Puzzles in International Finance: Portfolio Diversification and Debt Maturity

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Dedication

To my parents, brother and nephews

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Chapter 1

Introduction

The aim of this thesis is to contribute to the understanding of two features of international financial markets: the home bias in country portfolios and the existence of short term lending to finance long term projects in emerging markets. I discuss each in turn.

One of the empirical regularities standard models cannot explain is the strong bias of investor's portfolio towards domestic securities. This empirical regularity contradicts the large gains from diversification modern finance theory predicts. Theoretical models of portfolio choice in closed economy usually neglect the effect of the macroeconomic fluctuations since there is no reason , at least a priori, to think that these fluctuations affect investors differently. In open economies this simplification might produce important mistakes. In the international financial markets investors belonging to different countries face different macroeconomic conditions and therefore they take distinct investment decisions.

Nevertheless, the introduction of macroeconomic conditions in the international portfolio choice is not new (Baxter and Jerman, 1996). It is curious that according to standard RBC theory, real wages and return on capital are positive correlated. Clearly, this correlation implies investors should hold portfolios bias toward foreign securities. Thus, according to standard models of finance theory we have a home bias puzzle in international portfolios but, according to RBC theory the puzzle is worst than we think.

The new open macroeconomic theory, based on the assumptions of nominal stickiness on prices and wages, predicts important effects of demand shock in the overall economy. One of these important effects is the negative correlation between real wages and capital return produced by demand shocks when nominal wages are staggered. Clearly, this correlation implies that domestic capital hedges labor income fluctuations and therefore, goes in the direction to explain the home bias puzzle.

The first chapter of this thesis develops this idea. It tries to exemplify the mentioned effect on the portfolio composition through a simple model and after, it tries to test empirically the importance of this effect. In order to achieve this second goal I construct and estimate a structural VAR to identify a demand shock to see the qualitative and quantitative responses it generates on the correlation between real wages and share prices.

The second aspect of international financial markets I address in this the-

sis is the existence of short-term debt. Emerging markets need capital inflows to finance investment projects necessary to catch up with develop countries. Usually, the most important projects produce returns in the long run (like infrastructure). In order to finance these projects it is recommendable that loans feature the same maturity, otherwise, liquidity crisis may arise.

If we observe data on the East-Asian crisis of the late 90's we will observe that high levels of short term debt were present in all countries affected. Many authors have suggested a connection between short term borrowing and liquidity shortages that can lead to such crisis. Nevertheless, there have been very few attempts to understand the existence of short term borrowing.

This thesis argues that short term borrowing is the natural outcome in a financial market where international investors have market power. This market power can be though as the fact that nowadays most lending is done through financial intermediaries. These financial intermediaries compete in first place to get funds from investors and in second place to allocate capital in emerging markets[1]. This priority of financial intermediaries in the competition for deposits gives market power to the investors. Clearly, the question that arises is why do international investors would use their market power to lend in short term?

The assumption behind is that emerging markets offer good opportunities to invest. Entrepreneurs in emerging markets have highly profitable

[1]Emerging markets are always willing to admit more funds.

investment projects but usually they do not have the funds to finance them. International investors know these projects are profitable and thus, they want to invest. But, abundance of funds in the developed countries compete reducing the interest rates. Then, the surplus of the projects go mainly to the entrepreneurs. Thus, excessive flow of funds to emerging markets would reduce the benefit for international lenders.

Nonetheless, this is not what we observe. In reality emerging markets are very risky and investors are not willing to lend unless they face high interest rates. It means, it is the risk of emerging markets what drives capital flows and interest payments and not the usual high expected payoff of economies that are still catching up. Hence, without risk the interest rate are low, basically as the interest rates of developed countries. On the other hand, with risk, interest rates are high. Therefore, risky emerging markets enlarge the menu of assets available to lenders improving their welfare.

The connection should now be clear, international investors prefer emerging markets to bear risk because this enlarges their menu of assets and short term debt generates it own risk, the liquidity risk. I will argue in this thesis that lenders will induce market to finance projects with short term debt to achieve higher interest rates.

Chapter 2

Sticky Wages, Demand Shocks and International Portfolio Diversification

According to empirical findings there is little diversification in country portfolios. For instance, a representative U.S. investor held in 1989 more than the 90% of her portfolio composed of equity issued by American firms. A similar behavior is found for investors of the major capital markets in the world. Japanese investors held more than 95% of her portfolio composed of Japanese securities and for British investors more than 85% in British securities.[1]

This extreme bias towards domestic securities is difficult to reconcile with standard models in finance theory which predict large gains from diversifica-

[1]See French and Poterba (1991).

tion. For this reason this empirical finding has been labelled the *international diversification puzzle*. Many attempts to explain the puzzle have been done but none seems reasonable enough to produce portfolios with such large biases.

One of the most surprising findings (Baxter and Jermann (1993), the puzzle is worse than you think) is that in a standard RBC model the correlation between labor income and return to capital is positive which implies that we should observe foreign bias in the country portfolios. Indeed if the labor shares are bigger than the size of the country in the world, agents should short sell domestic capital. Nonetheless, the data does not support this argument. Bottazzi, Pesenti and Van Wincoop (1996) showed that for most of the OECD countries, the correlation between real wages and profit rates is negative, a result consistent with home bias in the portfolios.

The goal of this paper is to present an explanation for this negative correlation between real wages and returns to capital within the framework of a general equilibrium (two country) model. The idea is to combine nominal rigidities: sticky wages with demand shocks. We may expect that demand shocks rise prices and with sticky nominal wages the real wage falls. If employment does not expand too much labor income will be reduced. On the other hand the demand expansion plus the reduction of the real wage creates big opportunities for capital owners. Therefore, it is a good hedge against demand shocks to hold home securities. On the other hand if we have supply shocks the correlation will be positive as explained in the RBC literature.

The chapter is organized as follows: section 1 presents the basic model. Section 2 explains why the sign of the correlations depend on the shocks and the nominal rigidities. Section 3 computes the optimal portfolio for the extreme case of symmetric economies and numerically for the general case. Section 4 presents the empirical findings of a VAR system to identify the effect of demand shocks on the correlation of real wages and returns to capital. Section 5 concludes.

2.1 International portfolio diversification with sticky wages

Let the world be composed by two countries -home and foreign- populated by households and firms[2]. Time last for two periods. There is perfect capital mobility in the first period, hence there is a portfolio choice for the households to reduce consumption uncertainty. The sources of uncertainty are aggregate technology shocks in each country Z, Z^* and money supply shocks in each country M, M^*. Money in this economy is necessary to consume i.e. there is required that all transactions are backed with cash.

The timing of the model is the following: in the first period households choose the portfolio (amount of capital invested in home and foreign country and risk free bonds) and the nominal wages. In the second period the tech-

[2]To avoid confusions, the variables of the foreign country will have asterisks.

nology level and the money supply are known, then firms demand capital and the different varieties of labor available in its respective country, households demand real balances and consume.

The model features incomplete financial markets i.e. the bonds in the economy are not contingent on the state of the world. The model assumes that nominal wages are preset in the first period, before the uncertainty is solved (second period) and thus before the price levels are determined.

The next section will present the problems of firms and households for the home country. For the foreign country all equations are equivalent.

2.1.1 The firms

There is a continuum of identical perfectly competitive firms that live only in the second period. They produce a single final good (Y) using capital and a composite of specialized labor factor.

$$Y = f(K, L)Z \tag{2.1}$$

Where f is the production function[3], K is the amount of capital used by the firm and L is a composite of the different varieties of labor.

$$L = \left[\int_0^1 L_i^{\phi} di\right]^{1/\phi}$$

The parameter ϕ lies in the interval $(0, 1)$ and represents the degree of substitution between different varieties of labor. As mentioned before Z rep-

[3]The production function satisfies all the standard assumptions.

resents a technology shock that is country specific but aggregate to all firms within a country.

The problem of the firm is to maximize profits taking factor prices as given.

$$\max_{K,L_i} \quad Y - \int_0^1 \frac{w_i}{p} L_i di - (1+r)K$$

Where r is the rental rate of capital, w_i is the nominal wage rate for the labor factor i and p is the nominal price level of the final good.

Optimal choice of firms imply the following factor demands:

$$1 + r = f_K(K, L)Z \tag{2.2}$$

$$\frac{w_i}{p} = f_L(K, L)\left(\frac{L}{L_i}\right)^{1-\phi} Z \tag{2.3}$$

Thus, we have standard demand function for capital, its marginal product. The demand (and the marginal product) of labor is different of the standard because the assumption of specialized labor factor. The demand of labor depends on the degree of substitution of labor varieties ϕ.

2.1.2 The households

There is a continuum of households with identical preferences but different labor factors. Each household lives in both periods. They are initially endowed with an amount A of the final good and they have monopoly power

in their specific labor factor.

In the first period households choose their portfolio: they can invest in domestic (X) or foreign ($\hat{X}$) capital and they can lend or borrow a risk free bond (B). They also choose the nominal wage taking as given the specific labor demand they will face in the second period[4]. In the second period households demand real balances, consume and supply labor. Households get income from the return of their portfolios and labor income.

Formally, the problem of household i is:

$$
\begin{aligned}
&\max \;\; E\left\{ U(C_i) + V(\bar{L} - L_i) \right\} \\
&\qquad s.t. \\
&C_i + \frac{m_i}{p} + \frac{\hat{m}_i}{p^*} = (1+r)X_i + (1+r^*)\hat{X}_i + B_i + \frac{w_i}{p}L_i \\
&X_i + \hat{X}_i + qB_i = A \\
&L_i = \left(\frac{p}{w_i}\right)^{\frac{1}{1-\phi}} \Big[f_L(K,L)Z \Big]^{\frac{1}{1-\phi}} L \\
&\mu_i C_i \leq \frac{m_i}{p} \quad , \quad (1-\mu_i)C_i \leq \frac{\hat{m}_i}{p^*}
\end{aligned}
$$

Where C_i represents consumption of household i, $\bar{L}$ is the total amount of hours available for the household and L_i is the amount of hours that the household works. m_i/p is the demand of domestic real balances and $\hat{m}_i/p$ is the demand of foreign real balances. As mentioned before there is a cash requirement, thus μ_i represents the fraction of consumption that the household

[4]Once the wage is set they commit to work as much as firm requires.

purchases in the domestic country and $(1-\mu_i)$ the fraction in the foreign country. These fractions of consumption are totally backed by real balances of each country. The risk free bond is traded in the first period at the price q.

As usually assumed cash constraints will be binding. Then, we can rewrite the budget constraint of the household as:

$$C_i = \frac{1}{2}[(1+r)X_i + (1+r^*)\hat{X}_i + B_i + \frac{w_i}{p}L_i] \tag{2.4}$$

Note that we are not assuming that resources from seignorage are given back to the households trough government transfers. Implicitly we are assuming that resources from seignorage disappear from the economy[5].

An important assumption of the model is the separable utility function in consumption and leisure. This is done to neglect the effect of leisure risk in the hedging of consumption uncertainty.

The optimal conditions for the household's problem are:

$$E\left\{U'(C_i)\left[(1+r)-(1+r^*)\right]\right\} = 0 \tag{2.5}$$

$$E\left\{U'(C_i)\left[1-(1+r^*)q\right]\right\} = 0 \tag{2.6}$$

$$E\left\{\left[U'(C_i)\phi\frac{w_i}{p} - 2V'(\bar{L}-L_i)\right]L_i\right\} = 0 \tag{2.7}$$

Equations (2.5) and (2.6)[6] are standard portfolio conditions -marginal

[5]The government blow islands with those resources.

