

(46) $$c + pi = ey_e + (1-e)y_u,$$

(47) $$d = i + (1-\delta)d_{-1}.$$

Again we set $p = 1$. The parameters δ and ψ are chosen such that $c_{ss} = i_{ss} = \delta d_{ss}$.

Theoretical results. Taking the unconditional expectation of both sides of the budget constraint and rewriting gives

(48) $$\frac{E[d]}{d_{ss}} + \frac{E[c]}{c_{ss}} = y \equiv E\left[ey_e + (1-e)y_u\right].$$

Consequently, if $E[d] > d_{ss}$ due to uncertainty, then $E[d] < c_{ss}$ and vice versa.

If we take unconditional expectations on both sides of the …first-order conditions, then we get

(49) $$\left(1-\beta(1-\delta)\right)E\left[c^{-\gamma_c}\right] = \psi E\left[d^{-\gamma_d}\right].$$

Taking a second-order approximation of the terms inside the expectation, it can be shown that

(50) $$\frac{\left(\frac{E[d]}{d_{ss}}\right)^{\gamma_d}}{\left(\frac{E[c]}{c_{ss}}\right)^{\gamma_c}} \approx \frac{1+\gamma_d(\gamma_d+1)V[d]/\left(E[d]\right)^2}{1+\gamma_c(\gamma_c+1)V[c]/\left(E[c]\right)^2}.$$

Here $V[x]$ is the variance of x. To understand the implication of this formula suppose that $\gamma_c = \gamma_d$. If the variance of d scaled by its squared mean is larger (smaller) than the scaled variance of c, then $E[d]$ is larger (smaller) than its steady state value and $E[c]$ is smaller (larger) than its steady value. If $\gamma_c \neq \gamma_d$, then the same result holds except that additional scaling is required to quantify the relationship.

This reasoning does not reveal whether $E[d]$ or $E[c]$ increases as σ increases. The following proposition sheds light on this question by showing that according to the second-order perturbation policy rule $E[d]$ increases and $E[c]$ decreases with σ.

Proposition 4 *If the policy functions for* c, d, *and* i *are determined by equations 45 through 47, then the demand for durables increases with uncertainty according to the second-order perturbation approximation.*

Proof *Let* $g(z;\sigma)$ *be the policy function for nondurable consumption as a function of beginning-of-period resources,* z, *and the amount of uncertainty,* σ. *The second-order Taylor-series expansion of* $g(z;\sigma)$ *is given by*

(51) $$g(z;\sigma) \approx g(\bar{z};0) + \left.\frac{\partial g(z;\sigma)}{\partial z}\right|_{z=\bar{z},\sigma=0} (z-\bar{z}) + \left.\frac{\partial g(z;\sigma)}{\partial \sigma}\right|_{z=\bar{z},\sigma=0} \sigma$$

$$+ \left.\frac{\partial^2 g(z;\sigma)}{\partial z^2}\right|_{z=\bar{z},\sigma=0} \frac{(z-\bar{z})^2}{2} + \left.\frac{\partial^2 g(z;\sigma)}{\partial \sigma^2}\right|_{z=\bar{z},\sigma=0} \frac{\sigma^2}{2}$$

(52) $$+ \left.\frac{\partial^2 g(z;\sigma)}{\partial z \partial \sigma}\right|_{z=\bar{z},\sigma=0} (z-\bar{z})\sigma$$

$$= \bar{g} + \bar{g}_z (z-\bar{z}) + \bar{g}_\sigma \sigma + \frac{1}{2}\bar{g}_{zz}(z-\bar{z})^2 + \frac{1}{2}\bar{g}_{\sigma\sigma} + \bar{g}_{z\sigma}(z-\bar{z})\sigma$$

We have to show that $\bar{g}_{\sigma\sigma} > 0$. *To do this, we first …find expressions for* $\bar{g}_z$ *and* $\bar{g}_{zz}$. *It is well known that these do not depend on* σ.[8] *Consequently, they can be solved from the Euler equation for the case without uncertainty. This Euler equation is given by*

(53) $$0 = -g(z)^{-\gamma_c} + \begin{pmatrix} \psi\left[z - g(z)\right]^{-\gamma_d} \\ +\beta(1-\delta)(g(y+(1-\delta)[z-g(z)]))^{-\gamma_c} \end{pmatrix}.$$

Since this equation holds for all z, *the derivatives of both sides of the equation have to be also equal to each other for all* z. *Thus,*

(54) $$0 = \gamma_c g^{-\gamma_c - 1} g_z$$

$$-\psi\gamma_d [z-g]^{-\gamma_d - 1}(1-g_z)$$

$$-\beta(1-\delta)^2 \gamma_c (g(y+(1-\delta)[z-g]))^{-\gamma_c - 1}[1-g_z].$$

This equation also holds for all z, *thus*

[8] See for example, Schmitt-Grohe and Uribe (2004).

(55) $$0 = \gamma_c g^{-\gamma_c - 1} g_{zz}$$

$$-\gamma_c(\gamma_c + 1) g^{-\gamma_c - 2} g_z^2$$

$$+\psi\gamma_d [z - g]^{-\gamma_d - 1} g_{zz}$$

$$+\psi\gamma_d(\gamma_d + 1)[z - g]^{-\gamma_d - 2}(1 - g_z)^2$$

$$+\beta(1-\delta)^2 \gamma_c(\gamma_c + 1)\left((g(y + (1-\delta)[z - g]))^{-\gamma_c - 2}[1 - g_z]^2\right)$$

$$+\beta(1-\delta)^2 \gamma_c (g(y + (1-\delta)[z - g]))^{-\gamma_c - 1} g_{zz}$$

Evaluating this expression at $z = \overline{z}$ *and* $\sigma = 0$ *gives*

(56) $$\overline{g}_{zz} = ...$$

$$= \frac{\gamma_c(\gamma_c + 1) g^{-\gamma_c - 2} g_z^2 - \psi\gamma_d(\gamma_d + 1)[z - g]^{-\gamma_d - 2}(1 - g_z)^2}{\gamma_c g^{-\gamma_c - 1} + \psi\gamma_d [z - g]^{-\gamma_d - 1} + \beta(1-\delta)^2 \gamma_c \left(g(y + (1-\delta)[z - g])\right)^{-\gamma_c - 1}}.$$

The sign of $\overline{g}_{zz}$ *can be either positive or negative. That is, the policy function for non-durables can be either convex or concave.*[9] *We now return to the problem with uncertainty. Using the problem´s first-order conditions, we know that the function* $g(z;\sigma)$ *satisfies the following equation for all values of* z *and* σ *:*

(57) $$g(z;\sigma)^{-\gamma_c} = ...$$

$$... = \begin{array}{c} \psi\left[\hat{} z - g(z;\sigma)\right]^{-\gamma_d} \\ +\beta(1-\delta) E\left[g(y + \sigma\varepsilon_{+1} + (1-\delta)[z - g(z;\sigma)];\sigma)^{-\gamma_c} \right] \end{array}.$$

[9] If the policy function for nondurables is concave (convex), then the policy function for the investment in durables is convex (concave). This follows directly from the budget constraint.

To simplify the notation we suppress the arguments and we use subscripts to denote derivatives and timing. For example,

(58) $$g_\sigma = g_\sigma(z;\sigma) = \frac{\partial g(z;\sigma)}{\partial \sigma}$$

and

(59) $$g_{\sigma,+1} = g_\sigma\left(y+\sigma\varepsilon+(1-\delta)[z-g(z;\sigma)];\sigma\right) = \ldots$$

$$\ldots = \frac{\partial g(y+\sigma\varepsilon+(1-\delta)[z-g(z;\sigma)];\sigma)}{\partial \sigma}.$$

Differentiation of equation (57) with respect to σ *gives*

(60) $$0 = \left(\gamma_c g^{-\gamma_c-1} + \gamma_d \psi [z-g]^{-\gamma_d-1}\right) g_\sigma + \ldots$$

$$\ldots + \beta(1-\delta)E\left[\gamma_c g_{+1}^{-\gamma_c-1}\left(g_{\sigma,+1} + (\varepsilon_{+1}+(1-\delta)g_\sigma)g_{z,+1}\right)\right].$$

Differentiating this equation again gives

(61) $$0 = \left(-\gamma_c(\gamma_c+1)g^{-\gamma_c-2} + \gamma_d\psi(\gamma_d+1)[z-g]^{-\gamma_c-2}g_\sigma\right)g_\sigma + \ldots$$

$$+\left(\gamma_c g^{-\gamma_c-1} + \gamma_d\psi[z-g]^{-\gamma_d-1}\right)g_{\sigma\sigma} + \ldots$$

$$\ldots + \beta(1-\delta)E\left[\begin{pmatrix} \gamma_c g_{+1}^{-\gamma_c-1} \\ g_{\sigma\sigma,+1} + g_{z\sigma,+1}(\varepsilon_{+1}-(1-\delta)g_\sigma) \\ (\varepsilon_{+1}+(1-\delta)g_\sigma)(g_{z\sigma,+1} + g_{\sigma\sigma,+1}(\varepsilon_{+1}-(1-\delta)g_\sigma)) \\ +(1-\delta)g_{\sigma\sigma}g_{z,+1} \end{pmatrix}\ldots\right.$$

$$\ldots - (g_{\sigma,+1} + (\varepsilon_{+1}+(1-\delta)g_\sigma)g_{z,+1}) \times \ldots$$

$$\left.\ldots \times \gamma_c(\gamma_{c+1})g_{+1}^{-\gamma_c-2}(g_{\sigma\sigma,+1} + g_{z,+1}(\varepsilon_{+1}-(1-\delta)g_\sigma))\right]$$

Evaluating this equation at the steady state and using that $E\left[\varepsilon_{+1}^2\right] = 1$ *gives*

(62) $$0=\left(\gamma_c\overline{g}^{-\gamma_c-1}+\gamma_d\psi[\overline{z}-\overline{g}]^{-\gamma_d-1}\right)\overline{g}_{\sigma\sigma}$$

$$+\beta(1-\delta)\begin{pmatrix}\gamma_c g^{-\gamma_c-1}(\overline{g}_{\sigma\sigma}+\overline{g}_{zz})\\ -\gamma_c(\gamma_c+1)\overline{g}^{-\gamma_c-2}\overline{g}_z^2\end{pmatrix},$$

which implies that

(63) $$\overline{g}_{zz}=\frac{\beta(1-\delta)\left(\gamma_c(\gamma_c+1)\overline{g}^{-\gamma_c-2}\overline{g}_z^2-\gamma_c\overline{g}^{-\gamma_c-1}\overline{g}_{zz}\right)}{(1-\beta(1-\delta))\gamma_c\overline{g}^{-\gamma_c-1}+\gamma_d\psi[\overline{z}-\overline{g}]^{-\gamma_d-1}}.$$

If $\overline{g}_{zz}\le 0$*, then* $\left(\gamma_c(\gamma_c+1)\overline{g}^{-\gamma_c-2}\overline{g}_z^2-\gamma_c\overline{g}^{-\gamma_c-1}\overline{g}_{zz}\right)>0$*, which in turn implies that* $g_{\sigma\sigma}>0$*. But as mentioned above,* $\overline{g}_{zz}$ *could be positive. Suppose that* $\overline{g}_{zz}>0$*. By rewriting equation (55), we get*

(64) $$\gamma_c(\gamma_c+1)g^{-\gamma_c-2}g_z^2-\gamma_c g^{-\gamma_c-1}g_{zz}=\ldots$$

$$\ldots=\psi\gamma_d[z-g]^{-\gamma_d-1}g_{zz}+\ldots$$

$$\ldots+\psi\gamma_d(\gamma_d+1)[z-g]^{-\gamma_d-2}(1-g_z)^2+\ldots$$

$$\ldots+\beta(1-\delta)^2\gamma_c(\gamma_c+1)\left((g(y+(1-\delta)[z-g]))^{-\gamma_c-2}[1-g_z]^2\right)+\ldots$$

$$\ldots+\beta(1-\delta)^2\gamma_c(g(y+(1-\delta)[z-g]))^{-\gamma_c-1}g_{zz}.$$

From this equation, we directly get that $\left(\gamma_c(\gamma_c+1)\overline{g}^{-\gamma_c-2}\overline{g}_z^2-\gamma_c\overline{g}^{-\gamma_c-1}\overline{g}_{zz}\right)$ *and thus* $g_{\sigma\sigma}$ *are also positive when* $\overline{g}_{zz}>0$*. This completes the proof.* ■

Similar to the discussion in the previous section, we will consider the case with and without the non-negativity constraint on investment in durables. The analysis above considered the case without this constraint. Important for our analysis is that equation (48) also holds if the non-negativity constraint is present. This means that $E[c]$ and $E[d]$ still cannot move in the same direction when uncertainty increases. We provide no analytical results for this case, but there is one insight that immediately comes to mind. The demand for durables is an increasing function of income if there is no non-negativity constraint. If there is a non-negativity constraint then the flat part of this function for low values of income will convexify the function. Consequently, and increase in uncertainty will increase the demand for durables.

Parameter values. We use standard values for β, γ_c, γ_d and δ. In particular, $\beta = 0.99$, $\gamma_c = \gamma_d = 2$, $\delta = 0.025$. We normalize y equal to 2 and we choose ψ such that the steady state values of consumption and investment are equal to 1. As made explicit in assumption 3, we assume that the shocks are not serially correlated.

