Chapter 3

Strategic Interaction with Densities with City Size Adjustment

1. Introduction

In this chapter we continue to analyze the choice of urban growth controls in a framework where the resulting levels of the controls arise from a process of strategic competition among cities. The main distinction with respect to chapter 2 consists in introducing a new type of land use regulation, the density restriction in the city, which was supposed to be normalized to k in all active cities in the previous chapter. As it will be shown, densities act as quantity-type instruments, and they are a specific kind of urban population controls.

The use of some kind of restriction on density levels is a common feature in virtually all planning systems. Local governments sometimes establish a maximum allowed density level, to prevent cities from becoming excessively concentrated in small areas. In other instances, as is frequently the case in the US, minimum lot size regulations are used, by means of which local communities are said to discriminate against low income households. This type of regulation ultimately limits population density. In Spain, it is a common practice to directly plan densities through the establishment of specific ratios of building space to lot area, or via maximum height for buildings.

Despite the practical popularity of density restrictions, the problems of how density restrictions are set and how they affect the urban equilibrium land pattern in the city have not received much attention in the literature, both at the theoretical and at the empirical level. Most studies devoted to analyze urban growth controls have focused on the consequences of intervening in the land market either through taxes or through the establishment of city bounds. However, some studies do take into account some type of density restriction (mostly in the form of minimum or maximum lot sizes) from a theoretical perspective [Pasha (1992); Pasha (1996); Fujita and Tokunaga (1993)] or empirically (Fu and Somerville, 2001).

In this work, density levels are accounted for in two ways. First, the density level of the city is one of the decision variables of the local government. In particular, the restriction consists on a full determination of the capital to land ratio, rather than a maximum limitation on the structural density. In this sense, the density constraint can be considered more stringent than usually is in the literature. Secondly, density negatively enters the utility function of residents. It has been considered that local jurisdictions care about total land rents in the city. From this perspective, higher densities involve more housing units per unit of land, what increases land rents. However, since households regard density increases as negative, an increase in density implies a reduced utility level, what translates into a diminishment of the maximum payment an individual is willing to pay for a particular plot of land.

Density restrictions are normally used combined with other regulatory instruments. In Spain, for instance, the joint use of densities and city boundaries is very common. Theoretically, however, the effects of the combined utilization of these two instruments have not been clearly established. As it is pointed out in Brueckner (1996), density restrictions have been said to contribute to urban sprawl when they impose a maximum bound at the intensity of land use, particularly when heights are limited near the urban centers. If density restrictions lead to less dense cities, and this somehow causes urban sprawl, the use of such planning regulations would partially be at odds with the simultaneous use of city limits below the market level.

In the current chapter we focus on the use of density levels as the planning instrument used by local governments. Let us for the moment focus on city sizes as the control variables to highlight the distinct features here incorporated with respect with the assumptions made in chapter 2. Although we do not explicitly model the exogenous choices of densities and city sizes (i.e. cities actually compete with a single planning instrument, density levels), we do allow for some degree of interaction between the two types of instruments. In the previous chapter housing consumption as well as density k were fixed, and it was possible that land rents did not coincide with the agricultural value at the city border. These assumptions prevent households from substituting housing away when housing rents increase, thus provoking a relatively high negative effect of land use regulations on utility. We ameliorate this negative impact and include some degree of flexibility by imposing the condition that land rents equal at the edge of the city, and allowing for the possibility that density rises when local governments choose small city sizes. However, so that variations in k can no indefinitely counterbalance the negative effects of restrictions, a cost component increasing with density is included in the utility function.

The points in the paragraph above equally apply when densities are the strategic variables and city sizes are the endogenous or adjusting ones. The latter is the case that has been here developed.