[6]Notice that the price of bonds (q) is set one period in advance thus is not random.

utility of consumption is orthogonal to excess returns- and equation (2.7) is a variation of the usual condition that the marginal utility of consumption times the real wage equalize the marginal utility of leisure. In the present context, they are not equal but they must be orthogonal to the hours worked. This is because households can not choose the labor supply. The term ϕ that multiplies the marginal utility of consumption is a markup that households charge because their market power.

2.1.3 Market clearing conditions

As usual to compute prices in the economy it is necessary to clear the competitive markets. In this economy we have, in each country, competitive markets for consumption, capital, bonds and money. The market clear conditions for the home country are[7]:

$$\int_0^1 \mu_i C_i di + \int_0^1 (1-\mu_i^*) C_i^* di = Y \tag{2.8}$$

$$\int_0^1 X_i di + \int_0^1 \hat{X}_i^* di = K \tag{2.9}$$

$$\int_0^1 B_i di + \int_0^1 B_i^* di = 0 \tag{2.10}$$

$$\int_0^1 m_i di + \int_0^1 \hat{m}_i^* di = M \tag{2.11}$$

Equation 2.8 states that the consumption of domestic goods of the home country plus the consumption of *foreign* goods of the foreign country -it means in the home country- must be equal to the output produce in the

[7]For the foreign country we have to add asterisks to the variables as usual.

home country. Equation 2.9 affirms that domestic investment in the home country plus the *foreign* investment in the foreign country equals the amount of capital invested in the home country. Equation 2.10 is the condition of net zero supply bonds and equation 2.11 implies that the money demand of domestic and foreign residents equals money supply.

Notice form equations 2.8 and 2.11 that output in home country equals the amount of domestic real balances $Y = \frac{M}{p}$. This condition, that we call the cash constraint, will have an important roll in the next section.

Given the symmetry of the model across all the agents within a country we can drop the individual index i, thus all the variables will be aggregate country variables.

Equilibirum

$$Y = f(K, L)Z$$

$$f_K(K, L)Z = 1 + r$$

$$f_L(K, L)Z = \frac{w}{p}$$

$$C = \frac{1}{2}[(1 + r)X + (1 + r^*)\hat{X} + B + \frac{w}{p}L]$$

$$X + \hat{X} + qB = A$$

$$E\left\{U'(C)\left[(1 + r) - (1 + r^*)\right]\right\} = 0$$

$$E\left\{U'(C)\left[1 - (1 + r^*)q\right]\right\} = 0$$

$$E\left\{\left[U'(C)\phi\frac{w}{p} - 2V'(\bar{L} - L)\right]L\right\} = 0$$

$$X + \hat{X}^* = K$$

$$B + B^* = 0$$

$$\frac{M}{p} = Y$$

2.2 Conditional correlations of capital and labor income

With the model set up we are in conditions to compute the correlation between the rate of return on home capital and the non tradable labor income. When the uncertainty is solved and we begin the second period some variables like nominal wages and capital stock are already set and we can take them as given.

From labor demand equation and the cash constraint we have:

$$\frac{M}{p} = f(K,L)Z \qquad and \qquad \frac{w}{p} = f_L(K,L)Z$$

Then, by the implicit function theorem:

$$\begin{aligned} \frac{dL}{dM} &= \frac{f_L(K,L)}{wf_L(K,L) - Mf_{LL}(K,L)} \geq 0 \\ \frac{dL}{dZ} &= 0 \end{aligned}$$

The effect of a monetary expansion on the hours worked is positive. This is because money increases the demand of final output and firms response is to increase worked hours and rise prices. On the other hand, there is no effect of technology in hours because of the assumption of multiplicative productivity. This is not true in more general setups but here it only simplifies the algebra.

Given this effect on worked hours we can ask what is the effect of supply and demand shocks in the factor prices. The effect of a supply shock

(technology) is to enlarge the production set. This creates profits for the firm inducing a expansion in the output, increasing the demand for factors and reducing the price level. The demand for capital rises the rate of return until it pin down profits[8]. But since the wage is preset, the real wage is only affected by the reduction in prices. Hence the effect of a supply shock is positive for both factors prices implying a positive correlation between these.

$$\frac{dr}{dZ} = f_K(K, L) \geq 0$$

$$\frac{dw/p}{dZ} = f_L(K, L) \geq 0$$

On the other hand if the cycle is mainly driven by demand (money) shocks we will observe a negative correlation. In general a demand shock will increase the price level and will induce firms to demand more capital. This increases the capital rate of return while the increase in prices reduces the real wage.

$$\frac{dr}{dM} = f_{KL}(K, L)Z\frac{dL}{dM} \geq 0$$

$$\frac{dw/p}{dM} = f_{LL}(K, L)Z\frac{\partial L}{\partial M} \leq 0$$

These correlations are not sufficient to determine the optimal portfolio. As mentioned before, the optimal portfolio has a very important hedging component against non tradable labor income risk for which real wage is

[8]Remember that in equilibrium profits must be zero.

only one component.

Supply (technology) shocks have no effect on hours worked. Then, the implications for labor income will be unambiguously positive. Demand (money) shocks shift real wage and hours worked in opposite directions producing an ambiguous effect on labor income.

$$\frac{dwL/p}{dZ} = f_L(K,L)L \geq 0$$

$$\frac{dwL/p}{dM} = [f_{LL}(K,L)L + f_L(K,L)]Z\frac{dL}{dM} \lesseqgtr 0$$

Whether real wage or hours dominate depend on the elasticity of the labor demand (η).

$$\begin{aligned} f_{LL}(K,L)L + f_L(K,L) &= \left(1 - \left|\frac{f_L(K,L)}{f_{LL}(K,L)L}\right|\right) f_{LL}(K,L)L \\ &= (1-\eta)\, f_{LL}(K,L)L \end{aligned}$$

Then, demand (money) shocks will produce a negative correlation between the capital rate of return and labor income if and only if the elasticity of the labor demand is lower than 1.

$$(1-\eta)\, f_{LL}(K,L)L \leq 0 \quad \Rightarrow \quad 1-\eta \geq 0 \quad \Rightarrow \quad \eta \leq 1$$

According to empirical studies this elasticity in the U.S. is around 0.4.

"*Estimates of the own-price elasticity vary more substantially with the year, type of differencing used, and industry. They average -.50 for production hours, -.41 for production workers, and -.44 for nonproduction workers.*" (The Long-run Demand for Labor: Estimates from Census Establishmet Data, Dunne and Roberts, 1993)[9]

2.3 Optimal portfolios

The model presented in the previous section does not give analytical solutions in general but we can extract some conclusions from a second order approximation.

From the optimal condition (2.5) we have that the excess return must be orthogonal to the marginal utility of consumption. For instance if we expect home return to be higher than foreign the optimal portfolio will weight more home equity. This increases consumption (decreases our marginal utility) in periods where the excess return is higher.

For a second order Taylor expansion we have:

[9]In their paper, the authors, does not define the price elasticity with the absolute value of the percent change of labor demand for one percent increment in real wages. For that reason, to be consistent with the notation of this paper, we should consider the elasticities in absolute value.

$$U'(\bar{C})[\bar{r}-\bar{r}^*]+\frac{1}{2}U''(\bar{C})Cov(C,r-r^*)+\frac{1}{2}U'''(\bar{C})[\bar{r}-\bar{r}^*]Var(C)\approx 0 \quad (2.12)$$

This expression clearly shows the positive correlation between the consumption and the excess returns when $\bar{r}\geq\bar{r}^*$.

On the other hand if there is a non marketable component in the consumption it should be insured. First we should buy (short sell) the amount of capital that hedges this risk and then we should diversify with the remaining part. In the model of previous section there is non marketable labor income. Then the portfolio will have to insure the non tradable labor risk. I examine this case with the following examples, first a simplified version to obtained approximated solutions, and through numerical methods for a general case.

2.3.1 Solution for the symmetric case

For the special case where both countries are symmetric and the exogenous shocks are independent and identically distributed the expected return are equalized. These assumptions and equation (2.4) allow us to rewrite (2.12) as:

$$XVar(r)+Cov\left(\frac{w}{p}L,r\right)\approx\hat{X}Var(r^*)$$

Symmetry also implies $Var(r)=Var(r^*)$, $K=A$ and $B=0$ among other things. Then:

$$X+Cov\left(\frac{w}{p}L,r\right)/Var(r)\approx A-X$$

It is important to note that given the symmetry among countries the portfolio that minimizes the variance is $X = A/2$, but the optimal portfolio does not try to just minimize the variance but to hedge also against the non marketable risk.

$$X \approx \frac{1}{2}\left[A - Cov\left(\frac{w}{p}L, r\right)/Var(r)\right]$$

In the appendix is shown how to compute this portfolio for a Taylor approximation. The two extreme cases where only one supply or demand shocks matters produce the following results:

$$Var(M) \to 0 \quad ; \qquad h = \frac{1}{2}\left[1 - \frac{f_L(K,L)L}{f_K(K,L)K}\right]$$

Where h is defined as the share of domestic in total equity of the portfolio[10]. This portfolio has the property pointed by Baxter and Jermann (1993) that the portfolio should have short positions in domestic capital if the labor share exceeds the share of the country in the capital market[11].

Thus, as mentioned before, if shocks come from supply, the correlation between labor income and return to capital will be positive. Moreover because it hedging motive we should expect country portfolios to be foreign biased.

[10] A formal definition:

$$h \equiv \frac{X}{A - qB}$$

[11] In the present example 1/2.

On the other hand, if shocks come from the demand side and the correlation between labor income and capital returns is negative we should expect country portfolios that are home biased.

$$Var(Z) \to 0 \quad ; \qquad h = 1 - \frac{\eta}{2}$$

Where the labor demand elasticity η plays a very important roll again. If $\eta < 1$ then $h > 1/2$ consistent with a home bias in country portfolios. With the elasticity measured by Dunne and Roberts (1993) of 0.4 - 0.5, we will obtained a portfolio 75% - 80% biased towards domestic securities.

2.3.2 Solution for the general case

The solution of the system of equations for the general case depends on the functional forms of utility and production functions. In this section I will use standard functional forms to obtain a numerical solution for a calibration of the parameters consistent with those used in the literature.

Let's first begin with the functional forms for the utility and production. The production function will be characterized by a Constant Elasticity of Substitution (CES):

$$f(K, L) = \Big[\alpha K^{\mu} + (1 - \alpha) L^{\mu} \Big]^{1/\mu}$$

For the utility of consumption and leisure let's assume Constant Relative Risk Aversion(CRRA) functions:

$$U(C) = \frac{C^{1-\sigma} - 1}{1 - \sigma} \quad ; \qquad V(\bar{L} - L) = D \frac{(\bar{L} - L)^{1-\gamma} - 1}{1 - \gamma}$$

After the characterization of the utility and production functions through these functional forms, I will proceed with the election of the parameter values.

Table 2.1: Parameters for the baseline model of Optimal International Portfolio choice

Parameter	**Value**	**Parameter**	**Value**
σ	2	μ	-6
γ	0	α	0.2
ϕ	0.9	$S.D.(M)$	0.5
$\bar{L}$	10	$S.D.(Z)$	0.1

Most of the parameters are chosen from the literature on International Business Cycles. The parameters of the shocks standard deviations come from the VAR analysis of next section.

