Chapter 6

Conclusion and future work

We pointed out in the introduction, when discussing about the versatility of wavelets as a tool in signal processing, that they can be adapted to many problems of different scientific disciplines and especially in our field, computer vision and image processing. In this work, we have shown that wavelets have this well-known flexibility permitting us to explore the solution for several general problems as segmentation and classification extending these solutions to several applications. Problems presented in this work are related to texture analysis, a large area of study in itself. We have began this dissertation on an introductory work aimed at the segmentation of a specific kind of images, and later we changed the approach to explore the topics of texture classification and synthesis. Besides, in all the problems we have proposed an image model as a starting point to study and justify how to process them.

The first part of this thesis has dealt with the classification of marble images that can be seen as structural textures. The elements defining these textures are grains that must be segmented. This segmentation of marble samples in their grains is used for specialists to classify them. This is a technique with good classification results but with a cumbersome stage of segmentation that makes infeasible perform it by hand. In this work we have automated this task, delivering to the professional this segmentation and morphological parameters of each grains useful in the classification stage.

If we analyze these marble images we realize that to delimit each grain is a difficult task due to noise in the image, macles, weak boundaries etc. Professionals use the information that arises from the observation of the samples through polarized light in order to decide where grains are. We also take advantage from this fact, and use the image formation model offered by the Johannsen's law, which relates the incident and transmitted light intensities through uniaxial crystals (marble). From a sequence, we can calculate two parameters (amplitude and phase) intrinsic of each grain that is used to aid segmentation.

Several methods have been evaluated to achieve the segmentation. In this sense,

and due to the characteristics of grains that are closed regions, we conclude that watershed transform is the best solution for this problem. This transformation applied directly to the images gives an excessive over segmentation due basically to noise. Therefore a previous filtering stage is mandatory.

The filtering step was done initially with similar tools used in the segmentation, but once we started to study multiresolution schemes, this step was redefined and designed in terms of the wavelet transform. In this work we have proposed a new wavelet filtering approach whose results can be easily interpreted in terms of relevant elements of the image: noise, non-homogeneous illumination and contributions to crest and valleys. In this way, we can isolate the important information to our problem that are ridges, now free of disturbing elements, giving a good starting point for the segmentation step. The information obtained in the segmentation is now expressed as a graph to refine the results with the previous amplitude-phase information. This duality of the representation, as image or as a graph, permits to work with the best one according to each process.

The same idea of a partial reconstruction of a wavelet decomposition in order to extract only the necessary elements and the representation of the partial result as a graph can be easily extended to similar problems of image segmentation. As a future development we include some preliminary results on the segmentation of people in indoor scenes. In this case, segmentation is a starting point of a labeling process where any part (clothing) of the subject must be automatically described in natural language in pursuit of a global description of the subject.

Results show that our method achieves a correct segmentation for most grains in a variety of marble types, without any initial knowledge on their characteristics. Once the segmentation is done, the expert studies several parameters related to the morphology of each grain and the relation among the bulk of grains. With all this information and based on his knowledge and expertise he gives a source quarry for the sample.

The second part of this work has addressed a problem of color texture classification through multiresolution decomposition techniques. This is an important subject due to its implications in quality control, and image retrieval. Since our research is related to one real application, we approach this part as a general texture classification but then tend to center our attention on achieving practical solutions. The kind of images we face now are characterized by their high visual similarity, that is, a completely different problem with regard to most publications in this area.

We adopt a common strategy to study this problem with a decomposition stage and a feature extraction but adapting each step to our requirements. In this sense, our aim was to find an optimal combination of color representation, decomposition scheme plus base and number of levels, and discriminant features.

A conclusion that reveals our work and also reported for other authors is that the bases used in the decomposition do not play a significant role achieving similar percentages of success. However all the process can be tuned for a specific problem or set of images to slightly increase the classification rate. Texture and color are two properties that coexist at the images and their interrelations can be more or less strong depending on the images. In this sense, we have proposed three image models in which this interrelation has different weight, and then some spatial-chromatic features are more important than others. These models refer to how texture is embedded into color and how texture in each channel relates to texture of the other ones.

In most papers related to texture classification, the feature extraction step yields a lot of information that must be reduced. This reduction should be performed by classic methods of dimensionality reduction without any knowledge about the data. In this work, we propose models in such a way that images following one of these models need a specific set of features to be characterized. This idea has been supported by actual results showing that selecting the right features achieves the smallest classification error.

Future research will address the estimation of other features with a high level of interrelation among channels and levels of the decomposition. Also, we will try mutual information as other measure of dependence between the images of the decomposition. Then, we will apply these results to a more elaborate paint recognition problem. In this case, we want to complete classification results with other inputs as chromatic (spectral) information to arrive to the determination of the components of these paints.

Finally, we essay to characterize texture through multidimensional probability density functions extracted from pixel neighborhoods. This description is used to classify and also to synthesize similar textures. We propose a way to classify textures according with this probability model based on similarity measures over these density functions. We compare the proposed 'metric' with standard distance values. Our synthesis starts with this model as a base and then has evolved to a multiresolution scheme. Results of the last part are still preliminary and a deal of further work is still necessary.

Appendix A

Detailed results

A.1 Tiles

The tile case was studied in-depth in order to plan the strategy in later classification studies. In this case a lot of decomposition schemes, bases and decomposition levels were analysed arriving to some conclusion in order to reduce the number of possible trials in other problems.

Tables represent the percentage of successful classification, % symbol has been elided for the sake of a better presentation of data. The proofs done in this section refers to three models of tile: A (Du, Duero), B (Es, Esla), C (Tb, Tiber); letters are the name used in the previous explanation (Sec. 4.4), names in parentheses are the label used in tables first, and the commercial name of these tiles.

Nomenclature used to label rows and columns of the next tables comprise: number of components of the feature vector (#f), type of set used in the classification (l: learning, t: test), transformation (W: multiresolution analysis, WP: wavelet packets, T: à trous, number of levels in the decomposition as the number that follows the transform or labeled as #l, functions used to obtain the features (L2: energy, EN: entropy), bases in the decomposition (D: Daubechies, B: B-spline), features are represented a prefix to the transform (F.a: without prefix, F.a+F.b: CN, F.a+F.b+F.c: CC). If a table is not divided in models it means that results are totals.

