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Septiembre 2015

tesis doctoral realizada por Marta Sarmiento-Comesías

Structural Behaviour and Design Criteria of  
Spatial Arch Bridges.

# Structural Behaviour and Design Criteria of Spatial Arch Bridges.

Annexes

Volume 2 of 2

tesis doctoral realizada por:

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Barcelona, Septiembre 2015

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Departament d'Enginyeria de la Construcció

# TESIS DOCTORAL

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**DESIGNED EXAMPLES OF**

**SPATIAL ARCH BRIDGES**

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## 1. INTRODUCTION

This annex includes most of the Spatial Arch Bridges (SABs) which have been designed until the present date (July 2015), considering from tender designs for competitions which did not reach the construction project stage to built examples. More than a 100 spatial arches have been design up to the present date.

Not only bridges are included in this annex. Some arch studies and roofs conformed or supported by spatial arches are considered. They are also included since they have a similar structural behavior regarding the spatial arch definition given in this thesis (Chapter III), although the loads instead of coming from the deck come eg. from a roof, which will differ in the fact that live loads are not as important. However, the structural behavior under a uniformly distributed loading is equivalent.

The SABs definition is reminded in the following lines.

- SABs are defined as bridges in which vertical deck loads produce bending moments and shear forces not contained in the arch plane due to their geometrical and structural configuration. Moreover, the arch itself may not be contained in a plane.
- Under the global concept of “spatial arch bridges” we understand both, bridges supported by arch ribs and by shells.
- The previously given definition applies to SABs employing arch ribs.

The examples are given in table format, including:


- the name of the bridge,
- the authors,
- the year of construction, if it is built, and of design, if it is not,
- its location,
- its function, ie its use,
- nn image and
- references where information of the bridge can be found



The examples are separated into two tables according to the SABs classification in Chapter III. A into two main types:



- Spatial arch ribs: arches in which the cross-section of the arch has a width/span and depth/span ratios low enough for the arch to be accurately analysed with frame elements with 6 degrees of freedom per node
- Shell arches: arches in which the cross-section of the arch has a width/depth and width/span ratios large enough for requiring an analysis with shell elements. The arch is a roof-like structure.





## 2. EXAMPLES OF SPATIAL ARCH RIBS



| NAME   | YEAR           | AUTHOR                    | LOCATION   | FUNCTION                   | IMAGE AND ARTICLE REFERENCES   |
|--|----------------|---------------------------|--|----------------------------|--|
| <p><b>Alameda Bridge<br/>(La Peineta Bridge)</b></p> | <p>1991-95</p> | <p>Santiago Calatrava</p> | <p>Valencia, Spain<br/><i>Crosses:</i> Turia river</p> | <p>Road and footbridge</p> |  <p>P. Jodidio, 2003; <i>structurae</i></p> |

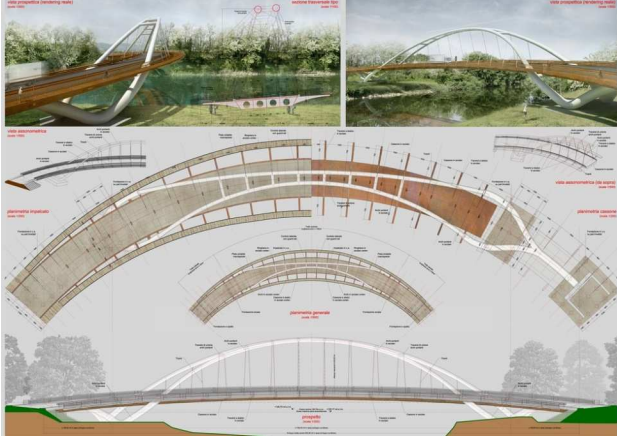
| NAME  | YEAR | AUTHOR                      | LOCATION  | FUNCTION    | IMAGE AND ARTICLE REFERENCES  |
|---|------|-----------------------------|---|-------------|---|
| <p><b>Arch bridge crossing the Brno-Vienna Expressway</b><br/><b>(Arch bridge across high-speed road R52 near Bratic)</b></p> | 1997 | Strasky, Husty and Partners | <p>Rajhrad, Czech Republic</p> <p><i>Crosses:</i> Brno-Vienna highway R52</p> | Road bridge |  <p>J. Strasky and I. Husty, 1997; J. Strasky, 1999; Strasky, 2000; Strasky, 2005; Strasky, Husty and Partners, Ltd/Projects</p> |
| <p><b>Arch Bridge in Abu Dhabi</b></p>  |      | Christian Menn              | Abu Dhabi, United Arab Emirates   |             |  <p>E. Brühwiler, 2009</p>  |


| NAME                         | YEAR | AUTHOR             | LOCATION  | FUNCTION    | IMAGE AND ARTICLE REFERENCES   |
|------------------------------|------|--------------------|---|-------------|--|
| <b>Artunduaga Bridge</b>     | 2008 | Arenas & Asociados | Basauri, Spain<br><i>Crosses:</i><br>Nervión River        | Road Bridge |  <p>J. J. Arenas de Pablo et al, 2011 (2)</p>   |
| <b>Bohlbach creek bridge</b> | 1932 | Robert Maillart    | Habkern, Switzerland<br><i>Crosses:</i><br>Bohlbach Creek | Road bridge |  <p>D. P. Billington, 1979, p60; D. P. Billington, 1997, pp154-155; M.Laffranchi and P.Marti, 1997; structurae</p> |

| NAME  | YEAR | AUTHOR                                   | LOCATION  | FUNCTION                            | IMAGE AND ARTICLE REFERENCES  |
|---|------|--|---|-------------------------------------|---|
| <p><b>Bridge across the Bacchiglione at Padua</b></p> |      | <p>Enzo Siviero<br/>Lorenzo Attolico</p> | <p><b>Padua.</b><br/><i>Connects: Via Vittorio Veneto and Via Isonzo</i><br/><i>Crosses: Bacchiglione River</i></p> | <p>Cycle-pedestrian<br/>Brisege</p> |  <p>E. Siviero and L. Attolico, 2010</p> |



| NAME                                       | YEAR        | AUTHOR                             | LOCATION  | FUNCTION | IMAGE AND ARTICLE REFERENCES  |
|--|-------------|------------------------------------|---|----------|---|
| <p><b>Bridge across the Olse River</b></p> | <p>2012</p> | <p>Strasky, Husty and Partners</p> | <p>connecting the<br/><b>Czech and<br/>Polish Tesin</b></p> |          |  <p>Strasky, Husty and Partners, Ltd/Projects</p> |



| NAME                                  | YEAR | AUTHOR  | LOCATION   | FUNCTION            | IMAGE AND ARTICLE REFERENCES  |
|---------------------------------------|------|---|--|---------------------|---|
| <b>Bridge across the Vltava River</b> | 2006 | Strasky, Husty and Partners                   | Most - Luční Jez, Ceske Budejovice (Budweis), Czech Republic<br><br><i>Crosses:</i> Vltava river | Pedestrian bridge   |  <p>J. Strasky, 2005; Strasky, Husty and Partners, Ltd/Projects</p>      |
| <b>Bridge over Galindo river</b>      |      | Javier Manterola from Carlos Fernández Casado | Bilbao, Vasqueland, Spain<br><br><i>Crosses:</i> Galindo river Mouth into Nervion river          | Road and footbridge |  <p>J. Manterola et al, 2005 and 2011; Bilbao en construcción, 2007</p> |



| NAME                                     | YEAR                              | AUTHOR   | LOCATION   | FUNCTION                 | IMAGE AND ARTICLE REFERENCES   |
|--|-----------------------------------|--|--|--------------------------|--|
| <p><b>Bridge over the Arno river</b></p> | <p>Project 2012<br/>Not built</p> | <p>BF Ingegneria and ACS ingegneri (Prato)</p> | <p>Figline, Italy<br/><i>Crosses:</i> River Arno</p> | <p>Pedestrian bridge</p> |  <p><a href="https://fckestructural.wordpress.com/2012/06/20/bienvenidos-al-nuevo-blod-de-fck-consultoria-estructural/">https://fckestructural.wordpress.com/2012/06/20/bienvenidos-al-nuevo-blod-de-fck-consultoria-estructural/</a><br/><a href="http://www.bfingegneria.altervista.org/">http://www.bfingegneria.altervista.org/</a></p> |


| NAME                                     | YEAR | AUTHOR                                | LOCATION  | FUNCTION                       | IMAGE AND ARTICLE REFERENCES   |
|--|------|---------------------------------------|---|--------------------------------|--|
| <p><b>Bridge over the Markkanaal</b></p> |      | <p>Zwarts &amp; Jansma Architects</p> | <p>Breda-Noord,<br/>Netherlands</p>   | <p>Road and<br/>footbridge</p> |  <p><a href="http://www.zwarts.jansma.nl/page/1557/nl">http://www.zwarts.jansma.nl/page/1557/nl</a></p> |
| <p><b>Bridge over the river Sile</b></p> |      |                                       | <p>Portegrandi,<br/>Venice, Italy</p> <p><i>Crosses:</i> River<br/>Sile</p> |                                | <p>Artuso et al, 2001</p>  |







| NAME   | YEAR             | AUTHOR   | LOCATION  | FUNCTION                 | IMAGE AND ARTICLE REFERENCES  |
|--|------------------|--|---|--------------------------|---|
| <p><b>Butterfly Bridge</b><br/>(Embankment Renaissance Bridge)</p> | <p>1998</p>      | <p>Engineer: Jan Bobrowski and Partners<br/><br/>Architect: Wilkinson Eyre</p> | <p>Bedford<br/><i>Crosses:</i> River Great Ouse</p> | <p>Pedestrian bridge</p> |  <p>M. Pollitt, 2000;<br/>Charlotte Community Design Studio-CCDS, 2008</p>   |
| <p><b>Celtic Gateway</b></p>                                       | <p>2003-2005</p> | <p>Gifford</p>   | <p>Holyhead, Wales, UK</p>                          | <p>Pedestrian bridge</p> |  <p><a href="http://www.gifford.uk.com/sectors-and-projects/bridges/project/celtic-gateway/">http://www.gifford.uk.com/sectors-and-projects/bridges/project/celtic-gateway/</a></p> |



| NAME   | YEAR      | AUTHOR   | LOCATION  | FUNCTION             | IMAGE AND ARTICLE REFERENCES   |
|--|-----------|--|---|----------------------|--|
| <b>Charvaux Footbridge</b>                   | 2000      | Architect Michel Roy<br>Structural engineer Marc Malinowsky        | Andrésey,<br>Yvelines, Ile de<br>France, France | Pedestrian<br>bridge |  <p data-bbox="1480 820 1928 847">Structurae; M. HelZel and I. Taylor, 2004</p> |
| <b>Churchill Way Footbridge, Basingstoke</b> | 2000-2003 | <b>Engineering:</b> Gifford<br><b>Architect:</b> Haskoll & Company | Basingstoke,<br>Hampshire                       | Pedestrian<br>bridge |    |


| NAME                           | YEAR                             | AUTHOR          | LOCATION                                | FUNCTION             | IMAGE AND ARTICLE REFERENCES   |
|--------------------------------|----------------------------------|-----------------|---|----------------------|--|
| <b>Citadelbridge. Y-Bridge</b> | Project<br>2008<br><br>Not built | NEXT Architects | Nijmegen,<br>Netherlands                | Pedestrian<br>bridge | <br><a href="http://www.nextarchitects.com/projects/1353">http://www.nextarchitects.com/projects/1353</a> |
| <b>Clyde Arch</b>              | 2006                             | Halcrow         | Glasgow, UK.<br>Crosses: River<br>Clyde | Pedestrian<br>bridge | <br><a href="http://www.puentemania.com/5399">http://www.puentemania.com/5399</a>                        |

| NAME                    | YEAR | AUTHOR   | LOCATION  | FUNCTION                              | IMAGE AND ARTICLE REFERENCES  |
|-------------------------|------|--|---|---------------------------------------|---|
| <b>Contreras Bridge</b> |      | J. Manterola<br>(Carlos Fernández Casado S.L.) | Madrid-<br>Valencia high<br>velocity<br>railway<br><br><i>Crosses:</i><br>Embalse de<br>Contreras | High<br>Velocity<br>Railway<br>bridge |  <p data-bbox="1458 855 1946 882">EIPSA: <a href="http://www.eipsa.net/es/inicio_es.asp">http://www.eipsa.net/es/inicio es.asp</a></p> <p data-bbox="1574 906 1832 933">Spanish works e-ACHE</p> |



| NAME   | YEAR                              | AUTHOR   | LOCATION   | FUNCTION                             | IMAGE AND ARTICLE REFERENCES   |
|--|-----------------------------------|--|--|--------------------------------------|--|
| <p><b>Cycling and Pedestrian Bridge over the 2ª Circular in Lisbon</b></p> | <p>Project 2009<br/>Not built</p> | <p>Impromptu Architects + Selahattin Tuysuz Architecture</p> | <p>Lisbon, Portugal</p>                                | <p>Cycling and Pedestrian Bridge</p> |  <p><a href="http://europaconcorsi.com/projects/109073-New-Cycling-and-Pedestrian-Bridge-over-the-2-Circular-in-Lisbon">http://europaconcorsi.com/projects/109073-New-Cycling-and-Pedestrian-Bridge-over-the-2-Circular-in-Lisbon</a><br/><a href="http://www.adapt-architects.com/project011.php">http://www.adapt-architects.com/project011.php</a></p> |
| <p><b>Dagu Bridge. “Sun and Moon Arches”</b></p>                           | <p>2005</p>                       | <p>T. Y. Lin International Group</p>                         | <p>Tianjin, China.<br/><i>Crosses:</i> Haihe River</p> | <p><b>Road Bridge</b></p>            |  <p>Han et al, 2007; Ma, 2010;<br/><a href="http://www.tylin.com/en/projects/dagu_bridge">http://www.tylin.com/en/projects/dagu_bridge</a></p>   |


| NAME  | YEAR | AUTHOR   | LOCATION                  | FUNCTION            | IMAGE AND ARTICLE REFERENCES  |
|---|------|--|---------------------------|---------------------|---|
| <b>De Gasperi Bridge</b>                        | 2009 | <p><b>Engineering:</b> Malerba, P. G., Galli, P. and Di Domizio, M..</p> <p><b>Design:</b> Metropolitana Milanese S.p.A.</p> | Milan Portello            | <b>Road Bridge</b>  |  <p>Malerba, 2010; Malerba et al, 2010 and 2011</p> <p><a href="http://en.structurae.de/structures/data/index.cfm?id=s0058452">http://en.structurae.de/structures/data/index.cfm?id=s0058452</a></p> |
| <b>Desdoblamiento del puente de la Peraleda</b> | 2005 | Estudio AIA  | Peraleda District, Toledo | Road and footbridge |  <p>Estudio AIA/Proyectos; R. Sánchez, 2005</p>   |



| NAME                      | YEAR | AUTHOR   | LOCATION                                 | FUNCTION           | IMAGE AND ARTICLE REFERENCES   |
|---------------------------|------|--|--|--------------------|--|
| <b>Douglas Footbridge</b> | 2008 | Andrea Menardo Atelier MESH+   | Lancashire, UK<br>Crosses: River Douglas | Pedestrian bridge  |  <p>Ateliermeshplus/Projects ; Archiportale</p>             |
| <b>Dragon Eco Bridge</b>  |      | <p><b>Structural Designer</b><br/>Tongji Architectural Design (Group) Co., Ltd.</p> <p><b>M&amp;E Engineers</b><br/>Zong Lianghai</p> <p><b>Architect:</b> Ding Jiemin</p> | Shenyang, China                          | <b>Road Bridge</b> |  <p>Castro, 2011<br/>Structural Awards 2014 (ISTRUCTE)</p> |



| NAME                    | YEAR | AUTHOR  | LOCATION  | FUNCTION             | IMAGE AND ARTICLE REFERENCES   |
|-------------------------|------|---|---|----------------------|--|
| <b>Dreiländerbrücke</b> | 2007 | <p><b>Structural engineering/ Consultants:</b><br/>Leonhardt, Andrä und Partner (LAP),<br/>Berlin</p> <p><b>Architects:</b> Feichtinger Architectes</p> | <p>Between <b>Weil am Rhein</b>,<br/>Lörrach<br/>(Landkreis),<br/>Baden-<br/>Württemberg,<br/>Germany and<br/><b>Huningue</b>,<br/>Haut-Rhin,<br/>Alsace, France</p> <p><i>Crosses:</i> Rhine<br/>River</p> | Pedestrian<br>bridge |  <p>Feichtinger Architects/ Projects; Leonhardt, Andrä und Partner/News, 2001; Feichtinger Architects , 2006; W. Zschokke, 2007; Le Moniteur des Travaux Publics et du Bâtiment, 2006, n. 5373 ; 2007, n5382, 5385,5404; Leonhardt, Andrä und Partner/News, 2008; Leonhardt, Andrä und Partner/Projekts, 2008</p> |


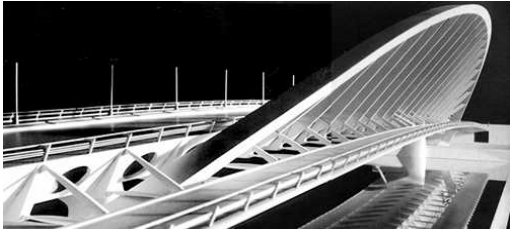




| NAME                     | YEAR | AUTHOR  | LOCATION   | FUNCTION            | IMAGE AND ARTICLE REFERENCES   |
|--------------------------|------|---|--|---------------------|--|
| <b>Elche bridge</b>      |      | Javier Manterola from Carlos Fernández Casado | Elche, Spain.<br><i>Crosses:</i><br>Vinalopó River                         | Road and footbridge |  <p>J. Manterola, 2005</p>                          |
| <b>Endarlatsa Bridge</b> |      | J. Manterola                                  | Between Navarra, Guipúzcoa and France.<br><i>Crosses:</i><br>Bidasoa River | Road Bridge         |  <p>J. Manterola et al, 2009, A. Vidondo, 2008</p> |



| NAME                 | YEAR      | AUTHOR  | LOCATION                           | FUNCTION               | IMAGE AND ARTICLE REFERENCES   |
|----------------------|-----------|---|------------------------------------|------------------------|--|
| <b>Europe bridge</b> | 1996-2000 | Architect: Calatrava<br>Structural Engineering: Greisch, Setec<br>TPI | Orléans<br>Crosses: Loire<br>river | Road and<br>footbridge |  <p data-bbox="1406 948 2002 1010">Photograph: Herrad Elisabeth Taubenheim (structurae).<br/>Del Forno, J. Y et al, 2001; Detry 2001</p> <p data-bbox="1391 1018 2018 1043"><a href="http://www.greisch.com/projet/pont_ouest_orleans-en.html">http://www.greisch.com/projet/pont_ouest_orleans-en.html</a></p> |


| NAME  | YEAR           | AUTHOR   | LOCATION  | FUNCTION                   | IMAGE AND ARTICLE REFERENCES  |
|---|----------------|--|---|----------------------------|---|
| <p><b>Father Bernatek's Footbridge over the River Vistula in Cracow</b></p> | <p>2010</p>    | <p>Promost Consulting Rzeszów,<br/>Consulting and Design Office<br/>Żółtowski and ZB-P Mosty Wrocław</p> | <p>Cracow, Poland<br/><i>Crosses:</i> River Vistula</p> | <p>Pedestrian bridge</p>   |  <p>(Flaga and Januskiewicz, 2011)</p>                                     |
| <p><b>Felipe II / Bach de Roda Bridge</b></p>                               | <p>1984-87</p> | <p>Santiago Calatrava</p>  | <p>Barcelona, Spain</p>                                 | <p>Road and footbridge</p> |  <p>Jodidio 2003; M. Torres, 2002; A. Tzonis, 2004; <i>structurae</i></p> |

| NAME                               | YEAR      | AUTHOR                       | LOCATION                                 | FUNCTION          | IMAGE AND ARTICLE REFERENCES   |
|------------------------------------|-----------|------------------------------|--|-------------------|--|
| <b>Friends Bridge</b>              | 1998      | Whitby Bird and Partners     | Lea Valley Park, Hackney Marshes, London | Pedestrian bridge |  <p>Ramboll Whitbybird/ Projects; structurae</p>  |
| <b>Gateshead millennium bridge</b> | 1998-2001 | Structural Designer: Gifford | Gateshead<br><i>Crosses:</i> Tyne River  | Pedestrian bridge |  <p>Atkins Bennet Engineering Design Consultants, 2001; D. Barker, 2001; Curran, 2003; Johnson and Curran, 2003; Sarmiento, 2008; S.Mehrkar-Asl, (s.f.); Gateshead Borough Council's dedicated bridge web site; structurae</p> |



| NAME                                      | YEAR    | AUTHOR             | LOCATION                      | FUNCTION        | IMAGE AND ARTICLE REFERENCES  |
|---|---------|--------------------|-------------------------------|-----------------|---|
| <b>Gennevilliers Port Railroad Bridge</b> | 2002    | Campenon Bernard   | Gennevilliers Harbour, France | Railroad bridge |  <p>Moussard et al, 2001</p>   |
| <b>Gentil Bridge</b>                      | 1987-88 | Santiago Calatrava | Paris, France                 | Not constructed |  <p>Jodidio, 2003; Santiago Calatrava. The Unofficial Website/ Bridges; A. Tzonis, 2004</p> |


| NAME  | YEAR        | AUTHOR                                   | LOCATION  | FUNCTION                     | IMAGE AND ARTICLE REFERENCES   |
|---|-------------|--|---|------------------------------|--|
| <p><b>Grand Wisata Overpass</b></p>             | <p>2007</p> | <p>PT Partono Fondas Eng. Consultant</p> | <p>Bekasi,<br/>Indonesien</p>   | <p>Road bridge</p>           |  <p>Supartono, 2009</p>                           |
| <p><b>Hacking Ferry Bridge (Ribble Way)</b></p> |             | <p>Wilkinson Eyre Architects</p>         | <p>Lancashire<br/><i>Crosses:</i> rivers<br/>Ribble and<br/>Calder<br/>confluence</p> | <p>Pedestrian<br/>bridge</p> |  <p>Firth and Kassabian, 2001; J. Eyre, 2002</p> |


| NAME                     | YEAR | AUTHOR  | LOCATION                     | FUNCTION    | IMAGE AND ARTICLE REFERENCES   |
|--------------------------|------|---|------------------------------|-------------|--|
| <b>Haneda Sky Arch</b>   |      |   | <b>Tokyo, Japan</b>          | Road bridge | <br><a href="http://homepage1.nifty.com/naomii/b/brg66e.htm">http://homepage1.nifty.com/naomii/b/brg66e.htm</a> |
| <b>Hulme Arch Bridge</b> | 1997 | Architect: Wilkinson Eyre Architects,<br>Structural engineer: Ove Arup and Partners | Manchester,<br>Great Britain | Road bridge | <br>N. Hussain and I. Wilson, 1999; Arup/Europe Project;<br>Arup/Bridges; structurae                           |


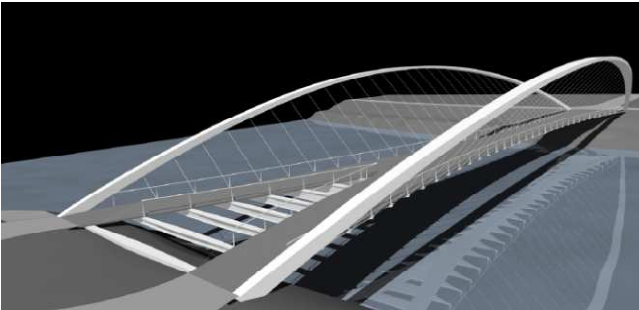
| NAME                      | YEAR | AUTHOR                            | LOCATION                           | FUNCTION               | IMAGE AND ARTICLE REFERENCES  |
|---------------------------|------|-----------------------------------|------------------------------------|------------------------|---|
| <b>James Joyce Bridge</b> | 2003 | <i>Design:</i> Santiago Calatrava | Dublin, Ireland<br>Crosses: Liffey | Road and<br>Footbridge |  <p data-bbox="1487 1134 1921 1267">M. Phillips and A. Hamilton, 2003;<br/>Architecture/Dublin bridges; Structurae;<br/>Aythor's own photographs.</p> |





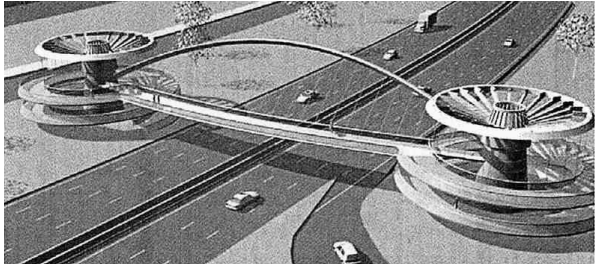

| NAME                               | YEAR      | AUTHOR   | LOCATION  | FUNCTION             | IMAGE AND ARTICLE REFERENCES  |
|------------------------------------|-----------|--|---|----------------------|---|
| <b>Juscelino Kubitschek bridge</b> | 2000-2002 | Designer Alexandre Chan<br>Structural engineer Mario Vila Verde                        | Brasilia,<br>Distrito<br>Federal, Brazil<br><i>Crosses:</i> Lake<br>Paranoá                 | Road bridge          |  <p data-bbox="1496 671 1912 699">F. Tarquis and P. Hue, 2005; <i>structurae</i></p>   |
| <b>Krickesteg</b>                  | 1994      | Structural engineering: IPP Prof.<br>Polonyi + Partner<br>Designer: Peter Freundenthal | Castrop-<br>Rauxel,<br>Recklinghausen<br>(Kreis), North<br>Rhine-<br>Westphalia,<br>Germany | Pedestrian<br>bridge |  <p data-bbox="1503 1246 1906 1273">T. Wolf, 2005, pp. 82, 138; <i>structurae</i></p> |


| NAME                        | YEAR    | AUTHOR             | LOCATION  | FUNCTION             | IMAGE AND ARTICLE REFERENCES  |
|-----------------------------|---------|--------------------|---|----------------------|---|
| <b>La Devesa Footbridge</b> | 1989-91 | Santiago Calatrava | Ripoll, Girona,<br>Catalunya,<br>Spain<br><br><i>Crosses:</i> Ter<br>river            | Pedestrian<br>bridge |  <p>D. J. Greenwold, 1999; P. Jodidio, 2003; Santiago Calatrava. The Unofficial Website/ Bridges; structurae</p> |
| <b>Landquart bridge</b>     | 1930    | Robert Maillart    | <b>Klosters,</b><br>Grisons,<br>Switzerland<br><br><i>Crosses:</i><br>Landquart river | Railroad<br>bridge   | M.Laffranchi and P.Marti, 1997; structurae  |



| NAME                       | YEAR | AUTHOR  | LOCATION  | FUNCTION             | IMAGE AND ARTICLE REFERENCES  |
|----------------------------|------|---|---|----------------------|---|
| <b>Leonardo's bridge</b>   | 2001 | <p><i>Structural engineering</i> Moelven<br/>Limtre AS<br/>Reinertsen Engineering AS</p> <p><i>Architecture</i> Selberg Arkitektkontor<br/>As</p> | <p>Ås, Akershus,<br/>Norway</p> <p>Crosses: E18<br/>Motorway<br/>[Norway]</p> | Pedestrian<br>bridge |  <p>K. Fritzen, 2003; V. Sand (s.f); BBC News, 2009; Selberg Architects; Leonardo Bridge Project Web</p> |
| <b>Lingotto footbridge</b> | 2005 | <p><i>Architectural design:</i> Hugh Dutton<br/>Associates</p> <p><i>Consulting engineer:</i><br/>FaberMaunsell</p>                               | Torino,<br>Piedmont, Italy  | Pedestrian<br>bridge | <p>J. Beideler, 2007; Bridge Design &amp; Engineering n° 42,<br/>2006; Le Moniteur des Travaux Publics et du Bâtiment:<br/>n5374, 2006; Engineering News-Record, 2006; structurae</p>       |

| NAME                      | YEAR            | AUTHOR  | LOCATION                                      | FUNCTION            | IMAGE AND ARTICLE REFERENCES   |
|---------------------------|-----------------|---|---|---------------------|--|
| <b>Logroño bridge</b>     | 2001-2003       | Javier Manterola from Carlos Fernández Casado | Logroño, Spain                                | Road and footbridge |  <p>J. Manterola, 2003; J. Manterola et al, 2005; Puentes y pasarelas de Logroño; Carlos Fernández Casado, S.L. en Realizaciones APTA &lt;web&gt;</p> |
| <b>Logroño Footbridge</b> | Still not built | Arenas & Asociados                            | Logroño, Spain.<br><i>Crosses:</i> Ebro River | Pedestrian bridge   |  <p>J. J. Arenas de Pablo et al, 2011 (4)</p>  |


| NAME                                     | YEAR             | AUTHOR   | LOCATION   | FUNCTION                     | IMAGE AND ARTICLE REFERENCES   |
|--|------------------|--|--|------------------------------|--|
| <p><b>Lohtorbrücke<br/>Heilbronn</b></p> | <p>Not built</p> | <p>Architect: Prof. Burkhardt<br/>Engineer: LAP</p>                              | <p>Heilbronn,<br/>Germany</p>                                      | <p>Pedestrian<br/>bridge</p> |  <p><a href="http://www.lap-consult.com/ingenieurbauwerke/kategorie/fuss-radwegbruecken/artikel/wettbewerb-lohtorbruecke-heilbronn.html">http://www.lap-consult.com/ingenieurbauwerke/kategorie/fuss-radwegbruecken/artikel/wettbewerb-lohtorbruecke-heilbronn.html</a></p> |
| <p><b>Lorca Footbridge</b></p>           | <p>2003</p>      | <p>J. Manterola, J. F. Revenga, M. A. Gil,<br/>A. L. Padilla, J. Muñoz-Rojas</p> | <p>Lorca, Spain.<br/><i>Crosses:</i><br/>Guadalentín<br/>river</p> | <p>Pedestrian<br/>bridge</p> |  <p>J. Manterola, 2003; J. Manterola, 2005</p>   |



| NAME                               | YEAR        | AUTHOR                            | LOCATION   | FUNCTION                   | IMAGE AND ARTICLE REFERENCES  |
|------------------------------------|-------------|-----------------------------------|--|----------------------------|---|
| <p><b>Los Niños footbridge</b></p> | <p>2005</p> | <p>Arenas y asociados</p>         | <p>Madrid, Spain<br/>Crosses: A-3</p>                | <p>Pedestrian bridge</p>   |  <p>J. J. Arenas, 2005, p343</p>   |
| <p><b>Main street bridge</b></p>   | <p>2006</p> | <p>Spiro N. Pollalis and HNTB</p> | <p>Columbus, Ohio, USA<br/>Crosses: Scioto river</p> | <p>Road and footbridge</p> |  <p><a href="http://www.hntb.com/expertise/bridges/main-street">http://www.hntb.com/expertise/bridges/main-street</a></p> |


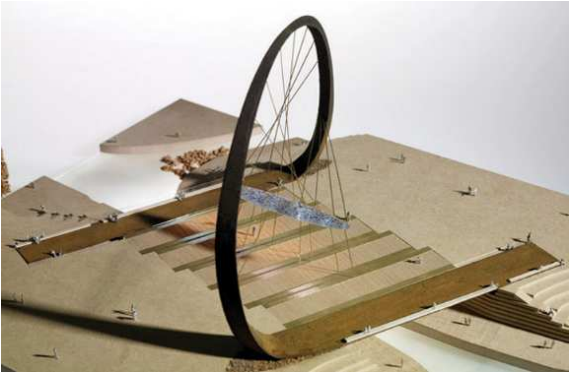
| NAME            | YEAR | AUTHOR    | LOCATION      | FUNCTION          | IMAGE AND ARTICLE REFERENCES  |
|-----------------|------|-----------|---------------|-------------------|---|
| Manrique Bridge |      | Calatrava | Murcia, Spain | Pedestrian bridge |  <p data-bbox="1646 730 1758 758">Structurae</p> |



| NAME                             | YEAR | AUTHOR             | LOCATION                                       | FUNCTION             | IMAGE AND ARTICLE REFERENCES  |
|----------------------------------|------|--------------------|--|----------------------|---|
| <b>Margaret Hunt Hill Bridge</b> | 2012 | Santiago Calatrava | Dallas, USA                                    | Road bridge          |  <p>Russel, 2012</p>   |
| <b>Merchants Bridge</b>          | 1995 | Ramboll Whitbybird | Manchester<br>Crosses:<br>Bridgewater<br>Canal | Pedestrian<br>bridge |  <p>Structurae; British Steel Web; Ramboll Whitbybird/Projects</p> |




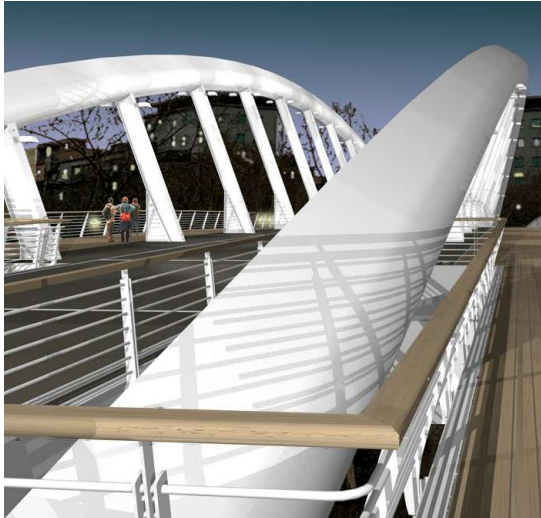
| NAME                             | YEAR        | AUTHOR   | LOCATION                       | FUNCTION                 | IMAGE AND ARTICLE REFERENCES  |
|----------------------------------|-------------|--|--------------------------------|--------------------------|---|
| <p><b>Miho Museum Bridge</b></p> | <p>1997</p> | <p><i>Architecture:</i> Pei Cobb Freed &amp; Partners Architects LLP</p> <p><i>Structural Engineering:</i></p> <p>Aoki Corporation</p> <p>Leslie E. Robertson &amp; Associates, R.L.L.P.</p> <p>Nakata &amp; Associates</p> <p>Whole Force</p> <p>Studio <a href="http://en.structurae.de/firms/d/ata/index.cfm?ID=f000110">http://en.structurae.de/firms/d/ata/index.cfm?ID=f000110</a></p> | <p>Shigaraki, Shiga, Japan</p> | <p>Pedestrian bridge</p> |  <p>L. Robertson, 2008</p> |


| NAME                                    | YEAR | AUTHOR  | LOCATION   | FUNCTION                    | IMAGE AND ARTICLE REFERENCES  |
|---|------|---|--|-----------------------------|---|
| <b>Nanning Bridge</b>                   | 2009 | Tung-Yen Lin  | <p>Nanning,<br/>Guangxi, China</p> <p><i>Crosses:</i><br/>Yongjiang<br/>River</p>  |                             |  <p>“Taking Flight”. Bridge Design &amp; Engineering. Rolling Programme</p> <p>Cheng et al, 2010</p> |
| <b>Nordsternpark Double Arch Bridge</b> | 1996 | <p><i>Structural engineering</i> : IPP Prof.<br/>Polonyi + Partner</p> <p><i>Architecture</i>: PASD Architekten<br/>Feldmeier + Wrede</p> | <p>Nordsternpark,<br/>Gelsenkirchen,<br/>North Rhine-<br/>Westphalia,<br/>Germany</p> <p>Crosses: Rhein-<br/>Herne-Kanal</p> | Cycle and pedestrian bridge |  <p>T. Wolf, 2005; structurae</p>   |


| NAME                      | YEAR            | AUTHOR                   | LOCATION  | FUNCTION              | IMAGE AND ARTICLE REFERENCES  |
|---------------------------|-----------------|--------------------------|---|-----------------------|---|
| <b>Observatory Bridge</b> | 2002            | Santiago Calatrava Valls | Liège, Liege,<br>Wallonia,<br>Belgium<br><br><i>Crosses: Albert<br/>Canal</i> | Road bridge           |  <p>Freyssinet Magazine, n209, 2000; Freyssinet cable-stayed structures, 2004, p54; Verlaine et al 2001; structuræ</p>   |
| <b>Olympic Bridge</b>     | 2007. Not built | McDowell+Benedetti       | London, UK  | Footbridge and Piazza |  <p><a href="http://www.mcdowellbenedetti.com/#/projects/265/">http://www.mcdowellbenedetti.com/#/projects/265/</a></p> |

| NAME                             | YEAR | AUTHOR           | LOCATION       | FUNCTION          | IMAGE AND ARTICLE REFERENCES   |
|----------------------------------|------|------------------|----------------|-------------------|--|
| <b>Olympic Stadium</b>           |      |                  | Athens, Greece | Roof              |  <p>structurae</p>  |
| <b>Painshill Park Footbridge</b> |      | Howard Humphreys |                | Pedestrian bridge |  <p>Littlehampton Welding Ltd. Architectural &amp; Structural Metalwork/ Bridges</p> |


| NAME                               | YEAR                              | AUTHOR   | LOCATION   | FUNCTION                             | IMAGE AND ARTICLE REFERENCES   |
|------------------------------------|-----------------------------------|--|--|--------------------------------------|--|
| <p><b>Palo Alto Footbridge</b></p> | <p>2015-07-24 still not built</p> | <p>Architects:<br/>64North Architecture, Bionic Landscape Architecture,<br/>Structural Engineer: HNTB Engineering<br/>Artist: Ned Kahn</p> | <p>San Francisco, USA.<br/><i>Crosses:</i> San Francisco Bay</p> | <p>Pedestrian and cyclist bridge</p> |  <p>Dezeen Magazine, 2015</p> |


| NAME                      | YEAR | AUTHOR       | LOCATION                                    | FUNCTION          | IMAGE AND ARTICLE REFERENCES  |
|---------------------------|------|--------------|---|-------------------|---|
| <b>Ponte della Musica</b> | 2011 | Buro Happold | Rome, Italy<br><i>Crosses</i> : River Tiber | Pedestrian bridge |  <p data-bbox="1603 922 1800 954">Liaghat et al 2011</p> |



| NAME                                    | YEAR      | AUTHOR               | LOCATION   | FUNCTION               | IMAGE AND ARTICLE REFERENCES   |
|---|-----------|----------------------|--|------------------------|--|
| <b>Port of Ondarroa Bridge</b>          | 1989-95   | Santiago Calatrava   | Ondarroa,<br>Vizcaya, Spain<br><br>Crosses:<br>Artibai | Road and<br>footbridge |  <p data-bbox="1462 914 1944 938">P. Jodidio, 2003; A. Tzonis, 2004; structurae</p> |
| <b>Port of Ouchy opening footbridge</b> | Not built | Lee Franck, Ove Arup | Port of Ouchy,<br>Switzerland                          | Footbridge             | Franck, 2011   |



| NAME  | YEAR               | AUTHOR   | LOCATION   | FUNCTION           | IMAGE AND ARTICLE REFERENCES  |
|---|--------------------|--|--|--------------------|---|
| <p><b>Reggio Emilia- A1<br/>Motorway Bridge</b></p> <p><b>Il casello<br/>autostradale</b></p> | <p><b>2006</b></p> | <p>Designer: Santiago Calatrava<br/>Checking engineering: De Miranda<br/>Associati</p> | <p>Reggio Emilia,<br/>Emilia-<br/>Romagna, Italy</p> | <p>Road Bridge</p> |  <p>M. Rando, 2010; E. Goberna, 2011<br/>Structurae; Comune di reggio Emilia; Km 129:<br/>Progetti per Reggio Emilia</p> |






| NAME   | YEAR               | AUTHOR  | LOCATION                                    | FUNCTION           | IMAGE AND ARTICLE REFERENCES  |
|--|--------------------|---|---|--------------------|---|
| <p><b>Reggio Emilia- A1 Motorway Bridge</b></p> <p><b>Ponti laterali</b></p> | <p><b>2006</b></p> | <p>Designer: Santiago Calatrava</p> <p>Checking engineering: De Miranda Associati</p> | <p>Reggio Emilia, Emilia-Romagna, Italy</p> | <p>Road Bridge</p> | <div style="text-align: center;">  <p>M. Rando, 2010;<br/>Structurae; Comune di reggio Emilia; Km 129:<br/>Progetti per Reggio Emilia</p> </div> |

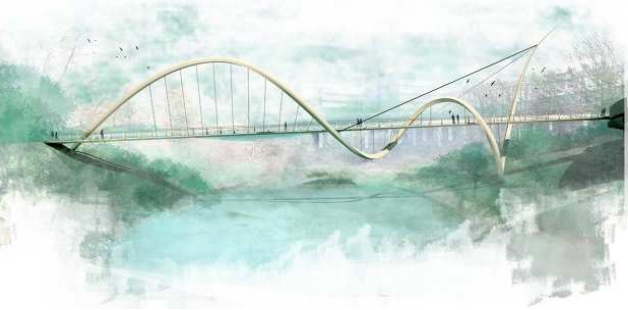

| NAME                               | YEAR        | AUTHOR  | LOCATION   | FUNCTION                 | IMAGE AND ARTICLE REFERENCES  |
|------------------------------------|-------------|---|--|--------------------------|---|
| <p><b>Ripshorst Footbridge</b></p> | <p>1997</p> | <p><i>Design:</i> Dr. Pelle, Ingenieurbüro für Bauwesen and Schlaich, Bergemann und Partner sbp gmbh</p> <p><i>Structural engineering:</i> Schlaich, Bergemann und Partner sbp gmbh</p> | <p>Oberhausen, North Rhine-Westphalia, Germany</p> | <p>Pedestrian bridge</p> |  <p>Schlaich Bergemann und Partner / Projects; structurae; Laffranchi, 1999; J.Schlaich and T. Moschner, 1999; J.Schlaich, 2005; J. Wolf, 2005</p> |



| NAME                                   | YEAR        | AUTHOR                          | LOCATION  | FUNCTION                     | IMAGE AND ARTICLE REFERENCES   |
|--|-------------|---------------------------------|---|------------------------------|--|
| <p><b>Riverside footbridge</b></p>     | <p>2008</p> | <p>Whitby Bird and Partners</p> | <p>Cambridge,<br/>Great Britain<br/><br/>Crosses: River<br/>Cam</p> | <p>Pedestrian<br/>bridge</p> |  <p>L. Debell, 2004; Ramboll Whitbybird/Press Releases;<br/>Ramboll Whitbybird/Projects; Cambridge County Council;<br/>Better Public Building Finalists; structurae</p> |
| <p><b>Rizhao Pedestrian Bridge</b></p> |             | <p>HHD_FUN Architects</p>       | <p>Rizhao, China</p>  | <p>Pedestrian<br/>bridge</p> |  <p><a href="http://www.archdaily.com/293031">http://www.archdaily.com/293031</a></p>   |

| NAME                           | YEAR      | AUTHOR                       | LOCATION   | FUNCTION    | IMAGE AND ARTICLE REFERENCES  |
|--------------------------------|-----------|------------------------------|--|-------------|---|
| <b>Roundabout crossing A-6</b> | Not built | Carlos Fernández Casado S.L. | Las Rozas,<br>Spain  | Road Bridge |  <p>L. Fernández Troyano, 2011</p>   |
| <b>Roundabout<br/>Ovotonde</b> |           | Zwarts & Jansma Architects   | Nijmegen,<br>Netherlands<br><br><i>Crosses:</i><br>highway A 325 | Road bridge |  <p><a href="http://www.zwarts.jansma.nl/page/1076/en">http://www.zwarts.jansma.nl/page/1076/en</a></p> |

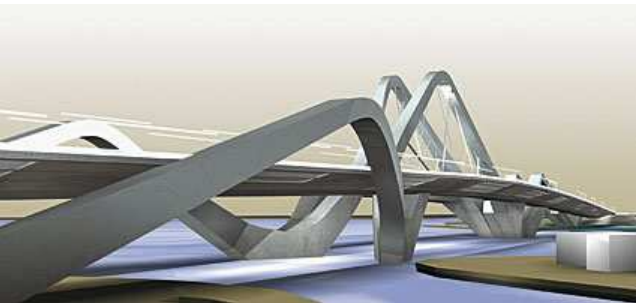

| NAME  | YEAR                                       | AUTHOR                             | LOCATION          | FUNCTION                     | IMAGE AND ARTICLE REFERENCES   |
|---|--|------------------------------------|-------------------|------------------------------|--|
| <p><b>Sackler bridge competition design</b></p> | <p>Project<br/>2005<br/><br/>Not built</p> | <p>FCK consultoría estructural</p> | <p>London, UK</p> | <p>Pedestrian<br/>bridge</p> |  <p><a href="https://fckestructural.wordpress.com/2012/06/20/bienvenidos-al-nuevo-blod-de-fck-consultoria-estructural/">https://fckestructural.wordpress.com/2012/06/20/bienvenidos-al-nuevo-blod-de-fck-consultoria-estructural/</a></p> |


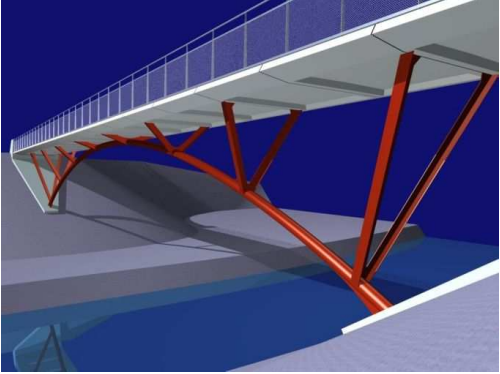
| NAME                          | YEAR      | AUTHOR  | LOCATION       | FUNCTION          | IMAGE AND ARTICLE REFERENCES  |
|-------------------------------|-----------|---|----------------|-------------------|---|
| <b>Saints Footbridge</b>      | 2012      | Architect: Moxon<br>Structural engineer:<br>Flint and Neill | StHelen's, USA | Pedestrian bridge |  <p>“Landmark bridge kicks off stadium opening”, <i>New Steel Construction</i>, February 2012</p>                    |
| <b>Salford Meadows Bridge</b> | Not built | ADAPT architects  | Manchester. UK | Pedestrian bridge |  <p><a href="http://www.adapt-architects.com/project029.php">http://www.adapt-architects.com/project029.php</a></p> |


| NAME   | YEAR              | AUTHOR                                      | LOCATION          | FUNCTION             | IMAGE AND ARTICLE REFERENCES  |
|--|-------------------|---|-------------------|----------------------|---|
| <b>Salford Meadows Bridge Competition</b>        | Not built<br>2014 |   | Manchester.<br>UK | Pedestrian<br>bridge |  <p><a href="http://www.oobe.co.uk/competitions/salford-meadows-bridge.html">http://www.oobe.co.uk/competitions/salford-meadows-bridge.html</a></p>      |
| <b>Salford Meadows Bridge Competition Winner</b> | Not built<br>2014 | Architect: Tonkin Liu<br>Engineer: Ove Arup | Manchester.<br>UK | Pedestrian<br>bridge |  <p><a href="http://www.ribacompetitions.com/salfordmeadowsbridge/winner.html">http://www.ribacompetitions.com/salfordmeadowsbridge/winner.html</a></p> |



| NAME   | YEAR        | AUTHOR                        | LOCATION  | FUNCTION           | IMAGE AND ARTICLE REFERENCES  |
|--|-------------|-------------------------------|---|--------------------|---|
| <p><b>Sanchinarro shopping mall access bridges</b></p> | <p>2003</p> | <p>J. J. Arenas</p>           | <p>Madrid, Spain</p>  | <p>Road bridge</p> |  <p>J. J. Arenas, 2005</p>   |
| <p><b>Schwandbach Bridge</b></p>                       | <p>1933</p> | <p><b>Robert Maillart</b></p> | <p>Schwandbach Creek,<br/>Switzerland</p> <p>Crosses:<br/>Schwandbach Creek</p> | <p>Road bridge</p> |  <p>D. P. Billington, 1979, pp174-182; D. P. Billington, 1997;<br/>M.Laffranchi and P.Marti, 1997; structurae</p> |


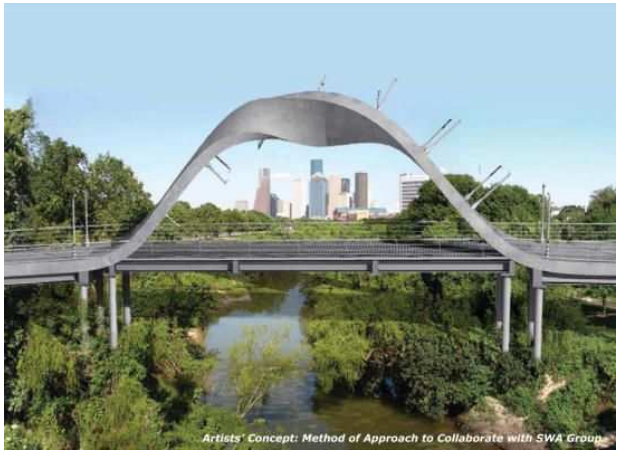


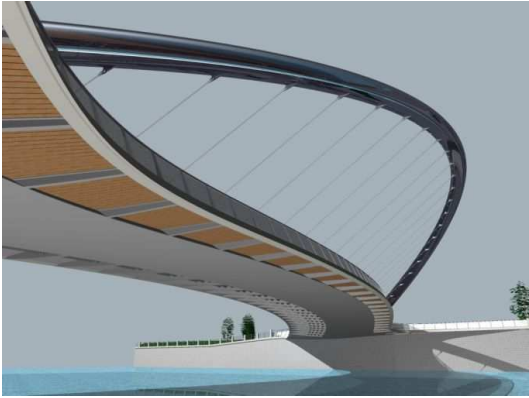
| NAME                              | YEAR                               | AUTHOR  | LOCATION                                     | FUNCTION           | IMAGE AND ARTICLE REFERENCES   |
|-----------------------------------|------------------------------------|---|--|--------------------|--|
| <p><b>Sheikh Zayed Bridge</b></p> | <p>Project 2004<br/>Built 2010</p> | <p><i>Design:</i> High Point Rendel<br/><br/>Architecture: Zaha Hadid<br/><i>Construction Engineering:</i> Buckland &amp; Tayler Ltd.</p> | <p>Between Abu Dhabi island and mainland</p> | <p>Road bridge</p> |  <p>H. Russel, 2006; structurae;</p>  |
| <p><b>Sixth Street Bridge</b></p> | <p>2012 Project</p>                | <p>HNTB with Michael Maltzan Architecture, AC Martin, and Hargreaves Associates</p>   | <p>Los Angeles</p>                           |                    |  <p><a href="http://www.archpaper.com/news/articles.asp?id=6262">http://www.archpaper.com/news/articles.asp?id=6262</a><br/>09.13.2012</p> |


| NAME                                   | YEAR | AUTHOR                      | LOCATION          | FUNCTION          | IMAGE AND ARTICLE REFERENCES  |
|--|------|-----------------------------|-------------------|-------------------|---|
| St James's Garden cycle and footbridge | 1995 | Ramboll Whitbybird          | Limehouse, London | Pedestrian bridge |  <p data-bbox="1543 879 1861 906">Ramboll Whitbybird/Projects</p>                  |
| Stress ribbon supported by arch STUDY  |      | Strasky, Husty and Partners |                   |                   |  <p data-bbox="1480 1342 1928 1369">Strasky, Husty and Partners, Ltd/ Studies</p> |



| NAME                                    | YEAR | AUTHOR                               | LOCATION   | FUNCTION             | IMAGE AND ARTICLE REFERENCES  |
|---|------|--------------------------------------|--|----------------------|---|
| <b>Stuttgart<br/>Cannstatter Straße</b> | 1977 | Jörg Schlaich<br>Leonhardt und Andrä | Bundesgartenschau, Stuttgart,<br>Germany<br><br><i>Crosses:</i><br>Cannstatter<br>street | Pedestrian<br>bridge | A. Holgate, 1997  |
| <b>Te Rewa Rewa</b>                     | 2010 | Novare Design                        | New Plymouth,<br>New Zealand<br><br><i>Crosses:</i><br>Waiwhakaiho<br>River              | Pedestrian<br>bridge |  <p data-bbox="1608 1321 1794 1353">Muluqueen, 2011</p> |

| NAME                      | YEAR            | AUTHOR             | LOCATION   | FUNCTION          | IMAGE AND ARTICLE REFERENCES  |
|---------------------------|-----------------|--------------------|--|-------------------|---|
| <b>Tier Garten Bridge</b> | 2000            | Stefan Polónyi     | <b>Dessau-Rosslau,</b><br>Saxony-Anhalt,<br>Germany<br><br><i>Crosses: Mulde River</i> | Pedestrian bridge |  <p>Structurae</p>                             |
| <b>Tirón Footbridge</b>   | Still not built | Arenas & Asociados | Haro, Spain  | Pedestrian bridge |  <p>J. J. Arenas de Pablo et al, 2011 (3)</p> |

| NAME                                      | YEAR                         | AUTHOR                     | LOCATION                    | FUNCTION  | IMAGE AND ARTICLE REFERENCES  |
|---|------------------------------|----------------------------|-----------------------------|---|---|
| <b>Tiszavirág híd<br/>(Mayfly Bridge)</b> | 2011                         |                            | <b>Szolnok,<br/>Hungary</b> | Pedestrian<br>bridge                              |  <p data-bbox="1509 820 1895 850"><a href="http://szolnokigyalogoshid.hu/blog/">http://szolnokigyalogoshid.hu/blog/</a></p>  |
| <b>Tolerance Bridge</b>                   | Project<br>2008<br>Not built | Artitsts: Elmgreen&Dragset | Houston, USA                | Sculpture/<br>Cycling and<br>Pedestrian<br>Bridge |  <p data-bbox="1599 1318 2002 1337"><small>Artitsts' Concept: Method of Approach to Collaborate with SWA Group</small></p> <p data-bbox="1637 1358 1767 1386">Brinn, 2011</p> |

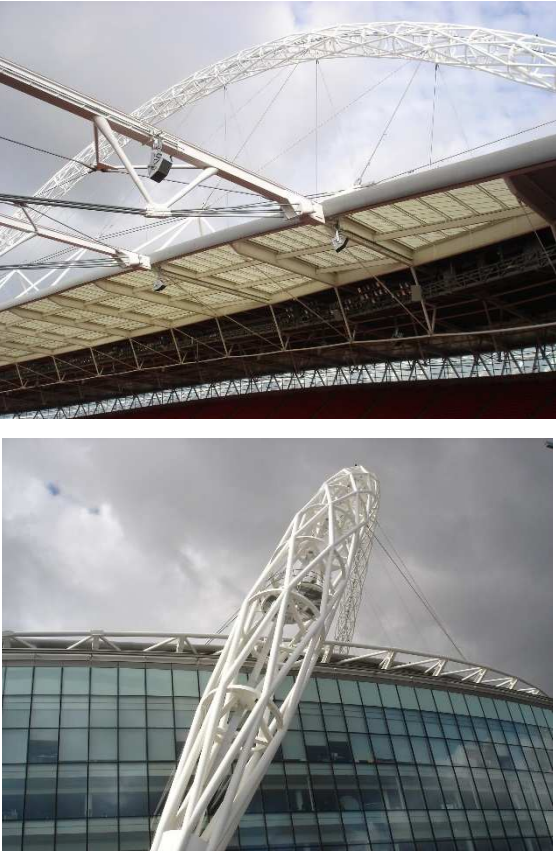
| NAME  | YEAR                            | AUTHOR                             | LOCATION  | FUNCTION                 | IMAGE AND ARTICLE REFERENCES  |
|---|---------------------------------|------------------------------------|---|--------------------------|---|
| <p><b>Tyrs' bridge across the Becva</b></p> | <p>2004<br/>Still not built</p> | <p>Strasky, Husty and Partners</p> | <p>Prerov, Czech Republic<br/><i>Crosses:</i> Becva river</p> | <p>Pedestrian bridge</p> |  <p>J. Strasky, 2004; Strasky, Husty and Partners, Ltd/ Competitions</p> |



| NAME                         | YEAR        | AUTHOR                             | LOCATION   | FUNCTION                     | IMAGE AND ARTICLE REFERENCES  |
|------------------------------|-------------|------------------------------------|--|------------------------------|---|
| <p><b>TZU Footbridge</b></p> | <p>1997</p> | <p>IPP Prof. Polonyi + Partner</p> | <p><b>Oberhausen,</b><br/>North Rhine-<br/>Westphalia,<br/>Germany</p> | <p>Pedestrian<br/>bridge</p> |  <p>structurae</p> |


| NAME  | YEAR        | AUTHOR   | LOCATION   | FUNCTION               | IMAGE AND ARTICLE REFERENCES   |
|---|-------------|--|--|------------------------|--|
| <p><b>Viaduct Crossing the Wirrbach River as Part of the Federal Road B88</b></p> | <p>2002</p> | <p>Leonhardt, Andrä und Partner (LAP),<br/>office Erfurt</p> | <p>Geschwenda,<br/>Thuringia,<br/>Germany.<br/><br/><i>Crosses:</i><br/>Wirrbach River</p> | <p>Road bridge</p>     |  <p><a href="http://www.lap-consult.com/projekt.php?sp=00015&amp;kat= 032">http://www.lap-consult.com/projekt.php?sp=00015&amp;kat= 032</a></p> |
| <p><b>Viaduct over Borough High Street at London Bridge</b></p>                   | <p>2011</p> | <p>Architect: Network Rail<br/><br/>Engineer: Atkins</p>     |  | <p>Railway viaduct</p> |    |





| NAME                  | YEAR | AUTHOR                        | LOCATION             | FUNCTION             | IMAGE AND ARTICLE REFERENCES   |
|-----------------------|------|-------------------------------|----------------------|----------------------|--|
| <b>Weinbergbrücke</b> | 2014 | Schlaich, Bergerman & Partner | Rathenow,<br>Germany | Pedestrian<br>bridge |  <p data-bbox="1458 727 1951 754">Schlaich, Bergermann und Partner web, 2014</p> <p data-bbox="1384 762 2024 858"> <a href="http://www.sbp.de/de/build/show/2718-Weinbergbr%C3%BCcke_Bundesgartenschau_2015_Havel_region">http://www.sbp.de/de/build/show/2718-Weinbergbr%C3%BCcke Bundesgartenschau 2015 Havel region</a> </p> |

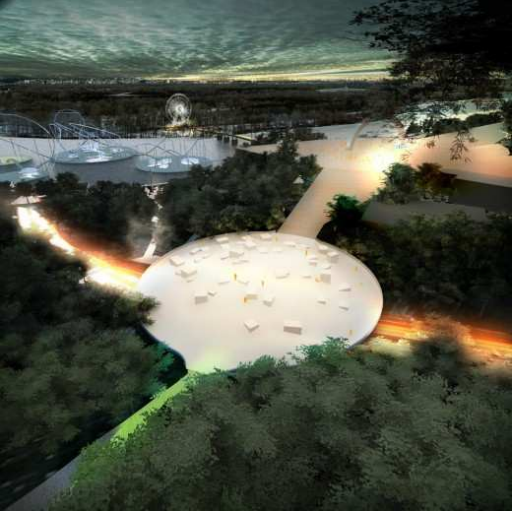
| NAME                   | YEAR | AUTHOR   | LOCATION   | FUNCTION | IMAGE AND ARTICLE REFERENCES  |
|------------------------|------|--|------------|----------|---|
| <b>Wembley Stadium</b> | 2007 | Architect: Foster+Partners<br>Structural Engineer: Mott Stadium Consortium | London, UK | Roof     |  <p data-bbox="1417 1265 1989 1326"> <a href="http://www.fosterandpartners.com/projects/wembley-stadium/">http://www.fosterandpartners.com/projects/wembley-stadium/</a> </p> |


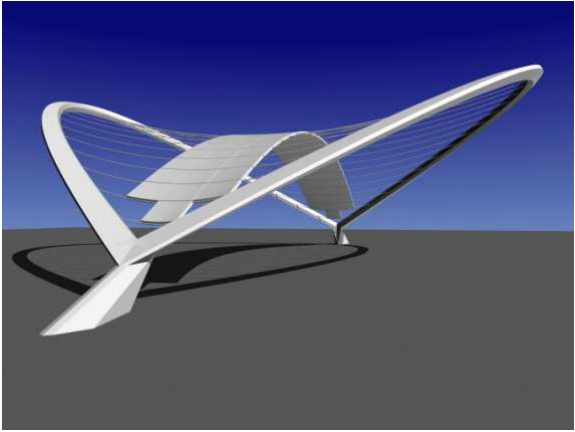
| NAME                            | YEAR | AUTHOR   | LOCATION  | FUNCTION          | IMAGE AND ARTICLE REFERENCES  |
|---------------------------------|------|--|---|-------------------|---|
| <b>Yarra Pedestrian Bridge</b>  | 2009 | Structural Engineer: Brown Consultants<br><br>Architect Grimshaw | Melbourne, Australi.<br><br><i>Crosses:</i> Yarra River | Pedestrian bridge |  <p data-bbox="1608 743 1794 770">Grimshaw, 2009</p>   |
| <b>York Milenium footbridge</b> | 2001 | Whitby Bird and partners   | York, UK<br><br><i>Crosses:</i> Ousa river              | Pedestrian bridge |  <p data-bbox="1384 1070 2024 1129">D. Mairs, 2001; Ramboll Whitbybird/ Projects; Lusas/Case; structurae; I. Firth, 2002; M. HelZel and I. Taylor, 2004</p> |

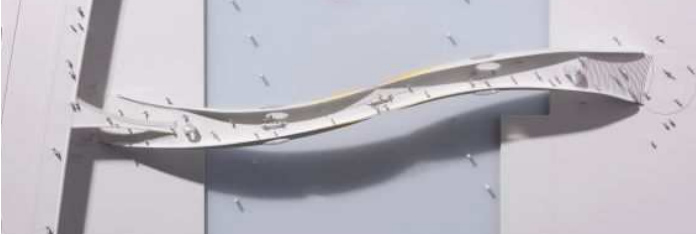

| NAME                     | YEAR | AUTHOR          | LOCATION                  | FUNCTION             | IMAGE AND ARTICLE REFERENCES   |
|--------------------------|------|-----------------|---------------------------|----------------------|--|
| <b>Ziggenbach bridge</b> | 1924 | Robert Maillart | Innerthal,<br>Switzerland | Pedestrian<br>bridge |  <p data-bbox="1413 815 1995 948">Photographer: Yoshito Isono,<br/>&lt;<a href="http://en.structurae.de/photos/index.cfm?JS=91933">http://en.structurae.de/photos/index.cfm?JS=91933</a>&gt;<br/>M.Laffranchi and P.Marti, 1997; structurae</p> |

### 3. EXAMPLES OF SPATIAL SHELL ARCHES



| NAME   | YEAR                      | AUTHOR  | LOCATION                        | FUNCTION               | IMAGE AND ARTICLE REFERENCES  |
|--|---------------------------|---|---------------------------------|------------------------|---|
| <p><b>Amphitheatre roof of a summer cinema, Karvina</b></p>                          | <p>Not built</p>          | <p>Strasky, Husty and Partners</p>  | <p>Karvina, Czech Republic</p>  | <p>Roof</p>            |  <p>Strasky, Husty and Partners, Ltd/Projects</p>  |
| <p><b>ARC International Wildlife Crossing Infrastructure Design: 'Landshape'</b></p> | <p>2007<br/>Not built</p> | <p>Zwarts &amp; Jansma Architects,<br/>OKRA Landscape Architects,<br/>IV-Infra and Sjef Jansen Plan ecology</p> | <p>Vail Pass, Colorado, USA</p> | <p>Animal Crossing</p> |  <p><a href="http://www.zwarts.jansma.nl/page/2863/en">http://www.zwarts.jansma.nl/page/2863/en</a></p> |



| NAME               | YEAR                           | AUTHOR | LOCATION      | FUNCTION             | IMAGE AND ARTICLE REFERENCES   |
|--------------------|--------------------------------|--------|---------------|----------------------|--|
| <b>Kiev Bridge</b> | Not built<br>(project<br>2011) | Maxwan | Kiev, Ukraine | Pedestrian<br>bridge |  <p data-bbox="1366 922 1912 954"><a href="http://www.maxwan.com/selected-projects/bridge/">http://www.maxwan.com/selected-projects/bridge/</a></p> |

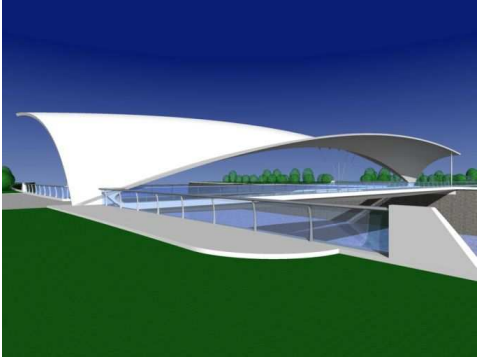

| NAME                       | YEAR      | AUTHOR                      | LOCATION         | FUNCTION          | IMAGE AND ARTICLE REFERENCES   |
|----------------------------|-----------|-----------------------------|------------------|-------------------|--|
| <b>Bridge of Peace</b>     | 2009-2010 | Michele De Lucchi           | Tbilisi, Georgia | Pedestrian bridge |  <p data-bbox="1301 831 1973 858"><a href="http://www.amdl.it/infrastructurepublic?p=the-bridge-of-peace">http://www.amdl.it/infrastructurepublic?p=the-bridge-of-peace</a></p> |
| <b>Congress hall study</b> | Not built | Strasky, Husty and Partners |                  | Roof              |  <p data-bbox="1402 1350 1872 1377">Strasky, Husty and Partners, Ltd/ Buildings</p>  |


| NAME                        | YEAR | AUTHOR   | LOCATION  | FUNCTION          | IMAGE AND ARTICLE REFERENCES  |
|-----------------------------|------|--|---|-------------------|---|
| <b>Leamouth footbridge</b>  | 2004 | <b>Design:</b> Strasky, Husty and Partners and Jan Kaplicky<br><br><b>Structural Engineering:</b><br>Strasky, Husty and Partners | Leamouth, London, UK<br><br>Crosses: River Lea where it joins with river Thames | Pedestrian bridge |  <p data-bbox="1384 646 1892 678">Strasky, Husty and Partners, Ltd/ Competitions</p>   |
| <b>Maasboulevard bridge</b> |      | Zwarts & Jansma Architects   |   | Pedestrian bridge |  <p data-bbox="1406 986 1870 1018"><a href="http://www.zwarts.jansma.nl/page/2382/en">http://www.zwarts.jansma.nl/page/2382/en</a></p> |



| NAME   | YEAR        | AUTHOR  | LOCATION  | FUNCTION                                      | IMAGE AND ARTICLE REFERENCES   |
|--|-------------|---|---|---|--|
| <p><b>Matadero and Invernadero Bridges</b></p> | <p>2011</p> | <p><b>Engineering:</b><br/>FHECOR Ingenieros Consultores</p> <p><b>Design:</b> MRío Arquitectos</p> | <p>Madrid, Spain.</p> <p><i>Crosses:</i><br/>Manzanares River</p> | <p>Pedestrian bridge</p>                      |  <p>H. Corres et al, 2011</p> |
| <p><b>Mixed-use Bridge for Amsterdam</b></p>   |             | <p>Architect: Laurent Saint-Val</p>   | <p>Amsterdam</p>  | <p>Cycle and pedestrian, habitable bridge</p> |  <p>Evolo, 2012</p>          |

| NAME                                      | YEAR                    | AUTHOR  | LOCATION              | FUNCTION          | IMAGE AND ARTICLE REFERENCES  |
|---|-------------------------|---|-----------------------|-------------------|---|
| <b>Pedestrian bridge in St. Helier</b>    | Project 2003. Not built | Strasky, Husty and Partners; design with Cezary Bednarski | St Helier, Jersey, UK | Pedestrian bridge |  <p data-bbox="1379 676 1895 756">Strasky, Husty and Partners, Ltd/Projects, 2003;<br/>I. Terzjiski and L. Odstrcilik, 2007</p>  |
| <b>Salford Meadows Bridge Competition</b> | Not built 2014          | InHold  | Manchester. UK        | Pedestrian bridge |  <p data-bbox="1384 1227 1890 1291">Sarmiento-Comesías et al, 2014;<br/><a href="http://inholdesign.wix.com/inholdbridgedesign">http://inholdesign.wix.com/inholdbridgedesign</a></p> |

| NAME                           | YEAR | AUTHOR                                      | LOCATION | FUNCTION          | IMAGE AND ARTICLE REFERENCES   |
|--------------------------------|------|---|----------|-------------------|--|
| Shell bridge study             |      | Strasky, Husty and Partners                 |          | Pedestrian bridge |  <p data-bbox="1415 767 1859 831">I. Terzijski and L. Odstreilík, 2007;<br/>Strasky, Husty and Partners, Ltd/ Studies</p> |
| Shell bridge study for IDABWIC | 2013 | Sarmiento-Comesías, Ruiz-Teran and Aparicio |          | Pedestrian bridge |  <p data-bbox="1464 1369 1809 1401">Sarmiento-Comesías et al, 2013</p>   |

| NAME                        | YEAR           | AUTHOR                | LOCATION   | FUNCTION           | IMAGE AND ARTICLE REFERENCES  |
|-----------------------------|----------------|-----------------------|--|--------------------|---|
| <p><b>Ponte Musmeci</b></p> | <p>1969-72</p> | <p>Sergio Musmeci</p> | <p>Potenza, Italy<br/>Crosses: Basento river</p> | <p>Road bridge</p> |  <p>Ponzo et al, 2013; Nicoletti, 1999</p> |



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**N2. ANNEXES RELATIVE TO  
CHAPTER IV**



## **N2.1. ARCH/STRUTS AND DECK/STRUTS JOINT STUDY**



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## **1. INTRODUCTION**

Firstly, in section 2, the deck and arch of an Inferior Deck Arch Bridge With Imposed Curvature (IDABWIC) have been analysed separately as balcony beams in order to see the effects of the different loads introduced by the hangers.