These parameters imply a labor demand elasticity of -0.4323, and labor income percentage of total income of 68.78%. Thus, with these assumptions the system of equations can be solved using numerical methods. The solution for the baseline model is presented in table 2.2:

Therefore, the model for the selected parameters seems to imply a bias towards domestic securities but a little lower than in actual portfolios. This result strongly depends on the assumption about the standard deviation of technology shocks over the standard deviation of monetary shocks, as can be seen in figure 2.1. This is the effect explained in the previous section about the correlation between labor and capital income conditioned on money or

Table 2.2: Solution for the baseline model of Optimal International Portfolio choice

Variables	**Values**
X	0.7986
$\hat{X}$	0.1925
$\hat{X}^*$	0.1931
w	0.3040
w^*	0.3073

The values of the solution were computed numerically with a matlab routine available upon request.

technology shocks.

2.4 Identifying the conditional correlations between real wages and equity prices: a VAR approach

In the previous sections, we have addressed the question of whether the finding of Bottazzi *et al.* (1996) on the negative correlation between real wages and real capital returns can be reconciled within a general equilibrium model. The answer seems to be satisfactory and the next step is to look at the data to see if it supports the theoretical vision.

In order to address this question I chose data on real wages and capital returns for the three countries that we include in the present study, United States, Japan and United Kingdom. The series chosen are: real wages defined as domestic hourly labor earnings divided by the domestic consumer price index. Real domestic share prices defined as domestic share prices divided by domestic consumer price index. Real foreign share prices defines as the weighted average between foreign share prices corrected withe the exchange rate and divided by the domestic consumer price index. The weights are chosen from the relative share in global portfolios. And the last series is the domestic consumer price index. Sources of the data can be found on the appendix.

With these series I construct a VAR for each country in order to identify the monetary (demand) shocks, and therefore, compute the conditional correlation between real wages and domestic and foreign share prices.

As a first step I analyze the order of integration of the series. I use Augmented Dicky Fuller (ADF) procedure to test for unit roots in the time series.

From the tables 2.4, 2.4 and 2.4, it can be seen that the time series of the analysis are integrated of order one, thus I will differentiate them. This structure allowed me to identify permanent monetary shocks as the only shocks that last on the long run on the consumer price index but vanishes for the real series.

Table 2.3: ADF test for the presence of unit roots in US variables

Variable	**levels**	**p-value**	**1st diff.**	**p-value**
Real wage	-2.0891	0.5468	-3.3072	0.0164
Domestic share price	-1.7198	0.7374	-8.4772	0.0000
Foreing share price	-2.2614	0.4517	-9.3109	0.0000
Domesic Consumer Pirce Index	-2.9568	0.1484	-3.1892	0.0228

Table 2.4: ADF test for the presence of unit roots in Japanese variables

Variable	**levels**	**p-value**	**1st diff.**	**p-value**
Real wage	-2.7251	0.2283	-3.2235	0.0208
Domestic share price	-1.7547	0.7213	-7.7348	0.0000
Foreing share price	-2.1900	0.4910	-8.5223	0.0000
Domesic Consumer Pirce Index	-1.7215	0.7364	-2.4820	0.1221

Thus, a structural VAR is estimated for each country (US, Japan and UK) on the four variables. After the estimation I impose long run restrictions to identify permanent monetary shocks as those that last in the long run only on the CPI. To fully identify the system I imposed other restrictions, but the permanent monetary shock does not rely on that. Therefore, we should only consider the fourth shock, and the fourth impulse response function.

Table 2.5: ADF test for the presence of unit roots in UK variables

Variable	**level**	**p-value**	**1st diff.**	**p-value**
Real wage	-2.7134	0.2355	-2.0142	0.2801
Domestic share price	-2.0750	0.5547	-8.8572	0.0000
Foreing share price	-2.0486	0.5692	-5.8486	0.0000
Domesic Consumer Pirce Index	-0.8361	0.9589	-2.7521	0.0681

In these graphs (2.2, 2.3, 2.4) can be seen the expected sign in the response of real wages and domestic share prices, the disturbing result is that the response of foreign share prices is of the same sign and, given the lack of error bands for cumulative impulse response functions, we can not say which are of bigger magnitude and therefore we can not be sure about the sign of the bias.

Variance decomposition

After identifying the permanent monetary shock, which implies the negative correlation between real wages and domestic share prices, it is important to check its impact in the total variance of the different variables.

For most countries the variance explained by the permanent monetary shock explains very little of real wages variances. For instance for the U.S. it is negligible up to the fourth decimal point, and for Japan and U.K. the effects doesn't reach a 10% for the first quarter. The following quarters the

variance explained increases very much for U.K. but given the reduced series for this country they are not conclusive. The other variables have a higher variance explained by permanent monetary shocks.

The most important variable to check is probably consumption, but given the reduced series of data it is not recommendable to include another variable. We can use real wage and share prices as a proxy of consumption.

Table 2.6: Variance decomposition for permanent monetary shocks: US variables

Quarter	**real wage**	**domestic equity**	**foreign equity**	**prices**
1	0.0000	0.0540	0.0408	0.4102
2	0.0000	0.0211	0.0160	0.3821
3	0.0000	0.0611	0.0495	0.3634
4	0.0000	0.0772	0.0634	0.3548
5	0.0000	0.0815	0.0672	0.3517
6	0.0000	0.0823	0.0679	0.3508
7	0.0000	0.0825	0.0680	0.3505
8	0.0000	0.0825	0.0680	0.3505

Table 2.7: Variance decomposition for permanent monetary shocks: Japan variables

Quarter	**real wage**	**domestic equity**	**foreign equity**	**prices**
1	0.0744	0.0037	0.0073	0.8770
2	0.2798	0.0001	0.0107	0.4223
3	0.2496	0.0021	0.0251	0.5170
4	0.2529	0.0034	0.0290	0.5195
5	0.2522	0.0039	0.0301	0.5232
6	0.2523	0.0041	0.0303	0.5235
7	0.2523	0.0042	0.0303	0.5237
8	0.2523	0.0042	0.0304	0.5238

2.5 Concluding remarks

One of the explanations about the portfolio diversification puzzle is that domestic capital produces a good hedge against non traded real wage risk. This is because in the data it is negatively correlated with domestic capital returns (measured with nonfinancial corporate profits). Traditional general equilibrium models generate positive comovements between these variables. Traditional models assume supply side shocks as the source of economic fluctuations.

The contribution of this paper is to explain how nominal rigidities, in the form of sticky wages, together with demand shocks may produce the negative

Table 2.8: Variance decomposition for permanent monetary shocks: UK variables

Quarter	**real wage**	**domestic equity**	**foreign equity**	**prices**
1	0.0648	0.0311	0.0859	0.5290
2	0.9797	0.0668	0.1538	0.2662
3	0.8610	0.0977	0.1878	0.2597
4	0.8521	0.1008	0.1900	0.2614
5	0.8517	0.1007	0.1896	0.2613
6	0.8516	0.1007	0.1897	0.2613
7	0.8516	0.1007	0.1897	0.2613
8	0.8516	0.1007	0.1897	0.2613

correlation observed in the data. The mechanism is through the loss of value in the nominal wage when prices increase.

We also search in the data the effects implied by the model. The answer is satisfactory in the response of the domestic variables. When the shock identified as permanent monetary increases prices, real wage decrease and domestic share prices increases. The disturbing effect is that foreign share prices increases as well.

A possible answer for this result is that the model assumes purchasing power parity, probably a depreciation in the currency as a consequence of a permanent monetary shock increases the domestic currency value of foreign

shares.

This possibility should be considered in further research. A probably good guess is that the effect on the portfolio composition of monetary shocks depends strongly on the degree of purchasing power parity deviations.

Figure 2.1: Sensibility of the bias to changes in the standard deviation of Z. For S.D.(M) = 0.5

