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DOCTORAL THESIS

Ph.D. Thesis:

Design, management, and simulation of mixed station-based and free-floating vehicle-sharing systems

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Doctoral Candidate:

Enrique Jiménez Meroño

Thesis Supervisor:

Francesc Soriguera Martí

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Abstract

This thesis provides tools, based on mathematical modelling, for the analysis, optimization and simulation, of vehicle-sharing systems. The thesis addresses various implementation levels of such systems, from their planning and design to their operational management.

Specifically, the problems addressed can be grouped into two blocks. The first block focuses on the strategical design of vehicle-sharing systems. This is, the long-term planning and the sizing of the fundamental elements of the system, namely: the fleet size, the required infrastructure, and the personnel. The second block, analyzes the day-to-day operational problems of systems already in operation and whose main design characteristics are known. Specifically, the optimization problem in the repositioning task assignment, is addressed.

The thesis develops solutions for the aforementioned problems trying to find an adequate balance between the mathematical complexity of the models and the level of detail and accuracy of the solutions. Parsimonious solutions are sought, aiming at facilitating its implementation in real systems with limited resources. Parsimonious models require modelling simplifications. This is the price to pay when modelling complex and advanced systems. This modelling philosophy has guided the most significant contribution of this thesis: the design and optimization models depicting mixed vehicle-sharing systems, with both free-floating and station-based layouts working simultaneously and complementing each other. The modelling of such systems represents an important milestone in the research on the strategical planning of vehicle-sharing systems.

For the strategical design problem, simplification has been achieved by using the method of continuous approximations. This allows the estimation of the costs and the causal effects of the decision variables in the system, without the need to solve computationally costly models. This simplification has provided sufficiently accurate and robust results, and has also allowed the development of a hitherto unexplored model for mixed vehicle-sharing systems, in which free-floating and vehicles in stations are used indistinctly. The optimization of this model and its subsequent analysis of results tells us under which circumstances it is more convenient to opt for a free-floating configuration, for a station-based one, or for a mixed system.

Regarding the day-to-day operational problems in the second block of the thesis, simplification is achieved by avoiding the common optimal routing solutions, based on mixed integer linear programming models. The proposed approach in the thesis is based on the optimal real-time pairwise matching of tasks to resources. The idea is to obtain a strategy less dependent on demand forecasts, since the assignment occurs in real time. This avoids the estimation of the future vehicle inventory level at stations. Results obtained by simulation show that the real-time optimal pairwise assignment strategy generally works better, unless the accuracy of the predictions is extremely high, which typically is unlikely. These results have been obtained thanks to the ad-hoc development of an agent-based simulator for vehicle-sharing systems, able to emulate the complex operation of mixed systems, with vehicles both free-floating and in stations, including the possibility of using electric vehicles. The simulator also allows to establish different priority levels for repositioning operations (e.g. recharging and relocation). The development of this simulator culminates the second block of the thesis.

With all this, the thesis presents a complete framework of tools which will help in the optimal design and operation of mixed vehicle-sharing systems, being the first time that this kind of systems are addressed at this level in the academic literature.

Resumen

Esta tesis provee de herramientas, basadas en la modelización matemática, para el análisis, optimización y simulación de sistemas de vehículos compartidos. La tesis trata varios niveles de implementación de estos sistemas, desde su fase de planificación y diseño hasta su gestión operativa.

Concretamente, los problemas tratados se pueden agrupar en dos bloques. El primer bloque pone foco en el diseño estratégico de sistemas de vehículo compartido. Es decir, la planificación a largo plazo y dimensionamiento de sus elementos fundamentales: flota, infraestructura, y personal. El segundo bloque considera los problemas operacionales del día a día de sistemas ya puestos en funcionamiento y cuyos principales elementos son conocidos. Específicamente, se trata el problema de la asignación óptima de tareas de reposicionamiento.

La tesis desarrolla soluciones para los problemas mencionados tratando de encontrar un equilibrio adecuado entre la complejidad matemática del modelo y el nivel de detalle y acierto de las soluciones. Se han utilizado modelos parsimoniosos con el objetivo de facilitar su implementación en sistemas reales con recursos limitados. Los modelos parsimoniosos requieren simplificaciones como precio a pagar al modelizar sistemas complejos y avanzados. Pero a cambio esta filosofía de modelización ha guiado hacia la contribución más significativa de la tesis: la introducción de modelos de diseño y optimización para sistemas mixtos de vehículos compartidos, en los cuales se contemplan tanto vehículos de flota libre como en estaciones y que funcionan indistintamente y se complementan. La modelización de estos sistemas mixtos supone un importante logro en la investigación de la planificación estratégica de sistemas de vehículos compartidos.

En cuanto al diseño estratégico del sistema, la simplificación se ha conseguido implementando la metodología de las aproximaciones continuas. Esto permite la estimación del coste y efectos de las variables de decisión del sistema sin necesidad de resolver modelos computacionalmente costosos. Esta simplificación ha ofrecido resultados suficientemente exactos y robustos, y además nos ha permitido desarrollar un modelo de diseño hasta ahora inexplorado para sistemas mixtos de vehículo compartido, en el que vehículos de flota libre y en estaciones se usan indistintamente. La optimización de ese modelo y su posterior análisis de resultados nos indica bajo qué circunstancias es más conveniente optar por un sistema de flota libre, con estaciones, o mixto.

Con respecto a los problemas de operativa diaria del segundo bloque de la tesis, la simplificación se consigue evitando las soluciones comunes de optimización de rutas basadas en modelos de programación mixta lineal i entera. La solución propuesta en la tesis se basa en el emparejamiento óptimo en tiempo real de tareas y recursos. La idea es obtener una estrategia menos dependiente de las predicciones de demanda. Esto evita la estimación del nivel de inventario de las estaciones a futuro. Los resultados obtenidos mediante simulación muestran que la estrategia de emparejamiento óptimo funciona mejor por regla general, salvo que el acierto de las predicciones sea altísimo, lo cual es poco frecuente. Estos resultados han sido obtenidos gracias al desarrollo ad-hoc de un simulador basado en agentes de sistemas de vehículo compartido, capaz de emular las operaciones complejas de los sistemas mixtos, e incluyendo la posibilidad de usar vehículos eléctricos. El simulador también ha permitido establecer distintos niveles de prioridad para las operaciones de reposicionamiento (p.ej. recarga y recolocación). Este simulador culmina el segundo bloque de la tesis.

Con todo esto, esta tesis presenta un marco completo de herramientas que ayudarán en la optimización del diseño y operación de sistemas mixtos de vehículos compartidos, siendo la primera vez que este tipo de sistemas son tratados a este nivel en la literatura académica.

Resum

Aquesta tesi proveeix d'eines, basades en la modelització matemàtica, per a l'anàlisi, optimització i simulació de sistemes de vehicles compartits. La tesi tracta diversos nivells d'implementació d'aquests sistemes, des de la seva fase de planificació i disseny fins a la seva gestió operativa.

Concretament, els problemes tractats es poden agrupar en dos blocs. El primer bloc posa focus en el disseny estratègic de sistemes de vehicle compartit. És a dir, la planificació a llarg termini i dimensionament dels seus elements fonamentals: flota, infraestructura, i personal. El segon bloc considera els problemes operacionals del dia a dia de sistemes ja posats en funcionament i els principals elements del qual són coneguts. Específicament, es tracta el problema de l'assignació òptima de tasques de reposicionament.

La tesi desenvolupa solucions per als problemes esmentats tractant de trobar un equilibri adequat entre la complexitat matemàtica del model i el nivell de detall i encert de les solucions. Per a trobar aquestes solucions, s'han utilitzat models parsimoniosos amb l'objectiu de facilitar la seva implementació en sistemes reals amb recursos limitats. Els models parsimoniosos requereixen simplificacions com a preu a pagar al modelitzar sistemes complexos i avançats. Però a canvi aquesta filosofia de modelització ha guiat cap a la contribució més significativa de la tesi: la introducció de models de disseny i optimització per a sistemes mixtos de vehicles compartits, en els quals es contemplen tant vehicles de flota lliure com en estacions i que funcionen indistintament i es complementen. La modelització d'aquests sistemes mixtos suposa un important assoliment en la recerca de la planificació estratègica de sistemes de vehicles compartits.

Per al cas de l'optimització del disseny estratègic del sistema, la simplificació s'ha aconseguit implementant la metodologia de les aproximacions contínues. Això permet l'estimació del cost i efectes de les variables de decisió del sistema sense necessitat de resoldre models computacionalment costosos. Aquesta simplificació ha ofert resultats prou exactes i robustos, i a més ens ha permès desenvolupar un model de disseny fins ara inexplorat per a sistemes mixtos de vehicle compartit, en el qual vehicles de flota lliure i en estacions s'usen indistintament. L'optimització d'aquest model i la seva posterior anàlisi de resultats ens indica sota quines circumstàncies és més convenient optar per un sistema de flota lliure, amb estacions, o mixt.

Respecte als problemes d'operativa diària del segon bloc de la tesi, la simplificació s'aconsegueix evitant les solucions comunes d'optimització de rutes basades en models de programació mixta lineal i sencera. La solució proposada en la tesi es basi en canvi en l'aparellament òptim en temps real de tasques i recursos. La idea és obtenir una estratègia menys dependent de les prediccions de demanda. Això evita la necessitat d'estimar el nivell d'inventari de les estacions a futur. Els resultats obtinguts mitjançant simulació mostren que l'estratègia d'aparellament òptim funciona millor per regla general, tret que l'encert de les prediccions sigui altíssim, la qual cosa és poc freqüent. Aquests resultats han estat obtinguts gràcies al desenvolupament ad hoc d'un simulador basat en agents de sistemes de vehicle compartit, capaç d'emular les operacions complexes dels sistemes mixtos, i incloent la possibilitat d'usar vehicles elèctrics. El simulador també ha permès establir diferents nivells de prioritat per a les operacions de reposicionament (p. ex. recàrrega i recol·locació). Aquest simulador culmina el segon bloc de la tesi.

Amb tot això, aquesta tesi presenta un marc complet d'eines que ajudaran en l'optimització del disseny i operació de sistemes mixtos de vehicles compartits, sent la primera vegada que aquest tipus de sistemes són tractats a aquest nivell en la literatura acadèmica.

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Contents

Abstract ii

Resumen iii

Resum iv

Agradecimientos v

Introduction 10
1. Vehicle-sharing: concept, classification, and main definitions 10
1.1. Evolution of vehicle-sharing. The dominance of one-way services. 11
1.2. Characteristics of vehicle-sharing systems 13
1.3. Station-based vs. Free-floating 14
1.4. Operations management: repositioning 15
2. Current state of the art and literature review 16
2.1. Demand estimation and profile characterization 17
2.2. Strategical and tactical design 18
2.3. Operative strategies: agency-based 19
2.4. Operative strategies: user-based 20
2.5. Simulation tools development 20
2.6. Side effects of vehicle-sharing 21
3. Thesis outline 21
3.1. Objectives 21
3.2. Methodologies 22
3.3. Delimitations 23
3.4. Contributions 24
3.5. Thesis structure: the papers 24
References 29

Paper I: A continuous approximation model for the optimal design of public bike-sharing systems 32
1. Introduction to public bike-sharing systems 34
2. Literature review and modelling framework 36
3. Model definition 38
3.1. Model overview and decision variables 38
3.2. Demand modelling 38

3.3. Generalized cost function 40
3.4. Extension to electrical bikes 48
3.5. On the mathematical properties of the generalized cost function 49
4. The Barcelona's Bicing case study: parameter estimation, model validation and system optimization 49
4.1. Parameter estimation for the Bicing system 49
4.2. Model validation 53
4.3. Optimization of the strategical design of a bike-sharing system: application to the Barcelona's Bicing 54
4.4. Sensitivity analysis and generalized results 59
5. Conclusions and further research 62
References 64
Paper II: A continuous approximation model for the optimal design of mixed free-floating and station-based car-sharing systems 66
1. Introduction 68
2. Literature review 69
3. Model definition 71
3.1. Model overview and decision variables 71
3.2. Demand modelling 72
3.3. Modelling agency costs, ZA 74
3.4. Modelling user costs, ZU 78
3.5. Objective function: Maximum profit vs Minimum total costs 79
4. Barcelona case study: parameter estimation, scenarios definition and system optimization results 80
4.1. Parameter estimation for the Barcelona case study 80
4.2. Definition of optimization scenarios 81
4.3. Optimization results 82
4.4. Robustness of optimal results 86
4.5. Sensitivity analysis with respect to potential demand density 87
4.6. On the share between FF and SB in mixed car-sharing systems 88
4.7. Sensitivity analysis with respect to on-street parking costs 91
4.8. Sensitivity analysis with respect to the unitary labor cost of repositioning 91
4.9. Decentralized vs centralized battery charging 92
5. Conclusions and further research 93
6. Acknowledgements 96
7. Appendix A. Demand served by a mixed car-sharing system 98
7.1. Appendix A.1. Demand served by the FF part of the mixed car-sharing system, λFF 99

7.2. Appendix A.2. Demand served by the SB part of the mixed car-sharing system, λSB 101
8. Appendix B. Demand parameter characterization from an origin/destination (O/D) trip demand matrix 102
9. Appendix C. Estimation of rebalancing rates 103
9.1. Appendix C.1. Baseline repositioning rate 103
9.2. Appendix C.2. Battery recharging repositioning rate for decentralized charging strategy 104
9.3. Appendix C.3. FF-SB compensation repositioning rate 104
9.4. Appendix C.4. Additional repositioning teams in centralized battery charging contexts, $kHUB$ 105
Paper III: Optimization of bike-sharing repositioning operations. A reactive real-time approach. 107
1. Introduction 109
2. Literature review 111
3. Preemptive versus reactive bike-sharing repositioning strategies 112
3.1. Common concepts 112
3.2. Real-time reactive pairwise task assignment optimization strategy 116
3.3. Preemptive routing optimization strategy 117
3.4. Mixed strategy 119
4. Case study in a simulated environment 119
4.1. Scenario definition in the simulation 120
4.2. Results 121
5. Conclusions 123
References 124
Paper IV: Forecasting demand at bike-sharing stations through machine learning techniques 126
1. Introduction and background 128
2. Machine learning methods to predict the inventory level at bike-sharing stations 129
2.1. Variables of the model 129
3. Machine learning regression methods 131
3.1. Random forest (RF) 131
3.2. Gradient booster (GB) 131
3.3. Artificial Neural Networks (NN) 132
4. Citi Bike NYC case of study 133
4.1. Effects of the time step duration in the prediction accuracy 135
4.2. Effects of spatial clustering in the prediction accuracy 136
5. Conclusions and further research 137
Paper V: Agent-based simulation of mixed vehicle-sharing systems 139
1. Introduction 141

2. Literature review 142
3. Agents in the simulation 145
3.1. Stations 145
3.2. Free-floating zones 145
3.3. Vehicles 146
3.4. Users 147
3.5. Repositioning teams 147
3.6. Auxiliary object types 148
4. Architecture of the simulator 148
4.1. Setup modules 149
4.2. Time step modules 151
4.3. Custom frequency modules 157
5. Input parameters to the simulator 158
6. Examples of application 160
7. Generalizability and scalability of the proposed simulation model 165
7.1. Simulating a different vehicle-sharing system (without modifying the agents' underlying behavioral rules) 165
7.2. Modifying the underlying behavioral rules of the active agents in the simulation. 166
7.3. Scalability and computational performance 166
8. Conclusions 167
References 168

Final remarks 169
1. Final remarks regarding the strategical design problem in the planning stage 169
2. Remarks about the repositioning problem in the operative stage 171
3. Future research lines for improvement 171

Introduction

Mobility services have experienced several changes during the last recent years, especially on the use of private vehicles for individual on-demand urban mobility. In particular, commercial vehicle-sharing services have grown exponentially. And nowadays they have become a popular transportation alternative in many cities around the world.

Academic research has not been immune to this trend. Many studies have been due to term, either from public or private initiative, addressing these systems under several points of view and focusing on different problems. This thesis is framed into these studies, and compiles five different works addressing the mathematical modelling, optimization, and simulation of vehicle-sharing systems, in order to provide tools for the proper design and management. All of these works are individual, but they have the common goal of obtain insights that allows to simplify the mathematical framework. And with that, to model more complex systems. In particular, the author proposes here the modelling of mixed station-based and free-floating vehicle-sharing systems, in which vehicles can be on stations or on street work simultaneously and complement to each other.

As far as the author is concerned, these modelling of mixed systems is a novelty in academic literature, and therefore represents the most important contribution of this thesis.

In this introduction chapter, a general overview of vehicle-sharing is provided, followed by an outline of the thesis, including summaries of objectives, findings, contributions, and results. Then, Paper I to Paper V are presented individually. Each one has its own focus, structure, methodology, and findings. And finally, the thesis ends with a global conclusions and discussion section, summarizing the most important findings, contributions, and discussing future developments and recommendations regarding mixed vehicle-sharing.

1. Vehicle-sharing: concept, classification, and main definitions

Mobility services can be split into two blocks according to the strategy for reducing trip costs: sharing the trip or sharing the vehicle. The trip-sharing block is based on the following concept: if more users make the same trip, the total cost of the trip will be split into more people, resulting in a lower cost. The main problem to design a system through this strategy is that users have different O/Ds and trip starting times, so they must be aggregated. That aggregation reduces the individual trip cost, but forces users to adapt their trips to the fixed routes, stops, and timetables. That results in a tradeoff between cost and flexibility. To this block belong scheduled public collective transport services and carpooling/ridesharing.

On the second block the tradeoff is different. In this case, there is no trip aggregation. All trips are individual trips. And the strategy lays in to chain as many individual trips as possible, to keep vehicles in use most of the time, since stopped vehicles decrease system efficiency and occupy a parking space. The problem here is that trips cannot be chained always without additional resources. If the destination point of one user is different than the origin point of the next one, the vehicle should be moved somehow to serve the second trip or force the second user to walk an additional access distance. Or if several users demand different trips at the same time, the fleet size must be increased or demand will be lost. This results in a tradeoff between operator cost and availability. Users will not need to adapt so much as with the previous strategy, but we should account that part of the demand cannot be serviced (on time, or ever). To this second block belong rentals, vehicles for hire with driver (e.g. taxis), and the mode addressed in this thesis: vehicle-sharing systems (car-sharing, bike-sharing, shared micromobility).

Understanding the previous distinction is fundamental to define and comprehend in a clear way the concept of vehicle-sharing: a transportation service based on short vehicle rentals for making individual trips. That way, a flexible on-demand transportation alternative is achieved with less costs than acquiring a private vehicle or than hiring a vehicle with its driver (e.g. taxis). In fact, vehicle-sharing can be understood as an evolution of rentals, in which the business model has been improved and adapted through new technologies in order to increase its flexibility and suitability for urban trips. Membership systems, geolocations, and mobile apps have eased a lot the process of search, reservation, and payment for the use, allowing users, for example, to spend less time in the renting process, making only one-way trips, or paying per minutes instead of an hourly or daily basis.

Based on the classification by Nansubuga & Kowalkowski (2021), we could find three types of vehicle-sharing systems:

- **Peer-to-peer vehicle-sharing (p2p)**. In these systems, users provide the fleet by sharing their own cars.
- **Corporate vehicle-sharing (B2B)**. In these systems, the customer are companies or communities, and services are only open to their members. In practice, these systems work as a long-term rental.
- **Commercial vehicle-sharing (B2C)**. These are the most common services and which are referred by default as vehicle-sharing systems. In B2C, the service is open to any user that registers.

B2C is the only one of the vehicle-sharing typologies that can be classified as a public non-collective transportation service. B2B services are not open to all users. And in p2p vehicle-sharing the offer is not controlled by the agency like in public transport systems. It should be instead classified as a shared-economy business. For these reasons, B2C vehicle-sharing is the object of study of this thesis in the field of transportation research.

1.1. Evolution of vehicle-sharing. The dominance of one-way services.

To understand the current characteristics of vehicle-sharing systems, it would be useful to have a look at the origin and evolution of these systems. This evolution has been different depending on the type of vehicle considered, but it has converged to similar system structures.

Bike-sharing has been historically an initiative of public administrations and municipalities in order to encourage people to use more sustainable transportation modes. The paradigms of the first generations of bike-sharing are good examples of this. Since the “White Bikes” in Amsterdam in the mid 60’s, to the “Bycyklen” system in Copenhagen in the 90’s, all were public-service oriented systems in which bikes were used almost for free. However, the main problem with those systems was the impossibility to identify and control users, which leaded to failures due to vandalism and thefts (Midgley, 2011).

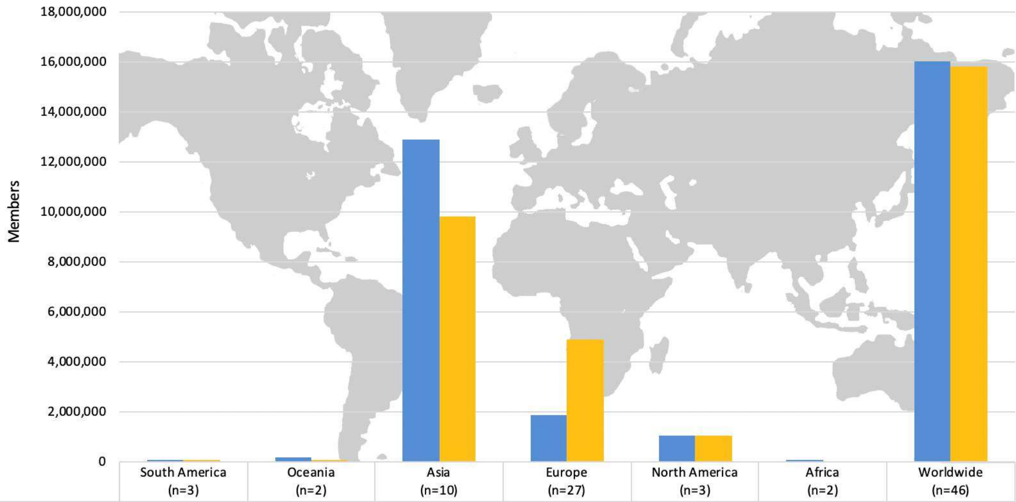
It is not until the third generation of bike-sharing in which that problem was solved. The “Vélo’v” and “Vélib” systems of Lyon and Paris, starting 2005 and 2007, are the main paradigms of this. In those systems, a membership system was implemented in which users had to register in advance and receive an electronic card. Bikes could be unlocked at stations by using the card. This solved many problems of vandalism, but not only that. Since the electronic lock and rental is more controlled, bike-sharing systems could easily operate in a one-way fashion. Since then, one-way vehicle-sharing has been the absolute trend, and allowed to popularize these systems. Users are no longer forced to make a return trip or to care about the bike once at destination. In any case, in this third generation, vehicle-sharing systems were still carried out by public initiative, because the occupancy of stations on-street has to be controlled (DeMaio, 2009; Parkes et al., 2013).

In more years, with the emergence of GPS tracking, a new boost and business model emerged. New systems, called as free-floating, were implemented. In these systems, bikes can be locked and unlocked directly by electronic card or even smartphone apps, without the need of a station, as the location of vehicles can be tracked by geolocation. These systems have been not necessarily been promoted and implemented by public initiative, but also by private investors. This also lead to the emergence of other micromobility systems, in which bikes are substituted by cheaper vehicles, such as electric scooters. Note that all free-floating systems are one-way systems by default, and that all new station-based systems are launched as one-way systems too.

Unlike bike-sharing, car-sharing expansion has been carried out mainly by private initiative. The first car-sharing service appeared in Zurich in 1948. The initiative was called "Selbstfahrergemeinschaft" (sefage in short) and consisted in a cooperative of members who paid for short car rentals. Those were times when it was difficult to afford a car, so sharing was a feasible way to use a car for occasional short trips. Using the Zurich initiative as a model, other European cities followed the lead. However, this car-sharing cooperative scheme soon became less attractive due to the economic prosperity and cost reduction of vehicle ownership. Since then, some few other attempts were carried out in this cooperative fashion, without any success (e.g. Procotip in Montpellier in 1971, Witkar in Amsterdam in 1973).

In the late 80's new car-sharing experiences started being successful again. These were round-trip cars-sharing initiatives, basically consisting on an evolution of car rental services, where users register once instead of each time they make a rental. Also, fares per hour or minute of rental were offered, instead of forcing users to make minimal day rentals. Near 200 car-sharing organizations appeared with these characteristics in more than 450 cities in Switzerland, Germany, Austria, the Netherlands, Denmark, Sweden, Norway, Great Britain, and Italy (Shaheen et al., 1999).

In spite of the relative success of some of those implementations, car-sharing was still not an option as an urban transportation alternative because of its round-trip performance. The paradigm changed completely with the emergence of one-way car-sharing, which allows the users to pick up the car at one point of the city and leave it off at a completely different location (Shaheen and Cohen, 2015).



	South America (n=3)	Oceania (n=2)	Asia (n=10)	Europe (n=27)	North America (n=3)	Africa (n=2)	Worldwide (n=46)
Roundtrip Members	12,892	156,000	12,902,000	1,872,591	1,065,606	16,800	16,025,889
One-Way Members	4,000	45,000	9,805,000	4,889,097	1,044,505	0	15,787,602

Fig. 1. Round-trip vs one-way car-sharing (from Shaheen and Cohen, 2020)

The first one-way car-sharing service was launched in 2008 in Ulm, Germany, by Car2Go, a subsidiary agency created by the car manufacturer Daimler AG. Soon, new one-way car-sharing services started in several other cities by different operators in a rapid expansion of the service. According to Shaheen and Cohen (2020), one-way car-sharing accounts roughly for the 50% of global membership and 40% of global fleets deployed. And these percentages are growing fast, as the 2018 one-way market share represented a 238% increase in membership and a 103% increase in fleets since 2016. Regionally, Europe has the largest percentage of one-way membership, representing more than 70% of the region's car-sharing membership (see Figure 1).

The summary is that the big qualitative step forward in vehicle-sharing development as a new urban transportation mode was the change from round-trip to one-way systems, independently of the type of vehicle. The original round-trip configuration required users to return the vehicles to the station from where they were picked up. This simplified operators' tasks, because they could plan stocks "simply" considering the expected demand for each station. However, this configuration is less convenient for the users, as not only it does force a return trip, but also makes the user pay for the rental even when the vehicle is stopped at destination.

For these reasons, since the implementation of one-way services, the popularity of vehicle-sharing systems has risen up. On the one hand, they became more attractive to users due to the easiness of rental and use. On the other hand, demand is usually imbalanced and vehicles tend to accumulate at specific zones while there is vehicle scarcity in others. This spatial imbalance in vehicle availability increases the operative costs due to the need of artificial repositioning and creates additional design difficulties.

1.2. Characteristics of vehicle-sharing systems

Vehicle-sharing systems consists of the following minimum elements:

- **Fleet**. They are the available vehicles to users. The type of vehicle is what usually determines the denomination of the service (i.e. car-sharing, bike-sharing).
- **Delimited service area**. It is the space where vehicles can be used. In some cases, users are able to ride outside from that area, but they always must return the vehicle inside. There can be reserved parking space for the vehicles (e.g. stations) or not.
- **Central control**. It is the system superstructure. It is responsible for taking operative decisions, manage memberships, and take care of the legal and commercial aspects of the business.
- **Operation and maintenance service (staff)**. They are employees which take care of the good condition of vehicles and/or relocations. The latter are also specifically called "**repositioning teams**". The tasks of all these employees is fundamental, since they are necessary to keep a good level of service and avoid collapse.

With respect to the design decisions, note that operators can take decisions on four main elements in order to serve demand and provide a good level of service. These decisions represent the main tradeoffs we can find in a vehicle-sharing system.

First, the fleet. The operator chooses which type of vehicles makes available to rent and its characteristics. The operator also finds here a first tradeoff between the fleet size and the demand served. The more vehicles there are, the more demand can be served, but a higher acquisition and maintenance costs.

Second, the stations. The operator can choose how many stations there will be in the system, if any. One of the most important strategical decisions when designing a vehicle-sharing system is to choose between a station-based (SB) or a free-floating (FF) system. In a SB system, vehicles must be parked at stations. Users are forced to reach a station to rent a vehicle and return it in another one. However, in FF systems there are no stations at all. So, users can freely rent and park vehicles anywhere inside the service region.

Third, the day-by-day operations, mostly relocation and recharging operations. The operator can hire staff in order to make reparations and/or relocate the vehicles to the most convenient locations. A bigger staff ensures more vehicles in the optimal locations and in perfect conditions. This avoids demand losses, but at the expense of a higher cost.

And fourth, the fare. Note that a lower fare would attract more users, but at the expense of needing subsidies or reducing agency profits to maintain the economic sustainability of the system. And that not forgetting other decisions such as fare integration with other transportation modes.

These tradeoffs are the main design issues explored by academic and industry research. The content of this thesis addresses the first three of them.

1.3. Station-based vs. Free-floating

Vehicle-sharing systems can have two different configurations depending on the allowed parking locations. These are free-floating and station-based vehicle-sharing.

In **free-floating** (FF) vehicle-sharing, the fleet is parked on the city streets. Typically, users can check the availability and location of nearby vehicles through a mobile app and reserve the desired vehicle to make their trips. Once the ride has been completed, users can return the vehicle at any available on-street parking spot inside the service region (typically corresponding with some administrative division).

In **station-based** (SB) vehicle-sharing, the fleet is distributed among parking lots (also called "stations") within the service region. Users can check the availability of vehicles in any station via a mobile app, and make a reservation or just drop-in. The trip finishes when the vehicle is returned to any of the designated parking locations. So, all trips need to be station-to-station.

Each vehicle-sharing configuration has some advantages and drawbacks. Ciari et al. (2014) mention some of them, namely:

- From the operator's point of view, FF vehicle-sharing may imply less infrastructure costs and a quicker implementation, since no installation is needed. However, in any case FF could also need collaboration with the local transportation authorities and its willingness to accept the implementation of the system.
- FF vehicles are equipped with geolocation devices, which allows the operator to track them at any time. This reduces the number of thefts or improper usage. Despite geolocation is not necessary in SB systems, these devices might also be installed in the vehicles to achieve vehicle tracking.
- FF vehicle-sharing presents operative problems in areas with low connectivity. This makes difficult to locate vehicles with precision or interfere with the locking system, which means reducing the level of service to the user.
- The existence of stations at predefined locations and with limited capacities (i.e. they might be full or empty) restricts some of the trips that could be done by users. Why from the user perspective this SB configuration is limiting, from the operator's perspective it could imply important benefits. This is because the locations and capacities of stations, if adequately planned, may limit the amount of vehicles' unbalance in the system, reducing the repositioning needs and their costs.
- SB systems provide additional protection to vehicles while parked. This reduces the probability of theft and vandalism.
- SB systems are better suited for the deployment of an electric vehicle fleet, because it is easier to install chargers at stations.
- The accessibility of free-floating systems depends on the vehicle availability in the service zone. They might offer a better accessibility during off-peak periods (with respect to the station-based equivalent) because available vehicles are widespread. However, this spatial accessibility will tend to be highly reduced in peak periods, when idle vehicles are scarce. This variability in the access distance to a vehicle along the day, which may present a high variance, penalizes the customer perception regarding the reliability of the system. In contrast the access distance in station-based systems is fixed and known, although the costs are higher in order to provide the same average accessibility than in the free-floating equivalent system.
- SB systems can be operated without reservation (i.e. drop-in) and still provide an acceptable level of

service, given that the system is designed so that the probability of empty stations is low. Eliminating the reservation time (required in free-floating) reduces the average service time and allows higher vehicle availability for the same fleet size.

Table 1 summarizes the main pros and cons of station-based and free-floating vehicle-sharing configurations.

Table 1. Pros and cons of free-floating and station-based configurations.

	Pros	Cons
Free-Floating (FF)	Less infrastructure costs (no stations).	Requires strong collaboration of the local transportation authority to accept on-street parking.
	Vehicle tracking all the time.	Vehicles prone to vandalism when parked on-street.
		Strong GSM connectivity needed in the whole service area.
	Better average accessibility for the same costs.	Accessibility varies throughout the day which may be perceived as a lack of reliability.
Station-based (SB)	Less rebalancing if the stations' location and capacity is planned carefully.	Requires multiple agreements with parking providers.
	Vehicle protection while parked.	Stations need to cover the whole service region, implying difficulties in finding adequate parking spots.
	Parking spots may be equipped more easily with charging devices.	The stations' density necessary to achieve high accessibility, may imply high costs and efforts.
	Reservation times not needed (drop-in operation) increasing the availability of the car fleet.	
	Longer term reservations may be implemented for an additional fare, increasing the reliability of the system	

1.4. Operations management: repositioning

Besides the infrastructure maintenance tasks, repositioning is the main and more complex operative issue in vehicle-sharing systems. These operations consist in the relocation of vehicles by the agency due to any operative reason. It is common to refer these operations as “rebalancing”, because they are necessary to correct the imbalanced spatial distribution of vehicles on one-way vehicle sharing systems. Recall that in one-way vehicle-sharing users are not forced to return the vehicles to their origins. This usually leads to a spatial imbalanced distribution of the vehicles, which can reduce its availability in some subregions and therefore the level of service offered. These operations are not necessarily are caused by demand imbalance, but also by another operative needs such as battery recharging in electric vehicles or reparation and maintenance.

Repositioning strategies can be divided into two groups: agency-based and user-based. Agency-based repositioning includes all strategies and operations in which hired employees perform the necessary tasks. These employees represent a direct cost, and therefore also a trade-off with the level of service offered (i.e. the availability of vehicles increases, but at the cost of a higher labor cost). On the contrary, user-based strategies rely on users for reducing the system imbalance or directly for performing the repositioning movements. These strategies include incentives (e.g. fare discounts) for renting/returning the vehicle at favorable locations, or allowing its battery recharge / refuel. Alternatively, the operating agency may force the vehicle returns at favorable locations (e.g. some stations or subzones delimited through electric fences in free-floating systems). Still, these strategies represent a cost for the agency because of the lack of revenue caused by the discounts or by demand losses if some trips are not allowed.

Because operating agency cannot ensure a full control of the fleet by only performing user-based strategies, the vast majority of the vehicle-sharing systems consider agency-based repositioning as the main repositioning approach, sometimes complemented by user-based strategies. In this context, the most common repositioning strategies are the following:

- **Repositioning in vehicles**: It consists in the relocation of the shared vehicles by carrying them in groups inside of a bigger repositioning vehicle (e.g. van, small truck, trailer). Note that this kind of repositioning is only possible if the shared vehicle is small, such as a bike or a scooter.
- **Repositioning without vehicles**: This strategy is common for car-sharing systems. In this case employees relocate the cars one by one, driving the car until the destination, and moving to the next task with a foldable electric scooter or bicycle. It has the additional difficulty of planning not only the vehicle relocations but also the staff relocations.

In addition, as some of the repositioning movements may be due to battery recharging / refueling or other maintenance tasks, where reaching the vehicle is critical, but its final location it is not, other repositioning frameworks can be of application, namely:

- **Hub repositioning**: In this strategy all vehicles are moved to a central hub to recharge or protect from vandalism, and then spread again over the service area. This strategy is more popular in micromobility systems like in shared electric scooters repositioning during night hours.
- **Battery swap**: This strategy can be performed for small electric vehicles (e.g. up to a motorbike). In this case, the staff moves to the vehicle and replaces the drained battery with a full on. The shared vehicle remains in the same place. This strategy is an option when the battery consumption is critical, but the vehicle location is not.

2. Current state of the art and literature review

Vehicle-sharing research focuses on several topics, and this state-of-the-art is structured around a categorical classification of these topics. The order of presentation tries to follow the development stages of a vehicle-sharing system. So, in the first categories we find studies which would be usually applied to the early stages, where the vehicle-sharing system is on its conception phase (e.g. market penetration studies). In the latter categories we group the studies suitable for systems already in operation (e.g. evaluation of fare strategies). Specifically, the categories considered are:

- **Demand estimation and its**: It includes all studies addressing the demand estimation of vehicle-sharing systems. Two main aspects can be considered in this sense. The first one is the overall amount of demand. This is, how many potential users are expected to use the system and which factors affect it. The second one relates the demand estimation to its temporal and spatial profiles and detailed behavior.
- **Strategical and tactical design and planning**: This category includes all the design problems which affect the long-term planning. The approach to these problems considers average parameters and addresses elements of the system which are rather difficult to change once established, such as the service area, the fleet characteristics and size (i.e. electrification of vehicles), or the configuration of the system (FF vs. SB).
- **Operative strategies - agency-based**: This group includes design and management problems which affect the day-by-day operations. The most important operative problem in vehicle-sharing systems is the repositioning optimization. This is how to relocate vehicles inside the service region in order to

achieve some goals (e.g. maximize service, recharge electric vehicles, maintenance, etc.). Reparation and maintenance operations are also included here, and therefore also the personnel relocation.

- **Operative strategies - user-based**: This group also addresses operative problems (i.e. repositioning optimization mainly), but in this case the approach focus on the modification of users' behavior through incentives or restrictions. This may include forbidding some type of trips, electric fences in FF systems or pricing strategies.
- **Simulation tools**: The research and development of all kind of simulation tools of vehicle-sharing systems belongs to this category. This includes standalone simulators (i.e. when the system is simulated without considering other transportation modes and alternatives) or macroscopic network simulators (i.e. including vehicle sharing options in multi-modal transportation simulators).
- **Side effects**: This category includes all the works in which the effects of vehicle-sharing in the overall mobility framework are studied. These are, for example, how vehicle-sharing systems imply a reduction in car ownership, or affects the modal share in a city or reduces greenhouse gases emissions.

Each category considers all the different types of vehicles (i.e. car-sharing, bike-sharing, micromobility, moto-sharing), since in many cases the type of vehicle is not much relevant. Not for instance that a strategic design methodology developed for defining the service area of a free-floating car-sharing system can be applied with little adaptation to a bike-sharing system too.

In the following subsections, the trends in research for each category are briefly explained. In addition, in each chapter of the thesis a deeper literature review is included, addressing the particular topic considered more specifically.

2.1. Demand estimation and profile characterization

Vehicle-sharing demand studies are a recurrent topic in the literature, since they provide a valuable source of information for later stages of planning and design. From these studies, two aspects can be highlighted. The first one is the demand share analysis. This includes the market penetration, the attitude of users towards vehicle-sharing, and in general, any possible study of how many users are prone to use the service and the key factors for its success. This is important to estimate the expected or potential demand of a system.

The second aspect to examine is the individual user behavior and its life cycle. This is, how users act under certain circumstances when using the system. These studies help to characterize the average usage patterns and to make realistic assumptions when modelling the system in the planning, design and operative stages.

Examples of these works are Zhao et al. (2015), Campbell et al. (2016), Faghih-Imani et al. (2017), Leister et al. (2018), Abolhassani et al. (2019), and Xu et al. (2020) for the bike-sharing case; Le Vine et al. (2014), Herrmann et al. (2014), De Luca & Di Pace (2014), Kopp et al. (2015), Zoepf & Keith (2016), Kortum et al. (2016), Sprei et al. (2019), and Becker et al. (2019) for car-sharing; and Liu et al. (2019) for micromobility.

The used methodology is very similar in all cases. All of them are based in data collection and analysis (Ferrero et al., 2018). The data collection part is mostly based on existing systems, and surveying. Surveys are especially important, because the simple observation of an existing system cannot provide an estimation of the potential demand, but only of the actually served demand without including unserved users. In some of the studies, data collection can also consider other transportation modes as proxies in the case of not having an existing vehicle-sharing system in the study area. This is the case of Dias et al. (2017), which includes also ride-sourcing modes in their study about car-sharing mode choice. Or the case of Cherry & Cervero (2007) and Fyhri & Fearnley (2015), who study the usage patterns of electric bicycles, and its conclusions have also been used by other authors for the study of shared e-bicycles.

With respect to the data analysis tools, we could find three big families: i) based on Logit and linear regression, ii) simulation, and iii) machine learning techniques. Each one has its pros and cons, and fits better depending on the purpose. Logit and linear regression models are usually used for the long-term estimation of demand, by studying the population characteristics of vehicle-sharing users. Simulation is used mainly when the spatial component of the problem is important at a microscopic level, such as for the station or vehicle location problems. And finally, machine learning techniques are used for short term usage predictions, in which the accuracy of these methods is higher than the other tools.

2.2. Strategical and tactical design

The expansion of one-way vehicle-sharing systems has had a repercussion on the research and on the related academic literature. Most of the methodological research appeared after 2010 (Martínez et al., 2012) as an answer to the problems encountered by newly implemented systems. Since then, lots of studies addressing the design of vehicle-sharing systems have been published, especially for the cases of bike-sharing and car-sharing.

Here, only works addressing the design at strategical and tactical levels are considerd. These are decisions taken in the medium or long term, such as the system layout (i.e. SB or FF), or the sizing of the fleet and repositioning resources. However, note that in vehicle-sharing systems most of the design problems are interrelated at different detail levels. So, it is possible to find strategic decisions which depend on the considered tactical or operational solution. For example, the fleet size would depend on the repositioning staff resources. If the fleet is reduced, more relocation operations are required to keep the same vehicle availability. But in turn, the staff repositioning cost depends heavily on the day-by-day relocation routes, whose optimization is an operational problem. For this reason, works in this category do not only focus on the strategical decisions, but also consider tactical and operational problems at the same time through approximations and hypotheses in their solutions. This has a heavy influence in methodology chosen.

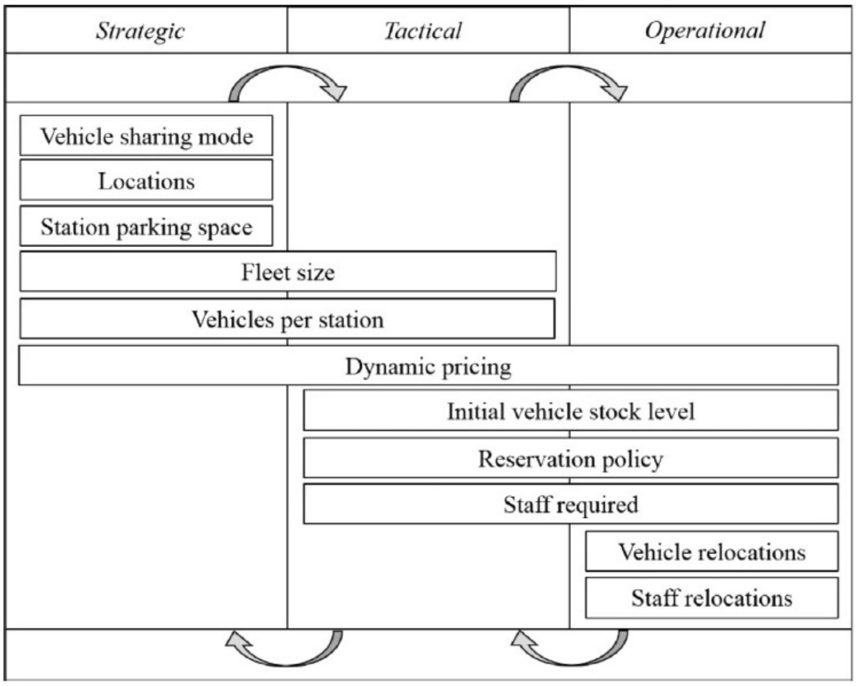


Fig. 2. Dependent decision problems in vehicle-sharing. (from Illgen and Höck, 2019)

In general, methodologies based on mixed integer programming (MIP) or mixed integer-linear programming (MILP) are the most popular ones when addressing the vehicle-sharing design optimization. These methodologies are of common use in classic logistics problems. Therefore, they fit well with the station location problem and the repositioning optimization problem, which can be considered adaptations of the facility location and pickup and delivery problems. In these cases, the strategic design model for the vehicle-sharing system is built on classical MIP or MILP formulations.

Despite this common approach, there are lots of differences between particular works. The focus and the definition of the objective function is one of them. Note that it is not possible to design a perfect system that completely satisfies all agents, and trade-offs arise. So, models are developed and adapted to achieve specific goals. For instance, Frade and Ribeiro (2015) propose a model to obtain a maximal coverage given a constrained cost, García-Palomares et al. (2012) considers the minimization of user costs, Shu et al. (2013) optimizes the system to serve a given demand, and Martínez et al. (2012) develops a model to maximize the system revenue. All of them for the bike-sharing case.

Other differences lay in the assumptions and approximations considered for solving the computational burden inherent to MIP and MILP. Boyacı et al. (2015) develops the concept of “virtual hub” in order to reduce the size of the problem. Other works, such as Yuan et al. (2019), Huang et al. (2018), Hua et al. (2019), Huang et al. (2020a) are further examples of different algorithms to find MILP solutions. Finally, we want to highlight here the work of Li et al. (2016a), which discretizes the space and treats each subarea as an infinite homogeneous plane where continuous approximation (CA) model equations are applied. The importance of this work for this thesis is huge, since CA is the methodological approach considered for the development of the strategic design models presented. The work of Li et al. (2016a) proves the suitability of CA to address this problem, achieving results with a relative error lower than 3% with respect to the MIP optimization solution, but with a computational time 100 times lower. In addition, they obtain further insights from the system through sensitivity analysis, which were overlooked by previous works.

2.3. Operative strategies: agency-based

The one-way configuration of vehicle-sharing systems carries inherently the need to deal with vehicle imbalance problems in the service region. These problems can be addressed in two ways: attempting an imbalance reduction (i.d. user-based strategies, explained in the next section), or compensating the imbalance through repositioning operations carried by hired employees or teams. This later strategy is the most common one, since it provides more control to the agency. Due to its complexity and importance, the repositioning optimization problem has become a recurrent topic in literature.

This problem has been partly addressed in the works presented earlier, as the repositioning intensity is part of the strategical design of vehicle-sharing systems. But here the approach is different. In the previous cases, the system design was unknown, and repositioning solutions were considered through its approximate average cost. However, in the operative management category the system design is given, and the objective is to optimize the specific tasks to be assigned to the repositioning teams for the following hours or days.

In any case, the methodological approach is very similar in both cases. The vast majority of research work addressing this problem is based on variants of MIP and MILP optimization problems. Again, differences between particular works lay in the objective function considered, the mathematical model to depict the operative of the system, or the heuristics and numerical methods to make its optimization solvable.

Literature makes here an important distinction according to the conditions of the system. Repositioning is called to be “static” when the system is assumed to be closed while vehicles are repositioned and the desired (i.e. the balanced) state is fixed. Examples of it are Alvarez-Valdes et al. (2016), Benchimol et al. (2011), Chemla et al. (2013), Bruglieri et al. (2014), Dell’Amico et al. (2014), Li et al. (2016b), Raviv et al. (2013), Schuijbroek et al. (2017), and Pal and Zhang (2017). In turn, repositioning is called “dynamic” when the relocation tasks take place while the system is in operation, so that users pick-up and return bikes, continuously modifying the system state. This approach requires a more complicated modelling, but it is more realistic, and represents the current trend in the literature. Some of the most significant articles reviewed for this thesis with respect to this topic are Nair and Miller-Hooks (2011), Contardo et al. (2012), Caggiani and Ottomanelli (2013), Nourinejad et al. (2015), Boyaci et al. (2017), Caggiani et al. (2018), Zhao et al. (2018b), Lei & Ouyang (2018), Shui & Szeto (2018), Repoux et al. (2019), and Osorio et al. (2021).

2.4. Operative strategies: user-based

Works in this group address operational problems too (especially the repositioning problem) but with a completely different approach. In this case, problems are dealt indirectly. The idea is that the actions taken by the operating agency affect user behavior in a way which cause the desired changes in the system. So, the principles behind these works and the used methodologies are completely different than in the previous sections. Despite being represented by a narrower group of articles, the range of solutions proposed is more diverse.

Probably the most popular approach in this group are solutions based on pricing and incentives. This is, to offer discounts or establishing a fare structure based on the rental and return locations, so that users can reduce the system imbalance through the use of the shared vehicles. Examples of this approach are proposed in Pfrommer et al. (2014), Haider et al. (2018), Zhang et al. (2019a), or in Stokkink & Geroliminis (2021).

Some other strategies are based on trip restrictions. For instance, in free-floating bike-sharing systems we can find a strategy called “electric fences”. Under this strategy, the operating agency divides the service region into subzones and could force users to return the bike only inside a specific subzone in order to allow them to lock the bike and complete the trip. Although not very common, this trip restriction strategy has been explored by works such as those by Zhang et al. (2019b) and Jia et al. (2022).

Finally, other strategies are simply policy changes with effects on the demand. Take as an instance, the study of Balac et al. (2014), where it was explored the option of paying on-street parking fees, which would be compensated by the improvement in vehicle accessibility and rentals.

