Three Essays on Imperfect Competition

A thesis presented

by

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To Aura

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Contents

Introduction

This thesis proposes three theoretical models underlining the importance of the business-stealing effects that arise when firms implement different strategies in imperfectly competitive markets. Firstly, we deal with secondary brands that are used by firms to steal from their rivals, customers from the low-end of the market. Secondly, we show that entry may be socially desirable when associated with a positive business-stealing effect - namely, when the market share stolen from the incumbent is replaced by a more efficient outcome. Thirdly and finally, we prove that loyalty rewarding pricing schemes are essentially business-stealing devices, and hence enhance competition. In the first two chapters of the thesis we use oligopoly models, while in the last one, competition is strengthen and the strategic effect disappears, by using a model of monopolistic competition.

The first chapter of the thesis presents a formal model that explains some of the reasons that lie behind firms' decisions to produce lower profile cheaper products called "secondary brands". Examples of secondary brands are private labels and fighting brands. In the first version of the model, when the rival in the horizontal space sells a high-quality product, we show that, for certain values of the parameters, production of secondary brands by intermediate size firms emerges as a non-cooperative equilibrium outcome. The very purpose of the secondary brands here is to attack the brand leader. These predictions are compatible with the empirical observations regarding manufacturers of private labels. In the second version of the model, with a low-quality rival, even market leaders may produce secondary brands but, this time, with a very different scope: to protect themselves from competition in the low-end of the market. This is in line with the scope of the fighting brands.

The second chapter provides a theoretical model that studies under what circumstances public intervention to encourage entry is desirable. Previous literature points to a general tendency for excessive entry in homogeneous product markets. This result is supported by the assumption that firms are symmetric or incumbents have some advantages. We claim that this may not be necessarily true. According to the World Bank, potential entrants that emerged in newly privatized markets from Eastern Europe were more efficient than the old incumbents. We propose to add to the theoretical literature on social efficiency of entry, by relaxing the symmetry assumption between incumbents and potential entrants and even allowing for certain advantages of the entrants. Under these conditions, previous results in the literature may not hold and moreover, may be reversed. We provide both models of quantity and price competition and also allow for foreign investors.

In the last chapter of the thesis, a joint research with Ramon Caminal, we present a version of the standard Hotelling model in order to analyze the effects of loyalty rewarding pricing schemes, like frequent flyer programs, on market performance. The main result is that these programs enhance competition (lower average prices and higher consumer surplus), even when firms cannot observe the history of purchases of newcomers. We also show that the form of commitment (coupons versus price level) is to some extent irrelevant, and that the incentives to introduce these programs decrease with the presence of exogenous switching costs. Various other issues are also discussed.

Each chapter is self-contained and suitable for independent reading.

Chapter 1
Two-Dimensional Competition in Asymmetric Duopolies: An Application to Secondary Brands

Producers of well-known brand names often decide to introduce vertical differentiation by launching lower profile cheaper products called ”secondary brands”. In imperfectly competitive markets, firms use secondary brands either to attack the rivals or to protect themselves from competition. This chapter presents a formal model that explains some of the reasons that lie behind firms’ decisions to adopt such practices.

1.1 Introduction

We start to motivate the discussion by mentioning a number of markets in which the phenomenon under consideration may be observed. Firstly, it is the case of some manufacturers producing secondary brands that will be further sold under retailers’ private labels. According to the Private Label Manufacturers’ Association (PLMA), ”Private label products encompass all merchandise sold under a retailer’s brand. That brand can be the retailer’s own name or a name created exclusively for the retailer”. Private labels are often called store brands, retailers’ brands or distributors’ brands. Over the last decade, private labels policies were extremely successful. In 2000, market share of store brands in the retailing sector reached 45% in volume and 43% in value in the U.K., 33% in volume and

27% in value in Germany, and 20% in volume and 14% in value in Spain[1]. Private labels leaders are Aldi, with 95% of its sales belonging to this category, Lidl (80%), Sainsbury (60%), Tesco's (40%) and Carrefour (30%)[2]. Manufacturers of store brands fall into three major classifications. Firstly, there are large national brand manufacturers that utilize their expertise to supply store brands. Secondly, some major retailers have their own manufacturing facilities and provide store brand products for themselves. Thirdly, there exists small manufacturers who specialize in particular product lines and concentrate on producing store brands almost exclusively[3]. We focus on the first category and notice that, while some national brands manufacturers refuse to supply retailers' brands, for others private labels supply is a profitable strategy. Dunne and Narasimhan (1999) and Quelch and Harding (1996) describe the following cases:

> ”In the 1980 Coke and Pepsi refused to supply Canadian grocers with a private label soft-drink. Eventually, the grocers found another source, Cott Corporation, which had started out as a regional brand but then redirected its strategies around retailers' needs. Cott became a considerable threat to the two big cola makers. Cott gained 30% of the Canadian retail soft-drink market by 1993, and it began to push into the U.S. and Europe. In 1994, Cott Corporation launched Classic Cola, a private label made for Sainsbury supermarkets in the United Kingdom.”

[1] Berges-Sennou, Bontems and Requillart (2004)

[2] Source: http://retailindustry.about.com/library/uc/02/uc_stanley4.htm

[3] Berges-Sennou (2002) and Dunne (1999) examine the choice among different manufacturers to supply private labels.

> ”Unilever’s Canadian subsidiary, Lever Ponds, welcomes private-labels agreements as a chance to take sales from the market leader in laundry detergents, Procter & Gamble.”

> ”Although Agfa was a household name in European film markets, it had only 2% of the market in North America. Kodak and Fuji dominated the category. While struggling to boost its market share, Agfa noticed that the rise of photofinishing outlets in supermarkets and drugstores was making retailers eager for good manufacturers of private-label film. Recognizing an opportunity to crack the market, Agfa Canada made a strategic commitment to serve retailers with private labels.”

> ”Heinz starts off by distinguishing between its core product, ketchup, and other products such as tomato sauces, soups and baked beans. Heinz produces private labels as well as branded versions of these other products, but it offers ketchup only under the Heinz name.”

Another example from European markets can be found on the French diary industry. The French multinational Danone is the world leader of fresh dairy products. But, in France, most retailer’s brands are produced by Senoble[4] - the third manufacturer of dairy goods on the French national market[5].

In all of these cases, it seems that private labels policies are adopted by manufacturers that are not brand leaders in their respective markets. In fact, Dunne and Narasimhan

[4] Source: http://www.senoble.fr

[5] Senoble also produces private labels for Spanish supermarket chains like, for example, Mercadona.

(1999) note: ”If your brand’s market share is relatively low or it runs second to the leading brand, you may be able to attack the market leader with a private label and gain market share”. This article proposes a formal game theoretical model that provides predictions compatible to these empirical observations. We show that, for certain values of the parameters, production of private labels by intermediate size firms emerges as a non-cooperative equilibrium outcome.

In general, private labels are perceived as similar to well-known brands but, of a lower quality and price[6]. In an empirical study, Sethuraman and Cole (1999) show that perceived quality differential is the most important variable influencing price premiums that consumers pay for national brands over store brands in grocery products. This difference in quality accounts for about 12 percent of the variation in price premiums across consumers and product categories[7].

The second example we thought of, when referring to secondary brands, is the case of fighting brands. This terminology is especially used in the management literature and refers to the situation when incumbents, in response to competition, expand their product lines, often including a lower-quality good. Johnson and Myatt (2003) provide interesting examples of fighting brands:

> ”Consider for instance the IBM Laser-Printer. A single version was initially

[6] A study undertaken by John Stanley Associates shows that private labels are up to 18% cheaper in Europe and up to 25% cheaper in the US than the brand leader.

[7] Today the gap in the level of quality between private label and brand name products has narrowed, especially due to the developement of ”Premium private labels” (Quelch and Harding (1996)). The biggest success story in premium private labels is the President’s Choice Decadent Chocolate Chip cookie (developed by Loblaw’s Canada), a high-quality item (with lots of butter and chips) that has become the best-selling cookie in many markets where it has been introduced (Hoch (1996)).

> sold, capable of printing ten pages per minute. The absence of a lower quality version suggests that IBM's gains from serving the low end of the market were not large enough to justify introducing a substitute product for its high-quality unit (which would have limited IBM's ability to extract surplus from high-value users). However, following Hewlett-Packard's entry into the market with its LaserJet IIP, a lower quality substitute for IBM's LaserPrinter, IBM needed to reevaluate its product line strategy [...]. In fact, IBM decided to introduce a fighting brand, the LaserPrinter E which was identical to its original LaserPrinter except for the fact that its software limited its printing to five rather than ten pages per minute."

> "In early 1991 Intel released the 486SX microprocessor. This chip was a modified version of the earlier 486, subsequently renamed 486DX. The sole difference was the omission of an internal floating point mathematics coprocessor, yielding an initial pricing of $258 relative to the 486DX price of $588. Interestingly, the industry literature recognized that the 486SX was a damaged version of the 486DX. [...] Apparently, the release of the 486SX followed the entry of Advanced Micro Devices (AMD) into the microprocessors market. The 486SX was therefore a fighting brand."

Finally, another case study comes from the airlines market where, the U.K. carrier, British Airways, introduced a fighting brand, Go, possibly in response to the incorporation of the Easy-Jet airlines - a fierce competitor in the low-cost segment of the market. The inferior quality of the fighting brands in the first two cases is obvious (a lower printing speed or a lower data processing speed due to a missing component)[8]. In the case of airlines, we may find the quality differential between the main carrier and the low-cost company in the ticket cancellation or changing conditions, restricted schedules, extra charges for weighty luggage or the lack of catering.

It is worth noting that, in general, the fighting brands producers are market leaders[9] and they use this strategy to protect their market share from competition. This is very different from the cases of private labels we described above where medium-size manufacturers use secondary brands to attack the brand leaders. Nevertheless, in all the examples presented so far (regarding both private labels and fighting brands), secondary brands are lower profile products manufactured by a brand name. This requires a model of vertical differentiation with two different qualities: a high one, for the products of the main brand and a low one, respectively, for the secondary brand products. However, competition in the horizontal space may come from different types of rivals. In the case of private labels manufacturers, in general, competition comes from the high-end of the market while in the case of fighting brands producers, rivals belong to the low-end of the market. On the purpose of covering a wide spectrum of real-life cases and answering a general question, we propose two versions of a model combining horizontal and vertical differentiation: in one case, the

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[8] In fact, Deneckere and McAfee (1996) who were the first to asses these study cases, called the fighting brands - "damaged goods". They brought a lot of evidence showing that manufacturers may intentionally damage a portion of their goods in order to price discriminate.

[9] See, for example, the press release of 6th of May 2005 at *http://press-releases.techwhack.com/415* entitled: "IBM Captures Number One Spot in Key High Speed Printer Segment". This article highlights that "IBM today announced that InfoTrends/CAP Ventures ranked IBM number one in the U.S. for 2004 in high-speed, black & white roll-fed printers".

secondary brand producer (who creates vertical differentiation) competes in the horizontal space with a high-quality rival and in the other, with a low-quality one.

The aim of this chapter is to study the incentives of a firm to produce secondary brands (hereinafter denoted SB) in circumstances as different as the ones described above. When the firm is a monopolist, the introduction of a SB brings in two opposite effects on profits. On the one hand, there is a positive effect due to price discrimination. It has two consequences. When the market is initially covered, the price discrimination effect makes rich consumers to pay a quality premium. Moreover, when the market is only partly covered, we deal with a demand expansion as some consumers, which were initially priced out of the market, can now afford to buy the cheaper alternative. On the other hand, there is a negative effect produced by an erosion on the firm's profits resulting from its own entry. This happens as some consumers, that used to buy the main brand, switch to the SB, which is cheaper. We find that, in the absence of costs of any kind, the first effect dominates and the monopolist will always produce the SB. In the case of duopoly, the introduction of a SB may also increase sales by stealing customers from a competitor that does not price discriminate. Hence, as compared to the monopoly case, the business-stealing effect gives new incentives for the introduction of a SB. In fact, we prove that, for some range of parameters, it may be efficient to launch a SB under duopoly even if, it is not efficient to do so under monopoly[10].

[10] In general, monopoly and duopoly are two different markets structures with little room for comparisons between themselves. Nevertheless, we may think of the international trade perspective and imagine monopoly as an autarchy and duopoly as a situation of free-trade between the two firms, potentially located in different countries.

For the duopoly case, we use a variation of Hotelling's (1929) spatial model where, in each of two cities located at a certain distance from each other, there is only one firm, each producing a brand good. There is horizontal differentiation between the two firms, which may be due to either geographical distance (which requires some transportation costs), or various switching costs buyers must incur when deciding to change the brand they are used to. Consumers live within the cities, so forming a population distribution consisting of two atoms. A central ingredient of the paper is the asymmetry in the population size between the cities. Consumers are heterogeneous both in the taste for quality and in the transportation cost. In order to price discriminate, only one of the firms decides to produce a secondary brand with the same horizontal characteristics as the initial brand but, of lower quality; take, for example, a good with a less sophisticated packaging sold under a different brand[11]. The rival firm produces a single brand of either high or low quality. This makes the difference between the two versions of the model we propose. Given this setting, the article provides the following results.

In the first version of the model, when the rival has a high quality - like in the case of private labels manufacturers - we obtain two types of equilibria. On the one hand, when the firm located in the smaller city is the one that potentially produces the SB, it will export to the other city either the main brand or the SB. This depends on the interaction between two

[11] Nowadays, the use of very attractive packaging to stimulate customers attention may be even more expensive than the goods themselves; therefore, giving up those sophisticated bags and fancy labels may bring huge cost savings to firms, which may be translated into important price lowerings. A good example of savings in packaging design is Caprabo's new store brand - AlCosto. All goods sold under this brand have the same design, which proves the choice of the company not to invest in expensive advertising ideas to create different designs for different products. Here, even though goods are mainly similar to those of most famous brands they may be seen to have a lower quality because of the simple fact that a customer can hardly differentiate whether he buys detergent, champoo or mineral water without a more careful inspection of the labels, which takes more time.

factors: price-discrimination and business-stealing. Price-discrimination increases profits from selling to own customers while business-stealing increases profits from selling to the rival. If the potential customer base (population in the city) of the SB producer is arbitrarily small, the business-stealing effect has a relatively high influence and the firm prefers to export the brand (and not the SB), which is more expensive, and hence, increase profits from selling to the rival. Conversely, if the potential customer base of the SB producer is not very small, but still smaller than the rival's (medium-size city) the firm should manufacture and partly export the SB. This is because now, business-stealing effect has a relatively lower influence on total profits and hence, price discrimination dominates. On the other hand, when the firm that potentially produces the SB is located in the big city, the model provides no equilibria in pure strategies.

In the second version of the model, when the rival has a low quality - like in the case of fighting brands - we obtain, apart from the two results of the previous case that still hold, a new equilibrium that characterizes the case when the firm producing the SB is located in the big city. Our model suggests that, in this case, SB should be produced in order to protect the firm from the competition in the low-end of the market. Here price discrimination is just an alternative to prevent some customers to buy from the rival.

Finally, a common element of the two versions of the model is that the potential customer base (the size of the city) determines which firm exports to/imports from the competitor. As in the paper of Garella and Martinez-Giralt (1989), always the firm located in the small city exports to the other one.

The remaining of this chapter is organized as follows. Section 1.2 briefly reviews the related literature. Section 1.3 deals with the benchmark case of monopoly and compares it with the social optimum. In section 1.4 we solve the main model with two firms and a high-quality rival and, consequently, find the equilibrium outcomes for different values of the parameters. Section 1.5 identifies the cases when incentives to launch the SB are higher in duopoly than in monopoly. Section 1.6 deals with some interesting cost considerations. Section 1.7 presents the second version of the model, with a low-quality rival. Section 1.8 gives some concluding remarks and is followed by the bibliography list. This chapter ends with an appendix.

1.2 Related literature

Our work has common elements with three strands in the economic literature. The first one is concerned with two-dimensional differentiation, the second one with multiproduct firms and finally, the third one is dealing with particular types of secondary brands.

The two-dimensional differentiation model used in this paper has been inspired from Garella and Martinez-Giralt (1989). We use the atom-distribution of population proposed in their model, but deal with multiproduct duopoly, while they consider single-product firms. Their result, claiming that small firms export to big ones, is valid in our model too. Nevertheless, we extend the analysis by introducing a new good of a different quality, which brings many new insights to the issue considered. A seminal paper on two-dimensional differentiation is the one of Neven and Thisse (1990), in which two single-product firms choose both characteristics: location and quality for their goods[12]. Our article is different from theirs in that location and quality are exogenous and, one firm may choose whether or not to launch a second good of lower quality[13]. Gilbert and Matutes (1993) consider a model of two horizontally differentiated firms, each producing two goods of different qualities. We propose a similar set-up, except for the asymmetry issue (in our model only one firm produces two goods). Nevertheless, our focus is completely different from theirs. They show that specialized production may be an equilibrium if firms can make ex-ante commitments to limit their production offerings, and if they can communicate this commitment to rivals. Finally, Degryse (1996) analyses a specific aspect of two dimensional-differentiation applied to banking. Namely, he assumes that the introduction of vertical differentiation between banks (the remote access) negatively affects the degree of horizontal differentiation. Moreover, they consider a symmetric model while for us, it is the asymmetry in market size that determines the existence or non-existence of different equilibria.

Regarding multiproduct firms, the classic contribution to monopoly pricing involving a quality-differentiated spectrum of goods of the same generic type is the one of Mussa and Rosen (1978). In their model, the seller offers a price-quality schedule and consumers are allocated to different qualities by a self-selection process. They find that, the monopolist sells a lower quality to all consumers, except for the richest one, as compared with what would be purchased in the social optimum. In our model, there are only two qualities (the

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[12] Dos Santos Ferreira and Thisse (1996) also address the issue of horizontal and vertical differentiation using the mathematical Launhardt model. This completly lacks of common elements with our model.

[13] Shaked and Sutton (1982) also allow firms to precommit to their quality levels, prior to the simultaneous choice of prices. Nevertheless, firms are again restricted to a single product and hence they cannot address the issue of product ranges we consider.

main brand and the SB) which are exogenously given. In the monopoly case, results go into the same direction: the social optimum requires more goods of high quality than are sold by the monopolist. The step of extending Mussa and Rosen (1978) to oligopoly has been taken by Champsaur and Rochet (1989). They consider two firms that commit to producing in chosen intervals of quality before competing in prices, and find that firms choose to offer nonoverlapping product lines, as this reduces the intensity of price competition. Hence, the product line offered by a given firm need not match the product line offered by a monopolist capable of offering the entire range of goods. Our set-up is similar to theirs, in that we explore a multiproduct firm in duopoly, but our focus is different. We introduce horizontal differentiation and condition the supply of the entire product line to the size of the potential market.

Finally, we shall mention some important references related to specific cases of secondary brands. Under this common denominator, our article managed to gather in the same pool two very different lines in industrial organization: private labels and fighting brands. Previous theoretical literature on private labels was mainly concerned with the analysis of vertical relationships between producers and retailers[14]. A central paper by Mills (1995) presented a model of retailer-manufacturer interaction in a vertical structure with retail competition between the manufacturer's major brand and the retailer's private label, substitute for the brand. The main arguments of this paper are that private label marketing is an instrument the retailer uses to capture part of the profit locked away by double marginaliza-

[14] Empirical research on private labels has mainly studied the factors explaining private labels market shares across product categories and/or retail chains (Dahr and Hoch, 1997; Hoch and Banerji, 1993; Raju, Sethuraman and Dhar, 1995). Hoch (1996) - focusing on the American food retailing industry, and Verhoef et al. (2002) - using data from Netherlands, discuss strategic options of brand manufacturers to respond to private labels.

tion, and that private labels improve the overall performance of distribution channels[15]. Our paper is completely different from this stream of literature, in that we are concerned with the strategic interaction between manufacturers. Therefore we do not model the vertical relationships between manufacturer and retailer, but rather assume a perfectly integrated vertical structure and deal with the issue of private labels from the point of view of the manufacturer. Moner-Colonques et al. (2004) consider the retailer's side of the story and examine the strategic reasons behind retailers' decisions whether to introduce a private label product when shelf space is scarce. Their key question is to identify what conditions make it profitable for retailers to replace a national brand with a private label. Their analysis is based on demand parameters that measure the cross-effects across brand types. Finally, Wolinsky (1987) argues that firms may sell both labeled and unlabeled products because consumers are imperfectly informed of producer's identity unless they see the label. Hence, buyers who strongly prefer a particular brand may be willing to pay a higher price for a labeled brand, while others may rather buy a cheaper unlabeled brand whose identity is uncertain. In his model there is no vertical differentiation (as both products have the same quality). Obviously, the last two papers we mentioned are different from ours both in the focus of the question and in the structure of the model.

The approach of fighting brands was extensively studied by Deneckere and McAfee (1996) followed by Johnson and Myatt (2003). The former authors showed that monopolists may intentionally damage a portion of their goods in order to price discriminate, as this may result in a Pareto improvement. The huge amount of evidence on ”damaged

[15] For more recent papers on vertical relationships see: Mills (1999), Bontems et al. (1999), Comanor and Rey (2000) and Gabrielsen and Sorgard (2001).

goods" brought by Deneckere and McAfee was consequently used by the latter authors to motivate their work that explores multiproduct quality competition. Johnson and Myatt compare cases when incumbent firms respond to entry by expanding their product line (fighting brands) with situations when products are removed from the market in face of entry (product line pruning). They only allow for competition from the low end of the market and show that, the incumbent will never choose to offer products that are of quality inferior to that of the entrants. The general intuition in their model is that, for a decreasing marginal revenue curve (which is the case in basic economics text-books), entry will drive the incumbent firm to contract its output (as reactions functions in Cournot competition have negative slopes). The incumbent will prefer to remove its products from the low-end of the market. This is because, in this model there is no horizontal differentiation and then, two goods of the same quality are perfect substitutes. Therefore, firms do not like competition in the same quality range and incumbents prefer to remain only with monopoly margins for high-quality goods. The case of producing fighting brands arises as an equilibrium in their model only for very special cases of demand structures that lead to increasing marginal revenue curves. Our contribution to this literature is threefold. Firstly, we show that, even when the entrant has a high-quality product, similar to that of the incumbent (like in the case of private labels manufacturers), it may result optimal for the incumbent to produce a SB of relatively lower quality, in order to attack the rival. The very purpose of the SB in this case, is very different from the one of fighting brands described by Johnson and Myatt, where they act as a defensive strategy against low-quality competition. Secondly, we introduce horizontal differentiation[16], which will make possible direct competition between similar quality goods. And thirdly, assuming asymmetry between the potential market size of the two firms will take to original results regarding the production of SB by different types of firms.

1.3 Benchmark: the monopoly case

Before presenting, in the main model, the incentives to produce a SB in an oligopolistic setting, we will start with the benchmark case of monopoly. We first compute the market equilibrium and then show its inefficiencies as compared to the social optimum. This preliminary section uses a model of a price-discriminating monopolist able to offer a range of products of different qualities, in the spirit of Mussa and Rosen (1978).