Quantitative results[10]. The results are summarized in Table 2.3 and Figure 2.3. The table gives some exact outcomes and Figure 2.3 plots the average investment in durables as a function of the amount of uncertainty. It plots the results for the economy with and without the non-negativity constraint. First consider the case when there is no non-negativity constraint. Consistent with the theoretical results we find that the average demand for durables is increasing in uncertainty. Moreover, quantitatively the impact of uncertainty on the demand for durables is substantially stronger than the results in

[10] Appendix A describes the procedure we follow to get the numerical results presented in this and the following section.

the two-period model. Qualitatively the results are similar for the two-period and the infinite-horizon model.

Table 2.3
Investment in durables and uncertainty: infinite-horizon model without bonds (% change from zero uncertainty)
A. Without non-negativity constraint

	i		c	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
$\sigma = 0$	0	0	0	0
$\sigma = 0.15$	0.011	0.039	-0.011	0.039
$\sigma = 0.50$	0.119	0.445	-0.119	0.445
$\sigma = 1$	0.484	1.863	-0.484	1.863
$\sigma = 1.25$	0.763	3.022	-0.763	3.022

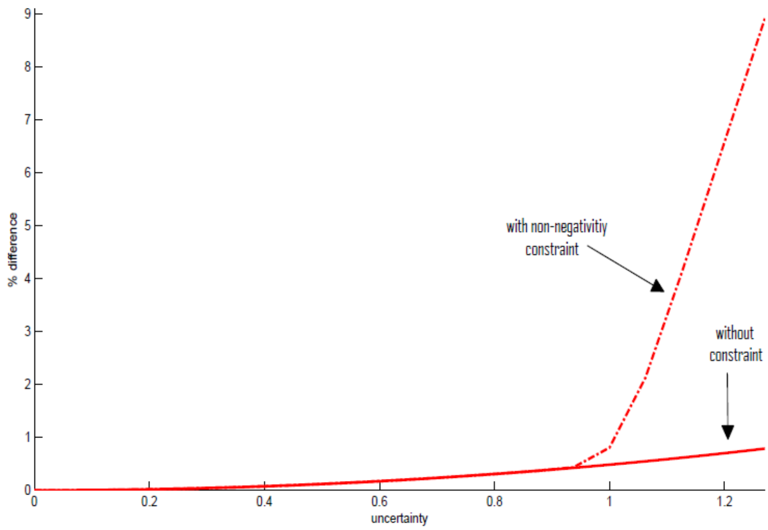
B. With non-negativity constraint

	i		c	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
$\sigma = 0$	0	0	1	1
$\sigma = 0.15$	0.011	0.039	-0.011	-0.039
$\sigma = 0.50$	0.119	0.445	-0.119	-0.445
$\sigma = 1$	0.807	2.052	-0.807	-2.052
$\sigma = 1.25$	8.245	9.538	-8.245	-9.538

Notes: The table reports the percentage difference between the chosen level at the indicated level of σ relative to the chosen level when σ is equal to 0.

This is not the case for the model with the non-negativity constraint. Whereas the demand for durables falls with an increase of uncertainty in the two-period it increases in the infinite-horizon model. Moreover, the demand increases sharply with uncertainty. For example, the average demand for durables when σ is equal to 1.2 is 6.58 percent above the average demand for durables when there is no uncertainty. But this difference is due to the focus on different objects. In the two-period model, we focus on the demand for durables by an agent that is not constrained. In the infinite-horizon model, we focus on the demand for durables averaged across both constrained and not constrained agents.[11] The sharp increase in the demand for durables when uncertainty increases is due to the direct positive effect of the non-negativity constraint on the demand for durables by the constrained agents. These agents would prefer a smaller level of investment in durables (namely a negative value), but are prevented from doing this by the non-negativity constraint.

Figure 2.3
Impact of uncertainty on durables investment: infinite-horizon model without bonds



Notes: This graph plots the difference between the average level of investment in durables chosen at the indicated level of uncertainty relative to the chosen level when there is no uncertainty. The average is across employed and unemployed agents where the latter could be constrained.

To illustrate that the results for the infinite-horizon are not that different from the two-period model we calculate the demand for durables by an employed agent. This agent is not constrained in his demand for durables, just like the agent in the two-period model

[11] Another difference is the effect that a build up buffer stock of bonds has on spending power, but that effect is eliminated by setting the bond price equal to 1.

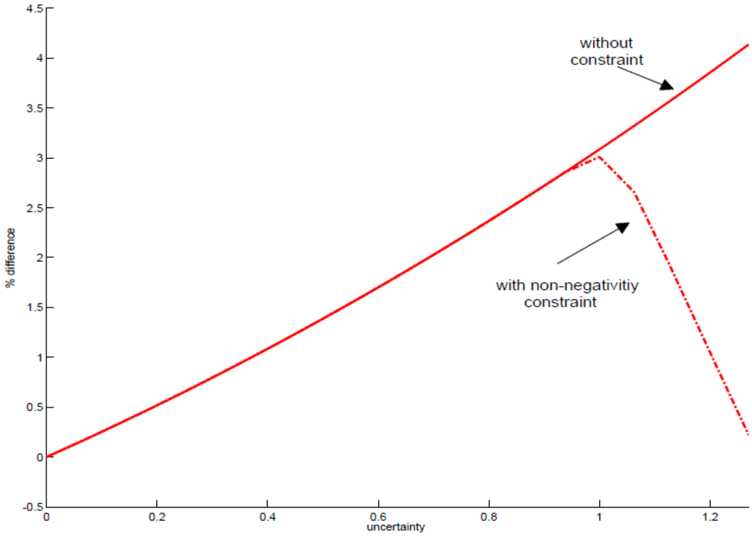
was not constrained in the first period. We cannot simply plot the demand for durables by an employed agent as a function of uncertainty. The reason is the following. In the infinite-horizon model, an increase in uncertainty not only increases uncertainty about future income realizations, but also increases the income of the employed agent (since we keep aggregate resources fixed). To make the comparison with the two-period model sensible, we keep the resources of the employed agent fixed.[12] As documented in Figure 2.4, the effect of uncertainty on the demand for durables by an unconstrained agent in the infinite-horizon model is very similar to the impact on the demand for durables by an unconstrained agent in the two-period model.

Alternative parameter values.

Assumption 4 *The process for* y *is determined by an asymmetric first-order Markov process. The switching probability is equal to* ρ_e *if the agent is employed (receives the high income realization) and equal to* ρ_u *when the agent is unemployed (receives the low income realization). This means that the unconditional probability of being employed is equal to* $\rho_u / \rho_e + \rho_u$. *The two possible realizations are* $y_u = y - \sigma\sqrt{(\rho_u / \rho_e)}$ *and* $y_e = y - \sigma\sqrt{(\rho_e / \rho_u)}$. *This implies that the unconditional mean is equal to* y *and the unconditional standard deviation is equal to* σ.

[12] There are only two realizations for income and our numerical solution only gives the outcome for these two values of income. Instead of reducing the income level, we can reduce the amount of beginning-of-period durables. Since income shocks are i.i.d., only the sum of available durables and income matters and a reduction in the stock of durables is like a reduction in income.

Figure 2.4
Impact of uncertainty on durables investment of an employed agent: infinite-horizon model without bonds

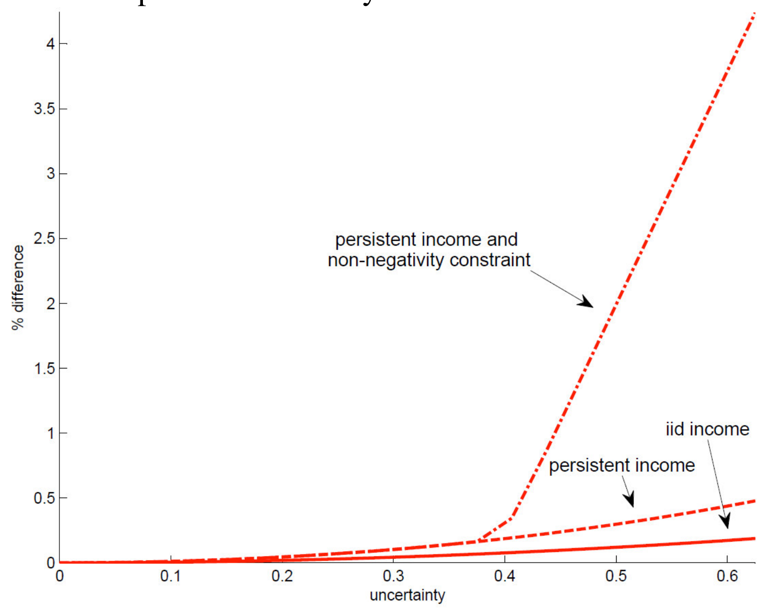


Notes: This graph plots the time path of the average stock of durables (across agents) relative to its pre-shock level when idiosyncratic uncertainty increases. Aggregate resources remain the same, so with perfect risk sharing there would be no change.

Parameter values. Up to this point we had considered $\rho_e = \rho_u = 0.5$. Now, we reduce the switching probability of the employed agent to more realistic levels, $\rho_e = 0.075$. This implies that the agent remains employed for longer periods and the unconditional probability of being employed increases from 0.5 to 0.869. We keep the same previous values for the rest of the parameters (i.e. $\beta = 0.99$, $\gamma_c = \gamma_d = 2$, $\delta = 0.025$).

Quantitative results. The results are summarized in Figure 2.5. Qualitatively the results are similar to those presented in Figure 2.3, namely that: a) the increase in uncertainty leads to more investment in durables; and b) the presence of a non-negativity constraint makes this type of purchases to increase sharply once the constraint is binding. There are, however, some quantitative differences. First, as a whole the economy accumulate more durables. Second, the non-negativity constraint becomes binding for smaller levels of uncertainty. This happens because for the same level of uncertainty unemployed agent´s income ($y-\sigma\sqrt{(\rho_u / \rho_e)} = y-2.582\sigma$) is much smaller than when the income process is iid ($y-\sigma$).

Figure 2.5
Impact of uncertainty on durable investment



Why does this economy invest more in durables if the probability of an employed agent of losing his job is smaller than in the case with iid income process? A comparison of the employed and unemployed agents between both economies is not direct: for a given level of uncertainty neither the level nor the persistence of income are the same. In order to make comparable the investment decisions in both economies we let the agent to be employed or unemployed for several periods.[13] By this way, the difference in income persistence is controlled. Additionally, we adjust the income of the employed and unemployed agent in order to make them identical in both economies.[14]

Figure 2.6
Impact of uncertainty on durable investment:
agent employed for 1000 periods

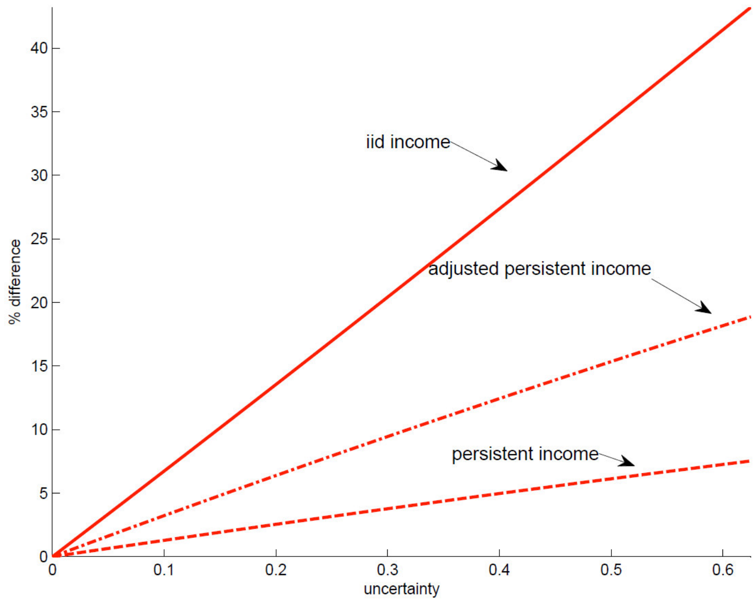


Figure 2.6 shows that an employed agent demands less durables in the economy with persistence in income, even after the adjustment in income. This reflect the fact that the probability of losing the job is much smaller ($\rho_e = 0.075$) than in the economy with iid income

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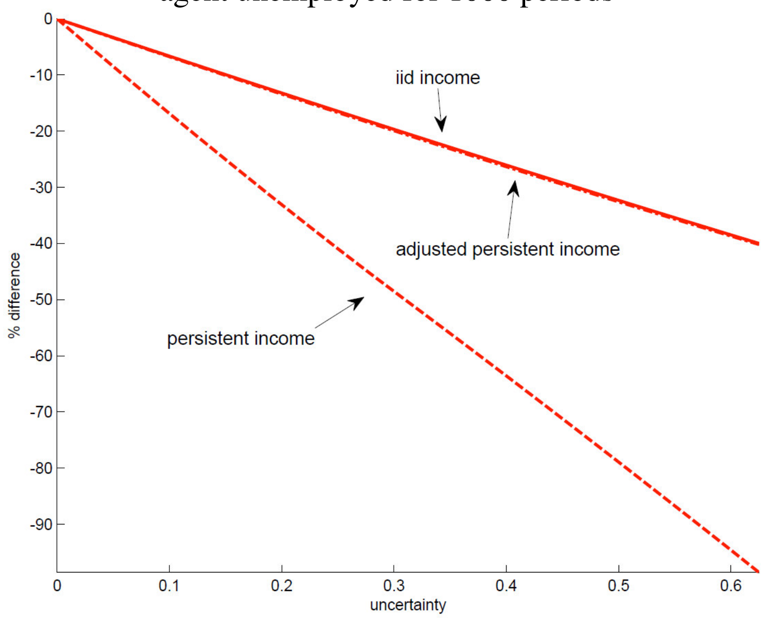
[13] Enough to ensure convergency in his demand for durable and non-durable goods.