		D	u	E	s	Т	b	
Features	#f	1	t	1	t	1	t	total
mean and variance	6	100.0	94.4	90.0	79.4	100.0	87.5	87.1
mean	3	93.8	76.3	83.8	63.8	82.5	79.4	-73.2
variance	3	98.8	90.0	90.0	72.5	73.8	70.0	77.5
energy of RGB	3	95.0	73.1	86.3	71.9	77.5	75.0	73.3

Table A.1: Classification ratios from simple features for models A, B and C.

		D	11	E	s	Т	b	
Scheme	#f	1	t	1	t	1	t	total
W1L2D2	12	100.0	93.1	97.5	84.4	100.0	88.8	88.8
W1L2D4	12	100.0	94.4	97.5	83.8	97.5	88.1	88.8
W1L2D6	12	100.0	95.6	95.0	85.0	95.0	84.4	88.3
W1L2D8	12	100.0	95.0	97.5	82.5	95.0	86.3	87.9
W1L2D10	12	100.0	95.6	96.3	81.9	97.5	86.3	87.9
W1L2D12	12	100.0	95.6	95.0	80.6	91.3	86.3	87.5
W1L2D14	12	100.0	96.3	97.5	81.3	88.8	88.1	88.6
W1L2D16	12	100.0	95.0	96.3	81.9	85.0	90.0	89.0
W1L2D18	12	100.0	95.0	96.3	82.5	85.0	88.8	88.8
W1L2D20	12	100.0	95.0	93.8	83.1	86.3	86.3	88.1
W2L2D2	21	100.0	93.8	97.5	82.5	98.8	89.4	88.6
W2L2D4	21	100.0	95.6	97.5	82.5	96.3	85.6	87.9
W2L2D6	21	100.0	95.0	96.3	82.5	92.5	86.9	88.1
W2L2D8	21	100.0	95.6	97.5	83.1	96.3	86.3	88.3
W2L2D10	21	100.0	94.4	96.3	83.8	97.5	90.6	89.6
W2L2D12	21	100.0	96.3	95.0	83.1	93.8	92.5	90.6
W2L2D14	21	100.0	95.0	95.0	84.4	96.3	88.8	89.4
W2L2D16	21	100.0	94.4	98.8	84.4	93.8	90.0	89.6
W2L2D18	21	100.0	95.0	95.0	85.6	95.0	88.1	89.6
W2L2D20	21	100.0	95.3	97.5	85.0	97.5	86.3	88.9
W3L2D2	30	100.0	95.6	95.0	78.8	96.3	87.5	87.3
W3L2D4	30	100.0	93.8	96.3	80.0	100.0	90.6	88.1
W3L2D6	30	100.0	96.3	97.5	78.8	100.0	87.5	87.5
W3L2D8	30	100.0	93.8	96.3	80.6	97.5	87.5	87.3
W3L2D10	30	100.0	93.8	96.3	82.5	98.8	88.8	88.4
W3L2D12	30	100.0	95.6	97.5	81.3	96.3	91.3	89.4
W3L2D14	30	100.0	95.0	98.8	84.4	97.5	89.4	89.6
W3L2D16	30	100.0	95.6	97.5	81.9	97.5	89.4	89.0
W3L2D18	30	100.0	93.1	96.3	81.9	96.3	88.8	87.9
W3L2D20	30	100.0	93.1	97.5	83.1	96.3	88.8	88.3
W4L2D2	39	100.0	95.0	97.5	82.5	100.0	88.1	88.5
W4L2D4	39	100.0	95.0	96.3	80.6	100.0	89.4	88.3
W4L2D6	39	100.0	95.6	97.5	76.9	100.0	87.5	86.7
W4L2D8	39	100.0	95.0	97.5	78.8	100.0	90.0	87.9
W4L2D10	39	100.0	95.6	97.5	81.3	100.0	90.0	89.0
W4L2D12	39	100.0	92.5	93.8	83.1	100.0	93.1	89.6
W4L2D14	39	100.0	93.1	97.5	83.1	100.0	88.8	88.3
W4L2D16	39	100.0	93.8	97.5	79.4	100.0	85.6	86.3
W4L2D18	39	100.0	93.8	96.3	79.4	100.0	90.6	87.9
W4L2D20	39	100.0	92.5	97.5	82.5	100.0	88.1	87.7
W5L2D2	48	100.0	94.4	95.0	81.3	98.8	86.9	87.5
W5L2D4	48	100.0	96.3	97.5	81.3	100.0	88.1	88.6
W5L2D6	48	100.0	95.0	96.3	78.8	100.0	86.9	86.9
W5L2D8	48	100.0	95.6	97.5	76.9	100.0	82.5	85.0
W5L2D10	48	100.0	96.3	97.5 06.2	80.6	100.0	88.1	88.3
W5L2D12	48	100.0	93.1	96.3 07 5	81.3	100.0	90.0	88.1
W5L2D14 W5L2D16	48	100.0	$94.4 \\ 93.1$	$97.5 \\ 97.5$	$79.4 \\ 80.6$	98.8 100.0	$91.3 \\ 88.8$	$\frac{88.4}{87.5}$
W5L2D16 W6L2D2	$\frac{48}{57}$	$\begin{array}{c} 100.0\\ 100.0\end{array}$	93.1 94.4	97.5 97.5	$\frac{80.6}{76.3}$	100.0 100.0	88.8 86.9	87.5 85.9
W6L2D2 W6L2D4	57 57	100.0 100.0	94.4 96.3	97.5 96.3	76.3 80.6	$100.0 \\ 100.0$	86.9 86.9	$85.9 \\ 87.9$
W6L2D4 W6L2D6	57	100.0 100.0	90.3 95.6	90.3 97.5	76.9	100.0 100.0	86.9	86.5
W6L2D6 W6L2D8	57	100.0 100.0	95.0 92.5	97.5 97.5	76.9	100.0 100.0	80.9	80.5 83.5
W6L2D8 W7L2D2	57 66	100.0 100.0	92.5 92.5	97.5 97.5	78.1 77.5	100.0 100.0	80.0 85.6	83.5 85.2
W7L2D2 W7L2D4	66	100.0 100.0		97.5 96.3	77.5 80.0		$85.0 \\ 87.5$	85.2 87.7
W (L2D4	00	100.0	95.6	90.5	80.U	100.0	61.0	01.1

Table A.2: Classification of the tree models using multiresolution analysis (Mallats algorithm) with energy features without removing mean and variance.

