Parameter	Cleaning procedure	Level	Exponent
ALPHA	'Island'	4.0/3.0	1.5
WIDTH	'Island'	1.3/0.3	1.5
LENGTH	'Island'	1.3/0.3	1.0
CONC	'Island'	1.3/0.3	/
DIST	'island'	1.3/0.3	1.5
Asymmetry	'classic'	4.0/3.0	2.0
NumberMountain	'classic'	3.5/2.7	/
MountainSize	'classic'	3.5/2.7	/

Table A.5: The table summarizes the optimal cleaning levels for the different image parameters.

The ***Number-Mountain*** distribution and ***Mountain-SIZE*** distribution are plotted for both, gammas and hadrons in Fig. A.28 and Fig. A.27.

The Number-Mountain distributions for gammas and hadrons are quite ***well separated***, but the one of the Mountain-SIZE parameter shows significant overlap and it is ***not useful for hard cuts***. However, it still improves the separation when used as an ***additional input*** to the LDA together with other parameters. This will be shown in the next section.

A.4.5 Improvement in gamma/hadron separation

As all of the new parameters have been now presented, we can proceed to check whether they give an improvement in discrimination of the total image dataset. The tool for this procedure is the ***LDA***. More parameters will be introduced, step by step, to see if an ***improvement*** can be achieved.

Improvement of the gamma/hadron separation by including the new parameters in the LDA

Now we include the new parameters, that were defined above, in the analysis and see if this increases the discrimination power. The new parameters are

- Parameters with ***weights*** (***exponents*** n) on q_i.
- Parameters obtained by ***different image cleaning*** procedures.
- ***Asymmetry***
- ***Number-Mountain***
- ***Mountain-SIZE***

Tab. summarizes the optimal cleaning levels for the different image parameters as obtained in the studies above. The image parameters of ***three datasets*** were calculated accordingly. These are the ***MC gamma dataset***, a ***(recorded) OFF*** data set and the ***Mkn 421 dataset*** (flares in 2001, 167 hours observation time).

The ***stepwise improvement*** is shown in ***four*** steps:

1. First the parameters ***WIDTH***, ***LENGTH***, ***CONC***, ***zenith angle*** $\cos\theta$, ***SIZE*** and some ***higher orders*** are included in the LDA input parameter list, as described in section A.3.5. The gives us the first nine inputs from input[0]...Input[8]:

$$\begin{aligned} Input[0] &= WIDTH \quad \text{(A.73)} \\ Input[1] &= LENGTH \\ Input[2] &= CONC \end{aligned}$$

$$Input[3] = SIZE$$
$$Input[4] = \cos\theta$$
$$Input[5] = WIDTH^2 \quad (A.74)$$
$$Input[6] = LENGTH^2 \quad (A.75)$$
$$Input[7] = CONC^2 \quad (A.76)$$
$$Input[8] = \frac{WIDTH * LENGTH}{SIZE} \quad (A.77)$$

2. Then the ***asymmetry angle*** α of Equ. A.71 and the its cosine were included.

$$Input[9] = \alpha \quad (A.78)$$
$$Input[10] = \cos\alpha$$

3. After this the mountain classification parameters ***Number-Mountain***, ***Mountain-SIZE*** and ***Mountain-Size/SIZE*** were included.

$$Input[11] = NumberMountain \quad (A.79)$$
$$Input[12] = MountainSize$$
$$Input[13] = MountainSize/SIZE$$

4. In the end the ***remaining rest*** was included, which give still ***small corrections*** and improve separation

$$Input[14] = Leakage \quad (A.80)$$
$$Input[15] = Leakage^2$$
$$Input[16] = Distance$$
$$Input[17] = AveragePedestalSigma$$

At each step the LDA was trained with the MC-gamma-dataset and the OFF-dataset which gives us as a result the ***discriminating power***. Then the trained LDA was applied to the dataset of ***Mkn 421*** (flares in 2001, 167 hours observation time). This yielded the number of ***excess events***, the ***background*** and the resulting ***significance (Li/Ma)***. The following static fix quality selection cut has been applied before the LDA procedure: $0.5 \leq DIST \leq 1.05$, $SIZE > 60\,PhE$, $Zenith\,angle < 50\,deg$, $WIDTH > 0.25$ and $LENGTH > 0.4$.

The results are presented in the Tab. A.6. In the first row the result of the ***simple static cut*** is shown. The next two rows show the results using classic image parameters with 1. ***linear input*** and 2. ***higher order input.*** The last ***four*** lines show the stepwise improvement by using parameters with weights and new parameters as listed before.