[14] We increase the amount of the beginning-of-period durables by $\sigma(1-\sqrt{\rho_e / \rho_u})$ and $\sigma(\sqrt{\rho_u / \rho_e}-1)$ for the employed and unemployed agents, respectively, in the persistent income economy.

process ($\rho_e = 0.5$). In the case of the unemployed agent (see Figure 2.7), the demand of durables is almost identical than in the economy with iid income process after the adjustment in income. Therefore, the higher demand for durables observed in the aggregate is explained by the fact that in the persistent income economy the agent spends more time as employed than in the case with iid income process.

Figure 2.7
Impact of uncertainty on durable investment:
agent unemployed for 1000 periods



2.3.2 Infinite-horizon with bonds

If there is no non-negativity constraint on the purchases of durables, then the agent's optimization problem is given by

(65) $$\upsilon(b_{-1}, d_{-1}, e) = \max_{c,b,i,d} \; u(c) + U(d) - P(b) + \beta E\left[\upsilon(b, d, e_{+1})\right]$$

s.t.

$$c + qb + pi = ey_e + (1-e)y_u + b_{-1}$$

$$d = i + (1-\delta)d_{-1}$$

The interest rate is fixed and does not depend on the amount of money invested ($b > 0$) or the amount borrowed ($b < 0$). The function $P(b)$ captures the idea that the cost (benefits) of borrowing (investing) will increase (decrease) if this activity increases. Such a property is not only realistic, but is also needed to keep the model well defined. An alternative is to let the interest rate charged depend on a function $P(b)$. This is more realistic, but our specification is easier to interpret. The literature often uses borrowing constraints to keep the problem well behaved, which would correspond to a particular discontinuous specification for $P(b)$.[15] Some authors use a formulation like ours to approximate the non-negativity constraint. In this case $P(b)$ is referred to as a penalty function. De Wind (2008) shows that many properties of models with a borrowing constraint are similar to those with a smooth penalty function. Here, we do not think of $P(b)$ as an approximation to a borrowing constraint. We use this formulation because it introduces bonds into the model with the least possible changes. For example, if $P(b)$ would have entered the budget constraint, then both the budget constraint and the first-order condition would have changed, whereas here only the first-order condition is affected. Introducing

[15] For example, the constraint that $b \geq 0$, is implemented by the function $P(b)$ that is equal to 0 if $b \geq 0$ and equal to infinity if $b < 0$.

bonds in this minimal way into the model, helps us to understand how the possibility to invest in bonds affects the relationship between uncertainty and investment in durables. Obviously, if bonds are introduced into the model in more complex ways, then these additional aspects may affect this relationship as well.

The solutions for c, d, i, and b satisfy the following set of equations:

$$pc^{-\gamma_c} = \psi d^{-\gamma_d} + \beta(1-\delta)E\left[p_{+1}c_{+1}^{-\gamma_c}\right],$$

$$qc^{-\gamma_c} = -\frac{\partial P(b)}{\partial b} + \beta E\left[c_{+1}^{-\gamma_c}\right],$$

$$c + qb + pi = ey_e + (1-e)y_u + b_{-1},$$

$$d = i + (1-\delta)d_{-1}.$$

We assume that the penalty function is given by

(66) $$P(b) = \frac{\eta_1 \exp(-\eta_0 b)}{\eta_0} \text{ with } \eta_0 > 0,\ \eta_1 > 0.$$

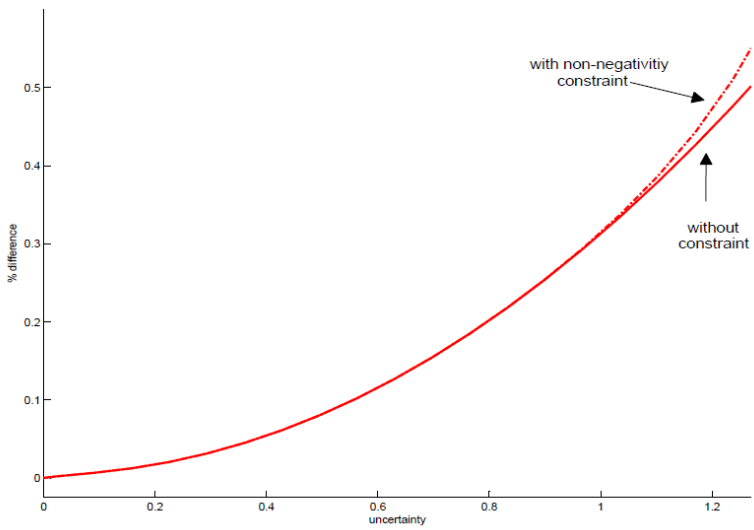
which means that the Euler equation for bonds is given by

(67) $$qc^{-\gamma_c} = \eta_1 \exp(-\eta_0 b) + \beta E\left[c_{+1}^{-\gamma_c}\right].$$

Parameter values. The parameters that also appear in the problem without bonds take on the same value. In our benchmark case, we set $q = 1$. This means that we do not have the additional channel discussed at the beginning of this section that resources increase if agents build up a buffer stock. We set η_1 equal to $(1-\beta)$, so that the steady state values of c and i are again equal to 1 and the steady state value of b is equal to 0.

Theoretical results. The advantage of the chosen parameter values related to the demand for bonds is that the theoretical results derived for the case without bonds carry over to the case with bonds. That is, the impact of an increase in uncertainty on the average demand of durables and nondurables must have the opposite sign. And according to the second-order approximation, the demand for durables should increase.

Figure 2.8
Impact of uncertainty on durables investment:
infinite-horizon model with bonds



Notes: This graph plots the time path of average investment in durables (across agents) relative to its pre-shock level when idiosyncratic uncertainty increases. Aggregate resources remain the same, so with perfect risk sharing there would be no change.

Quantitative results. Figure 2.8 plots the average investment in durables as a function of the amount of uncertainty. It plots the results for the economy with and without the non-negativity constraint. Some exact corresponding numbers are given in Table 2.4.

Table 2.4
Investment in durables and uncertainty: infinite-horizon model with bonds (% change from zero uncertainty)
A. Without non-negativity constraint

	i		c		b	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
$\sigma = 0$	0	0	0	0	0	0
$\sigma = 0.15$	0.012	0.017	-0.012	-0.017	0.005	0.044
$\sigma = 0.50$	0.082	0.114	-0.082	-0.114	0.038	0.342
$\sigma = 1$	0.312	0.397	-0.312	-0.397	0.150	1.474
$\sigma = 1.25$	0.486	0.541	-0.485	-0.540	0.239	2.255

B. With non-negativity constraint

	i		c		b	
	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$	$\gamma = 2$	$\gamma = 10$
$\sigma = 0$	0	0	0	0	0	0
$\sigma = 0.15$	0.012	0.017	-0.012	-0.017	0.005	0.044
$\sigma = 0.50$	0.082	0.114	-0.082	-0.114	0.038	0.342
$\sigma = 1$	0.314	0.397	-0.314	-0.397	0.150	1.474
$\sigma = 1.25$	0.524	0.541	-0.523	-0.540	0.243	2.255

Notes: The table reports the percentage difference between the chosen level at the indicated level of σ relative to the chosen level when σ is equal to 0. For bonds (which are zero when σ = 0), the table reports the actual difference.

First consider the case without the non-negativity constraint. Qualitatively, the results are similar to the case without bonds. But this means that the results are quite different from the results in the two-period model. For the case without bonds, we find that uncertainty increases the demand for durables in both the two-period and the infinite-horizon model. For the case with bonds, we find that uncertainty decreases the demand for durables in the two-period model, but increases the demand for durables in the infinite horizon model. To understand the difference let's consider the budget constraint:

(68) $$c + qb + pi = ey_e + (1-e)y_u + b_{-1}.$$

When considering the two-period model, we consider an increase in uncertainty keeping the value of b_{-1} fixed. If an increase in uncertainty leads to an increase in the value of b, then both c and i can decrease. When considering the infinite-horizon model, we focus on the unconditional means. This means that the value of b_{-1} is not kept fixed. If agents respond to the increase in uncertainty by building a buffer stock of bonds, then both b and b_{-1} increase. If $q < 1$, then this leads to additional spending power. If $q = 1$, then this is not the case, but the increase in b does not lead to a reduction in $c + pi$ as happens in the two-period model.

Now consider the case with the non-negativity constraint. The graph shows that imposing the constraint has only a small impact on the results when the agent can invest in bonds. The reason is that with bonds the agent can insure himself quite well against negative shocks. If uncertainty is high enough he would choose negative investment levels when unemployed, but these negative values are not as low as those chosen by the unemployed agent in the model without bonds. Consequently, the non-negativity constraint has much less of an impact.

General equilibrium. The fact that we do not impose general equilibrium in the bond market is not that problematic here as in the two-period model. The reason is the following. An increase in uncertainty does lead to an increase in the demand for bond. But this increases both the average value of bonds in the right-hand side of the budget constraint and the left-hand side. More precisely, since $q=1$ we have that

(69) $$E[c]+E[i]=2.$$

If we impose equilibrium in the bond market we get that $E[b]=0$, which means that the sum of the two expenditure components is still equal to two. The values of $E[c]$ and $E[i]$ could still be different when equilibrium is imposed, but we found only very minor changes.

2.4. Infinite-horizon business cycle models

In the previous section, we analyzed the long-term impact of an increase in uncertainty. We found that an increase in uncertainty leads to an increase in the demand for durables even if agents can invest in bonds. If investment in durables is restricted to be non-negative, then an increase in uncertainty leads to an even larger increase in the demand for durables. Although such a non-negativity constraint reduces the demand for durables of the employed agent, it is more than offset by the direct upward effect that unemployed agents cannot sell their durables.

The analysis of the previous section ignores transition dynamics. A key aspect of the analysis in the previous section is that the change in the average demand for durables and the change in the average demand for nondurables must have the opposite sign. This holds in the economy without bonds and in the economy with bonds when the bond price is equal to one.[16] We found that durables increased and nondurables decreased.

But the analysis of the previous section compares steady states, that is, it focuses on the long-term effects of an increase in uncertainty. In this section, we consider a model in which the economy switches between a low-uncertainty and a high-uncertainty regime. With this model, we can study what happens on impact and in the subsequent periods when the economy switches to the high-uncertainty regime. In particular, we analyze the responses of the average amount of nondurable consumption and investment in durable, where the average is across all agents in the economy.

Aggregate output is the same in both regimes, but agents face a higher probability of being unemployed in the high-uncertainty regime. Since aggregate output is the same across time, the average amount of nondurable consumption and investment in nondurables still has to move in opposite directions if the agents cannot borrow. In contrast, if the agent can borrow, then there is no condition that the demand for durables and nondurables should move in the opposite direction, at least not in the short run.

2.4.1 The model

If the agent can invest in bonds, his optimization problem is given by

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[16] If the bond price is less than one, then the interest income is on average positive and this extra income only provides an extra reason for the demand for durables to increase.

(70) $\upsilon(b_{-1}, d_{-1}, e, r) = \max_{c,b,i,d} \; u(c) + U(d) - P(b) + \beta E\left[\upsilon(b, d, e_{+1}, r_{+1})\right]$

s.t.

$c + qb + pi = ey_e + (1-e)y_u + b_{-1}$

$d = i + (1-\delta)d_{-1}$

The economy without bonds is the same, except that the option to invest in bonds is no longer present.

The model without and the model with bonds are identical to the corresponding models considered in the previous section, except that the transition probabilities and the value of y_e depend on the regime r. There are two regimes, a low-uncertainty and a high-uncertainty regime. The value of y_u is kept constant. The probability a low-income worker to become a high-income worker is also kept constant. The main difference between the two regimes is the probability of switching from the high individual income state to the low income state. In the low-uncertainty regime, this probability is equal to 0.05 and in the high regime this occurs with probability 0.1. By concentrating the increase in uncertainty on the largest group of workers that also have the most purchasing power we give the increase in uncertainty the best possible chance of having a substantial effect.

As in Krusell and Smith (1998), we choose the transition probabilities such that the fractions of high-income and low-income workers immediately jump to the new steady state levels when a regime change occurs. We are interested in a pure increase in uncertainty, not in a change in aggregate resources. Consequently, if the economy switches to the high-uncertainty regime and the fraction of low-income agents increases, the level of y_e actually increases.