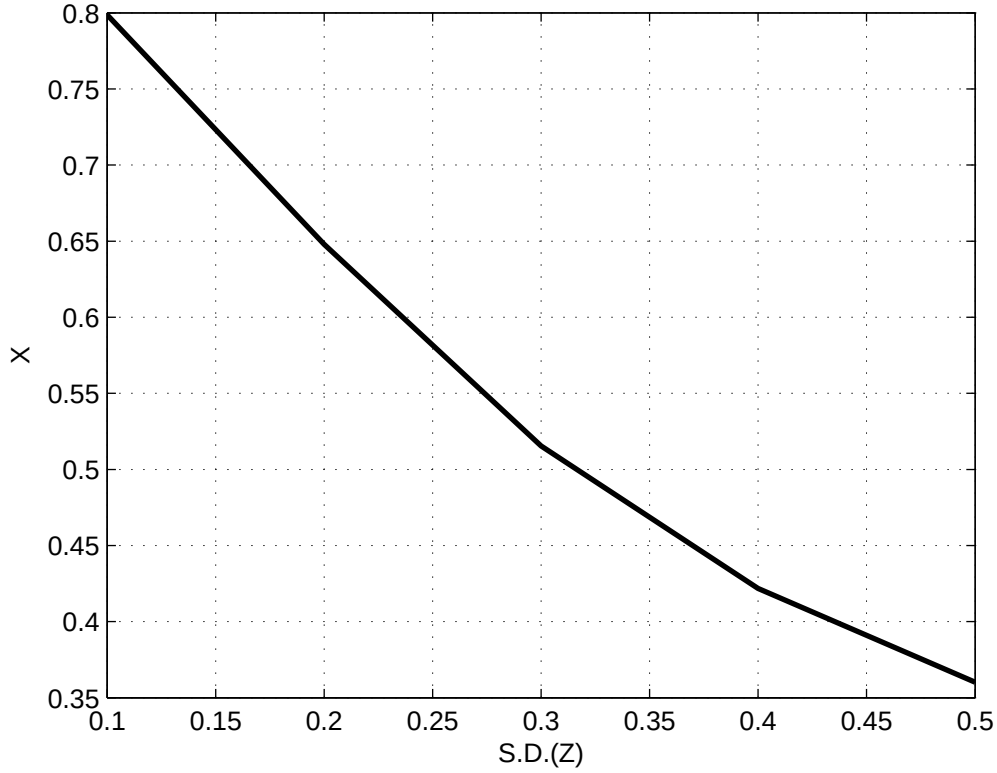
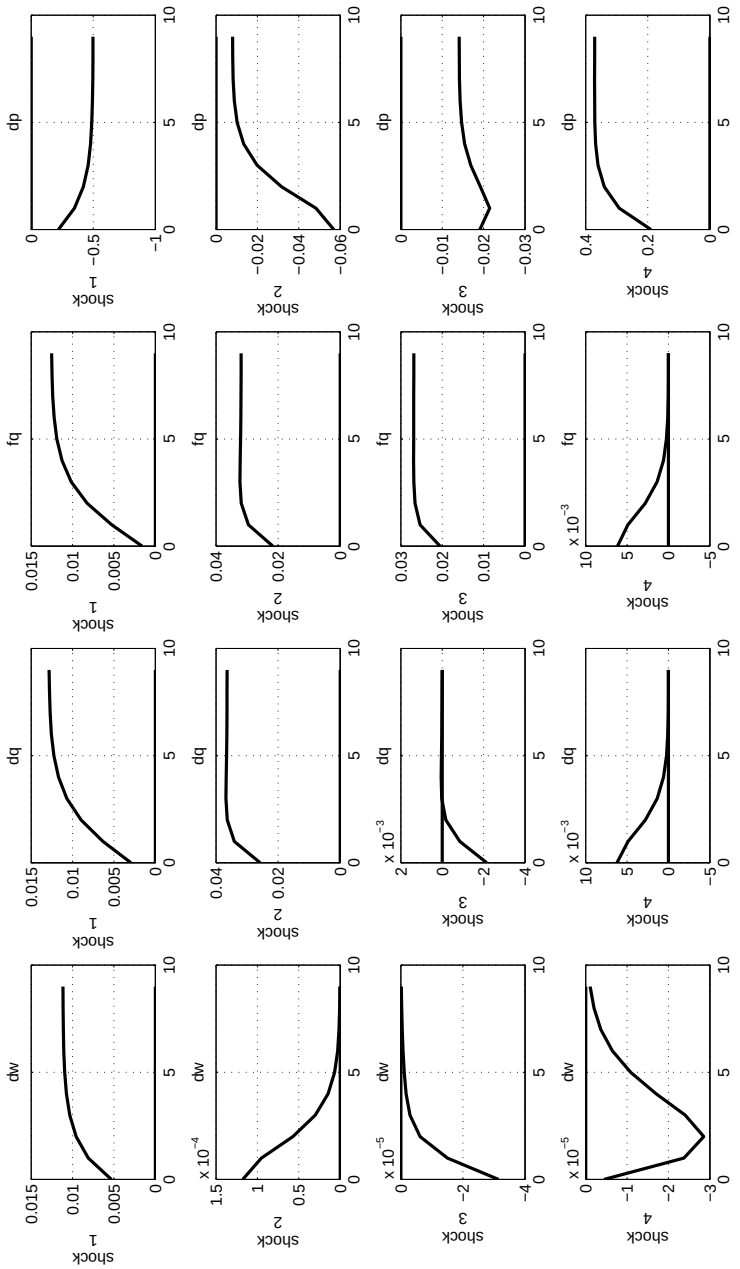
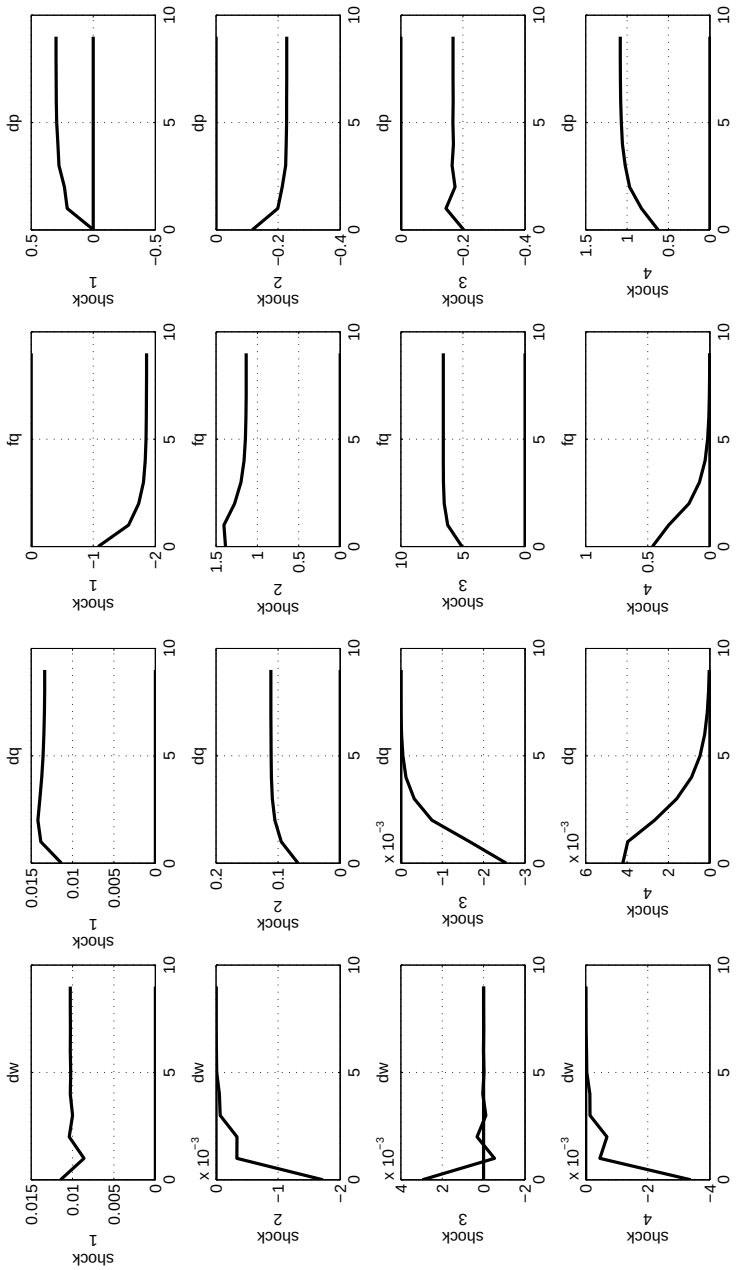


Figure 2.2: Cumulative impulse response functions for U.S.



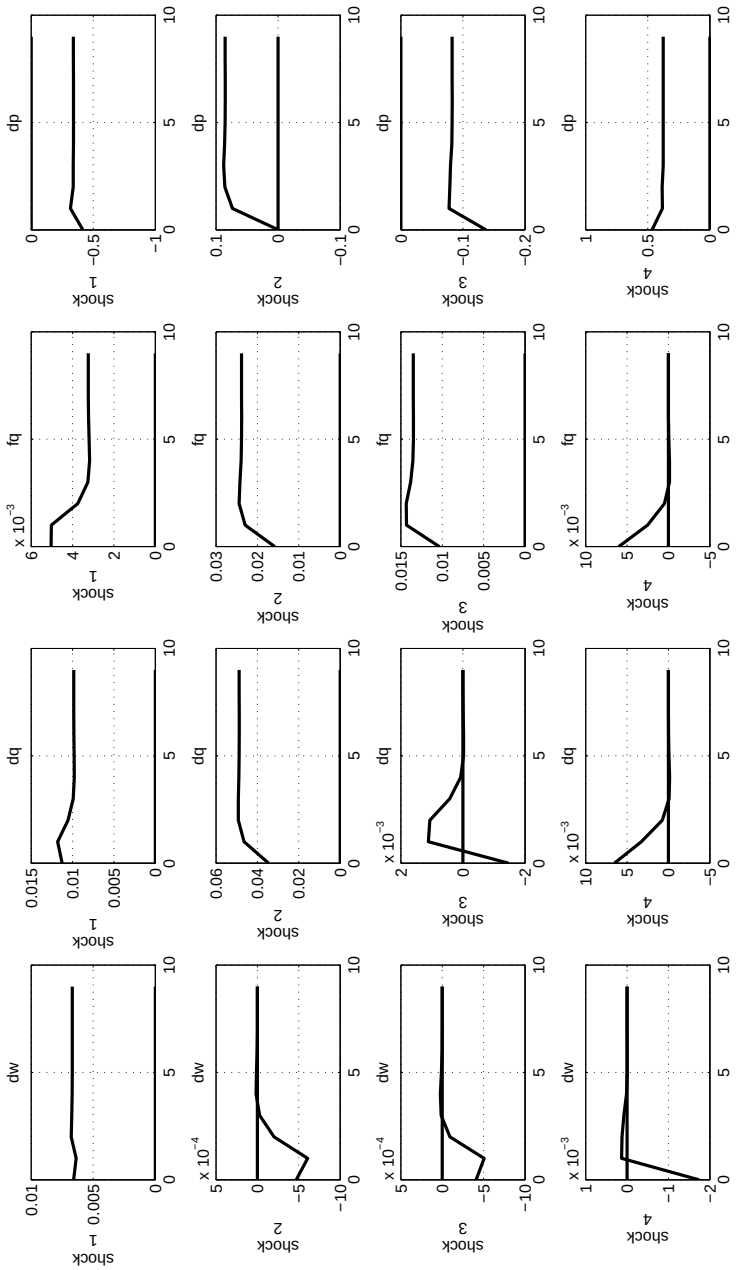
dw.- Domestic real wage. dq.- Domestic share prices. fq.- Foreign share prices. dp.- Domestic consumer price index

Figure 2.3: Cumulative impulse response functions for Japan



dw.- Domestic real wage. dq.- Domestic share prices. fq.- Foreign share prices. dp.- Domestic consumer price index

Figure 2.4: Cumulative impulse response functions for U.K.



dw.- Domestic real wage. dq.- Domestic share prices. fq.- Foreign share prices. dp.- Domestic consumer price index

Chapter 3

Short Term Debt, Country Risk Premia and Hedge Funds

A particular feature of the East Asian crisis was the excessive amount of short term borrowing[1]. Since then, there has been some research in this area trying to explain why this kind of borrowing is dangerous. More recently, there has been some work seeking to explain the rationale behind the existence of short term debt. That is, if short term debt is bad, why do we observe it? In this paper, we will present a possible explanation to answer this question.

When countries borrow short term, they are exposed to the risk of a *self fulfilling crisis*[2]. When creditors believe countries will not be able to honor

[1]See, for example Krugman [18].

[2]See for instance Cole and Kehoe [9] and Chang and Velasco [8]. Also, Rodrik and Velasco [28] show empirically the importance of short term debt in predicting crises.

their debt, they do not roll over the loans, and then countries effectively cannot pay. Hence, bad expectations may produce a crisis. The literature has concentrated in explaining why short term debt is "bad", but very few attempts on why does short term debt exist in the first place[3].

Other authors have focused on the implementation of policies that rule out the possibility of these crises, going from regulation of capital flows to the proposal of a new set of contracts. In a survey by Rogoff [29], he analyzes the potential role of international institutions, such as an international lender of last resort or an international bankruptcy court. He also shows the effect of controls on capital flows (both inflows and outflows). In this same direction, Chamon [7] proposes a way to eliminate the coordination failure by allowing the investors to promise to lend only if enough other investors do so as well. These proposals mainly concentrate on how to avoid the crises *given* the debt structure and does not question the debt structure itself[4].

It should be clear that countries are not interested in taking *self fulfilling crisis* risk, mainly because of the losses observed when there is such a crisis, but also because the high interest rates that are needed to attract funds. Thus, it is surprising that we observe short term borrowing given the monopoly power any debtor has on her debt structure and therefore on her risk.

[3]There are few exceptions. See, for example, Broner et al [5].

[4]A notable exception is Rogoff's proposal suggesting changes in the financial structure towards more equity rather than debt.

This paper is aimed on explaining this particular feature of the financial structure. In particular, given that debtors are negatively affected by short term debt, the question we want to address is "*Is it in the interest of creditors that countries finance long term projects with short term debt? If so, how do they accomplish it?*"

In order to answer these questions, we model a small country with long term investment projects to be financed by specialized risk-averse foreign investors. The economy produces many non-tradable intermediate goods that are in turn used to produce a tradable final good. The technology of the final good is crucial. We assume that there is a *critical mass* of inputs required to have positive production of the final good. This assumption creates a linkage between intermediate firms that will in turn affect the optimal debt structure that firms take.

If this project is financed with long term debt (i.e. if the project and the debt have the same maturity), there is no risk[5]. This together with the abundance of foreign resources competing on the project, imply zero premium for investors, i.e. they will get the world risk free rate. Thus, in this scenario all the surplus of the project goes to the entrepreneurs.