Table A.3: Classification ratios for the three models: Du, Es, Tb. The first two columns are number of features used for classification purposes and the number of decomposition levels. Next columns represent different bases (*e.g* D2 is Daublechies, and so on). The number inside the table are the classification percentage (% is suposed in all the results, it has been omitted for clearity)

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$												
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	#f	#l	D2	D4	D6	D8	D10	D12	D14	D16	D18	D20
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							D.,					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	10	4	0.0.1	04.4	05.0			05.0	00.0	05.0	05.0	05.0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$												
$\begin{array}{cccccccccccccccccccccccccccccccccccc$												
$\begin{array}{cccccccccccccccccccccccccccccccccccc$												
$\begin{array}{cccccccccccccccccccccccccccccccccccc$											93.8	92.5
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							96.3	93.1	94.4	93.1		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$					95.6	92.5						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	00	1	92.5	95.0								
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							Es					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	12	1	84.4	83.8	85.0	82.5		80.6	81.3	81.9	82.5	83.1
$\begin{array}{cccccccccccccccccccccccccccccccccccc$												
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	30											
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	39	4	82.5	80.6		78.8	81.3				79.4	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	48	5	81.3	81.3	78.8	76.9	80.6	81.3	79.4	80.6		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	57	6	76.3	80.6		78.1						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	66	7	77.5	80.0								
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$												
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$												
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$												
$\begin{array}{cccccccccccccccccccccccccccccccccccc$												
$\begin{array}{cccccccccccccccccccccccccccccccccccc$											90.6	88.1
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							88.1	90.0	91.3	88.8		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$					86.9	80.0						
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	66	7	85.6	87.5								
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$												
21 2 88.6 87.9 88.1 88.3 89.6 90.6 89.4 89.6 89.6 88.9 30 3 87.3 88.1 87.5 87.3 88.4 89.4 89.6 89.6 89.9 88.3 39 4 88.5 88.3 86.7 87.9 89.0 89.6 88.3 86.3 87.9 88.3 39 4 88.5 88.3 86.7 87.9 89.0 89.6 88.3 86.3 87.9 87.7 48 5 87.5 88.6 86.9 85.0 88.3 88.1 88.4 87.5 57 6 85.9 87.9 86.5 83.5 87.9 87.5			_									
30 3 87.3 88.1 87.5 87.3 88.4 89.4 89.6 89.0 87.9 88.3 39 4 88.5 88.3 86.7 87.9 89.0 89.6 88.3 86.3 87.9 87.7 48 5 87.5 88.6 86.9 85.0 88.3 88.1 88.4 87.5 57 6 85.9 87.9 86.5 83.5 83.5												
39 4 88.5 88.3 86.7 87.9 89.0 89.6 88.3 86.3 87.9 87.7 48 5 87.5 88.6 86.9 85.0 88.3 88.1 88.4 87.5 57 6 85.9 87.9 86.5 83.5 83.5												
48 5 87.5 88.6 86.9 85.0 88.3 88.1 88.4 87.5 57 6 85.9 87.9 86.5 83.5												
$57 ext{ } 6 ext{ } 85.9 ext{ } 87.9 ext{ } 86.5 ext{ } 83.5 ext{ }$											87.9	87.7
							88.3	88.1	88.4	87.5		
66 7 85.2 87.7					86.5	83.5						
	66	7	85.2	87.7								

Table A.4: Classification ratios for the \dot{a} trous algorithm (energy features). In this experiment we have varied the number of levels and base of decomposition.

			Du			\mathbf{Es}			$^{\mathrm{Tb}}$			total	
#f	#1	B0	B1	B2	B0	B1	B2	B0	B1	B2	B0	B1	B2
6	1	93.1	93.8	93.1	81.9	85.0	83.1	88.8	86.3	87.5	87.9	88.4	87.9
9	2	94.4	94.4	93.8	85.0	83.8	83.8	91.3	93.8	93.8	90.2	90.7	90.5
12	3	94.4	95.6	94.4	81.3	84.4	80.6	86.9	95.6	89.4	87.5	91.9	88.1
15	4	94.4	93.8	93.8	82.5	84.4	82.5	86.9	95.6	91.3	87.9	91.3	89.2
18	5	94.4	94.4	93.1	80.0	82.5	81.9	88.8	83.8	92.5	87.7	86.9	89.2
21	6	93.8	93.1	91.9	81.9	81.9	80.6	86.3	86.3	93.8	87.3	87.1	88.8
24	7	91.9	92.5	90.0	82.5	80.6	80.0	86.9	85.6	90.0	87.1	86.2	86.7

	W1	W2	W3	WP2	$_{\rm WP1,2}$
#f	12	21	30	48	60
with	mean a	und vari	ance		
D2	88.8	88.6	87.3	83.1	88.6
D4	88.8	87.9	88.1	85.0	87.5
D6	88.3	88.1	87.5	85.9	86.2
with	out mea	an and a	variance		
D2	84.6	84.8	83.2	80.8	76.9
D4	85.9	85.4	82.3	77.7	82.9
D6	85.8	85.4	83.8	80.9	83.4

Table A.5: Classification ratios for wavelet and wavelet packet decompositionschemes with and without illumination information.

Table A.6: Classification ratios for the three models and for learning and test sets. Features comes from the \dot{a} trous algorithm for different strategies exploring color.