Parameter values. Most parameters take on the same values as in the previous section. Only parameter values associated with the regime switch are different. The economy stays in the same regime with probability 7/8 and, thus, switches to the other regime with probability 1/8. The average across y_u and y_e is equal to 2 in both regimes. We set y_u and the probability a low-income worker to become high-income worker equal to 0.3 and 0.5, respectively, and as mentioned above both values remain constant across regime changes. Let $\eta_{e,hi}$ and $\eta_{e,lo}$ be the fraction of high-income agents in the high-uncertainty and the low-uncertainty regime, respectively. These are given by the following steady state equations:

(71) $$\eta_{e,hi} = 0.5(1-\eta_{e,hi}) + (1-0.1)\eta_{e,hi},$$

(72) $$\eta_{e,lo} = 0.5(1-\eta_{e,lo}) + (1-0.05)\eta_{e,lo}.$$

The value of $\eta_{e,hi}$ is equal to 0.8333 and the value of $\eta_{e,lo}$ is equal to 0.9091. As mentioned above, we choose transition probabilities such that there is an instantaneous adjustment to these levels if a regime change occurs. Since aggregate output is fixed, the value for $y_{e,hi}$ can be solved from

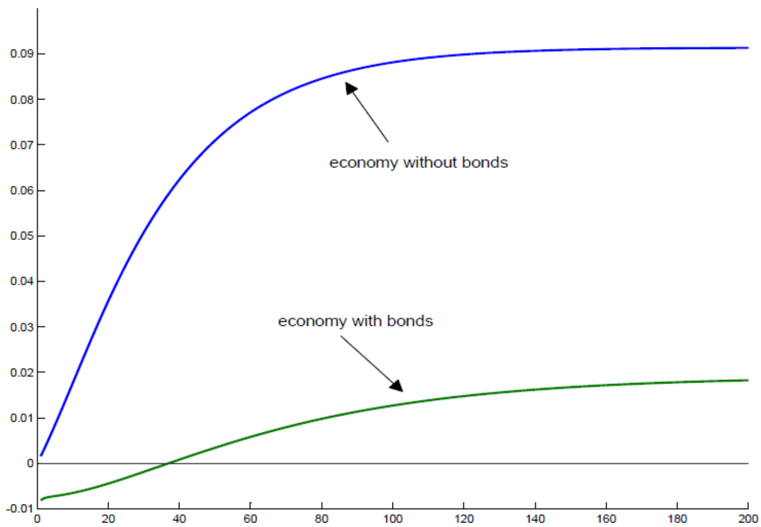
(73) $$2 = y_e\eta_{e,hi} + y_u(1-\eta_{e,hi})$$

and the value for $y_{e,lo}$ can be solved from

(74) $$2 = y_e \eta_{e,lo} + y_u (1 - \eta_{e,lo}).$$

Quantitative results. Figures 2.9 and 2.10 display the key results. Figure 2.9 reports the results for the level of durables and Figure 2.10 reports the results for the level of investment. Each figure displays the percentage change of the indicated variable relative to its pre-switch level when the economy changes from the low-uncertainty to the high-uncertainty regime.[17] The figures report the results for the economy with and without bonds.

Figure 2.9
Increase in idiosyncratic uncertainty and % change in stock of durables



Notes: This graph plots the response over time of the level of durables when the economy switches from the low-uncertainty to the high-uncertainty economy. The numbers are expressed as percentage difference relative to the pre-switch value.

[17] The graph gives the results when there is no further regime change. The policy rules are calculated, however, under the assumption that a regime change occurs with probability 1/8.

In the economy without bonds, the switch to the regime with higher uncertainty leads to an increase in the demand for durables. This is not surprising. The same happened in the static model and the infinite-horizon model without aggregate uncertainty.

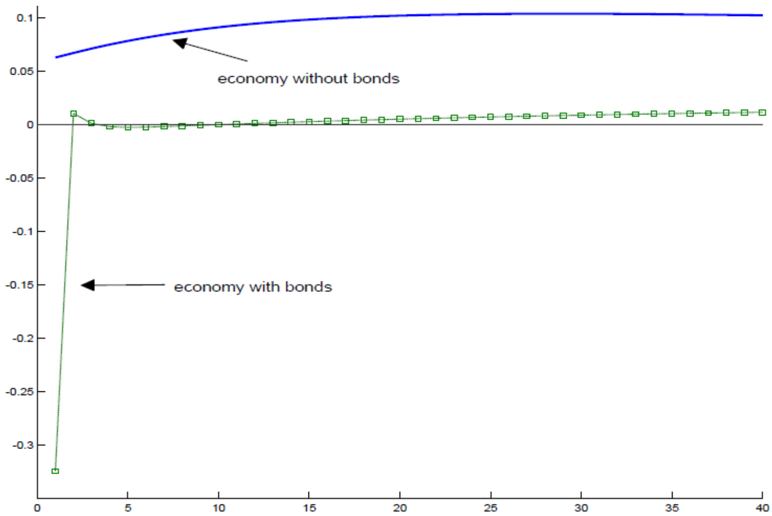
More interesting is that in the economy with bonds, the switch to the regime with higher-uncertainty leads to a reduction in the demand for durables. Figure 2.9 shows that the desired stock of durables is below the pre-switch level for almost 40 periods. Not shown, the demand for nondurables also decreases. At some point, the reduction turns into an increase. This is not surprising. If the economy reaches a new steady state, then the extra savings cancels against the extra redemptions and there can be no change in total aggregate consumption.[18]

The graph suggests that a very simple model can generate the desired result, i.e., a prolonged decrease in the demand for durables when uncertainty increases. Unfortunately, there are several reasons why this result is not that great. Although the increase in uncertainty is substantial, the reduction in the stock of durables is only 0.1 percent. As shown in Figure 2.10, investment in durables drops by a bit more than 0.3% on impact. This is not a trivial drop, but investment only drops on impact. That is, the adjustment to the desired lower stock of durables occurs in one period. But most problematic is that one cannot expect this result to survive in general equilibrium. In this particular example, the aggregate

[18] Here we assume again that interest rate on savings is equal to 0. If the interest rate was positive, then there would be in the limit only upward pressure on the demand for durables.

demand for bonds increases (which makes the reduction in both consumption commodities possible).

Figure 2.10
Increase in idiosyncratic uncertainty and % change in investment in durables



Notes: This graph plots the response over time of the level of durables when the economy switches from the low-uncertainty to the high-uncertainty economy. The numbers are expressed as percentage difference relative to the pre-switch value.

2.5. Conclusions

In this study we evaluate the impact of increases in income volatility on the demand for durable goods using both two-period and infinite horizon models. In these models agents are subject to idiosyncratic risk against which they cannot perfectly insure. In some cases agents can use bonds in order to protect against income shocks. Notice that durables do not always serve as a perfect substitute for bonds because in some cases we add the existence of a non-negativity constraint on this type of goods.

Our results indicate that in most of the cases increases in income uncertainty leads to an increase in the demand for durables even if agents can invest in bonds. This result is due to the fact that durables are also used as a saving asset. There are, however, two exceptions to this general result. First, under the two-period model with bonds, agents reduce the consumption of durables in order to increase bond holdings which serve as a buffer stock. Second, the transition from a low-uncertainty regime to a high-uncertainty regime results in a temporal reduction of investment in durables. Yet, these two specific results are not expected to remain in a general equilibrium setup when the price of bonds increases to keep the aggregate net supply equal to zero.

We plan to extend this simple framework in the following ways. First, we want to use a general equilibrium setup such that the price of the bond and the relative price of durables are determined in equilibrium. Second, we will incorporate the following features of durable goods: indivisibility and down payment requirements. Removing the fixed price assumption on the relative price of durables might mitigate our results because the demand for durables and non-durables react in opposite directions in most of the cases considered in our analysis. On the contrary, the fact that one needs to make a significant investment in order to acquire a durable good - down payment requirements - is likely to cause agents to decrease more significantly their purchases of this type of good when uncertainty increases.

3. Great Moderation Debate: Should We Worry about Using First-Order Approximated Policy Functions?

3.1 Introduction

The Great Moderation - the phenomenon of a substantial decline in the volatility of many macroeconomic indicators in the US (and other developed countries) over the last three decades - and its sources are an important topic on the current academic and political agenda. Although most of the leading macroeconomists agree about its existence and (rough) timing, there is still disagreement about the key factors explaining its existence. One potential reason for this lack of consensus, as indicated by Canova and Gambetti (2010), is the use of various econometric techniques in this type of studies.

Here, we focus on the reliability of first-order approximated policy functions (or solutions) of dynamic stochastic general equilibrium (DSGE) models when evaluating the sources of the Great Moderation. This type of approximation is commonly used in Bayesian estimations of structural models as it describes well the dynamic of the model around a particular equilibrium. However, the presence of significant nonlinearities in the model and the prevalence of large shocks reduce its accuracy. Since the standard analysis of the Great Moderation relies on the estimation of first-order approximations during periods of high volatility –periods during which both, nonlinearities and shocks are more likely to occur - the suitability of this type of approximations is questionable.

This study aims at determining the effects of using first-order approximations when evaluating the sources of the Great Moderation. For this purpose we implement the following simulation exercise: First, we generate macroeconomic series which exhibit volatility levels similar to those observed during the high volatility period for the US economy (1954:3 - 1984:4). These series are obtained using second-order approximations of a New Keynesian model and two sets of parameter values. Both sets are obtained from the estimation of a New Keynesian model, where we use actual US data for a low volatility period (1985:1 – 2007:3). Three parameters are, however, chosen in a way that they allow us to reproduce the higher volatility observed in the period 1954:3 - 1984:4. These parameters are those that define either the monetary policy rule or the standard deviations of the perturbation terms of the shocks that affect the economy. As explained below, these two sets of parameters represent the two most common hypotheses explaining the Great Moderation. Second, using the simulated series, we estimate a New Keynesian model but employ its first-order approximation, as it is standard in Bayesian estimations. Finally, we compare the parameter estimates with the actual parameter values used to generate the series. This comparison allows us to evaluate the importance of omitting the second-order terms of the policy functions in the evaluation of the sources behind the Great Moderation.

Our results show that when high volatility data are generated according to the "good policy" hypothesis, omitting the second-order terms of the approximation could lead to an erroneous misinterpretation of the sources of the Great Moderation. More precisely, we estimate a reduction in the volatility of the technology shock when the actual variation in the volatility of the data generating process is only due to changes in the coefficients of the monetary policy rule. However, we do not encounter such problems when the variations in the macroeconomic volatility between the periods 1954:3 - 1984:4 and 1985:1 – 2007:3 are due to a reduction in the volatility of the shocks affecting the economy (in other words, when we generate the data according to the "good luck" hypothesis).

So far, few studies that incorporate estimations of DSGE models depart from the use of first-order approximations. For instance, An (2007), using a similar New Keynesian model and the same macroeconomic variables, does not find significant differences between parameters estimated using first and second-order approximations. However, the sample used in his study only includes the low macroeconomic volatility period. Fernández-Villaverde and Rubio-Ramírez (2005) also do not find significant posterior statistics differences when they estimate the first and second-order approximations of a simple version of the Neoclassical model using information for the period 1964-2003. In contrast, our study considers a model with more nonlinearities and a sample of higher volatility level. Both elements might lead to significant biases in the parameter estimates generated by omitting the second-order terms.[1]

[1] Another references are An and Schorfheide (2007) and Fernández-Villaverde and Rubio-Ramírez (2007).

The remainder of the chapter is organized as follows. In the next section we discuss the potential problems related to the use of approximations. Section 3.3 presents the New Keynesian model and the macroeconomic indicators used in the simulation. Section 3.4 describes the simulation exercise while section 3.5 shows the main results. Section 3.6 concludes and outlines possible avenues for future research.

3.2. Second-order approximations: a simple example

The use of first-order approximations of a model, together with the Kalman filter, is the most common way to fit models to the data. It does not require high computational power and there exist many toolkits that facilitate their implementation. On the contrary, second-order approximations are much less popular as their implementation requires much higher computational power. Since they are not widely known, in this section we use a simple model to evaluate the composition and the role of the second-order terms of this type of approximation. Additionally, we use a simulation exercise to analyze the effects resulting when omitting these terms from the estimation of the structural parameters.

3.2.1 A simple model

Let´s consider the following optimization problem of a representative agent:

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$$

subject to: $C_t + Q_t A_{t+1} = A_t + Y_t$

where C_t and Y_t are the levels of consumption and income, respectively, in period *t*. A is a risk free asset and Q is its price. The income level, Y, follows the subsequent exogenous process: $Y_t = \rho Y_{t-1} + (1-\rho)\bar{Y} + \eta\sigma\varepsilon_t$.The stochastic term, $\eta\sigma\varepsilon$, is composed by a zero-mean normal perturbation, ε, with a standard deviation equal to 1. Following the notation of Schmitt-Grohe and Uribe (2004), σ is considered as a scale factor and is equal to zero in steady state. $\eta\sigma$ defines the variance-covariance matrix of the perturbation term.

In equilibrium, the risk free asset has a zero net supply ($A_t = 0$, $\forall t$), and therefore $C_t = Y_t$, $\forall t$. Given that the income, and therefore the consumption, is exogenous, the only variable to be determined in equilibrium is the price of the asset. The analytical expression for this price is obtained using the Euler equation derived from the optimization problem and a specific form of the utility function (in this case we choose a constant relative risk aversion (CRRA) utility function). Thus, Q_t can be represented as:

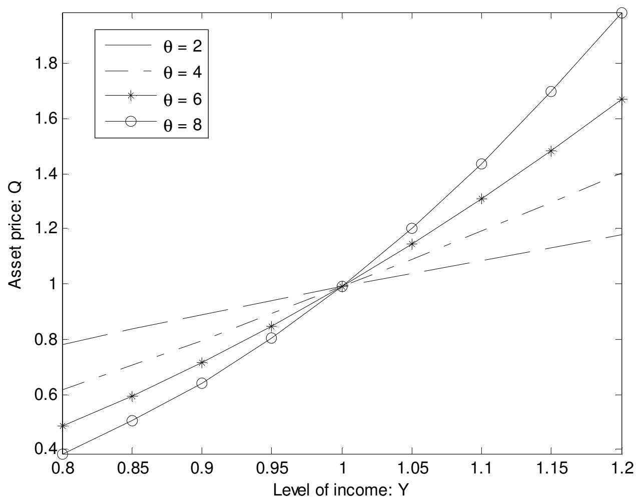
$$Q_t = \beta E_t\left(\frac{U'(C_{t+1})}{U'(C_t)}\right) = \beta E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\theta} = \beta E_t\left(\frac{Y_t}{\rho Y_t + (1-\rho)\bar{Y} + \eta\sigma\varepsilon_{t+1}}\right)^{\theta}$$

Figure 3.1 represents the value of Q_t for different levels of income around its steady state ($\bar{Y} = 1$), and different values of θ and ρ.[2] As we can see, the higher the level of income the more increases the

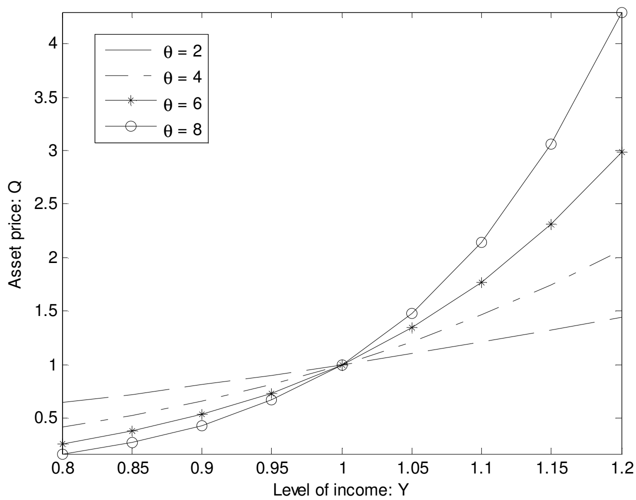
[2] We consider β=0.9901 and ε equal to zero.

price of the asset in order to keep its net supply equal to zero. Additionally, prices react stronger for higher degrees of risk aversion (θ) and lower persistency in the income process (ρ), reflecting a more convex policy function.