On a next step, in section 2.6, an IDABWIC model has been analysed to see which forces are more interesting to introduce in arch and deck in order to diminish the internal forces.

Finally, in section 3, a series of IDABWIC models with different cross-sections and different hanger/arch and hanger/deck joint conditions have been analysed and compared in order to establish which is the best hanger/arch and hanger/ deck configuration in order to diminish the arch internal forces.

## **2. UNCOUPLED STUDY OF AN ARCH AND DECK OF AN INFERIOR DECK ARCH BRIDGE WITH IMPOSED CURVATURE**

The objective of this study is to analyse the effect of the torques and bending moments introduced by hangers on arch and deck in order to determine which hanger joint configuration is most interesting to employ in order to reduce bending and torsional moments on arch and deck.

Finite element (FE) frame models of the arch and deck fixed at abutments have been studied separately.

### **2.1 Loading**

Hangers introduce single concentrated loads on arch and deck. We have studied the effect of the introduction of hanger bending moments both as a single moment at span center to exemplify the effect of single loads and as uniform loads to understand the effect on the whole arch or deck.

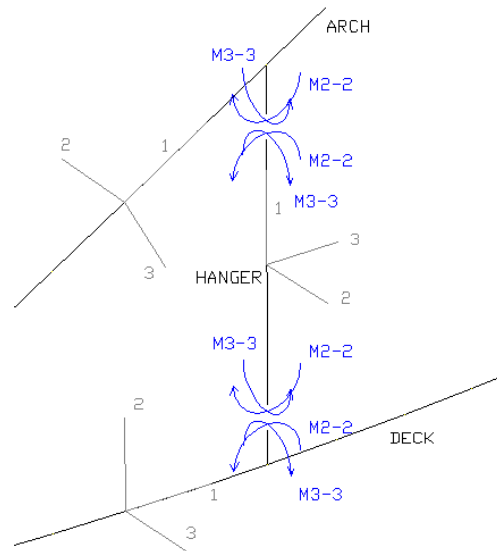
### **2.2 Axis definition**

The hanger local axis employed on the model are all parallel to each other and the global axis, ie: only perpendicular to the bridge plan alignment at the span center. This means a significant change if the local axis of hangers were perpendicular to the plan alignment, ie: joints and hangers cross-sections would be not parallel to each other, but each of them oriented perpendicularly to the plan axis curve.

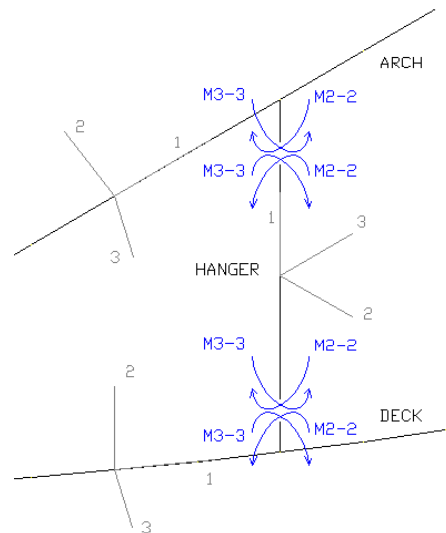
When oriented as the latter axis described, transverse positive hanger bending moments (M3-3, Figure 2-1) on hangers produce negative torques on arch and positive ones on deck. Longitudinal positive hanger moments (M2-2 Figure 2-1) produce positive bending moments loading out of the deck arch plane, ie: with their axis contained on the arch plane and outward to the curve, and negative ones on the arch.

However, when oriented parallel to each other, both bending moments introduce a mixture of bending moments and torques which have non-negligible effects (Figure 2-2). This has been studied for equal torque and moments values.





**Figure 2-1: Positive moments according to hanger local axis (perpendicular and tangencial to plan alignment) acting on hangers and transmitted to arch and deck**



**Figure 2-2: Positive moments according to hanger local axis (parallel to global axis) acting on hangers and transmitted to arch and deck**

We should note that for the hanger/arch joints the definition of their orientation is much more complex.

When oriented all of them parallel to the global axis or only perpendicular to the plan alignment, both bending moments produce on the arch a torque and bending moments in and out of the arch plane. To produce a pure torque and bending, the joints and hanger orientation should be perpendicular to the curve alignment of the plane that contains the arch (or its approximation). The hanger cross section employed at arch joints would be then different than the one employed at deck joints. On a real arch, this would complicate calculations and construction unnecessarily.

We have introduced both, loading according to local and global axis, on arch and deck. Always considering the positive loading value according to the hanger and deck axis, ie: positive

torques cause the arch or deck to turn down and outwards the curve, positive longitudinal bending causes compressions on the upper fiber and positive transverse bending causes compressions on the inner fiber.

### 2.3 Analysis of the deck as balcony beam

In the present section a balcony beam is analysed without considering the action of hangers.

In the present section, the balcony-beam is studied under different loads.

#### 2.3.1 Mechanical properties and frame model definition:

*Reference deck*

$$A = 0,1431\text{m}^2$$

$$J = 0,0615\text{m}^4$$

$$I_2 = 0,2517\text{m}^4$$

$$I_3 = 0,0196\text{m}^4$$

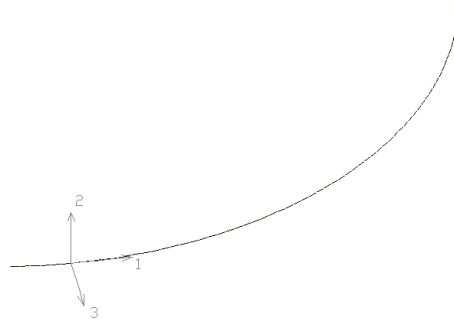


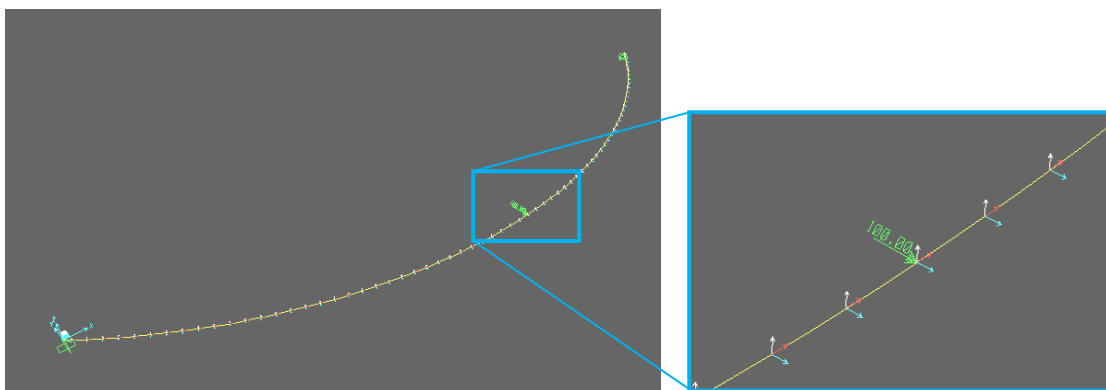
Figure 2-3: Deck local axis definition (1 tangent to deck and 1-2 plane on the deck horizontal plane)

#### 2.3.2 Single moment loads at span center

*Load is named after the load which would transmit the hanger if it is not released (ie: name corresponds to the hangers' non-released moments)*

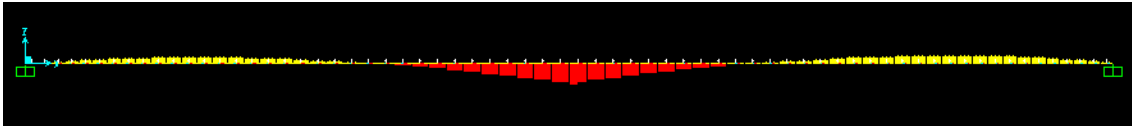
##### 2.3.2.1 Load definition

$$M_2 = 100\text{kN}\cdot\text{m}$$



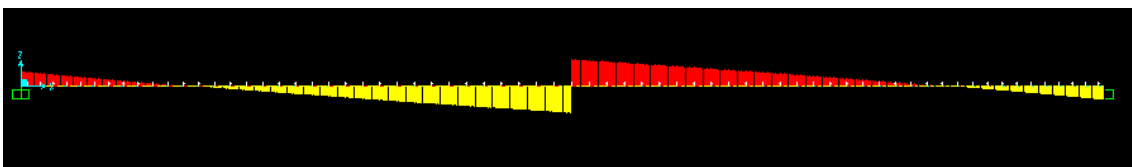
### 2.3.2.2 Internal forces on deck

Torsion: very low (aprox 10% of the one caused by M3)



M2-2=0

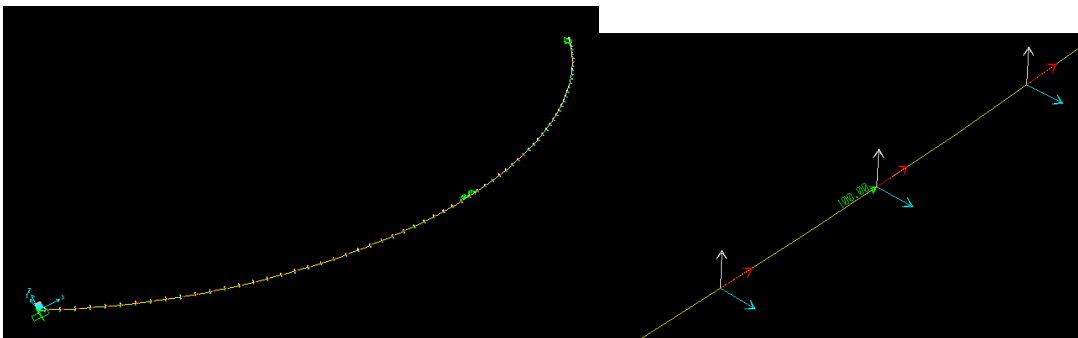
M3-3



N=0

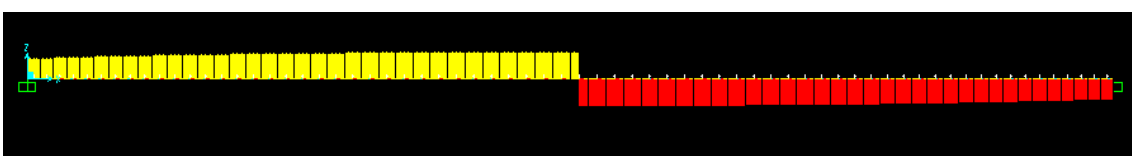
### 2.3.2.3 Load definition

M3=100kN·m



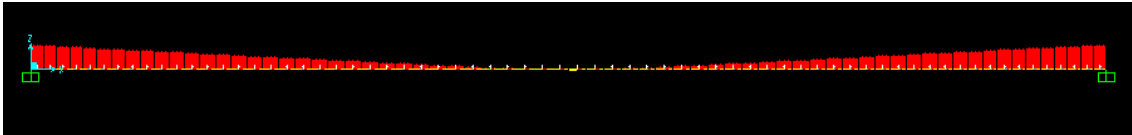
### 2.3.2.4 Internal forces on deck

Torsion



M2-2=0

M3-3



N=0

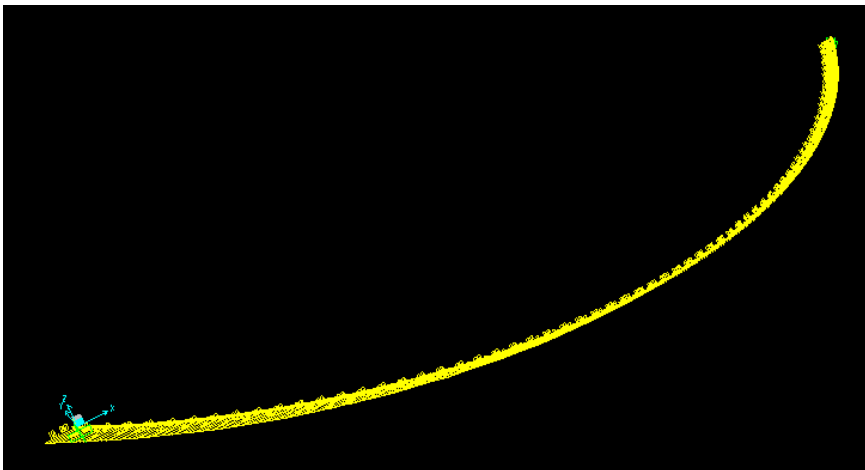
### 2.3.3 Uniformly distributed bending moments and torques

*We will compare global with local definition, in order to see if not considering hanger axis perpendicular to the plan alignment introduces an important error or it is negligible.*

#### 2.3.3.1 Defined on global axis

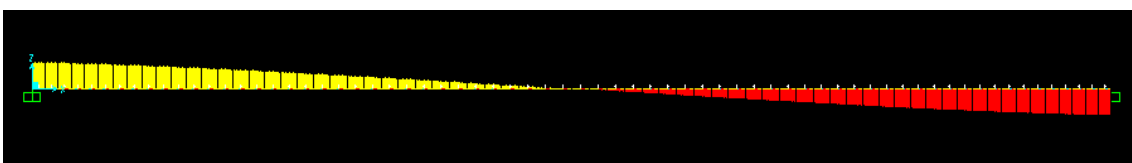
##### 2.3.3.1.1 Load definition

$$m3G=mx=10\text{kN}\cdot\text{m}/\text{m}$$



##### 2.3.3.1.2 Internal forces on deck

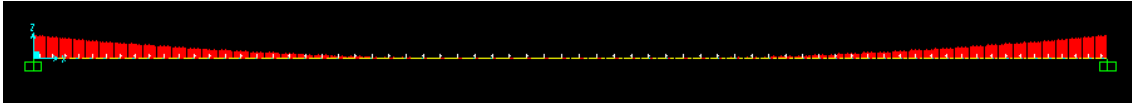
Torsion (max=417 kN·m)



This torsion distribution is equivalent to the one produced by a vertical loading on the deck.

$$M2-2=0$$

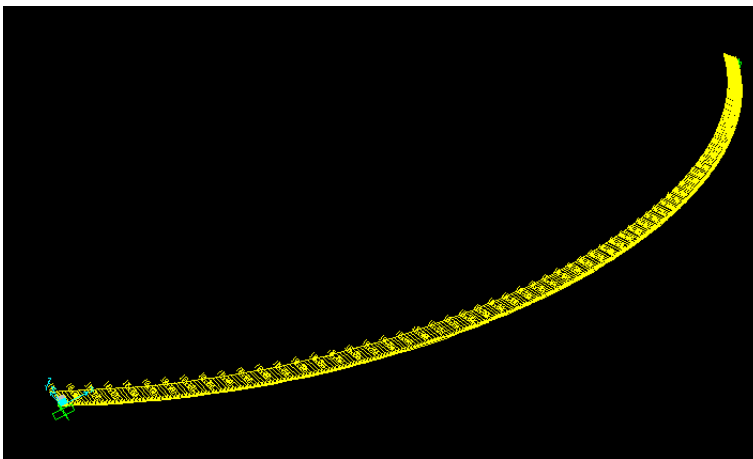
$$M3-3 \text{ (max= -362kN}\cdot\text{m)}$$



$$N=0$$

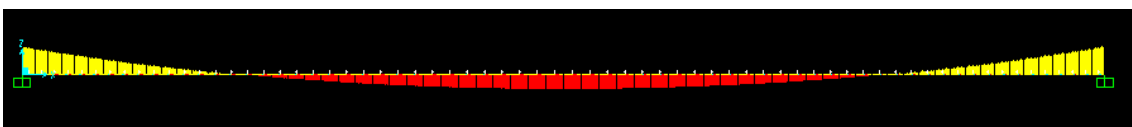
### 2.3.3.1.3 Load definition

$$m2G=-my=10\text{kN}\cdot\text{m/m}$$



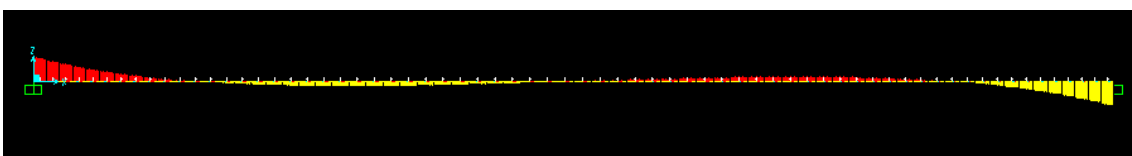
### 2.3.3.1.4 Internal forces on deck

$$\text{Torsion (max=129 kN}\cdot\text{m)}$$



$$M2-2=0$$

$$M3-3 \text{ (max= 56kN}\cdot\text{m)}$$



$$N=0$$

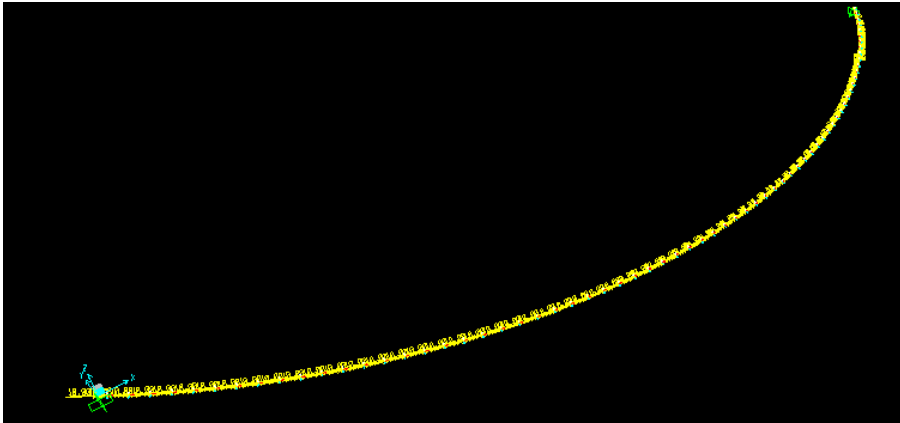
Defined on local axis

### 2.3.3.1.5 Load definition

These are the loads which would be introduced by the fixed hangers if the joints were orientated perpendicularly to the plan alignment.

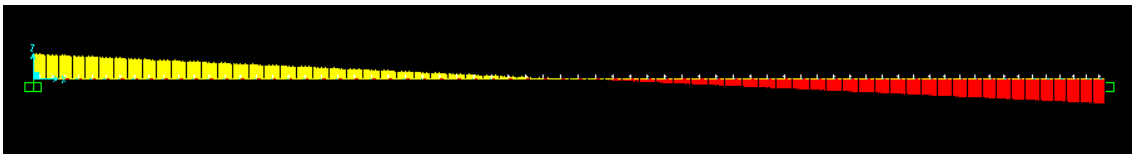
$$T_q = m_3 L = m_1 = 10 \text{ kN} \cdot \text{m/m}$$

We are exclusively introducing a torque



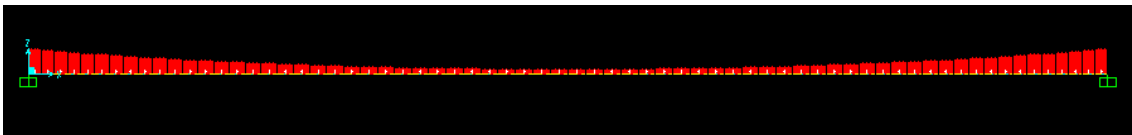
### 2.3.3.1.6 Internal forces on deck

Torsion (max= 469kN·m)



$$M_2 = 0$$

M3-3 (max= -230kN·m)

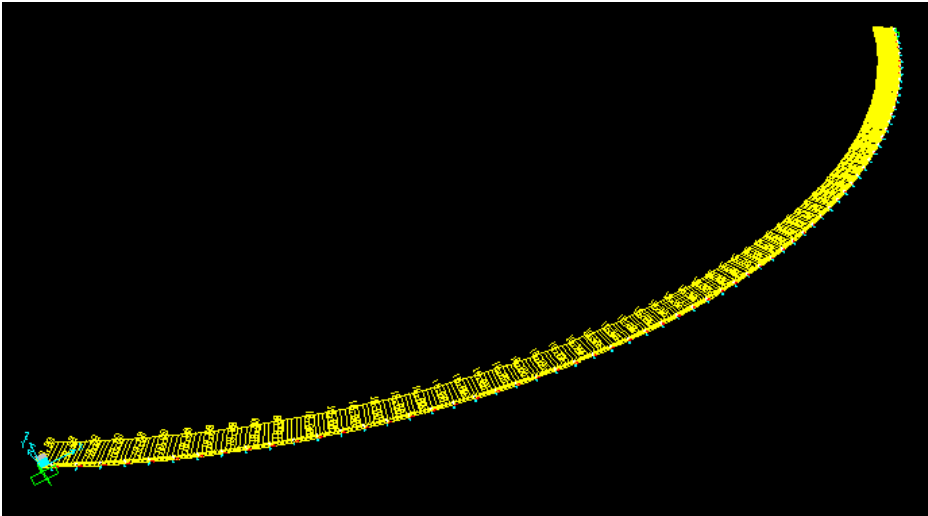


$$N = 0$$

### 2.3.3.1.7 Load definition

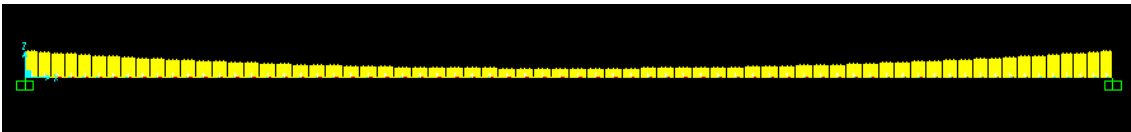
$$M_q = m_2 L = m_3 = 10 \text{ kN} \cdot \text{m/m}$$

We are exclusively introducing a bending moment



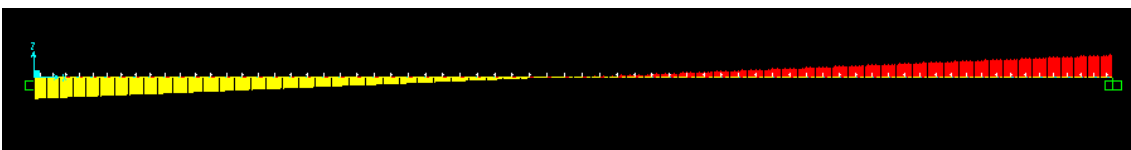
### 2.3.3.1.8 Internal forces on deck

Torsion (max=0,6 kN·m)



$$M_2 = 0$$

M3-3 (max=1kN·m) very low



$$N = 0$$

## 2.4 Analysis of the arch as balcony beam

### 2.4.1 Mechanical properties and frame model definition

Reference arch

$$J = 0,0215\text{m}^4$$

$$I_3 = 0,0108\text{m}^4$$

$$A = 0,0914\text{m}^2$$

$$I_2 = 0,0108\text{m}^4$$

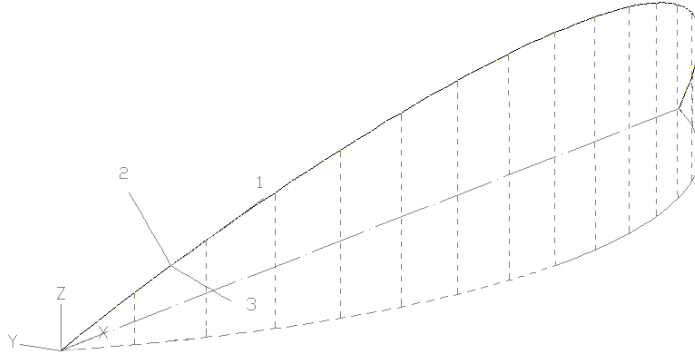


Figure 2-4: Arch local axis definition (1 tangent to the arch and 1-2 plane on the arch plane<sup>1</sup>)

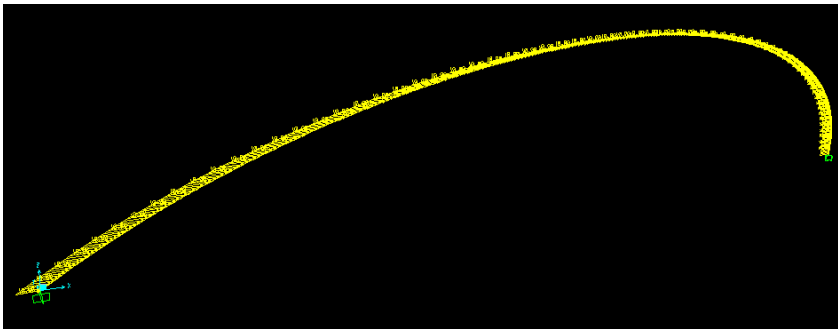
### 2.4.2 Uniformly distributed bending moments and torques

We will compare global with local definition, in order to see if not considering hanger axis perpendicular to the plan alignment introduces an important error or it is negligible.

#### 2.4.2.1 Defined on global axis

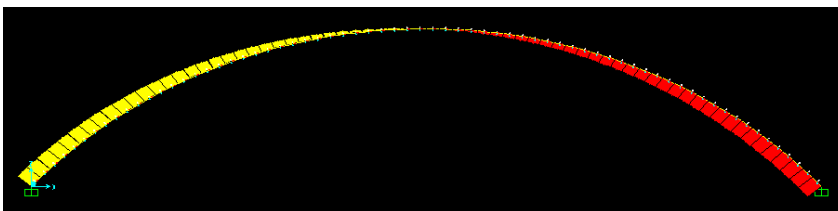
##### 2.4.2.1.1 Load definition

$$-m_3G = m_x = 10\text{kN} \cdot \text{m/m}$$



##### 2.4.2.1.2 Internal forces on deck

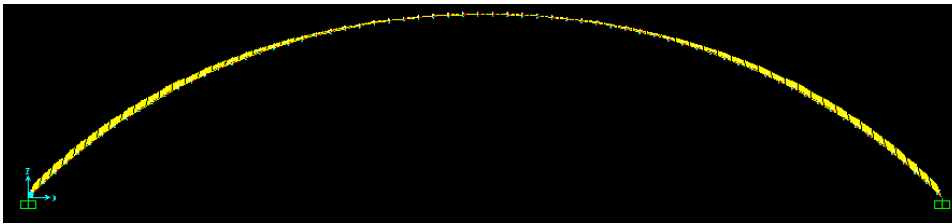
Torsion (max=326 kN·m)



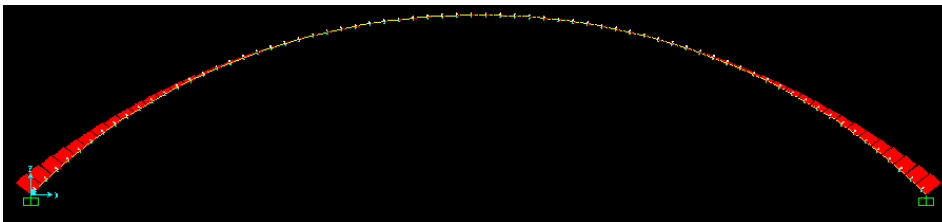
<sup>1</sup> If it were contained on a plane, which is not the case



M2-2(max=489)



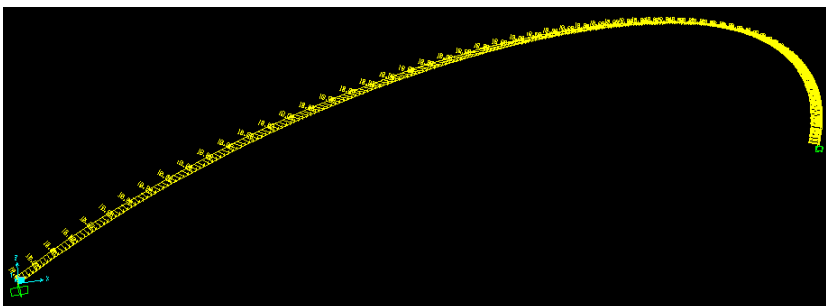
M3-3 (max= -112kN·m)



N=0

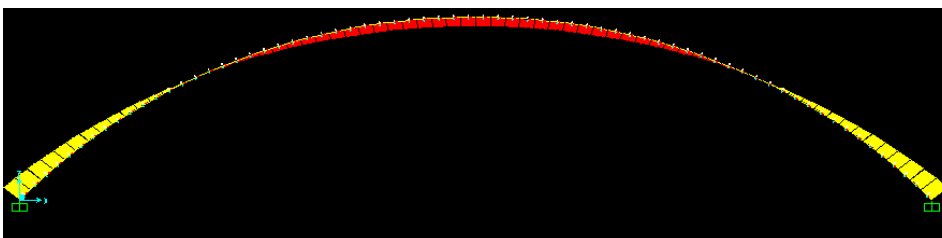
#### 2.4.2.1.3 Load definition

$$-m2G=-my=10\text{kN}\cdot\text{m}/\text{m}^2$$



#### 2.4.2.1.4 Internal forces on deck

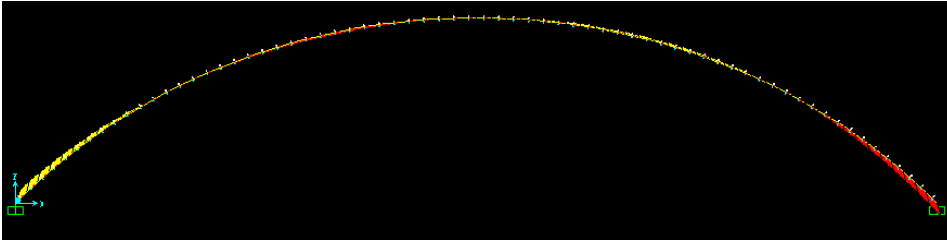
Torsion (max= 122kN·m)



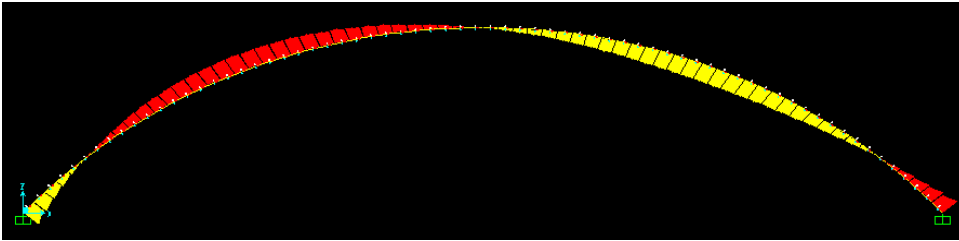
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<sup>2</sup> It is introduced as negative on global axis because its projection is positive on local ones

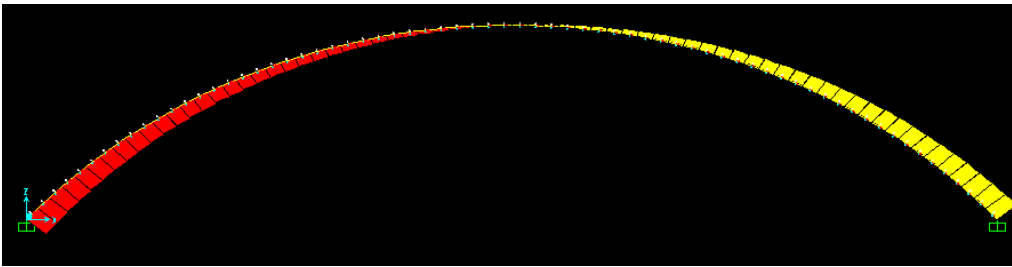
M2-2(max=84)



M3-3 (max= 12kN·m)



N (max=6)



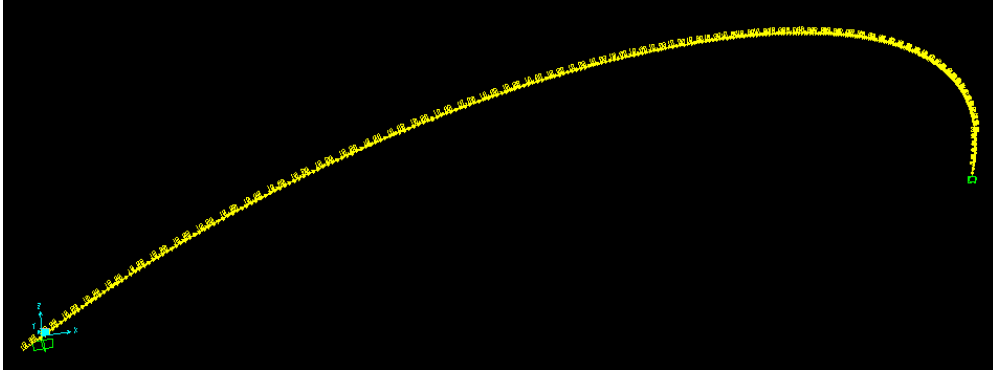
#### 2.4.2.2 Defined on local axis

##### 2.4.2.2.1 Load definition

These are the loads which would be introduced by the fixed hangers if the joints were orientated perpendicularly to the plan alignment.

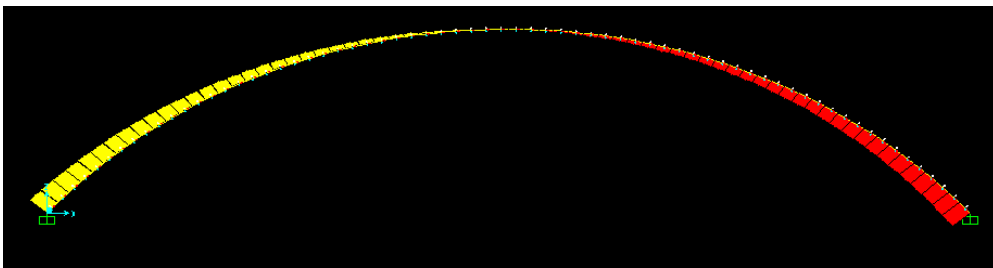
$$T_q = -m3L = m1 = 10 \text{ kN} \cdot \text{m/m}$$

We are exclusively introducing a torque

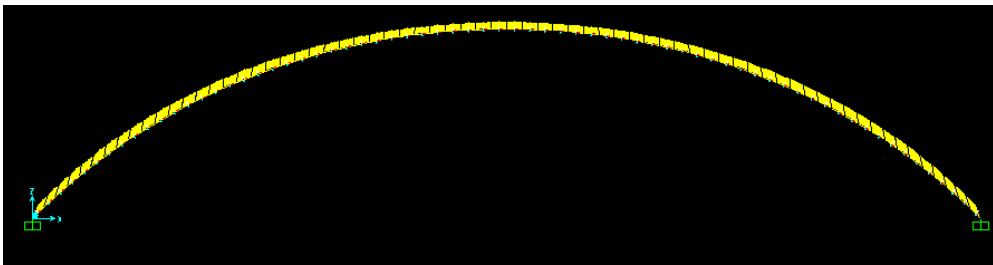


#### 2.4.2.2.2 Internal forces on deck

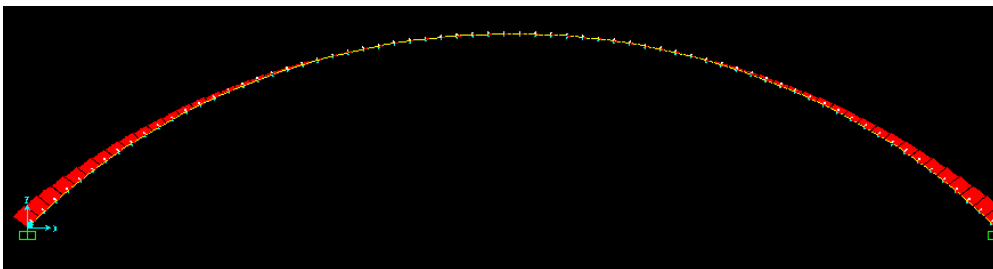
Torsion (max= 413kN·m)



M2-2 (max= 309kN·m)



M3-3 (max= -67kN·m)

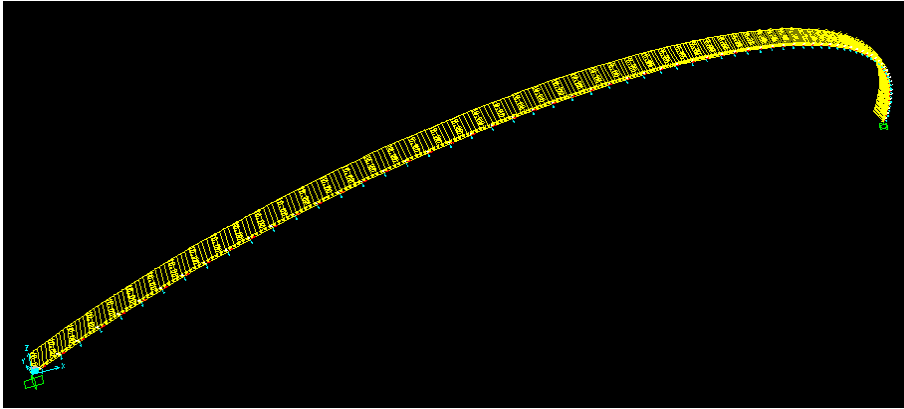


N=0

Load definition

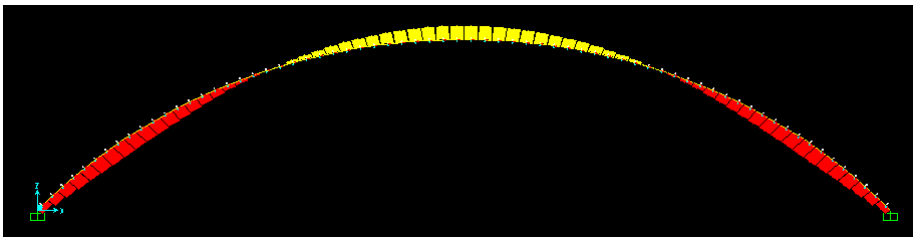
$$M_q = -m_2 L = m_3 = 10 \text{ kN} \cdot \text{m/m}$$

We are exclusively introducing a bending moment

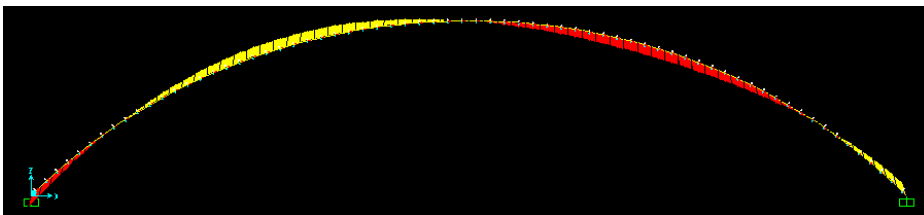


2.4.2.2.3 Internal forces on deck

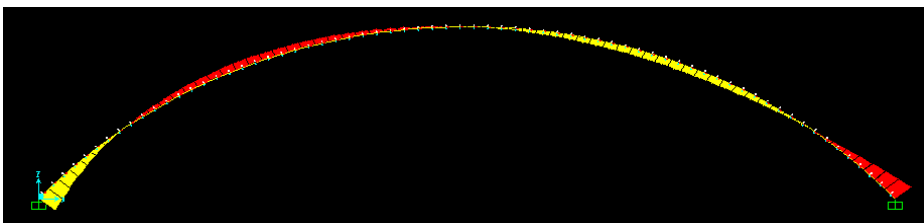
Torsion (max=2,6kN·m)



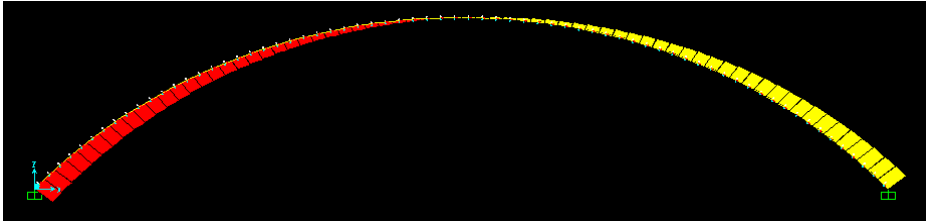
M2-2=14



M3-3 (max=42) very low



N (max=9)



## 2.5 Analysis of the results

As it is already known for balcony-beams (Aparicio, 1978) and the results attest, torsion and bending moments are coupled. On a balcony beam contained in a plane, a positive torque ( $T_q$ ), ie: which tends to turn it outwards and down, not only produces a torsion ( $T_1$ ), but also causes a negative bending 3-3 moment ( $M_1$ ), ie: tensions at the upper fibers of the deck cross section. And a positive bending 3-3 moment ( $M_q$ ) causes a positive torsion ( $T_2$ ), apart from the expected bending moment ( $M_2$ ).

At the arch, a positive uniform torque ( $T_q$ ) produces torsion and bending moments both in the arch plane (negative  $M3-3_1$ ) and out of the arch plane (positive  $M2-2_1$ ). The only possible explanation is that the arch is not contained in a plane. This torsion/ $M2-2$ / $M3-3$  coupling also takes place for bending moments loading ( $M_q$ ). The non-planarity effect seems therefore not negligible under torque or bending loading. However, when comparing arch geometries contained in a plane or not under vertical loadings we can conclude that the effect of non-planarity is negligible for internal forces and displacements, even for fixed hangers (see chapter IV).

**The axial force variation that takes place when releasing or fixing the hanger joints is caused by the change in the hanger shear forces transmitted to arch and the deck through the joints.**

The torsion/bending moments coupling, but is related to the torsion/bending stiffness relationship of the balcony beam.

At the deck, the maximal bending moment  $M_1$  value under a positive torque ( $T_q$ ) is half the maximal torsional moment value  $T_1$  and  $T_2$  is  $0,6 \cdot M_2$ . For the same value of  $M_q$  as  $T_q$ ,  $T_1$  is 400 times bigger than  $M_2$  for the chosen cross-section, being its torsional rigidity approximately 30 times larger than its bending one.

At the arch, the maximal total bending moment value  $M_1$  is 0,8 times the maximal torsion value  $T_1$ , whereas under  $M_q$ ,  $T_2$  is 0,06 times  $M_2$ . For the same value of  $M_q$  as  $T_q$ ,  $T_1$  is 9 times bigger than  $M_2$  for the cross section employed, which has a torsional rigidity only 2 times larger than its bending one.

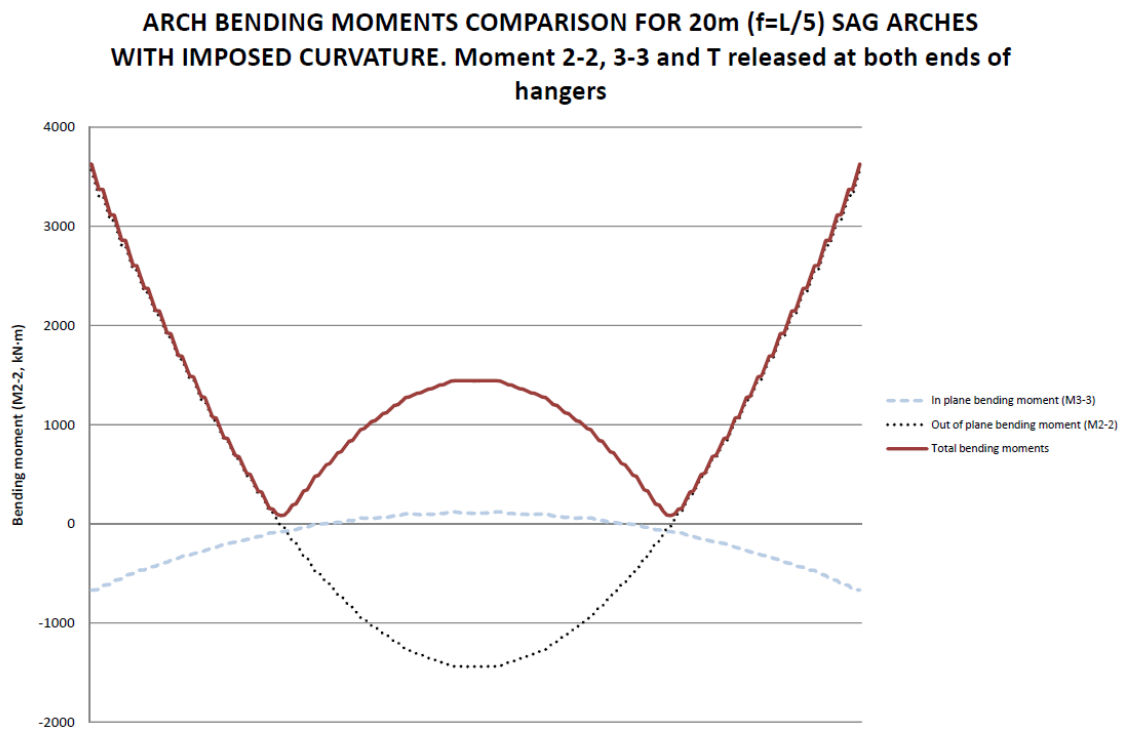
This maximal values comparison indicates the importance of the torsional and bending stiffness relationship. We have proved what it could already be stated intuitively: the higher the torsional stiffness of a balcony beam, the higher the influence of  $M3-3$  hanger joints and the lower the

M2-2 influence (see Figure 2-1). However, to state some kind of more exact proportional behaviour, a further research is required.

## 2.6 Relationship with the hanger joint study

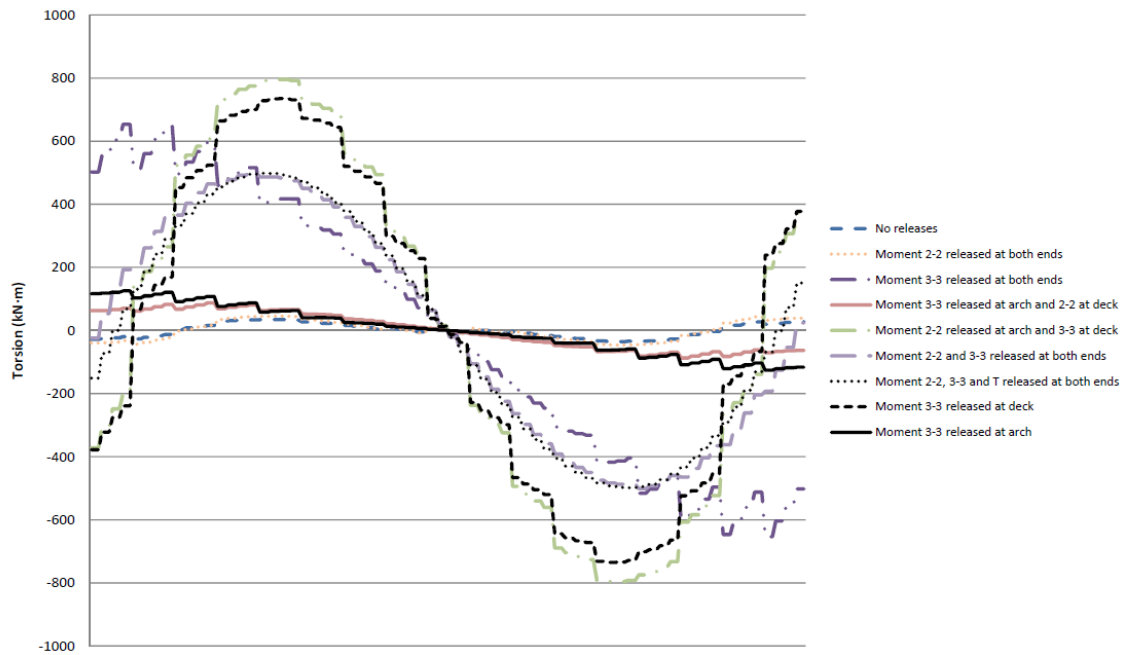
The behavior of an IDABWIC under vertical loading is analysed to, firstly, evaluate what is expected to be more convenient at hanger/arch and hanger/deck joints in order to reduce arch internal forces.

According to the results in Figure 2-5 and Figure 2-6, it is interesting to introduce negative torques at arch, to reduce both M2-2 and M3-3 at springings (see section number 2.4 inclined non-planar arch balcony beam results: positive torques introduce negative M3-3 and positive M2-2 at arch springings, and we want to introduce opposite moments to compensate the ones generated by the vertical load). Positive M3-3 at hangers will generate negative torques at arch. Vertical loads introduce positive M3-3 at hangers (see Figure 3-13). Therefore, it is interesting to fix M3-3 at hanger/arch joints in order to obtain this positive bending moments, which will compensate bending and torsion at arch.



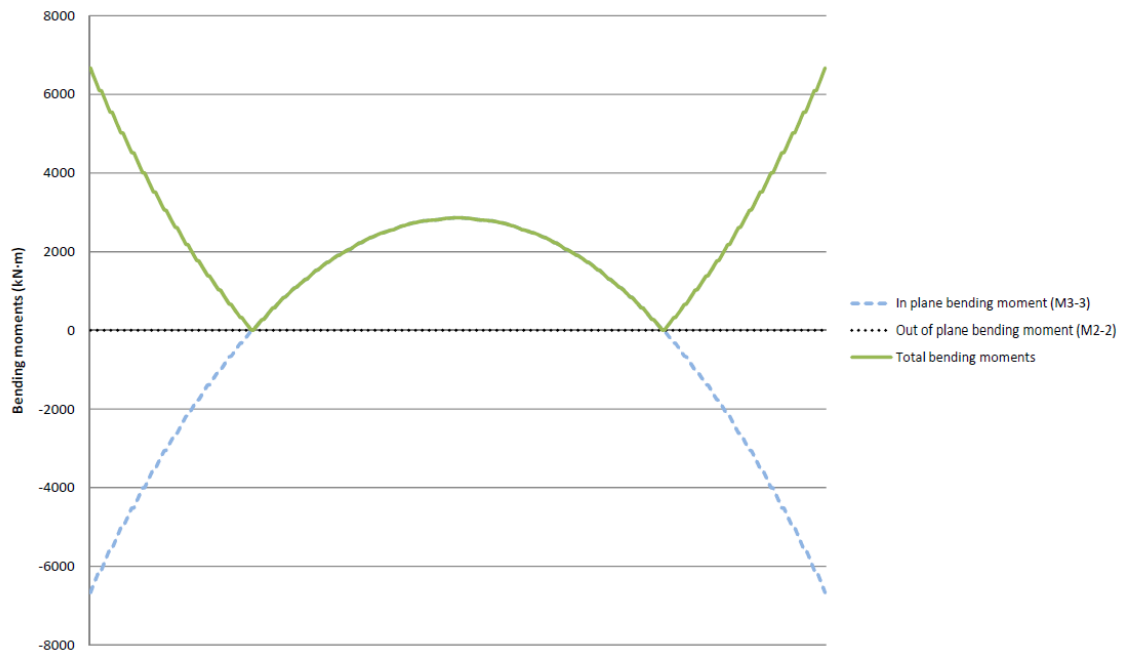
**Figure 2-5: Arch bending moments under a uniform vertical deck loading of 10kN/m, for model (2) with the moments 2-2, 3-3 and torsion released at both ends of hangers**

**ARCH TORSION MOMENTS COMPARISON FOR 20m (f=L/5) SAG ARCHES WITH IMPOSED CURVATURE**



**Figure 2-6: Arch torsion moments comparison under a uniform vertical deck loading of 10kN/m, for model (2) with the moments 2-2, 3-3 and torsion released at both ends of hangers**

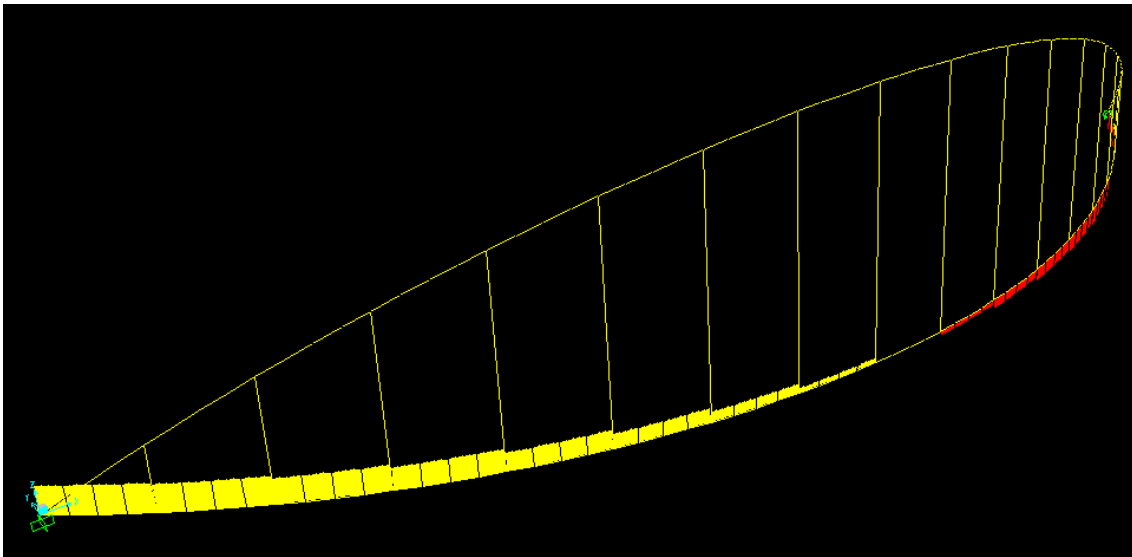
**DECK BENDING MOMENTS FOR 20m (f=L/5) SAG ARCHES WITH IMPOSED CURVATURE. Moments 2-2, 3-3 and torsion released at both ends of hangers**



**Figure 2-7: Deck bending moments under a uniform vertical deck loading, for model (2) with the moments 2-2, 3-3 and torsion released at both ends of hangers**

To compensate the 2-2 bending moments produced at abutments we need to introduce positive moments and to compensate the span center ones, negative ones. We can achieve such a

distribution at abutments by introducing a positive uniform torque on the deck. This would be transmitted by positive M3-3 at hangers, so it is interesting to fix M3-3 at hanger/deck joints.



**Figure 2-8: under a uniform vertical deck loading, for model (2) with the moments 2-2, 3-3 and torsion released at both ends of hangers**

A positive uniform torque produces a torsion behaviour on the deck shaped as the one produced by a vertical load. Therefore, a negative torque would be necessary to compensate it. If we want to employ a deck cross-section with low torsional rigidity, it will be better to release M3-3 at hanger-deck joints.

However, we will be always working with closed cross-sections which have a good behaviour to torsion and we find more interesting to use de joint configuration which enhances the antifunicularity behaviour of the arch, and reduces the bending too on the deck.

In conclusion, the most favourable joint conditions will be to release M2-2 and fix M3-3 at both hanger ends.

The following study will give more light on the behaviour of the hangers under vertical load for different joint configurations.

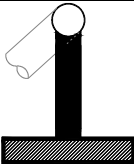
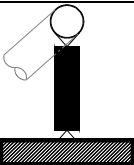
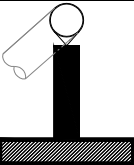
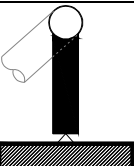

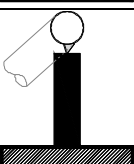
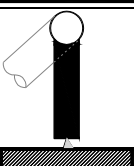


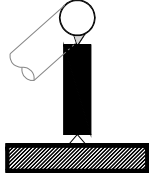
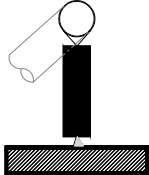
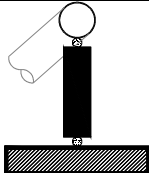
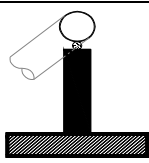
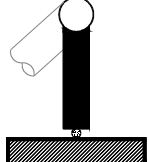
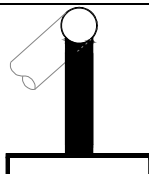
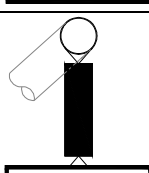
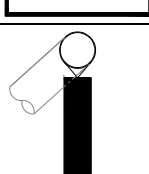
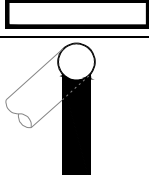
### 3. STUDY OF HANGER/DECK AND HANGER/ARCH JOINTS

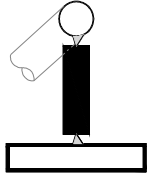
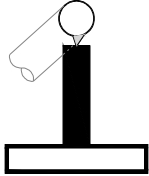
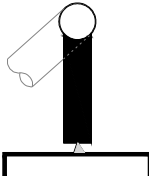
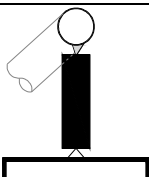
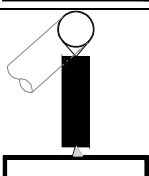
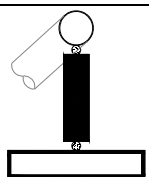
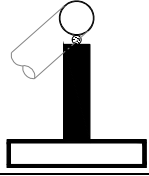
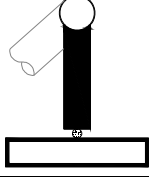
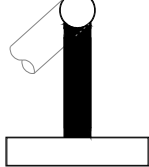
Different cross-sections for the arch, deck and hangers, and hnger/deck and hanger/arch joints (Table 3-1) have been studied in order to determine which is the best joint configuration in each case.

