Universitat Autònoma de Barcelona

Essays on the Spanish Macroeconomy

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A mis padres y mi hermano, por estar siempre ahí.

Abstract

This doctoral thesis tries to shed some light on two topics of current relevance in Spanish macroeconomics. The social insurance system of unemployment and retirement, and the problem of tax evasion. My focus on these topics is mainly practical, and always with an eye on the policy implications that this work can have. The dissertation is divided in three chapters, first on the social insurance design of unemployment and retirement policies, and the second and third focused the steady state and business cycle implications that VAT evasion have on fiscal policy.

In the first chapter I develop a framework to study life cycle unemployment and retirement decisions together, in order to have some foundation on how the interactions between these two phenomena should be taken into account when taking decisions about changing the public social insurance system currently active in Spain. This model allows us to study nontrivial effects on unemployment, retirement decisions, savings and average pensions of common policies such as changes in replacement rates in unemployment insurance, or of retirement pensions. I use this framework to find optimal policies in unemployment and retirement. The novelty of this article is that in the presence of unemployment risk, a positive steady state replacement rate in pensions benefits is optimal, together with sizable unemployment benefits. Moreover, this framework is also useful to study the impact that unemployment and retirement policies have on inequality.

In the second chapter we look at a prevalent problem of the Spanish economy: tax evasion. We extend previous macroeconomic frameworks on the shadow economy with consumption tax evasion. The model presented here is the first one to jointly tackle tax evasion on both labor income tax and value added/consumption tax. The consequences for taxation of this model are important, and we show that it may be infeasible to implement tax shifts implying significant reductions of income tax in favour of higher excises on consumption. Our model produces Laffer curves in which revenues fall quickly when tax rates are raised above very reasonable levels. The limits imposed by tax evasion are stricter for VAT, for which the *slippery slope* of the Laffer curve starts roughly at a mere rate of 10%.

On the third chapter we extend the theoretical framework of tax evasion and explore its properties on the business cycle. With a fairly simple modelling of the underground economy, using a TFP shock that affects both the legal an il-

legal sector symmetrically, we are able to produce a countercyclical evolution of the shadow economy, fact that has been pointed out by the empirical literature. The reason is the opportunity cost in terms of time that participating in the underground economy imply, and it may be not worth the trouble at times when the productivity is high.

Chapter 1

Life Cycle Unemployment and Retirement

1.1 Introduction

The objective of this paper is to understand the relationship between two important labor market phenomena at older ages: unemployment and retirement. The linkage between the two arises since reaching a certain age, workers become eligible for a public pension, and depending on the size of it, they may choose to stay in the labor market or retire. These incentives are more salient if unemployment benefits are low, making job market search more costly. The distance to retirement, i.e., the number of years remaining until a worker is eligible for Social Security affects how agents fare in the labor market, from a supply and demand perspective. From the demand side, there are fewer incentives to hire a worker if the expected labor relationship is going to be shorter, and in the case of existing employees, they are normally more expensive to lay off due to employment protection legislations. On the supply perspective, older workers will, on average, have higher levels of wealth accumulated, making them more reluctant to accept the first offer and bargain for higher wages. However, there are other institutional factors conditioning choices. The design of the Social Insurance (SI) scheme that the government provide, such as unemployment insurance (UI) and Social Security (SS) retirement pensions may actually encourage them to give up job search and the labor market, either by going to unemployment and claiming benefits, or retiring instead of waiting to the statutory age. These intertwining forces are commonplace in the developed world, where older unemployed find it difficult to come back to labor market. This has been documented by Hahn (2009) or Shimer (2012) for the US.

Let us have a look at the empirical evidence for the case of Spain, and understand why the phenomenon of unemployment is very dependent on age. Figure 1.1 shows the average quarterly probability that an unemployed worker

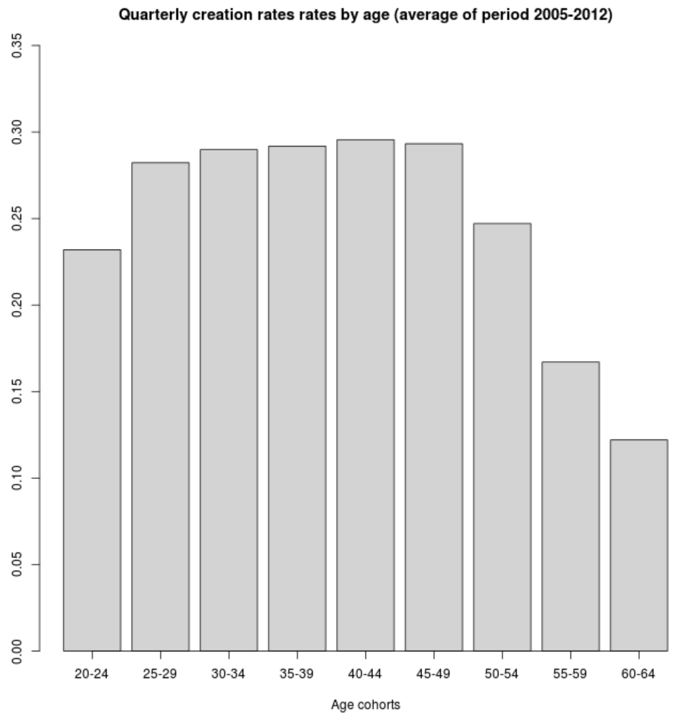


Figure 1.1: Probability of job creation by ages in Spain. Source: Muestra Continua de Vidas Laborales, wave 2012 (MCVL 2012).

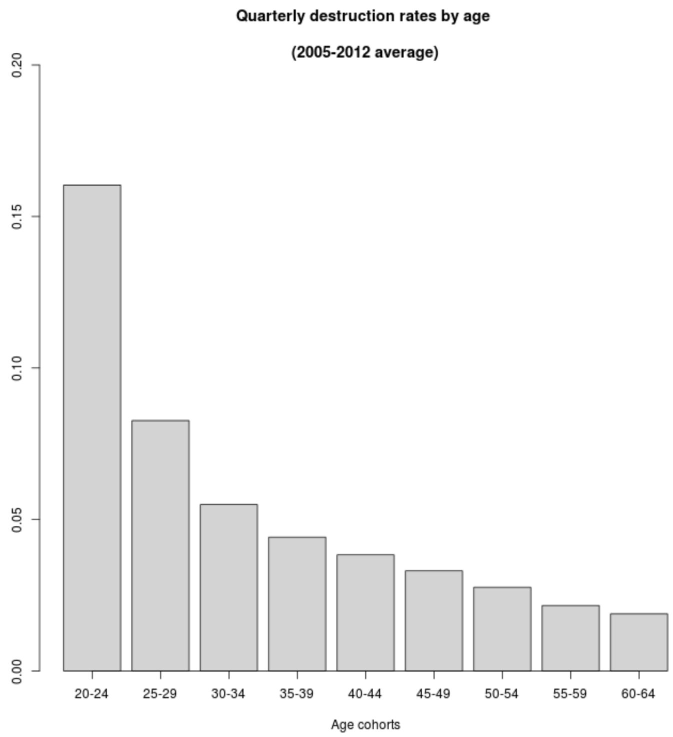


Figure 1.2: Separation rates by age cohorts in Spain. Source: MCVL 2012.

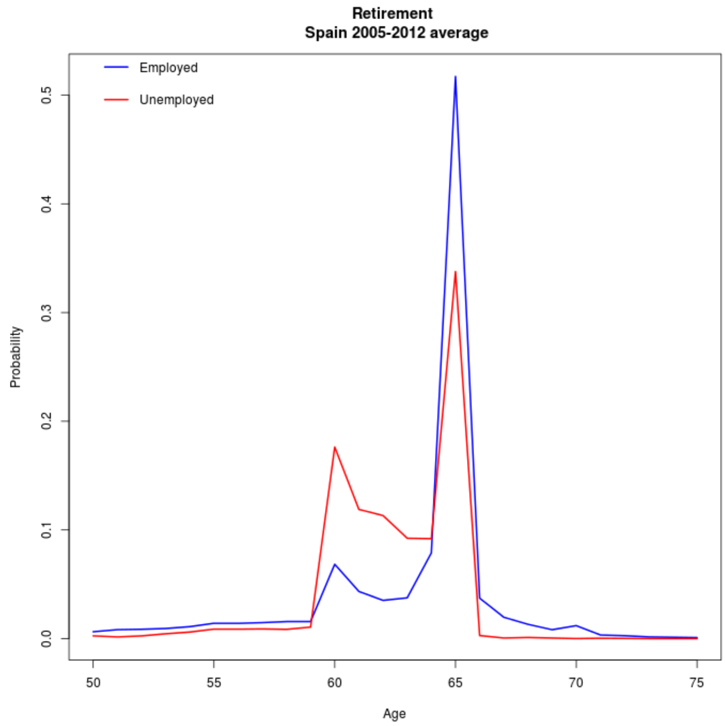


Figure 1.3: Probability of retirement by labor market status and age. Source: MCVL 2012.

finds a job in Spain, by different ages, whereas the probability that a worker gets separated from her current employer in a quarter is displayed in Figure 1.2. These two flows, referred as entry and exit, shape the life cycle profile of unemployment. The direct consequence of job creation falling with age is that the duration of an unemployment spell increases with age, for the reasons explained above. In Spain, an unemployed person aged above 50 can spend roughly around a year and a half to find another job on average. The situation for individuals without education in this cohort is much more dismal.[1] [2]

Figure 1.3 shows the probability of retiring for employed and unemployed workers for different ages. As we can see, an important proportion of unem-

[1]These figures are elaborated using The *Muestra Continua de Vidas Laborales* (MCVL) a very rich (4% of the population) sample of the registries in the Spanish Social Security. For the individuals provided in the sample, all their labor history (time intervals for each labor contract that an individual has had, periods earning unemployment benefits, dates of retirement, whether they retire or not). This allows us to ellaborate robust metrics of the labor market flows in Spain.

[2]The prospects of finding a job are so bleak for cohorts of old unemployed workers in Spain, especially those without education that were working in the construction sector, that the government implemented a non-contributive subsidy called the *ayuda familiar* for those who have not found a job, have run out of unemployment benefits, and are not eligible for retirement, which lasts virtually until they retire.

ployed population retire at 60, which is the *early* retirement age in Spain, whereas their employed counterparts prefer to wait until *normal* age at 65. This is an important piece of evidence of the influence that labor market status can have on the choice of exiting the labor market, and why we ought to study unemployment and retirement jointly.[3]

Governments ought to take these issues into account when designing an optimal SI system, since retirement benefits have an important role to smooth consumption in two ways. First, its existence allows agents to smooth consumption maybe by leaving the labor market earlier, and leaving room for younger workers. Second, once they are retired, pensions complement agents' savings, do not forcing them to save that much, for instance, if they have lost their job, and their more urgent need is to maintain consumption now rather than saving for retirement. This "insurance" mechanism that pension benefits imply in case of low savings caused by a bad individual history of unemployment is a novelty of this paper. The direct consequence of this is that a positive replacement rate in pensions *is* indeed optimal.[4]

To analyze the importance of these issues we build a model that has life cycle unemployment, savings and endogenous retirement choice, calibrated to the Spanish economy. With it, we see that once we take into account unemployment and retirement together, policies of removing SS benefits are no longer optimal. What is more, we observe that given the model specified, systems with social insurance parameters similar to those existent in Spain are optimal.

The structure of the paper is the following. In section 1.2 we review what the literature has done on life cycle unemployment and retirement. Section 1.3 lays out the model, and 1.4 we explain the equilibrium. In section 1.5 we talk about the calibration strategy. Section 1.6 gives some numerical results of the model. and explore its possibilities for optimal policy. Section 1.8 concludes.

1.2 Literature

Traditionally, the literature on retirement has focused more on explaining to what extent financial incentives (or the lack thereof), arising from US public Social Security (SS) benefits determine retirement decisions of agents, giving rise to patterns in hazard rates of retirement characterised by "spikes" in early

[3]This is by no means a unique phenomenon in Spain. These patterns are common in most developed countries. See Blöndall and Scarpetta (1999).

[4]The replacement rate is the proportion that an individual would get when retired of a weighted average of her lifetime earnings.

and normal retirement ages determined by law.[5]

In one of the first works into the topic, Diamond and Gruber (1998) studied the declining trend in labor market participation of older workers, and argued that the cause is the increase in the SS funds. They calculate *synthetic* earning histories for median workers and their accumulated SS wealth, and using these variables find *implicit* tax rates on continuing working for another year. Their finding is that these "taxes" peak at 62 and 65, concluding that retirement is caused by this institutional discouragement to work additional years, since between ages 62 to 65, worker's pension does not grow in an actuarial fashion when postponing retirement an extra period. Coile and Gruber (2001) also support the idea of SS affecting retirement, yet their results are different. Constructing the SS wealth on *true* work histories instead of median workers, and using the methodology of the "peak value" of the SS benefits developed by Stock and Wise (1988), they show that Diamond and Gruber's implicit taxes are overestimated and the reason for the spikes must be that agents are financially constrained, such that they retire as soon as the minimum benefits are available. Krueger and Pischke (1992) use a natural experiment arising from a legal change in the US occured in 1977 that meant a significant reduction in SS benefits for those born after 1916. They calculate hypothetical SS wealth based on average wage histories for individuals surveyed in the March sample of the Current Population Survey from 1976 to 1988, and see how SS wealth, and its growth by working an additional year affect labor market participation of older workers, and if there are different effects for individuals affected for the reform or not. They found that only a sixth of the continuous decline of participation came from the benefit reform.[6]

It has not been until recently that empirical works have looked into the issue of interactions between unemployment and retirement. Coile and Levine (2007, 2011) in a series of papers have documented some interesting relationships. First, older unemployed approaching statutory age are more likely to retire than their employed counterparts, in the spirit of what we find in figure 1.3. Second, in the presence of financial shocks affecting 401(k) plans may have forced some *defer* retirement to compensate for the wealth loss. Third, others, after facing a labor market shock decide to *anticipate* it given the bleak perspectives of returning to the market. Fourth, unemployed agents tend to rationally use state-provided UI to design pathways to retirement that imply voluntarily spending some time unemployed before retiring. This is done by a non-trivial 40% of workers in countries such as Japan. Incentives like these have should have implications on the design of unemployment policies

[5] Stock and Wise (1988) study instead the incentives provided by the private pension funds of workers.

[6] Those born at 1920 with a labor history earning average wages would earn roughly 20% less that those unaffected.

for workers of all ages, and especially those close to retirement. García Pérez et al. (2010) studied empirically these institutional interactions for the case of Spain, and discover significant evidence of their existence, exacerbated by the little monitoring that public employment services do on unemployed workers. Thus, there is need for a model that attempt to capture these non-trivial effects.[7]

Another branch of study are the theories of the labor market participation over the life cycle. There are various papers whose objective is why unemployment, job creation and job destruction evolve the way they do over the life cycle. Hahn (2009), in one of the first papers on equilibrium life cycle unemployment, introduces age into the standard search and matching framework of Mortensen and Pissarides (1994) to study the efficiency "age-wise" of the market outcome, and to analyze what are the impacts of changing unemployment subsidies. Chéron et al. (2013), with a similar model, tries to answer normative questions such as the optimal age design of unemployment policies. Esteban-Pretel and Fujimoto (2011) give an explanation of why job destruction is decreasing over age cohorts, using a model in which firms learn probabilistically the type/productivity of the worker. Menzio, Telyukova and Visschers (2012) propose a directed search model that explains job creation and job destruction over the life cycle. However, these papers do not take into account the choice of retirement.

Large scale OLG general equilibrium models have been extensively used in the evaluation of SS reforms, pension scheme changes, delays in statutory ages of retirment and so forth. Conesa and Krueger (1999) study the transitional effects of moving from a Pay-As-You-Go (PAYG) to a fully funded system. De Nardi, İmrohoroğlu and Sargent (1999) try to see how the US SS system would fare compared to other different policy scenarios, provided the current growing life expectancy. Hugget and Ventura (1999) see what are the steady state distributional effects of moving from current US SS system to a two-tier SS where a minimum income is guaranteed for those with low contributions. For Spain, Díaz-Giménez and Díaz-Saavedra (2009) or Sánchez Martín (2010) assess the effect that a delay in the statutory retirement age would have on the sustainability of SS. However, the drawback of all these models is that they do not consider "extensive margin reactions, i.e., impact that these policies would have on unemployment and all adjustments are in the amount of hours supplied". One classical conclusion of these studies is that the optimal replacement rate of pensions, defined as the proportion of the lifetime earnings received once retired, should be zero. We claim that under our assumptions, and taking into account unemployment risk, this is not true, and we find that a positive steady state replacement rate is optimal.

[7] See Coile and Levine (2007).

The implications of including assets in models dealing with unemployment, either in search or matching frameworks have also been studied. Andolfatto (1996) studies the macroeconomic implications of including labor markets frictions à la Mortensen Pissarides into an RBC model. This makes hours fluctuate more than wages, thus solving the counterfactual conclusion of RBC models of hours and productivity being strongly correlated. Den Haan, Ramey and Watson (2000), using a similar model to Andolfatto (1996), how a frictional labor market magnifies the impact of aggregate shocks, and make them more persistent. Krusell, Mukoyama and Şahin (2010) draw a bridge between Aiyagari type of models of incomplete markets and idisyncratic shocks, and models with labor market frictions and aggregate shocks. The resulting model has the positive feature that wages behave very similar to the standard Mortensen-Pissarides framework, "endogenizing" the idiosyncratic shocks, but life cycle features of Aiyagari models are preserved. These models again do not take into account the retirement choice.