		D	u	E	s	T	Э	total
Scheme	#f	1	t	1	t	1	t	test
CNT2L2B1	12	100,0	94, 4	96, 3	86,9	100,0	86,9	89,4
CNT3L2B1	21	98,8	91,3	92,5	87,5	97,5	83,1	87,3
CNT4L2B1	33	100,0	91,9	95,0	86,3	98,8	83,1	87,1
CCT2L2B1	18	100,0	95,0	91,3	82,5	100,0	91,9	89,8
CCT3L2B1	24	100,0	95, 6	97,5	87,5	100,0	95,0	92,7
CCT4L2B1	30	100,0	93,8	96, 3	85,6	98,8	81,3	86,9
T3L2B1RGBgris	6	100,0	97,5	92,5	77,5	85,0	80,0	85,0
T3L2B1sMV	12	100,0	91, 9	87,5	82,5	91,3	74,4	82,9
CCT3L2B1	24	100,0	$95,\! 6$	97,5	87,5	100,0	95,0	92,7
CCKLP+MT3L2B1	24	100,0	$95,\! 6$	93,8	86,9	100,0	95,0	92,5
KLE+MT3L2B1	12	100,0	96, 3	92,5	81,9	100,0	85,0	87,7
KLP+MT3L2B1	12	100,0	$95,\! 6$	92,5	78,1	100,0	86,3	86,7
		-						
T2L2B1(R)	5	100,0	$95,\! 6$	90,0	80,6	87,5	86,3	87,5
T2L2B1(G)	5	100,0	96, 9	90,0	82,5	83,8	81,9	87,1
T2L2B1(B)	5	100,0	96, 9	92,5	82,5	86,3	80,6	86,7
T3L2B1(R)	6	100,0	96, 3	91,3	78,1	90,0	83,8	86,1
T3L2B1(G)	6	100,0	98,1	92,5	78,1	85,0	80,6	85,6
T3L2B1(B)	6	100,0	96, 3	92,5	80,0	87,5	75,0	83,8
T4L2B1(R)	7	100,0	93,8	91,3	76,9	91,3	77,5	82,7
T4L2B1(G)	7	100,0	97,5	92,5	78,1	90,0	74,4	83,3
T4L2B1(B)	7	100,0	96, 9	92,5	80,0	87,5	70,0	82,3
CNT2L2B1(R)	6	100,0	96, 9	91,3	85,0	90,0	86,3	89,4
CNT2L2B1(G)	6	100,0	$95,\! 6$	93,8	85,6	85,0	78,8	86,7
CNT2L2B1(B)	6	100,0	$95,\! 6$	92,5	85,6	80,0	74,4	85,2
CNT3L2B1(R)	9	100,0	98,1	92,5	84,4	87,5	83,1	88,5
CNT3L2B1(G)	9	100,0	97,5	95,0	86,3	85,0	78,1	87,3
CNT3L2B1(B)	9	100,0	96, 9	93,8	84,4	87,5	72,5	84,6
CNT4L2B1(R)	13	100,0	96, 3	92,5	84,4	92,5	76,3	85,7
CNT4L2B1(G)	13	98,8	88,8	95,0	84,4	91,3	71,9	81,7
CNT4L2B1(B)	13	98,8	88,1	95,0	85,0	88,8	70,0	81,0
KLE+MT3L2B1(C1)	6	100,0	97,5	92,5	77,5	88,0	80,0	85,0
KLE+MT3L2B1(C2)	6	98,8	96, 3	93,8	78,8	90,0	81,3	85,5
KLE+MT3L2B1(C3)	6	100,0	97,5	91,3	79,4	95,0	86,9	87,9

A.2 Paints

Table A.7: Classification ratios for paints at $\times 500$ magnification and for learning and test sets. Features comes from the à *trous* algorithm for different strategies exploring basically relation among levels and channels.

Features	#f	1	t
T2L2B1, grey, without mean and variance	3	54.7	29.2
T3L2B1, grey, without mean and variance	4	65.6	32.3
T4L2B1, grey, without mean and variance	5	64.1	21.9
CNT2L2B1, grey, without mean and variance	4	54.7	29.2
CNT3L2B1, grey, without mean and variance	7	65.6	32.3
CNT4L2B1, grey, without mean and variance	11	64.1	21.9
CNT2L2B1	12	89.1	57.3
CNT3L2B1	21	96.9	64.6
CNT4L2B1	33	96.9	57.3
CCT2L2B1	18	100.0	72.9
CCT3L2B1	24	100.0	63.5
CCT4L2B1	30	100.0	68.8

A.3 Marble

Table A.8: Classification ratios for marble images and for learning and test sets. Features comes from the \dot{a} trous algorithm for different strategies exploring basically relation among levels and channels.

Features	#f	1	t
mean and variance	2	29.20	15.25
T2L2B1, without mean and variance	3	48.60	26.40
T3L2B1, without mean and variance	4	57.10	34.70
T4L2B1, without mean and variance	5	66.65	36.10
T2L2B1	4	43.05	26.40
T3L2B1	7	59.70	33.35
T4L2B1	11	63.90	37.50
CNT2L2B1, without mean and variance	12	52.75	27.75
CNT3L2B1, without mean and variance	21	61.10	34.70
CNT4L2B1, without mean and variance	33	66.65	36.10
CNT2L2B1	18	52.80	25.00
CNT3L2B1	24	63.90	36.10
CNT4L2B1	30	83.30	45.80
CNT4L2B0	30	65.25	47.20
CNT4L2B2	30	79.15	37.50
CNT4L2B3	30	80.55	38.85

A.4 Brodatz

Table A.9: Classification ratios for Brodatz images and for learning and test sets. Features comes from the \dot{a} trous algorithm for different strategies exploring basically relation among levels and channels. Two set of images have been used: 111 images and 55 images.

		111 iı	nages	55 images	
Features	#f	1	t	1	t
mean and variance	2	32.45	23.50	54,90	43,90
T3L2B1, without mean and variance	4	48.10	38.30	92,85	87,75
CNT3L2B1, without mean and variance	7	59.50	47.65	96,70	93,55
T3L2B1	4	74.70	61.30	92,95	83,40
CNT3L2B1	7	78.45	66.05	$93,\!65$	86,25

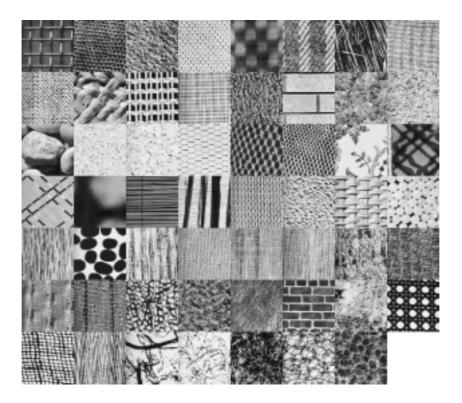


Figure A.1: 55 subimages from the Brodatz album [13] (images are regions of 160×160 of a big one of 640×640).