In general, any user-based rebalancing strategy could be useful, but they must adequately weight the trade-off between the imbalance reduction and the costs of the strategy. Note that, in most cases, this cost is not only the direct agency cost of the strategy (i.e. due to incentives as a fare reduction) but also the user cost due to more annoyance to users, such as increasing the access or egress distances or the revenue loss if demand is lost due to the restrictions.

2.5. Simulation tools development

As far as the authors are concerned, simulation of vehicle-sharing systems has been quite unexplored in the academic literature. Only a few authors have developed and published simulation frameworks either as standalone simulators or as an auxiliary tool for specific purposes, usually the spatial demand prediction.

In the first group we include the works of Lopes et al. (2014) for station-based car-sharing systems, and Fernandez et al (2020) and Angelelli et al. (2022) for station-based bike-sharing systems. These works represent excellent starting points for vehicle-sharing systems simulation research, since they describe in detail the involved elements. In particular, Lopes et al. (2014) develops a complete simulation framework for station-based car-sharing system, including users, repositioning activities, and maintenance activities. Moreover, their simulation framework includes setup modules for stations, vehicles, and O/D demand. In turn, Fernandez et al (2020) focuses on modelling the users’ behavior, and develops an event-based simulator in which events correspond to the milestones in the users’ life cycle. Different user types are defined, with different behavior (i.e. checking the app information or not, accepting the system recommendations or not) which are randomly generated. The model is applied to the simulation of the station-based bike-sharing system of Madrid. In this work, however, repositioning teams are not modelled as agents. Repositioning operations are considered as system inputs, with little margin to interact with the rest of the simulation or to test different repositioning algorithms. In contrast, the work of Angelelli et al. (2022) specifically focuses on the modelling of repositioning agents. Authors develop an algorithm in which repositioning teams decide which operations are necessary at each station based on the forecasted vehicles' inventory level. In this work, users are not defined as agents, and vehicle requests and returns at stations are generated as individual stochastic processes.

The second group is more common. In the context of bicycle sharing systems, Kek et al. (2009), Romero et al. (2012), Caggiani & Ottomanelli (2013), Caggiani et al. (2018), Jian et al. (2016), and Jin et al. (2022) are examples of the use of microsimulation to predict the number of requests and returns of bicycles at stations for any design purpose. In turn, Wang et al. (2010), Nair & Miller-Hooks (2011), Clemente et al. (2013), Jorge et al. (2014), Nourinejad et al. (2015), and Boyacı et al. (2017, 2019), and Huang et al. (2018, 2020a) are examples of the same problem and similar methodologies, but applied to car-sharing systems. Note that the latter works do not simulate a whole system, but in most cases only the demand generated.

In spite of the differences between these simulation approaches, is it possible to find some commonalities in the used methodologies. All of them consider agent-based simulation frameworks in which each element of the system behaves independently, and in most of the cases user generation is modelled as a non-homogeneous Poisson process. These works proof that it can be considered that vehicle-sharing demand fulfills the Poisson assumptions. These are: i) the number of events in an interval is a positive integer. ii) the events occur independently, which is consistent with mostly individual trips. iii) two events do not occur at exactly the same instant, so demand requests should be modelled as consecutive events. iv) the average rate of events is constant and independent of any occurrences. Note that demand varies with time, and therefore a realistic modelling should divide the analysis period in shorter time spans of constant demand (i.e. one hour).

2.6. Side effects of vehicle-sharing

Finally, in this group includes all research works dealing with the impacts of vehicle-sharing systems and with all the aspects of the system other than their planning, design and operation. This includes a multitude of topics, being the environmental aspects and the interaction with other transportation modes the most prominent ones.

The specific case of car-sharing is carefully analyzed by many works due to its risks and uncertainty from the sustainable mobility point of view. Note that car-sharing implementations have shown an effect in the reduction of car-ownership. However, they are a strong competitor of public transport and bike trips. So, it is unclear which of both effects will prevail. Examples of studies addressing this are Firnkorn & Müller (2011), Ampudia-Renuncio et al. (2018), From et al. (2019), and Sprei et al. (2019), for the case of free-floating vehicle-sharing. While Namazu & Dowtalabadi (2018) and Carrone et al. (2020) does a similar task with station-based car-sharing.

The effects of bike-sharing are less studied, but still we can find examples such as Wang et al. (2019), evaluating the impact of private-initiative free-floating bike-sharing. It observed that the implementation of these systems increases the overall usage of cycling modes. But that sometimes leads to undesirable effects (such as the increase of traffic accidents, or the excessive occupation of public space in critical zones) if the city was not properly adapted with regulations and infrastructure.

3. Thesis outline

3.1. Objectives

The general objective of this thesis is to provide new tools, namely optimization and simulation models, to help the design and management of one-way mixed vehicle-sharing systems. These are systems in which both a SB and FF layouts work simultaneously and complement to each other. As far as the author is concerned, this is the first time that these systems are modelled in literature. So, any progress in this way represents a contribution to academic research.

These models and tools are also expected to address these systems on two different stages. The first one is the strategic planning stage. In this stage we are expecting to obtain implementation guidelines and size the elements of the system (fleet, infrastructure, personnel) on long term. The second one is the day-by-day stage, in which we consider an already implemented and performing system, whose elements are given and fixed, and we want to operate the system in the most efficient way.

Despite on each one of the works composing the thesis the problem addressed is different and the goals and objectives are specific, in all cases models and solutions were developed seeking the following principles:

- Insights. The problems addressed require complex modelling and optimization techniques. However, an excess of complexity can difficult to extract clear insights and develop general strategies and recommendations. Therefore, models and tools seek simplicity and results avoid to focus in the details, but instead, attempt to extract general conclusions about the causal relationships in the problem.
- Reproducibility. In line with the previous, an excess of complexity also goes against the practical application of the methods proposed. Many vehicle-sharing systems, even the bigger ones, do not have the capability of implementing many of the strategies proposed in the academic literature. Even in some extreme cases, it is difficult for researchers to follow the proposals from other authors. For this same reason, the simulation tool has been developed, to test all the models proposed here and prove that they are ready for implementation. In addition, al the models are developed and presented in a comprehensive way in order to help the future research of upcoming authors.
- Realism. All the models presented are applied to real-life cases of study, in order to prove the obtainability of inputs and estimate the order of magnitude of the obtained solutions. This also helps to identify the strong and weak points of each model.

3.2. Methodologies

The methodological approaches used in the thesis vary from chapter to chapter in order to find the modelling and optimization tools that best fit with the detail level and characteristics of the problem addressed. This section presents an overview of the methodologies used to address each one of the previous defined objectives. In any case, note that each methodology is developed deeper in the corresponding thesis chapter.

The first block of the thesis (i.e. the first two objectives and contributions detailed previously) address the strategic design of vehicle-sharing systems. For this block, models are developed with the purpose of obtaining the overall optimization of the system design, its insights, and some implementation recommendations, such as the order of magnitude of the main decision variables. For these objectives, robustness is more valuable than a high level of detail, since knowing the average performance suffices to obtain these insights and make long term decisions. Therefore, the focus is set on obtaining reliable relationships between the main decision variables and the KPIs of a system. Continuous approximations fit these purposes. It requires strong simplifications but allows obtaining very clear trade-offs and insights. This methodology has proven to be an adequate way to model transportation systems and logistics problems in a simple way, which allows easier optimizations.

The next block of the thesis addresses an operational research problem such as the vehicle repositioning optimization. Note that now, the detail is important. The main objective of this contribution is to develop an operative strategy, so that the optimization models require to consider when and in which stations to assign the repositioning tasks. The methodology here consists in developing different optimization algorithms which belong to two different families. The first one is a MILP-based routing algorithm. In this case, the solution obtained will be a chain of tasks for each repositioning team for some pre-defined time horizon. The advantage of this methodology is that the optimization takes into account all tasks in the chain. So, it is expected to reach an optimal or close to optimal solution for the period considered. However, it also has two important drawbacks: i) computational cost and ii) heavy reliance on demand forecast, as the method needs to know accurately how vehicles will move to ensure that future tasks are optimal. The second repositioning optimization algorithm is based on a pairwise task assignment. It considers a pool of repositioning teams and a pool of potential tasks, and the algorithm pairs them optimally. Unlike the routing algorithm, the method does not take into account future tasks when assigning the present one. So, solution is theoretically worse than the previous over the time horizon considered. However, it has the advantage of a really simple implementation, cheap computational cost, and less reliance on demand forecast, since the tasks are assigned in real time. Both optimization models are compared through simulated cases of study.

Also in the second block of the thesis, the fourth objective and contribution deals with obtaining short-term demand predictions at stations, once the system is in operation. Machine learning regression methods are used to that end. Note that there are many factors that intervene and affect the demand of a particular station, and it would be complex to determine the exact relationship between demand and all its depending factors. However, note that for every particular case of study a lot of historical data is available. Given this context, machine learning regression methods are a very suitable solution for this problem. They do not require to know the detailed relationships between all the variables (in fact, they do not provide it), but they are able to identify patterns between inputs and outputs of the model, and return accurate predictions.

The last work in the block consists in the development of an agent-based simulator for vehicle-sharing systems. This simulator allows testing the strategies developed in the previous contributions, especially the strategies concerning to repositioning operations. For this reason, the elements of the system (e.g. users, stations, vehicles) and their behavior need to be simulated individually. The most suitable methodology for developing a simulation framework with these characteristics is an agent-based simulation coded through object-oriented programming. Using this methodology, users, vehicles, stations, and repositioning teams are created as individual agents, sharing their attributes with the other agents of the same type.

3.3. Delimitations

As stressed in the previous sections, the methods and contributions compiled in the present thesis are developed in a way so that they complement each other. Take as an example the level of detail considered, and note that the lack of detail of the macroscopic strategic design models is mitigated by the agent-based simulation tool and by the operative optimization models.

Still, there are delimitations in this thesis that must be considered. Actually, some of them do not belong to the scope of the thesis, because they have more to do with political decisions than with technical evaluations. For instance, this is the case of the influence of vehicle-sharing in goals such as incentivize sustainable transportation, reduce car ownership, or manage land use. Note that the present thesis does not give recommendations on which of these objectives to pursue, but only on the most appropriate solution for fulfilling it when decided.

With respect to technical and methodological delimitations, probably the most important has to do with user behavior modelling. In the thesis user behavior has been considered an imput and modelled in a simple homogeneous way in most of the instances. This allows to simplify the resulting models, although it must be acknowledged that vehicle-sharing user behavior can be more complex. Consider, for example, the development of endogenous demand models in which the number of potential users depends on the level of service offered. If the average availability of vehicles is low, it could happen that some users abandon the system in the long term and the potential demand considered for design would be reduced. This effect could be weaker or stronger depending on the interactions with other transportation modes, because shared-vehicle modes compete with public and private transportation alternatives. A proper way to study these interactions should be through macroscopic simulation and demand modelling.

In another order of events, note also that rebalancing strategies in this thesis are focused in the artificial movements carried out by the operator. However, it is also possible to develop user-based strategies as an alternative or in combination, in which user behavior is controlled (i.e. electric fences) or incentivized (i.e. pricing strategies with discounts depending on the origin and destination of the trip) in order to reduce imbalance and the unfavorable trips. The development of these strategies requires, of course, the modeling of this complex user behavior.

Finally, another unaddressed aspect in this thesis is the station design and location optimization. At the strategic level the system is treated as a whole, not considering the topology of the service area or any individual treatment of stations or subzones. Therefore, it would be advisable in the future to complement these works with facility location models that would provide a way to delimit the service region and optimize the location of stations, charging stations, workshops and vehicle depots.

These delimitations are outlined through the thesis in order to guide future research works in this field.

3.4. Contributions

Each one of five works in this thesis provide a different tool to model vehicle-sharing systems which is a contribution by itself. They are summarized below. Note that they can be split into two blocks depending on the particular problem addressed.

The first block aims to provide models to optimize the strategic design of vehicle-sharing systems. This means, sizing the fleet, stations, and personnel when a system is to be implemented or big changes are planned over the layout. Two models are included in this block of the thesis:

1. An analytical model to optimize the design of bike-sharing systems. This model considers that the system can be designed on a station-based (SB) or free-floating (FF) fashion, but this decision is given and it is an input to the model.
2. An analytical model to optimize the design of a mixed car-sharing system. Unlike the previous, this model considers that both, SB and FF, configurations can work simultaneously. Therefore, the decision of opting for one or another, or the particular mixture between them, will be a result of the optimization.

The second block of the thesis deals with operative problems. In particular, the main objective is the optimization of the repositioning tasks. This is, how to make the relocation of vehicles by agency employees in the most efficient way. Note that this problem needs a much deeper detail level, such as knowing the individual inventory level of each station. Three models and tools are included to solve this repositioning optimization problem:

3. A discrete model and optimization algorithm to assign repositioning tasks to vehicles. The development of this model is the main contribution of this block. Still two additional complementary tools are developed in order to prove its suitability.
4. A regression model based on machine learning techniques to forecast vehicle-sharing demand at stations. The result of this model feeds the previous repositioning optimization model.
5. An agent-based simulation model, which is developed and programmed in order to implement and test the optimization algorithms developed in the thesis.

Through all these tools and models, the thesis provides a global understanding and overall solutions for the design and management of mixed station-based and free-floating vehicle-sharing systems.

3.5. Thesis structure: the papers

This thesis core is composed by five different individual works presented here as scientific journal papers with the purpose of its publication. (From now on, they will be referred as Papers).

Some of them have been already published, and others have been submitted and are under the review. They have not been altered in any aspect, with the exception of some layout and formatting-specific issues. Hence, some notation may differ from chapter to chapter of the thesis, and some unavoidable repetitions of background information can be found.

All papers can be read sequentially or independently. However, it is advisable to follow the current order of chapters or, at least, to follow the order inside a same thematic block (e.g. Paper I with Paper II addressing the strategic design problem; and Paper III with Paper IV and Paper V addressing the repositioning optimization problem). In order to minimize the aforementioned repetitions, different cases and perspectives (i.e. systems based on different vehicles, or different cases of study) are presented in the different papers. The next section presents and summarizes these papers.

3.5.1. Paper I: A continuous approximation model for the optimal design of public bike-sharing systems

Paper I introduces and develops a first version of a macroscopic parsimonious model based on the continuous approximations methodology and adapted to the case of bike-sharing systems. The model consists in a series of equations defining all user and agency costs in the system as a function of its main decision variables. Battery recharging constraints are included when part of the fleet consists of electric bicycles. Through this model, the main trade-offs are identified, and the optimal values for the strategical design variables of the system can be derived (i.e. the number of bicycles, the number of stations and the required intensity of rebalancing operations). The model considers both station-based and free-floating configurations in order to be compared.

The results obtained are based on real data from a case of study based in Barcelona. The optimization was made through the minimization of the generalized costs (including users and agency costs). The unconstrained optimization is considered, yielding the optimal design from a social perspective. In addition, other optimization scenarios are defined in which some minimum service thresholds are imposed in order to relax agency costs without incurring in a big user cost increase, and thus providing an acceptable level of service.

Thanks to the characteristics of the continuous approximations methodology, the optimization of the model is computationally cheap, and sensitivity analysis can be done with relative ease in order to obtain relevant insights. With respect to the properties of the model, results show that the optimal designs are robust for all the considered inputs to the model. All results exhibit small elasticities with respect to unitary costs and technological parameters, so that deviations from the optimal system design do not imply a large increase in the total system cost. Given this robustness, the model is a useful tool to obtain a first approach to the optimum system design.

In addition, the paper specifically analyzes those inputs whose value may vary largely in different implementations, in order to provide some policy and design recommendations for these particular contexts. This is the case of the possible large variation in the average demand density, whose analysis proves that economies of scale exist in bike-sharing systems. Another interesting case is the particular value of the acquisition cost of bikes, because a significant variation does affect the trade-off between the size of the bike fleet and the number of hired repositioning teams. Different implementation contexts exist where the relative cost of capital and human assets may vary largely. For instance, if bicycle acquisition costs are very cheap with respect to repositioning teams, this could result in the deployment of huge bicycle fleets while repositioning operations are kept to a minimum. Under these conditions, bike accumulation problems are prone to appear, and the recommendation would be to address bicycle parking policies (e.g. implement a station-based configuration) or analyze the possibility of imposing a maximum fleet size for bike-sharing systems and force the operating agency to reposition bicycles in order to maintain the required level of service.

Station-based and free-floating system configurations are compared in the paper, showing that free-floating systems achieve a better average level of service for the same agency costs. However, free-floating configurations imply other risks, such as bicycle clogging central areas of the city and the reduction of their useful life due to exposition to theft and vandalism. These factors would eventually increase system costs, since it is shown that elasticity of costs is higher for free-floating than for station-based systems. And finally, it is also observed that the implementation of electric bike-sharing can be unfeasible with a free-floating configuration, while it does not imply further restrictions on station-based ones.

As a general conclusion to the paper, it can be said that the continuous approximation model presented in Paper I provides a good approximation to the optimum system design, it provides valuable insights, and a better understanding of the system. Those features make the model very appropriate for this stage of design.

Paper I has been published in:

- *Sustainable Cities and Society*, 2020, vol. 52, p. 101826.

3.5.2. Paper II: A continuous approximation model for the optimal design of mixed free-floating and station-based car-sharing systems

In Paper II the previous parsimonious model is adapted to the case of car-sharing and some extensions are included. The most significant add-on is that station-based and free-floating configurations are no longer considered exclusive. Instead, the design allows a mixed configuration, in which both cars on-street and in stations can be used indistinctly. In this way, it would be possible to exploit the advantages of each configuration and mitigate some of its inherent problems, like the electric-vehicle recharge in free-floating configurations, or the lack of accessibility in station-based systems.

In addition, the model presents other changes in the formulation for being adapted to the car-sharing context. This includes the modification of the objective function (i.e. it includes the maximization of the operating agency profit in addition to the minimization of the user plus agency costs), the change in the repositioning operations approximation to deal with one team – one vehicle approach for car-sharing, and new recharging options and constraints to apply when a fleet of electric cars is considered.

Similar to Paper I, the model in Paper II has been applied to a case study taking the parameters from the city of Barcelona. Note that the model optimization in this case does not only return the most important design parameters (i.e. fleet size, stations, repositioning teams), but also if it is more convenient to opt for a station-based, free-floating, or mixed configuration.

Results prove that the profitability of mixed car-sharing systems mainly depend on three factors. First, leaving part of the potential demand unserved (15-35% in most of the analyzed scenarios). This allows increasing vehicle utilization rates and reducing rebalancing needs, which yields in higher profit. Second, the station coverage. In all cases it has been observed that the number of parking stations to be used should be as large as possible, until complete station-based coverage is reached. The benefits of using more stations are higher than their possible penalties and costs, especially if the vehicle fleet is electric and relies on stations for battery recharge. And third, the fraction of users preferring to park in stations, which would be a consequence of the parking availability on streets, and which determines the share between the station-based and free-floating layouts in the system together with the station coverage. The optimal configuration for each case will be a system in which the fleet size and its distribution is adapted to the users' parking preferences in order to minimize repositioning needs.

Finally, it must be noted that the proposed designs are robust, and deviations could be accepted without implying severe penalties. So, the model presented is a useful foundation for the analysis of mixed car-sharing systems from a system wide perspective and in analytical terms.

Paper II has been submitted to:

- *Transportation Research Part C: Emerging Technologies*, 2023

In addition, parts of the content have been presented at:

- XIV Transport Engineering Conference (CIT 2021) meeting, Burgos, Spain, 2021.

3.5.3. Paper III: Optimization of bike-sharing repositioning operations: A reactive real-time approach.

Paper III addresses the problem of repositioning task optimization in station-based bike-sharing systems. This is to assign specific vehicle relocation tasks to particular repositioning teams in order to improve the level of service offered without increasing the agency costs (i.e. constant number of employees and working shifts).

The paper develops an innovative task assignment strategy based on real-time pairwise optimization between tasks and teams. According to this strategy, when a repositioning team finishes its current task, a pairwise assignment optimization algorithm is run between all the repositioning teams and possible tasks. This strategy is compared to optimal routing optimization strategies based on MIP techniques, which is the prevailing approach in the academic literature. The a priori advantages of pairwise assignment are a very reduced computational cost, and a lower dependency to inventory level predictions, since it is observed in real-time and the method does not require to plan tasks hours in advance. However, unlike routing-based strategies, it does not consider the the chain of possible tasks during the optimization horizon.

On a simulated case of study based in Barcelona, the performance of three strategies were compared: i) the proposed real-time pairwise assignment optimization, ii) the MIP-based routing approach solution, and iii) a mixed strategy, which uses the real-time pairwise optimization as a complement to a precalculated optimal route. The level of service offered in each scenario is measured by the number of users not finding available bikes at their origin or parking slots at their destination. Results show that, in general, the real-time pairwise assignment strategy is a better strategy than those based on preemptive routing optimization. The benefit of the real-time assignment increases when the accuracy of the demand forecast and the estimation of the inventory level drops. Even if the accuracy of the demand predictions is good, the real-time assignment can be preferable due to its simplicity of implementation and low computational cost. And, in any case, it improves the performance of MIP-based solutions if included as a complement in a mixed strategy.

Paper III has been submitted to:

- *EURO Journal on Transportation and Logistics*, 2023.

In addition, parts of the content have been presented at:

- III Campus del Foro de Ingeniería del Transporte, Cercedilla, Madrid, Spain, 2019.
- EURO Working Group of Transportation (EWGT) meeting, Barcelona, Spain, 2019.
- EIT Urban Mobility. Doctoral Training Network Annual Forum 2020. (Online).

3.5.4. Paper IV: Forecasting demand at bike-sharing stations through machine learning techniques

Paper IV is a short work that complements the Paper III by addressing the problem of demand prediction at bike-sharing stations. As observed previously, the accuracy of these predictions is crucial for the success of preemptive routing optimization strategies for repositioning. Forecasting methods are needed to predict the inventory level at stations when planning the repositioning operations, and also to estimate the benefit obtained by performing each repositioning task. Note that, it will be more efficient to move vehicles to stations with a high vehicle net loss, than to stations where more returns are expected. In Paper IV a regression model is developed to address such demand predictions and three machine-learning non-linear methods are used to solve it, namely: i) random tree, ii) gradient boost, and iii) artificial neural networks. The paper describes the model variables and the calibration processes of the different methods. The performance of the three methods is assessed in their application to a case of study based in the New York City bike-sharing system. The methods are not only compared in terms of accuracy, but also with respect to the easiness to calibrate the algorithms. Under these criteria, results show that the calibrated Neural Network algorithm yielded the best performance in the analyzed case study. In spite of this, differences are small, and the Random Forest method also yielded very good accuracy under most conditions with much less calibration effort. This means that the latter could be an advisable option when a quick simple prediction is required.

Paper IV has been submitted to:

- *Data Science for Transportation*, 2023.

In addition, parts of the content have been presented at:

- XV Transport Engineering Conference (CIT 2023) meeting, La Laguna, Spain, 2023.

3.5.5. Paper V: Agent-based simulation of vehicle-sharing systems

In Paper V a modular agent-based simulator is developed and described. The simulator is able to emulate the detailed behavior of a vehicle-sharing system. The proposed simulation framework is used to simulate two different systems, namely: i) a mixed car-sharing system, and ii) a station-based bike-sharing system. The first simulation is complementary to the car-sharing model presented in Paper II. The latter simulation experiment was used to obtain the simulated results of Paper III.

The description of the simulator is detailed enough to allow its replicability. It includes a complete portrayal of all agents, necessary inputs, and all the functions implemented in a modular structure. This modular structure implies that each aspect is treated separately, so that each module can be replaced or modified without affecting the others. Thanks to that, the depiction of the simulator model can be generalized to any vehicle-sharing context or adapted by other researchers to test their own work with ease.

Finally, and in addition to the description of the simulation framework, some of the results of the implementations are shown as an example of possible applications. These results show that the generation of the potential demand, the trips served, the trips lost, and the repositioning tasks, emulate well a real system with an affordable computational time. Therefore, this simulation framework can be used to analyze the performance of vehicle-sharing systems and support the decision-making process from the strategical design to the operative analysis, by performing efficiently a wide range of experiments.

Paper V has been submitted to:

- *Journal of Simulation*, 2023.

In addition, previous versions of the content have been presented at:

- XIII Transport Engineering Conference (CIT 2018) meeting, Gijón, Spain, 2018.

3.5.6. Summary of publications

As detailed in each of the previous subsecions, the papers conforming the core of this thesis have been published or submitted for publication to international peer-reviewed journals. Table 2 summarizes the scientific papers that compose the present thesis.

In addition to this academic contribution, the ideas of this thesis have also been presented in four international major congresses and a minor scientific campus. Three of these presentations resulted in peer-reviewed Procedia papers. These Procedia papers are also referenced in Table 2, but they are not included in this thesis document, since they present early stages of the same ideas developed in the main chapters.

Table 2. Summary of the publications

Chapter	Title	Authors	Journal	Status
Chapter 2	A continuous approximation model for the optimal design of public bike-sharing systems	Francesc Soriguera Enrique Jiménez	Sustainable Cities and Society	Published (2020, vol. 52, p. 101826)
Chapter 3	A continuous approximation model for the optimal design of mixed free-floating and station-based car-sharing systems	Enrique Jiménez Francesc Soriguera	Transportation Research Part C	Resubmitted (2023)
Chapter 4	Optimization of bike-sharing repositioning operations: A reactive real-time approach.	Enrique Jiménez Francesc Soriguera	EURO Journal on Transportation and Logistics	Submitted (2023)
	A new dynamic repositioning approach for bike sharing systems	Enrique Jiménez Francesc Soriguera	Transportation Research Procedia	Published (2020, vol. 47, p. 227-234)
Chapter 5	Forecasting demand at bike-sharing stations through machine learning techniques	Jaume Torres Enrique Jiménez Francesc Soriguera	Data Science for Transportation	Under review (2023)
Chapter 6	Agent-based simulation of vehicle-sharing systems	Enrique Jiménez Francesc Soriguera	Journal of Simulation	Under review (2023)
	A simulation model for public bike-sharing systems	Francesc Soriguera Víctor Casado Enrique Jiménez	Transportation Research Procedia	Published (2018, vol. 33, p. 139-146)

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Paper I

A continuous approximation model for the optimal design of public bike-sharing systems

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A continuous approximation model for the optimal design of public bike-sharing systems.

Francesc Soriguera and Enrique Jiménez

Abstract— During the last decade, public bike-sharing systems have gained momentum and popularity. Many cities worldwide have put their trust in bike-sharing to promote bicycle use and move towards more sustainable mobility. This paper presents a parsimonious model from which to derive the optimal strategical design variables for bike-sharing systems (i.e. the number of bicycles, the number of stations and the required intensity of rebalancing operations). This requires an integrated view of the system, allowing the optimization of the trade-off between the costs incurred by the operating agency and the level of service offered to users. The approach is based on the modelling technique of continuous approximations, which requires strong simplifications but allows obtaining very clear trade-offs and insights. The model has been validated using data from Bicing in Barcelona, and the results prove, for example, the existence of economies of scale in bike-sharing systems. Also, station-based and free-floating system configurations are compared, showing that free-floating systems achieve a better average level of service for the same agency costs. In spite of this, the performance of free-floating systems will tend to deteriorate in the absence of a strong regulation. Furthermore, if electrical bikes are used, results show that battery recharging will not imply an active restriction in station-based configurations. In conclusion, the proposed modeling approach represents a tool for strategic design in the planning phase and provides a better understanding of bike-sharing systems

Keywords— bike-sharing, electric bike, facility location problem, rebalancing, optimization, continuous approximation, Bicing-Barcelona.

1. Introduction to public bike-sharing systems

Cities around the world envisage huge potential in cycling as a sustainable alternative to motorized individual mobility. The bicycle, as an urban transportation mode, accounts for a marginal modal share in many cities, while most of the urban trips could be done efficiently, in terms of time and costs, by cycling (Heinen el al., 2010). In such contexts, cycling could reduce motorized traffic and curtail pollutant emissions (Cao and Shen, 2019), promoting an environmentally sustainable and socially equitable transportation system, together with a healthier way of life (Jain and Tiwari, 2016).

Public bike-sharing programs stand out as one of the most ambitious initiatives taken by transportation authorities to promote cycling in cities. The bike-sharing concept is simple: take the bike for your trip and leave it behind for others when finished. Benefits for the user are multiple, including the release of all the burdens of ownership (i.e. investment, maintenance, storage, etc.) and the liberty and flexibility of one-way trips, not worrying about the bicycle once at the destination. Bike-sharing also provides a convenient alternative to walking for the first- and last-mile segments in multimodal trips (Lu et al., 2018). Pioneer implementations (e.g. *White Bikes* program in Amsterdam, The Netherlands (1965); *Vélos Jaunes* in La Rochelle, France (1974*); Green Bike Scheme* in Cambridge, UK (1993); or *Bycyklen* in Copenhagen, Denmark (1995)) allowed understanding that in order to reduce the exposure to theft and vandalism, both the user and the bike needed to be clearly identified. This, together with technological progress, gave rise to the currently most accepted framework of public bike-sharing systems, based on bicycle docking at stations and electronic membership cards (known as 3rd generation or station-based systems; see DeMaio (2009) or Shaheen et al. (2010) for an extensive review of the past, present and future of bike-sharing programs). In this type of systems, it is only at stations where members can pick-up or return bicycles. Implementations frequently referred to are Wuhan (90,000 bikes) or Hangzhou (78,000), being the largest station-based systems in the world, and also Paris *Velib* (20,600), Barcelona *Bicing* (6,000), or Montreal *Bixi* (5,200). More recently, the introduction of GPS devices and advanced locks in shared bikes entailed new opportunities for the original concept of free-floating bike-sharing (i.e. station-less). Since 2015, free-floating initiatives have appeared around the world, and with special intensity in China. For example, *Mobike* and *Ofo*, the two largest operators in China, rolled out 280,000 shared bikes in Shanghai and 200,000 in Beijing as of June 2017; by the end of 2017, these fleets had raised to 2.35 and 1.5 million respectively with a total of 15 operating companies. This explosive growth implied that globally, 150 Chinese cities were served by free-floating systems in 2017, with an intense competition between over thirty suppliers and more than 80 million registered users (Wang et al., 2019). Outside China, free-floating initiatives have been much more restrained, including *SocialBicycles (SoBi)* mainly across the USA, or *Call-a-bike* in Germany (Reiss and Bogenberger, 2016). In these free-floating systems (which represent the 4th generation of the bike-sharing concept), bikes are spread out over a clearly limited service region. Bikes can be locked to an ordinary bicycle rack (or to any solid frame, or standalone), eliminating the need for stations. All the technology and functionalities of the station are integrated into the bikes, which are equipped with a built-in GSM communication module allowing reservations to be made through a mobile app. Currently, bikes are locked/unlocked by scanning the QR code printed on the bike, which replaces the outdated built-in keypads and membership PINs. Also, bicycles generally have an integrated GPS device to prevent theft and allowing real-time tracking of the bicycle. In comparison with station-based systems, free-floating configurations may save on the start-up cost by eliminating the cost of constructing stations, especially if the technological bike upgrades do not imply a significant extra-cost. Also, free-floating systems become more convenient for users because of shorter access distances, especially at the destination, but also at the origin if bikes are somehow uniformly distributed throughout the service region. From the user perspective, free-floating also eliminates the worries about the shortage of a vacant spot at the destination station. In spite of all these benefits, the lack of stations may also create problems, like bicycles clogging pedestrian walkways and overflowing into residential neighborhoods, thus disturbing residents. In fact, these problems have already appeared in the huge Chinese implementations, where authorities are debating and starting to implement new regulations for such systems (Wang et al., 2019). Regulation, together with the slowdown in the increase of the number of registered users in China is causing the collapse of some operating companies and is forcing others to withdraw the market. By April 2018, the number of operating companies in China decreased from 30 to 17. Despite this consolidation phase of the free-floating market, the current momentum of bike-sharing systems is a reality. Today, there are 1,975 cities around the world operating bike-sharing schemes, with approximately 15 million bikes in use (Bike-sharing World Map, 2019).

Electrical bike-sharing programs are not experiencing the same success yet. While bike-sharing programs are growing in most large cities around the world, electric-bikes are only used in small pilots, which generally do not become large-scale implementations. *BiciMAD*, the bike-sharing system in Madrid (Spain), is possibly the only exception, with 2,000 e-bikes and 165 stations. This situation is probably due to the higher investment costs of e-bikes and charging docks at stations, together with a shorter tradition of electrical bike technology. In spite of this, electrical bikes might have potential for improving the efficiency of bike-sharing systems, not only because users exhibit a higher willingness to use electric bikes (Martínez et al., 2012), but also because the demand asymmetry towards down-sloping trips could be reduced, especially in hilly cities, alleviating the need for artificial rebalancing of the system.

Despite the popularization of the bike-sharing concept during the last decade, the system design is not free of difficulties. The main operational problem is the system's imbalance, caused by random and asymmetric demands (i.e. variable requests vs returns at different zones / stations). This gives rise to situations in which there are no available bicycles in a particular zone / station, while others are jam-packed. This last situation is more problematic in station-based systems because the user faces the impossibility of returning the bike in already full stations. System imbalance is addressed in the operating phase by artificially rebalancing bicycles among stations (or distribution zones, in case of free-floating systems). In addition to this, in the planning phase, the number, location and size of stations or distribution zones, and the size of the bicycle fleet, are critical decisions that affect rebalancing operations and steer the success of the implementation.

Optimal system design, in terms of the selection of the previous decision variables, should respond to the optimization of the trade-off between users' and operating agencies' costs. On the one hand, from the users' perspective, the service level is defined by the availability and accessibility of bicycles (i.e. in terms of the access distance), both at the origin and destination of the trip. This includes the ease of bicycle return. In general, the level of service increases with the number of bikes and with the number and size of stations. On the other hand, operating agencies aim to limit the investment and maintenance costs of bikes and stations, and the operational costs of bike repositioning within the system. The viability of a system depends on the ability to provide a reasonably good level of service while keeping costs affordable.

Nevertheless, many preliminary studies and guides used in practice for introducing bike-sharing programs tend to give only general qualitative recommendations for the strategic system design (García-Palomares et al., 2012). Take as an example the Spanish methodological guide (IDAE, 2007). In this context, new designs tend to be based on previous experience from other implementations, and trial and error. Bad system design implies long access times and high probabilities of full / empty stations, discouraging the demand for the system, or excessive operating costs. In both scenarios the system is condemned to failure.

In order to improve the decision-making process in the planning and design of public bike-sharing systems, it is needed a simple and quantitative optimization framework accounting for the main costs and existing trade-offs. The present paper proposes a strategic design methodology for bike-sharing systems based on continuous approximations (CA). The method provides the optimal system design, in terms of the number of stations, total parking slots, size of the bicycle fleet and required repositioning level, from where to anticipate agency costs and the level of service offered. The CA approach requires simplification of the system by considering all variables as continuous. This allows analytical modeling of the system structure, unveiling the main insights and trade-offs, which makes this approach suitable for planning purposes.

The structure of the remainder of the paper is as follows: the next section, Section 2, summarizes previous research on vehicle-sharing systems and justifies the proposed macroscopic modeling approach. Section 3 presents, in detail, the model formulation. Next, in Section 4, the Barcelona's *Bicing* system is selected as a case study. The analysis of the *Bicing* system includes the parameter estimation, the validation of the model and the optimization results. In addition, a sensitivity analysis is performed in order to generalize the obtained results. Finally, the paper ends with the conclusions section, acknowledgments and reference list.

2. Literature review and modelling framework

Bike-sharing systems have attracted increasing attention from the scientific community in the last years. While it was rather difficult to find any specific research work on bike-sharing before 2010 [Martínez et al. (2012)], today there is a vast amount of scientific literature dealing with this topic. Most of the methodological research appeared as an answer to the problems encountered by operating agencies running the bike-sharing systems implemented a few years earlier. The main operational problem is the fleet imbalance, and many research studies face this problem from the operations research perspective. Many times, this consists of defining the optimal routes and repositioning movements that light trucks should perform in order to rebalance the system with minimum cost. A number of studies address the static case, where the system is considered closed and the desired (i.e. the balanced) state is fixed (see Alvarez-Valdes et al., 2016; Benchimol et al., 2011; Chemla et al., 2013; Dell'Amico et al., 2014; Li et al., 2016a; Raviv et al., 2013) for station-based configurations and Pal and Zhang, 2017, for free-floating systems). Fewer studies (Nair and Miller-Hooks, 2011; Contardo et al., 2012; Caggiani and Ottomanelli, 2013) for station-based configurations and Caggiani et al. (2018) for free-floating systems, focus on the more complex dynamic case, where repositioning activities take place while the system is in operation, so that users pick-up and return bikes, continuously modifying the system state. In all these works, the target inventory level at each station is assumed known, although setting this optimal level is a challenging task, as it should consider the interactions among the inventory levels at different stations (Datner et al., 2017). In general, these studies borrow concepts from previous research on one-way car-sharing and formulate mathematical optimization programs based on classical problems of operations research, like routing and inventory optimization. Differences among them lay in the objectives, constraints and solution techniques used. Alvarez-Valdes et al. (2016) presents a detailed review of such differences.

All the previous studies address the optimization of repositioning operations. The strategic design of the system (i.e. free-floating *vs* station-based configuration, or the number and size of stations) and the tactical decisions (i.e. size of the bike fleet and the repositioning level proposed) are considered given and fixed, although they can be sub-optimal. There are relatively few studies in the literature focusing on the strategic design of public bike-sharing systems. This strategic approach requires an integrated view, considering the trade-off between the user costs and the investment, maintenance and operational costs incurred by the promoting agency (Yuan et al., 2019). Because the number of stations, the number of bikes and the number of required rebalancing operations are dependent decision variables, an integrated design framework needs to be proposed. Neglecting part of the system costs, as in García-Palomares et al. (2012), where the proposed GIS based solution only considers user costs, or in Lin and Yang (2011); Lin et al. (2013); Yuan et al. (2019) where repositioning operations are not considered, oversimplifies the problem. Notice that some authors claim that the redistribution of bicycles over a day can be neglected in the planning phase since this is an operational decision, while only long-term decisions affecting facility investments (bikes, stations, bike lanes), should be considered. The issue here is that the rebalancing level selected has an impact on the strategic design of the system, since there exists a trade-off between the bicycle fleet size and the required repositioning level in order to achieve a particular level of service. More rebalancing saves on bike investment and vice versa.

Other research studies dealing with the strategic design of bike-sharing systems are built on classic facility location problems, where each customer is assigned to one facility with an operational cost that increases with the distance. The design goal is to determine the best locations to build facilities to balance the trade-off between facility investment and operational costs. The typical way to approach location problems is to discretize the space and solve the location design with integer programming techniques. Again, differences appear with respect to the main objective of the optimization, the assumptions considered (included in the mathematical program through the constraints), and the solution techniques used, being the problem generally NP-Hard (Campbell et al., 2002). For instance, Frade and Ribeiro (2015) propose a linear program to obtain a maximal covering solution given a constrained implementation cost. Çelebi et al. (2018) propose an integrated set-covering and queuing model to determine stations' location and their capacity. Martínez et al. (2012) formulate a mixed-integer linear program to maximize bike-sharing system revenue, whose solution is obtained through a branch and bound algorithm. Shu et al. (2013) use a network flow model, which is solved through linear programming, to determine the optimal number of bicycles, docking positions and rebalancing movements necessary to serve a given demand. In all these cases, the number of stations is assumed given, and the repositioning strategy is periodic every 24h. More general frameworks try to minimize the overall system cost. For instance, Yuan et al. (2019) using a mixed integer linear programming (MILP) model, and Lin et al (2013) a hub location inventory model. In both works bike rebalancing between stations is neglected. In addition, Lin et al (2013) shows that the computational complexity of the formulation requires the application of a greedy heuristic to find near-optimal solutions. In fact, the excessive computational burden is a major drawback of integer programming approaches when facing real-world problems (i.e. medium to large scale problems). Nevertheless, from the planning perspective this is not prohibitive since near-optimal solutions, which can be achieved in a reasonable amount of time, suffice. The fact that mathematical programming solutions are "black boxes" allowing neither to gain insights into the system behavior, nor into the existing trade-offs and sensitivity to parameters, is a much more troubling issue.

Continuous approximations (CA) overcome some of these difficulties. The complexity of the problem is simplified, and because the approach is based on analytical modeling of the problem structure, it is able to yield elegant insights. Daganzo and Newell (1986) developed a CA approach to solve location problems, and later Daganzo (2005) showed that CA yield location designs which are very close to the discrete true optimum. The method has been applied to a number of transportation problems (see Ansari, et al., 2017, for a comprehensive review) and in particular to one-way car-sharing systems, as in Daganzo (2010), where the near optimal design is obtained assuming a uniform demand level or in Li et al. (2016b) allowing dynamic and heterogeneous demands, and considering electrical vehicles' charging times. In both studies, only static rebalancing is considered, being in the last case periodic every 24h. This fixed rebalancing period limits the possibility of increasing the frequency of rebalancing operations to reduce the fleet size. This is the reason why in Li et al. (2016b) it is found that the system design is not affected significantly by rebalancing costs. This could be different if the rebalancing period was defined as a decision variable, allowing intensive repositioning within short periods (i.e. continuous rebalancing) leading to a smaller vehicle fleet.

The present paper develops a parsimonious CA model to be used as a comprehensive design framework on how to economically deploy a public bike-sharing system able to provide reliable service to stochastic trip demands in an urban area. A generalized cost function (enclosing user plus agency costs) is defined in terms of the main decision variables. These are the number and overall size of stations (only in a station-based configuration), the number of bikes, the repositioning period, and the accepted probability of no-service (i.e. system unavailability during peak periods). Minimization of the generalized cost function results in the optimal values for the decision variables that define an optimal system design, without falling into the complexities, details and data greediness of disaggregate models. Different scenarios are modelled, analyzed and compared, including station-based *vs* free-floating configurations, and the possibility of adopting electric bikes. This modeling approach should be seen as a tool for the strategic design in the planning phase. Afterwards, more detailed models could be used for fine-tuning (e.g. exact location and size of each station, repositioning algorithm and scheduling, or routing of the repositioning trucks). Nonetheless, the direct implementation of the results of the proposed approximate model should not be disregarded, as easiness in the implementation stage has a value in real life (Ansari, et al., 2017).

3. Model definition

3.1. Model overview and decision variables

The model is defined over a service region of area R, where all demand is generated. The number of bicycle docking stations is introduced into the model by the station density, Δ [stations/km^2], being one of the decision variables. So, ΔR represents the total number of stations, which implicitly divides the service region into sub-regions, (i.e. the influence area of every single station). In other words, every station is associated with the demand generated in its corresponding sub-region. Assuming fairly uniform distribution of stations in R, sub-regions result of similar size and of fairly convex shape. Stations can be real or virtual. In a station-based model, the station is a real fixed depot that users access in order to take or return the bicycle. In a free-floating model, stations are virtual and bicycles can be freely parked throughout the sub-region.

Demand is composed of requests (demand of bikes at the origin) and returns (demand of parking slots at the destination). Because demand is random and generally asymmetric, the number of requests does not need to coincide with the number returns in a given sub-region and for a period of time. This implies system imbalance and leads to service problems. Users may not find any available bike near their origin, or may not find any available parking spot at their destination (in station-based configurations). These problems could be addressed in the planning phase of the system as follows:

- *Bike repositioning*. Consider that for every period of duration h [hours] employees with light trucks could move a number of bikes from nearly full to nearly empty sub-regions. The duration h is a decision variable that determines the repositioning intensity in the system.
- *Increasing the fleet size*. The operating agency could decide to reduce the repositioning intensity (i.e. increase the repositioning period h) and still offer the same level of service to users (i.e. the same probability of finding an available bicycle and an available parking spot) by providing an additional number of bicycles and parking spots. These should be enough to account for the demand imbalance during the extended repositioning period. To this end, the number of bikes in service, m, and the total number of parking slots, M, are decision variables.
- *Allowing no-service situations.* It could be accepted that a fraction, P_e, of users do not find available bicycles at origin, especially during peak periods. Also, a fraction P_f of users may not find available parking slots at destination (only in case of station-based systems). On the one hand, this causes a penalty to users. On the other hand, it reduces agency costs. P_e and P_f, are the probabilities of empty and full stations respectively, and are considered as decision variables.

The objective of the model is to optimize these strategic decision variables (i.e. station density (Δ), fleet size (m), overall number of parking slots (M), repositioning period (h), and no-service probabilities (P_e and P_f), in order to minimize the overall system costs, composed of the agency costs (fleet, stations and repositioning) and the user costs (access and no-service penalties).

3.2. Demand modelling

3.2.1. On demand uncertainty, endogeneity and heterogeneity

Demand generation and trip distribution in bike-sharing systems has been analyzed by a number of research works in the last decade. These works generally exploit the trips' database from some bike-sharing implementation, and try to identify the contextual factors affecting the overall demand generation and trip distribution for bike-sharing systems. This framework allows applying typical demand modelling methodologies to bike-sharing systems. Some examples include Campbell et al. (2016) where the selected case studies are the large-scale bike-sharing systems in Beijing, China. This work depicts a multinomial choice model and concludes that mixed land use patterns, favoring short trips, positively impact bike-sharing demand. Results also show that electrical bikes are way more tolerant to longer trip lengths. Also, Faghih-Imani et al. (2017); Noland et al. (2016) analyze the socio-economic, land use, terrain elevation and infrastructural factors affecting the demand of bike-sharing systems, using data from the Barcelona, Seville and New York implementations. The analysed data corroborates that demand is higher in places with high population density, with available cycling infrastructure, and near busy subway stations, employment and activity centres, and points of interest. In contrast, demand is lower in higher elevation regions, and in zones with poor coverage and availability of the bike-sharing system. Finally, recent studies point out that bike-sharing demand is negatively influenced by extreme weather conditions (Ashqar, et al., 2019; Kutela and Teng, 2019; Scott and Ciuro, 2019).

Despite the previous demand modelling efforts, in the proposed model the average demand for the bike-sharing system is considered as an exogenous input. The justification of such simplification follows the reasoning in Daganzo (2010). The system design should target a demand level that is expected to materialize at some point in the future. In addition, results show that optimal designs are robust, so that they are near-optimal for a broad range of demand levels. In this context, and assuming the planning phase of the system, to include complex and uncertain demand models to account for endogeneity (i.e. demand as a function of the system design, while the system design is also a function of the demand level), as is done for instance in Romero et al. (2012); Martínez el al. (2012), would over-complicate the model and the interpretation of results.

In addition, the consideration of dynamic and heterogeneous demand profiles (i.e. the inclusion of the detailed evolution of demand in time and space) would turn CA into a very complex optimization problem, as shown in Li et al. (2016b). The obtained insights could then be somehow blurred, because the causality of some results would be more difficult to identify. Furthermore, the model would become data intensive since detailed origin / destination matrixes would be needed. For instance, Garcia-Palomares et al. (2012); Frade and Ribeiro (2015); Li et al. (2016b) require O/D demands for small zones (e.g. 500 m wide) and for every “time step", which needs to be short enough to capture peak demands (i.e. a few hours). This level of detail in the demand characterization is virtually impossible to be robustly predicted during the planning process and for the whole duration of the useful life of the system. In light of this, and because the aim of this research is to obtain a parsimonious model which provides very clear insights, a uniform average demand level is assumed. It must be stressed at this point that this does not mean neglecting demand fluctuations in time and space. The model, as described in the next section, captures random demand variations and the spatial imbalance of requests versus returns.

3.2.2. Demand density characterization

We define λ [trips/km²·h] as the average demand density over the service region of area R. Then, λR represents the average number of requests in the whole system per unit time. Because of the conservation of the number of bicycles, this needs to be equal to the average number of returns per unit time considering the whole service area, R. However, considering a particular location, r ($r \in R$), the density of requests, $\lambda_{q(r)}$, is generally different than the density of returns, $\lambda_{t(r)}$, leading to system imbalance. The demand imbalance density at location r can be expressed as a fraction of the average demand density, λ. This is:

$$\lambda_{t(r)} - \lambda_{q(r)} = \varphi_{(r)}\lambda \qquad [\text{imbalance movements/km}^2\cdot\text{h}] \qquad (1)$$

where $\varphi_{(r)}$ is a dimensionless variable defining the imbalance level at location r. Because of conservation of the number of bikes over the whole service region R:

$$\int_R \left(\lambda_{t(r)} - \lambda_{q(r)}\right) dr = 0 \Longrightarrow \int_R \ \varphi_{(r)}\, dr = 0 \tag{2}$$

This implies that:

$$\int_{R_t} \varphi_{(r)}\, dr = -\int_{R_q} \varphi_{(r)}\, dr \tag{3}$$

where R_t is the partition of R where $\lambda_{t(r)} > \lambda_{q(r)}$ (i.e. $\varphi_{(r)} > 0$; the density of returns is higher than the density of requests), and R_q is the complementary of R_t in R (i.e. $\lambda_{q(r)} > \lambda_{t(r)}$; $\varphi_{(r)} < 0$; more requests than returns). A third partition of R might exist including self-balanced sub-regions, if any.