Figure 3.1
Asset price policy function
(a) $\rho = 0.5$



(b) $\rho = 0.0$



3.2.2 The approximated policy function

Since most of the models used in economics are too complex to get an exact representation of their policy functions, it is common in the literature to use linear approximations of these functions. We use *perturbation methods* to get the first and second-order approximations for Q_t.[3] The resulting approximations have the following representations:

First-order policy function: $Q_t = \beta + a(Y_t - \bar{Y})$

Second-order policy function:

$$Q_t = \beta + a(Y_t - \bar{Y}) + \frac{1}{2}b(Y_t - \bar{Y})^2 + \frac{1}{2}c\sigma^2$$

where:

$$a = \theta\beta(1-\rho)\bar{Y}^{-1}$$

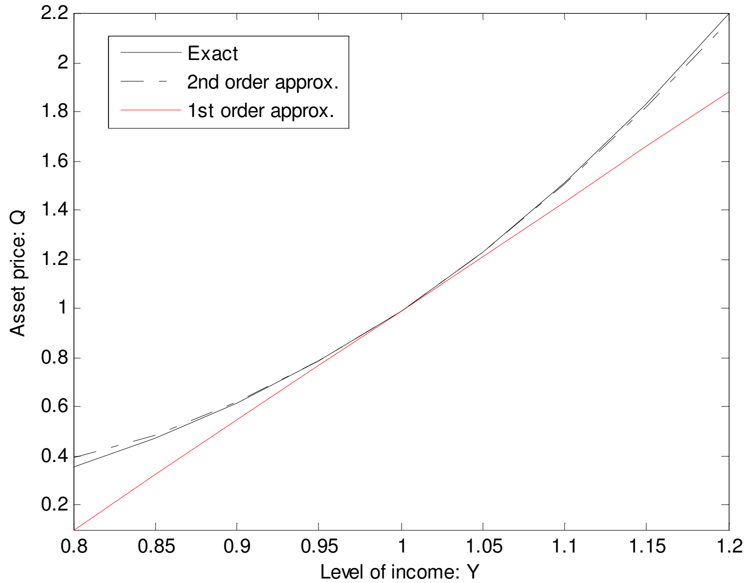
$$b = \theta\beta(1-\rho)[\theta(1-\rho)-(1+\rho)]\bar{Y}^{-2}$$

$$c = \theta\beta(1+\theta)\eta^2 \qquad \forall\theta > 0$$

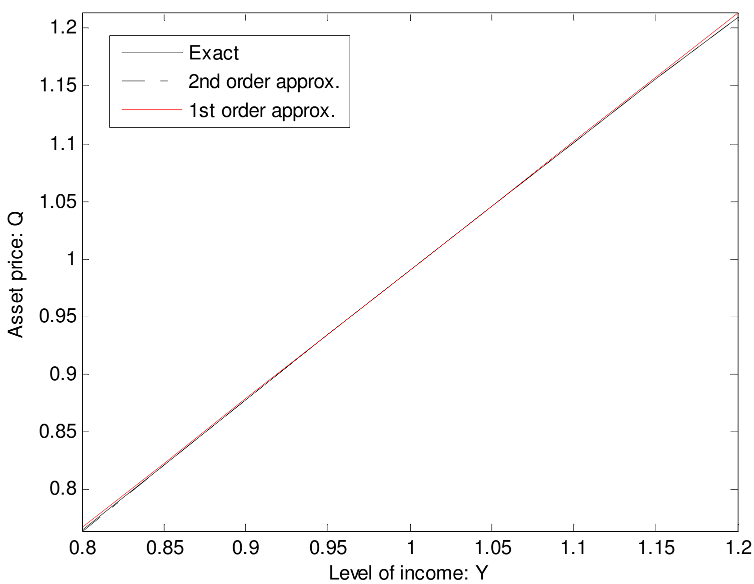
As we can see in Figure 3.2, the benefits of using second-order instead of first-order approximation depend on the curvature of the actual policy function (determined by the parameters θ and ρ) as well as on the deviation of Y from its steady state value, $\bar{Y}$, (which is determined by σ).

[3] The reader should refer to Judd (1998) for a detailed presentation of perturbation methods or to Schmitt-Grohe and Uribe (2004) for a paper-version exposition of this technique. In addition, Schmitt-Grohe and Uribe provide Matlab programs to obtain second-order approximations.

Figure 3.2
1^{st} and 2^{nd} order approximations
(a) $\theta = 6$



(a) $\theta = 1.5$



3.2.3 Simulation exercise and Bayesian estimation

How does the omission of the second-order terms (of the approximation) affect the parameter estimates?

To answer this question we simulate a series of the asset price using its second-order approximation. Then, we estimate two structural parameters (θ and ρ) of the model using standard Bayesian procedures.[4] These procedures rely on using first-order approximations and the Kalman filter to compute the conditional likelihood function. We assume that only the asset price, Q, is observable. The simulated series for this variable contains 500 observations and is generated using four different sets of parameter values for θ, ρ and σ. The rest of parameters have the following values: $\bar{Y} = 1$, $\eta = 1$, $\beta = 0.9901$.

Table 3.1 shows that omitting the second-order terms affects significantly the estimated parameters.[5] In all cases the true parameter values are out of the confidence interval defined by the mean plus/minus two standard deviations of the posterior distribution.[6] As expected, the size of the biases is related to the two

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[4] We assume that the values of $\bar{Y}$, η and β are known. Additionally, because the only "observable" series used in the estimation is Q, it is not possible to identify θ, ρ and σ separately. Therefore, we also assume that σ is known.

[5] The Metropolis-Hasting algorithm searches for sequences of draws from the posterior distribution. Each of these sequences originally are composed by 5000 draws from which the first 1500 are discarded. From the remaining draws, only one out of 10 are kept to reduce serial correlation. We consider uniform prior distributions for θ and ρ for the intervals [3 ; 15] and [0.05 ; 0.70], respectively.

[6] We also generate series of the asset price using the first order approximation and implement the Bayesian estimation. For this case, each parameter's posterior

following factors: the size of the curvature (measure by θ and ρ) and the magnitude of the volatility (σ).

Table 3.1
Posterior distributions

	θ				ρ			
	Mean	Std.	Conf. Interv.*		Mean	Std.	Conf. Interv.*	
Case A: θ=4, ρ=0.5, σ=0.1	5.36	0.52	4.33	6.39	0.62	0.03	0.55	0.69
Case B: θ=10, ρ=0.5, σ=0.1	19.44	0.48	18.49	20.39	0.72	0.01	0.69	0.74
Case C: θ=4, ρ=0.1, σ=0.1	4.98	0.31	4.36	5.59	0.19	0.04	0.11	0.28
Case D: θ=4, ρ=0.5, σ=0.2	7.31	0.35	6.62	8.00	0.69	0.01	0.66	0.71

*Confidence intervals = posterior mean +/- 2 posterior standard deviations.

3.3. Model and data

We use a small version of the New Keynesian model similar to the one used by Canova (2009). Despite its simplicity, this type of model fits relatively well the dynamic of the output gap, inflation and interest rates observed in the US. The following sub-sections contain the description of the model and the data used in the simulation exercise.

3.3.1 The model

The version of the New Keynesian model that we use is composed by three equations: an Euler equation, a Phillips curve and an empirical monetary policy rule. In this model households' preferences are additive in consumption and leisure. Firms have monopolistic power in the goods market and have to deal with the presence of stickiness in prices. The production function is linear in the labor input, but does not include capital. The monetary policy

mean lays inside of the confidence interval formed by the actual parameter value plus/minus one standard deviation of the posterior distribution.

rule is defined by a backward Taylor rule. The stochastic part of the model is constituted by three shocks: a demand shock, a technology shock and a monetary policy shock. The first two are first-order autoregressive processes while the latter is white noise.[7]

The linearized version of this model is composed by the following equations:

(1) $$y_t = E_t y_{t+1} - \frac{1}{\sigma} E_t \left[i_t - \pi_{t+1} \right] + e_{1t}$$

(2) $$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(\gamma+\sigma) y_t - ...$$

$$... - \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}(\gamma+1) e_{2t}$$

(3) $$i_t = \alpha_i i_{t-1} + (1-\alpha_i)\alpha_\pi \pi_{t-1} + (1-\alpha_i)\alpha_y y_{t-1} + e_{3t}$$

y, π, i represent the output gap, inflation and the interest rate. The parameters that define the behavior of the private sector are the risk aversion (σ), the inverse of the elasticity of labor supply (γ) and the degree of price stickiness, in a Calvo staggered price setting, (α). The monetary policy rule is described by three coefficients: the degree of interest rate persistency (α_i), the response of the interest rate to lagged inflation (α_π) and lagged output gap (α_y). β defines the intertemporal discount factor. e_{1t}, e_{2t} and e_{3t} represent the demand, technology and monetary policy shocks, respectively.

[7] See Canova (2009) for an exposition of the features of this type of model and alternative variations.

Our interest lies in matching the second moments of the output gap, inflation and the interest rate. Notice that severe identification problems arise with more complicated versions of the model if their estimation only includes the previously mentioned macroeconomic indicators. Therefore, we rely on this simple version of the New Keynesian model which suffices our purposes. Additionally, we would like to point out that Canova (2009) shows that despite its simplicity, this type of model fits relatively well the dynamic of the output gap, inflation and interest rates observed in the US.

3.3.2 The data

We use US quarterly data on the GDP index (normalized by the size of the working age population), CPI inflation and the effective Federal funds rates for the period 1954:3 – 2007:3. The source for this information is the FREDII databank of the Federal Reserve Bank of St. Louis. All series are filtered by the Baxter and King filter.[8] We denote as output gap the resulting filtered series of the logarithm of the normalized GDP index.[9]

Following Gali and Gambetti (2009), we divide our sample in a period of high volatility, which includes the observations between 1954:3 and 1984:4 (called “pre-1984” period), and a period of low volatility, which includes the observations of the period 1985:1 – 2007:3 (called “post-1984” period). Table 3.2 displays the standard deviations for each of the macroeconomic series during both

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[8] The volatilities of the series do not change significantly by using other filters such as Hodrick- Prescott filter.
[9] Because of the simplified nature of the model we use this statistically computed measure.

periods. As we can see, the reduction in the level of volatility in all the indicators is quite significant, and reflects what is called the Great Moderation.

Table 3.2
Great Moderation: standard deviations (%)

	pre-1984	post-1984	% variation
Fed interest rates	0.387	0.230	-40.7
Inflation	0.438	0.228	-48.0
Output gap	1.740	0.791	-54.5

Note: "pre-1984" represent the period 1954:3-1984:4. "post-1984" represents the period 1985:1 – 2007:3. Series filtered by the Baxter and King filter.

3.4. Simulation exercise

The main simulation exercise implemented in this study has the following steps.

Step 1: Obtaining the baseline parameter values

We obtain the baseline parameter values by estimating the model summarized by equations (1)-(3) during the period of low volatility ("pre-1984"). We use Bayesian techniques and the first-order approximation to estimate the posterior distributions of 11 of the 12 parameters of the model.[10] First-order approximations serve probably as better approximation for this period than for the period of higher volatility. In fact, An (2007) does not find significant differences when estimating this type of model for a similar

[10] We fix the intertemporal discount factor to 0.9837 in order to reduce some identification problems.

volatility period (1987-2002) using first and second-order approximations.

Table 3.3 contains some posterior distribution statistics of the parameters of the model.[11] The posterior means of these parameters define our baseline values.