The remaining of the current chapter organizes as follows. The next section describes the basic model, with the specific features that concur here. Density enters both as a decision variable to local planners and as an argument in the utility function of households. The equilibrium conditions are derived for the more general non-regulated scenario. Then section 3 considers the case where the land market is intervened through the use of population controls. It analyzes the case where cities decide simultaneously upon one single type of instrument, density levels, describing the equilibrium conditions and the main relationships among the variables, with an stress on the effects on welfare. Finally, the main conclusions are highlighted in section 4.

2. The model with a disutility of density

This chapter follows the basic features of the theoretical model introduced in chapter 2, and therefore several of the assumptions previously introduced in the description of the model apply here as well. The main differences arise as a result of incorporating the possibility that there are different density levels among cities.

There are 3 cities, $i = 1, 2, 3$. Cities 1 and 2 may impose land use regulations, while city 3 accommodates all potential diverted residents. Cities are supposed to be linear, with a width of 1, and the CBD located at an extreme of the segment. All households commute to the CBD to work and, as a result, they incur in some transportation costs, $T(r)$, which are linear and increasing with distance, that is $T(r) = tr$, with $t > 0$. Residents are mobile households that receive an exogenous annual income Y. There is no heterogeneity among households due to differences in income. Income is fully spent within each period and it pays for transportation costs, $T(r)$; for a composite good that includes all other non-housing private goods, z, with $P_z = 1$; and for the housing rent. All households consume an exogenous size $\overline{s}_i = 1$. To live at a certain distance from the CBD, they must pay a rent $R(r)$ per unit of housing to absentee landowners. Then, the housing bid-rent of any household can be expressed as

$$R_i(r) = \frac{Y - T(r) - z(\overline{s}_i, k_i, u)}{\overline{s}_i},$$

and this housing demand function simplifies to

$$R_i(r) = Y - tr - z(k_i, u).$$

Notice that the consumption of private goods now depends upon density as well, because this variable affects negatively the citizens' utility. Since the households' size has been exogenously fixed to 1, we can concentrate on the remaining variables. We assume an additively separable specification of utility, as is commonly done in agency and other economic models. More specifically, the utility of a citizen of city i ($i = 1, 2, 3$) is given by

$$U_i(1, z_i, k_i) = v(z_i) - c(k_i),$$

where $v(\cdot)$ is an increasing and concave function, and $c(\cdot)$ is an increasing and convex function: for all z and all k,

$$v'(z) > 0, \quad v''(z) \leq 0, \quad c'(k) > 0, \quad c''(k) \geq 0$$

2.1 Characterization of the equilibrium

As in the previous chapter, the third city plays a passive role. In order to characterize the equilibrium, we can equivalently assume that cities fix either the density or the size (see below). So let us assume that density is the choice variable. Given density levels k_1, k_2 and k_3, exogenously chosen, the equilibrium will result from the following properties:

1. Given that citizens have perfect mobility, the final utility levels will be equal across cities.

2. The land rent at the city boundary equals the agricultural land rent, normalized here to zero.

3. Supply equals demand in the housing market.

4. All N citizens find accommodation.

Notice that condition 2 did not hold in the version of the model provided in chapter 2, except for the passive city 3. Given 1 above, there will exist a certain equilibrium utility level u such that

$$U_i(1, z_i, k_i) = u, \quad i = 1, 2, 3$$

With our specification of utility functions:

$$v(z_i) = u + c(k_i), \quad i = 1, 2, 3$$

That is,

$$z_i = v^{-1}[u + c(k_i)], \quad i = 1, 2, 3$$

Assuming perfect competition, the housing supply function is given by:

$$R_i(r) = \frac{L_i(r)}{k_i} + P,$$

where P is the cost of one unit of capital. Therefore, condition 3 of equilibrium in the housing market requires:

$$L_i(r_i) = k_i\,[Y - P - t\,r_i - z_i], \quad i = 1, 2, 3$$

Now, condition 2 implies

$$L_i(r_i) = 0, \quad i = 1, 2, 3$$

that is,

$$Y - P = t\,r_i + z_i, \quad i = 1, 2, 3$$

Finally, to accommodate all population it is needed that:

$$k_1\,r_1 + k_2\,r_2 + k_3\,r_3 = N.$$

Concluding, the equilibrium is characterized by the following system of equations:

$$\left.\begin{aligned} t\,r_1 + v^{-1}[u + c(k_1)] &= Y - P \\ t\,r_2 + v^{-1}[u + c(k_2)] &= Y - P \\ t\,r_3 + v^{-1}[u + c(k_3)] &= Y - P \\ k_1\,r_1 + k_2\,r_2 + k_3\,r_3 &= N \end{aligned}\right\}$$

Where r_1, r_2, r_3, and u are endogenous variables, and the remaining variables are exogenous, i.e. treated as parameters.

It can be shown that, given a set of exogenous variables for which the system has a solution, this solution is unique. As a matter of fact, we might interchange the roles of the endogenous variables r_1 and r_2 with the exogenous variables k_1 and k_2, because, given fixed values of the remaining exogenous variables, there is a one-to-one correspondence between these two pairs of variables.

To analyze the density-setting game between cities 1 and 2, we are interested in performing comparative statics, that is, sensitivity analysis of the endogenous variables with respect to the exogenous ones. To do this, we proceed differentiating the equilibrium identities with respect to the exogenous variables. To make easier the visualization of the next equations, we will use some shorthand notation. Given $i \in \{1, 2, 3\}$, let $w_i(u, k_i) = (v^{-1})'[u + c(k_i)]$, and let us write it simply as w_i, with the understanding that it is a function of both u and k_i. Let us also write c_i' to denote $c'(k_i)$.

We will see next that the Implicit Function Theorem can be applied to this system of equations around any solution, and it implies that there exist smooth functions of the endogenous variables in terms of the exogenous ones. Differentiating those functions and expressing it in matrix notation, we obtain:

$$\begin{pmatrix} t & 0 & 0 & w_1 \\ 0 & t & 0 & w_2 \\ 0 & 0 & t & w_3 \\ k_1 & k_2 & k_3 & 0 \end{pmatrix} \begin{pmatrix} \partial r_1/\partial k_1 & \partial r_1/\partial k_2 \\ \partial r_2/\partial k_1 & \partial r_2/\partial k_2 \\ \partial r_3/\partial k_1 & \partial r_3/\partial k_2 \\ \partial u/\partial k_1 & \partial u/\partial k_2 \end{pmatrix} + \begin{pmatrix} w_1\, c_1' & 0 \\ 0 & w_2\, c_2' \\ 0 & 0 \\ r_1 & r_2 \end{pmatrix} = 0$$

Notice that the determinant Δ of the jacobian with respect to the endogenous variables (the matrix we have to invert to solve for the derivatives) is always nonzero:

$$\Delta = -t^2\ [k_1\, w_1 + k_2\, w_2 + k_3\, w_3] < 0$$

This implies that solutions are locally unique, and the Implicit Function Theorem is always applicable around any solution to the equilibrium conditions. The inverse of the jacobian with respect to the endogenous variables is

$$\frac{t}{|\Delta|} \begin{pmatrix} k_2\, w_2 + k_3\, w_3 & -k_2\, w_1 & -k_3\, w_1 & t\, w_1 \\ -k_1\, w_2 & k_1\, w_1 + k_3\, w_3 & -k_3\, w_2 & t\, w_2 \\ -k_1\, w_3 & -k_2\, w_3 & k_1\, w_1 + k_2\, w_2 & t\, w_3 \\ t\, k_1 & t\, k_2 & t\, k_3 & -t^2 \end{pmatrix}$$