1.3.1 Market equilibrium

We study the optimal product choice of a monopolist that potentially sells a brand good of quality $q > 0$, and a SB of quality 0. In what follows, subscripts a and 0 refer to the main brand and the SB, respectively. We denote by p_a and p_0 the prices of the two goods. For the moment, fixed and variable costs are normalized to zero, for simplicity. Later in this chapter, more precisely in section 1.6, we come back to this assumption and analyse the potentially interesting issues regarding the introduction of different types of costs in our model.

[16] Horizontal differentiation may be due to: location (manufacturers sell from different points or airlines companies have the hub airports in different cities), preference for certain brand or the need to stick to a specific brand due to the existence of switching costs (it costs time to change to a different software for example).

Consumers differ in their taste for quality, θ, which is a random variable uniformly distributed in the interval [0,1]. The type of every consumer is private information, therefore first degree price discrimination is ruled out. The monopolist only knows the distribution of types. The willingness-to-pay for a good of quality zero is denoted by R and is the same for everybody.

Consumers enjoy the following utilities when purchasing the SB and the brand a, respectively:

$$\begin{aligned} U_0 &= R - p_0 \\ U_a &= R + \theta q - p_a \end{aligned}$$

If neither good is achieved utility is equal to zero.

The SB is preferred to brand a, if θ is less than or equal to the threshold value θ^*, given by: $\theta^* = \frac{p_a - p_0}{q}$. The following table shows consumers' choice between brand a and SB:

Table 1.1: Consumers' choice between brand a and SB

$\theta^* < 0$ ($p_0 > p_a$)	All consumers prefer brand a
$0 \leq \theta^* \leq 1$ ($p_a - q \leq p_0 \leq p_a$)	Consumers with $\theta \leq \theta^*$ prefer the SB
$\theta^* > 1$ ($p_0 < p_a - q$)	All consumers prefer the SB

Then, the problem of the monopolist may be written as follows:

$$\begin{aligned} \underset{p_0, p_a}{Max}\, \pi &= p_0\theta^* + p_a(1-\theta^*) = p_a - \frac{(p_a - p_0)^2}{q} \\ & s.t. \begin{cases} p_0 \leq R \\ \theta^* \in [0,1] \end{cases} \end{aligned}$$

Some comments are needed on the two restrictions of the optimization problem. The monopolist is indifferent between any $p_0 > R$ and $p_0 = R$ (as demand for the SB is zero in both cases). Hence, without loss of generality we may assume $p_0 \leq R$. The second restriction is equivalent to $p_0 \in [p_a - q, p_a]$. This is sufficient as now, the monopolist is indifferent between any $p_0 < p_a - q$ and $p_0 = p_a - q$ (as $\theta^* = 1$) and between any $p_0 > p_a$ and $p_0 = p_a$ (as $\theta^* = 0$).

In the intervals considered, the objective function is increasing in the price of the SB, which takes us to a corner solution:

$$p_0 = R$$

Substituting this value into the objective function we obtain a one-variable restricted optimization:

$$\begin{aligned} \underset{p_a}{Max}\,\pi &= p_a - \frac{(p_a - R)^2}{q} \\ & s.t.\; p_a \in [R, R+q] \end{aligned}$$

It has an interior solution:

$$p_a = R + \frac{q}{2}$$

With these prices demand is equally shared between the two goods ($\theta^* = \frac{1}{2}$).

The following proposition summarizes the results:

Proposition 1 *The prices set by the monopolist are:* $p_0 = R$ *and* $p_a = R + \frac{q}{2}$. *Consumers with* $\theta \in [0, \frac{1}{2}]$ *buy the secondary brand, while those with* $\theta \in [\frac{1}{2}, 1]$ *buy the main brand. Profits obtained by the monopolist are:* $\pi = R + \frac{q}{4}$.

Intuitively, proposition 1 says that the monopolist will always introduce the SB. This is so, as the positive effect of price discrimination dominates the negative effect of profit erosion, as described in the introduction. Let us measure the two effects in the case of our model. In order to do so, we need to compute the equilibrium outcome in the situation where no SB is sold in the market and compare it with the result of proposition 1.

If the monopolist produces only the high quality brand, he faces the following demand:

$$D = \begin{cases} 0, \text{ if } p_a > R + q \\ 1 - \frac{p_a - R}{q}, \text{ if } R + q \geq p_a > R \\ 1, \text{ if } p_a \leq R \end{cases}$$

Given this demand function, the optimal price is set at:

$$p_a = \begin{cases} \frac{R+q}{2}, \text{if } q > R \\ R, otherwise \end{cases} \rightarrow D = \begin{cases} \frac{1}{2} + \frac{R}{2q}, \text{if } q > R \\ 1, otherwise \end{cases}$$

Notice that, in this case, if $q > R$ only part of the demand is served (only rich consumers can afford to buy the brand), while the rest remains with nothing.

The following table shows total and partial profits[17] in both cases: with and without SB.

[17] By partial profits we mean that part of the profits obtained from selling only one of the two goods.

Table 1.2: Monopolist's profits

	Brand a	SB	Total
With SB	$\frac{R}{2}+\frac{q}{4}$	$\frac{R}{2}$	$R+\frac{q}{4}$
No SB	$\begin{cases} \frac{(R+q)^2}{4q}, \text{if } q>R \\ R, \text{if } q<R \end{cases}$	-	$\begin{cases} \frac{(R+q)^2}{4q}, \text{if } q>R \\ R, \text{if } q<R \end{cases}$

A simple comparison gives us a measure of the two effects created by the introduction of the SB. On the one hand, price discrimination has a positive effect on profits for two reasons. Firstly, the price of the brand increases when two goods are offered (rich consumers will have to pay a quality premium) and secondly, overall demand increases as well (new consumers can afford to buy when there is an alternative on the market). On the other hand, some consumers that used to buy the high-quality brand, will switch to the SB, which is cheaper. This creates a negative effect, of erosion on profits due to firm's own entry. Notice that, demand for the brand and profits from selling the brand decrease in the case with SB as compared with the case without SB. Nevertheless, the positive effect dominates and total profits increase with the launch of a SB.

1.3.2 Welfare analysis

First Best

In the first best case, the social planner seeks to maximize total surplus. As there are no costs in our model, total surplus equals gross consumer surplus, which attains its maximum when all consumers purchase the high quality good. Therefore, we can say that the equilibrium is not efficient as compared to the first best. The monopolist sells too little of the quality good. Our model, then, is an example of underprovision of quality, like in Mussa and Rosen.

Second Best

If the social planner cannot allocate resources as in the first best case, but may dispose of a second best intervention tool such as the right to prohibit the SBs, situation changes as follows.

If the production of SBs is allowed, total surplus is:

$$TS^{allow} = \int_0^{\frac{1}{2}} Rd\theta + \int_{\frac{1}{2}}^1 (R + q\theta)d\theta = R + \frac{3}{8}q$$

Alternatively, if the SBs is prohibited, total surplus is now:

$$TS^{prohibit} = \begin{cases} \int_{1-(\frac{1}{2}+\frac{R}{2q})}^1 (R + q\theta)d\theta = R(\frac{3}{4} + \frac{3R}{8q}) + \frac{3}{8}q, \text{ if } q > R \\ \int_0^1 (R + q\theta)d\theta = R + \frac{q}{2}, \text{ if } q < R \end{cases}$$

Summing up, we can state that:

- If $R < \frac{2}{3}q \rightarrow TS^{prohibit} < TS^{allow}$
- If $R > \frac{2}{3}q \rightarrow TS^{prohibit} > TS^{allow}$

The SBs should be allowed when the quality differential is relatively high, which is the same as to say that consumers are relatively poor (their reservation price, R, is lower). Then, market equilibrium is second best socially efficient for $R < \frac{2}{3}q$.

Hence, as compared to the first best the efficiency of the equilibrium is ambiguous. Prohibition of SBs brings in two opposite effects. On the one hand this is good for welfare as the demand for the high-quality brand increases. On the other hand this is bad for welfare, as overall demand is lowered, as some poor consumers are left out of the market. If R is relatively high, the first (positive) effect dominates and, therefore, SBs should be prohibited[18]. Otherwise, production of SBs should be allowed.

1.4 The duopoly case

In order to represent the strategic interaction between two horizontally differentiated manufacturers, we shall consider the following variation of Hotelling's (1929) spatial model that was used in Garella and Martinez-Giralt (1989).

1.4.1 The model

Two cities, A and B, are located at a distance L from each other. In each of the cities there is only one firm, called A and B, respectively. Each firm produces a brand good (brand a and brand b, respectively) of the same quality, $q > 0$. Consumers are concentrated in the two cities, so forming a population distribution consisting of two atoms[19]. We denote by S the size of city A and normalize to 1 the size of city B. In each atom, consumers are differentiated according to their taste for quality, θ, which is a random variable uniformly distributed in the interval $[0, 1]$. Reservation price is R and transportation cost per unit

[18] Notice that, in the particular case $R > q$ (market is completely covered when only the brand is offered), the fact that prohibition is optimal is immediate, as the second effect (demand reduction) does not exist.

[19] The reason for departing from the standard Hotelling model in what concerns consumers horizontal distribution is that our approach better suits the purpose of this paper. We mainly look for those customers in city B that buy the products of firm A, reason for which it is better and sufficient to consider two atoms. A uniform distribution of the population in the horizontal space would unnecessarily complicate the model (demands would be areas instead of segments) without bringing new insights into the results.

of distance, θ, for all consumers. The fact that transportation cost is proportional to the taste for quality is reasonable if we consider that the opportunity cost of travelling time is higher for rich people (those who value more the quality). An alternative interpretation of horizontal differentiation is the existence of some switching costs that consumers have to incur if they move from their most preferred brand.

In order to benefit from the ”vertical distribution” of consumers, only one of the firms, let it be firm A, for example, is considering to supply a secondary brand - which is a product similar to the main brand but, of a lower quality - which we normalize to zero.

Finally, we assume constant marginal costs, equal to zero, for the production of all goods (the two brands and the SB). We do not differentiate variable costs according to quality because we want to capture the optimality of a low quality good due to price discrimination and strategic interaction. Furthermore, we will assume that the only costs involved in the launch of the main brand, but not in that of the SB, are the fixed costs (like, for example, advertising expenditures), which, in a static model, are sunk.

In what follows, we will write the utilities derived by consumers from purchasing every of the goods. We denote by $U(i,j)$, the utility of consuming good j, with price p_j, when living in city i ($i =$A,B and $j = 0$ for the secondary brand, $j = a$ for brand a, and $j = b$ for brand b).Then,

Consumers in A have*:*

$$
\begin{aligned}
U(A,0) &= R - p_0 \\
U(A,a) &= R + \theta q - p_a \\
U(A,b) &= R + \theta q - p_b - \theta L
\end{aligned}
$$

And, consumers in B have:

$$
\begin{aligned}
U(B,0) &= R - p_0 - \theta L \\
U(B,a) &= R + \theta q - p_a - \theta L \\
U(B,b) &= R + \theta q - p_b
\end{aligned}
$$

For computational purposes, in what follows we have to focus the analysis to either $q < L$ or $q > L$[20]. We will assume $q < L$. This seems more reasonable if we take into account that, in the extreme case of very low L, brands would be almost substitutes, while we are interested in horizontally differentiated firms. Nevertheless, the opposite case, $q > L$, gives similar results[21].

The following tables show consumers' choices between pairs of goods in both cities:

Table 1.3: Consumers' choice between brand a and *brand* b

	CITY A	**CITY B**
All consumers prefer brand b if	$p_a > p_b + L$	$p_a > p_b$
All consumers prefer brand a if	$p_a < p_b$	$p_a < p_b - L$
Consumers prefer brand a only if	$\theta \geq \theta^a = \frac{p_a - p_b}{L}$	$\theta \leq \theta^b = \frac{p_b - p_a}{L}$

[20] This is because, the difference $L - q$ determines the sign of θ^1, defined in Table 4.

[21] Computational details are available upon request.

Table 1.4: Consumers' choice between brand b and the SB

	CITY A	**CITY B**
All consumers prefer brand b if	$p_0 > p_b + L - q$	$p_0 > p_b$
All consumers prefer SB if	$p_0 < p_b$	$p_0 < p_b - L - q$
Consumers prefer SB only if	$\theta \geq \theta^1 = \frac{p_0 - p_b}{L-q}$	$\theta \leq \theta^2 = \frac{p_b - p_0}{L+q}$

The choice between brand a and the SB is the same in both cities and has been described in *Table 1.1*, in the monopoly case. Let us now define the demands for the three goods given the prices of the other goods. For the simplicity of presentation, we consider pairs of prices and then take all possible combinations among them. Moreover, we assume that markets are completely covered. This means that, in equilibrium, consumers can afford to buy the good they prefer. The condition on R ensuring this, is[22]: $R \geq \frac{(L+q)(2+s)}{3}$.

If SB were not sold in the market, the demand of brand a, given p_b and, alternatively, the demand of brand b, given p_a, are defined taking into account the information in *Table 1.3*. If the price of brand a were $p_b + L$ the consumer in city A facing the highest transport cost is indifferent between buying either brand. Hence, if $p_a > p_b + L$ brand a gets no demand at all. As p_a decreases, firm A starts to serve brand a to consumers in city A (those with $\theta \geq \theta^a$) until $p_a = p_b$ when all consumers in city A are captured. If the price of brand a continues to decrease, consumers from city B with low transport cost will start buying the brand a (those with $\theta \leq \theta^b$). Finally, if p_a keeps on decreasing below $p_b - L$, brand a will be preferred to brand b by the whole market. Summarizing, we can write the demand for brand a, D_a, and the corresponding demand for brand b, D_b, as follows:

[22] A sufficient condition to have markets completly covered is to impose the lower bound on R be the maximum equilibrium price: see Appendix 1.A.1.

$$D_a = \begin{cases} S+1, & p_a < p_b - L \\ S+\theta^b, & p_a \in [p_b - L, p_b] \\ S(1-\theta^a), & p_a \in [p_b, p_b + L] \\ 0, & p_a > p_b + L \end{cases} \qquad D_b = \begin{cases} 0, & p_b > p_a + L \\ 1-\theta^b, & p_b \in [p_a, p_a + L] \\ S\theta^a + 1, & p_b \in [p_a - L, p_a] \\ S+1, & p_b < p_a - L \end{cases}$$

Notice that, for any pair of prices, demands must sum to $S+1$, which is the whole population of the two cites.

In equilibrium, firm A would never choose $p_a < p_b - L$, as from choosing $p_a = p_b - L$, would get the same. For the same reason, firm B would never choose $p_b < p_a - L$ (which is the same as $p_a > p_b + L$), as from choosing $p_b = p_a - L$, would get the same. Therefore we are left with two possible intervals to be analyzed: $p_a \in [p_b - L, p_b]$ and $p_a \in [p_b, p_b + L]$.

A similar analysis may be done to value consumers' choices between SB and brand b. We get the demand of SB given the price of brand b and the demand of brand b given the price of SB, respectively:

$$D_0 = \begin{cases} S+1, & p_0 < p_b - q - L \\ S+\theta^2, & p_0 \in [p_b - q - L, p_b] \\ S(1-\theta^1), & p_0 \in [p_b, p_b + L - q] \\ 0, & p_0 > p_b + L - q \end{cases} \qquad D_b = \begin{cases} S+1, & p_b < p_0 + q - L \\ 1+S\theta^1, & p_b \in [p_0 + q - L, p_0] \\ 1-\theta^2, & p_b \in [p_0, p_0 + q + L] \\ 0, & p_b > p_0 + q + L \end{cases}$$

For the same reasoning as above, we can eliminate the extreme intervals from the analysis. Then, the interesting intervals are: $p_0 \in [p_b - q - L, p_b]$ and $p_0 \in [p_b, p_b + L - q]$.

All possible combinations among the intervals defined for p_a and p_0 as functions of p_b, lead to the following four cases.

1. $p_a \in [p_b - L, p_b]$ and $p_0 \in [p_b - q - L, p_b]$
2. $p_a \in [p_b - L, p_b]$ and $p_0 \in [p_b, p_b + L - q]$
3. $p_a \in [p_b, p_b + L]$ and $p_0 \in [p_b - q - L, p_b]$

4. $p_a \in [p_b, p_b + L]$ and $p_0 \in [p_b, p_b + L - q]$

Moreover, the condition $p_0 \in [p_a - q, p_a]$, which has been explained in the monopoly case, has to be always fulfilled.

Firms choose the three prices simultaneously. Then, we have to solve the following system of optimizations for each of the four combinations of price intervals described above.

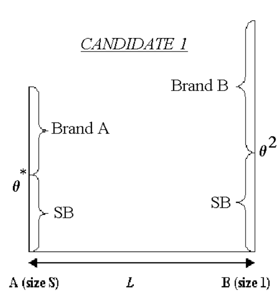
$$\begin{cases} \underset{p_0, p_a}{Max}\ \pi_a = p_0 D_0 + p_a D_a \\ \underset{p_b}{Max}\ \pi_b = p_b D_b \\ s.t. \textit{Equilibrium prices lie within the intervals considered} \end{cases}$$

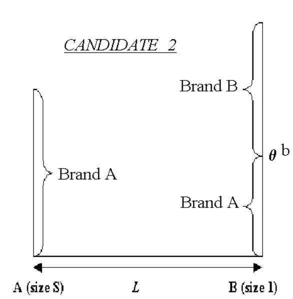
The first order conditions corresponding to every case are written in the appendix.

1.4.2 Results

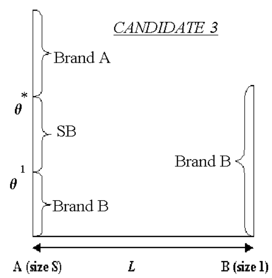
A systematic analysis of all possible price intervals (see Appendix 1.A.1.), leads to four possible demand structures, every of which gives a candidate to a Nash equilibrium of the simultaneous price game.

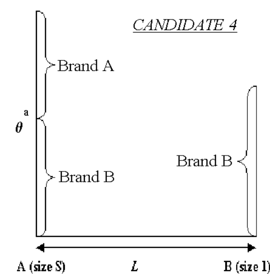
- If $S \leq 1$ (city A is smaller) the big city imports from the small one, either the SB (candidate 1) or the brand (candidate 2).





- If $S \geq 1$ (city A is bigger) again the big city imports from the small one.





Remark 1 *If an equilibrium in pure strategies exists it always has to be that the firm located in the small city exports to the other one.*

This confirms the results obtained by Garella and Martinez-Giralt (1989) in the case of single-product firms. The significant contribution of this paper with respect to this result is that here, the imported good may be either the main brand or the secondary brand.

In what follows we have to check which of the four solutions described above is a Nash equilibrium. In the Appendix (1.A.2.) we compute the values of the parameters for which neither of the two firms finds profitable to deviate to other intervals.

The following Proposition summarizes our findings:

Proposition 2 *If $S < 1$,*

- *Candidate 1 is a Nash equilibrium if $\frac{q}{L} < H_1(S)$.*

- *Candidate 2 is a Nash equilibrium if $\frac{q}{L} < H_2(S)$.*

If $S > 1$, there is no Nash equilibrium in pure strategies.

Proof. See appendix 1.A.2. ■

The functions $H_1(S)$ and $H_2(S)$ are defined in the Appendix (1.A.2.). We need to impose the positiveness of this functions, in order to ensure the existence of an equilibrium for at least some values of the parameters. We find that $H_1(S) > 0$ for $S > 0.46$ and $H_2(S) > 0$ for $S < 0.46$. Notice that, the intervals of S required for the existence of each equilibrium do not overlap. This is shown in Figure 1.1 and takes to the following corollary:

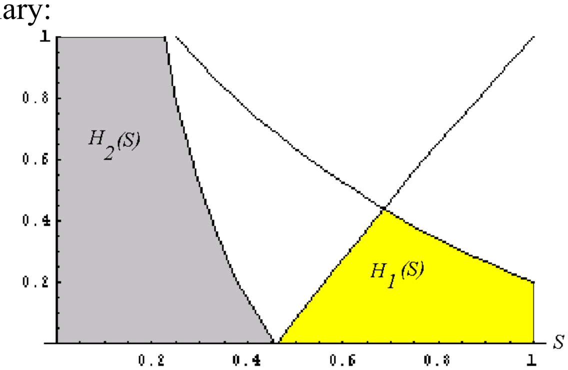


Figure 1.1

Corollary 1 *If an equilibrium exists it is unique.*

When the firm that exports is the one that potentially produces the SB ($S < 1$), we observe that the good sold "abroad" may be either the brand or the SB. Let us discuss on the values of S for each of the two equilibria of this case. Before starting the analysis, it is worth mentioning that, in equilibrium, business-stealing effect, that is the fraction of consumers in city B that buy a product of firm A, is the same for both candidates 1 and 2, namely $\frac{1-S}{3}$ (see Appendix 1.A.1.). This expression is decreasing with S. On the one hand, if S is relatively high (but still less than 1) we are in the case of candidate 1. Here, business-stealing effect is relatively low. Therefore, as S decreases, the business-stealing effect increases, and firm A wants to deviate to the structure given by candidate 2, where it can sell the brand (which is more expensive) in the city B. On the other hand, if S is relatively low, we are in the case of candidate 2, and the business-stealing effect is relatively high. But, as S increases, the business-stealing effect decreases and firm A wants to deviate to candidate 1, in order to benefit of the advantages of price discrimination by selling the SB. In sum, if business-stealing effect is high it prevails, and firm A wants to sell the expensive brand in the other city in order to maximize profits, while when it is low, price discrimination effect prevails and firm chooses to introduce the SB.

Notice that, the existence of the equilibrium for $S < 1$ requires relatively high values of L with respect to q. This restriction is even stronger than our initial assumption of sufficiently high horizontal differentiation (as compared to the quality gap: $q < L$).

When the firm producing the SB is bigger in size than the other firm ($S > 1$), there is no equilibrium in pure strategies[23]. This is because, given the optimal prices of firm A in the case of candidate 3, firm B always wants to deviate to the demand structure given by candidates 1 or 4 and given the optimal prices of firm B in the case of candidate 4, firm A always wants to deviate to the demand structure 3. In other words, firm A wants to deviate

[23] Our conjecture is that, if firm B were allowed to introduce a SB too, an equilibrium may exist as well for the case when city A is bigger. The equilibrium would be similar to the one given by candidates 1 or 2, in this case with firm B, which is smaller, exporting to city A either tha main brand or the SB. Nevertheless, the computational complexities pushed us to restrict to only one firm producing a SB. However, real life decisions regarding the launch of a SB are not made simultaneously by different firms, hence it is worth studying the static process of a firm introducing a SB at a given moment.

to the case with SB, while firm B doesn't like the demand structure where it has to compete with the SB in the city of the rival.

1.4.3 Welfare analysis

If the social planner could implement the first best, it would impose local regulated monopolies for the brands and no SBs in the market. Therefore equilibrium is inefficient as compared to the first best.