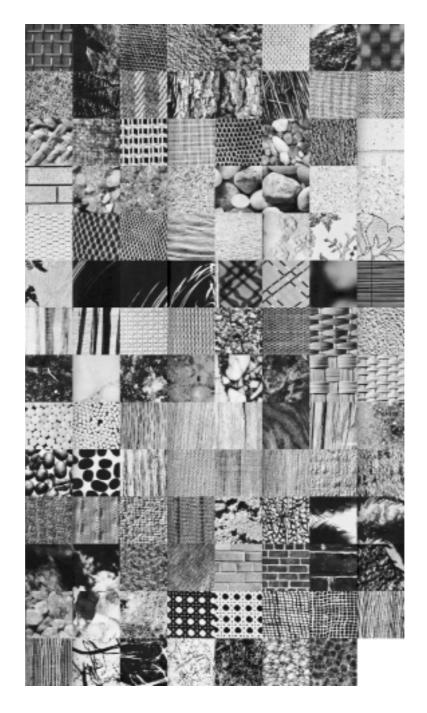


Figure A.2: 111 subimages from the Brodatz album [13] (images are regions of 160×160 of a big one of 640×640).

Bibliography

- Ali N. Akansu and Richard A. Haddad. Multiresolution Signal Decomposition: Transforms, Subbands, and Wavelets. Academic Press, Inc., 1992.
- [2] Fritz Albregtsen, Helene Schulerud, and Luren Yang. Texture classification of mouse liver cell nuclei using invariant moments of consistent regions. In 6th International Conference on Computer Analysis of Images and Patterns (CAIP'95), pages 496-502, Prague, 1995.
- [3] Aurelio Álvarez Pérez. Clasificación automatizada de mármoles mediante procesamiento digital de imagen. Entretiens d'archéologie et d'histoire, Les marbres blancs des Pyrénées:71-85, 1995.
- [4] Aurelio Álvarez Pérez, B. Obelic, A. Puig, and D. Haye. Determination of provenance of marbles used in mediterranean area. PACT, (45), 1994.
- [5] Aurelio Álvarez Pérez, F. Plana, and A. Puig. Determinación automatizada de parámetros petrográficos, mediante tratamiento de imagen. aplicación a la clasificación de mármoles. *Boletín de la sociedad española de mineralogía*, (14):39– 45, 1991.
- [6] Ramon Baldrich Caselles, Maria Vanrell Martorell, and Juan José Villanueva Pipaón. Texture-colour features for tile classification. In European Symposium on Industrial Laser and Inspection, volume 3826 of EuroOpto Series, pages 124– 135, Germany, June 1999.
- [7] Ziv Bar-Joseph, Ran El-Yaniv, Dani Lischinski, and Werman Michael. Texture mixing and texture movie synthesis using statistical learning. *IEEE Transac*tions on Visualization and Computer Graphics, 7(2):120–135, April–June 2001.
- [8] C. Beucher. Watersheds of functions and picture segmentation. In IEEE Int. Conf. on Acoustic Speech and Signal Processing'82 (ICASSP), pages 1928–1931, Paris, 1982.
- [9] Chris Bishop. Neural Networks for Pattern Recognition. Oxford University Press, 1995.
- [10] F.Donald Bloss. Introducción a los métodos de Cristalografía Óptica. Omega, 1985.

- [11] C. R. Boukouvalas. Colour Shade Grading and its Applications to Visual Inspection. PhD thesis, University of Surrey, 1996.
- [12] Alan Conrad Bovik, Marianna Clark, and Wilson S. Geisler. Multichannel texture analysis using localized spatial filters. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, PAMI-12(1):55-73, 1990.
- [13] P. Brodatz. Textures: A Photographic Album for Artists and Designers. Dover, 1966.
- [14] T. Caelli and D. Reye. On the classification of image regions by colour, texture and shape. *Pattern Recognition*, 26(4):461–470, 1993.
- [15] Jean-François Cardoso. Blind signal separation: statistical principles. Proceedings of the IEEE, 9(10):2009-2025, 1998.
- [16] Klaus Castleman. Digital Image Processing. Prentice-Hall, Inc, 1979.
- [17] R. R. Coifman and D. Donoho. Translation-invariant de-noising, in Wavelets and Statistics, pages 125–150. Lecture Note in Statistics, Springer-Verlag, 1995.
- [18] R. R. Coifman and D. L. Donoho. Translation-invariant de-noising. Technical report, Department of Statistics, 1995.
- [19] T.M. Cover and J.A. Thomas. Elements of Information Theory. New York: Wiley, 1991.
- [20] Ingrid Daubechies. Ten Lectures on Wavelets. SIAM, Philadelphia, 1992.
- [21] J.G. Daugman. Quadrature-phase simple-cell pairs are appropriately described in complex analytic form. Journal of the Optical Society of America, 10(7):375– 377, 1993.
- [22] J. D. De Bonet. Multiresolution sampling procedure for analysis and synthesis of texture images. In *Computer Graphics. ACM SIGGRAPHP*, pages 229–238, 1997.
- [23] Ingrid Denne. Microscopic evaluation of effect colors. approach to color match. In 6th Congresso Internacional de Tintas, Sao Paulo, September 1999.
- [24] Panjwani D.K. and G. Healey. Markov random field models for unsupervised segmentation of textured color images. *IEEE Transactions on Pattern Analysis* and Machine Intelligence, 17(10):939-954, October 1995.
- [25] David L. Donoho. De-noising by soft-thresholding. IEEE Transactions on Information Theory, 41(3):613-627, 1995.
- [26] David L. Donoho and Iain M. Johnstone. Adapting to unknown smoothness via wavelet shrinkage. Journal of the American Statistical Association, 90(432):1200-1224, 1995.