Then, the average imbalance density (i.e. per unit area and unit time) over the partition R_q is expressed as:

$$\frac{\int_{R_q}(\lambda_{t(r)} - \lambda_{q(r)})dr}{R_q} = \frac{\int_{R_q} \varphi_{(r)}\lambda dr}{R_q} = \Phi_q \lambda \tag{4}$$

where $\Phi_q \stackrel{\text{def}}{=} \frac{\int_{R_q} \varphi_{(r)} dr}{R_q}$. Note that $\Phi_q < 0$ by definition.

Equivalently, the average imbalance density over R_t is:

$$\frac{\int_{R_t}(\lambda_{t(r)} - \lambda_{q(r)})dr}{R_t} = \frac{\int_{R_t} \varphi_{(r)}\lambda dr}{R_t} = \Phi_t \lambda \tag{5}$$

where $\Phi_t \stackrel{\text{def}}{=} \frac{\int_{R_t} \varphi_{(r)} dr}{R_t}$, $\Phi_t > 0$ by definition.

Given these definitions and according to the previous conservation equations, $\Phi_t R_t = -\Phi_q R_q$ holds, and the average imbalance density over R can be defined as:

$$\Phi = \frac{\Phi_t R_t + |\Phi_q| R_q}{R} = \frac{2\Phi_t R_t}{R} = \frac{2|\Phi_q| R_q}{R} \tag{6}$$

3.3. Generalized cost function

Strategic decision variables steer the overall performance of the system from medium to long term. Once selected, their values are difficult to modify because it would imply the redesign of the whole system. In the proposed model, optimal values (i.e. m^*, M^*, Δ^*, h^*, P_e^* and P_f^*) are found by minimizing a generalized cost function. This represents the generalized cost of the system (i.e. agency plus users' costs) as a function of the decision variables and parameters that define a particular scenario. Because m and M can be expressed in terms of the other decision variables, and P_f will be assumed fixed, as discussed later, the optimization involves 3 degrees of freedom. The optimization is formulated in its Lagrangian form (i.e. unconstrained optimization), where each term of the users' and agency costs in the objective function is weighted by its respective unitary cost, prorated per unit time (i.e. units of [€/h]). The general form of the generalized cost function considered is:

$$C = C_I + C_O + C_R + C_A + C_{NS} \qquad [€/h] \tag{7}$$

where C_I stands for the infrastructure costs, C_O for the operative costs (excluding repositioning), C_R for the repositioning costs, C_A for the users' access cost, and C_{NS} for the no-service penalty. The following sections are devoted to derive each term of the generalized cost function, C. Figure 1 schematically describes the composition of the generalized cost function and its dependency to the decision variables and parameters included in each part.

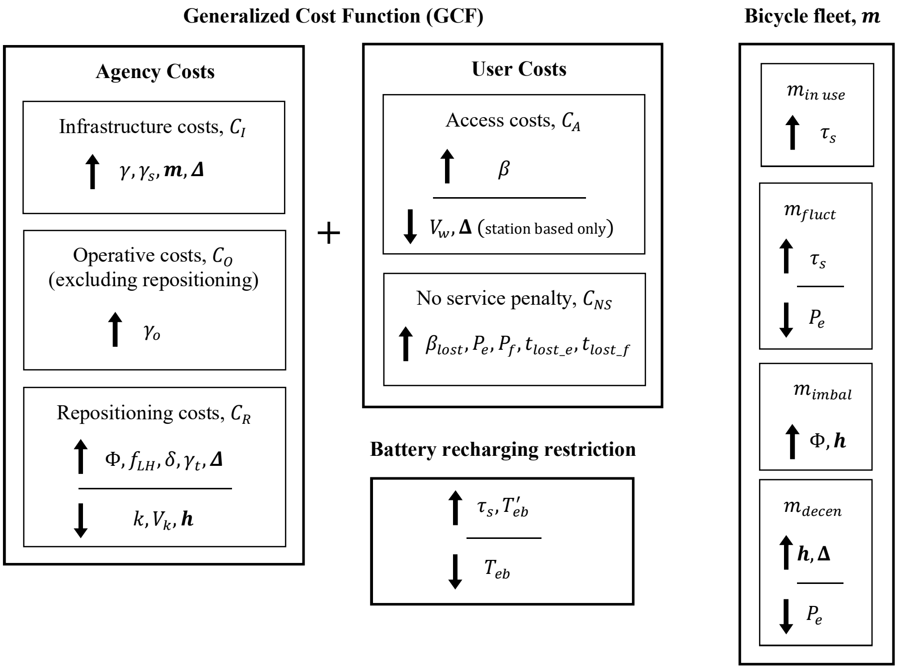

Fig. 1. Schematic description of the composition of the Generalized Cost Function (GCF).
Note: 1) The notation for all the variables and parameters is defined in the following sections and summarized in Tables 1 and 2; 2) Decision variables are in bold characters. Non-bold characters represent parameters of the model; 3) Upside arrows indicate positive dependency. Downside arrows indicate negative dependency; 4) In station-based systems, the overall number of bicycle parking slots, M, exhibits the same dependencies than the bicycle fleet, m.

3.3.1. Infrastructure costs, C_I

Infrastructure costs account for the infrastructure investments made on the system installation. These include bicycles (m) and stations (ΔR), in station-based systems. Their unitary costs, prorated per unit time, are represented by two cost factors: γ [€/h·bike], the cost of acquiring and renewing bicycles; and γ_s [€/h·station] the construction cost of the stations. In free-floating systems, $\gamma_s = 0$, because there are no stations. Then:

$$C_I = \gamma m + \gamma_s \, \Delta R \qquad [€/h] \qquad (8)$$

In Equation 8 and for station-based systems, it is assumed that the cost of stations is independent of the station size (i.e. number of bike parking slots). This considers that the fixed cost per station (e.g. kiosk, power, communications...) is much higher than the marginal cost of adding additional slots. In spite of this, it is true that different station sizes imply different urban space consumption. However, the consumption of urban space is not included in the infrastructure cost of the system since the allocation of urban space to different transportation modes represents a higher level policy decision in cities. We assume no restrictions in this sense.

3.3.1.1. Bicycle fleet size estimation, m

The bicycle fleet size, m, can be expressed in terms of the other decision variables, by considering a maximum accepted probability of empty stations, P_e. In order to accomplish this service level, the fleet size is defined as the sum of the four terms described next.

Average number of bicycles simultaneously being used, $m_{in\ use}$

This term assures that there are enough available bikes to cover (on average) the demand. By using Little's equation (Little, 1961), this term is obtained as the average demand rate times the average service time (i.e. the average time each bicycle is used per customer). The average demand density is λ, which translated to the number of trips per unit time in the whole service area results in λR [trips/h]. The average travel time, τ_s [h], is an input parameter. Then:

$$m_{in\ use} = \lambda R \tau_s \tag{9}$$

Safety bicycle stock to account for demand fluctuations, m_{fluct}

Demand is random and subject to fluctuations in time. A safety stock of bicycles is considered in order to cover a possible excess of requests during peak times. This additional stock can be calculated as a probability level multiplied by the standard deviation of the random phenomenon. Assuming the demand rate as a Poisson random variable (Alvarez-Valdes et al., 2016; Li et al., 2016b; Lin et al.; 2013), the mean and the variance of the number of requests coincide. So, the standard deviation is expressed as $\sqrt{\lambda R \tau_s}$. The probability level is given by the inverse of the standard normal cumulative probability function, F, evaluated for the probability $(1 - P_e)$, one-sided, P_e being the probability of no service at the origin of the trip. Then:

$$m_{fluct} = F^{-1}_{(1-P_e)} \sqrt{\lambda R \tau_s} \tag{10}$$

Additional bicycle stock to account for the average imbalance of the system, m_{imbal}

The average imbalance of the system implies that during h (i.e. the rebalancing period) some trips are not balanced, and therefore some stations may experience bicycle scarcity. An additional stock needs to be provided to account for this issue. This value is calculated by integrating the net change in the number of bicycles per unit area and unit time over R_q (see Equation 4), realizing that the negative value is only a convention meaning more requests than returns, so that additional bike stock is needed. Obviously, in R_t the same amount of additional parking slots will be needed. Equation 4 defines the average net change density (i.e. imbalance movements per unit area and unit time). Considering the region R_q and in one repositioning interval, h, the additional bike stock required would be:

$$m_{imbal} = R_q \Phi_q \lambda h \tag{11}$$

Additional bicycle stock to account for the decentralization of the system, m_{decen}

Decentralization accounts for the variation of demand in space. A decentralized system (i.e. ΔR independent sub-regions) implies that having enough vehicles to serve the overall demands in the whole region R does not imply that enough vehicles are available at the sub-regions where they are needed. So fleet size must be enlarged considering this phenomenon.

The additional stock to account for the decentralization of the system is proportional to the standard deviation of the net change in the number of vehicles in any sub-region. Assuming independency between requests and returns and considering again a Poisson process (i.e. variance equal to the mean), this additional stock can be computed as:

$$Var\left(\lambda_{t(r)} - \lambda_{q(r)}\right) = Var\left(\lambda_{t(r)}\right) + Var\left(\lambda_{q(r)}\right) = 2\lambda \tag{12}$$

This variance is computed in terms of a density (i.e. per unit time and unit area). Then, in one repositioning interval, h, and in the influence area of one sub-region, $\frac{1}{\Delta}$, this is $2\lambda h(1/\Delta)$. The standard deviation is defined as the root square of the variance, and this value needs to be multiplied by the number of sub-regions (i.e. ΔR). Again, the safety stock considered is weighted by a probability level $F^{-1}_{(1-P_e)}$, being F^{-1} the inverse of the one-sided standard normal cumulative probability density function. Then we finally obtain:

$$m_{decen} = F^{-1}_{(1-P_e)} R\sqrt{2\lambda h \Delta} \tag{13}$$

Summary

The total required bicycle fleet size is the sum of the average number of bikes to serve the peak demand, plus safety stocks to account for random demand fluctuations, average imbalance and decentralization of the system. This is:

$$m = m_{in\ use} + m_{fluct} + m_{imbal} + m_{decen} \tag{14}$$

$$m = \lambda R\tau_s + F^{-1}_{(1-P_e)}\sqrt{\lambda R\tau_s} + R_q\Phi_q\lambda h + F^{-1}_{(1-P_e)} R\sqrt{2\lambda h \Delta} \tag{15}$$

3.3.1.2. Estimation of the overall size of stations, M

In station-based systems, the overall number of available parking slots at stations, M, needs to be defined in the planning phase. M must be equal to the fleet size, m, plus the additional slots to account for all the fluctuations. This includes the demand fluctuations in subzones with more returns than requests, the average imbalance in R_t, and the additional slots to account for decentralization of the system, where more returns than requests happen. The derivation of these four terms is equivalent to the estimation of the fleet size, but considering the probability of not finding an available parking slot at destination, P_f.

$$M = m + F^{-1}_{(1-P_f)}\sqrt{\lambda R\tau_s} + R_t\Phi_t\lambda h + F^{-1}_{(1-P_f)} R\sqrt{2\lambda h \Delta} \tag{16}$$

Note that the third term in Equation 16 is equivalent to $R_q|\Phi_q|\lambda h$ (see Equation 6).

3.3.2. Operative costs (excluding repositioning), C_O

Operative costs include bicycle maintenance and repair, control and system administration. It is considered that these costs are related to bicycle usage and are assumed to be proportional to the demand. λR is the average number of trips served per unit time, and γ_o is defined as the unitary operative cost per trip, considering the prorated cost of all the concepts involved. Then, the operative costs are formulated as:

$$C_O = \gamma_o \lambda R \qquad [€/h] \tag{17}$$

This approach neglects the possibility of economies of scale with respect to operative costs. This means that the extrapolation of γ_o values to very different demand levels in relation to from where they were estimated must be done with caution.

3.3.3. Repositioning costs, C_R

In order to balance the system to its original configuration, for every period h the imbalanced bicycles are moved to a more convenient location. This set of tasks is performed by agency employees and implies an additional operative cost. Due to its modeling complexity this part of the operative cost is analyzed here separately.

The average number of repositioning operations (i.e. bikes to be moved) is equal to the average system imbalance plus the average system decentralization (see Section 3.3.1.1 devoted to fleet size estimation for a description of these concepts). This is generally an overestimation because some compensation between both phenomena could exist. Because all sub-regions experience, to some degree, imbalance and decentralization, the assumptions of this rebalancing model imply that all sub-regions are visited by repositioning trucks every period h. In practice, stations with few bikes to be rebalanced would not be visited until the amount of repositioning operations to be performed was significant. In contrast, other stations may be visited several times during h. In this context, the rebalancing period h, must be understood as the inverse of the average number of visits per station and unit time. In addition, repositioning operations are considered to be continuously performed during h. So, the interpretation of h should not be that every h units of time all the sub-regions are found with their optimal number of bicycles. This is not the case. What happens is that during a period h all the imbalance movements have been performed. It should be understood that when a sub-region is left with its optimal number of bicycles, it may start being imbalanced immediately thereafter.

Repositioning is assumed to be performed with trucks (or vans with a trailer) with a capacity of k bikes, and with one driver. Trucks visit stations (or sub-regions) and load or unload bikes, depending on if there is an excess (i.e. in R_t, where $\varphi_r > 0$) or a shortage of bikes (i.e. in R_q, where $\varphi_r < 0$). Sub-regions are left with their optimal number of bikes. If trucks fill up, they need to travel to sub-regions with a shortage of bikes. If trucks become empty, they need to travel the opposite way. With such assumptions, repositioning trucks perform two types of "trips" between sub-regions. There are peddling trips, where trucks are visiting contiguous stations (i.e. either loading or unloading bikes, with partly full trucks), and line-haul trips where trucks travel full from sub-regions in R_t to sub-regions in R_q or return empty in the opposite direction.

3.3.3.1. Line-haul distance

Because of the truck capacity restriction, k, and because of the demand patterns leading to the systematic bicycle imbalance of different zones, generally far apart, full and empty line-haul trips appear. The average distance of these line-haul trips, $E_{[LH]}$, depends on the particular distribution of the imbalance levels over the service region R (i.e. the spatial distribution of $\varphi_{(r)}$), and can be approximated by the distance between the centers of gravity of the partitions R_q and R_t with respect to $\varphi_{(r)}$. Furthermore, $E_{[LH]}$ can be expressed as a fraction of the diameter of R, as proposed in Larson and Odoni (1981). In this context, the expected line-haul travel distance, $E_{[LH]}$, can be generalized as:

$$E_{[LH]} = f_{LH}\sqrt{R} \qquad \text{[km/line-haul trip]} \tag{18}$$

where $\sqrt{\mathrm{R}}$ approximates the "diameter" of the service region, assuming a fairly compact and convex region, and f_{LH} is the parameter between 0 and 1 which needs to be derived from the spatial distribution of $\varphi_{(r)}$.

The number of line-haul trips between sub-regions in R_t to sub-regions in R_q (i.e. full trips) can be obtained as the average imbalance in the system during a period h, over the capacity of the repositioning trucks. This number needs to be doubled, because there are an equivalent number of return (empty) trips. Note that the decentralization of the system, which leads to random imbalance levels for some stations and periods of time, does not contribute to the number of line-haul trips, as this phenomenon is addressed by peddling trips between stations. Considering that the average imbalance of the system can be expressed as $R_q\Phi_q\lambda h$, the number of line-haul trips is:

$$\#\, line - haul\ trips = \frac{2R_q\Phi_q\lambda h}{k} \tag{19}$$

Therefore, the total line-haul distance is obtained as:

$$D_{[LH]} = \frac{2R_q\Phi_q\lambda h f_{LH}\sqrt{R}}{k} \qquad \text{[km]} \tag{20}$$

3.3.3.2. Peddling distance

The total peddling distance can be computed as the solution to the transportation problem, where the objective is to minimize the cost of distributing a product (i.e. the bicycles) from a number of sources (or origins) to a number of destinations. Daganzo and Smilowitz (2004) developed simple approximations to the solution, useful to predict the performance of complex logistic systems in the planning stages. According to this reference, the average peddling distance per visited station, $E_{[PD]}$, is proportional to the inverse of the root-square of the density of the points to visit [visits/km^2].

Peddling distance in station-based systems

In station-based systems, bikes are found only at stations, so that the density of the points to visit by the rebalancing trucks is precisely the density of stations, Δ. Therefore:

$$E_{[PD\_station]} = \frac{1.1}{\sqrt{\Delta}} \qquad \text{[km/visited station]} \tag{21}$$

where 1.1 is the proportionality constant derived in Daganzo and Smilowitz (2004). Note that $\frac{1}{\sqrt{\Delta}}$ could be interpreted as an approximation to the "diameter" of the sub-region, assuming fairly compact and convex sub-regions. Considering that the total number of stations to visit is ΔR, the total peddling distance in station-based systems can be expressed as:

$$D_{[PD\_station]} = 1.1R\sqrt{\Delta} \qquad \text{[km]} \tag{22}$$

Peddling distance in free-floating systems

In free-floating systems, bicycles can be found at any location within the service region. This means that repositioning trucks need to pick-up bikes one by one in sub-regions with bikes in excess. In contrast, it is assumed that bikes are delivered all together in a central location of sub-regions with a bicycle deficit. The total peddling distance is approximated, as previously, using the formulae derived in Daganzo and Smilowitz (2004). However, with the free-floating assumptions, the density of points to visit is different when picking-up with respect to when delivering bikes.

In sub-regions in R_t (i.e. bikes in excess), the number of idle bicycles, $m_{i(h)}$, candidates for being picked-up just at the end of the repositioning period, can be expressed as:

$$m_{i(h)} = (m - \lambda R\tau_s)\frac{R_t}{R} + R\sqrt{2\lambda h\Delta} + R_q\Phi_q\lambda h \tag{23}$$

The first term in Equation 23 stands for the equilibrium number of idle vehicles. Note that only sub-regions in R_t are considered. The second term accounts for the expected number of vehicles in excess due to decentralization effects (see Equation 13 with a probability level equal to one standard deviation from the mean). The third and last term of Equation 23 stands for the average imbalance of the system, which is positive in R_t. Then the expected peddling distance per picked-up bicycle, $E_{[PD\_floating\_pickup]}$, is:

$$E_{[PD\_floating\_pickup]} = 1.1\sqrt{\frac{R_t}{m_{i(h)}}} \qquad \text{[km/pick-up]} \tag{24}$$

The total picking-up peddling distance, $D_{[PD\_floating\_pickup]}$ is obtained by multiplying the expected distance in Equation 24, times the average number of bikes to be rebalanced. This is:

$$D_{[PD\_floating\_pickup]} = \left(1.1\sqrt{\frac{R_t}{m_{i(h)}}}\right) \cdot \left(R\sqrt{2\lambda h\Delta} + R_q\Phi_q\lambda h\right) \qquad \text{[km]} \tag{25}$$

Bike delivery is done just at a single location for each sub-region, so that the total peddling distance in delivering rebalanced bikes is equivalent to the station-based case, just considering that the number of sub-regions in R_q where the bicycles are to be delivered is ΔR_q. Then:

$$D_{[PD\_floating\_deliver]} = 1.1R_q\sqrt{\Delta} \qquad \text{[km]} \tag{26}$$

The total peddling distance when rebalancing free-floating bike-sharing systems, $D_{[PD\_floating]}$, is simply obtained as the sum of the expected distances in picking-up and delivering bikes (i.e. Equations 25 and 26).

3.3.3.3. Repositioning teams

The time required to rebalance the system is invested in traveling between different sub-regions and in loading/unloading bicycles to/from the trucks. This total repositioning time can be expressed per unit time, and this would correspond to the number of required repositioning teams. This is formulated as:

$$\#\,repo\ teams = \left[\frac{D_{[LH]}+D_{[PD]}}{V_k} + \left(R\sqrt{2\lambda h\Delta} + R_q\Phi_q\lambda h\right)2\delta\right]\frac{1}{h} \tag{27}$$

Where V_k is the average travelling speed of repositioning trucks in the service region, so that the first term in Equation 27 reflects the total travelling time. The second term in brackets is the average number of bikes to be rebalanced, and δ is the average time required to load or unload one bike to/from the truck. The factor 2 affecting δ accounts for the fact that all bikes need to be loaded and unloaded. Take into account that in free-floating systems δ will be higher, since the economies of scale of picking-up bikes at a single location (i.e. the station) are lost. Finally, both terms are divided by h, the repositioning period, to express the total repositioning time per unit time. It is interesting to note that the terms of Equation 27 accounting for the average imbalance of the system are independent of h, while those accounting for the decentralization decrease with h.

3.3.3.4. Repositioning costs

Repositioning costs account for the depreciation, maintenance and operation of repositioning trucks, including the labor, which is the main cost. γ_t [€/h] is the prorated cost per unit time of one repositioning team, so that repositioning costs are obtained as:

$$C_R = \gamma_t \cdot \#repo\ teams \qquad \text{[€/h]} \tag{28}$$

3.3.4. User access cost, C_A

The accessibility of the system is determined by the users' average access time to bicycles. It is assumed that users access the bicycle location by walking. Then, the access cost can be calculated by considering the average access distance [km], the walking speed V_w [km/h], and the value of time β [€/h], which monetizes the access time. The cost per customer needs to be multiplied by the average demand rate λR to account for all users in the system.

$$C_A = (E[access\ origin] + E[access\ destination]) \cdot \frac{\beta\lambda R}{V_w} \qquad \text{[€/h]} \tag{29}$$

The expected access distance, at the origin but also at the destination (i.e. from where the bike is left to the desired final destination), depends on whether the system is configured as station-based or free-floating. On the one hand, in station-based systems the user walks to the nearest station with available bikes. So, the access distance depends on the station density. The same happens at the destination. On the other hand, in the free-floating case the user walks to the nearest available bicycle. So, the access distance at the origin depends on the available bicycle density (i.e. the idle fleet, m_i, per unit area). Also, it is assumed that the user leaves the bike just at the door of his destination, so that the access costs at destination are null in the free-floating scenario.

Considering a L1 metric and assuming fairly convex sub-regions, the average access distance can be approximated by half (i.e. ¼ in each of the 2 dimensions) the average diameter of the influence region of one station, or of one idle bicycle, for free floating configurations and assuming uniformly distributed idle bicycles in the sub-region (Larson and Odoni, 1981). This is:

$$\text{Station-based:} \quad E[access\ origin] = E[access\ destination] = \frac{1}{2\sqrt{\Delta}} \quad \text{[km]} \tag{30}$$

$$\text{Free-floating:} \quad E[access\ origin] = \frac{1}{2} \cdot \sqrt{\frac{R}{m_i}} \quad \text{[km]} \tag{31}$$

Where the average idle fleet, m_i, is estimated as the total fleet minus the average number of bicycles in use:

$$m_i = m - \lambda R \tau_s \tag{32}$$

Note that the idle bicycle fleet is variable during the repositioning period, h, because of the imbalance and decentralization phenomena. Therefore, the average accessibility level of free-floating systems is not constant in time and space.

Under the previous assumptions, the access distance in free-floating systems does not depend on the density of sub-regions, Δ, but only on the number of available bicycles. This means that while the reduction of Δ implies a reduction of agency costs, there is no associated user cost. In the absence of trade-off, Δ is minimized for a situation where the whole service region forms a single sub-region. This result is not realistic since in such a case, the assumption of uniform distribution of available vehicles in the sub-region would not hold, and rebalancing operations would be greatly underestimated. For this reason, it is necessary to set a minimum number of sub-regions in the free-floating scenario, which translates to a minimum Δ as a restriction in the optimization. This restriction will be binding. $\Delta_{min} = 1.5$ [sub-regions/km^2] is selected considering that the uniformity distribution is acceptable in sub-zones of $\frac{1}{\Delta_{min}} = 0.67$ [km^2] leading to a worst case average access distance of approximately 400 m (the maximum access distance would be the double of this) when the bicycle availability is minimum (i.e. one single bike in the sub-region). This is the standard for the worst accessibility level of most bike-sharing systems (Lin & Yang, 2011; Garcia-Palomares et al., 2012) and also the usual standard accepted for public transportation systems [TRB (2013)]. Consider that Millward et al. (2017) found that the often-used walking range is shorter than 600m, and very few exceed 1,200 m.

3.3.5. No-service penalty, C_{NS}

This term of the generalized cost function aims to take into account the extra costs users perceive when they do not find any available bicycle, or any free parking slot (in station-based systems). When a user faces this no-service situation, he might decide to wait, to go to another sub-region, or to use another transportation mode. In any case, this implies a penalty to the user. These user costs are modeled considering the decision variables P_e and P_f, the probability of empty or full stations, respectively, affecting the total demand of the system, λR. Then, the overall no-service penalty in the system can be formulated as:

$$C_{NS} = \lambda R \beta_{lost}\left(P_e t_{lost\_e} + P_f t_{lost\_f}\right) \quad (33)$$

where β_{lost} is the users' perceived value of lost time (i.e. in general $\beta_{lost} > \beta$) and t_{lost_e}, t_{lost_f} are the costs of no-service (in units of time), either due to an empty or full station. By definition, P_f is null in free-floating systems, because there are no stations. In station-based systems, P_f depends on the total number of bicycle parking slots, M. Because P_f decreases with M, and there is no agency cost associated with M (recall that it has been assumed a fixed cost per station, independent of their size), the optimization procedure would lead $P_f \rightarrow 0$ and $M \rightarrow \infty$. To avoid this situation, P_f is considered fixed at 1%. This is consistent with the previous assumption of no (or very little) restriction in terms of urban space consumption, and from the user perspective it eliminates the more penalizing situation of not knowing what to do with the bicycle once at the destination.

3.4. Extension to electrical bikes

Station-based sharing systems are especially suited for the use of electrical bikes. Battery recharging is the only additional requirement, and this task can be easily performed while e-bikes are docked at stations. The previous model for station-based bike-sharing systems can be easily extended to the e-bikes context by considering that e-bikes and e-stations might be more expensive and by including the battery recharging restriction. This can be done by imposing, over the whole system, that the average battery consumption per unit time has to be lower than the average battery recharging per unit time. Otherwise, the overall battery level in the system would be reduced with time, and eventually would fail in serving the demand. Note that this does not mean that during a particular peak period the overall battery level could not be reduced, but that on average this deficit will be recovered. This assumes 24/7 operating period, the worst case situation, because batteries cannot use closed periods to recharge.

On the one hand, T_{eb} is defined as the available usage time of a fully charged e-bike. This can be estimated as D_{eb}/V_{eb}, the ratio of the maximum distance covered with a fully charged e-bike over the average cycling speed in an urban environment. On the other hand, T'_{eb} is defined as the time required to fully charge the e-bikes' battery from empty. Therefore T_{eb}/T'_{eb} is the time available for using the e-bike per unit time charging. With these definitions the previous battery recharging restriction can be expressed as:

$$\lambda R \tau_s < m_i \frac{T_{eb}}{T'_{eb}} \quad (34)$$

Recall from Equation 32 that m_i is the average number of idle bicycles (i.e. recharging at stations). Substituting Equation 32 into Equation 34, the battery recharging restriction is simplified to:

$$m > \lambda R \tau_s \left(1 + \frac{T'_{eb}}{T_{eb}}\right) \quad (35)$$

If the fleet size (see Equation 15) does not fulfill this restriction, then additional e-bikes would be needed just to provide enough battery level. Otherwise, the battery restriction is not binding, meaning that turning the system into electric does not imply additional fleet. Note that this last situation will happen if the additional fleet required to account for the temporal demand fluctuations and the various sources of stations' imbalance represents a fraction larger than $\frac{T'_{eb}}{T_{eb}}$ of the number of bicycles in use (i.e. $\lambda R \tau_s$). Or in other words, that the most restrictive case in terms of battery recharging would be a system with very low imbalance and with uniform demands.

3.5. On the mathematical properties of the generalized cost function

The generalized cost function (GCF) derived in the previous sections acts as an objective function, and will be minimized with respect to the decision variables in order to obtain the optimal system design. The continuous approximations modeling framework allows ensuring that the proposed GCF is well behaved. The GCF is continuous and differentiable, being the sum of continuous and differentiable functions. Moreover, the optimization problem for each decision variable could be expressed as:

$$GCF^* = min\{GCF = Ax^a + Bx^{-b}: 0 < x \leq \infty;\ a, b > 0\} \tag{36}$$

Where x is the decision variable, and the optimum is achieved for some x^*. Problems of this form are known as generalized EOQ optimizations, a terminology inherited from the logistics discipline where the economic order quantity (EOQ) is the order quantity that minimizes the total holding and ordering costs. The properties of this type of optimization have been thoroughly studied (e.g. Daganzo, 2005), proving the existence of a single and robust minimum. The robustness property, which means that the curvature of the GCF near the optimum is low, implies that the sensitivity of the optimal values of the decision variables to variations in the input parameters is equally low. This is a nice property for an optimization problem in the planning stage of a system, because it ensures reliable solutions although many inputs being highly uncertain. It can be proved that the GCF is more robust for lower values of the powers a and b.

4. The Barcelona's Bicing case study: parameter estimation, model validation and system optimization

Since 2007, *Bicing*, the bike-sharing system in the city of Barcelona (Spain), has been operative. *Bicing* is a station-based system where members are identified using a membership card. Today it covers all the city districts, and its 106,430 members perform approximately 1.5 million trips every month (www.bicing.cat).

4.1. Parameter estimation for the Bicing system

Table 1 presents the main design variables of the *Bicing* system, and Table 2 summarizes the parameter estimation in the context of Barcelona's *Bicing* bike-sharing system. The unitary costs estimated in Table 2 are slightly higher than those reported in Frade and Ribeiro (2015) for the city of Coimbra in Portugal.

Table 1. Barcelona *Bicing*'s system design variables

Variable description	Notation	Units	Value	Source
Available fleet size	m	[bikes]	5 236	Data from the Barcelona's *Bicing* web service[1]
Average bike daily usage	–	[trips/bike·day]	9.52	Total daily demand / available fleet size
Total number of parking slots	M	[slots]	10 246	Data from the Barcelona's *Bicing* web service
Slots / fleet ratio	M/m	[slots/bike]	1.96	
Stations density	Δ	[stations/km^2]	8.20	402 stations in 49 km^2
Repositioning period	h	[hours]	8.39	On average the repositioning frequency of stations is 2.86 times/day. This includes 10% of the stations that are visited more than 6 times/day and 20% that are not visited. h is obtained as the inverse of the average repositioning frequency. Data from the Barcelona's *Bicing* web service.
Repositioning operations	–	[bikes/day]	13 164	Data from the Barcelona's *Bicing* web service. Confirmed by Alonso et al. (2015).
Number of repositioning teams	#repo teams	-	24	Barcelona's *Bicing* hires 115 workers to perform repositioning operations (López, 2009). The estimation of the #repo teams assumes 24/7 system operation and an annual working shift of 1826 h.
Probability of empty stations	P_e	-	0.1355	Estimated from data provided by the Barcelona's *Bicing* web service, from where the fraction of the operating time when a particular station is full or empty can be obtained. The average probability is simply computed as a weighted average of these fractions, where the weights are the average daily demand of each station.
Probability of full stations	P_f	-	0.1247	

Table 2. Input parameters for the Barcelona case study.

	Parameter description	Notation	Units	Value	Source
Demand	Area of the service region	R	[km^2]	49	Barcelona's *Bicing* website (www.bicing.cat)
	Average demand density	λ	[trips/h·km^2]	42.37	Data from the Barcelona's *Bicing* web service. This corresponds to an annual demand of 18.2 million trips.
	Average imbalanced demand fractions	Φ_t	-	0.129	Estimated from data provided by the Barcelona's *Bicing* web service (see Section 4.1). $\Phi = \frac{\Phi_t R_t + \lvert\Phi_q\rvert R_q}{R}$ (See Equation 6).
		Φ_q	-	-0.108	
		Φ	-	0.118	
	Imbalance partitions of R	R_t	[km^2]	0.458·R	Estimated from data provided by the Barcelona's *Bicing* web service (see Section 4.1)
		R_q	[km^2]	0.549·R	
	Average service time	τ_s	[min]	13.28 (s)[2] 20.78 (f)	Barcelona's *Bicing* website (www.bicing.cat). In free-floating systems, a max. reservation time of 15min is considered. In this context, bicycles are on hold 7.5min on average. This is added to the average service time.
Users	Average walking speed	V_w	[km/h]	3.6	Catalonia's mobility observatory[3]
	Users' average value of time	β	[€/h]	11.4	Official value used for transport investment appraisal in Barcelona [*ATM - Autoritat del Transport Metropolità*][4]
	Users' average value of time lost	β_{lost}	[€/h]	26.7	$\beta_{lost} = 2.34\beta$ (Asensio and Matas,2008) for an average trip (94% commuters with scheduled arrival time) in the Barcelona region.
	No-service penalty (empty)	t_{lost_e}	[min]	10.2	Average users' wait at an empty station (*Ajuntament de Barcelona*, 2007)

[1] Barcelona's *Bicing* web service provides data regarding the location (x, y, z coordinates) and size of stations, and their real time bike occupancy (with per minute updates). http://opendata-ajuntament.barcelona.cat/data/ca/dataset/bicing. Data was extracted on May 7th, 2014.

[2] (s) stands for station-based systems; (f) stands for free-floating systems.

[3] *Mobilitat. Generalitat de Catalunya*. http://mobilitat.gencat.cat/es/serveis/mitjans_de_transport/a_peu/

[4] This is approximately 80% of the average hourly income in Spain.

	No-service penalty (full)	t_{lost_f}	[min]	20.4	Post-process waits feel longer than pre-process delays. A factor of 2 is considered (Maister, 1984).
Agency unitary costs	Bicycle unitary cost and depreciation	γ	[€/h·bike]	0.0279 (s)	*Bicing* bicycles cost 400€ each and they have a useful life of 1.7 years (López, 2009). This short lifespan of bicycles is in accordance with the arguments and data provided in Martinez et al. (2012), where it is asserted that in systems under operation, bicycle theft and vandalism has proven to be high. In addition, γ includes the fact that 4% of the bikes are not available due to maintenance (www.bicing.cat).
				0.0549 (f)	In free-floating systems, bikes incorporate all the functionalities of stations. Free-floating bikes cost 786€ each (Lee, 2017). Same previous assumptions are considered.
	Station unitary cost and depreciation	γ_s	[€/h·station]	0.311	11 M€ for 402 stations in the *Bicing* system, considering a useful life of 10 years (López, 2009).
	Operative cost per trip	γ_o	[€/trip]	0.6369	Operative cost of the *Bicing* system (excluding repositioning) in 2010 was 7.1 M€. Demand during the same period was 11.1 million trips (López, 2009).
	Cost of a repositioning team (truck + labor)	γ_t	[€/h·team]	22.8	The total repositioning cost (trucks + labor) of the *Bicing* system during 2010 was of 3.2 M€, for a total of 115 workers (López, 2009). γ_t is obtained considering an annual working shift of 1826 h and an efficiency factor of 2/3 (i.e. 1/3 of the available hours are lost times and inefficiencies).
Repositioning	Capacity of a repositioning truck	k	[bikes]	32	Truck capacity in Barcelona's *Bicing* system.
	Average cruising speed of repositioning trucks	V_k	[km/h]	20.6	Average cruising speed for motorized vehicles in the city of Barcelona (*Ajuntament de Barcelona*, 2016)
	Line-Haul distance repositioning parameter	f_{LH}	-	0.339	$f_{LH}\sqrt{R}$ is the L1 distance between the centers of gravity of partitions R_t and R_q in R, where each station location is weighted by its respective unbalance level $\varphi_{(r)}$. Data from the Barcelona's *Bicing* web service.
	Time to load / unload one bike	δ	[seconds]	37.5 (s)	In Barcelona's *Bicing* system it takes 20 min to load / unload one truck (i.e. 32 bikes)[5].
				63.75 (f)	Pick-up in free-floating systems δ_p=90s (Pal and Zhang, 2017, dispersed bicycles). Delivery operation is equivalent to station-based systems δ_d=37.5s. δ represents the average value.
Electrical bikes	Electrical Bicycle depreciation	γ_{eb}	[€/h·bike]	0.0838	Electrical *Bicing* pilot in Barcelona. Electrical bikes cost 1,200€ each (*Ajuntament de Barcelona*, 2014; *La Vanguardia*, 2014). Useful life and maintenance rate are assumed to be the same as conventional bikes (i.e. 1.7 years and 4%).
	Electrical Station depreciation	γ_{s_eb}	[€/h·station]	0.4921	From the stations' investment cost in the electrical *Bicing* pilot (1.94M€ for 45 stations), and considering a useful life of 10 years (*Ajuntament de Barcelona*, 2014; *La Vanguardia*, 2014).
	Operative cost per electrical bike trip	γ_{o_eb}	[€/trip]	1.1061	The total operative cost of the electrical *Bicing* pilot (excluding repositioning) was expected to be of 2.2 M€ and the total demand of 2 million trips for a 3.5 year period (*Ajuntament de Barcelona*, 2014; *La Vanguardia*, 2014).
	Usage time of a fully charged e-bike	T_{eb}	[hours]	2.7	Computed as D_{eb}/V_{eb}, the ratio of the maximum distance covered with a fully charged e-bike (40 km) over the average cycling speed in urban environment (15 km/h).
	Time required to fully charge the e-bike battery from empty	T'_{eb}	[hours]	2	(*Ajuntament de Barcelona*, 2014)

[5] Personal communication with operating agency representatives.

4.1.1. Imbalance parameter estimation in the planning phase of a bike-sharing system

From all the input parameters to the model, those related to the demand imbalance (i.e. Φ_t, Φ_q, R_t and R_q) are probably the most difficult ones to estimate in the planning phase of a bike-sharing system. While the average demand density, λ, can be set as a policy goal (i.e. the demand level expected to materialize in the future and for which the system is going to be optimally designed, as discussed in Section 3.2.1), the imbalance parameters depend on the origin / destination structure of trips, which is difficult to anticipate.

Previous research on the asymmetry of bike sharing trips has shown that most of the users do not perform closed trip chains using the bike-sharing system (i.e. origin-destination + destination-origin trip chains). For example, Zhao et al. (2015) concluded, for a case study in Nanjin (China), that only 40% of the trips belong to closed trip chains. In addition, Ehrgott et al. (2012); Winters et al. (2011) show that even in closed trip chains, the selected routes are not necessarily symmetrical, because users tend to select routes away from traffic, pollution, debris and poor road maintenance. Cyclists also tend to avoid long steep sections in the selection of the best route, and this generally depends on the direction of the trip.

In spite of this, the spatial asymmetry in the individual usage of bike-sharing systems does not necessarily contribute to the system's imbalance. Note that the asymmetric behavior of one user could be balanced by the opposite behavior of another user. It is the generalized asymmetry of the O/D structure of trips which leads to imbalance situations. Two main factors have been identified to contribute to bike-sharing imbalance. First, the existence of segregated land-use urban patterns with a clearly asymmetrical temporal distribution of trips. For instance, the morning commute creates a scarcity of bicycles at residential areas and a surplus in activity and employment centers, or transit stations (Noland et al., 2016; Zhao et al., 2015). In the absence of rebalancing, such areas would be imbalanced and poorly served during most of the day, until the evening commute compensates the situation. In contrast, mixed land-use patterns contribute to a higher and more balanced bike-sharing demand (Campbell et al., 2016). Second, the gradient in the terrain elevation also contributes to bike-sharing imbalance, as users tend to avoid uphill trips (Ehrgott et al., 2012; Winters et al., 2011). The higher areas in the bike-sharing service region tend to exhibit a scarcity of bicycles and poor service, contributing to lower demands (Faghih-Imani et al., 2017).

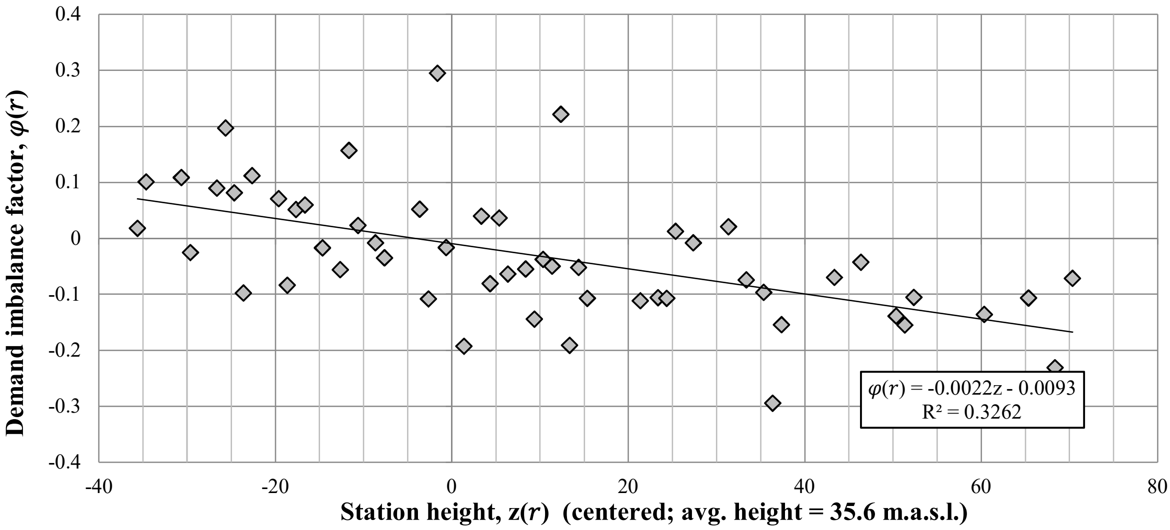

Fig. 2. Relationship between the demand imbalance factor and the stations' height.
Note: Data extracted from the *Bicing* web service on May 7th, 2014.

In such context, the distribution of stations' height can be used as a proxy to approximate the system's demand imbalance. Figure 2 shows the relationship between the stations' height, $z_{(r)}$, and their imbalance level, $\varphi_{(r)}$, for the Barcelona's *Bicing* system. Results corroborate that one the main causes of the system imbalance in bike-sharing systems is that uphill trips are less desirable, so that bicycles tend to "precipitate" to lower elevation stations. Given this behavior, the height distribution across the service region could be used as a first approximation to estimate these imbalance parameters in the absence of more detailed data in the planning phase. For instance, one should expect higher imbalance levels for higher average slopes[6] in the service region. In addition, the area of the partition of R with heights above the mean could be assimilated to R_q (i.e. more requests than returns), and the complementary would be assimilated to R_t (i.e. more returns than requests).

In flat cities, where the gradient in the terrain elevation is negligible, it is the land-use urban pattern what steers the system's demand imbalance. In such cases, the demand imbalance of bike-sharing systems will be similar to that of other on-demand transportation systems, such as taxi or car-sharing systems. The demand imbalance for these systems can then be used to estimate the imbalance parameters in the planning phase of bike-sharing systems.

4.2. Model validation

The proposed optimization framework consists in minimizing a generalized cost function, composed of the main costs of a bike-sharing system (i.e. agency + users costs). In Section 3, these costs have been modeled in terms of the decision variables (i.e. subject to optimization) and some parameters. In addition, in order to reduce the number of degrees of freedom in the optimization (i.e. the number of independent decision variables), relationships between variables have been developed. This allowed establishing dependent decision variables[7] (i.e. fleet size, m; number of parking slots, M; and number of repositioning teams) expressed as a function of independent design variables (i.e. density of stations, Δ; repositioning period, h; and no-service probabilities, P_e and P_f).

In order to validate the previous modelling approach two issues must be addressed. First, an accurate estimation of the model parameters; and second, the validation of the several models composing the generalized cost function. On the one hand, the parameters' estimation process has been presented in Table 2, noting that this is not free of difficulties, especially for those parameters which are subjective for each individual (e.g. different values and perceptions of time: β, β_{lost}, t_{lost_f}). For such parameters, the validation of their estimations always involves a large degree of uncertainty (Wheat and Batley, 2015). On the other hand, the validation of the models requires empirical observations of both, dependent and independent variables, in addition to the estimated parameters. This allows comparing the observed with the predicted dependent variables and establishing the models' accuracy. Note that the validation of the model does not require optimal designs, but just empirical observations from a particular case study. However, this validation is not always possible, because some of the variables are difficult to observe, especially those related to the user before entering the system. For instance, in the present models' validation emulating the *Bicing* system in Barcelona, the modeling of the access distance could not be validated, because observed values of the distance covered by users when accessing the system are not available. In contrast, results of the validation of the model for the bicycle fleet size, m, for the number of station-based parking slots, M, and for the repositioning model, are shown in Table 3. It can be seen that the

[6] The average slope in the Barcelona *Bicing* service region is 1.5%. The average slope is computed as $\frac{P_{90th}(z_{(r)})-P_{10th}(z_{(r)})}{\sqrt{R}}$, where $P_{i\,th}$ refers to the i'th percentile.

[7] m is just one possible selection as a dependent variable. This is consistent with the explicit formulation for m (see Equation 15). In spite of this, any of the decision variables could be considered the dependent variable, and expressed in terms of the others with more or less complexity. For instance, it is particularly easy to obtain $P_e = f(\Delta, h, m)$.

comparisons between observed and predicted values result in relative errors below 13%, an indicator of the goodness of the model at this aggregate level.

Table 3. Fleet size, station-based parking slots and repositioning models validation with Barcelona *Bicing*'s observed data

Variable description	Notation	Units	Model results*	Observed data	Relative error
Available fleet size	m	[bikes]	5 622	5 236	7.37 %
Average bike daily usage	–	[trips/bike·day]	8.86	9.52	- 6.87 %
Total number of parking slots	M	[slots]	10 976	10 246	7.12 %
Slots / fleet ratio	M/m	[slots/bike]	1.95	1.96	- 0.23 %
Repositioning operations	–	[bikes/day]	13 621	13 164	3.47 %
Number of repositioning teams	#repo teams	–	21	24	- 12.5 %

* Considering the *Bicing's* design variables: $\Delta = 8.20$ [stations/km^2]; $h = 8.39$ [h]; $P_e = 0.1355$; $P_f = 0.1247$; and all the parameters as in Table 2.

4.3. Optimization of the strategical design of a bike-sharing system: application to the Barcelona's Bicing

Optimization of the system consists in the minimization of the generalized cost function so that optimal decision variables are obtained and some optimal key performance indicators can be derived. Different optimization scenarios are analyzed. First, a Lagrangian (i.e. unrestricted) optimization is performed. The Lagrangian scenario results in minimum overall costs, considering both user and agency costs. This global and unrestricted optimization depends on the unitary costs considered, which act as weighting factors for the several concepts in the generalized cost function (see Section 3.3). In particular, results depend on β [€/h], the users' value of time. The Lagrangian approach considers an average β, estimated for a particular society, so that the obtained solution is optimal from the social point of view.

However, in many cases the budget to deploy bike-sharing systems is limited, and generally not enough to provide this social optimum level of service. It is typical in these situations that policy makers define the minimum level of service the system should provide (i.e. a service standard). Given a limited budged, such restrictions are binding, and then the optimization consists in minimizing the agency costs while providing this minimum accepted level of service. These scenarios are referred to as "standards" optimization.

The defined standard's scenario imposes limited accessibility and availability of the service. Regarding accessibility, instead of minimizing users' access costs, the station density is set to $\Delta = 8.20$ stations/km^2, corresponding to an average access distance of 175m (see Equation 30). Regarding the availability, instead of minimizing no-service penalties, a default percentage of not served users is accepted. This is introduced in the model by fixing $P_e = 0.1355$, meaning that, on average, 13.55% of the demand will not find any available bike within the desired sub-region. These service standards correspond to the current level of service offered by Barcelona's *Bicing* system.

In Tables 4 and 5, the results of the system optimization in the Lagrangian and standards scenarios are presented. Both, station-based and free-floating configurations are analyzed. Table 4 focuses on system design and Table 5 on system costs.