Table 3.3
Posterior distribution statistics: "pre-1984" period

	Symbol	Mean	Std.dv.
Coef. risk aversion	σ	1.569	0.050
Inv. elasticity labour supply	γ	2.626	0.032
Price stickiness	α	0.797	0.011
T.R.: interest smoothing	α_i	0.887	0.006
T.R.: inflation	α_π	1.924	0.077
T.R.: output	α_y	0.728	0.034
Autoreg coeff. demand shock	ρ_1	0.831	0.007
Autoreg coeff. tech. shock	ρ_2	0.832	0.008
Std. dv. demand innov.	σ_1	0.231	0.021
Std. dv. tech. innov.	σ_2	0.293	0.023
Std. dv. monetary innov.	σ_3	0.092	0.005

Step 2: Parameter values for the high volatility period

Our objective is to generate series of the output gap, inflation and interest rate using second-order approximations that replicate the higher volatilities of the actual macroeconomic indicators for the "pre-1984" period.[12] In principle we use the baseline parameter values, except for three parameters. In one case, these three parameters are the standard deviations of the perturbation terms of the three exogenous shocks. When using this set of parameters, the

[11] We use uniform prior distributions for the Bayesian estimation. Appendix A contains the specific characteristic of these distributions.
[12] We use the algorithm by Schmitt-Grohe and Uribe (2004) to get the second-order approximations.

reduction in the volatilities of the macroeconomic indicators between the "pre-1984" and "post-1984" periods is caused by less severe shocks affecting the economy, explanation for the Great Moderation that is called the "good luck" hypothesis.[13]

In the other case, the three parameter values that deviate from the baseline case are the coefficients of the Taylor rule. When using this set of parameters, the Great Moderation is explained by a change in the way how the monetary policy was conducted. In the literature about the Great Moderation, this explanation for the reduction in the volatility of the macroeconomic indicators is called the "good policy" hypothesis.[14]

We simulate series of the output gap, inflation and interest rate using different values for each of the three parameters previously mentioned in each of the cases. The selected parameter values are those that minimize the sum of the squared difference between the variance of each simulated series and the actual one.[15] For the parameters related to the "good luck" hypothesis (column 2 of Table 3.4), all standard deviations of the perturbation terms are much higher than in the baseline case. With respect to the

[13] Studies supporting this hypothesis are Sims and Zha (2006), Primiceri (2005), Canova and Gambetti (2005) and Stock and Watson (2003).

[14] See Clarida et. al. (2000), Lubik and Schorfheide (2004) and Boivin and Giannoni (2006) for studies supporting this hypothesis.

[15] The minimization procedure consists of evaluating the sum of the squared difference between the variance of the simulated series and the actual ones over a three dimensional grid of points. For the case of the standard deviations of the perturbation terms of the shocks, each dimension of the grid goes from 0.01 to 1. The dimensions of the grid employed to determine the values of the coefficients of the Taylor rule are the following: from 0.75 to 0.99 for the degree of interest persistency, from 1.01 to 2.5 for the response of the interest rate to the inflation, and from 0 to 1.5 for the response of the interest rate to output gap.

parameters related to the “good policy” hypothesis (see column 3) we observe the following: the coefficients of lagged inflation and lagged output gap in the Taylor rule decreases significantly. In particular, the response of the interest rate to lagged inflation is close to one.

Table 3.4
Simulation exercise: sets of parameter values

	Symbol	Baseline [1/] (1)	High volatility [2/] Good Luck (2)	High volatility [2/] Good Policy (3)
Intertemporal discount factor	β	0.984	0.984	0.984
Coef. risk aversion	σ	1.569	1.569	1.569
Inv. elasticity labour supply	γ	2.626	2.626	2.626
Price stickiness	α	0.797	0.797	0.797
T.R.: interest smoothing	α_i	0.887	0.887	0.876
T.R.: inflation	α_π	1.924	1.924	1.015
T.R.: output	α_y	0.728	0.728	0.008
Autoreg coeff. demand shock	ρ_1	0.831	0.831	0.831
Autoreg coeff. tech. shock	ρ_2	0.832	0.832	0.832
Std. dv. demand innov.	σ_1	0.231	0.500	0.231
Std. dv. tech. innov.	σ_2	0.293	0.340	0.293
Std. dv. monetary innov.	σ_3	0.092	0.140	0.092

[1/] Posterior means obtained from the estimation of the New Keynesian model using data for the "pre-1984" period.
[2/] Highlighted numbers indicate those parameters used to match the volatility levels for output gap, inflation and the interest rate observed for the "post-1984" period.

How well do these sets of parameter values replicate the volatility of the three macroeconomic indicators used in this study? Table 3.5 shows the standard deviations of the data using both sets of parameter values. In general, the simulated series overestimate the level of the volatility of the interest rate and inflation but are close to the level of the volatility of the output gap. However, these simulated series are close to match the relative reduction of the actual series between the “pre-1984” and “post-1984” periods. The only exception is the interest rate under the “good policy” hypothesis. This result might imply that the “good policy” hypothesis by itself cannot account for the reduction in the volatility of all the series. Therefore, it could require the variation of some private sector parameters or the volatility of some of the shocks. These alternatives are, however, currently not considered.

Table 3.5
Simulation data: standard deviations (in %)

	"Good luck" parameter values			"Good policy" parameter values		
	pre-1984	post-1984	% variation	pre-1984	post-1984	% variation
Fed interest rates	0.739	0.370	-50.0	0.417	0.370	-11.3
Inflation	0.637	0.348	-45.3	0.756	0.348	-53.9
Output gap	1.503	0.744	-50.5	1.724	0.744	-56.8

Note: 2nd order approximation was employed to simulate the data for the "Pre-1984" period. For the "Post-1984" period, first-order approximation is used instead. Each series contains 20 000 observations.

Step 3: Estimation of the model using first-order approximations

Using simulated series of the output gap, inflation and interest rate, we estimate the New Keynesian model presented in the previous section using only first-order approximation, as most studies do that implement Bayesian or likelihood estimations. As mentioned in the introduction, the goal of this study is to evaluate if we can rely on this type of estimation for periods of high volatility. In particular, the design of this simulation experiment allows us to determine if omitting the second-order terms of the policy functions could bias the analysis about the sources of the Great Moderation.

The following section contains the results of this estimation.

3.5. Results

We start this section presenting the posterior distribution statistics obtained from the simulation exercise. In particular, we evaluate if these estimates significantly differ from the actual parameter values used to generate the data. Additionally, we implement counterfactual and Impulse-Response Function (IRFs) analyses to determine whether potential biases in the parameter estimates affect the evaluation of the sources of the Great Moderation and the dynamic of the economy, respectively.

3.5.1 Posterior distribution estimates

Tables 3.6 shows the posterior mean and standard deviation for each of the parameters of the model. When the simulated data correspond to the "good luck" hypothesis (column 1 of Table 3.6), we find that three parameters lie outside of the confidence interval defined by the mean of each posterior distributions plus/minus 2 posterior standard deviations. These parameters are the coefficient of the lagged output gap in the Taylor rule (α_y), the autoregressive coefficient of the technology shock (ρ_2) and the standard deviation of the perturbation term of this shock (σ_2). Despite the presence of these biases, the comparison of the estimates, which correspond to our simulated "pre-1984" sample and the parameter estimates obtained for the "post-1984" sample (column 3)[16], highlights the reduction in the volatility of the shocks as the source of our simulated Great Moderation. In particular, the standard deviations

[16] For easy comparison the estimates of Table 3.3 are included in column 3 of Table 3.6.

related to the perturbation term of the demand and the monetary policy shocks decrease between these two samples (from a posterior mean of 0.499 and 0.145 to 0.231 and 0.092, respectively). We also observe a reduction in the persistence of the technology shock (from 0.883 to 0.832). With respect to the Taylor rule, the estimated persistence of the interest rate (α_i) and the response of the interest rate to lagged inflation (α_π) are not statistically different from their estimates for the "post-1984" sample (which are the actual values used to generate the data). The only potential source of misidentification when identifying the source of the Great Moderation constitutes the relatively high value of the coefficient that measures the response of the interest rate to the lagged output gap (from 0.832 in the "pre-1984" sample to 0.728 in the "post-1984" sample). However, as the counterfactual and the IRF analyses show, this reduction does not have a significant impact on the volatility of the economy.

Table 3.6
Posterior distribution statistics: "pre-1984" and "post-1984" periods estimates

		(1)				(2)				(3)	
		"Pre-1984" period								"Post-1984" period	
		Good luck				Good policy					
	Symbol	True	Mean	Std.dv.		True	Mean	Std.dv.		Mean	Std.dv.
Coef. risk aversion	σ	1.569	1.632	0.125		1.569	1.235	0.077	*	1.569	0.050
Inv. elasticity labour supply	γ	2.626	2.604	0.180		2.626	2.625	0.178		2.626	0.032
Price stickiness	α	0.797	0.799	0.007		0.797	0.847	0.006	*	0.797	0.011
T.R.: interest smoothing	α_i	0.887	0.890	0.005		0.876	0.880	0.003		0.887	0.006
T.R.: inflation	α_π	1.924	1.786	0.117		1.015	1.066	0.022	*	1.924	0.077
T.R.: output	α_y	0.728	0.832	0.051	*	0.008	0.002	0.002	*	0.728	0.034
Autoreg coeff. demand shock	ρ_1	0.831	0.831	0.009		0.831	0.965	0.004	*	0.831	0.007
Autoreg coeff. tech. shock	ρ_2	0.832	0.883	0.012	*	0.832	0.983	0.004	*	0.832	0.008
Std. dv. demand innov.	σ_1	0.500	0.500	0.019		0.231	0.061	0.005	*	0.231	0.021
Std. dv. tech. innov.	σ_2	0.340	0.293	0.019	*	0.293	0.404	0.012	*	0.293	0.023
Std. dv. monetary innov.	σ_3	0.140	0.145	0.003		0.092	0.094	0.002		0.092	0.005

Note: * indicates that the true parameter value is outside of the confidence interval defined by the posterior mean +/- 2 posterior standard deviations.

When the simulated data correspond to the "good policy" hypothesis, the results are quite different. In this case, almost all the posterior estimates are significantly different from the actual values of the underlying parameters (see column 2 in Table 3.6).[17] In particular, the value of the standard deviation of the perturbation term of the technology shock, σ_2, is over-estimated (the posterior mean is 0.404 while the actual value is 0.293). This result is particularly interesting because it could lead to errors when determining the sources of the Great Moderation. Thus, after having evaluated the posterior estimates obtained from the "pre-1984" and "post-1984" samples, a researcher could erroneously conclude that the reduction of the macroeconomic volatility between these two periods is not only due to the change in the monetary policy rule but also to a reduction in the volatility of one of the shocks hitting the economy. In fact, the counterfactual and the IRF analyses reconfirm this erroneous conclusion.

3.5.2 Counterfactual exercise

Using the previous parameter estimates, we implement the same counterfactual exercise that Boivin and Giannoni (2006) employ to determine the sources of the Great Moderation. In so doing, we can determine the extent to which the statistically significant biases obtained for some parameter estimates could lead to erroneous

[17] The exceptions are the following three parameters: the inverse elasticity of the labour supply, the Taylor rule parameter that controls the degree of smoothness of the interest rate, and the standard deviation of the perturbation term of the monetary policy shock.

conclusions about the validity of the “good luck” or “good policy” hypotheses.

The counterfactual exercise consists in calculating the variation in the volatilities of the simulated series for a given set of parameter estimates. This potential variation is due to the use of two posterior estimates (obtained for the “pre-1984” and “post-1984” periods) for a subset of parameters. All other parameters are fixed at the value of their estimates obtained for either the “pre-1984” period or the “post-1984” period.

For this exercise we consider three subsets of parameters: the “stochastic processes” (SH) parameters, which include the autocorrelation coefficients of the exogenous shocks and the standard deviations of their perturbation terms; the “monetary policy” (MP) parameters, which contain the three coefficients of the Taylor rule; and the “private sector” (PS) parameters, which consist of the coefficient of risk aversion, the inverse of the elasticity of labor supply and the price stickiness.

The result of this counterfactual exercise for the parameter estimates related to the “good luck” hypothesis can be found in Table 3.7. In the absence of any biases in the estimation, the result is pretty clear: only changes in the SH parameters can explain the reduction in the volatilities of the macroeconomic series. This result is precisely observed in rows 1-4 of Table 3.7. In each of these rows the only parameter values that change are the ones for the SH parameters. The PS and MP parameters correspond either to their values obtained for the “pre-1984” period (which we denote as "pre") or to their values for the "post-1984" period (denoted as "post"). Rows 5-8 show the same type of estimation, but this time only the MP parameters change when holding the others constant. The values contained in these rows show that changes in the parameter estimates of the Taylor rule have very low impact on the volatility of the series.[18]

Table 3.7
Standard deviations (in %) of Output gap, Inflation and Interest Rate in Counterfactual Experiments: the "Good policy" hypothesis

Row	Parameter combination	Sd Output gap	Sd Inflation	Sd Interest rate
	Total reduction	-51.4	-47.1	-51.5
	A. importance of the shocks			
1	mp (pre) - ps (pre)	-52.4	-46.2	-51.6
2	mp (pre) - ps (post)	-52.6	-46.5	-52.6
3	mp (post) - ps (pre)	-50.5	-46.2	-51.7
4	mp (post) - ps (post)	-51.8	-47.0	-51.4
	B. importance of monetary policy			
5	sh (pre) - ps (pre)	0.4	-0.4	2.4
6	sh (pre) - ps (post)	0.6	-1.1	-1.4
7	sh (post) - ps (pre)	4.4	-0.4	2.2
8	sh (post) - ps (post)	2.4	-2.0	0.9

We repeat this exercise but this time using the parameter estimates related to the "good policy" hypothesis. As Table 3.8 shows, the results are not in line with what we expected, namely that changes in the values of the MP parameters are the only factor behind the reduction in the volatilities of the macroeconomic series. First, rows 1-4 show that changes in the values of the SH parameters explain entirely the reduction in the volatilities of these series. Second, changes in the MP parameters reduce only the standard deviation of the output gap, and in some cases the standard deviation of

[18] Similar results are obtained when the PS parameter are the ones that change.

inflation. However, they lead to an increase in the volatility of the interest rate (see rows 5-8). Therefore, we find that the biases in the parameter estimates obtained using data generated according to the "good policy" data are not only statistically significant but could also cause erroneous conclusions about the sources of the Great Moderation.