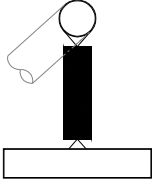
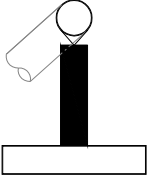
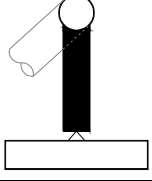
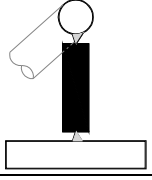
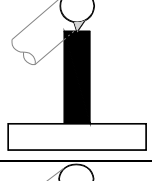
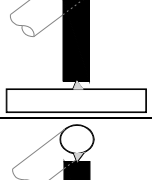
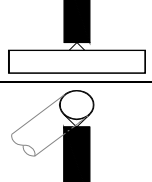
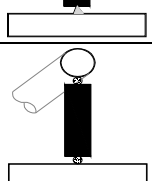
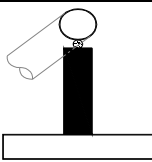

#### 3.1 Mechanical properties and configurations of joints studied:

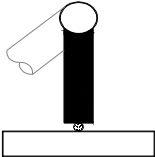
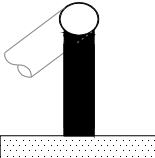
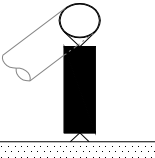
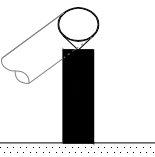
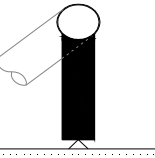
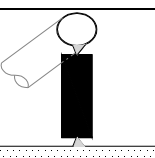
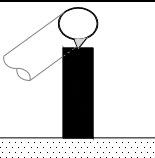
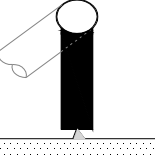
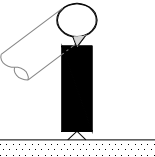
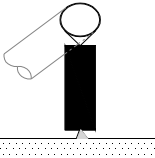
**Table 3-1: Cross-section values employed for the comparison of the behaviour of different superior arch bridges with imposed curvature models with different hanger joints configuration**

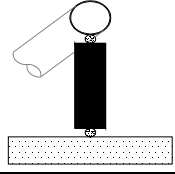
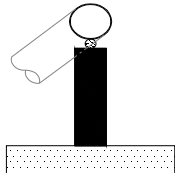
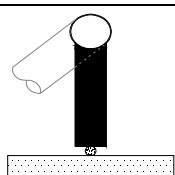
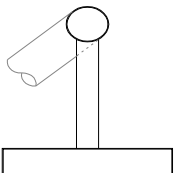
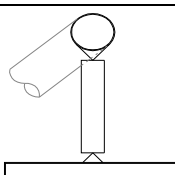
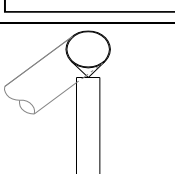
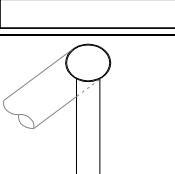
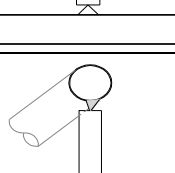
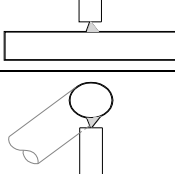
| LEGEND NUMBER    | ARCH  | DECK  | HANGERS  | HANGER JOINTS               | SYMBOL  |             |
|------------------|---|---|--|-----------------------------|---|-------------|
| <b>Model (1)</b> | <i>Reference arch</i><br>$A= 0,0914m^2$<br>$J= 0,0215m^4$<br>$I_2= 0,0108m^4$<br>$I_3= 0,0108m^4$ | <i>Rigid deck</i><br>TABL.RIG.TORS.VERT<br>$A= 0,1431m^2$<br>$J= 10m^4$<br>$I_2= 0,2m^4$<br>$I_3= 0,2m^4$ | H.I3.0<br>$A= 0,0304m^2$<br>$J= 1,097 \cdot 10^{-3}m^4$<br>$I_2= 7,34 \cdot 10^{-4}m^4$<br>$I_3= 7,34 \cdot m^4$ | No releases                 |    | Model (1.1) |
|                  |   |   |  | Moment 2-2 released         |    | Model (1.2) |
|                  |   |   |  | Moment 2-2 released at arch |   | Model (1.3) |
|                  |   |   |  | Moment 2-2 released at deck |  | Model (1.4) |
|                  |   |   |  | Moment 3-3 released         |  | Model (1.5) |
|                  |   |   |  | Moment 3-3 released at arch |  | Model (1.6) |
|                  |   |   |  | Moment 3-3 released at deck |  | Model (1.7) |

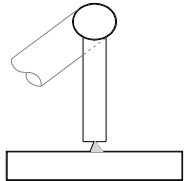
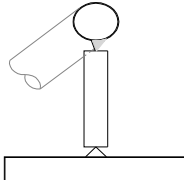
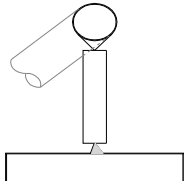
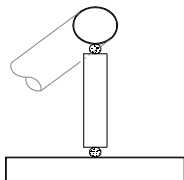
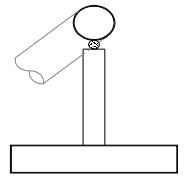
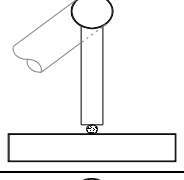
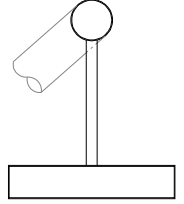
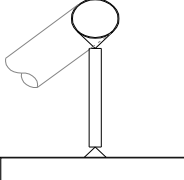
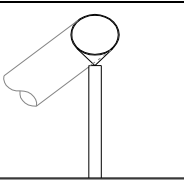
|                  |  |  |  |   |   |              |
|------------------|--|--|--|---|---|--------------|
|                  |  |  |  | Moment 3-3 released at arch and 2-2 at deck |    | Model (1.8)  |
|                  |  |  |  | Moment 2-2 released at arch and 3-3 at deck |    | Model (1.9)  |
|                  |  |  |  | Moment 2-2 and 3-3 released                 |    | Model (1.10) |
|                  |  |  |  | Moment 2-2 and 3-3 released at arch         |    | Model (1.11) |
|                  |  |  |  | Moment 2-2 and 3-3 released at deck         |   | Model (1.12) |
| <b>Model (2)</b> | Reference arch<br>A= 0,0914m <sup>2</sup><br>J= 0,0215m <sup>4</sup><br>I2= 0,0108m <sup>4</sup><br>I3= 0,0108m <sup>4</sup> | Rigid deck<br>TABLRIGTORS<br>A= 0,1431m <sup>2</sup><br><b>J= 10m<sup>4</sup></b><br>I2= 0,2m <sup>4</sup><br><b>I3= 0,02m<sup>4</sup></b> | A= 0,0304m <sup>2</sup><br>J= 1,097·10 <sup>-3</sup> m <sup>4</sup><br>I2= 7,34·10 <sup>-4</sup> m <sup>4</sup><br><b>I3= 7,34·m<sup>4</sup></b> | No releases                                 |  | Model (2.1)  |
|                  |  |  |  | Moment 2-2 released                         |  | Model (2.2)  |
|                  |  |  |  | Moment 2-2 released at arch                 |  | Model (2.3)  |
|                  |  |  |  | Moment 2-2 released at deck                 |  | Model (2.4)  |

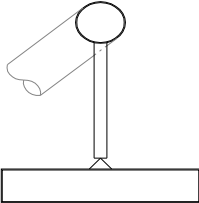
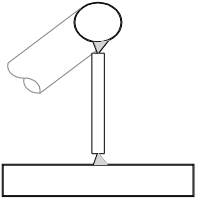
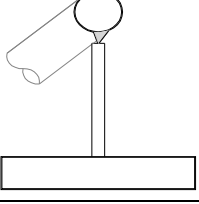
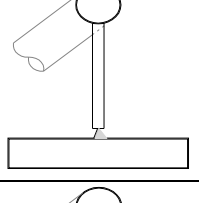
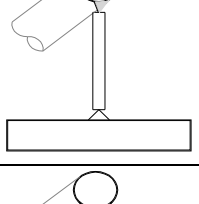
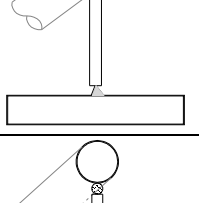
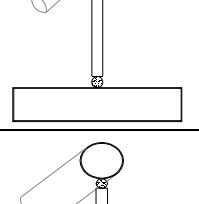
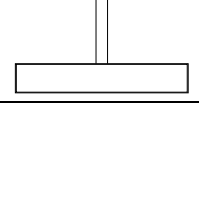
|                  |   |   |  |   |   |              |
|------------------|---|---|--|---|---|--------------|
|                  |   |   |  | Moment 3-3 released                         |    | Model (2.5)  |
|                  |   |   |  | Moment 3-3 released at arch                 |    | Model (2.6)  |
|                  |   |   |  | Moment 3-3 released at deck                 |    | Model (2.7)  |
|                  |   |   |  | Moment 3-3 released at arch and 2-2 at deck |    | Model (2.8)  |
|                  |   |   |  | Moment 2-2 released at arch and 3-3 at deck |   | Model (2.9)  |
|                  |   |   |  | Moment 2-2 and 3-3 released                 |  | Model (2.10) |
|                  |   |   |  | Moment 2-2 and 3-3 released at arch         |  | Model (2.11) |
|                  |   |   |  | Moment 2-2 and 3-3 released at deck         |  | Model (2.12) |
| <b>Model (3)</b> | <i>Reference arch</i><br>$A= 0,0914\text{m}^2$<br>$J= 0,0215\text{m}^4$ | <i>Reference deck</i><br>$A= 0,1431\text{m}^2$<br>$J= 0,0615\text{m}^4$ | $A= 0,0304\text{m}^2$<br>$J= 1,097 \cdot 10^{-3}\text{m}^4$<br>$I_2= 7,34 \cdot 10^{-4}\text{m}^4$ | No releases                                 |  | Model (3.1)  |

|  |  |  |                        |   |   |              |
|--|--|--|------------------------|---|---|--------------|
|  | $I_2 = 0,0108m^4$<br>$I_3 = 0,0108m^4$ | $I_2 = 0,2517m^4$<br>$I_3 = 0,0196m^4$ | $I_3 = 7,34 \cdot m^4$ | Moment 2-2 released                         |    | Model (3.2)  |
|  |  |  |                        | Moment 2-2 released at arch                 |    | Model (3.3)  |
|  |  |  |                        | Moment 2-2 released at deck                 |    | Model (3.4)  |
|  |  |  |                        | Moment 3-3 released                         |    | Model (3.5)  |
|  |  |  |                        | Moment 3-3 released at arch                 |   | Model (3.6)  |
|  |  |  |                        | Moment 3-3 released at deck                 |  | Model (3.7)  |
|  |  |  |                        | Moment 3-3 released at arch and 2-2 at deck |  | Model (3.8)  |
|  |  |  |                        | Moment 2-2 released at arch and 3-3 at deck |  | Model (3.9)  |
|  |  |  |                        | Moment 2-2 and 3-3 released                 |  | Model (3.10) |
|  |  |  |                        | Moment 2-2 and 3-3 released at arch         |  | Model (3.11) |

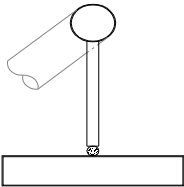
|                  |  |  |  |   |   |              |
|------------------|--|--|--|---|---|--------------|
|                  |  |  |  | Moment 2-2 and 3-3 released at deck         |    | Model (3.12) |
| <b>Model (4)</b> | Reference arch<br>A= 0,0914m <sup>2</sup><br>J= 0,0215m <sup>4</sup><br>I2= 0,0108m <sup>4</sup><br>I3= 0,0108m <sup>4</sup> | Larger torsional rigidity, but low bending rigidity around 3-3 axis:<br><i>DECK.FLEX.RT</i><br>A= 0,1431m <sup>2</sup><br>J= 0,08m <sup>4</sup><br>I2= 0,2517m <sup>4</sup><br>I3= 0,001m <sup>4</sup> | A= 0,0304m <sup>2</sup><br>J= 1,097·10 <sup>-3</sup> m <sup>4</sup><br>I2= 7,34·10 <sup>-4</sup> m <sup>4</sup><br>I3= 7,34·m <sup>4</sup> | No releases                                 |    | Model (4.1)  |
|                  |  |  |  | Moment 2-2 released                         |    | Model (4.2)  |
|                  |  |  |  | Moment 2-2 released at arch                 |    | Model (4.3)  |
|                  |  |  |  | Moment 2-2 released at deck                 |   | Model (4.4)  |
|                  |  |  |  | Moment 3-3 released                         |  | Model (4.5)  |
|                  |  |  |  | Moment 3-3 released at arch                 |  | Model (4.6)  |
|                  |  |  |  | Moment 3-3 released at deck                 |  | Model (4.7)  |
|                  |  |  |  | Moment 3-3 released at arch and 2-2 at deck |  | Model (4.8)  |
|                  |  |  |  | Moment 2-2 released at arch and 3-3 at deck |  | Model (4.9)  |

|                  |   |   |  |                                     |   |              |
|------------------|---|---|--|-------------------------------------|---|--------------|
|                  |   |   |  | Moment 2-2 and 3-3 released         |    | Model (4.10) |
|                  |   |   |  | Moment 2-2 and 3-3 released at arch |    | Model (4.11) |
|                  |   |   |  | Moment 2-2 and 3-3 released at deck |    | Model (4.12) |
| <b>Model (5)</b> | <i>Reference arch</i><br>A= 0,0914m <sup>2</sup><br>J= 0,0215m <sup>4</sup><br>I2= 0,0108m <sup>4</sup><br>I3= <b>0,0108m<sup>4</sup></b> | <i>Reference deck</i><br>A= 0,1431m <sup>2</sup><br>J= 0,0615m <sup>4</sup><br>I2= 0,2517m <sup>4</sup><br>I3= 0,0196m <sup>4</sup> | Rigid hangers<br>800pipe<br>A= 0,0609m <sup>2</sup><br>J= 9,15·10 <sup>-4</sup> m <sup>4</sup><br>I2= 4,58·10 <sup>-4</sup> m <sup>4</sup><br>I3= 4,58·10 <sup>-4</sup> m <sup>4</sup> | No releases                         |    | Model (5.1)  |
|                  |   |   |  | Moment 2-2 released                 |   | Model (5.2)  |
|                  |   |   |  | Moment 2-2 released at arch         |  | Model (5.3)  |
|                  |   |   |  | Moment 2-2 released at deck         |  | Model (5.4)  |
|                  |   |   |  | Moment 3-3 released                 |  | Model (5.5)  |
|                  |   |   |  | Moment 3-3 released at arch         |  | Model (5.6)  |

|                  |   |   |  |   |   |              |
|------------------|---|---|--|---|---|--------------|
|                  |   |   |  | Moment 3-3 released at deck                 |    | Model (5.7)  |
|                  |   |   |  | Moment 3-3 released at arch and 2-2 at deck |    | Model (5.8)  |
|                  |   |   |  | Moment 2-2 released at arch and 3-3 at deck |    | Model (5.9)  |
|                  |   |   |  | Moment 2-2 and 3-3 released                 |    | Model (5.10) |
|                  |   |   |  | Moment 2-2 and 3-3 released at arch         |   | Model (5.11) |
|                  |   |   |  | Moment 2-2 and 3-3 released at deck         |  | Model (5.12) |
| <b>Model (6)</b> | <i>Reference arch</i><br>A= 0,0914m <sup>2</sup><br>J= 0,0215m <sup>4</sup><br>I2= 0,0108m <sup>4</sup><br><b>I3= 0,0108m<sup>4</sup></b> | <i>Reference deck</i><br>A= 0,1431m <sup>2</sup><br>J= 0,0615m <sup>4</sup><br>I2= 0,2517m <sup>4</sup><br>I3= 0,0196m <sup>4</sup> | Rigid hangers<br>400pipe<br>A= 0,0239m <sup>2</sup><br>J= 8,64·10 <sup>-4</sup> m <sup>4</sup><br>I2= 4,32·10 <sup>-4</sup> m <sup>4</sup><br>I3= 4,32·10 <sup>-4</sup> m <sup>4</sup> | No releases                                 |  | Model (6.1)  |
|                  |   |   |  | Moment 2-2 released                         |  | Model (6.2)  |
|                  |   |   |  | Moment 2-2 released at arch                 |  | Model (6.3)  |

|  |  |  |  |   |   |              |
|--|--|--|--|---|---|--------------|
|  |  |  |  | Moment 2-2 released at deck                 |    | Model (6.4)  |
|  |  |  |  | Moment 3-3 released                         |    | Model (6.5)  |
|  |  |  |  | Moment 3-3 released at arch                 |    | Model (6.6)  |
|  |  |  |  | Moment 3-3 released at deck                 |   | Model (6.7)  |
|  |  |  |  | Moment 3-3 released at arch and 2-2 at deck |  | Model (6.8)  |
|  |  |  |  | Moment 2-2 released at arch and 3-3 at deck |  | Model (6.9)  |
|  |  |  |  | Moment 2-2 and 3-3 released                 |  | Model (6.10) |
|  |  |  |  | Moment 2-2 and 3-3 released at arch         |  | Model (6.11) |



|                  |   |   |   |   |   |              |
|------------------|---|---|---|---|---|--------------|
|                  |   |   |   | Moment 2-2 and 3-3 released at deck         |  | Model (6.12) |
| <b>Model (7)</b> | <i>Reference arch</i><br>A= 0,0914m <sup>2</sup><br>J= 0,0215m <sup>4</sup><br>I <sub>2</sub> = 0,0108m <sup>4</sup><br><b>I<sub>3</sub>= 0,0108m<sup>4</sup></b> | <i>Rigid deck</i><br>TABLRIGTORS<br>A= 0,1431m <sup>2</sup><br><b>J= 10m<sup>4</sup></b><br>I <sub>2</sub> = 0,2m <sup>4</sup><br><b>I<sub>3</sub>= 0,02m<sup>4</sup></b> | A= 0,0304m <sup>2</sup><br>J= 1,097·10 <sup>-3</sup> m <sup>4</sup><br>I <sub>2</sub> = 7,34 m <sup>4</sup><br><b>I<sub>3</sub>= 7,34·m<sup>4</sup></b> | No releases                                 |   | Model (7.1)  |
|                  |   |   |   | Moment 2-2 released                         |   | Model (7.2)  |
|                  |   |   |   | Moment 2-2 released at arch                 |   | Model (7.3)  |
|                  |   |   |   | Moment 2-2 released at deck                 |   | Model (7.4)  |
|                  |   |   |   | Moment 3-3 released                         |   | Model (7.5)  |
|                  |   |   |   | Moment 3-3 released at arch                 |   | Model (7.6)  |
|                  |   |   |   | Moment 3-3 released at deck                 |   | Model (7.7)  |
|                  |   |   |   | Moment 3-3 released at arch and 2-2 at deck |   | Model (7.8)  |
|                  |   |   |   | Moment 2-2 released at arch and 3-3 at deck |   | Model (7.9)  |
|                  |   |   |   | Moment 2-2 and 3-3 released                 |   | Model (7.10) |
|                  |   |   |   | Moment 2-2 and 3-3 released at arch         |   | Model (7.11) |
|                  |   |   |   | Moment 2-2 and 3-3 released at deck         |   | Model (7.12) |

### 3.2 Torsion release discussion

We must note that hanger/arch and hanger/deck torsion releases have not been studied, but they will influence the bridge behaviour too.

When releasing M2-2 and M3-3, torsion is always unavoidably released too. However, the difference between releasing torsion or not is absolutely negligible regarding the shape of the internal forces distribution in the arch (see from Figure 3-3 to Figure 3-5). However, it is of course not negligible if we want to calculate the efforts for the bridge dimensioning, because the error committed is of 5%.

### 3.3 Local axis definition

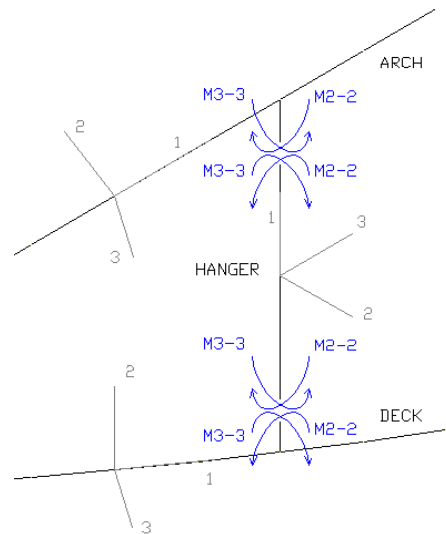


Figure 3-1: Local axis definition

### 3.4 Axial forces behaviour for different hanger joints configurations

#### 3.4.1 A first intuitive approach

Intuitively, a deck with a very rigid to torsion cross section (models (1), (2) and (7)) will cause higher axial forces on the arch because the shear forces transmitted by hangers ( $V_{2-2_H}$ ), due to a fixed transverse bending moment ( $M_{3-3_H}$  at joints), will be bigger.

The higher the deck's torsional stiffness, the higher the influence of the transverse bending moment ( $M_{3-3_H}$ ) transmitted by hangers. It can be proved so (Figure 3-2).

Therefore, an important value is expected to be the relationship of the deck torsional rigidity and the hangers transverse flexural rigidity ( $J_D/I_{3-3_H}$ ).

The deck bending rigidity will be an important value too, depending on its relationship with the torsional rigidity, as observed at section 3.5.1.1. Therefore a model with equal torsional and bending deck rigidity (7) has also been studied, expecting that fixing  $M_{3-3_H}$  loses importance in front of other cases with the same high torsional rigidity of the deck.

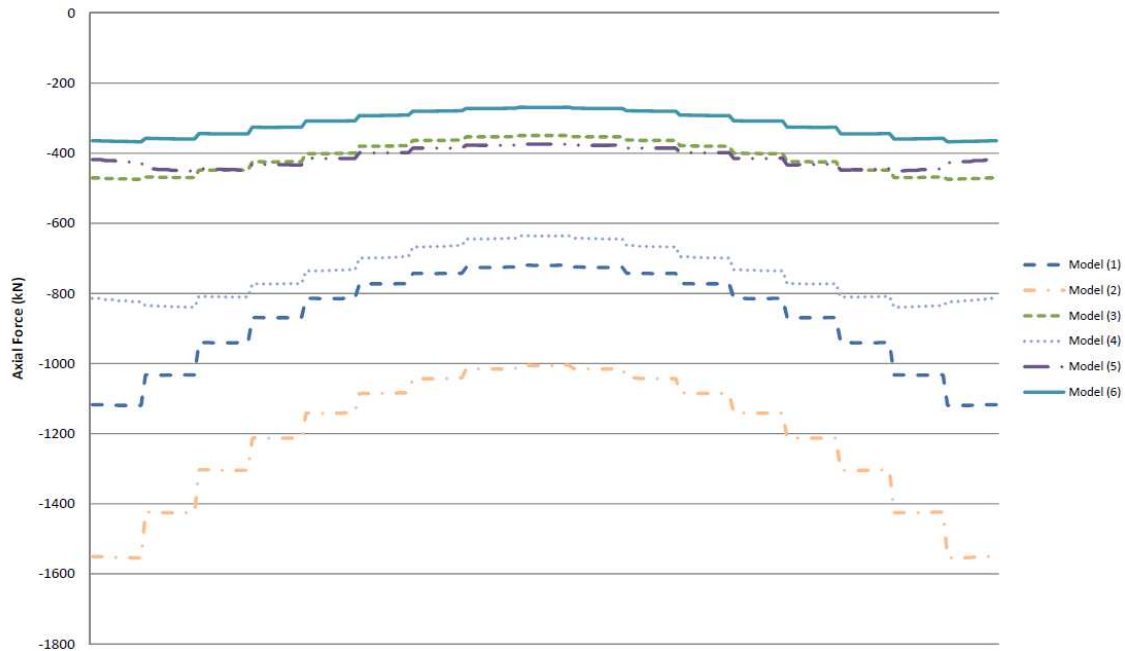
When the deck torsional rigidity is low, we expect torsional bending moments in the deck to decrease, and therefore shear forces 2-2 in the hangers are expected to diminish too. Consequently, the axial forces at the arch will be lower. The influence of the transversal bending moment ( $M_{3-3_H}$ ) transmitted by hangers and the associated shear force  $V_{2-2_H}$ , diminishes. Fixing  $M_{3-3_H}$  or not at hanger joints is expected to be less important for those models ((3), (4), (5) and (6)).

We want to prove this intuitions with the thorough parametrical analysis presented on the following sections.

### 3.4.2 Influence of the deck cross-section bending and torsional rigidity

Arch axial forces are higher for deck cross sections with a high torsional rigidity. We must note the difference between (1) and (2): for a same torsional deck stiffness, the arch with lower bending stiffness (2) has a higher axial force (Figure 3-2).

Increasing a bit the torsional rigidity of the deck cross-section and diminishing the flexural rigidity greatly increases the axial forces in the arch (compare (3) with (4) at Figure 3-2).



**Figure 3-2: Arch axial forces comparison for the different models analysed with M2-2 released at both ends**

### 3.5 Decks with a large torsional rigidity (models (1), (2) and (7))

Models (1), (2) and (7) employ a very large theoretical torsional rigidity of the deck cross-sections, so as to be able to consider the deck cross-section “infinitely” rigid to torsional bending moments.

The type of behaviour is divided into two big groups (see from Figure 3-3 to Figure 3-5) when employing what can be considered as a deck with infinite torsional rigidity:

- M3-3 fixed at hanger/deck joints and
- M3-3 released at those joints

When M3-3 is fixed at hanger/deck joints, the whole arch is under compression and the deck is tensioned (see Figure 3-3).

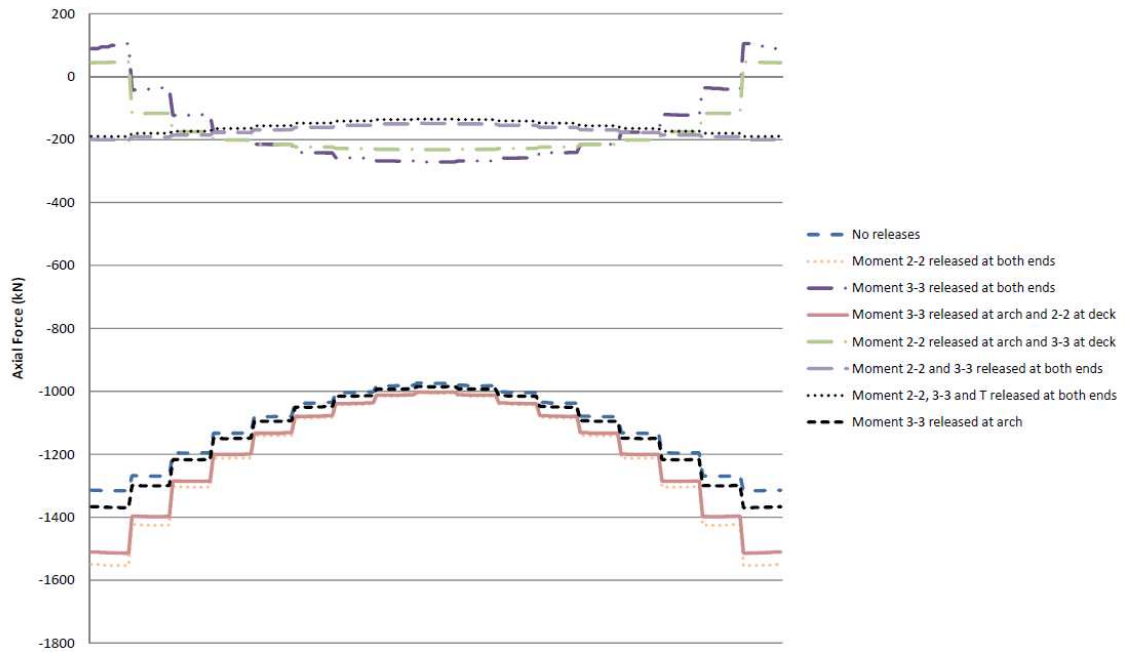
When released, the axial forces diminish and the behaviour at the abutments and springings changes completely. The arch is slightly tensioned at springings (see Figure 3-3).

Bending moments have been analysed too for cases (2) and (7). and torsional moments for model (2).

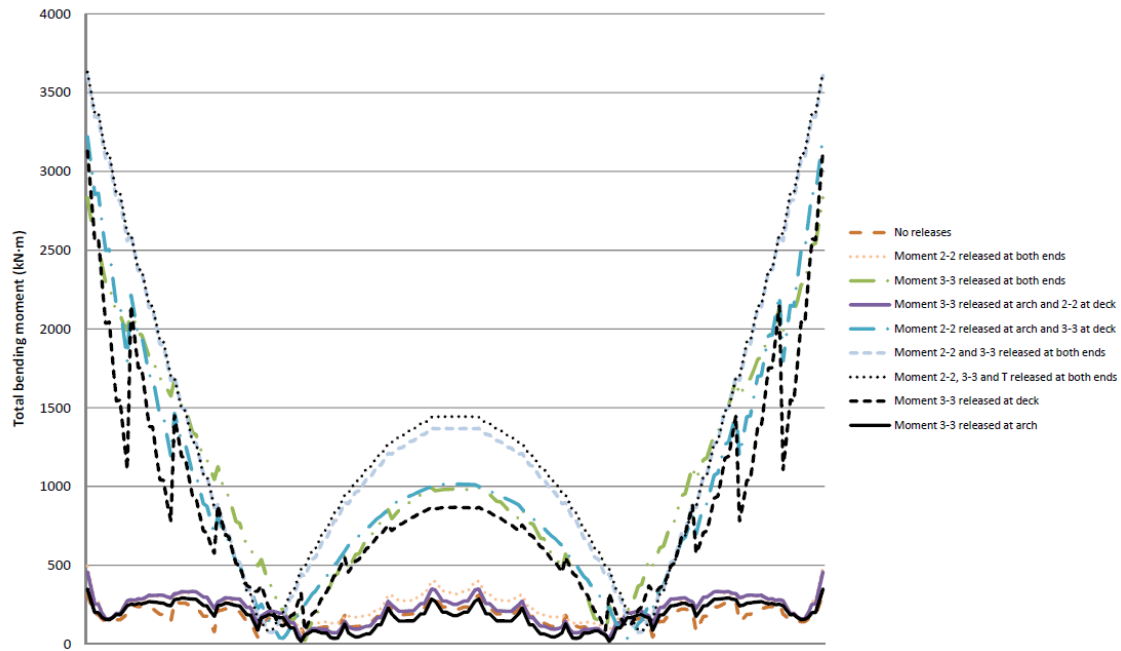
Bending moments are minimal at springings for fixed hangers and at span center for M3-3 released at hanger/arch joints. For these configurations and M2-2 released at both ends, they acquire nearly equal values (see Figure 3-4).

Torsions are minimal for fixed hangers, but as long as M3-3 is not released at hanger/deck joints they maintain low values for other configurations too (see Figure 3-5).

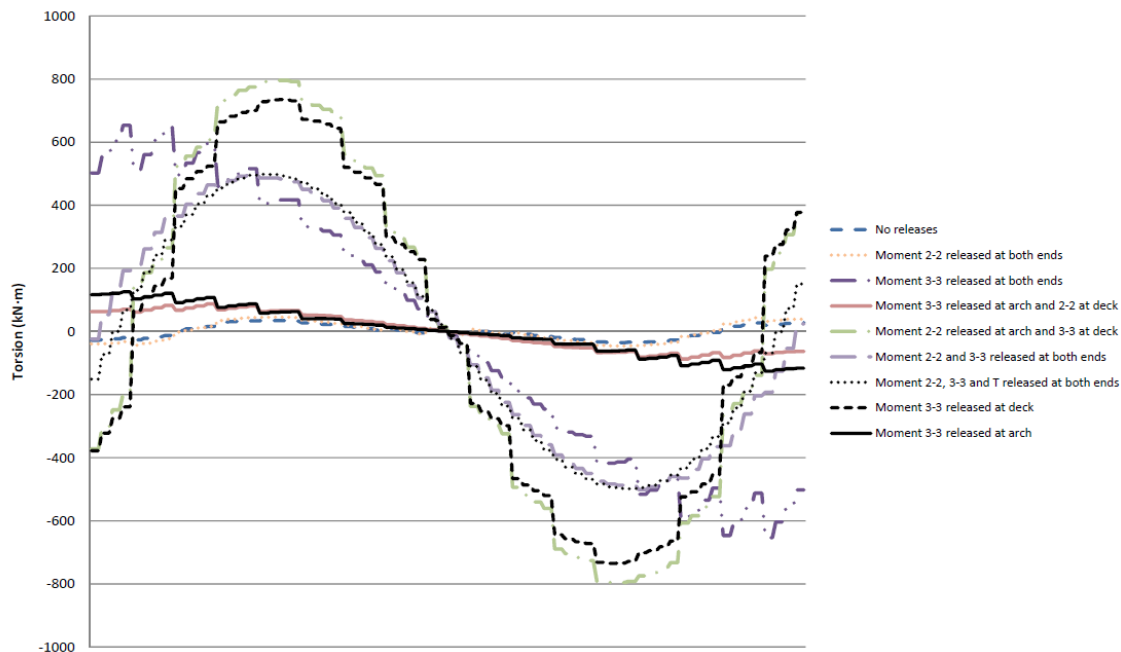
Therefore, if we want to tend to antifunicularity any of these configurations with M3-3 fixed at hanger/deck joints will be adequate. It is logical that axial forces are higher, because we are enhancing the behavior as an arch.



**Figure 3-3: Arch axial forces comparison for cross-section (2)**



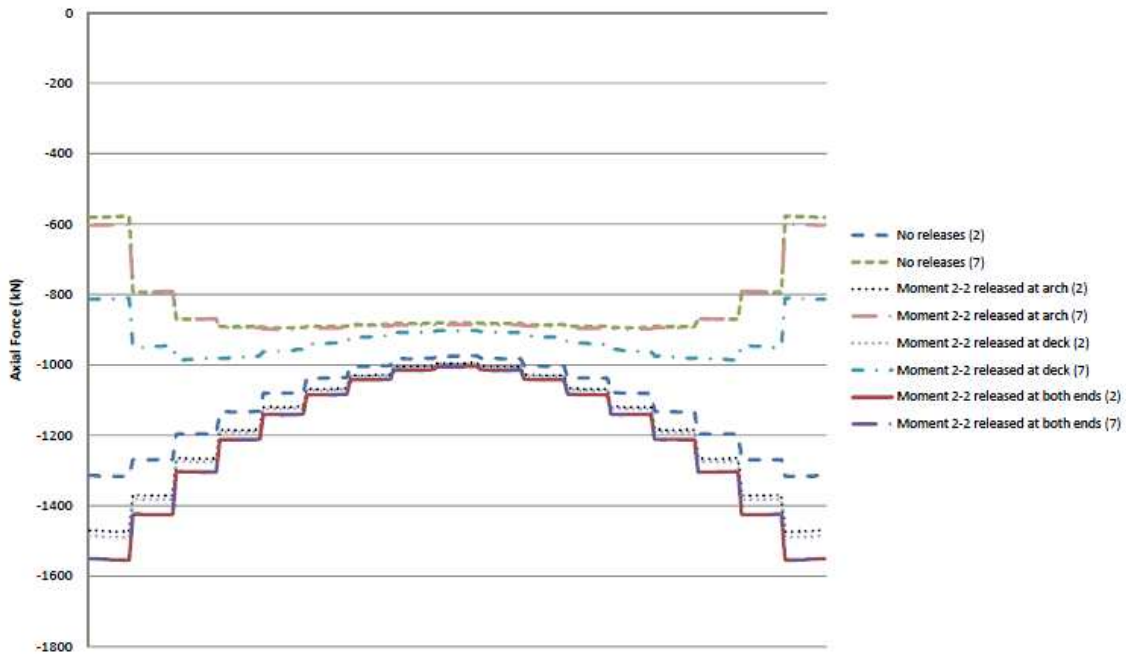
**Figure 3-4: Arch total bending moments comparison for cross-section (2)**



**Figure 3-5: Arch torsions comparison for cross-section (2)**

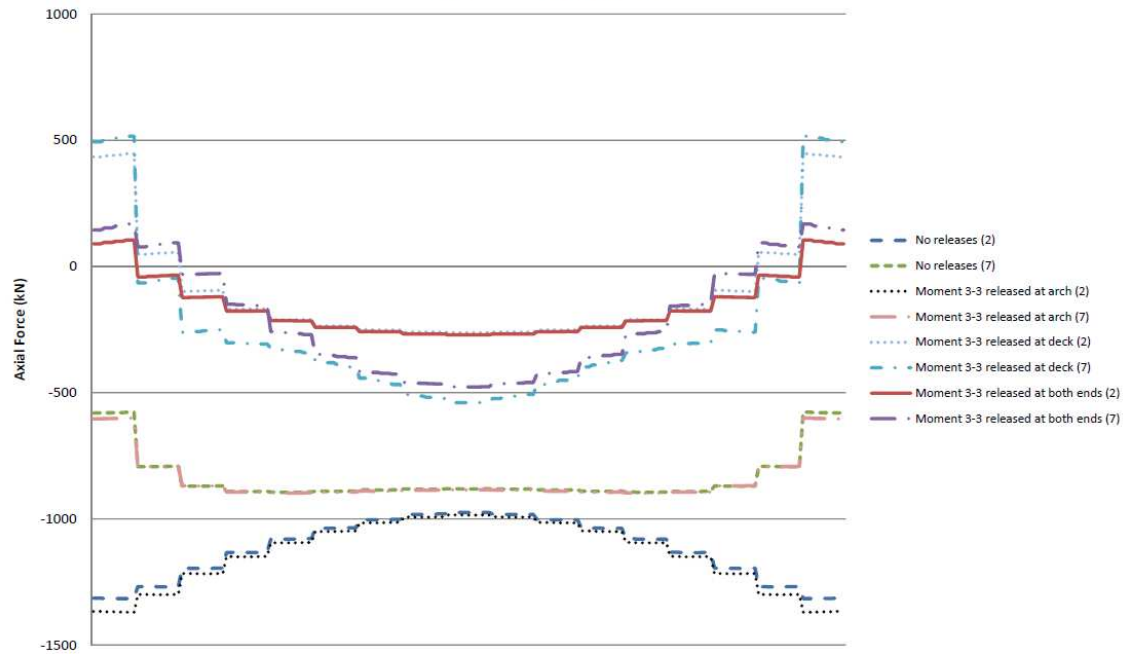
If we study a model with a deck cross-section with a large torsional bending rigidity and with hangers with a very large longitudinal bending rigidity (7), we observe (Figure 3-6) that the shape of axial forces distribution in the arch is similar to the one obtained for the models with fixed hangers and with a low torsional rigidity deck cross section (section 3.6, from Figure 3-15 to Figure 3-21). Therefore, the influence of releasing M2-2 is a matter more of the deck torsional stiffness/hanger stiffness relationship, rather than each one of them separately. This is noticed at extremes, where the hangers are stiffer (due to their shorter length).

However there are still two differenced groups: M3-3 released at hanger/deck joints or not. Therefore, this does not depend on the hanger and deck I3-3 relationship, which is the same for models (2), (3), (5), (6) and (7), but on the deck torsional rigidity (J).



**Figure 3-6: Arch axial forces comparison for hangers very rigid longitudinally and transversally and only rigid transversally**

**ARCH AXIAL FORCES COMPARISON FOR 20m ( $f=L/5$ ) SAG ARCHES WITH IMPOSED CURVATURE**



**Figure 3-7: Arch axial forces comparison for hangers very rigid longitudinally and transversally and only rigid transversally**

It is highly remarkable that for case (7), when fixing M2-2, the axial compression on the arch decreases, when the opposite was expected (Figure 3-6 compared with section 3.4.1).

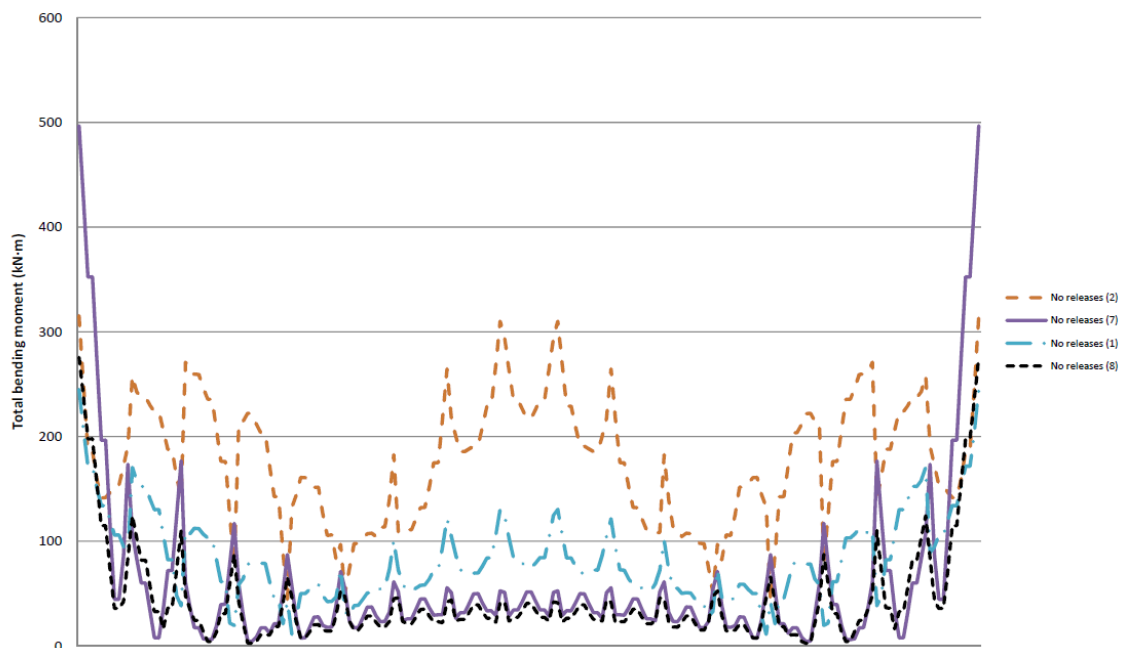
For both cases it is recommendable to release M2-2 at arch, so as to obtain higher compressions at springings, where higher bending will take place. Fixing M2-2 for hangers with high I2-2 rigidity greatly diminishes the total bending moment, except at springings, where it increases a lot (Figure 3-8).

We have compared the shear forces at hangers of models (2) and (7) with M3-3 released at hanger/deck joints. At model (7) V3-3 is 1,3 times higher than at model (2). However, V2-2 increases even more and is 1,5 higher. Consequently, axial forces at arch decrease.

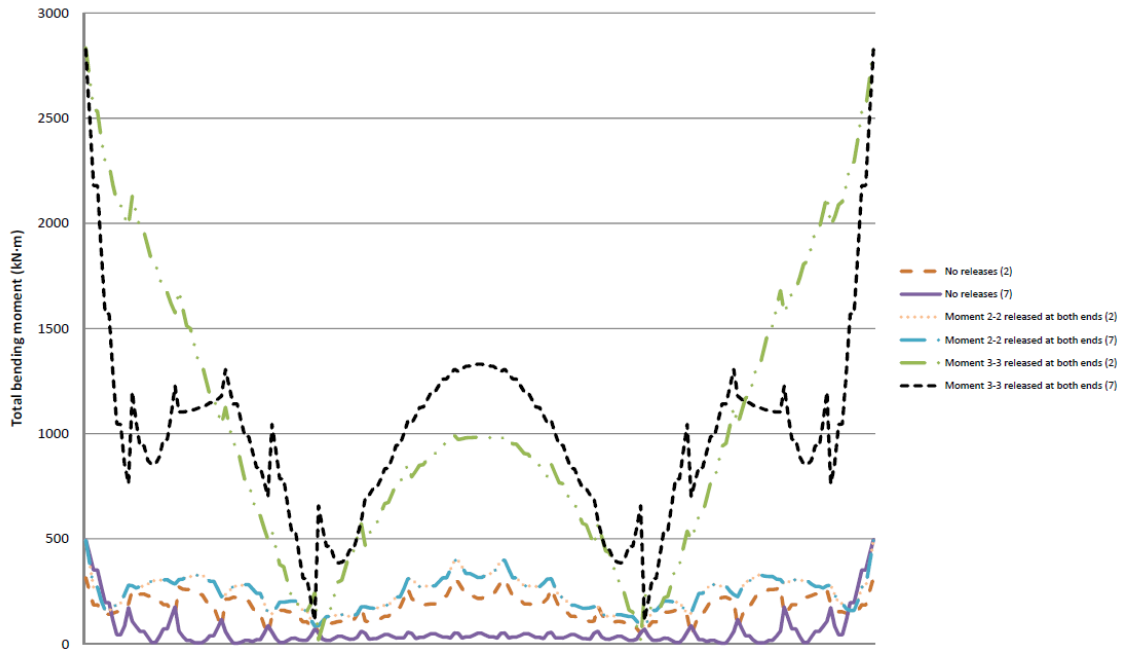
Therefore, to tend to antifunicularity, without causing additional arch compressions, but on the contrary, releasing them, we should not only give transverse rigidity to the system but also longitudinal bending stiffness (Figure 3-8).

When giving I2-2 bending rigidity to hangers it will be important not only to fix M2-2, but also M3-3. If they are pinned transversally, moments will be even bigger than for hangers with low I2-2 (Figure 3-9).

Releasing M3-3 at arch has a similar effect as releasing M2-2. Releasing it at deck is never recommendable. When employing hangers with a high I2-2, axial forces at springings diminish a lot and the result is a non-uniform axial forces distribution at arch (Figure 3-7) and a loss of arch behaviour (high moments at Figure 3-10).



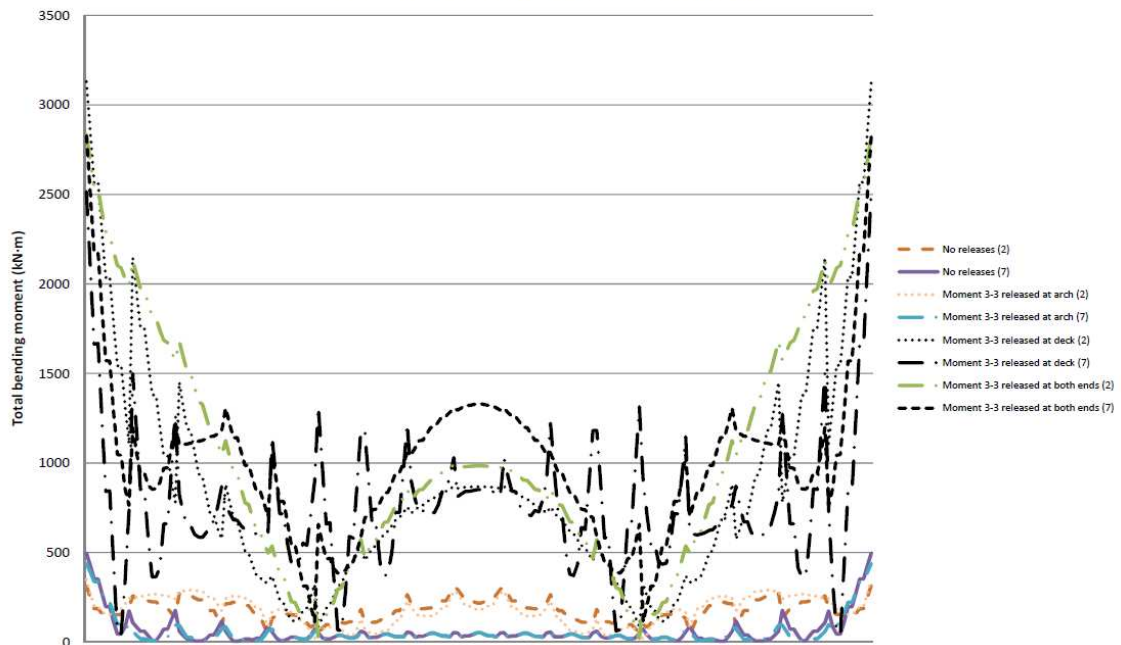
**Figure 3-8: Arch total bending moments comparison for different models with fixed hangers**



**Figure 3-9: Arch total bending moments comparison for different hangers' joints conditions and different hanger I2-2 rigidities**

When releasing M3-3 at deck and employing hangers with high I2-2, it is remarkable that the effect of single loads introduced by hangers is highly accentuated (Figure 3-10).

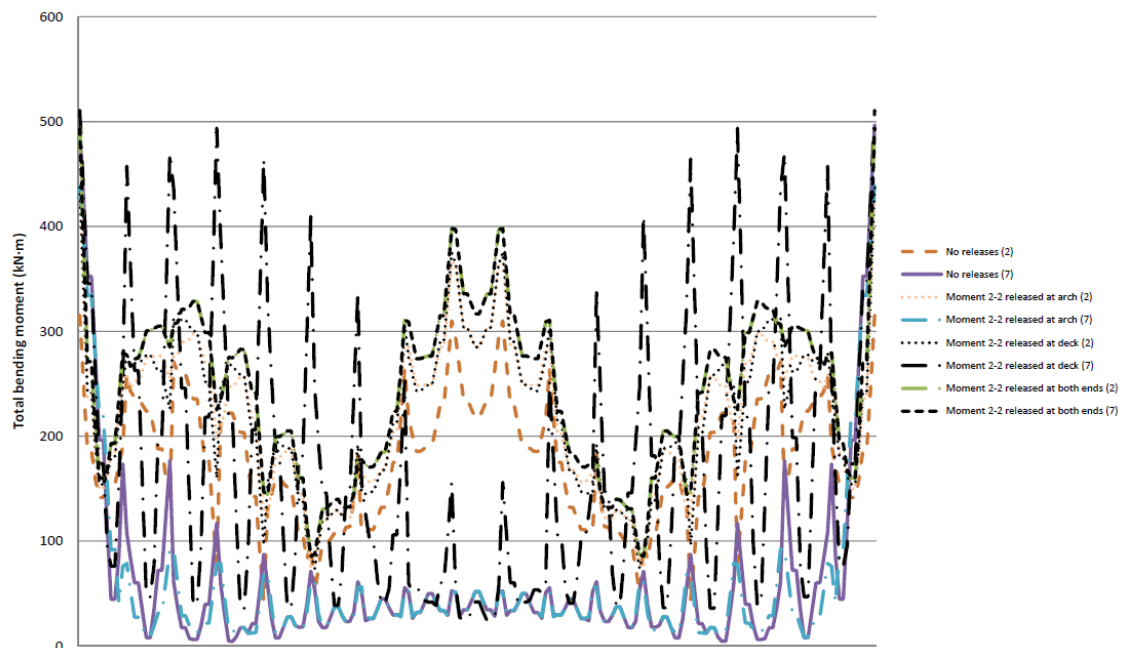
**ARCH TOTAL BENDING MOMENTS COMPARISON FOR 20m (f=L/5) SAG ARCHES WITH IMPOSED CURVATURE**



**Figure 3-10: Arch total bending moments comparison for different hangers' joints conditions and different hanger I2-2 rigidities**



**ARCH TOTAL BENDING MOMENTS COMPARISON FOR 20m (f=L/5) SAG  
ARCHES WITH IMPOSED CURVATURE**



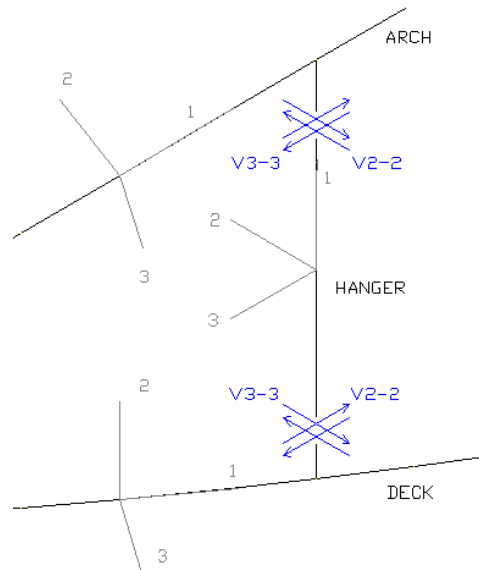
**Figure 3-11: Arch total bending moments comparison for different hangers' joints conditions and different hanger I2-2 rigidities**

**3.5.1 Explanation of the cause of this behaviour for model (2) and (7)**

If not released at joints, hangers will transmit vertical loads, shear forces and bending moments to arch and deck. The value of the 6 internal forces they can transmit changes when releasing one of them.

The change of the bending moments transmitted by the hangers when they are either fixed or released, does not transmit axial forces to the neither the arch nor deck. What causes the axial forces variation is the change of the shear forces at the hanger.

Positive V3-3 values tension the deck and compress the arch and the opposite happens with V2-2 positive values (Figure 3-12).



**Figure 3-12: Positive shear 3-3 and 2-2 forces.**

When M2-2 is released at both hanger ends, V2-2 negative shear forces value increases, especially at the extreme stiffer (due to their shorter length) hangers. This increases axial tensions at deck and compressions at the arch.

V3-3 shear positive forces values decrease, mainly at shorter, hence stiffer, hangers at extremes.

Therefore, for every model, whatever the mechanical properties of the sections employed for deck and hangers, releasing M2-2 gives the maximal axial forces on arch. However, when analysing the bending moments (see from Figure 3-8 to Figure 3-11), we can observe that eliminating the longitudinal hanger/deck interaction is not the best solution to minimize them. This means axial forces increase, but not because we are enhancing the antifunicular behaviour, but because we are increasing the horizontal rigidity of the system and, therefore, increasing the horizontal forces on the arch, which cause a decrease on the balcony beam forces, but lead too to an increase on the forces on the arch plane.

If M3-3 is released at hanger/deck joints, V2-2 becomes positive and larger in value and positive V3-3 increases greatly. This effect takes place at extreme hangers (not at central ones where both shear forces are small and negative). Consequently the deck has compression axial forces at its abutments and tensions at span center, and the arch is tensioned at springings.

When releasing M3-3 at hanger/arch joints, V3-3 shear forces hardly change and negative V2-2 shear forces remain negative and increase a bit in value but diminish considerably at extreme hangers compared to M2-2 released. Therefore, axial forces increase a bit at arch and deck, and the arch is all under compression and the deck tensioned, rather in the same manner as when hangers are fixed or M2-2 is released.

In conclusion, only releasing M3-3 at hanger/deck joints causes an important change in behaviour and it is due to the sign change of V2-2 at extreme stiff hangers.

The associated forces M3-3 and V2-2 transmitted by the hangers rule the arch and deck behaviour.

### 3.5.1.1 M3-3 behaviour of hangers:

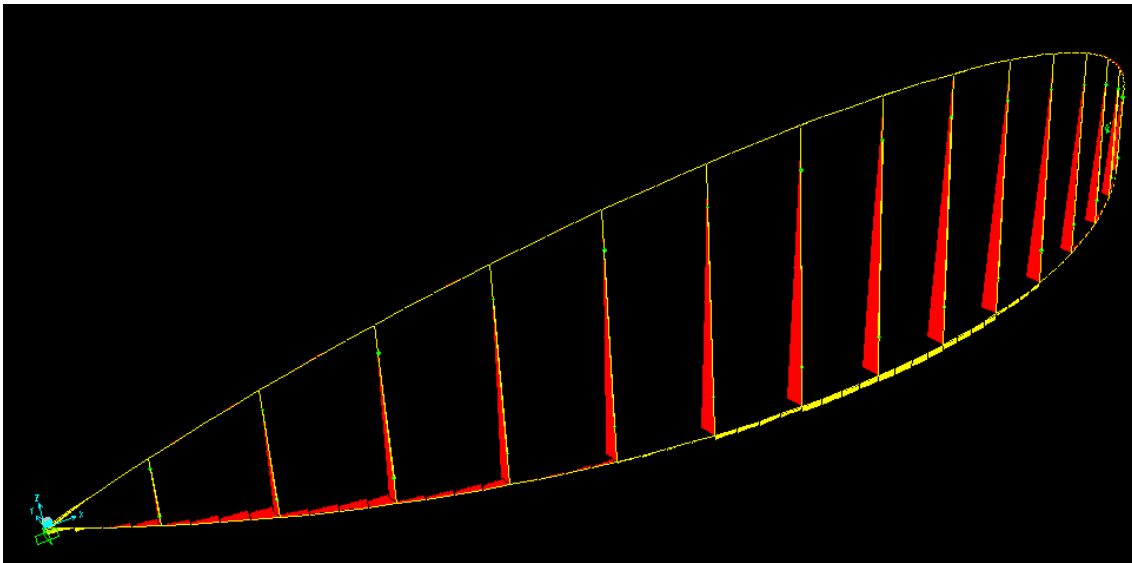


Figure 3-13: M3-3 under vertical loads for model (2) with hanger joints with M2-2 released at both ends and the rest of internal forces fixed

When we employ *hanger joints with M2-2 released at both ends and the rest of internal forces fixed*, the maximal value of M3-3 at hangers under vertical uniform loading of 10 kN/m for model (2) is obtained at the connection of the span center hangers with the deck and has 2000kN·m value (Figure 3-13).

The maximal value at the connection with the arch is much lower, with a value of 24 kN·m and is reached at the span center hangers too. However it can be considered constant at all hangers (21 kN·m is the lowest, at extreme hangers).

This joint configuration is the best one to diminish deck bending moments.

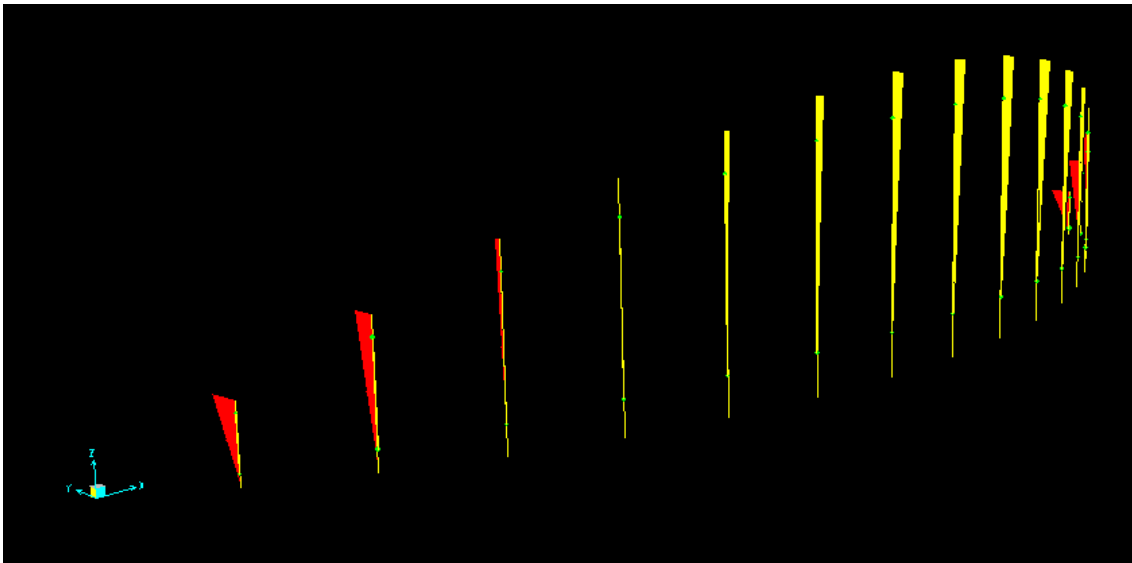
When the *hangers are completely fixed*, the behaviour of M3-3 at the hangers is very similar, and only slightly lower (maximal value is 1880 kN·m, obtained too at the connection of the span center hangers with the deck).

However, the maximal value at the connection with the arch is much higher, with a value of 91 kN·m and is reached at the second most extreme hanger. The lowest value is 17,6kN·m and is reached at the span center hangers. This joint configuration is then more favourable to diminish arch bending and torsional moments.

When the *bending and torsional moments are released at hanger/arch joints and only M2-2 is released at hanger/deck joints*, the M3-3 behaviour at hangers is very similar to hanger joints with M2-2 released at both ends and the rest of internal forces fixed. But, of course, with no bending moments transmitted to the arch.

When the *bending and torsional moments are released at hanger/deck joints and only M2-2 is released at hanger/arch joints*, the M3-3 behaviour at hangers is completely different to the cases exposed before (Figure 3-14), with greater values transmitted to the arch, and none to the deck. The extreme hangers transmit the maximal positive moment of 351 kN·m and the span center hangers the maximal negative one, of 140kN·m. Therefore, the hangers are introducing

positive torques at span center, which might vary the bending only slighter (because torque loads have only a small influence on the bending at span center). They are, however, introducing positive ones at springings, which will increase it.



**Figure 3-14: M3-3 at hangers under vertical loads for model (2) with bending and torsional moments released at hanger/deck joints and only M2-2 released at hanger/arch joints**

We can see at Figure 3-4 that the total bending moments on the arch behave as expected.

Maximal total bending moments are obtained for pinned hangers and are quite high too when M3-3 is released at hanger/deck joints.

For model (2), employing fixed hangers minimizes total bending moments at arch springings. The influence of short hangers at extremes can be clearly observed in Figure 3-14.

When M3-3 is released at hanger/arch joints, the lowest bending moments are obtained at the arch span center and very low ones too at springings. It is one of the best configurations to minimize total bending moments on the arch. The arch tends, therefore, to the antifunicular, resisting only axial compressions, which are nearly uniform at every cross section.

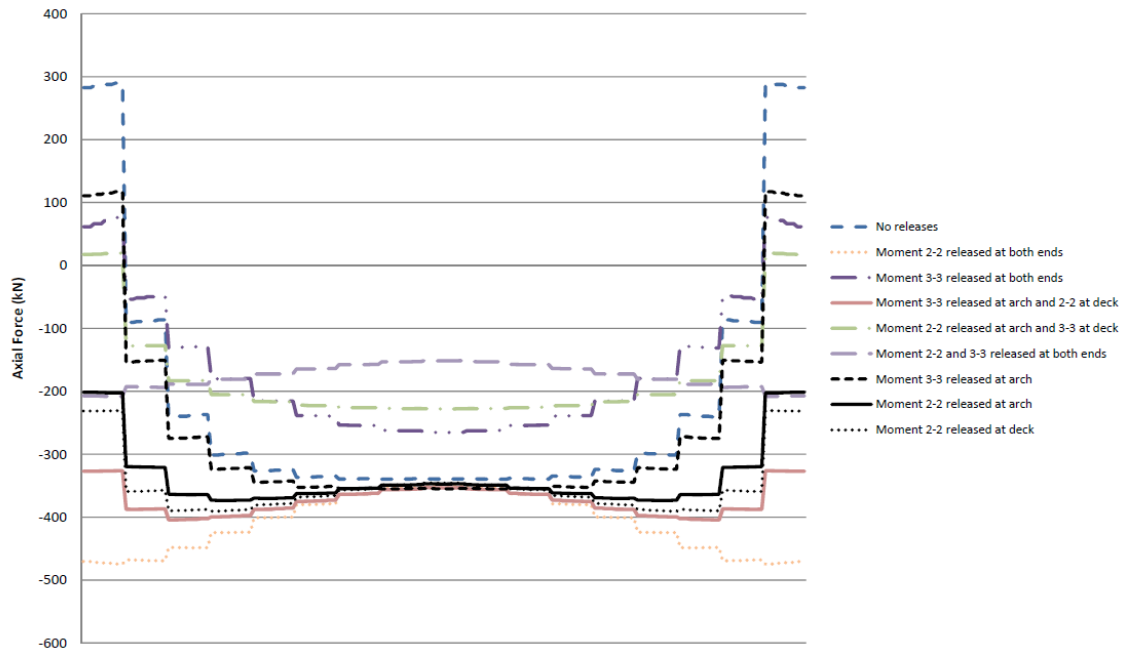
It is highly remarkable that for case (7) we would have expected V3-3 to increase when fixing M2-2 and causing the axial compression on the arch to increase too. However, it is not so, but quite the opposite, in spite of not diminishing as much as for model (2). This is because, if we compare the models with M2-2 released at hanger/deck joints and all other internal forces fixed to the model with fixed hangers for model (7), positive V3-3 at hangers nearly doubles but V2-2 more than doubles. When both effects are added they lead to axial forces in the arch to decrease.