- [27] Benoît Duc, Stefan. Fischer, and Josef Bigün. Face authentication with gabor information on deformable graphs. *IEEE Transactions on Image Processing*, 8(4):505-516, April 1999.
- [28] Richard O. Duda and Peter E. Hart. Pattern Classification and Scene Analysis. John Wiley and Sons, Inc, 1973.
- [29] Graham D. Finlayson, Subho S. Chatterjee, and Brian V. Funt. Color angular indexing. In European Conference on Computer Vision, pages 16–27, April 1996.
- [30] M. Flickner, H. Sawhney, W. Niblack, J. Sahley, Q. Huang, B. Dom, M. Gorhani, J. Hafner, D. Lee, D. Petkovic, D. Steele, and P. Yanker. Query by image and video content: The qbic system. *IEEE Computer*, 28(9):23–32, September 1995.
- [31] A. Gagalowicz, S. De Ma, and C. Tournier-Lasserve. Efficient models for color textures. In Eighth International Conference on Pattern Recognition, pages 412–414, 1986.
- [32] Rafael C. Gonzalez and Paul Wintz. Digital Image Processing. Addison-Wesley, second edition, 1987.
- [33] H. Greenspan, C. Carson, S. Belongie, and J. Malik. Region-based image querying. In CVPR'97 Workshop on Content-Based Access of Image and Video Libraries, 1997.
- [34] Thor O. Gulsrud and John Håkon Husøy. Optimal filter for detection of clustered microcalcifications. In 15th International Conference on Pattern Recognition (ICPR 2000), volume 1, pages 508–511, Barcelona, September 2000.
- [35] Allan Hanbury and Frédéric Gardeux. A quantitative description of wood texture. In *Quality Control by Artificial Vision (QCAV'2001)*, pages 200–205, Le Creusot, France, May 2001.
- [36] Robert M. Haralick, K. Shanmugam, and Its'Hak Dinstein. Textural features for images classification. *IEEE Transactions on Systems, Man and Cybernetics*, SMC-3(6):610-621, 1973.
- [37] G. Healey and L. Wang. Illumination-invariant recognition of texture in color images. Journal of the Optical Society of America, 12(9):1877–1883, September 1995.
- [38] D. Heeger. Model for the extraction of image flow. Journal of the Optical Society of America A, 4(8):1455–1471, 1996.
- [39] D.J. Heeger and J.R. Bergen. Pyramid-based texture analysis/synthesis. In Computer Graphics. ACM SIGGRAPHP, pages 229–238, Los Angeles, USA, August 1995.

- [40] N. Herz and M. Waelkens. Classical marble: geochemistry, technology, trade. In NATO ASI Series. Series E: Applied Sciences, 153. Proceedings of the NATO advanced research workshop on marble in ancient Greece and Rome, pages 231– 379, Lucca, Italy, 1988.
- [41] Lyndon Hill. Phase correlation. http://www.ee.surrey.ac.uk/Personal/L. Hill/pc.html, 2000.
- [42] Yu ichi Ohta, Takeo Kanade, and Toshiyuki Sakai. Color information for region segmentation. *Computer Graphics and Image Processing*, 13(3):222-241, 1980.
- [43] I. IEC, J. SC, and W. ITU-T. Jpeg 2000 part i final committee draft version 1.0, 2000.
- [44] Charles E. Jacobs, Adam Finkelstein, and David H. Salesin. Fast multiresolution image querying. In *Computer Graphics. ACM SIGGRAPHP*, pages 277–286, Los Angeles, USA, August 1995.
- [45] A. Jain and S. Bhattacharjee. Text segmentation using gabor filters for automatic document processing. 5(3):169–184, 1992.
- [46] G. Jain, A.and Healey. A multiscale representation including opponent color features for texture recognition. *IEEE Transactions on Image Processing*, 7(1):124– 128, 1998.
- [47] H. Li, B. S. Manjunath, and S. K. Mitra. Multisensor image fusion using the wavelet transform. *Graphical models and image processing: GMIP*, 57(3):235– 245, 1995.
- [48] H. Li, B. S. Manjunath, and S. K. Mitra. Multisensor image fusion using the wavelet transform. *Graphical models and image processing: GMIP*, 57(3):235– 245, May 1995.
- [49] S. Z. Li. Markov Random Field Modeling in Computer Vision. Springer-Verlag, Tokyo, 1995.
- [50] J. Liang and T. W. Parks. A translation-invariant wavelet representation algorithm with applications. *IEEE Transactions on Signal Processing*, 44(2):225– 232, 1996.
- [51] Josep Lladós Canet, Gemma Sánchez, and Enric Martí Godia. A string-based method to recognize symbols and structural textures in architectural plans, pages 91–103. Lecture Notes in Computer Science, Springer-Verlag, 1998.
- [52] A. M. López, W. Niessen, J. Serrat, K. Nicolay, B. Ter Haar Romeny, J. J. Villanueva, and M. Viergever. Frontiers in Artificial Intelligence and Applications: Pattern Recognition and Applications, chapter New Improvements in the Multiscale Analysis of Trabecular Bone Patterns, pages 251–260. IOS Press -Ohmsa, 2000.

- [53] Antonio Manuel López Peña. Multilocal Methods for Ridge and Valley Delineation in Image Analysis. PhD thesis, UAB, Facultat de ciències, Departament d'informàtica, August 1999.
- [54] Antonio Manuel López Peña, David Lloret, Joan Serrat i Gual, and Juan José Villanueva Pipaón. Multilocal creaseness based on the level set extrinsic curvature. Computer Vision and Image Understanding, 77(2):111–144, February 2000.
- [55] Antonio Manuel López Peña, Felipe Lumbreras Ruiz, Joan Serrat i Gual, and Juan José Villanueva Pipaón. Evaluation of methods for ridge and valley detection. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 21(4):327–335, 1999.
- [56] Felipe Lumbreras Ruiz. Reconocimiento y clasificación de mármoles. Master's thesis, UAB, Facultat de ciències, Departament d'informàtica, September 1993.
- [57] Felipe Lumbreras Ruiz, Ramon Baldrich Caselles, Maria Vanrell Martorell, Joan Serrat Gual, and Juan José Villanueva Pipaón. *Recent research developments* in optical engineering, chapter Multiresolution colour texture classification of ceramic tiles, pages 213–228. Research Signpost, 1999.
- [58] Felipe Lumbreras Ruiz, Ramon Baldrich Caselles, Maria Vanrell Martorell, Joan Serrat Gual, and Juan José Villanueva Pipaón. Multiresolution colour texture representations for tile classification. In VIII National Symposium on Pattern Recognition and Image Analysis (SNRFAI'99), pages 227–234, Bilbao, Spain, May 1999.
- [59] Felipe Lumbreras Ruiz, Joan Serrat Gual, Ramon Baldrich Caselles, Maria Vanrell Martorell, and Juan José Villanueva Pipaón. Colour texture recognition through multiresolution features. In *Quality Control by Artificial Vision* (QCAV'2001), pages 114–121, Le Creusot, France, May 2001.
- [60] Felipe Lumbreras Ruiz and Joan Serrat i Gual. Segmentation of petrographical images of marbles. Computers and Geosciences, 22(5):547-558, 1996.
- [61] Felipe Lumbreras Ruiz and Joan Serrat i Gual. Wavelet filtering for the segmentation of marble images. Optical Engineering, 35(10):2864–2872, October 1996.
- [62] Stephan Mallat. A theory for multiresolution signal decomposition: The wavelet representation. *IEEE Transactions on Pattern Analysis and Machine Intelli*gence, 11(7):674-694, 1989.
- [63] Stéphane Mallat. A wavelet tour of signal processing. Academic press, 1998.
- [64] Stephane G. Mallat and Sifen Zhong. Characterization of signals from multiscale edges. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 14(7):710-732, 1992.