Results in the Lagrangian scenario show that the optimal level of service, from the social point of view, should be significantly higher than the current level of service offered by Barcelona's *Bicing* system. In this social optimum configuration, all user costs are reduced. This includes lower access distances to stations (i.e. higher station density), and extremely low probabilities of no-service, either because of empty or full stations (i.e. larger bicycle fleet and many more parking slots at stations). The optimal repositioning period is also shorter than the current one, implying higher reposition intensity. This, together with the increased decentralization (i.e. due to many more stations), implies that more repositioning trucks are necessary. Obviously, this improved level of service increases the agency cost (i.e. by 33%), with the objective of achieving a greater reduction in user cost (i.e. -71%). Overall, the total (i.e. social) cost of the system is reduced by approximately 45%.

Table 4. Optimization results for the Barcelona's *Bicing* case study. Decision variables and system design.

	Variable	Units	Lagrangian		Standards (given Δ, P_e)		Barcelona's Bicing observed data
			Station-Based	Free-Float	Station-Based	Free-Float	
Decision Variables	Density of stations – Δ	stat/km^2	$20.65^{40.0}_{10.5}$	$1.50^{2.5}_{1.5}$	**8.20**	**1.50**	**8.20**
	Repositioning period – h	hours	$6.81^{23.9}_{2.3}$	$8.30^{23.6}_{3.1}$	$10.77^{62.2}_{2.1}$	$17.66^{54.8}_{5.5}$	**8.39**
	Empty Stations – P_e	-	$0.0061^{0.042}_{0.001}$	$0.0015^{0.016}_{0.001}$	**0.1355**	**0.1355**	**0.1355**
	Full Stations – P_f	-	**0.01**	-	**0.01**	-	**0.1247**
Bicycle fleet	Available fleet size – m	units	$14\,761^{26\,533}_{10\,057}$	$6\,542^{11\,630}_{4\,062}$	$6\,460^{20\,890}_{4\,243}$	$5\,450^{11\,907}_{2\,843}$	**5236**
	Avg. # of vehicles in use	units	460	719	460	719	-
	Stock for demand fluctuations	units	54	80	24	30	-
	Stock for avg. imbalance	units	827	1 008	1 309	2 145	-
	Stock for decentralization	units	13 420	4 735	4 668	2 556	-
	Bicycles under maintenance	units	590	262	258	218	**764**
	Bicycle usage	trips/day	3.38	7.62	7.71	8.41	**9.52**
Stations	Total number of stations – ΔR	units	$1\,012^{1\,960}_{515}$	74^{123}_{74}	**402**	74	**402**
	Total number of slots – M	units	$28\,084^{54\,798}_{17\,619}$	-	$17\,684^{55\,656}_{9\,538}$	-	**10 246**
	Ratio slots/fleet – M/m	-	1.90	-	2.74	-	**1.96**
Access	Avg. access distance (origin + destination)	km	$0.220^{0.309}_{0.158}$	$0.046^{0.065}_{0.034}$	**0.349**	$0.051^{0.083}_{0.033}$	**0.349**
Repositioning	Repositioning rate	veh/h	906.73	313.21	515.11	252.92	**548.5**
	Total repositioning time	hours/h	21.15	13.28	12.30	10.73	-
	Total time lost due to inefficiencies	hours/h	10.75	6.64	6.15	5.37	-
	# Repositioning teams	units	33^{56}_{19}	20^{29}_{15}	19^{44}_{12}	17^{24}_{13}	**24**
	Avg. team performance	veh/ team·h	$27.48^{28.46}_{26.31}$	$15.66^{15.68}_{14.92}$	$27.11^{28.28}_{25.50}$	$14.88^{16.23}_{14.88}$	-
	Avg. daily visits per station	-	3.52	2.89	2.23	1.36	**2.86**

* Recall that the *Bicing* system serves an area $R = 49$ km^2, with an average demand of 49 832 trips/day (i.e. $\lambda = 42.37$ trips/h·km^2), and an average imbalance $\Phi = 0.118$ (see Table 2).

** Values in bold represent inputs to the model and observed data. Model results are in non-bold characters.

*** Upper and lower bounds are shown beside the optimal values for each variable. These bounds are defined as the maximum deviation from the optimal values so that the increase in total system cost is <5%, assuming that all other variables are kept at their optimal value.

Table 5. Optimization results for Barcelona's *Bicing* case study. Costs:

	Type of cost	Units	Lagrangian		Standards (given Δ, P_e)		Barcelona's *Bicing* current costs
			Station-Based	Free-Float	Station-Based	Free-Float	
Agency Costs	Infrastructure costs	€/h	758.93	359.16	319.46	299.20	294.24
	IC - Bikes	€/h	444.31	359.16	194.44	299.20	169.22
	IC - Stations	€/h	314.62	-	125.02	-	125.02
	Operation costs	€/h	1 322.42	1 322.42	1 322.42	1 322.42	1 322.42
	Repositioning costs	€/h	424.12	303.48	281.21	245.32	310.70
User Costs	Access costs	€/h	1 447.06	301.58	2 295.54	334.58	2 295.54
	No service penalty	€/h	245.61	13.83	1 465.51	1 277.02	3 627.48
Total Costs	Total agency costs	€/h	2 573.06	1 985.05	1 923.10	1 866.94	1 927.35
		M€/year	22.54	17.39	16.85	16.35	16.88
	Total users' costs	€/h	1 692.67	315.41	3 761.04	1 611.60	5 923.02
	Total costs	€/h	4 265.73	2 300.46	5 684.14	3 478.54	7 850.37
Avg. Costs	Generalized cost per trip[1]	€/trip	2.05	1.11	2.74	1.68	3.78
	Single Fare[2]	€/trip	1.24	0.96	0.93	0.90	0.93
	Annual Fare[3]	€/year	211.78	163.38	158.29	153.66	158.64[4]

[1] Average cost per trip, including agency and users' costs.
[2] Average agency cost per trip. This is equivalent to the fare users need to pay in the absence of subsidies.
[3] Assuming 147.3 trips/member·year. This is the average annual usage per member in Barcelona's *Bicing* system.
[4] Currently, Barcelona's *Bicing* system is subsidized. Members pay only around 32% of this cost (i.e. 47.16 €/member·year)

The standards scenario aims to optimize the system design by considering the current level of service offered by the *Bicing* system[8]. This is to minimize the agency cost by only optimizing the trade-off between the size of the bicycle fleet and the intensity of the repositioning operations. The results show that although the optimal design is achieved for a somewhat larger fleet and lower repositioning level, the overall benefits in total agency costs with respect to the current design are insignificant. This means that the actual design of the *Bicing* system in Barcelona is adequate, accepting the provided level of service as a standard. In spite of this, note that the no-service penalty at the destination, P_f, could be reduced by adding more parking slots at critical locations. An increase of 73% in the overall size of stations would reduce the probability of full stations from the current 12.5% to 1%, almost eliminating this important penalty for users. This would significantly improve the level of service offered. The main cost of such decision would be the increase in urban space consumption. The distribution of urban space to different transportation modes and activities should be the object of a higher level of debate in the city planning.

In addition, results show that optimal designs are robust, as predicted by the generalized EOQ structure of the optimization problem. This means that deviations from the optimal values do not imply sharp increases in the overall costs. This can be observed from the results in Table 4, by realizing the significantly wide ranges of near-optimal designs for which the overall cost does not increase more than 5% with respect to the optimal.

Finally, Tables 4 and 5 also allow comparing the station-based configuration, currently implemented in the *Bicing* system, with the free-floating alternative. The results show that free-floating configurations are able to provide a better level of service, in terms of a reduced access distance, with a smaller bicycle fleet and lower repositioning costs. The benefits of the free-floating configuration are more evident when the accessibility to the system needs to be high (i.e. Lagrangian scenario). This is because in the station-based configuration many

[8] The service standards considered include a maximum accepted access distance and a maximum no-service probability at the origin (i.e. due to empty stations). However, the no-service probability at the destination (i.e. due to full stations) is not set as a service standard. This is because the cost of urban space consumption is not considered here, and there is no cost directly related to M, the total number of parking slots in the system.

stations are needed to achieve the required higher accessibility, and this also implies a larger bicycle fleet because of the higher decentralization. In contrast, when a low accessibility level suffices (e.g. in the standards scenario), the benefits of free-floating diminish. This is because the penalties of station-based are less in this case, and because a minimum number of sub-regions is also necessary in free-floating, to ensure, to some extent, a uniform distribution of bicycles throughout the service region. This is why in the standards scenario the agency costs are similar in station-based and free-floating configurations.

In spite of the clear benefit in the total costs of free-floating systems, they suffer from several important drawbacks. First, the accessibility of the system is given by the number of idle bicycles dispersed through the service region, and because this number changes with time, the access distance will vary significantly during the day. This means that during peak demand periods, the access distance will grow due to the lower bicycle availability. This variability in the level of service, and its deterioration in peak hours, penalizes the user perception of free-floating systems. In contrast, in station-based systems, the accessibility is given by the density of stations, which is constant with time. Second, in station-based systems the precise location of stations can be strategically selected in order to mitigate the average imbalance of the system by avoiding locating stations at the highest or lowest elevations in the service region. The ability to influence the origins and destinations of the trips with the station location is lost in free-floating systems, and therefore the average imbalance is expected to increase. This would imply an increase in the overall costs of the system (see Figure 4). Third, because bicycles lose the protection of stations when not used, and can be left in isolated zones inside the service region, they are more exposed to vandalism, deterioration and theft. This will probably reduce the useful life of bicycles, which is translated into a further increase in the prorated bicycle cost. And fourth, free-floating configurations are not suitable for the implementation of electric bike-sharing since the lack of stations complicates the battery recharging process.

4.3.1. Optimization results considering electrical bicycles

Tables 6 and 7 show the results of the system optimization in case of using electrical bikes in Barcelona's *Bicing* context. In order to facilitate the comparison, the previous results, using traditional mechanical bicycles (i.e. from Tables 4 and 5), are partially reproduced. Only station-based configurations are considered since this allows the battery recharging process. With respect to using mechanical bicycles, electrical bikes will imply higher infrastructure costs (of bicycles and stations), and also higher operative costs (see the electrical bicycle section of Table 2). In addition, the battery consumption restriction (see Equation 35) needs to be considered. The users' behavior, in terms of the overall demand and O/D structure of the trips, is assumed to not be affected by the adoption of electrical bicycles. This means, for instance, that the effects of electrical propulsion on the average duration of the trip (τ_s) and the average imbalance of the system (Φ) are not considered.

Results show that battery recharging does not represent an active restriction in electrical bike-sharing systems. This is because the minimum required fleet that allows a sufficient battery level to keep the system running is well below the optimal fleet required to account for demand fluctuations, system imbalance and decentralization. It can be concluded that station-based sharing systems are well posed for the implementation of electrical vehicles.

Secondly, it can be seen that agencies would confront the increase in the unitary cost of bicycles and stations (i.e. the infrastructure costs) by increasing the repositioning intensity (i.e. labor costs mainly). This implies a reduction of the bicycle fleet and station density, compensated by a reduction of the repositioning period. In addition, fewer bikes and stations also imply a reduction in the optimal level of service offered to users. In conclusion, the increase in the system's cost is shared between the operating agency and the users of the system. Therefore, if electrical bicycles do not confer additional benefits, which are not considered in the present model, there is no reason to implement them. The potential of electrical bike-sharing could be realized if electrical bicycles were able to modify user behavior. There is evidence that electrical bikes increase the attractiveness of cycling and reduce the aversion to both, distance and uphill trips (Jones et al., 2016; Fyhri and Fearnley, 2015).

In bike-sharing systems, this could imply an increase of demand (λ), an increase of the average trip time (τ_s), and a reduction of the average imbalance (Φ) (Campbell et al., 2016). The reduction of the average imbalance directly implies a reduction in the total costs of the sharing system. In addition, the slight increase in average costs per trip due to a higher service time could be largely compensated by the increase in demand due to the existing economies of scale of bicycle sharing systems, as discussed in the next section. In conclusion, electrical bicycles could modify the behavior of bike-sharing users so that the increase in infrastructure costs would be compensated by a larger and more balanced demand.

Table 6. Optimization results for the electrical bicycle sharing system. Decision variables and system design.

	Variable	Units	Lagrangian		Standards (given Δ, P_e)	
			Traditional	Electrical	Traditional	Electrical
Decision Variables	Density of stations – Δ	stat/km^2	20.65	13.44	**8.20**	**8.20**
	Repositioning period – h	hours	6.81	2.68	10.77	4.36
	Empty Stations – P_e	-	0.0061	0.0101	**0.1355**	**0.1355**
	Full Stations – P_f	-	**0.01**	**0.01**	**0.01**	**0.01**
Bicycle fleet	Available fleet size - m	units	14 761	7 129	6 460	3 982
	Avg. # of vehicles in use	units	460	460	460	460
	Stock for demand fluctuations	units	54	50	24	24
	Stock for avg. imbalance	units	827	326	1309	529
	Stock for decentralization	units	13 420	6 294	4 668	2 969
	Min. required fleet due to the battery recharging restriction	units	-	804	-	804
	Bicycles under maintenance	units	590	285	258	159
	Bicycle usage	trips/day	3.38	6.99	7.71	12.52
Stations	Total number of stations - ΔR	units	1012	658	**402**	**402**
	Total number of slots - M	units	28 084	13 809	17 684	10 836
	Ratio slots/fleet - M/m	-	1.90	1.94	2.74	2.72
Access	Avg. access distance (origin + destination)	km	0.220	0.273	**0.349**	**0.349**
Repositioning	Repositioning rate	veh/h	906.73	1 130.59	515.11	740.37
	Total repositioning time	hours/h	21.51	28.00	12.30	18.02
	Total time lost due to inefficiencies	hours/h	10.75	14.00	6.15	9.01
	# Repositioning teams	units	33	43	19	28
	Avg. team performance	veh/ team·h	27.48	26.29	27.11	26.44
	Avg. daily visits per station	-	3.25	8.94	2.23	5.51

** Only station-based configurations are considered when using electrical bikes.
** These results consider Barcelona's *Bicing* system parameters shown in Table 2. In particular, a service area $R = 49$ km^2, with an average demand of 49 832 trips/day (i.e. $\lambda = 42.37$ trips/h·km^2), and an average imbalance $\Phi = 0.118$.
*** Values in bold represent inputs to the model. Model results are in non-bold characters.

Table 7. Optimization results for the electrical bicycle sharing system. Costs.

	Type of cost	Units	Lagrangian		Standards (given Δ, P_e)	
			Traditional	Electrical	Traditional	Electrical
Agency Costs	Infrastructure costs	€/h	758.93	984.90	319.46	566.92
	IC - Bikes	€/h	444.31	660.90	194.44	369.10
	IC - Stations	€/h	314.62	324.00	125.02	197.82
	Operation costs	€/h	1 322.42	3 299.71	1 322.42	3 299.71
	Repositioning costs	€/h	424.12	640.09	281.21	411.90
User Costs	Access costs	€/h	1 447.06	1 793.71	2 295.54	2 295.54
	No service penalty	€/h	245.61	283.58	1 465.51	1 465.51
Total Costs	Total agency costs	€/h	2 573.06	4 924.70	1 923.10	4 278.53
		M€/year	22.54	43.14	16.85	37.48
	Total users' costs	€/h	1 692.67	2 077.29	3 761.04	3 761.04
	Total costs	€/h	4 265.73	7 001.99	5 684.14	8 039.58
Avg. Costs	Generalized cost per trip[1]	€/trip	2.05	3.37	2.74	3.87
	Single Fare[2]	€/trip	1.24	2.37	0.93	2.06
	Annual Fare[3]	€/year	211.78	405.34	158.29	352.16

[*] These results consider Barcelona's *Bicing* cost parameters shown in Table 2.
[1] Average cost per trip, including agency and user costs.
[2] Average agency cost per trip. This is equivalent to the fare users need to pay in the absence of subsidies.
[3] Assuming 147.3 trips/member·year. This is the average annual usage per member in Barcelona's *Bicing* system.

4.4. Sensitivity analysis and generalized results

A sensitivity analysis of the model, with respect to the input parameters that can significantly vary between different implementations or that can evolve with time, allows extending the previous results and obtaining some general conclusions and recommendations for the planning of bicycle sharing systems. All the analyses in this section refer to the optimization results obtained in the Lagrangian scenario (i.e. min. total costs).

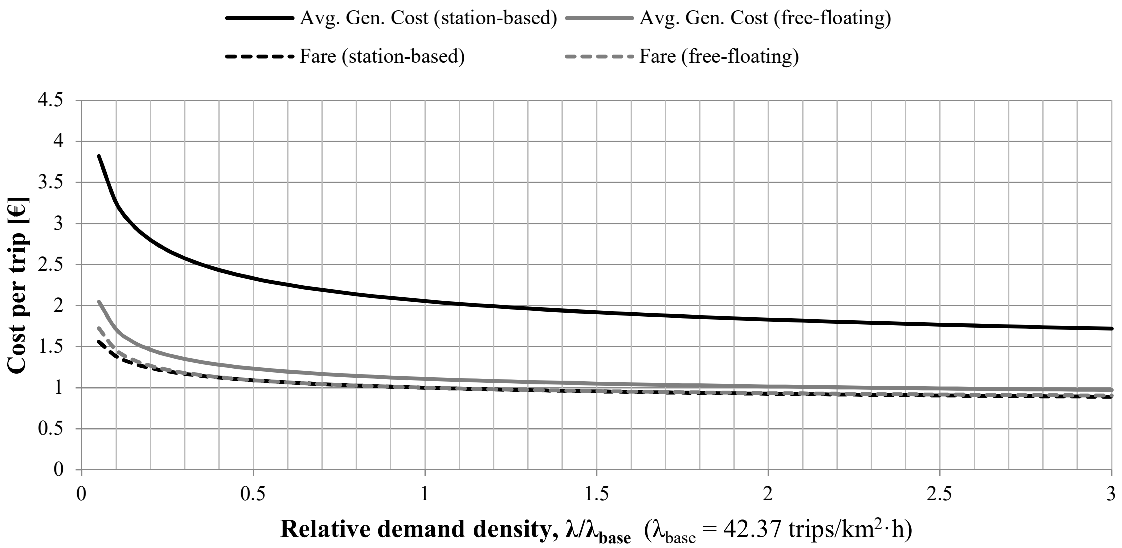

Fig. 3. Economies of scale of bicycle sharing systems

Figure 3 shows that bicycle sharing systems exhibit economies of scale. This means that the average cost per trip is reduced with an increase of the overall demand of the system. Economies of scale are due to the higher utilization of bikes and stations (i.e. pooling effect). Repositioning and operative costs are proportional to demand and do not contribute to the economies of scale. Economies of scale are particularly intense for low demand. For average demand densities lower than 20 trips/km^2·h, the average cost per trip grows rapidly, and the viability and competitiveness of bicycle sharing systems is jeopardized. In contrast, the sensitivity of the system design variables for higher demand is lower but still significant. As an example, if the demand increases by a factor of 3 (with respect to the base demand of Barcelona's *Bicing* system of λ = 42.37 trips/km^2·h) the fleet size and the station density is multiplied only by 2. Overall, the total costs increase by a factor of 2.55. This results in a reduction of the average costs by a factor of 0.85. Economies of scale are very similar in station-based and free-floating configurations. From Figure 3 it can also be seen that the average generalized cost of free-floating systems is lower than the station-based equivalent for all demand levels. This is due to a better average level of service in the free-floating configuration (i.e. lower user costs) for the same agency costs.

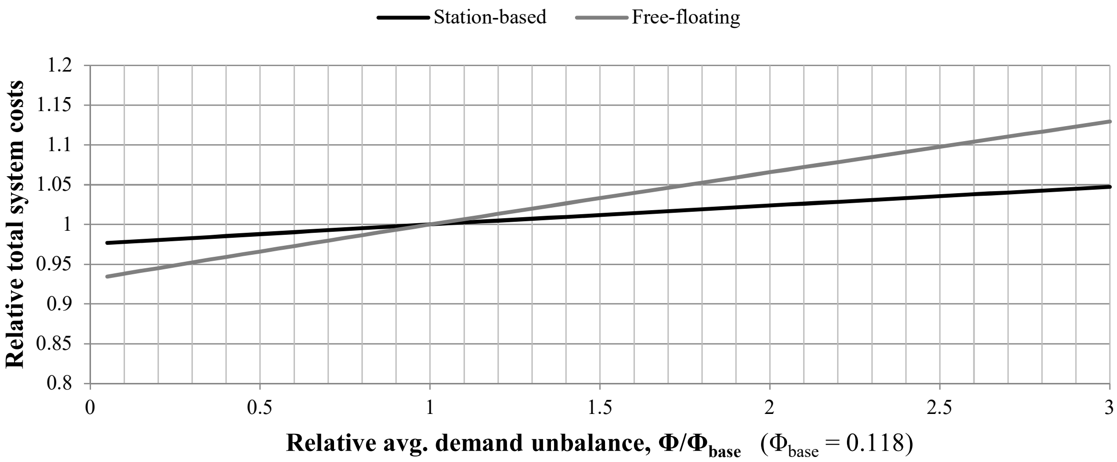

Fig. 4. Effects of the average demand imbalance in total costs of bike-sharing systems

The effect of the average demand imbalance, Φ, on the costs of bike-sharing systems is analyzed in Figure 3. It is shown that system costs increase with Φ. This is because more bicycles and more repositioning are needed to compensate for the higher system imbalance. The elasticity of total costs with respect to Φ is 0.02 for station-based configurations and 0.07 for free-floating systems, for all Φ levels (i.e. the slope of the linear relationship between the relative increase in Φ and the relative increase in total costs). This means that free-floating configurations are more sensitive to the increase of Φ than their station-based equivalents. Taking into account that free-floating systems tend to higher Φ values because the location of the stations cannot help in modulating the demand, the increase in costs due to this factor in free-floating systems is more likely.

Next, Figure 5 analyzes the effects of the acquisition cost of bikes on the design of public bike-sharing systems. This shows the high elasticity of the fleet size for low bicycle acquisition costs. This analysis is especially relevant since there is an extremely high variability in the reported acquisition cost of bicycles. While for the Barcelona *Bicing* system bikes cost 400 € each (López, 2009) and the first generation of bikes with enhanced ICT capabilities in order to be used in free-floating systems costs 786 € each (Lee, 2017), today, the bicycle costs reported for the huge Chinese free-floating implementations are as low as 40 €/bike (Lee, 2017). With these costs, which represent 10% or less of the considered cost for bicycles, the size of the fleet would be multiplied by a factor of 4 or even more. The increase of the bicycle fleet implies a reduction of the repositioning intensity. This means that if bicycle costs are very low (especially with respect to the repositioning costs) systems will tend to have huge bike fleets and very little repositioning. This is, for instance, what happened in the Chinese free-floating implementations. Such large bike fleets create problems in the urban environment because of bicycles clogging pedestrian walkways and green zones. The conclusion is that bicycle parking regulations should be addressed when implementing free-floating bike-sharing systems. In addition, in such contexts a maximum fleet size could also be established. This would force the operating agency to consider repositioning operations.

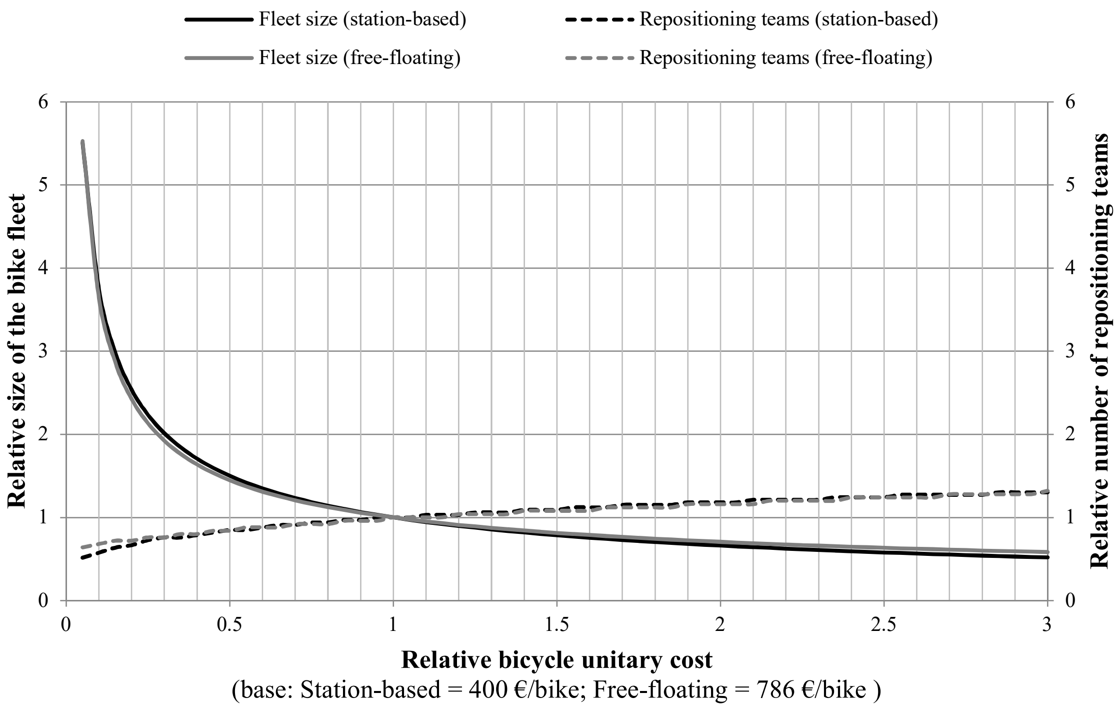

Fig. 5. Effects of bicycle unitary costs on the size of the bike fleet and repositioning intensity (station-based configuration).

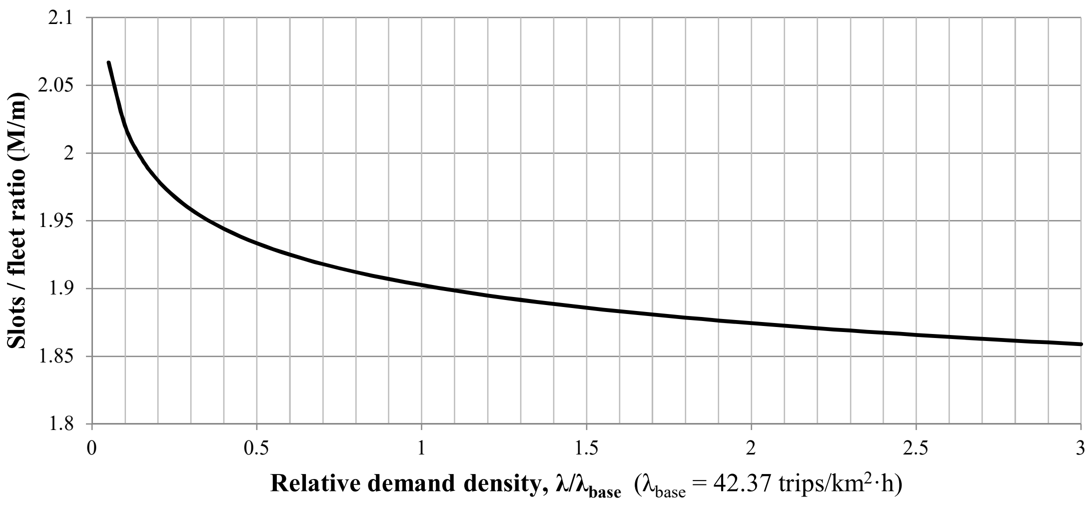

Fig. 6. Sensitivity of the parking slots to bicycles ratio (M/m) in optimal station-based bicycle sharing systems.
Note: A probability of full stations $P_f = 0.01$ is considered.

Finally, Figure 6 analyzes the parking slots to bicycles ratio, M/m. This ratio is of much interest when planning station-based bike-sharing systems, as it is frequently used to determine the overall capacity of stations in the system. This ratio determines the average probability of full stations at the destination. Results show that the optimal ratio is quite stable for different demand levels. Values range from 1.85 to 2.05, for a 1% probability of full stations. The lower values are obtained for higher demands. As a general guideline, $M/m = 1.9$ would be acceptable for a wide range of demand levels where bike-sharing systems are viable. This recommendation is valid given the social optimum bike fleet, m (i.e. from the Lagrangian optimization). In standards scenarios, where the bike fleet can be sub-optimal (e.g. due to a higher accepted probability of empty stations, P_e), M/m ratios of around 1.9 would lead to an equally sub-optimal overall number of parking slots, M. In such a case, this ratio would imply a similar accepted probability of full stations, P_f. If the consumption of urban space is not critical, larger M/m ratios are encouraged in these scenarios (e.g. $M/m = 2.74$ for Barcelona's *Bicing* standards scenario).

5. Conclusions and further research

The continuous approximations model for bicycle sharing systems presented in this paper allows obtaining optimal system designs, in terms of the size of bicycle fleet, the number of stations, the total number of parking slots and the required rebalancing intensity. The model also allows comparing station-based with free-floating configurations and assessing the battery recharging restriction when using electrical bikes. The proposed analytical model is validated against Barcelona's *Bicing* system, obtaining relative errors below 13% in the design variables and performance indicators.

For the particular case of Barcelona's *Bicing*, the optimization shows that the current level of service offered is sub-optimal from the social point of view. This is considering the trade-off between user and agency costs, where user costs are obtained from the monetization of the access time and penalties due to empty or full stations. The results show that by increasing the level of service, the benefits for users largely compensate for the increase in agency costs. In spite of this, if the current *Bicing* level of service is accepted as a standard, for instance due to budget limitations, the actual system design is adequate in terms of the number of bicycles, stations and repositioning level. Nevertheless, the possibility of full stations could be virtually eliminated by adding more parking slots. 1.9 parking slots per bicycle are recommended in optimal configurations. However, if the system design is sub-optimal, with a limited level of service and bicycle fleet, keeping this ratio implies an equally sub-optimal probability of full stations.

Generalizing the results, it can be said that optimal designs are robust for all the inputs of the model, in the sense that the results exhibit small elasticities in all unitary costs and technological parameters. Also, deviations from the optimal system design do not imply a large increase in the total system cost. In spite of this, the sensitivity of the model has been analyzed with respect to those inputs whose value might vary largely in different implementations. The average demand density is one of such inputs. It is proven that economies of scale exist in bicycle sharing systems. The average cost per trip is reduced with growing demand. For low demand levels (e.g. less than 20 trips/km^2·h) the average cost per trip increases significantly and the viability and potential of bike-sharing systems is compromised.

The acquisition cost of bikes could also vary largely between different implementations and this significantly affects the trade-off between the size of the bike fleet and the repositioning level. Especially if bicycle cost is very low, the size of the fleet can greatly increase. Actually, this happened in the recent Chinese implementations of free-floating bike-sharing systems, where the reported cost of bikes is as low as 40 €/bike. Huge fleets are spread throughout cities while repositioning operations are kept to a minimum, if not inexistent. This creates problems of bicycle accumulation in the central areas of these cities. The recommendation in this context should be to address bicycle parking policies and analyze the possibility of imposing a maximum fleet size for bike-sharing systems. This would force the operating agency to reposition bicycles in order to maintain the required level of service. This situation brings into question the suitability of free-floating bike-sharing configurations. Free-floating can provide a better average level of service with lower agency costs, in comparison to an equivalent station-based configuration. This benefit is particularly significant when the access distance to bicycles is desired to be especially short. However, free-floating configurations may imply other problems: bicycles tend to clog up city centers; the level of service deteriorates during peak hours (i.e. longer walking distance to bikes) because of a reduction in the number of idle bicycles; bicycles are more exposed to theft and vandalism and their useful life is reduced; and the average imbalance of the system could increase in the absence of the station regulatory effect. All these effects would eventually increase system costs. For instance, note that system costs increase with the average demand imbalance, and elasticity is higher for free-floating than for station-based systems.

Free-floating bike-sharing configurations are also not suitable for the implementation of e-bikes, unless solar battery recharging technology is developed at a competitive cost for e-bikes. Meanwhile, electrical bike-sharing will need to rely on recharging stations. The results obtained in this paper show that in station-based systems, the usage of electrical bikes does not imply an increase of the bike fleet due to battery recharging restrictions. The possibility of mixed systems (i.e. free-floating but with some stations to allow battery recharging) is not analyzed here and it is left for further research. In this mixed configuration, repositioning movements could not only be used to rebalance bicycle locations, but also to rebalance the available battery levels in the system. In general, the usage of electric bicycles implies an increase in the infrastructure and operative costs of bike-sharing systems. In spite of this, their potential resides in their ability for increasing cycling attractiveness and reducing users' aversion to uphill trips. These would be translated into higher and more balanced demands, contributing to compensating for the increase in cost when using electrical bikes.

Actually, user behavior regarding bicycle sharing systems is a topic of rising interest (Abolhassani, et al., 2019; Godavarthy and Rahim Taleqani, 2017; Mattson and Godavarthy, 2017; Reynaud et al., 2018) and still requiring further research. This means not only developing endogenous demand models able to estimate bike-sharing demand for a given level of service, or to assess the effects on demand of using electrical bicycles, but also to investigate pricing policies to promote self-rebalancing trips or to assess the potential benefits and penalties of allowing bicycle reservations.

We conclude that the CA model presented in this paper and its results provide valuable insights and a better understanding of bicycle sharing systems. In spite of this, it is necessary to highlight that the simplifications made in order to formulate a parsimonious model (e.g. uniform demand, average imbalance, average repositioning) imply that the obtained results should be considered only in aggregate terms. This means that the proposed model could be used in order to obtain a first approach in the design of bicycle sharing systems (i.e. in the planning phase). Other model types could be used for the fine tuning of this first approach solution in the tactical and operational phases. For instance, station locations and their specific size and optimal inventory levels could be determined using discrete methods from the operations research field (e.g. solution methods for the facility location problem), with the inputs of more or less detailed O/D demands. Also, due to the importance of relocating strategies in overall costs, these should be addressed in detail in the operational phase of the system. This implies designing strategies to efficiently solve the dynamic "transportation problem", also a problem with a long tradition in the field of operations research. In this context, the proposed CA model should not only be considered as a fast alternative to obtain approximate solutions, but could also be integrated into these exact and heuristic solution approaches.

Finally, the operational optimization of sharing systems can also be addressed using simulation-optimization techniques. Bicycle sharing operations could be emulated and the simulation could be fed with real-time information in order to obtain a real-time management tool. At this operational level the CA approach can also play a role, increasing the responsiveness of the system in a dynamic decision-making environment with respect to using only discrete exact models.

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Paper II

A continuous approximation model for the optimal design of mixed free-floating and station-based car-sharing systems

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A continuous approximation model for the optimal design of mixed free-floating and station-based car-sharing systems

Enrique Jiménez and Francesc Soriguera

Abstract— One-way car-sharing systems have become a popular urban transportation mode in many cities worldwide. First pilot implementations were originally station-based, so that trips needed to be station-to-station. These created implementation difficulties in many cities and higher infrastructure costs for operating agencies. The appearance of free-floating on-street systems eased these limitations and implied an important growth of car-sharing implementations. In spite of these, free-floating systems are not free of planning and operative difficulties, which if not addressed carefully, might imply the economical unsustainability of the system. The idea of mixed systems, where a free-floating system and a station-based system complement each other is new. The objective is to exploit the potentialities of both designs, while limiting their respective drawbacks. This paper presents a parsimonious model from which to derive the optimal strategical design variables for mixed car-sharing systems (i.e. the vehicle fleet size, the number of stations and the required intensity of rebalancing operations). This requires an integrated view of the system, allowing the optimization of the trade-off between the costs incurred by the operating agency and the level of service offered to users. The approach is based on the modelling technique of continuous approximations, which requires strong simplifications but allows obtaining very clear trade-offs and insights. The model has been applied to a case study taking parameters from the city of Barcelona. Results prove, the profitability of mixed car-sharing systems, which in particular contexts is higher than pure free-floating or pure station-based systems on their own. Furthermore, if electrical cars are used, results show that battery recharging will not imply an active restriction in to the system configuration. In conclusion, the proposed modelling approach represents a tool for strategic design in the planning phase and provides a better understanding of car-sharing systems.

Keywords— car-sharing, electric vehicles, facility location problem, rebalancing, optimization, continuous approximation, Barcelona.

1. Introduction

Car-sharing services are playing a key role in the new urban mobility frameworks. The idea behind car-sharing is to make available a fleet of cars to users for making a trip without necessarily having to own the vehicle. From the user perspective, car-sharing provides the benefits of on-demand transportation (i.e. travel when and where the user wants) at a lower cost resulting from a more intensive use of the vehicle, spreading the costs of vehicle depreciation and operation over many users. In comparison to mass public transportation, car-sharing is much more flexible not relying on routes and schedules, and can be cheaper than taxi (where the user needs to pay for the driver) or than private vehicle ownership (a highly underused capital asset) for certain demand levels (Daganzo and Ouyang, 2019).

Car-sharing initiatives have been explored since the 60s, but they became especially successful since 2010s, when technological development allowed for one-way trips. One-way car-sharing allows users to rent a car without the need of making a return trip and neither paying for the rental when the vehicle is stopped at destination. One-way operation highly increased the attractiveness and popularity of the car-sharing services. According to Shaheen and Cohen (2020), one-way car-sharing accounts roughly for the 50% of global membership and 40% of global fleets deployed worldwide. And these percentages are growing fast, as the 2018 one-way market share represented a 238% increase in membership and a 103% increase in fleets since 2016.

One-way vehicle sharing systems open the possibility of being implemented either in a free-floating or in a station-based configuration, adding a new dimension on the complexity of the design and management of these systems. In free-floating car-sharing (FF), the fleet of cars is deployed on the city streets. Users can check the location of nearby available cars through a mobile app and reserve the desired vehicle to make their trip. Once the ride has been completed, users can return the car by parking it on any available on-street parking spot inside a designed service region (typically the whole municipality area). The works by Kortum et al. (2016), Sprei et al. (2019), and Fromm et al. (2019), provide an overall overview on free-floating car-sharing systems and evaluate empirical data on their usage in different cities of Europe and North America. Similarly, Ampudia-Renuncio et al. (2018; 2020) provide a spatial evaluation of the trip flows throughout the whole service area in Madrid, using real usage data of free-floating car-sharing systems collected from the different operators. In contrast, in station-based car-sharing (SB), the fleet is located at specific parking lots or stations. Users can also check the availability of cars via a mobile app, and can make a reservation, although walk-in to find a car is also a possibility. At the end of the trip, users must return the car to any of the designated parking locations. So, in practice, all trips are station-to-station.

Each configuration has some advantages and drawbacks, as reported in Ciari et al. (2014) and Kopp et al. (2015). From the operating agency's point of view, FF requires less infrastructure, as parking stations are not needed. This could also represent cost savings, as generally off-street parking is private and the agency would need to pay for its usage, while on-street parking is public, and might be subsidized for the use of car-sharing systems. Also, FF service is easier to implement as it is not dependent on the agreements to recruit parking spots at stations. However, municipalities and local administrations could be reluctant to concede the usage of public space to FF car-sharing initiatives, and this could limit the number of FF cars in service. From the customers' perspective, accessibility is the key factor. In the case of SB car-sharing, there might not be stations near the location of the trip origin or destination. Hence, customers might need to invest more time to access the system, or even discard the trip if the access or egress distance is too big. In spite of this, it is easier for SB systems to ensure a good distribution and car availability, since the stations distribution and their vehicle capacity is governed by the operating agency. In contrast, FF services are usually less reliable, as vehicles may largely accumulate in some regions, and be scarce in others. Artificial repositioning of vehicles is extremely expensive and usually cannot completely compensate these situations. Given this context, FF car-sharing is perceived as an unreliable service by users, who do not take for granted that they will be able to find a car when needed. This means that FF car-sharing service is generally limited to urban regions with a good coverage of public transportation options, where the lack of reliability of the system can be compensated by other transportation modes. If there is no other viable alternative, the usage of FF car-sharing is discouraged.

Since both SB and FF systems have different pros and cons, it is crucial during the planning phase to decide which one would fit better to a particular context. Moreover, it could be interesting to design mixed configurations, combining the advantages of both. For example, SB systems with a limited number of stations can improve their coverage by including a number of FF cars. Or otherwise, if the allowed FF vehicle fleet is limited by the municipalities, FF systems could operate with an extra fleet by including cars in stations. With this purpose, the present paper develops a macroscopic model for the planning of a mixed car-sharing system, which includes a FF and a SB layer working simultaneously. Under this configuration, users can rent cars either in stations or on-street, and return them indistinctly as desired.

The proposed model includes the estimation of the demand served by the system, which considers that cars can be spatially and temporally imbalanced, but they will keep serving trips as long as there is still potential demand to be served in a given area and time. The model also estimates the required number of repositioning operations based on rebalancing and battery recharging needs. Note that the model considers the use of electrical vehicles, where battery recharging can be performed either with domestic installations at stations or with superchargers in a central battery charging hub.

The methodology used is based on continuous approximations. This methodology is able to depict clearly the insights and trade-offs of the system under analysis, and it has been successfully used before in car-sharing design problems (Li et al., 2016; Lei and Ouyang, 2018; Daganzo and Ouyang, 2019), obtaining optimal solutions at a very low computational cost without significantly losing accuracy and robustness in the results. Therefore, continuous approximations represent an adequate methodology to evaluate the performance and make strategic decisions in the design phase of a mixed car-sharing system.

The rest of the paper is structured as follows. Section 2 describes the current state-of-the-art regarding car-sharing design in the scientific literature. Section 3 formulates the model, including the definition of the decision variables, the formulation of costs, and the estimation of the served demand by the system. The model also includes the constraints for the battery recharge of electric vehicles. Next, in Section 4, the model is applied to a case of study for the city of Barcelona, Spain. Optimization results are analyzed, including their robustness and sensitivity to the main parameters. Finally, the paper ends with the conclusions and references.

2. Literature review

The present paper deals with the strategic planning of mixed car-sharing systems, which encompasses the study and optimization of the fundamental decision variables of the system, namely: the number of stations, the required number of parking spots, the dimensioning of the vehicle fleet and the definition of the repositioning resources. These decisions are strategic and define the system for the medium to long term.

Most of the research in vehicle sharing systems address the strategies to perform vehicle repositioning operations, which define a day-to-day operative problem. See Jorge and Correia (2013 and Ferrero et al. (2020), which provide a deep literature review of the research field up to date. The vehicle repositioning problem fits into the operations research discipline, which is usually addressed with mixed integer programming (MIP) and its variations, which are developed in order to reduce the computational burden of this methodology. For example, Boyacı et al. (2015) uses a multi-objective mixed integer linear programming (MILP) which includes vehicle charging requirements to define an optimal vehicle repositioning framework for car-sharing systems. The concept of “virtual hub” is introduced in order to solve real-size problems with an extremely large number of relocation variables. The model is applied to the specific case of the city of Nice, being a very complete application putting into practice most of the previous research to date. Other examples are the works by Nourinejad et al. (2015), Boyacı et al. (2017), Gambella et al. (2018), Zhao et al. (2018), Ma et al. (2019), and

Huang et al. (2020b), which develop different repositioning optimization algorithms to solve the relocation operational problem with variations of mathematical programming.

As noted by Illgen and Höck (2019), this existing literature for car-sharing systems, mainly based on operations research methodologies, have influenced and directed the methodological approaches to address the strategic planning problem of car-sharing systems towards mathematical programming. Take as an example the works by Huang et al. (2018) and Huang et al. (2020a) proposing a mixed-integer non-linear programming model (MINLP) to solve the car-sharing station location and capacity problem. A logit model is used to represent the non-linear demand variation by considering the differences in utility of car-sharing versus private cars. The main conclusion drawn is that pricing and parking space rental costs are key factors that influence the profitability of car-sharing operators. In turn, Hua et al. (2019) incorporates demand uncertainty by developing a stochastic model, which is solved by multi-stage linear programming (MSLP), including long-term charging infrastructure decision together with the real-time fleet operation policies. The model is applied to a numerical experiment using data from New York City. Results show that the demand density affects the fleet size, number of stations, and relocation resources, but optimal designs keeps the no-service rate quite constant. It is found that it is usually better to leave more demand unserved than relying intensively on repositioning teams. These conclusions are in accordance with the results obtained in the present paper for mixed car-sharing systems.

One common drawback in models based on MIP and its variations is that, despite they can reach optimal solutions to the relocation problem, they suffer from high computational burden, and their applications are limited to manageable sizes. To address this issue, Lei and Ouyang (2018) propose a hybrid modeling framework for the rebalancing problem in large bike-sharing systems, where a continuous approximation approach is used to solve the local routing inside the multiple subregions and a reduced size discrete model is used to address the line-haul problem between subregions. The hybrid model is able to produce a good solution for large-scale instances in a short computation time. Such approach has been subsequentially used in Osorio et al. (2021) to model the optimal rebalancing and on-board charging of shared electric scooters in large-scale systems, and in Jiang et al. (2020) to address the optimal bicycle allocation and investment strategies in a system with various bike-sharing companies in competition. In turn, Li et al. (2016) and He et al. (2017) propose a different approach where repositioning costs are considered as a linear approximation instead of a MIP-based solution, simplifying the strategical design problem. The work of Li et al. (2016) uses continuous approximations (CA) to model the car-sharing system. To deal with the spatial variability of the demand, the service area is discretized, and each subarea is treated as an infinite homogeneous plane where the CA model cost equations are applied. Numerical experiments for a particular case study show that the CA solution is able to estimate the total system cost very efficiently and with reasonable accuracy. On average, the relative error of the solutions is below 3% with respect to the MIP counterpart and the computational time is 100 times lower. In addition, they obtain further insights from the system through sensitivity analysis, which were overlooked by previous works.

A similar approach has been chosen in the present paper to analyze the optimal planning of mixed car-sharing systems and discuss in which cases the FF and SB configurations are adequate and to what extent. A CA model is developed, where the vehicle repositioning operations cost is adapted to a linear equation. Based on the recommendations of Bruglieri et al. (2014) repositioning staff are assumed to travel between operations with foldable bikes, which can be carried in the car trunk when performing the repositioning movement. The CA solution proposed by Daganzo and Smilowitz (2004) for the transportation problem between homogeneously, but randomly distributed supply and demand points in a region of arbitrary shape is used to estimate the overall distance covered by repositioning teams. To the authors' best knowledge, this is the first time that the design of mixed car-sharing systems is specifically addressed and analyzed.

3. Model definition

3.1. Model overview and decision variables

The mixed car-sharing model is defined over a service region $\mathcal{R}$, whose area is R [km^2], where there is a free-floating (FF) and a station-based (SB) layer operating simultaneously (see Figure 1). The SB layer relies on parking stations which are distributed over $\mathcal{R}$. Parking stations are not exclusively used by the car-sharing system, and it is considered that the operating agency just rents parking slots at existing parking stations (i.e. there is no implementation cost of stations). The demand coverage area of each parking station is defined as c [km^2], which is directly obtained from the maximum distance that users are willing to walk to access one vehicle, a [km]. a is an input to the model, and considering the L1 metric (i.e. Manhattan Distance) we have: $c = 2a^2$ (i.e. the area of the rhombus centered at the station with diagonals of length equal to $2a$). In turn, Δ_{SB} [stations/km^2] is defined as the density of parking stations over $\mathcal{R}$, so that $\Delta_{SB}R$ is the number of parking stations in the system. Note that, $\Delta_{SB}c$ is the fraction of the service area covered by the SB system, and it is assumed that the coverage areas of different parking stations do not overlap, so that $\Delta_{SB}c \leq 1$. Regarding the FF layer, the service region is divided into subzones, r ($r \in \mathcal{R}$) acting like virtual stations. The area of each subzone is c, so that if there was at least one available vehicle in each subzone, the whole population in $\mathcal{R}$ would have access to FF car-sharing vehicles, implying a complete coverage of the FF car-sharing.

In addition to the previous definitions, some user behavioral assumptions are needed in order to formulate the model. These are summarized below:

- At the origin of the trip, users prefer the FF service, if available. This is due to the better accessibility of the on-street FF vehicles.
- If there is not any FF available vehicle within an access distance a, then users will opt for the SB service, only if their location falls inside the coverage area of one SB station. Otherwise, the trip is not served and it is lost.
- At destination, users will park the vehicle either FF (i.e. on-street) or SB (i.e. at a parking station). This will depend on the on-street parking availability in $\mathcal{R}$, and on the SB coverage. In any case, it is assumed that users are always able to park the car within a distance a of their final destination, despite this may involve an additional time looking for parking.