### **3.6 Decks with a usual torsional rigidity (models (3), (4), (5) and (6))**

The behaviour of the arch and deck changes completely, compared to the one described when using a deck cross section with high torsional rigidity.

There are not two clearly differenced groups. In general, axial forces at extremes are lower than at span center and are even tensioned for many joint configurations (Figure 3-5).

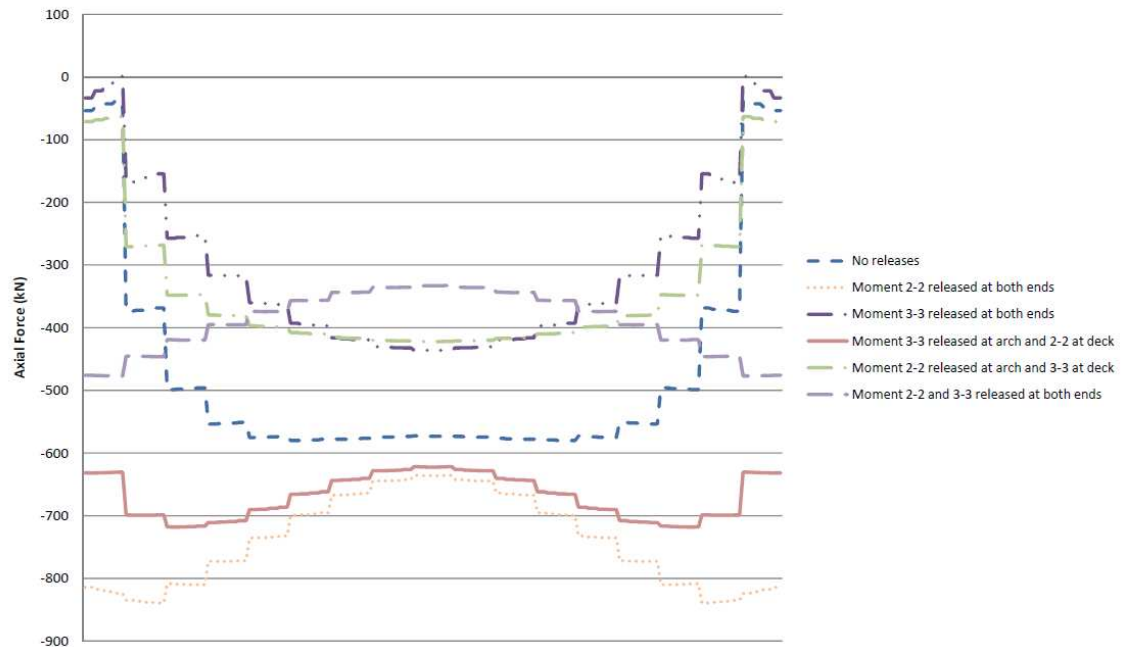
### ARCH AXIAL FORCES COMPARISON FOR 20m (f=L/5) SAG ARCHES WITH IMPOSED CURVATURE



**Figure 3-15: Arch axial forces comparison for model (3)**

Just by giving a small increase to the torsional rigidity value of the deck cross section and a lower bending moment rigidity (Figure 3-16) we obtain much more higher axial forces.

### ARCH AXIAL FORCES COMPARISON FOR 20m (f=L/5) SAG ARCHES WITH IMPOSED CURVATURE



**Figure 3-16: Arch axial forces comparison for model (4)**

The solution to obtain a more uniform distribution of axial forces at arch and deck is to release M2-2.

The cases of fixed hangers (Figure 3-20) or M3-3 released at hanger/arch joints, which gave good results for decks very rigid to torsion, causes tensions when employing a deck which is not as rigid to torsion.

We can appreciate on Figure 3-20, that the worst case is model (5). This is logical because it is the case in which the relationship between the hangers' longitudinal bending stiffness with the deck torsional stiffness is higher (being the rigidity relationship  $I2-2_H/J_D$ ). This effect is, however, only remarkable at extremes, where short hangers are stiffer. It is a similar effect to the observed on the comparison of models (2) and (7) on section 3.5.

In contrast, the model with the lowest  $I2-2_H/J_D$  and  $I3-3_H/I3-3_D$  relationship presents the highest axial compressions in the arch, because it enhances the hanger-deck transverse behaviour and minimizes the longitudinal one.

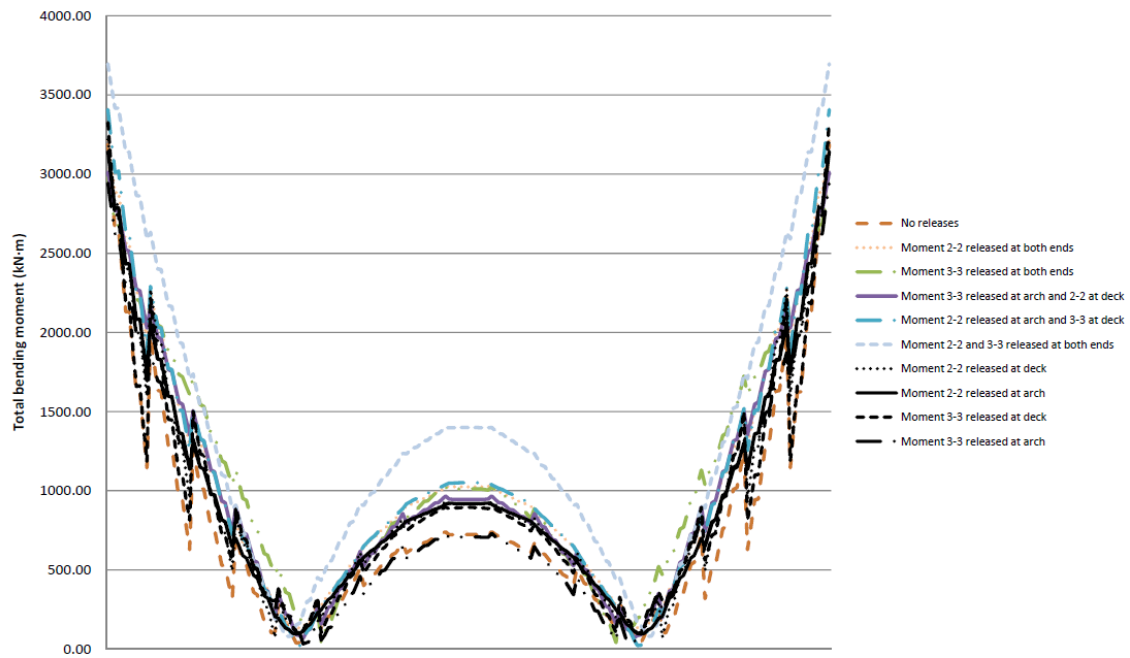
This is a very important conclusion. However, when analysing the bending moments, we can observe that eliminating the longitudinal hanger/deck interaction is not the best solution to minimize them. This means, as already observed for 3.5.1, that axial forces increase, but not because we are enhancing the antifunicular behaviour, but because we are increasing the horizontal rigidity of the system and, therefore, increasing the horizontal forces on the arch, which cause a decrease on the balcony beam forces, but lead too to an increase on the forces on the arch plane. This is the same which could be observed for model (2) compared to (7) (see from Figure 3-8 to Figure 3-11). Therefore, the described effect is the same whatever the deck torsional rigidity.

### **3.7 Comparison of models from (1) to (7)**

The behaviour of models (1) to (7) of Table 3-1 is compared in figures from Figure 3-17 to Figure 3-23.

For model (3), whereas axial forces behaviour was very different at springings for different hanger joints' configurations, for bending moments the influence of employing different hanger joint conditions is very low.

Although the difference is not important with other link conditions, the lowest bending moments are obtained for M3-3 released at hanger/arch joints (Figure 3-17).

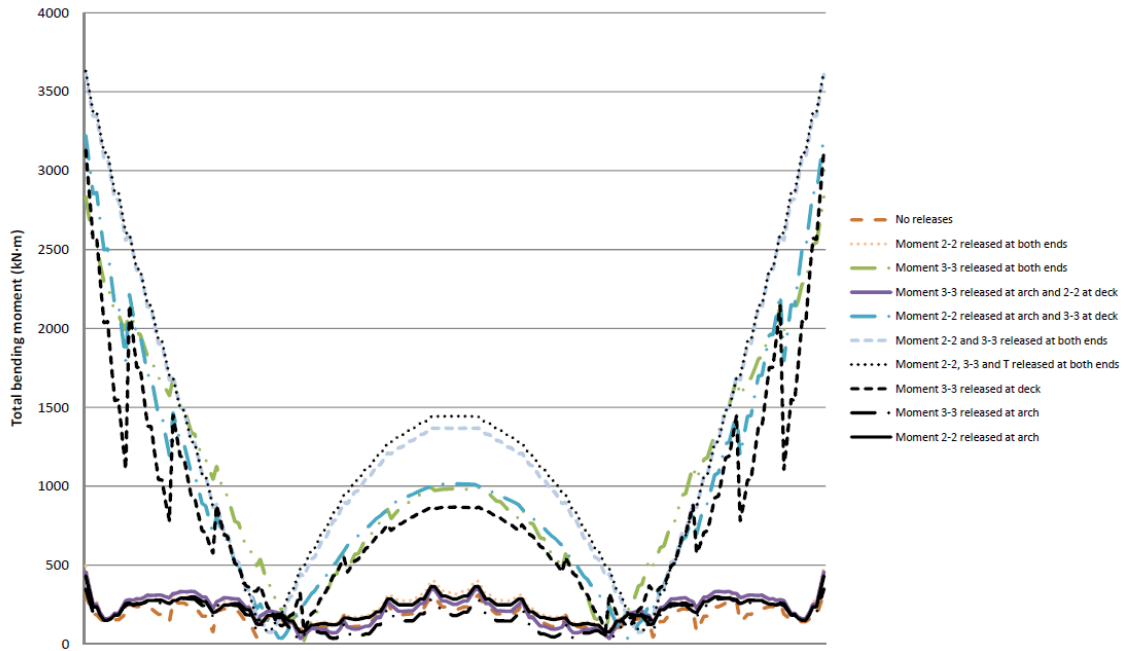


**Figure 3-17: Arch total bending moments comparison for model (3)**

For model (2) bending moments for different hanger joints' configurations are classified in two different big groups just as axial forces. If M3-3 is released at deck bending moments increase a lot. However, when releasing not only M3-3 at hanger/deck joints, but M2-2 too, moments increase even more and this could be even considered a third group (Figure 3-18).

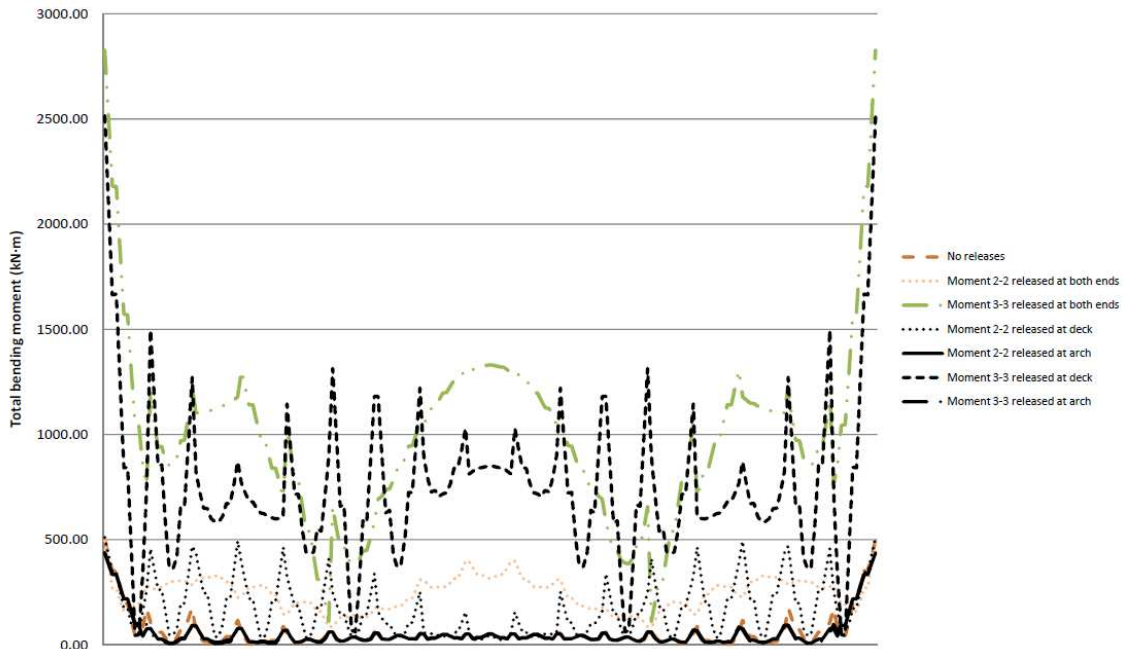
In contrast to model (3) the hanger joints' configuration has an important influence on bending moments, both at springings and span center.

Comparing it with model (3), it is only interesting to give a high torsional rigidity to the deck if we fix M3-3 on hanger/deck joints. When M3-3 is fixed, total bending moments values are much lower than for model (3). They are similar for different hanger link conditions, just as it happened with axial forces. The minimal total bending moments at springings is obtained for completely fixed hangers and at span center for only M3-3 released at hanger/arch joints.



**Figure 3-18: Arch total bending moments comparison for model (2)**

For model (7), bending moments for M2-2 released at hanger/arch joints coincide with those of the case of M3-3 released at hanger/arch joints (Figure 3-19). These configurations give the minimal total bending moments and they are much lower than for model (2). Therefore, giving longitudinal rigidity to the hangers reduces the bending moments and leads to antifunicularity.



**Figure 3-19: Arch total bending moments comparison for model (7)**

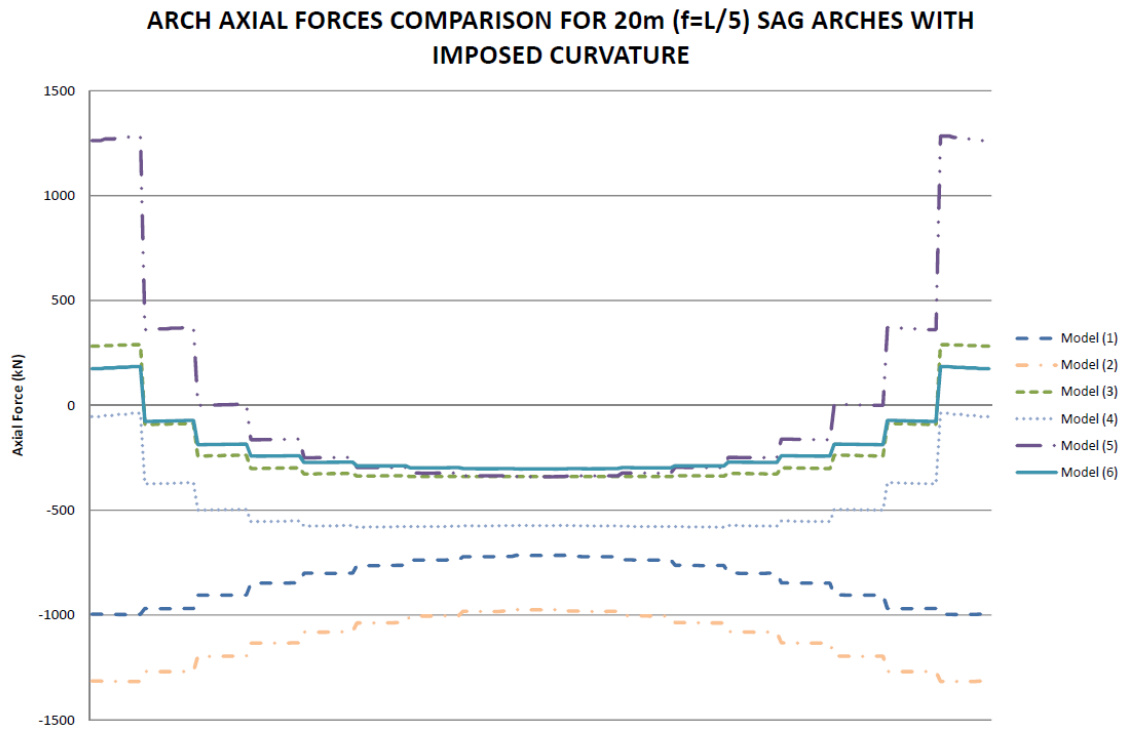
Whatever the mechanical properties of the cross-sections employed, the lowest bending moments are obtained for M3-3 released at hanger/arch joints. In spite of not giving the



maximal axial forces on arch, these link conditions are the best ones to tend to the antifunicular arch.

If we increase the torsional rigidity of the deck cross-section and fix M3-3 on hanger/deck joints, the total bending moment diminishes. And if M3-3 is released at hanger/arch joints with all other internal efforts fixed, increasing the hangers' cross-sectional longitudinal rigidity helps to minimize the total bending moments on the arch, and nearly cancel them.

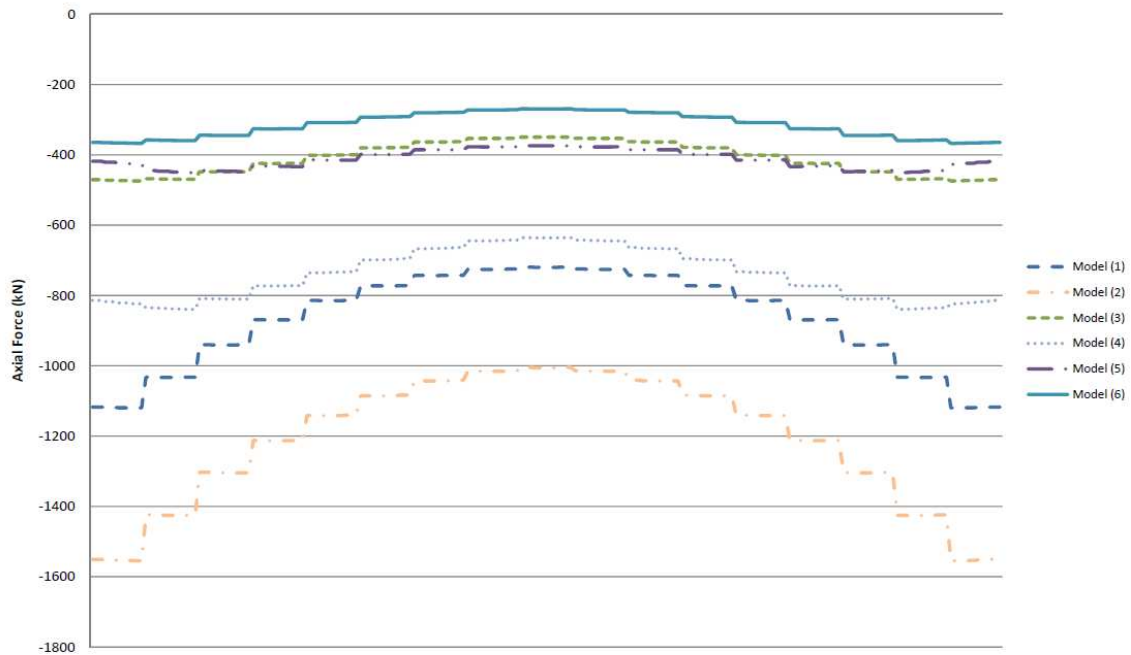
This effect can also be observed in the balcony-beam study at the beginning of this annex.



**Figure 3-20: Arch axial forces comparison of different models with completely fixed hangers**

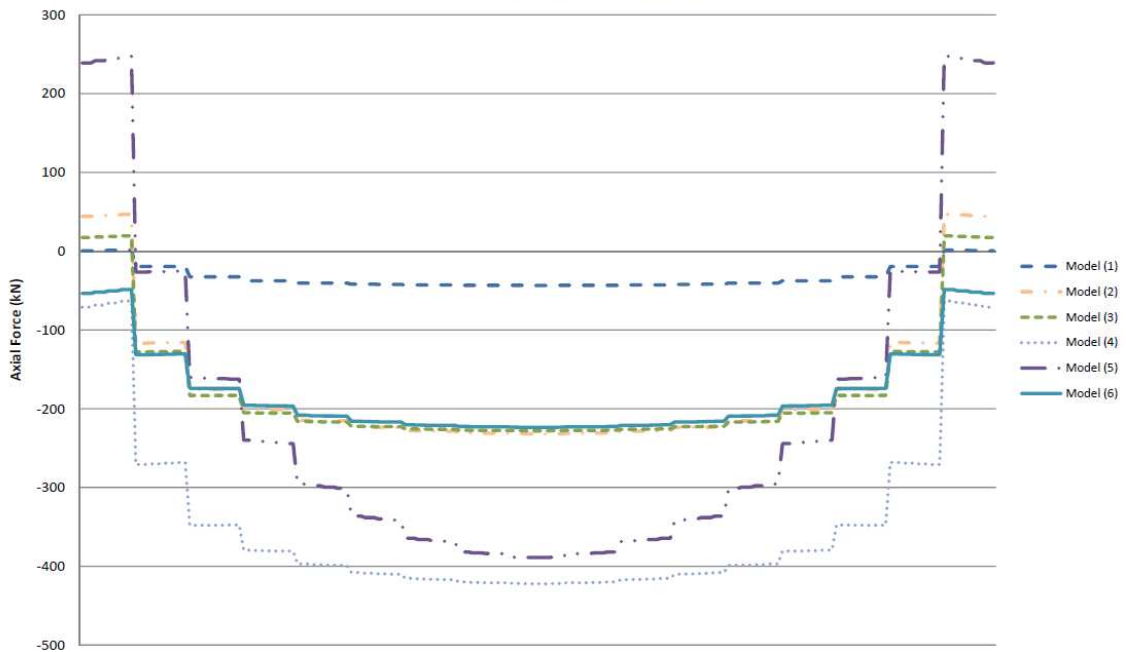
Releasing M2-2 is the only way that the axial force distribution has the same shape, in spite of employing different cross sections of deck and hangers (Figure 3-21). This is because the hangers have a higher longitudinal rigidity in proportion to the deck and compared to models (1) or (2).

**ARCH AXIAL FORCES COMPARISON FOR 20m (f=L/5) SAG ARCHES WITH IMPOSED CURVATURE**



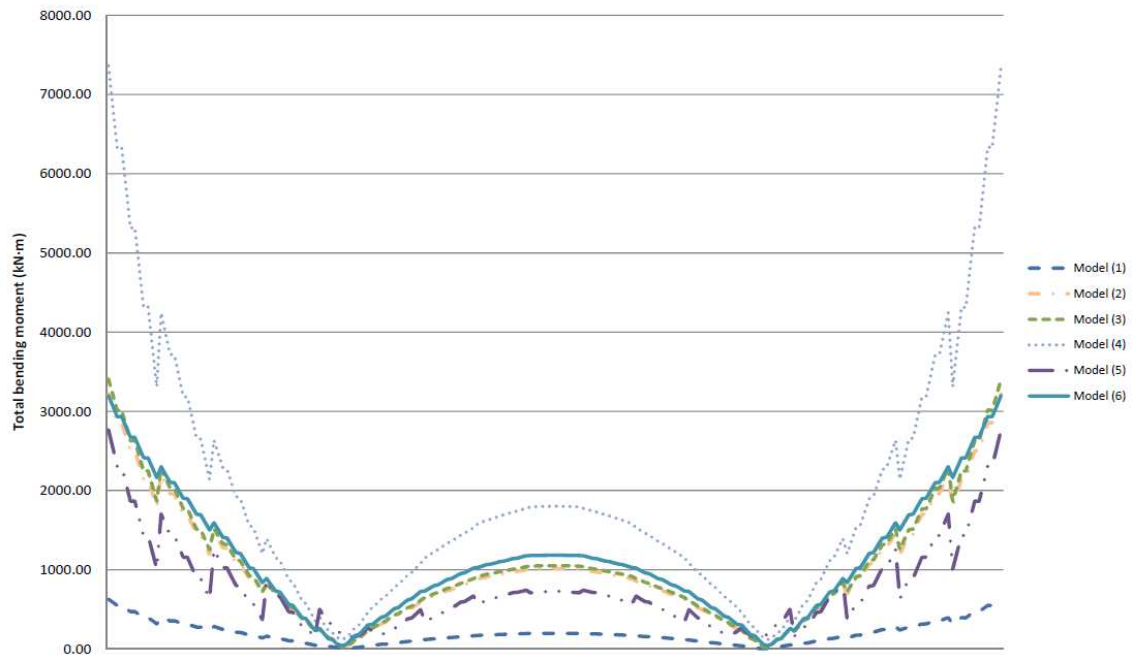
**Figure 3-21: Arch axial forces comparison of different models with M2-2 released at both ends of hangers**

**ARCH AXIAL FORCES COMPARISON FOR 20m (f=L/5) SAG ARCHES WITH IMPOSED CURVATURE**



**Figure 3-22: Axial forces comparison for M3-3 released at hanger/deck joints and M2-2 at hanger/arch joints**

**ARCH TOTAL BENDING MOMENTS COMPARISON FOR 20m (f=L/5) SAG  
ARCHES WITH IMPOSED CURVATURE**



**Figure 3-23: Total bending moments comparison for M3-3 released at hanger/deck joints and M2-2 at hanger/arch joints**

Note the importance of the deck longitudinal bending rigidity ( $I_{3-3_D}$ ) in order to diminish the total bending moment on the arch. Although it causes a decrease on the axial force on the arch too, it makes it more uniform.

On the present section the torques and moments on arch and deck caused by hangers on the arch and deck for an arch bridge under vertical deck loading have been analysed. This has been done for different hangers' joint configuration for models (1), (2), (3) and (6).

For models with hangers with a low longitudinal rigidity axial forces are higher at springings than at span center, unless M3-3 is released at deck.

On the contrary, for models with hangers with a high longitudinal rigidity, axial forces are higher at span center than at springings, unless M2-2 is released at both ends of hangers.

Minimal bending moments for model (2) on arch springings are obtained for fixed hangers and at span center, for M3-3 released at hanger/arch joints.

For model (7) they are obtained for M2-2 released at hanger/arch joints and the rest of forces fixed.

However, for cases with a high longitudinal rigidity of hangers very similar bending moments' results are obtained for the cases of fixed hangers, M2-2 released at hanger/arch joints, M3-3 released at hanger/arch joints and M2-2 released at hanger/arch joints. Therefore, fixing the hangers is the best configuration to tend to antifunicularity whatever the mechanical properties of cross-sections employed, if the deck has a high torsional rigidity.

### 3.7.1 Sign of torsion and moments transmitted by hangers on an arch bridge with vertical loading

According to the axis system on Figure 3-1:

A positive M3-3 on hanger causes a positive torsion on arch and a negative one on deck.

A positive M2-2 on hanger causes a positive M3-3 on arch and a negative one on deck.

It is interesting to introduce negative torques at arch, to reduce in plane bending moments (M3-3) and out of plane bending moments (M2-2) at springings and positive ones to reduce M2-2 and M3-3 at span center. According to this, it is favourable for the arch to have positive M3-3 on hangers at span center and negative ones at springings.

The torques and moments on arch and deck caused by each hangers' joint configuration have been analysed for different models (1), (2), (3) and (6) (from Table 3-2 to Table 3-5). We have marked their effect on the arch with the following legend:

|  |   |
|--|---|
|  | No effect because no torques are introduced |
|  | Favourable for the arch's springings        |
|  | Favourable on the whole arch                |
|  | Unfavourable                                |

| <i>MODEL (1)</i>                   | <i>Sign of torsion caused on arch</i>          | <i>Sign of moment caused on arch</i> | <i>Sign of torsion caused on deck</i> | <i>Sign of moment caused on deck</i> |
|------------------------------------|--|--------------------------------------|---------------------------------------|--------------------------------------|
| <i>No releases</i>                 | Negative                                       | Negative                             | Positive                              | Negative                             |
| <i>Moment 2-2 released</i>         | Negative*                                      | Null                                 | Positive                              | Null                                 |
| <i>Moment 2-2 released at arch</i> | Negative*                                      | Null                                 | Positive                              | Negative                             |
| <i>Moment 2-2 released at deck</i> | Negative                                       | Negative                             | Positive                              | Null                                 |
| <i>Moment 3-3 released</i>         | Null   | Negative                             | Null                                  | Negative                             |
| <i>Moment 3-3 released at arch</i> | Null   | Negative                             | Positive                              | Negative                             |
| <i>Moment 3-3 released at deck</i> | Positive at span center/Negative at springings | Negative                             | Null                                  | Negative                             |

\*Exception on shortest hangers (nearest to springings) and one at L/3 aprox

**Table 3-2: Sign of torsion and moments transmitted by hangers on an arch bridge with vertical loading for model (1)**

| <b>MODEL (2)</b>                                   | <i>Sign of torsion caused on arch</i>          | <i>Sign of moment caused on arch</i> | <i>Sign of torsion caused on deck</i> | <i>Sign of moment caused on deck</i> |
|--|--|--------------------------------------|---------------------------------------|--------------------------------------|
| <i>No releases</i>                                 | Negative                                       | Negative                             | Positive                              | Negative                             |
| <i>Moment 2-2 released</i>                         | Negative*                                      | Null                                 | Positive                              | Null                                 |
| <i>Moment 2-2 released at arch</i>                 | Negative*                                      | Null                                 | Positive                              | Negative                             |
| <i>Moment 2-2 released at deck</i>                 | Negative                                       | Negative                             | Positive                              | Null                                 |
| <i>Moment 3-3 released</i>                         | Null   | Negative                             | Null                                  | Negative                             |
| <i>Moment 3-3 released at arch</i>                 | Null   | Negative                             | Positive                              | Negative                             |
| <i>Moment 3-3 released at deck</i>                 | Positive at span center/Negative at springings | Negative                             | Null                                  | Negative                             |
| <i>Moment 3-3 released at arch and 2-2 at deck</i> | Null   | Negative                             | Positive                              | Null                                 |
| <i>Moment 2-2 released at arch and 3-3 at deck</i> | Positive at span center/Negative at springings | Null                                 | Null                                  | Negative                             |
| <i>Moment 2-2 and 3-3 released</i>                 | Null   | Null                                 | Null                                  | Null                                 |
| <i>Moment 2-2 and 3-3 released at arch</i>         | Null   | Null                                 | Positive                              | Negative                             |
| <i>Moment 2-2 and 3-3 released at deck</i>         | Positive at span center/Negative at springings | Negative                             | Null                                  | Null                                 |

\*Exception on shortest hangers (nearest to springings) and one at L/3 aprox

**Table 3-3: Sign of torsion and moments transmitted by hangers on an arch bridge with vertical loading for model (2)**

| <b>MODEL (3)</b>                   | <i>Sign of torsion caused on arch</i>           | <i>Sign of moment caused on arch</i> | <i>Sign of torsion caused on deck</i>          | <i>Sign of moment caused on deck</i> |
|------------------------------------|---|--------------------------------------|--|--------------------------------------|
| <i>No releases</i>                 | Positive at span center/Negative at springings  | Negative                             | Positive at span center/Negative at abutments  | Negative                             |
| <i>Moment 2-2 released</i>         | Positive at span center*/Negative at springings | Null                                 | Positive                                       | Null                                 |
| <i>Moment 2-2 released at arch</i> | Positive at span center*/Negative at springings | Null                                 | Positive at span center*/Negative at abutments | Negative                             |
| <i>Moment 2-2 released at deck</i> | Positive at span center/Negative at springings  | Negative                             | Positive                                       | Null                                 |
| <i>Moment 3-3 released</i>         | Null  | Negative                             | Null   | Negative                             |
| <i>Moment 3-3 released at arch</i> | Null  | Negative                             | Positive at span center/Negative at abutments  | Negative                             |
| <i>Moment 3-3 released at deck</i> | Positive at span center/Negative at springings  | Negative                             | Null   | Negative                             |

\*Longer range of positive span center values. Only 3 hangers of negative torsion near to springings and abutments.

**Table 3-4: Sign of torsion and moments transmitted by hangers on an arch bridge with vertical loading for model (3)**

| <b>MODEL (6)</b>                   | <i>Sign of torsion caused on arch</i>           | <i>Sign of moment caused on arch</i> | <i>Sign of torsion caused on deck</i>          | <i>Sign of moment caused on deck</i> |
|------------------------------------|---|--------------------------------------|--|--------------------------------------|
| <i>No releases</i>                 | Positive at span center/Negative at springings  | Negative                             | Positive at span center/Negative at abutments  | Negative                             |
| <i>Moment 2-2 released</i>         | Positive at span center*/Negative at springings | Null                                 | Positive                                       | Null                                 |
| <i>Moment 2-2 released at arch</i> | Positive at span center*/Negative at springings | Null                                 | Positive at span center*/Negative at abutments | Negative                             |
| <i>Moment 2-2 released at deck</i> | Positive at span center/Negative at springings  | Negative                             | Positive**                                     | Null                                 |
| <i>Moment 3-3 released</i>         | Null  | Negative                             | Null   | Negative                             |
| <i>Moment 3-3 released at arch</i> | Null  | Negative                             | Positive at span center/Negative at abutments  | Negative                             |
| <i>Moment 3-3 released at deck</i> | Positive at span center/Negative at springings  | Negative                             | Null   | Negative                             |

\*Longer range of positive span center values. Only 3 hangers of negative torsion near to springings and abutments.

\*\*Except the shortest hangers (nearest to abutments)

**Table 3-5: Sign of torsion and moments transmitted by hangers on an arch bridge with vertical loading for model (6)**

### 3.7.2 Influence of the hangers' bending rigidity

Although they have a much more low transverse rigidity than hangers employed at (3), hangers employed at (5) give approximately the same axial force. This means that this transverse rigidity value could be considered as “infinitely” rigid already.

Employing hangers with a less rigid cross-section leads to slightly lower axial forces. Although the change is not very important at span center, it is drastically different at springings (Figure 3-2 model (6) with hangers with half the diameter than (5))

At first, we could think that employing hangers with multiaxial symmetry helps obtaining a more uniform distribution of axial forces on the arch (Figure 3-2 model (5)). However, this is misleading, because it depends on the relationship of the hanger/deck stiffness and on the joint configuration too. Nevertheless, it is important to highlight that the longitudinal hanger/deck interaction is not negligible. An adequate hanger joint configuration, giving the hangers a certain longitudinal rigidity, can help as to uniformize internal forces on the arch.

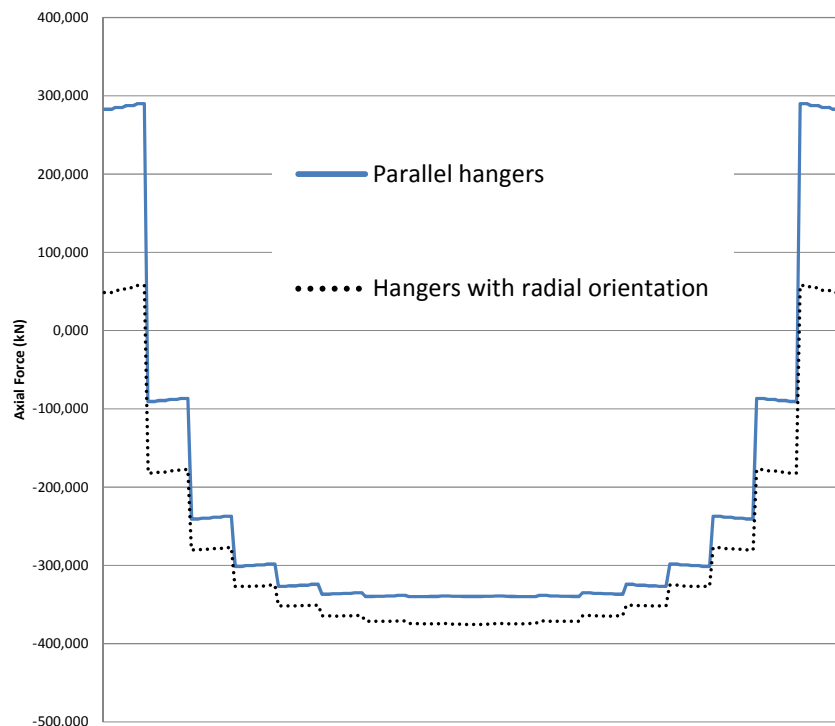
### 3.8 Hanger orientation

The hangers largest rigidity axis has been orientated perpendicularly or radially as explained in section 2.2 (see Figure 2-1 and Figure 2-2).

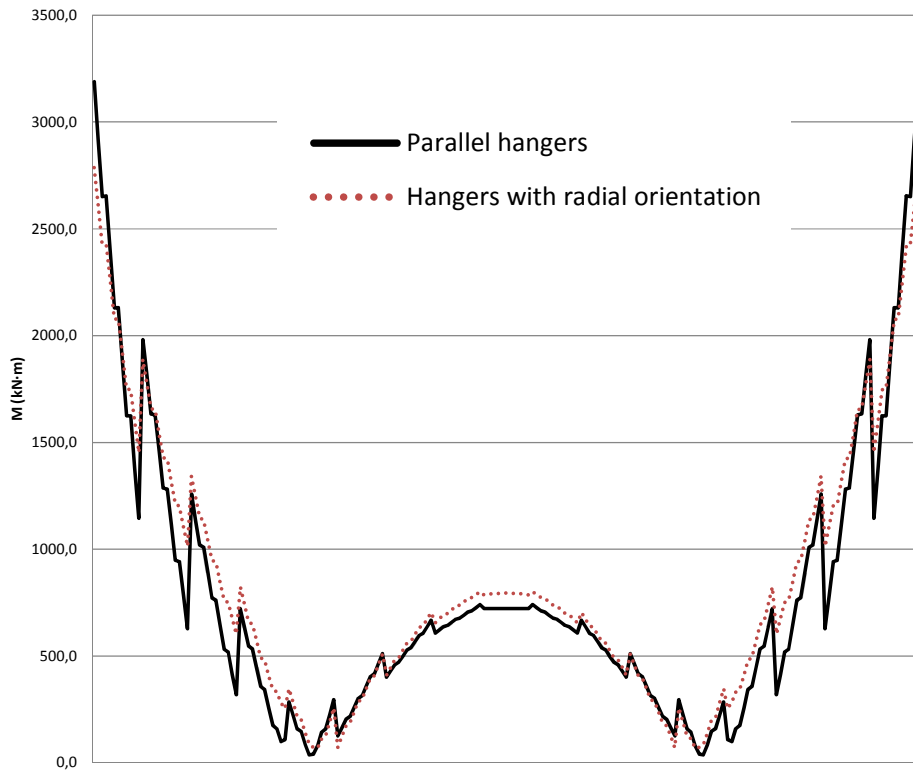
The model employed for this study is model (3) (see Table 3-1), with the hangers fixed at both ends, ie: at arch and deck.

The results of internal forces have been compared (see figures from Figure 3-24 to Figure 3-27).

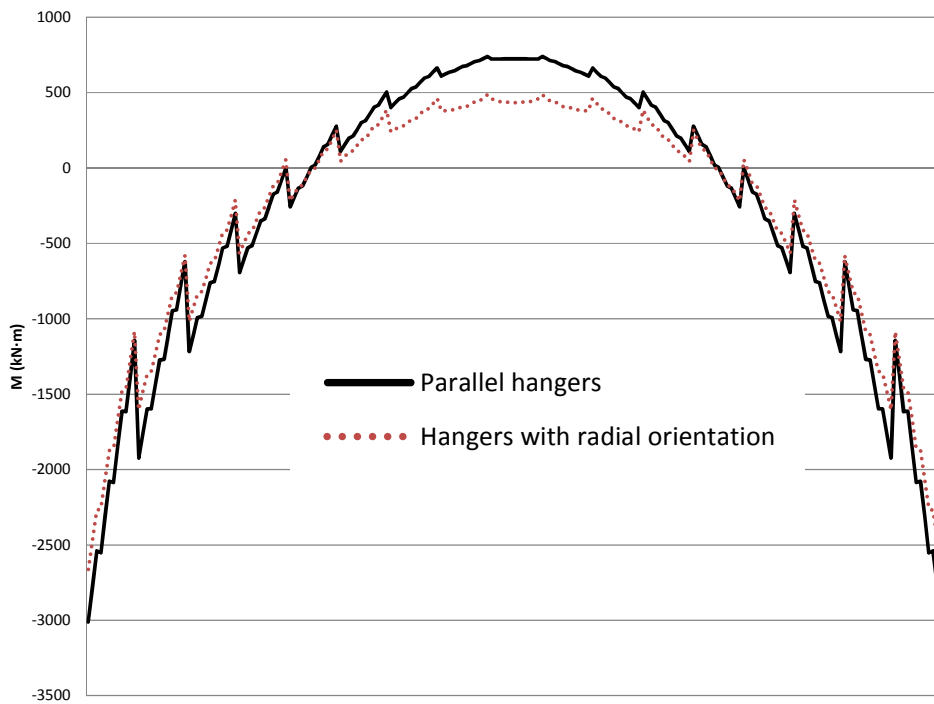
- Arch axial compression forces increase with hangers with a radial orientation (Figure 3-24).
- Arch total bending moments diminish at extremes and increase at the arch crown with hangers with a radial orientation (Figure 3-25).
- Arch in-plane bending moments diminish with hangers with a radial orientation (Figure 3-26).
- However, the out-of-plane behavior in general, seems better controlled with the hangers parallel orientation (Figure 3-27).



**Figure 3-24: Arch axial forces comparison for different hanger orientation**

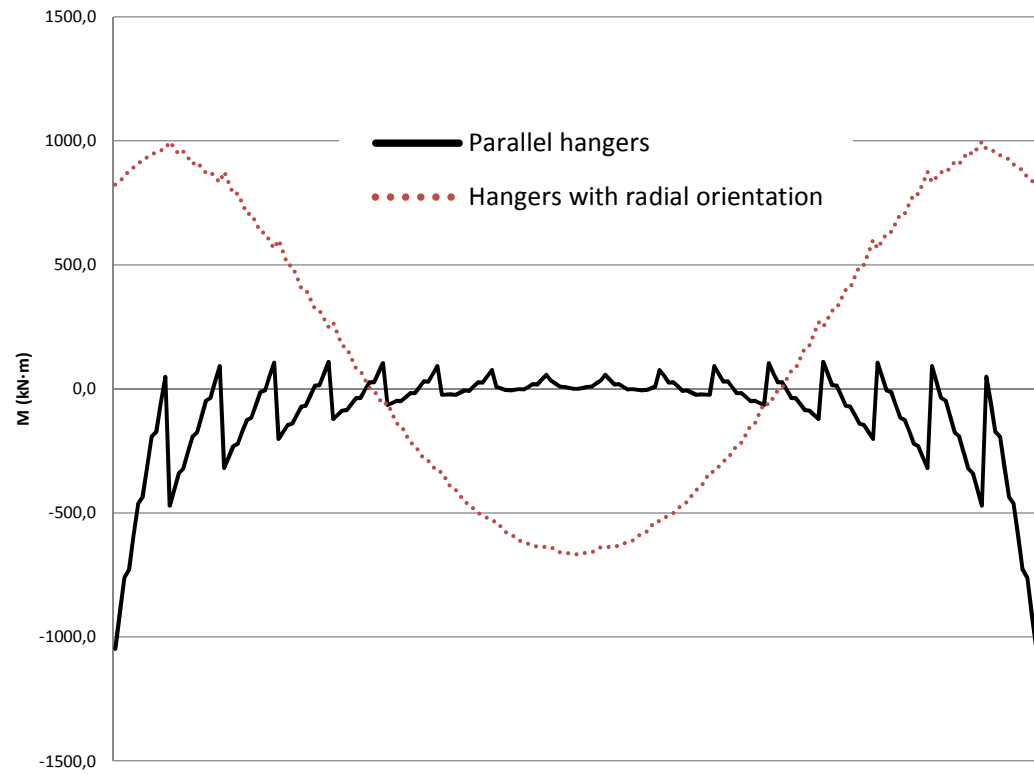


**Figure 3-25: Arch total bending moments comparison for different hanger orientation**



**Figure 3-26: Arch in-plane bending moments comparison for different hanger orientation**





**Figure 3-27: Arch total bending moments comparison for different hanger orientation**

### 3.9 Summary

The maximal axial forces, at both arch and deck, for every model are obtained when releasing M2-2 at both hanger ends (Figure 3-29). Shear forces transmitted by hangers increase, tensioning the deck and compressing the arch. Longitudinal bending and torsions are diminished through the transmission of transverse bending moments.

Longitudinal bending moments transmitted by hangers produce tension axial forces on the arch and compressions on the deck due to the change of shear forces. This effect is particularly enhanced at short extreme hangers, which have a higher stiffness, due to their smaller length.

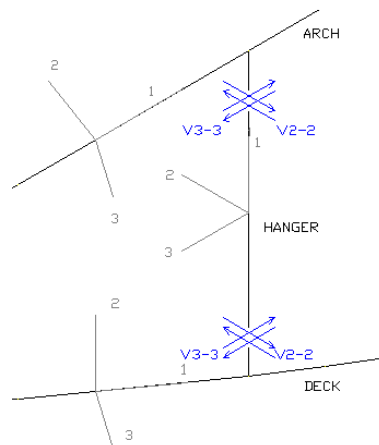
If not released at joints, hangers will transmit vertical loads, shear forces and bending moments to arch and deck. The value of the 6 internal forces they can transmit changes when releasing one of them.

The change of the bending moments transmitted by the hangers when they are either fixed or released, does not transmit axial forces to the neither the arch nor deck. What causes the axial forces variation is the change of the shear forces at the hanger.

Positive V3-3 values tension the deck and compress the arch and the opposite happens with V2-2 positive values (Figure 3-28).

Negative transverse shear forces values (V2-2) compensate out of plane bending moments (M2-2) and positive ones in plane bending moments (M3-3).

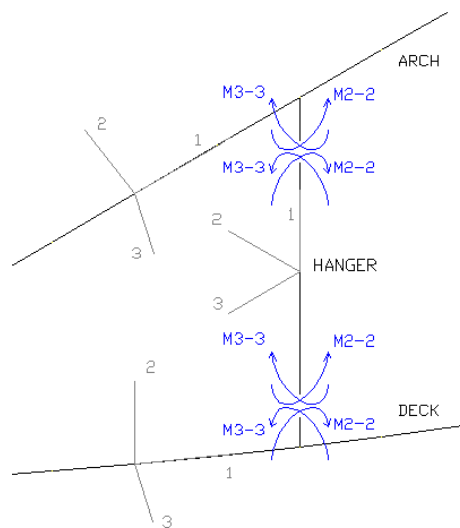
The most interesting is to diminish out of plane bending moments and increase axial forces, to enhance the arch behaviour. Therefore, we will be interested in hanger joint configuration which introduce negative  $V2-2$ .



**Figure 3-28: Positive shear forces at hangers and their transmission to arch and deck**

The definition of the bending moments at arch is the following:

- $M3-3$  are in plane bending moments
- $M2-2$  are out of plane bending moments



**Figure 3-29: Positive bending moments at hangers and their transmission to arch and deck**

According to the axis system on Figure 3-29:

- A positive  $M3-3$  on hanger causes a positive torsion on arch and a negative one on deck.
- A positive  $M2-2$  on hanger causes a positive  $M3-3$  on arch and a negative one on deck.

It is interesting to introduce negative torques at arch, to reduce in plane bending moments ( $M3-3$ ) and out of plane bending moments ( $M2-2$ ) at springings and positive ones to reduce  $M2-2$  and  $M3-3$  at span center. According to this, it is favourable for the arch to have positive  $M3-3$  on hangers at span center and negative ones at springings.

Longitudinal bending moments transmitted by hangers produce tension axial forces on the arch and compressions on the deck due to the change of shear forces. This effect is particularly enhanced at short extreme hangers, which have a higher stiffness, due to their smaller length.

When M2-2 is released at both hanger ends, V2-2 negative shear forces value increases, especially at the extreme stiffer (due to their shorter length) hangers. This increases axial tensions at deck and compressions at the arch. Shear forces transmitted by hangers increase, tensioning the deck and compressing the arch. Longitudinal bending and torsions are diminished through the transmission of transverse bending moments.

Therefore, for every model, whatever the mechanical properties of the sections employed for deck and hangers, releasing M2-2 gives the maximal axial forces on arch. However, when analysing the bending moments, we can observe that eliminating the longitudinal hanger/deck interaction is not the best solution to minimize them. This means axial forces increase, but not because we are enhancing the antifunicular behaviour, but because we are increasing the horizontal rigidity of the system and, therefore, increasing the horizontal forces on the arch. This horizontal forces increase not only causes a decrease on the balcony-beam forces, but also leads to an increase on the forces on the arch plane.

When employing what can be considered a deck with infinite torsional rigidity, the type of behaviour is divided into two big groups: M3-3 fixed at hanger/deck joints and M3-3 released at those joints.

When M3-3 is fixed at hanger/deck joints, the whole arch is under compression and the deck is tensioned.

When released, the axial forces diminish and the behaviour at the abutments and springings changes completely. The arch is slightly tensioned at springings.

As stated, the cases of fixed hangers or M3-3 released at hanger/arch joints give good results for decks very rigid to torsion. However, these configurations cause tensions when employing a deck which is not as rigid to torsion.

### **3.10 Conclusions**

- The maximal axial forces, at both arch and deck, for every model are obtained when releasing M2-2 (bending moments with transverse axis, Figure 3-29) at both hanger ends.
- The minimal total bending moment in the arch, when employing a rigid to torsion cross section, is obtained with M3-3 (bending moments with longitudinal axis, Figure 3-29) fixed at hanger/deck joints; and
- The maximal total bending moment in the arch corresponds to hangers pinned at both ends.

There are two important facts (apart from the joint configuration) to control the internal forces, as far as this study is concerned:

- 1) The torsional rigidity value of the deck cross-section controls the axial force magnitude in the arch

- 2) The bending stiffness of the hangers in relationship to the deck stiffness controls the shape of the axial forces because it controls the forces at arch abutments, due to the higher stiffness of short hangers

The vertical bending rigidity of the cross-section of the deck, does not have as much influence as the torsional one on axial forces. However, this cannot be fully assured, since the most rigid to bending section is 10 times bigger than the lowest one studied, whereas the torsional rigidity proportion of the studied sections is 100.

Moreover, it might help us to control bending.

## **REFERENCES**

APARICIO, A. C. "Los puentes curvos del enlace de Santamarca de la autopista de la Paz, en Madrid". *Hormigón y acero* n132, pp304-325, ATEP, Madrid, 1978



**N2.2. SIMPLIFIED MODEL OF A  
PINNED HANGER WITH  
SPRINGS MODELLING THE  
STIFFNESS OF THE ARCH**



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# 1. INTRODUCTION

This annex is a broadened and further detailed version of the section 4.2 of chapter IV.

A simple analytical model of a single pinned hanger (Figure 1) of a bridge with an arch inclined an angle  $\alpha$  with respect to the hanger has been studied. The model has three springs:  $K_1$ , for the in-plane stiffness of the inclined arch;  $K_2$ , for the balcony-beam stiffness of the arch and  $K_3$ , for the balcony-beam stiffness of the deck.

The axial stiffness of the hanger is  $K_H = EA_H / L_H$

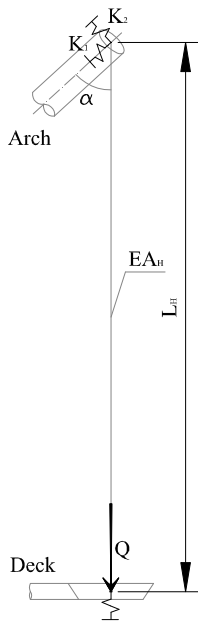


Figure 1

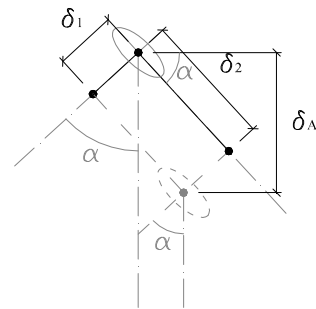


Figure 3

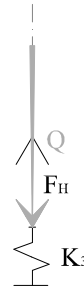


Figure 4

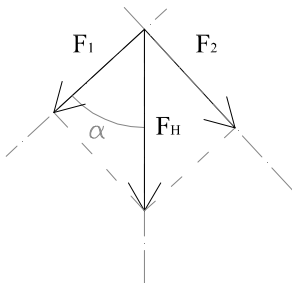


Figure 2

## 2. EQUATIONS DEVELOPMENT AND ANALYSIS

The model leads to the following equations:

$K_1$ : Axial stiffness of the arch

$K_2$ : Balcony – beam stiffness of the arch

$K_3$ : Vertical stiffness of the deck (flexural and torsional stiffness of the deck)

$K_H$ : Axial stiffness of the hanger ( $E A_H / L_H$ )

$$\delta_1 = \frac{F_1}{K_1} \quad \text{Eq. 1}$$

$$\delta_2 = \frac{F_2}{K_2} \quad \text{Eq. 2}$$

$$F_1 = F_H \cdot \cos\alpha \quad \text{Eq. 3}$$

$$F_2 = F_H \cdot \sin\alpha \quad \text{Eq. 4}$$

$$\delta_H = \delta_D - \delta_A = \frac{F_H}{K_H} \quad \text{Eq. 5}$$

$$\delta_D = \frac{Q - F_H}{K_3} \quad \text{Eq. 6}$$

$$\delta_A = \delta_2 \cdot \sin\alpha + \delta_1 \cdot \cos\alpha \quad \text{Eq. 7}$$

$$F_1 = \frac{Q}{\frac{1}{\cos\alpha} \left( 1 + \frac{K_3}{K_2} \sin^2\alpha + \frac{K_3}{K_1} \cos^2\alpha + \frac{K_3}{K_H} \right)} \quad \text{Eq. 8: Force in the plane of the arch}$$

The axial load taken by the arch ( $F_1$ , see Eq. 8) will increase with increases of  $K_1$ ,  $K_2$  or  $K_H$  or with decreases of  $\alpha$  or  $K_3$ .

$$F_2 = \frac{Q}{\frac{1}{\sin\alpha} \cdot \left( 1 + \frac{K_3}{K_2} \sin^2\alpha + \frac{K_3}{K_1} \cos^2\alpha + \frac{K_3}{K_H} \right)}$$

**Eq. 9: Force perpendicular to the plane of the arch**

$$F_D = Q \cdot \left( 1 - \frac{1}{1 + \frac{K_3}{K_2} \sin^2\alpha + \frac{K_3}{K_1} \cos^2\alpha + \frac{K_3}{K_H}} \right)$$

**Eq. 10: Force taken by the deck**

$$F_H = \frac{Q}{1 + \frac{K_3}{K_2} \sin^2\alpha + \frac{K_3}{K_1} \cos^2\alpha + \frac{K_3}{K_H}}$$

**Eq. 11: Force taken by the hanger**

From the analysis of the equations 8 to 10, we can observe:

- The non-dimensional term  $(K_3/K_1) \cdot \cos^2\alpha$  is much smaller than the rest, since the axial stiffness of the arch is significantly larger than the balcony beam stiffness of the arch. Therefore, the forces will not change significantly with variations of  $K_1$ .
- The non-dimensional term  $(K_3/K_H) \cdot \cos^2\alpha$  is much smaller than the rest when employing rigid hangers (profiles) and this term can be neglected, the forces taken by each element will then hardly vary with  $K_H$ . In the case of employing flexible hangers (stay-cables), this term cannot be neglected.
- For low  $K_H$  values, if we increase it,  $F_1$  and  $F_2$  will increase, therefore  $F_H$  will increase too and  $F_D$  will decrease. For a given arch  $K_1$  and  $K_2$  remain constant and, therefore,  $\delta_A$  increases (Eqs. 1, 2 and 7). However, we can observe that for a given deck,  $K_3$  remains constant, and, therefore,  $\delta_D$  decreases (Eq. 6). We can note that  $\delta_H$  decreases too, because  $F_H$  increases less than  $K_H$ .
- For high  $K_H$  values, the influence on the arch forces ( $F_1$  and  $F_2$ ) if we increase the axial stiffness of the hangers ( $K_H$ ) is so small, that they would remain constant.
- If we want to achieve antifunicularity, we necessarily need the condition  $F_2=0$ , in order to achieve no balcony-beam forces on the arch, which would cause bending moments.
- For a given  $\alpha$ , obtaining antifunicularity by playing with the different elements' stiffness is impossible when employing pinned hangers (Eq. 9), because  $F_2=0$  necessarily implies  $F_1=0$ , which means there is no arch behaviour.
- If we want to achieve null bending moments, we need to work with the hanger-arch and hanger-deck joint conditions.

Regarding displacements:

$$\begin{aligned}
\delta_A &= \frac{F_2}{K_2} \cdot \sin\alpha + \frac{F_1}{K_1} \cdot \cos\alpha \\
&= \frac{Q \cdot \sin^2\alpha}{\left(K_2 + K_3 \cdot \sin^2\alpha + \frac{K_2 \cdot K_3}{K_1} \cos^2\alpha + \frac{K_2 \cdot K_3}{K_H}\right)} \\
&+ \frac{Q \cdot \cos^2\alpha}{\left(K_1 + \frac{K_2 \cdot K_3}{K_1} \cdot \sin^2\alpha + K_3 \cdot \cos^2\alpha + \frac{K_2 \cdot K_3}{K_H}\right)} = \\
&= \frac{Q \cdot (K_1 \cdot \sin^2\alpha + K_2 \cdot \cos^2\alpha)}{\left(K_1 \cdot K_2 + K_1 \cdot K_3 \cdot \sin^2\alpha + K_2 \cdot K_3 \cdot \cos^2\alpha + \frac{K_1 \cdot K_2 \cdot K_3}{K_H}\right)}
\end{aligned}$$

**Eq. 12**

When diminishing the hanger stiffness ( $K_H$ ), ie: for example, employing flexible hangers, the arch displacements ( $\delta_A$ ) will decrease, but we should not forget of analysing how it might affect the deck displacements (Eq. 12).

In order to diminish the arch displacements ( $\delta_A$ ), the most efficient way when employing pinned hangers according to this simple analysis is to increase the stiffness of the deck (Eq 12).

$$\text{If } K_1 = K_2 \rightarrow \delta_A = \frac{Q}{\left(K_1 + K_3 + \frac{K_1 \cdot K_3}{K_H}\right)}$$

It can be logically proved:

$$\text{If } \alpha = 0 \rightarrow \left\{ \begin{array}{l} F_1 = \frac{Q}{\left(1 + \frac{K_3}{K_1} + \frac{K_3}{K_H}\right)} \\ F_2 = 0 \end{array} \right\}$$

Which is the classical vertical arch.

$$\text{If } \left\{ \begin{array}{l} K_H \gg K_3 \\ K_1 = K_2 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} F_1 = \frac{Q \cdot \cos \alpha}{\left(1 + \frac{K_3}{K_2}\right)} \\ F_2 = \frac{Q \cdot \sin \alpha}{\left(1 + \frac{K_3}{K_2}\right)} \end{array} \right\}$$

$$\text{If } \left\{ \begin{array}{l} K_1 \gg K_2 \\ K_H \gg K_3 \\ \alpha \gg 45^\circ \end{array} \right\} \rightarrow \left\{ \begin{array}{l} F_1 = \frac{Q \cdot \cos \alpha}{\left(1 + \frac{K_3}{K_2} \sin^2 \alpha\right)} \\ F_2 = \frac{Q \cdot \sin \alpha}{\left(1 + \frac{K_3}{K_2} \sin^2 \alpha\right)} \end{array} \right\}$$

Therefore, it is interesting to employ a very rigid deck and flexible archs and hangers, in order to diminish the balcony beam force on the arch ( $F_2$ ). At the same time, this will diminish the arch axial force ( $F_1$ ) too.

Given a high  $K_3$ , if we want to diminish the force on deck ( $F_D$ ), it will be interesting to increase the hanger stiffness ( $K_H$ ).

Differentiating Eq. 8, we obtain:

$$\frac{\partial F_1}{\partial \alpha} = \frac{-Q \cdot \sin \alpha \cdot \left[1 + \frac{K_3}{K_2} (1 + \cos^2 \alpha) - \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H}\right]}{\left(1 + \frac{K_3}{K_2} \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H}\right)^2}$$

**Eq. 13**

And considering Eq. 8:

$$\frac{Q \cdot \cos \alpha}{F_1} = \left(1 + \frac{K_3}{K_2} \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H}\right)$$

**Eq. 14**

we obtain:

$$\frac{\partial F_1}{\partial \alpha} = -\frac{F_1^2 \cdot \sin \alpha}{Q \cdot \cos^2 \alpha} \cdot \left[ 1 + \frac{K_3}{K_2} (1 + \cos^2 \alpha) - \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H} \right]$$

**Eq. 15**

$\frac{\partial F_1}{\partial \alpha}$  is always negative, since the non-dimensional term  $(K_3/K_1) \cdot \cos^2 \alpha$  is significantly smaller than the rest.

The larger the curvature of an imposed curvature bridge, the larger the angle  $\alpha$ . This results in a larger reduction of the axial force taken by the arch (Eq. 15).

Differentiating Eq. 9, we obtain:

$$\frac{\partial F_2}{\partial \alpha} = -\frac{\cos \alpha}{\sin \alpha} \cdot \frac{\partial F_1}{\partial \alpha}$$

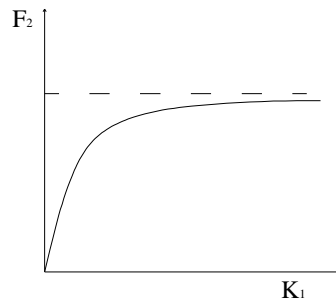
**Eq. 16**

$$\frac{\partial F_2}{\partial K_1} = \frac{Q \cdot K_3 \cdot \cos^2 \alpha \cdot \sin \alpha}{K_1^2 \cdot \left( 1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H} \right)^2} = \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\partial F_1}{\partial K_1}$$

**Eq. 17**

The higher the hangers' stiffness ( $K_H$ ) or the balcony beam stiffness of the arch ( $K_2$ ), the higher the influence of the axial stiffness of the arch ( $K_1$ ) on the balcony beam forces on the arch.