This paper attempts to draw a theoretical bridge between two strands of macroeconomics hitherto disconnected, namely life cycle unemployment, and Social Security models of retirement. The contribution is twofold. First, it lays out a model that studies life cycle equilibrium unemployment model with retirement and in which contributions by firms and workers are used to sustain UI and pensions, allowing to see what are the implications of different unemployment or retirement policies, or alternative tax structures on firms and workers on aggregate variables such as life cycle unemployment or welfare. Second, it gives us an instrument to do normative analysis taking into account the effects on the extensive margin of the labor market, and decision to exit it at later ages.

Close to this study is Hairault, Langot and Sopraseuth (2010). They empirically show that the distance to retirement affects employment probability, and propose a search model capable of explaining the decline in employment rate at ages close to retirement as workers search less intensively, and propose, a movement towards a more actuarially fair pension system, where benefits are more closely linked to contributions. Hairault, Langot and Zylberberg (2013) provide a stylised model of unemployment and retirement to explain why unemployed retire earlier than their employed counterparts. However, theirs is a partial equilibrium analysis whereby unemployment and retirement transfers are not funded from contributions by either firms or workers.

1.3 Model

The framework here is similar to Esteban-Pretel and Fujimoto (2011), where unemployment by age is modelled as different segmented labor submarkets in which workers of a specific type cannot by substituted by others. This avoids the externality of workers of different ages searching more or less and thus making more difficult to find a job for the other agents. It is just a simplification coming from a reasonable assumption that young workers cannot be perfectly substituted by their old counterparts, and vice-versa.[8]

In this model we incorporate a SI scheme that provides workers with both unemployment benefits in case of job loss, which are proportional to wages paid to those working at that age, and they are granted an old age pension that is linked to their earning history. There are restrictions on the age at individuals can retire, being able to first do it at the *early retirement*, albeit subject to penalties, and a *normal retirement* age at which they are granted the full pension, which is a fraction (replacement) of the average earning history.[9].

These idiosyncratic histories include a dimension in heterogeneity that will have consequences on savings and the choice on when to retire, apart from obviously the pensions they will get. This also allows us to have a say about the inequality (in income, pensions, savings) that different policies may produce. Agents are able to save part of their income in a given period, and get paid a fix return r on it, by means of a storage technology. Therefore, these assets are not used in the production function of the firm. Next we develop a detailed explanation of how the social security works in this model. In order to do this we have to introduce a rather cumbersome but necessary notation on the labor histories, but this will be simplified later on, and the social security "account" of an agent will be simplified to a single state variable.[10]

Time is discrete and goes forever. Agents live for a deterministic number of periods, agents do not die stochastically for simplicity. The unit of time is a year. An agent's age is denoted by $i \in \{i_0, i_1, \ldots, I\}$, i.e., they enter in the labor market unemployed at age i_0, and die with certainty at I. In our analysis $i_0 = 20$ and $I = 80$. We abstract from population dynamics, all age cohorts have the same constant size normalized to unity, and when a cohort

[8] One could think that these workers have been educated intensively in some skills that may become obsolete as time passes.

[9] In real life the calculation is not that simple. Normally later years in the life account more, an there may be minimum or maximum pensions. Here in the model we abstract from these, and set a simple average.

[10] There is no extra theoretical difficulty in having an equilibrium in the asset market, but computationally it is very complex. The state space considerably grows, and grid searches become infeasible. For more details, see the computational appendix.

dies, another of equal size replaces it. For this reason, and provided that there is no inactivity in this model, unemployment level and unemployment rate can be used interchangeably. Individuals can transfer resources forward in time saving an amount a at an exogenous interest rate r.[11]

Individuals can be either be employed, unemployed, or retired. When an agent of age i get together with a firm, the output is given by $F(i)$, an exogenous function that determines the life cycle profile of productivity. Hence, age is the only determinant of the output of a match. This function is explained on section 1.5. There is no intensive margin in hours, agents either work or not. Out of this $F(i)$ the worker gets a wage income subject to taxation, and firms get the output minus wages (there are no other costs of production). When unemployed, agents earn benefits. Both wages and unemployment benefits are explained in section 1.3.2.

The *labor status* of a specific worker j at age i is given by the function

$$L_j(i) = \begin{cases} 1 & \text{if employed} \\ 0 & \text{if unemployed} \end{cases} .$$

The earnings of the agents are wages if employed, or the perceived unemployment benefits otherwise:

$$l_j(i) = \begin{cases} w(i,\cdot) & \text{if} \quad L_j(i) = 1 \\ b_u(i,\cdot) & \text{if} \quad L_j(i) = 0 \end{cases} .$$

Functions $w(i,\cdot)$ and $b_u(i,\cdot)$ can have any form. The *history* of a worker up to age i, recording all her employment status and earnings is given by the 2-dimensional i-tuple:

$$H_j(i) = \Big\{ \{L_j(i_0), l(i_0)\}, \{L_j(i_1), l(i_1)\}, \ldots, \{L_j(i), l(i)\} \Big\}.$$

The set of all possible histories up to i is denoted by $\mathcal{H}(i)$. In equilibrium there will be a distribution of agents over all possible employment histories $\widetilde{\Psi}(H_j(i))$.

The challenge posed by this formulation is that the size of all possible sets of histories is infeasible in computational terms. However, we can "compress" all the information that the history conveys into a single continuous variable measuring the current value of the SS account. To have this, the calculations linking retirement benefits to a given labor history has to be simple enough such that we do not need to track previous histories. Let $\widetilde{\Phi} : \mathcal{H}_j(i) \to \mathbb{R}^+$ be this function, assigning to each labor market history realization a value of the SS account,

[11] This is the same as saying that agents are borrowing constrained in a small open economy.

$$\widetilde{\Phi}(H_j(i)) = \frac{1}{i} \sum_{i=i_0}^{i} \Big\{ L_j(i)w(i) + (1 - L_j(i))b_u(i) \Big\} = \phi_j(i),$$

which is basically the average of the gross earnings she made until previous period. This average will rise if they are working, or will fall if they spend periods unemployed, since $b_u(i) \leq w(i)$. Function $\widetilde{\Phi}(\cdot)$ is surjective, i.e., there may be multiple histories generating the same SS values $\phi_j(i)$. However, given that we can get today's account as a weighted average between yesterday account and today's earnings,

$$\begin{aligned}\widetilde{\Phi}(H_j(i)) &= \left(\frac{i-1}{i}\right) \widetilde{\Phi}(H_j(i-1)) + \left(\frac{1}{i}\right) \Big(L_j(i)\, w(i) + (1 - L_j(i))b_u(i)\Big) \\ &= \Phi\Big(\phi_j(i-1), L_j(i)\Big),\end{aligned}$$

there is no need of keeping track of all possible histories, just only one continuous variable.

The equilibrium distribution of the economy is given by $\Psi(s, i, i^{ret}, a, \phi)$ where s is the current state of the agent in the labor market, i.e., whether they are employed, unemployed or retired; i is the age, i^{ret} is the age at which the agent retired, and ϕ is the amount contributed to her future pension, the so-called SS account. For convenience in the exposition of the model, we can define the distributions of employed, unemployed and retired as

$$\begin{aligned} e(i, a, \phi) &= \Psi(s = \text{employed},\ i,\ \emptyset,\ a\ ,\ \phi) \\ u(i, a, \phi) &= \Psi(s = \text{unemployed},\ i,\ \emptyset,\ a\ ,\ \phi) \\ r(i, i^{ret}, a, \phi) &= \Psi(s = \text{retired},\ i,\ i^{ret},\ a\ ,\ \phi). \end{aligned}$$

The variable indicating the age of retirement, i^{ret}, is only defined for agents that have actually retired. Once they are retired, it remains constant throughout the life of the agent. It is important to keep track of the age at which an agent retires because it is a key determinant of the retirement pension she will receive.

Labor market frictions are modelled à la Mortensen and Pissarides (1994). Let us assume the existence of a homogeneous mass of vacant firms willing to engage in production, big enough so that each one lacks market power. Searching for workers is costly for firms, and they have to pay c to maintain an open vacancy. There are different labor sub-markets indexed by $z = (i, a, \phi)$. Vacant jobs and unemployed come together according to a matching function $m(u(z), v(z))$ representing frictional labor market arrangements. It is assumed to be constant returns to scale, so that we can characterize the number of matches by the *market tightness* $\theta(z) = v(z)/u(z)$. Using this matching function, the probability that a firm meets an unemployed is $m(u(z), v(z))/v(z) = q(\theta(z)) = q(z)$, and the probability that an unemployed finds a firm is $m(u(z), v(z))/v(z) = p(\theta(z)) = p(z)$. Firms produce when met with one worker, i.e., labor relationships are restricted to firm worker pairs. We assume no on-the-job search.[12]

Job destruction is an exogenous function $\lambda(\cdot)$, which will be calibrated to match the decreasing profile in age shown in Figure 1.2. For the US, similar results can be seen in Lise, Meghir and Robin (2013) or Menzio et al. (2013). We chose to keep the problem simple and not to look at endogenous separations.[13]

On the government side, there is a single budget that pays the unemployment benefits and retirement pensions, and it is financed by a proportional tax on labor income τ_w. This means that both employed and unemployed agents pay for social insurance. The government has to balance its budget every period, that is, there is no debt.[14]

1.3.1 Value Functions

Agents

Let $W(i, a, \phi)$ and $U(i, a, \phi)$ be the value of being employed and unemployed when up to age i your savings are a and you have contributed ϕ to your SS account. Also, define $R(i, i^{ret}, a, \phi)$ as the value of being retired at age i, when you retired at i^{ret}, have an amount of assets a saved, and a SS account worth of ϕ. Workers decide today how much to save, and whether or not they will be retired tomorrow[15].

Agents are borrowing constrained, i.e., $a \geq 0$. The value function of a worker is given by

[12] The reason to asume that there are different submarkets for each z is just to simplify computation.

[13] To endogenously explain separation rates we require at least an idiosyncratic shock on productivity, which would add another state variable to the model.

[14] The assumption that unemployed also pay taxes is used without loss of generality, since we could define their benefits as the net income of taxes.

[15] The timing of the retirement decision is not relevant, but this alternative is more realistic than choosing contemporaneously whether to be retired, when workers need to communicate this decision to their employers and the government beforehand.

$$
\begin{aligned}
W(i,a,\phi) = \max_{a'} \Big\{ & u\big((1-\tau_w)\, w(i,a,\phi) + (1+r)a - a'\big) - B \\
& + \beta\, C_{WR}\big(i+1, a', \phi'_w\big) \Big\} \,,
\end{aligned} \tag{1.1}
$$

being the optimal choice of a worker between staying in the market and retiring, if entitled to do so, given by

$$
C_{WR}\big(i{+}1, a', \phi'\big) = \begin{cases} \max\Big\{ C_W\big(i+1, a', \phi'_w\big)\,,\ R(i+1\,, i+1\,, a'\,, \phi'_w)\Big\} & \text{if } i+1 \geq i_e \\ C_W\big(i+1, a', \phi'_w\big) & \text{otherwise}\,, \end{cases}
$$

with i_e being the *early* retirement age set by law, from which you are entitled to retire, albeit subject to penalties whenever you retire before the *normal* retirement age, that is: $i^{ret} \in [i_e, i_r)$.[16]

The expected value function of staying in the market for a worker is given by

$$
C_W\big(i+1, a', \phi'_w\big) = \big(1 - \lambda(i+1)\big) W(i+1, a', \phi'_w) + \lambda(i+1) U(i+1, a', \phi'_w)\,.
$$

Note that this value is uncertain, because with a probability $\lambda(i{+}1)$ the agent will be unemployed. ϕ'_w is the following value of an agent's social security account, provided that she was employed today. What (1.1) tells us is that if the worker is older than the early retirement age, she can choose whether to stay in the market next year or to retire. The second case is when they are still not entitled to do so. Parameter B represent the disutility of staying in the labor market, which is assumed to be the same for a worker than for unemployed. The policy function reflecting optimal saving decisions of workers is given by $a' = g^a_w(i, a, \phi)$.[17]

The retirement decision of (1.1) can be expressed in the following retirement policy function

$$
g^r_w(i,a,\phi) = \begin{cases} 1 \quad \text{if} \quad i \geq i_r \quad \text{and} \quad R(i,i,a,\phi) \geq C_W(i,a,\phi) \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise}\,. \end{cases} \tag{1.2}
$$

[16] The *i's* in subindices are parameters (i_e, i_r), and i^{ret} is a choice variable.

[17] Having different disutilities of being employed and unemployed would have been more realistic, for instance, smaller for those unemployed. However, this would imply that for some subset of policies, there may be voluntary unemployment, and this issue is out of the scope of this paper.

Similarly, the value of being unemployed is given by the equation

$$U(i,a,\phi) = \max_{a'} \Big\{ u\big((1-\tau_w)\, b_u(i,a,\phi) + (1+r)a - a'\big) - B \\ + \beta C_{UR}\big(i+1, a', \phi'_u\big) \Big\} , \tag{1.3}$$

$$C_{UR}\big(i{+}1, a', \phi'_u\big) = \begin{cases} \max \Big\{ C_U\big(i+1, a', \phi'_u\big) \, , \, R(i+1, i+1, a', \phi'_u) \Big\} & \text{if } i+1 \geq i_e \\ C_U\big(i+1, a', \phi'_u\big) & \text{otherwise} \end{cases}$$

with $C_{UR}(\cdot)$ the continuation value of an unemployed choosing between waiting for a job or retiring. Parameter ϕ'_u reflects tomorrow's social security account value, given that today the agent was unemployed. Function $b_u(i,a,\phi)$ is the unemployment benefit. The rule for setting it will be explained in 1.3.2. The *continuation value of unemployment* is

$$C_U\big(i+1, a', \phi'_u\big) = \big(1 - \lambda(i+1)\big)\, p(i+1, a', \phi'_u)\, W(i+1, a', \phi'_u) \\ + \big[1 - (1 - \lambda(i+1)\big)\, p(i+1, a', \phi'_u)\big]\, U(i+1, a', \phi'_u) \, .$$

Notice this value depends on the probability at which unemployed meet vacants $p(\cdot)$. Newly formed matches can also be destroyed right away with probability $\lambda(i+1)$. As before, define the retirement decision of unemployed as

$$g^r_u(i,a,\phi) = \begin{cases} 1 \quad \text{if} \quad i \geq i_e \quad \text{and} \quad R(i,i,a,\phi) \geq C_U(i,a,\phi) \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise} \, , \end{cases} \tag{1.4}$$

and the saving decision $a' = g^a_u(i,a,\phi)$.

Pensions

Individual SS account evolves following this rule

$$\phi'_s = \left(\frac{i-1}{i}\right)\phi + \left(\frac{1}{i}\right) l_s(i) \quad l_s \in \{w(i,a,\phi), b_u(i,a,\phi)\}, \quad s \in \{w, u\} \, ,$$

which is a weighted average of past and current earnings, be it as employee $w(i,a,\phi)$, or your unemployment compensation $b_u(i,a,\phi)$. Variable ϕ, together with the age at which agents choose to retire i^{ret} determine the penalty that it is applied to your SS account, expressed in functional form as

$$\eta(i) = \begin{cases} \eta_0 + (1-\eta_0)\left(\frac{i-i_e}{i_r-i_e}\right) & \text{if } i \in [\, i_e \,,\, i_r) \\ \\ 1 & \text{otherwise} \,. \end{cases}$$

This penalty is simply indicating that agents retiring earlier than standard age $i \in [i_e, i_r)$ see their pension reduced by a fraction $\eta(i)$. This fraction starts from $\eta_0 \in [0, 1)$ and increases linearly to no penalty when retiring at i^{ret}.

Lastly, define the value of being retired at age i, when retiring at $i^{ret} \in [i_e, I)$, having savings a and SS account of ϕ:

$$R(i,\, i^{ret},\, a,\, \phi) = \max_{a'} \left\{ u\big(b_r(i^{ret}, \phi) + (1+r)a - a'\big) + \beta R(i+1, i^{ret}, a', \phi) \right\}, \tag{1.5}$$

where $b_r(i^{ret}, \phi)$ is the pension entitlement, which is the product of

- ψ_r: the pensions replacement rate. This is the main policy parameter
- $\eta(i^{ret})$ The penalty for early retirement
- ϕ: The individual's SS account

Similarly to the other cases, saving decisions of the retired are reflected in the policy function $g_r^a(i, i^{ret}, a, \phi)$.