Table 3.8
Standard deviations (in %) of Output gap, Inflation and Interest Rate in Counterfactual Experiments: the "Good policy" hypothesis

Row	Parameter combination	Sd Output gap	Sd Inflation	Sd Interest rate
	Total reduction	-70.1	-63.2	-50.8
	A. importance of the shocks			
1	mp (pre) - ps (pre)	-37.8	-51.5	-70.2
2	mp (pre) - ps (post)	-43.0	-45.4	-68.3
3	mp (post) - ps (pre)	-56.0	-82.8	-74.0
4	mp (post) - ps (post)	-58.7	-75.7	-72.7
	B. importance of monetary policy			
5	sh (pre) - ps (pre)	-32.6	32.5	54.5
6	sh (pre) - ps (post)	-23.1	15.3	50.8
7	sh (post) - ps (pre)	-52.4	-52.9	34.9
8	sh (post) - ps (post)	-44.2	-48.6	29.7

3.5.3 Impulse-Response Functions (IRFs)

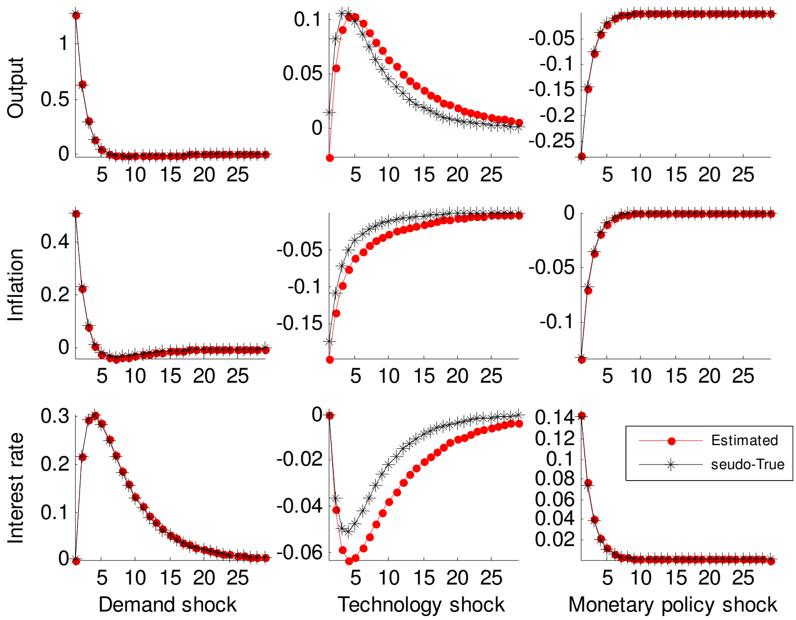
Finally, we calculate IRFs using the parameters estimates and the first-order approximation and compare them with those obtained using the true parameter values and the correct specification of the approximation.[19] The aim of this exercise is to determine how the

[19] Properties such as symmetry, scalability and path-independency of the IRFs for linear processes are not shared with nonlinear processes. In our calculations past history of the variables and shocks are fixed at their steady state level. We simulate various initial shocks and trace out the response distributions along the horizon. We did not find significant differences with respect to the responses obtained using first order policy functions, at least for shocks between plus/minus two standard deviations.

biases in the parameters and the use of a lower order approximation affect the dynamic of the economy.

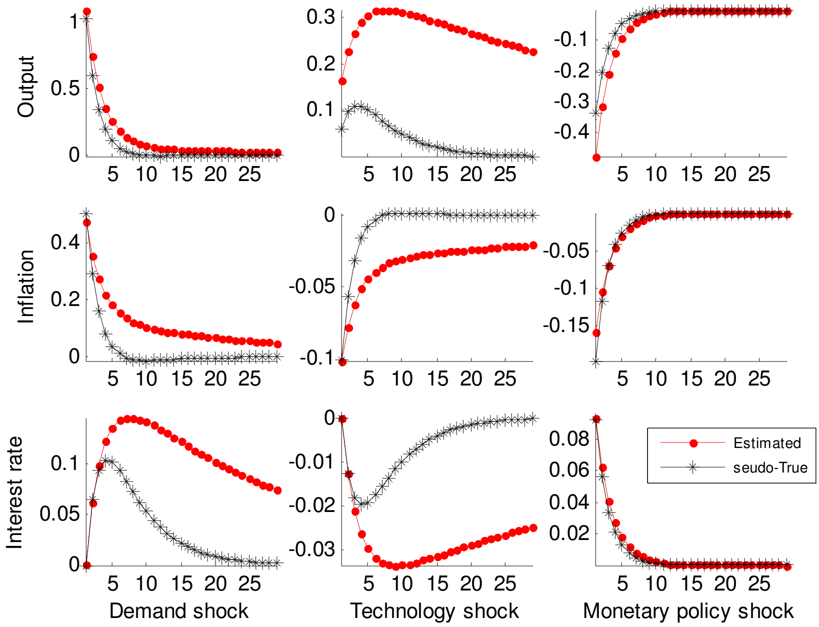
Figure 3.3 shows the IRFs related to the "good luck" hypothesis. The responses of the three macroeconomic variables to each of the three shocks do not alter significantly when we use the estimated parameter values and the actual ones (and the first and second-order approx. policy functions, respectively).

Figure 3.3
Impulse-Response Functions for the "good luck" case



This result, however, does not remain once we use estimates related to the "good policy" hypothesis. In particular, as Figure 3.4 shows, both demand and technology shocks have more persistent effect on the output gap, inflation and interest rate than when using the actual parameter values and correct policy functions. This result is due to the over-estimation of the autoregressive coefficients of these shocks.

Figure 3.4
Impulse-Response Functions for the “good policy” case



Therefore, the biases in the parameter estimates obtained when the data are generated according to the “good policy” hypothesis not only alter our analysis of the sources of the Great Moderation but also have significant impact on the dynamic of the model.

3.6. Conclusions

This chapter provides evidence of potential problems related to the use of first-order approximations in the estimation of DSGE models. This evidence is based on a simulation exercise which consists on the following steps. First, we generate data for three macroeconomic series (the output gap, inflation and interest rate) using second-order policy functions of a New Keynesian model. The simulated data are created to reproduce the high volatility observed during the period 1954:3 – 1984:4, and are obtained using parameter values that reflect the two competing hypothesis behind the Great Moderation: 1) the “good luck” hypothesis which postulates that the volatility level of the shocks affecting the economy determine the high volatility level of the macroeconomic series; and 2) the “good policy” hypothesis which stresses the way how monetary policy was conducted as the main cause behind the observed volatility level. Second, we estimate the structural parameters of the model considering only first-order policy functions as commonly used in Bayesian estimations of structural models. Finally, we compare the parameter estimates and the actual parameter values in order to determine whether omitting the second-order terms of the policy function could lead to erroneous conclusions about the sources of the Great Moderation.

Our results show that when high volatility data are generated according to the “good policy” hypothesis, omitting the second-order terms of the approximation could lead to an erroneous misinterpretation of the sources of the Great Moderation. This is not the case when the data are generated according to the “good luck” hypothesis.

There are some interesting issues not addressed in this study. Firstly, in light of the evidence presented in this study, an estimation of the DSGE model using second-order approximations is required to evaluate the sources of the Great Moderation by using

DSGE models. Secondly, numerical methods can be employed to get a more accurate description of the policy functions. In so doing, it is possible to directly evaluate the areas in the parameter space where first-order (or higher order) policy functions lead to a bad approximation. Finally, the results presented in this study are probably model-specific. Therefore the estimation of more complete models, such as the model by Smets and Wouters (2007), are required to test the potential problems highlighted in this study.

To conclude, the popularity that second or higher order approximations could reach in macroeconomics will depend on the existence of relevant questions where their use provides significantly better answers than first-order approximations. It will also hinge on the advances of computational algorithms and computing power which will allow for a reduction of the costs associated with their estimation. This study provides an example of an important topic where it is worth to invest in this type of estimations.

Appendix

Appendix of chapter 1

Appendix A: Prior distributions of structural parameters

	Symbol	Distribution	Mean	Std.
Share of capital in production	α	Normal	0.30	0.05
Inv. Elasticity of Intertemporal substitution	σ_c	Normal	1.50	0.38
Fix cost in production	Φ	Normal	1.25	0.13
Adjust cost of investment	S''	Normal	4.00	1.50
Habits in consumption	η	Beta	0.70	0.10
Wage stickiness	ξ_w	Beta	0.50	0.10
inv. Elast. labor supply	σ_l	Normal	2.00	0.75
Price stickiness	ξ_p	Beta	0.50	0.10
Wage indexation	ι_w	Beta	0.50	0.15
Price indexation	ι_p	Beta	0.50	0.15
Capital utilization elasticity	ψ	Beta	0.50	0.15
Taylor rule: response to inflation	r_π	Normal	1.50	0.25
Taylor rule: response to lagged interest rate	ρ_R	Beta	0.75	0.10
Taylor rule: response to changes in output	$r_{\Delta y}$	Normal	0.13	0.05
Trend growth rate	γ_bar	Normal	0.40	0.10
Steady state of inflation	π _bar	Gamma	0.63	0.10
Steady state of hours worked	l _bar	Normal	0.00	2.00
Steady state of nominal int rate	r _bar	Gamma	1.15	0.30
Autocorrelation coef. Price Mk up shock	ρ_p	Beta	0.50	0.20
Autocorrelation coef. Wage Mk up shock	ρ_w	Beta	0.50	0.20
Autocorrelation coef. Product. Shock	ρ_a	Beta	0.50	0.20
Autocorrelation coef. Risk premium shock	ρ_b	Beta	0.50	0.20
Autocorrelation coef. Government shock	ρ_g	Beta	0.50	0.20
Autocorrelation coef. Investment-Specific shock	ρ_q	Beta	0.50	0.20
Autocorrelation coef. Monet policy shock	ρ_r	Beta	0.50	0.20
Correlation Government and productivity shocks	ρ_{ga}	Normal	0.50	0.25
Std Price Mk up innovation	σ_p	Inv. Gamma	0.10	2.00
Std. Wage Mk up innovation	σ_w	Inv. Gamma	0.10	2.00
Std. Product. Innovation	σ_a	Inv. Gamma	0.10	2.00
Std. Risk premium innovation	σ_b	Inv. Gamma	0.10	2.00
Std. Government innovation	σ_g	Inv. Gamma	0.10	2.00
Std. Inv. Specific innovation	σ_q	Inv. Gamma	0.10	2.00
Std. Monet policy innovation	σ_r	Inv. Gamma	0.10	2.00
Gain - no inflation	$g^{non\pi}$	Uniform	0.00	1.00
Gain - inflation	g^{π}	Uniform	0.00	1.00
Std. measurement error on expectations	σ_{exp}	Inv. Gamma	0.10	2.00

Note: for uniform distributions the values assigned as mean and standard deviation correspond to the range of the domain.

Appendix of chapter 2

Appendix A: On the calculation of the wealth-income distribution*

In the infinite-horizon model considered in this study there are two states variables: wealth (w) and current income (y). Wealth is defined either as the stock of durable goods (net of depreciation) or as the sum of the stocks of durable goods (net of depreciation) and bonds.

In order to determine the unconditional means of the different variables of the model (e.g. investment in durables, stock of durables, nondurable consumption and stock of bonds) we need to calculate the stationary unconditional distribution of (w_t, y_t) pairs, $\lambda_t(w, y) = \Pr(w_t = w, y_t = y)$.

Considering the optimal policy function $w' = g(w, y)$ and the exogenous Markov chain P that determines y, the law of motion for the distribution λ_t is defined as:

$$\Pr(w_{+1} = w', y_{+1} = y') = \sum_{w_t} \sum_{y_t} \Pr(w_{+1} = w' \mid w_t = w, y_t = y)$$

$$\cdot \Pr(y_{t+1} = y' \mid y_t = y).\Pr(w_t = w, y_t = y)$$

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* The content of this appendix is based on Ljungqvist and Sargent (2004).

or

$$\lambda_{t+1}(w',y') = \sum_{w}\sum_{y}\lambda_t(w,y)\Pr(y_{t+1}=y' \mid y_t = y).I(w',y,w)$$

$I(w',y,w)$ is an indicator function that for three consecutives nodes on the grid of w (w_{i-1}, w_i and w_{i+1}) adopts the following form:

$$I(w',y,w) = \begin{cases} 0, \quad \text{if } g(w_i,y) < w_{i-1} \\ \dfrac{g(w_i,y)-w_{i-1}}{w_i - w_{i-1}}, \quad \text{if } w_{i-1} \le g(w_i,y) < w_i \\ 1, \quad \text{if } \mathrm{g}(w_i,y) = w_i \\ \dfrac{g(w_i,y)-w_{i-1}}{w_i - w_{i-1}}, \quad \text{if } w_i < g(w_i,y) \le w_{i+1} \\ 0, \quad \text{if } g(w_i,y) > w_{i+1} \end{cases}$$

The indicator function $I(w',y,w)$ identifies the proportion of t states w, y that are sent into w' at time *t+1*. The previous equation can be rewritten as:

$$\text{(A1)} \quad \lambda_{t+1}(w',y') = \sum_{y}\sum_{\{w:w'=g(w,y)\}}\lambda_t(w,y)P(s,s')$$

The stationary distribution λ is the one that solves the previous equation ($\lambda_{t+1} = \lambda_t$). It is possible to obtain this stationary distribution by iterating (A1). An alternative is to create a Markov chain that describes the solution of the optimum problem, then to compute an invariant distribution from the left eigenvector associated with a unit eigenvalue of the stochastic matrix. This procedure implies the following steps:

1. Map the pair of vectors (w, y) into a single state vector x. Because in our exercises y could only take two values (y_e and y_u), x adopt the following form:

   $$x' = [(w_1, y_e), (w_2, y_e), \ldots, (w_N, y_e), (w_1, y_u), (w_2, y_u), \ldots, (w_N, y_u)];$$

   where N represents the total number of grid points used for w. Thus, x is a vector Nx2.