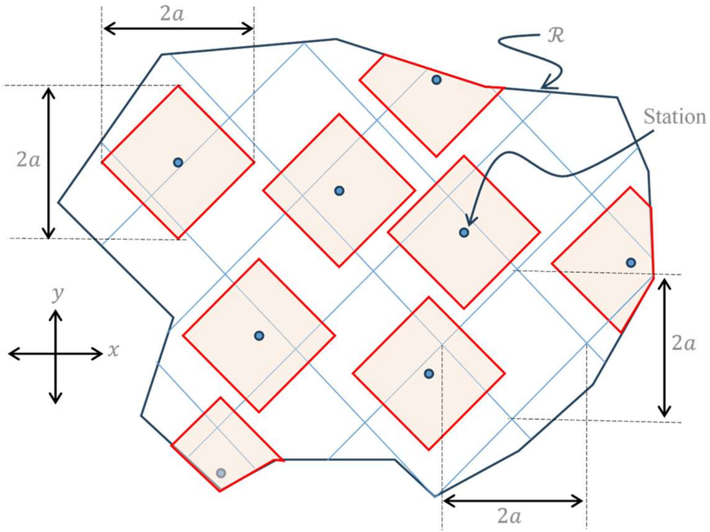

Figure 1. Sketch of the FF and SB layers over a service region $\mathcal{R}$. (Note: It is assumed an infinite grid of orthogonal streets in the x, y directions).

The particular planning of the system affects the overall amount of demand served, generally with a trade-off with the system costs. Five planning decision variables steer the performance of the mixed car-sharing system, which correspond to the five degrees of freedom of the model (see Table 2). These are:

- The vehicle fleet, composed of the FF fleet size (m_{FF}) and the SB fleet size (m_{SB}).
- The repositioning periods, h_{FF}, h_{SB} [hours] for the FF and SB systems respectively. The repositioning period is defined as the average time while the repositioning teams need to rebalance the whole system (i.e. move the vehicles from where they are in excess to where they are scarce). They define the required number of repositioning teams.
- The coverage of the SB system, defined in terms of the density of parking stations, Δ_{SB} [stations/km^2]. The fraction of $\mathcal{R}$ covered by the SB system is $\Delta_{SB}c \in [0,1]$.

Table 2. Summary of decision variables in the model.

Decision Variable	**Definition**		**Notation**	**Units**
Stations' density	Number of SB parking stations per unit area		Δ_{SB}	[stations/km^2]
Fleet size	Number of vehicles in the system	Total	m	[cars]
		Free-floating	m_{FF}	
		Station-based	m_{SB}	
Repositioning period	Average time needed to rebalance the whole system	Free-floating	h_{FF}	[h]
		Station-based	h_{SB}	

Finally, it is considered that the vehicle fleet can be composed (totally or in part) by electrical vehicles. Battery charging of electrical vehicles may take place at regular parking stations (i.e. decentralized battery charging) or at a centralized battery charging location (i.e. at a charging hub). In any case, the battery recharging of the electrical vehicles in the fleet will represent a constraint in the model. This constraint will limit the feasible relationship between the number of electric cars, their battery autonomy, the number and type of battery chargers required, and the minimum number of repositioning employees.

The optimal design of the mixed car-sharing system (i.e. the selection of the optimal values for the decision variables) will be obtained from the maximization of the profit of the operating agency. Profits directly result from the difference between revenues and agency costs, which will determine the objective function of the mathematical optimization. In addition, the previous results will be compared to those obtained from the minimization of total costs (i.e. the sum of agency costs and user cost). This last optimization takes into account the users' access cost to the system and the penalties users incur when not finding an available vehicle, so that the resulting design could be considered as optimal from the social point of view. The previous optimization frameworks require to formulate the demand served and the agency and users' costs as functions of the decision variables. These formulations are addressed in the next sections.

3.2. Demand modelling

We define λ_{max} [trips/km^2·h] as the average potential demand density for the system. This parameter is an input to the model, and represents the total attracted demand for car-sharing systems in case of complete coverage and vehicle availability. Therefore, λ_{max} is the maximum number of car-sharing trips that could be generated per unit time and area. We define λ_{FF} as the demand actually served by the FF service, which is the first preference of users. Generally, $\lambda_{FF} < \lambda_{max}$ because a fraction of the demand will be lost due to FF vehicle unavailability at a given time and subzone. Considering the previous behavioral assumptions, if one trip cannot be served by the FF layer and if the origin of the trip is within the coverage area of one station, the trip could be served by the SB layer. Therefore, the potential demand density for the SB part of the car-sharing system would be:

$$\lambda_{SB\_max} = (\lambda_{max} - \lambda_{FF})\Delta_{SB}c \tag{1}$$

Recall that $\Delta_{SB}c$ is the fraction of the service region covered by the SB system. Again, probably not all the potential demand for the SB system will be served, and the actual demand served by the SB system will be $\lambda_{SB} < \lambda_{SB\_max}$. Given these definitions, the total demand lost due to the vehicle unavailability would be:

$$\lambda_{lost} = \lambda_{max} - \lambda_{FF} - \lambda_{SB} \tag{2}$$

One part of these lost trips occurs due to the lack of FF service in regions without SB coverage. This part is:

$$\lambda_{FF\_lost} = \lambda_{max} - \lambda_{FF} - \lambda_{SB\_max} \tag{3}$$

The other part is lost due to the lack of SB vehicle availability at stations. This second part is:

$$\lambda_{SB\_lost} = \lambda_{SB\_max} - \lambda_{SB} \tag{4}$$

Recall that all λ's are defined in terms of average demand densities (i.e. trips per unit time and unit area; [trips/h·km^2]).

Regarding vehicles' returns, in a mixed car-sharing system once users reach their destination, they can park the vehicle either on-street (FF), or in a parking station (SB) if the final destination is within a station's coverage area. We define P_{SB} as the average fraction of users who park SB when they have both parking options available (i.e. FF and SB). $P_{SB} \in [0,1]$ is an input parameter and depends on the on-street parking availability in $\mathcal{R}$. If on-street parking is largely available, $P_{SB} \to 0$, as on-street parking would be more accessible and preferred. In contrast, if on-street parking is scarce $P_{SB} \to 1$, as users would move towards parking stations given the difficulty of finding an on-street parking spot. With these definitions, the FF returns demand density is:

$$\lambda_{t\_FF} = (\lambda_{FF} + \lambda_{SB})(1 - \Delta_{SB}cP_{SB}) \tag{5}$$

and the SB returns demand density is:

$$\lambda_{t\_SB} = (\lambda_{FF} + \lambda_{SB})\Delta_{SB}cP_{SB} \tag{6}$$

Note that if on-street parking availability is extremely scare, it might happen that $\lambda_{t\_SB} \gg \lambda_{SB}$, which means that the SB returns are larger than the SB requests, implying a significant net movement of vehicles from FF to SB systems. Specifically, this would happen if the results of the optimization yield:

$$\lambda_{SB} \gg \frac{\lambda_{FF}\Delta_{SB}cP_{SB}}{1-\Delta_{SB}cP_{SB}} \tag{7}$$

If this happens, the rebalancing of the FF system would be too costly, resulting in the "vanishing" of the FF fleet. In such context, the car-sharing system will turn to be 100% SB, and the FF part of the system would be disregarded (i.e. $m_{FF} = 0;\ \lambda_{FF} = 0;\ k_{FF} = 0$).

In contrast, if on-street parking is largely available and $\lambda_{t\_SB} \ll \lambda_{SB}$, vehicles will accumulate in the streets, and the system would turn to be 100% FF. In such context, if electrical vehicles are used, battery charging facilities need to be provided on-street or at a single or various centralized charging hubs.

In any case, pure FF and pure SB systems are just particular extreme cases of the mixed car-sharing system modelled here. Table 3 summarizes the demand modelling variables.

Table 3. Summary of demand modelling variables.

Variable	Definition[1]		Notation	Estimation
Average potential demand density	Maximum density of trips susceptible to use the mixed car-sharing system		λ_{max}	Input to the model
Average potential SB demand density	Maximum density of trips susceptible to use the SB part of the system		λ_{SB_max}	Eq. 1
Average served demand density	Density of trips actually served by the mixed car-sharing system	Free-floating	λ_{FF}	Appendix A.1
		Station-based	λ_{SB}	Appendix A.2
Average returns demand density	Density of returns in the mixed car-sharing system	Free-floating	λ_{t_FF}	Eq. 5
		Station-based	λ_{t_SB}	Eq. 6
Average lost demand density	Lost demand due to lack of FF service and limited SB coverage.		λ_{FF_lost}	Eq. 3
	Lost demand due to lack of available SB cars in parking stations		λ_{SB_lost}	Eq. 4

[1]All demand variables are defined in terms of densities (i.e. per unit time and area; [trips/h·km^2])

In Appendix A, the demand modelling variables presented in Table 3 are derived from the demand input parameters to the model. These inputs are λ_{max}, the average potential demand density, and two dimensionless parameters defining its temporal and spatial variability. For the temporal variability, we define CV_λ_{max}, its coefficient of variation. For the spatial variability, we define the average demand imbalance parameter Φ, which represents the fraction of vehicle requests not balanced with the corresponding returns. See Appendix A for further details on such definitions.

3.3. Modelling agency costs, Z_A

Agency cost include all the costs incurred in the operating of the mixed car-sharing system. This includes the infrastructure costs, Z_{IC} (i.e. fleet costs and parking costs), operative costs (excluding repositioning), Z_O, and repositioning costs Z_R. Total agency costs are expressed as:

$$Z_A = Z_{IC} + Z_O + Z_R \tag{8}$$

All costs, Z, are depicted in monetary units per time unit (e.g. [€/h]).

3.3.1. Infrastructure costs, Z_{IC}

Infrastructure costs account for all the investments made to acquire and renew the vehicle fleet and to cover the parking costs. These costs can be estimated as:

$$Z_{IC} = \gamma(m_{FF} + m_{SB} + k_{FF} + k_{SB}) + \gamma_{S\_FF}(m_{FF} + k_{FF}) + \gamma_{S\_SB}(m_{SB} + k_{SB}) \tag{9}$$

In Equation 9, γ, is the unitary depreciation cost of one car in the system and γ_{S_FF}, γ_{S_SB}, are the renting costs of one parking spot on-street and in a parking station, respectively. If the fleet is composed of different types of vehicles with different costs (e.g. electrical and combustion engine vehicles), the parameter γ must be a weighted average, taking into account their relative frequency. The same concept applies to the parking spots with different costs (e.g. equipped with charging devices or not). All cost parameters are expressed per unit time.

k_{FF}, and k_{SB}, are the number of repositioning teams in the FF and SB systems, respectively. Note that each team relocates one vehicle at a time, so that the total number of car-sharing vehicles in the system is $m_{FF} + m_{SB}$ (i.e. vehicles available for service) plus $k_{FF} + k_{SB}$ (i.e. vehicles being repositioned and not available). Equation 9 also considers that there must be, at least, as many parking spots as vehicles in the whole fleet.

3.3.2. Restriction in vehicle fleets due to electrical vehicles' battery charging

If the car-sharing vehicle fleet is composed of electric vehicles (totally or in part), battery charging needs to be considered in the model. This is included in the form of a restriction between the electrical fleet size, number of parking spots equipped with recharging infrastructure and repositioning operations. This battery recharging restriction is based on the following assumptions and definitions:

- The fraction of the electric vehicles in the fleet is defined as e, which is an input to the model. Electrical vehicles are uniformly distributed in the fleet, so that they can serve either FF or SB trips indistinctly.
- In one trip, an electrical vehicle consumes a battery time of $\tau_c + \tau_p$ [hours], where τ_c is the average vehicle circulating time and τ_p is the average parking time. Note that the average service time, τ_s, includes the circulation time, the parking time and also the average reservation time of the vehicle while it is stopped.
- The available vehicle usage time for an electrical vehicle with a battery charged to 80% is T_{ev} [hours]. The necessary time to recharge the 80% of one battery is T'_{ev} [hours]. Therefore, the battery recharging rate is T_{ev}/T'_{ev}. Battery recharging time is approximately linear until 80%. Both, T_{ev} and T'_{ev} are inputs to the model.
- There is no battery charging equipment on-street. Battery charging operations are carried out either at SB stations (i.e. decentralized charging) or at a battery charging hub (i.e. centralized charging).

Given these assumptions, the battery recharging condition to be included into the model must ensure that the total battery consumption rate is lower or equal than the total battery recharging rate, so that the battery level of electrical vehicles in the system is maintained in the long term. The total battery consumption rate is $e(\lambda_{SB} + \lambda_{FF})R(\tau_c + \tau_p)$. In turn, the total battery recharging rate is obtained as the total number of vehicles being recharged times the charging efficiency, T_{ev}/T'_{ev}. This is:

$$e(\lambda_{SB} + \lambda_{FF})R(\tau_c + \tau_p) \le (Num.\, vehicles\ recharging)\frac{T_{ev}}{T'_{ev}} \tag{10}$$

or equivalently:

$$(Num.\,vehicles\ recharging) \geq e(\lambda_{SB} + \lambda_{FF})R(\tau_c + \tau_p)\frac{T'_{ev}}{T_{ev}} \tag{11}$$

i) Decentralized battery charging at SB parking stations

For the decentralized battery charging context, the total number of vehicles recharging at parking stations, are the electrical fraction of idle SB vehicles, $e \cdot m_{i\_SB}$, which is estimated as the total SB electrical fleet minus the number of SB electrical vehicles being used. This is:

$$e \cdot m_{i\_SB} = e\left(m_{SB} - \lambda_{SB}R(\tau_c + \tau_p)\right) \tag{12}$$

Substituting Equation 12 into Equation 11 we obtain:

$$e\left(m_{SB} - \lambda_{SB}R(\tau_c + \tau_p)\right) \geq e(\lambda_{SB} + \lambda_{FF})R(\tau_c + \tau_p)\frac{T'_{ev}}{T_{ev}} \tag{13}$$

Which simplifies to the following restriction to m_{SB}:

$$m_{SB} \geq \left[\lambda_{SB}\left(1 + \frac{T'_{ev}}{T_{ev}}\right) + \lambda_{FF}\frac{T'_{ev}}{T_{ev}}\right]R(\tau_c + \tau_p) \tag{14}$$

Note that the restriction in Equation 14 does not depend on e. This means that if the previous restriction is not fulfilled, it is necessary either to increase the battery charging efficiency, T'_{ev}/T_{ev}, or to increase the SB vehicle fleet, m_{SB}, over the optimal value.

ii) Centralized battery charging in a central hub.

For the centralized battery charging context, the minimum number of vehicles recharging in Equation 11 needs to be equal to the minimum number of charging slots at the central hub, H. So that:

$$H \geq e(\lambda_{SB} + \lambda_{FF})R(\tau_c + \tau_p)\frac{T'_{ev}}{T_{ev}} \tag{15}$$

In addition, these vehicles will not be available for service until returned to the SB of FF systems. So, for the centralized charging context, the vehicle fleet needs to be increased by H vehicles.

3.3.3. Operative costs (excluding repositioning), Z_O

The operative cost includes all the cost related to vehicle usage, such as maintenance, cleaning, and fuel consumption. All these costs are simplified into a single parameter, γ_e [€/trip], so that the total operative costs per unit time are:

$$Z_O = \gamma_e(\lambda_{FF} + \lambda_{SB})R \tag{16}$$

Again, different fleet configurations might imply different operative cost per trip (e.g. fuel consumption is different than electric consumption). In these cases, γ_e must be obtained as a weighted average.

3.3.4. Repositioning costs, Z_R

Due to its special relevance and modelling complexity, repositioning costs are treated separately from the rest of operative costs. Repositing operations are aimed to move vehicles from where they are in excess to where there is a deficit, according to the following modelling assumptions:

- Repositioning is done by agency employees. Every employee moves one car at a time. Once the car is repositioned at the desired location, the employee moves to the next task with an electric scooter. The operation is concluded once the whole cycle is complete.
- Repositioning costs are estimated separately for FF and SB systems. FF repositioning operations groups those operations where the vehicle is taken from the streets, independently of where it is going to be left. In turn, SB repositioning considers the vehicles taken from parking stations.
- Repositioning is complete every h_{FF} units of time for the FF part of the system, and every h_{SB} units of time for the SB part. h_{FF} and h_{SB} are decision variables in the optimization of the system design. On average, each station (real or virtual) is visited once during the repositioning period. Actually, this is an idealization of the model. In practice, some of them will be visited several times and others will not be visited at all during a particular period.
- The number of repositioning employees working simultaneously, k, is such that all tasks are completed during one repositioning period. k is the sum of those teams working in FF and SB repositioning operations (i.e. $k = k_{FF} + k_{SB}$). k_{FF} and k_{SB} are obtained from the number of FF and SB repositioning operations to perform during h_{FF} and h_{SB}.

Given these assumptions, repositioning cost can be expressed as the average number of repositioning employees working simultaneously, k, multiplied by the unitary labor cost of the hired staff, C_t, [€/employee·hour]. This is:

$$Z_R = C_t(k_{FF} + k_{SB}) \tag{17}$$

The required number of employees (either for the FF or SB systems) is equal to the total time necessary to perform the repositioning tasks per unit time. Considering the repositioning rate (i.e. the number of repositioning operations per unit time), we have:

$$k = Repositioning\ rate \cdot Time\ required\ per\ operation \tag{18}$$

In turn, the total repositioning rate needs to account for: *i)* repositioning movements to compensate the spatial demand imbalance and system decentralization, referred as baseline repositioning operations; *ii)* battery recharging repositioning operations for electrical vehicles; and *iii)* repositioning movements to maintain the FF and SB vehicle fleets stable and compensate net flows between them. These will be analyzed separately in Appendix C.

3.4. Modelling user costs, Z_U

Users' costs include the access to the system (i.e. Z_{AC}, the cost of the time required to walk to the nearest available car, and from the parking location to the final destination) and the no-service penalties (i.e. Z_{NS}, the cost of aborting a car-sharing trip).

$$Z_U = Z_{AC} + Z_{NS} \tag{19}$$

There is a trade-off between agency costs and users' costs, as access costs and no service penalties increase with the deterioration of the level of service offered by the system. Next sections formulate users' costs in terms of the planning decision variables.

3.4.1. Access cost, Z_{AC}

The system's accessibility is determined by the average access distance to an available vehicle at the origin of the trip, D_O, and the average egress distance from the parking location to the users' final destination, D_D. It is assumed that users reach the vehicle location by walking at v_w speed. Access and egress distances imply a cost for the user in terms of the time invested. This cost is monetized using the value of time, β [€/h], which is an input parameter to the model. Then, the total access costs are formulated as:

$$Z_{AC} = \left[D_{O\_FF}\lambda_{FF} + D_{D\_FF}\lambda_{t\_FF} + D_{O\_SB}\lambda_{SB} + D_{D\_SB}\lambda_{t\_SB}\right]\frac{1}{v_w}\beta R \tag{20}$$

The average access and egress distances may be different for FF and SB systems. In the SB system, vehicles are located at stations. The maximum distance a user is willing to walk to access a station, is defined as a [km], which conforms the coverage area of each station (i.e. a rhombus whose diagonals length is equal to $2a$ centered at the station). Then, assuming uniformly distributed origins / destinations, the average access / egress distance to / from a station, in L1 metric, is estimated as $2a/3$.

$$D_{O\_SB} = D_{D\_SB} = \frac{2}{3}a \tag{21}$$

Note that we assume that the coverage areas of different parking stations do not overlap, which eliminates the possibility of shorter access distances due to an increased density of stations.

In contrast, in the FF system, vehicles are randomly located on-streets. Then, the average access distance depends on the density of idle FF vehicles. The number of FF idle vehicles, m_{i_FF}, is estimated as the total available FF fleet minus the average number of FF cars in use. This is:

$$m_{i\_FF} = m_{FF} - \lambda_{FF} R \tau_s \tag{22}$$

Assuming that idle FF vehicles are uniformly distributed over the service region, the influence area of each one would be R/m_{i_FF}. Then, considering the L1 metric, the average access distance is estimated as half the diameter of the influence area of the idle vehicle. This is:

$$D_{O\_FF} = min\left[\frac{2}{3}a, \frac{1}{2}\sqrt{\frac{R}{m_{i\_FF}}}\right] \quad (23)$$

Note that in this case, the access distance is also restricted to a maximum of $2a/3$. This will happen when there is an average of less than one car per subzone. For the average FF egress distance, it is assumed as $2a/3$, because it is considered that all users are able to park within the subzone of their final destination.

$$D_{D\_FF} = \frac{2}{3}a \quad (24)$$

3.4.2. No-service penalty, Z_{NS}

If a trip cannot be completed due to the lack of resources, the user will suffer an economic penalty. This loss is incurred by the time spent on the attempt for the car-sharing trip, the cost (e.g. the fare) of the transportation alternative used, and the annoyance produced. Penalties are not the same for each no-service situation. Users assume that FF service is less reliable and that they might not find an available vehicle nearby when needed. In contrast, the SB service is perceived as more robust, and users always expect to find an available vehicle at stations. This means that the unitary no-service penalty, γ_{FF_lost}, applied to λ_{FF_lost} is in general smaller than γ_{SB_lost}, applied to λ_{SB_lost}.

Then, the total no-service penalties per unit time are obtained as the lost demand per unit time multiplied by the no-service penalty factors. This is:

$$Z_{NS} = \left(\gamma_{FF\_lost}\lambda_{FF\_lost} + \gamma_{SB\_lost}\lambda_{SB\_lost}\right)R \quad (25)$$

3.5. Objective function: Maximum profit vs Minimum total costs

For the mathematical optimization of the strategical design variables in the mixed car-sharing system (see Table 2), the objective function consists in maximizing the profit of the operating agency. Profit is obtained as the difference between the revenues from fares and the agency costs of the system. This is:

$$Max[F(\lambda_{FF} + \lambda_{SB})R - Z_A] \quad (26)$$

Where the fare, F, is an input to the model.

In addition, an optimization scenario is set where the objective function responds to the minimization of the total costs, including the agency costs to provide the service and the users' costs derived from the access to the vehicle and from the penalties due to not served demand, as expressed in Equation 27. In such case, the optimization yields the optimum level of service from the social perspective, which is derived from the monetization of users' penalties and costs, and accounts for the unreliability cost of the system.

$$Min[Z_A + Z_U] \quad (27)$$

4. Barcelona case study: parameter estimation, scenarios definition and system optimization results

4.1. Parameter estimation for the Barcelona case study

The city of Barcelona is considered for a particular application of the proposed mixed car-sharing planning methodology. The required inputs to the model have been estimated from various sources and are presented in Table 4.

Table 4. Input parameters for the Barcelona case study.

	Parameter description	Notation	Units	Value	Source
Demand inputs	Area of the service region	R	[km^2]	39.19	The service area is central Barcelona. Demand data was provided by Inlab (2019) as a O/D matrix with 228 zones. Only trips longer than 1.5 km were considered, and a carsharing market penetration of 0.5% of the total number of trips (Firnkorn and Müller, 2011; Bruglieri et al., 2014; Le Vine et al., 2014; Kopp et al., 2015)
	Predominant trip generation area fraction	R_q/R	[-]	0.53	
	Predominant trip attraction area fraction	R_t/R	[-]	0.47	
	Average potential demand density	λ_{max}	[trips/h·km^2]	9.77	
	Standard temporal deviation	$std(\lambda_{max})$	[trips/h·km^2]	0.977	
	Request imbalance	Φ_q	[-]	0.215	
	Return imbalance	Φ_t	[-]	0.241	
	Fraction of station returns	P_{SB}	[-]	0.43	EMEF 2019 (Autoritat del Transport Metropolità, 2020)
City inputs	Average circulating time	τ_c	[min]	11.1	Result of dividing the average trip distance, d_c, by the average car speed, v. d_c is obtained from the from the O/D of the trips considered (see Demand inputs above). Euclidean distance*1.15 is considered between any O/D.
	Average parking time	τ_p	[min]	6.6	Survey conducted in nineteen major European cities (Conduent, 2016)
	Average service time	τ_s	[min]	27.7	Result of adding the average circulating time, the average parking time, and the average reservation time. Note that τ_r is defined as the maximum reservation time. $\tau_s = \tau_c + \tau_p + \tau_r/2$.
	Average car speed in the city	v	[km/h]	15.3	This is estimated as 2/3 of the average speed measured in Barcelona. The 1/3 reduction considers delays at intersections. (Ajuntament de Barcelona, 2018)
	Average repositioning speed (using electric scooter)	v_k	[km/h]	8.8	(Liu et al., 2019)
User behavioral inputs	Maximum access distance	a	[km]	0.4	(Transportation Research Board, 2013)
	Average walking speed	v_w	[km/h]	3	(Generalitat de Catalunya, 2017)
	Users' average value of time	β	[€/h]	11.4	Official value used for transport investment appraisal in Barcelona (Asensio and Matas, 2008)
	Users' no service penalty (FF)	γ_{FF_lost}	[€/trip]	7.93	Considered as the average taxi fare in central Barcelona (Autoritat Catalana de la Competencia, 2018; Institut Metropolità del Taxi, 2020)
	Users' no service penalty (SB)	γ_{SB_lost}	[€/trip]	2.50	(Herrmann et al., 2014; Ampudia-Renuncio et al., 2018), considering 80% avg. public transportation fare (Transport Metropolità de Barcelona, 2020) + 20% Avg. taxi fare in Barcelona (see above).

Agency costs and policies	Prorated vehicle's acquisition cost	γ	[€/car·h]	0.33 (electric car) 0.23 (ICE car)	Seat Mii market cost (SEAT, 2020), considering a useful life of 5 years, a residual value of 50%, an unavailability ratio of %5 due to maintenance and repairs (Bösch et al., 2018), and average insurance costs.
	Renting cost of a SB parking slot	γ_{S_SB}	[€/parking·h]	0.25 (no charger) 0.30 (Wallbox) 1.18 (fast charger)	Long-term renting cost of a parking slot in Barcelona (B:SM, 2020; SABA, 2020) plus the installation of charging infrastructure (Wallbox, 2020; Schroeder and Traber, 2012; Zhang et al., 2018).
	Average cost per parking (FF)	γ_{S_FF}	[€/parking·h]	0	It is assumed that on-street parking is subsidized.
	Average operative cost per trip	γ_e	[€/trip]	1.37 (electric car) 1.45 (ICE car)	Energy and fuel consumption are estimated according to trip distance, the vehicle SEAT Mii technical specifications, and the price index in Catalonia (Idescat, 2017). Cost of administrative control and maintenance are adapted from Bösch et al. (2018) to the currency and power purchase in Spain.
	Average cost per repositioning worker	C_t	[€/worker·h]	21.54	Labor cost of 14.32 €/h according to Idescat (2017). 33% of the working time is considered lost or ineffective for repositioning. The prorated acquisition cost of scooters is included but it is residual [0.04 €/h].
	Average time spent on fixed repositioning operations	δ	[min]	6	Authors' own estimation.
	Average autonomy of electric vehicles with 80% of battery charge	T_{ev}	[h]	13.07	SEAT Mii technical specifications (SEAT, 2020).
	Average recharging time of electric vehicles to reach 80% of battery charge.	T'_{ev}	[h]	4 (Wallbox) 1 (fast charger)	
	Charging hub location	d_{HUB}	[km]	0	In case of centralized battery charging, the hub location is considered to be near the center of the service region.
	Maximum reservation time	τ_r	[min]	20	Taken from ShareNow policies in Madrid (ShareNow, 2022).
	Average fare	F	[€/min]	0.27	

4.2. Definition of optimization scenarios

Table defines six scenarios for the Barcelona case study. In all of them, the vehicle fleet is completely composed of electrical vehicles (i.e. $e = 1$). This is the more demanding case in terms of the battery charging restriction. In general, battery charging is decentralized at stations, except for Scenario 0, where this is not possible as there are no stations (i.e. Scenario 0 is a pure FF system). In this scenario, the battery charging happens at a centralized hub. Scenarios 1, 2 and 3 represent mixed FF and SB car-sharing systems with different coverage rates of the SB system (i.e. different number of SB parking stations are used). Scenario 4 is a pure SB system, where it is not possible to park on-street. In all the previous scenarios, the objective function responds to the maximization of the operating agency profits. Only in Scenario 5, a mixed car-sharing system equivalent to Scenario 2, the minimization of total costs is considered (i.e. social optimum).

Note that the density of SB stations, Δ_{SB}, which was defined as a decision variable, is actually used as an input defining the different scenarios. This is because there is not a direct cost associated to the number of parking stations, but just to the number of parking slots used, which depends on the vehicle fleet. In contrast, more parking stations could yield benefits to the system by increasing the SB coverage and serving more potential demand. This implies that any optimization trying to maximize profit would yield the maximum number of available parking stations as optimal. This available number of parking stations depends on each particular implementation, and defines a particular SB coverage. Three possible SB coverage levels are analyzed here.

Table 5. Definition of optimization scenarios

Id.	SB coverage	FF fleet limitation (m_{FF})	Fraction of e-vehicles (e)
Scenario 0 (Pure FF system)	SB coverage = 0 % $\Delta_{SB} = 0$ [stations/km^2] Number of SB parking stations = 0	-	$e = 1$ (100% electrical fleet) Battery charging = centralized at hub
Scenario 1	SB coverage = 33 % $\Delta_{SB} = 1.04$ [stations/km^2] Number of SB parking stations = 41	-	$e = 1$ (100% electrical fleet) Battery charging = decentralized at stations
Scenario 2 (Baseline Scenario)	SB coverage = 66 % $\Delta_{SB} = 2.08$ [stations/km^2] Number of SB parking stations = 82		
Scenario 3	SB coverage = 100 % $\Delta_{SB} = 3.13$ [stations/km^2] Number of SB parking stations = 122		
Scenario 4 (Pure SB system)	SB coverage = 100 % $\Delta_{SB} = 3.13$ [stations/km^2] Number of SB parking stations = 122	$m_{FF} = 0$	
Scenario 5 (Social optimum)	SB coverage = 66 % $\Delta_{SB} = 2.08$ [stations/km^2] Number of SB parking stations = 82	-	

4.3. Optimization results

Note that the model has features that could potentially lead to non-convexity problems. The first one is the multicausal estimation of the repositioning movements of the system. Depending on the case, repositioning can be determined by the spatial imbalance (which depends on h_SB, h_FF), by the recharging needs, or by the SB-FF imbalance (which do not depend on h_SB, h_FF). So, depending on the case, the tradeoff changes and h_SB, h_FF could be more or less influential in the final solution. In addition to that, the SB-FF imbalance could happen in two directions, becoming favorable or unfavorable to the repositioning. Therefore, the optimal configuration could tend to force this compensation instead of addressing the demand served tradeoffs. And finally, the battery recharging requirements could be also critical and force the system to meet the requirements by eliminating some tradeoffs (i.e. impossibility to increase the FF/SB fleet ratio).

Given that, the optimization has been solved through a custom algorithm implemented into the numeric computing platform MATLAB (version R2021a). This algorithm decomposes the global problem into smaller convex problems. Each one corresponding to a different performance case. The smaller problems are in turn solved through another custom algorithm which uses the *fminsearch* (which is based on the simplex search method (Lagarias et al., 1998). Then, the solutions of all cases are compared and the best one is chosen. In addition to that, it is recommended the implementation of warning flags in order to a manual examination. Since in some scenarios the convergence could be difficult and it will require a finer tuning of the algorithm (i.e. tolerances, solution ranges).

The results obtained are presented in Tables 5, 6 and 7.

Table 6. Optimization results for the Barcelona's mixed car-sharing case study. Decision variables and KPIs.

	Variable		Units	Scenario 0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
				Pure FF	33% SB	66% SB	100% SB	Pure SB	66% SB
Decision Variables	Stations	Density of stations - Δ_{SB}	[stat/km^2]	0.00	1.05	2.09	3.11	3.11	2.09
		Number of stations - $\Delta_{SB}R$	[stations]	0	41	82	122	122	82
		SB parking slots used[1]	[parking slots]	0	40	62	79	230	253
	Fleet size	Free-Floating fleet - m_{FF}	[cars]	321	260	137	101	0	106
		Station-Based fleet - m_{SB}	[cars]	0	33	55	71	226	253
		Total available vehicle fleet[2]	[cars]	333	300	199	179	230	365
	Repositioning	Repo teams - k	[employees]	9.1	11.5	11.5	10.6	5.9	9.2
		FF repo period - h_{FF}	[hours]	-	25.0	15.4	12.4	-	17.4
		SB repo period - h_{SB}	[hours]	-	9502.6	203.3	290.5	53.0	33.2
Vehicle Fleet	FF fleet size	Vehicles in use	[cars]	130	130	117	100	0	104
		Idle vehicles	[cars]	191	130	20	1	0	2
		Total FF fleet size - m_{FF}	[cars]	321	260	137	101	0	106
	SB Fleet size	Vehicles in use	[cars]	0	9	26	48	124	48
		Idle vehicles	[cars]	0	24	81	24	102	204
		Min. fleet size (recharge)[3]	[cars]	0	33	45	60	104	61
		Total SB fleet size - m_{SB}	[cars]	0	33	55	71	226	253
	Spare vehicles	Recharging at hub	[cars]	10	0	0	0	0	0
		Being repositioned	[cars]	3	8	8	7	4	6
		In maintenance	[cars]	17	15	10	9	12	18
		Total spare vehicles	[cars]	29	23	18	16	15	24
	Average number of daily trips per vehicle[4]		[trips/car·day]	14.29	16.96	26.43	30.21	19.79	15.34
Repositioning	Repositioning rate	Baseline FF	[operations/h]	19.72	27.08	32.54	33.33	0.00	28.86
		Baseline SB	[operations/h]	0.00	0.17	2.94	4.96	18.51	9.62
		FF=>SB repositioning	[operations/h]	0.00	0.00	0.00	0.00	0.00	9.62
		SB=>FF repositioning	[operations/h]	0.00	23.96	32.54	33.33	0.00	0.00
		Keep EVs' battery charge	[operations/h]	6.36	6.36	5.73	4.88	0.00	5.10
		Total FF repositioning rate	[operations/h]	13.36	3.12	0.00	0.00	0.00	28.86
		Total SB repositioning rate	[operations/h]	0.00	23.96	32.54	33.33	18.51	0.00
		Total repositioning rate	[operations/h]	13.36	27.08	32.54	33.33	18.51	28.86
	Repositioning metrics	Avg. repo time / operation	[h/operation]	0.31	0.28	0.24	0.21	0.21	0.21
		Moving repositioning time	[h/h]	6.06	7.67	7.68	7.05	3.91	6.10
		Time lost in repositioning	[h/h]	3.03	3.83	3.84	3.53	1.95	3.05
		Total repositioning time	[h/h]	9.10	11.50	11.53	10.58	5.86	9.15
		Average team performance	[ops/employee·h]	2.17	2.35	2.82	3.15	3.16	3.15
User	Users' average access distance		[m]	200	246.5	266.7	266.7	266.7	266.7
	Users' avg. usage time (incl. reservation) - τ_s		[minutes]	27.6	27.6	27.6	27.6	27.6	27.6

[1] This corresponds to the total SB fleet size, m_{SB}, plus the number of SB vehicles being repositioned.
[2] This corresponds to the total FF + SB fleet size, (i.e. $m_{FF} + m_{SB}$) plus the average number vehicles being repositioned.
[3] This corresponds to the minimum number of SB vehicles in order to fulfill the battery charging restriction for electrical vehicles.
[4] Daily or annual results assume that the peak hourly demand considered happens during 13h/day. During the remaining 11h, off-peak demand is on average 30% of the peak demand. These magnitudes are consistent with the daily mobility demand for the city of Barcelona. All days are considered equal during the year.
[5] Only battery recharging repositioning operations to the hub exist in this scenario.

Table 7. Optimization results for the Barcelona's mixed car-sharing case study. Served demand results.

		Variable	Units	Scenario 0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
				Pure FF	33% SB	66% SB	100% SB	Pure SB	66% SB
Demand	Free-Floating demand	FF demand served - λ_{FF}	[trips/h·km^2]	7.19	7.20	6.48	5.52	0.00	5.77
			[trips/day]	4765.28	4766.19	4294.89	3655.91	0.00	3821.98
			[%]	73.64	73.66	66.37	56.50	0.00	59.07
		FF demand lost[1] - λ_{FF_lost}	[trips/h·km^2]	2.58	2.57	1.09	4.25	0.00	4.00
			[trips/day]	1705.50	1133.93	719.00	10.77	0.00	875.27
			[%]	26.36	19.22	14.34	0.29	-	18.63
		Transferred to SB system[2] (i.e. Potential SB demand - λ_{SB_max})	[trips/h·km^2]	0.00	0.86	2.20	4.23	9.77	2.68
			[trips/day]	0.00	570.66	1456.88	2804.10	6470.78	1773.52
			[%]	0.00	8.82	22.51	43.33	100.00	27.41
	Station-based demand	SB demand served - λ_{SB}	[trips/h·km^2]	0.00	0.50	1.46	2.65	6.88	2.68
			[trips/day]	0.00	328.43	964.15	1754.16	4557.80	1773.52
			[%]	-	57.55	66.18	62.56	70.44	100.00
		SB demand lost - λ_{SB_lost}	[trips/h·km^2]	0.00	0.37	0.74	1.59	2.89	0.00
			[trips/day]	0.00	242.23	492.73	1049.93	1912.98	0.00
			[%]	-	42.45	33.82	37.44	29.56	0.00
	Total demand	Served	[trips/h·km^2]	7.19	7.69	7.94	8.17	6.88	8.45
			[trips/day]	4765.28	5094.62	5259.04	5410.07	4557.80	5595.51
			[%]	73.64	78.73	81.27	83.61	70.44	86.47
		Lost	[trips/h·km^2]	2.58	2.08	1.83	1.60	2.89	1.32
			[trips/day]	1705.50	1376.16	1211.74	1060.71	1912.98	875.27
			[%]	26.36	21.27	18.73	16.39	29.56	13.53

[1] This corresponds to the FF demand not served by the FF system, and outside the coverage of the SB system. So, this demand is lost.
[2] This corresponds to the FF demand not served by the FF system, and inside the coverage of the SB system. So, this constitutes the potential demand for the SB system.

Results of the optimization in the different scenarios show that it is difficult for car-sharing systems to serve a high percentage of the potential demand. This is because leaving part of the demand not served (i.e. mainly the temporal and spatial peak demand fluctuations) results in a dramatic reduction of fleet and repositioning costs. Optimal fleets tend to be restrained, reducing idle times and yielding a higher utilization rate of vehicles (i.e. 14-30 trips/vehicle·day; see Table 6). The optimal percentage of the potential demand to serve depends on the configuration of the system. It can be seen (see Table 7) that mixed car-sharing systems (i.e. FF+SB) perform better than pure FF or pure SB systems in this sense. Mixed systems are able to serve 78-83% of the potential demand while maximizing the profit of the operating agency. In contrast, a pure FF system would only serve around 73% of the demand, and a pure SB system a 70%. The lower demand served in the case of a pure FF system is a result of considering a complete electrical vehicle fleet. The absence of stations forces the system to rely on a battery charging hub, implying additional repositioning operations and additional costs at the hub. These costs exhibit higher marginal costs with respect to the demand served in this pure FF scenario, so that the optimal percentage of the potential demand to serve is lower. In contrast, mixed or pure SB systems rely on decentralized battery charging using chargers located at parking stations, not incurring in the previous additional costs.

In addition, when we compare Scenario 2 with its analogous Scenario 5, it is observed that results are very similar independently of the objective function. Of course, one returns a slightly better profit result, while the other a lesser generalized cost. But they are in a close range, because the benefit of serving additional trips is in this case quite close in terms of additional revenue or user cost reduction. However, we highlight here that in each case the optimization focuses on serving a different type of demand. In Scenario 5, 100% of the SB demand is served in order to avoid the more expensive penalties at stations. While in Scenario 2, more FF demand is served because the revenue is the same and the cost for increasing the FF fleet is cheaper than the SB counterpart.

Table 8. Optimization results for the Barcelona's mixed car-sharing case study. Costs.

	Variable		Units	Scenario 0	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario5
				Pure FF	33% SB	66% SB	100% SB	Pure SB	66% SB
Agency	Infrastructure costs - Z_{IC}	Fleet costs	[€/h]	110.05	99.15	65.66	59.10	75.99	120.34
		FF parking costs	[€/h]	0.00	0.00	0.00	0.00	0.00	0.00
		SB parking costs	[€/h]	0.00	11.35	16.98	21.08	57.78	64.65
		Hub parking costs	[€/h]	7.51	0.00	0.00	0.00	0.00	0.00
	Operative costs - Z_O		[€/h]	386.30	413.00	426.32	438.57	369.48	453.60
	Repositioning costs - Z_R		[€/h]	130.62	165.16	165.52	151.91	84.20	131.41
Users	Access costs - Z_{AC}		[€/h]	528.17	610.95	630.67	648.78	546.58	671.02
	No service penalties - Z_{NS}	FF demand lost	[€/h]	252.29	167.74	106.36	1.59	0.00	129.48
		SB demand lost	[€/h]	0.00	113.66	231.21	492.66	897.63	0.00
Total	Total agency costs - Z_A		[€/h]	634.48	688.66	674.48	670.66	588.45	769.99
			[M€/year]	3.91	4.25	4.16	4.14	3.63	4.75
	Total user's costs - Z_U		[€/h]	780.46	892.35	968.24	1143.04	1444.20	800.50
	Total costs - Z		[€/h]	1414.94	1581.01	1642.72	1813.70	2032.66	1570.49
Per trip	Generalized cost per trip[1]		[€/trip]	5.02	5.24	5.28	5.67	7.54	4.74
	Revenue neutral fare[2]		[€/trip]	2.25	2.28	2.17	2.10	2.18	2.33
	Profit[3]		[€/h]	712.91	751.85	812.51	859.04	700.26	812.14
			[€/trip]	2.53	2.49	2.61	2.68	2.60	2.45
			[M€/year]	4.40	4.64	5.01	5.30	4.32	5.01

[1] Average cost per trip, including agency and users' costs.
[2] Average agency cost per trip. This is equivalent to the fare users need to pay in the absence of subsidies and profits.
[3] Profit is computed according to the considered fare of 0.27 [€/min]. See Table 4.

The obtained results are grounded on the assumption that leaving a significant fraction of the demand unserved, does not affect the overall potential demand. This assumption is acceptable if considering that car-sharing systems complement other urban mobility options, so that the unreliability of the system does not penalize its potential demand, as the trip can be served by other transportation modes, maybe less convenient (e.g. public transportation) or more expensive (e.g. taxi system), but still available.

Regarding the performance of mixed car-sharing systems, results show that the demand served by the SB system grows together with the stations' coverage of the service area. In particular, if the station's coverage is low, the SB fleet is just the minimum to ensure the battery recharging at stations and the demand served by the SB system is marginal. Therefore, it is recommended to include as many parking stations as possible in the system so that the SB system reaches a significant coverage of the service area and increases the overall amount of demand served. In spite of this, with the increase of the SB coverage and subsequent increase of the potential demand for the SB system, the non-served demand rates at stations grow, because there are not many idle cars at all stations to serve the increased demand fluctuations. Actually, note that the total fleet size is reduced with the increase of the SB coverage in mixed car-sharing systems. This is because the inclusion of SB fleet brings additional parking costs at stations. This smaller fleet is partially compensated by more repositioning operations.

Baseline repositioning movements grow with the demand served. It can also be seen in Table 6 that the optimal system designs aim to minimize repositioning operations by compensating baseline repositioning with an equivalent amount of FF – SB compensation movements. For the particular case of the Barcelona case study, with $P_{SB} = 0.43$, there is a net movement of vehicles from the FF to the SB systems, so that repositioning teams need to compensate them with SB=>FF movements. Note that only Scenario 1, with low SB coverage, requires FF repositioning (i.e. vehicles taken from streets), as in all other scenarios these movements are already performed by users. Repositioning movements due to the battery charging restriction slightly decrease with the SB coverage, as there are more vehicles already recharging at stations (i.e. a higher SB fleet). In all scenarios these battery recharging movements do not require additional repositioning, as other sources of repositioning are enough to account for this, except for Scenario 0 (pure FF) where battery charging movements always imply additional repositioning, as vehicles need to be moved to the battery recharging hub.

Regarding the system costs, for all scenarios the optimal agency cost per trip (i.e. the revenue neutral fare) is between 2.1 – 2.3 [€/trip] (see Table 1). With the considered fares, this yields a profit of 2.5 – 2.7 [€/trip]. The overall profit of the system will depend on its capacity to serve higher fractions of the potential demand. This means that mixed systems with higher SB coverage will be overall more profitable, as they are able to serve higher fractions of the potential demand. Operative costs are the most important agency costs, implying around 65% of total agency costs, although they do not imply important planning risks, as they are proportional to the demand served and directly compensated by the fares paid by users. The remaining 35% of agency costs is shared by fleet, parking and repositioning costs, being parking costs only significant in pure SB systems (i.e. 10% of total agency costs). In mixed systems, SB parking only accounts for 5% of the total agency costs in the worst case of complete SB coverage, because the system tends to exploit the FF system with subsidized on-street parking.

Finally, user costs can be seen as a proxy for the level of service offered, and include the average access distance to an available vehicle and the probability of the user not finding any available vehicle nearby (i.e. the fraction of not served demand). The average access distance is constant for all scenarios and equal to 266m, which is determined by the uniform probability of having a vehicle inside the catchment area. This means that for all scenarios, optimal vehicle fleets are small and idle vehicles are few. So, on average each user will have access to at most one vehicle. In turn, no service penalties grow with the amount of demand not served, particularly with the SB demand not served, as it is assumed that users perceive the SB system as more reliable with higher no-service penalties.

4.4. Robustness of optimal results

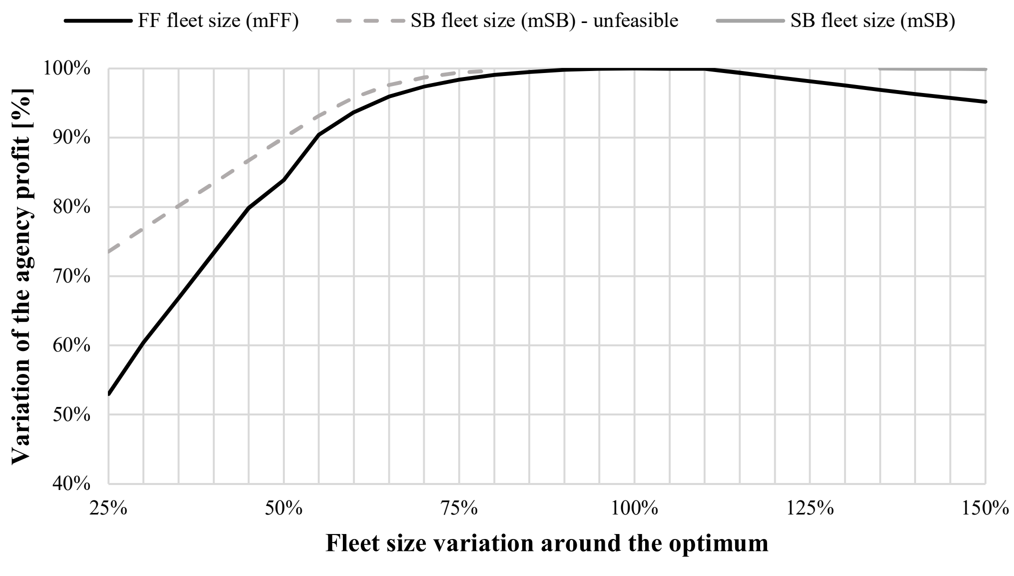

Figure 2. Robustness of the optimal design of a mixed car-sharing system to the fleet size. (Note: Scenario 2 is taken as the baseline scenario).

Robustness is a desirable property in any optimization framework. A robust optimal solution means that deviations in the selection of the optimal decision variables imply smaller deviations in the result of the objective function. Figure 2 shows that the results obtained for the optimal design of a mixed car-sharing system, are robust with respect to the size of the vehicle fleet, which represents the main decision variable in the system. See, for instance, that variations of 25% of the optimal fleet size, imply variations in agency profit of less than 5%. It can also be seen that sensitivity is higher if fleet size is reduced under the optimum value. In this case, a reduction of 50% of FF fleet can reach almost a 20% reduction of the profit. In the case of reduction of the optimal SB fleet, one must bear in mind that this could lead to unfeasible designs in terms of the battery recharging requirements. So, the recommendation is that it is safer to exceed the optimum fleet size than not reaching it.