Firms

Let $J(i, a, \phi)$ and $V(i, a, \phi)$ be the value of a firm that has a worker or an idle vacancy in the (i, a, ϕ) submarket, respectively. Firms are risk neutral and they are different entities with respect to the workers/unemployed, i.e., they keep the proceeds of their activity. The value of having a vacancy posted is

$$\begin{aligned} V(i, a, \phi) = & -c + \beta\big(1 - \lambda(i+1)\big)\, q\big(i+1, g_u^a(\cdot), \phi\big)\big(1 - g_u^r(i+1, a, \phi)\big) J\big(i+1, g_u^a(\cdot)\,,\, \phi\big) \\ & + \beta\left[1 - (1 - \lambda(i+1))\, q(i+1, a', \phi)\right] V(i,\, g_u^a(\cdot)\,,\, \phi^u)\,, \end{aligned}$$

where $c > 0$ is the per period vacancy cost. With some probability $(1 - \lambda(i))q(i, \cdot)$ the vacant is matched with a worker, the firm will start producing, otherwise the position remains unfilled. We assume without loss of generality that matches might be destroyed at any period, including their inception. Free entry will attract entrants to the market until there are no gains in opening more vacancies in any labor submarket, $V(i,a,\phi)=0 \quad \forall\, i,a,\phi$. The previous equation gives us the *Vacancy Posting Condition*:

$$c \geq \beta\,(1-\lambda(i))\,q(i,a,\phi)\,\big(1-g_u^r(i,a,\phi)\big)\,J(i,a,\phi) \qquad \forall\,\{i,a,\phi\}\,, \tag{1.6}$$

which holds with strict inequality whenever the firm decides not to hire a worker for that age. This will happen if the vacancy cost outweigh the value of having a job filled by a worker.

For those firms with a job filled, the value function will be

$$J(i,a,\phi) = \pi(i,a,\phi) \\ + \beta\big(1-\lambda(i+1)\big)\,(\,1-g_w^r(i+1,g_w^a(\cdot),\phi_w')\,)\,J\big(i+1,g_w^a(\cdot),\phi_w'\big)\,, \tag{1.7}$$

$\pi(i,a,\phi)=F(i)-w(i,a,\phi)$ the profits a firm makes in sector (i,a,ϕ). As we will see just below at section 1.3.2, wages will only depend upon age. The value of a job are current profits plus the expected discounted value of having the job filled tomorrow. Notice that with probability $\lambda(i+1)$ the match is destroyed and the firm goes vacant, which in equilibrium has value zero.

1.3.2 Wage Bargaining

When a worker and a firm happen to be matched, both sides have a bilateral monopoly power because any part is better off entering into a production relationship than splitting. Hence, wages are the outcome of a bilateral bargaining between the worker and the firm over the surplus of the match. $U(i,a,\phi)$ is the *threat* point of the worker as he can always get this value when unemployed. Function $V(i,a,\phi)$ is the firm's threat point. Let μ be the bargaining weight of the worker. Assuming that free entry will drive the value of the vacancy to zero at all ages, and taking as given the SS scheme, wages are obtained solving

$$w(i,a,\phi) = \text{argmax}\Big\{\,\big(W(i,a,\phi;w)-U(i,a,\phi)\big)^{\mu}\,J(i,a,\phi;w)^{1-\mu}\,\Big\}\,.$$

We will call this the full bargaining wage. There are two computational problems with solving this problem. On one hand, to account for the the consumption-smoothing effects that unemployment and pensions that are being studied here, agents need to be risk averse, which makes the above problem not to have a close-form solution for wages, greatly complicating the fix point algorithm. On the other hand, if a non-linear solution method is applied on the nested fix point, equilibrium wages may be negative for given guesses of taxes or market tightness which forces us to deal with corner solutions in wages, making the solution infeasible. This problem persists even if agents bargained over their finite lifetime surplus, but they behaved as if they were risk neutral, simpler case for which a close-form solution exists.[18]

There are two solutions to this problem. First is to assume fixed wages. The problem with this is that we want to do policy with this model and see how welfare changes with unemployment policies, and these are expected to have effects on wages, hence employment and taxes. Another solution is to solve a "partial" Nash bargaining in which wages are determined bargaining only about today's output (future is ignored), and where risk aversion is ignored. This will obviously be a rough approximation to what the true wages would be, but it will be easy to solve and wages will react to policies.[19]

The modified wage bargaining is given by solving:

$$w(i,a,\phi) = \text{argmax}\Big\{ \big(w(i,\phi) - b_u(i,a,\phi)\big)^{\mu} \big(F(i) - w(i,a,\phi)\big)^{1-\mu} \Big\} \,,$$

which gives rise to the following wage equation

$$w(i,a,\phi) = \mu F(i) + (1-\phi) b_u(i,a,\phi) \,.$$

Assuming that unemployment compensation is a fraction (replacement) of wages paid to similar workers, $b_u(i,a,\phi) = \psi_u w(i,a,\phi)$ with $\psi_u \in [0,1]$, the wage equation becomes

$$w(i,a,\phi) = w(i) = \left(\frac{\mu}{1-\psi_u(1-\mu)} \right) F(i) \,. \tag{1.8}$$

1.3.3 Government

The two replacement rates, ψ_u for unemployment benefits, and ψ_r for retirement pensions, together with legal ages of retirement i_e and i_r and the functional form of early retirement penalties $\eta(i)$ determine all the welfare state in this model. From the revenue side, the government's SI budget is financed with a single tax on a labor income and unemployment compensations τ_w. Those retired do not pay τ_w on benefits received. The SI restricted to a balanced budget set of policies, which is represented by the equation below,

$$\begin{aligned} \tau_w & \left(\int w(i)\, \mathrm{d}e(i,a,\phi) + \int b_u(i)\, \mathrm{d}u(i,a,\phi) \right) \\ & = \int b_u(i)\, \mathrm{d}u(i,a,\phi) + \int b_r(i^{ret},\phi)\, \mathrm{d}r(i,i^{ret},a,\phi) \,, \end{aligned} \tag{1.9}$$

[18] The algorithm used to solve the model is described in detail in section 1.9.

[19] This is equivalent to a reduced-form model in which wages are fixed but shifted with unemployment policies.

where in the left hand side there are revenues, coming from contributions of employed and unemployed, and in the right hand side costs, both from unemployment benefits and retirement pensions, being the system organized in a single budget.

1.4 Equilibrium

A stationary recursive equilibrium in this economy is composed by value functions $W(i, a, \phi)$, $U(i, a, \phi)$, $J(i, a, \phi)$ and $R(i, i^{ret}, a, \phi)$; policy functions $g_w^a(i, a, \phi)$, $g_u^a(i, a, \phi)$, $g_r^a(i, i^{ret}, a, \phi)$, $g_w^r(i, a, \phi)$ and $g_u^r(i, a, \phi)$, wages $w(i)$, benefits $b_u(i)$ and $b_r(i^{ret}, \phi)$, a measure of the agents in the economy $\Psi(s, i, i^{ret}, a, \phi)$, a tax rate τ_w and market tightness $\theta(i, a, \phi)\ \forall\,(i,\ i^{ret},\ a, \phi) \in \big\{\{i_0, I\}\times\{i_e, I\}\times R^+ \times R^+\big\}$ such that, given preferences, matching and productive technology, and government policies ψ_u and ψ_r:

- Wages satisfy (1.8).
- Agents behave accordingly to value functions (1.1), (1.3), (1.7) and (1.5).
- Retirement policy functions are determined by (1.2) and (1.4).
- $\theta(i, a, \phi)$ satisfies (1.6).
- $\Psi(\cdot)$ is consistent with labor histories and retirement choices produced by $\lambda(i)$, $\theta(i, a, \phi)$, (1.2) and (1.4)
- On the government side, given the choices of the agents, τ_w is such that (1.9) is balanced.

1.5 Calibration

The calibration is intended to match Spanish labor market and retirement patterns. We have to set a vector of parameters, selecting a matching technology, utility and the age profile of of successful matches. First, let us look at the functional forms.

For the matching function, I take the specification of Den Haan, Ramey and Watson (2000):[20]

[20] The authors offer a "microfoundation" for this function. Suppose J channels, understood as the place where workers and firms go to find a job or fill its vacants, are created. Each unemployed u, goes randomly to one of these places, and each vacant firm v too. The probability a worker meets a firm is then v/J, and that a firm meets an unemployed is u/J. Thus, total matches are uv/J. Assuming that $J = (u^\gamma + v^\gamma)^{1/\gamma}$ gives us the matching function.

$$m\big(u(i,a,\phi),v(i,a,\phi)\big) = \frac{u(i,a,\phi)\,v(i,a,\phi)}{\big(u(i,a,\phi)^\gamma + v(i,a,\phi)^\gamma\big)^{1/\gamma}}\,. \tag{1.10}$$

The associated meeting rates for vacant jobs and unemployed are:

$$q(i,a,\phi) = \frac{m\big(\,u(i,a,\phi),v(i,a,\phi)\,\big)}{v(i,a,\phi)} = \left(\frac{1}{1+\theta(i,a,\phi)^\gamma}\right)^{\frac{1}{\gamma}}$$
$$p(i,a,\phi) = \frac{m\big(\,u(i,a,\phi),v(i,a,\phi)\,\big)}{u(a,\phi)} = \left(\frac{1}{1+\theta(i,a,\phi)^{-\gamma}}\right)^{\frac{1}{\gamma}}\,.$$

Functions $q(i,a,\phi)$ and $p(i,a,\phi)$ are written in terms of the market tightness of each sub-markets since this meeting function is CRS. The advantage over the standard Cobb-Douglass counterpart is that the meeting rates are guaranteed to produce an interior solution for $q(\cdot)$ and $p(\cdot)$, without having to deal with truncations in $\theta(\cdot)$.[21]

For the match output $F(i)$ I assume a quadratic function, increasing at the early ages of the career of an agent, and decreasing afterwards, being a reduced form for how technological change makes skills of older individuals less productive. The explicit functional form is given by

$$F(i) = \max\Big\{ -f_0(i-(f_1-20))^2 + 1, 0\Big\}\,,$$

where $F(i)$ is represented in Figure 1.4. Let us explain the function briefly. f_0 is a parameter that determines how quick the productivity increases and decreases, and f_1 determines the age at which wages are maximum, whose value is normalised to 1. The values set are $f_0 = 0.00007$ and $f_1 = 40$. The first one is chosen so that when agents are 70 years old they are not productive, and the second determines that the maximum productivity is reach at age 40. The max operator ensures that the productivity of the agents does not turn negative. This concave shape of the life cycle productivity is widespread in OLG models used in SS reform such as Conesa and Krueger (1999) and Díaz-Giménez and Díaz-Saavedra (2009).

Now, it remains to give values to i_e, i_r, μ, β, γ, c, B, $\lambda(i)$ and the two policy variables ψ_u and ψ_r. From these parameters, we calibrate γ and B to match moments from the data, the remaining ones are took from the literature or directly from observable values. We will first explain the free parameters chosen. Early retirement is set at $i_e = 60$, and normal to $i_r = 65$, as it is in Spain. Bargaining power of the agents is assumed $\mu = 0.5$, as it is standard in the literature. see for instance, Mortensen and Pissarides (1994). The discount

[21] Despite values are always in $[0,1]$, sometimes there is a corner solution for (1.6), implying no job creations especially for old ages, and infinite durations of unemployment. This will not occur if the vacancy posting condition is low enough.

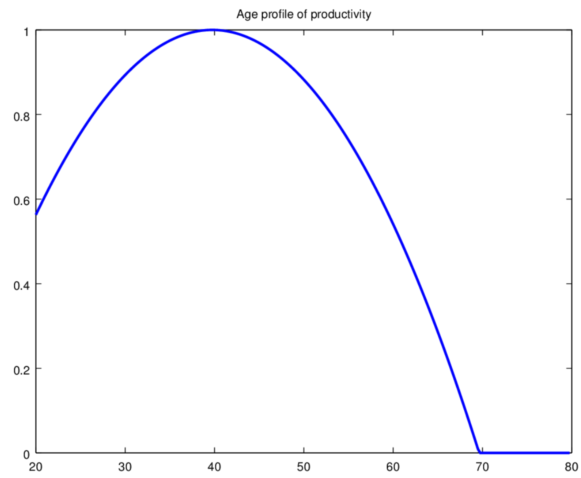


Figure 1.4: Match output by age

factor together with the interest rate r determine life cycle savings, we set $\beta = 1$, and r to the average annual real rate for the period 2005-2012 which is roughly 2%. We average all these years to have a good representation of the long term interest rate. Separation rate $\lambda(i)$ is calibrated as an exponentially decreasing function of age, chosen to fit its data counterpart shown in figure 1.2:

$$\lambda(i) = \begin{cases} 0.2 & \text{if } i = 20 \\ 0.985\,\lambda(i-1) & \text{otherwise}\,. \end{cases}$$

In Spain unemployment benefits are given as a proportion of past contributions, and this proportion is decreasing with the spell of unemployment, and they are awarded only if you have spent a certain time employed. To simplify this setting, we take the average spell of unemployment in Spain, which is roughly a year, and calculate the proportion that an unemployed would get, which is $\psi_u = 0.59$. For the pension replacement rate, we follow Boldrin et al. (1999) and set a replacement rate of $\psi_r = 0.5$. These two parameters will represent our benchmark scenario.

In the literature, the cost of posting vacancies is normally calibrated together with the elasticity of the matching function to replicate some moments of the data, especially job creation and job destruction, see for instance Pérez and Osuna (2012). In the model here the two parameters in question are γ and c, however, here job destruction is exogenous. This means that there is a one-to-one relationship between unemployment and job creation, so that we cannot target job creation and job destruction to calibrate the model, mean-

Variable	Model	Data
Av. retirement age	63.946	63.357
Av. unemployment	0.17014	0.1689

Table 1.1: Target and model outcomes

Parameter	Description	Value
μ	Nash bargaining	0.5
β	Discount	1
r	Returns of savings	0.02
c	Vacancy cost	0.1
B	Disutility of work	0.66667
γ	Elasticity of matching function	0.53333
i_e	Early retirement age	60
i_r	Normal retirement age	65
ϕ_0	Early retirement penalty	0.6
ψ_u	Unemployment repl. rate	0.59
ψ_r	Retirement repl. rate	0.5

Table 1.2: Calibration summary

ing that the model is over-identified. To solve this issue we fix the cost of posting a vacancy c. There is little direct evidence on the costs of maintaining a vacancy in Spain. We set it at $c = 0.1$.[22]

The approach we take for the remaining two parameters is to calibrate them jointly such that the model produces moments similar to the observed counterparts in the data. Parameter γ governs job creation, so adjusting it we can match unemployment rate between 1999 and 2013 in Spain, as an estimate of its steady state value. For B, the disutility of work/unemployment, I set it such that the average retirement age matches the Spanish data, i.e., $i^{ret} = 63.36$. They are both set together to minimize the sum of the quadratic deviations between data and model. The two target moments from the data, and its model counterparts are shown results are shown in table 1.1, and the summary of all the parameters is given in table 1.2.[23]

[22]see, for example, Andolfatto (1996).

[23]Normal retirement age has been delayed progressively from 2013, and from 2027 onwards it will be 67, but we are going to stick to the previous system because we have the data to calibrate the model, and since our interest is the steady state, there is no more insight using this new statutory age.

1.6 Results

Here we present some of the results of the benchmark model above described some comparative statics changing the unemployment benefits ψ_u and the pension replacement rate ψ_r. We will analyze the effects on taxes, average assets and pensions, their dispersions and the welfare of the new entrants of the model. As this model only looks at the steady state, this is the important variable for doing policy comparisons.

In Table 1.3 we show a summary of the effects of changing unemployment insurance, and results are worth some comments. The row of welfare of the newbors is expressed as a proportion (over 1) of the welfare of the benchmark case. First, reducing unemployment benefits shrinks private savings. The reason of this is that without policies that provide support in case of job loss, agents are unable to save, because their income is so small in these unfortunate situations that they will be consuming all their unemployment benefits, or running down their previous savings. As we will see in the following section, there will be a "bliss" point on unemployment benefits. Second, increasing unemployment benefits does *not* reduce savings. The reason for this has to be that agents are financially constrained for their retirement. Third, UI increase the average pension because in the case of job loss agents contribute greater amounts (since unemployment benefits also contribute to the agent's SS account), producing higher pensions.

Fourth, there are interesting second order effects, more obvious in the case of assets. Increasing unemployment benefits makes the distribution of assets to become more disperse. The reason for this is simple. This distribution aggregates over all ages, so the reason why there is a higher variance is because there is more "distance" between the savings of the young, which are none, and the savings of workers at mid age, which save more. Another effect, which is less evident, is that increasing unemployment benefits slightly reduces the variance in pensions. The reason for this is that high unemployment benefits diminish the difference that worker and an unemployed contribute to the pension. Conversely, lower unemployment benefits exacerbate the effect of a bad labor history in future pension benefits through the lower contributions they make.