Let us study the second best case, when the only tool of the social planner would be to prohibit the production of SBs. The intuition behind the analysis of welfare is straightforward. We focus on the case of candidate 1, which is the only equilibrium with secondary brands. The production of SB brings in two types of inefficiencies: consumption of SB instead of the main brand and transportation costs, whereas the prohibition of SB eliminates the first effect. And the business-stealing effect is the same (the demand stolen from the rival is the same both when the exported good is the SB and the main brand) which means that transportation cost is the same. Hence, SBs should always be prohibited in the case of candidate 1. This extreme result may be relaxed with the introduction of a cost differential between different quality goods. Section 6 comments on this possibility.

1.5 Incentives to produce SB under different market structures

As proved in section 3, demand for the SB in the monopoly case is half of the market ($\frac{S}{2}$). The demand for SB in duopoly (candidate 1) has been computed in the Appendix (1.A.1.):

$$D_0 = \frac{S}{2} + \frac{1-S}{3}$$

This is more than half of the market. The second fraction in the expression above represents business ”stolen” from the rival. Then, intuitively, it is possible that a firm in duopoly has higher incentives to launch a SB than a monopolist. The two different market structures (monopoly and duopoly) may be interpreted, for example, from an international trade perspective. If the two firms were located in two different countries, each one would be a monopolist when free trade is not allowed and a duopolist otherwise.

Let us compare firm A’s profits in monopoly and duopoly (candidate 1) respectively.

$$\begin{aligned} \pi^{monopoly} &= S(R+\frac{q}{4}) \\ \pi_a^{duopoly} &= \frac{(L+q)(1+2S)^2}{9} + \frac{Sq}{4} \end{aligned}$$

If $R < \frac{(L+q)(1+2S)^2}{9S}$, then $\pi^{monopoly} < \pi_a^{duopoly}$. If some fixed cost, F, were to be covered for the production of SB, it might happen that $\pi^{monopoly} < F < \pi_a^{duopoly}$. Then, the duopolist can cover the fixed cost, while the monopolist cannot.

Taking into account that the existence of an equilibrium of type 1 requires a lower bound for the reservation price ($R \geq \frac{(L+q)(2+S)}{3}$), we still have to check that the set of parameters, for which the two inequalities regarding R must hold simultaneously, is not empty. Indeed, the set of R, for which $\frac{(L+q)(2+S)}{3} \leq R < \frac{(L+q)(1+2S)^2}{9S}$ is never empty.

Hence, there exists values of the parameters (R, F, L, q and S) for which a SB will be launched in duopoly even if, it will be not, in monopoly.

1.6 Cost considerations

In order to study firms' incentives to launch secondary brands we used a model without any kind of costs. In this section we come back to cost considerations and introduce a cost differential between the SB and the quality good. The cost analysis is undertaken for the monopoly case, but this may be extended to the case of duopoly.

In section 1.3, we found that the monopolist chooses to serve half of the demand with the SB and the other half with the brand. This situation is socially inefficient as, according to the first best, the monopolist should sell to everybody the high quality good. Nevertheless, if the higher-quality good had a relatively higher cost of production, the social optimum may change in the direction of producing at least some SB. The purpose of this section is to compare the bias between the market equilibrium and the social optimum in two different situations: with and without cost differential.

Let us denote by c, the unit cost of producing the main brand and normalize to zero the production cost of the SB. Then, monopolist's profits would be:

$$\pi = p_0\theta^* + (p_a - c)(1 - \theta^*)$$

The equilibrium prices are, in this case: $p_0 = R$ and $p_a = R + \frac{q+c}{2}$.

At these prices, demand for the SB is: $\theta^* = \frac{1}{2} + \frac{c}{2q}$ (or 1 if $c > q$), which is higher than in the case with no cost differential. This is obvious, as the SB is now relatively cheaper to produce. The residual demand goes to the brand: $D_a^{equil} = 1 - \theta^* = \frac{1}{2} - \frac{c}{2q}$ (or 0 if $c > q$).

Total surplus is now:

$$TS = \int_0^{\theta^*} Rd\theta + \int_{\theta^*}^1 (R + q\theta)d\theta - c(1-\theta^*) = R + \frac{q}{2}(1-(\theta^*)^2) - c(1-\theta^*)$$

In the first best case, the optimum is reached for: $\theta^* = \frac{c}{q}$ (or 1 if $c > q$). This makes demand for the brand: $D_a^{opt} = 1 - \frac{c}{q}$.

Then, the social optimum requires the production of more of the main brand than it is really produced in the market:

$$D_a^{opt} = 1 - \frac{c}{q} > D_a^{equil} = \frac{1}{2}(1 - \frac{c}{q})$$

To conclude, one can say that the brand is still insufficient ($D_a^{opt} - D_a^{market} = \frac{1}{2} - \frac{c}{2q} \geq 0$) but "less insufficient" than in the case without cost differential. Remember that, when the two goods were produced at no costs, the difference $D_a^{opt} - D_a^{market} = 1 - \frac{1}{2} = \frac{1}{2}$.

Let us discuss the implications of introducing the cost differential in the duopoly model. Remember that two inefficiencies characterized the equilibrium in the case of candidate 1. On the one hand, the amount of brand goods sold in the market was insufficient. As in the monopoly case, this bias will decrease with the introduction of a cost differential. On the other hand, as the amount of SB products will increase, the transportation costs of consumers from city B buying the SB may increase too, which is bad for welfare. Hence, it is not obvious whether a quality cost differential will "improve" the bias between market equilibrium and social optimum in duopoly.

1.7 Low-quality rival

In the model presented so far, we assumed that horizontal competition takes place between brands of the same quality. This resembles the example of two manufacturers, each producing a brand good of a common quality, which is higher than the quality of a secondary brand potentially produced by one of them.

In this section we will assume that the rival in the horizontal competition comes from the low-end of the market, like in the examples of fighting brands. The model we use is a slightly modified version of the one presented in section 1.4. Everything is the same except that now, firm B produces a good of quality 0. Following an identical reasoning as in the case of high-quality rival, we obtain the same four candidates to equilibrium(see Appendix 1.A.3. for a full description of the equilibrium in this case).

Even the four candidates are the same, the existence criteria of the possible equilibria change because now, the price intervals required by each demand structure are different The following proposition summarizes the results of this case:

Proposition 3 *If* $S < 1$,

- *Candidate 1 is a Nash equilibrium if* $\frac{q}{L} < H_1^*(S)$.
- *Candidate 2 is a Nash equilibrium if* $\frac{q}{L} < H_2^*(S)$.

If $S > 1$,

- *Candidate 3 is a Nash equilibrium if* $\frac{q}{L} < H_3(S)$.

- *Candidate 4 is never a Nash equilibrium.*

Figures 1.2 and 1.3 show the parameters intervals required by the existence of each equilibria. Again, for a given size S, the equilibrium is unique.

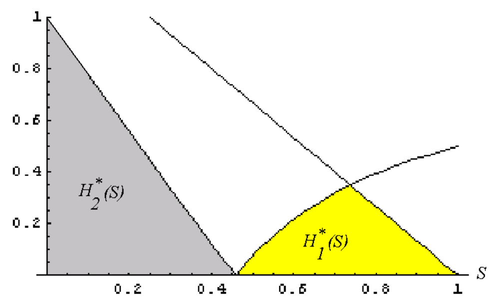


Figure 1.2

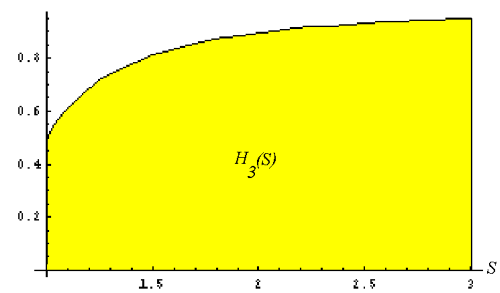


Figure 1.3

For the case $S < 1$ the behavior of the two firms is similar to the main model presented in section 1.4. Nevertheless, in the case of low-quality rival, the model provides an equilibrium even for $S > 1$. For a wide range of the parameters an equilibrium given by candidate 3 exists. If the firm potentially producing the SB is located in the bigger city, in most of the cases it will indeed produce the SB in order to protect itself from competition as the rival ”invades” its market. This is very much in line with the case of fighting brands.

Summarizing, the most important difference between the two versions of the model is that, in the first one, competition comes from the high-end, while in the second one, from the low-end of the market. This makes an important distinction in the purpose of launching secondary brands in each of the two cases. In the first version of the model, secondary brands are produced by medium-sized manufacturers in order to attack the brand leader.

In the second version, the very purpose of the secondary brands is to protect the market leaders from competition by low-quality rivals.

Finally, a few comments are due to give some insights on the introduction of fixed or variable costs to this version of the model. Here, as well as in the case of high-quality rival and for the same values of the parameters, the incentives to launch a SB in duopoly may be higher than in monopoly, if some fixed costs are needed[24]. Indeed, the very purpose of the fighting brands was to compete with the rivals and therefore, they were not produced under monopoly.

The introduction of a variable cost differential between the main brand and the SB has been already studied in section 1.6 for the case of monopoly. We claimed that, the existence of a strictly positive variable cost for the high-quality product will improve the bias between the market equilibrium and the social optimum (both measured in the amount of brand good sold). At the end of section 1.6, we also discussed this issue for the case of duopoly. Remember that, in the first version of the model, the only equilibrium with SB was given by candidate 1. We concluded that, for the case of candidate 1, it is not obvious whether the variable cost differential will improve the bias between the market equilibrium and the social optimum, as an increase in the production of SB (because it is relatively cheaper to produce) may also increase total transportation costs due to the fact that more consumers may wish to travel in order to buy the SB. In the second version of the model, apart from candidate 1, we have a new equilibrium with SB, namely the one given by candidate 3. Here, even if the production of SB may increase with the introduction of a

[24] Section 1.5 provides a full description of this issue; the analysis here is identical.

variable cost differential, this will not affect the total transportation costs, as the additional amount of SB goods is not sold abroad, as in the case of candidate 1. Hence, it seems that the bias between the market equilibrium and the social optimum is improved in the case of candidate 3 as compared to the case of candidate 1.

1.8 Conclusions

This chapter explains why some brand manufacturers refuse to supply secondary brands, while for others, this is a profitable strategy. Real life examples of secondary brands are private labels and fighting brands. Empirical evidence shows that market share matters in firms' decisions to such policies. We use two versions of a model of two-dimensional differentiation in an asymmetric duopoly and obtain the following results. In the first version of the model, with horizontal competition between similar brands, we find that intermediate size firms produce secondary brands in order to attack the rival. This is in line with the empirical evidence regarding private labels. In the second version of the model, when competition comes from the low-end of the market (like in the case of fighting brands), even brand leaders produce secondary brands, in order to protect themselves from the rivals.

From the social point of view, the equilibrium with secondary brands is inefficient for two reasons: firstly, because the social optimum requires more brand products than are sold in equilibrium and secondly, because of the transportation costs incurred by consumers to buy the SB in the other city. If a cost differential between the SB and the brand good is introduced, the market equilibrium may be improved as compared to the social optimum.

Finally, if some fixed costs are needed, the incentives of a firm to launch a SB in duopoly (when free-trade is allowed between the cities) may be higher than in monopoly (in a situation of autarchy), due to the business stealing effect.

1.9 References

Bergés-Sennou, F., 2002, ”Who Will Produce the Private Label?”, *mimeo*, INRA

Bergés-Sennou, F., P. Bontems and V. Requillart, 2004, ”Economics of Private Labels: A Survey of Literature”, *Journal of Agricultural and Food Industrial Organization*, 2(1), 1-23.

Bontems, P., S. Monier-Dilhan and V. Réquillart, 1999, ”Strategic Effects of Private Labels”, *European Review of Agricultural Economics,* 26(2), 147-165.

Champsaur, P. and J.-C. Rochet, 1989, ”Multiproduct Duopolists”, *Econometrica*, 57(3), 533-557.

Comanor, W. and P. Rey, 2000, ”Vertical Restraints and the Market Power of Large Distributors”, *Review of Industrial Organization*, 17(2), 135-153.

Dahr, S.K. and S.J. Hoch, 1997, ”Why Store Brand Penetration Varies by Retailer”, *Marketing Science*, 16(3), 208-227.

D'Aspremont, C., J.J.Gabszewicz and J.-F. Thisse, 1979, ”On Hotelling's Stability in Competition”, *Econometrica* 47, 1145-1150.

Degryse, H., 1996, ”On the Interaction between Vertical and Horizontal Product Differentiation: An Application to Banking”, *The Journal of Industrial Economics*, 44(2), 169-186.

Deneckere, R.J. and R.P. McAfee, 1996, ”Damaged Goods”, *Journal of Economics and Management Strategy*, 5(2), 149-174.

Dos Santos Ferreira, R., J.F. Thisse, 1996, ”Horizontal and Vertical Differentiation: The Launhardt Model”, *International Journal of Industrial Organization,* 14(4), 485-506.

Dunne, D., 1999, ”Should grocery manufacturers supply private labels?”, University of Toronto.

Dunne, D. and C. Narasimhan, 1999, ”The New Appeal of Private Labels”, *Harvard Business Review*, 77(3), 41-52.

Gabrielsen, T. and L. Sorgard, 2001, ”Private Labels, Price Rivalry and Public Policy”, University of California, Santa Barbara.

Garella, P.G. and X. Martinez-Giralt, 1989, ”Price Competition in Markets for Dichotomous Substitutes”, *International Journal of Industrial Organization,* 7(3), 357-367.

Gilbert, R.J. and C. Matutes, 1993, ”Product Line Rivalry with Brand Differentiation”, *The Journal of Industrial Economics*, 41(3), 223-240.

Hotelling, H., 1929, ”Stability in Competition”, *The Economic Journal,* 39, 41-57.

Hoch, S.J., 1996, ”How Should National Brands Think about Private Labels?”, *Sloan Management Review*, 77(3), 89-102.

Hoch, S.J. and S. Banerji, 1993, ”When Do Private Labels Succeed?”, *Sloan Management Review*, 34(4), 57-167.

Johnson, J.P. and D.P.Myatt, 2003, ”Multiproduct Quality Competition: Fighting Brands and Product Line Pruning”, *American Economic Review*, 93(3), 748-774.

Mills, D.E., 1995, ”Why Retailers Sell Private Pabels”, *Journal of Economics and Management Strategy* 4(3), 509-528.

Mills, D.E., 1999, ”Private Labels and Manufacturer Counterstrategies”, *European Review of Agricultural Economics* 26(2), 125-145.

Moner-Colonques, R., J. J. Sempere-Monerris and A. Urbano, 2004, ”Private Labels vs National Brands: A Strategic Approach”, University of Valencia, Spain.

Mussa, M. and S. Rosen, 1978, ”Monopoly and Product Quality”, *Journal of Economic Theory,* 18(2), 301-317.

Neven, D. and J.F. Thisse, 1990, ”On Quality and Variety Competition”, in: J.J. Gabszewicz, J.F. Richard and L. Wolsey. (eds.), *Economic decision making: Games, econometrics, and optimization. Contributions in honour of Jaques Drèze* (North-Holland, Amsterdam), 175-199.

Quelch, J.A. and D. Harding, 1996, ”Brands versus Private Labels: Fighting to Win”, *Harvard Business Review*, 74(1), 99-109.

Raju, J.S., R. Sethuraman and S. Dhar, 1995, ”The Introduction and Performance of Store Brands”, *Management Science*, 41(6), 957-978.

Sethuraman, R. and C. Cole, 1999, ”Factors Influencing the Price Premiums that Consumers Pay for National Brands over Store Brands”, *Journal of Product and Brand Management,* 8(4), 340-351.

Shaked, A. and J. Sutton, 1982, ”Relaxing Price Competition Through Product Differentiation”, *Review of Economic Studies,* 49(1), 3-13.

Verhoef, P., E. Nijssen and L. Sloot, 2002, ”Strategic Reactions of National Brand Manufacturers towards Private Labels. An Empirical Study in the Netherlands”, *European Journal of Marketing*, 36(11/12), 1309-1326.

Wolinsky, A., 1987, ”Brand Names and Price Discrimination”, *The Journal of Industrial Economics,* 35(3), 255-268.

1.A Appendix to Chapter One

1.A.1 Equilibrium Candidates

We solve the restrained optimization problem in each of the four cases and obtain the candidates to equilibrium. Case 1 and Case 4 take each to two possible candidates respectively, while cases 2 and 3 give no additional results. Hence, in total, we will deal with four candidates to equilibrium.

Case 1. $p_a \in [p_b - L, p_b]$ *and* $p_0 \in [p_b - q - L, p_b]$

In city A, both brand a and SB are always preferred to brand b, so demand is shared between SB and brand a. For the demand structure in city B we identify two situations:

i) If $\frac{q}{L} \leq \frac{p_a - p_0}{p_b - p_a}$ then $\theta^b \leq \theta^2 \leq \theta^*$. Demand in city B is shared between the SB and brand b.

Then, we have to solve the following system of first order conditions:

$$\begin{aligned} \underset{p_0,p_a}{Max}\, \pi_a &= p_0 D_0 + p_a D_a = p_0(S\theta^* + \theta^2) + p_a S(1-\theta^*) \\ \underset{p_b}{Max}\, \pi_b &= p_b D_b = p_b(1-\theta^2) \end{aligned}$$

$$s.t. \left\{ \begin{array}{c} p_0 \in [p_a - q, p_a] \\ p_a \in [p_b - L, p_b] \\ p_0 \in [p_b - q - L, p_b] \\ \frac{q}{L} \leq \frac{p_a - p_0}{p_b - p_a} \end{array} \right.$$

The equilibrium prices and demands are listed in table 1.5:

Table 1.5: Results for candidate 1

	PRICE	DEMAND
SB	$p_0 = \frac{(L+q)(1+2S)}{3}$	$D_0 = \frac{S}{2} + \frac{1-S}{3} = \frac{2+S}{6}$
Brand a	$p_a = \frac{(L+q)(1+2S)}{3} + \frac{q}{2}$	$D_a = \frac{S}{2}$
Brand b	$p_b = \frac{(L+q)(2+S)}{3}$	$D_b = 1 - \frac{1-S}{3} = \frac{2+S}{3}$

The equilibrium profits are:

$$\begin{aligned}
\pi_a &= \frac{(L+q)(1+2S)^2}{9} + \frac{Sq}{4} \\
\pi_b &= \frac{(L+q)(2+S)^2}{9}
\end{aligned}$$

We have to check now that prices are interior to their domains:

1. $p_0 \in [p_a - q, p_a]$ is always fulfilled for the whole set of parameters

2. $p_a \in [p_b - L, p_b] \rightarrow S \leq \frac{2L-q}{2L+2q}$

3. $p_0 \in [p_b - q - L, p_b] \rightarrow S \leq 1$

4. $\frac{q}{L} \leq \frac{p_a - p_0}{p_b - p_a}$ is always fulfilled for the whole set of parameters

Condition 3 is implied by condition 2, hence we can give it up. Then, we are left with one condition that has to be fulfilled by the parameters, in order to ensure the existence of interior solutions in this case, namely:

$$S \leq \frac{2L-q}{2L+2q} \tag{1.1}$$

ii) If $\frac{q}{L} \geq \frac{p_a - p_0}{p_b - p_a}$ then $\theta^b \geq \theta^2 \geq \theta^*$. Now, demand in city B is shared among the three goods.

Here, we have to solve the following problem:

$$\begin{aligned}
\underset{p_0,p_a}{Max}\,\pi_a &= p_0D_0 + p_aD_a = p_0(S\theta^* + \theta^*) + p_a(S(1-\theta^*) + \theta^b - \theta^*) \\
\underset{p_b}{Max}\,\pi_b &= p_bD_b = p_b(1-\theta^b) \\
& s.t. \begin{cases} p_0 \in [p_a - q, p_a] \\ p_a \in [p_b - L, p_b] \\ p_0 \in [p_b - q - L, p_b] \\ \frac{q}{L} \geq \frac{p_a - p_0}{p_b - p_a} \end{cases}
\end{aligned}$$

In this case, we have a corner solution: $p_0 = p_a$. Then, in equilibrium, no SB is sold in the market.

Prices and demands for the two brands are listed in table 1.6:

Table 1.6: Results for candidate 2

	PRICE	DEMAND
Brand a	$p_a = \frac{L(1+2S)}{3}$	$D_a = S + \frac{1-S}{3} = \frac{1+2S}{3}$
Brand b	$p_b = \frac{L(2+S)}{3}$	$D_b = 1 - \frac{1-S}{3} = \frac{2+S}{3}$

The equilibrium profits are:

$$\begin{aligned}
\pi_a &= \frac{L(1+2S)^2}{9} \\
\pi_b &= \frac{L(2+S)^2}{9}
\end{aligned}$$

Again, we have to check that prices are interior:

1. $p_a \in [p_b - L, p_b] \rightarrow S \leq 1$

2. $\frac{q}{L} \geq 0$ is always fulfilled for the whole set of parameters

Now, the only condition to ensure interior prices in this case is:

$$S \leq 1 \tag{1.2}$$

Case 2. $p_a \in [p_b - L, p_b]$ *and* $p_0 \in [p_b, p_b + L - q]$

Remember that, in each case, we also have to impose: $p_0 \in [p_a - q, p_a]$. Then, case 2 is reduced to $p_a = p_0 = p_b$. No SBs would be sold in the market and each firm would serve its own city with the brand. This combination of prices was feasible in case 1 but, nevertheless, was not optimal. It is not an equilibrium as firm A could profitably deviate by slightly decreasing p_0 to attract some customers from the rival.

Case 3. $p_a \in [p_b, p_b + L]$ *and* $p_0 \in [p_b - q - L, p_b]$

In city A, SB is always preferred to brand b. The only possible structure is the one where demand is shared between the SB and brand a. A necessary condition for its existence is: $\theta^* > \theta^a$, which is equivalent to $\frac{q}{L} < \frac{p_a - p_0}{p_a - p_b}$. Notice that, this is always true for $p_0 < p_b$ as $\frac{q}{L} < 1$ and $\frac{p_a - p_0}{p_a - p_b} > 1$.

In city B, brand b is always preferred to brand a, so demand is shared between the SB and brand b. For this structure to exist we need $\theta^* > \theta^2$ which is again true for the intervals considered.

Then, demands are the same as in the case of candidate 1, but restrictions are different:

$$\underset{p_0,p_a}{Max}\,\pi_a = p_0(S\theta^* + \theta^2) + p_a S(1-\theta^*)$$

$$\underset{p_b}{Max}\,\pi_b = p_b(1-\theta^2)$$

$$s.t.\begin{cases} p_0 \in [p_a - q, p_a] \\ p_a \in [p_b, p_b + L] \\ p_0 \in [p_b - q - L, p_b] \end{cases}$$

Solution is the same, but equilibrium prices have to fulfill different conditions to ensure interior solutions, namely:

1. $p_0 \in [p_a - q, p_a]$ is always fulfilled for the whole set of parameters

2. $p_a \in [p_b, p_b + L] \rightarrow S \in [\frac{2L-q}{2L+2q}, \frac{8L-q}{2L+2q}]$

3. $p_0 \in [p_b - q - L, p_b] \rightarrow S \leq 1$

Combining 2. and 3. we get:

$$\frac{2L-q}{2L+2q} \leq S \leq 1 \tag{1.3}$$

Notice that, both *Case 1-i)* and *Case 3* give the same solution, hence we can state that, in sum, a necessary condition for candidate 1 to be en equilibrium is $S \leq 1$ (which is the union of the intervals given by (1.1) and (1.3)).