4.5. Sensitivity analysis with respect to potential demand density

This section analyzes the effect of the magnitude of the potential demand density in the optimal design and profitability of a mixed car-sharing system. Figure 3 shows that the agency cost, user's cost and profit per trip are insensitive to the magnitude of the potential demand. This results from the system serving almost the same percentage of the potential demand with the same average number of trips/vehicle·day for the different demand levels (see Figure 4). Therefore, mixed car-sharing systems do not exhibit significant economies or diseconomies of scale. Figure 4 also shows that the fleet size and the overall agency profit grow proportionally to the potential demand.

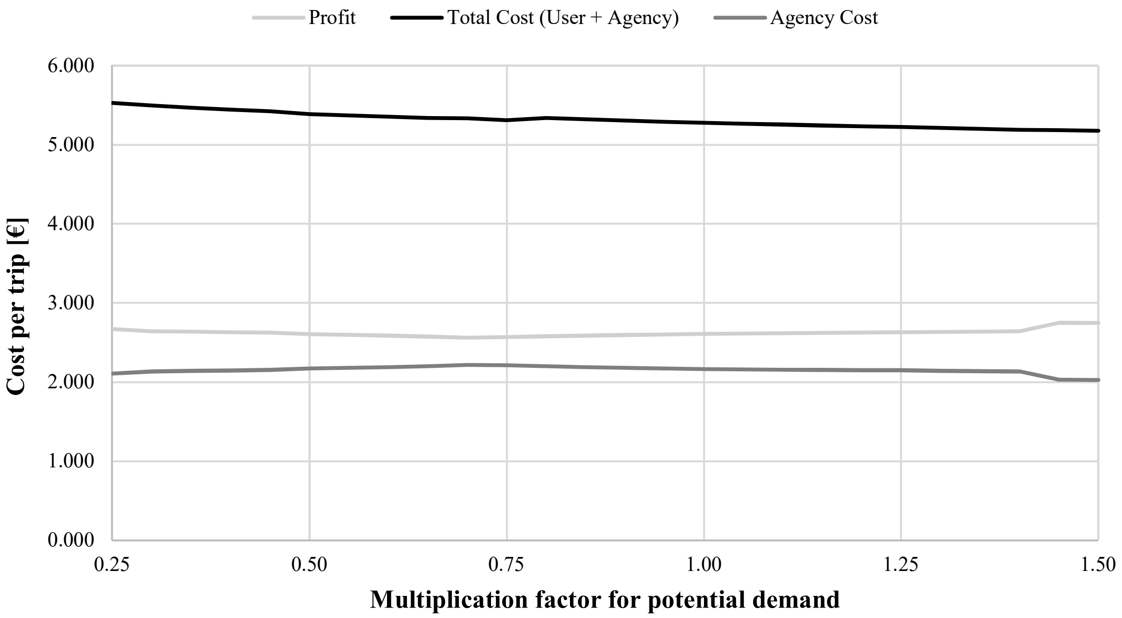

Figure 3. Sensitivity of system costs to different potential demand densities. (Note: Scenario 2 is taken as the baseline scenario; Default demand density was 9.77 trips/h·km^2).

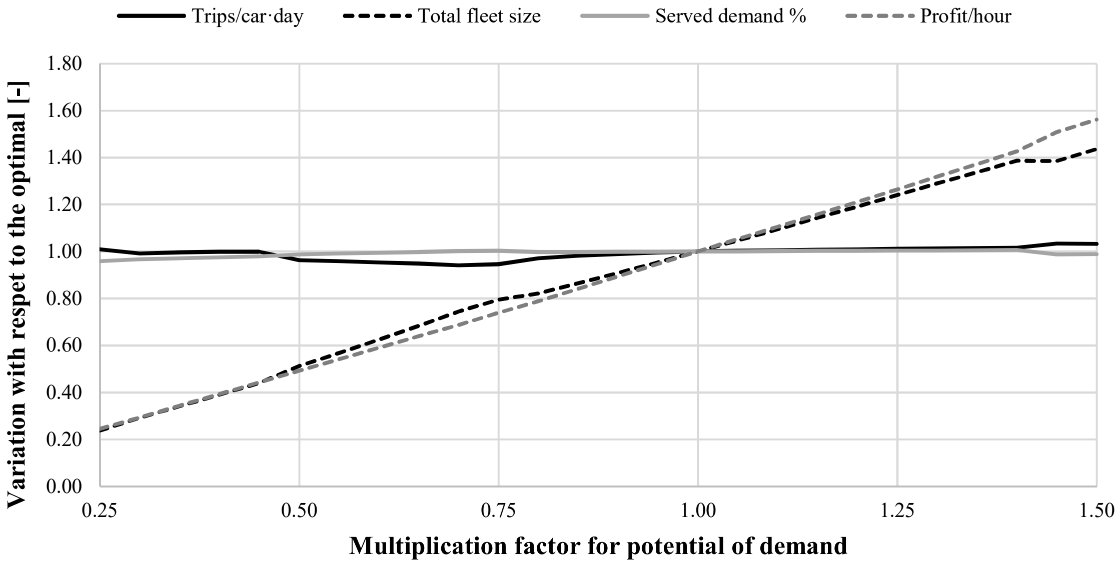

Figure 4. System performance sensitivity to different potential demand densities. (Note: Scenario 2 is taken as the baseline scenario; Default demand density was 9.77 trips/h·km^2).

4.6. On the share between FF and SB in mixed car-sharing systems

The main factors determining the share between FF and SB in a mixed car-sharing system are *i)* the SB stations' coverage and *ii)* the behavior of users when returning the vehicle. To a lesser extent, the parking cost (either on-street or in stations), does also play a role. This section analyzes these factors and effects on the system configuration.

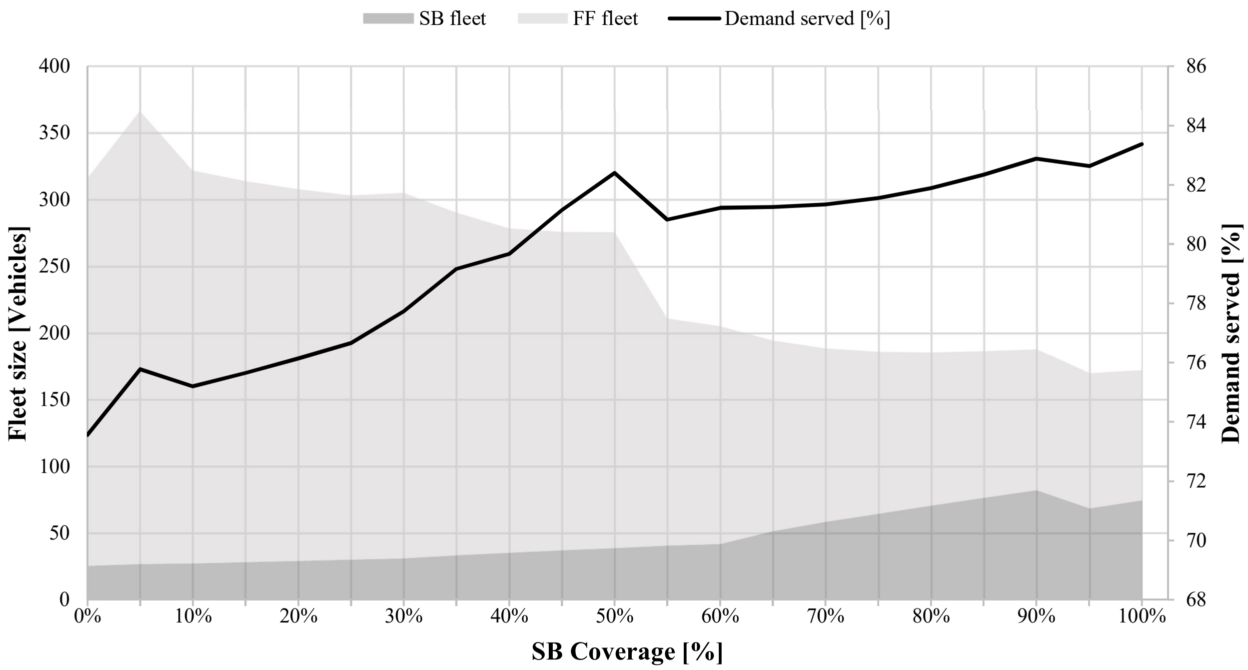

Figure 5. Vehicle fleet and percentage of the potential demand served as a function of SB stations' coverage.

4.6.1. The effect of SB stations' coverage

As SB coverage grows, the optimal SB vehicle fleet also grows, while FF fleet decreases (see Figure 5). Overall, the total vehicle fleet decreases with the increase of the SB coverage, due to the higher SB parking costs. Still, the percentage of the potential demand served grows, due to an increased vehicle repositioning intensity.

FF and SB systems play different roles depending on the SB coverage. If SB coverage is low, the objective of the SB system is only to provide enough battery availability to the vehicle fleet. This happens until SB coverage of 60% (see Figure 7) where the SB fleet is the minimum necessary to fulfill the battery charging requirement (i.e. $m_{SB}/m_{SB\_min} = 1$). Note that the particular threshold value of the SB coverage until when this happens depends on the users' parking behavior upon returning the vehicle (i.e. the fraction of SB returns when a parking station is available, which for the present case study is $P_{SB} = 0.43$). Lower values of P_{SB} would imply a smaller SB system, so that the SB fleet would be the minimum for a wider range. Figure 7 also shows that for growing SB coverages, the fraction of the FF vehicle fleet which are idle is progressively reduced, so that when the SB coverage is complete, all the FF vehicles are always in use. This means that the FF vehicle availability decreases when SB coverage grows, because a growing part of the demand can be transferred to the SB system.

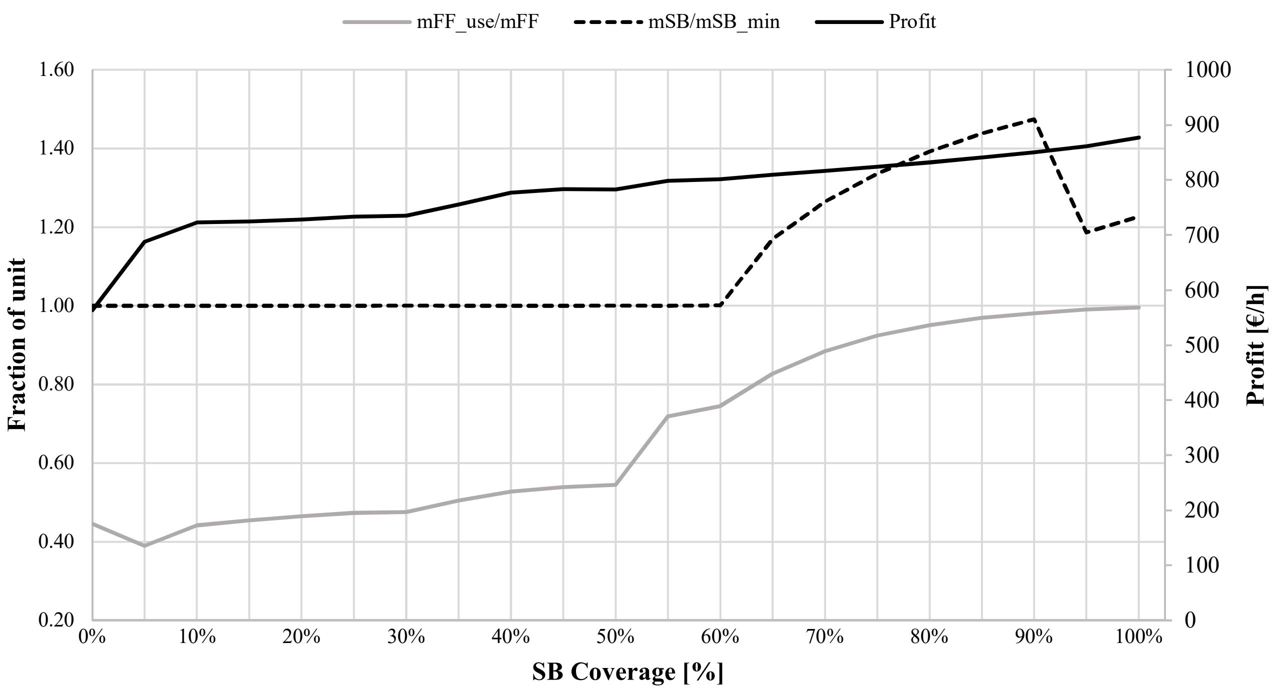

Figure 6. Performance of the FF and SB systems as a function of SB stations' coverage.

4.6.2. The effect of users' behavior when returning the vehicle, P_{SB}

The fraction of users returning the vehicle at parking stations (when their final destination is within the coverage area of one station), P_{SB}, is an important parameter to consider in any implementation, as, like the SB coverage, it plays a fundamental role in the share and performance of the FF and SB systems. If P_{SB} is low and most users return the vehicles on-street, the SB system will be marginal and devoted only to the battery recharge (see Figure 7 and Figure 8) regardless of the existing SB coverage. Note that Figure 7 and Figure 8 represent a 66% SB coverage as Scenario 2 is taken as the reference for the analysis. If P_{SB} grows, the relative importance of the SB system in terms of vehicle fleet and demand served will also grow, always in accordance with the existing SB coverage. This means that, if the SB coverage is large, the shifting towards the SB system will be more important, and the FF will be reduced to the minimum expression only to serve regions without stations.

At the limit, if the SB coverage is complete and $P_{SB} = 1$, the FF system would be inexistent. In contrast, if the SB coverage is low, the effects of P_{SB} will be less noticeable.

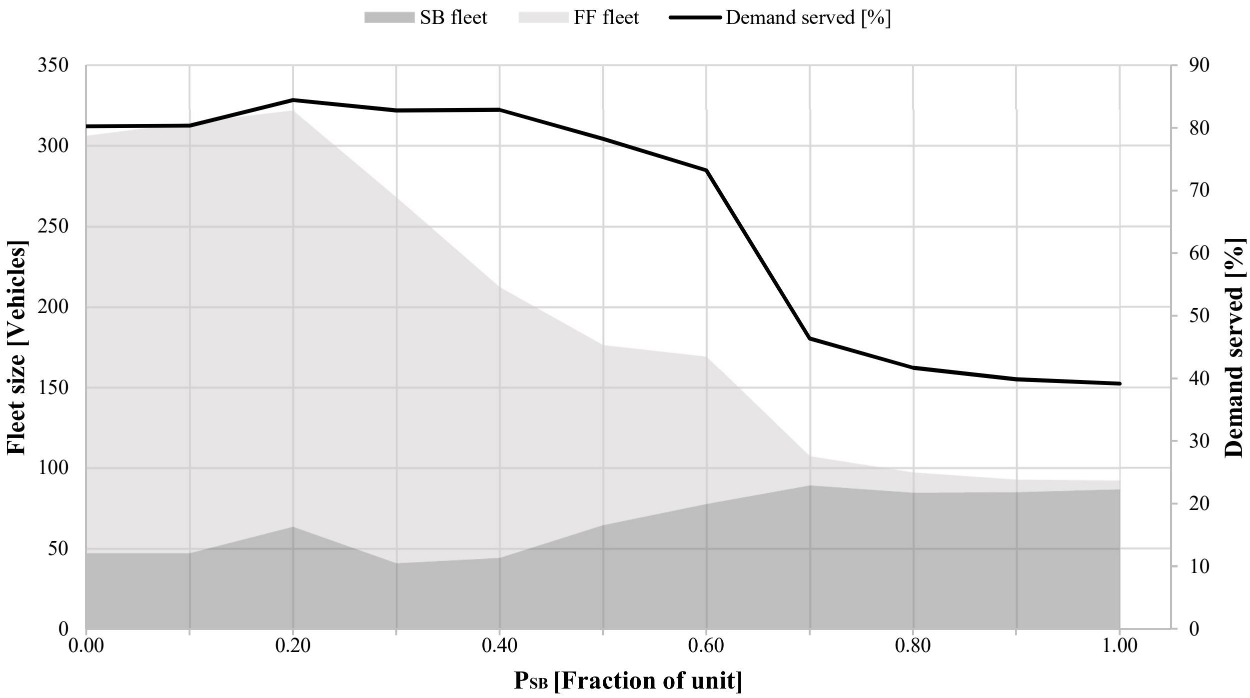

Figure 7. Vehicle fleet and percentage of the potential demand served as a function of the fraction of users returning the vehicle at parking stations (Note: Scenario 2 taken as the baseline scenario).

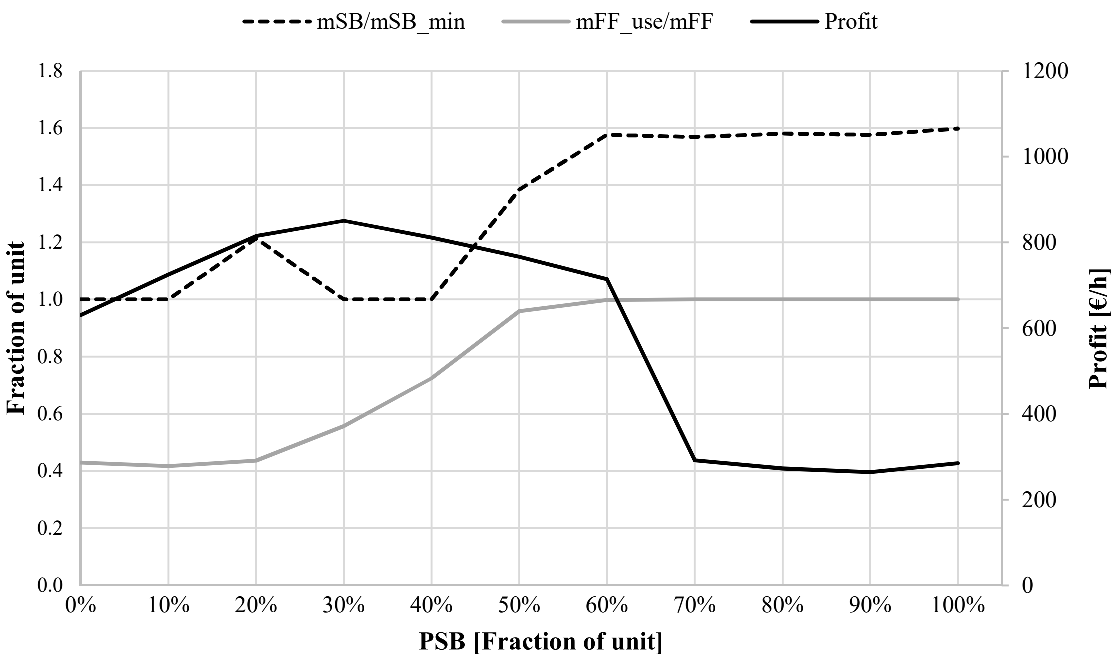

Figure 8. Performance of the FF and SB systems as a function of the fraction of users returning the vehicle at parking stations (Note: Scenario 2 taken as the baseline scenario).

Maximum profit happens around $P_{SB} = 0.3$, which results to be a robust optimum for all the contexts. This value represents the users' parking behavior which allows to primarily use the most profitable FF system (i.e. less parking costs) while reducing much of the repositioning movements to parking stations for charging batteries, as these movements are already done by users.

4.7. Sensitivity analysis with respect to on-street parking costs

The parameters considered for the Barcelona case study assume a completely subsidized on-street parking for the FF car-sharing vehicles, while the cost of SB parking is 0.30 €/h. This section analyzes the effects of this subsidy, and what would change if on-street parking was not free of charge.

Figure 9 shows that the effects of the on-street parking costs are detrimental for a wide range of values. If the cost of on-street parking grows, fleets are slightly reduced, yielding a similar reduction in the amount of demand served and in the resulting profits. Still, quite insignificant considering that parking costs are usually claimed by operating agencies as one of the main drawbacks for the implementation of car-sharing systems in cities. This result, in combination with the results of Balac et al. (2017), where through a simulation approach was found that car-sharing systems usage could be benefited from the increase of parking fees, denies the belief that subsidized parking is strictly necessary for the profitability of car-sharing systems.

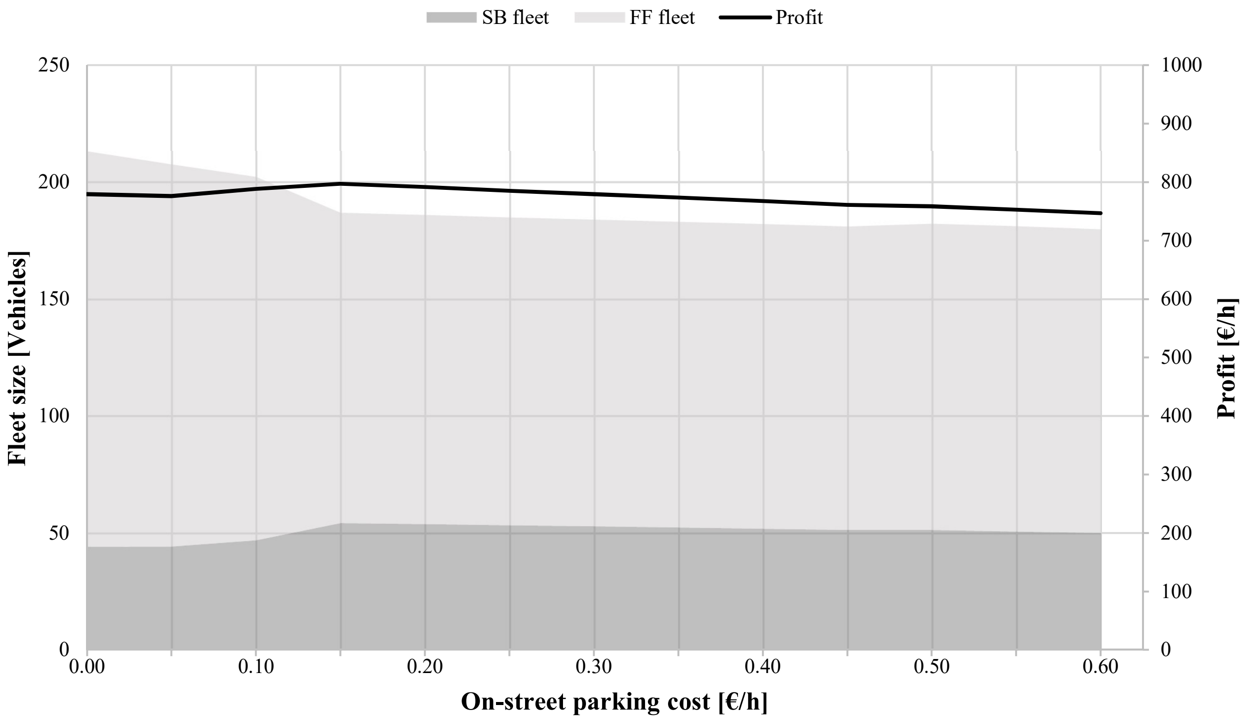

Figure 9. Fleet size and profit as a function of the rental cost of on-street parking (Note: Scenario 2 taken as the baseline scenario).

4.8. Sensitivity analysis with respect to the unitary labor cost of repositioning

Repositioning cost is another relevant agency cost which steers the design and performance of car-sharing systems. Figure 10, analyses the sensitivity of the system design and performance for varying labor costs of repositioning teams. It is seen that if repositioning costs are very small (e.g. 50% of the considered default value), it is worth having additional vehicle fleet to serve a higher fraction of the demand and increase profit. Note that this additional fleet allows to serve more trips, which are not directly balanced by the users, implying a higher likelihood of requiring a rebalancing movement. If the labor cost of repositioning grows, the optimal number of repositioning teams would be reduced, implying less capacity of performing repositioning movements. The system design responds to this situation by reducing the available vehicle fleet, implying a smaller fraction of the potential demand served, and a slightly reduced profit.

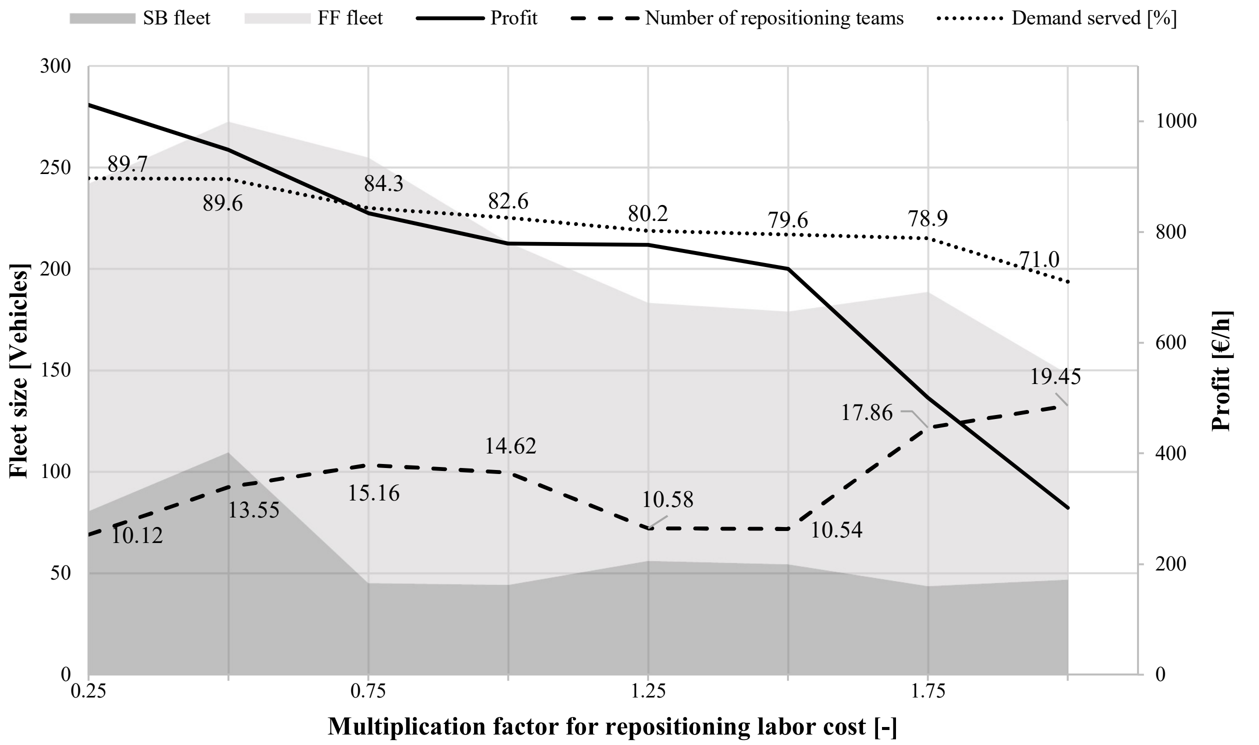

Figure 10. Fleet size and profit as a function of the labor cost of repositioning. (Note: Scenario 2 taken as the baseline scenario. Default labor cost was 21.54 €/h).

4.9. Decentralized vs centralized battery charging

Two battery charging configurations have been analyzed: *i)* decentralized battery charging at stations, and *ii)* centralized battery charging at a recharging hub. Recall that recharging at the hub always imply additional repositioning movements (and costs), as these cannot be done directly by users. In contrast, the decentralized SB battery recharging implies a minimum SB fleet to fulfill the battery charging requirement (i.e. obviously this is not possible in a pure FF system). Results show (see Figure 11) that overall, systems relying in a recharging hub serve slightly less demand. This is because the additional repositioning costs in this case are compensated by slightly smaller vehicle fleets. It can also be seen that if the SB coverage is low, it is better to use a recharging hub, while if the SB coverage is high, decentralized battery charging is the best option in terms of agency profit. The breakpoint between these situations is around a SB coverage of 35%.

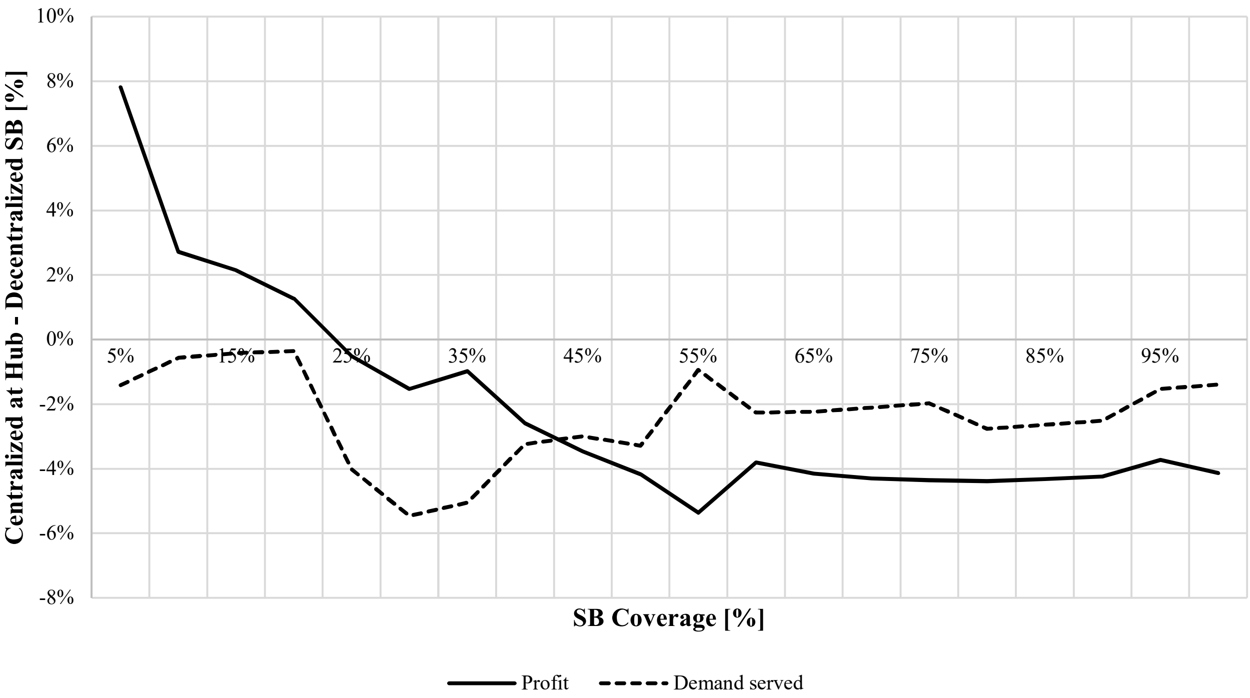

Figure 11. Difference in the amount of demand served and profit between the two battery recharging strategies considered. (Note: Scenario 2 taken as the baseline).

5. Conclusions and further research

An analytical planning model for a one-way mixed free-floating (FF) and station-based (SB) car-sharing system has been developed. This is based on the modeling of the strategical variables of the system and their relevant trade-offs, using continuous approximations. This analytical approach requires to simplify reality to some extent, and to obviate some details of operation. However, results provide valuable insights, valid in a wide range of contexts and which constitute the foundations for further research. This concluding section summarizes the main trade-offs that should be taken into account in the planning of mixed one-way vehicle sharing systems, as well as the insights obtained from the optimization of the model for the Barcelona case study. Parameters have been estimated for the city of Barcelona, although they could be valid for any city of similar size and socio-economical context.

On the optimal demand to serve. In order to ensure the economical sustainability of the system and maximize the operating agency profit, it is advisable to leave 15-25% of the potential demand not-served. This percentage grows in case of pure FF systems, and it is consistent with other optimization works addressing demand responsive transportation systems (Militão & Tirachini, 2021). This allows increasing the vehicle utilization rates and reducing rebalancing needs, which yields reduced agency costs and higher profit. Trying to serve higher fractions of the potential demand implies negative marginal profits. Serving demand fractions above 85% of the potential demand, results in huge agency costs and extreme over-dimensioning of the vehicle fleet size and repositioning operations, so that profit rapidly decreases, which could even turn into loses. It should be recognized that, from the user perspective, this fraction of non-served demand penalizes the reliability of the system. Users will only accept this level of service if other reliable one-way public transportation options are available, in order to substitute car-sharing, if necessary. If car-sharing is only competing with the private car, especially in contexts where public transportation services are limited, this lack of reliability of the system may eliminate car-sharing as a transportation option. If this is the case, a smart structure of fares and reservation policies in order to be able to charge a premium to ensure service availability is necessary for the system being profitable.

On the share between FF and SB systems. The possibility of car-sharing systems operating efficiently in a mixed fashion, where both parking on-street and at stations are possible, depends on the system design and on the users' parking behavior in the city. Two variables steer the share between both systems. On the one hand, the SB coverage. This is the number of parking stations used by the system and their spatial distribution over the service region. The number of parking stations to be used should be as large as possible, until complete SB coverage is reached. The benefits of using more stations are higher than their possible penalties and costs, especially if the vehicle fleet is electric and relies on stations for battery recharge. On the other hand, P_{SB}, the fraction of users who decide to park in a station when this option is available, largely determines the share between FF and SB systems. Note that low P_{SB} implies plenty of available on-street parking. In such context, vehicles will not return to stations, and the system will tend to be a pure FF system. Stations might be there only for recharging the batteries of electrical vehicles, when needed. In turn, moving the vehicles to stations for battery recharge would be the main task of repositioning teams. In contrast, if P_{SB} is large, it implies that on-street parking is scarce and users rely on parking stations to leave the car when reaching their destinations. In this context, it is even more important to ensure complete SB coverage, so that the system would tend to be mainly a SB system. A small FF system might be there only to serve regions outside the SB coverage (if any), and to save on parking costs, if on-street parking was subsidized, as far as the parking savings are larger than the cost of repositioning FF vehicles. Mixed systems prevail in between, as shown in Figure 12. In spite of this, note that in many contexts mixed car-sharing systems consist on a FF fleet, which is continuously in use, and the system relies on the SB fleet to provide additional vehicle availability to users.

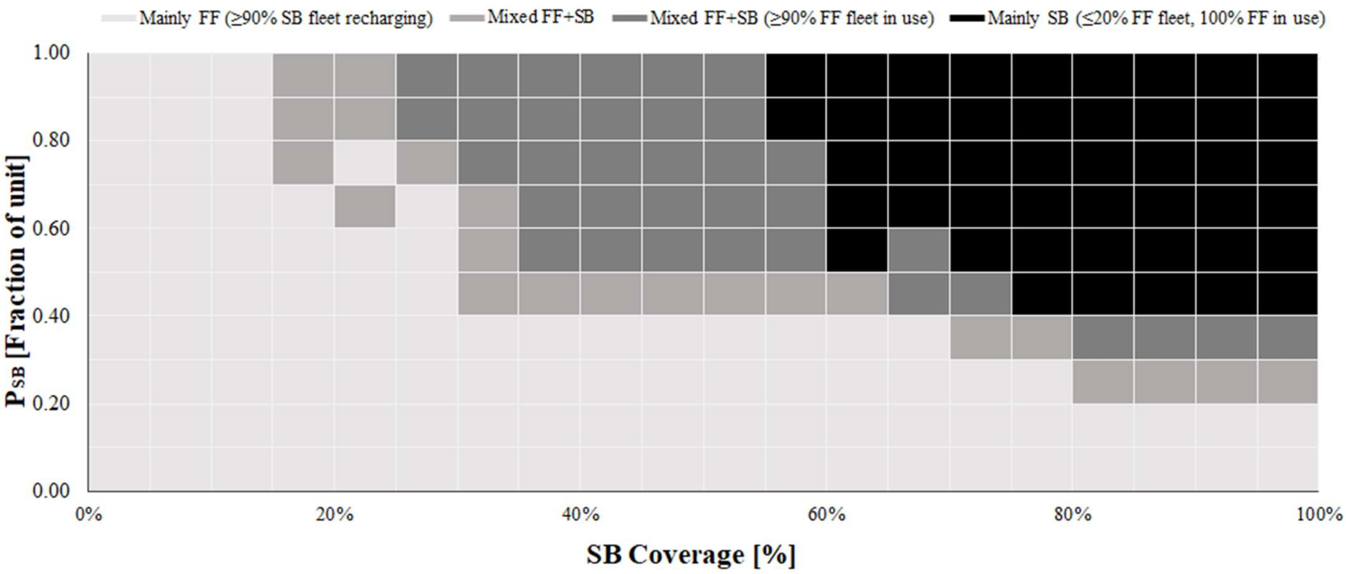

Figure 12. Difference in the amount of demand served and profit between the two battery recharging strategies considered. (Note: Scenario 2 taken as the baseline).

On electrical vehicle fleets. Using partial or total electrical vehicle fleet does not imply significant additional restrictions and costs to a mixed car-sharing system. EVs are slightly more expensive and require battery recharging infrastructure and some additional repositioning. However, these costs are compensated by the lower operating costs of EVs. Battery recharging is better performed in a decentralized fashion at stations, provided that SB coverage is significant (i.e. > 35%). Otherwise, battery recharging must be performed at a recharging hub.

In conclusion, an adequate design of the car-sharing system in terms of the vehicle fleet and SB coverage, yields a system where the expensive rebalancing operations are minimized. This is achieved by serving moderate fractions of the potential demand, while balancing FF and SB systems so that the repositioning required to maintain the system stable and keep the battery levels in the FF vehicles is mainly user based and fulfilled with minimum costs. Given these optimal designs, mixed car-sharing system can operate on revenue neutral fares of around 2 [€/trip], which given the proposed typical fares, could yield significant profit.

Considering the parameters estimated for the Barcelona case study, and particularly a potential demand of roughly 6250 [trips/day] with 82 parking stations over a service area of 39 km^2 (i.e. SB coverage of 66%), an optimal design would consist of a fleet of 200 vehicles (i.e. roughly 150 vehicles FF and 50 vehicles SB), with 11.5 repositioning teams. 80% of the potential demand could be served (i.e. around 5200 [trips/day]), implying an average of 26.4 [trips/vehicle·day]. To implement and operate this system would cost around 4 million € per year, and considering a fare of around 5 [€/trip] the system would yield a profit of around 5 million €/year.

The sensitivity of system costs and profit to suboptimal designs is small. This means that the proposed designs are robust, and deviations could be accepted without implying severe penalties. However, it should be noted that this sensitivity is larger when the deployed resources are below the optimal values. This means that special care should be devoted to avoid being excessively conservative in the fleet size deployment, knowing that over-sized fleets almost do not penalize profit.

Regarding the sensitivity of optimal designs to potential demand, the vehicle fleet size and the number of required repositioning teams grow almost proportionally with the potential demand for the system. This yields a very slight increase in the fraction of the potential demands served, with an equally very slight increase in the agency profit per trip. So, economies of scale in mixed car-sharing systems are mild, and can be considered insignificant. Specifically, the elasticity of profit with respect to demand is roughly 0.05 (i.e. if the potential demand grows 1%, the profit per trip would grow 0.05%). Similarly, optimal designs and profits are quite insensitive to input cost parameters, like the parking cost (on-street or station-based) or the labor costs of repositioning teams. This means that the obtained results are robust, and can be generalized to many possible contexts.

The present paper represents the foundation for the analysis of mixed car-sharing systems from a system wide perspective and in analytical terms. Clearly, it represents a simplification of reality, which is essential to obtain global insights regarding the optimization of the system from the planning perspective. Further research should be addressed to mitigate the differences between the model and reality, in order to test the model and to obtain the order of magnitude of the errors committed. This could include the relaxation of the assumption of spatially uniform demand level, including variable demands in central regions and periphery, and a discrete modelling approach to account for the particular behavior of individual customers, vehicles, trips and repositioning operations. Simulation is the best tool to accomplish these objectives. Therefore, further research should be devoted to develop a simulation model for one-way shared vehicle systems. An agent-based simulation seems an adequate approach, as agents are clearly identified: customers, vehicles and repositioning teams. Such simulation model would also help in the analysis and optimization of the system at the operational level. For instance, current relocation strategies still exhibit potential for improvement.

Another direction for further research could develop a customers' behavioral model in one-way vehicle sharing systems. This should include a more precise characterization of the user behavior in terms of the origin / destination imbalance and parking behavior at destination given the characteristics of a particular city. Also, a demand model could be formulated in order to calibrate the effects on demand of different pricing strategies or different competitive contexts between transportation alternatives. Demand for the system could then be treated as an endogenous variable, instead of exogenous and assumed given. These analyses should be based on data from current pilot implementations (revealed preferences) or in stated preference surveying. This analysis is critical, being the demand sensitivity to unreliable service, and the parking behavior at destination (on-street or station-based) the important factors which determine the optimal design of the system.

Finally, further research could also address the potential of new technologies, operational platforms, and management strategies (smart pricing, user incentives, bookings, etc.) to reduce the need for relocation movements, further promoting user-based relocations.

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7. Appendix A. Demand served by a mixed car-sharing system

λ_{max} [trips/km^2·h] is defined as the average potential demand density of the system, and it is an input to the model. However, trip demand is not uniform over time and space. In order to characterize this variability, the model includes some other demand inputs. The temporal variation of demand density is characterized by its standard deviation std_λ_{max}, which can be expressed in terms of the coefficient of variation, CV_λ_{max}, as:

$$CV\_\lambda_{max} = \frac{std\_\lambda_{max}}{\lambda_{max}} \quad (A.1)$$

Trip generation and attraction rates are not uniform in space either. In one-way vehicle-sharing systems, users can return the vehicle at destinations different than their origins, which generate potential demand imbalance. In order to model this demand imbalance, we define $\lambda_{q(r)}$ and $\lambda_{t(r)}$, representing the density of requests and returns, respectively, in the subzone r, $r \in \mathcal{R}$. Then, the demand imbalance level in r is defined as a fraction of the average potential demand by the dimensionless variable $\varphi_{(r)}$. This is:

$$\lambda_{t(r)} - \lambda_{q(r)} = \varphi_{(r)}\lambda_{max} \quad (A.2)$$

By definition, and considering the conservation of vehicles over the service region $\mathcal{R}$, we have:

$$\frac{\sum_{\forall r} \lambda_{t(r)} c_{(r)}}{R} = \frac{\sum_{\forall r} \lambda_{q(r)}\, c_{(r)}}{R} = \lambda_{max} \quad (A.3)$$

Where $c_{(r)}$ represents the area of subzone r, where $R = \sum_{\forall r} c_{(r)}$. In fact, we consider subzones of similar size, so that $c_{(r)} \approx c$.

From Equation 10 we can derive that:

$$\sum_{\forall r} \left(\lambda_{t(r)} - \lambda_{q(r)}\right) c_{(r)} = \lambda_{max} \sum_{\forall r} \varphi_{(r)} c_{(r)} = 0 \Rightarrow \sum_{\forall r} \varphi_{(r)} c_{(r)} = 0 \quad (A.4)$$

If we define $\mathcal{R}_q$ as the partition of $\mathcal{R}$ where requests are larger than returns (i.e. $\forall r \in \mathcal{R}_q$, $\varphi_{(r)} < 0$), and $\mathcal{R}_t$ as the partition of $\mathcal{R}$ where returns are larger than requests (i.e. $\forall r \in \mathcal{R}_t$, $\varphi_{(r)} > 0$), then, from Equation A.4 we derive:

$$\sum_{\mathcal{R}_t} \varphi_{(r)} c_{(r)} = -\sum_{\mathcal{R}_q} \varphi_{(r)} c_{(r)} \quad (A.5)$$

In addition, we can define Φ_q and Φ_t as the average imbalance level in partitions $\mathcal{R}_q$ and $\mathcal{R}_t$ as:

$$\Phi_t = \frac{\sum_{\mathcal{R}_t} \varphi_{(r)} c_{(r)}}{R_t}; \ \Phi_q = -\frac{\sum_{\mathcal{R}_q} \varphi_{(r)} c_{(r)}}{R_q} \quad (A.6)$$

Where R_t and R_q are respectively the areas of the partitions $\mathcal{R}_t$ and $\mathcal{R}_q$. This means that:

$$\Phi_t R_t = \Phi_q R_q \tag{A.7}$$

Note that Φ_q is defined positive (see Equation A.6). Finally, we define the average demand imbalance level over $\mathcal{R}$ as the dimensionless parameter Φ:

$$\Phi = \frac{\Phi_t R_t + \Phi_q R_q}{R} \tag{A.8}$$

In conclusion, the potential demand for the mixed car-sharing system and its temporal and spatial variability is characterized by three input parameters: λ_{max}, CV_λ_{max}, and Φ. Appendix B provides reference on how to obtain these parameters if an origin/destination (O/D) trip demand matrix is available.

7.1. Appendix A.1. Demand served by the FF part of the mixed car-sharing system, λ_{FF}

The density of trips served by the FF part of the car-sharing system depends on two factors. First, on the potential demand density, λ_{max}. And second, on the capacity of the system to serve these trips. If λ_{max} is smaller than the capacity of the system, then $\lambda_{FF} = \lambda_{max}$. Otherwise, the FF system will work at full capacity, and some potential demand will be lost. This means that the estimation of λ_{FF} requires determining the capacity of the FF system.

In ideal conditions (i.e. uniformly distributed constant demand; uniformly distributed vehicle fleet), if all FF vehicles were being used, the maximum density of trips that could be served would be $m_{FF}/R\tau_s$, where m_{FF} is the FF vehicle fleet size and τ_s is the average usage time of vehicles. However, this maximum capacity is reduced due to temporal, and spatial demand variations leading to non-uniform distribution of vehicles. The capacity reduction of the FF system due to these phenomena is included into the model through three terms: CR_{FF_temp}, CR_{FF_imb}, and CR_{FF_spat}, corresponding respectively to: *i)* temporal fluctuations in the demand; *ii)* average demand imbalance; and *iii)* spatial system decentralization.

So, λ_{FF} is obtained as the minimum of the two limiting factors: the potential demand and the real FF system capacity. This is:

$$\lambda_{FF} = \min\left\{\lambda_{max}, \frac{m_{FF}}{R\tau_s} - CR_{FF\_temp} - CR_{FF\_imb} - CR_{FF\_spat}\right\} \tag{A.9}$$

If anything, Equation A.9 is on the conservative side, since it accounts for a demand spike and an unfavorable traveling pattern happening at the same time (Daganzo and Ouyang, 2019; Barrios, 2011).

i) Capacity reduction due to temporal demand fluctuations, CR_{FF_temp}

λ_{max} is defined as the average potential demand density. However, demand density is a variable subject to fluctuations. This implies that, even if λ_{max} is larger than the capacity of the system, there might be some periods with significant lower demands, when all demand is served and some capacity is lost. CR_{FF_temp} accounts for this low demand fluctuations in the estimation of the FF system capacity. The capacity reduction CR_{FF_temp} is estimated as the probability of a fluctuation which makes the demand lower than the capacity of the system in ideal conditions, multiplied by the average capacity lost in this case. The average capacity loss is determined by the temporal standard deviation of maximum potential demand, std_λ_{max}. Assuming that demand follows a Poisson distribution (in accordance with previous works; see Chapter 7 of [Daganzo and Ouyang, 2019]), and that the Normal distribution is an excellent approximation to the Poisson for large values of the average, we can estimate CR_{FF_temp} as:

$$CR_{FF\_temp} = F_{\left[\frac{\frac{m_{FF}}{R\tau_s}-\lambda_{max}}{std\_\lambda_{max}}\right]} \cdot std\_\lambda_{max} \tag{A.10}$$

Where $F_{[\ ]}$ is the standard Normal cdf, and therefore, $F_{\left[\frac{\frac{m_{FF}}{R\tau_s}-\lambda_{max}}{std\_\lambda_{max}}\right]}$ is the probability of the potential demand density being $\leq \frac{m_{FF}}{R\tau_s}$.

ii) Capacity reduction due to average demand imbalance, CR_{FF_imb}

Demand imbalance implies a temporary deficit of vehicles in subregion $\mathcal{R}_q$ and an equivalent surplus in subregion $\mathcal{R}_t$. This deficit of vehicles yields some lost demand in $\mathcal{R}_q$, which will be compensated by the additional demand served in $\mathcal{R}_t$ due to the surplus of vehicles. Therefore, the non-uniform distribution of vehicles will not imply an overall loss in the number of trips served, unless the vehicle availability in $\mathcal{R}_t$, considering the surplus, exceeds the average potential demand, λ_{max}. In this last case, some vehicles will remain idle in $\mathcal{R}_t$, implying a reduction in the capacity of the system with respect to that in ideal conditions. This capacity reduction to account for the vehicle imbalance, $CR_{FF_{imb}}$, is obtained as the probability of the density of vehicle availability in $\mathcal{R}_t$ being larger than λ_{max}, multiplied by the average loss. Considering the repositioning periods, h_{FF}, h_{SB} [hours] for the FF and SB systems respectively, namely the decision variables defining the average time while the repositioning teams need to rebalance the whole system (i.e. move the vehicles from where they are in excess to where they are scarce), then the maximum excess of vehicles in $\mathcal{R}_t$ just before the end of h_{FF} would be: $\lambda_{FF}\Phi_q R_q h_{FF}$. This magnitude needs to be divided by 2, to obtain the average during the whole period h_{FF}, and divided by $R\tau_s$ to express it in terms of a demand density. Finally, considering the Poisson-Normal approximation for the demand distribution, we have:

$$CR_{FF\_imb} = F_{\left[\frac{\lambda_{FF}(1+\Phi_t)-\lambda_{max}}{std\_\lambda_{max}}\right]} \cdot \frac{\lambda_{FF}\Phi_q R_q h_{FF}}{2R\tau_s} \tag{A.11}$$

Where, $F_{\left[\frac{\lambda_{FF}(1+\Phi_t)-\lambda_{max}}{std\_\lambda_{max}}\right]}$ is the probability of the potential demand density being $\leq \lambda_{FF}(1+\Phi_t)$, the demand that could be served in $\mathcal{R}_t$ considering the vehicle surplus.

iii) Capacity reduction due to spatial decentralization, CR_{FF_spat}

Finally, it must be considered that a one-way car-sharing system is a spatial decentralized system. This means that, in addition to the average demand imbalance, there might be random fluctuations in the returns and requests at a given FF subzone, r. These random fluctuations can be characterized by the standard deviation of the difference between the number of returns, $n_{t(r)}$, and the number of requests, $n_{q(r)}$ in the subzone. This is:

$$std\left(n_{t(r)} - n_{q(r)}\right)_{FF} = \sqrt{Var\left(n_{t(r)} - n_{q(r)}\right)_{FF}} = \sqrt{2Var\left(n_{(r)}\right)_{FF}} = \sqrt{2\lambda_{FF}h_{FF}c} \tag{A.12}$$

Equation A.12 assumes independence between requests and returns and the Poisson distribution of demand, so that the variance is equal to the mean. This implicitly assumes a similar overall spatial distribution of requests and returns. There might be specific cases and times of the day with different spatial distribution of demand (e.g. spatially distributed requests and concentrated returns, or vice versa). However, in big systems (i.e. in big cities) the complexity of the spatial demand distribution with multiple generation and attraction focusses, works well for the proposed simplifying assumptions. In summary, we consider that such assumptions represent an adequate trade-off between modeling simplicity and representativity. Similar approaches have already been used previously (see Daganzo and Ouyang, 2019).