The effects of changing pension replacement rates are in table 1.4. Given that wages are not influenced by retirement benefits by the simplification explained in section 1.3.2, the impact are smaller. We see that giving higher pensions agents slightly save less. The table shows that increasing ψ_r makes agent retire later, but the dynamics of these policies are more complex as we will see in figure 1.8(b). What it is indeed interesting it that, in terms of welfare, the value $\psi_r = 0.5$ seem to be better. We will explore this result in

ψ_u	0.5	0.59	0.7
τ	0.255936	0.273979	0.292627
$mean(i^{ret})$	65.4674	63.8414	63.1767
$mean(a)$	0.306648	0.622534	0.919155
$var(a)$	0.0932029	0.296905	0.383561
$mean(\phi)$	0.255014	0.276847	0.303675
$var(\phi)$	0.000180812	0.000108601	7.46605e-05
W. entrants	0.999461	1	1.00073

Table 1.3: Effects of changing unemployment benefits

ψ_r	0.35	0.5	0.65
τ	0.219261	0.273979	0.329985
$mean(i^{ret})$	63.8201	63.8414	65.703
$mean(a)$	0.655916	0.622534	0.564308
$var(a)$	0.358055	0.296905	0.214392
$mean(\phi)$	0.189563	0.276847	0.359021
$var(\phi)$	5.67693e-05	0.000108601	0.000214105
W. entrants	0.999942	1	0.999941

Table 1.4: Effects of changing pension replacement rate

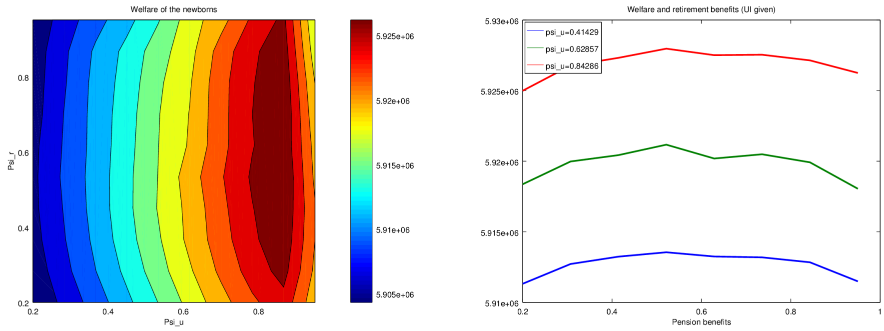
section 1.7.

1.7 Optimal policy

In this section we look at an optimal policy question of the Social Insurance system. What is the combination of policies that a government would choose to maximise the welfare of the agents? The measure of aggregate welfare of a benevolent dictator would be sum of the utility of a certain set of agents $\widetilde{S} \subset S$. The problem could be formulated as follows

$$\mathcal{W}(\psi_u, \psi_r) = \max_{\{\tau_w, \psi_u, \psi_r\}} \sum_{s \in \widetilde{S}} \Big\{ e(s)W(s) + u(s)U(s) + r(s)R(s) \Big\}$$
$$\text{subject to} \quad \text{SS budget balance (1.9)}\,,$$

where $\widetilde{S}$ is the set of agents which the social planner consider when designing the policy. To measure whether a policy is good or bad, we analyze its impact on the newborn individuals, (our $\widetilde{S}$), which are the ones that are going to bear the welfare impact of a policy change. For this we try all the possible policy combinations, calculate how individuals born in a world with these policies would fare and find the best. This problem amounts to maximise social welfare given that SS budget balances. Obviously, these value functions in the problems arise from optimal behaviour of agents, i.e., they satisfy equations (1.1), (1.3) and (1.5). Results are in figures 1.5, 1.6 and 1.7. UI and pension size are represented in the x and y axis respectively.



(a) Welfare as a function of ψ_u (horizontal) and ψ_r (vertical)

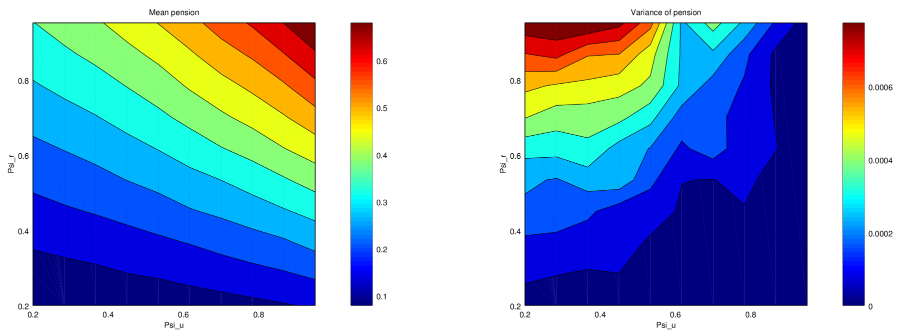
(b) Welfare as function of ψ_r, given some ψ_u

Figure 1.5: Welfare of newborns

As we can clearly see, for a given level of retirement benefits, there is an optimal unemployment replacement ratio of around 85% of wages. This level is indeed independent of how generous pensions are. The reason for this is the simplification we made in the wage bargaining, making wages independent of retirement policies. For retirement is optimal to set a replacement rate of roughly 50%. This is an important result because, to our knowledge, this is the only paper that can achieve an optimum positive replacement rate in pension benefits, i.e., in other models it is always optimal to replace a PAYG by a fully funded system. The reason of this result is that unemployment risk is unaccounted for in previous models. As we have pointed out other times iin the paper, when agents lose their job, they cannot save for their retirement, and therefore a newborn agent is willing to give up income bearing a higher taxation in exchange for a greater security in her retirement income. Tax rates implied by these welfare policies are showed in figure 1.8(a). Unemployment benefits, above a certain threshold, roughly $\psi_u = 0.5$, require taxes to grow more than linearly to sustain the budget. This is the cost-side of the insurance benefits they produce, they raise wages since unemployment benefits are the threat point in the bargaining, and hence discourage firms to hire, which means that the extra costs coming from higher benefits and a higher unemployed mass, have to be supported by fewer individuals that have to pay more. The marginal benefit of higher ψ_u becomes negative around 85% and giving more of it is suboptimal.

The impact of these policies in retirement ages are not trivial. First, let us explain what are the trade-offs involved for both unemployment and retirement policies. When unemployment benefits increase, there are two opposite effects in place. First of all, those who lose their jobs contribute more during those periods, having a better SS account, which may encourage agents to withdraw earlier from the labor market. On the other side, more generous benefits reduce the financial constraint faced by older unemployed, thus encouraging them to wait and see if they can get an opportunity in the job market. When retirement benefits increase, the effects unambiguously make agents to retire earlier.

This is reflected in graph 1.8(b). The age of retirement is twin-peaked, and the period that agents stay in the market is maximum when either there are very low SS policies in both unemployment and retirement, and when there are medium levels of protection in both policies. In the first case, when no protection, it makes sense that agents stay longer in the labor market, since they do not have either pension nor a mechanism to save when unemployed, and hence they have to spend more time on average in the market in order to contribute for their accounts. As long as the benefits increase, agents exit the labor market earlier. The second case, with pensions around $\psi_r \in (0.6, 0.8)$ and unemployment benefits in $\psi_u \in (0.4, 0.65)$, agents retire later. In this zone, the dominating effect is that unemployment benefits make agents stay longer in the labor market, thus increasing the chances of getting another employment, and hence, delaying, on average, the retirement age.



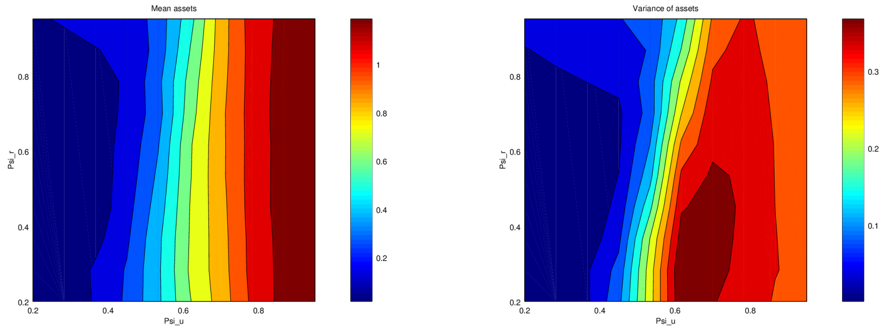
(a) Mean pension

(b) Variance of pensions

Figure 1.6: Effects on pensions

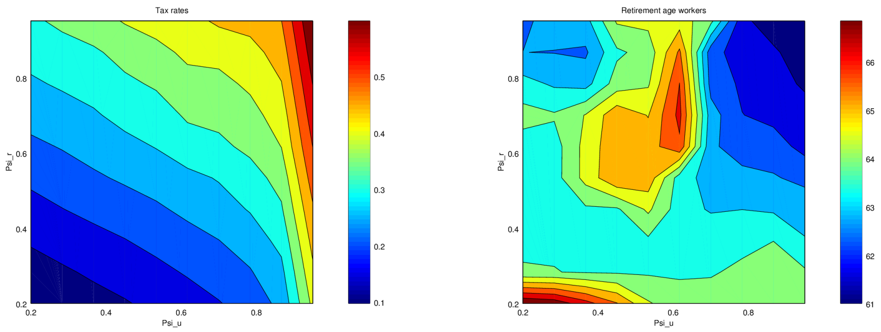
1.8 Conclusions

In this article we provide a new macroeconomic framework to study life cycle unemployment and retirement decisions altogether. It is composed of a life



(a) Mean Assets (b) Variance of assets

Figure 1.7: Effects on assets



(a) Tax rate (b) Average retirement age

cycle search and matching model with endogenous retirement. The individual labor history of the agents determine the pension they shall get, and it is adjusted by two variables. First, the amount they have thus far contributed, denoted by ϕ, and second, the age at which they retire, i^{ret}. If they retire before the *normal retirement age*, they face penalties in their benefits. Agents are not idiosyncratically heterogeneous in the model, per se, but the labor histories produced by a lifetime of shocks indeed are, which produces interesting effects on the distributions of assets, pensions and age of retirement. The way ϕ is defined, it allow us to simplify the whole labor history into a a single continuous variable, which makes the problem tractable. In spite of the technical simplifications assumed, we believe is rich enough to use it to perform policy experiments.

As an example application of it, we search for an optimal design of the Social Security system, understood in this model as the unemployment benefits and the retirement pension replacement rate. We find that unemployment benefits are optimal, and should be sizeable (they should pay roughly 85% of what workers would get being in the labor market), and more importantly, a positive replacement rate (50% of their average lifetime earnings) is found to be optimal in the model, which is a novelty compared to the standard literature, and should encourage researchers that call for the removal of PAYG systems to study the issue taking into account the risk of unemployment and how pensions provide agents with additional ways to improve their welfare.

1.9 Appendix: Computational strategy

The computational problem in this article is solved with a *nested* fixed point approach, i.e., finding a fix point one equilibrium variable at a time. This method is widely used in life cycle models in which, even though the number of equations is finite (after discretising continuous variables) it is so high that is infeasible in computational terms. Let us explain the algorithm in detail for this model. We have two variables in which we have to find an equilibrium: taxes and market tightness

- Discretize the SS account $\phi \in \{0, \phi_1, \ldots, \phi_H\}$
- Discretize assets $a \in \{0, a_1, \ldots, a_H\}$
- take a guess of the tax rate τ_w^0:
  - take guess of market tightness $\theta^0(\cdot)$:
    * Solve the value and policy functions given guesses τ_w^0 and $\theta^0(\cdot)$
    * Simulate the distribution of agents $\Psi(\cdot\,, r^0, \tau_w^0, \theta^0)$ given those optimal choices. For doing this, we start from an arbitrary distribution of population, say, 16% of the population is unemployed for all age cohorts until age 65, from that point on, everybody is retired. For each single individual of the economy, we simulate labor market shocks, and according to these shocks and the policy functions, they evolve in the labor market. We do these simulations 60 periods, so the initial distribution assumed is independent, and $\Psi(\cdot)$ is entirely generated from the model.
  - If the guess of θ^0 is correct, calculating $\theta^1 = f(r^0, \tau_w^0, \theta^0)$ from (1.6) should give us $\theta^1 \simeq \theta^0$. If instead $\theta^1 \neq \theta^0$ we update our guess and solve again the model with θ^1. We repeat until until $\theta^i \to \theta^*$
- We check the tax that balances the budget τ_w^1, given by equation (1.9). If $\tau_w^1 \simeq \tau_w^0$, we are done. If not, we go back again and solve with τ^1, and repeat until convergence $\tau^i \to \tau^*$
- Convergence of τ and θ are achieved when $||\tau^i - \tau^{i-1}|| < \delta^\tau$, where δ^τ is the convergence criterion for taxes, and the same for θ. Equilibrium variables are updated using a convex combination between the previous and new value, to avoid overshoot, for example: $\tilde{\tau}^i = \alpha\tau^i + (1-\alpha)\tau^{i-1}$, where α is the weighting factor of the new value in updating the guess.

Chapter 2

The Macroeconomics of VAT Evasion

(joint with Alejandro Forcades Pujol)

2.1 Introduction

In recent times, international observers such as the European Union have been recommending southern European economies with fiscal straits to undertake tax reforms consisting in increasing the fiscal pressure on indirect taxation together with a reduction in income taxes, policy commonly referred as a tax shift. This is mentioned for instance in the 2012 Annual Growth Survey.[1]

Apart from being a requirement for achieving a harmonization in fiscal policies and the eventual creation of a fiscal union in the Eurozone, the economic rationale for such reforms are the claimed improvements in efficiency. The reason is that taxes on consumption do not affect the intensive margin decision of labor supply or saving, thereby improving economic growth, which hopefully, will bring budgets back to balance. For the case of Spain, quantitative evaluations from both European institutions and from experts from the Spanish government praise the benefits that such reforms would have. From the former, Orsini et al. (2014) study the macroeconomic implications of a revenue-neutral tax cut on labor income of the low skilled workers of 1% GDP, compensated with a raise in VAT. They argue that such a measure would reduce unemployment by 1 percentage point (pp). In march 2014, a commission of experts from the Spanish Treasury issued a report looking into this issue. They argued that an increase in consumption tax of 1 pp, coupled with a reduction of both 0.7 pp in labor tax and 0.3 pp in capital tax, will produce a

[1]By indirect taxation we mean consumption taxes. Throughout the article we refer as Value Added Tax (VAT), sales tax and consumption tax indistinctively. The full text of the Survey can be accessed at `http://ec.europa.eu/europe2020/pdf/annualgrowthsurveyen.pdf`

grow of GDP of 0.3 pp, and in employment of 0.2 pp in a three-year period. This conclusion is also frequent in the literature of optimal fiscal policy. Correia (2010) finds for the US that eliminating the current progressive income tax, and establishing a flat tax on consumption would reduce inequality and improve the welfare of the currently poor.[2]

These studies rest on the assumption that agents will pay these tax increases religiously, which is very unrealistic. Tax evasion is a widespread phenomenon in all the developed world. Estimates from Schneider et al. (2007, 2013) show that the extent of the informal sector in Europe range from 10 to almost 30% of the GDP. This informal sector, also dubbed *shadow economy*, is commonly divided in two parts. First, we have the *undeclared work*, which are all the labor activities that do not pay income tax, and second, there is *under-reporting*, which is the practice normally done by small business by which they only declare a part of the sales, thus reducing their profit figures. In the first case we are talking about income tax evasion. The second case is a bit more complex because it involves corporate income tax and VAT evasion. Let us look at the incentives to undertake these two kinds of illegal activities.[3]

On the income side, the reasons to evade are evident. Not paying the corresponding labor income tax could be understood as a lottery for the agent. If she is not inspected by the government, the agent is rewarded with your untaxed income, and in the case of not being that lucky, the agent will have to pay the amount due plus an administrative sanction. Hence agents evade income until the marginal benefit of evading equals the marginal cost, which is the tax plus penalty.

In terms of modelling, for VAT evasion we will opt for a simple representation in which firms are selling a *legal* good for which the agents will have to pay VAT, and another *illegal* good without it. In this scenario, VAT evasion arises from a double incentive, both for consumers and firms, to evade this tax. From the consumer perspective, if the the good that pays VAT and the good that does not are reasonably good substitutes, the agent will have an incentive to spend at least part of their their income on these cheaper untaxed goods. In the extreme case in which the goods were perfect substitutes, agents will spend all of their income in the untaxed goods. We will disregard this case as uninteresting in practical terms. Agents ristk nothing when ackquiring goods without VAT, this is a theoretical assumption and a good representation of

[2]The document elaborated by the fiscal experts is: *Comisión de Expertos para la Reforma del Sistema Tributario Español.* Full text: `http://www.fettaf.com/system/application/documentos/documentosvarios/InformeExpertosReformaSistemaTributarioEspanol.pdf`

[3]Our focus is restricted to developed economies. Developing countries face the problem of lacking a well functioning system to screen and collect money from tax payers. For such issues of *state capacity*, see Besley and Persson (2007).

reality. For example, in Spain, firms are responsible for collecting the VAT originated from their activities, and in case of suspicion, governments will inspect their accounts to check that they are doing so, and if they are not, they will be prosecuted and fined.[4] [5]

On the firm side, incentives are more complex. Business have to comply with a profit/corporate income tax. One way to avoid this disbursement to the state is simply selling part of their production underground, which will purportedly reduce their revenues and profits, thus diminishing the tax base. Agents are aware that they are buying these goods under the counter, so they will not pay the VAT that firms are complied to collect, which makes a double fraud, both in profit tax and VAT. The existence of a demand for these goods justify its supply, given the preferences we assume agents will want these untaxed good, so if the firms do not participate in the market another entrant firm would do it, so not selling illegal goods means losing money for them.

There are plenty of empirical studies looking at the issue of tax evasion. An interesting macroeconomic fact has been frequently found is that, all else equal, the higher the tax rates, the more sizeable the shadow economy becomes. We report here some of this evidence. Table 2.1 shows the regression results of a couple of studies, the first column taking VAT gap as dependent variable, and the second column using the whole shadow economy, taken from Schneider (2010). We see in both cases that higher tax rates reduce compliance. This is not the only study that looks into this, it has also been shown by Tanzi(1983), Mare (1996), Bovi (1999) and Giles (1999), among others.[6]

Coming back to the policy recommendations, this countercyclical property of tax evasion makes the suggestion of raising VAT more unreasonable, given the current recession that Spain is wading through. Such a policy would cause a reduction in revenues in two ways. First, in bad economic times there is more tax evasion, and second, the increase in taxes will provide further in-

[4] In this paper we do not want to explain *why* does tax evasion exist, but rather *how* does it change. Hence, we assume that agents derive utility from consuming illegal goods. This will create a demand for illegal goods, which in turn will encourage firms to supply these goods. A study attempting to explain why evasion exist would have to take into account factors such as the driving factors of *tax morale* of the individuals, and the *capacity* that the state has to indeed enforce the established taxes. Despite being an extremely interesting topic, these issues are out of the scope of this study.