The higher the deck stiffness ( $K_3$ ), the axial stiffness of the arch ( $K_1$ ) or the curvature (measured as  $\alpha$ ), the lower the influence of the axial stiffness of the arch ( $K_1$ ) on the balcony beam forces on the arch. The  $K_1$ - $F_2$  relationship is of the type:



$$\frac{\partial F_2}{\partial K_2} = \frac{Q \cdot \sin^2 \alpha \cdot K_3}{K_2^2 \cdot \left( 1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H} \right)^2} = \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\partial F_1}{\partial K_2}$$

**Eq. 18**

We want to control  $\delta_2$ , therefore we will analyze the influence on it of the variation of different variables.

$$\frac{\partial \delta_2}{\partial K_1} = \frac{1}{K_2} \cdot \frac{\partial F_2}{\partial K_1} = \frac{Q \cdot K_3 \cdot \cos^2 \alpha \cdot \sin \alpha}{K_1^2 \cdot K_2 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H}\right)^2}$$

**Eq. 19**

$$\frac{\partial \delta_2}{\partial K_1} > 0 \rightarrow \uparrow K_1 \rightarrow \uparrow \delta_2$$

$$\left\{ \begin{array}{l} \uparrow K_1 \\ \uparrow K_3 \end{array} \right\} \rightarrow \downarrow \frac{\partial \delta_2}{\partial K_1}$$

$$\text{For } K_1 \gg \rightarrow \frac{\partial F_2}{\partial K_1}, \frac{\partial \delta_2}{\partial K_1} \cong \text{constant (0)}$$

Given a high enough axial stiffness of the arch, it does not influence the balcony beam forces or displacements of the arch.

$$\uparrow K_3 \rightarrow \downarrow \frac{\partial \delta_2}{\partial K_1}$$

$$\frac{\partial \delta_2}{\partial K_2} = \frac{Q \cdot \sin \alpha \cdot \left[ K_3 - K_2 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H}\right) \right]}{K_2^3 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H}\right)^2}$$

**Eq. 20**

$$\frac{\partial \delta_2}{\partial K_2} \begin{array}{l} ? \\ > 0 \\ < 0 \end{array}$$

$$\frac{\partial \delta_2}{\partial K_2} = \frac{Q \cdot \sin \alpha}{K_2^2 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H}\right)} \cdot \left[ \frac{K_3 \cdot \sin^2 \alpha}{K_2 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H}\right)} - 1 \right]$$

$A$   $B$

We can assure  $A > 0$ , but we do not know  $B$ .

$$\text{If } \left[ \frac{K_3 \cdot \sin^2 \alpha}{K_2 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H}\right)} - 1 \right] > 1 \rightarrow K_3 \cdot \sin^2 \alpha$$

$$> K_2 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha + \frac{K_3}{K_H}\right) \rightarrow B > 0 \rightarrow \frac{\partial \delta_2}{\partial K_2} > 0$$

If we suppose  $K_1 = K_3$ , we want to prove whether:

$$K_3 \cdot \sin^2 \alpha > K_2 \cdot \left(1 + \frac{K_3}{K_2} + \frac{K_3}{K_H}\right)$$

This can only be true for an imposed bridge with a lot of curvature, a very low  $K_2$  and a very rigid deck. The two first statements together do not have much sense and  $K_2$ , therefore we can consider:

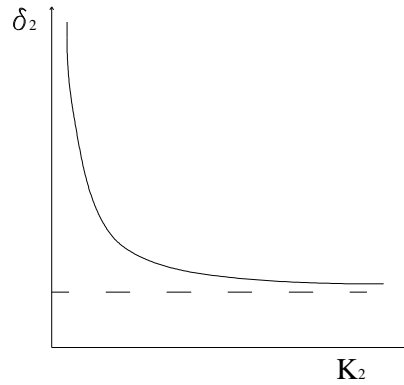
$$\frac{\partial \delta_2}{\partial K_2} < 0$$

Therefore, if we increase the balcony beam stiffness of the arch, the arch displacements diminish.

But we should note that in general our first hypothesis is not true and  $K_2 < K_1$ , so for very rigid decks, it might be so that:

$$\frac{\partial \delta_2}{\partial K_2} > 0$$

However, in general we can consider the  $K_2$ - $\delta_2$  relationship to be as following:



The higher the stiffness of the arch (both  $K_1$  and  $K_2$ ), the deck's stiffness ( $K_3$ ) or the hangers' stiffness ( $K_H$ ), ie: employing rigid hangers, the lower the influence of  $K_2$  on displacements.

Given a high enough  $K_2$  value, displacements will not vary any more when increasing it.

$$\lim_{K_2 \rightarrow \infty} \frac{\partial \delta_2}{\partial K_2} = 0$$

$$\lim_{K_1 \rightarrow \infty} \frac{\partial \delta_2}{\partial K_2} = Q \cdot \sin \alpha \cdot \frac{K_3 \cdot \sin^2 \alpha - K_2 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_H}\right)}{K_2^3 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_H}\right)^2} \begin{matrix} ? \\ > 0 \\ < \end{matrix}$$

$$\lim_{K_H \rightarrow \infty} \frac{\partial \delta_2}{\partial K_2} = Q \cdot \sin \alpha \cdot \frac{K_3 \cdot \sin^2 \alpha - K_2 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha\right)}{K_2^3 \cdot \left(1 + \frac{K_3}{K_2} \cdot \sin^2 \alpha + \frac{K_3}{K_1} \cos^2 \alpha\right)^2} \begin{matrix} ? \\ > 0 \\ < \end{matrix}$$

$$\lim_{K_3 \rightarrow \infty} \frac{\partial \delta_2}{\partial K_2} = \frac{Q \cdot \sin^3 \alpha}{K_2^3 \cdot \left(\frac{\sin^2 \alpha}{K_2} + \frac{\cos^2 \alpha}{K_1} + \frac{1}{K_H}\right)^2} > 0$$



### 3. CONCLUSIONS

The most relevant conclusions of this analytical model are the following:

- If  $F_2$  is represented as a function of the balcony beam stiffness of the arch ( $K_2$ ), for given values of  $\alpha$ ,  $K_3$ ,  $K_1$  and  $K_H$ , it can be observed that there is a bound for the balcony beam stiffness of the arch ( $K_2$ ) from which the contribution to the resistance of the arch ( $F_2$ ) does not increase.
- Increasing  $K_2$  enhances both the arch ( $F_1$ ) and the balcony beam ( $F_2$ ) mechanisms.
- Obtaining an antifunicular arch, by modifying the stiffness of the different elements of the system, is impossible because  $F_2=0$  (no balcony-beam forces in the arch) implies  $F_1=0$  (no arch behaviour). Therefore, if we want to eliminate the bending moments we need to work with the hanger-arch and hanger-deck joint connections or to employ an additional external system (such as the stay cables in Galindo Bridge, Figure 2) which prevents out-of-plane displacements in the arch.

We should note that, in reality, the systems will not be as simple as the models described before, since (1) the distribution of internal forces depends on the arch and deck individual behaviour (2) the hangers have different length and stiffness along the bridge and therefore transmit different forces.

N3. ANNEX RELATIVE TO V. A:

FIGURES OF THE STRUCTURAL  
BEHAVIOUR OF THE GEOMETRY AND  
BEARING CONDITIONS STUDY OF  
SPATIAL ARCH BRIDGES WITH A  
SUPERIOR DECK



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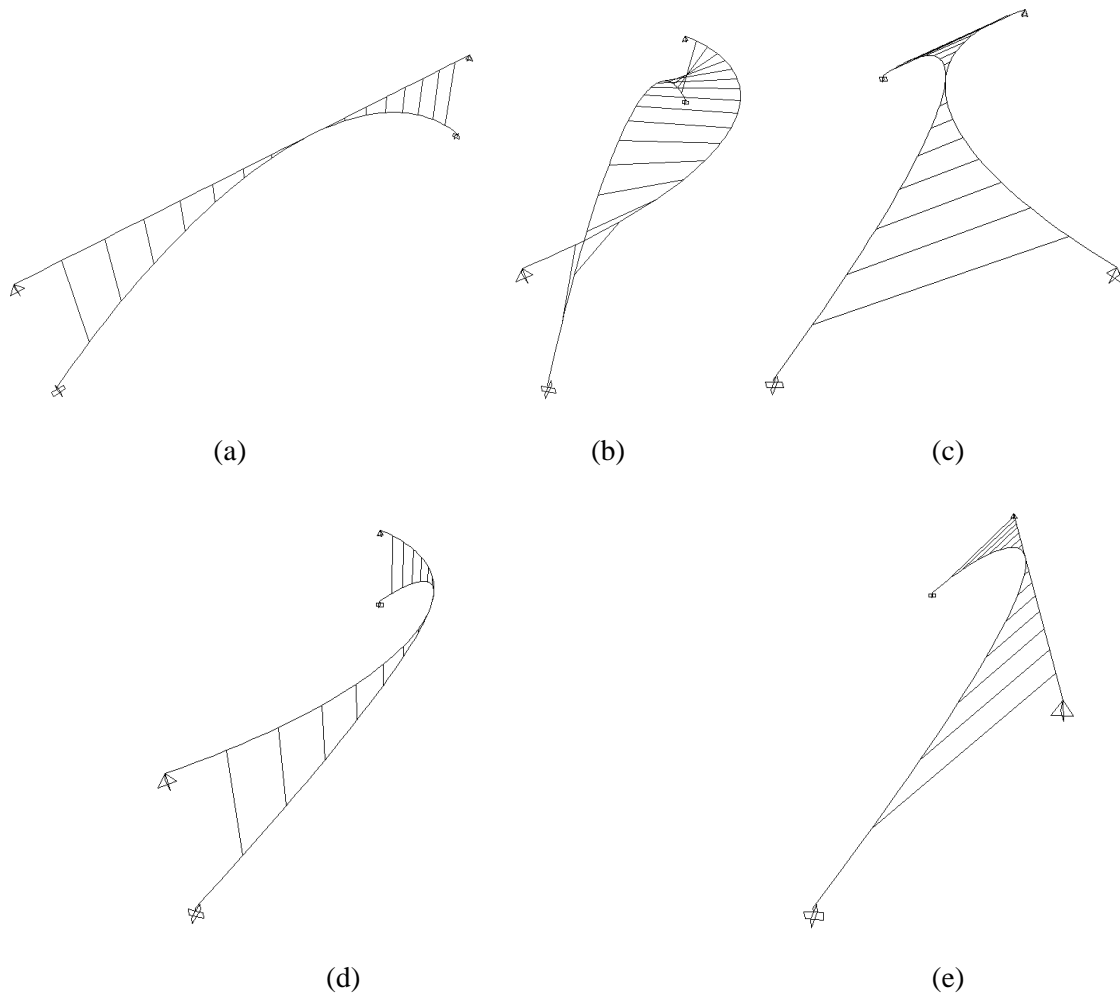
## 1. INTRODUCTION

This annex supports the main conclusions drawn in Chapter VA with data and results shown in Figures and Tables. The number of figures and tables was too high to include it in the chapter. Hence, the full results are detailed in this annex.

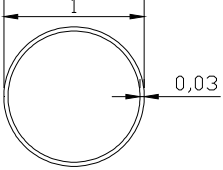
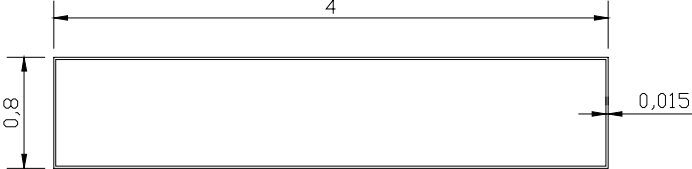
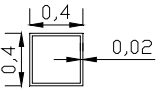
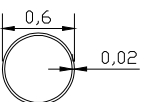
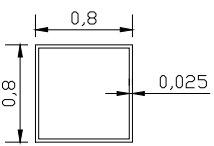
This annex is meant for the deeply interested reader, so that all the results of the research conducted for chapter VA are available and the comments in the aforementioned chapter can be verified.

Due to the large amount of figures and tables in this annex, an index of figures and another index of tables is presented at the end of the annex.

## 2. GEOMETRY AND CROSS-SECTIONS



**Figure 2-1- Studied bridge geometries. (a) Vertical planar arch bridge with superior straight deck (reference model); (b) Vertical planar arch with superior curved deck; (c) Arch and deck with symmetrical curvature in plan; (d) Arch and deck with coincident curvature in plan (imposed curvature); (e) Arch curved in plan with superior straight deck (both contained in the same plane)**

|        | Model 1  | Model 2   | Model 3   |
|--------|--|---|---|
| ARCH   |  |   |   |
| DECK   |  |   |   |
| STRUTS |   |  |  |

**Table 2-1: Cross-sections for different models**

Those bridge geometries have been studied for different hypothesis of the boundary conditions of the deck at its ends. The bending moments at both support sections are released.

- Longitudinal displacements (ie: tangential to the curve in plan) may be free or restrained (*f.l.d.* or *r.l.d.*).
- The twisting rotation may be free or restrained (*f.t.r.* or *r.t.r.*).

In every case study the arch springings are fixed and the struts are fixed to both arch and deck.

The analysis of the results is structured in two main sections:

- Structural response under a uniform vertical load
- Structural response under temperature variation

### 3. STRUCTURAL RESPONSE UNDER A UNIFORM VERTICAL LOAD

Different superior deck geometries and for different struts' rigidities have been studied in front of a uniform load of 10kN/m on the whole deck (Figure 3-1) and we have compared their structural response.

Results are divided into two hypotheses: free or restrained longitudinal movements at deck abutments.

Struts have always been considered fixed, unless a pinned struts hypothesis is specifically commented, in order to compare to the fixed struts case.

The arch is defined like a bent parabolic arch bridge (chapter IV). We have considered its spatial configuration with no simplifications for the frame model analysis. When analysing the results in this section we will refer to in-plane and out-of-plane forces, although the arch is not contained in a plane. We are referring to an approximation plane given by three points: arch springings and arch span center.

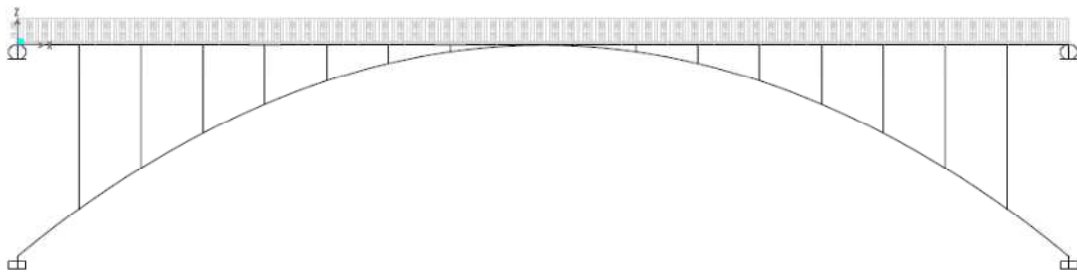


Figure 3-1: Load system: 10kN/m on the deck

#### 3.1 PLANAR VERTICAL ARCH WITH A STRAIGHT SUPERIOR DECK

On the following figures we can see the behaviour of model (a) with longitudinal displacements free at deck connection with the abutments and for cross-sections correspondent to model 1 of Table 2-1 as examples of the response under a 10kN/m uniform load on deck (without self-weight nor permanent loads).

We should pay special attention to the axial forces, because they show the force distribution between the arch and the deck. For grounds with low horizontal resistance, obtaining a bridge configuration which diminishes the arch axial force value will be important too, in order to decrease the horizontal forces transmitted at the arch springings to the ground.

Please refer to section 2 or employ the bookmark for the deck abutments boundary conditions nomenclature employed in the figures.



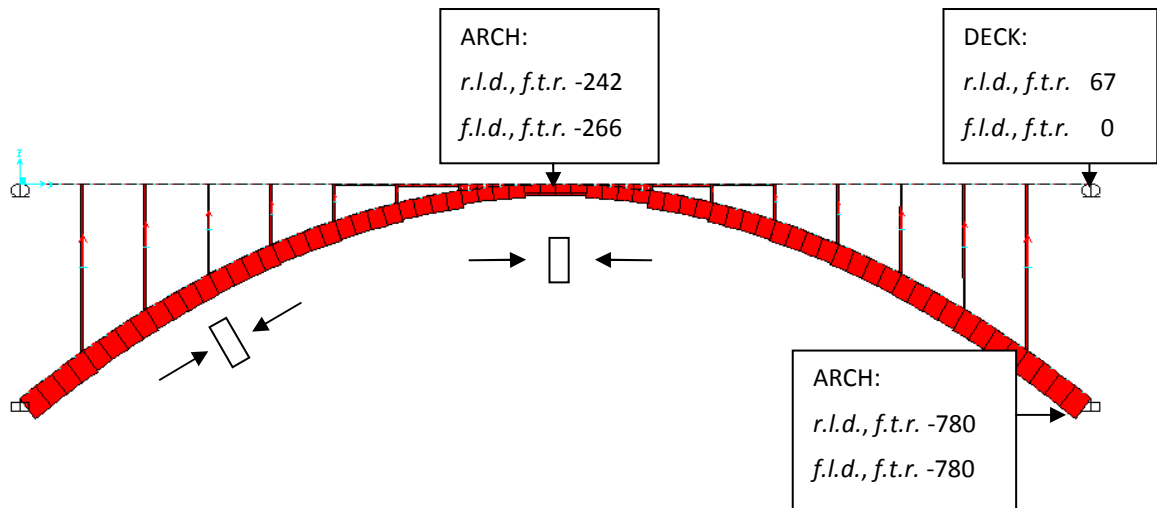


Figure 3-2- Model (a). Axial forces (in kN, compressions  $N < 0$ ) for different boundary conditions, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is the r.l.d. and f.t.r. case.

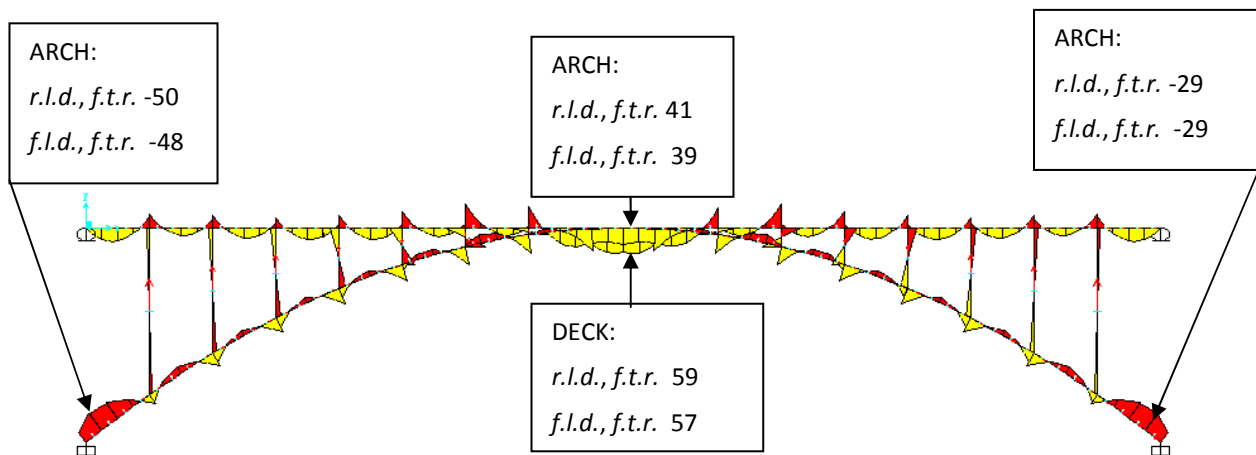


Figure 3-3- Model (a). M3-3 bending moments (in  $\text{kN}\cdot\text{m}$ ) for different boundary conditions of the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is the f.l.d. and f.t.r. case.

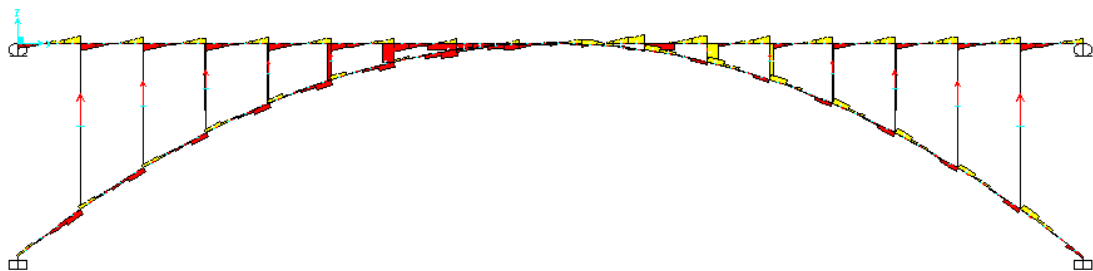


Figure 3-4- Model (a). Shear forces.

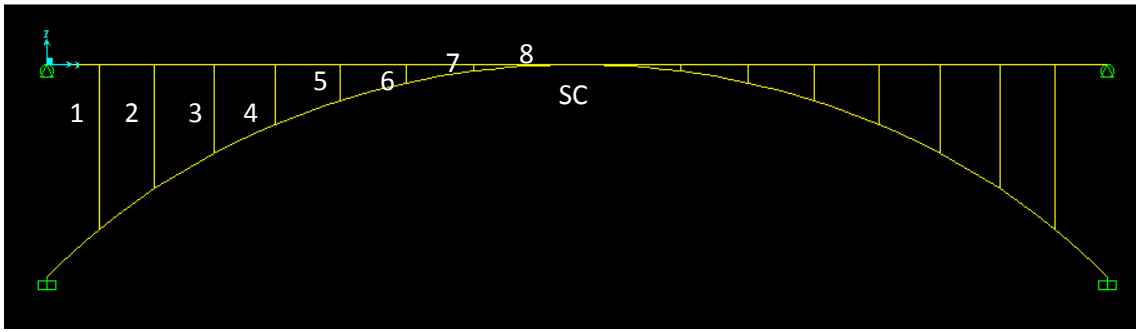


Figure 3-5: Numbering of elements

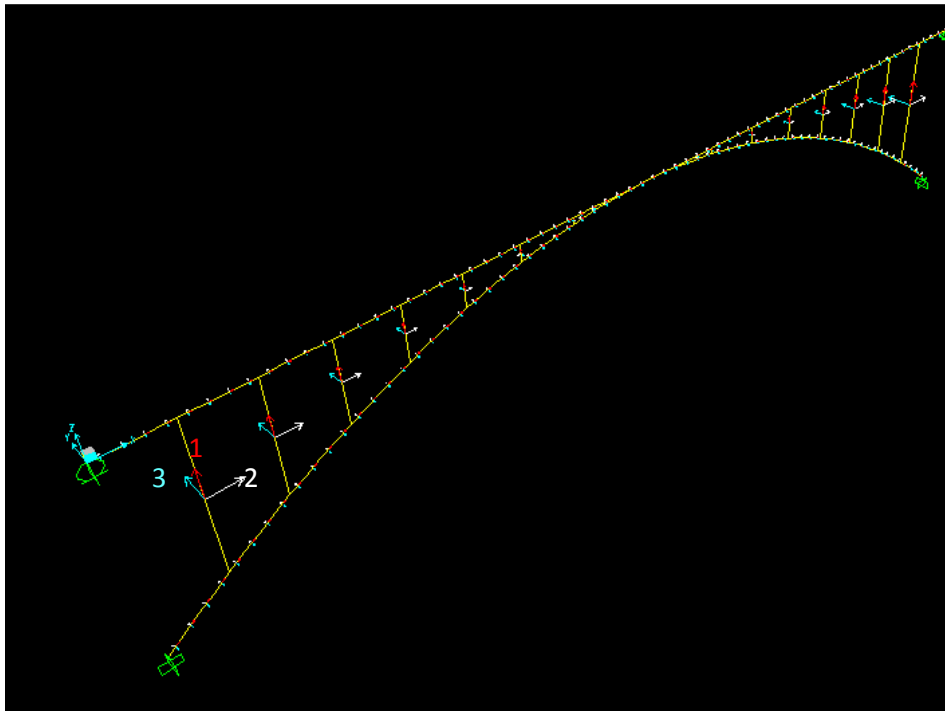


Figure 3-6: Model (a). Local axis at struts

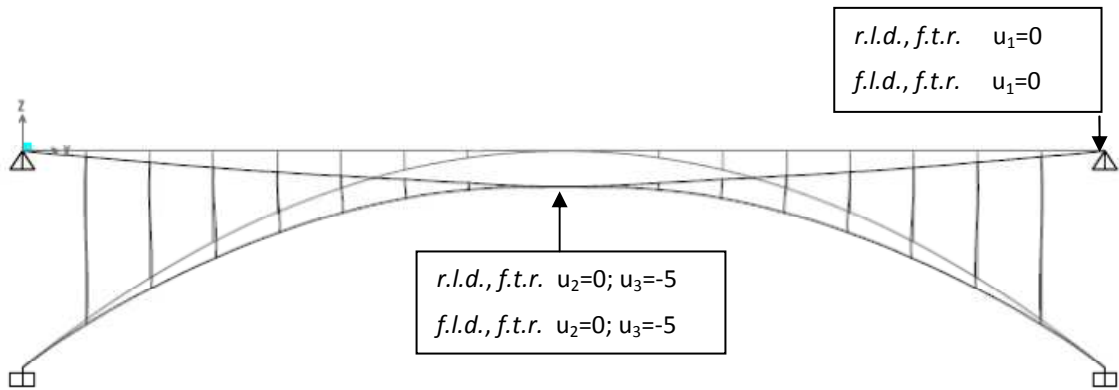
| Element  | Axial force (kN) | Shear 2-2 (kN) | Shear 3-3 (kN) |
|--|------------------|----------------|----------------|
| <b>Struts</b>  |                  |                |                |
| 1  | -65,6            | -0,77          | 0              |
| 2  | -59,8            | -1,54          | 0              |
| 3  | -58,4            | -3,07          | 0              |
| 4  | -59,5            | -6,71          | 0              |
| 5  | -60,3            | -17,43         | 0              |
| 6  | -63,3            | 53,71          | 0              |
| 7  | -61,6            | -137,20        | 0              |
| 8  | -35,4            | -133,05        | 0              |
| <b>Arch and deck axial forces at span center (kN)</b>              |                  |                |                |
| SC arch  |                  | -266,2         |                |
| SC deck  |                  | -352,7         |                |
| Sum of arch and deck axial forces                                  |                  | -618,9         |                |
| <b>Total horizontal reactions at abutments and springings (kN)</b> |                  |                |                |
| R <sub>x</sub> arch  |                  | 618,8          |                |
| R <sub>x</sub> deck  |                  | 0              |                |
| Sum of arch and deck horizontal reactions                          |                  | 618,8          |                |

**Table 3-1: Model (a). Shear forces at struts and span center under q10. Free deck longitudinal movements. See the numbering of struts in Figure 3-5**

If we observe the shear forces at struts (Table 3-1 and Table 3-2), we note that they are almost all negative. Hence, struts transmit tensions to the arch. If we pin the struts, the compressions in the arch increase only slightly at springings (from 780,2kN to 783,6kN), but more than double at span center (from 266,2 to 623,2kN). If we add the axial forces of arch and deck at span center for the fixed strut case we obtain a similar value to the pinned case value at arch span center (-618,9kN).

| Element  | Axial force (kN) | Shear 2-2 (kN) | Shear 3-3 (kN) |
|--|------------------|----------------|----------------|
| <b>Struts</b>  |                  |                |                |
| 1  | -65,6            | -0,81          | 0              |
| 2  | -59,8            | -1,62          | 0              |
| 3  | -58,4            | -3,22          | 0              |
| 4  | -59,6            | -7,07          | 0              |
| 5  | -60,4            | -18,40         | 0              |
| 6  | -63,5            | -56,96         | 0              |
| 7  | -61,9            | -146,36        | 0              |
| 8  | -34,7            | -142,88        | 0              |
| <b>Arch and deck axial forces at span center (kN)</b>              |                  |                |                |
| SC arch  |                  | -241,6         |                |
| SC deck  |                  | -309,9         |                |
| Sum of arch and deck axial forces                                  |                  | -551,5         |                |
| <b>Total horizontal reactions at abutments and springings (kN)</b> |                  |                |                |
| R <sub>x</sub> arch  |                  | 618,6          |                |
| R <sub>x</sub> deck  |                  | -66,7          |                |
| Sum of arch and deck horizontal reactions                          |                  | 551,9          |                |

**Table 3-2: Model (a). Shear forces at struts and span center under q10. Restrained deck longitudinal movements at abutments. See the numbering of struts in Figure 3-5**

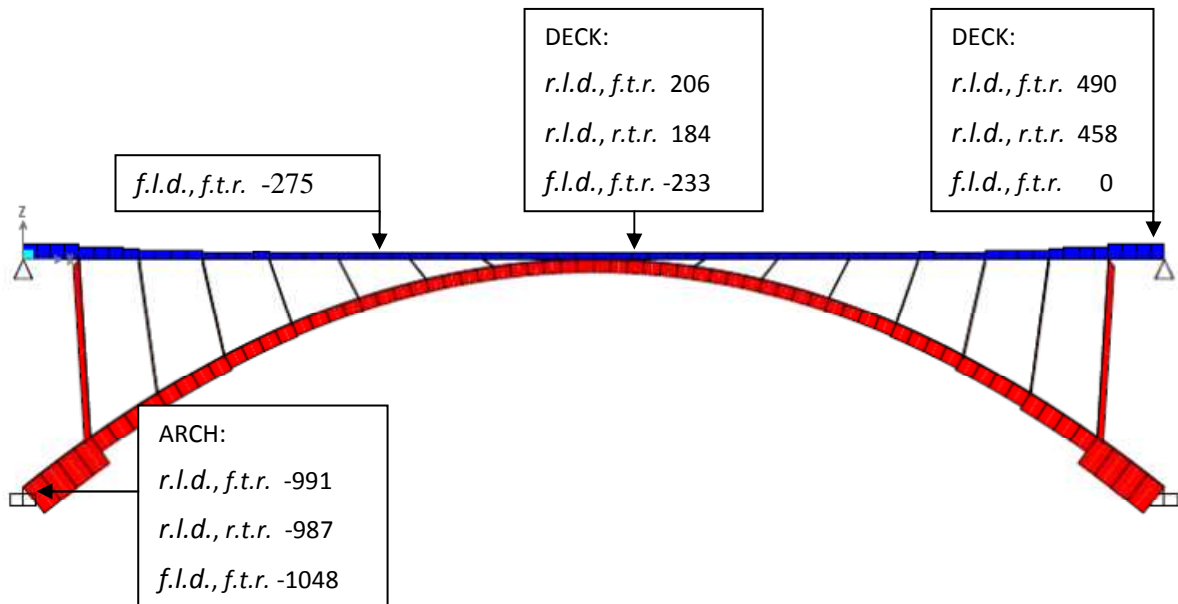


**Figure 3-7-Model (a). Displacements (in mm) for different boundary conditions of the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is the r.l.d. and f.t.r. case.**

When restraining the longitudinal displacements at the deck connection with abutments the forces hardly vary (compare Table 3-2 with Table 3-1). The deck is more tensioned (tensions at extremes appear because of the movement restrain, before they were only transmitted by the fixed struts) and axial compression forces at deck and arch span center diminish (Table 3-2), but the variations are negligible.

### 3.2 SPATIAL ARCH BRIDGES WITH A SUPERIOR DECK

On the following figures (from Figure 3-8 to Figure 3-24) we can see the behaviour of a superior deck spatial arch bridge models from b to e (Figure 2-1).



**Figure 3-8- Model (b). Axial forces (in kN, compressions  $N<0$ ) for different boundary conditions, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is the r.l.d. and f.t.r. case.**

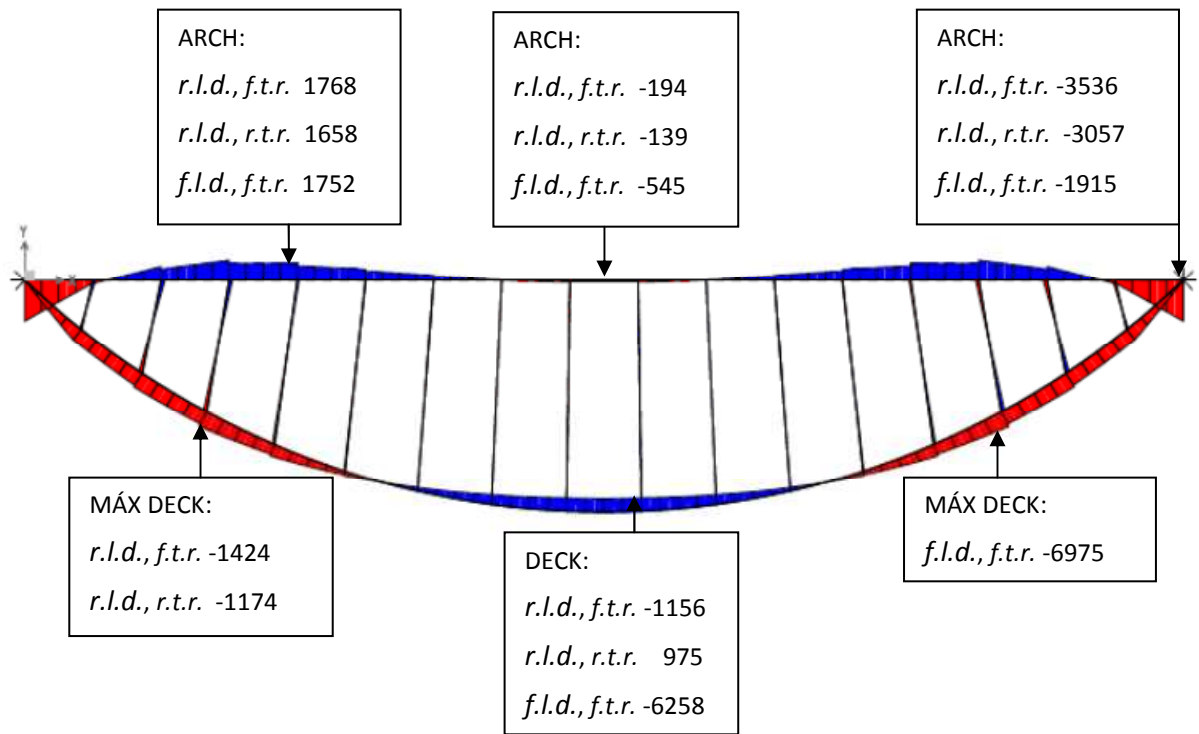


Figure 3-9- Model (b). M2-2 bending moments (in kN·m) for different boundary conditions of the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is plotted for the restrained longitudinal displacements (r.l.d.) and restrained twisting rotations (r.t.r.) case

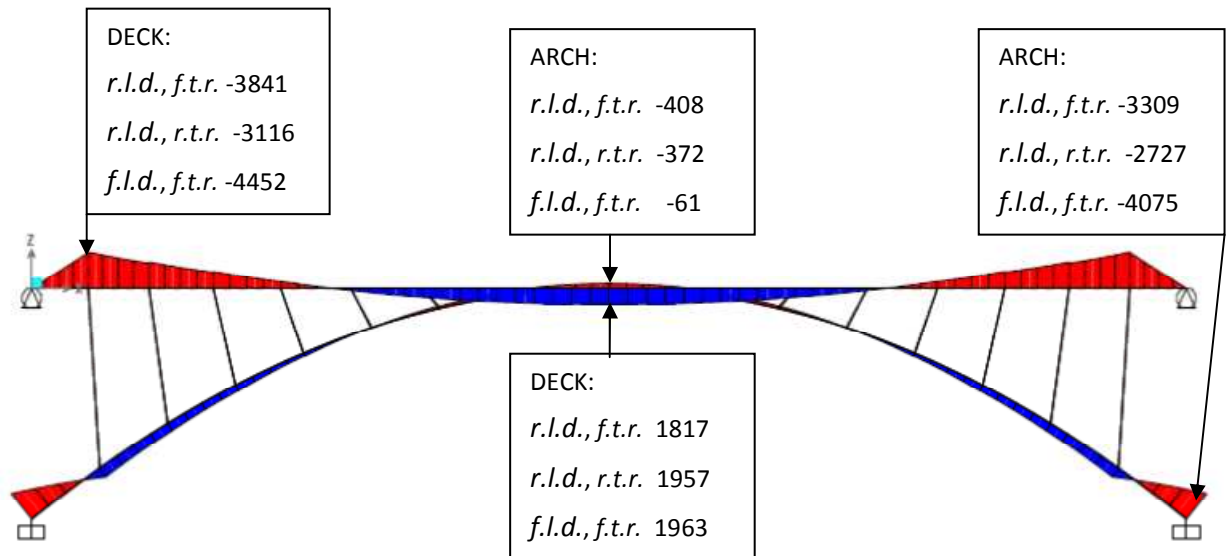


Figure 3-10: Model (b). M3-3 bending moments (in kN·m) for different boundary conditions of the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is the f.l.d. and f.t.r. case.

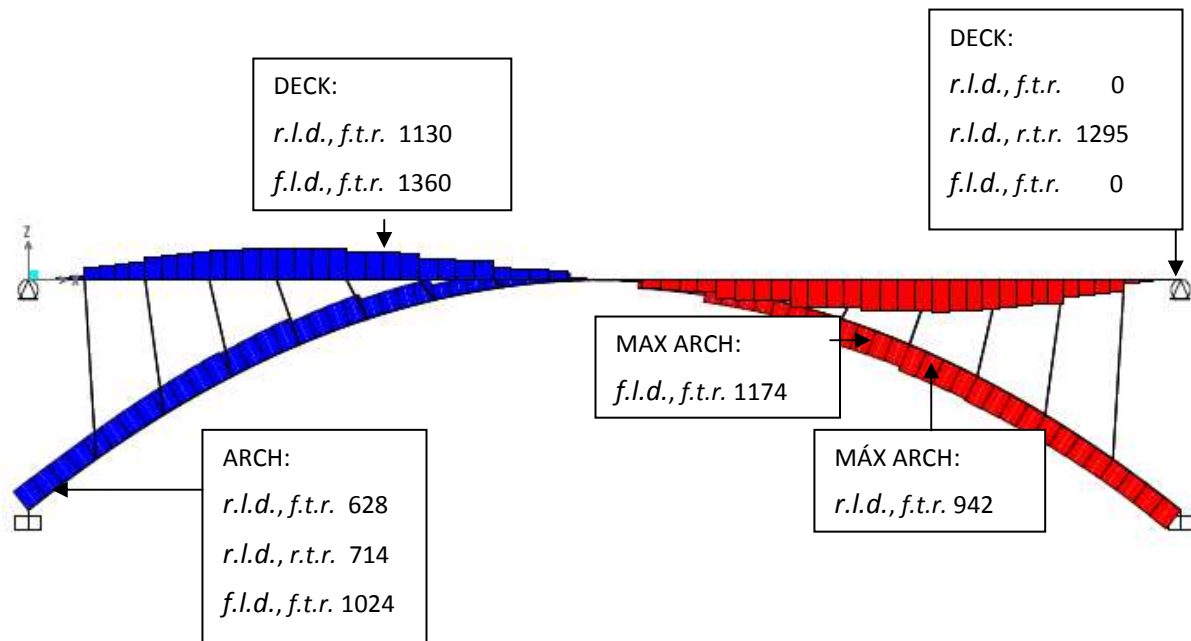


Figure 3-11- 1. Model (b). Torsional moments (in kN·m) for different boundary conditions of the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is the f.l.d. and f.t.r. case.

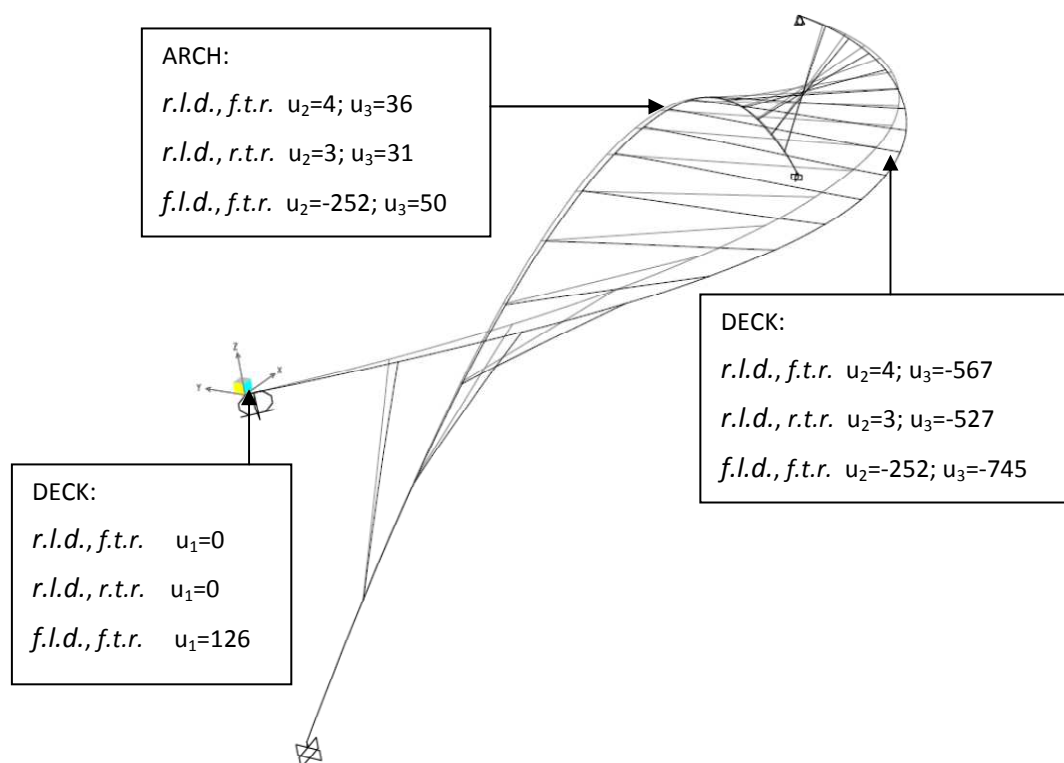
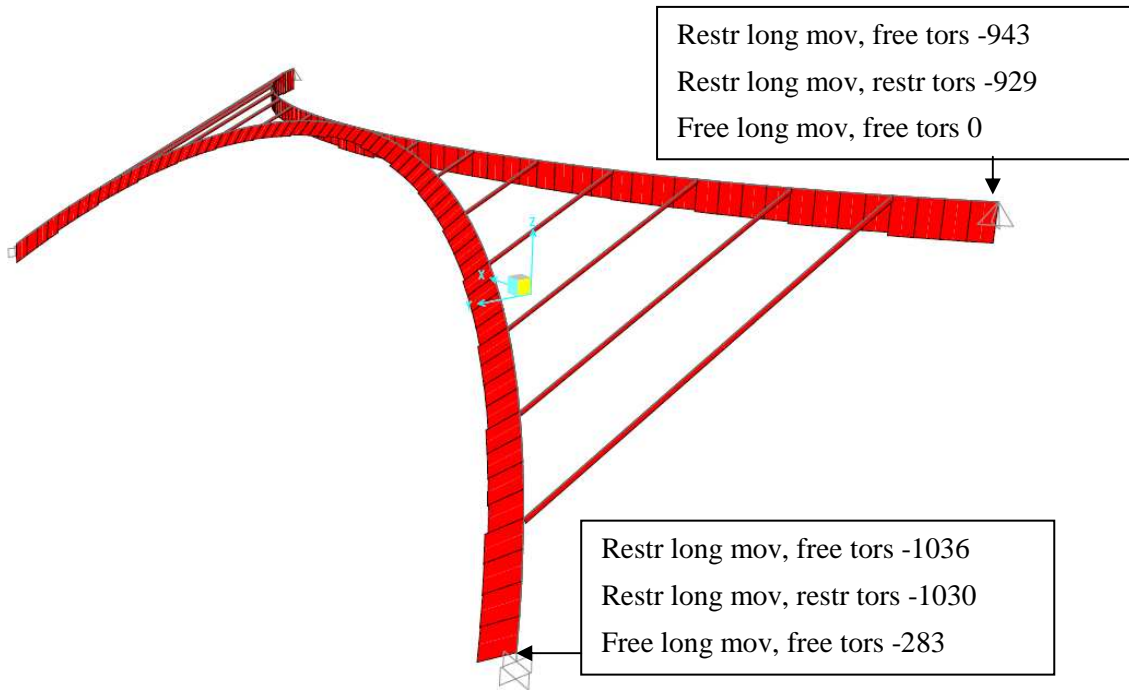
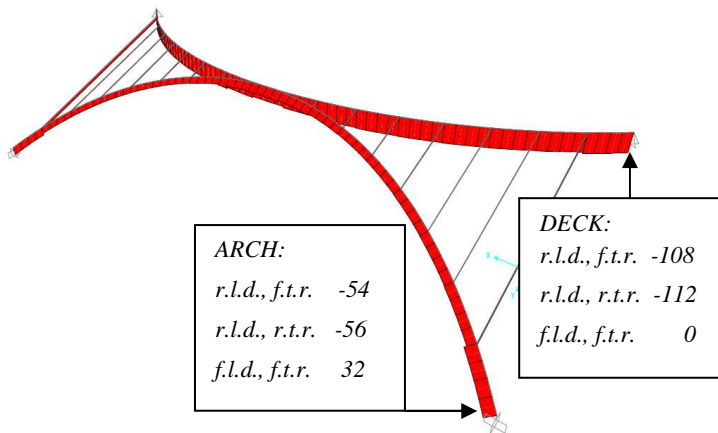


Figure 3-12: Model (b). Displacements (in mm) for different boundary conditions of the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is the f.l.d. and f.t.r. case.



**Figure 3-13: Model (c). Axial forces (in kN, compressions  $N < 0$ ) for different boundary conditions, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is the r.l.d. and f.t.r. case.**



**Figure 3-14: Model (c). Axial forces (in kN, compressions  $N < 0$ ) for different boundary conditions of the deck abutments, under a  $\Delta T=25^\circ\text{C}$  at both arch and deck. The diagram employed to show the values is the r.l.d. and f.t.r. case. (As a reference value the axial force in the deck ends for model (a) with r.l.d. is -8927 kN)**

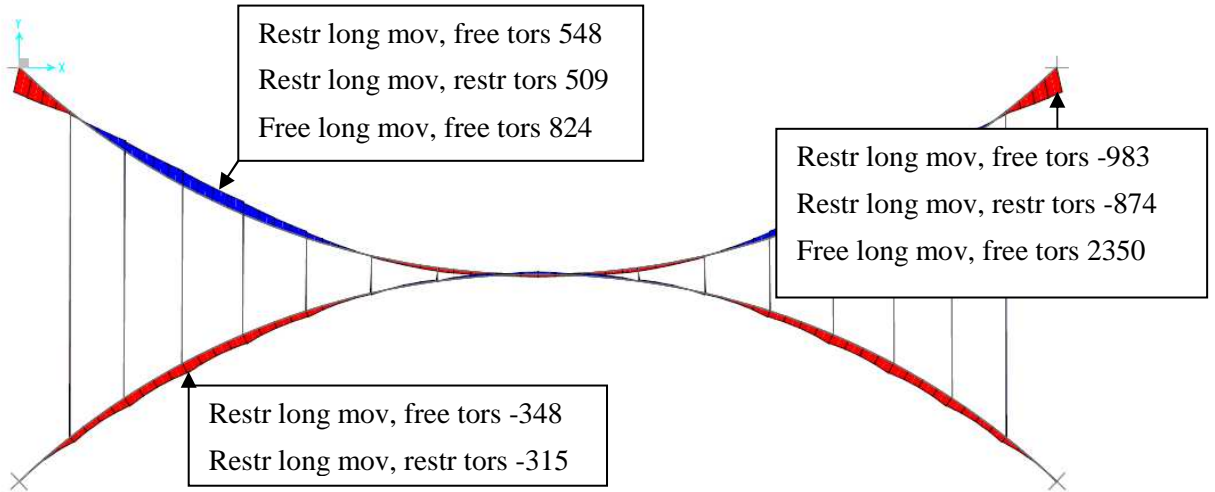


Figure 3-15- Model (c). M2-2 bending moments (in kN·m) for different boundary conditions of the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is plotted for the restrained longitudinal displacements (r.l.d.) and restrained twisting rotations (r.t.r.) case

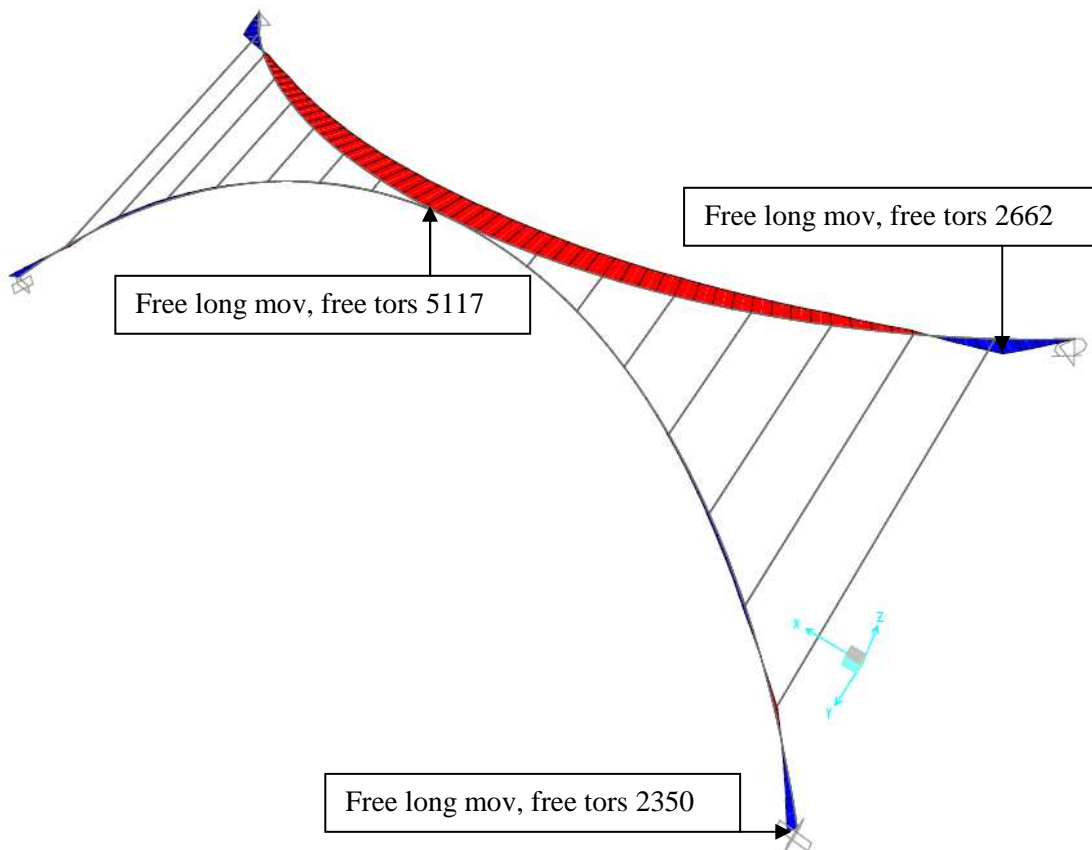


Figure 3-16- Model (c). M2-2 bending moments (in kN·m) for different boundary conditions of the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is plotted for the free longitudinal displacements (f.l.d.) and free twisting rotations (f.t.r.) case



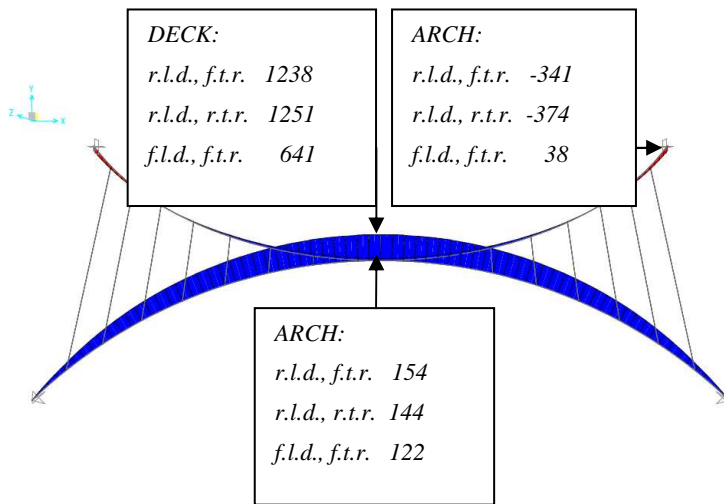


Figure 3-17: Model (c). M2-2 bending moments (in kN·m) of for different boundary conditions of the deck abutments, under a  $\Delta T=25^{\circ}\text{C}$  at both arch and deck. The diagram employed to show the values is plotted for the restrained longitudinal displacements (r.l.d.) and free twisting rotations (f.t.r.) case.

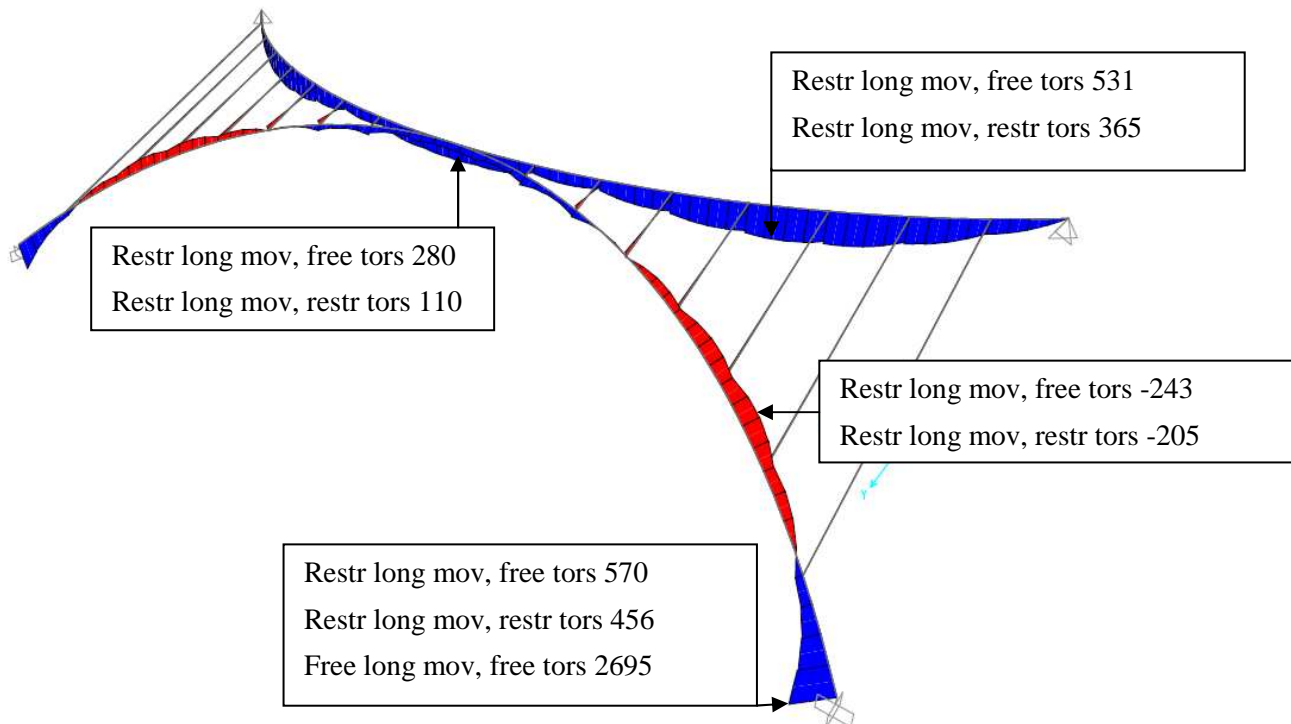


Figure 3-18: Model (c). M3-3 bending moments (in kN·m) for different boundary conditions of the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The diagram employed to show the values is the r.l.d. and f.t.r. case.

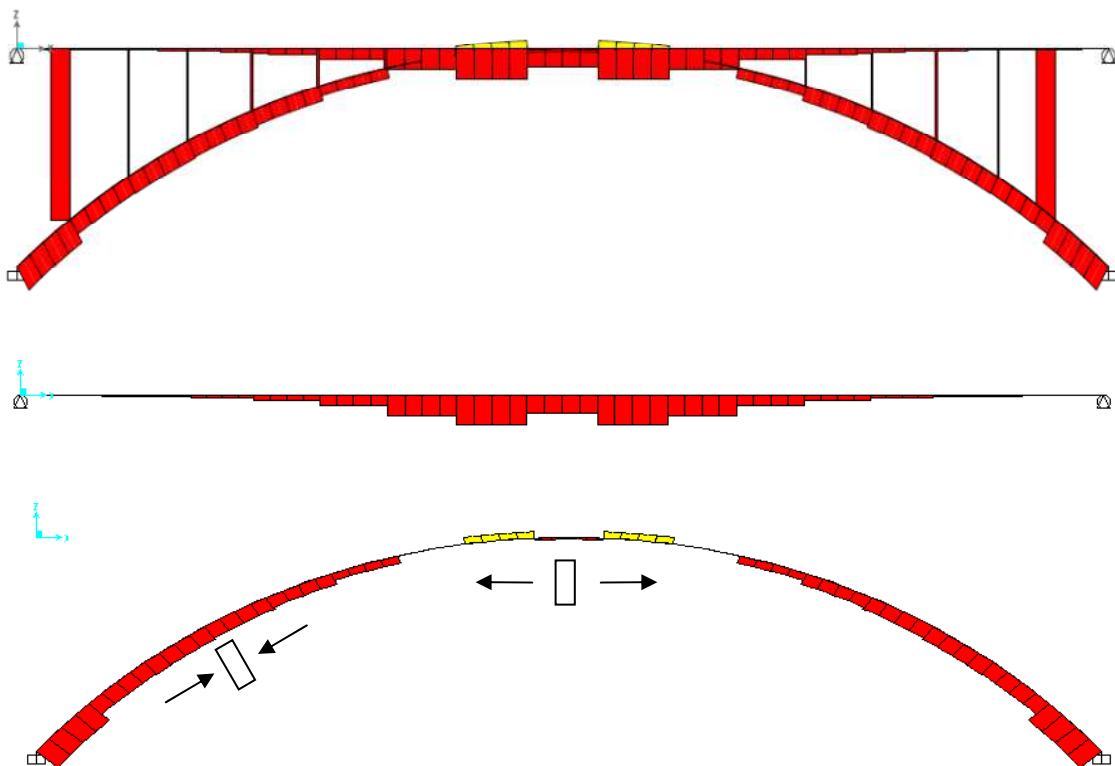
On the following figures (from Figure 3-19 to Figure 3-24) we can see the behaviour of a superior deck arch bridge with a curved deck and arch curved in plan coincident in plan with the deck (model (d)), with longitudinal displacements free at deck connection with the abutments and for cross-section correspondent to model 1 of Table 2-1.

If we add the axial forces in the deck and arch span center (Table 3-13) we obtain 472kN, which is approximately the same value as the horizontal component of the reaction at arch springings (484kN), lower than the value obtained for model (a). This value gives an idea of the arch behavior component.

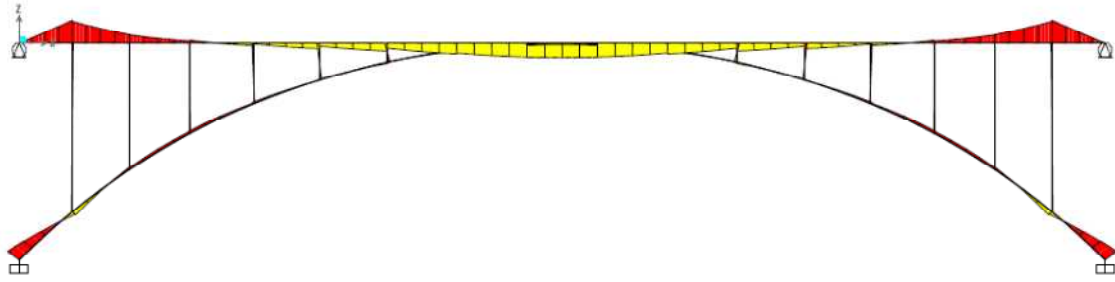
Under a vertical uniform loading, horizontal movements outwards the plan curve take place. They produce tensions on deck and transverse bending moments (with vertical axis, Figure 3-22).

Horizontal displacements of the arch depend on:

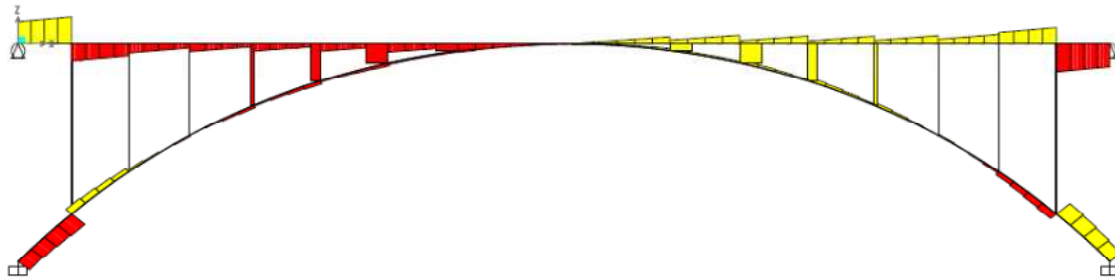
- the tension axial forces on the deck and therefore on the tension rigidity of the deck
- the transverse bending moments (with a vertical axis), ie: on the transverse flexural and axial rigidity of the deck (rigidity as an arch).



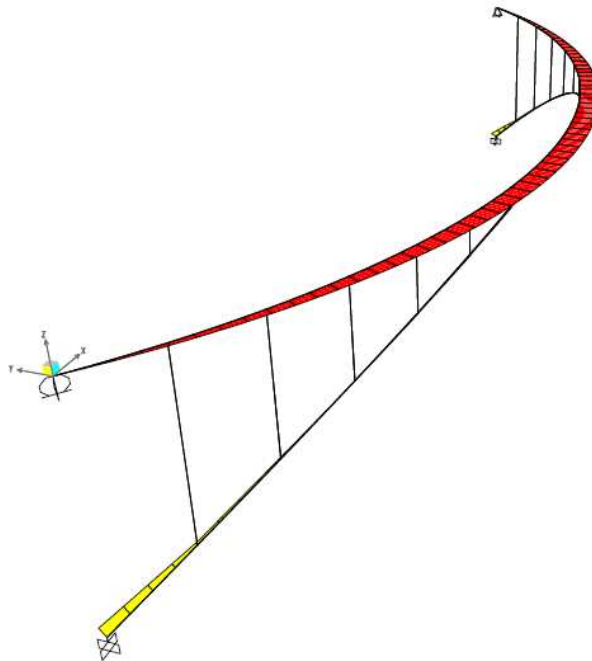
**Figure 3-19- Model (d). Arch and deck with equal plan curvature. [1] Axial force; [2] Axial forces in deck; [3] Axial forces in arch**



**Figure 3-20- Model (d). Arch and deck with equal plan curvature. Bending moment of the arch in the plane of the arch and balcony beam moment of the deck.**



**Figure 3-21- Arch and deck with equal plan curvature. Shear 2-2. Model (d)**



**Figure 3-22- Model (d). Arch and deck with equal plan curvature. Balcony beam bending moments of arch and transverse arch moment of deck.**

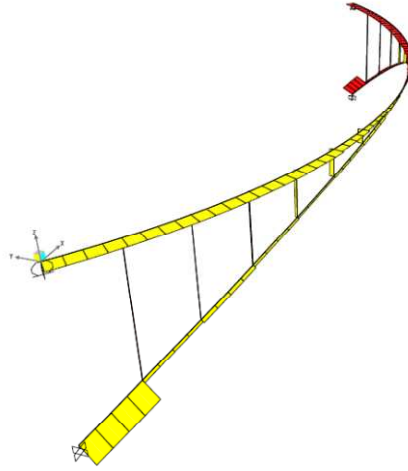


Figure 3-23- Arch and deck with equal plan curvature. Shear 3-3. Model (d)

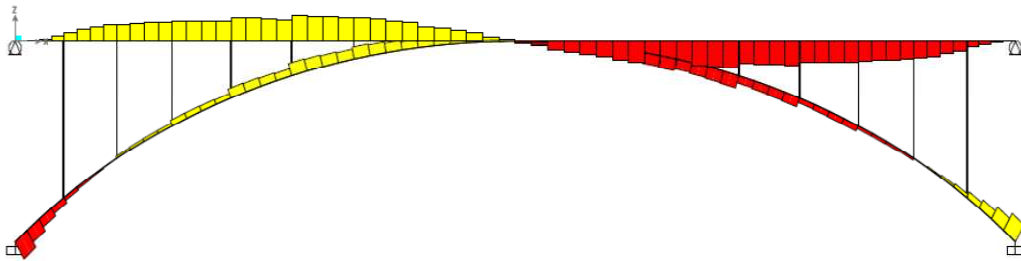


Figure 3-24- Arch and deck with equal plan curvature. Torsion. Model (d)

On the following figures (from Figure 3-25 to Figure 3-28) we can see the comparison between different geometries.