- [65] K.V. Mardia, J.T. Kent, and J.M. Bibby. *Multivariate Analysis*. Academic Press, 1997.
- [66] F. Meyer and C. Beucher. Morphological image segmentation. Journal of Visual Communication and Image Representation, 1(1):21-46, 1990.
- [67] Yves Meyer. Wavelets: algorithms & applications. SIAM, Philadelphia, 1993.
- [68] Paul Niggli. Rock and mineral deposits. W.A. Freeman. San Francisco, 1954.
- [69] Henryk Palus. The Colour Image Processing Handbook, chapter 4, pages 67–90. Chapman & Hall, 1998.
- [70] J. A. Peñaranda, L. Briones, and J. Florez. Colour machine vision system for process control in ceramics industry. In Proc. of SPIE: New image Processing Techniques and Applications: Algorithms, Methods and Components, volume 3101, pages 182–192, 1997.
- [71] A. Pentland, R. Picard, and S. Sclaroff. Photobook: Content-based manipulation of image databases. Int. J. of Computer Vision, 18(3):233-254, 1996.
- [72] Kris Popat. Conjoint Probabilistic Subband Modeling. PhD thesis, Massachusetts Institute of Technology, September 1997.
- [73] Kris Popat and Rosalind W. Picard. Cluster-based probability model and its application to image texture processing. *IEEE Transactions on Image Processing*, 6(2):268–284, February 1997.
- [74] J. Portilla, Navarro R., Nestares O., and A. Tabernero. Texture synthesisby-analysis based on a multiscale early-vision model. *Optical Engineering*, 35(8):2403-2417, August 1996.
- [75] Javier Portilla and Eero P. Simoncelli. A parametric texture model based on joint statistics of complex wavelet coefficients. *International Journal of Computer Vision*, 40(1):49–71, October 2000.
- [76] Albert Pujol Torras, Felipe Lumbreras Ruiz, Xavier Varona Gómez, and Juan José Villanueva Pipaón. Locating people in indoor scenes for real applications. In 15th International Conference on Pattern Recognition (ICPR2000), volume 4, pages 632–635, Barcelona, September 2000.
- [77] Trygve Randen. Filter and Filter Bank Design for Image Texture Recognition. PhD thesis, Norwegian University of Science and Technology, Stavanger College, 1997.
- [78] Trygve Randen and John Håkon Husøy. Filtering for texture classification: a comparative study. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 21(4):291–310, April 1999.
- [79] Todd R. Reed and J.M. Hans Du Buf. A review of recent texture segmentation and feature extraction techniques. CVGIP: Image Understanding, 57(3):359– 372, May 1993.

- [80] A. Said and W. Pearlman. A new, fast, and efficient image codec based on set partitioning in hierarchical trees. *IEEE Transactions on Circuits and Systems* for Video Technology, 6(3):243-250, June 1996.
- [81] Javier Sánchez Pujades. Diseño e implementación de un lenguaje de programación multiparadigma para la visión por computador. PhD thesis, UAB, Facultat de ciències, Departament d'informàtica, September 1996.
- [82] Diego Santa-Cruz and Touradj Ebrahimi. A study of jpeg 2000 still image coding versus other standards. In Proc. of the X European Signal Processing Conference (EUSIPCO'00), pages 673–676., Tampere, Finland, September 2000.
- [83] Michael Schröder and Alex Dimai. Texture information in remote sensing images: A case study. In Worshop on Texture Analysis (WTA'98), Freiburg, October 1998.
- [84] David W. Scott. Multivariate Density Estimation. John Wiley and Sons, Inc, 1992.
- [85] Jean Serra. An introduction to mathematical morphology. Computer Vision, Graphics and Image Processing, 35(3):283-305, 1986.
- [86] L. Shafarenko, M. Petrou, and J. Kittler. Automatic watershed segmentation of randomly textured color images. *IEEE Transactions on Image Processing*, 6(11):1530-1544, November 1997.
- [87] J. M. Shapiro. Embedded image coding using zerotrees of wavelet coefficients. *IEEE Transactions on Acoustics, Speech and Signal Processing*, 41(12):3445–3462, 1993.
- [88] Eero P. Simoncelli, W.T. ans Adelson E.H. Freeman, and D. J. Heeger. Shiftable multi-scale transforms. *IEEE Transactions on Information Theory*, 38(2):587– 607, March 1992.
- [89] Jean-Luc Starck and Albert Bijaoui. Filtering and deconvolution by the wavelet transform. *Signal Processing*, 35(3):195–211, February 1994.
- [90] Eric J. Stollnitz, Tony D. DeRose, and David H. Salesin. Wavelets for computer graphics: A primer, part 1. *IEEE Computer Graphics and Applications*, 15(3):76-84, May 1995.
- [91] B.J. Super and A.C. Bovik. Shape from texture using local spectral moments. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 17(4):333–343, April 1995.
- [92] Funiaki Tomita and Saburo Tsuji. Computer analysis of Visual Textures, chapter 5. Structural Texture Analysis, pages 71–171. Kluwer academic publishers, 1990.
- [93] M. Tuceryan and A.K. Jain. Handbook of Pattern Recognition and computer Vision, chapter 2. Texture Analysis, pages 207–248. C. H. Chen, L. F. Pau, P. S. P. Wang (eds.), World Scientific Publishing Co., second edition, 1998.