[5] It is important here to bear in mind how VAT operates in real world. Abstracting from intermediate production, VAT is a tax that is collected by the firms and transferred to the government. That is why in fiscal policy it is said that VAT is *neutral* to the firm in terms of costs. However, if we contemplate the possibility of evasion, it will not be, as we will see.

[6] The *VAT gap* is a measure of VAT evasion obtained from national accounts. It measures the difference between the theoretical revenues the government expects (from the tax base) and the actual revenue raised, and the fraction of disparity between the two is called VAT gap.

	Dependent variables	
	VAT evasion	Shadow economy
Unemployment rate	0.90*** (0.30)	0.63*** (5.96)
Indirect tax rate	0.74* (0.44)	0.31*** (3.85)
Income tax	-	0.37*** (4.30)
Sample:	EU-27 (2000-2011)	39 OECD (1998-2010)

Table 2.1: Consumption Tax Compliance vs. Tax Rates. Source: Schneider (2010) and European Commission (2013).

centives to evade, making the currently strained budget unsustainable. As we shall see in section 2.5, our model with tax evasion in consumption and income produces Laffer curves on both VAT and income tax.[7]

The contribution of this paper is twofold. First, it lays out a theoretical model where both direct and indirect tax evasion coexist as a result of optimal decision of agents. Up to our knowledge, there is no study that combines both. Such a model is able to shed light on the steady state effects that changes in taxation will have on the amount evaded by the agents, both on consumption and income, allowing us to see how governments are limited when setting fiscal policies in the presence of tax evasion. Our model produces the positive relationship between tax evasion and tax rates observed in the data. Second, when we extend the model to the business cycle, we are able to replicate the cyclical properties of tax evasion with very little structure. Moreover, this paper is used as a foundation when designing an optimal fiscal system in the presence of tax evasion. In Forcades and Pino (2015) we study what would be the optimal tax system in Spain chosen by benevolent planner that acknowledge the existence of tax evasion.

The layout of the paper is the following. We look at what the literature has done in the field of tax evasion in section 2.2. In section 2.3 we lay out the foundation for the model of tax evasion, and in 2.4 we explore the calibration strategy followed to make our model resemble the Spanish economy. In section 2.5 we present some comparative static results showing how tax evasion limits the ability of the governments to raise taxes, problem specially acute for the case of VAT. Finally, section 2.6 concludes.

[7]The *Laffer curve* is a hypothetical relationship between revenues and tax rates that has an inverted U-shape. The term owes its name to Arthur Laffer, American economist who supposedly sketched the curve on a napkin.

2.2 Literature review

There is a vast literature talking about tax evasion, approaching the problem from a variety of angles. Earlier studies tackled the issue from a microeconomic perspective, dealing with models in which agents/firms optimally choose whether to evade/not to reveal their types in order to pay less to the government. The literature started with the seminal paper of Alligham and Sandmo (1972), that provided a simple yet appealing explanation on tax evasion borrowing from models of economics of crime and optimal portfolio theory. There have been multiple extensions to their framework, but grosso modo, tax evasion is modelled as a risky asset, and at the end risk aversion is what drives tax evasion, a rather limited result to explain the wide differences in tax evasion across countries. Cremer and Pestieu (2001) study the optimal tax mix between commodity and income taxation in the presence of moral hazard. When individual characteristics such as income or labor endowment are unobservable to the government, and setting different taxes on different commodities plays a role in encouraging agents to truthfully reveal their type.[8]

Looking more specifically into the issue of consumption tax evasion, there is not that much done. McLaren (1998), in the spirit of "black markets" where the goods are smuggled into a country, uses a model of sales tax evasion where government choose the effort to curtail these activities. He shows that assuming a feasible plan of taxes that induce black markets, there exists always another plan that does not induce black markets and yields as much utility as the black market equilibrium, in the spirit that lowering taxes on tobacco can do away with cigarette smuggling.

There are also papers studying the profit tax evasion behaviour on firms. Bayer and Cowell (2009) look at the strategic interdependence of firms in a same industry when reporting profits. For instance, if one firm declares lower profits than their competitors in the industry, it can expect to be audited with a higher likelihood. These interdependencies shape how a government should design optimal audit rules. There are even experimental approaches to the tax mix problem under the problem of tax evasion, which are part of a new economic discipline known as behavioural public finance. Watrin and Ullmann (2008) show that even when tax rates and enforcement efforts are the same in both income and sales tax, there is lower compliance when the tax payment decision is framed as a consumption tax, rather than an income tax.

There is a growing literature dealing with tax evasion from a macroeconomic perspective. Busato and Charini (2004) and Busato, Chiarini and Rey (2012) study the general equilibrium implications of income tax evasion. They

[8]For a historical review of the literature, see Alm (2012).

show that tax evasion is welfare improving for agents so long as it helps them to smooth the effect of distortionary fiscal policies. Also, tax evasion in their article helps to improve the fit of the RBC model to macroeconomic variables such as the hours worked. They do not contemplate the possibility of VAT evasion in their study. In a model in which untaxed consumption goods are bought with cash, Nicolini (1998) studies what would be the optimal monetary policy in this framework. He claims that even though there are theoretical grounds to set a positive inflation rate to "tax" the informal sector, the welfare gain of it would be very small. Caballé and Panadés (2004) look into the opposite causality, that is, they analyze the impact on tax evasion of inflation. Their novelty is the dynamic setting, in which tax inspections occur depending on the previous decisions to evade. This gives rise to a non-degenerate distribution of assets, even though agents have identical income. The delay of inspection will increase tax evasion in economies with high inflation. Orsi, Raggi and Torino (2013) use Busato and Chiarini (2004) model and take it to a more quantitative level, adding a rich set of shocks in order to structurally estimate the Italian economy and be able to find the size of the underground economy. Their conclusion is that, from mid eighties to 2000s tax evasion has not stop increasing, coinciding with a period of continuous increases in income tax in Italy. They do not account for consumption tax evasion either.

2.3 Model

Here we develop a macroeconomic model with tax evasion in both income and consumption. Our work is close to Ihrig and Moe (2004) and Busato and Chiarini (2004) in terms of the modelling of income tax evasion. From the consumption tax evasion, our model borrows from the above mentioned works of Virmani (1989) and Nicolini (1998). Our approach to model the incentives of the firms is novel.

2.3.1 Agents

There is a continuum of agents that live indefinitely, which chooses how much to consume, work, and invest. The difference with respect to standard models is that they are able to do these choices evading taxes. On the labor income side, they can work in an underground sector in which they do not pay labor income tax, and consume a good that does not pay VAT. Obviously, there is a risk of incurring in these illegal activities. If the worker is caught not declaring its income, she will have to pay a the corresponding tax τ^n on these activities, plus a administrative surcharge s^n over the tax rate. For example, if income tax is 10%, and s^n is 1.5, the agent will have to pay an income tax of 15% over her undeclared income. Agents are inspected with probability p^n. Let us denote w^i and n^i, $i \in \{M, U\}$, the wages and hours worked in the market ($i = M$) and underground ($i = U$) labor market. Therefore their expected income from illegal activities is given by

$$p^n(1 - s^n\tau^n)w^n n^U + (1 - p^n)w^U n^U \, .$$

Moreover, as we will see explicitly in section 2.4, there is also an extra disutility of working in the illegal sector, apart from the leisure that the agent is giving up. The economic reason is that this underground labor market does not provide the same protections than its market counterpart, like not having unemployment benefits, health insurance or a publicly provided pension. The technical reason will be to match reasonable levels of the underground labor market.

On the consumption tax side, agents can buy goods with VAT, c^V, and without VAT, c^{NV}, being p^{NV} the relative price of the illegal good with respect to its market counterpart. Unlike the labor tax evasion, the government does not inspect whether the agents buy or not the good with VAT. Instead, inspection will occur on the firm side, which makes sense since firms are the "collectors" of VAT. The difference between c^V and c^{NV} is that agents have to spend time "shopping" for the latter.[9]

The rate at which they transform time into c^{NV} is given by

$$c^{NV} = f(n^S) = A^S n^S \, ,$$

where n^S is the amount of time spent shopping, and A^S is the linear technology or rate of transformation between time and goods. As we have said in the introduction, it is not an extremely unrealistic assumption. In real life one can expect that not all of the entrepreneurs will be willing to get into trouble selling without VAT, either because their moral standards, or just the deterrence that the penalties produce. We intend to capture the time would-be tax offenders spend going around stalls, trying to find a crooked seller willing to offer c^{NV}. In this chapter we can understand this parameter just as a calibration requirement, to make the consumption of the illegal good to fit reasonable ratios. This parameter will be more important in Chapter 3, when we study the business cycle properties of tax evasion.

Agents drive utilty from aggregate consumption, which is like a basket of the two goods. To combine them we use a CES aggregator in the following way

[9]This, at least to our knowledge, what happens in Spain. The government has no means to know whether a good is sold with or without VAT, and if it had, it would be very complicated.

$$c_t = \left[(c_t^V)^\rho + \phi(c_t^{NV})^\rho\right]^{\frac{1}{\rho}} ,$$

with parameter ϕ governing the weight of the consumption without VAT in the consumption aggregator. Its value will be below one, since we assume that agents like c^{NV} less than c^V. This parameter will be used to match the observed levels of consumption with and without VAT. Parameter ρ determines the degree of substitutability between the two types of consumption goods. When $\rho = 1$ we are in the case of perfect substitutes, and when it approaches negative infinity the goods became perfect complements (Leontief utility). It is worth doing some remarks about this. There is no economic reason to assume that there should be some degree of complementarity between the goods, people do not usually like to consume goods that do not pay taxes, they do it because they have to. We do it just as a mathematical simplification that allow us to have interior solutions for the consumption of both goods. Moreover, as we will see in the calibration section, this complementarity is very small.

We assume that all agents pool their income together, so they do no not have to keep a state variable to know whether the agent has been inspected in the current period. This is equivalent to saying that there exists a representative agent that gets an *average* income weighted by the probabilities that the labor income tax is inspected. This agent maximize their discounted lifetime utility, that depends on consumption (which is an aggregate of consumption with and without VAT) and leisure, choosing demand for legal and illegal consumption, and the supply of hours in the market and underground, that is

$$\max_{\{c_t^V, c_t^{NV}, n_t^M, n_t^U, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) ,$$

subject to the budget constraint

$$\begin{aligned}(1+\tau_t^c)c_t^V + p_t^{NV} c_t^{NV} + i_t = \; & (1-\tau_t^n)\, w_t^M n_t^M + (1-\tau_t^k) r_t k_t + \\ & (1-p^n)\, w_t^U n_t^U + p^n(1-s^n\tau_t^n)\, w_t^U n_t^U + \pi_t ,\end{aligned} \tag{2.1}$$

the law of motion for capital

$$i_t = k_{t+1} - (1-\delta)k_t , \tag{2.2}$$

the time constraint

$$l_t + n_t^M + n_t^U + n_t^S \leq 1 , \tag{2.3}$$

the appropriate non-negativity constraints, $c_t, k_{t+1} \geq 0$, initial condition for capital k_0, and a shopping time condition for goods purchased without consumption tax

$$n_t^S = c_t^{NV}/A^S \,. \tag{2.4}$$

As we can see from (2.1), the investment good is expressed in the price of the good with VAT. The households first-order conditions can be summarized in the following system of equations, given by the MRS between the good with and without VAT

$$\frac{U_{c_t^V}(\cdot)}{U_{C_t^{NV}}(\cdot)} = \frac{(1+\tau_t^c)}{p_t^{NV}}\,, \tag{2.5}$$

the MRS between consumption of VAT good and market labor

$$\frac{U_{c_t^V}(\cdot)}{U_{n_t^M}(\cdot)} = \frac{(1+\tau_t^c)}{(1-\tau_t^n)w_t^M}\,, \tag{2.6}$$

the MRS between legal and illegal work

$$\frac{U_{n_t^M}(\cdot)}{U_{n_t^U}(\cdot)} = \frac{(1-\tau_t^n)w_t^M}{(1-p^n s^n \tau_t^n)w_t^U}\,, \tag{2.7}$$

and the Euler equation

$$U_{c_t^V}(\cdot) = \beta\big(1+(1-\tau_{t+1}^k)r_{t+1}-\delta\big)U_{c_{t+1}^V}(\cdot)\,, \tag{2.8}$$

where $U_i(\cdot)$ is the marginal utility with respect to variable i, $\frac{\partial U(\cdot)}{\partial i}$.

2.3.2 Firms

Firms are competitive, risk neutral and maximize expected profits. We assume that firms operate two production functions, one that use legal work (for which the agents will have to pay income tax), and another production function that uses illegal work. The novel part is that, from this total production, they decide what to sell with VAT and what without it. Let us explain in detail the rationale behind this.

Suppose that the firms have the legal activities in the front door, and a sweatshop running in the back door, so to speak. Whatever that comes out of these two production functions is treated as total production, and it is not distinguishable one from another. Once the production is done, in the counter of the shop, the sales assistant surreptitiously asks the client whether she wants to buy the good without VAT and save some money. If the agent say yes, then the sales assistant wraps the good in a paper that says c^{NV}, and the production becomes the good without VAT. The agent pays p^{NV} for it.

If the agent's morale standards are higher, the assistant says sorry and wrap the good in a c^V paper, and charges $(1+\tau^c)$ for the good. Even though both goods are produced exactly in same way, they are not the same good. The intuition behind is that if the consumer has some problems with the good and want to ask for a refund, the shop is not going to offer it if the good was c^{NV}, because it is as if this transaction never occurred. Similarly for customer service, post-sale service, etc. This leads us to model c^V and c^{NV} as different goods. The proportion of sales that the firm chooses to offer without VAT is denoted with $\theta_t \in [0,1]$. All the sales done without VAT will not enter into the accounting of the firm, and will not be taken into account when the firm has to pay the profit tax. So VAT evasion for the firm plays two roles. First, it allows to satisfy the demand agents have for c^{NV} and second, saves them some money in the profit tax, obviously under case that they have not been caught.

The government is obviously aware that some firms evade taxes, and inspects them with probability p^c. Recall that firms are responsible for collecting the VAT, and if they operate according to the law, this tax will be neutral for them. However, if they happen to sell c^{NV}, they will be in trouble when caught by the government, that will demand all the corresponding VAT of c^{NV}, which the firm indeed does not have it because it was not collected from the consumers, and will have to disburse it from its own profits. Similarly to the case of the agents, the government will also demand an administrative sanction s^c on top of the standard VAT rate. In terms of the model, this amount is $(1-s^c\tau_t^c)\theta_t p_t^{NV}(Y_t^M + Y_t^U)$. We will consider that in case of inspection of the firm, the government does not care that they are operating with illegal work n^u, since this activity is already prosecuted on the agent's side. On top of this surcharge on the unpaid VAT, if the firm is inspected, all the revenues generated by sales without VAT will be taken into account when paying the profit tax. This effect is reflected in π^i and π^{ni}, which we will see below in equations (2.9) and (2.10), denoting profits when inspected and not inspected, respectively.[10]

Specifically, the problem that firms solve is given by

$$\max_{\theta_t, n_t^M, n_t^U, k_t} \left\{ p^c \, \pi_t^i + (1-p^c)\, \pi_t^{ni} \right\}$$

subject to

[10]The fact that in case of inspection they ignore n^U is just a simplification, and in such a case the government would also prosecute the entepreneurs for this illegal activity.

$$
\begin{aligned}
Y_t^M &= f(k_t, n_t^M) = A_M\,(k_t)^\alpha (n_t^M)^{1-\alpha} \\
Y_t^U &= f(k_t, n_t^U) = A_U\,(n_t^U)^\sigma \\
Y_t &= Y_t^M + Y_t^U \\
\theta_t &\in [0,1]
\end{aligned}
$$

$$\pi^i = (1-\tau^\pi)\Big[(1-\theta_t)Y_t + (1-s^c\tau_t^c)\theta_t p_t^{NV} Y_t - w_t^M n_t^M - w_t^U n^U - r_t k_t\Big] \tag{2.9}$$

$$\pi^{ni} = (1-\tau^\pi)\Big[(1-\theta_t)Y_t - w_t^M n_t^M - w_t^U n_t^U - r_t k_t\Big] + p_t^{NV}\theta_t Y_t\,. \tag{2.10}$$

We model the illegal production Y^U under the assumption that it is labor intensive, and that the capital that is used is mainly fixed, such as the building of the firm and so on, allowing us to write the production function in the above form. This simplification has been used in the literature of informal/underground economy, such as Ihrig and Moe (2004) and Busato and Chiarini (2012), among others. There is evidence for including this simplificaiton. Orsi, Raggi and Torino (2014) structurally estimate the share of capital in the sector employing illegal work, and find it to be smaller than its legal counterpart. Moreover, it is intuitive that installing or changing the productive capacity for the illegal sector is more noticeable than hiring some workers (it requires bringing machines, or buying machines), and this may attract the attention of the state, leading to the firm eventually being caught. The first order necessary conditions for firm optimality gives us the following equations for the price of the illegal consumption good, and the usual pricing equations of the productive factors,

$$p_t^{NV} = \frac{(1-\tau_t^\pi)}{p^c(1-\tau^\pi)(1-s^c\tau_t^c) + (1-p^c)}\,, \tag{2.11}$$

$$r_t = \alpha A^M k_t^{\alpha-1}(n_t^M)^{1-\alpha} \tag{2.12}$$

$$w_t^M = (1-\alpha)A^M k_t^\alpha (n_t^M)^{-\alpha} \tag{2.13}$$

$$w_t^U = \sigma A^U (n_t^U)^{\sigma-1}\,. \tag{2.14}$$

It is worth giving a comment about the equilibrium price p^{NV}. As it is seen from (2.11), its value is determined by parameters of the economy such as tax rates and the strictness of inspections, meaning that the price does not depend on quantities. This is the result of having a linear choice in the proportion of evaded goods. The firm is faced by an arbitrage condition, and sets a price that just compensate it for the risk taken in the activity. The inverse demand of the firm, $p^{NV}\left(C^{NV}\right)$, is a flat function, and all adjustments to achieve the equilibrium will occur in the demand side. We thought of different structures for the choice of tax evasion in the firm side that would not give rise to this phenomenon, but all of them implied more complicated modelizations, in which the extra complications implied outweighed the minor problem of having a fixed price.