2. Define the Markov chain M on x_t:

   $$M = \Pr[(w_{t+1} = w', y_{t+1} = y') \,|\, (w_t = w, y_t = y)]$$

   $$= \Pr(w_{t+1} = w' \,|\, w_t = w, y_t = y).\Pr(w_{t+1} = w' \,|\, y_t = y)$$

   $$= I(w', y, w) P(s, s').$$

   where M is a vector 2Nx2N.

3. Calculate the stationary distribution π of the Markov process M using the eigenvector associated with a unit eigenvalue of this matrix. There is a result which says that for any Markov transition matrix M, there exists a vector π such that:

(A2) $\pi' = \pi' M$

Rearranging equation (A2) yields,

(A3) $(I - M)\pi = 0$

Therefore, π is the eigenvector associated with matrix M. The fact that M is a stochastic matrix (i.e., all its elements are nonnegative and the sum of the elements of each row equals one) guarantees that M has at least one unit eigenvalue, and that there is at least one eigenvector π that satisfies equation (A3).

4. Finally, "unstack" vector x and use π to get the stationary distribution λ:

$$\lambda(w_i, y_e) = \Pr(w_t = w_i, y = y_e) = \pi(i)$$

and

$$\lambda(w_i, y_u) = \Pr(w_t = w_i, y = y_u) = \pi(N + i)$$

For $i = 1, ..., N$.

An additional alternative to get the stationary distribution $\lambda(w, y)$ is to simulate the model for a significant number of periods. After discarding the first observations to remove any bias due to initial conditions, find the stationary distribution by counting the number of periods the model enters in each pair (w, y).

The results that we present in this study are generated using both approaches: simulation and using the eigenvector associated with a unit eigenvalue of M. The latter approach can be used in all the cases covered in this study with the exception of the economy with bonds and non-negativity constraint in investment of durables. This case requires the use of three state variables (durables, bonds and income), and as a consequence, the definition of the stochastic matrix M becomes more tedious. For this specific case, we relying only on simulation.

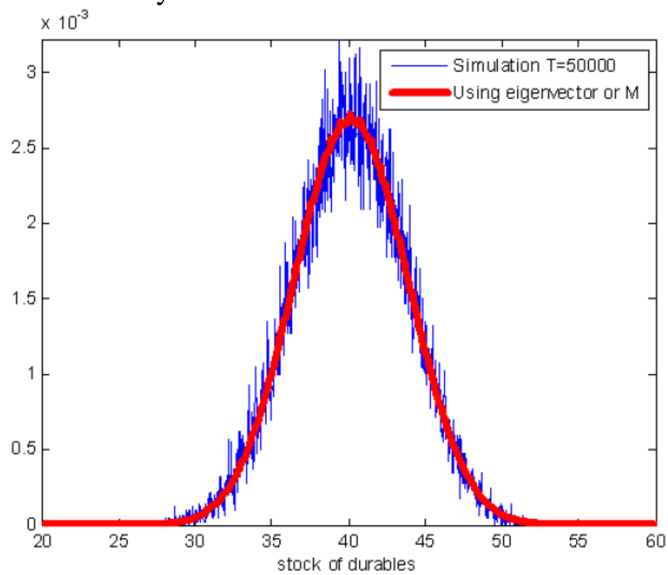
In order to illustrate the importance of using an accurate procedure to get the distribution of wealth-income, we consider the case of an economy without bonds and no non-negativity constraint (as in section 3.1). For the parameter values $\beta = 0.99$, $\gamma_c = \gamma_d = 2$, $\delta = 0.025$, and $\sigma = 1$, we simulate the model for different number of periods. After discarding the first 10 000 observations, we calculate the unconditional mean of the stock of durables. We also use the eigenvector associated with a unit eigenvalue of M, as described previously. Table A1 summarized the results of this exercise:

Table A1
Unconditional mean of the stock of durables

Method	$E(d)$	$(E(d)/d_{ss} - 1)x100$
Using eigenvector of M	40.193	0.484
Simulation T=500 000	40.193	0.484
T=100 000	40.162	0.405
T= 50 000	40.138	0.346
T= 20 000	40.251	0.628

Additionally, Figure A1 shows the distributions of the stock of durables obtained under simulation and using the eigenvector of M.

Figure A1
Stationary distribution of the stock of durables



Notice that even though the income process y is iid, it is required to simulate the economy for a significant number of periods (i.e. half a million) to get close to the results obtained using the eigenvector of M. Considering that the differences obtained in some of the models covered in this chapter are very small, accuracy in the simulation procedure is an important requirement.

Appendix of chapter 3

Appendix A: Prior distributions used in the Bayesian estimations

Prior distributions: Uniform distribution (Min,Max)

	Symbol	Min	Max
Coef. risk aversion	σ	0.30	5.0
Inv. elasticity labour supply	γ	0.10	5.0
Price stickiness	α	0.10	0.90
T.R.: interest smoothing	α_i	0.05	0.95
T.R.: inflation	α_π	1.01	5.0
T.R.: output	α_y	0.001	3.0
Autoreg coeff. demand shock	ρ_1	0.10	0.99
Autoreg coeff. tech. shock	ρ_2	0.10	0.99
Std. dv. demand innov.	σ_1	0.001	1.0
Std. dv. tech. innov.	σ_2	0.001	1.0
Std. dv. monetary innov.	σ_3	0.001	1.0

Bibliography

Adam, Klaus and Mario Padula. (2011). “Inflation Dynamics and Subjective Expectations in the United States.” Economic Inquiry 49, 13-25.

An, Sungbae. (2007). “Bayesian Estimation of DSGE Models: Lessons from Second-Order Approximations.” Mimeo.

An, Sungbae and Frank Schorfheide. (2007). “Bayesian Analysis of DSGE models.” Econometric Reviews 26, 113-172.

Ang, Andrew, Geert Bekaert and Min Wei. (2007). "Do macro variables, asset markets, or surveys forecast inflation better?" Journal of Monetary Economics 54, 1163–1212.

Binder, Michael and Hashem Pesaran. (1997). "Multivariate Linear Rational Expectations Models: Characterization of the Nature of the Solutions and Their Fully Recursive Computation." Econometric Theory 13, 877–888.

Blanchard, Olivier and Charles Kahn. (1980). "The Solution of Linear Difference Models under Rational Expectations." Econometrica 48, 1305–1312.

Boivin, Jean and Marc P. Giannoni. (2006). “Has Monetary Policy Become More Effective?.” The Review of Economics and Statistics 88, 445–462.

Branch, William and George Evans. (2006). "A simple Recursive Forecasting Model." Economics Letters 91, 158–166.

Calvo, Guillermo. (1993). "Staggered Prices in a Utility-Maximizing Framework." Journal of Monetary Economics 12, 383-398.

Canova, Fabio. (2009). “What Explains the Great Moderation in the US? A Structural Analysis.” Journal of the European Economic Association 7, 697-721.

Canova, Fabio and Luca Gambetti. (2010). "Do Expectations Matter? The Great Moderation Revisited." American Economic Journal: Macroeconomics 2, 183-205.

Canova, Fabio and Luca Sala. (2009). "Back to square one: Identification issues in DSGE models". Journal of Monetary Economics 56, 431-449.

Clarida, Richard, Jordi Gali and Mark Gertler. (2000). "Monetary Policy Rule and Macroeconomic Stability: Evidence and Some Theory." Quarterly Journal of Economics CXV, 147-180.

Carboni, Giacomo and Martin Ellison. (2009). "The Great Inflation and the Greenbook." Journal of Monetary Economics 56, 831-841.

Carceles-Poveda, Eva and Chryssi Giannitsarou. (2007). "Adaptive Learning in Practice." Journal of Economic Dynamics and Control 31, 2659-2697.

Carroll, Christopher and Wendy Dum. (1997). "Unemployment Expectations, Jumping (S,S) Triggers, and Household Balance Sheets." NBER Macroeconomics Annual 12, 165-230.

Christiano, Lawrence. (2002). "Solving Dynamic Equilibrium Models by a Methods of Undetermined Coefficients." Computational Economics 20, 21–55.

Coibion, Olivier and Yuriy Gorodnichenko. (2010). "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts." Mimeo.

De Wind, Joris. (2008). "Punishment Functions." Unpublished manuscript, University of Amsterdam.

Del Negro, Marco and Stefano Eusepi. "Fitting Observed Inflation Expectations." Mimeo.

Del Negro, Marco and Frank Schorfheide. (2008). "Forming Priors for DSGE Models (and How it Affects the Assessment of Nominal Rigidities)." Journal of Monetary Economics 55, 1191-1208.

Erceg, Christopher and Andrew Levin. (2003). "Imperfect Credibility and Inflation Persistence." Journal of Monetary Economics 50, 915-944.

Evans, George and Seppo Honkapohja. (1995). "Local Convergence of Recursive Learning to Steady States and Cycles in Stochastic Nonlinear Models." Econometrica 63, 195-206.

Evans, George and Seppo Honkapohja. (2001). Learning and Expectations in Macroeconomics. Princeton University Press.

Fernández-Villaverde, Jesús and Julio Rubio-Ramírez. (2005). "Estimating Dynamic Equilibrium Economies: Linear versus Nonlinear Likelihood." Journal of Applied Econometrics 20, 891-910.

Fernández-Villaverde, Jesús and Julio Rubio-Ramírez. (2008). "How Structural are Structural Parameters?" NBER Macroeconomics Annual 2007.

Gali, Jordi and Luca Gambetti. (2009). "On the Sources of the Great Moderation." American Economic Journal: Macroeconomics 1, 26-57.

Honkapohja, Seppo and Kaushik Mitra. (2003). "Learning with Bounded Memory in Stochastic Models." Journal of Economic Dynamics and Control 27, 1437—1457.

Ireland, Peter. (2007). "Changes in the Federal Reserve's Inflation Target: Causes and Consequences." Journal of Money, Credit and Banking 39, 1851-82.

Judd, Kenneth. (1998). Numerical Methods in Economics. The MIT Press.

Kimball, Miles S. (1995). "The Quantitative Analytics of the Basic Neomonetarist Model." Journal of Money, Credit, and Banking 27, 1241–77.

Krusell, Per and Anthony Smith Jr. (1998). "Income and Wealth Heterogeneity in the Macroeconomy." Journal of Political Economy 106, 867-896.

Ljungqvist and Sargent. (2004). Recursive Macroeconomic Theory. MIT press.

Lubik, Thomas and Frank Schorfheide. (2004). "Testing for Indeterminacy: An application to US monetary Policy." American Economic Review 94, 190-217.

Mankiw, Gregory, Ricardo Reis, Justin Wolfers. (2003). "Disagreement about Inflation Expectations." NBER Macroeconomics Annual 18, 209-248.

Marcet, Albert, and Juan Pablo Nicolini. (2003). "Recurrent Hyperinflations and Learning." American Economic Review 93, 1476—1498.

Milani, Fabio. (2007). "Expectations, Learning and Macroeconomic Persistence." Journal of Monetary Economics 54, 2065–2082.

Nunes, Ricardo. (2010). "Inflation Dynamics: The Role of Expectations." Journal of Money, Credit and Banking 42, 1161-1172.

Orphanides, Athanasios and John C. Williams. (2005a). "Inflation-scares and Forecast-based Monetary Policy." Review of Economic Dynamics 8, 498-527.

Orphanides, A. and J.C. Williams. (2005b.) "Imperfect Knowledge, Inflation Expectations, and Monetary Policy." In The Inflation-Targeting Debate, edited by Ben S. Bernanke and Michael Woodford, 201-246. University of Chicago Press.

Primiceri, Giorgio. (2005). "Time Varying Structural VAR and Monetary Policy." Review of Economic Studies 72, 453-472.

Roberts, John M. (1997). "Is Inflation Sticky." Journal of Monetary Economics 39, 173-196.

Roberts, John M. (1998). “Inflation Expectations and the Transmission of Monetary Policy.” Finance and Economics Discussion Series, 43, Board of Governors of the Federal Reserve System.

Sargent, Thomas. (1999). The Conquest of American Inflation. Princeton University Press.

Sargent, Thomas, Noah Williams and Tao Zha. (2006). “Shocks and Government Beliefs: The Rise and Fall of American Inflation.” American Economic Review 96, 1193-1224.

Sbordone, Argia. (2007). “Inflation persistence: Alternative interpretations and policy implications.” Journal of Monetary Economics 54, 1311–1339.

Schmitt-Grohe, Stephanie and Martin Uribe. (2004). “Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function.” Journal of Economic Dynamics and Control 28, 755-775.

Sims, Christopher. (2002). "Solving Rational Expectations Models." Computational Economics 20, 1-20.

Slobodyan, Sergey and Rafael Wouters. (2009a). “Learning in an estimated medium-scale DSGE model.” CERGE-EI Working Papers WP396.

Slobodyan, Sergey and Rafael Wouters. (2009b). “Estimating a medium-scale DSGE model with expectations based on small forecasting models.” Mimeo.

Smets, Frank and Rafael Wouters. (2007). “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach.” The American Economic Review 97, 586-606.

Sims, Christopher and Tao Zha. (2006). “Were there Regime Switches in US Monetary Policy?” American Economic Review 96(1), 54-81.

Stock, James and Mark Watson. (2003). “Has the Business Cycle Changed and Why?” NBER Macroeconomics Annual 2002.

Uhlig, Harald. (1999). “A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily.” In Computational Methods for the Study of Dynamic Economies, edited by Ramon Marimon and Andrew Scott, 30-61. Oxford University Press.

Xu, Shaofeng. (2010). “Idiosyncratic Income Risk and Housing Tenure Decision.” Unpublished manuscript, University of California at Davis.