- [94] M. Unser. Texture clarification and segmentation using wavelet frames. IEEE Transactions on Image Processing, 4(11):1549–1560, November 1995.
- [95] Krüger V. and G. Sommer. Real-time face tracking using gabor wavelet networks. In 15th International Conference on Pattern Recognition (ICPR2000), volume 1, Barcelona, September 2000.
- [96] I. Vajda. Theory of Statistical Inference and Information. Dordrecht, The Netherlands: Kluwer, 1989.
- [97] G. Van de Wauwer, S. Livens, P. Scheunders, and D. Van Dyck. Color texture classification by wavelet energy correlation signatures. In *ICIAP'97*, volume 1, pages 327–334, 1997.
- [98] Marina Vannucci. Nonparametric density estimation using wavelets, 1998. Discussion Paper 95–26, ISDS, Duke University.
- [99] Maria Vanrell. Identificació de les dimensions d'un espai de representació de textures basat en un model computacional de percepció preatentiva. PhD thesis, UAB, Facultat de ciències, Departament d'informàtica, 1996.
- [100] Maria Vanrell i Martorell. Anàlisi granulomètrica de textures: Aplicaió a la classificació de la pneumoconiosi. Master's thesis, UAB, Facultat de ciències, Departament d'informàtica, 1992.
- [101] L. Vincent and P. Soille. Watersheds in digital spaces: an efficient algorithm based on immersion simulations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 13(6):583-598, 1991.
- [102] E.E. Wahlstrom. *Optical Crystallography*. John Wiley and Sons, Inc, fourth edition, 1969.
- [103] Brian A. Wandell. The synthesis and analysis of color images. IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI-9(1):2-13, January 1987.
- [104] Harry Wechsler. Computational vision. Academic press, 1990.
- [105] M. V. Wickerhauser and R. R. Coifman. Entropy based methods for best basis selection. *IEEE Trans. on Inf. Theory*, 38(2):719–746, 1992.
- [106] Wickerhauser:1994. Adapted Wavelet Analysis from Theory to Software. A. K. Peters, 1994.

Publications

- Albert Pujol, Jordi Vitrià, Felipe Lumbreras, Juan José Villanueva. "Topological Principal Component Analysis for face encoding and recognition". *Pattern Recognition Letters*, Vol 22, No. 6–7, pp. 769–776. April 2001.
- Felipe Lumbreras, Ramon Baldrich, Maria Vanrell, Joan Serrat, Juan José Villanueva. "Multiresolution colour texture classification of ceramic tiles". *Recent research developments in optical engineering*. Research Signpost Ed., Vol 2, pp. 213–228. (Índia), 1999. ISBN: 81–7736–007–6.
- Antonio Manuel López, Felipe Lumbreras, Joan Serrat, Juan José Villanueva. "Evaluation of methods for ridge and valley detection". *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol 21, No. 4, pp. 327–335. April 1999.
- Antonio Manuel López, Felipe Lumbreras, Joan Serrat. "Creaseness from Level Set Extrinsic Curvature". Lecture Notes in Computer Science 1407. Computer Vision – ECCV'98, Eds. H. Burkhardt and B. Neumann, Springer-Verlag. Vol 2, pp. 156 – 169. 1998. ISBN 3–540–64613–2.
- A. López, F. Lumbreras, A. Martínez, J. Serrat, X. Roca, X.Varona, J. Vitriá. "Capítulo 3 – Técnicas software para la visión". *Aplicaciones de la visión por computador a la industria*, Ed. Centro de Visión por Computador. (Barcelona), october 1997. ISBN 84-922529-3-6.
- Felipe Lumbreras, Joan Serrat. "Wavelet filtering for the segmentation of marble images". *Optical Engineering*, Vol 35, No. 10, pp. 2864 – 2872. October 1996.
- Felipe Lumbreras, Joan Serrat. "Segmentation of petrographical images of marbles". Computers and Geosciences, Vol 22, No. 5, pp. 547 – 558. 1996.
- Josep Lladós, Felipe Lumbreras, Vicente Chapaprieta, Joan Queralt. "ICAR: Identity Card Automatic Reader". Sixth International Conference on Document Analysis and Recognition(ICDAR'2001). Seattle, Washington, USA. September 10-13, 2001.

- M. Vanrell, F. Lumbreras, A. Pujol, R. Baldrich, J. Lladós, J.J. Villanueva. "From Colour Normalisation to Colour Constancy by Background Information". International Conference on Image Processing (ICIP'2001). Thessaloniki, Greece. October 7–10, 2001.
- F. Lumbreras, J. Serrat, R. Baldrich, M. Vanrell, J.J. Villanueva. "Color Texture Recognition through Multiresolution Features". *Quality Control by Artificial Vision (QCAV'2001)*. Vol. 1, pp. 114–121. Le Creusot, France. May 21–23, 2001.
- F. Lumbreras, X. Roca, D. Ponsa, J. Martínez, S. Sánchez, C. Antens, J.J. Villanueva. "Visual Inspection of Safety Belts". Award for Best Poster. Quality Control by Artificial Vision (QCAV'2001). Vol. 2, pp. 526–531. Le Creusot, France. May 21–23, 2001.
- A. Pujol, F. Lumbreras, X. Varona, J.J. Villanueva. "Locating People in Indoor Scenes for Real Applications". 15th International Conference on Pattern Recognition (ICPR 2000). Vol. 4, pp. 632–635. Barcelona. September 3–7, 2000.
- F. Lumbreras, R. Baldrich, M. Vanrell, J. Serrat, J.J. Villanueva. "Multiresolution Colour Texture Representations for Tile Classification". VIII National Symposium on Pattern Recognition and Image Analysis (SNRFAI'99). Vol. 1, pp. 145–152. Bilbao. May 12–14, 1999.
- A. Pujol, F. Lumbreras, X. Varona, J.J. Villanueva. "Template Matching through Invariant Eigenspace Projection". VIII National Symposium on Pattern Recognition and Image Analysis (SNRFAI'99). Vol. 1, pp. 227–234. Bilbao. May 12–14, 1999.
- J. Lladós, F. Lumbreras, X. Varona. "A Multidocument Platform for Automatic Reading of Identity Cards". VIII National Symposium on Pattern Recognition and Image Analysis (SNRFAI'99). Vol. 1, pp. 271-278. Bilbao, May 12-14, 1999.
- J. Sánchez, F. Lumbreras, M. Molina. "Fuse Boxes Inspection System". Workshop on European Scientific and Industrial Collaboration (WESIC'98). pp. 291– 295. Girona, June 10–12, 1998.
- A. López, F. Lumbreras, J. Serrat. "Creaseness from level sets extrinsic curvature". Fifth European Conference on Computer Vision (ECCV'98). Vol 2, pp. 156–169. Freiburg, Germany. June 2–6, 1998.