2.3.3 Government

The role of the government is residual in this model. Its only function is to collect taxes. Revenues come from four taxes: consumption taxes τ^c, labor income taxes τ^n, capital taxes τ^k and the profit tax for the firms τ^π. Moreover, it also raises revenues from the penalties that it imposes on tax evaders, both agents and firms. This revenue is used in unproductive government consumption. The expression of the revenue is

$$\begin{aligned} rev_t = & \underbrace{\tau_t^c\left(c_t^V + p_t^{NV} p^c s^c \theta_t Y_t\right)}_{\text{VAT}} + \underbrace{\tau_t^n\left(w_t^M n_t^M \ + \ p^n s^n w_t^U n_t^U\right)}_{\text{IT}} + \underbrace{\tau_t^k r_t k_t}_{\text{capital tax}} \\ & \underbrace{+\tau^\pi\left[\left((1-\theta_t) + p^c(1 - s^c\tau_t^c)\theta_t p_t^{NV}\right) Y_t - r_t k_t - w_t^M n_t^M - w_t^U n_t^U\right]}_{\text{Profit tax}}, \end{aligned} \tag{2.15}$$

where in each tax, we have first, the legal revenues collected from the corresponding tax, and after the part of the revenues caught and recovered in inspections. As we said before, when the firm is caught evading, it is also forced to take into account the illegal revenues made from selling without VAT at the calculation of their payment of profit tax. This is given by $p^c(1 - s^c\tau_t^c)\theta_t p_t^{NV}$, as it can be seen in equation (2.15).[11]

2.3.4 Equilibrium

A Competitive Equilibrium in this economy are sequences of quantities $\left\{c_t^V, c_t^{NV}, n_t^M, n_t^U, n_t^S, k_{t+1}, \theta_t\right\}_{t=0}^{\infty}$ and prices $\left\{r_t, w_t^M, w_t^U, p_t^{NV}\right\}_{t=0}^{\infty}$ such that, given prices, agents decisions satisfy first order conditions (2.5) to (2.8), for firms, equations (2.11) to (2.14) hold, and markets clear, that is

[11] The assumption of useless government consumption is innocuous. The revenues could go back to the agents as transfers, or in the form of a public good. We checked both cases and it did not alter our results, and excluded for simplicity.

Market for the good with VAT

$$c_t^V + g_t + i_t = (1 - \theta_t)\, Y_t \,, \tag{2.16}$$

market for the good without VAT

$$c_t^{NV} = \theta_t \, Y_t \,, \tag{2.17}$$

and labor market

$$l_t + n_t^M + n_t^U + n_t^S = 1 \,. \tag{2.18}$$

2.4 Calibration

In this section we describe the functional forms and parameters assigned to the model. Our objectivein this exercise is to make it resemble the Spanish economy. The vector of parameters that we need to calibrate is $\left(\phi, \rho, \beta, \gamma, \alpha, \tau^c, \tau^n, \tau^k, p^c, p^n, s^c, s^n, A^S, \Phi_l, \Phi_u\right)$. The first two of them are free parameters. Values for β, γ, α, s^c and s^n will be taken from the literature. Parameters τ^c, τ^n, τ^k, p^c, p^n, A^S, Φ_l and Φ_u will be calibrated to match certain steady state moments of the data. For reference, all the calibrated parameters are reported on Table 2.2. We will start from the agent side. The representative consumer is assumed to have the following preferences:

$$U(c_t, l_t) = log(c_t) + \Phi_l \frac{(1 - n_t^M - n_t^U - n_t^S)^{1-\gamma}}{1-\gamma} - \Phi_u n_t^U \,,$$

where the aggregate consumption, c_t, is a basket that aggregates both taxed and untaxed goods, with an aggregator as explained in section 2.3,

$$c_t = \left[(c_t^V)^\rho + \phi(c_t^{NV})^\rho\right]^{\frac{1}{\rho}} \,.$$

For calibrating ρ we follow Benhabib et al. (1991) and Ragan (2005), which use a similar aggregator in the context of home production to combine market and homemade goods. Their estimates for the substitutability parameters range from 0.8 and 0.9. Provided the explanation we have given above about the nature of how substitutable are the two goods between each other (they are nearly the same good), we set slightly higher value, $\rho = 0.95$. We have experimented with different values of the parameter and this choice is without loss of generality.

On the labor supply side, the parameter γ is set to 4, which implies a Frisch elasticity of labor supply around 0.5, which is a standard value in the literature; specifically, is the average value between King and Rebelo (1999) and Trabandt and Uhlig (2011). We set the discount to $\beta = 0.96$, implying a long run interest rate of 4%, which is the average 10-year government bond yield since the introduction of the Euro. We use a discount factor $\delta = 0.07$, taken from Díaz-Giménez and Díaz-Saavedra (2009). Capital share is set to $\alpha = 0.3734$, from the Spanish National Accounts in 2009-2012, see Gollin (2002). The underground output elasticity of labor, σ, is set to the same value as in the labor market, i.e., $\sigma = 1 - \alpha$. We have no available data in order to estimate such parameter, so we prefer to set it to a standard value. This parameter affects the profits that the firm earns from illegal activities. For this value of σ, the ratio of profits to GDP is a 4%. Effective tax rates are estimated following Mendoza et al. (1994), except profit tax that is set to 0.25, which is the rate that most firms pay in Spain.

We choose Φ_l equal to 0.13909 to reach a value of n^M in the model that matches the share of time allocated to work in the market in Spain, which is a 32.2%, following the Working Time Survey, published by the INE in (1996). The extra disutility of the underground labor, Φ_u, is set to 1.7091 to match a share of underground labor over the total labor equal to 21.9%, which is the the average value over 1997-1998 of this ratio for Spain, target that we obtain from Scheneider (2000).[12]

The parameter of the shopping-time A^S, is set to 3.8636 in order to match a ratio of evadable consumption to total consumption equal to 23.25%, the average value of the period 2008-2011 estimated by the study of the Taxation and Customs Union Directorate for the EU-Comission (2013). It estimates the share of evaded consumption or VAT gap from National Accounts data and Input-Output Tables.

Finally, let us have a word about the parameters determining the penalties of tax evasion. Governments are very reluctant to disclose information about tax inspections and procedures, since such information may encourage tax offenders to outsmart the system. To set the probability of being caught evading income tax, we use data on the number of inspected tax filings. We set p^N to 0.1, which is simply a proportion of inspected tax filings with respect to the number of registered firms. We obtained this information from the *Ministerio de Trabajo. Dirección General de la Inspección de Trabajo y Seguridad Social*. The number of firms is taken from the INE. We use data on labor inspections on firms, even though we have maintained throughout the whole article that income tax evasion is done on the agent side. This is still true, but we have not find any data on the number of inspections on the agent side. Moreover, the data we use is about *labor* inspections, so it has to be the same as if we have data on the inspections on Income Tax filings. . This is the same procedure that Busato and Chiarini (2004) use. We set the same value for p^c, since we have been unable to find any data on VAT inspections. For the surcharges in tax evasions, a good representation of the Spanish tax law is to set s^i, equal to 1.5.

[12]The Working Time survey is elaborated by the Instituto Nacional de Estadística.

Parameter	Description	Value
δ	Depreciation	0.0782
β	Discount factor	0.96
ρ	Elasticity of substitution	0.95
ϕ	Weight of c^{NV}	0.78
Φ_l	Leisure param.	0.13909
Φ_u	Extra disutility of working underground	1.7091
γ	Elasticity of leisure	4
α	Share of capital in Y^M	0.3734
σ	Labor parameter in Y^U	0.6266
A^S	Shopping time	3.8636
p^c	Prob. of being caught evading VAT	0.1
s^c	Surcharge when caught evading VAT	1.5
p^n	Prob. ef caught evading income tax	0.1
s^n	Surcharge when evading income tax	1.5
τ^c	consumption tax	14.80%
τ^n	income tax	37.54%
τ^k	capital tax	24.95%
τ^π	profit tax	0.25%

Table 2.2: Calibration parameters

2.5 Laffer Curves

In this section, we move on to see some comparative statics in terms of taxes, and analyze what happens with revenues in both consumption and labor taxation. We compare these results with the model without evasion, in which we take a standard model with one consumption good that always pay taxes, and only one production function that employs only legal work, whose returns pay diligently its income tax.[13]

All results reported in this section are taken from the steady state of the model. In Figures 2.1(a) and 2.1(b) we plot total revenues when changing labor income and consumption taxes, respectively. Both graphs compare the case with and without evasion. The remaining taxes that are not varying are set to its estimated values. Tax rates are indicated in the x-axis, and revenues in the y-axis. Blue plots for the case of tax evasion, and green when there is not.

As we can see, for the case of no tax evasion, there is no slippery slope in

[13] In this case, since profits are zero, τ^π becomes irrelevant.

the Laffer curve of consumption tax, i.e., the government can increase taxation to whatever level it wants, and revenues will continue increasing. On the other hand, Laffer curve of labor income tax equals to zero for $\tau_t^n = 1$. These two forms of the Laffer curve are standard in the literature, e.g., Trabandt and Uhlig (2011). However, we can see that under tax evasion both Laffer curves peak at a rate between 0 and 1. Therefore, evasion limits tax policies the government can set in order to raise revenues. For income tax, the peak of the Laffer curve occurs before, specifically, roughly above a tax rate equal to 30% revenues fall. What is more surprising is the case of consumption tax, where taxes above 10% decrease revenues because agents give up c^V for its illegal counterpart c^{NV}. These two graphs are our word of warning for policy reforms calling for a total substitution of income taxation in favor of indirect taxation.

Now, we focus on the labor income tax revenues and consumption tax (and profit tax) revenues. We observe that labor income tax revenues start falling around a 40% tax rate when tax evasion is considered, see Figure 2.1(c). In the case of consumption tax, Figure 2.1(d) displays the consumption tax revenues, the red line is the addition of both consumption and profit tax revenues. We can see that profit tax losses are an important element when we analyze the effect of consumption tax evasion on government revenues. Specifically, consumption tax revenues decrease beyond a tax rate of 22%, but it does around 12% when profit tax revenues are taken into account.

We present in Figure 2.2 Laffer curves for a alternative Frisch elasticity equal to 0.5. In this case, Laffer curves of labor income tax shift out their peaks around 5-10 percentage points of tax rates. For example, in Figures 2.1(a) and 2.2(a), we observe that, in the case of non-evasion, Laffer curve peak at around 0.75 and, for lower elasticity of labor supply, it does around 0.85. Hence, as expected, government is able to set higher taxes in order to raise revenues due to the lower elasticity of labor supply.[14]

In the standard model of one consumption without tax evasion, when taxes are raised on labor income, there are the two well know income and substitution effect. By the income effect, you are poorer because you earn less in real terms, and you must work more hours in order to keep a constant income. The substitution effect, makes you work less hours because the marginal productivity of labor (wages net of taxes) is lower, or conversely, the opportunity cost of your time, that is, leisure, is relatively cheaper. These effects are what drive the standard Laffer curve on income tax. In the case of income tax evasion, there is now another tradeoff: if income tax increases, workers will start deviating hours to the underground market, making the (double) substitution

[14]We need to adjust some parameters so as to work with an economy that resembles the previous one. Therefore, we set Φ_l equal to 0.1390, ϕ to 0.78, Φ_u to 1.7091, and A^S to 3.8636.

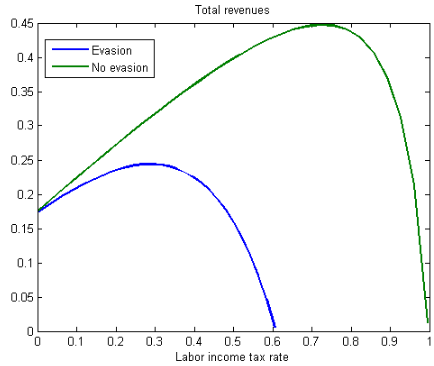
effect overtake the income effect at lower tax rates.

Thus far, nothing is novel. Our contribution is to pinpoint also a tradeoff in consumption side. Under the standard model, without consumption tax evasion, revenues with consumption tax are always rising. There is nothing that actually precludes τ^c from being greater than 1 (as it may be the case in special goods like tobacco), even thought this may not be optimal. The reason is that there is not a substitution effect like in the case of labor. Obviously higher taxes reduce the consumption (moving upwards along the demand curve) but the effect is generally not strong enough to reduce revenues. This is the efficiency reason that economic studies use as an argument for shifting towards consumption taxation. Our model adds a tradeoff to increasing consumption tax evasion: the illegal market. If consumption taxes are raised, individuals will devote time to find imperfect yet cheaper alternatives to the highly taxed consumption good, reducing the capacity to raise revenues of the government, and producing a similar Laffer curve in consumption taxes. Our claim is that in countries where these policies are most advocated, i.e., southern European economies, it is where tax evasion is more widespread and therefore where more ineffectual these policies will be. We are not saying that tax increases will not be necessary, our contribution is that they will not be miraculous. Efforts in these countries should be focused on curtailing tax evasion, which will increase revenues even without modifying the fiscal system, and in a latter stage, undertake fiscal reforms that raise revenues.

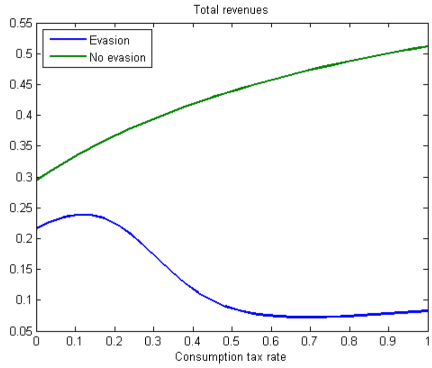
2.6 Conclusion

In this article, we extend previous macroeconomic frameworks on shadow economy with consumption tax evasion. The model presented here is the first one to jointly tackle both labor income and consumption tax evasions. We employ this modellization to explore what are the fiscal policy limitations that tax evasion imposes on the taxes that a government can set. These consequences are indeed significant. In particular, we produce Laffer curves in which revenues fall quicker than in the standard model without tax evasion when tax rates are pushed beyond reasonable levels, specially for the case of consumption tax in which the slippery slope of the Laffer curve starts roughly at a mere 10%, rate way below the 20% that most advanced economies have. These new Laffer curves should lead to better policy design for countries in which tax evasion is widespread. This is very important since current works on fiscal policy using DGSE models ignore tax evasion.