The final ALPHA plot for the ***Mkn 421*** test sample (167 hours of observation) can be seen in Fig. A.29. An OFF-data sample with lower statistics is also plotted in the same figure to ***demonstrate*** that the background estimation by fitting a polynomial is in agreement with the OFF-data distribution. The ALPHA distribution is wider than the previous using classical image parameters because much more ***additional low energy events*** (which have an unsharper ALPHA distribution) were found. The final result are about 16000 +- 250 excess events with a significance (Li/Ma) of about 83 sigma.

Fig. A.30 shows the output of the LDA. Fig. A.31 displays the cut efficiency of the ***final*** version plotted against zenith angle and energy. The low energy cut-efficiency improved quite significantly and it's almost flat in zenith angle response. The average cut efficiency (after trigger and 'precut') is now as high as 80 %. To ***avoid confusion*** it has to be mentioned that the ***precut on DIST*** is ***dependent*** on the exponent n=1.5. For this

Step	Disk. power	Excess events	Backg.	Sig.
static cuts	/	5942.9+-134	3869	47.9
classic parameters linear	0.678	11389+-189	9681	60.8
classic parameters quad.	0.736	12592+-204	9158	67.3
1. Parameters quad.	0.755	14486+-256	11760	70.4
2. Plus asymmetry	0.766	16079+265	10886	77.8
3. Plus mountains	0.769	15636+-242	9275	80.0
4. Plus rest	0.777	16182+-245	9060	82.8

Table A.6: The table presents the stepwise increase of discriminating power, excess events and significance while reducing the background by including the new image parameters. The first line contains the results of the static cuts. The next two lines show the numbers for classic HILLAS parameters using linear and higher order inputs in the LDA. The next four steps show the increase in discrimination of hadrons and gammas by using image parameters with weights and including new parameters. In the first step the parameters ***WIDTH***, ***LENGTH***, ***CONC***, ***zenith angle*** $\cos\theta$, ***SIZE*** and some ***higher orders*** are used as inputs for the LDA. In the next three step the list is expanded by the asymmetry, mountain classifications and in the end by parameters that correct for correlations.

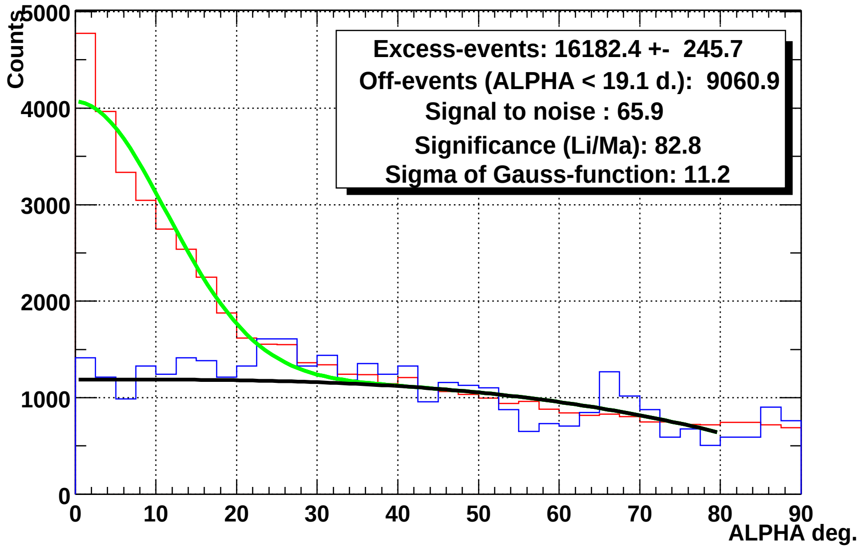


Figure A.29: The ALPHA plot of the final LDA cut. The plot shows the final result with the maximum significance of 82.8 sigma that achieved for the dataset of Mkn 421 of 167 hours of observation time.

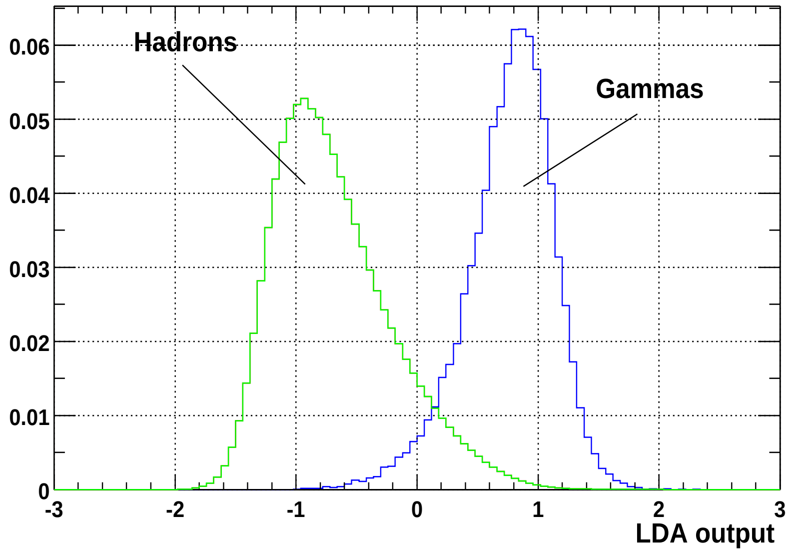


Figure A.30: The histogram shows the output distributions (normalized to one) of the LDA for (recorded) hadrons and (MC) gammas after introducing all possible improvements.