The matrix of the derivatives of the endogenous variables with respect to the exogenous ones is found by multiplying this inverse matrix by the jacobian of the system with respect to the exogenous variables. The result is

$$\begin{pmatrix} \partial r_1/\partial k_1 & \partial r_1/\partial k_2 \\ \partial r_2/\partial k_1 & \partial r_2/\partial k_2 \\ \partial r_3/\partial k_1 & \partial r_3/\partial k_2 \\ \partial u/\partial k_1 & \partial u/\partial k_2 \end{pmatrix} =$$

$$\frac{t}{|\Delta|} \begin{pmatrix} -w_1\, [t\, r_1 + k_2\, w_2\, c_1' + k_3\, w_3\, c_1'] & w_1\, [-t\, r_2 + k_2\, w_2\, c_2'] \\ w_2\, [-t\, r_1 + k_1\, w_1\, c_1'] & -\, w_2\, [t\, r_2 + k_1\, w_1\, c_2' + k_3\, w_3\, c_2'] \\ w_3\, [-t\, r_1 + k_1\, w_1\, c_1'] & w_3\, [-t\, r_2 + k_2\, w_2\, c_2'] \\ t\, [t\, r_1 - k_1\, w_1\, c_1'] & t\, [t\, r_2 - k_2\, w_2\, c_2'] \end{pmatrix}$$

Of all the partial derivatives, the only ones that unambiguously have a sign are

$$\frac{\partial r_1}{\partial k_1} < 0 \quad \text{and} \quad \frac{\partial r_2}{\partial k_2} < 0$$

This is not news, since we already mentioned that k_i and r_i are variables that can be substituted one by another. The sign indicates the direction of the change of the endogenous variables when an adjustment occurs: higher chosen densities imply smaller city sizes, and lesser densities larger cities.

3. The density-setting game

Assume now that cities 1 and 2 are deciding their respective density levels (equivalently, city size) so as to maximize the total revenues from property taxes. We therefore take the aggregate land rents as their objective functions. The expression for the total land rents in city i is given by

$$\begin{aligned}\Pi_i(k_1, k_2) = \int_0^{r_i} L_i(r)\, dr = \\ \int_0^{r_i} k_i\, [Y - P - z_i - t\, r_i]\, dr = \\ k_i\, r_i\, [Y - P - z_i] - \frac{t\, r_i^2\, k_i}{2} = \\ k_i\, r_i\, [Y - P - z_i - t\, r_i] + \frac{t\, r_i^2\, k_i}{2} = \frac{1}{2}\, t\, r_i^2\, k_i\end{aligned}$$

The first order conditions for the Nash equilibrium maximization problem are:

$$\frac{\partial \Pi_i}{\partial k_i} = \frac{1}{2}\, t \left[2\, r_i\, k_i \frac{dr_i}{dk_i} + r_i^2 \right] = 0,$$

where $r_i = r_i(k_1, k_2)$ is the function found from the equilibrium conditions.

The first order conditions can be simplified to

$$\begin{aligned}2\, k_1 \frac{\partial r_1}{\partial k_1} + r_1 = 0 \\ 2\, k_2 \frac{\partial r_2}{\partial k_2} + r_2 = 0.\end{aligned}$$

Let us consider the first order condition for city 1. We have:

$$t\,r_1 = 2\,t\,k_1\left(-\frac{\partial r_1}{\partial k_1}\right),$$

and substituting the derivative of the equilibrium conditions we found above:

$$t\,r_1 = \frac{2\,k_1\,w_1\,[t\,r_1 + k_2\,w_2\,c_1' + k_3\,w_3\,c_1']}{k_1\,w_1 + k_2\,w_2 + k_3\,w_3}.$$

Putting together the terms that affect $t\,r_1$,

$$[k_2\,w_2 + k_3\,w_3 - k_1\,w_1]\;t\,r_1 = [k_2\,w_2 + k_3\,w_3]\;2\,k_1\,w_1\,c_1',$$

or,

$$t\,r_1 = \left(\frac{k_2\,w_2 + k_3\,w_3}{k_2\,w_2 + k_3\,w_3 - k_1\,w_1}\right)\;(2\,k_1\,w_1\,c_1')$$