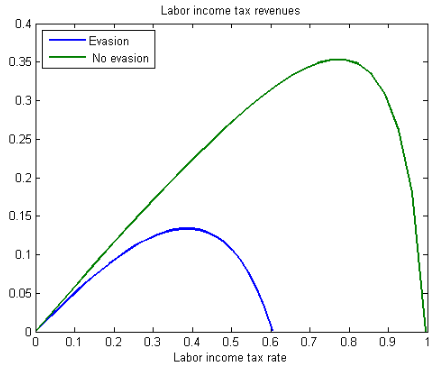
Consequently, our paper casts doubt on the possibility of implementing tax reforms suggested by international institutions and scholars, implying signifi-



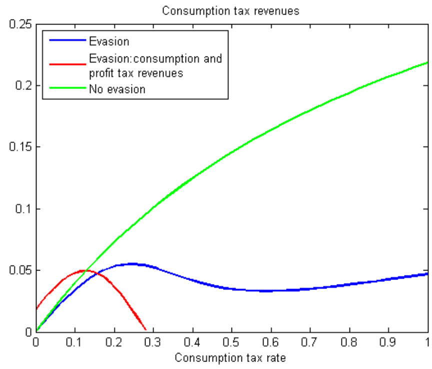
(a) Laffer curve of labor income tax



(b) Laffer curve of consumption tax



(c) Laffer curve for income tax revenues

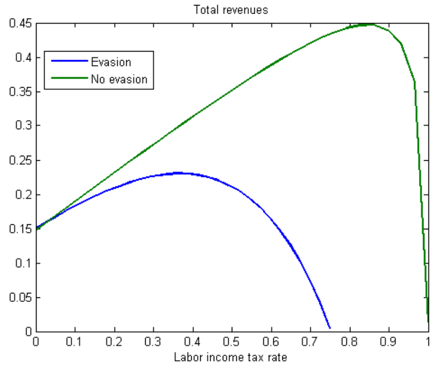


(d) Laffer curve for consumption tax revenues

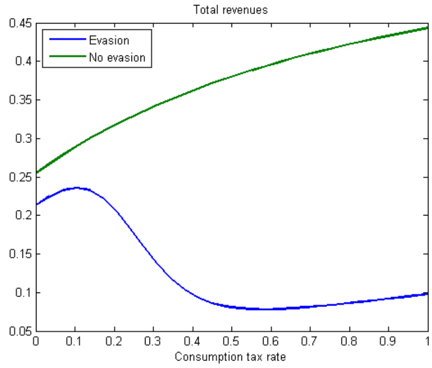
Figure 2.1: Laffer Curves

cant reductions of income tax, or its elimination altogether, in favor of higher rates over consumption. Therefore, extreme and well-known tax results in the literature of optimal fiscal policy will disappear; namely, Coleman (2000) result of taxing only consumption, and not taxing income (from capital and labor) at all, will not be feasible. Therefore tax evasion should be another key variable to be taken into account when doing fiscal policy design, and not only income tax evasion should be taken into account, but also consumption tax evasion. The structure developed in this paper is the building block for Forcades and Pino (2015), where we take this framework and attempt the normative question of studying what would be the optimal tax mix (between consumption, labor, and capital) given the fiscal limitations that tax evasion imposes.

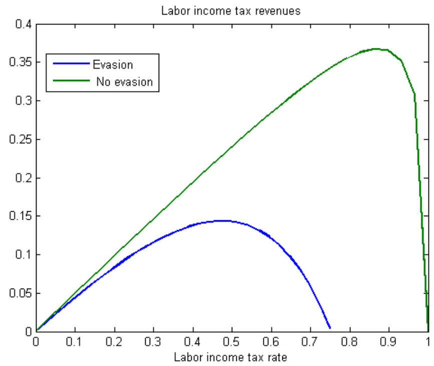
An important limitation of this article is that tax evasion is taken as given. In reality, in dire economic conditions, governments in need to raise revenues will be aware of this and may attempt to reduce tax evasion in order to increase tax revenues, endogenizing the design of how much should tax evasion



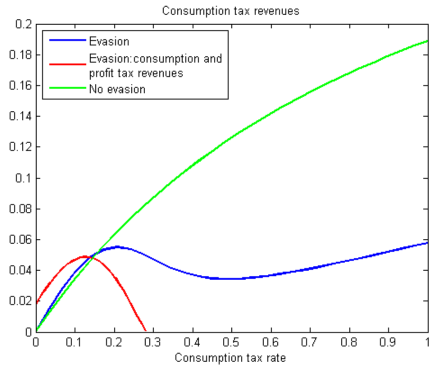
(a) Laffer curve of labor income tax



(b) Laffer curve of consumption tax



(c) Laffer curve for income tax revenues



(d) Laffer curve for consumption tax revenues

Figure 2.2: Laffer Curves for alternative Frisch elasticity (0.5)

should be prosecuted, in which taxes should inspection focus their efforts, and so on. From the insights that we extract in our model, prosecuting more VAT evasion may be more effective, since as we saw from Section 2.5, there is a lot of potential revenue lost by not being able to prosecute consumption tax, which can increase revenues even at tax rates $\tau^c > 1$. An interesting study would be to see what are the optimal evasion “prosecution” in order to maximize revenue. This can be a strategy to increase revenues in circumstances where a government lack the political “capital” to undertake tax reforms. It would be interesting to study this in a heterogeneous agent framework, because that curtailing tax evasion may be harmful for individuals relying on the underground economy in order to make a living, and policies to reduce fiscal deficit may possibly be incompatible with optimality solutions that take into account the welfare of the society and to some degree, its level of inequality.

On the other hand, we disregard here capital income tax evasion, which is probably a form of evasion practiced by the top end of the income distribution. Policies attacking this type of evasion may, on top of the beneficial revenue effects, inequivocally reduce inequality, since the richer part of the population gets most of its income from returns to capital. The drawback of these policies is that the supply of capital is much more elastic than labor supply, and savings may flee the country when more stringent policies are put in place. Lastly, there is an important family of taxes left out of the study, which are property taxes (on land or housing). Taxes on such goods, whose demand is very inelastic, (at least in the short run), are more difficult to evade, and if revenues fall short, they should be taken into account instead of increasing easily evadable taxes. These interesting topics are left for future work.

Chapter 3

Business Cycle and VAT Evasion

(joint with Alejandro Forcades Pujol)

3.1 Introduction

The Great Recession have had a terrible economic impact on Europe, especially in southern economies such as Portugal, Spain and Italy. The GDP fell, in real terms, between 3 and 5% in 2009, the most severe year of the crisis, and they are just starting to recover. Fiscal policies devised to counteract the effect of the recession have left governments public accounts in a bleak state. Governments are searching for ways to reverse these debt positions to avoid credit downgrades that may risk lengthening the recession even longer. Tax increases have occurred on most of Europe. For instance, Spain raised its general Value Added Tax (VAT) from 18% to 21% in 2012. The recommendations to raise taxes have also been coming from international institutions such as the European Union, advising increases in the fiscal pressure on indirect taxation together with a reduction in income taxes, policy commonly referred as a tax shift. This is mentioned for instance in the 2012 Annual Growth Survey.[1].

The economic foundation for such reforms are the expected improvements in efficiency that lowering income taxes have. Taxes on consumption do not affect the intensive margin decision of labor supply or saving, thereby improving investment and growth, which eventually will raise revenues. For Spain, quantitative studies have shown the benefits of these reforms. This conclusion is also frequent in the literature of optimal fiscal policy, and not only for European countries, but also for the United States. Correia (2010) finds for the US that eliminating the current progressive income tax, and establishing

[1]Full text `http://ec.europa.eu/europe2020/pdf/annualgrowthsurveyen.pdf`

a flat tax on consumption would reduce inequality and improve the welfare of the currently poor.[2]

Another issue is the evolution of the shadow economy along the business cycle. The empirical literature has pointed out that it is countercyclical, e.g., shadow economy increases in downturns. Figure 3.1 shows the Hodrick-Prescott-filtered cyclical components of the GDP (solid line) and the shadow economy (dashed) for some European countries. It is visually clear that tax evasion grows during the troughs of the business cycle, and decreases in its peaks. One possible cause is that agents, in an attempt to smooth consumption, and given their lower income during crises, have to incur in some illegal activities to achieve it. This can done by buying some goods without VAT, or working in an underground sector, because during crises legal firms reduce their activity. Our novelty comes in the former breed of shadow economy, i.e., consumption tax evasion. We will explain the mechanisms in detail in section 3.2, but to give a tease of how the model works, we model this good without VAT in a way that it becomes an *inferior good*, ceteris paribus, the lower the income, the higher the consumption. Up to our knowledge, we have not seen inferior goods on a dynamic macroeconomic model. The way we achieved it was to assume that, in order to acquire the good without VAT, agents will have to spend time *shopping* for them. This is not an unearthly assumption, in reality not all the firms are willing to get into trouble for selling goods under the counter, and agents seeking these illegal substitutes will be likely to spend time shopping them around.

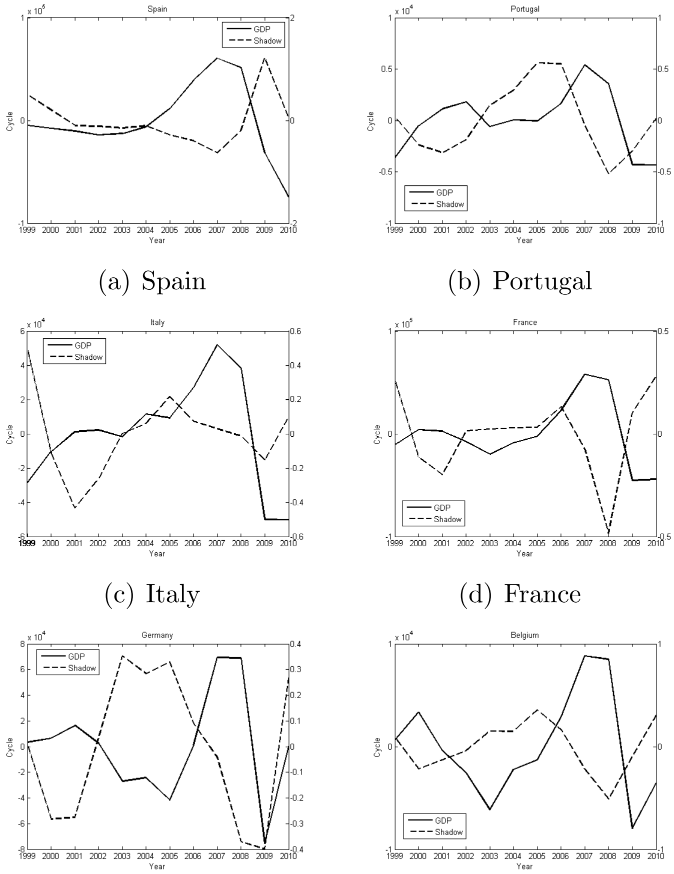
3.2 Model

We lay out here a business cycle model with VAT and income tax evasion. The foundation is identical to the model used in the prevous chapter, only adding a real business cycle shock in the legal and illegal production of goods.

3.2.1 Agents

There is continuum of agents that live indefinitely, which chooses how much to consume, work, and invest. Moreover, they can choose how much to evade either working in an underground sector not paying income tax, and/or consuming an illegal good that does not pay VAT. If the worker is caught not declaring its income, she will have to pay a the corresponding income tax

[2] See for instance Orsini et al. (2014) and report made by the Spanish Commision for the Reform of the Fiscal System in Spain. Full text: `http://www.fettaf.com/system/application/documentos/documentosvarios/InformeExpertosReformaSistemaTributarioEspanol.pdf`



(a) Spain (b) Portugal

(c) Italy (d) France

(e) Germany (f) Belgium

Figure 3.1: Cyclical components: GDP and the shadow economy. Source: our calculations using data from Schneider (2013).

τ^n, plus a proportional penalty s^n over the tax rate. For instance, if income tax is 10%, and s^n is 1.5, the agent will have to pay an income tax of 15%. This event occurs with probability p^n. Denote w^i and n^i, $i \in \{M, U\}$, the wages and hours worked in the market ($i = M$) and underground ($i = U$) labor market. Agents expected income from illegal activities is given by $p^n(1 - s^n\tau^n)w^n n^U + (1 - p^n)w^U n^U$.

On the consumption tax side, agents can buy goods with VAT, c^V, and without c^{NV}, being p^{NV} the relative price of the illegal good with respect to the legal counterpart. Government inspection occurs on the firm side, which are the "collectors" of VAT. The difference between c^V and c^{NV} is that agents have to spend time "shopping" for the latter. The rate at which they transform time into c^{NV} is given by $c^{NV} = f(n^S) = A^S n^S$, where n^S is the amount of time spent shopping, and A^S is the linear technology or rate of transformation between time and goods. It is reasonable to assume some time to find illegal goods for sale, not all the shops will be willing to get into trouble selling without VAT. As we will see, this feature trick makes the good without VAT inferior, and thus its consumption will be countercyclical.

What agents really care about in her utility is an aggregate consumption, which is like a basket of the two goods. To combine them we use a CES aggregator in the following way

$$c_t = \left[(c_t^V)^\rho + \phi(c_t^{NV})^\rho\right]^{\frac{1}{\rho}}$$

where ϕ determines the weight of the consumption without VAT in the aggregator. Its value will be below one, since we assume that agents like c^{NV} less than c^V. This parameter will be used to match the observed levels of consumption with and without VAT. Parameter ρ determines the degree of substitutability between the two types of consumption goods. When $\rho = 1$ we are in the case of perfect substitutes, and when it approaches negative infinity the goods became perfect complements (Leontief utility). It is worth doing some remarks about this. There is no economic reason to assume that there should be some degree of complementarity between the goods, people do not usually like to consume goods that do not pay taxes, they do it because they have to. We do it just as a mathematical simplification that allow us to have interior solutions for the consumption of both goods. Moreover, as we will see in the calibration section, this complementarity is very small.

We assume that all agents pool their income together, so they do no not have to keep a state variable to know whether the agent has been inspected in the current period. This is equivalent to saying that there exists a representative agent that gets an *average* income weighted by the probabilities that LIT is inspected. This agent maximize their expected discounted lifetime utility, that depends on consumption (which is an aggregate of consumption with and without VAT) and leisure, choosing demand for legal and illegal consumption, and the supply of hours in the market and underground, that is

$$\max_{\{c_t^V, c_t^{NV}, n_t^M, n_t^U, k_{t+1}\}_{t=0}^{\infty}} \mathrm{E}\left[\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)\right]$$

subject to

$$\begin{aligned}(1+\tau_t^c)c_t^V + p_t^{NV} c_t^{NV} + i_t = {} & (1-\tau_t^n)\, w_t^M n_t^M + (1-\tau_t^k) r_t k_t + \\ & (1-p^n)\, w_t^U n_t^U + p^n (1 - s^n \tau_t^n)\, w_t^U n_t^U + \pi_t\end{aligned} \tag{3.1}$$

the law of motion for capital,

$$i_t = k_{t+1} - (1-\delta) k_t \tag{3.2}$$

the time constraint

$$l_t + n_t^M + n_t^U + n_t^S \leq 1 \tag{3.3}$$

the appropriate non-negativity constraints, $c_t, k_{t+1} \geq 0$, initial condition for capital , k_0 and a shopping time condition for goods purchased without consumption tax

$$c_t^{NV} = A^S n_t^S \tag{3.4}$$

As we can see from (3.1), the investment good is expressed in the price of the good with VAT. The households first-order conditions can be summarized in the following system of equations, given by the MRS between the good with and without VAT

$$\frac{U_{c_t^V}(\cdot)}{U_{C_t^{NV}}(\cdot)} = \frac{(1+\tau_t^c)}{p_t^{NV}} \tag{3.5}$$

the MRS between consumption of VAT good and market labor

$$\frac{U_{c_t^V}(\cdot)}{U_{n_t^M}(\cdot)} = \frac{(1+\tau_t^c)}{(1-\tau_t^n)w_t^M} \tag{3.6}$$

the MRS between legal and illegal work

$$\frac{U_{n_t^M}(\cdot)}{U_{n_t^U}(\cdot)} = \frac{(1-\tau_t^n)w_t^M}{(1-p^n s^n \tau_t^n)w_t^U} \tag{3.7}$$

and the Euler equation

$$U_{c_t^V}(\cdot) = \beta \mathrm{E}\left[\left(1+(1-\tau_{t+1}^k)r_{t+1}-\delta\right)U_{c_{t+1}^V}(\cdot)\right] \tag{3.8}$$

where $U_i(\cdot)$ is the marginal utility with respect to variable i, $\frac{\partial U(\cdot)}{\partial i}$.

3.2.2 Firms

Firms are competitive, risk neutral and maximize expected profits. We assume that firms operate two production functions, one that use legal work (for which the agents will have to pay income tax), and another production function that uses illegal work. The framework is exactly the same as Forcades and Pino (2015), i.e., firms have two different production functions, legal Y^M and illegal Y^U, using declared and undeclared work, respectively. Once both productions are done, they are indistinguishable from one another, and treated as total production. It is at the moment of the sale when the good is sold legally or illegally depending on whether the client wants to buy the good with or without VAT, with the obvious advantage for the buyer of not paying the tax. If the agent accepts, then the agent is acquiring c^{NV} (without knowing whether it came from Y^M or Y^U), and paying p^{NV} for it. If the agent's morale standards are higher, she will rather choose c^V, and will be charged $(1+\tau^c)$ for the good. Even though both goods are produced exactly in same way, they are not the same good. The intuition behind is that if the consumer has some problems with the good and want to ask for a refund, the shop is not going to offer it if the good was c^{NV}, because it is as if this transaction never occurred. Similarly for customer service, post-sale service, etc. This leads us to model c^V and c^{NV} as different goods. The proportion of sales that the firm chooses to offer without VAT is denoted with $\theta_t \in [0, 1]$. All the sales done without VAT will not enter into the accounting of the firm, and will not be taken into account when the firm has to pay the profit tax. So VAT evasion for the firm plays two roles. First, it allows to satisfy the demand agents have for c^{NV} and second, saves them some money in the profit tax, obviously under case that they have not been caught.