Case 4. $p_a \in [p_b, p_b + L]$ *and* $p_0 \in [p_b, p_b + L - q]$

In city B, brand b is always preferred to both brand a and the SB, so all consumers buy brand b. In city A, we distinguish two cases:

i) If $\frac{q}{L} \leq \frac{p_a - p_0}{p_a - p_b}$ then $\theta^1 \leq \theta^a \leq \theta^*$. In this case, demand in city A is shared among the three goods.

We have to solve the following problem:

$$\begin{aligned}
\underset{p_0,p_a}{Max}\ \pi_a &= p_a D_a + p_0 D_0 = p_0 S(\theta^* - \theta^1) + p_a S(1 - \theta^*) \\
\underset{p_b}{Max}\ \pi_b &= p_b D_b = p_b(1 + S\theta^1) \\
& s.t. \begin{cases} p_0 \in [p_a - q, p_a] \\ p_a \in [p_b, p_b + L] \\ p_0 \in [p_b, p_b + L - q] \\ \frac{q}{L} \leq \frac{p_a - p_0}{p_a - p_b} \end{cases}
\end{aligned}$$

The equilibrium prices, demands and profits are presented below:

Table 1.7: Results for candidate 3

	PRICE	DEMAND
SB	$p_0 = \frac{(L-q)(1+2S)}{3S}$	$D_0 = \frac{S}{2} - \frac{S-1}{3} = \frac{2+S}{6}$
Brand a	$p_a = \frac{(L-q)(1+2S)}{3S} + \frac{q}{2}$	$D_a = \frac{S}{2}$
Brand b	$p_b = \frac{(L-q)(2+S)}{3S}$	$D_b = 1 + \frac{S-1}{3} = \frac{2+S}{3}$

$$\pi_a = \frac{(L-q)(1+2S)^2}{9S} + \frac{Sq}{4}; \pi_b = \frac{(L-q)(2+S)^2}{9S}$$

We have to check now that prices are interior to their domains:

1. $p_0 \in [p_a - q, p_a]$ is always fulfilled for the whole set of parameters

2. $p_a \in [p_b, p_b + L] \rightarrow S \geq \frac{2L-2q}{2L+q}$

3. $p_0 \in [p_b, p_b + L - q] \rightarrow S \geq 1$

4. $\frac{q}{L} \leq \frac{p_a - p_0}{p_a - p_b}$ is always fulfilled for the whole set of parameters

Condition 2. is implied by condition 3. hence we can give it up. Then, prices are interior solutions as long as:

$$S \geq 1 \tag{1.4}$$

ii) If $\frac{q}{L} \geq \frac{p_a - p_0}{p_a - p_b}$ then $\theta^1 \geq \theta^a \geq \theta^*$. In this case, demand in city A is shared between the two brands.

The optimization problem is, as follows:

$$\begin{aligned} \underset{p_a}{Max}\ \pi_a &= p_a D_a = p_a S(1 - \theta^a) \\ \underset{p_b}{Max}\ \pi_b &= p_b(1 + S\theta^a) \\ & s.t. \begin{cases} p_a \in [p_b, p_b + L] \\ \frac{q}{L} \geq \frac{p_a - p_0}{p_a - p_b} \end{cases} \end{aligned}$$

Equilibrium prices, demands and profits are presented in what follows:

Table 1.8: Results for candidate 4

	PRICE	DEMAND
Brand a	$p_a = \frac{L(1+2S)}{3S}$	$D_a = S - \frac{S-1}{3} = \frac{1+2S}{3}$
Brand b	$p_b = \frac{L(2+S)}{3S}$	$D_b = 1 + \frac{S-1}{3} = \frac{2+S}{3}$

$$\pi_a = \frac{L(1+2S)^2}{9S}; \pi_b = \frac{L(2+S)^2}{9S}$$

Again, we have to check that prices are interior:

1. $p_a \in [p_b, p_b + L] \rightarrow S \geq 1$

2. $\frac{q}{L} \geq \frac{p_a - p_0}{p_a - p_b}$ $\rightarrow$ is always fulfilled as zero demand for the SB means infinitely high p_0.

Then, the only condition on the parameters that has to be fulfilled in order to have interior solution is:

$$S \geq 1 \tag{1.5}$$

Summary of 1.A.1.:

In the analysis above, there emerged four different demand structures, denoted by candidate 1, 2, 3 and 4, respectively. Each of these leads to a candidate to a Nash equilibrium in the respective price intervals. The first two candidates require $S \leq 1$, while the last two, the opposite: $S \geq 1$.

1.A.2 Nash Equilibria

We find conditions on the parameters of the model to ensure that every of the four candidates to equilibrium be a Nash solution. Table 1.9 lists the expressions of profits that were computed in the first part of the appendix and will be used throughout this proof.

Table 1.9: Profits for each candidate

	For any prices	*In equilibrium*
CANDIDATE 1		
π_a	$\frac{p_a(p_0-p_a+q)S}{q}+p_0(\frac{p_b-p_0}{L+q}+\frac{S(p_a-p_0)}{q})$	$\frac{(L+q)(1+2S)^2}{9}+\frac{Sq}{4}$
π_b	$\frac{p_b(L+p_0-p_b+q)}{L+q}$	$\frac{(L+q)(2+S)^2}{9}$
CANDIDATE 2		
π_a	$\frac{p_a(p_b-p_a)q-L((S+1)(p_a-p_0)^2-p_aqS)}{Lq}$	$\frac{L(1+2S)^2}{9}$
π_b	$\frac{p_b(L+p_a-p_b)}{L}$	$\frac{L(2+S)^2}{9}$
CANDIDATE 3		
π_a	$\frac{p_a(p_0-p_a+q)S}{q}+p_0(\frac{p_b-p_0}{L-q}+\frac{(p_a-p_0)}{q})$	$\frac{(L-q)(1+2S)^2}{9S}+\frac{Sq}{4}$
π_b	$p_b(1+\frac{S(p_0-p_b)}{L-q})$	$\frac{(L-q)(2+S)^2}{9S}$
CANDIDATE 4		
π_a	$\frac{p_a(p_b-p_a+L)S}{L}$	$\frac{L(1+2S)^2}{9S}$
π_b	$p_b(1+\frac{S(p_a-p_b)}{L})$	$\frac{L(2+S)^2}{9S}$

I. Assume candidate 1 is a Nash equilibrium.

Then, the following conditions must hold simultaneously:

i) $S<1$

ii) $\pi_a^1(p_0^*,p_{a,}^*p_b^*)\geq\pi_a^i(p_0^i,p_{a,}^ip_b^*)$ where p_b^* is the equilibrium price of firm B in the case of candidate 1 and p_0^i and $p_{a,}^i$ maximize profits to firm A when demands are given by candidate i ($i=2,3,4$), under the corresponding restrictions (for the existence of candidate i).

iii) $\pi_b^1(p_0^*,p_{a,}^*p_b^*)\geq\pi_b^i(p_0^*,p_{a,}^*p_b^i)$ where p_0^* and p_a^* are the equilibrium prices of firm A in the case of candidate 1 and p_b^i maximizes profits to firm B when demands are given by candidate i ($i=2,3,4$), under the corresponding restrictions (for the existence of candidate i).

Let us find the values of the parameters for which inequalities ii) and iii) hold.

Firstly, given the equilibrium price of firm B in the case of candidate 1, assume firm A wants to deviate to the demand structure given by candidate 2. By maximizing profits in this case, firm A would choose: $p_a^2 = p_0^2 = \frac{2L(1+2S)+q(2+S)}{6}$. With these values, price restrictions given by candidate 2 are always fulfilled. Now, in order for firm A not to have incentives to deviate to structure given by candidate 2, we need to impose: $\pi_a^1(p_0^*, p_{a,}^* p_b^*) > \pi_a^2(p_0^2, p_{a,}^2 p_b^*)$. This inequality holds for $\frac{q}{L} < f_1(S) = \frac{8S^2+5S-4}{(S+2)^2}$. We repeat the argument considering now deviations of firm A to the structure given by candidate 3. The optimum prices chosen by A in this case are: $p_a^3 = \frac{L(5+S)+q(2+S)}{6}$ and $p_0^3 = \frac{L(5+S)+q(-1+S)}{6}$. These prices are interior to the domain imposed by the existence of candidate 3 if $\frac{q}{L} < \frac{1-S}{5+S}$. When this condition holds, firm A never wants to deviate to the structure given by candidate 3. If it doesn't hold, firm A has to choose a corner solution, namely, $p_0^3 = p_b^*$. For this value, firm A neither wants to deviate. Hence, there are no restrictions imposed by incentives to deviate to candidate 3. Finally, deviations to the demand structure given by candidate 4 are never optimal, following a similar argument as in the previous case.

Secondly, given the equilibrium prices of firm A in the case of candidate 1, assume firm B wants to deviate to the demand structure given by candidate 2. By maximizing profits in this case, firm B would choose: $p_b^2 = \frac{4L(2+S)+q(5+4s)}{12}$. This value never fulfills the last restriction given by structure 2, namely: $\frac{q}{L} > \frac{p_a-p_0}{p_b-p_a}$. Then, we have a corner solution: $p_b^2 = p_a^* + \frac{L}{2}$. With this value, always $\pi_b^1(p_0^*, p_{a,}^* p_b^*) > \pi_b^2(p_0^*, p_{a,}^* p_b^2)$ and firm B never wants to deviate to the demand structure given by candidate 2. If firm B wants to deviate to the case of candidate 3, the optimal price should again be a corner, namely $p_b^3 = p_0^*$. For this value, $\pi_b^1(p_0^*, p_{a,}^* p_b^*) > \pi_b^3(p_0^*, p_{a,}^* p_b^3)$ if $\frac{q}{L} < f_2(S) = \frac{11-8S}{7+8S}$. Finally, if firm B wants to deviate to the case of candidate 4, the optimal price should be again a corner but, in this case, deviations are never optimal.

Summarizing, if we denote by $H_1(S) = Min\{f_1(S), f_2(S)\}$, candidate 1 is a Nash equilibrium if:

$$\frac{q}{L} < H_1(S)$$

<u>*II. Assume candidate 2 is a Nash equilibrium.*</u>

Then, the following conditions must hold simultaneously:

i) $S < 1$

ii) $\pi_a^2(p_0^*, p_{a,}^* p_b^*) \geq \pi_a^i(p_0^i, p_{a,}^i p_b^*)$ where p_b^* is the equilibrium price of firm B in the case of candidate 2 and p_0^i and $p_{a,}^i$ maximize profits to firm A when demands are given by candidate i ($i = 1, 3, 4$), under the corresponding restrictions (for the existence of candidate i).

iii) $\pi_b^2(p_0^*, p_{a,}^* p_b^*) \geq \pi_b^i(p_0^*, p_{a,}^* p_b^i)$ where p_0^* and p_a^* are the equilibrium prices of firm A in the case of candidate 2 and p_b^i maximizes profits to firm B when demands are given by candidate i ($i = 1, 3, 4$), under the corresponding restrictions (for the existence of candidate i).

Firstly, given the equilibrium price of firm B in the case of candidate 2, assume firm A wants to deviate to the demand structure 1. The optimal prices chosen by firm A, would be: $p_a^1 = \frac{2L(1+2S)+3q(1+S)}{6}$ and $p_0^1 = \frac{2L(1+2S)+3qS}{6}$. These values always fulfill restrictions given by the existence of candidate 1. Moreover, in order to avoid deviations to structure 1, we need to impose $\pi_a^2(p_0^*, p_{a,}^* p_b^*) > \pi_a^1(p_0^1, p_{a,}^1 p_b^*)$. This inequality holds for: $\frac{q}{L} < \frac{-8S^2-5s+4}{9S(1+S)}$. Furthermore, firm A will neither deviate to structure 3, nor to structure 4.

Secondly, restriction imposed by no deviation of firm B are dominated by the previous restrictions regarding firm A.

Hence, if we denote by $H_2(S) = \frac{-8S^2-5S+4}{9S^2+9S}$, candidate 2 is a Nash equilibrium for:

$$\frac{q}{L} < H_2(S)$$

III. Assume candidate 3 is a Nash equilibrium.

Then, the following conditions must hold simultaneously:

i) $S > 1$

ii) $\pi_a^3(p_0^*, p_{a,}^* p_b^*) \geq \pi_a^i(p_0^i, p_{a,}^i p_b^*)$ where p_b^* is the equilibrium price of firm B in the case of candidate 3 and p_0^i and $p_{a,}^i$ maximize profits to firm A when demands are given by candidate i ($i = 1, 2, 4$), under the corresponding restrictions (for the existence of candidate i).

iii) $\pi_b^3(p_0^*, p_{a,}^* p_b^*) \geq \pi_b^i(p_0^*, p_{a,}^* p_b^i)$ where p_0^* and p_a^* are the equilibrium prices of firm A in the case of candidate 3 and p_b^i maximizes profits to firm B when demands are given by candidate i ($i = 1, 2, 4$), under the corresponding restrictions (for the existence of candidate i).

We will prove that candidate 3 will never be a Nash equilibrium. This is because, given the optimal prices of firm A in the case of candidate 3, firm B always wants to deviate to the demand structures given by either candidate 1 or candidate 4.

Firstly, following a similar argument as in the case of the first two candidates, we proved that firm B will deviate to the demand structure given by candidate 1 unless: $\frac{q}{L} < g_1(S) = \frac{1-5S+4S^2}{1+13S+4S^2}$. The condition imposed by non-deviation to structure 2 is dominated by the previous restriction on the parameters, hence it is unnecessary.

Now, the optimal price firm B would choose if it were to deviate to structure 4 is: $p_b^4 = \frac{(4L-q)(2+S)}{12S}$. If this value satisfies the restrictions required by the existence of candidate 4 then, it will always happen that: $\pi_b^3(p_0^*, p_{a,}^* p_b^*) < \pi_b^4(p_0^*, p_{a,}^* p_b^4)$ and hence, firm B will deviate. We have that p_b^4 is interior if: $\frac{q}{L} < g_2(S) = \frac{4(S-1)}{7S+2}$. As $g_1(S) < g_2(S)$ for $S > 1$ then, it will always be that $\frac{q}{L} < g_2(S)$, as long as $\frac{q}{L} < g_1(S)$.

Summarizing, we proved that, if $\frac{q}{L} < \frac{1-5S+4S^2}{1+13S+4S^2}$, firm B deviates to the demand structure given by candidate 4. Otherwise, firm B deviates to structure 1.

IV. Assume candidate 4 is a Nash equilibrium.

Then, the following conditions must hold simultaneously:

i) $S > 1$

ii) $\pi_a^4(p_0^*, p_{a,}^* p_b^*) \geq \pi_a^i(p_0^i, p_{a,}^i p_b^*)$ where p_b^* is the equilibrium price of firm B in the case of candidate 4 and p_0^i and $p_{a,}^i$ maximize profits to firm A when demands are given by candidate i ($i = 1, 2, 3$), under the corresponding restrictions (for the existence of candidate i).

iii) $\pi_b^4(p_0^*, p_{a,}^* p_b^*) \geq \pi_b^i(p_0^*, p_{a,}^* p_b^i)$ where p_0^* and p_a^* are the equilibrium prices of firm A in the case of candidate 4 and p_b^i maximizes profits to firm B when demands are given by candidate i ($i = 1, 2, 3$), under the corresponding restrictions (for the existence of candidate i).

Following a similar reasoning as in the previous case, we proved that candidate 4 will never be a Nash equilibrium as the following inequality holds for the whole set of parameters:

$$\pi_a^4(p_0^*, p_{a,}^* p_b^*) < \pi_a^3(p_0^3, p_{a,}^3 p_b^*)$$

In other words, given the optimal prices of firm B in the case of candidate 4, firm A always wants to deviate to the demand structure given by candidate 3.

Summary of 1.A.2.:

- Candidate 1 is a Nash equilibrium for $\frac{q}{L} < H_1(S) = Min\{\frac{8S^2+5S-4}{(S+2)^2}, \frac{11-8S}{7+8S}\}$
- Candidate 2 is a Nash equilibrium for $\frac{q}{L} < H_2(S) = \frac{-8S^2-5S+4}{9S^2+9S}$
- Candidates 3 and 4 are never Nash equilibria.

We found two equilibria for $S < 1$. Nevertheless, notice that, $H_1(S) > 0$ for $1 > S > 0.46$ while $H_2(S) > 0$ for $0 < S < 0.46$. The intervals for S in each of the two cases do not overlap. This ensures the uniqueness of the equilibrium for $S < 1$.

1.A.3 Equilibrium in the case of low-quality rival

$$\text{Candidate 1}\begin{cases} p_0 \in [p_a - q, p_a] \\ p_0 \in [p_b - L, p_b] \\ \begin{cases} p_a \in [p_b, p_b + q + L] \\ \text{or } p_a \in [p_b - q - L, p_b] \\ \& \frac{q}{L} \leq \frac{p_a - p_0}{p_b - p_a} \end{cases} \end{cases}$$

	Price	*Demand*
SB	$p_0 = \frac{L(1+2S)}{3}$	$D_0 = \frac{S}{2} + \frac{1-S}{3}$
Brand a	$p_a = \frac{L(1+2S)}{3} + \frac{q}{2}$	$D_a = \frac{S}{2}$
Brand b	$p_b = \frac{L(2+S)}{3}$	$D_b = 1 - \frac{1-S}{3}$

$$\text{Candidate 2}\begin{cases} p_0 \in [p_a - q, p_a] \\ p_0 \in [p_b - L, p_b] \\ p_a \in [p_b - q - L, p_b] \\ \frac{q}{L} \geq \frac{p_a - p_0}{p_b - p_a} \end{cases}$$

	Price	*Demand*
SB	-	-
Brand a	$p_a = \frac{(L-q)(1+2S)}{3}$	$D_a = S + \frac{1-S}{3}$
Brand b	$p_b = \frac{(L-q)(2+S)}{3}$	$D_b = 1 - \frac{1-S}{3}$

$$\text{Candidate 3}\begin{cases} p_0 \in [p_a - q, p_a] \\ p_0 \in [p_b, p_b + L - q] \\ p_a \in [p_b, p_b + L] \\ \frac{q}{L} \leq \frac{p_a - p_0}{p_0 - p_b} \end{cases}$$

	Price	*Demand*
SB	$p_0 = \frac{L(1+2S)}{3S}$	$D_0 = \frac{S}{2} - \frac{S-1}{3}$
Brand a	$p_a = \frac{L(1+2S)}{3S} + \frac{q}{2}$	$D_a = \frac{S}{2}$
Brand b	$p_b = \frac{L(2+S)}{3S}$	$D_b = 1 + \frac{S-1}{3}$

$$\text{Candidate 4}\begin{cases} p_0 \in [p_a - q, p_a] \\ p_0 \in [p_b, p_b + L] \\ p_a \in [p_b, p_b + L + q] \\ \frac{q}{L} \geq \frac{p_a - p_0}{p_0 - p_b} \end{cases}$$

	Price	*Demand*
SB	-	-
Brand a	$p_a = \frac{(L+q)(1+2S)}{3S}$	$D_a = S - \frac{S-1}{3}$
Brand b	$p_b = \frac{(L+q)(2+S)}{3S}$	$D_b = 1 + \frac{S-1}{3}$

Following an identical algorithm as in the case of high-quality rival, we find the following Nash equilibria:

1. Candidate 1 is a Nash equilibrium if $\frac{q}{L} < H_1^*(S) = Min\{\frac{8S^2+5S-4}{9S(1+S)}, \frac{4(1-S)}{3}\}$;

2. Candidate 2 is a Nash equilibrium if $\frac{q}{L} < H_2^*(S) = \frac{-8S^2-5S+4}{(2+S)^2}$;

3. Candidate 3 is a Nash equilibrium if $\frac{q}{L} < H_3(S) = \frac{-(4+4S+S^2-18S^3)+\sqrt{16-122S-120S^2-28S^3+325S^4}}{18S^2(1+S)}$

4. Candidate 4 is never a Nash equilibrium as firm A always wants to deviate to the demand structure given by candidate 3.

Chapter 2
Entry Bias in a Market with Asymmetric Costs

2.1 Introduction

A recent World Bank study[25] supports the idea that, in general, new entrants are more efficient than incumbents in markets from Eastern European transition economies[26]. Efficiency is measured by profit-to-sales ratio in three types of firms: state-owned enterprises, privatized firms and ”de novo”. The first two types may be considered as incumbent firms, while the last one represents the new entrants. For each type of firm, profit-to-sales ratio was divided into different categories: negative, zero or positive. The study shows that the proportion of firms with profit-to-sales ratio negative or zero is much bigger for the first two types of firms (state-owned and newly privatized) as compared to the last one (new entrants).

Moreover, the same study has noticed: ”After a decade of reform and the substantial privatization of the relatively large state-owned enterprises [...] the private sector in South Eastern Europe industries may still be insufficient to induce commensurate competitively structured markets”. The first policy recommendation given by this report to the countries in the region is to facilitate the entry of new competitors in order to increase business rivalry.

25 H.G. Broadman et al., 2004, ”Building Market Institutions in South Eastern Europe”

26 The report is focused on 8 countries in the region, namely: Albania, Bosnia and Herzegovina, Bulgaria, Croatia, Macedonia, Moldova, Romania and Serbia and Montenegro.

Hence, besides the fact that entrants are more efficient in the markets from the region, it has been observed that entry is insufficient and therefore should be encouraged.

There is plenty of evidence of public intervention to support entry of new firms in order to enhance competition. The European Bank for Reconstruction and Development is the largest investor in Eastern European region and uses its close relationship with governments to promote policies that bolster the business environment. Numerous cases are described in the press release of the Bank: ”EBRD boosts competition in Polish consumer finance”, ”EBRD supports competition in Estonian pension market” or ”Bank‘s investment will help promote competition and stimulate customer choice in the Czech Republic telecoms industry by facilitating the entry of a new international mobile operator in the market”.

The aim of this chapter is to provide a theoretical model to study under what circumstances public intervention to encourage entry is desirable. Surprisingly, previous literature points to a general tendency for excessive entry in homogeneous product markets. In a model with simultaneous entry by many identical firms, Mankiw and Whinston (1986) find the fundamental forces that lie behind this entry bias[27]. This requires a policy to stop entry rather than to facilitate it. Excessive entry was found typically by assuming that firms are symmetric or incumbents have some advantages[28]. Using our motivation from the tran-

[27] Suzumura and Kiyono (1987) also find excessive entry, but using a quasi-Cournot model.

[28] Apart from Mankiw and Whinston (1986), also Von Weizsäcker (1980) and Perry (1984) use models with many symmetric firms. They identify excessive entry in markets with economies of scale. Klemperer (1988) also finds excessive entry but he assumes that incumbents have certain advantages given by the switching costs consumers have to incur if they want to buy from the entrant. In a more recent paper, Nachbar, Petersen and Hwang (1998), using some intuition from Klemperer show that entry may become ”less excessive” if some of the incumbent’s costs are sunk, because this reduces accomodation (or, equivalently, business-stealing).

sition period in Eastern Europe, we claim that this may not be necessarily true. According to the World Bank study mentioned previously potential entrants that emerged in newly privatized markets were more efficient than the old incumbents. We propose to add to the theoretical literature on social efficiency of entry, by relaxing the symmetry assumption between incumbents and potential entrants and even allowing for certain advantages of the entrants[29]. Under these conditions, previous results in the literature may not hold and moreover, may be reversed.