reason the average cut efficiency after precut here ***cannot*** be compared directly with the value obtained at the beginning when using classical HILLAS parameters (the average cut efficiency after precut was around 70 %, but the sample was smaller due to a different definition of DIST with $n = 1.0$). Apparently the cut efficiency increased only by 13 % while the excess events increased by 30 %, which is ***no contradiction***. The precut on DIST has always been chosen to result in a ***maximally large*** significance.

Fig. A.32 shows the effective area according to the LDA selection cut (which was optimized for maximal significance). In the calculation of the effective areas the telescope trigger efficiency is also included. This is why the collection area for energies below 1 TeV is rather small while the cut efficiency is still very high, approximately 60%. The effective areas for the ***lowest zenith angle*** are ***above 55 000*** m^2 (below 1 TeV). They reach almost ***95 000*** m^2 for the largest zenith angle of ***45°.***

A.4.6 Conclusion about the introduction of new image parameters and new image cleaning algorithms

It has been shown that the introduction of ***exponents*** on the charge in each pixel gave clear improvements of the ***separation power*** of the image parameter set in case of the parameters ***ALPHA***, ***WIDTH*** and ***LENGTH*** while the introduction of weights ***depending on the noise*** in each pixel ***did not*** improve the discrimination.

New image parameters like the ***asymmetry angle*** of the a shower, ***mountains*** and leakage where clearly able to further reduce the background.

A ***significant improvement*** came from the usage of a ***different cleaning algorithm***, the ***'island'*** cleaning which allowed to ***decrease*** the cleaning level and thus keep more information for the separation. This improves the separation for small energy events.

Altogether (also by using the LDA as separation tool) it was possible to ***double the significance*** of the detected signal and to almost triple the amount of ***signal (excess) events*** (always at maximum significance).

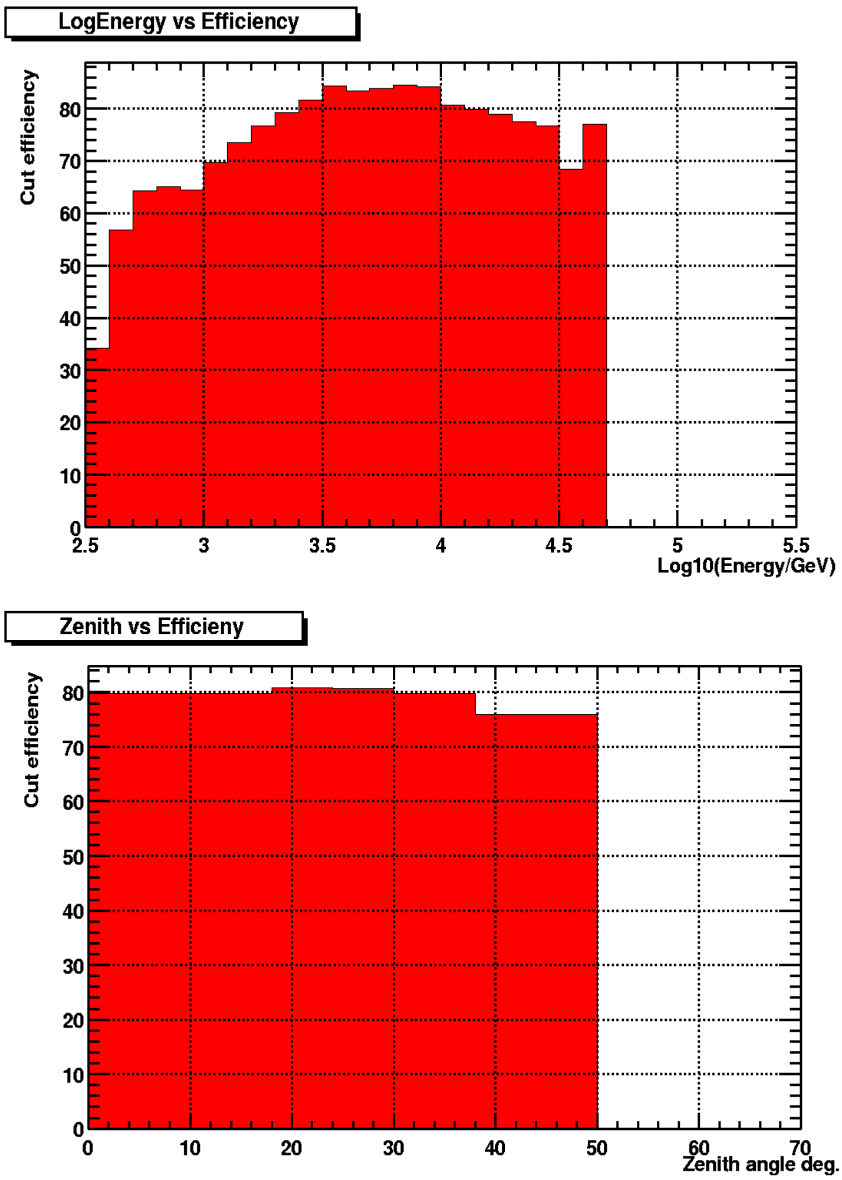
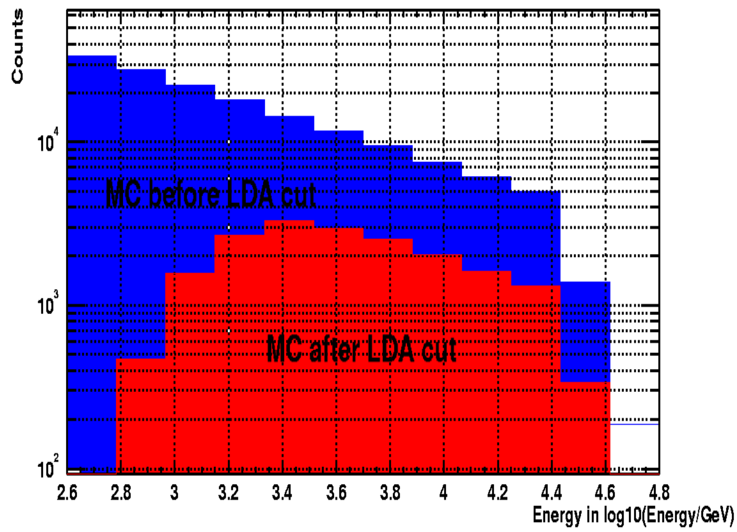