The government will inspect firms with probability p^c to see whether they are complying with their VAT duties, which they are of collecting. If firms operated legally, VAT should be neutral to them, they just collect the tax and give it to the government. However, if they are caught selling c^{NV}, they will be asked for the corresponding VAT of c^{NV}, which the firm indeed does not possess because it was not collected from the consumers, and will have to pay it from its own revenues. On top of that, the government will also ask for an administrative sanction s^c on top of the standard VAT rate. In terms of the model, this amount is $(1 - s^c\tau_t^c)\theta_t p_t^{NV}(Y_t^M + Y_t^U)$. We will consider that in case of inspection of the firm, the government does not care that they are operating with illegal work n^U, since this activity is already prosecuted on the agent's side. On top of this, if the firm is inspected, all the revenues generated by sales without VAT will be taken into account when paying the profit tax. This effect is reflected in π^i and π^{ni}, which we will see below in equations (3.9) and (3.10), denoting profits when inspected and not inspected, respectively. Therefore, firms solve

$$\max_{\theta_t, n_t^M, n_t^U, k_t} \left\{ p^c\, \pi_t^i + (1 - p^c)\, \pi_t^{ni} \right\}$$

subject to

$$\begin{aligned}
Y_t^M &= f(k_t, n_t^M) = e^{z_t} A_M\,(k_t)^{\alpha} (n_t^M)^{1-\alpha} \\
Y_t^U &= f(k_t, n_t^U) = e^{z_t} A_U\,(n_t^U)^{\sigma} \\
Y_t &= e^{z_t}(Y_t^M + Y_t^U) \\
\theta_t &\in [0, 1]\,,
\end{aligned}$$

where z_t is the aggregate shock on the economy, which follow the autoregressive process $z_t = \psi z_{t-1} + \varepsilon_t$. This shock is the same for both production sectors. The profits are defined as

$$\pi^i = (1-\tau^\pi)\Big[(1-\theta_t)Y_t + (1-s^c\tau_t^c)\theta_t p_t^{NV} Y_t - w_t^M n_t^M - w_t^U n^U - r_t k_t\Big] \tag{3.9}$$

$$\pi^{ni} = (1-\tau^\pi)\Big[(1-\theta_t)Y_t - w_t^M n_t^M - w_t^U n_t^U - r_t k_t\Big] + p_t^{NV}\theta_t Y_t\,. \tag{3.10}$$

We model the the illegal production Y^U as labor intensive. This simplification has been used in the literature of informal/underground economy, such as Ihrig and Moe (2004) and Busato and Chiarini (2012), among others. Orsi, Raggi and Torino (2014) structurally estimate the share of capital in the sector employing illegal work, and find it to be smaller than its legal counterpart. It is intuitive that installing or changing the productive capacity for the illegal sector may attract the attention of the state, leading to the firm eventually being caught.

The government raises revenues from consumption taxes τ^c, labor income taxes τ^n, capital taxes τ^k and the profit tax for the firms τ^π. Moreover, it also raises revenues from the penalties that it imposes on tax evaders, both agents and firms. This revenue is unproductive, is used in a useless expenditure. The expression of the revenues is

$$\begin{aligned} rev_t = {} & \underbrace{\tau_t^c\big(c_t^V + p_t^{NV} p^c s^c \theta_t Y_t\big)}_{\text{VAT}} + \underbrace{\tau_t^n\big(w_t^M n_t^M + p^n s^n w_t^U n_t^U\big)}_{\text{IT}} + \underbrace{\tau_t^k r_t k_t}_{\text{capital tax}} \\ & \underbrace{+\tau^\pi\Big[\big((1-\theta_t) + p^c(1-s^c\tau_t^c)\theta_t p_t^{NV}\big)Y_t - r_t k_t - w_t^M n_t^M - w_t^U n_t^U\Big]}_{\text{Profit tax}}, \end{aligned} \tag{3.11}$$

which are obviously affected by the business cycle through the production aggregator $Y_t = e^{z_t}(Y_t^M + Y_t^U)$. The FOC of the share of the goods without VAT gives us its price:

$$p_t^{NV} = \frac{(1-\tau_t^\pi)}{p^c(1-\tau^\pi)(1-s^c\tau_t^c) + (1-p^c)} \tag{3.12}$$

The price of the productive factors are given by the standard equations, being the wage of the underground sector ty only novelty.

$$r_t = \alpha e^{z_t} A^M k_t^{\alpha-1}(n_t^M)^{1-\alpha}\,, \tag{3.13}$$

$$w_t^M = (1-\alpha)e^{z_t} A^M k_t^{\alpha}(n_t^M)^{-\alpha}\,, \tag{3.14}$$

$$w_t^U = \sigma e^{z_t} A^U (n_t^U)^{\sigma-1}\,. \tag{3.15}$$

3.2.3 Equilibrium

A Competitive Equilibrium in this economy are sequences of quantities $\left\{c_t^V, c_t^{NV}, n_t^M, n_t^U, n_t^S, k_{t+1}, \theta_t\right\}_{t=0}^{\infty}$ and prices $\left\{r_t, w_t^M, w_t^U, p_t^{NV}\right\}_{t=0}^{\infty}$ such that, given prices, a given sequence of taxes $\{\tau_t^c, \tau_t^n, \tau_t^k, \tau_t^\pi\}$, agents decisions satisfy first order conditions (3.5) to (3.8); for firms, equations (3.12) to (3.15) hold, and markets clear,

market for the good with VAT

$$c_t^V + g_t + i_t = (1 - \theta_t)\, Y_t \tag{3.16}$$

market for the good without VAT

$$c_t^{NV} = \theta_t\, Y_t \tag{3.17}$$

and labor market

$$l_t + n_t^M + n_t^U + n_t^S = 1\,. \tag{3.18}$$

3.3 Calibration

The calibration of this chapter follows the setup laid out in previous chapter for the static model, but adds the parameters related to the business cycle. These are the parameters for the autorregressive process followed by the the productivity shock, and the variance of its innovations. We follow Prescott (1986) for this, and we set the persistence parameter, ρ, equal to 0.979 and the standard deviation of the innovation, σ_ε, to 0.0072. We briefly explain the the calibration used. A summary is presented in Table 3.1.

We have to calibrate the vector $\left(\phi, \rho, \beta, \gamma, \alpha, \tau^c, \tau^n, \tau^k, p^c, p^n, s^c, s^n, A^S, \Phi_l, \Phi_u\right)$. The first two of them are free parameters. Values for β, γ, α, s^c and s^n will be taken from the literature. Parameters τ^c, τ^n, τ^k, p^c, p^n, A^S, Φ_l and Φ_u will be calibrated to match steady state moments from the Spanish economy. On the agent side, the representative consumer has the following preferences:

$$U(c_t, l_t) = log(c_t) + \Phi_l \frac{(1 - n_t^M - n_t^U - n_t^S)^{1-\gamma}}{1-\gamma} - \Phi_u n_t^U\,,$$

where the aggregate consumption, c_t, is the aggregate consumption (both taxed and untaxed goods) with an aggregator

$$c_t = \left[(c_t^V)^\rho + \phi(c_t^{NV})^\rho\right]^{\frac{1}{\rho}}\,.$$

For ρ, we follow Benhabib et al. (1991) and Ragan (2005), which use a similar aggregator for combining market and homemade goods. Their estimates for the substitutability parameters range from 0.8 and 0.9. In our framework we face more substitutable goods, thus we set a slightly higher value, $\rho = 0.95$. We have experimented with different values of the parameter and this choice is without loss of generality.

On the labor supply side, the parameter γ is set to 4, which implies a Frisch elasticity of labor supply around 0.5, a standard value in the literature, see King and Rebelo (1999) and Trabandt and Uhlig (2011). We set the discount to $\beta = 0.96$, implying a long run interest around 4%, which is roughly the average 10-year government bond yield since the introduction of the Euro. We use a discount factor $\delta = 0.07$, taken from Díaz-Giménez and Díaz-Saavedra (2009). Capital share is set to $\alpha = 0.3734$, calculated from the Spanish National Accounts in 2009-2012, see Gollin (2002). We set the underground output elasticity of labor equal to its legal counterpart, that is, $\sigma = 1 - \alpha$, given the lack of data to estimate it. Effective tax rates are estimated following Mendoza et al. (1994), except profit tax that is set to 0.25, the rate that most firms pay in Spain. Parameter Φ_l is calibrated to 0.13909 to match a 32.2% share of legal work reported by the Working Time Survey, published by the INE in (1996). The extra disutility of the underground labor, Φ_u, is similarly set to 1.7091 to get a share of underground worked hours over total labor equal to 21.9%, average value for Spain, obtained from Scheneider (2000). Parameter A^S is set to 3.8636, achieving a ratio of evaded over total consumption of 23.25%, value estimated by the Taxation and Customs Union Directorate (2013).

We specify the probability of being caught evading income tax, p^N to 0.1, which is simply a proportion of inspected tax filings with respect to the number of registered firms. We use data on labor inspections on firms, even though we have maintained throughout the whole article that income tax evasion is done on the agent side. This is still true, but we have not find any data on the number of inspections on the agent side. We set the same probability for inspection of consumption evasion p^c, since we have been unable to find any data on VAT inspections. For the surcharges in tax evasions, a good representation of the Spanish tax law is to set s^i, equal to 1.5.[3]

3.4 Business Cycle Results

We move on to see how our model performs in the business cycle. Table 3.2 reports second moments from the model. We HP-filter the log-series produced by the model, to make our results comparable with the literature. We take simulations that are $t = 1000$ periods longs, and calculate standard deviations

[3]Information about tax filings comes from the *Ministerio de Trabajo. Dirección General de la Inspección de Trabajo y Seguridad Social.* The number of firms is taken from the INE.

Parameter	Description	Value
δ	Depreciation	0.0782
β	Discount factor	0.96
ρ	Elasticity of substitution	0.95
ϕ	Weight of c^{NV}	0.78
Φ_l	Leisure param.	0.13909
Φ_u	Extra disutility of working underground	1.7091
γ	Elasticity of leisure	4
α	Share of capital in Y^M	0.3734
σ	Labor parameter in Y^U	0.6266
A^S	Shopping time	3.8636
ψ	Autocorrelation of shock	0.979
σ_ε	Standard deviation of shock	0.0072
p^c	Prob. of being caught evading VAT	0.1
s^c	Surcharge when caught evading VAT	1.5
p^n	Prob. ef caught evading income tax	0.1
s^n	Surcharge when evading income tax	1.5
τ^c	consumption tax	14.80%
τ^n	income tax	37.54%
τ^k	capital tax	24.95%
τ^π	profit tax	0.25%

Table 3.1: Calibration parameters

Variable (j)	σ_j (%)	σ_j/σ_y	$corr(j,Y)$
Y	1.2665	-	-
Y^M	1.344	1.0612	0.98767
Y^U	1.4873	1.1743	0.53835
Y^U/Y	3.7577	2.967	-0.83721
N^M	0.35714	0.28199	0.94552
N^U	1.2615	0.99604	-0.64699
i	5.5754	4.4022	0.82974
C	0.79358	0.6266	0.95752
C^V	1.4936	1.1793	0.98258
C^{NV}	2.2273	1.7586	-0.99911
θ	1.5491	2.7554	-0.99767

Table 3.2: Model results. Standard deviations and correlatons with respect to output.

and correlations for each simulation . We do 1000 simulations, and report the average across simulations. Column 1 shows standard deviations (per cent) of the variables. Column 2 reports the relative deviations with respect to total output. Column 3 shows the correlations of the variable in question with respect to total output.

Given the parameters used for the shock process, which are the values used by the literature, our model produces a higher volatility than standard models. It is very difficult to achieve these levels of volatility of investment with a standard model without recurring to non-standard calibrations. Here, with a very simple two sector model of consumption and income tax evasion, we obtain this hard-to-get result. For instance, in the models used by the literature investment fluctuates around 3 times more than output, whereas our model produces 4.4 times higher volatility of investment. For a reference study, see Hansen and Wright (1992). More interestingly, the variables that fluctuate most are those related with illegal activities. We can see that even though C^V and C^{NV} are very volatile, aggregate consumption is not, since they move in opposite directions. This is the case because both consumptions are highly substitutable.

The variables that are countercyclical are those related with illegal activities. There are two reasons for this. First, on the consumption tax evasion side, shopping for the illegal good requires time, and when the economy is in the middle of an expansion, labor productivity, and hence wages, are higher. This makes the opportunity cost of shopping for the illegal good (which are the foregone wages by not being working in the market) very expensive, and instead of spending time looking for c^{NV}, agents work more and start buying the more expensive c^V good. This is a demand effect. Thus firms see that the business of c^{NV} is no longer profitable, so θ falls. Second, on the income tax evasion side, a good shock increases the productivity of both production functions, but since the legal sector uses capital, its productivity increases more, and so do their wages. Hence, agents are encouraged to move to the legal sector by its higher wages, which results in the illegal hours falling. Y^U does not fall, and the reason is that the increase of e^z is higher than the decrease of n^σ. However, the share of illegal production is indeed reduced over the cycle. We can visually observe these effects on figure 3.2, where we plot scatterplots of the shocks and the variable of interest across a point in time for all simulations. The solid black lines are a linear regression of the simulated points, for comparison.

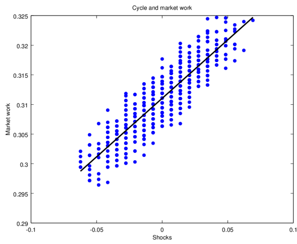
These high volatility on investment can be a good explanation on why investment falls abruptly on depressions. An important point will obviously be that the bleak economic expectations discourage investment. However, as a consequence of unemployment and the decreases in personal income, individuals may be willing to work in the underground economy, much labor intensive and with lower wages. These choices may even delay the "theoretical" recovery of the economy. If we observe decreases of GDP per capita Y^M, this may not be a recession of the magnitude observed in he figures, it may be less intense since Y^U could possibly be growing. Take for instance the Spanish economy. With an unemployment rate around 20% and without prospects to go down, there are no significant signs of social unrest. This is generally attributed to the idiosincrasies of the Spanish character, but we believe that a more reasonable claim is that many people are *de facto* employed, thus making the situation more dramatic. This is something positive if we think in static terms of welfare of the population, but if we think in dynamic terms, the private and public sector are foregoing many all the investments in education, RD, and so on, that will probably be key for growing in the future years, thus procrastinating the exit of the recession with the underground economy may be dynamically inefficient. This is an issue that the Spanish government has failed thus far to address, and we hope it gets addressed very soon so that the Spanish economy does not sacrifice its future growth.

3.5 Conclusion

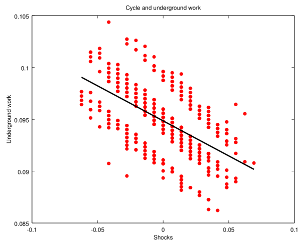
In this chapter, we extend our previous macroeconomic frameworks on shadow economy with business cycless. Up to our knowledge, the model presented here is the first approach to study the cyclical effects with a setup that combines labor income and consumption tax evasions.

Two main results arise when we explore the cyclical properties of our framework by introducing an aggregate productivity shock that affects equally both the declared and undeclared sectors. First, we find that consumption, investment, and labor are more volatile and their correlations with output are lower compared to the standard model. Therefore, including labor income and consumption tax evasions improves the ability of the standard RBC models to match key statistics of data, and hints that tax evasion may be an underlying motive of why we observe high divestments in times of crisis, for instance. Second, we are able to produce a countercyclical evolution of the shadow economy, an empirical regularity that has been pointed out by the literature, without assumming asymmetrical shocks on the legal and underground economy. This is based on on the microfounded assumption that acquiring illegal goods have an opportunity cost of being in the labor market, that becomes too expensive to bear in times of high productivity.

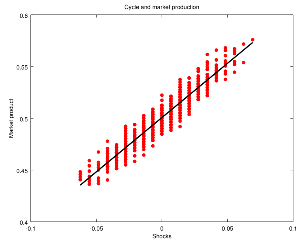
There are limitations to our study that would be interesting to relax in future work in order to have a deeper understanding of the business cycle effects of tax evasion. As we said in the previous chapter, this model assume that agents pool their income together, so we are unable to say anything about the relationship between tax evasion and inequality. Suppose that in times of crisis most of the individuals are going to the black market to keep alive. If this was the case, tax evasion on recessions would act reducing inequality. If, on the contrary, the opposite is true (hypothesis that seems more supported by the data), and in times of crisis only the poor lose their jobs and have to go underground to make a living, tax evasion may be exacerbating the already existing inequality. If this is true, for instance, unemployment benefits could be used to reduce tax evasion. Encouraging workers to stay in the legal market and actively search for a job will be probably worth doing if, by this means governments are able to reduce the the underground economy and the duration of recessions.



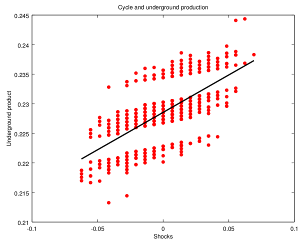
(a) Shock and n^M



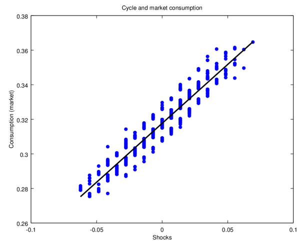
(b) Shock and n^U



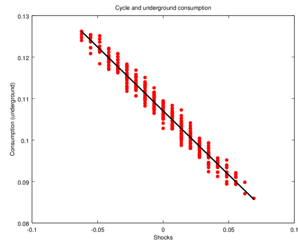
(c) Shock and Y^M



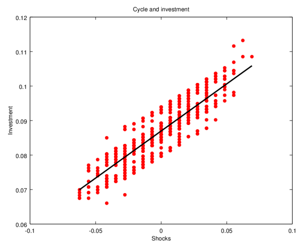
(d) Shock and Y^U



(e) Shock and C^V



(f) Shock and C^{NV}



(g) Shock and i

Figure 3.2: Cyclical components: GDP and the shadow economy.

Chapter 4

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