But, the project can also be financed with *short term* debt (that is debt maturity shorter than the project: there is *maturity mismatch*). The exis-

[5]In other words, we are assuming there is no production risk. This assumption can be easily removed.

tence of production linkages eliminates the monopoly power of the borrowers on their risk. If a large fraction of firms borrow short term, the possibility of a crisis on those firms will produce a collapse of the final good sector reducing the demand for all intermediates. Thus, even when borrowing fully in long term, a firm can be risky. The liquidity risk acts like an *entry barrier* limiting the amount of resources competing for the project. This allows international investors to extract part of the surplus (otherwise taken by entrepreneurs). Then, the investor's portfolio is composed by both risk free investment in the world's safe technology (the country is small and does not exhaust resources) and risky investment in the country.

This paper will argue that this last portfolio (composed partly by short term debt) is preferred by the lenders to the one composed by long term debt. Notice however that this does not imply that creditors *like* crises; they just like the possibility of it. On the other hand, entrepreneurs prefer to avoid maturity mismatches. However, high enough term premium[6] can induce entrepreneurs to finance their project with short term debt. In other words, an equilibrium with a project fully financed by short term debt might exist.

Which equilibrium would be observed? We will argue that if, when deciding the investment creditors can coordinate, they will choose to finance these projects with short term debt. This coordination can be done through some international financial intermediary, e.g. a hedge fund. However, coor-

[6]We refer to term premium as the difference between long term and short term interest rate.

dination is not perfect. The existence of a hedge fund is not enough to avoid the possibility of a crises: investors can run on the hedge fund, forcing the hedge fund to run on the country.

The paper is organized as follows: Section 2 presents a model of debt maturity choice in a competitive bond market. Section 3 does welfare comparisons between equilibria for both agents and for the economy as a whole. Section 4 introduces a financial intermediary (a hedge-fund) to obtain a unique equilibrium. Section 5 concludes.

3.1 A model of optimal debt structure in a competitive bond market

Let the economy be composed by a continuum of firms that produce intermediate goods to be sold to a final sector. The intermediate goods are differentiated and non-traded whilst the final good is traded. In addition, there is a continuum of international lenders that behave perfectly-competitive. All lenders and entrepreneurs live for three periods: 1, 2, 3.

Each entrepreneur maximizes her utility function $E\{U(\cdot)\}$. She has an illiquid investment project that lasts two periods. She invests an amount K_i in period 1 and the project yields $f(K_i)$ in period 3. The production function f is a continuous, concave, twice-differentiable function that satisfies Inada conditions. We assume that in period 3, once debts are paid, she consumes.

If, for any reason at period 2, she has to disinvest part of the capital, she will only get a fraction φ of it. The remaining fraction is destroyed due to costly disinvestment.

On the other hand, the representative lender is a risk averse investor with wealth W. She can invest in a risk-free storage technology that we assume pays 1, or lend in two maturities: short term loans (D) which last one period and long term loans ($K - D$) which last two periods (charging respectively net rates r_S and r_L). If she decides to lend short term, at period 1 she will have another decision to take: She will decide whether to roll over the debt or to use these resources to invest in the risk-free technology. Finally, in period 2 she consumes. Thus, the lender's problem is to maximize her expected utility $E\{V(\cdot)\}$, where V satisfies all desirable properties.

The final sector uses different inputs denoted by Z_i. Moreover, the final sector can only produce if there is a *sufficiently large amount of inputs available*.

Formally, let $N = \{i : Z_i > 0\}$, then the final good production is

$$Y = \begin{cases} \frac{1}{\alpha}\left[\int_0^1 Z_i^{\alpha} di\right]^{\frac{1}{\alpha}} & , \quad m(N) \geq \mu \\ 0 & , \quad m(N) < \mu \end{cases} \tag{3.1}$$

Where $m(.)$ ia the measure of the set, μ is the minimum amount of inputs required to have positive production. This creates a linkage between firms to insure that in case of financial distress, all firms are affected. Ir there are

not enough intermediate firms finishing their projects, the final good sector can not produce and the economy faces a crisis.

Definition 1 *Let $\bar{K}$ be the level of capital for which $f'(K) = 1$.*

The amount of capital invested in the country will never exceed $\bar{K}$. This would be the capital invested if there were no issues of liquidity. For any level of capital higher than this threshold, the return would be lower than the risk-free technology.

Assumption 1 *Available resources for entrepreneurs are not scarce, i.e. $\bar{K} < W$.*

This assumption will imply that competition among entrepreneurs will not exhaust surplus.

Assumption 2 *International investors are specialized: they invest a non negligible fraction of their portfolio in the country.*

Notice that lenders are risk averse. If lenders invest a negligible fraction of their wealth in a country, they would behave as risk neutral. The specialized investors assumption prevents this from happening.

The timing of the problem is as follows

- At $T = 1$, lenders and entrepreneurs decide the amount to be invested and the debt structure.

- At $T = 2$, lenders holding short term debt decide to rollover or not their debt.
- At $T = 3$, entrepreneurs sell their intermediate good to the final output sector and debt is repaid.

Definition 2 ***(Competitive bond market equilibrium)*** *The equilibrium will be characterized by a set of prices* $\{P_i, r_{Si}, r_{Li}\}\ \forall i$, *and quantities* $\{D_i, K_i\}\ \forall i$, *such that:*

(*i*) *Given* $\{P_i\}$, *final sector firms maximize profits.*
(*ii*) *Given* $\{r_{Si}, r_{Li}\}$, *lenders maximize utility.*
(*iii*) *Given* $\{r_{Si}, r_{Li}\}$ *and final sector demand, entrepreneurs maximize utility.*
(*iv*) *Markets clear.*

3.1.1 Final good sector

At $T = 3$, the final good sector firms buy inputs to produce the only tradable good in the economy (Y). We will use the price of the final good as the numeraire, i.e. $P_Y = 1$. the problem of the final good sector is

$$\max_{\{Z_i\}} \left\{ Y - \int_0^1 P_i Z_i di \right\}$$

where P_i is the price of intermediate i, and Y is the production as defined in Equation 3.1.

The first order conditions are:

$$Z_i: \quad \frac{1}{\alpha}\left[\int_0^1 Z_j^{\alpha} dj\right]^{\frac{1-\alpha}{\alpha}} Z_i^{\alpha-1} - P_i = 0; \ \ \forall i$$

Let G_i be the revenue function of intermediate i, i.e. $G_i = P_i Z_i$. The first order condition implies[7]:

$$G_i = \frac{1}{\alpha}\left[\int_0^1 Z_j^{\alpha} dj\right]^{\frac{1-\alpha}{\alpha}} Z_i^{\alpha}$$

3.1.2 Roll over decision

At $T = 2$, lenders have to decide if they roll over the debt or not.

Let us begin the analysis by analyzing the possibility of a run on a single firm. Denote β_i as the fraction of lenders that at period 1 decide to run on firm i. Again, we refer to "run" as deciding not to renew their short term contracts from period 1 to period 2.

If the revenue of firm i after paying the long term bond holders, is still capable of paying the short term bond holders that did not run $(1-\beta_i)$, then a run was definitely not optimal: The β_i investors that ran should not have run, i.e. $\beta_i = 0$[8].

[7]This expression is valid whenever production of intermediate is completed. In some cases, this will not be the case. If production is not completed, G_i would be equal to the residual value of the firm.

[8]Actually, they are indifferent since from period 1 to period 2, the project yields as the safe technology. However, we assume that there is some small cost of running.

On the other hand, suppose that firm i is not capable of repaying the $1-\beta_i$ bond holders that did not run. In this case, a run was optimal, and all investors should have done it, ($\beta_i = 1$). Lemma 1 states this formally[9].

Lemma 1 *For each firm i, there cannot be partial runs: either all investors run or none of them do. In particular*

$$G(K_i - \beta_i(1+r_{Si})D_i/\varphi) - (1+r_{Li})(K_i - D_i) \geq (1-\beta_i)(1+r_{Si})D_i \Rightarrow \beta_i = 0$$
$$G(K_i - \beta_i(1+r_{Si})D_i/\varphi) - (1+r_{Li})(K_i - D_i) < (1-\beta_i)(1+r_{Si})D_i \Rightarrow \beta_i = 1$$

Let us now consider the possibility of runs on different firms. Lemma 1 implies that each firm is either fully attacked or is not attacked at all. It is easy to see that for some levels of short term debt,

$$G(K_i - (1+r_{Si})D_i/\varphi) - (1+r_{Li})(K_i - D_i) \geq 0 \tag{3.2}$$

i.e. firm i is able to pay its long term bond holders even in the case of a full run on its short term obligations. Clearly, for levels of debt that satisfy 3.2, firm i will not be attacked[10].

[9]Notice that the short term interest rate from period 1 to period 2 is always zero since uncertainty is fully resolved in period 1

[10]Notice that $G()$ depends on intermediate goods prices. Prices can be low enough so that Equation 3.2 is never satisfied

On the other hand, whenever

$$G(K_i) < (1 + r_{Si})D_i + (1 + r_{Li})(K_i - D_i), \tag{3.3}$$

firm i will be fully attacked. Lemma 2 states these two results formally.

Lemma 2 *For each firm i,*

- $G_i(K_i - (1 + r_{Si})D_i/\varphi) - (1 + r_{Li})(K_i - D_i) \geq 0 \Rightarrow Prob(\beta_i = 1) = 0$
- $G_i(K_i) < (1 + r_{Si})D_i + (1 + r_{Li})(K_i - D_i) \Rightarrow Prob(\beta_i = 1) = 1$

For further notation, we will refer to $\tilde{D}_i$ as the level of short term debt that satisfies 3.2 with equality, i.e. as the maximum level of short term debt for which firm i will be attacked with probability zero[11].

Note that Lemma 2 does not cover all possible levels of short term debt. What can we say about these other levels of short term debt? Entrepreneurs with these levels of short term debt will have positive probability of runs. We will assume for these firms that the attack decision follows a sunspot.

Let $B \subseteq [0, 1]$ be the set of firms that are attacked, i.e. $i \in B$ only if $\beta_i = 1$. Furthermore, denote S to be the set of firms that for any possible "attack" set B, satisfy 3.2. In other words, $S = \{i : \forall B, \ D_i \leq \tilde{D}_i\}$. Finally, denote $R = [0, 1] - S$ as the set of firms that are not in S.

[11] Clearly $\tilde{D}_i$ depends on the demand of the final good sector.

Notice that a possible attack can be anything going from the empty set to the set R[12]. Moreover, if a firm i is not part of an attack B, but for this event firm i satisfies 3.3, then B should have probability zero. In order to satisfy these conditions we will take a simple and often-used distribution focusing on attack to all firms or no attack at all.

Assumption 3 *Lenders will decide to run or not following a sunspot: With probability π lenders will attack all firms, and with probability $1-\pi$ they will not attack at all.*

$$Prob(B = \emptyset) = 1 - \pi$$
$$Prob(B = R) = \pi$$

This is not the only distribution consistent with lemma 2.

3.1.3 Optimal Debt Choice

We are now capable of analyzing the problem of lenders and entrepreneurs. Let us begin by defining r_i as the weighted average return rate of investment in firm i, i.e.

Definition 3 *Let's define the average interest rate for entrepreneur i as the weighted average of the short term and long term interest respectively.*

$$r_i = \frac{D_i}{K_i} r_{Si} + \frac{K_i - D_i}{K_i} r_{Li}$$

[12]It should be clear to see that $B \cap S \neq \emptyset$ implies Prob(B)=0.

Moreover let R_1 and R_2 be a partition of the set R ($R_1 \cup R_2 = R$ and $R_1 \cap R_2 = \emptyset$) . R_1 will be composed of firms such that $i \in R_1$ if and only if $\tilde{D}_i < D_i \leq \varphi K$. Naturally, $i \in R_2$ if and only if $D_i \geq \varphi K$.