Figure A.31: The upper plot shows the cut-efficiency vs log(Energy/GeV) and the lower plot shows the γ cut efficiency vs zenith angle (in degrees). The LDA has a quite good cut efficiency for energies smaller than 1 TeV. Above 500 GeV the cut efficiency is still better than 60 % to 65 %. Above 10 TeV the efficiency decreases slightly because very big showers (high energy) are always truncated at the camera border. The cut efficiency is measured after the trigger and after applying a filter cut of $0.5 \leq DIST \leq 1.05$, $SIZE > 60\,PhE$, $Zenith\,angle < 50\,deg$ and a two-next-neighbour software trigger. The lower plot shows an average cut efficiency of 80 % for a cut that results in maximal significance. The DIST pre-cut is dependent on the exponent n. For this reason the improvement shown here seems smaller than it actually is, compared to Fig. A.12.



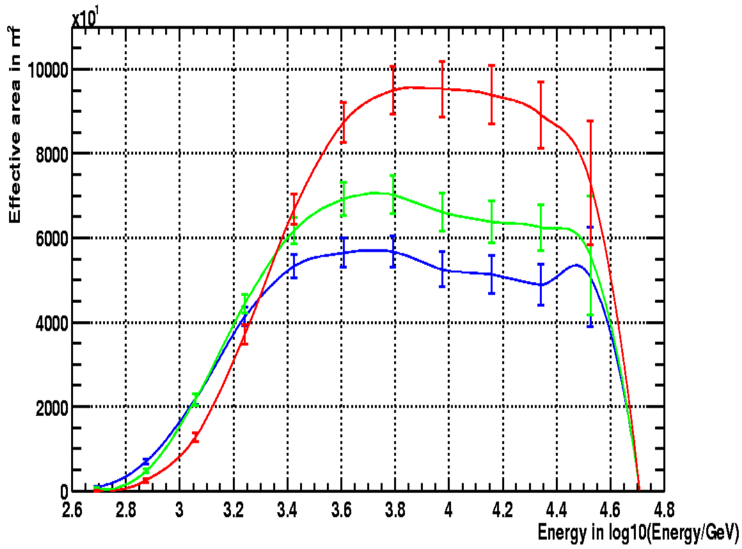


Figure A.32: The lower plot shows the effective areas calculated for the LDA selection cut using a power law spectrum with a spectral index of $\alpha = 2.9$ for three zenith angles. The blue curve is for a 12°, the green curve for a 32° and the red curve for a 45° zenith angle. The upper plot shows the simulated MC events before cut (the blue curve) and the triggered events after LDA cut (the red curve).