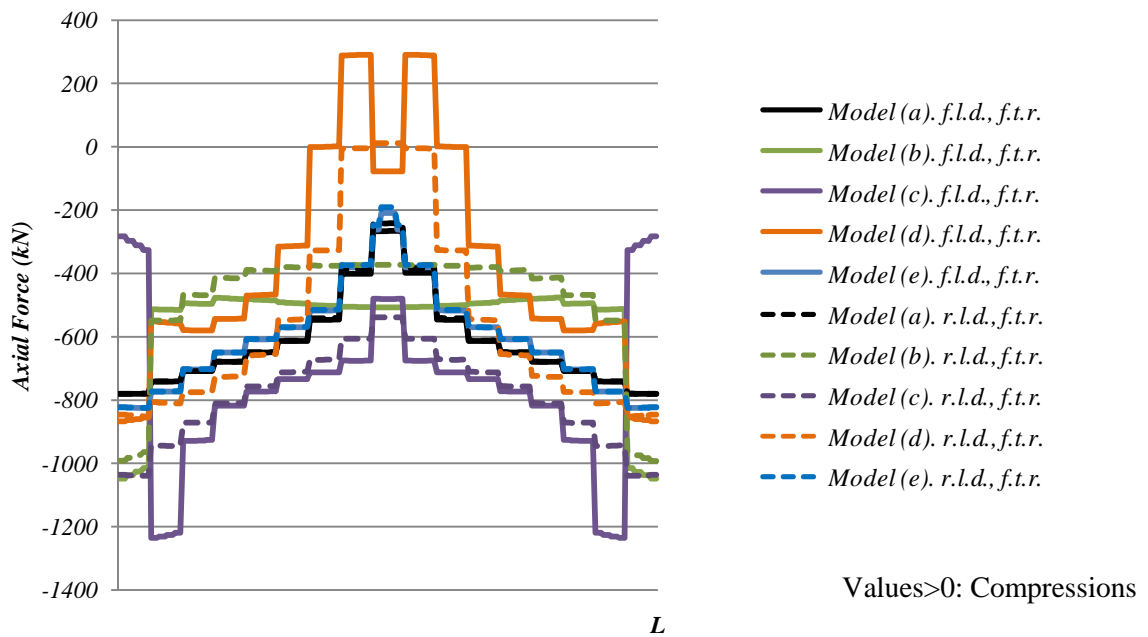
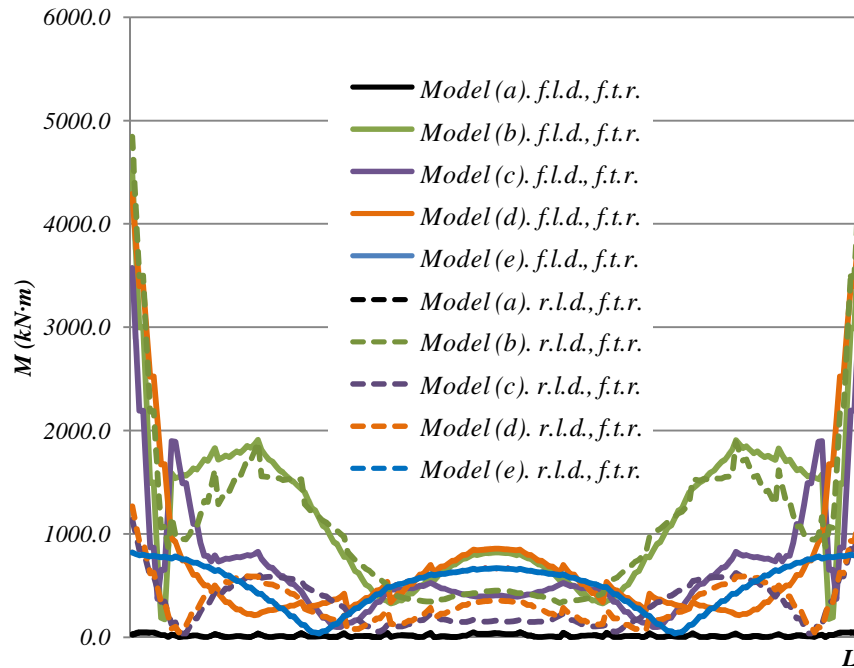
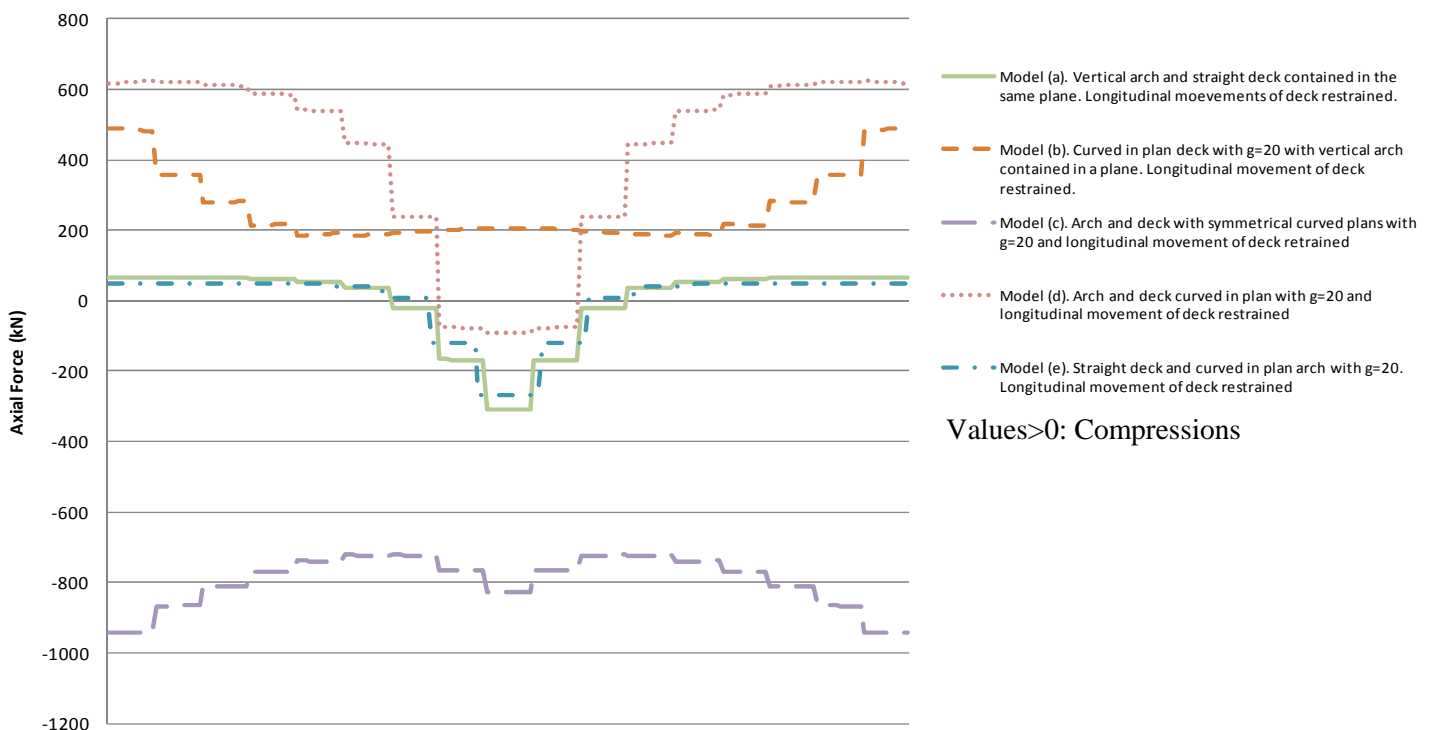


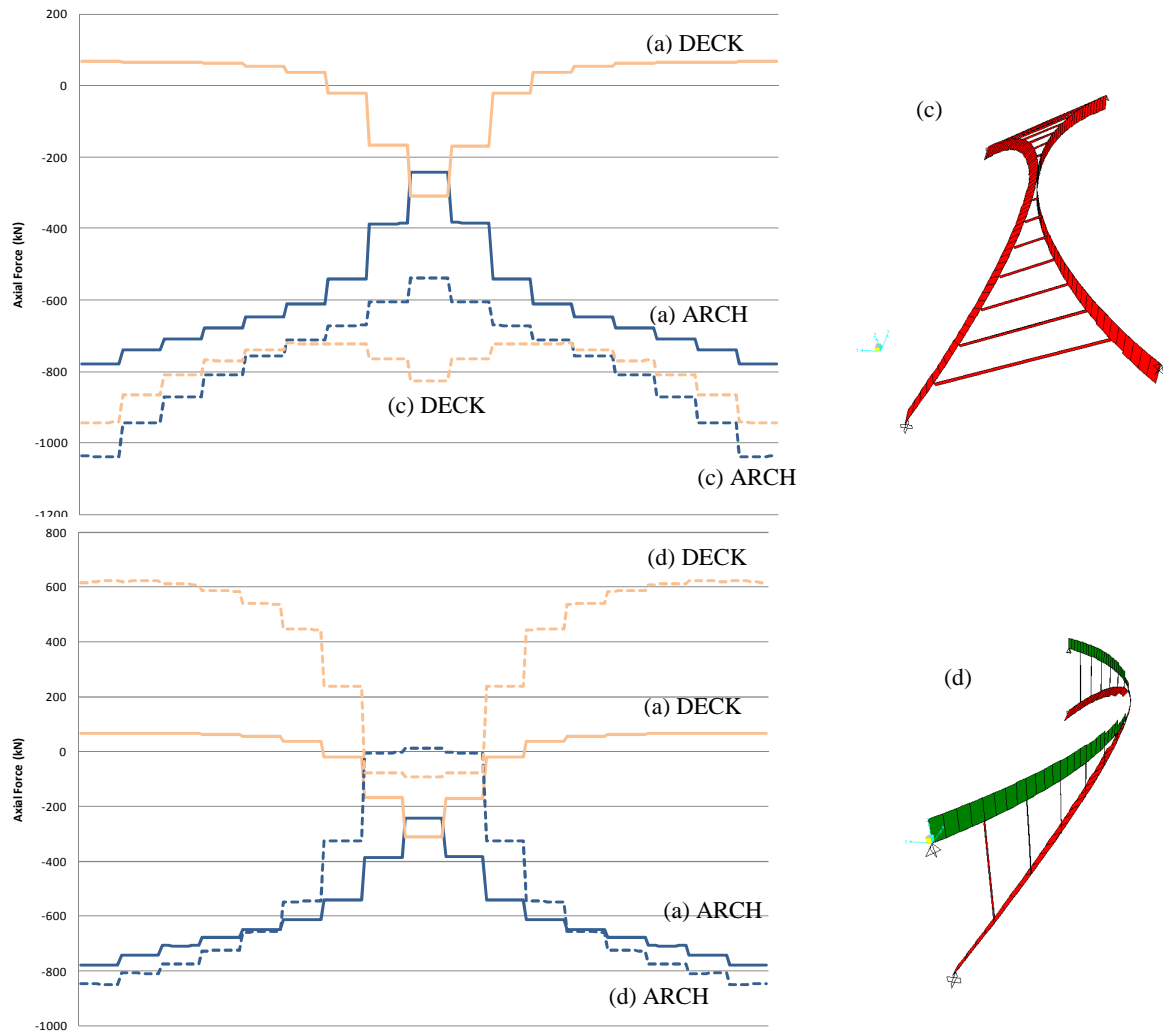
Figure 3-25: Comparative diagram of the arch axial forces (in kN, compressions  $N < 0$ ) of the different geometries and the different boundary conditions at the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The abscissas are the arch length from 0 to  $L_A$



**Figure 3-26: Comparative diagram of arch total bending moments ( $M = \sqrt{M_{22}^2 + M_{33}^2}$ , in kN·m) of the different bridge geometries and the different boundary conditions at the deck abutments, under a vertical deck loading  $q=10\text{kN/m}$ . The abscissas are the arch length from 0 to  $L_A$**



**Figure 3-27: Deck axial forces comparison under  $q_{10}$  for different geometries in superior deck arch bridges. Fixed struts. Longitudinal displacements of deck restrained at the connection of deck with abutments. The abscissas are the deck length from 0 to  $L_D$**



**Figure 3-28: Comparative diagram of the axial forces (in kN, compressions  $N < 0$ ) of model (a) with models (c) and (d) with r.l.d. at the deck abutments, under a vertical deck loading  $q = 10 \text{ kN/m}$ . The abscissas are the bridge length from 0 to  $L_{\text{bridge}}$**

### 3.2.1 Axial forces. Bearing conditions: restrained longitudinal displacements at deck abutments

Due to the employment of fixed joints at struts/arch and struts/deck, axial forces are distributed between the deck and arch in superior deck arch spatial bridges. This is due to the Vierendel truss effect. When releasing torsional and bending moments the deck's axial force is zero, because there is no Vierendel truss effect.

Axial forces in the arch depend on the axial forces and shear forces taken by the struts. Depending on the inclination of the strut in relationship to the arch plane each effect will be more or less predominant.

Extreme struts might take high forces because they are nearer to the fixed abutments and springings, although struts at span center might concentrate higher forces on those models in which they are shorter and, therefore, stiffer.

The deck is working as if it was completely fixed because the arch is fixed and the struts too, so the first strut is fixing the deck.

When comparing forces of arch bridges with a curved in plan deck (b, c and d) with the vertical planar arch with straight deck (a), we have to bear in mind that the loads for a curved deck are slightly higher because it is longer than a straight one for the same span. However, the differences caused by this fact are negligible as we will observe in each of the following commented models.

The difference of behaviour for the different geometries can be clearly seen by observing the axial forces distribution (from Figure 3-25 to Figure 3-67).

For the classical vertical superior deck arch bridges the axial forces taken by the deck are very low (Figure 3-2).

When employing a vertical planar arch for a superior curved deck (model (b), Figure 2-1), tensions at deck increase and compressions in arch decrease greatly (Figure 3-29). This is logical because the in-plane component of the loads on arch diminishes. However, at springings axial forces increase slightly (Figure 3-29).

At the span center, where the struts are nearly horizontal axial compressions in the arch plane are mainly transmitted through V2-2 shear forces in struts instead of axial forces (Table 3-3). The total value of the axial forces projection in the arch plane and V2-2 is lower than the axial forces transmitted by the struts to the arch in model (a). Therefore the axial forces in general diminish, except at span center and springings where the value is higher. In some cases the struts are even tensioned (struts 6 to 8 Table 3-3).

V2-2 shear forces in struts at extremes produces out-of-plane forces on the arch. For struts (1) and (2) this will compensate part of the ones produced by the struts' compression forces, for (4) and (5) it will add to them and from (6) to (8) they will compensate part of the forces produced by the tensions.

V3-3 shear forces in central struts are lower than for model (a) and with opposite sign (note that local axis are rotated, so we have to compare positive V2-2 of model (a) with negative V3-3 of model (b), Figure 3-6 and Figure 3-31), so they will contribute to increase compressions in the arch, although their values are negligible.

If the same solution was employed, but the position in which the arch and deck crossed in plan gave lower inclinations of struts, axial forces would not increase as much at span center (see chapter V.B section 2).

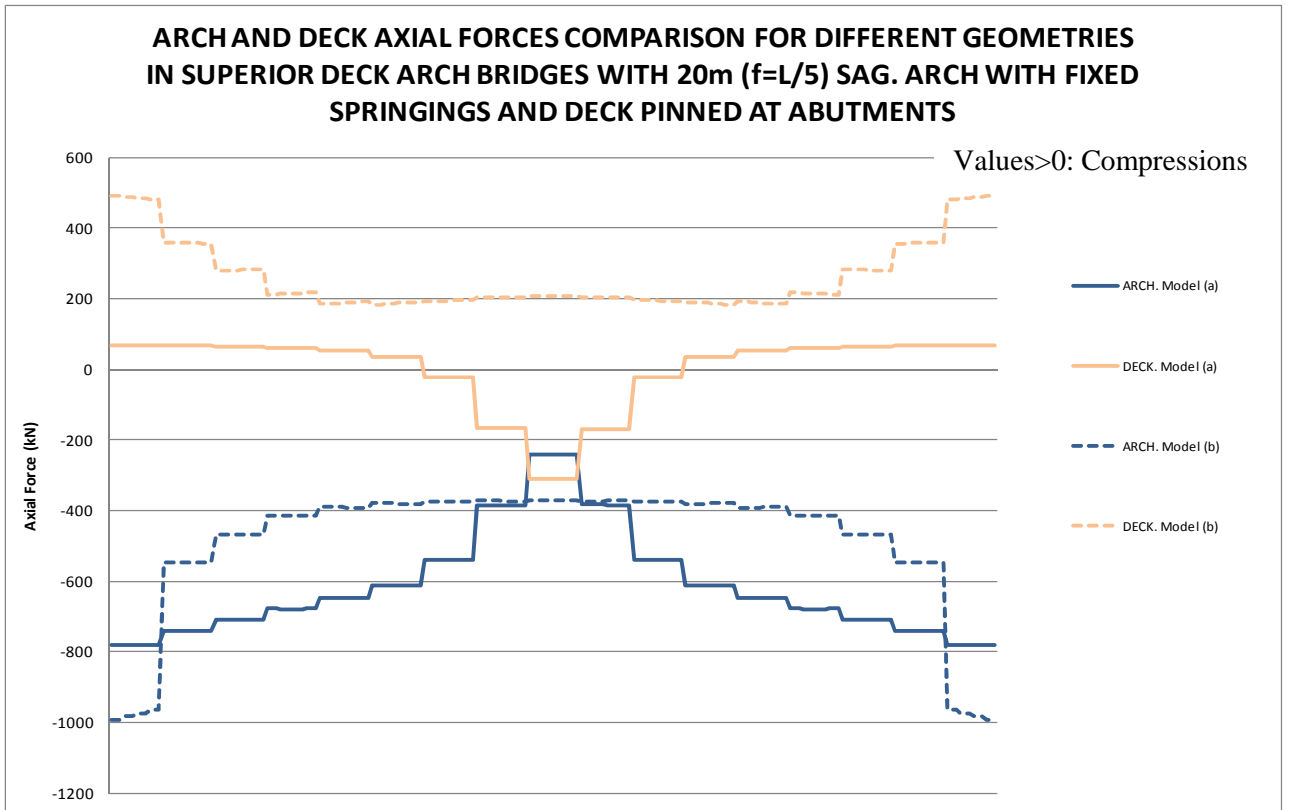


Figure 3-29: Models (a) and (b). Arch and deck axial forces comparison under a vertical uniformly distributed loading. Restrained longitudinal movements at deck abutments. *The abscissas are the bridge length from 0 to  $L_{bridge}$*

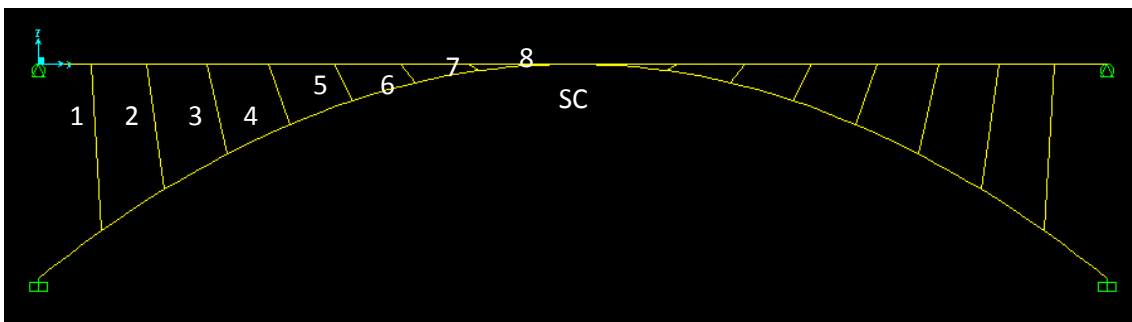


Figure 3-30: Model (b). Numbering of elements



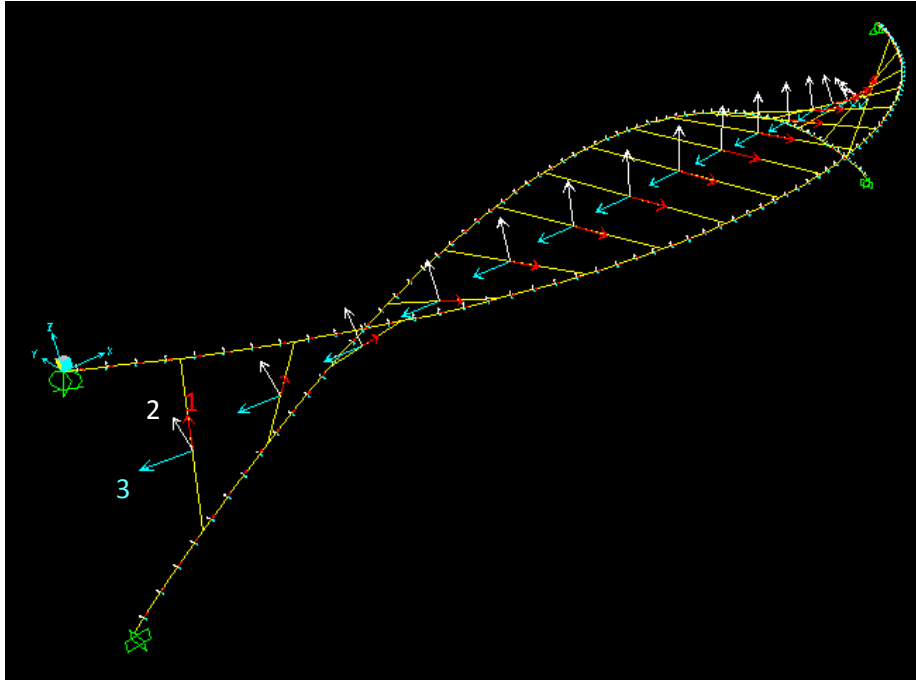


Figure 3-31: Model (b). Local axis at struts

| Element   | Axial force (kN) | Shear 2-2 (kN) | Shear 3-3 (kN) |
|---|------------------|----------------|----------------|
| <b>Struts</b>   |                  |                |                |
| 1   | -785,23          | 17,54          | 24,50          |
| 2   | -142,47          | 10,43          | 45,59          |
| 3   | -126,11          | -2,85          | 41,22          |
| 4   | -62,14           | -13,78         | 22,11          |
| 5   | -19,53           | -20,20         | 7,97           |
| 6   | 5,81             | -23,55         | 1,42           |
| 7   | 19,34            | -25,19         | -0,55          |
| 8   | 26,90            | -25,87         | -0,38          |
| <b>Arch and deck axial forces at span center (kN)</b> |                  |                |                |
| SC arch   |                  | -372,42        |                |
| SC deck   |                  | 206,39         |                |
| Sum of arch and deck axial forces                     |                  | -166,03        |                |
| % variation of the sum in relationship to model (a)   |                  | -69,9          |                |

Table 3-3: Model (b). Shear forces at struts and span center under a vertical uniformly distributed loading. Restrained longitudinal movements at deck abutments.

If arch and deck have an opposite curvature sign, both deck and arch are compressed (Figure 3-32). Compressions in arch increase in comparison with the vertical planar arch with straight deck (Figure 3-32). In fact the antifunicular could have opposite sign curvature (*J. Jorquera, 2009*), the same one (*J. Schlaich et al 1999*) and can sometimes have different curvature signs (*J. Jorquera, 2009*).

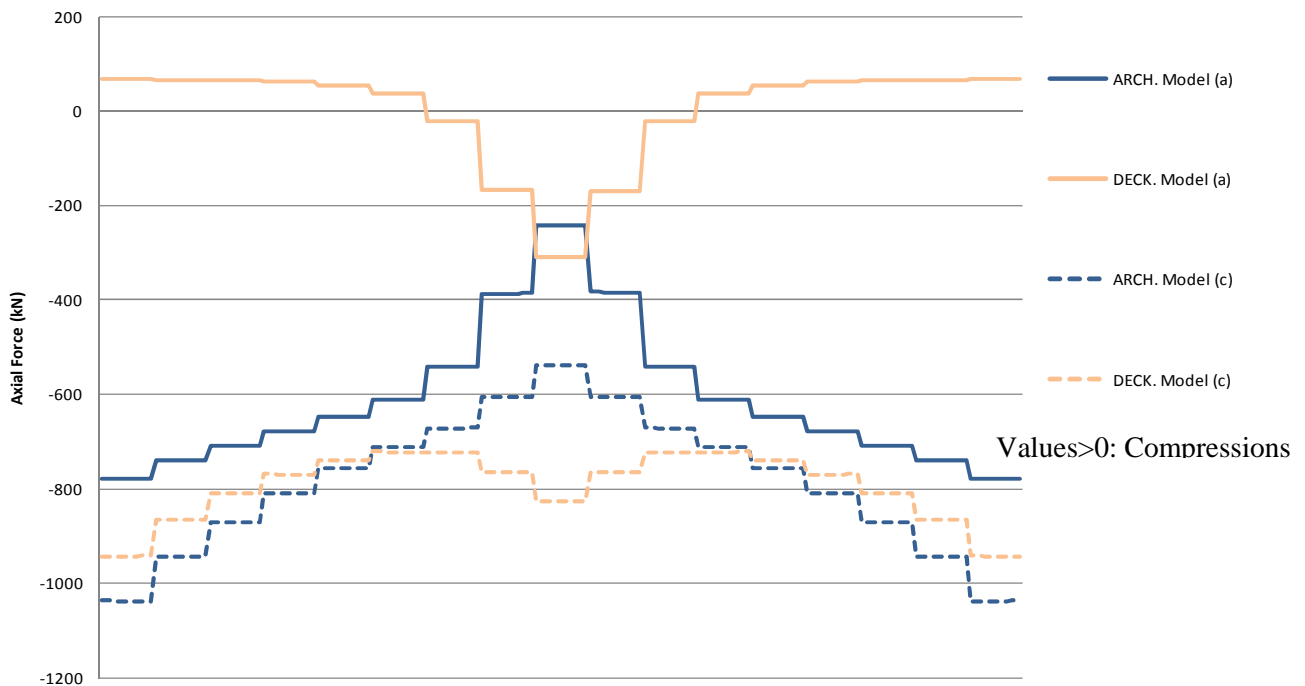
Since the deck has curvature in plan, it will work like a balcony beam. Therefore, the vertical displacements will increase in comparison with model (a). This will cause higher compressions on struts, so higher compressive vertical loads compared to model (a) will be transmitted to the

arch. However, we must highlight that extreme struts are tensioned, although the rest are compressed.

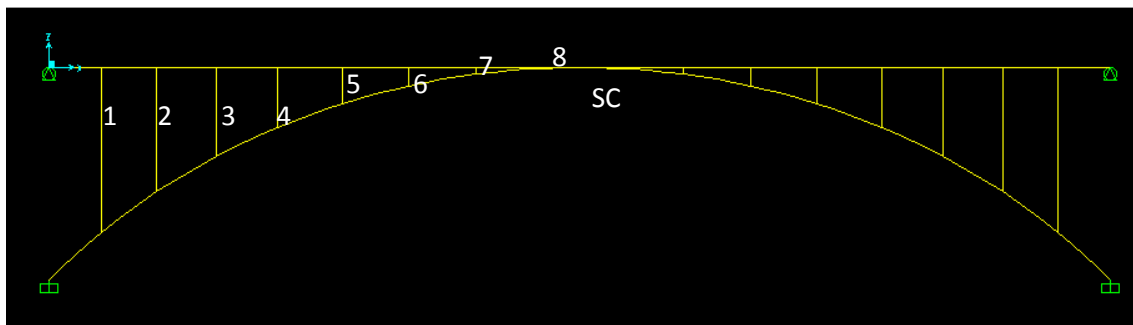
We have to note again that local axis are rotated, so we have to compare positive V2-2 of model (a) with negative V3-3 of model (c) (Figure 3-6 and Figure 3-34).

V3-3 shear forces at struts 3, 4 and 7 (

Figure 3-33) are smaller than the equivalent ones of model (a). At strut 8 V3-3 is a bit larger and at struts 5 and 6 it has an opposite sign. It would be expected that V3-3 transmitted by strut 8 caused tensions in the arch that diminish the axial compression forces. However, this effect is not high enough to counteract the higher axial forces transmitted by the struts (Table 3-4 compared to Table 3-1).



**Figure 3-32: Models (a) and (c). Arch and deck axial forces comparison under under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments. The abscissas are the bridge length from 0 to  $L_{bridge}$**



**Figure 3-33: Model (c). Numbering of elements**

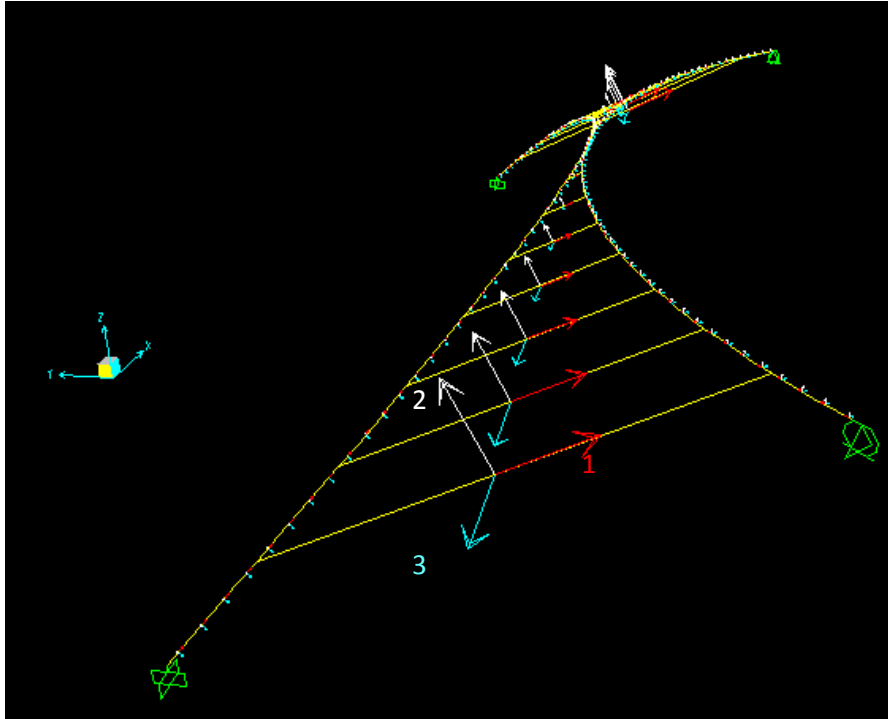


Figure 3-34: Model (c). Local axis at struts

| Element   | Axial force (kN) | Shear 2-2 (kN) | Shear 3-3 (kN) |
|---|------------------|----------------|----------------|
| <b>Struts</b>   |                  |                |                |
| 1   | -135,37          | -0,035         | 1,03           |
| 2   | -113,50          | -1,24          | 1,47           |
| 3   | -102,88          | -3,41          | 2,05           |
| 4   | -99,81           | -8,48          | 2,83           |
| 5   | -93,11           | -21,64         | 4,16           |
| 6   | -82,21           | -52,26         | 10,25          |
| 7   | -82,83           | -55,02         | 48,11          |
| 8   | -65,89           | -5,77          | 63,65          |
| <b>Arch and deck axial forces at span center (kN)</b> |                  |                |                |
| SC arch   |                  | -538,22        |                |
| SC deck   |                  | -825,94        |                |
| Sum of arch and deck axial forces                     |                  | -1364.16       |                |
| % variation of the sum in relationship to model (a)   |                  | 147,4          |                |

Note: Shear 3-3 forces are concentrated on the shortest central struts

Table 3-4: Model (c). Shear forces at struts and span center under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments.

When the arch and deck have the same curvature sign (model (d)) the arch is compressed and the deck tensioned (Figure 3-19, Figure 3-24 and Figure 3-28). This is due to shear forces transmitted by struts. The deck could work as funicular in its plan.

In comparison with the vertical planar arch bridge with straight deck (model (a), Figure 3-35), tensions in the deck increase and compressions in the arch increase at springings and diminish greatly at span center.

When restraining the longitudinal movements at deck abutments, due to the fact that the struts are fixed at arch and deck, the model (d) becomes a curved Vierendel truss.

V2-2 shear forces produce tensions in the arch, except at span center, and out-of-plane tipping torques.

At struts 1 and 2 V2-2 sign changes (Table 3-5) in comparison with model (a) (Table 3-1), this shear forces will compress the arch. V2-2 at struts from 4 to 7 are higher than for model (a). Those, together with the fact that the compressive load in the arch plane diminishes due to the inclination of the arch, will decrease the axial compression forces in the arch.

However, we must note that the fact that the deck has curvature in plan will cause it to work like a balcony beam. Therefore, the vertical displacements will increase in comparison with model (a). This will cause higher compressions on struts, so higher compressive vertical loads compared to model (a) will be transmitted to the arch.

V3-3 shear forces produce compressions on arch, except at springings and out-of-plane counterbalancing forces. However, V2-2 shear forces are higher than V3-3 and V3-3 effect might be negligible.

All in all, these internal forces cause the arch axial compressions to increase for the extreme thirds of the span. However, they decrease on the central third of the arch span, in comparison with model (a).

Conceptually, if the deck was infinitely stiff in the horizontal direction (ie: to axial forces and transverse bending moments) or had some kind of stiff transverse supports, horizontally it would react as if horizontal transverse (y global direction, axis of Figure 3-20) displacements were prevented and the compressions on arch and deck would increase, the same would happen when increasing the transverse stiffness of the arch. This is because increasing the bending stiffness increases the internal forces, as seen for the inferior deck case for a simple model with pinned hangers (chapter IV). These will increase both in- and out-of-plane arch forces. An increase of in-plane forces increases the axial forces on the arch.

For all the struts, the stiffest axis is set perpendicular to the line joining the deck abutments. For a curved in plan deck, only at span center is this the most effective orientation (annex N3.1), with axis 3 perpendicular to the plan curvature.

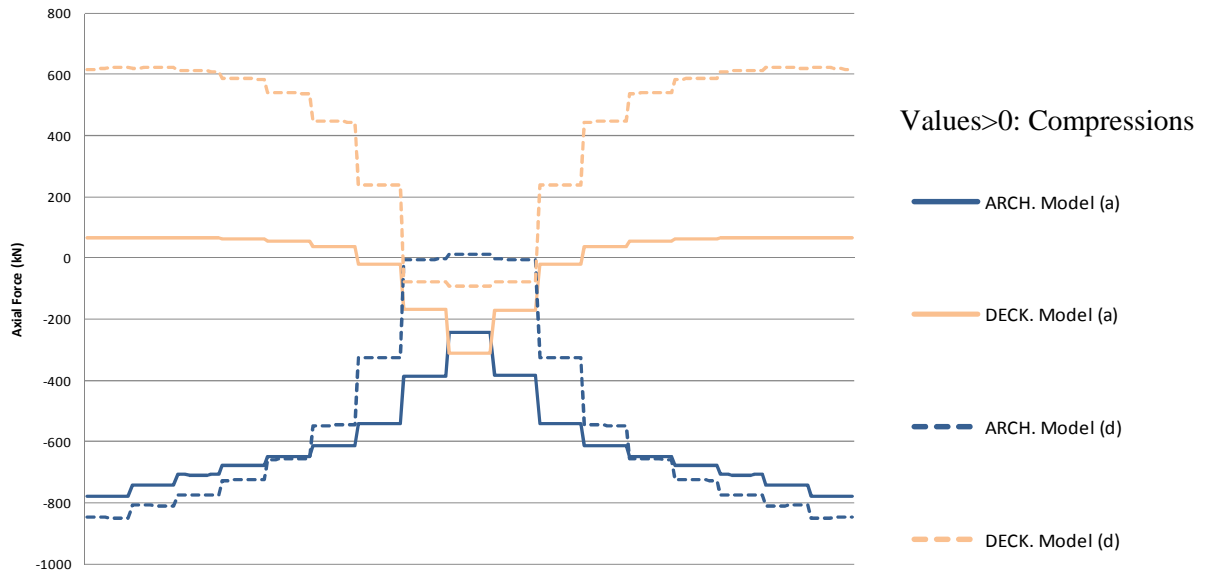


Figure 3-35: Models (a) and (d). Arch and deck axial forces comparison. *The abscissas are the bridge length from 0 to  $L_{bridge}$*

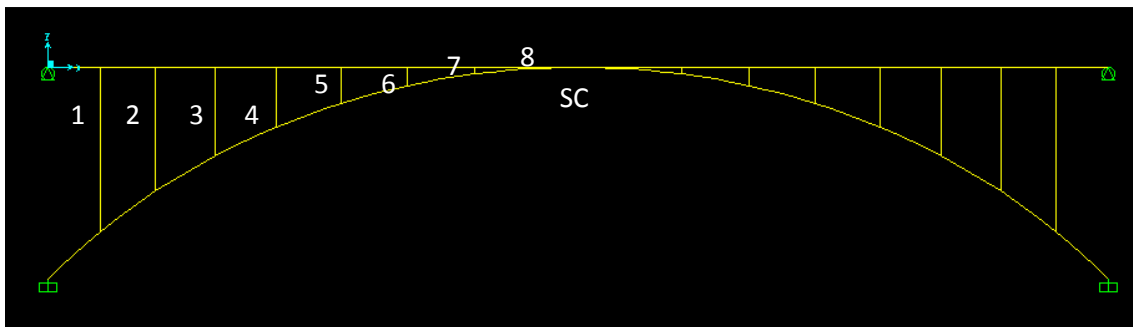


Figure 3-36: Model (d). Numbering of elements

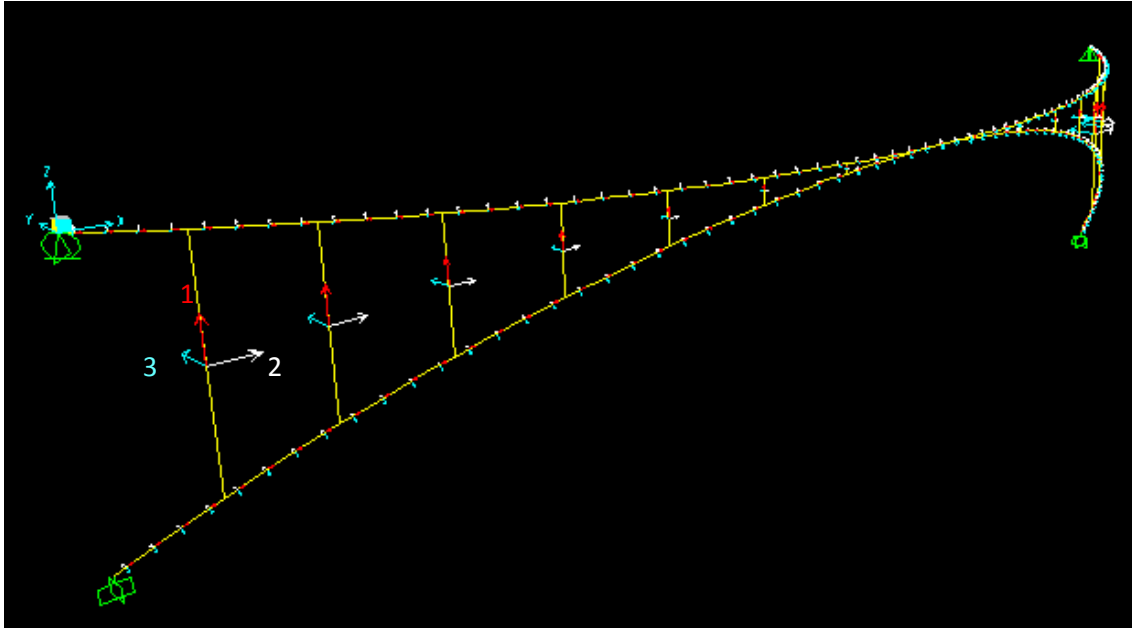


Figure 3-37: Model (d). Local axis at struts

| Element   | Axial force (kN) | Shear 2-2 (kN) | Shear 3-3 (kN) |
|---|------------------|----------------|----------------|
| <b>Struts</b>   |                  |                |                |
| 1   | -79,7            | -8,6           | -2,9           |
| 2   | -55,8            | -9,3           | 5,3            |
| 3   | -66,9            | -11,0          | 23,6           |
| 4   | -71,2            | -20,4          | 57,5           |
| 5   | -73,0            | -60,6          | 101,6          |
| 6   | -76,0            | -186,3         | 107,4          |
| 7   | -60,7            | -311,3         | 45,9           |
| 8   | -19,9            | -14,3          | -3,1           |
| <b>Arch and deck axial forces at span center (kN)</b> |                  |                |                |
| SC arch   |                  | 11,1           |                |
| SC deck   |                  | -91,2          |                |
| Sum of arch and deck axial forces                     |                  | -80,1          |                |
| % variation of the sum in relationship to model (a)   |                  | -85,5          |                |

Table 3-5: Model (d). Shear and axial forces at struts and span center under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments.

If we compare models (c) and (d), we observe that, except for the tensioned strut 1, all the others have higher compression values for model (c) (Table 3-4 and Table 3-5). Moreover, as we have already stated, all of it is transmitted to the arch as an in-plane a compressive load, whereas for model (d) only a part of it is projected in the arch plane. Therefore, it is logical that the arch of model (c) has to resist higher axial forces.

When employing a curved arch to support a straight superior deck (model (e), Figure 2-1), compressions decrease slightly in the arch (Figure 3-38) in comparison with the vertical planar arch bridge with straight deck, except at springings, where they increase slightly.

We have to note again that the local axes are rotated, so we have to compare positive V2-2 of model (a) with negative V3-3 of model (c) (Figure 3-6 and Figure 3-40).

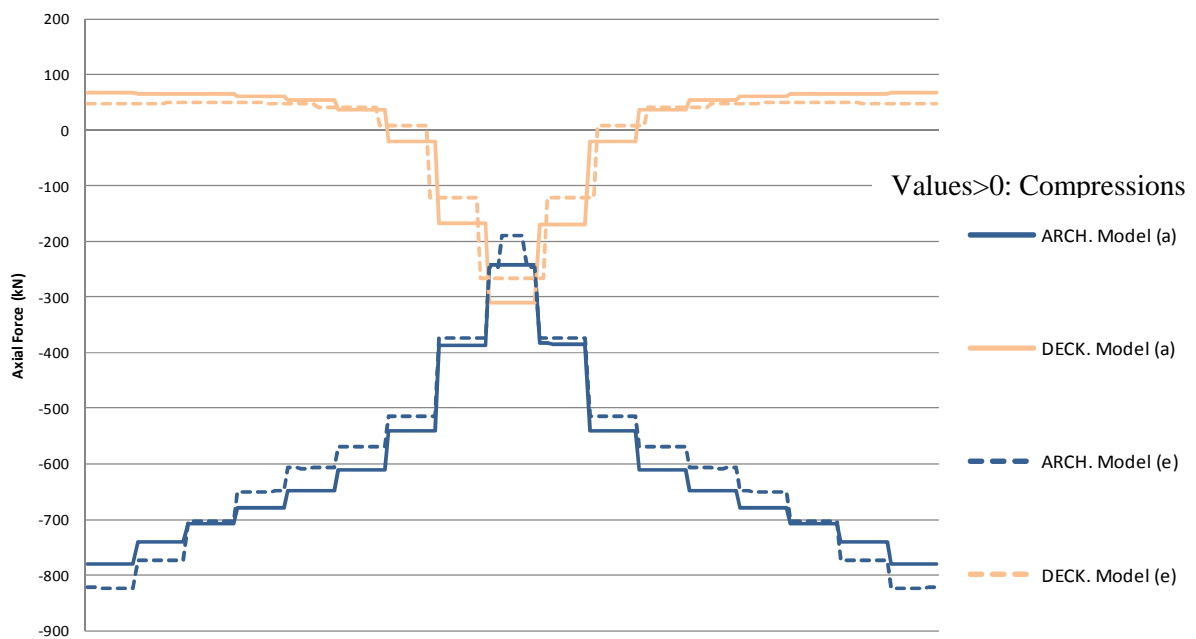
V2-2 shear forces in the struts introduce out-of-plane forces on the arch.

V3-3 at struts 1 to 3 (Figure 3-39) have an opposite sign than the equivalent ones of model (a), so they will contribute to increase compression axial forces. At struts 4 to 7 they are smaller than for model (a), so they will cause lower tensions in the arch.

Except for strut 8, axial forces at struts are smaller for model (a) than for (e) (Table 3-6 compared to Table 3-2). The opposite would be expected, because the deck is the same in (a) and (e), so the loads are exactly the same. We could think that the same vertical forces will appear and that their projection as axial forces on the struts would be lower. However, the deformations do not only depend on the deck, but on the whole bridge system. An inclined deck is less rigid than a vertical one, since it is acting as balcony beam too. This leads to higher deck deformations, which produce higher axial forces even on inclined struts.

According to the axial forces increase, it would be expected that the arch compression axial forces increase for model (e) in comparison with (a) except at span center. Nevertheless, this is not so, they only increase at springings, because of the influence of V3-3 shear forces in the struts.

If we pin the struts, axial forces at arch springings increase a bit, from 822,7kN to 830,0kN. At span center they increase too, 209,1 to 212,0kN. This is because when pinning the struts the axial forces, which the struts take, increase and they transmit no V3-3 to the arch that might compensate arch compressions.



**Figure 3-38: Models (a) and (e). Arch and deck axial forces comparison under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments. *The abscissas are the bridge length from 0 to  $L_{bridge}$***

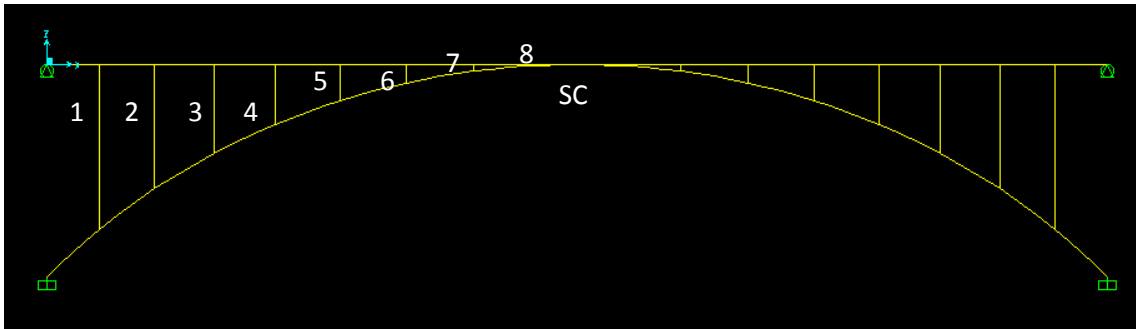


Figure 3-39: Model (e). Numbering of elements

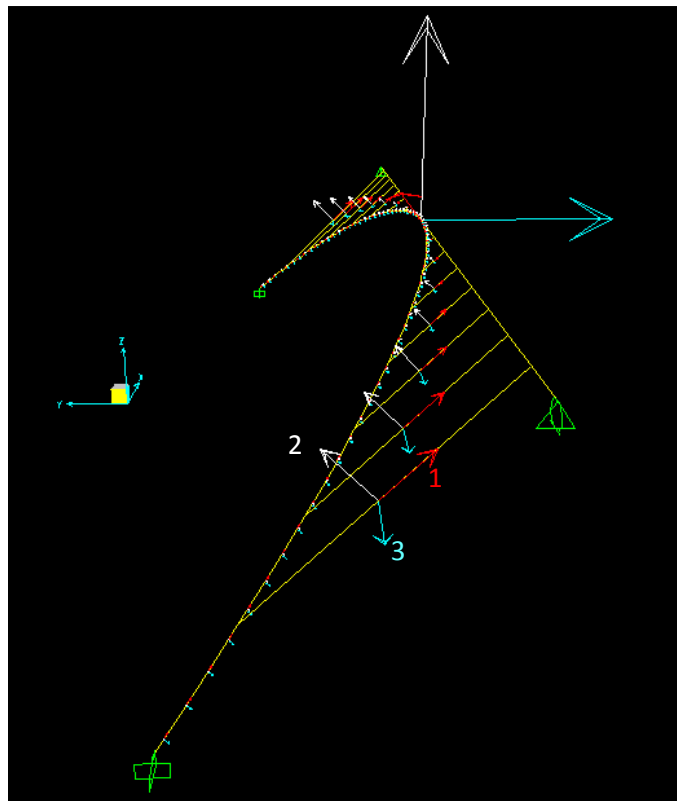


Figure 3-40: Model (e). Local axis at struts



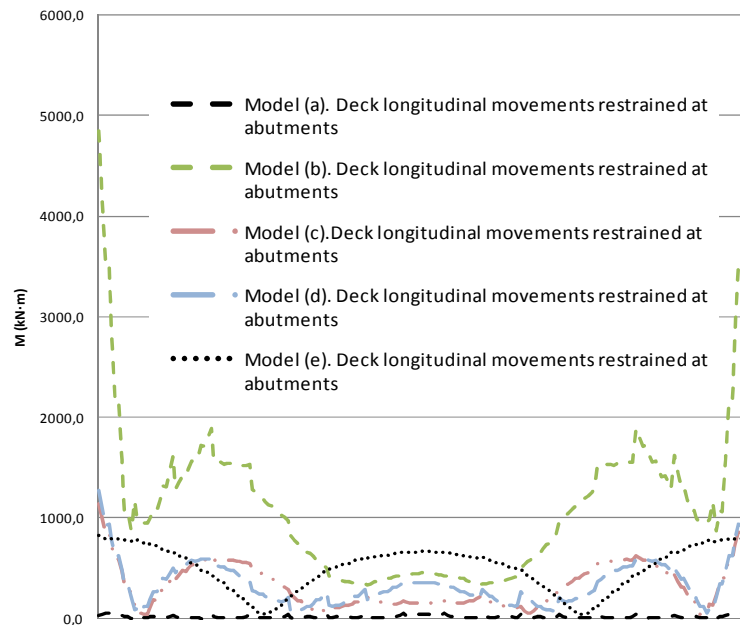
| Element   | Axial force (kN) | Shear 2-2 (kN) | Shear 3-3 (kN) |
|---|------------------|----------------|----------------|
| <b>Struts</b>   |                  |                |                |
| 1   | -70,10           | 5,77           | -0,63          |
| 2   | -103,49          | 6,14           | -0,74          |
| 3   | -88,11           | 5,43           | -0,45          |
| 4   | -79,90           | 1,69           | 1,05           |
| 5   | -75,05           | -8,29          | 7,00           |
| 6   | -73,00           | -21,81         | 33,18          |
| 7   | -80,64           | -9,06          | 128,38         |
| 8   | -48,19           | -3,72          | 146,94         |
| <b>Arch and deck axial forces at span center (kN)</b> |                  |                |                |
| SC arch   |                  | -190,4         |                |
| SC deck   |                  | -297,9         |                |
| Sum of arch and deck axial forces                     |                  | -488,3         |                |
| % variation of the sum in relationship to model (a)   |                  | -11,5          |                |

**Table 3-6: Model (e). Shear forces and axial forces at struts and span center under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments.**

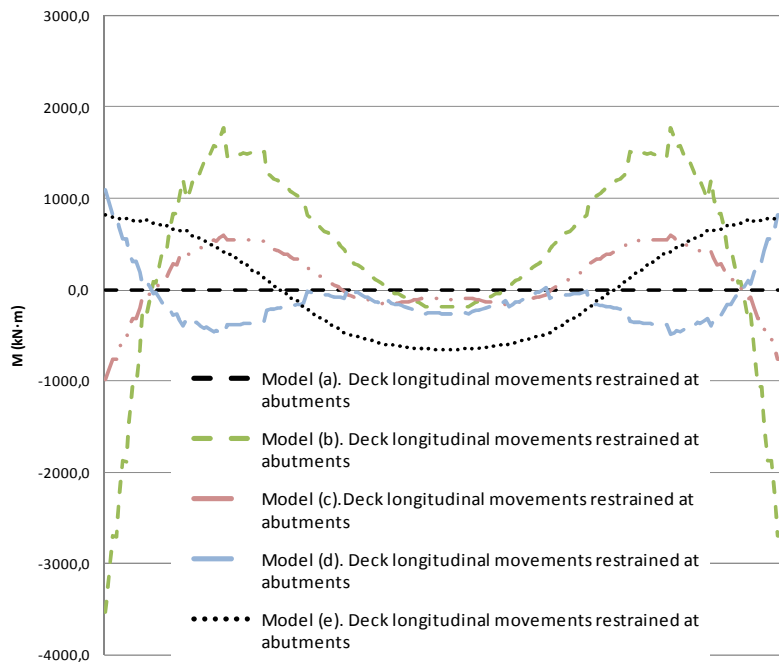
If we need to employ a curved deck, we have to bear in mind that model (b) will give the lowest axial forces in most of the length of the arch, and (c) the highest. Model (d) will suffer tensions at the arch's span center.

### 3.2.2 Bending moments. Bearing conditions: deck longitudinal displacements restrained at abutments

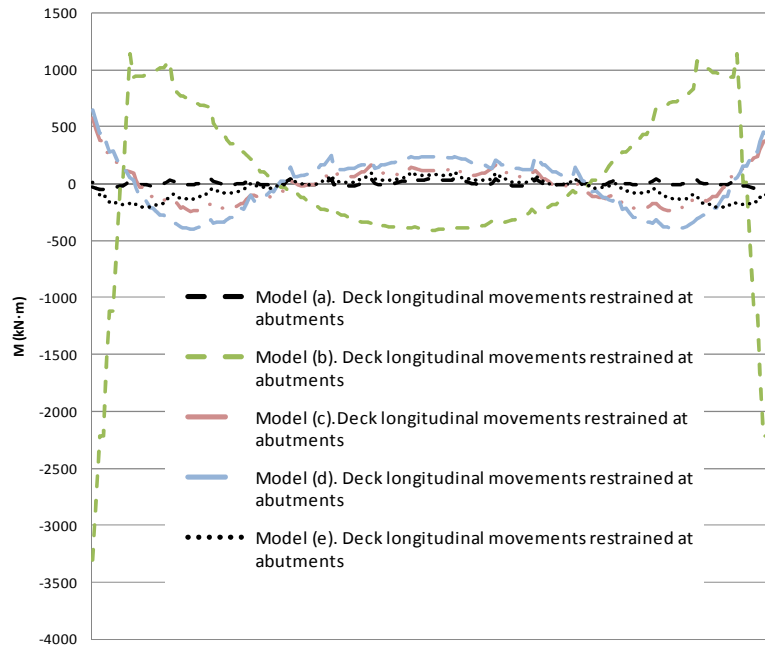
In figures from Figure 3-41 to Figure 3-43 the comparison of the bending moments for the different studied geometries under a uniformly distributed vertical load  $q=10\text{kN/m}$  can be observed.



**Figure 3-41: Arch total bending moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments. *The abscissas are the arch length from 0 to  $L_A$***



**Figure 3-42: Arch out-of-plane bending moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments. *The abscissas are the bridge length from 0 to  $L_{bridge}$***



**Figure 3-43: Arch in-plane bending moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments. The abscissas are the bridge length from 0 to  $L_{bridge}$**

Model (b) has the highest total and out-of-plane bending moments at  $L/4$  and extremes (Figure 3-41 and Figure 3-42). This is due to the high axial forces transmitted by the struts (Table 3-3)

Very high out-of-plane moments (Figure 3-42) are caused by the important inclination of the struts. It is however remarkable that in-plane bending moments increase greatly too (Figure 3-43).

If the same solution was employed, but the position in which the arch and deck cross in plan and the arch/deck vertical distance gave lower inclinations of struts, it would give much lower bending moments and higher axial forces.

Out-of-plane forces in model (b) are also caused by axial and 2-2 shear forces (mainly by 2-2 shear forces at extremes and by axial forces at span center). For struts (1) and (2) this will compensate part of the ones produced by the struts' compression forces, which are very high. For (4) and (5), it will add to them and from (6) to (8) they will compensate part of the forces produced by the tensions.

When pinning the struts in model (b) M2-2 out-of-plane bending moments in arch increase from  $-1915\text{kN}\cdot\text{m}$  at springings, for the fixed struts case, to  $-2771\text{kN}\cdot\text{m}$  for the pinned struts case, and from  $-544,5\text{kN}\cdot\text{m}$  to  $1892,6\text{kN}\cdot\text{m}$  at span center. This is because, when pinning the struts, the axial forces that struts are taking increase (Table 3-7).

| Element | Axial force (kN) |
|---------|------------------|
| 1       | -1230,3          |
| 2       | -83,7            |
| 3       | -175,0           |
| 4       | -96,6            |
| 5       | -19,1            |
| 6       | 34               |
| 7       | 57,4             |
| 8       | 74,2             |
| SC arch | 351,1            |
| SC deck | 305,4            |

**Table 3-7: Model (b). Pinned struts. Axial forces at struts and span center under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments.**

Solutions with curved deck and arch, (c) and (d), have very similar bending moments. These are the solutions which, regarding the behaviour of the whole arch, give the lowest total bending moments (Figure 3-41) and out-of-plane bending moments (Figure 3-42).

Total bending moments are higher for (d) (Figure 3-41) at span center because both in-plane and out-of-plane bending moments are higher (Figure 3-42 and Figure 3-43).

V2-2 shear forces in the struts produce out-of-plane tipping forces on the arch in model (c).

Out-of-plane forces on model (d) are produced by axial forces and 2-2 and 3-3 shear forces. V2-2 produces tensions on arch, except at span center and out-of-plane tipping forces.

Out-of-plane bending moments have opposite sign for (c) than for (d) except at span center, as expected (Figure 3-42). They are only slightly higher in value for (d) than for (c) at extremes and span center. At  $L/4$  they are higher for (c).

Important torsions are transmitted at springings and  $L/3$  for (c) with opposite sign to (d) (Figure 3-59), these compensate out-of-plane moments at span center.

In-plane bending moments work best at springings for arch and deck symmetrical in plan (Figure 3-43).

Model (e) has the highest total and out-of-plane bending moments at span center.

In model (e), V2-2 produces out-of-plane tipping forces on the arch, except at springings and at the span center, where they are stabilizing forces.

On the one hand, the arch introduces bending moments of vertical axis on the deck for models with a curved deck. The dimensions of the deck are higher transversally, so these bending moments will be resisted by the highest dimension of the deck.

On the other hand, the arch helps the deck to resist bending moments with radial axis. This arch contribution can be measured by a coefficient X defined below, which compares the deck difference between the main positive and negative moments with isostatic moments. The values obtained for each model for both cases restrained or free deck longitudinal movements are exposed in Table 3-8.

However, the arch will introduce bending moments with vertical axis on the deck through fixed struts, the total bending moments distribution in arch and deck can be seen from Figure 3-44 to Figure 3-58.

$$X = \frac{M_{DECK}}{M_{ISOST}} \quad (1)$$

$$\begin{aligned} \text{Total moment of the deck} \equiv M_{DECK} &= \frac{-(M_{negleft} + M_{negright})}{2} + M_{pos} \\ M_{SIST} &= \frac{M_{negizquierda} + M_{negderecha}}{2} + M_{positivo} \end{aligned} \quad (2)$$

$$\text{Isostatic moment} \equiv M_{ISOST} = \frac{q \cdot L_{DECK}^2}{8} \quad (3)$$

$$X' = 1 - X \quad (4)$$

From Table 3-8 it is concluded that the maximal arch help for resisting deck for bending moments with radial axis is obtained when restraining the tangential longitudinal movements at deck abutments.

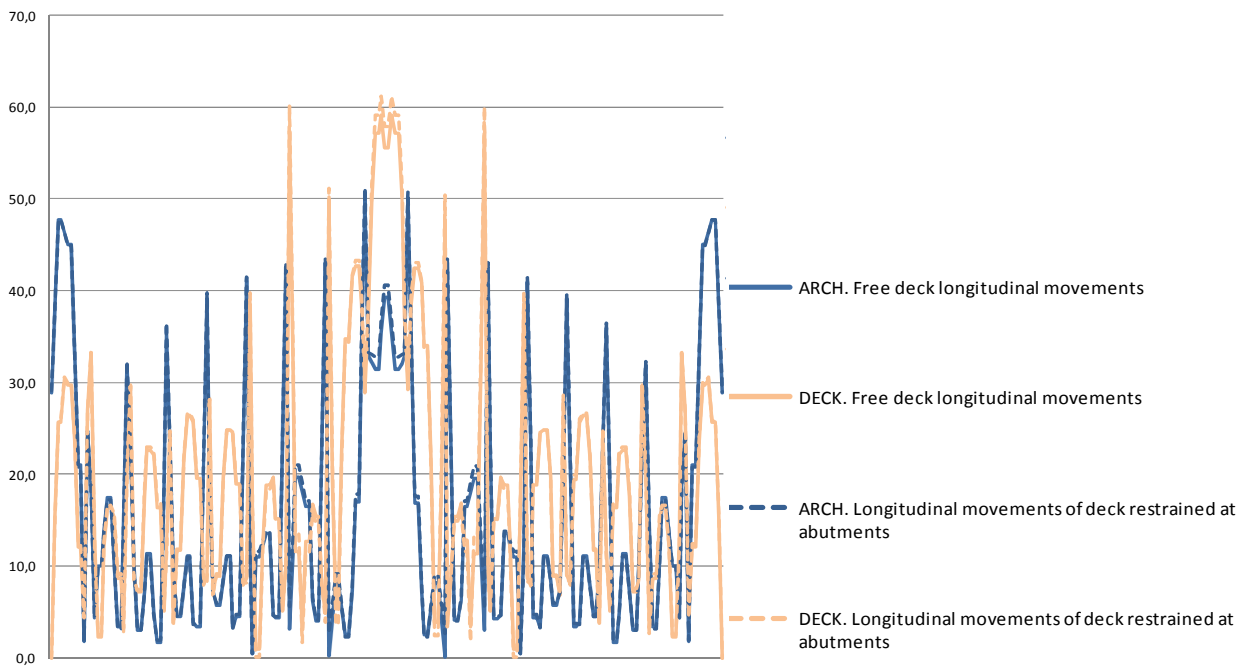
For models with a curved deck there is an incredibly big difference of the deck total bending moments values when the deck longitudinal movements are restrained or not at abutments (from Figure 3-45 to Figure 3-57). For models with a curved deck ((b), (c) and (d)), restraining them diminishes a lot out-of-plane moments in the arch. When employing a straight deck the differences are negligible (Figure 3-44 and Figure 3-58). Therefore it is always highly recommendable to restrain them.

The model with curved deck in which the arch offers a maximal help to the deck regarding bending moments is model (c) (Table 3-8), closely followed by model (d).