Intuitively, in the symmetric case - best characterized in the literature by Mankiw and Whinston (1986) - marginal entry is more desirable to the entrant than it is to society because of the output reduction entry causes in other firms (business-stealing effect). We introduce a new effect: the asymmetry in costs between incumbent and potential entrant. When incumbent is more efficient than the entrant, excessive entry is even more severe, as the part of profit ”stolen” from the incumbent is replaced by a less efficient outcome. However, when the asymmetry is reversed and the entrant is more efficient, the profit ”stolen” from the incumbent is replaced by a better outcome. Then, from a certain level of cost advantage for the entrant, the marginal contribution of entry is higher for the society than for the entrant firm and, consequently, entry may become insufficient. In other words, entry by a less efficient firm increases average cost of the industry, which is bad for social welfare, whereas entry by a more efficient firm decreases average cost, which is good for welfare. The following figures clearly show our contribution to the literature. We denote

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[29] Other papers that deal with issues of asymmetries in the entry process are Ashiya (2000) and Thomas (2002). The former author allows for the simultaneous existence of both stronger and weaker potential entrants relative to the incumbent. He shows that an incumbent firm may allow entry of a weak firm and use it to alter the strong firm’s entry decision. The latter paper studies how Bertrand competition is affected by allowing firms to have different entry costs.

by Pm and Pd, monopoly and duopoly prices respectively, by Qm, the level of production in monopoly and, by Qi and Qe, the production levels of the incumbent and of the entrant respectively, if entry occurs.

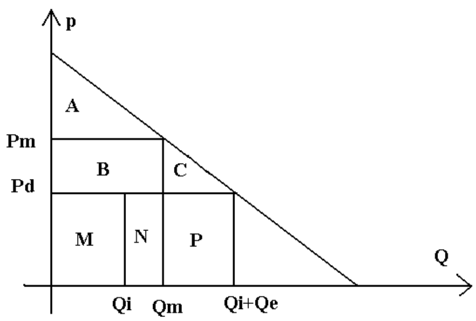

Figure 2.1

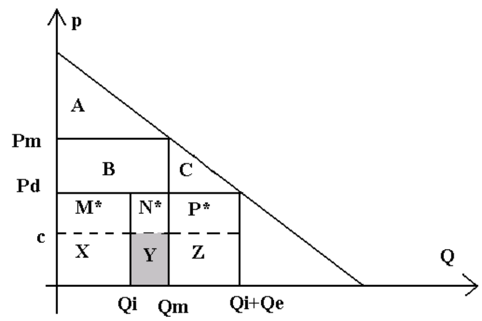

Figure 2.2

Figure *2.1* represents the symmetric case. For simplicity we take unit costs equal to 0. The gain in social welfare produced by entry is: $\Delta W = C + P$, and the profit of the entrant is: $\pi = N + P$. Mankiw and Whinston (1986) have shown that, under imperfect competition and business-stealing effect, the following holds:

$$C < N \tag{2.6}$$

This means that, there are cases when a positive set-up cost may be covered by the entrant but not by the change in welfare. This leads to excessive entry.

Figure 2.2 gives the intuition of the model we propose. When the incumbent is more efficient than the entrant (formally we take entrant's unit cost equal to c and incumbent's unit cost equal to 0), the gain in social welfare when entry occurs is: $\Delta W = C + P^* - Y$, while entrant's profit is: $\pi = N^* + P^* = N - Y + P^*$. If (2.6) holds, we can say that excessive entry still prevails (the gain in social welfare is still lower than entrant's profits).

This situation may be reversed when the entrant is more efficient than the incumbent (now we take incumbent‘s unit cost equal to c and entrant’s unit cost equal to 0). In this case, the social gain from entry is: $\Delta W = C + P^* + Y + Z$, while the profit of the entrant is: $\pi = N^* + P^* + Y + Z$. Notice that entrant’s profit is the same as in the symmetric case but the gain in welfare is higher with the area Y. If this area is sufficiently high (more exactly when $C > N^*$) - which means that the entrant is sufficiently more efficient than the incumbent - the entry bias is reversed: marginal entry is more desirable to the society than it is to the entrant firm itself, as it reduces considerably the average cost of the industry.

To resume, we state that, when the entrant is less efficient or a little bit more efficient than the incumbent markets may exhibit excessive entry, while in the case when the entrant is sufficiently more efficient than the incumbent, industries might be characterized by insufficient entry. Consequently some public intervention is needed.

As a policy instrument to correct the entry biases described above, we propose a subsidy/tax that brings the level of entry to its optimal value and does not depend on the entrant’s marginal cost. The only element we need to construct the policy is the set-up cost. This is industry-specific and, to our judgement, may be easier observed, by the social planner, than the marginal cost - which is rather firm-specific. Other authors that have studied different public policies for entry promotion and deterrence are: Dixit and Kyle (1985) and Kim (1997). The first paper, motivated by the Boeing-Airbus example, deals with the game between two governments that aim at regulating entry. The second article shows that public regulation aimed at preventing excess entry is globally suboptimal as it induces the incumbent to behave strategically against the government. Obviously, the focus of these papers is completely different from ours.

Finally, we examine the implications of product differentiation for the nature of entry biases. Previous literature on the topic used models of monopolistic competition and sustained that variety is good for social welfare. Spence[30] (1976) and Dixit and Stiglitz (1977) argued that entry is insufficient with respect to the social optimum. In order to study the effects of asymmetries between incumbent and potential entrant we use the two-firms model of product differentiation proposed by Sigh and Vives (1984). Remember that, in the homogeneous goods case, we proved that business-stealing by an entrant has a negative effect on welfare if the entrant is relatively less efficient than the incumbent (which results in excessive entry), and a positive effect if the entrant is sufficiently more efficient (insufficient entry). Now, if we sum up to the model the effect of product diversity, which is good for welfare, we expect to have a bigger area of parameters where entry is insufficient. We find that, even when the entrant is less efficient than the incumbent, for sufficiently high differentiation between products, insufficient entry may arise. The bias tends to what previous models of monopolistic competition predicted.

The remaining of this chapter is organized as follows. Section 2.2 deals with entry optimality when there is asymmetry between incumbent and potential entrant. Section 2.3 describes the optimal policy to regulate entry for both national and foreign investors. Section 2.4 shows that our analysis is robust to product differentiation and section 2.5 concludes the paper. We end with the list of references.

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30 See also Koenker and Perry (1981) that generalize Spence's model.

2.2 Cost asymmetry and entry bias

We aim here at providing simple, yet general conditions under which entry in an industry is excessive, insufficient or optimal. The analysis compares the levels of entry which are market efficient and socially optimal and results depend on the cost asymmetry between incumbent and potential entrant. In what follows, we will refer to the difference between market equilibrium and social optimum as, the entry bias. Excessive entry means positive bias while insufficient entry - negative one. When entry is optimal, there is no bias.

We consider a homogeneous product market with one incumbent and one potential entrant, denoted by i and e respectively. Should the entrant decide to get into the market it must incur a fixed set-up cost of $F \geq 0$. If entry is considered socially efficient/inefficient, government may subsidize/tax a proportion, σ, of the entry cost.

The cost function is linear for both firms and the differential between their marginal costs is c. Let us normalize incumbent's marginal cost to 0 and take entrant's marginal cost as c. The parameter c is allowed to take negative values with the interpretation that, if $c < 0$, the entrant is more efficient than the incumbent. In order to ensure strictly positive quantities in equilibrium, we restrict the analysis to the interval $c \in (-1, \frac{1}{2})$.

The inverse demand function is: $p = 1 - Q$, where Q is the aggregate quantity produced in the industry. If entry occurs, the two firms play a simultaneous Cournot game and produce quantities q_i and q_e, respectively. If not, incumbent acts as a monopolist in the market. Given these setting, firms' profits are, respectively:

$$\pi_i = pq_i$$
$$\pi_e = pq_e - cq_e - (1-\sigma)F$$

W is the social welfare computed as the sum between consumers' surplus and firms' profits. Superscripts d and m denote duopoly and monopoly situations respectively.

$$W^d = \int_0^Q (1-x)dx - cq_e - F$$
$$W^m = \int_0^Q (1-x)dx$$

Table 2.1 contains information on the equilibrium quantities, price, profits and welfare for both monopoly and duopoly.

Table 2.1: Equilibrium and welfare results

	Monopoly	Cournot
q_i	$q_i = \frac{1}{2}$	$q_i = \frac{1+c}{3}$
q_e	-	$q_e = \frac{1-2c}{3}$
Q	$Q = \frac{1}{2}$	$Q = \frac{2-c}{3}$
p	$p = \frac{1}{2}$	$p = \frac{1+c}{3}$
π_i	$\pi_i = \frac{1}{4}$	$\pi_i = \left(\frac{1+c}{3}\right)^2$
π_e	-	$\pi_e = \left(\frac{1-2c}{3}\right)^2 - (1-\sigma)F$
W	$W^m = \frac{3}{8}$	$W^d = \frac{11c^2-8c+8}{18} - F$

The results listed in Table 2.1 will be used in the following two subsections in order to compute the market efficient and the socially optimal levels of entry, respectively.

2.2.1 Market efficient level of entry

In this subsection we will determine the market efficient level of entry, with the purpose of comparing it with the socially optimal level of entry and find out sufficient conditions for entry to be excessive, insufficient or optimal.

Definition 1 *The market efficient level of entry, denoted by c^m, is the threshold value of marginal cost differential up to which entrant's profits are positive.*

It is determined by the free-entry condition:

$$\pi_e = 0$$

This is a quadratic equation in c, but, the only root belonging to our interval of analysis is:

$$c^m = \frac{1 - 3\sqrt{(1-\sigma)F}}{2}$$

Firms enter as long as $c < c^m$ and stay out otherwise. Notice that, we look for the level of entry not in terms of number of firms, as usual. Instead we consider levels of technologies of a given firm, expressed here by marginal costs.

2.2.2 The socially optimal level of entry

In order to decide whether entry is socially desirable or not, we compare the levels of welfare in monopoly and oligopoly respectively.

Definition 2 *The socially optimal level of entry, denoted by c^s, is the threshold value of cost differential up to which entry is socially desirable.*

For values of c below c^s entrant's technology is relatively good and entry is socially desirable (government may subsidize entry), whereas in the opposite case, technology is relatively costly, entry is socially inefficient and, as a consequence, it should be taxed.

Formally, entry is socially desirable as long as duopoly welfare is higher than monopoly one. In order to compute c^s, we need to solve the following quadratic equation:

$$W^d(c^s) = W^m$$

The only solution that belongs to our interval of analysis is:

$$c^s = \frac{8 - 3\sqrt{1 + 88F}}{22}$$

If $c \leq c^s$ entry is socially desirable, as $W^d(c) > W^m$, whereas when the opposite holds, monopoly is socially better. Notice that, as compared to the market efficient level of entry, c^s does not depend on σ. This means that the socially optimal level of entry is independent on the government intervention in the issue of entry.

2.2.3 Discussion of entry optimality

In this section, we will compare the two thresholds found in the previous subsections. As long as the market equilibrium is different from the social optimum ($c^m \neq c^s$), there exist an entry bias. If this bias is positive, namely if $c^m > c^s$, we say that entry is excessive as, in equilibrium, more firms want to enter than is socially efficient. In the opposite case, if

$c^m < c^s$, then the bias is negative and there is insufficient entry: according to social criteria, too few firms enter in equilibrium. Remember the expressions of the two thresholds we aim to compare:

$$c^m = \frac{1 - 3\sqrt{(1-\sigma)F}}{2}; c^s = \frac{8 - 3\sqrt{1+88F}}{22}$$

Let us plot the curves c^s and c^m, which are decreasing functions of the entry cost, F.

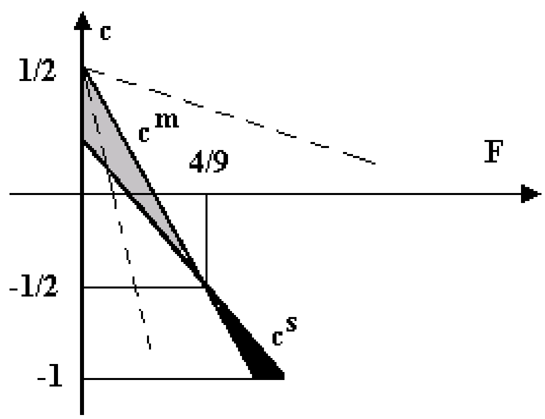


Figure 2.3:. Entry bias in a Cournot model

If we allow for entry regulation, curve c^m will move to the right or to the left with respect to the initial position, rotating around the point $(0, \frac{1}{2})$. As σ increases, curve c^m moves to the right until it gets to be parallel to the horizontal axis and, as σ decreases, curve c^m moves to the left, until it gets to coincide with the vertical axis. The socially efficient level of entry, c^s does not depend on σ. For a clear presentation of entry biases, let us ignore for the moment the entry subsidy/tax. For the present analysis we assume $\sigma = 0$, and leave for section 2.3 the discussion of entry regulation. The coordinates of the crossing point between the two curves are $(\frac{4}{9}, -\frac{1}{2})$.

The two colored regions formed by the curves c^m and c^s represent the sets of parameters where there exist an entry bias. Let us discuss each one in turn. The light region corresponds to the case $c^m > c^s$. We are in a situation of excessive entry: more firms would be willing to enter than is socially efficient. This case corresponds to relative low values of entry cost. Conversely, if $c^s > c^m$ we are in the dark region which corresponds to a situation of insufficient entry. In terms of entry cost, it is relatively high in this region, which makes entry less desirable from the firm's point of view. Outside the two regions discussed above, entry is exactly optimal. In the upper part it is neither socially desirable nor market efficient, whereas in the lower region entry is both socially desirable and market efficient.

We have found combinations of the two parameters of the model (fixed cost and the level of asymmetry in marginal costs between the entrant and the incumbent) for which entry biases may exist. The following proposition characterizes them.

Proposition 4 *Entry in a Cournot model with asymmetric costs is:*

i) excessive, if $\frac{8-3\sqrt{1+88F}}{22} < c < \frac{1-3\sqrt{(1-\sigma)F}}{2}$

ii) insufficient, if $\frac{1-3\sqrt{(1-\sigma)F}}{2} < c < \frac{8-3\sqrt{1+88F}}{22}$,

iii) optimal, otherwise

Notice that, when firms are symmetric (the case $c = 0$) if there is a bias, it is necessarily of excessive entry (within the light region), which confirms the results in Mankiw and Whinston (1986).

Let us now study the effect of entry regulation. As the level of subsidy increases, firms find themselves more and more interested to enter and the dark region of insufficient entry gets smaller and smaller until it disappears. On the contrary, when the subsidy decreases, it is more likely that firms are unwilling to enter as they are not able to cover by themselves entry costs. In this case, the region of insufficient entry becomes bigger.

In the next section we will find the optimal entry policy.

2.3 Entry regulation

In this section we look for the optimal policy of the government regarding entry. If we are in any other point (combination of c and F) outside the colored regions there is no conflict between government's interest and firm's action: both prefer either entry or non-entry. If we are in the light region (excessive entry), the government's interest is to stop entry (impose a tax), while if we are in the dark region (insufficient entry), it is in the interest of the government to encourage entry (by giving a subsidy).

The government can determine the optimal fiscal policy regarding entry even without knowing the value of cost differential between firms, c, by making the market threshold value equal to the socially optimal threshold:

$$c^m(\sigma) = c^s$$

This condition gives the optimal subsidy/tax to entry for any value of the entry cost, F:

$$\sigma = \frac{-2 + 33F - 2\sqrt{1 + 88F}}{121F}$$

With this tax/subsidy government manages to induce firms to do what is socially optimal. Notice that, σ is an increasing function of F. This is because, as we look at Figure 2.3, we observe that, the higher F the lower the region of excessive entry and the higher the region of insufficient entry. In other words, the more expensive it is to enter, the higher have to be the incentives to make firms to enter. Hence, for low values of F, we need an entry tax (negative σ) and for high values of F we need an entry subsidy (positive σ).

Figure 2.4, representing σ as a function of F shows that it is indeed a tax for $F < \frac{4}{9}$ and a subsidy otherwise[31].

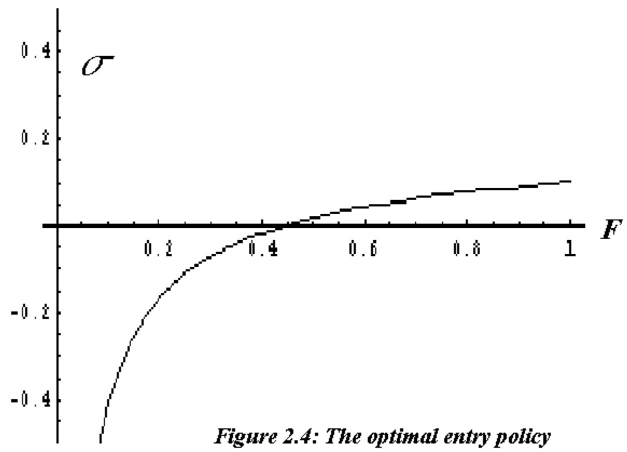


Figure 2.4: The optimal entry policy

This result is extremely important: it says that information on the entrant's marginal cost is irrelevant when deciding the optimal entry policy. Knowing the entry cost F (which is industry-specific) is sufficient to achieve this goal. No firm-specific information (like entrant's cost) is needed.

31 Remember that the curves c^m and c^s cross for $F = \frac{4}{9}$.

2.3.1 Foreign investor

In the model considered so far, the entire profit of the entrant is part of the national welfare. Nevertheless, this is not the case when the entrant is a foreign investor, as it happened in many countries in Eastern Europe, after the fall of the communism. National investors may lack of either funds or knowledge to start up a new firm, at least in the transition period. Therefore, we think it is worthwhile to extend the model and allow for the case when the entrant is a foreign firm and its profits will not be part of the national welfare. The duopoly welfare in the case of foreign investment, denoted by W^f, can be directly computed using the information in Table 2.1:

$$W^f = W^d - \pi^e = \frac{2 + c^2}{6} - \sigma F$$

Notice that, in this case, welfare depends on σ, which is the amount of entry subsidy/tax. In the initial case, W^d was independent of σ, as this was a direct transfer between two members of the same economy, namely the government and the entrant. Having computed W^f, we can now proceed and find the new threshold giving the socially optimal level of entry, which we denote c^f. The steps to follow are the same as in section 2.2. The result is the solution to the equation $W^f(c^f) = W^m$, namely:

$$c^f = -\frac{1}{2}\sqrt{1 + 24\sigma F}$$

The difference with respect to c^s is that c^f depends now on the entry policy, σ.

The threshold determining the market efficient level of entry, c^m is the same as in section 2.2.1., namely:

$$c^m = \frac{1 - 3\sqrt{(1-\sigma)F}}{2}$$

Figure 2.5 is similar to Figure 2.3 and represents the new entry bias with a foreign investor:

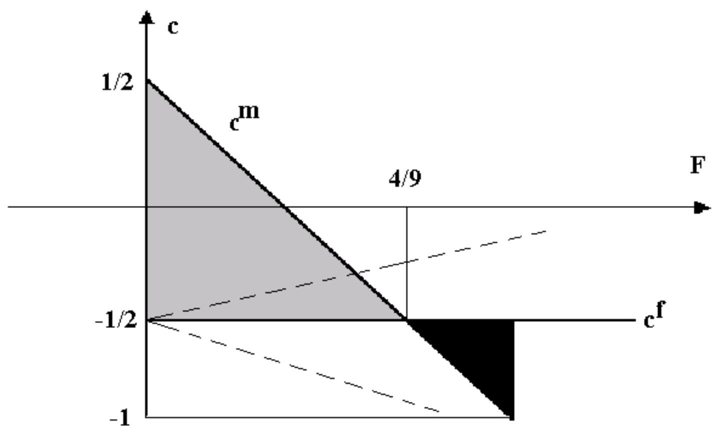


Figure 2.5: Entry bias with foreign investor

Again, this figure is drawn for $\sigma = 0$. Interestingly, the two curves cross in the same point as in figure 2.3. But now, not only c^m but also c^f varies with σ. More precisely, c^f rotates around the point $(0, -\frac{1}{2})$. The movement is upward when σ is negative and decreases and downward respectively, when σ is positive and increases. Nevertheless, irrespective of the sign of σ, the socially optimal level of entry is always negative. This means that, entry is socially desirable only for very efficient entrants relative to the incumbents.

The optimal entry policy is determined as in the case of national entrant, by solving the following quadratic equation in σ:

$$c^m(\sigma) = c^f(\sigma)$$

Between the two solutions, we will choose the one that gives σ as an increasing function of F, because, as stated in section 2.3, and also showed in Figure 2.5, the more expensive it is to enter (the higher F), the higher have to be the incentives to stimulate firms to enter.

Surprisingly, the optimal subsidy/tax to be paid/charged to the foreign entrant is exactly the same as the one imposed on the national entrant, namely:

$$\sigma = \frac{-2 + 33F - 2\sqrt{1 + 88F}}{121F}$$

Even though the optimal policy is the same, there are two differences with respect to the model with national entrant. First, the total welfare is lowered in the amount of entrant's profits and second, the two biased regions are bigger in the case of foreign investor. Geometrically, the two colored areas increase in figure 2.5 as compared to figure 2.3 because the curve c^m does not change and, moreover, c^f may be obtained by rotating downward the curve c^s around the crossing point of the two thresholds, that does not change either. Intuitively, this is because, for $\sigma = 0$, the foreign investor pays the entire entry cost (which makes entry more insufficient, as the incentives to enter decrease), or is not taxed at all (which makes entry more excessive, as the incentives to enter increase).

2.4 Optimal entry and product differentiation

Going back to the initial case with a national entrant, in this section we will check the robustness of the model by allowing for product differentiation with both quantity and price competition. To model product differentiation, we follow the paper of Singh and

Vives (1984). There are two firms in the industry, producing quantities q_i and q_e, and charging prices p_i and p_e, respectively. One of the firms is an incumbent (i) and the other a potential entrant (e). Demands come from the maximization problem of a representative consumer with separable utility function:

$$u(q_i, q_e) = q_i + q_e - \frac{q_i^2}{2} - \frac{q_e^2}{2} - \gamma q_i q_e$$

With this utility function, inverse demands are linear and given by:

$$\begin{aligned} p_i &= 1 - q_i - \gamma q_e \\ p_e &= 1 - q_e - \gamma q_i \end{aligned}$$

The parameter γ represents the degree of product differentiation, ranging from zero when the goods are independent to one when the goods are perfect substitutes. First, we assume that firms compete in quantities. Cost considerations are identical to the homogeneous product case, but we abstract from policy issues. Hence, there is no σ in this model. We only aim at checking the existence of different entry biases in this type of models.