Finally denote X_{Si} (X_{Li}) as the amount of debt invested short (long) term in firm i.

Lenders' problem

Lenders will solve the following problem:

$$\max_{\{x_S, x_L\}} \left\{ (1-\pi) V \left(W + \int_0^1 [x_{Si} r_{Si} + x_{Li} r_{Li}] di \right) + \pi V \Big(W + \right.$$
$$\int_S [x_{Si} r_{Si} + x_{Li} r_{Li}] di + \int_{R_1} \left[x_{Si} r_{Si} + x_{Li} \left(\frac{G_i(K_i - (1+r_{Si}) D_i/\varphi)}{K_i - D_i} - 1 \right) \right] di$$
$$\left. + \int_{R_2} \left[x_{Si} \left(\frac{\varphi K_i}{D_i} - 1 \right) + x_{Li}(-1) \right] di \right) \Big) \Big\}$$

It is easy to show that the first order conditions of this problem imply that lenders will charge the following interest rates:

$$\forall i \in S, \quad r_i = 0 \tag{3.4}$$
$$\forall i \in R_1, \quad r_i = \frac{\pi}{1-\pi} \frac{V'(C_R)}{V'(C_\emptyset)} \left[\frac{K_i - D_i - G(K_i - D_i/\varphi)}{K_i} \right] \tag{3.5}$$
$$\forall i \in R_2, \quad r_i = \frac{\pi}{1-\pi} \frac{V'(C_R)}{V'(C_\emptyset)} (1-\varphi) \tag{3.6}$$

Where C_B is the payoff for the lenders when the event $B \in \{\emptyset, R\}$ occurs.

Entrepreneurs' problem

What is the optimal response of entrepreneurs? Notice entrepreneurs act as monopolistic competitive in their own intermediate good. They maximize

the following problem

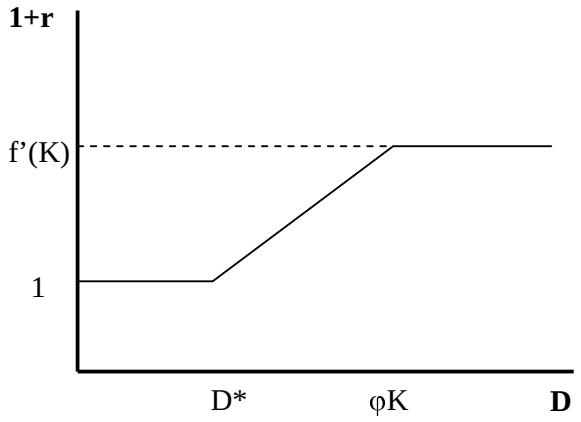
$$\max_{K_i, D_i} \left\{ (1-\pi) U(G(K_i) - (1+r_i)K_i) + \pi U(\mathbf{I}_{\{S\}}(G(K_i) - (1+r_i)K_i)) \right\}$$

s.t. Equations (3.4) to (3.6).

$\mathbf{I}_{\{S\}}$ is an index function that takes the value of one whenever $i \in S$.

Whenever feasible, entrepreneurs will choose to be in S, interest rate is zero ($r_i = 0$) and there is no attack with certainty. Symmetry of the problem implies that either all entrepreneurs are in S or none of them are.

Figure 3.1: Interest rate charged by different levels of short term debt for a given K



The dashed line represents the interest rate when S is not feasible. Continues line are interest rates for different levels of short term debt when S is feasible.

Suppose first that $\forall i \in S$, i.e. every entrepreneur satisfies Equation 3.2

(with $r_S = r_L = 0$). If this is the case, all entrepreneurs will produce a positive amount of intermediates, ($Z_i > 0$), and thus, there is positive production (see Equation 3.1).

Moreover, the entrepreneur's problem maximization and symmetry imply

$$G'(K) = f'(K) = 1$$

i.e. $K = \bar{K}$.

Furthermore, notice that $G_i = \frac{1}{\alpha} f(\bar{K})$, $\forall i$. It should be easy to see that there is a positive level of short term debt $\tilde{D}$ such that Equation 3.2 is satisfied. So, S was indeed feasible, therefore they choose to be in S and we have an equilibrium.

On the other hand, suppose that S is not feasible. Moreover assume $m(R_2) > 1 - \mu$, where $m(R_2)$ denotes the measure of the set R_2. In this case, whenever $B = R$, then $Y = 0$. Thus, the revenue function is $G_i = \varphi K_i - D_i$(in other words, the residual value).So, S is indeed not feasible and every entrepreneur i is indifferent about her debt structure[13]. The first order condition of the problem now imply

$$K_i : \left[\int_0^1 Z_j^{\alpha} dj\right]^{\frac{1-\alpha}{\alpha}} Z_i^{\alpha-1} f'(K_i) - (1 + r_i) = 0$$

[13]To see that substitute $G_i = \varphi K_i - D_i$ in Equation 3.5 and notice it becomes identical to 3.6

By symmetry among entrepreneurs, we have: $Z_i = Z_j$. This condition and the lenders supply of funds give the solution for the debt market:

$$r = \frac{\pi}{1-\pi} \frac{V'(W-(1-\varphi)K)}{V'(W+rK)}(1-\varphi) \tag{3.7}$$

$$f'(K) = 1 + r \tag{3.8}$$

Equations 3.7 and 3.8 define the second equilibrium in the economy. Notice there is an investment level lower than $\bar{K}$.

Proposition 1 *There are two equilibria in the model:*

- $m(S) = 1$, $D \leq \tilde{D}$
- $m(R_2) > 1 - \mu$, $D \geq \varphi K$

Proof.

See Appendix. ■

Proposition 1 shows the two equilibria in the model. In the chart below equilibria are characterized: the respective interest rates, utility levels, debt amounts and investment amounts in each of the two equilibria. In the Appendix, we show that there are no other equilibrium.

Characterization of equilibria

Equilibrium E0

$$r = r_S = r_L = 0$$

$$f'(K) = 1$$

$$D \leq \tilde{D}$$

$$U(G(K) - K)$$

$$V(W)$$

Equilibrium E1

$$r = \frac{\pi}{1-\pi} \frac{V'(W - (1-\varphi)K)}{V'(W + rK)} (1-\varphi)$$

$$r_S = \frac{r}{1-\varphi} \left(1 - \varphi \frac{K}{D}\right)$$

$$r_L = \frac{r}{1-\varphi}$$

$$f'(K) = 1 + r$$

$$D \geq \varphi K$$

$$(1-\pi)U(G(K) - (1+r)K)$$

$$(1-\pi)V(W + rK) + \pi V(W - (1-\varphi)K)$$

3.2 Equilibria and welfare comparisons

We have seen that when deciding the maturity structure, there exists multiple equilibria: the project can be financed with long term debt (E0) or with short term debt ($E1$)[14]. We now turn to answer the question: which equilibrium does each agent prefer? It should be obvious to see that entrepreneurs prefer the equilibrium with long term debt ($E0$). The project will be completed with probability one, it will generate resources, interest rates are zero and there is no default.

But, what do lenders prefer? One can think that they should also prefer equilibrium $E0$. There is no possibility of crisis, and investors always seek to avoid crises. Don't they? Let us compare the perceived utility in both equilibria. In equilibrium $E0$, when they invest only in long term debt, lenders have only one investment opportunity: both long term debt and the risk free technology pay 1. On the other hand, when they finance the project investing in short term debt, lenders are investing in two assets: Short term debt (K) and risk free technology ($W-K$). But notice, that they can always replicate their investment in $E0$, by fully investing their wealth in the risk free technology: they *optimally* choose not to. Therefore, their utility must be higher in $E1$.

In particular, the perceived utility of the lenders in the case of long term financing ($E0$) is $V_0 = V(W)$ while in the case of $E1$ is $V_1 = (1-\pi)V(W+$

[14]Remember we have assumed without loss of generality that $D = 0$ in $E0$ and $D = K$ in $E1$

$rK) + \pi V(W - (1 - \varphi)K)$. It should be clear that $V_1 > V_0$: otherwise they could always set $K = 0$ and thus $V_1 = V_0$. Proposition 2 states this point.

Proposition 2 *Lenders prefer to finance the investment projects with short term debt. In other words, they prefer Equilibrium* $E1$ *to Equilibrium* $E0$.

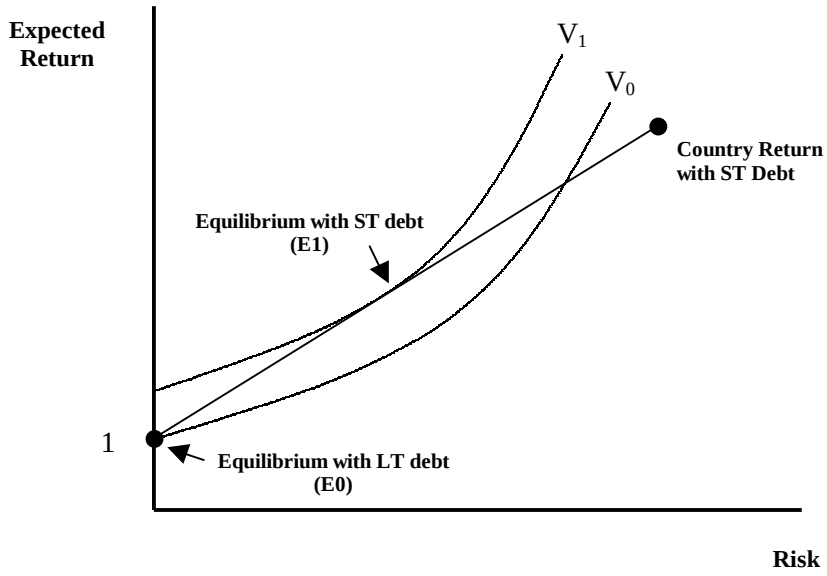
Proof.

See Appendix. ■

Figure 3.2 gives an intuition for this result. We plot an expected return - risk graph and compare the utility in both equilibria. V_0 denotes the utility perceived by lenders when investing long term. Clearly, this type of debt bears no risk and thus the equilibrium point lies on the vertical axis. When one includes short term debt (with a high return and high risk), the average return will be the point marked as return with short term debt. Then, V_1 would be the utility of the lenders when investing K in short term and $W - K$ in the safe technology. It is easy to see that $V_1 > V_0$.

Why are lenders better off in an equilibrium where crises are possible? Notice that in equilibrium $E0$, given that $W > K$ (i.e. resources are not scarce) lenders cannot extract any surplus from the entrepreneurs. Why? If lenders wish to charge more than 1 on the debt, the extra $W - K$ will immediately flow to the project and thus, the equilibrium rates will again be the risk free ones.