Since the fraction is larger than one, we have that

$$t\,r_1 > 2\,k_1\,w_1\,c_1' > k_1\,w_1\,c_1'.$$

Proceeding in the same manner for city 2, we find that

$$t\,r_2 > 2\,k_2\,w_2\,c_2' > k_2\,w_2\,c_2'.$$

Now if we look at the expressions for the derivatives of the endogenous equilibrium variables with respect to k_1, we can see that, at the Nash equilibrium values, and given the previous inequalities, the derivatives of these variables all have a well-defined sign:

$$\frac{\partial r_2}{\partial k_1} < 0; \quad \frac{\partial r_3}{\partial k_1} < 0; \quad \frac{\partial u}{\partial k_1} > 0$$
$$\frac{\partial r_1}{\partial k_2} < 0; \quad \frac{\partial r_3}{\partial k_2} < 0; \quad \frac{\partial u}{\partial k_2} > 0$$

One first result arising from the signs above is the inverse relationship between density and city size generalizes for all cities in the system at the equilibrium levels. Thus increasing density in city 1 reduces the city size in all cities in the system, not only the own.

More interesting is the way in which density affects utility in equilibrium. In particular, notice how the Nash equilibrium displays suboptimal levels of density. That is, the social welfare would be increased if both cities were to increase their choices of density (equivalently, if both cities were to decrease their size r_i). This result holds irrespective of the specification of the utility over wealth (v) or the density disutility (c) functions.

Such result seems at odds with the ones obtained in the previous chapter. In the supply-restriction model introduced in chapter 2 there were no amenity effects associated to the urban growth controls, and thus no positive effects linked to restricting city sizes. It was then shown that larger city sizes raise the equilibrium utility level of the system of cities. Even though in those models smaller densities (and consequently larger city sizes) positively contribute to the utility of residents, here it is found instead that utility could be increased by diminishing city sizes (or compacting cities). The main difference with respect to the equilibrium conditions we used before is that we have allowed here the possibility that city sizes adjust to changes in density, which was not possible in the setting of chapter 2.

The interpretation goes as follows. In principle, density increases cause utility to decrease. If city borders can adjust to counterbalance this impact, then more compacted city sizes allow for larger transportation savings, which increases utility. When accounting for this substitution effect between city size and density, local governments end up choosing density levels too low compared with the optimal ones. When competition in densities takes place, the equilibrium solution yields too stringent or too low restrictions depending on whether city sizes can be adjusted or not. Thus, if we think that it is relatively easy to modify city boundaries, then competition between jurisdictions results in cities that are not constrained enough. Instead, if the planning system is rather inflexible and city boundaries are difficult to adjust, and agents know that, then competition provokes too stringent restrictions.

4. Conclusions

The main contribution of this chapter consists in providing another extension to the theoretical literature devoted to explaining and evaluating urban growth controls in the presence of strategic competition. Several distinct features are present. The first one comes from contemplating the case where density restrictions can be used as regulatory instruments, and maximum bounds to city size are determined endogenously and negatively vary with densities. Also, densities negatively affect the utility of residents. We then investigate into the interactions occurring between these two variables, and identify some differences with respect to the previous analyses.

Density restrictions as strategic decision variables are analyzed, and are shown to act as strategic substitutes. In fact their role can be interchanged with that of city boundaries as planning instruments. We find that when cities compete with densities, the chosen values are always too low compared to the social optimum, even when accounting for the negative effect that increased densities infringe on households. These results differ from the ones obtained in a previous chapter, but they can be reconciled when accounting for the negative interaction that exists between city size and density, and the differences in the adjustment possibilities of some of the variables. The emergence of asymmetric cities could be possible if differences in the *flexibility* of some variables would exist across cities, even in the absence of heterogeneity among agents.