Profits for the incumbent and the entrant respectively are, as follows:

$$\begin{aligned} \pi_i &= q_i p_i \\ \pi_e &= q_e(p_e - c) - F \end{aligned}$$

If entry occurs, in the equilibrium of the Nash-Cournot game, firms choose the following quantities:

$$
\begin{aligned}
q_i &= \frac{2-\gamma+\gamma c}{4-\gamma^2} \\
q_e &= \frac{2-\gamma-2c}{4-\gamma^2}
\end{aligned}
$$

If entry does not occur, the incumbent is a monopolist and he chooses, as in section 2.2:

$$q_i = \frac{1}{2}$$

2.4.1 Market efficient level of entry

As in section 2.2, we determine the threshold value of marginal cost differential up to which entry is efficient, by imposing the zero profit condition: $\pi_e = 0$.

In equilibrium, entrant's profit is:

$$\pi_e = \frac{(2c+\gamma-2)^2}{(\gamma^2-4)^2} - F$$

Then, the market efficient level of entry is:

$$c^m(F,\gamma) = 1 - \frac{\gamma + \sqrt{F}(4-\gamma^2)}{2}$$

2.4.2 Socially efficient level of entry

Entry is socially efficient as long as welfare is higher in duopoly than in monopoly. We find the threshold value of c, up to which this condition holds.

Social welfare equals the utility of the representative consumer minus total costs:

$$W(q_i, q_e) = (q_i + q_e - \frac{q_i^2}{2} - \frac{q_e^2}{2} - \gamma q_i q_e) - c * q_e - F$$

In order to find the socially optimal level of entry we proceed as in the homogeneous case, by solving the following equation in c,

$$W_d(c^s) = W_m$$

In equilibrium, duopoly welfare is:

$$W_d = -\frac{c^2(\gamma^2 - 12) + 2c(\gamma - 2)^2 + 2(\gamma - 2)^2(-3 - \gamma)}{2(\gamma^2 - 4)^2} - F$$

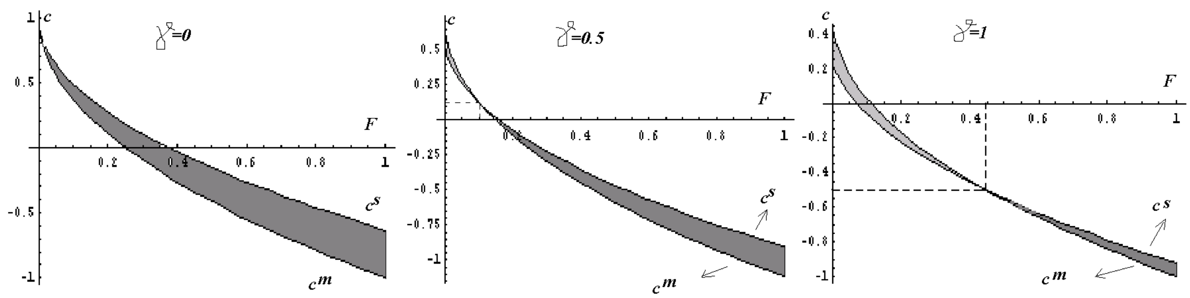
Monopoly welfare is the same as in the homogeneous case: $W_m = \frac{3}{8}$.

Then, the socially optimal level of entry is:

$$c^s(F, \gamma) = \frac{-24 + 16\gamma + 2\gamma^2 - 2\gamma^3 + \sqrt{\gamma^6(1 - 8F) + 8\gamma^4(20F - 1) + 16\gamma^2(1 - 56F) + 1536F}}{2(\gamma^2 - 12)}$$

2.4.3 Results for quantity competition

Before formally state the result, let us draw some intuition from the following simulations. We draw the curves $c^m(F)$ and $c^s(F)$ for different values of γ. Consumers like differentiation (a low γ), therefore, we expect to find more often insufficient entry, as compare to the case of homogeneous goods. The light regions represent areas of excessive entry, while the dark regions, areas of insufficient entry.



Figures 2.6: Entry bias in a differentiated product market with quantity competition

The case $\gamma = 0$ corresponds to independent goods (maximum level of differentiation). We observe that entry is insufficient for the whole range of c we consider. This means that the positive effect of product diversity dominates business-stealing, even when the entrant is very inefficient. Indeed, there is more insufficient entry than in the case of homogeneous goods.

The other extreme case, $\gamma = 1$, corresponds to perfect substitutes. Here, the product diversity effect disappears and the result is identical to what we have obtained in the homogeneous product case, analysed in section 2.2.

Let us denote by c^* the value of the cost differential where the curves c^s and c^m cross. Explicitly,

$$c^*(\gamma) = \frac{2 - 3\gamma}{2}$$

Excessive entry may arise above c^* and insufficient entry below c^*. The following proposition states the formal results:

Proposition 5 *Entry in a market with differentiated products is:*

i) excessive, when $c^s(F,\gamma) < c < c^m(F,\gamma)$

ii) insufficient, when $c^s(F,\gamma) > c > c^m(F,\gamma)$,

iii) optimal, otherwise.

Moreover, if c^* *is the value where the two curves cross, the higher the product differentiation (the lower* γ*), the higher* c^* *(the lower the region of excessive entry and the higher the region of insufficient entry):*

$$\frac{\partial c^*}{\partial \gamma} < 0$$

Proposition 5 says that, in this setting, the magnitude and sign of the entry bias is the result of two factors: the level of asymmetry between the technologies of the two firms and the level of product differentiation. On the one hand, for a given level of asymmetry between firms, the more differentiated goods are, the better entry is for social welfare and hence, more often insufficient entry may arise. And, on the other hand, for a given level of differentiation between the products of the two firms, the more efficient the entrant is relative to the incumbent, the better entry is for social welfare and hence, the larger the region of insufficient entry. Hence, results go into the same direction as in our homogeneous product model. For the particular case $\gamma = 1$, we have $c^* = -\frac{1}{2}$, which was the result obtained in section 2.2.

2.4.4 Price competition

Some words need to be said for the case when firms compete in prices instead of quantities. Following Singh and Vives (1984) we know that, in terms of total surplus, the Bertrand equilibrium dominates the Cournot one. This means that, entry is more desirable for society in the Bertrand case - curve c^s moves upward. Moreover, as Bertrand competition is more aggressive than the Cournot one, profits diminish and firms will have fewer incentives to enter - curve c^m moves downward. If we bring together the two effects, we find that, in the case of price competition, the areas of excessive entry should diminish with respect to what we had in the case of quantity competition. Actually, for $\gamma = 0.5$, the area of excessive entry disappears completely and, for all values of c, the entry bias is that of insufficient entry[32]:

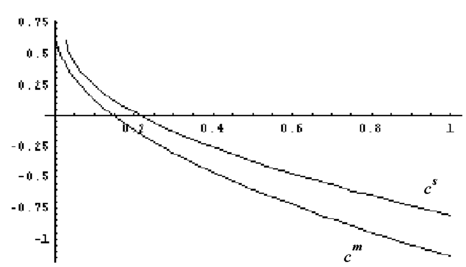


Figure 2.7: Product differentiation and price competition

2.5 Conclusions

Previous articles have shown that free entry may be socially inefficient. Many authors have pointed to a general tendency for excessive entry in homogeneous product markets when

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[32] Algebra for this case is available upon request.

firms are symmetric or incumbents have certain advantages. We identify situations when entrants are relatively more efficient than incumbents, in which case results of the previous literature may be reversed. Covering a wide range of asymmetries between firms we characterize the entry bias between the market equilibrium and the social optimum. We show that, when the entrant is less efficient or just a little more efficient relative to the incumbent, markets exhibit excessive entry, while in the case when the entrant is sufficiently more efficient, industries are characterized by insufficient entry. Furthermore, we find an innovative tool to regulate entry which does not require firm-specific information. Finally, we extend the analysis to product differentiation and characterize entry biases in the model proposed by Singh and Vives (1984) where, again, we introduce cost asymmetries between firms. Even though the regions of entry bias are changed by the effect of product diversity, results follow the same tendencies as in the homogeneous product case.

2.6 References

Ashiya, M., 2000, ”Weak entrants are welcome”, *International Journal of Industrial Organization*, 18(6), 975-984.

Broadman, H.G., J. Anderson, C.A. Claessens, R. Ryterman, S. Slavova, M. Vagliasindi and G.A. Vincelette, 2004, ”Building Market Institutions in South Eastern Europe”, *The World Bank, 29301, Directions in Development*

Dixit, A.K. and Kyle, A.S., 1985, ”The Use of Protection and Subsidies for Entry Promotion and Deterrence”, *American Economic Review*, 75(1), 139-152.

Dixit, A.K. and Stiglitz, J.E., 1977, ”Monopolistic Competition and Optimum Product Diversity”, *American Economic Review*, 67(3), 297-308.

Kim, Jaehong, 1997, ”Inefficiency of subgame optimal entry regulation”, *RAND Journal of Economics*, 28(1), 25-36.

Klemperer, P. ,1988, ”Welfare effects of entry into markets with switching costs”, *The Journal of Industrial Economics*, 37(2), 159-165.

Koenker, R. W. and Perry, M.K., 1981, ”Product differentiation, monopolistic competition, and public policy”, *The Bell Journal of Economics*, 12(1), 217-231.

Mankiw, N.G. and Whinston, M.D., 1986, ”Free entry and social inefficiency”, *RAND Journal of Economics,* 17(1), 48-58.

Nachbar, J.H., Petersen, B. C. and Hwang, I., 1998, ”Sunk costs, accommodation, and the welfare effects of entry”, *The Journal of Industrial Economics*, 46(3), 317-332.

Perry, M.K., 1984, ”Scale economies, imperfect competition, and public policy”, *The Journal of Industrial Economics*, 32(3), 313-333.

Singh, N. and Vives, X., 1984, ”Price and Quantity Competition in a Differentiated Duopoly”, *RAND Journal of Economics*, 15(4), 546-54.

Spence, M., 1976, ”Product Selection, Fixed Costs, and Monopolistic Competition”, *Review of Economic Studies*, 43(2), 217-235.

Suzumura, K. and Kiyono, K., 1987, ”Entry Barriers and Economic Welfare”, *Review of Economic Studies*, 54(1), 157-167.

Thomas, C., 2002, ”The effect of asymmetric entry costs on Bertrand competition”, *International Journal of Industrial Organization*, 20(5), 589-609.

von Weizsäcker, C.C., 1980, ”A welfare analysis of barriers to entry”, *Bell Journal of Economics*, 11(2), 399-420.

Chapter 3
Are Loyalty-Rewarding Pricing Schemes Anti-Competitive?

3.1 Introduction

In some markets sellers discriminate between first time and repeat buyers. In some cases they provide a coupon along with the good, which can be used as a discount in the next purchase of the same product (repeat-purchase coupons). In other cases sellers set a high initial price and commit to lower prices for subsequent purchases (sports clubs, mortgages). Most airlines have set up frequent-flyer programs (FFPs) that offer registered travelers free tickets or free class upgrades after a number of miles has been accumulated.[33] Similar programs are also run by car rental companies, supermarket chains and other retailers.

The specific details vary substantially from one market to another, but in all these examples repeat buyers receive a better treatment than first-time buyers. What is less obvious is to what extent firms commit to the price of repeat purchases. For instance, in the case of FFPs, travelers may gain the right to "buy" a ticket at zero price, but they can also use these miles to upgrade the ticket, in which case the net price is left ex-ante undetermined.[34] In

[33] Frequent flyer programs seem to be more popular than ever. In fact, according to The Economist (January 8th, 2005, page 14) "the total stock of unredeemed frequent-flyer miles is now worth more than all the dollar bills in circulation around the world". The same article also mentions that unredeemed frequent flyer miles are a non-negligible item in some divorce settlements!

The reader can visit www.webflyer.com for more detailed information on the volume and specific characteristics of some of these programs.

[34] Airlines also impose additional restrictions, like blackout dates, which are sometimes modified along the way.

the case of repeat-purchase coupons, discounts can take various forms (proportional, fixed amount or even more complex), and again there is no specific commitment to a particular price.

What are the efficiency and distributional effects of those loyalty rewarding programs? Do they enhance firms' market power? Should competition authorities be concerned about the proliferation of those schemes?

Many economists and policy analysts seem to believe that these programs are anti-competitive, in the sense that they benefit firms and hurt consumers. Those believes are probably based on a combination of casual observations, a limited amount of systematic empirical evidence, and fragmented economic intuition. From an empirical point of view, the attention has been focused on the air transport industry. FFPs were first introduced by major US airlines right after deregulation and they were interpreted as an attempt to isolate themselves from competition. Early empirical studies could only find weak evidence of the influence of loyalty programs on the pattern of repeat purchases.[35] More recently, Lederman (2003) has reported significant effects of FFPs on market shares, after controlling for other relevant factors. In particular, she shows that enhancements to an airline's FFP, in the form of improved partner earning and redemption opportunities, are associated to increases in the airline's market share. Moreover, those effects are larger on routes that depart from airports at which the airline is more dominant. She interprets these results as indicating that FFP reinforces firms' market power.

[35] See Sharp and Sharp (1997).

On the other hand, various pieces of economic intuition might have played a role in forming those believes. Cairns and Galbraith (1990) showed that, under certain circumstances, FFP-type of policies could be an effective barrier to entry. Banerjee and Summers (1987) and Caminal and Matutes (1990) (CM, from here after) analyzed the effect of various loyalty-rewarding policies in duopoly models and their results have been interpreted as analogous to those of the exogenous switching costs literature (Klemperer, 1995). It is immediate that because of these policies consumers are partially locked-in, and hence they may remain loyal even when switching is ex-post efficient. Their effect on consumer welfare is less straightforward. Banerjee and Summers (1987) did show that lump-sum coupons are likely to be a collusive device and hence consumers would be better off if coupons were forbidden. However, Caminal and Matutes (1990), argued that the specific form of the loyalty program might be crucial. In particular, if firms are able to commit to the price they will charge to repeat buyers, then competition is enhanced and prices are reduced. However, in their model lump-sum coupons tend to relax price competition, which was a result very much in line with those of Banerjee and Summers (1987). Hence, the desirability of such programs from the point of view of consumer welfare seemed to depend on the specific details, which in practice may be hard to interpret.

In this chapter we attempt to improve our understanding of these issues. We try to make progress by introducing four innovations. Firstly, we extend the standard Hotelling model to allow for an arbitrary number of symmetric firms. In fact, most of the attention is devoted to the case of a large number of monopolistically competitive firms. In a dynamic oligopoly model, the price chosen by an individual firm affects future prices set by rival firms. Obviously, in a duopoly such a strategic commitment effect is magnified, but as the number of firms increases this effect vanishes. Hence, it is worthwhile studying the limiting case (monopolistic competition) where such an effect has been shut down completely. However, even when the dynamic strategic effect is absent, policies that reward consumer loyalty create an intertemporal link. Thus, in a monopolistically competitive framework we can still focus on the time inconsistency problem faced by individual firms and the commitment value of alternative pricing schemes.

Our second innovation has to do precisely with the set of commitment devices. When firms cannot or do not want to (perhaps because of uncertainty about future demand or costs) commit to future prices, then we show that a simple discounting rule can actually be sufficient to implement the ex-ante optimal plan. Thirdly, we ask about the interaction between endogenous and exogenous switching costs. In particular, we ask whether firms have more or less incentives to introduce loyalty rewarding schemes whenever consumers are already partially locked-in for exogenous reasons. In other words, we ask whether endogenous and exogenous switching costs are complements or substitutes. Fourthly, we extend the analysis to an overlapping generations framework, where firms are unable to distinguish between former customers of rival firms and consumers that have just entered the market. Finally, we discuss more informally some other issues, like the role of firms' relative sizes, barriers to entry, or firms' incentives to join a partnership.

The main result of this paper is that loyalty rewarding pricing schemes are basically a business-stealing device, and as a result they enhance competition: average prices are reduced and consumer welfare is increased. The introduction of a consumer loyalty program is a dominant strategy for each firm (provided these programs involve sufficiently small administrative costs) but in equilibrium all firms loose (prisoner's dilemma). This result is compatible with the empirical evidence reported by Lederman (2003), which has been summarized above.[36] In fact, in the asymmetric oligopoly version of the model it can be shown that the introduction of loyalty programs by all firms raises the market share of the large firm. As in the symmetric case, those programs benefit consumers and hurt firms, although the reduction in profits is relatively smaller for larger firms.

In our benchmark two-period model firms compete for a single generation of consumers. Each firm produces a different variety of the good. Each consumer derives utility only from two varieties and consumers are symmetrically distributed among all possible pairs. Those consumers that derive utility from a particular pair are uniformly located in the $[0,1]$ interval, and relative distances to the extremes represent their relative valuations (transportation costs). Consumers derive utility from the same pair of varieties in both periods, but their location in each period are independent draws. When the number of varieties goes to infinity the model resembles the case of monopolistic competition. Hence, the current price of an individual firm does not affect its rivals' future prices. In this case, the best deal that a firm can offer to their potential consumers in the first period includes a commitment to a price equal to marginal cost for repeat purchases. The reason is that such a rule maximizes the joint payoffs of an individual firm and its first period consumers, since it induces consumers to repeat purchase from the same supplier every time their reservation

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[36] To the best of our knowledge there is no systematic evidence on the effect of loyalty programs on firm profitability. Lederman (2003) constructs an index of the average fare charged by each airline. These indeces do not seem to include the zero price tickets used by frequent flyers. She shows that an enhacement of the airline's FFP raises its own average fare, which is again compatible with the predictions of our model.

price is above the firm's opportunity cost. The rents created by committing to a price equal to marginal cost for repeat purchases can be appropriated by the firm through a higher first period price, and hence this will be part of the firm's optimal strategy. [37] This result is analogous to the one obtained by Crémer (1984) in a model of experience goods.[38] However, from a social point of view, those commitment strategies distort the ex-post allocation of consumers and average transportation costs increase. As a result, a firm that commits to marginal cost pricing to repeat buyers also set lower average prices, in order to compensate for the increase in transportation costs. Nevertheless, the firm obtains higher profits because a larger fraction of first period consumers remain loyal in the second period. Hence, the introduction of such a pricing scheme by an individual firm has a negative externality on other firms by making it harder for them to attract new customers in the second period, which induces them to set lower prices. As a result, the ability to discriminate between new customers and repeat buyers increases transportation costs but enhances competition (consumers are better off).

If firms can only set in the first period the second period price of repeat purchases but not the second period price charged to newcomers then they face a time inconsistency problem. The reason is that a higher second period price for newcomers makes the offer of rival firms in the first period less attractive and increases the firm's first period market share. If firms set the second period price for newcomers disregarding its effect on the first period market share then they will choose a lower price than in the case they enjoy

[37] First period consumers buy a bundle: one unit of the good plus an option to buy a unit of the good in the second period at a predetermined price.

[38] See also Bulkley (1992) for a similar result in a search model, and Caminal (2004) in cyclical goods model.

full commitment power. However, such a time inconsistency problem does not alter the qualitative properties of the equilibrium. In either case firms set a price equal to marginal cost for repeat purchases and in equilibrium average prices are lower than in the case firms cannot price discriminate between newcomers and repeat buyers.

Under some circumstances firms may not be able or may not wish to commit to the price for repeat purchases. For instance, perhaps the specific characteristics of the future good are not well known in advance, or more generally there is uncertainty about demand or costs conditions. In a model with certainty we show that commitment to a simple discounting rule (a combination of proportional and lump-sum discounts) is equivalent to committing to future prices for both repeat buyers and newcomers. Therefore, as a first approximation, coupons are actually equivalent to price commitment. In other words, the focus of the previous literature on lump-sum coupons was highly misleading, especially in combination with the strategic commitment effect present in duopoly models.

We also pay attention to the interactions between exogenous and endogenous switching costs. As discussed in Klemperer (1995), switching consumers often incur in transaction costs (closing a bank account) or learning costs (using for the first time a different software). These kinds of switching costs are independent of firms' decisions. If firms can use loyalty rewarding pricing schemes then average prices and firm profits decrease with the size of these exogenous switching costs. The same result arises when firms cannot discriminate between repeat buyers and newcomers, although the mechanism is completely different. We also show that the presence of exogenous switching costs reduces firms' incentives to introduce artificial switching costs. That is, when consumers are relatively immobile for exogenous reasons the ability of loyalty rewarding pricing schemes to affect consumer behavior is reduced.

Finally, we embed the benchmark model in an overlapping generations framework in order to consider the more realistic case where firms cannot distinguish between consumers that just entered the market and consumers with a history of purchases from rival firms. More specifically, firms set for each period a price for repeat buyers (those who bought in the past from the same supplier) and a regular price (for the rest). We show that there is a stationary equilibrium with features similar to those of the benchmark model. In particular, average prices are also below the case in which firms cannot price discriminate between newcomers and repeat buyers. The main difference with the benchmark model is that firms set the price for repeat buyers above marginal cost (but below the regular price). The reason is that the regular price is not only the instrument to collect the rents generated by a reduced price for repeat purchases, but is also the price used to attract consumers who previously bought from rival firms. Hence, firms are not able to fully capture all these rents and hence are not willing to maximize the efficiency of the long-run customer relationship.

This chapter is organized as follows. The next section presents the benchmark model. Section 3.3 contains a preliminary discussion of the main effects. A more formal analysis of the benchmark model can be found in Section 3.4. The next section considers the form of commitment, discounting versus price level. In Section 3.6 we study the interaction between endogenous and exogenous switching costs and in Section 3.7 we present the overlapping generations framework and the main results. Section 3.8 discusses possible extensions and Section 3.9 includes some concluding remarks. The paper closes with the Appendix preceded by the list of references.

3.2 The benchmark model

This is essentially a two-period Hotelling model extended to accommodate an arbitrary number of firms and, in the limit, it can be interpreted as a model of monopolistic competition.

There are n firms (we must think of n as a large number) each one produces a variety of a non-durable good. Varieties are indexed by i, $i = 1, ..., n$. Demand is perfectly symmetric. There is a continuum of consumers with mass $\frac{n}{2}$. Each consumer derives utility only from two varieties and the probability of all pairs is the same. Thus, the mass of consumers who have a taste for variety i is 1, and $\frac{1}{n-1}$ have a taste for varieties i and j, for all $j \neq i$. Consumers are also heterogeneous with respect to their relative valuations. In particular, a consumer who has a taste for varieties i and j, is located at $x \in [0, 1]$, which implies that her utility from consuming one unit of variety i is $R - tx$, and her utility from consuming one unit of variety j is $R - t(1 - x)$. Those consumers who value varieties i and j are uniformly distributed on $[0, 1]$. Thus, if $n = 2$ then this is the classic Hotelling model. If $n > 2$ firm i competes symmetrically with the other $n-1$ firms. If n is very large the model resembles monopolistic competition, in the sense that each firm: (i) enjoys some market power, and (ii) is small with respect to the market, even in the strong sense that if one firm is ejected from the market then no other firm is significantly affected. As usual we also assume that R is sufficiently large, so that all the market is served in equilibrium.