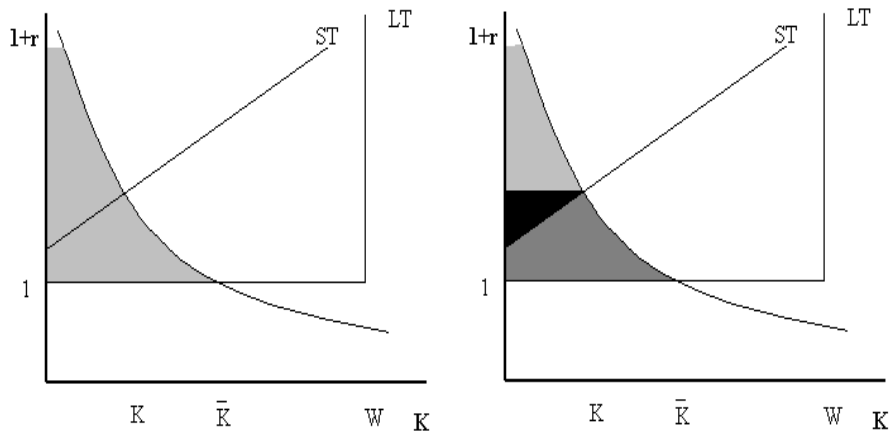
Figure 3.2: Lenders prefer financing projects with Short Term Debt



On the other hand, when they finance the project with short term debt, lenders have optimally chosen to place K in the project and $W - K$ in the risk free technology, although the latter pays strictly less than the project. Why don't these resources flow into the project? Clearly, the reason is that the project now bears risk. Thus, the risk of a crisis is acting as an "entry barrier" preventing other lenders from placing their resources in the entrepreneurs risky project. This allows lenders to *extract surplus* from the entrepreneur making this equilibrium more attractive.

It is important to notice that lenders prefer the equilibrium debt level

Figure 3.3: Increase in Lenders Surplus and Dead-Weight Loss



where a crisis is *possible* ex ante, but clearly they want the crisis *not* to happen ex post.

Figure 3.3 shows another interpretation of this result. The line $f'(k)$ is the demand of funds of entrepreneurs and the lines ST and LT are the supply of funds by the lenders. As in a basic supply and demand model, we can calculate and compare surplus from each agent. In the graph on the left, the equilibrium when the project is financed with long term debt, all the surplus goes to the entrepreneurs (gray area). On the other hand, the graph on the right shows the equilibrium when the project is financed with short term debt. Here the surplus of entrepreneurs is clearly reduced but, there is positive surplus for the lenders. Thus, lenders prefer this equilibrium.

What can we say from a social point of view? It should be straightforward to see that equilibrium $E0$, i.e. the one with long term debt is socially optimal. On one hand, in the first scenario, the project takes place and it generates new resources with probability 1, while in the second it only generates resources with probability $1 - \pi$: for the latter case, a planner intervention could improve both agents by reallocation of resources. Moreover, notice that in equilibrium $E0$ the invested capital equals the optimal level of the entrepreneurs. On the other hand, when the project is financed with short term debt, K is smaller than the optimal level of capital, even though lenders have available resources: in this case, there in *under-investment* in the economy.

Figure 3.3 also shows this point. The graph on the left shows the total surplus of the project when financed by long term debt (the gray area). On the other hand, the graph on the right shows the surplus when the project is financed by short term debt. There is a significant dead-weight loss (dark gray area) due to the possibility of crises and the corresponding reduction in investment. Clearly, from the surplus areas we can see long term debt is socially preferred.

3.3 Departing from competitive bond markets

The model presented in the previous sections has two equilibria. One is preferred by the lenders ($E1$) and the other preferred by the entrepreneurs ($E0$). Unless we depart from the competitive bond market, we cannot say much more. In particular, we cannot say which of the two equilibria actually occurs.

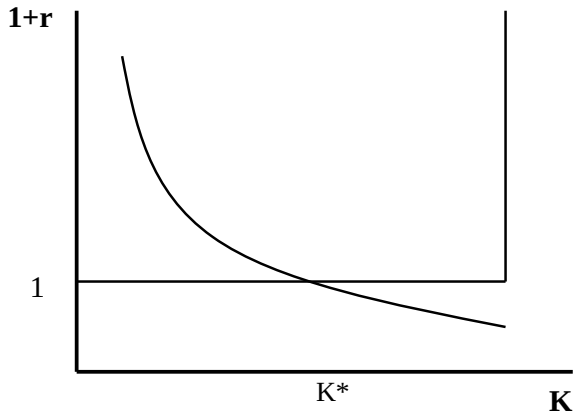
Coordinated entrepreneurs

Let us first assume that entrepreneurs can coordinate to assure their preferred outcome. How do entrepreneurs coordinate? The natural way would be through the government. The government can impose taxes on short term debt, or can act as a lender of last resort.

Coordinated entrepreneurs would maximize their utility subject to Equation 3.4 to Equation 3.6. Clearly, the choice of entrepreneurs is to be in S, i.e. to run *safe* projects.

Note that despite coordination, the outcome is the same as in the competitive bond market equilibrium. This is due to the perfectly elastic supply of funds under long term financing. The equilibrium is showed in figure 3.4.

Figure 3.4: Coordinated Entrepreneurs choose long term financing

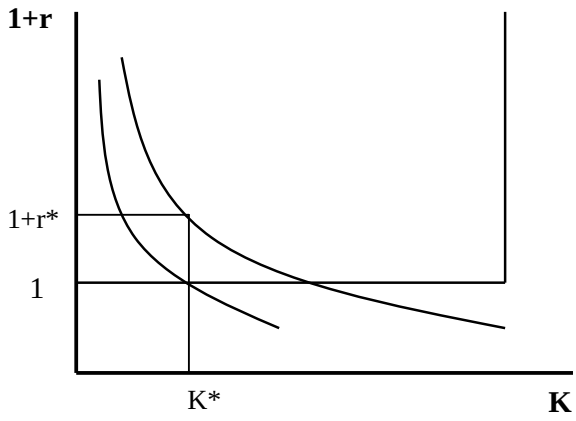


Coordinated lenders

The first way of coordination that one would think of is perfect coordination. Lenders will aim to extract monopoly profits from entrepreneurs. The outcome in this scenario would be long term borrowing (no risk) and they would charge an interest rate higher than the world's risk free. Moreover, the capital inflows would be very low. Formally, lenders maximize their problem (See Section 3.1.3) subject to entrepreneurs' demand $f'(K) = 1 + r$. The equilibrium is showed in figure 3.5.

It is easy to see that this is the classic cartel problem of coordination. Unless there is some technology to avoid defection, this outcome will not be sustained in equilibrium: lenders have incentives to deviate. Since each lender is negligible and does not affect the market interest rate, she will de-

Figure 3.5: Perfect coordination of lenders induce monopoly interest rates and under-investment



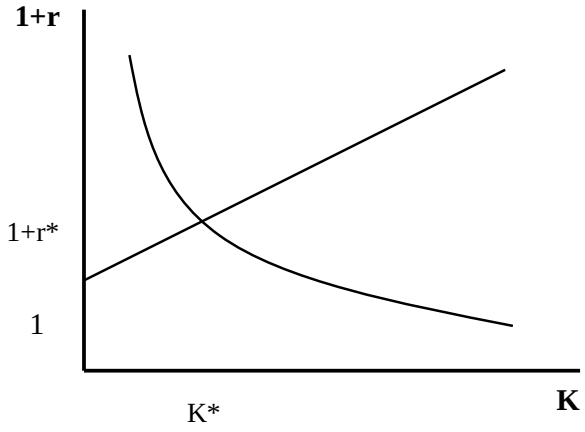
cide to invest more than the monopoly quantity.

Without a technology to deter defection, the coordination itself should assure that lenders will not deviate. This implies that the contract has to consider not only entrepreneurs demand but also that lenders cannot do better lending directly to entrepreneurs. Therefore, the contract should yield a Nash Equilibrium.

The only Nash equilibria in this model are the ones obtained in the previous sections: one with short term debt ($E1$) and the other with long term debt ($E0$). Since coordinated lenders that can not avoid defection should choose a Nash equilibrium, they choose the one in which they are better off:

$E1$. See figure 3.6.

Figure 3.6: Coordination of lenders induce short term investment



We think that this scenario resembles the late 90's financial markets situation (high levels of short term debt). However, it is difficult to think of a coordination as described above. On the other hand, we did observed that a significant fraction of the Asian flows came through financial intermediaries, such as hedge funds. We will present a model of optimal debt maturity with a hedge fund and show that it replicates the imperfect coordination result.

3.3.1 Equilibrium with hedge funds

Nowadays, most international investments are made through some type of financial intermediary, e.g. investment banks, hedge funds, etc. In particular,

several authors have stressed the importance of such agencies in triggering some recent crises such as the East Asian one.

Through a hedge fund, lenders will be able to guarantee the existence of the equilibrium *preferred* by them ($E1$).

A hedge fund will be an agency that offers a particular *contract* to the lenders. A contract is composed of four items: a return rate r, an investment amount K, the probability of repayment q[15], and a fee charged by the agency for managing the funds λ. In other words, define a contract as $\mathbf{H_j} = \{r_j, K_j, q_j, \lambda_j\}$.

The timing is similar to the one in the competitive bond market. However, there is an extra period: $T = 0$ where hedge funds compete. Without loss of generality, we will assume there are two hedge funds competing for funds à la Bertrand.

- At $T = 0$, hedge funds compete to attract lenders' resources fully specifying all characteristics of contracts.
- At $T = 1$, hedge funds and entrepreneurs negotiate the debt structure[16]
- At $T = 2$, lenders decide to roll over or not the loans.

[15]Choosing q is equivalent to choose between long term debt ($q = 1$) or short term debt ($q = 1 - \pi$).

[16]Notice that, since the contract was already fully specified in period 0 by the hedge fund and the lenders, the entrepreneur basically has no decision to take in this period. She can only decide to take or to reject the hedge fund's offer.

- At $T = 3$, production takes place and debts are repaid.

In order to attract lenders to invest through the hedge funds, these will necessarily offer contracts in which lenders have no incentives to deviate. In addition, they should meet entrepreneur's demand of capital that will be given by $f'(K) = 1 + r$. Thus, hedge funds will only offer contracts that satisfy either $E0$ or $E1$.

Notice that to run or not is *not* a decision variable for the hedge fund, i.e. its existence does not avoid the possibility of a self fulfilling crisis[17]. However, given that it is (fully) leveraged, and that the investors can always ask their money from the fund, whenever investors run on the hedge fund, the latter has to run on the entrepreneurs.

Formally, a particular hedge fund ($\mathbf{H}_1$) solves the following problem:

$$\max \begin{cases} q\lambda_1[rK] & , \quad \mathbf{H}_1 \succ_L \mathbf{H}_2 \\ q\lambda_1[rK/2] & , \quad \mathbf{H}_1 \sim_L \mathbf{H}_2 \\ 0 & , \quad \mathbf{H}_1 \prec_L \mathbf{H}_2 \end{cases}$$

s.t.

$$f'(K) = 1 + r$$

$$V_{\mathbf{H}_1} = \begin{cases} (1-\pi)V(W + (1-\lambda_1)rK) + \pi V(W - (1-\varphi)K) & , \quad q = 1 - \pi \\ V(W) & , \quad q = 1 \end{cases}$$

[17] In other words, we are assuming an *open-end* fund.

$$r \quad = \quad \begin{cases} \frac{\pi}{1-\pi}\frac{V'(W-(1-\varphi)K)}{V'(W+(1-\lambda_1)rK)}(1-\varphi) & , \quad q = 1-\pi \\ 0 & , \quad q = 1 \end{cases}$$

where $\mathbf{H}_1 \succ_L \mathbf{H}_2$ means lenders prefer (get higher utility) contract of hedge fund 1 than contract of hedge fund 2. Hedge funds are constrained by the entrepreneurs' demand and the supply of funds by lenders[18].

Clearly, the competition among hedge funds will drive the fee to zero. Moreover, it will imply the occurrence of Equilibrium $E1$.