Note that $\lambda_{FF} h_{FF} c$ is the mean of the number of requests and returns in one subzone and during a repositioning period (i.e. before the system is rebalanced). In order to express this standard deviation in terms of a density (i.e. by notation $std(t-q)$), we need to divide Equation A.12 by the relevant area (i.e. c) and period of time (i.e. h_{FF}). We obtain:

$$std(t-q)_{FF} = \sqrt{\frac{2\lambda_{FF}}{h_{FF} c}} \qquad (A.13)$$

System decentralization will imply lost trips in subzones with a deficit of vehicles, and additional trips in subzones with a surplus, unless there is not enough demand to use these vehicles. This means that decentralization only implies an effective capacity reduction if the potential demand is $\leq \lambda_{FF} + std(t-q)_{FF}$.

In order to determine the capacity reduction in this case, note that due to the random fluctuations from system decentralization, half of the FF subzones in $\mathcal{R}$ (i.e. $R/2c$) will have a deficit of vehicles and the other half will have an equivalent surplus. So, the maximum number of imbalanced vehicles in the system at the end of the repositioning period is $\frac{R}{2c}\sqrt{2\lambda_{FF} h_{FF} c}$. The average number during the repositioning period is half of this, and in order to express the average decentralization in terms of a density, it needs to be divided by the service area, R, and service time, τ_s. So, the average capacity loss due to system decentralization is $\frac{\sqrt{2\lambda_{FF} h_{FF} c}}{4c\tau_s}$.

Therefore, we have that:

$$CR_{FF\_spat} = F_{\left[\frac{\lambda_{FF} + std(t-q)_{FF} - \lambda_{max}}{std\_\lambda_{max}}\right]} \cdot \frac{1}{\tau_s}\sqrt{\frac{\lambda_{FF} h_{FF}}{8c}} \qquad (A.14)$$

Where, again, $F_{[\;]}$ is the standard Normal cdf, and, $F_{\left[\frac{\lambda_{FF} + std(t-q)_{FF} - \lambda_{max}}{std\_\lambda_{max}}\right]}$ is the probability of the potential demand being $\leq \lambda_{FF} + std(t-q)$.

7.2. Appendix A.2. Demand served by the SB part of the mixed car-sharing system, λ_{SB}

The estimation of the density of trips served by the SB part of the car-sharing system is analogous to that of the FF part. However, recall that the maximum potential demand for the SB system is λ_{SB_max}, which considers the part of the demand already served by the FF system and the possible limited coverage of SB parking stations. The equivalent equations to determine λ_{SB} for the SB system are:

$$\lambda_{SB} = \min\left\{\lambda_{SB\_max}, \frac{m_{SB}}{R\tau_s} - CR_{SB\_temp} - CR_{SB\_imb} - CR_{SB\_spat}\right\} \quad (A.15)$$

Where:

$$CR_{SB\_temp} = F_{\left[\frac{\frac{m_{SB}}{R\tau_s} - \lambda_{SB\_max}}{std(\lambda_{SB\_max})}\right]} \cdot std(\lambda_{SB\_max}) \quad (A.16)$$

$$CR_{SB\_imb} = F_{\left[\frac{\lambda_{SB}(1+\Phi_t) - \lambda_{SB\_max}}{std(\lambda_{SB\_max})}\right]} \cdot \frac{\lambda_{SB}\Phi_q R_q h_{SB}}{2R\tau_s} \quad (A.17)$$

$$CR_{SB\_spat} = F_{\left[\frac{\lambda_{SB} + std(t-q)_{SB} - \lambda_{SB\_max}}{std(\lambda_{SB\_max})}\right]} \cdot \frac{1}{\tau_s}\sqrt{\frac{\lambda_{SB} h_{SB}}{8c}} \quad (A.18)$$

and:

$$std(t-q)_{SB} = \sqrt{\frac{2\lambda_{SB}}{h_{SB}c}} \quad (A.19)$$

8. Appendix B. Demand parameter characterization from an origin/destination (O/D) trip demand matrix

In the proposed continuous approximations model for the mixed car-sharing system, the potential demand for the system and its temporal and spatial variability is characterized by three input parameters: λ_{max}, CV_λ_{max}, Φ_q and Φ_t, as described in Appendix A. If detailed demand data is available (e.g. from real data of an already implemented system, or from the outputs of a demand estimation analysis), these will be generally in the from of an origin / destination (i.e. O/D) matrix. This Appendix B provides reference on how to obtain the potential demand parameters of the model from an O/D demand matrix.

Consider a particular subzone, r ($r \in \mathcal{R}$), and a given time step, t ($t \in [1,T]$). $n_q(r,t)$, defines the number of generated trips (i.e. requests), and $n_t(r,t)$, the number of attracted trips (i.e. returns) in the (r,t) space·time region. $n_q(r,t)$ and $n_t(r,t)$ are the data included in an O/D matrix at time t. Then the average potential demand density over $\mathcal{R}$ in the period $[1,T]$, is obtained as:

$$\lambda_{max}(t) = \frac{\sum_{r\in\mathcal{R}} n_q(r,t)}{R \cdot t} = \frac{\sum_{r\in\mathcal{R}} n_t(r,t)}{R \cdot t} \quad (A.20)$$

Where $\lambda_{max}(t)$ is the average demand density over the service region in time step *t*. Then:

$$\lambda_{max} = \frac{\sum_{t\in[1,T]} \lambda_{max}(t)}{T} \quad (A.21)$$

Regarding the temporal variation, we define std_λ_{max}, as:

$$std\_\lambda_{max} = std\left[\frac{\sum_{r\in\mathcal{R}} n_q(r,t)}{R\cdot t}\right] \approx std\left[\frac{\sum_{r\in\mathcal{R}} n_t(r,t)}{R\cdot t}\right] \tag{A.22}$$

Then:

$$CV\_\lambda_{max} = \frac{std\_\lambda_{max}}{\lambda_{max}} \tag{A.23}$$

Finally, regarding the spatial distribution, we define $R_q(t)$ as the area of the partition of the service region $\mathcal{R}$, where there are more requests than returns. In turn, $R_t(t)$, is the area of the partition of $\mathcal{R}$ with more returns than requests, in both cases obtained from the O/D matrix at time period *t*. The average area of these partitions is computed as the weighted average considering all the O/D matrixes from $t = 1 \dots T$. Note that $R_t + R_q \le R$.

$$R_q = \frac{\sum_{t\in[1,T]} R_q(t)\cdot\lambda_{max}(t)}{\sum_{t\in[1,T]}\lambda_{max}(t)} \tag{A.24}$$

$$R_t = \frac{\sum_{t\in[1,T]} R_t(t)\cdot\lambda_{max}(t)}{\sum_{t\in[1,T]}\lambda_{max}(t)} \tag{A.25}$$

Then, the average imbalance parameters, Φ_q and Φ_t, are obtained as:

$$\Phi_q = abs\left|\frac{\sum_{t\in[1,T]}\sum_{r\in\mathcal{R}_q}\left(n_t(r,t)-n_q(r,t)\right)}{\sum_{t\in[1,T]}\lambda_{max}(t)\cdot R_q(t)}\right| \tag{A.26}$$

$$\Phi_t = \frac{\sum_{t\in[1,T]}\sum_{r\in\mathcal{R}_t}\left(n_t(r,t)-n_q(r,t)\right)}{\sum_{t\in[1,T]}\hat{\lambda}_{max}(t)\cdot R_t(t)} \tag{A.27}$$

Note that Φ_q is defined positive.

9. Appendix C. Estimation of rebalancing rates

The total repositioning rate is composed of three components: *i)* repositioning movements to compensate the spatial demand imbalance and system decentralization, referred as baseline repositioning operations; *ii)* battery recharging repositioning operations for electrical vehicles; and *iii)* repositioning movements to maintain the FF and SB vehicle fleets stable and compensate net flows between them. These will be analyzed separately in the next subsections.

9.1. Appendix C.1. Baseline repositioning rate

The number of baseline operations generated by imbalance and decentralization is equal to the average number of cars out of position at the end of the repositioning period. For the system imbalance this is: $\lambda_{FF}\Phi_q R_q h_{FF}$ for the FF, and $\lambda_{SB}\Phi_q R_q h_{SB}$ for the SB layers of the system. And for the system decentralization we have:

$\frac{R}{2c}\sqrt{2\lambda_{FF}h_{FF}c}$ for the FF, and $\frac{R}{2c}\sqrt{2\lambda_{SB}h_{SB}c}$ for the SB. See Appendix A for the details of these estimations. These total number of operations must be divided by h_{FF}, h_{SB} respectively in order to express them per unit time. So, we obtain:

$$FF\ baseline\ repositioning\ rate = \lambda_{FF}\Phi_q R_q + \frac{R}{2}\sqrt{\frac{2\lambda_{FF}}{h_{FF}c}} \tag{A.28}$$

$$SB\ baseline\ repositioning\ rate = \lambda_{SB}\Phi_q R_q + \frac{R}{2}\sqrt{\frac{2\lambda_{SB}}{h_{SB}c}} \tag{A.29}$$

9.2. Appendix C.2. Battery recharging repositioning rate for decentralized charging strategy

In decentralized charging contexts (note that the case of centralized hub battery charging is analyzed in Appendix C.4, battery recharging of electrical vehicles takes place only at stations and might imply additional repositioning movements (i.e. moving FF vehicles with low battery level to a SB battery recharging spot and also moving SB vehicles with full battery to provide FF service). Recall that the battery consumption by FF electrical vehicles at any given instant is $e\lambda_{FF}R(\tau_c+\tau_p)$. This battery level is made available by providing electrical vehicles with at least 80% of the battery level, which means that they need to be made available at a rate of $e\lambda_{FF}R(\tau_c+\tau_p)/T_{ev}$. These vehicles are provided by SB→FF movements, so that the minimum number of SB repositioning operations in order to provide enough battery level is:

$$SB\ battery\ repo\ rate = \frac{e\lambda_{FF}R(\tau_c+\tau_p)}{T_{ev}} \tag{A.30}$$

An equivalent number of FF→SB movements is necessary to move the FF vehicles with low battery level to a SB battery recharging spot, so that the minimum number of FF repositioning operations in order to maintain enough battery level is:

$$FF\ battery\ repo\ rate = \frac{e\lambda_{FF}R(\tau_c+\tau_p)}{T_{ev}} \tag{A.31}$$

In decentralized battery charging context, these operations can be done at the same time while performing the baseline repositioning, and will only imply additional movements when the baseline repositioning rate is not enough. The additional repositioning costs for centralized battery charging systems is analyzed separately in Appendix C.4.

9.3. Appendix C.3. FF-SB compensation repositioning rate

Finally, repositioning needs to compensate the net flow of FF→SB or SB→FF trips in order to keep the number of FF and SB vehicles approximately constant in the long term. The number of these compensation movements per unit time is obtained as the difference between request and return densities for each subsystem, multiplied by the area of the service region. This is:

$$
\begin{aligned}
&FF \rightarrow SB\ repositioning\ rate = (\lambda_{SB} - \lambda_{t\_SB})R \quad if\ \lambda_{SB} - \lambda_{t\_SB} > 0 \\
&SB \rightarrow FF\ repositioning\ rate = (\lambda_{FF} - \lambda_{t\_FF})R \quad if\ \lambda_{FF} - \lambda_{t\_FF} > 0
\end{aligned}
\tag{A.32}
$$

Therefore, these values must be the minimum number of repositioning operations in each subsystem. This number of operations could be achieved at the same time they compensate imbalance and decentralization and provide battery level. For instance, $SB \rightarrow FF$ compensation operations may account for the FF baseline operations (totally or in part) and for providing battery level to the FF fleet. Equivalently, $FF \rightarrow SB$ operations may account for the SB baseline operations and for moving vehicles with low battery level to the charging stations.

So, considering all these repositioning concepts, the minimum repositioning rate that the system must ensure is:

$$
FF\ repositioning\ rate = Max \begin{bmatrix} FF\ baseline - (SB \rightarrow FF); \\ FF\ battery - (SB \rightarrow FF); \\ FF \rightarrow SB; \end{bmatrix} \tag{A.33}
$$

$$
SB\ repositioning\ rate = Max \begin{bmatrix} SB\ baseline - (FF \rightarrow SB); \\ SB\ battery - (FF \rightarrow SB); \\ SB \rightarrow FF; \end{bmatrix} \tag{A.34}
$$

Regarding the time required for each repositioning operation, employees spend time in three tasks: *i)* travel with a scooter from their current position to the vehicle to be repositioned; *ii)* perform fixed operations (i.e. folding the scooter, checking and cleaning the car, reporting, refueling when necessary); and *iii)* travel with the car to the desired location. The optimal distances to travel by repositioning employees result from the solution of the transportation problem considering a density of origins/destinations of $\Delta_{FF} = 1/c$ and Δ_{SB}. According to Daganzo and Smilowitz (2004) approximate solution to the transportation problem, the distance between two consecutive visits is $1.1/\sqrt{\Delta}$, where Δ is the density of origins/destinations. Note that $1/\sqrt{\Delta}$ would be approximately the diameter of the coverage zone of each origin/destination. Given this solution, and considering v and v_k as the car and scooter average speed in the city, respectively, the average time spent per repositioning operation is:

$$
Time\ required\ per\ FF\ operation = \frac{1.1\sqrt{2}a}{v} + \frac{1.1\sqrt{2}a}{v_k} + \delta \tag{A.35}
$$

$$
Time\ required\ per\ SB\ operation = \frac{1.1}{\sqrt{\Delta_{SB}} \cdot v} + \frac{1.1}{\sqrt{\Delta_{SB}} \cdot v_k} + \delta \tag{A.36}
$$

Where δ is the required time to perform the fixed operations and $\sqrt{\Delta_{FF}} = \sqrt{1/c} = 1/\sqrt{2}a$.

9.4. Appendix C.4. Additional repositioning teams in centralized battery charging contexts, k_{HUB}

The previous section considered the case of decentralized battery charging, where regular repositioning movements can provide battery availability to the FF system. However, if the battery charging happens at a centralized hub, all the repositioning movements are to be done by repositioning employees, to and from the charging hub. In such case, the previous battery recharging repositioning rate must be disregarded, and instead additional repositioning teams, k_{HUB}, need to be considered according to Equation 18, where the additional repositioning rate, and the time required per hub operation are:

$$Hub\ repositioning\ rate = \frac{e(\lambda_{FF} + \lambda_{SB})R(\tau_c + \tau_p)}{T_{ev}} \tag{A.37}$$

Note that in this centralized case hub battery charging must account also for the battery consumed by the SB system demand.

$$Time\ required\ per\ hub\ operation = 2\left(\frac{\sqrt{R}}{2} + d_{HUB}\right)\left(\frac{1}{v}\right) + \delta \tag{A.38}$$

Where d_{HUB} is the distance from the location of the recharging hub to the center of the service region, from where the expected distance to a random location in the service region is $\sqrt{R}/2$. This total repositioning distance is multiplied by 2 (i.e. go and return) and by the inverse of the average travelling speed by car. Still, δ units of time are required to perform fixed operations.

Paper III

Optimization of bike-sharing repositioning operations: A reactive real-time approach.

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Optimization of bike-sharing repositioning operations: A reactive real-time approach.

Enrique Jiménez and Francesc Soriguera

Abstract— One of the critical issues in the operation of vehicle-sharing systems is the optimization of the fleet repositioning movements. Repositioning implies the artificial movement of vehicles from places where they accumulate to others in which they are scarce. This yields a higher vehicle availability, without over dimensioning the vehicle fleet and while increasing the vehicle utilization rates. In the particular case of bike-sharing systems, repositioning implies to deploy a fleet of small trucks or vans able to move groups of bicycles from one location to another, with the purpose of maximizing the users' level of service while minimizing the operating agency costs. This repositioning optimization problem has been previously addressed in the operations research field through Mixed Integer Programing (MIP) and its variants, generally facing two limitations. First, its high computational cost, which prevent achieving direct solutions in realistically large systems. So, it has been necessary to develop heuristics and approximations. And second, its reliance and sensitivity to demand forecasts, with its inherent level of uncertainty. Aiming to overcome these weaknesses, this paper presents a strategy based on a real-time pairwise assignment between repositioning trucks and tasks, in order to optimize the bike-sharing repositioning operations. On a simulated case study, the proposed strategy has been implemented and compared to the MIP-based routing approach. Results show that the proposed real-time pairwise assignment method is able to improve the performance of the repositioning operations, especially in scenarios where the demand forecast is not accurate. Being based on real-time information, the proposed strategy is flexible enough to solve unpredictable situations. So, the proposed strategy can be implemented as an alternative to MIP-based solutions, or as a complementary strategy for dynamic real-time adaptation of static long-term solutions.

Keywords— bike-sharing, vehicle-sharing, repositioning, optimization, routing, real-time assignment.

1. Introduction

Vehicle relocation is a key issue in the design and operation of vehicle-sharing systems, either at strategic or operational levels. Its importance lays in the fact that, to some extent, O/D demand is usually spatially imbalanced. If no action is taken, vehicles would be scarce in zones where trip generation is predominant and would accumulate at specific attracting destinations, causing overall service limitations. This problem can be even more intense in bike-sharing systems, because the spatial imbalance of demand is magnified by the predominance of downhill trips.

Relocation strategies (also called rebalancing strategies) can be classified into two groups. The first group of strategies are called user-based or demand-oriented strategies. They rely on users for taking and returning the bikes into more favorable positions from the system perspective. One possible way to achieve this is by incentivizing users with pricing strategies (i.e. offering discounts if the bike is returned to stations with more risk to become empty or taken from full stations). Examples of this approach are proposed in Pfrommer et al. (2014), Haider et al. (2018), Zhang et al. (2019a), or in Stokkink & Geroliminis (2021). Incentives might be substituted by trip restrictions in critical situations. For instance, in free-floating systems (i.e. where bicycles are taken and returned on-street), the operating agency divides the service region into subzones and could force the user to return the bike only inside a specific subregion. This can be achieved through "electric fences" so that the user will be only able to lock the bike and complete the return inside some predefined subregions. This strategy plays the same role as "full stations" in station-based bike-sharing systems (i.e. where trips are station to station) and prevents that some zones become clogged with bicycles. Although not very common, this trip restriction strategy has been explored by works such as those by Zhang et al. (2019b) and Jia et al. (2022).

In spite of promoting user-based relocations, operating agencies usually need to rely on agency-based strategies. In this case, the agency deploys repositioning teams, which perform artificial movements to achieve some desired fleet distribution. In the case of bike-sharing, the repositioning team is typically formed by an employee with a small truck or van, which allows moving several bikes on a single trip. Agency-based repositioning strategies are expensive, as they are mainly steered by labor cost. However, the operating agency, has full control of the repositioning movements, and the repositioning teams can also address bike maintenance. For this reason, the majority of bike-sharing systems in the world have their own fleet of repositioning trucks.

Agency based bicycle repositioning has been a hot research topic since the popularization of one-way bike-sharing systems in the early 00's. In the scientific literature, the bike-sharing repositioning optimization problem has been mainly addressed through optimal routing methodologies (see Section 2 for a literature review). Bicycle pick-up and delivery tasks are assigned to repositioning trucks conforming the stops of a route. So, the problem can be seen as an adaptation of classic logistics problems, such as the one-commodity pickup and delivery problem, which is frequently confronted in the context of operations research by using mixed integer programming (MIP) optimization strategies. These methodologies, while suited given the problem nature, do have some drawbacks. First, they imply a high computational cost, which increases (not polynomially) with the size of the system. This means that it is hard to scale the problem to big systems or to increase the complexity of the solution without incurring in astronomical computation times. Researchers explore different heuristics and approximations in order to simplify the problem up to be solvable in a reasonable computing time, but there is no method in the literature able to cover efficiently all the study cases.

A common simplification of the repositioning problem is to consider that operations are carried out only when the system is closed and the bicycle unbalance is known and fixed (e.g. repositioning only during night hours). In the literature, this is called "static" repositioning. This may not be enough in many systems, as it would require and extremely over-dimensioned fleet size to avoid unbalance problems during the operation period. Dynamic repositioning is needed in these situations, which implies performing repositioning operations while the system is providing service and the bicycle distribution is changing because of users' trips. Dynamic repositioning requires to forecast users' demand, in order to decide which stations are likely to experience unbalance problems and must be visited by a repositioning truck during the coming hours. In fact, all optimal rebalancing strategies should consider the forecasted demand to some extent, as the utility of relocation movements depends on how many users will rent a bike on a station or zone that otherwise would have been empty and how many users will return it on a station or zone that otherwise would have been full. If the expected demand for some empty (or full) zones is negligible, the utility of moving bikes there is also negligible, because that relocation movement will not result in more demand served.

However, demand prediction is a challenging issue, always implying a high degree of uncertainty, especially in the case of non-recurrent circumstances (e.g. extreme meteorology, temporal changes on the system infrastructure, special events, …). The possibility of multiple competing shared-vehicle companies over a given service region makes such demand forecast even more challenging (Jiang et al. (2020)). And finally, predicting actual requests and returns in station-based vehicle-sharing systems in order to determine the vehicle inventory level at stations, implies the added difficulty that the station may become full or empty, and therefore not able to accommodate some of the potential demand. Demand at a given station is also affected by the possibility of the nearby stations becoming full or empty and diverting some of their potential demand. In conclusion, there could exist situations in which, even if the routing optimization algorithm yields an exact solution for the dynamic repositioning problem, in practice, that solution fails because users' demand behaves different than predicted.

Some vehicle-sharing systems may opt for including the possibility of trip reservations, and use the reservation information to better predict the future inventory of the stations, as proposed in Repoux et al. (2019). In spite of this, it would be still advisable to develop repositioning strategies which mitigate their dependence to demand forecasts. With this purpose, the present paper suggest a reactive approach which can complement the current routing optimization solutions. The fundamentals of the proposed method lie in that repositioning teams, instead of following precalculated routes, decide in real time which will be the next task to perform, according to the observed current system status. This strategy does not strongly rely on demand forecasts for long time horizons, since it observes the system when a new task is to be assigned. This allows the repositioning teams to monitor the non-recurrent events and the second-order effects of demand diverted from nearby full/empty stations. This approach may represent a huge advantage in circumstances with a high demand uncertainty, which can compensate the sub-optimal routing resulting from not considering the future tasks in the assignment process.

In this paper, the reactive approach is formulated and tested in order to determine in which circumstances becomes more effective than the existing alternatives. In order to do so, different repositioning strategies are applied to a simulated case study. The first one is a pure reactive assignment strategy, where repositioning tasks are assigned just when the repositioning team becomes idle from the previous task. The optimal task assignment considers the current needs and positions of all teams in the system. The second one is a pure preemptive routing solution, in which all tasks and repositioning movements are planned in advance, at the beginning of the day and according to the expected demand prediction. Finally, the third tested strategy is a mixed method that combines both approaches. It plans in advance the expected route for the whole operation period using the routing optimization method, but after each task is finished, it evaluates the utility of the following task and the possible alternatives, to check if other tasks could improve the performance of the system. A similar approach to this mixed strategy has been tested in Angelelli et al. (2022), where results show that real-time reevaluations could improve the initial repositioning task schedule. Note that this mixed strategy encompasses the previous ones. If the demand forecast was very accurate, the mixed strategy would not improve the preemptive routing. In contrast, if demand behavior deviates much from the expected, the mixed strategy will be equivalent to the real-time reactive strategy. In practice, operating agencies may opt for one or other on convenience, according to their specific context, or deploy the more flexible mixed strategy that combines both approaches.

The rest of the paper is structured as it follows. Section 2 describes the current state-of-art of bike-sharing repositioning optimization. Section 3 presents the basic ideas and the detailed formulation of each repositioning optimization considered. Section 4 defines the case study, which is based on the bike-sharing system deployed in the city of Barcelona (Spain), and shows the obtained results from a simulated environment in which all models were applied. Finally, the paper ends with the conclusions, acknowledgements, and references.

2. Literature review

The vast majority of research works addressing the bike-sharing artificial repositioning optimization (i.e. agency-based, with small trucks or vans) are based on variants of MIP optimization problems. Different works focus on specific parts of the problem, like the definition of the objective function, the development of the mathematical model to depict the operative of the system and its constraints, or the development of heuristics, algorithms, and numerical methods to make the optimization solvable. This section reviews the most significant contributions on these aspects.

With respect to the objective function in the optimization, two main groups of models are found. The first group includes models that consider a goal on the inventory level at all stations (i.e. the optimum bike distribution to be met), and minimize the routing cost of the trucks to achieve it. Thus, this problem is equivalent to the one-commodity pickup-delivery problem, in which the supplies will be the number of bikes on stations exceeding the optimum, and demands will be the number of bikes under the optimum at the rest of the stations. The advantage of these models is that they simplify the degrees of freedom of the problem, because they consider known the inventory level to be achieved. Research works in this group include those by Chemla et al. (2013), Dell'Amico et al. (2014), Pal & Zhang (2017) and Bulhões et al. (2018). It can be argued that the limitation of these models is the lack of flexibility, as repositioning teams must visit a large number of stations to achieve the optimal bike distribution. Usually, they do not consider that it might be advisable to achieve a suboptimal distribution if the penalty is compensated by a reduction of the repositioning costs (i.e. not visiting all stations). In any case, the optimal inventory level must be defined previously, and sometimes it is not detailed how this is achieved.

The second group of models are those which include in the objective function the minimization of no-service penalties. The objective is then to reduce the number of situations in which a user does not find a bike at the origin of the trip or a parking spot at the destination. In this case, it is not necessary to establish a goal in the inventory level, but a relationship (generally stochastic) between the inventory level at each station and the expected number of no-service situations. There are several proposals in the literature to establish such relationship. For instance, Nair & Miller-Hooks (2011), and Alvarez-Valdes et al. (2016), use the Skellam probability distribution to estimate the expected inventory level of the stations at the end of the operating period, and according to this, they estimate the expected number of no-service situations. This approach has been adapted and used in the present paper. Instead, Raviv et al. (2013), Gast et al. (2015), and Schuijbroek et al. (2017) estimate the expected number of not served users by using Markovian chains. In turn, Caggiani & Ottomanelli (2013), Jian et al. (2016), Caggiani et al. (2018), and Dattner et al. (2019) opt for a microsimulation approach to predict the evolution of the system. Note that, in all cases, these estimations rely on demand forecasting (i.e. on the expected number of bike requests and returns at given stations or zones). Therefore, any model should take into account some uncertainty and inaccuracy on these forecasts, and its effects on the solution should always be a matter of concern. With respect to demand forecasting, methods based on neural networks or similar data-based approaches, are usually recommended, as in Caggiani & Ottomanelli (2013) and Caggiani et al. (2018). Their advantage is that they can recognize usage patterns even if the relationship with its causal effects is complex or unknown. This usually suits the demand prediction problem in bike-sharing systems, since operators have big datasets available, which can be used to train the algorithms, even if it is way less clear which parameters influence the system usage and how. In any case, it is also advisable to complement these data-based methods with empirical demand studies such as those of Reiss & Bogenberger (2015), Faghih-Imani et al. (2017), and Reynaud et al. (2018).

Regarding the formulation of the optimization model, we recommend the work of Raviv et al. (2013) to understand the differences between the two main families of formulations for this problem. The first family consists of arc-indexed routing formulations, where its solution determines which arcs are covered by which vehicles, and eventually defining which stations are visited. The second family of formulations consist of time-indexed algorithms, where it is directly determined which stations are visited and also when. Time-indexed formulations offer more possibilities, since they allow trucks to visit the same station more than once, using stations to make transshipments of bikes by synchronizing the visits of different trucks, or extending the problem to dynamic repositioning. However, time-indexed formulations are harder to solve, since they introduce the time discretization as a new decision variable. Many research works take one of these typical formulations as their baseline, adapting it to its particular case study. For example, Caggiani et al. (2018) addresses the case of free-floating bike-sharing dynamic rebalancing. The main difference with respect to station-based systems is that free-floating rebalancing requires a previous definition of subzones in the service area (which may be seen as virtual stations) before optimizing the relocation tasks. This is done through clustering methods. Li et al. (2016) further complicates the problem by developing a model in which there are several types of bikes (e.g., bikes with one, two, or three seats and those with a child-seat). The proposed solution is based on a hybrid Genetic Algorithm (for the routing optimization) and a greedy heuristic to determine the number of bikes loaded and delivered.

Finally, we mention the methods used to reduce the problem size and complexity to face the optimization of real size problems and make it solvable. Schuijbroek et al. (2017) divides the whole system into several clusters, to reduce the problem size and make it computable, although the optimality for the whole system is not assured. A similar approach based on a clustering procedure is proposed in Boyaci et al. (2017). In turn, Lei & Ouyang (2018) and Osorio et al. (2021), use continuous approximations to find a solution on local areas, in combination with a discrete formulation to solve a reduced size problem for the line-haul routes. This allows reducing the computational burden successfully. Finally, Shui & Szeto (2018) adopt a rolling time horizon approach to decompose the dynamic rebalancing problem into a subset of several static rebalancing problems. Route optimization on each one is done through an Artificial Bee Colony algorithm.

3. Preemptive versus reactive bike-sharing repositioning strategies

In this section, the algorithms and formulations which define the considered repositioning strategies are developed. As a previous step before defining each strategy, some general concepts are outlined.

3.1. Common concepts

3.1.1. Problem background and notation

The bike-sharing repositioning problem consists in assigning tasks to the repositioning vehicles with the objective of minimizing the no-service penalties. Penalties are evaluated during the operation period, typically a daily cycle of 24 hours. Time is discretized into time steps, which make up the time step vector, $\Gamma = (0, \dots, t, \dots, T)$. Each component of the time step vector, t, represents the time elapsed since the beginning of the operational period (i.e. $t = 0$). Time steps are defined so that at every t some repositioning task ends. This means that new tasks are only assigned at the beginning of these time steps, and only one task per vehicle at most. The duration and number of time steps are decision variables, as it corresponds to the number of repositioning operations, not known in advance.

Since this problem is faced at the operational level, the main strategic parameters of the bike-sharing system are assumed to be known. We define the system as consisting of a set of S stations (indexed i for the origin of the trip and j for the destination; these stations may be real in case of station-based systems or virtual for free-floating systems), V repositioning vehicles (indexed v), and a total fleet size of B bikes in the system. Note that S, V, and B are the main strategic decision variables defining the level of service and costs of the bike-sharing system. In particular, V, is considered given, so that the overall repositioning costs during the analyzed period will be constant for all the considered repositioning strategies. In fact, these decision variables exhibit strong interrelationships, and must be wisely determined during the planning phase of the system. Soriguera & Jiménez (2020) provide a methodology for the optimal design of public bike-sharing systems at the strategical level.

Each station j has an inventory level at time t, $b_{j,t}$, and a maximum capacity k_j. Analogously, repositioning vehicle, v, carries a number of bikes $b_{v,t}$ at time t and has a capacity k_v. Note that inventory levels include the time index t, since the number of bikes at stations and in repositioning vehicles will vary with time. The location of all stations is known, and therefore, the trip duration between any pair of stations i, j can be estimated. This is represented by the trip duration matrix D_{ij}. Additionally, we define the position of the repositioning vehicle v at time t as $x_{v,t}$, which is assumed to be the location of the nearest station to vehicle v at time step t.

The initial conditions for the inventory level at stations and repositioning vehicles ($b_{j,0}$, $b_{v,0}$) are known, because they can be directly observed at the beginning of the repositioning period. In addition, the demand forecast is defined in terms of $b_{j,t}^{ret}$, $b_{j,t}^{req}$, representing the forecasted number of bikes to be returned or requested, respectively, at station j from time t and up to the end of the operating period, T. So, by definition, $b_{j,0}^{ret}$, $b_{j,0}^{req}$, will be the total forecasted returns and requests at station j during the whole period of analysis. Note that the repositioning movements assigned at time $\hat{t} < t$, will be included in the demand predictions, $b_{j,t}^{ret}$, $b_{j,t}^{req}$ if the repositioning task has not been already completed at time t. Once the repositioning task is completed, the corresponding bicycle movements will already be included in the station inventory level.

The decision variables for the problem are those defining all the tasks in the repositioning system. This includes the repositioning vehicle trip tensor, $X_{i,j,v,t}$, consisting of 0's and 1's, and depicting if a trip from station i to station j is assigned to vehicle v at time step t (i.e. $X_{i,j,v,t} = 1$) or not (i.e. $X_{i,j,v,t} = 0$). Also, the bike-movement matrix, $Y_{j,v,t}$, which depicts how many bikes are considered by vehicle v on the task assigned at station j at time t. Note that a positive value for $Y_{j,v,t}$ means that bicycles are left at the station (i.e. increase in the inventory level at the station) and a negative value means that bikes are taken from the station (i.e. reduction of the inventory level). And finally, the time step vector, Γ, that depicts the duration and number of time steps until the end of the considered operative period, usually 24 hours.

3.1.2. Objective function and estimation of the no-service expected penalties

The final objective for all the considered repositioning strategies is to minimize the no-service penalties at the end of the operating period. No-service penalties represent the cost users incur when not finding a bike at their origin or an available parking slot at destination. For all the strategies, we define the no-service penalty function, $Z_{NSP}\left(b_{j,t}\right)_{j,t}$ [€], as the expected no-service penalty at station j, from time step t to the end of the period, T, given an existing inventory level $b_{j,t}$. By definition, this is the integral of the no-service penalty incurred in a particular scenario, $z_j(b)$, multiplied by the probability of that scenario to happen, $P\left[b_{j,T} = b \middle| b_{j,t}\right]$, as expressed in Equation (1).

$$Z_{NSP}\left(b_{j,t}\right)_{j,t} = \int P\left[b_{j,T} = b \middle| b_{j,t}\right] \cdot z_j(b) \cdot db \tag{1}$$

Note that b represents the potential number of bikes at the station (or zone) and it is not constrained between 0 and k_j. Actually, when b is outside the $\left[0, k_j\right]$ range it represents trips that cannot be accommodated, because of the lack of bicycles ($b < 0$), or because of the lack of parking spots ($b > k_j$), generating no-service penalties. Specifically, if there exist enough demand, the no-service penalties, $z_j(b)$ [€] increase lineally with b when it is less than zero or higher than capacity, at a rate β_e and β_f, respectively. β_e and β_f [€/trip] represent the unitary penalty for the user when finding the station empty at the origin of the trip, or full at the destination. This leaves us with a piecewise linear $z_j(b)$, as shown in Equation (2).

$$z_j(b) = \begin{cases} -b \cdot \beta_e & b < 0 \\ 0 & 0 \leq b \leq k_j \\ (b - k_j) \cdot \beta_f & b > k_j \end{cases} \tag{2}$$

In Equation (1), the different scenarios are defined by the potential inventory levels at the end of the operating period ($b_{j,T}$). This is a simplification which allows determining the probability of a particular scenario using only the aggregate prediction of bicycle returns and requests from t to T (i.e. $b_{j,t}^{ret}$, $b_{j,t}^{req}$) as described in the coming paragraphs. However, this may underestimate the expected no-service penalty if the future evolution of returns and requests, yields temporary periods where the station is full or empty. Therefore, if predictions of the detailed time evolution of demand were available, it would be advisable to evaluate Equation (1) for all the intermediate time steps and aggregate the final result.

With respect to the probability of achieving a particular potential inventory level at the end of the period, $P\left[b_{j,T} = b \middle| b_{j,t}\right]$, it is assumed that follows a Normal probability distribution with mean $\bar{b}_{j,t|T}$ and variance $\sigma_{j,t|T}^2$ (see Equations (3) to (5)).

$$P \sim Normal\left[\bar{b}_{j,t|T}, \sigma_{j,t|T}^2\right] \tag{3}$$

$$\bar{b}_{j,t|T} = b_{j,t} + b_{j,t}^{ret} - b_{j,t}^{req} \tag{4}$$

$$\sigma_{j,t|T}^2 = b_{j,t}^{ret} + b_{j,t}^{req} \tag{5}$$

Recall that $b_{j,t}$ is the observed inventory level at station j at time t. In turn, $b_{j,t}^{ret}$ and $b_{j,t}^{req}$ are demand forecasts, which can be assumed to follow a Poisson distribution (Alvarez-Valdes et al. (2016); Li, Ma et al. (2016); Lin et al. (2013)). The difference between Poisson random variables (e.g. in Equation (4)) yields a new random variable following a Skellam distribution, whose mean is the difference of the means of the original variables, and whose variance is their sum. The Skellam distribution might be well approximated by the Normal distribution if at least one of the original means is large (i.e. $b_{j,t}^{ret} > 20, b_{j,t}^{req} > 20$) which usually is the case. Take into account that this estimation of $P\left[b_{j,T} = b \middle| b_{j,t}\right]$ depends on forecasted variables (i.e. $b_{j,t}^{ret}$ and $b_{j,t}^{req}$), whose estimation methods might range from simplistic estimations considering only the aggregated daily long term average, to advanced data driven methods considering the possible time evolution and space correlation. In any case, their accuracy cannot be taken for granted and would always be an issue.

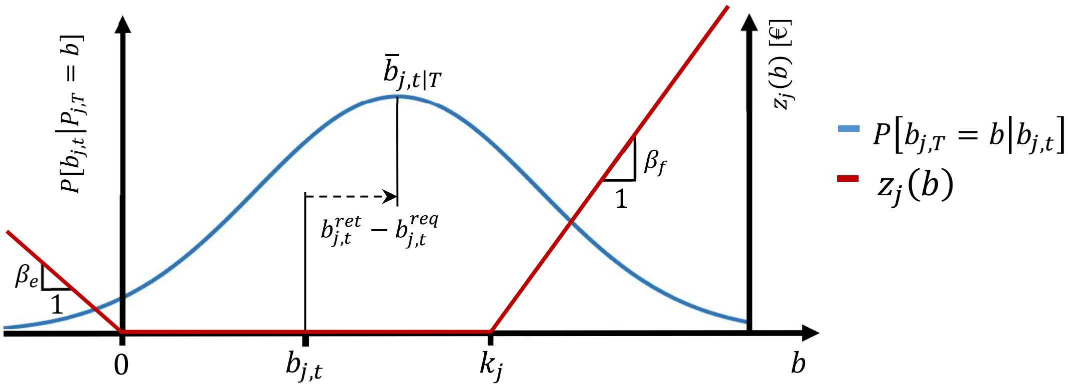

Fig. 1. Penalty function $z_j(b)$ (in red) and potential inventory level probability distribution $P[b_{j,t}|b_{j,T}=b]$ (in blue).

Figure 1 illustrates the definition of $z_j(b)$ and $P[b_{j,T}=b|b_{j,t}]$ for a station j that receives on average more returns than requests (i.e. $b_{j,t}^{ret}-b_{j,t}^{req}>0$). With these definitions, $Z_{NSP}(b_{j,t})_{j,t}$ is a convex function defining a well posed minimization problem. This is the case because the expected no-service penalties will be higher if $b_{j,t}$ is closer to the boundaries $[0,k_j]$. Note also that the expected imbalance, $b_{j,t}^{ret}-b_{j,t}^{req}$, will move the optimum inventory, $b_{j,t}^{*}$, closer to zero or to the capacity of the station, k_j, depending on if it is negative or positive (i.e. more returns or more requests).

3.1.3. Optimal distribution of bicycles over the service region, $b_{j,t}^{*}$

Having defined the overall objective function (i.e. minimizing the no-service penalties over the operation period), the optimal inventory level at each station (or sub-region in free-floating systems) can be determined. This is to find the distribution of bikes over stations, $b_{j,t}^{*}$, that minimizes the no-service penalty cost over the whole system. The problem is formulated as it follows:

$$b_{j,t}^{*} \;\Big|\; \min_{b_{j,t}} \sum_{j\in S}\left[Z_{NSP}(b_{j,t})_{j,t}\right] \tag{6}$$

Subject to:

$$\sum_{j\in S} b_{j,t}^{*} \le B \tag{7}$$

$$b_{j,t}^{*} \ge 0 \tag{8}$$

$$b_{j,t}^{*} \le k_j \tag{9}$$

Restriction in Equation (7) limits the total available bicycle fleet, while Equations (8) and (9) set the feasible number of bicycles at each station. Note that the optimal distribution of bikes is time-dependent. So, in order to improve the repositioning performance, it is advisable to keep updating the optimal inventory level during the process, for instance, anytime the repositioning tasks are assigned.

3.2. Real-time reactive pairwise task assignment optimization strategy

This new repositioning strategy assigns tasks to vehicles in real time, at time step t, just after the previous task has been completed. This means that the positions and the currently assigned tasks of all the repositioning vehicles are known, as well as the inventory level of all stations at time t.

This strategy relies on an optimization process based on a utility matrix $U_{j,v,t}$. The size of this matrix computed at time t is SxV, and it contains the utility of each combination of station-vehicle assignment. Utility here is defined as the reduction of no-service penalties due to repositioning, minus the repositioning cost (i.e. the monetization of the duration of the repositioning task). This represents an efficient assignment of the repositioning resources. At time t, each element of the matrix will be calculated as described in Equation (10).

$$U_{j,v,t} = Z_{NSP}\left(b_{j,t}\right)_{j,t} - Z_{NSP}\left(b_{j,v,t}^{end}\right)_{j,t} - c_v \cdot \tau_{j,v,t} \tag{10}$$

Where,

$b_{j,t}$ is the inventory level of the station j previous to the task.
$b_{j,v,t}^{end}$ is the inventory level of the station j once the task has been performed by vehicle v.
c_v is the cost of the repositioning vehicle per unit time.
$\tau_{j,v,t}$ is the duration of the task corresponding to vehicle v visiting station j.

And,

$$\tau_{j,v,t} = \delta \cdot abs\left\|b_{j,t} - b_{j,v,t}^{end}\right\| + D_{x_{v,t},j} \tag{11}$$

$$b_{j,v,t}^{end} = \begin{cases} \max\{b_{j,t}^*, b_{j,t} - (k_v - b_{v,t})\} & if\ b_{j,t}^* < b_{j,t}\ (take\ bicycles) \\ \min\{b_{j,t}^*, b_{j,t} + b_{v,t}\} & if\ b_{j,t}^* \geq b_{j,t}\ (leave\ bicycles) \end{cases} \tag{12}$$

Where, δ is the unitary time spent loading or unloading one bike to the truck, and $D_{x_{v,t},j}$ is the trip duration from the observed current position of the vehicle, $x_{v,t}$, to station j.

Note from Equation 12 that, $b_{j,v,t}^{end}$ depends on v, since each vehicle carries a different number of bikes at time t. $b_{j,v,t}^{end}$ will be the closest to the optimal inventory level for station j that can be achieved taking into account the vehicles' capacity restriction. This capacity restriction implies that the maximum number of bicycles that can be taken from a station is the number of empty slots in the vehicle, and that the maximum number of bicycles that can be left are those carried by the vehicle. In both cases, these depend on the considered vehicle, v.

Once the utility matrix $U_{j,v,t}$ is defined, the pairwise matching algorithm to assign tasks to vehicles at time step t, is run according to the following steps:

1. The optimal bicycle distribution, $b_{j,t}^*$, is calculated for the whole system (see Section 0).
2. The utility matrix $U_{j,v,t}$ is calculated for all vehicle-station pairs. For the idle repositioning vehicles, their current position and inventory level is considered. For busy repositioning vehicles, it is considered their position and inventory level at the end of their current task. Considering busy vehicles is extremely important, as it means that not only the idle vehicles are candidates for each task, but the whole repositioning fleet. The optimal solution achieved will be better the larger the set of vehicles considered.
3. All repositioning vehicles are assigned pairwise to the task which maximizes the total utility of the

system. Note that these assignments are only tentative at this point.
4. Tasks assigned to busy repositioning vehicles are discarded.
5. Tasks assigned to idle repositioning vehicles are considered final. The time step vector, Γ, is updated with the new components $t+\tau_{j,v,t}$, where $\tau_{j,v,t}$ are the durations of the final assigned tasks, computed as in Equation 11. If vehicle v, whose location was station i, is finally assigned to visit station j at t, then $X_{i,j,v,t}=1$. Any other value in the trip matrix will be zero. The number of bicycles moved within the task will be $Y_{j,v,t}=b_{j,v,t}^{end}-b_{j,t}$. And finally, the demand predictions, $b_{j,t}^{ret}$ and $b_{j,t}^{req}$, at assigned stations are updated for times until $t+\tau_{j,v,t}$ to account for the bicycle movements in the assigned tasks.
6. Idle teams are set to perform their final assigned task, and are set as busy until time step $t+\tau_{j,v,t}$, when the task will be completed. Once repositioning vehicles reach the assigned station, j, at time $\hat{t}$ ($\hat{t}\in(t,t+\tau_{j,v,t})$), the optimal inventory level at the station, $b_{j,\hat{t}}^{*}$, is updated to account for bicycle requests and returns in the period $(t,\hat{t})$. The final number or bicycles taken/left at the station, $b_{j,v,\hat{t}}^{end}$, is updated accordingly (see Equation (12)).

For solving the optimal pairwise assignment, the MATLAB function "matchpairs" has been used. This solver function is based on the algorithm developed by Duff & Koster (2001) to solve the linear assignment problem. This solution has been proved to be computationally quick and effective.

3.3. Preemptive routing optimization strategy

This strategy is based on the MIP-related methodologies, predominant in the literature, to address the vehicle-sharing repositioning problem. Conceptually, it is different from the previous reactive strategy, because instead of assigning tasks in real time, the objective is to design in advance the optimal routes for all trucks and for the whole period (e.g. 24 hours). This means that the optimization takes into account all the expected tasks in the period, and therefore, the solution could be better that only taking into account the next task to be assigned. However, the dynamic application of this strategy strongly relies on the demand forecast. Note that at the time of the assignment, only the status of the system at the beginning of the day ($b_{j,0}$, $b_{v,0}$) is known. This implies that errors in the demand forecast could wash out any potential benefit resulting from a more efficient routing.

The MIP formulation of this routing optimization problem yields a computational complexity that prevents finding an exact solution for real size problems. This limitation was already acknowledged in Raviv et al. (2013) and Ho & Szeto (2014) analyzing similar approaches. In addition, considering the time-step vector as a decision variable adds additional difficulty with respect to the previous works. In order to reach an approximate solution, a feasible seed solution is calculated first. This starting seed is obtained by applying the previous reactive pairwise task assignment optimization strategy from the beginning of the operation period and assuming the forecasted inventory levels at the stations as the actual ones. This is by computing the stations' and vehicles' inventory level after each time step as in Equations (17) and (18). This process returns a feasible first approach solution in a reasonable computation time, which allows speeding up the MIP solvers. The problem is further simplified by considering a constant duration for all the tasks, τ. τ is set as the maximum duration of the tasks assigned in the seed solution. This means that all the vehicles complete one task each time step, so that the number of repositioning tasks is set in advance. Then, the time step vector, Γ, is not a decision variable any more, simplifying the problem by one degree of freedom.