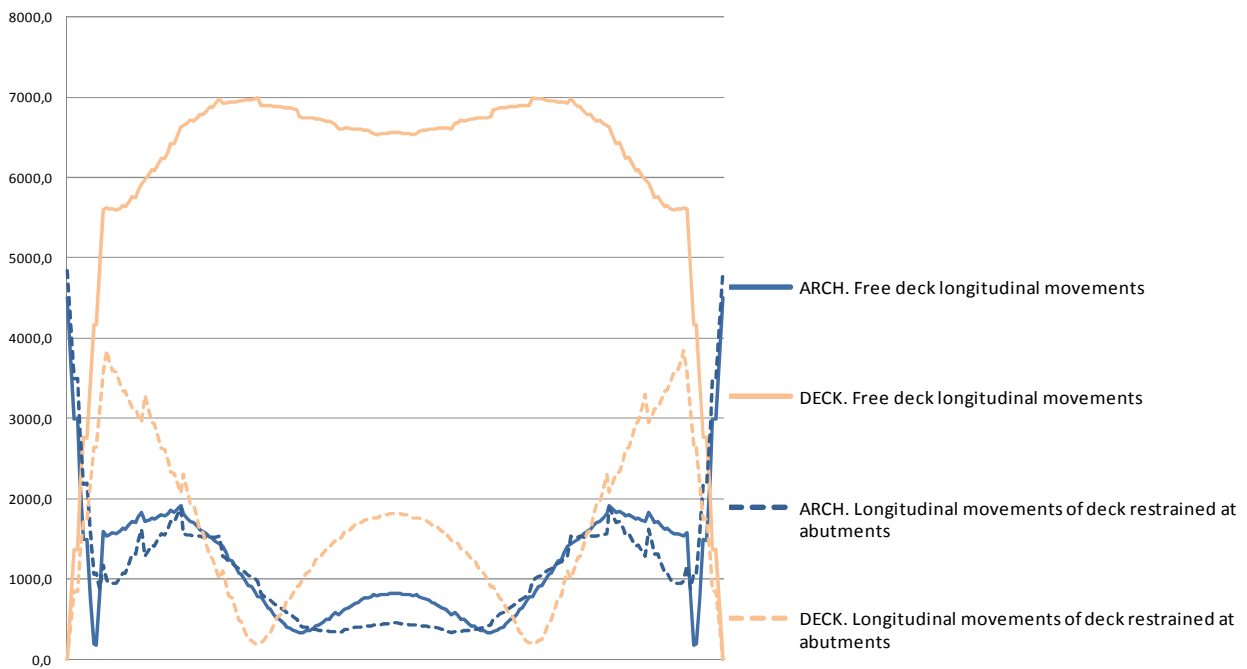
For model (d) with restrained longitudinal tangential deck movements a higher Vierendel effect can be appreciated.

| <b>Model</b>                              | <b>q</b> | <b>L<sub>DECK</sub></b> | <b>Misost</b> | <b>Mneg l</b> | <b>Mneg r</b> | <b>Mpos</b> | <b>Msyst</b> | <b>X</b>      | <b>X'</b>   |
|---|----------|-------------------------|---------------|---------------|---------------|-------------|--------------|---------------|-------------|
| (a) long mov restrained at deck abutments | 10       | 100                     | 12500         | 51,2          | 51,2          | 61,1        | 112,3        | <b>0,0090</b> | <b>0,99</b> |
| (a) long mov free at deck abutments       | 10       | 100                     | 12500         | 49,3          | 49,3          | 59          | 108,3        | <b>0,0087</b> | <b>0,99</b> |
| (b) long mov restrained at deck abutments | 10       | 110,3468                | 15220,52      | 3845.8        | 3845.8        | 1817.2      | 5663         | <b>0,3721</b> | <b>0,63</b> |
| (b) long mov free at deck abutments       | 10       | 110,3468                | 15220,52      | 4521,9        | 4521,9        | 1962,7      | 6484,6       | <b>0,4260</b> | <b>0,57</b> |
| (c) long mov restrained at deck abutments | 10       | 110,3468                | 15220,52      | 0             | 0             | 531,3       | 531,3        | <b>0,0349</b> | <b>0,97</b> |
| (c) long mov free at deck abutments       | 10       | 110,3468                | 15220,52      | 0             | 0             | 2933,3      | 2933,3       | <b>0,1927</b> | <b>0,81</b> |
| (d) long mov restrained at deck abutments | 10       | 110,3468                | 15220,52      | 0             | 0             | 636         | 636          | <b>0,0418</b> | <b>0,96</b> |
| (d) long mov free at deck abutments       | 10       | 110,3468                | 15220,52      | 2132,6        | 2132,6        | 1316,5      | 3449,1       | <b>0,2266</b> | <b>0,77</b> |
| (e) long mov restrained at deck abutments | 10       | 100                     | 12500         | 0             | 0             | 948,3       | 948,3        | <b>0,0759</b> | <b>0,92</b> |
| (e) long mov free at deck abutments       | 10       | 100                     | 12500         | 0             | 0             | 947,6       | 947,6        | <b>0,0758</b> | <b>0,92</b> |

**Table 3-8: Models (a) to (e). Contribution of the arch to the deck resistance of bending moments with radial axis under a vertical uniformly distributed loading q10.**



**Figure 3-44: Model (a): Arch and deck total bending moments comparison. The ordinates are the total bending moments (kN-m) and the abscissas are the bridge length from 0 to  $L_{bridge}$  under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments.**



**Figure 3-45. Model (b): Arch and deck total bending moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments. The ordinates are the total bending moments (kN-m) and the abscissas are the bridge length from 0 to  $L_{bridge}$**

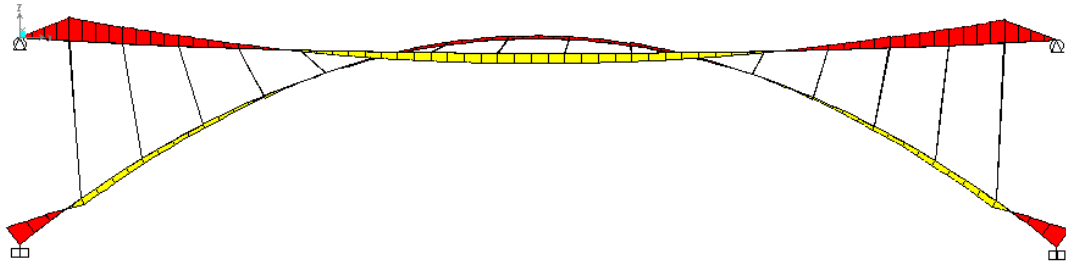


Figure 3-46: Model (b). Longitudinal tangential movements free at deck abutments. M3-3 bending moments: Deck balcony beam bending moments and arch in-plane bending moments under a vertical uniformly distributed loading  $q_{10}$ . 3D view

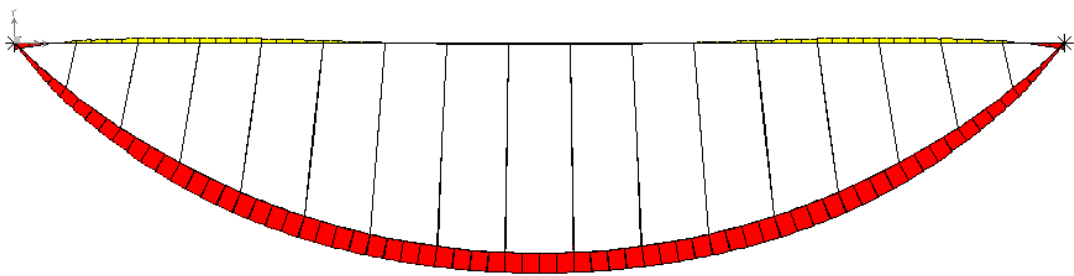


Figure 3-47: Model (b). Longitudinal tangential movements free at deck abutments. M2-2 bending moments: Deck vertical axis bending moments and arch out-of-plane bending moments under a vertical uniformly distributed loading  $q_{10}$ . Plan view

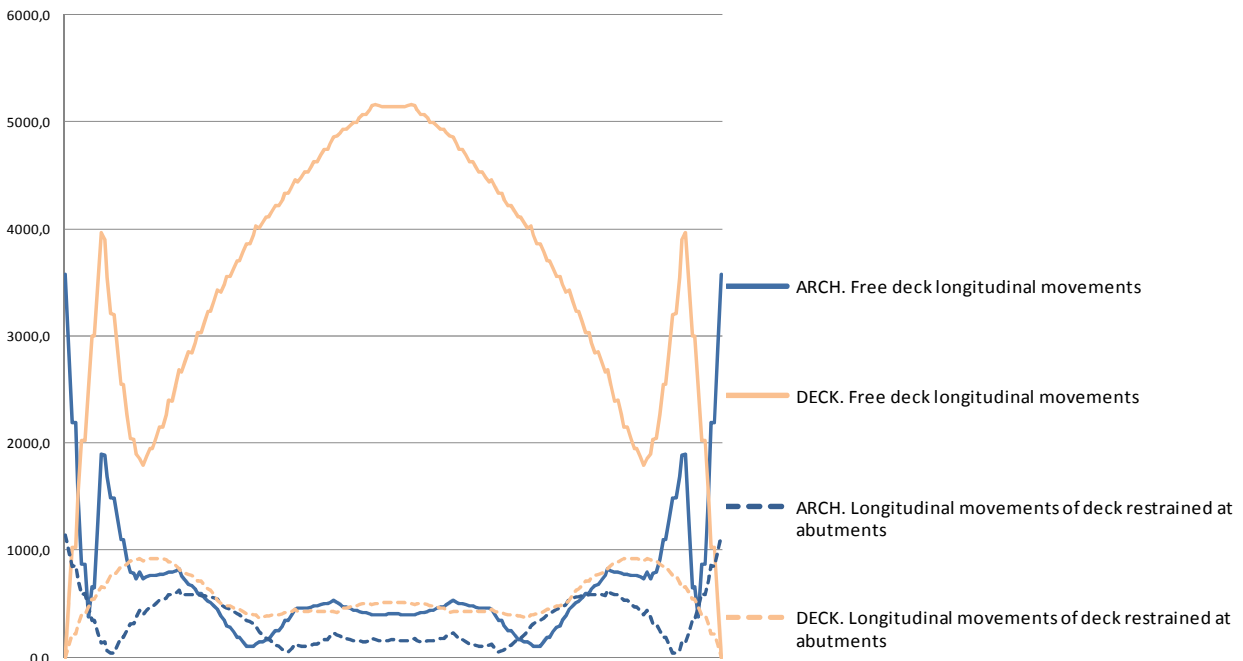
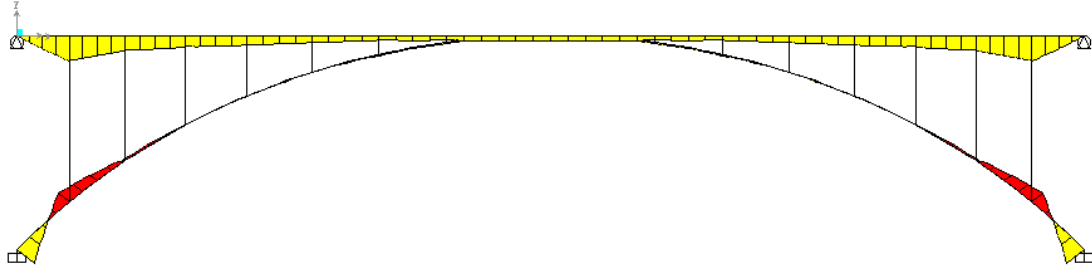
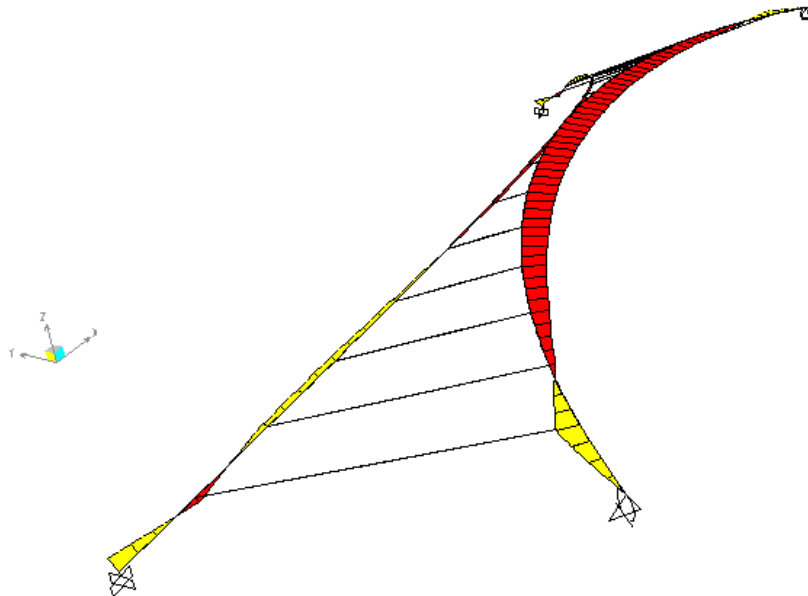


Figure 3-48. Model (c): Arch and deck total bending moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments. *The ordinates are the total bending moments (kN-m) and the abscissas are the bridge length from 0 to  $L_{bridge}$*

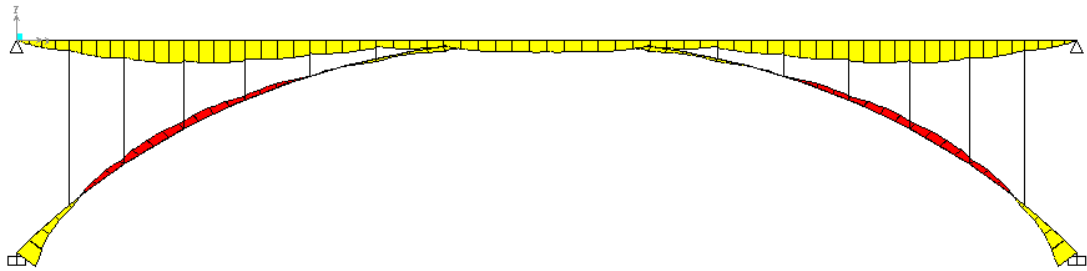




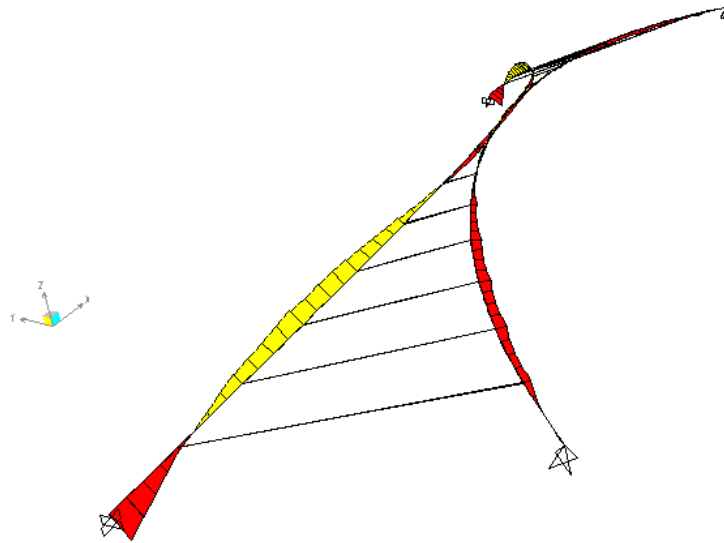
**Figure 3-49: Model (c). Longitudinal tangential movements free at deck abutments. M3-3 bending moments: Deck balcony beam bending moments and arch in-plane bending moments under a vertical uniformly distributed loading  $q_{10}$ . Longitudinal view**



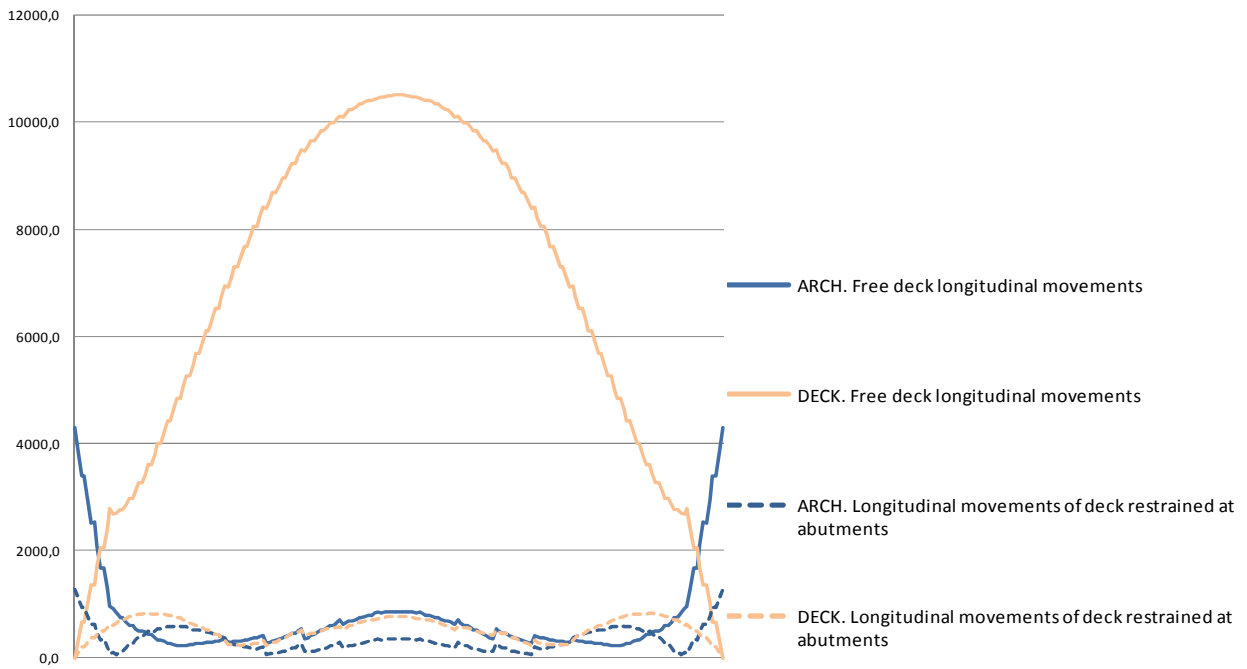
**Figure 3-50: Model (c). Longitudinal tangential movements free at deck abutments. M2-2 bending moments: Deck vertical axis bending moments and arch out-of-plane bending moments under a vertical uniformly distributed loading  $q_{10}$ . 3D view**



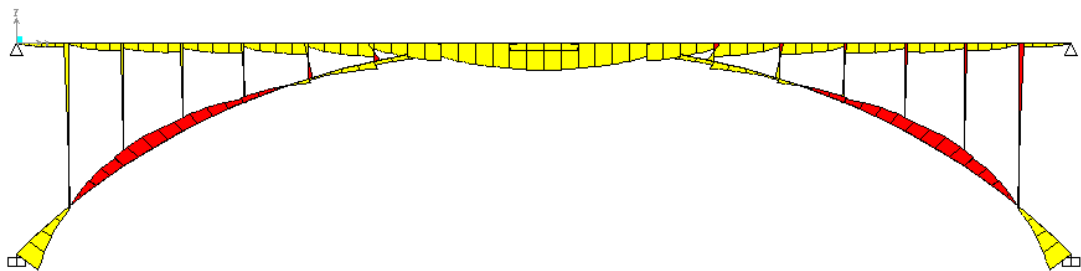
**Figure 3-51: Model (c). Longitudinal tangential movements of deck restrained at abutments. M3-3 bending moments: Deck balcony beam bending moments and arch in-plane bending moments under a vertical uniformly distributed loading  $q_{10}$ . Longitudinal view**



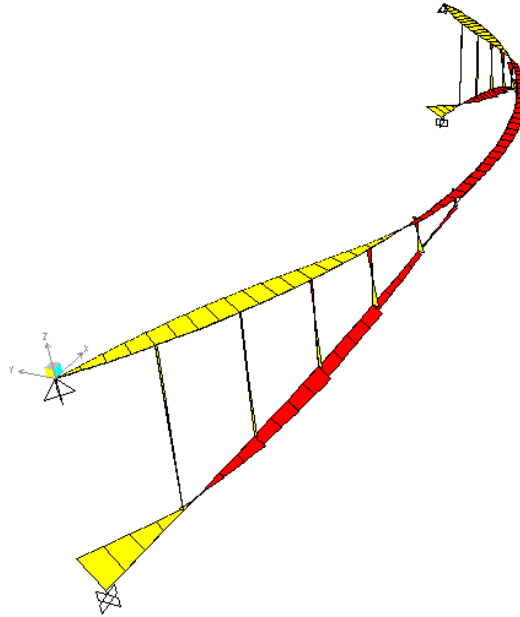
**Figure 3-52: Model (c). Longitudinal tangential movements of deck restrained at abutments. M2-2 bending moments: Deck vertical axis bending moments and arch out-of-plane bending moments under a vertical uniformly distributed loading  $q_{10}$ . 3D view**



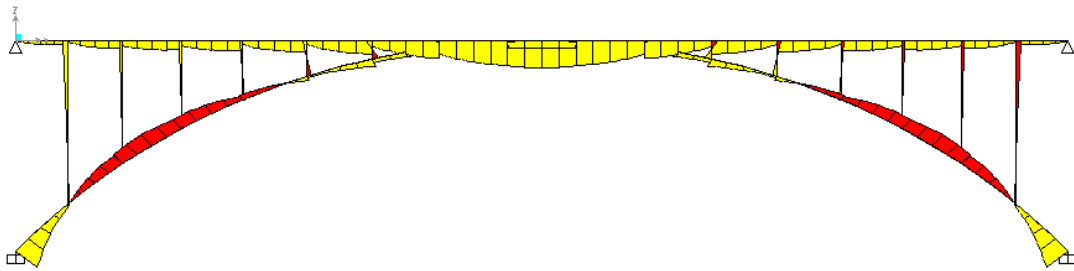
**Figure 3-53: Model (d): Arch and deck total bending moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments. The ordinates are the total bending moments (kN·m) and the abscissas are the bridge length from 0 to  $L_{bridge}$**



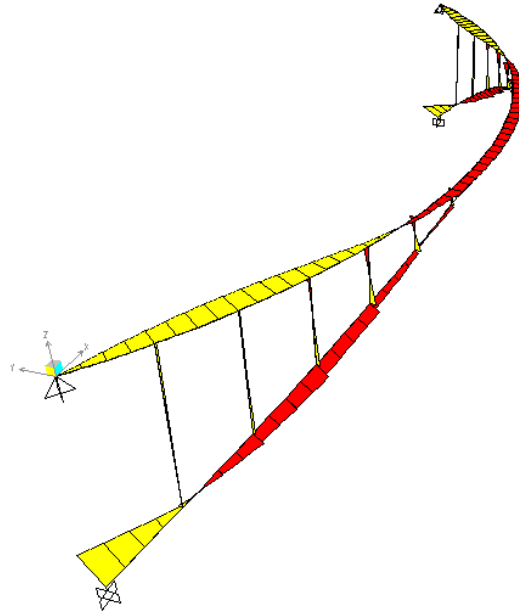
**Figure 3-54: Model (d). Longitudinal tangential movements free at deck abutments. M3-3 bending moments: Deck balcony beam bending moments and arch in-plane bending moments under a vertical uniformly distributed loading  $q_{10}$ . Longitudinal view**



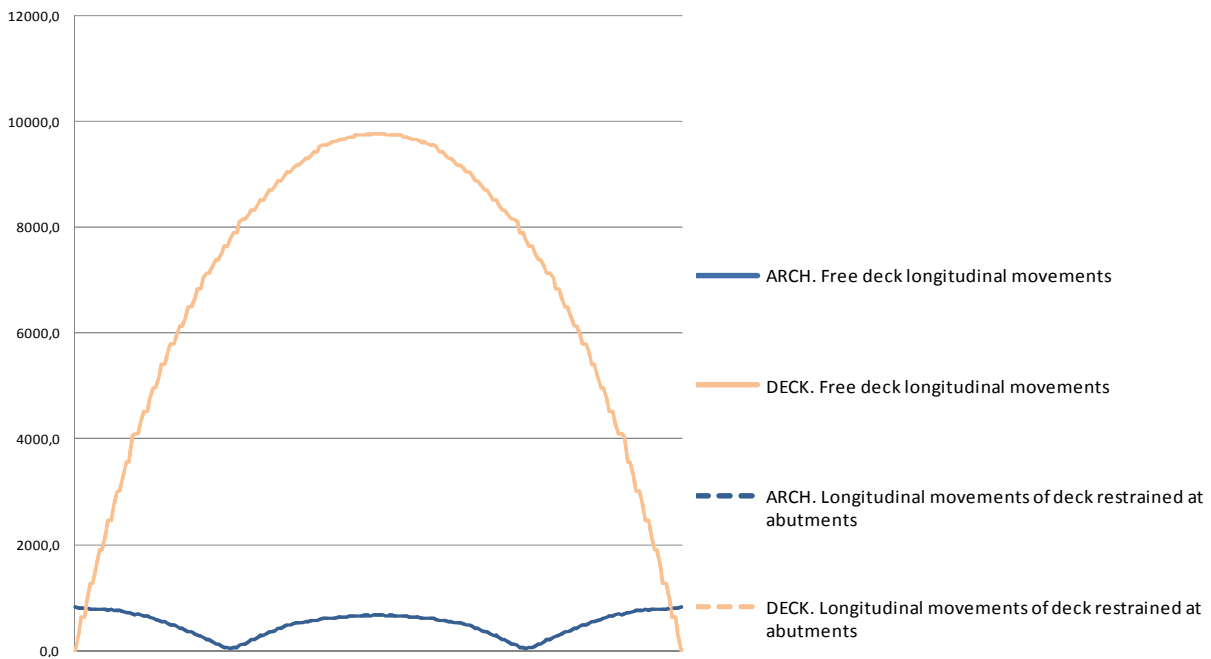
**Figure 3-55: Model (d). Longitudinal tangential movements free at deck abutments. M2-2 bending moments: Deck vertical axis bending moments and arch out-of-plane bending moments. 3D view**



**Figure 3-56: Model (d). Longitudinal tangential movements of deck restrained at abutments. M3-3 bending moments: Deck balcony beam bending moments and arch in-plane bending moments. Longitudinal view**



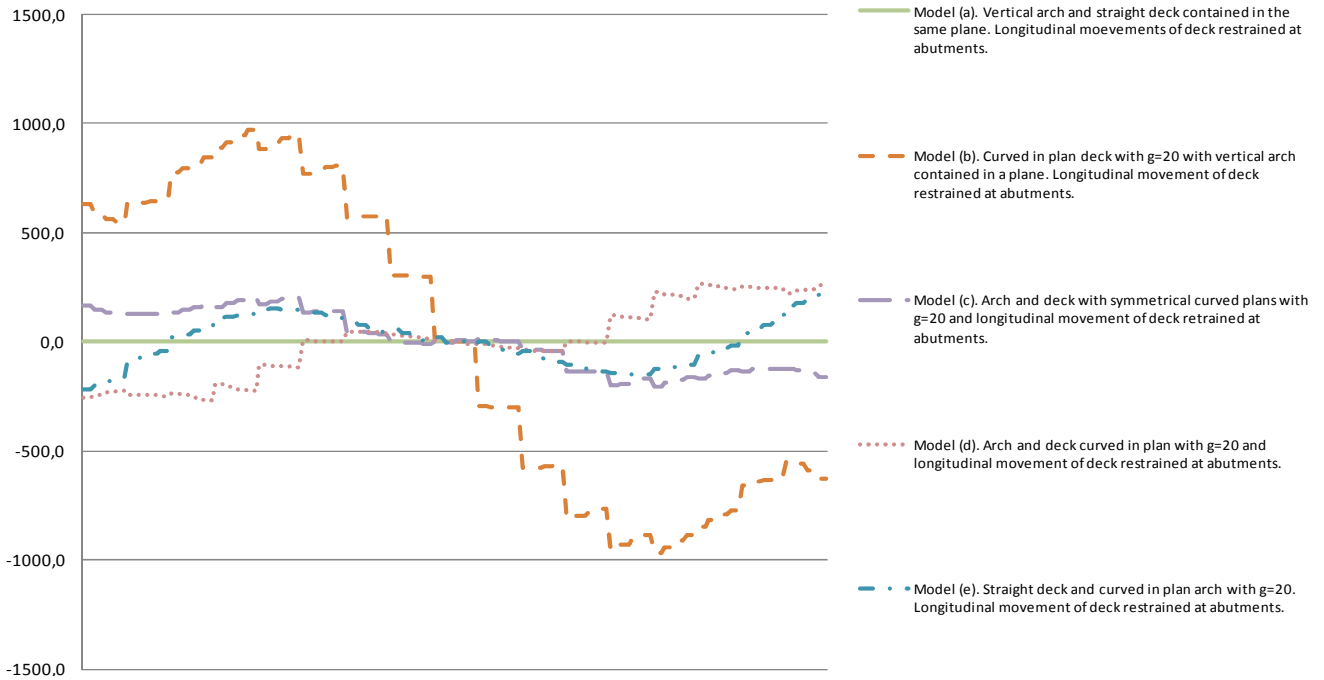
**Figure 3-57: Model (d). Longitudinal tangential movements of deck restrained at abutments. M2-2 bending moments: Deck vertical axis bending moments and arch out-of-plane bending moments.**  
3D view



*Note: differences between the two bearing hypothesis are so small that they cannot be appreciated graphically.*

**Figure 3-58: Model (e): Arch and deck total bending moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Restrained longitudinal movements at deck abutments. The ordinates are the total bending moments (kN·m) and the abscissas are the bridge length from 0 to  $L_{bridge}$**

### 3.2.3 Torsional moments. Bearing conditions: restrained longitudinal displacements at deck abutments



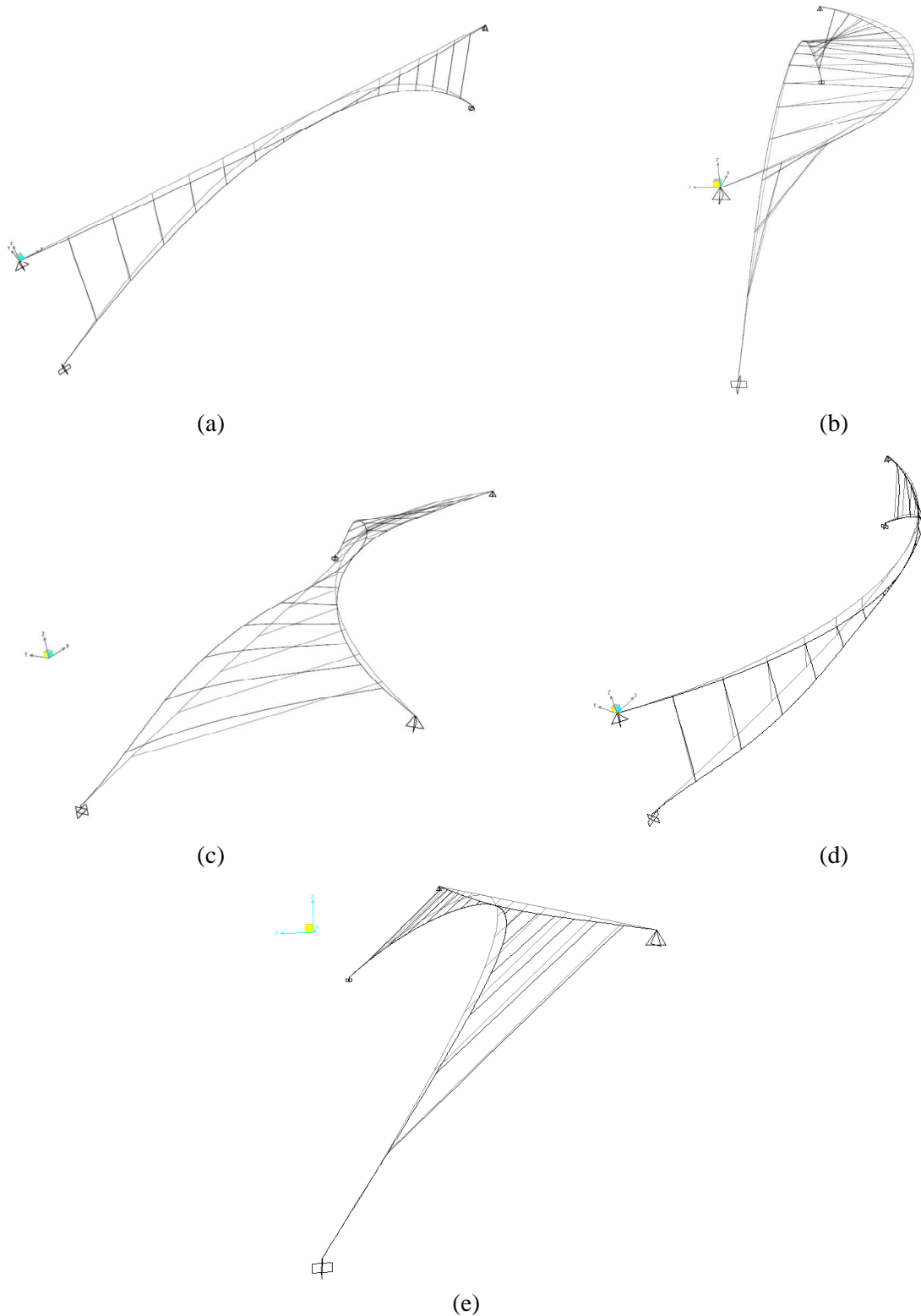
**Figure 3-59: Models (a) to (e). Torsional moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Deck longitudinal movements restrained at abutments. The ordinates are the torsional bending moments ( $kN\cdot m$ ) and the abscissas are the deck length from 0 to  $L_D$**

Comparing Figure 3-42 and Figure 3-59, the relationship between bending and torsional moments can be appreciated. Where we have important torques, there are also important bending moments and vice versa, since they are coupled:

$$\frac{dM}{ds} = -V - \frac{T}{R}$$

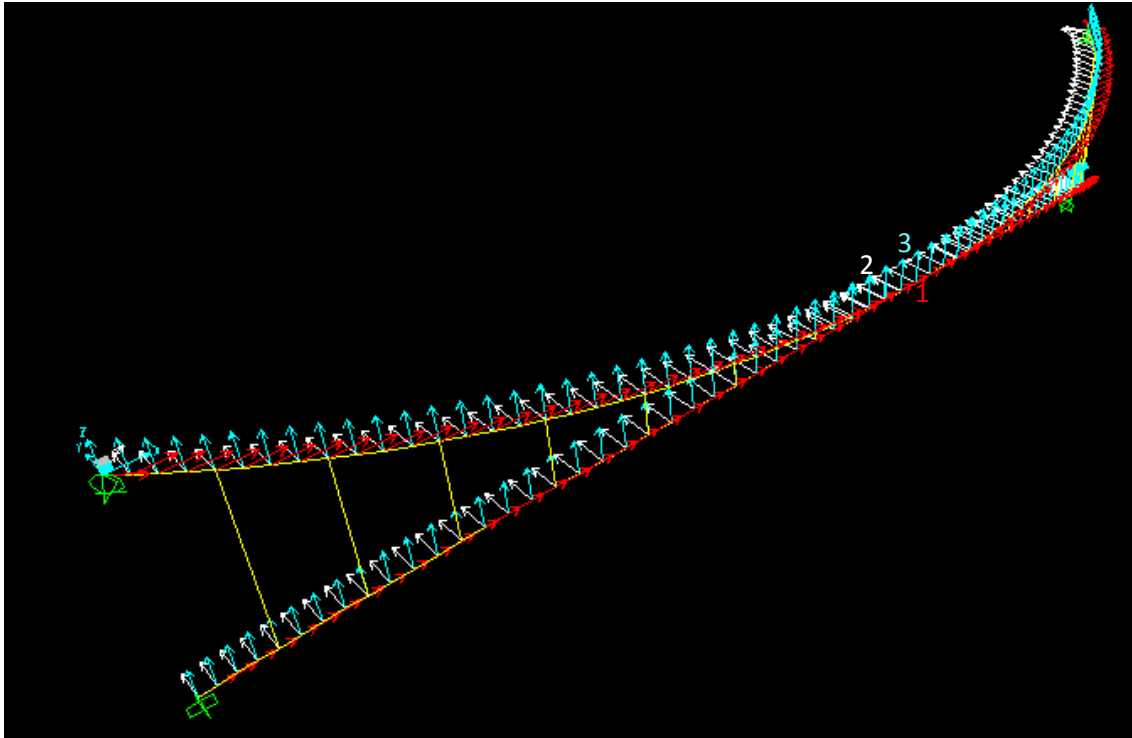
### 3.2.4 Displacements. Bearing conditions: restrained longitudinal displacements at deck abutments

The deformed shape of the different models can be seen on the following Figure 3-60.



**Figure 3-60- Deformed shapes of the studied cases under a vertical 10kN/m uniform load on the deck with restrained longitudinal movements at deck abutments. (a) classical vertical arch contained in a plane with straight superior deck; (b) vertical arch contained in a plane with curved superior deck; (c) Both arch and deck curved and symmetrical in plan; (d)Both arch and deck curved and coincident in plan; (e) Straight deck and curved in plan arch**

On the following Table 3-9 is given the value of the displacements for the different models at span center. The direction nomenclature is employed according to the axes which are parallel to the global ones, ie: 1-2 contained in the plane that contains the deck, with 1 parallel to the direction defined by the abutments and 2 perpendicular to it and positive in the inwards direction of the arch curve (and of the deck curve for model (b)). 3 is vertical and positive upwards. As an example we can see the axis of joints for model (d) on Figure 3-61.



**Figure 3-61: Model (d). Joints axis**

In comparison with the free longitudinal displacements at deck abutments, displacements, both horizontal and vertical, diminish greatly for model (b), (c) and (d). For models (a) and (e) they do not change.

Horizontal displacements change their sign for model (b) at both arch and deck, ie: when restraining the longitudinal displacements the deck moves inwards its curvature and the arch moves in the same direction. These displacements diminish greatly as expected from the boundary conditions.

It is remarkable that restraining longitudinal displacements is a way to achieve lower vertical displacements too. Therefore, regarding vertical displacements under a vertical uniform load it is more favourable to restrain the deck longitudinal movements.

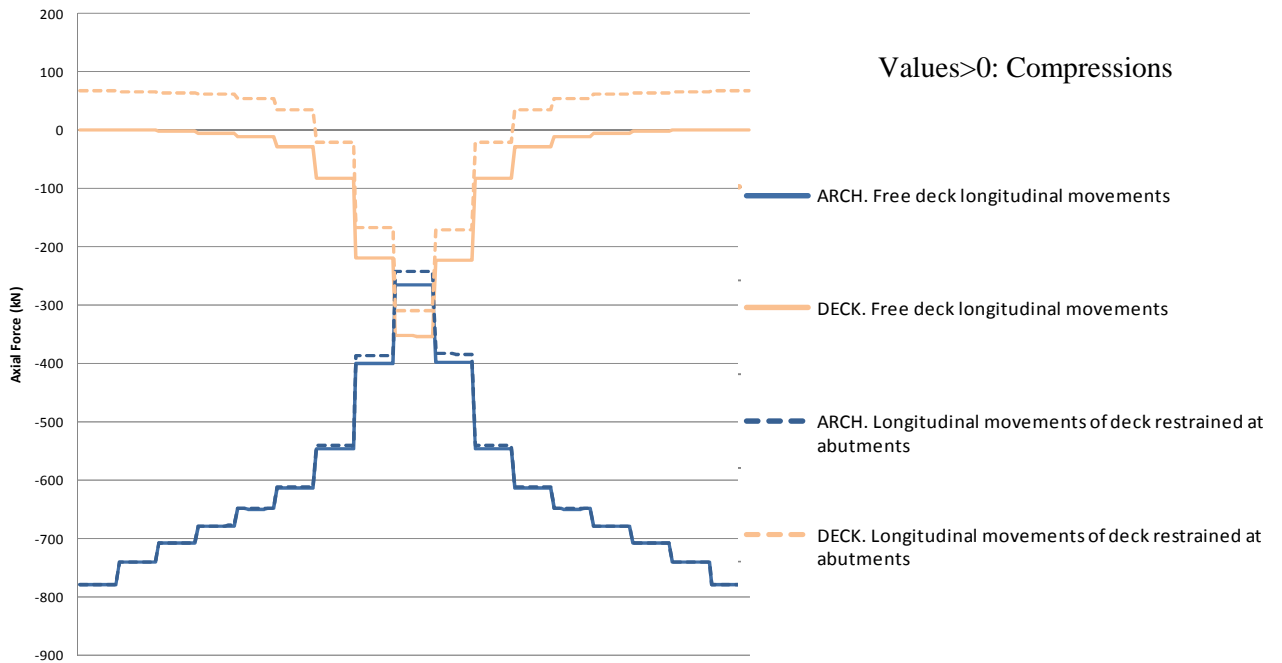


| Model |      | $u_1$ longitudinal displacements (mm) | $u_2$ horizontal displacements (mm) | $u_3$ vertical displacements (mm) |
|-------|------|---------------------------------------|-------------------------------------|-----------------------------------|
| (a)   |      | 0                                     | 0                                   | -5                                |
| (b)   | Arch | 0                                     | 4                                   | 36                                |
|       | Deck | 0                                     | 4                                   | -567                              |
| (c)   |      | 0                                     | -2,3                                | -25                               |
| (d)   |      | 0                                     | -4                                  | -42                               |
| (e)   |      | 0                                     | -207                                | -221                              |

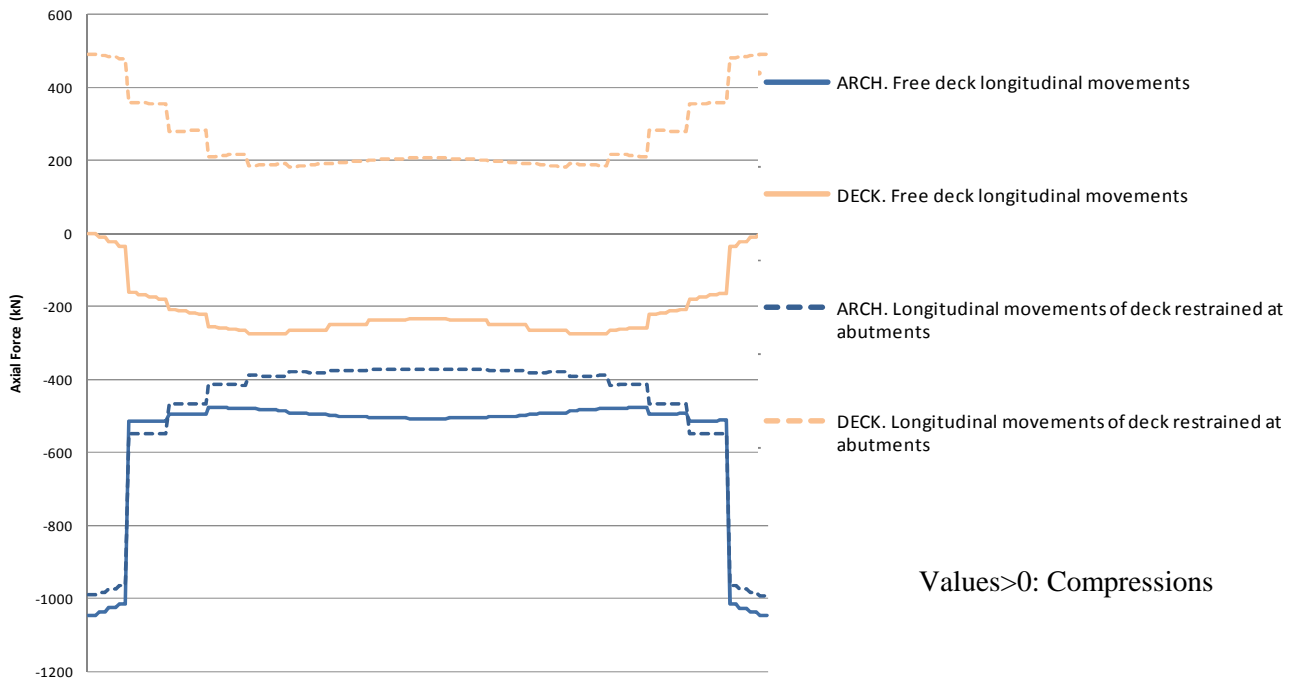
**Table 3-9: Models (a) to (e). Fixed struts. Restrained longitudinal displacements at deck abutments. Displacements at deck and arch span center under a vertical uniformly distributed loading  $q_{10}$ .**

### 3.2.5 Axial forces. Bearing conditions: free longitudinal displacements at deck

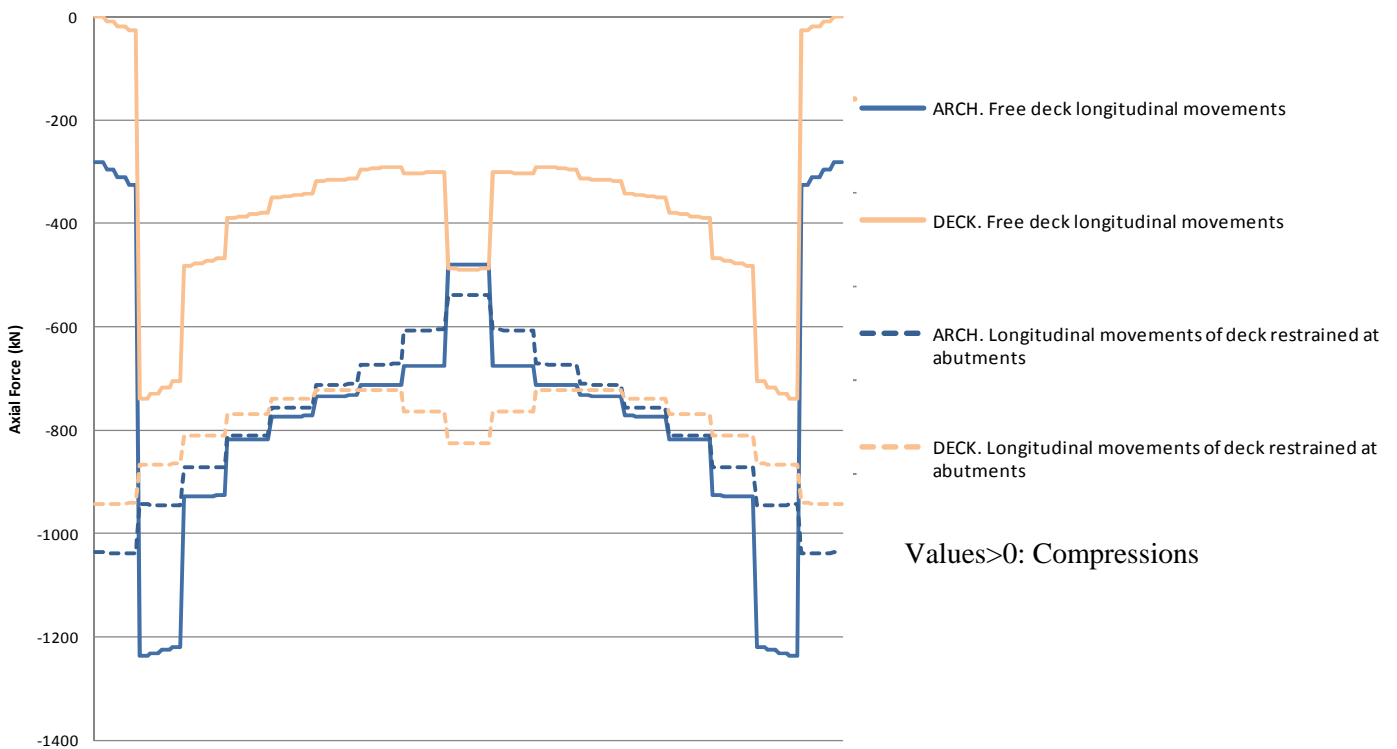
On the following figures (from Figure 3-62 to Figure 3-66) we can see the comparison of the axial forces at arch for each model with free or restrained longitudinal movements of the deck at abutments.



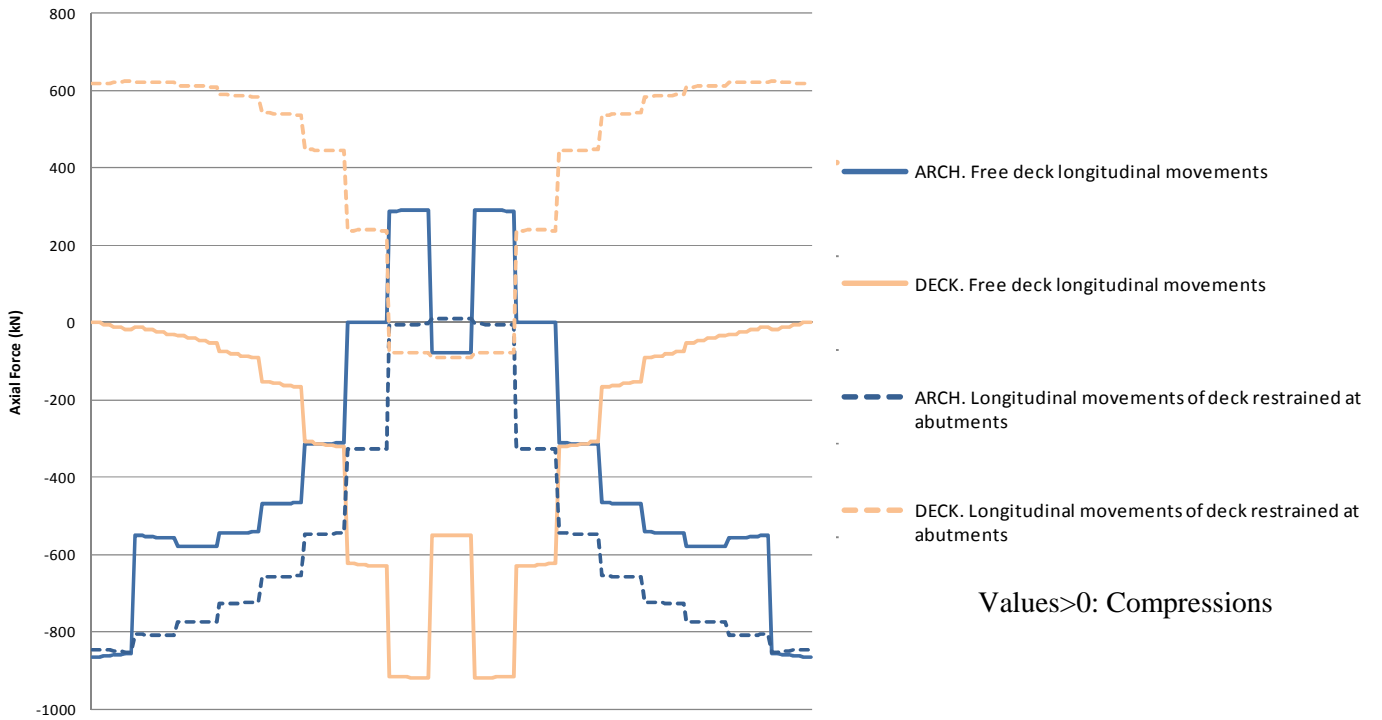
**Figure 3-62: Model (a). Axial forces in arch and deck under a vertical uniformly distributed loading  $q_{10}$ . Comparison of deck longitudinal movements restrained or free. The abscissas are the bridge length from 0 to  $L_{bridge}$**



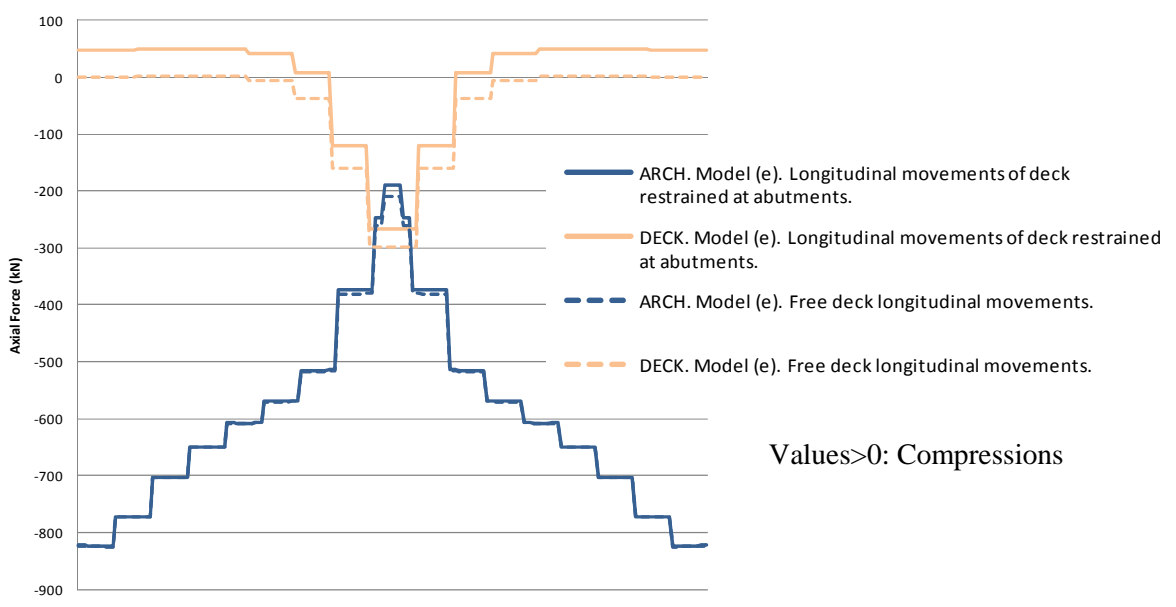
**Figure 3-63: Model (b). Axial forces in arch and deck under a vertical uniformly distributed loading  $q_{10}$ . Comparison of deck longitudinal movements restrained or free. *The abscissas are the bridge length from 0 to  $L_{bridge}$***



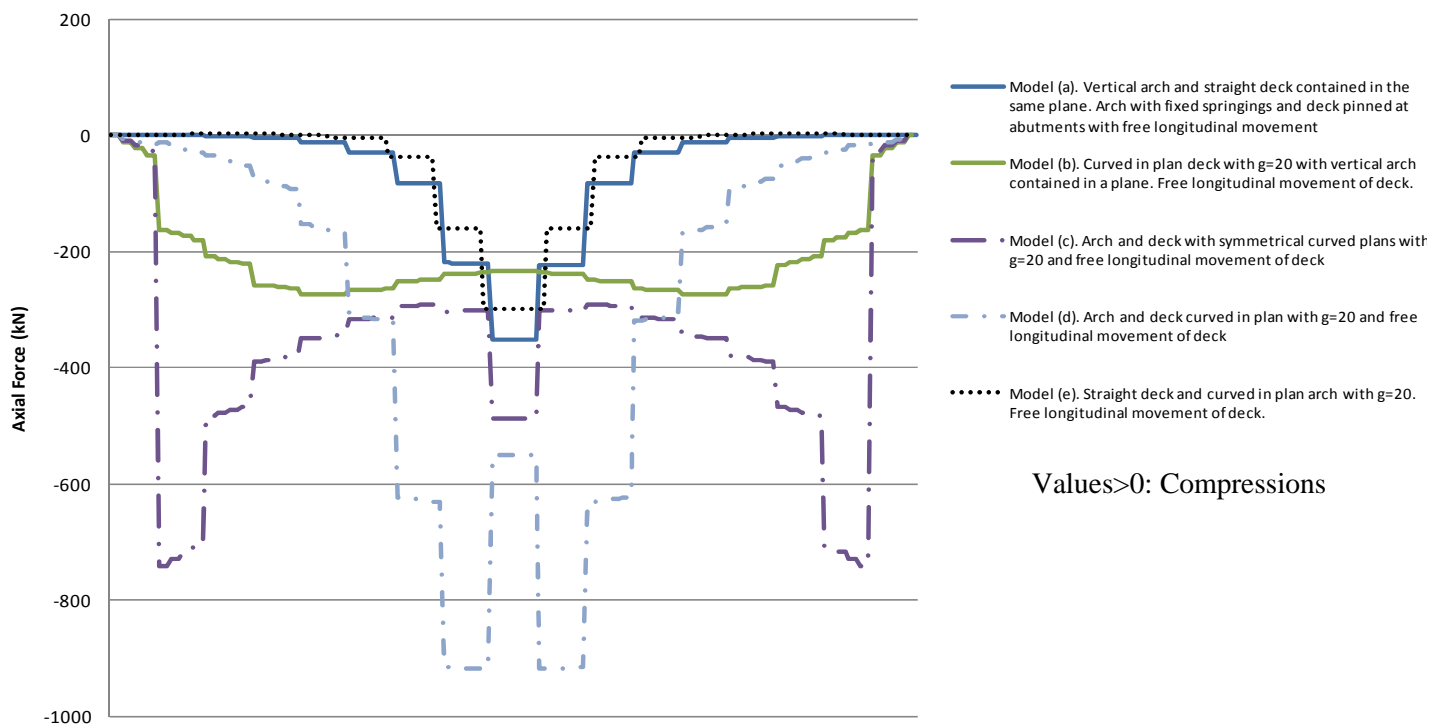
**Figure 3-64: Model (c). Axial forces in arch and deck under a vertical uniformly distributed loading  $q_{10}$ . Comparison of deck longitudinal movements restrained or free. *The abscissas are the bridge length from 0 to  $L_{bridge}$***



**Figure 3-65: Model (d). Axial forces in arch and deck under a vertical uniformly distributed loading  $q_{10}$  of deck longitudinal movements restrained or free. The abscissas are the bridge length from 0 to  $L_{bridge}$**



**Figure 3-66: Model (e). Axial forces in arch and deck under a vertical uniformly distributed loading  $q_{10}$ . Comparison of deck longitudinal movements restrained or free. The abscissas are the bridge length from 0 to  $L_{bridge}$**



**Figure 3-67: Deck axial forces comparison under  $q_{10}$  for different geometries in superior deck arch bridges. Fixed struts. Longitudinal displacements of deck free at the connection of deck with abutments. The abscissas are the deck length from 0 to  $L_D$**

On the following tables from Table 3-10 to Table 3-13 we can see the axial and shear forces at struts and at arch and deck span center and they can be compared with the results of tables from Table 3-3 to Table 3-6, which show the same results for the equivalent models with free longitudinal deck movements at abutments.

| Element  | Axial force (kN) | Shear 2-2 (kN) | Shear 3-3 (kN) |
|--|------------------|----------------|----------------|
| <b>Struts</b>  |                  |                |                |
| 1  | -65,6            | -0,77          | 0              |
| 2  | -59,8            | -1,54          | 0              |
| 3  | -58,4            | -3,07          | 0              |
| 4  | -59,5            | -6,71          | 0              |
| 5  | -60,3            | -17,43         | 0              |
| 6  | -63,3            | 53,71          | 0              |
| 7  | -61,6            | -137,20        | 0              |
| 8  | -35,4            | -133,05        | 0              |
| <b>Arch and deck axial forces at span center (kN)</b>              |                  |                |                |
| SC arch  |                  | -266,2         |                |
| SC deck  |                  | -352,7         |                |
| Sum of arch and deck axial forces                                  |                  | -618,9         |                |
| <b>Total horizontal reactions at abutments and springings (kN)</b> |                  |                |                |
| $R_x$ arch   |                  | 618,8          |                |
| $R_x$ deck   |                  | 0              |                |
| Sum of arch and deck horizontal reactions                          |                  | 618,8          |                |

**Table 3-10: Model (a). Shear forces and axial forces at struts and span center under a vertical uniformly distributed loading  $q_{10}$ . Longitudinal displacements free at deck abutments.**

When restraining the longitudinal displacements at the deck connection with abutments the forces hardly vary for model (a) when comparing them to the free longitudinal deck movements

case. The deck is more tensioned (tensions at extremes appear because of the movement restrain, before they were only transmitted by the fixed struts) and axial compression forces at deck and arch span center diminish, but the variations are negligible.

| Element | Axial force (kN) | Shear 2-2 (kN) | Shear 3-3 (kN) |
|---------|------------------|----------------|----------------|
| 1       | -995,9           | 14,33          | -2,72          |
| 2       | -59,2            | 10,39          | 7,59           |
| 3       | -113,5           | -1,85          | 3,50           |
| 4       | -64,7            | -13,43         | -6,74          |
| 5       | -18,2            | -21,11         | -11,51         |
| 6       | 13,5             | -25,58         | -10,77         |
| 7       | 32,0             | -27,97         | -7,20          |
| 8       | 41,8             | -29,00         | -2,48          |
| SC arch | -507,2           | -5,99          | 0              |
| SC deck | -233,3           | 0              | 2,67           |

**Table 3-11: Model (b). Shear forces and axial forces at struts and span center under a vertical uniformly distributed loading q10. Longitudinal displacements free at deck abutments.**

For model (b) 3-3 shear forces at extremes increase a lot when longitudinal displacements are restrained at abutments, as expected because shear forces appear mainly at extremes to prevent the movements. At span center, since movements are smaller, shear 3-3 forces are smaller.

| Element | Axial force (kN) | Shear 2-2 (kN) | Shear 3-3 (kN) |
|---------|------------------|----------------|----------------|
| 1       | 1277,7           | -2,27          | 0,59           |
| 2       | -436,8           | -4,67          | 2,88           |
| 3       | -183,2           | -8,33          | 2,86           |
| 4       | -85,0            | -16,23         | 1,12           |
| 5       | -91,7            | -34,50         | -2,56          |
| 6       | -76,7            | -70,00         | -7,92          |
| 7       | -79,2            | -55,40         | 20,54          |
| 8       | -70,5            | -1,59          | 190,96         |
| SC arch | -480,5           | -4,66          | 4,67           |
| SC deck | -488,4           | 11,72          | 0              |

Note: Shear 3-3 forces are concentrated on the shortest central struts

**Table 3-12: Model (c). Shear forces and axial forces at struts and span center under a vertical uniformly distributed loading q10. Longitudinal displacements free at deck abutments.**

When restraining longitudinal movements at deck abutments (Table 3-4) and comparing the results to the free longitudinal displacements hypothesis in model (c) (Table 3-12), the 3-3 shear forces do not increase as much as they did for model (b).

V2-2 shear forces decrease.

Axial forces decrease as they did for model (b): Extreme struts are compressed, whereas for free deck longitudinal displacements they were tensioned.

| Element | Axial force (kN) | Shear 2-2 (kN) | Shear 3-3 (kN) |
|---------|------------------|----------------|----------------|
| 1       | -582,0           | 10,15          | -7,14          |
| 2       | 36,5             | 2,20           | -1,02          |
| 3       | -48,1            | -12,42         | 12,88          |
| 4       | -58,4            | -42,81         | 39,77          |
| 5       | -67,2            | -117,85        | 80,05          |
| 6       | -75,0            | -286,64        | 94,92          |
| 7       | -43,4            | -283,83        | 14,08          |
| 8       | -3,3             | 367,40         | 36,26          |
| SC arch | -77,6            | -6,09          | 2,93           |
| SC deck | -550,6           | 0              | 9,27           |

*Shear 2-2: Important change of sign from 7 to 8.*

**Table 3-13: Model (d). Shear forces and axial forces at struts and span center under a vertical uniformly distributed loading  $q_{10}$ . Longitudinal displacements free at deck abutments.**

When restraining longitudinal movements at deck abutments and comparing the results to the free longitudinal displacements hypothesis in model (d), the 3-3 shear forces increase at L/4 (Table 3-13 compared with Table 3-5).

V2-2 shear forces decrease.