This model is related to the "spokes" model of Chen and Riordan (2004). The main difference is that in their model all consumers have a taste for all varieties. In particular, a consumer located at x, $x \in \left[0, \frac{1}{2}\right]$, in line i pays transportation cost tx if she purchases from firm i, and $t(1-x)$ if she buys from any firm $j \neq i$. Hence, firms are not small with respect to the market, in the sense that an individual firm is able to capture the entire market by sufficiently lowering its price. Thus, their model can be interpreted as a model of non-localized oligopolistic competition, rather than a model of monopolistic competition.

Each consumer derives utility from the same pair of varieties in both periods, although her location is randomly and independently chosen in each period. Marginal production costs is $c \geq 0$.

Both firms and consumers are risk neutral and neither of them discount the future. Thus, their total expected payoff at the beginning of the game is just the sum of the expected payoffs in each period.

3.3 Preliminaries

Let us consider the case $t = 1$ and $c = 0$ and suppose that only one firm can discriminate in the second period between old customers (those who bought from that firm in the first period) and newcomers (those consumers who patronized other firms), while the rest cannot tell these two types of consumers apart. In equilibrium non-discriminating firms will set in both periods the price of the static game, i.e., if we let subscripts denote time periods then we have $\overline{p}_1 = \overline{p}_2 = 1$. Let us examine the alternatives of the firm which is able to price discriminate. In case such a firm does not use its discriminatory power, then it will find it optimal to imitate its rivals and set $p_1 = p_2 = 1$. It will attract a mass of consumers equal to one half in each period, and hence it will make profits equal to $\frac{1}{2}$ in the first period, and the same in the second, i.e., $\frac{1}{4}$ from repeat buyers and $\frac{1}{4}$ from new customers.

Suppose instead that the discriminating firm commits in the first period to a pair of prices (p_1, p_2^r) , where p_1 is the price charged for the first period good, and p_2^r is the price charged in the second period only to repeat buyers. In this case we are assuming that the ability to commit is only partial, since the firm cannot commit to the second period price for newcomers . In fact, the discriminating firm will also charge to new customers in the second period a price $p_2^n = 1$, since the market is fully segmented and the firm will be on its reaction function. The firm's commitment is an option for consumers, who can always choose to buy in the second period from rival firms. Thus, p_1 is in fact the price of a bundle, one unit of the good in the first period plus the option to repeat trade with the same supplier at a predetermined price.

We can now ask what is the value of p_2^r that maximizes the joint payoffs of the discriminating firm and its first period customers. Clearly, the answer is $p_2^r = 0$, i.e., marginal cost pricing for repeat buyers. In other words, the optimal price, from the point of view of the coalition of consumers and a single firm, is the one that induces consumers to revisit the firm if and only if consumers' willingness to pay in the second period is higher than or equal to the firm's opportunity cost. Moreover, the discriminating firm will in fact be willing to set $p_2^r = 0$ because it can fully appropriate all the rents created by a lower price for repeat buyers. More specifically, if the firm does not commit to the price for repeat buyers then a consumer located at x who visits the firm in the first period will obtain a utility

$U^{nc} = R - 1 - x + R - 1 - \frac{1}{4}$. That is, she expects to pay a price equal to 1 in both periods, but expected transportation costs in the second period are $\frac{1}{4}$. Instead, if the firm commits to $p_2^r = 0$ then the same consumer gets $U^c = R - p_1 - x + R - \frac{1}{2}$. That is, in the first period she pays the price p_1 but in the second period with probability 1 the consumer will repeat supplier (maximum transportation cost is equal to the price differential) and pays the committed price $p_2^r = 0$ and the expected transportation cost $\frac{1}{2}$. Hence, independently of their current location, consumers' willingness to pay have increased by $\frac{3}{4}$ because of the commitment to marginal cost pricing for repeat buyers ($U^c - U^{nc} = \frac{7}{4} - p_1$). Hence, the first period demand function of the discriminating firm has experience an upwards parallel shift of $\frac{3}{4}$. Thus, if the firm were to serve half of the market (like in the equilibrium without price discrimination) then $p_1 = \frac{7}{4}$. As a result, profits would be equal to $\frac{7}{8}$ in the first period (which is higher than the level reached in the absence of discrimination, $\frac{3}{4}$) and $\frac{1}{4}$ in the second period (from newcomers). In fact, the optimal first period price is even lower, $p_1 = \frac{13}{8}$, which implies that the first period market share is higher than one half, and total profits are equal to $\frac{145}{128}$ (profits increase by $\frac{17}{128}$ because of the commitment to $p_2^r = 0$).

The intuition about the incentives to commit to marginal cost pricing for repeat buyers is identical to the one provided by Crémer (1984).[39] Note, however, that the average price paid by a loyal consumer at the discriminating firm is lower, $\frac{13}{16}$ instead of 1, and no consumer is made worse off. The main reason behind lower average prices is that under commitment to p_2^r consumers incur into higher expected transportation costs. Also, profits are higher mainly because the firm is able to retain in the second period a larger proportion

39 See also Bulkley (1992) and Caminal, (2004) for the same result in different set-ups.

of first period customers, which enhances consumers' willingness to pay in the first period. Summarizing, when a single firm commits to the price for repeat buyers then, on the one hand, consumer surplus increases and, on the other hand, this creates a negative externality to rival firms (a business stealing effect).[40]

Most of these intuitions will be present below in the analysis of games where all firms are allowed to price discriminate between old customers and newcomers. Strategic complementarities will exacerbate the effects described in this section and as a result consumers will be better off than in the absence of price discrimination although overall efficiency will be reduced (higher transportation costs).

At this point it is important to note that the result about marginal cost pricing only holds under specific circumstances. Our benchmark model includes some special assumptions. One of them is that the first period price is paid only by a new generation of consumers who have just entered the market and face a two-period horizon. As a result, all the rents created by marginal cost pricing in the second period can be fully appropriated by the firm through the first period price. This is why the firm is willing to offer a contract that includes marginal cost pricing in the second period. In Section 3.7 we discuss in detail the importance of this assumption. For now it may be sufficient to think of the case that a fraction of first period revenues are taxed away. In this case, the firm cannot fully appropriate

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[40] In fact, the firm would like to sell the option to buy in the second period at a price equal to marginal cost, separately from the first period purchase. However, transaction costs associated to such a marketing strategy could be prohibitive. Ignoring those transaction costs, the firm would charge a price equal to $\frac{3}{4}$ for the right to purchase at a price equal to zero in the second period and a price $p_1 = 1$ for the first period purchase. The entire potential customer base would buy such an option and hence total profits would be $\frac{5}{4}$ which is above the level reached by selling the option to first period buyers only, $\frac{145}{128}$.

all the rents and as a result p_2^r will be set above marginal costs, but below the price charged to newcomers.

3.4 Symmetric commitment to the price of repeat purchases

3.4.1 The full commitment game

Let us start with a natural benchmark. Suppose that each firm sets simultaneously in the first period the three prices (p_1, p_2^r, p_2^n) , where the notation has been introduced in the previous section. If we denote with bars the average prices set by rival firms, then second period market shares among repeat buyers and newcomers, x_2^r, x_2^n, are given respectively by:

$$x_2^r = \frac{t + \overline{p}_2^n - p_2^r}{2t} \tag{3.7}$$

and

$$x_2^n = \frac{t + \overline{p}_2^r - p_2^n}{2t} \tag{3.8}$$

Finally, the first period market share, x_1, is given by:

$$p_1 + tx_1 + x_2^r \left(p_2^r + \frac{tx_2^r}{2} \right) + (1 - x_2^r) \left(\overline{p}_2^n + \frac{t\,(1 - x_2^r)}{2} \right) = \tag{3.9}$$

$$= \overline{p}_1 + t\,(1 - x_1) + x_2^n \left(p_2^n + \frac{tx_2^n}{2} \right) + (1 - x_2^n) \left(\overline{p}_2^r + \frac{t\,(1 - x_2^n)}{2} \right)$$

The optimization problem of a firm consists of choosing (p_1, p_2^r, p_2^n) in order to maximize the present value of profits:

$$\pi = (p_1 - c)\, x_1 + x_1 x_2^r \,(p_2^r - c) + (1 - x_1)\, x_2^n \,(p_2^n - c) \tag{3.10}$$

The next proposition summarizes the result (some computational details are given in the Appendix):

Proposition 6 *There is a unique symmetric Nash equilibrium of the full commitment game, which is described in the second column of Table 3.1.*

The first column of Table 3.1 (see Appendix) shows the equilibrium of the game in which firms cannot discriminate between repeat buyers and newcomers. In this case all prices in both periods are equal to $c + t$, all market shares are equal to $\frac{1}{2}$, and hence total surplus is maximized (the allocation of consumers is ex-post efficient). If we compare the first two columns we note that:

Remark 2 *In the equilibrium under full commitment consumers are better off and firms are worse off than in the absence of commitment.*

Finally, when firms can discriminate between repeat buyers and newcomers total surplus is lower because of the higher transportation costs induced by the endogenously created switching costs.

Thus, the possibility of discriminating between repeat buyers and newcomers makes the market more competitive with average prices dropping far below the level prevailing in the equilibrium without discrimination. Firms offer their first period customers an 'efficient' contract, in the sense of maximizing their joint payoffs, which includes a price equal to marginal cost for their repeat purchases in the second period. Such loyalty rewarding scheme exacerbates the fight for customers in the second period and induces firms to charge relatively low prices for newcomers. Since firms make zero profits from repeat purchases but also low profits out of second period newcomers, their fight for first period customers is only slightly more relaxed than in the static game. The other side of the coin is that consumers' valuation of the option included in the first period purchase is relatively moderate. All this is reflected in first period prices which are only slightly above the equilibrium level of the static game.

It is important to note that p_2^n is above the level that maximizes profits from newcomers in the second period (see below). The reason is that by committing to a higher p_2^n the firm makes the offer of their rivals less attractive, i.e., from equation 3.9 we have that $\frac{dx_1}{dp_2^n} > 0$.

3.4.2 The partial commitment game

In the real world sometimes firms sign (implicit or explicit) contracts with their customers, which include the prices prevailing in their future transactions. However, it is more difficult to find examples in which firms are able to commit to future prices that apply to new customers.

Let us consider the game in which firms choose (p_1, p_2^r) in the first period, and p_2^n is selected in the second period after observing x_1 and $\overline{p}_{2.}^r$.

The next result shows that the equilibrium strategies of Proposition 6 are not time consistent (intermediate steps are specified in the Appendix).

Proposition 7 *There is a unique subgame perfect and symmetric Nash equilibrium of the partial commitment game, which is described in third column of Table 3.1.*

The equilibrium of the partial commitment game also features marginal cost pricing for repeat buyers, since the same logic applies. However, the equilibrium value of p_2^n is now lower than in the equilibrium of the full commitment game. The reason is that p_2^n is chosen in the second period in order to maximize profits from second period newcomers. Hence, firms disregard the effect of p_2^n on the first period market share. In this case, since firms obtain higher profits from newcomers this relaxes competition for first period customers, which is reflected in higher first period prices. As a result:

Remark 3 *In the equilibrium of the partial commitment game consumers are better off and firms are worse off than in the absence of commitment.*

Remark 4 *Both, consumers and firms are better off under partial commitment than under full commitment.*

Thus, the time inconsistency problem does not have a significant impact on the pro-competitive effect of commitment to the price for repeat purchases. Moreover, each firm benefits from expanding its own commitment capacity but it prefers that its rivals enjoy as little commitment power as possible.

Our model can be easily compared with the duopoly model analyzed in CM. In fact, the only difference is that the current model considers many firms and each one does not have an influence on the future behavior of their rivals. In other words, the strategic commitment effect is missing. As a result, firms wish to commit to marginal cost pricing for repeat buyers since this is the best deal it can offer to their customers. Instead, in the equilibrium of the duopoly game, firms commit to a price below marginal cost for repeat buyers. The reason is that even though duopolistic firms also benefit from committing to marginal cost pricing in the second period, there is an additional effect, which has to do with the fact that they can influence the price that their rivals charge to newcomers. In particular, if a firm sets p_2^r below marginal costs then, on the one hand, it reduces the rents generated by the customer relationship but, on the other hand, it induces the rival to set a lower $\overline{p}_2^n$, which makes the offer of the original firm more attractive to first period consumers.

3.5 Commitment to a linear discount

There might be many reasons why firms may wish to avoid committing to a fixed price for repeat buyers. For instance, there may be uncertainty about cost or demand parameters. In fact, in some real world examples we do observe firms committing to discounts for repeat buyers while leaving the net price undetermined. In this section we consider firms' commitment to linear discounts for repeat buyers instead of commitment to a predetermined price.

Suppose that in the first period firms set (p_1, v, f), where v and f are the parameters of the discount function:

$$p_2^r \equiv (1 - v)\, p_2 - f \tag{3.11}$$

Thus, v is a proportional discount and f is a fixed discount. In the second period firms set the regular price, p_2.

We show that there exist an equilibrium of this game that coincides with the symmetric equilibrium of the full commitment game of Section 3.4.1. Thus, in our model a linear discount function is a sufficient commitment device. By fixing the two parameters of the discount function firms can actually commit to the two prices, p_2^r and p_2^n.

More specifically, in the second period firms choose p_2 in order to maximize second period profits:

$$\pi_2 = x_1 x_2^r \left(p_2^r - c\right) + \left(1 - x_1\right) x_2^n \left(p_2 - c\right)$$

where p_2^r is given by equation 3.11. The first order condition characterizes the optimal price:

$$x_1 \left(1 - v\right) \left(x_2^r - \frac{p_2^r - c}{2t}\right) + \left(1 - x_1\right) \left(x_2^n - \frac{p_2 - c}{2t}\right) = 0$$

If other firms set the prices given by Proposition 6, and $x_1 = \frac{1}{2}$, then it is easy to check that it is optimal to set those same prices provided $v = \frac{4}{5}$ and $f = \frac{2}{15}t - \frac{4}{5}c$. Thus, using such a pair of (v, f) a firm can implement the desired pair of second period prices. Consequently, given that other firms are playing the prices given by Proposition 6, the best response of an individual firm consists of using such a linear discount function and the value of p_1 given also in Proposition 6, which results in $x_1 = \frac{1}{2}$. The next proposition summarizes this discussion.

Proposition 8 *There exist an equilibrium of the linear discount game that coincides with the equilibrium of the full commitment game.*

Hence, in our model there is no difference between price commitment and coupon commitment, at least as long as firms can use a combination of proportional and lump-sum coupons. In practice, this may not be so easy and firms may prefer using exclusively one type of coupons for simplicity. If this is the case firms will attempt to use the type of coupons that minimizes the scope of the time inconsistency problem, which depends on parameter values. For instance, if c is approximately equal to $\frac{t}{6}$ then proportional discounts alone will approximately implement the payoffs of the full commitment game. In a broad set of parameters, proportional discounts are better than lump-sum discounts at approximating full commitment strategies. The reason is that with proportional discounts firms can always set the value of either p_2^r or p_2^n, although it is generally impossible to hit both values. In contrast, with lump-sum discounts both prices will be far away from their target values. In other words, lumps-sum discounts alone are a very bad instrument of commitment to future prices. We illustrate this point in the Appendix for the case where rival firms are playing the equilibrium strategies of the full commitment game. Thus, at least in this two-period framework, firms will not have incentives to introduce lump-sum discounts. Hence, the emphasis of the existing literature on this type of loyalty-rewarding schemes was probably misleading. In the model of CM firms prefer committing to p_2^r than committing to a lump-sum discount. Our point here is that if commitment to p_2^r is not feasible or desirable (because of uncertainty, for instance) then still firms would prefer proportional (or, even better, linear) discounts, over lump-sum discounts.

In order to compare the role of lump-sum coupons under oligopoly and under monopolistic competition, in the Appendix we compute the symmetric equilibrium of the game with lump-sum coupons, i.e. firms set in the first period (p_1, f) and p_2 in the second. In this case we have that $p_2^r = p_2 - f$. It turns out that in equilibrium $f > 0$, firm profits are below the equilibrium level of the static game, but above the level obtained in the equilibrium of both the partial and the full-commitment games. Thus, firms would be better off if they were restricted to use lump-sum coupons instead of being allowed to commit to prices for repeat buyers. The reason is that lump-sum coupons are a poor commitment device and hence the business stealing effect is very moderate but present. Under oligopoly (CM) firms are better off in the coupon equilibrium, just because of the strategic commitment effect; that is, coupons imply a commitment to set a high regular price in the future which induces other firms to set higher future prices. It is such Stackelberg leader effect that makes coupons a collusive device.[41]

3.6 Interaction between endogenous and exogenous switching costs

Suppose that consumers incur an exogenous cost s if they switch suppliers in the second period. Let us assume that s is sufficiently small, so that optimal strategies are given by interior solutions. If firms can use loyalty rewarding pricing schemes, what is the effect of exogenous switching costs on market performance? Does such a natural segmentation of the market increases or decreases firms' incentives to introduce artificial switching costs?

Let us introduce exogenous switching costs in the partial commitment game of Section 3.4.2. That is, firms choose (p_1, p_2^r) in the first period, and p_2^n is selected in the second

[41] In the Appendix we discuss in more detail the intuition behind the difference between the duopoly and the monopolistic competition cases.

period after observing x_1 and $\overline{p}_2^r$. The only difference is that now, those consumers that switch suppliers in the second period pay s. Therefore, second period market shares become:

$$x_2^r = \frac{t + \overline{p}_2^n + s - p_2^r}{2t}$$

$$x_2^n = \frac{t + \overline{p}_2^r - s - p_2^n}{2t}$$

Similarly, first period market shares are implicitly given by:

$$p_1 + tx_1 + x_2^r \left(p_2^r + \frac{tx_2^r}{2} \right) + (1 - x_2^r) \left(\overline{p}_2^n + s + \frac{t(1 - x_2^r)}{2} \right) =$$

$$= \overline{p}_1 + t(1 - x_1) + x_2^n \left(p_2^n + s + \frac{tx_2^n}{2} \right) + (1 - x_2^n) \left(\overline{p}_2^r + \frac{t(1 - x_2^n)}{2} \right)$$

Proposition 9 *The unique subgame perfect and symmetric Nash equilibrium of the partial commitment game with exogenous switching costs includes* $p_1 = c + \frac{9t}{8} + \frac{s^2 - 2st}{8t}$, $p_2^r = c$, $p_2^n = c + \frac{t}{2} - \frac{s}{2}$. *As a result,* $x_1 = \frac{1}{2}$, $x_2^r = \frac{3}{4} + \frac{s}{4t}$, $x_2^n = \frac{1}{4} - \frac{s}{4t}$. *Total profits per firm is* $\pi = \frac{5t}{8} + \frac{s^2 - 2st}{8t}$, *and consumer surplus per firm is* $CS = R - c - \frac{29t^2 - 6st + 5s^2}{32t}$.

Hence, exogenous switching costs do not affect the price for repeat buyers but they reduce p_1 and p_2^n. Therefore, they reduce average prices and firm profits. The intuition goes as follows. For the same reasons as in Section 3.4, firms have incentives to commit to marginal cost pricing for repeat buyers. However, because of the exogenous switching costs, in the second period firms find it more difficult attracting consumers who previously bought from rival firms. As a result, they choose to set a lower second period regular price and nevertheless the fraction of switching consumers decreases. Since second period profits from newcomers are reduced, firms are more willing to fight for consumers in the first period and hence they find it optimal to set a lower first period price. Thus, even though consumers are partially locked-in for exogenous reasons and hence the market is even more segmented, profits fall.

Note, however, that in the absence of price discrimination, since all consumers change location, then again profitability decreases with switching costs.[42] However, the mechanism is quite different. In in the absence of price discrimination, switching costs affect prices through two alternative channels. On the one hand, in the second period a firm with a higher first period market share finds it profitable to set a higher price in order to exploit its relatively immobile customer base. As a result, first period demand will be more inelastic, since consumers expect that a higher market share translates into a higher second period market price and hence respond less to a price cut. This effect pushes first period prices upwards. On the other hand, firms make more profits in the second period out of their customer base, so incentives to increase the first period market share are higher. This effect pushes prices downwards. It turns out that the second effect dominates.

Therefore, the presence of price commitment affects the impact of exogenous switching costs. If firms commit to the second period price for repeat buyers, then this is equivalent to a commitment not to exploit locked-in consumers. Hence, the price sensitivity of

[42] This result holds under both monopolistic competition and duopoly (Klemperer, 1987).

first period consumers is unaffected. Nevertheless, firms' incentives to fight for first period market share increase in both cases, which turns out to be the main driving force.

Let us now turn to the question of how exogenous switching costs affect the incentives to introduce loyalty rewarding pricing schemes. Suppose that committing to the price of repeat purchases involves a fixed transaction cost. For instance, these are the costs airlines incur running their frequent flier programs (associated to advertising the program, recording individual purchases, etc.). The question is how the maximum transaction cost firms are willing to pay is affected by s.

The main intuition can already be obtained by considering the case of large switching costs. If s is sufficiently large then consumers will never switch in the second period, i.e., $x_2^r = 1$, $x_2^n = 0$. In this case, it is redundant to introduce endogenous switching costs, since they do not affect consumer allocation in the second period, which implies that consumers and firms only care about $p_1 + p_2$ and not about the time sequence. Hence, in this extreme case, it is clear that the presence of exogenous switching costs leaves no room for loyalty rewarding pricing schemes.

For low values of s the comparative static result provides a similar insight. As s increases, consumers switch less frequently and hence the effectiveness of price commitment to induce consumer loyalty is reduced. More precisely, if no other firm commits to p_2^r the net gain from committing to $p_2^r = c$ decreases with s. Similarly, if all other firms commit to $p_2^r = c$ the net loss from not committing also decreases with s (See Appendix 3.A.5 for details). In other words, exogenous and endogenous switching costs are imperfect substitutes.

3.7 An overlapping generations framework

In many situations firms may find it difficult to distinguish between consumers that have just entered the market and consumers who have previously bought from rival firms. In order to understand how important was this assumption in the analysis of the benchmark model we extend it to an infinite horizon framework with overlapping generations of consumers, in the same spirit as Klemperer and Beggs (1992).[43]

Time is also a discrete variable, but now there is an infinite number of periods, indexed by $t = 0, 1, 2, ...$ Demand comes from overlapping generations of the same size. Each generation is composed of consumers who live for two periods and have the same preference structure as the one described in Section 3.2. Thus, besides the larger number of periods, the main difference with respect to the benchmark model is that in this section we assume that firms are unable to discriminate between first period consumers and second period consumers that previously patronized rival firms. Firms set for each period two different prices: p_t, the price they charge to all consumers who buy from the firm for the first time, and p_t^r, they price they charge to repeat buyers.