Proposition 3 *Assume there is free entry of hedge funds in the economy. Hedge funds do not charge any fee for managing the funds, i.e.* $\lambda = 0$. *Moreover, the unique equilibrium is:* $, D = K < \bar{K}$ *and* $r = \frac{\pi}{1-\pi}\frac{V'(W-(1-\varphi)K_i)}{V'(W+rK_i)}(1-\varphi)$. $(E1)$

Proof.

See Appendix. ■

Obviously, the result of a zero transaction fee is not a realistic result. In reality, hedge funds do charge a fee for their operations. This happens basically because of imperfect information. Given that hedge fund has some information that is not accessible (or accessible at a high cost) to individual lenders, the latter are willing to pay some kind of fee. Another explanation

[18]Remember that lenders can have one of two supply schedules (See Equations 3.4 and 3.6) depending on the probability q, which in turn depends on the debt maturity.

would be that through the hedge fund, lenders can exploit increasing returns, sharing a possible high fee among many agents.

3.4 Concluding remarks

This paper presents a model of optimal debt maturity. If long term projects are financed with short term debt there are possibilities of (*self-fulfilling*) liquidity crises. These crises cause very much damage to the global economy, specially to emerging market entrepreneurs, the borrowers. This is precisely the reason that makes so puzzling the existence of short term debt. If borrowers suffer from it, why don't they avoid it?

The assumption of production complementarities is the novelty feature of the model. This assumption destroys the monopoly power that borrowers have on their risk of liquidity. If enough firms are financed with short term debt, the possibility of a liquidity crisis will affect the whole economy, including the firms with long term borrowing.

In this paper, we have shown a situation in which short term borrowing can be an equilibrium. Nevertheless, the assumptions of the paper do not exclude the possibility for long term borrowing as an equilibrium. Hence, we have multiple equilibria. The natural question to ask is *What are the welfare effects of each equilibrium?*

Long term borrowing ensures projects are completed and that the optimal amount of capital is invested. This is a Pareto efficient allocation. But, precisely for this reason, lenders will compete to invest their money, reducing the interest rates and, therefore, destroying any possibility to get part of the surplus the projects generate.

On the other hand, short term lending may produce the abandonment of profitable projects because of expectational shocks. This will produce lower investment and higher interest rates. Clearly this is not efficient and reduces borrowers welfare. But, surprisingly it improves lenders welfare. Short term borrowing has risk but has a higher return as well. Therefore, investing in short term debt enlarges the menu of assets for international investors.

The final part of the paper shows a possible rationale for the excessive short term debt observed in emerging markets. Given the positive effect on the foreign investors, they will try to coordinate in order to achieve the short term equilibrium. Thus, we model a different view of financial markets, where there is more than only competitive bond markets, there are financial intermediaries that compete with each other to manage lenders' money. Once this stage is over, they negotiate with entrepreneurs. The result is that only short term debt can be an equilibrium.

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Appendix A

Data and Optimal Portfolios of Chapter 2

A.1 Data

The data used in this paper comes from OECD Statistics and the book INVESTMENTS by Bodie, Kane and Marcus:

Hourly Earnings for OECD countries Hourly earnings correspond to seasonally adjusted average total earnings in manufacturing and private sector paid per employed person per hour, including overtime pay and regularly recurring. The Data used is for United States, Japan and United Kingdom. The data is on quarterly frequency and the period is from 1970 to 2004, except for United Kingdom that is from 1990 to 2004.

Share Prices for OECD Countries and Major Non-Member Economies

Share prices data refer most frequently to "all shares". Monthly data are averages of daily quotations, quarterly and annual data are averages of monthly figures. The Data used is for United States, Japan and United Kingdom. The data is on quarterly frequency and the period is from 1970 to 2004.

Exchange Rates for OECD Countries and Major Non-Member Economies The exchange rates present daily averages of spot rates quoted for the US dollar on national markets expressed as national currency unit per US dollar. The Data used is for United Stares, Japan and United Kingdom. The data is on quarterly frequency and the period is from 1970 to 2004.

Consumer Price Indices (CPI) for OECD Countries Consumer Prices Indices (CPI) are a general measure of inflation. The Data used if or United States, Japan and United Kingdom. The data is on quarterly frequency and the period is from 1970 to 2004.

Global Equity Market Capitalization and Global Portfolio Shares, 1997 Data from INVESTMENTS, Bodie, Kane and Marcus, 1999. The Data used is for United States, Japan and United Kingdom.

The weights of each country in world markets are:

U.S 47.84%

Japan 12.93%

U.K. 9.28%

A.2 Optimal portfolios

This section presents the approximation to obtain the optimal portfolio of section 2.3.1.

$$X = \frac{1}{2}\left[A - Cov\left(\frac{w}{p}L, r\right)/Var(r)\right]$$

$$h = \frac{1}{2}\left[1 - \frac{\frac{1}{A}Cov\left(\frac{w}{p}L, rK\right)\frac{1}{K}}{Var(rK)\frac{1}{K^2}}\right]$$

$$h = \frac{1}{2}\left[1 - \frac{Cov\left(Lf_L(K,L)Z, Kf_K(K,L)Z\right)}{Var(Kf_K(K,L)Z)}\right]$$

When the only relevant shock is the supply (technology) we get:

$Var(M) \to 0$

$$h = \frac{1}{2}\left[1 - \frac{Lf_L(K,L) * Kf_K(K,L)Var(Z)}{(Kf_K(K,L))^2 Var(Z))}\right]$$

$$h = \frac{1}{2}\left[1 - \frac{Lf_L(K,L)}{Kf_K(K,L))}\right]$$

Thus, when the labor income is more than 2 times the capital income, we should observe that investors short sell domestic capital.

When the only relevant shock is the demand (money) we get:

$Var(Z) \to 0$

For a first order Taylor approximation:

$$h = \frac{1}{2}\left[1 - \frac{Cov\left(\bar{L}\bar{f}_L + \bar{f}_L * (L - \bar{L}) + \bar{L}\bar{f}_{LL} * (L - \bar{L}), K\bar{f}_K + K\bar{f}_{KL} * (L - \bar{L})\right)}{Var(K\bar{f}_K + K\bar{f}_{KL} * (L - \bar{L}))}\right]$$

$$h = \frac{1}{2}\left[1 - \frac{(\bar{f}_L + \bar{L}\bar{f}_{LL}) * Var(L)}{K\bar{f}_{KL} * Var(L)}\right]$$

Given that, $Kf_{KL} = Kf_{LK}$=$Kf_{LK} = -Lf_{LL}$, we obtain:

$$h = \frac{1}{2}\left[1 + \frac{\bar{f}_L + \bar{L}\bar{f}_{LL}}{L\bar{f}_{LL}}\right]$$

$$h = \frac{1}{2}\left[1 + 1 - \eta\right]$$

Thus, for inelastic labor demands we will observe a home bias in the international portfolios.

$$h = 1 - \frac{\eta}{2}$$

Appendix B

Proofs Propositions Chapter 3

Proof of Proposition 1

We will prove this result in three steps:

First, assume that $\{[0,1] - \{i\}\} \subseteq S$, then $\forall j \neq i,\ \ Z_j > 0$. Moreover, $K_j = \bar{K}$. Thus, given that $\forall i,\ \ K_i \leq \bar{K},\ \ f(K_i) > K_i$, we have:

$$G(K_i) = \frac{1}{\alpha}[f(\bar{K})]^{1-\alpha}[f(K_i)]^{\alpha} > \frac{1}{\alpha}\bar{K}^{1-\alpha}K_i^{\alpha} > K_i$$

Then, $\tilde{D}_i$ is strictly positive. Hence, S is feasible for entrepreneur i and it will be her choice. Therefore, $S = [0,1]$ is an equilibrium.

Second, assume that $m(R_2) > 1 - \mu$, then whenever $B = R$, the revenue of each firm is its residual value.

$$G(K_i - (1 + r_{Si})\tilde{D}_i) = \varphi K_i - (1 + r_{Si})\tilde{D}_i = (1 + r_i)K_i$$

Which implies

$$\tilde{D}_i = \frac{\varphi - 1 - r_i}{1 + r_{Si}}K_i < 0$$

Therefore, S is not feasible for any entrepreneur. Moreover, the interest rates are determined by equations 3.5 and 3.6. But notice, whenever $i \in R_1$, $G_i = \varphi K_i - D_i$, then both equations are equivalent. Therefore entrepreneurs are indifferent between being in R_1 or R_2. Thus, $m(R_2) > 1 - \mu$ is an equilibrium.

Finally, we have to show that there are no more possible equilibria. Note the problem is symmetric, then if S is not feasible for some agents it must be not feasible for every entrepreneur. Thus, we have to check if there are no equilibria whenever S is not feasible and $m(R_2) \leq 1 - \mu$. In this case, the problem of an entrepreneur in R_1 is:

$$\begin{aligned} &\max_{K,D} \{(1-\pi)U(G(K) - (1+\rho(K,D))K\} \\ &\quad s.t. \\ &\rho(K,D) = \frac{\pi}{1-\pi}\frac{V'(C_R)}{V'(C_\emptyset)}\left[\frac{K - D - G(K - D/\varphi)}{K}\right] \end{aligned}$$

The first order conditions for this problem are:

$$\begin{aligned} D: &\quad -\rho_D K \leq 0, \quad D \geq 0 \\ K: &\quad G'(K) - 1 - \rho(K,D) - \rho_K K = 0 \end{aligned}$$

Thus, $D = 0$ and $G'(K) = 1$ is the solution of this problem. Then, If everybody follow this policy we have:

$$G'(K) = f'(K) = 1 \Rightarrow K = \bar{K}$$

This implies $\rho < 0$. This is a contradiction because lenders can always do better investing in the safe technology and hence, there are no equilibria under these conditions. ■

Proof of Proposition 2

Computing a second order Taylor expansion for the lender's utility we have:

$$\begin{aligned} V(W) = \quad & (1-\pi)V(W+rK) + \pi V(W-(1-\varphi)K) + \\ & \Big[(1-\pi)V'(W+rK)r - \pi V'(W-(1-\varphi)K)(1-\varphi) \Big] (0-K) + \\ & \frac{1}{2} \Big[(1-\pi)V''(W+r\tilde{K})r^2 + \pi V''(W-(1-\varphi)\tilde{K})(1-\varphi)^2 \Big] (0-K)^2 \end{aligned}$$

Where, $\tilde{K} \in (0, K)$. Notice that the lender's first order condition is:

$$(1-\pi)V'(W+rK)r - \pi V'(W-(1-\varphi)K)(1-\varphi) = 0$$

Thus, the problem becomes:

$$\begin{aligned} & V(W) - (1-\pi)V(W+rK) - \pi V(W-(1-\varphi)K) \\ & = \Big[(1-\pi)V''(W+r\tilde{K})r^2 + \pi V''(W-(1-\varphi)\tilde{K})(1-\varphi)^2 \Big] (0-K)^2 < 0 \end{aligned}$$

And, clearly, this implies:

$$V(W) < (1-\pi)V(W+rK) + \pi V(W-(1-\varphi)K)$$

And thus $V_0 < V_1$. ■

Proof of Proposition 3

Let's solve this in two parts.

First, suppose one of the hedge funds offer to lenders $q = 1$, then the reaction of the other hedge fund will offer $q = \pi$ and will charge the lenders the increase in utility minus a small fraction. Therefore, it is not an equilibrium to offer $q = 1$.

Second, given that $q = \pi$, the only variable to choose for the hedge funds are the fees that they will charge to the lenders. Clearly, because of the assumption of constant (zero) marginal cost, Bertrand competition will imply a zero fee. ■