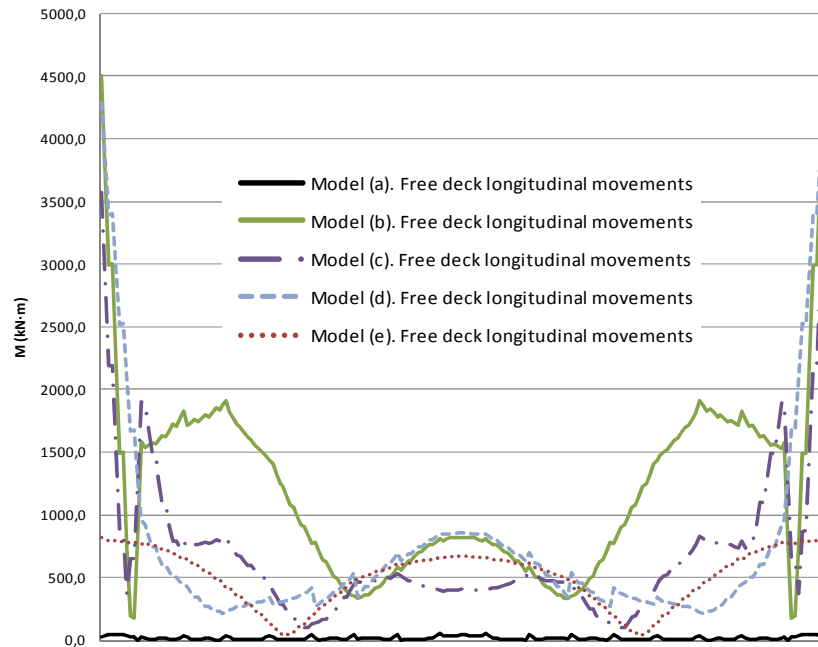
Axial forces decrease a lot in extreme struts and the arch, the arch is even tensioned at span center.

When restraining longitudinal movements at deck abutments and comparing the results to the free longitudinal displacements hypothesis in model (e), the 3-3 shear forces increase slightly.

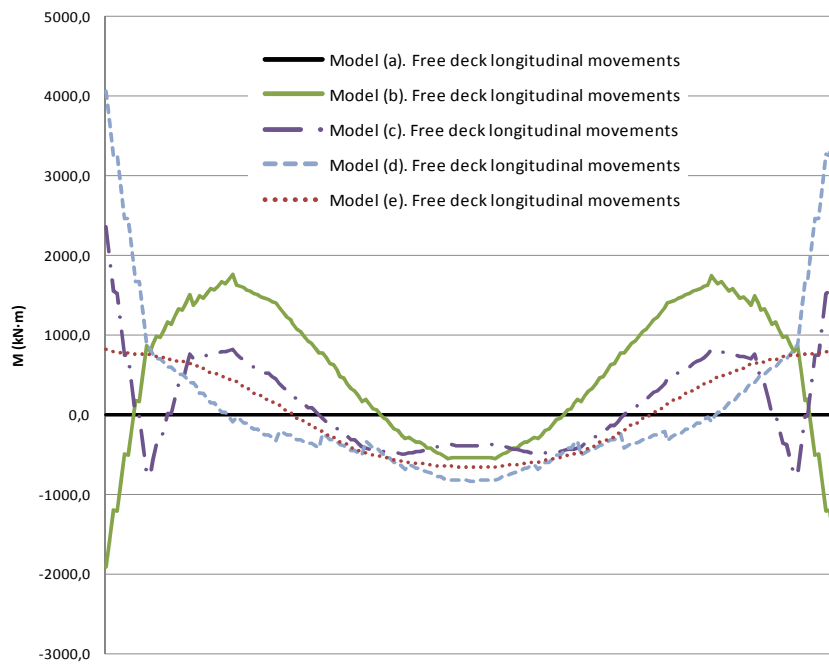
V2-2 shear and axial forces hardly vary.

### 3.2.6 Bending moments. Bearing conditions: free deck longitudinal displacements

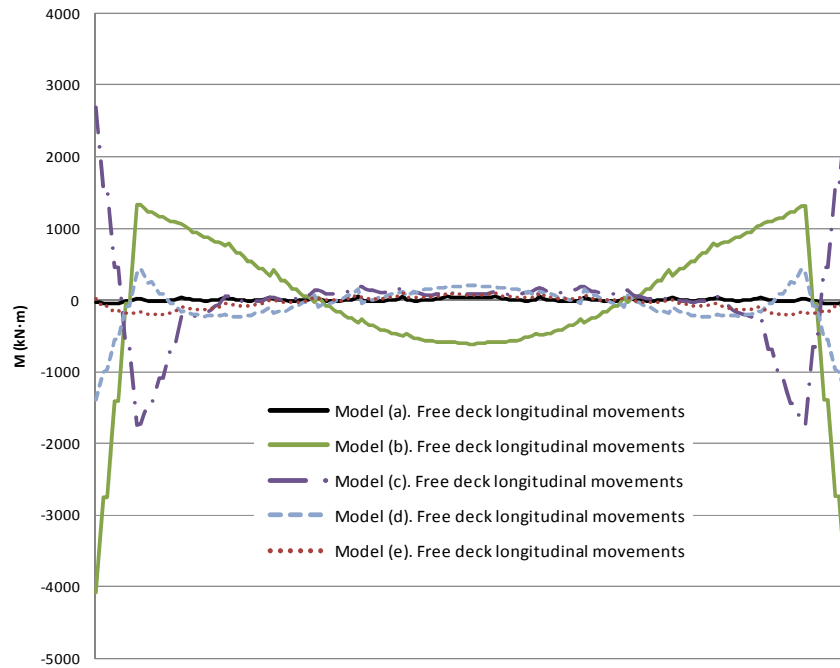
In figures from Figure 3-68 to Figure 3-71 the comparison of the bending moments for the different studied geometries under a uniformly distributed vertical load  $q=10\text{kN/m}$  can be observed.



**Figure 3-68: Models (a) to (e). Arch total bending moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Longitudinal displacements free at deck abutments. The abscissas are the arch length from 0 to  $L_A$**

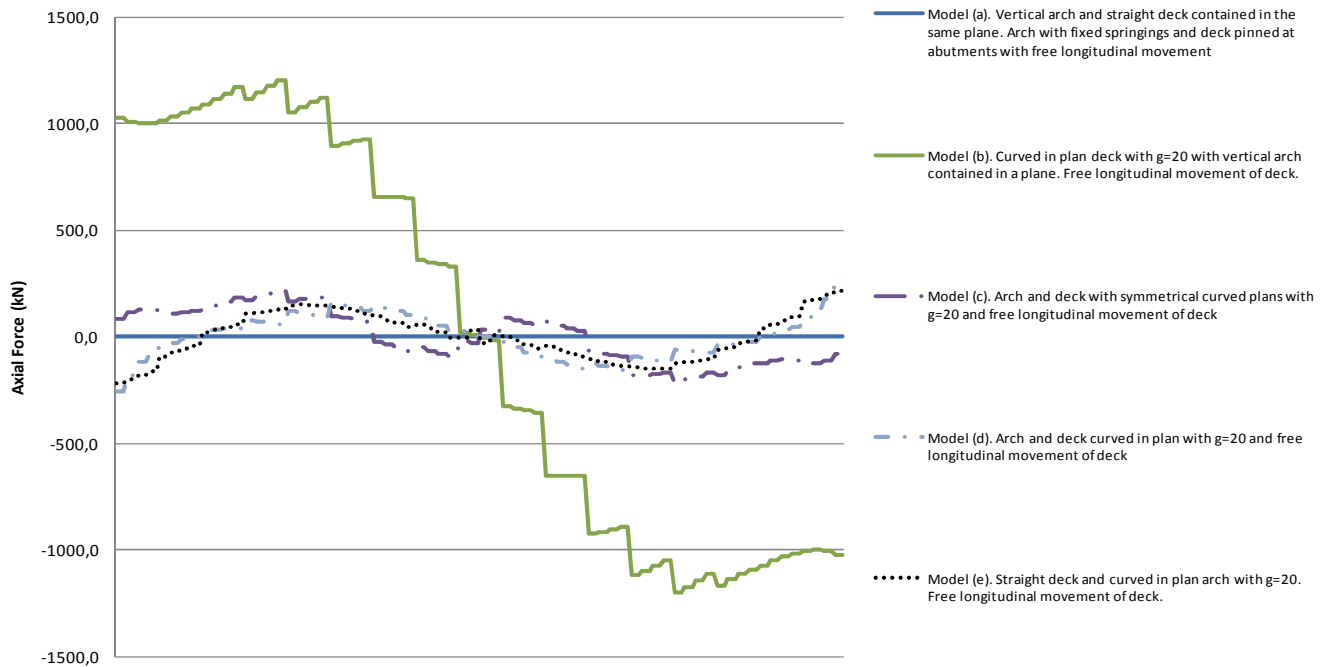


**Figure 3-69: Models (a) to (e). Arch out-of-plane bending moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Longitudinal displacements free at deck abutments. The abscissas are the arch length from 0 to  $L_A$**



**Figure 3-70: Models (a) to (e). Arch in-plane bending moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Longitudinal displacements free at deck abutments. The abscissas are the arch length from 0 to  $L_A$**

### 3.2.7 Torsional moments. Bearing conditions: free deck longitudinal displacements



**Figure 3-71: Models (a) to (e). Arch torsional moments comparison under a vertical uniformly distributed loading  $q_{10}$ . Longitudinal displacements free at deck abutments. The abscissas are the arch length from 0 to  $L_A$**



3.2.8 Displacements. Bearing conditions: free longitudinal displacements at deck abutments

The definition of the local axis is the same as the one in section 3.2.4.

| Model |      | $u_1$ longitudinal displacements (mm) | $u_2$ horizontal displacements (mm) | $u_3$ vertical displacements (mm) |
|-------|------|---------------------------------------|-------------------------------------|-----------------------------------|
| (a)   |      | 0                                     | 0                                   | -5                                |
| (b)   | Arch | 0                                     | -252                                | 50                                |
|       | Deck | 0                                     | -252                                | -745                              |
| (c)   |      | 0                                     | -151                                | -185                              |
| (d)   |      | 0                                     | -333                                | 354                               |
| (e)   |      | 0                                     | -207                                | -221                              |

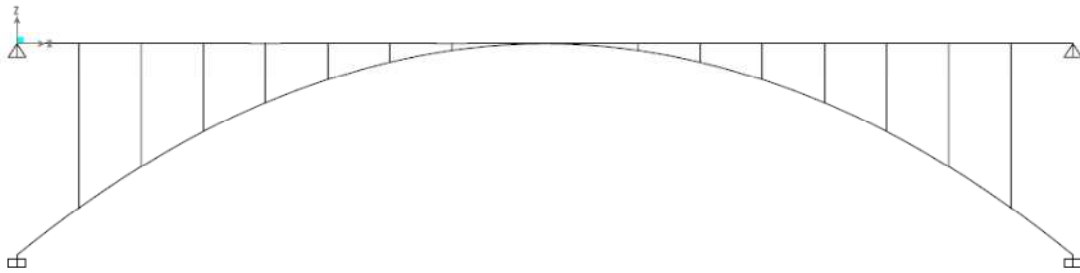
**Table 3-14: Models (a) to (e). Fixed struts. Free longitudinal displacements at deck abutments. Displacements at deck and arch span center under a vertical uniformly distributed loading  $q_{10}$ .**

## 4. STRUCTURAL RESPONSE UNDER TEMPERATURE VARIATION

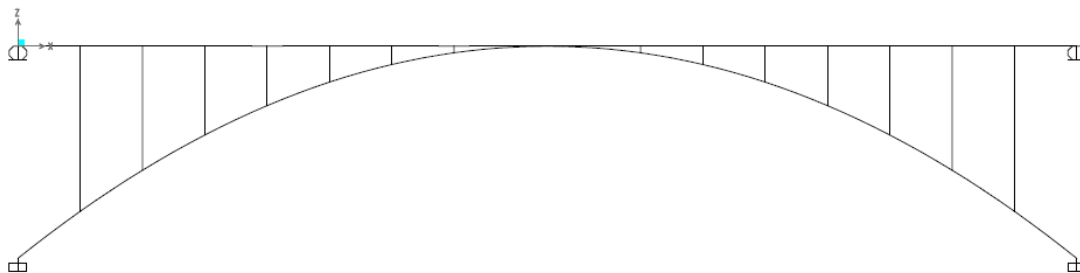
### 4.1 RELEASED VERSUS FIXED LONGITUDINAL MOVEMENTS

Different superior deck geometries have been studied. First, in front of a  $25^{\circ}\text{C}$  temperature variation on deck, and, afterwards, for this same temperature variation in both arch and deck. Each case has been studied for both joint cases: fixed (Figure 4-1(a)) or free (Figure 4-1 (b)) movement on the deck longitudinal direction. In both cases the arch springings are clamped.

(a)

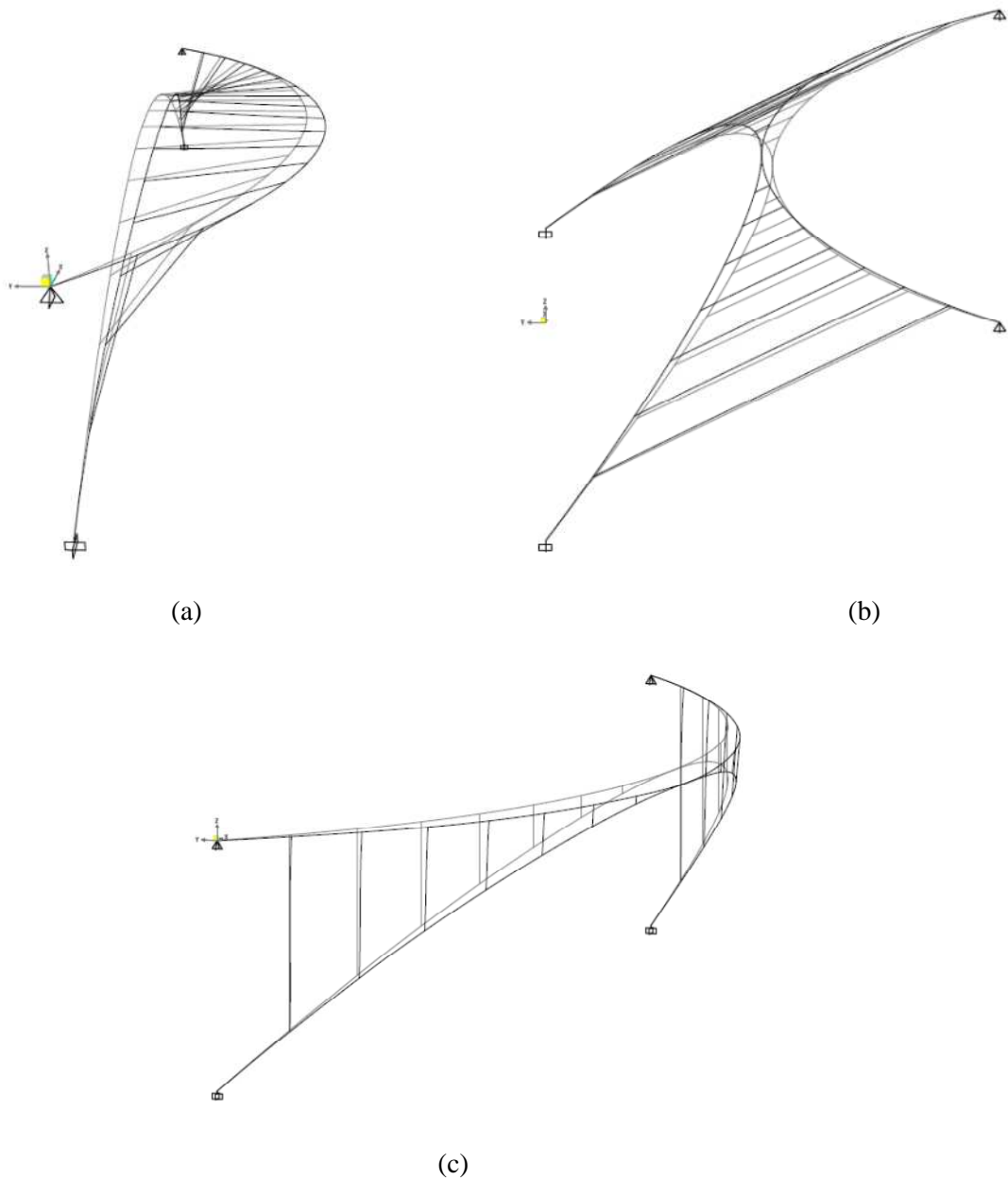


(b)

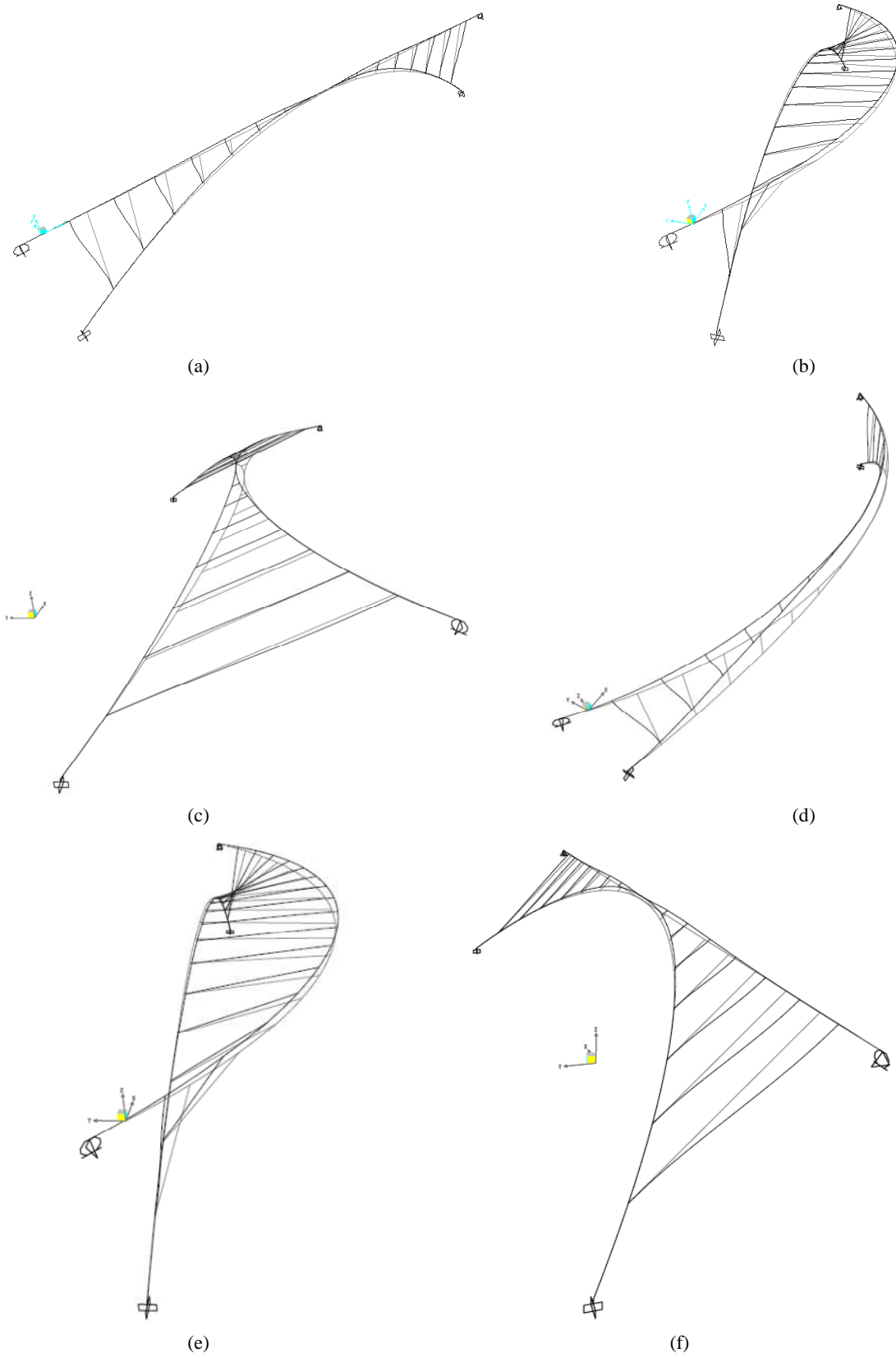


**Figure 4-1: Longitudinal displacements (a) restrained at deck abutments (b) free at deck abutments**

On the following figures (Figure 4-2 and Figure 4-3) the different studied geometries and their deformed shapes can be observed.

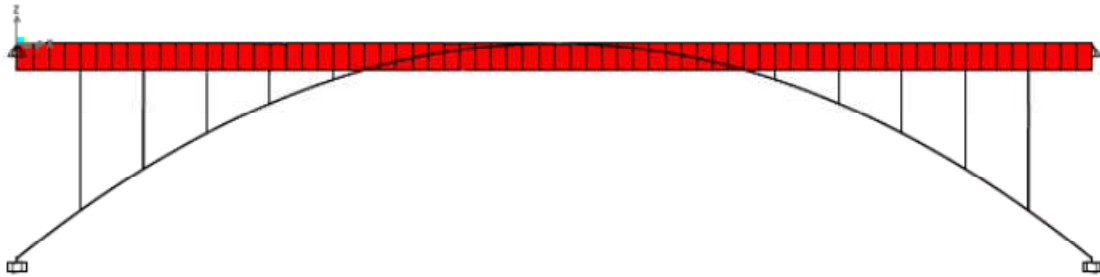


**Figure 4-2- Deformed shape under a 25°C temperature variation. Abutments pinned and horizontal displacements fixed at deck abutments. (a) Model b: vertical arch contained in a plane with curved superior deck; (b) Model c: Both arch and deck curved and symmetrical in plan; (c) Model d: Both arch and deck curved and coincident in plan;**



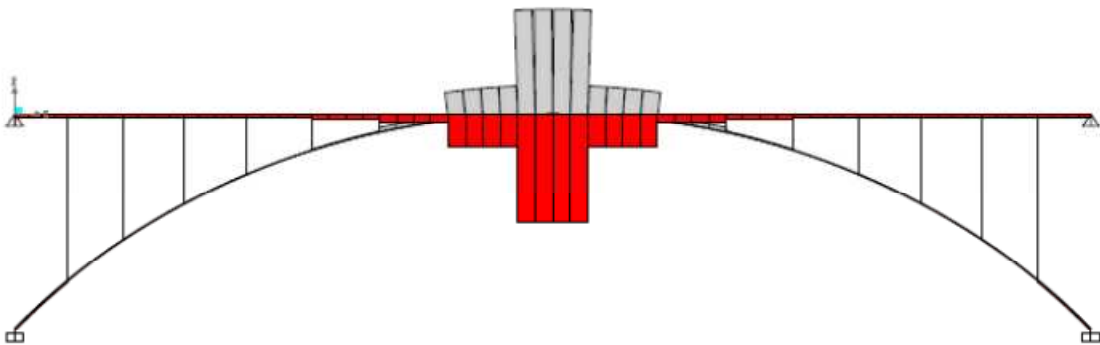
**Figure 4-3- Deformed shape under a 25°C temperature variation. Abutments pinned and horizontal displacements released. (a) Classical vertical arch contained in a plane with straight superior deck; (b) vertical arch contained in a plane with curved superior deck; (c) Both arch and deck curved and symmetrical in plan; (d) Both arch and deck curved and coincident in plan; (e) Vertical arch and curved deck; (f) Straight deck and curved in plan arch**

In Figure 4-4 we can see the structural behaviour of a superior deck vertical arch bridge contained in a plane for cross-section correspondent to model 1 of Table 2-1 with abutments pinned and horizontal displacements fixed under a temperature variation of 25 °C on the deck.

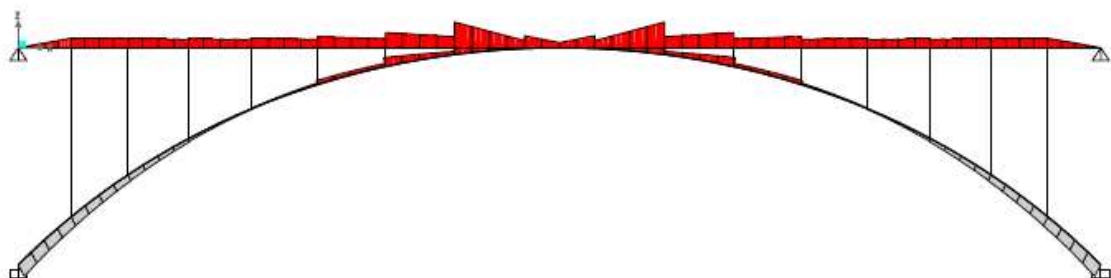


**Figure 4-4: Model (a). Superior straight deck vertical arch bridge contained in a plane. Axial forces**

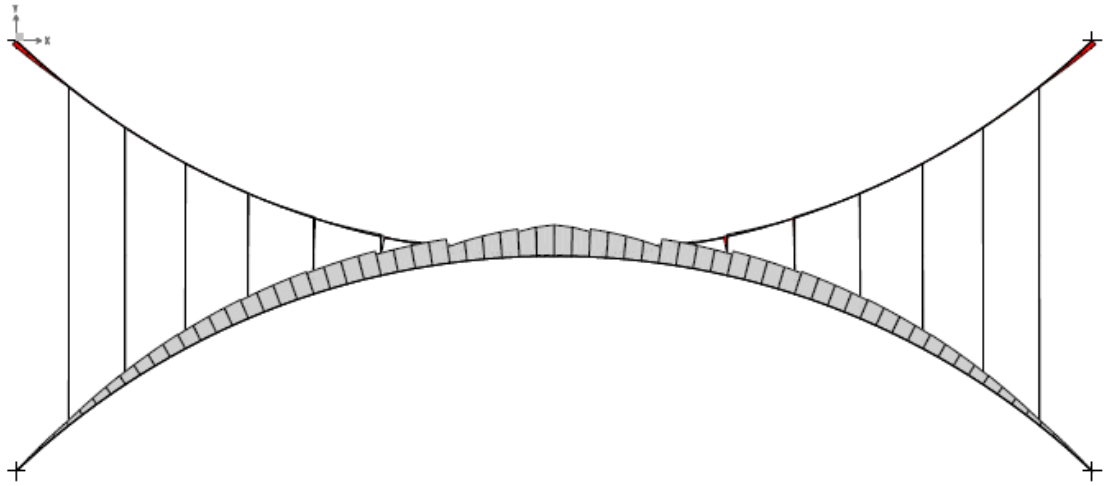
On the following figures (from Figure 4-5 to Figure 4-8) we can see the behaviour of a superior deck arch bridge with deck and arch symmetrically curved in plan(model c, Figure 2-1) for cross-section correspondent to model 1 of Table 2-1 with the abutments pinned and displacements fixed.



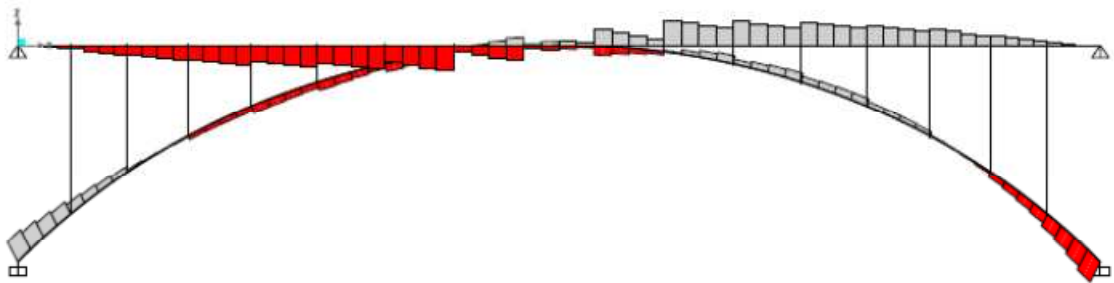
**Figure 4-5- Model (c). Superior deck arch bridge with deck and arch symmetrically curved in plan. Axial forces.**



**Figure 4-6- Model (c). Bending moments 3-3**

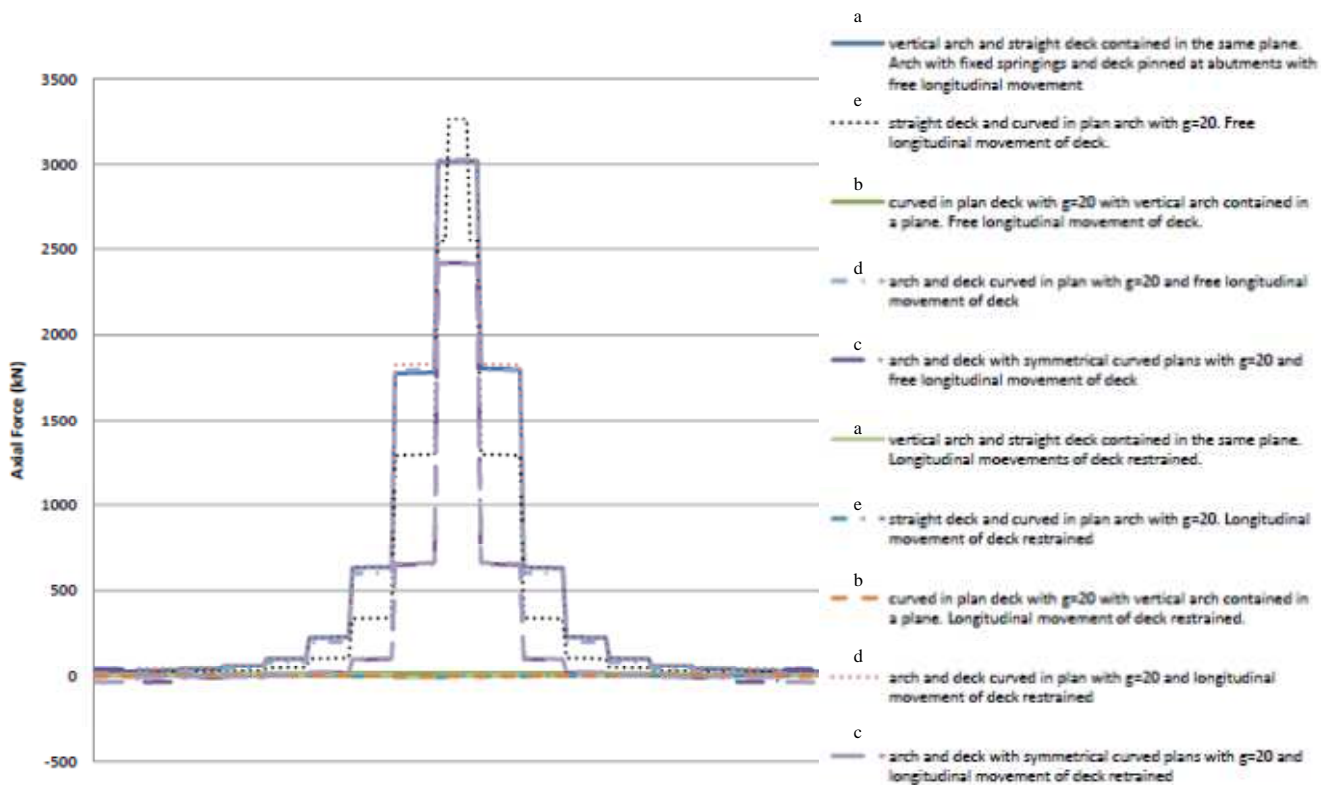


**Figure 4-7- Model (c). Bending moments 2-2 (plan view)**



**Figure 4-8- Model (c). Torsional moments**

On the following figures we can see the comparison between different geometries (from Figure 4-9 to Figure 4-13).

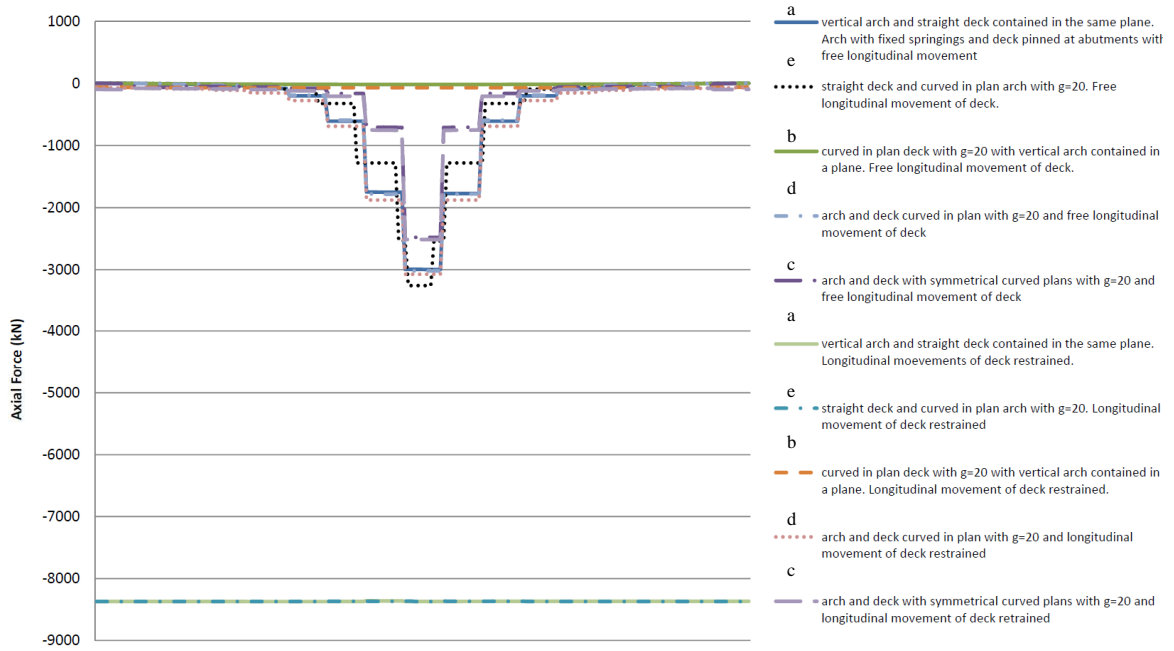


**Figure 4-9: Arch axial forces under a 25°C temperature variation on arch and deck. Comparison for different geometries and different support conditions at deck abutments. *The abscissas are the arch length from 0 to  $L_A$***

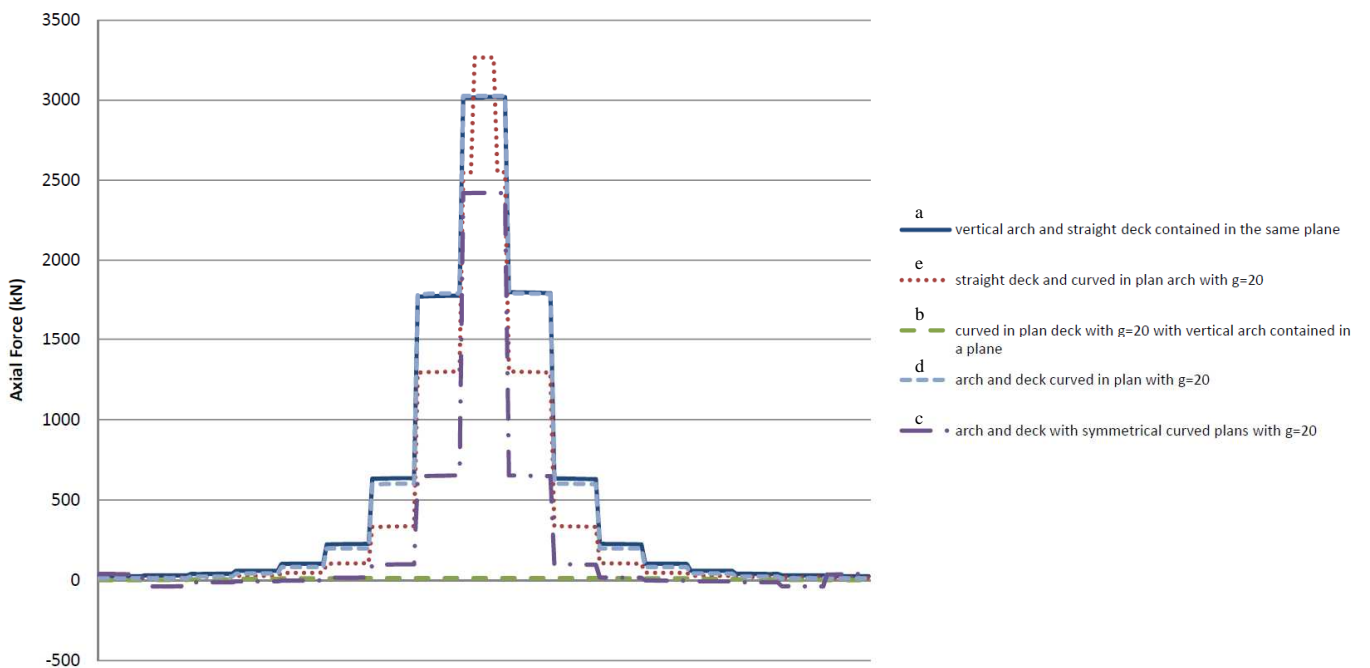
In all the cases, when the temperature is increased on the deck, the arch is tensioned and the deck is compressed. It is surprising that the axial forces in the arch for the case in which arch and deck have a symmetrical plan have the same sign as for the case in which arch and deck have a coincident plan.

Temperature variations cause negligible axial forces on the arch and deck for the case of a classical planar vertical arch with a curved superior deck (Figure 4-10 and Figure 4-11). For the rest of cases axial forces do appear on the arch if the deck has a free longitudinal movement. Maximal axial forces are obtained for the case of a straight in plan deck with a curved in plan arch. When thinking of the deformations (Figure 4-3) these results are completely logical.

When arch and deck are both curved in plan it makes no difference releasing longitudinal movements at abutments or not. However, if one (arch or deck) is straight in plan, restraining the longitudinal displacements of the abutments eliminates the axial forces caused on the arch by the temperature variation (Figure 4-9).

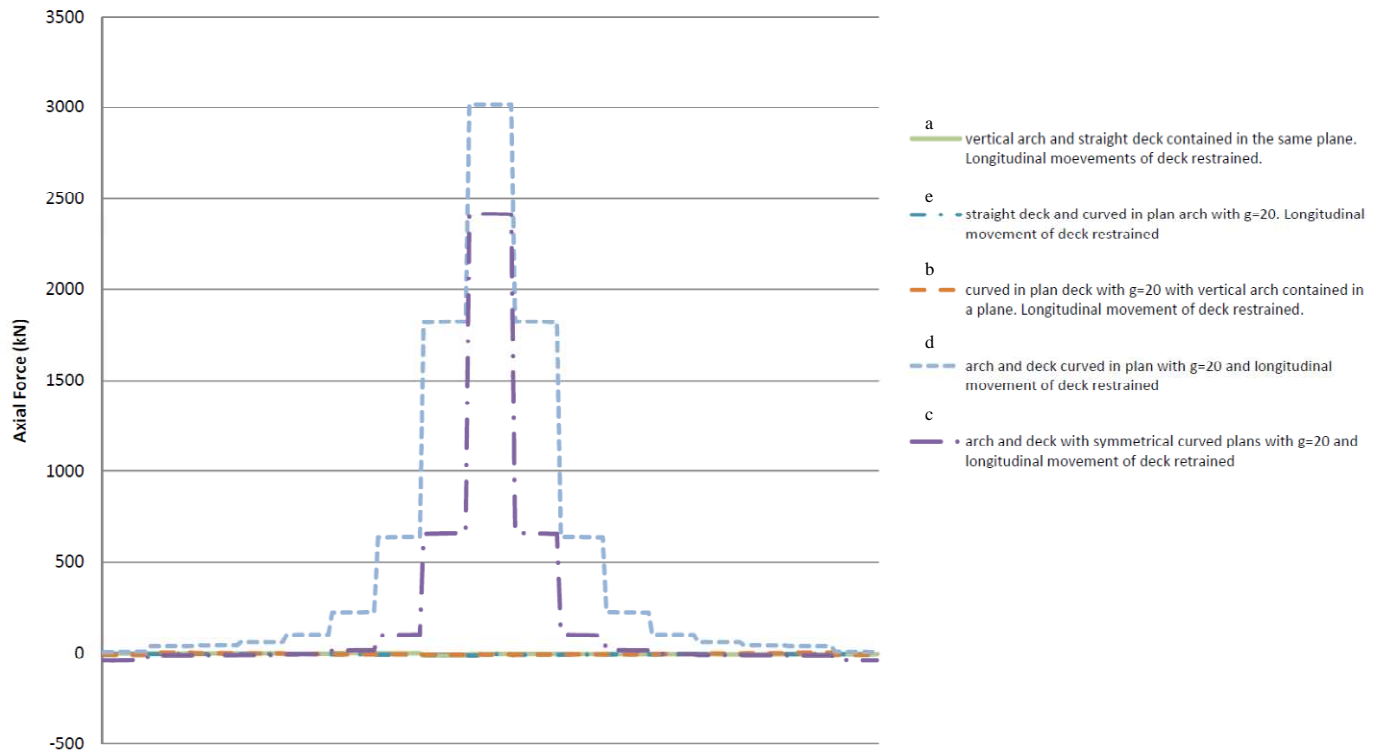


**Figure 4-10 Deck axial forces under a 25°C temperature variation on arch and deck. Comparison for different geometries and different support conditions at deck abutments. The abscissas are the deck length from 0 to  $L_D$**

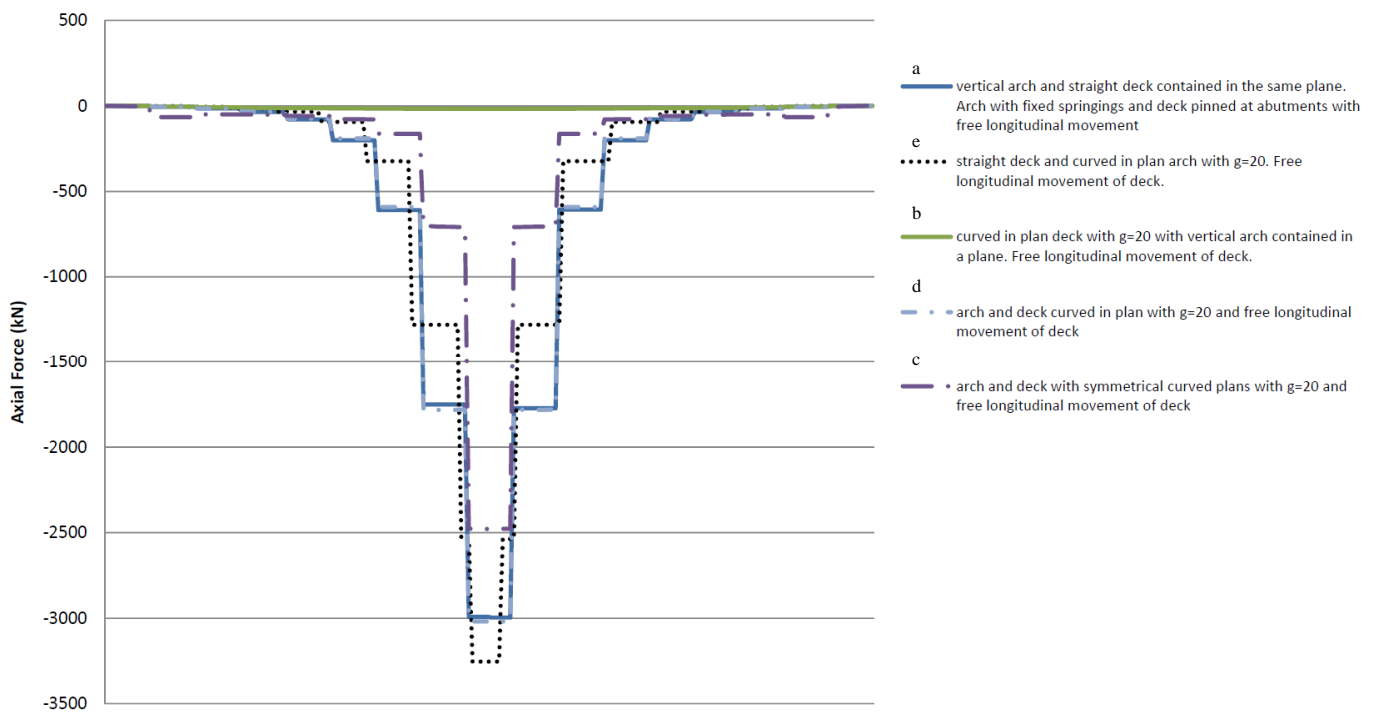


**Figure 4-11: Arch axial forces under a 25°C temperature variation on arch and deck. Comparison for different geometries with free longitudinal movements at deck abutments. The abscissas are the arch length from 0 to  $L_A$**



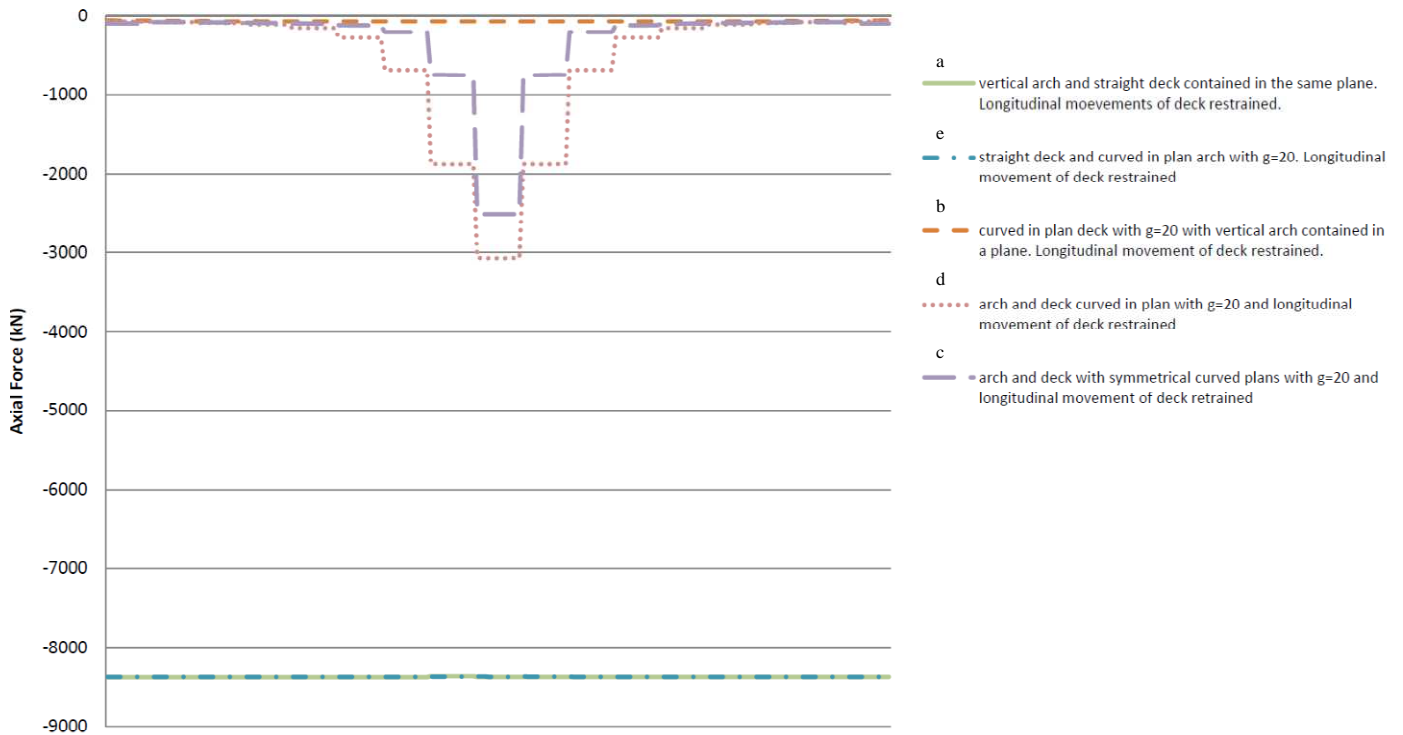


**Figure 4-12: Arch axial forces under a 25°C temperature variation on arch and deck. Comparison for different geometries with restrained longitudinal movements at deck abutments. The abscissas are the arch length from 0 to  $L_A$**



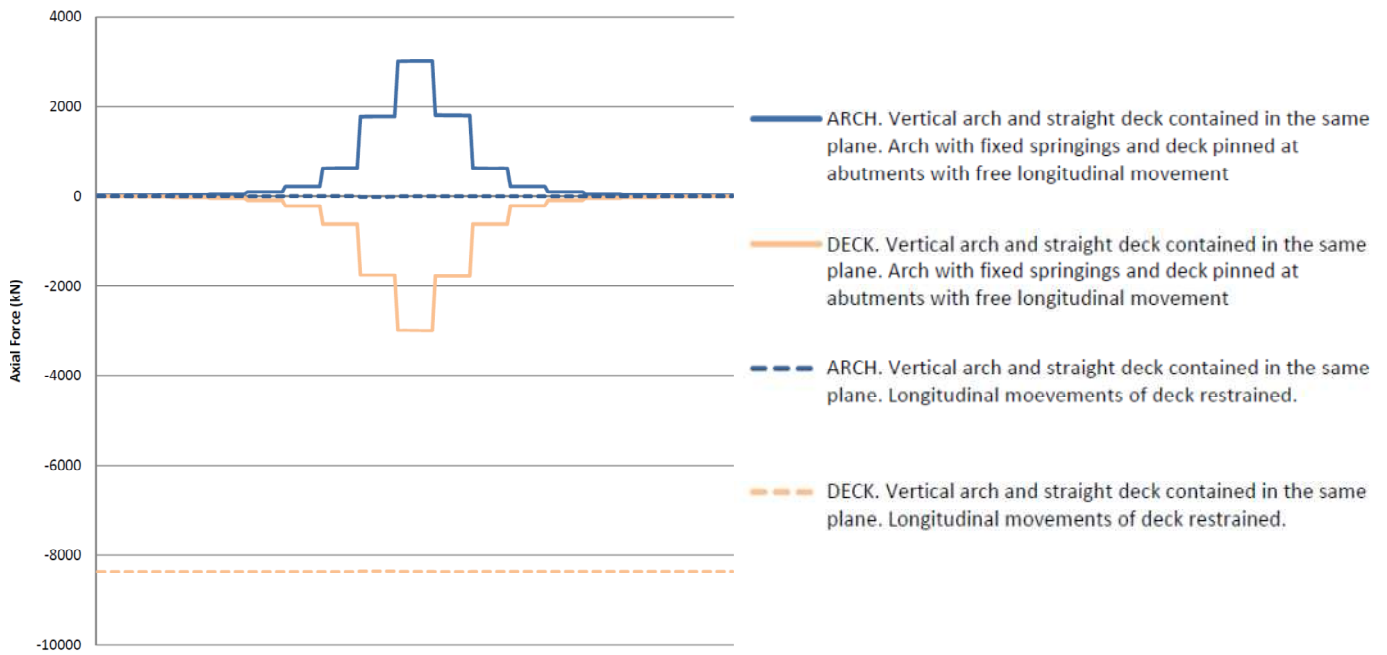
**Figure 4-13 Deck axial forces under a 25°C temperature variation on arch and deck. Comparison for different geometries with free longitudinal movements at deck abutments. The abscissas are the deck length from 0 to  $L_D$**

On the first two cases of Figure 4-13 the axial forces are lower than for the horizontal restrained displacement case (Figure 4-14). The other cases have approximately the same results as when restrained (Figure 4-13 and Figure 4-14).

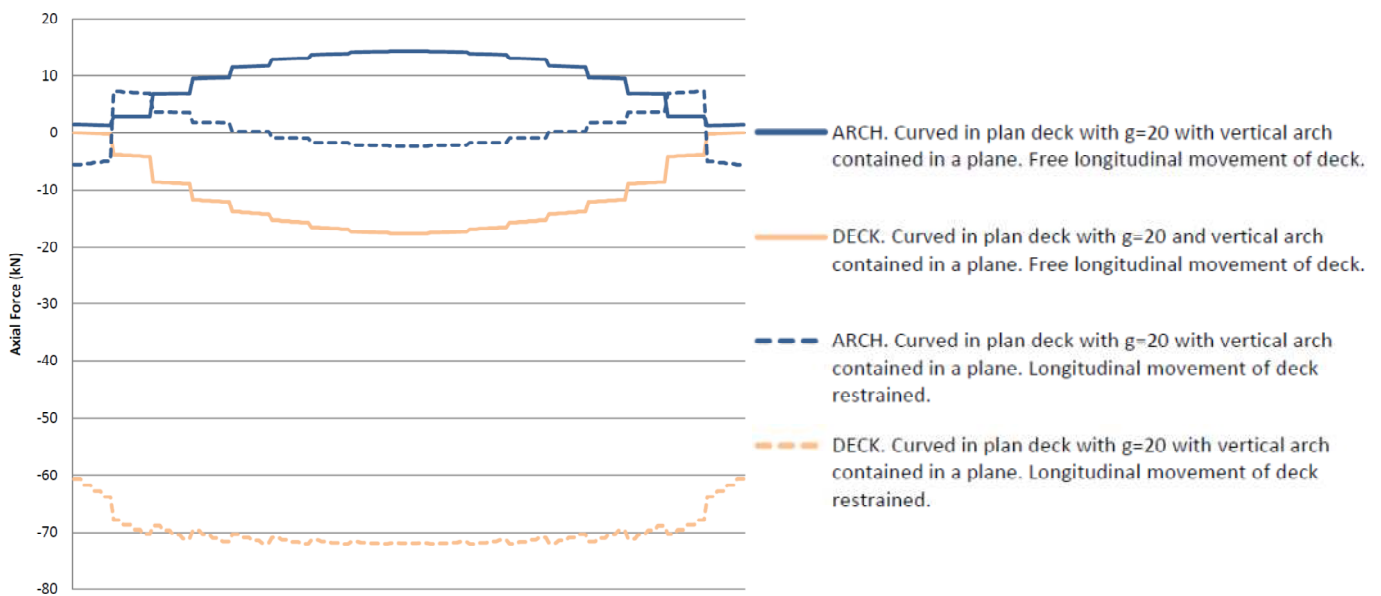


**Figure 4-14: Deck axial forces under a 25°C temperature variation on arch and deck. Comparison for different geometries with restrained longitudinal movements at deck abutments. The abscissas are the deck length from 0 to  $L_D$**

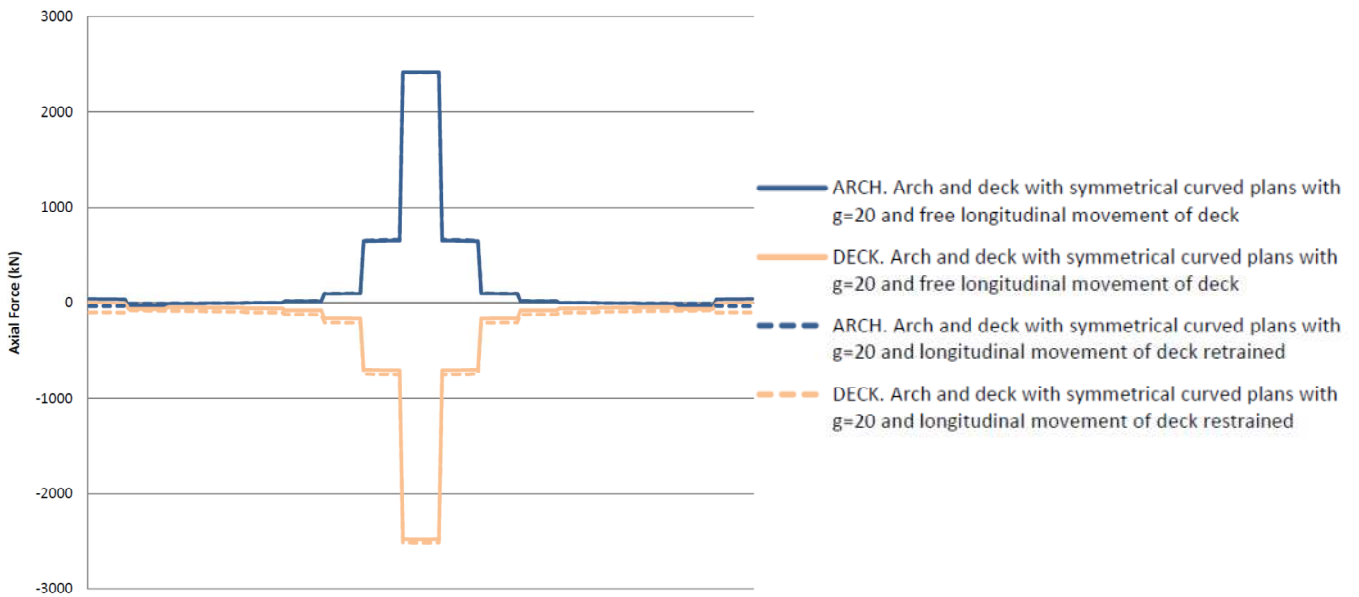
We can see the comparison between released or not horizontal displacement for every geometry case on the following figures, from Figure 4-15 to Figure 4-19.



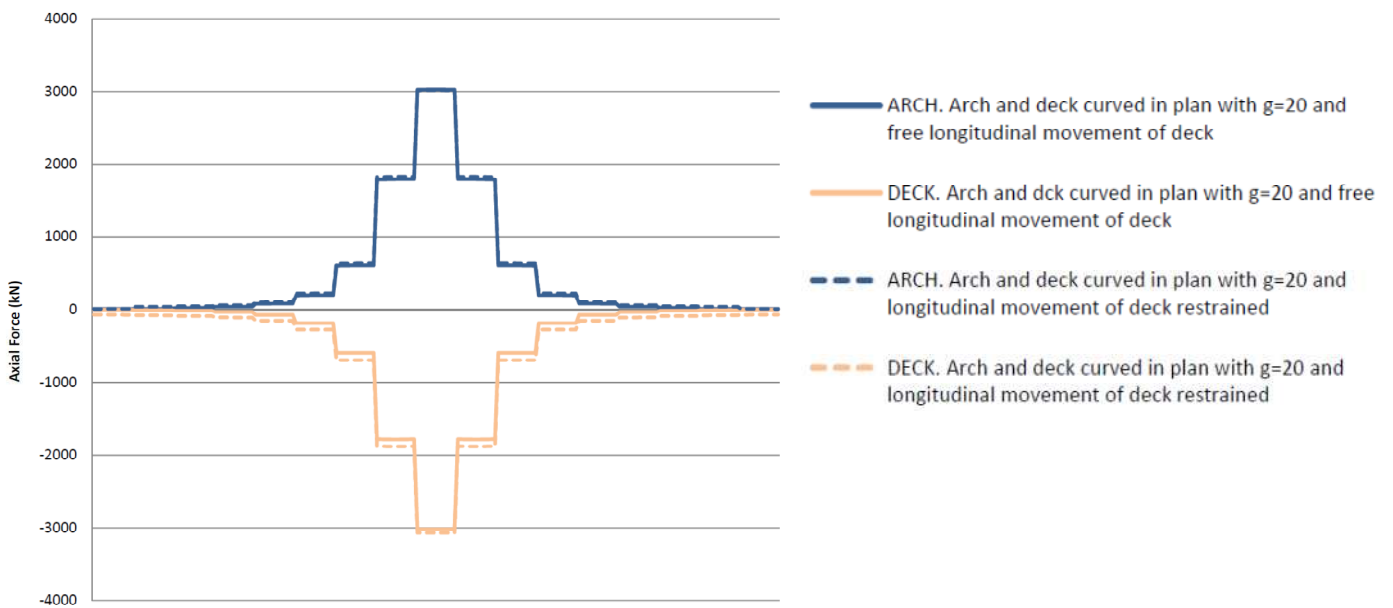
**Figure 4-15: Model (a).** Arch and deck axial forces under a 25°C temperature variation on arch and deck. Comparison for different support conditions at deck abutments. *The abscissas are the bridge length from 0 to  $L_{bridge}$*



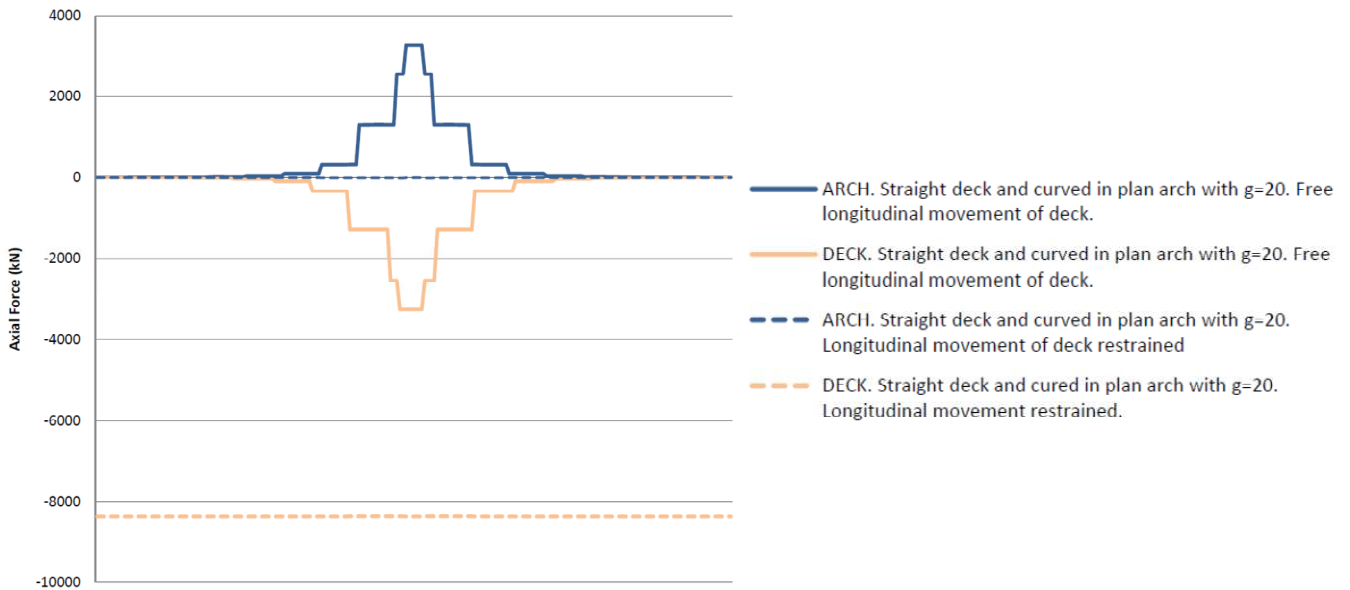
**Figure 4-16: Model (b).** Arch and deck axial forces under a 25°C temperature variation on arch and deck. Comparison for different support conditions at deck abutments. *The abscissas are the bridge length from 0 to  $L_{bridge}$*



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**Figure 4-19: Model (e). Arch and deck axial forces under a 25°C temperature variation on arch and deck. Comparison for different support conditions at deck abutments. The abscissas are the bridge length from 0 to  $L_{bridge}$**

Whatever the studied model, axial forces on arch and deck are similar when releasing horizontal displacements.

Fixing longitudinal displacements works correctly for curved decks but, not for straight decks, as experience on other bridge types commonly suggest.

When employing a straight deck with fixed abutments, temperature variations cause important axial forces on deck, whereas when curved in plan it has more space to expand and the axial forces greatly diminish. This means that employing fixed abutments in curved in plan arch bridges is advantageous. This is specially interesting in seismic areas, where it is necessary to restrain movements at deck abutments.

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