Thus, profits in period t are given by:

$$\pi_t = (p_t - c)\left[x_t + (1 - x_{t-1})\, x_t^n\right] + x_{t-1}\left(p_t^r - c\right) x_t^r$$

where x_t, x_t^r, x_t^n, as in previous section, stand for period t market shares with young consumers, old consumers who bought from the firm in the last period, and old consumers who did not buy from the firm in the last period, respectively, which are given by:

[43] See also To (1996) and Villas-Boas (2004).

$$x_t = \frac{1}{2t}\left\{\overline{p}_t - p_t + x^n_{t+1}\left(p_{t+1} + \frac{x^n_{t+1}}{2}\right) + \left(1 - x^n_{t+1}\right)\left(\overline{p}^r_{t+1} + \frac{1 - x^n_{t+1}}{2}\right) - \right.$$

$$\left. -x^r_{t+1}\left(p^r_{t+1} + \frac{x^r_{t+1}}{2}\right) - \left(1 - x^r_{t+1}\right)\left(\overline{p}_{t+1} + \frac{1 - x^r_{t+1}}{2}\right)\right\} \tag{3.12}$$

$$x^r_t = \frac{t + \overline{p}_t - p^r_t}{2t} \tag{3.13}$$

$$x^n_t = \frac{t + \overline{p}^r_t - p_t}{2t} \tag{3.14}$$

These equations are analogous to equations 3.9, 3.7, and 3.8, respectively. The firm's payoff function in period 0 is:

$$V_0 = \sum_{t=0}^{\infty} \beta^t \pi_t \tag{3.15}$$

where $\beta \in (0, 1)$ is the discount factor. We will focus later on the limiting case of $\beta \to 1$.

Let us first deal with the full commitment case. Thus, given the sequence of current and future prices set by the rivals, $\{\overline{p}_t, \overline{p}^r_t\}_{t=0}^{\infty}$, the price for repeat buyers set in the past, p^r_0, and the past market share with young consumers, x_{-1}, an individual firm chooses $\left\{p_t, p^r_{t+1}\right\}_{t=0}^{\infty}$ in order to maximize 3.15. We focus on the stationary symmetric equilibria, for the limiting case of $\beta \to 1$. The result is summarized below (See Appendix for details):

Proposition 10 *In the unique stationary symmetric equilibrium* $c + t > p > c + \frac{t}{2} > p^r > c$.

Thus, the flavor of the results is very similar to the one provided by the benchmark model. Firms have incentives to discriminate between repeat buyers and newcomers, which creates artificial switching costs, and nevertheless consumers are better off than in the absence of such discrimination. The reason is that treating repeat buyers better than newcomers has only a business stealing effect and as a result the market becomes more competitive, in the sense that average prices are lower than in the absence of such discrimination (i.e., in the equilibrium of the static game).

The main difference with respect the benchmark model is that in the current set up p^r is set above marginal cost. In the two-period model p_1 was exclusively the instrument used by the firm to collect the rents created by setting a lower price to repeat buyers in the second period. Since an individual firm could fully appropriate all these rents, it was also willing to commit to marginal cost pricing in the second period, which maximizes the joint surplus of the firm and its customers. In the current framework, the regular price p_t is not only paid by young consumers but also by old newcomers. Thus, if p_t increases in order to capture the rents created by a lower p^r_{t+1} then the firm looses from old newcomers. As a result, the firm does not find it profitable to maximize the joint surplus of the firm and young consumers and set the price for repeat purchases equal to marginal cost. Nevertheless, such a price is still lower than the regular price.

In this section we have dealt so far with the case of unlimited commitment capacity. It would be probably be more realistic to grant firms a somewhat more limited commitment power. Firms can sometimes sign long-run contracts with current customers, but it is much more unlikely that they can commit to future prices for newcomers. Thus, alternatively, we could have assumed that in period t firms can set their regular price, p_t, and the price to be charged in the next period to repeat buyers, p^r_{t+1}. We conjecture that the Markov equilibria of such partial commitment game differs from the one of the full commitment game. The reason is twofold. First, under partial commitment firms set p_t after x_{t-1} has already been determined. This is analogous to the game of Section 3.4.2. Thus, firms do not take into account that a higher p_t makes the offers of their rivals less attractive and hence it raises $x_{t-1.}$ Hence, under partial commitment regular prices will tend to be lower. Second, under partial commitment demand by young consumers becomes more elastic. A lower p_t implies a larger x_t, which implies that the firm's incentives to attract in period $t+1$ old consumers that are currently trading with its rivals are reduced. As a result, p_{t+1} will be expected to be higher, which in turn increases x_t further. Therefore, the higher elasticity of demand induces firms to set lower regular prices. Hence, both effect push regular prices downwards.

On the other hand, lower regular prices implies that firms are less able to capture the rents associated to reduced prices for repeat buyers, which will tend to raise the price for repeat purchases. That is, we conjecture that, under partial commitment, the stationary symmetric equilibrium will be characterized by a lower p and a higher p^r than under full commitment. As it occurred in Section 3.4, restricting firms ability to commit to future prices for newcomers has a quantitative effect on equilibrium prices, but the main qualitative features of equilibrium are independent of it.

In this section firms must set a uniform price to old and young newcomers. Alternatively, firms could offer a menu of contracts and let these two types of consumers separate themselves. The contract targeted to old newcomers could simply offer a single price for the current transaction, p_t^n. The contract targeted to the young can include a price for the current transaction, p_t, and a price for the next period in case the customer remains loyal, p_{t+1}^r. Two incentive-compatibility constraints must be satisfied, implying that neither type has incentives to imitate the other type. A preliminary exploration of this case indicates that firms will find it profitable to separate these two types, and that one of the incentive constraints will be binding. As a result, the equilibrium prices of the full commitment game for the two period model, cannot be part of the stationary equilibrium of the model with overlapping generations. In other words, the equilibrium of the game where firms can offer a menu of contracts in each period is somewhere in between the equilibrium described in this section and the one of Section 3.4.1. Hence, no new insights are obtained.

3.8 Discussion

In this section we comment on the role of various assumptions and consider different extensions.

3.8.1 Consumer horizon

If we let consumers live for more than two periods, then consumers might be able to accumulate claims to different loyalty programs (might join more than one FFP). This could reduce the potential lock-in effect of loyalty programs. However, if rewards are properly designed, if they are a convex function of the number of purchases, then the same qualitative effects should be obtained.[44]

3.8.2 Heterogeneous patterns of repeat purchases

Let us consider the two-period game of Section 3.1 with the following variation. There are two types of consumers: frequent flyers, who purchase in both periods, and occasional flyers, who only purchase in one period. In order to maintain total demand constant we let first period occasional flyers to be replaced in the second period by a different generation of the same size. First, if firms cannot discriminate between these two types of consumers then p_2^n will be higher than in Proposition 6. As a result, profits from newcomers in the second period who are frequent flyers will be lower, and hence competition for frequent flyers in the first period will be relaxed. Nevertheless, in the first period frequent flyers will be sensitive to the commitment to a lower price for repeat purchases and hence their willingness to pay will be higher than that of occasional travelers. Hence, firms may be able to discriminate between these two type of consumers. For instance, they could separate them by offering a business class and a tourist class, both with the same quality, but with only one of them associated with loyalty rewards.

[44] Fernandes (2001) studies a model where consumers live for three periods. Unfortunately, he restricts attention to concave rewards. In particular, consumers obtain a lump-sum coupon with the first purchase, which must be used in the next purchase with the same supplier. In this extreme example, consumers' lock-in effects are minimized.

3.8.3 Partnerships

Recently airlines have formed FFP partnerships. On the one hand, those partnership enhance the FFP program of each partner by expanding earning and redemption opportunities. On the other hand, they may affect the degree of rivalry. Those observers that interpret FFP as enhancing firms' market power have a hard time understanding the formation of partnerships of domestic airlines who compete head to head in the same routes. In their view those partnerships appear to increase airline substitutability.[45] In contrast, we claim that FFP are business-stealing devices. Hence, partnership between directly competing firms may relax competition by colluding on less generous loyalty rewards. We are currently working on a 3-firm version of our benchmark model in order to analyze firms' incentives to form partnership and their effects on market performance.

3.8.4 Entry

In this paper we have characterized loyalty programs as a business-stealing device provided there is sufficient competition (the market is fully served). However, in markets where there is room for entry, incumbents may use loyalty programs as a barrier to entry. The existence of a large share of consumers with claims to the incumbents' loyalty program may be sufficient to discourage potential entrants.

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45 See Lederman (2003), Section VI.

3.8.5 Relative sizes

Consider the two-period model for $n = 3$, and assume that varieties 1 and 2 are produced by the same firm (L) and variety 3 is produced by an independent firm (S). Unlike in the benchmark model suppose that over time consumers shift location and the pair of varieties that they derive utility from. This feature could capture fairly well the fact that a large airline's FFP offers higher redemption opportunities.[46] Preliminary results indicate that, as in the symmetric large-n case, all firms loose with the introduction of loyalty programs (with the commitment to the second-period price for repeat buyers), but the large firm looses relatively less, because its market share increases as consumers attach a higher value of the large firm's program. Those predictions are compatible with the empirical evidence reported in Lederman (2003). She finds that enhacements to an airline's FFP are associated to increases in the airline's market share, and this effect is larger for more dominant airlines. She interprets this result as indicating that FFP reinforces firms' market power. According to our model this interpretation is only correct in relative terms: large airlines are relatively protected from the pro-competitive effects of FFP, but all airlines loose with the introduction of FFPs.

3.9 Concluding remarks

The answer we provide to the title question is rather sharp. Loyalty rewarding pricing schemes are essentially a business stealing device, and hence they reduce average prices and increase consumer welfare. Such a pro-competitive effect is likely to be independent of

[46] In a more general model a large firm would also offer higher earning opportunities.

the form of commitment (price level versus discounts). Therefore, competition authorities should not be particularly concerned about these pricing strategies. If anything, perhaps authorities should promote and even subsidize the introduction of this kind of programs.

From an empirical point of view there are many important questions that need to be posed. In the real world, we observe a high dispersion in the size and characteristics of loyalty rewarding pricing schemes. What are the factors that explain those cross-industry differences? One possible answer is transaction costs. Discriminating between repeat buyers and new consumers can be very costly sometimes, as sellers need to somehow keep track of individual history of sales. Those transaction costs are likely to vary across industries, both in absolute value and also relative to the mark up. This might explain some fraction of the cross-industry variations in loyalty-rewarding pricing schemes. Unfortunately, it is not obvious which proxies of industry-specific transaction costs are available.

3.10 References

Banerjee, A. and Summers, L. (1987), On frequent flyer programs and other loyalty-inducing arrangements, H.I.E.R. DP no 1337.

Bulkley, G. (1992), The role of loyalty discounts when consumers are uncertain of the value of repeat purchases, International Journal of Industrial Organization, 10, 91-101.

Cairns, R. and J. Galbraith (1990), Artificial compatibility, barriers to entry, and frequent-flyer programs, Canadian Journal of Economics, 23 (4), 807-816.

Caminal, R. (2004), Pricing ‘Cyclical’ Goods, mimeo Institut d’Anàlisi Econòmica, CSIC.

Caminal, R. and Matutes, C. (1990) Endogenous Switching Costs in a Duopoly Model, International Journal of Industrial Organization, 8, 353-373.

Chen, Y. and M. Riordan (2004), Vertical Integration, Exclusive Dealing and Ex Post Cartelization, mimeo University of Columbia.

Crémer, J. (1984) On the Economics of Repeat Buying, The RAND Journal of Economics,15 (3) Autumn, 396-403.

Fernandes (2001), Essays on customer loyalty and on the competitive effects of frequent-flyer programmes, PhD thesis, European University Institute.

Klemperer, P. (1995) “Competition when Consumers have Switching Costs: An overview with Applications to Industrial Organization, Macroeconomics, and International Trade” The Review of Economic Studies,62 (4) October, 515-539.

Klemperer, P. (1987), The competitiveness of markets with switching costs, The RAND Journal of Economics 18 (1), 138-150.

Klemperer, P. and A. Beggs (1992), Multi-Period Competition with Switching Costs, Econometrica, vol. 60, 651-666.

Lederman, M. (2003), Do Enhancements to Loyalty Programs Affect Demand? The Impact of International Frequent Flyer Partnerships on Domestic Airline Demand, mimeo MIT.

Narasimhan, C. (1984), A Price Discrimination Theory of Coupons, Marketing Science 3, 128-147.

Sharp, B. and A Sharp (1997), Loyalty programs and their impact on repeat-purchase loyalty patterns, International Journal of Research in marketing 14 (5), 473-486.

To, T. (1996), Multi-Period Competition with Switching Costs, Journal of Industrial Economics 44, 81-88.

Villas-Boas, M. (2004), Dynamic Competition with Experience Goods, mimeo University of California, Berkeley.

3.A Appendix to Chapter Three

3.A.1 Proposition 6

The first order conditions of the firm's optimization problem are given by:

$$\frac{d\pi}{dp_1} = x_1 - \frac{M}{2t} = 0$$

$$\frac{d\pi}{dp_2^r} = x_1 x_2^r - \frac{x_2^r M}{2t} - \frac{x_1 \left(p_2^r - c\right)}{2t} = 0$$

$$\frac{d\pi}{dp_2^n} = \left(1 - x_1\right) x_2^n + \frac{x_2^n M}{2t} - \frac{\left(1 - x_1\right)\left(p_2^n - c\right)}{2t} = 0$$

where $M \equiv p_1 - c + x_n^r \left(p_2^r - c\right) - x_2^n \left(p_2^n - c\right)$ and x_2^r, x_2^n and x_1 are given by equations 3.7-3.9 in the text. In a symmetric equilibrium we have that $x_1 = \frac{1}{2,}, x_2^r = 1 - x_2^n$. Plugging these conditions on the first order conditions and solving the system we obtain the strategies stated in the proposition.

If we denote the elements of the Hessian matrix by H_{ij}, then evaluated at the first order conditions we have that $H_{11} = -\frac{1}{t}$, $H_{22} = -\frac{17}{18t}$, $H_{33} = -\frac{13}{18t}$, $H_{12} = -\frac{5}{6t}$, $H_{13} = H_{23} = 0$. Hence, the matrix is negative semidefinite and second order conditions are satisfied.

3.A.2 Proposition 7

In the second period the firm chooses p_2^n in order to maximize second period profits, which implies that:

$$p_2^n = \frac{t + \overline{p}_2^r + c}{2}$$

After plugging this expression in equation 3.9, the firm chooses (p_1, p_2^r) in order to maximize 3.10. The first order conditions are:

$$\frac{d\pi}{dp_1} = x_1 - \frac{M}{2t} = 0$$

$$\frac{d\pi}{dp_2^r} = x_1 x_2^r - \frac{x_2^r M}{2t} - \frac{x_1 \left(p_2^r - c\right)}{2t} = 0$$

Evaluating these conditions at a symmetric equilibrium and solving we obtain the strategies stated in the proposition.

The elements of the Hessian matrix evaluated at the first order conditions are $H_{11} = -\frac{1}{t}$, $H_{12} = -\frac{3}{4t}$, $H_{22} = -\frac{13}{16t}$. Hence, second order conditions are satisfied.

3.A.3 The commitment capacity of lump-sum coupons

Suppose that other firms have set $\overline{p}_2^r = c$ and $\overline{p}_2^n = c + \frac{2t}{3}$. Then the best response in the first period is to set exactly these prices. Instead, consider a firm that arrives at the second period with $x_1 = \frac{1}{2}$ and a lump-sum coupon f. Then such a firm would choose p_2 in order to maximize:

$$\pi_2 = \frac{1}{2} \left\{ \left(p_2 - f - c\right) x_2^r + \left(p_2 - c\right) x_2^n \right\}$$

where

$$x_2^r = \frac{t + \overline{p}_2^n - p_2 + f}{2t}$$

$$x_2^n = \frac{t + \overline{p}_2^r - p_2}{2t}$$

If f is large, then the solution includes $x_2^n = 0$ and the outcome is dominated from the ex-ante point of view by $f = 0$. If f is not too large the solution is interior and the ex-post optimal prices will be given by:

$$p_2^r \equiv p_2 - f = \frac{2t}{3} + c - \frac{f}{2}$$

$$p_2^n \equiv p_2 = \frac{2t}{3} + c + \frac{f}{2}$$

Thus, as f increases p_2^r gets closer to the optimal ex-ante response, but p_2^n is driven further away from its ex-ante optimal value. Therefore, there is no value of f that allows the firm to commit to a pair of prices close to the best response.

3.A.4 Equilibrium with lump-sum coupons

For arbitrary prices and market shares the second period optimization problem provides the following first order condition:

$$p_2 = \frac{t + c + \overline{p}_2 + 2x_1 f - (1 - x_1)\overline{f}}{2}$$

In the first period, firms choose (p_1, f) in order to maximize first period profits. The first order conditions are:

$$\frac{d\pi}{dp_1} = x_1 - \frac{M}{2t + \frac{\left(f+\overline{f}\right)\left(2f+\overline{f}\right)}{4t}} = 0$$

$$\frac{d\pi}{df} = -\frac{x_1\left(1-x_1\right)\left(2f+\overline{f}\right)}{2t} + M\frac{\overline{p}_2 + t - c + f\left(2-4x_1\right) + \overline{f}\left(1-3x_1\right)}{8t^2 + \left(f+\overline{f}\right)\left(2f+\overline{f}\right)} = 0$$

where $M \equiv p_1 - c + x_n^r\left(p_2 - f - c\right) - x_2^n\left(p_2^n - c\right)$. If we evaluate these conditions at the symmetric allocation, then we have that $p_1 = c+t$, $p_2 = c+\frac{4t}{3}$, $f = \frac{2t}{3}$. Thus, profits are $\pi = \frac{8t}{9}$, and consumer surplus per firm is $CS = R - c - \frac{43t}{36}$.

If we compare the equilibrium under monopolistic competition and duopoly (CM) then we observe that both coupons and second period prices are the same in both games, but the first period under duopoly is $p_1 = c + \frac{13t}{9}$, which is far above the first period price of the monopolistic competition equilibrium. The intuition is the following. Under duopoly the elasticity of the first period demand with respect to the first period price is higher than under monopolistic competition. The reason is that a higher first period market share (because of a lower first period price) induces the rival firm to set a lower second period price, since it has more incentives to attract new customers. Such a lower expected second period price makes the first period offer of the rival firm more attractive, which in turn reduces the increase in first period market share. As a result, such a reduction in the price elasticity of demand induces firms to set a higher first period price.

Strategic commitment has two separate effects of different signs on the level of coupons, and it turns out that they cancel each other. On the one hand, a higher coupon induces the rival firm to set a lower second period price, which has a negative effect on second period profits. Hence, duopolistic firms would tend to set lower coupons. On the other hand, a higher coupon involves a commitment to set lower prices for repeat buyers, which increases first period demand. If the first period price is higher then the increase in first period profits brought about by a higher coupon is exacerbated. Hence, through this alternative channel, duopolistic firms would tend to set higher coupons. In our model both effects cancel each other and the level of coupons is the same under both duopoly and monopolistic competition and therefore, the level of second period prices is also the same.

3.A.5 The substitutability between endogenous and exogenous switching costs

Suppose that only one firm can commit to p_2^r. Then, analogously to Klemperer (1987), non-discriminating firms set:

$$p_1 = c + t - s + \frac{s^2}{2t}$$

$$p_2 = c + t$$

and make profits:

$$\pi = t - \frac{s}{2} + \frac{s^2}{4t} \tag{3.16}$$

The discriminating firm will optimally set:

$$p_1 = c + \frac{13t}{8} + \frac{13s^2 - 20st}{32t}$$

$$p_2^r = c$$

$$p_2^n = c + t - \frac{s}{2}$$

As a result profits will be:

$$\pi^c = \frac{145t}{128} + \frac{-1312st^3 + 920s^2t^2 - 72s^3t + 81s^4}{2048t^3} \tag{3.17}$$

The net benefit from committing (the difference between 3.17 and 3.16) decreases with s (provided s is not too large).

Suppose now that all firms commit and set the equilibrium strategies of Proposition 4. If one firm does not commit then it will optimally set:

$$p_1 = c + \frac{431t^4 - 104t^3s + 178t^2s^2 + 27s^4}{520t^3 + 48t^2s + 72ts^2}$$

$$p_2 = c + \frac{161t^3 - 23t^2s + 11s^2t - 21s^3}{260t^2 + 24st + 36s^2}$$

As a result profits will be:

$$\pi^{nc} = \frac{1221t^4 - 372t^3s + 190t^2s^2 - 52ts^3 + 37s^4}{2080t^3 + 192t^2s + 288ts^2} \tag{3.18}$$

The net loss from not committing (the difference between profits obtained in the equilibrium of Proposition 9 and 3.18) decreases with s.

3.A.6 Proposition 10

The first order conditions with respect to p_t and p_t^r are respectively:

$$\beta^t \left\{ x_t + (1 - x_{t-1})\, x_t^n - (p_t - c)\, \frac{2 - x_{t-1}}{2t} + \left[(p_t^r - c)\, x_t^r - (p_t - c)\, x_t^n \right] \frac{dx_{t-1}}{dp_t} \right\} +$$

$$+\beta^{t-1} \left\{ (p_{t-1} - c)\, \frac{dx_{t-1}}{dp_t} \right\} = 0$$

$$\beta^t \left\{ x_{t-1} \left[x_t^r - \frac{p_t^r - c}{2t} \right] + \left[(p_t^r - c)\, x_t^r - (p_t - c)\, x_t^n \right] \frac{dx_{t-1}}{dp_t^r} \right\} + \beta^{t-1} \left\{ (p_{t-1} - c)\, \frac{dx_{t-1}}{dp_t^r} \right\} = 0$$

From equations 3.12 to 3.14:

$$\frac{dx_{t-1}}{dp_t} = \frac{x_t^n}{2t}$$

$$\frac{dx_{t-1}}{dp_{t-1}} = -\frac{1}{2t}$$

$$\frac{dx_{t-1}}{dp_t^r} = -\frac{x_t^r}{2t}$$

If we set $\beta = 1$ and evaluate equations x and x in a symmetric equilibrium ($x_t = \frac{1}{2}, x_t^r = 1 - x_t^n$) we get:

$$t\,(2 - x^r) - \frac{3}{2}\,(p - c) + (p + p^r - 2c)\, x^r\,(1 - x^r) = 0 \tag{3.19}$$

$$t + p - 2p^r + c - \frac{p + p^r - 2c}{2t^2}\,(t + p - p^r) = 0 \tag{3.20}$$

where

$$x^r = \frac{1}{2} + \frac{p - p^r}{2t}$$

If $p^r = c$, the value of p that satisfies equation 3.19 is in the interval $\left(c + \frac{t}{2}, c + t\right)$. Also, p increases with p^r for all $p^r > c$. On the other hand, the equation implicitly characterized by equation 3.20 goes through the points $(p^r = c, p = c + t)$ and $\left(p^r = p = c + \frac{t}{2}\right)$ and is decreasing in this interval. Therefore, there is a solution of the system in this interval, which proves the proposition.

Table 3.1

	No commitment	Full commitment	Partial commitment
p_1	$c+t$	$c+\frac{10}{9}t$	$c+\frac{9}{8}t$
p_2^r	$c+t$	c	c
p_2^n	$c+t$	$c+\frac{2}{3}t$	$c+\frac{1}{2}t$
x_1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
x_2^r	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{3}{4}$
x_2^n	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{4}$
π	t	$\frac{11}{18}t$	$\frac{5}{8}t$
CS	$R-c-\frac{5}{4}t$	$R-c-\frac{33}{36}t$	$R-c-\frac{29}{32}t$