Given these considerations, the formulation of the preemptive routing optimization problem is the following:

$$\max_{X,Y}\sum_{\forall t\in\Gamma}\sum_{\forall j\in S}\left[Z_{NSP}\left(b_{j,t}\right)_{j,t}-Z_{NSP}\left(b_{j,t}^{end}\right)_{j,t}\right] \qquad (13)$$

Subject to:

$$b_{j,t}^{end} = b_{j,t} + \sum_{\forall v} Y_{j,v,t} \tag{14}$$

$$0 \leq b_{j,t}^{end} \leq k_j \tag{15}$$

$$b_{j,t+\tau} = max\{0, min[b_{j,t}^{end} + (b_{j,t}^{ret} - b_{j,t+\tau}^{ret}) - (b_{j,t}^{req} - b_{j,t+\tau}^{req}), k_j]\} \tag{16}$$

$$b_{v,t}^{end} = b_{v,t} - \sum_{\forall j} Y_{j,v,t} \tag{17}$$

$$0 \leq b_{v,t}^{end} \leq k_v \tag{18}$$

$$\sum_{\forall j} X_{x_{v,0},j,v,0} = 1 \tag{19}$$

$$\sum_{\forall i,j,t} X_{i,j,v,t} \leq 1 \tag{20}$$

$$\sum_{\forall i} X_{i,j,v,t} = \sum_{\forall i} X_{j,i,v,t+\tau} \tag{21}$$

$$\tau_{v,t} = \delta \cdot abs \left\| \sum_{\forall j} Y_{j,v,t} \right\| + \sum_{\forall i,j} X_{i,j,v,t} \cdot D_{i,j} \tag{22}$$

$$\tau_{v,t} \leq \tau \tag{23}$$

$$abs\|Y_{j,v,t}\| \leq \sum_{\forall i} X_{i,j,v,t} \cdot k_j \tag{24}$$

Equation (13) is the objective function, which aims to maximize the savings in the no-service penalties. These savings are defined as the difference between the expected penalty cost before and after a particular repositioning task, determined from the respective inventory levels at the station $b_{j,t}$ and $b_{j,t}^{end}$, considering the whole operative period and all the stations. Note that the repositioning cost of each operation is not considered in the objective function. This responds to the fact that the time step vector is not a decision variable any more, and it is assumed that all vehicles perform one task in each time step. Therefore, the repositioning costs are constant and independent from the particular routing solution obtained. Recall that the decision variables are the repositioning vehicle trip tensor, $X_{i,j,v,t}$, and the bike-movement matrix $Y_{j,v,t}$, which define the repositioning operations in the period considered. Equation (14) calculates the inventory level at the station after the repositioning tasks are performed. Note that if there is no visit by the repositioning teams, the inventory level will be kept constant. Constraint (15) ensures that the inventory level at stations after any visit stay always between 0 and the stations' capacity. Equation (16) updates the inventory level of the station for the next time step according to the forecasted demand (i.e. requests and returns). This inventory level is also limited between 0 and the station's capacity. Note that by definition, $b_{j,t}^{ret} > b_{j,t+\tau}^{ret}$ and $b_{j,t}^{req} > b_{j,t+\tau}^{req}$. Equation (17) updates the inventory level of the repositioning vehicles, also limited by their capacity boundaries in constraint (18). Constraints (19)-(21) define the routing problem. Constraint (19) ensures that each vehicle starts one trip from its observed location, $x_{v,0}$, at the beginning of the day. Constraint (20) sets that each vehicle is assigned at most to one task every time step. Constraint (21) ensures that vehicles start each task at the ending location of the last one. Equation (22) calculates the duration of each task. Constraint (23) limits the duration of each route not exceeding the considered maximum duration of each task, τ. And finally, constraint (24) it is included to ensure that $Y_{j,v,t}$ only takes values different than zero if a task has been assigned on $X_{i,j,v,t}$.

The previous optimization problem was solved applying a guided diving heuristic by using the MATLAB solver function "intlinprog", developed after the works of Danna et al. (2005) and Berthold (2006).

3.4. Mixed strategy

This strategy is a combination of the previous two. Its application procedure will be as follows:

1. At the beginning of the operation period, the preemptive routing strategy is run, and all the repositioning vehicles will get a schedule of assigned tasks for the whole period.
2. The forecasted inventory level for all the stations will take into account these planned repositioning tasks.
3. When a repositioning vehicle, v, finishes one task, the next scheduled task, which consists in visiting station j', is considered as "tentative". The planned bike movements are discounted from the inventory level forecast.
4. The real-time reactive pairwise task assignment optimization strategy is run, with the restriction that only tasks with a duration $\tau_{v,j,t} \leq \tau$ are considered as feasible. The process yields an assigned task consisting in a visit to station j, where j may be different than j'. This task is also considered as tentative.
5. If $U_{v,j,t} > U_{v,j',t}$ the new task from the real-time reactive algorithm is finally assigned. Otherwise, the previous planned task is the final assigned task.
6. Inventory level forecasts on stations are updated according to the task finally assigned.

Note that the conditions to modify the originally scheduled task (i.e. step 5) imply that the new task must yield a larger utility and must not exceed the duration of the original task. This last condition ensures that the vehicle can still perform the following scheduled tasks within the operational period.

4. Case study in a simulated environment

The performance of each model is evaluated through simulation experiments. The simulation is constructed based on the station-based bicycle sharing system operating in Barcelona, Spain (i.e. called "*Bicing*"). The selected service area for the simulation considers the central area of Barcelona, with an extension of 39 km^2 containing 347 stations, with varying capacities (e.g. $k_j = 12 \div 54$). The bicycle fleet is composed of 4838 bikes, and the system relies on 13 repositioning trucks with a capacity of 16 bikes each. All of them are continuously working during the operation period of 24 hours with the chosen repositioning strategy.

The observed daily demand for the system consists of an average of 34 840 trips/day, which are non-uniformly distributed in time and space. In 53% of the stations there are more requests than returns (i.e. generation areas) while in 47% of the stations happens the opposite (i.e. attraction areas). On average, 11.8% of the demand is not balanced (i.e. one bicycle request without a compensating bicycle return, or vice versa). Demand follows the same time distribution as the overall mobility demand in Barcelona, being the peak hour between 18-19h. Demand has been input to the simulation in the form of O/D matrixes every minute.

Regarding the simulation of the no-service situations, it is considered that one trip is not served at the origin of the trip if there is not a station with available bikes within a maximum walking distance of 400m. At the destination, the no-service penalty is incurred if the user does not find an available parking spot within 400m of the desired destination. The estimated penalty cost of a trip not served at the origin (i.e. empty stations) is estimated to be 1.9 €/trip, considering the user annoyance of having to look for an alternative mode of transportation. In turn, the penalty if the no service happens at destination (i.e. full stations) is estimated according to the additional time spent by users looking for a station to return the bike, being 3.9 €/trip the obtained average value.

Table 1 summarizes all the parameters and inputs defining the simulation model. Further reference regarding the characterization of the *Bicing* system can be found in Soriguera & Jiménez (2020).

Table 1. Summary of simulation parameters and inputs

	Parameter description	**Units**	**Value**
Demand inputs	Area of the service region	[km^2]	39.19
	Total demand	[trips/day]	34 840
	Trip attraction area fraction	[-]	0.47
	Trip generation area fraction	[-]	0.53
	Average trip imbalance	[-]	0.118
User behavioral inputs	Maximum access distance	[km]	0.4
	Average walking speed	[km/h]	3
	Average cycling speed in the city	[km/h]	15.3
	Users' no service penalty at origin, β_e	[€/trip]	1.9
	Users' no service penalty at destination, β_f	[€/trip]	3.9
System and repositioning inputs	Number of stations	[stations]	347
	Capacity of stations, k_j	[bikes]	12 - 54
	Available bicycle fleet size	[bikes]	4838
	Number of repositioning teams	[trucks]	13
	Capacity of repositioning teams, k_v	[bikes/truck]	16
	Average speed of the repositioning vehicles	[km/h]	20.6
	Average cost per repositioning worker, c_v	[€/team·h]	21.54
	Average unitary time spent on picking up or delivering one bike, δ	[min/bike]	0.625

4.1. Scenario definition in the simulation

Each simulation consists of three demand cycles of 24 hours. The first two cycles are warming up cycles, and results are taken only from the third one. 4 simulation scenarios are considered. Scenario 0 is the baseline scenario in which there is no artificial rebalancing. This scenario is considered as the worst possible case in terms of the level of service provided. Scenarios 1 to 3 consider respectively the 3 different repositioning optimization strategies (i.e. real-time reactive assignment, preemptive route optimization, and mixed strategy) using a naive demand forecast. This forecast assumes that for each station the number of requests and returns simply follow the average daily demand rate for that zone of the service area. The mean absolute percentage error (MAPE) for the demand estimations using this forecasting method was 36%. The MAPE is computed as the aggregate difference between the predicted and the actual requests and returns at every station j and time period t, for the whole service region and operation period, over the actual number of requests and returns at the same stations and time periods.

All scenarios were simulated three times with different seeds for the randomly generated trips, which fulfil the global O/D demand patterns (i.e. same demand attraction and generation zones). Results depict the average performance of the three simulations on each scenario.

Table 2. Summary of scenarios

Id.	Repositioning strategy	Demand forecast
Scenario 0	No artificial rebalancing	-
Scenario 1	Real-time pairwise assignment	Average demand in the subzone. (MAPE: 36%)
Scenario 2	Preemptive routing optimization	
Scenario 3	Mixed strategy	

4.2. Results

The main results and KPIs of the performed simulations are shown in Table 3. Results show that all strategies provide savings on user costs over 80% with respect to the no-repositioning scenario. But the performance of each strategy is different, being the mixed strategy the best one and the real-time reactive assignment performing better that the preemptive routing strategy. It is significant to notice that the real-time reactive assignment strategy (Scenario 1) results in a lesser fraction of empty and full stations than the other two strategies.

Table 3. Summary of KPIs

KPI		Units	Scenario 0	Scenario 1	Scenario 2	Scenario 3
No-service at the origin of the trip		[Users]	7 415	3 767	4 059	3 785
		[%]	21.28	10.80	11.64	10.86
No-service at the destination of the trip		[Users]	5 814	1 035	1 225	988
		[%]	24.11	3.72	4.45	3.56
Avg. travel time increase		[Min.]	14.72	6.09	4.97	5.49
Avg. egress distance increase		[Meters]	210.51	189.99	170.82	181.92
Avg. stations empty		[%]	18.19	2.20	4.70	2.31
Avg. stations full		[%]	29.33	4.07	5.69	4.22
Estimated no-service penalty cost	Origin	[€]	14 088	7 157	7 711	7 192
	Destination	[€]	60 749	4 471	4 319	3 846
	Total	[€]	74 837	11 628	12 030	11 039
	Savings[1]	[€]	-	63 208	62 806	63 798
		[%]	-	84.46	83.92	85.25

[1] Savings with respect to Scenario 0 (without artificial rebalancing).

Figure 2 shows the spatial comparison of the stations that were full or empty at some point during the operation period for each strategy. The comparison is made between Scenario 0 and Scenario 3. It is clearly seen how in

Scenario 0 (no rebalancing), bicycles "precipitate" from the higher to the lower parts of the city. Note that the South-East border of the service region corresponds to the Mediterranean coast, so that the downslope in Barcelona goes from the North-West to the South-East parts of the city. Rebalancing (e.g. Scenario 3 with mixed rebalancing) mostly solves this system unbalance.

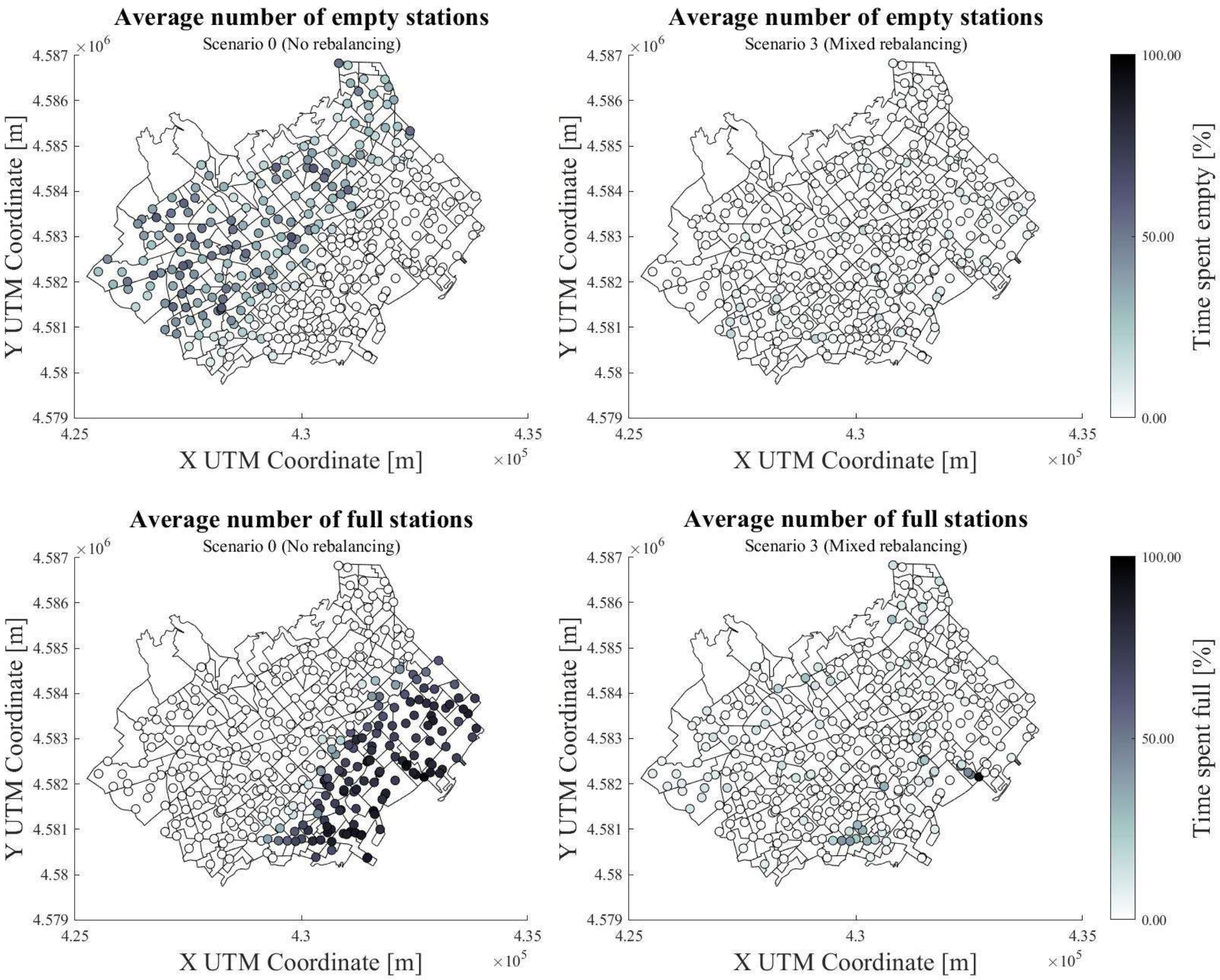

Fig. 2. Average number of full and empty stations during the day on Scenario 0 (no rebalancing) and Scenario 3 (mixed strategy).

Presumably, the performance of the repositioning strategies considered varies with the accuracy of the demand prediction. To examine this effect, new scenarios were simulated considering demand predictions with varying accuracies. In some scenarios, errors were artificially introduced in the demand predictions, while in others part of the future demand was considered to be perfectly known. As a result, MAPE of the demand predictions in the new sets of simulated scenarios varies between 16-68%. Results of the performance of the different repositioning strategies in these new scenarios are summarized in Figure 3, which depicts the total no-service penalty cost of the system as a function of the accuracy of the demand prediction.

Results show that the real-time pairwise optimization assignment (i.e. Strategy 1) is less reliant on the accuracy of the demand prediction than strategies 2 and 3, which both include the preemptive routing optimization. When the error in the demand prediction increases, the performance of routing optimization strategies worsens more than the real-time pairwise assignment. It seems clear that any possible disadvantage of the pairwise optimization (i.e. not considering the potential following tasks in order to chain them in an optimal route) becomes compensated by the advantage of a more adequate task assignment by observing the system in real time.

In contrast, the preemptive routing strategy overperforms the real-time pairwise assignment optimization when the demand prediction errors are low. However, even if the accuracy of the demand prediction is high, the real-time pairwise assignment strategy still returns good results. This implies that, if its implementation simplicity and low computational cost are considered as relevant factors, this strategy would be very recommendable in all contexts.

The performance of the mixed strategy is also noticeable. It looks clear that the real-time component included in the strategy helps to slightly improve the estimated route performance. This can be seen by the slightly reduced no-service penalty cost of the mixed strategy with respect to the routing strategy, for all accuracy levels of the prediction. The behavior of the mixed strategy relies on how restrictive is the criterion to adapt the route when new real-time information is considered. Note that for the current formulation of the mixed strategy, only the next task is evaluated, and this constrains which alternative tasks can be taken as a replacement. So, only small changes in the preemptive route are expected. If the evaluation and replacement criteria was less restrictive, the number of possible alternative tasks would grow, and the performance of the mixed strategy would become closer to the pure real-time pairwise assignment strategy. This would be advisable when the error in the demand forecast is high, although real-time pairwise assignment is always preferable if the accuracy of demand forecasts is expected to be high.

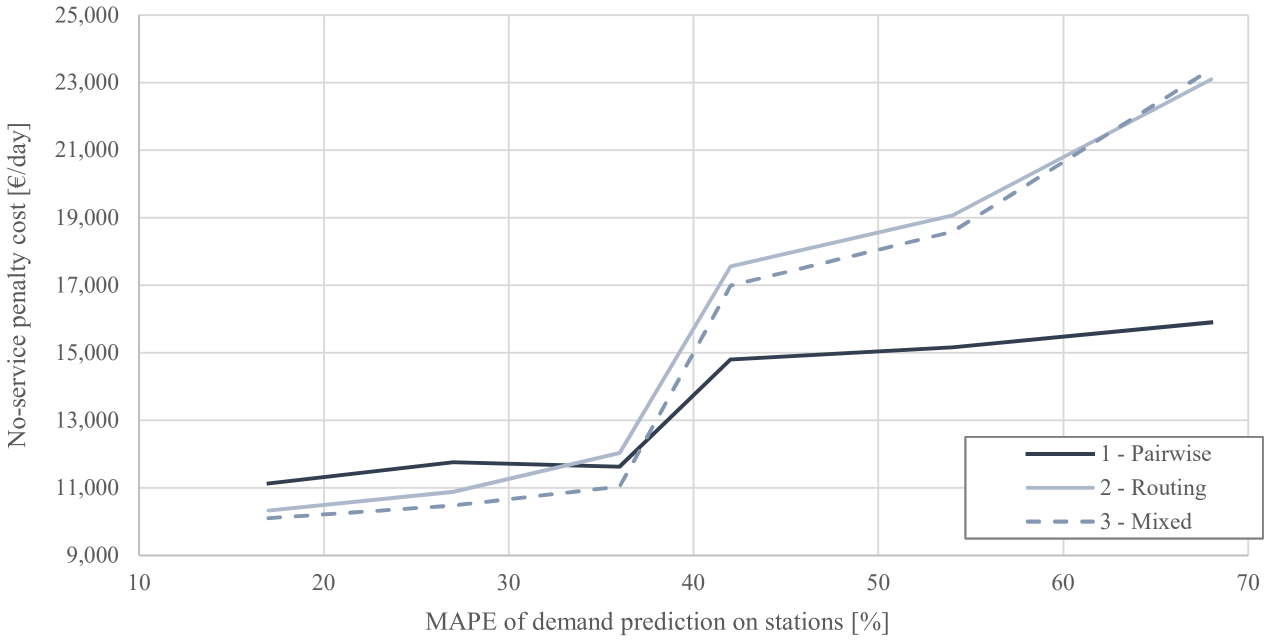

Fig. 3. Performance of different repositioning strategies as a function of the mean absolute percentage errors in demand prediction.

5. Conclusions

A new bike-sharing repositioning strategy has been developed. It is based on real-time pairwise assignment between tasks and repositioning teams. The strong points of the proposed strategy are twofold. First, the goodness of the strategy is less dependent on the forecasts of the inventory level at stations, which are always uncertain. And second, the implementation of the method is simple and involves a low computational cost in comparison with strategies based on routing optimization.

The results obtained through simulation experiments show that, in general, the real-time pairwise assignment strategy is a better strategy than those based on preemptive routing optimization. The benefit of the real-time assignment increases when the accuracy of the demand forecast and the estimation of the inventory levels are low, but even if the accuracy of the demand predictions is relatively good, the real-time assignment can be preferable due to its simplicity of implementation and low computational cost. In any case, the real-time pairwise assignment strategy can be implemented in a mixed strategy with the preemptive routing optimization, improving the performance of the latter.

The sub-optimal performance of preemptive routing repositioning strategies is due to the errors in the prediction of the bicycles inventory level at stations. Such errors, usually neglected in the related literature, are due to the existence of non-recurrent unpredictable events, or more often to the “second-order” effects where the predicted demand at a particular station, which is full or empty, is diverted to other nearby stations leading to other stations becoming full or empty and generating a highly unpredictable context. Preemptive routing strategies do not take these situations into account, while the reactive real-time pairwise assignment optimization can directly observe these unexpected situations.

Preemptive routing optimization algorithms could be further improved, as well as the accuracy of demand forecasting methods. However, the gains might be marginal while the computational cost and implementation difficulties can increase from significant to astronomical. Note that the difference in the computational time for the strategies proposed in this paper is already huge. Reaching a solution in the preemptive routing optimization strategy took from several minutes and up to hours, and in some cases, the algorithm did not even find a better solution than the seed. In contrast, the reactive pairwise optimization takes less than a second to reach the solution.

Another reason which favors reactive assignment strategies in front of preemptive routing strategies is the possible inclusion of different types of repositioning tasks. Generally, the analysis of vehicle-sharing repositioning operations in the scientific literature focusses only on solving the vehicle imbalance. But in practice, there are other reasons for repositioning teams to visit stations, such as the maintenance and repair of bicycles and parking slots. The inclusion of such maintenance tasks would yield higher uncertainty in the task predictions, so that real-time reactive strategies would be in a better situation to face them.

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Paper IV

Forecasting demand at bike-sharing stations through machine learning techniques

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Forecasting demand at bike-sharing stations through machine learning techniques

Jaume Torres, Enrique Jiménez and Francesc Soriguera

Abstract—The main operative problem of bike-sharing systems is the unbalanced distribution of bicycles over the service region, resulting in zones where bicycles are scarce and zones where bicycles accumulate. In order to provide an acceptable level of service, the operator needs to carry out artificial repositioning movements, which are costly. Bike-sharing repositioning optimization solutions have been developed, which rely on the estimation of the expected number of requests and returns at each location. Errors in this prediction are directly transferred to suboptimal repositioning solutions. For this reason, the development of methodologies able to forecast bike-sharing demand with accuracy is an issue of much concern. This paper deals with this problem using machine learning regression methods, which return demand predictions from inputs such as historical and meteorological data. Three different machine learning regression techniques have been analyzed (i.e. random forest, gradient booster, and artificial neural networks) and applied to a case study based on the New York City bike-sharing system. The paper describes the variables of the models and their calibration processes. Results are analyzed to determine which one of the three techniques and under what conditions is the most accurate. Comparison is not only made in terms of accuracy, but also with respect to the easiness to calibrate the algorithms. In cases where accuracy is similar for all methods, the easiest one would be the most advisable to implement.

Keywords— bike-sharing, station-based, demand prediction, machine learning regression.

1. Introduction and background

Repositioning operations are a fundamental part of vehicle-sharing operative management. Repositioning aims to relocate vehicles within the service region in order to achieve various objectives (e.g. recharge batteries of electric vehicles, perform maintenance operations, maximize the demand served, or improve the vehicle availability). In the particular case of station-based bike-sharing systems, repositioning operations are carried out by vans or small trucks, which move groups of bikes between stations with the main objective of reducing the number of empty and full stations, and therefore improving the level of service offered. Since this phenomenon is caused by demand imbalance in the service region, these relocation operations are also commonly called rebalancing operations.

The efficient planning of repositioning operations is important. Repositioning is costly, and to a large extent, it can determine the success or failure of the vehicle-sharing system. Lack of repositioning, or its poor performance, results in the frustration of users when left without the possibility of picking-up a bike at the origin of their trip because of empty stations, or when unable to return it near their destination due to the lack of available parking slots. Given the importance of the problem, several authors have proposed repositioning strategies and algorithms able to maximize the service provided without incurring in excessive repositioning costs. Examples are the works by Schuijbroek et al., (2017), Caggiani et al. (2018), Lei and Ouyang (2018), and Zhang et al. (2021). Despite these works follow different approaches to solve the repositioning optimization problem, all of them have one thing in common: they rely on an accurate forecast of the expected usage (i.e. requests and returns) at the vehicle-sharing stations. This forecast is fundamental on all models, because it is used to define the utility of each repositioning movement. Clearly, it would be more profitable to address stations with high demands and which are likely to become full or empty during the operation period. On the contrary, it does not matter much if a station is empty or full when the expected demand during the following hours is negligible. Predicting the bicycle inventory level at stations is one of the key variables if repositioning takes place while the system is in operation (i.e. “dynamic” repositioning, as opposed to “static” repositioning which takes place only when the system is closed to users). Take as examples the works by Caggiani et al. (2018), and Zhang et al. (2021), which use neural network techniques to update the predicted inventory levels of the system at different times during the repositioning optimization process.

The most recent literature regarding the short-term usage prediction of vehicle-sharing systems relies on advanced regression techniques based on machine learning methods. Works by Yang et al. (2016), and Xu et al. (2018) can be considered the starting point of this approach. Specifically, Yang et al. (2016) opts for a random forest method for a station-based bike-sharing system, while Xu et al. (2018) uses long short-term memory neural networks for the case of a free-floating system. Several other authors have contributed with their own ad-hoc methodologies based on different machine learning applications, as in Lin et al. (2018a), Guo et al. (2019), Boufidis et al. (2020), Sathishkumar and Cho (2020), Yang et al. (2020), Li et al. (2021; 2023). We refer to Abouelela et al. (2023) for an excellent and up to date literature review on the application of machine learning techniques to big data from vehicle-sharing systems.

The present work is built on the same approach and proposes three alternative machine learning algorithms to make short-term predictions of requests and returns at the New York City bike-sharing stations from January 15th to 29th, 2019. The usage dataset from the whole year 2018 has been used to train the models. The methods focus on the prediction up to a minimum horizon of one day (i.e. the next day) as this is critical to plan the repositioning operations overnight. In spite of this, results are analysed up to a horizon of 15 days. In all cases, predictor variables are related to the type of calendar day and meteorology, being the most influential factors in the determination of bike-sharing usage (Kim et al., 2012; El-Assi et al., 2017; Mattson and Godavarthy, 2017;

Yoon et al., 2017; Lin et al., 2018b; Shen et al., 2018; Durán-Rodas et al., 2020; Wessel, 2020). Note that the usage data of the system is a biased approximation for its demand. Potential demand might be truncated at the station level (e.g. diverted to a nearby station or lost) when the station is empty of vehicles or parking spots. Gamelli et al. (2020) and Wang et al. (2023) analyse in detail such demand truncation at the station level. If the interest resides in the potential demand prediction, it is worth mentioning that in the present paper, the clustering of nearby stations in the prediction might attenuate such difference.

In summary, the main objectives of the research are twofold. First, to explore the model variables and to establish an adequate treatment methodology which allows obtaining the best calibration of the models. Second, to compare the accuracy of the predictions obtained with the different methods. Previous research in Boufidis et al. (2020) suggests that accuracy differences are small. In such case, the most convenient method would be that with a simpler calibration and implementation.

The rest of the paper is structured as it follows. Section 2 describes the methodology of the analysis. This includes the definition of the considered variables and its data treatment, together with the description of the calibration process of the three machine learning methods considered. Next, Section 3 introduces the case of study based on the New York City station-based bike-sharing system and analyses the results of application of the three methods. Finally, the paper ends with the conclusions section and reference list.

2. Machine learning methods to predict the inventory level at bike-sharing stations

2.1. Variables of the model

All the proposed models will consider up to 13 variables. The first two, are the dependent variables, namely the trips generated (i.e. requests) and the trips attracted (i.e. returns) at every station. These are the variables to forecast by using some other predictor variables. Predictors are classified as it follows: *i)* identifier of each station in the system (1 variable); *ii)* time and calendar related predictors (4); and *iii)* weather related predictors (4). Table 1 details all these variables used in the machine learning algorithms.

Table 1. Variables considered in all the models.

Class	Variable	Type	Range
Dependent	Requests (generated)	Continuous	Positive
	Returns (attracted)	Continuous	Positive
ID	Station ID	Identifier	Number of stations
Time & calendar related	Season	Discrete	0 - 3
	Day of the week	Discrete	0 - 6
	Hour of the day	Discrete	0 - 23
	Minute of the hour	Discrete	0 – 59
	Holiday	Binary	0 - 1
Weather related	Temperature level	Discrete	0 - 5
	Wind speed level	Discrete	0 - 5
	Rain level	Discrete	0 - 5
	Snow level	Discrete	0 - 5

2.1.1. Dependent variables: requests and returns at bike-sharing stations

The proposed model predicts the number of requests (i.e. trips generated; picked-up bicycles) and the number of returns (i.e. trips finished; bikes left) for every station in the system and for a given time-step. The model will use historical data of these dependent variables to learn. Usage data is generally available from the systems operating worldwide, although it might be organized slightly differently, with more or less variables reported and with different time aggregations. Typically, the shorter time aggregations are of 1 minute, and they usually do not go above 5 minutes, as this information is used to monitor and inform users about the real-time inventory level at stations. In any case, the time-step of the prediction must be larger than the time aggregations of this historical input data.

The usage data preprocessing consists in a few steps. First, a data cleaning process, consisting in detecting and deleting extreme outliers and possible errors. Take as an example the common case of stations without demand along the day, which usually means that the station is closed or out of order. Second an aggregation process to fulfil the desired time-step of the regression method. This only applies in case that the selected time-step is larger than the time granularity of the available data. And third, a standardization process consisting in subtracting the mean and dividing by the standard deviation, in order to obtain a standard variable with zero mean and unit variance.

1.1.1. Time and calendar related predictors

Time predictors describe when the bike is requested from, or returned to, the station. All bike-sharing datasets include the minute, hour, day, month and year of each request and return. The calendar variables (i.e. day, month and year) are processed as follows:

- Day of the month: It is discretized into seven categories corresponding to the day of the week (i.e. Monday, to Sunday).
- Month and year: They are aggregated into a single variable (i.e. season) and discretized into four categories (i.e. winter, spring, summer, autumn).

Such discretization of calendar variables responds to the typical usage behaviour of bike-sharing systems, with different daily patterns and seasonality (Younes et al., 2020; Abouelela, et al., 2023).

In addition, the “hour” and “minute” time variables are aggregated into the time-step selected for the machine learning method. The selected time-step may range from the refreshing time of the monitored bike-sharing data (usually, 1-5 minutes), to longer time-steps (e.g hourly or daily predictions) which could provide higher accuracy in the predictions due to the increase of the sample size of the dependent variables.

Finally, the “holiday” variable is included into the database in order to take into account calendar bank holidays.

2.1.2. Weather related predictors

Four weather predictors are considered: temperature, wind speed, rain and snow precipitation. Data is obtained from the meteorological registers in the city where the system is operating. Meteorological measurements can be considered as continuous variables. However, user behaviour is not sensitive to small variations of meteorological variables, so that discretization into different levels would fit better the purpose of predicting bike-sharing usage. For example, the decision to use or not the system can be affected by if it rains or it does not. But, under rainy conditions, users will not distinguish the nuance of a few millimetres more or less of rainfall. The perception of windy conditions could be similar.

In an attempt to model adequately this behaviour, six ranges are defined for each meteorological variable, corresponding to different intensity degrees. For precipitation variables (i.e. rain and snow), a binary transformation (i.e. rains or not) could also be a possibility. In spite of this, it has been found that the approach with five ranges yields better results. The reason for this it has been identified to be related to the heterogeneity of precipitation conditions in time and space. Episodes with low precipitation levels and only in certain regions of the city can be classified as rainy conditions, but still with many users which are not affected. Therefore, if these variables are treated as binary, the results are more prone to errors.

3. Machine learning regression methods

Three regression methods have been considered and compared in the present work. These are: *i)* random forest, *ii)* gradient booster, and *iii)* artificial neural networks. In this section the calibration process for each of them is presented in order of complexity. Random forest is the simplest method and needs little calibration, while artificial neural networks is the most complex and needs a bigger calibration effort. Gradient booster method lies in between.

3.1. Random forest (RF)

One of the advantages of RF is its easiness to implement. It is only needed to characterize the number of estimators (i.e. the number of decision trees randomly created) in order to get an accurate model. For the purpose of this work, an accuracy analysis has been done for different number of estimators. As observed in Fig. 1, the marginal gain in the accuracy decreases with the number of estimators. Accuracy strongly grows when adding a few estimators, but the marginal gain is null over 100 estimators. In this case, overestimating the number of estimators does not imply a deterioration of the results, but only an increase of the computational time. This increase in the computational time is notorious when surpassing 100 estimators. In conclusion, 100 estimators are considered to be an adequate number for the RF method.

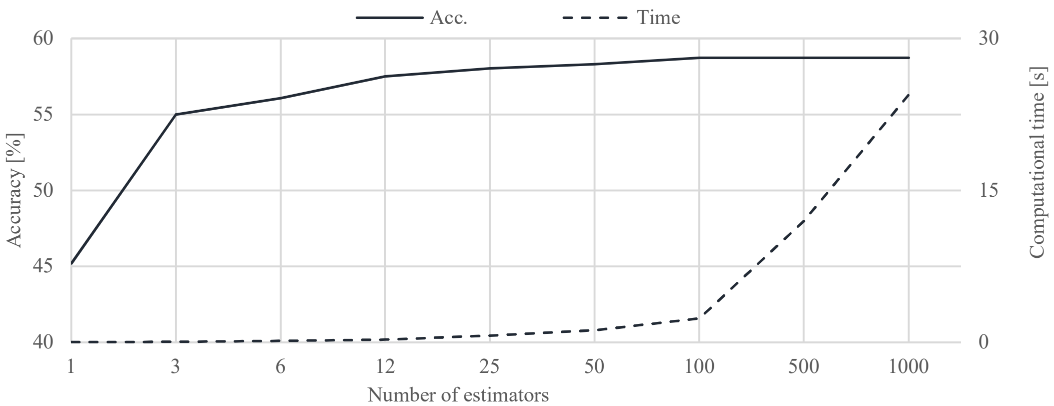

Fig. 1. Accuracy analysis for parameter tuning in RF.
(Note. Accuracy is defined as the complementary of the mean relative error).

3.2. Gradient booster (GB)

Unlike the RF technique, the GB method creates the predictors sequentially so that each one can learn from the errors in the previous iterations. This means that these algorithms are prone to overfitting if they are not properly controlled thorough regularization techniques. This is achieved by the calibration of two parameters. The first one is again the maximum number of estimators. Unlike the RF, in the GB an overestimation of this parameter can be detrimental for the results due to the overfitting. The second calibration parameter is the learning rate, which shrinks the update rule of the algorithm. The lower the learning rate, the larger will be the improvement in the model generalization capabilities.

A calibration analysis has been carried out to determine an adequate value for these parameters. Results are shown in Fig. 2. The best accuracy has been achieved with 500 estimators and a learning rate of 0.2. Higher values lead to overfitting problems, and accuracy actually slightly decreases when the learning rate increases.

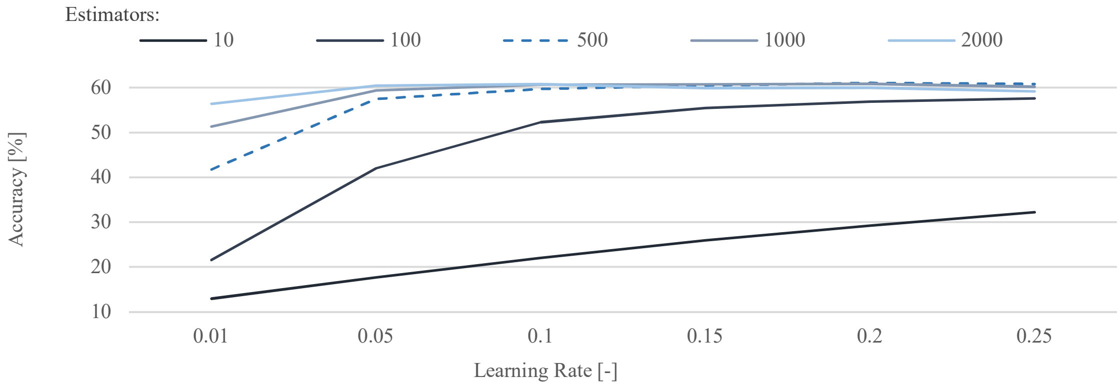

Fig. 2. Accuracy analysis for parameter tuning in GB with different number of estimators.

3.3. Artificial Neural Networks (NN)

NN are a much more complex method in relation to the previous RF and GB methods, because it is not only needed to calibrate a few parameters, but also to build the structure of the algorithm itself. On the one hand, this partly softens the "black box" effect of previous methods, since the analyst has more control on the structure of the algorithm and its parameters, which might yield improvements to obtain a better accuracy. On the other hand, this increases the complexity of the model as it requires additional calibration efforts, without eliminating the risk of overfitting.

In order to define and calibrate the NN algorithm for the problem being analyzed, different layouts with a different number of hidden layers between inputs and outputs, and with different activation functions (e.g. linear, tangent hyperbolic - tanh, rectified linear unit - relu), have been tested to see which one yields the best accuracy. For each proposed layout, the number of epochs (i.e. the number of times the algorithm processes the same data) has been monitored to reach an adequate trade-off between under- and overfitting.

Table 2. NN structure for test and calibration.

Layout	Parameters	Input	Hidden 1	Hidden 2	Hidden 3	Output	Epochs	Accuracy [%]
Layout 1	Neurons	n_inputs	n_inputs	-	-	1	25	24.75
	Act. Func.	-	tanh	-	-	relu		
Layout 2	Neurons	n_inputs	2×n_inputs	-	-	1	25	25.24
	Act. Func.	-	tanh	-	-	relu		
Layout 3	Neurons	n_inputs	n_inputs	n_inputs/2	-	1	50	61.52
	Act. Func.	-	tanh	relu	-	linear		
Layout 4	Neurons	n_inputs	n_inputs	n_inputs	-	1	50	61.32
	Act. Func.	-	tanh	relu	-	linear		
Layout 5	Neurons	n_inputs	2×n_inputs	n_inputs	-	1	50	61.15
	Act. Func.	-	tanh	relu	-	linear		
Layout 6	Neurons	n_inputs	2×n_inputs	n_inputs/2	-	1	50	61.05
	Act. Func.	-	tanh	relu	-	linear		
Layout 7	neurons	n_inputs	n_inputs	n_inputs	n_inputs	1	30	60.98
	Act. Func.	-	tanh	relu	relu	linear		
Layout 8	Neurons	n_inputs	2×n_inputs	n_inputs	n_inputs/2	1	30	60.76
	Act. Func.	-	tanh	relu	relu	linear		
Layout 9	Neurons	n_inputs	n_inputs	n_inputs/2	n_inputs/4	1	40	50.54
	Act. Func.	-	tanh	relu	relu	linear		
Layout 10	Neurons	n_inputs	n_inputs	2×n_inputs	n_inputs	1	30	61.54
	Act. Func.	-	tanh	relu	relu	linear		

Table 2 summarizes all the experiments, showing that layouts L-1 and L-2, which are based on a single layer, do not have enough training power to achieve acceptable accuracies. Two or three layers are required to predict quite accurately the output. Layouts L-3, L-4, L-5 and L-10 outperform the others in terms of accuracy, being L-

3 the most convenient for the analyzed problem when considering the resulting accuracy and complexity of the NN layout.

4. Citi Bike NYC case of study

The previous machine learning regression methods have been applied to a case study based in the New York City bike-sharing program (i.e. Citi Bike NYC), as in Cantelmo et al. (2020). Datasets were retrieved from the available historical database. The whole year 2018 data has been used for training the models, and data from January 15th to 29th, 2019 has been used for testing the accuracy of the results. By the time of the analysis, Citi Bike NYC consisted of 706 stations and 12.000 bikes. Fig. 3 shows the spatial configuration of the system, while Fig. 4 illustrates the temporal variability of its usage.

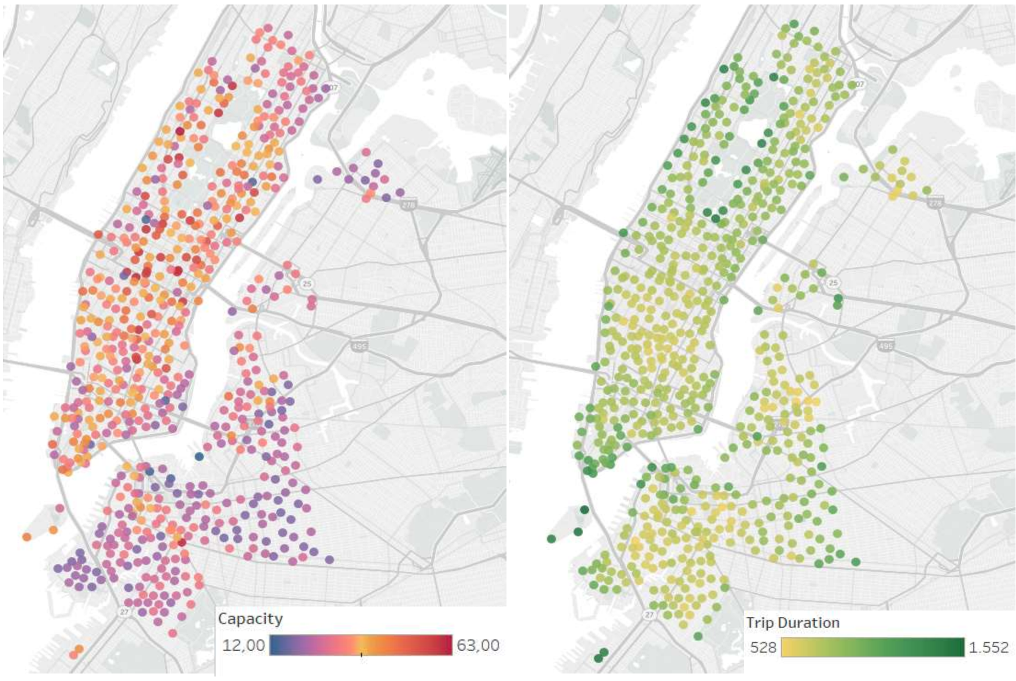

Fig. 3. Stations' capacity (left) and originated trips' duration [s] (right) in the New York City bike-sharing program.

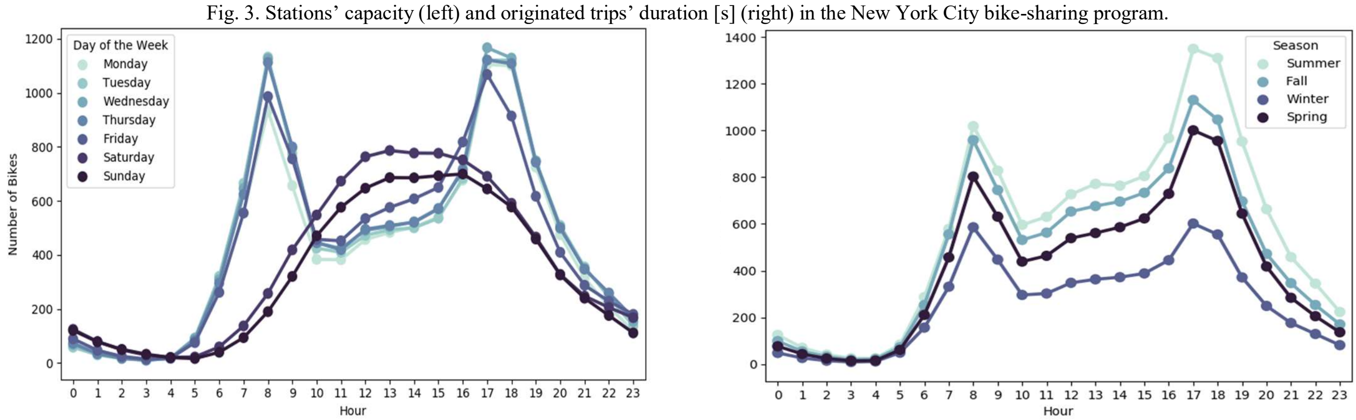

Fig. 4. Average hourly usage showing the daily and seasonal variability of the usage in the New York City bike-sharing program.

Two time-steps have been defined for the data aggregation and prediction. In general, the time-step is set to 1 hour, which ensures significant change in the stations' inventory levels and an adequate granularity for the application of the results to the planning of repositioning operations. However, in stations with very low average demands (i.e. less than 2 requests or returns per hour), the time-step is increased to 3 hours to obtain a significant amount of data. Table 3 shows the results obtained for regular and for low demand stations. In turn, Fig. illustrates the prediction for a particular station.

Table 3. Usage prediction results for regular and low demand stations.

		Regular stations (1h aggregation period)						**Low demand stations** (3h aggregation period)					
		Random Forest (RF)		**Gradient Boosting (GB)**		**Neural Network (NN)**		**Random Forest (RF)**		**Gradient Boosting (GB)**		**Neural Network (NN)**	
		Ret.	**Req.**	**Ret.**	**Ret.**	**Req.**	**Ret.**	**Ret.**	**Req.**	**Ret.**	**Req.**	**Ret.**	**Req.**
Working days	Avg. demand	9.3	8.9	9.3	3.4	3.1	3.4	3.4	3.1	3.4	3.1	3.4	3.1
	Avg. Error	3.6	2.8	3.5	1.8	1.7	1.8	1.8	1.7	1.8	1.7	1.8	1.6
	Accuracy (%)	60.8	67.5	62.2	48.3	46.0	47.1	48.3	46.0	47.1	46.8	48.3	47.5
	Max. Error	38.2	18.7	37.6	11.1	8.5	9.7	11.1	8.5	9.7	6.9	10.1	6.5
Weekends & holiday	Avg. demand	4.3	4.2	4.3	2.6	3.0	2.6	2.6	3.0	2.6	3.0	2.6	3.0
	Avg. Error	2.7	2.5	3.7	2.1	1.6	2.0	2.1	1.6	2.0	1.7	2.1	1.8
	Accuracy (%)	38.6	39.9	15.1	19.7	45.2	21.7	19.7	45.2	21.7	42.1	18.8	40.9
	Max. Error	29.4	24.7	31.3	7.4	6.4	8.0	7.4	6.4	8.0	5.6	6.8	5.9
Rainy days	Avg. demand	5.7	5.7	5.7	3.4	3.1	3.4	3.4	3.1	3.4	3.1	3.4	3.1
	Avg. Error	3.3	2.6	3.7	1.6	1.4	1.5	1.6	1.4	1.5	1.4	1.5	1.4
	Accuracy (%)	42.0	54.0	34.3	53.2	55.9	56.2	53.2	55.9	56.2	55.1	55.9	56.0
	Max. Error	38.2	24.7	37.7	11.1	8.5	9.7	11.1	8.5	9.7	6.9	10.1	6.5
Peak Hours	Avg. demand	20.2	27.7	20.2	7.1	4.9	7.1	7.1	4.9	7.1	4.9	7.1	4.9
	Avg. Error	7.5	6.2	6.8	3.2	2.1	2.8	3.2	2.1	2.8	1.8	2.8	1.8
	Accuracy (%)	63.0	77.7	66.2	55.3	58.0	60.0	55.3	58.0	60.0	63.8	60.3	64.5
	Max. Error	25.2	18.7	23.5	11.1	8.5	10.1	11.1	8.5	10.1	6.5	9.7	6.5
Overall	Avg. demand	7.5	7.3	7.5	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1	3.1
	Avg. Error	3.3	2.8	3.6	1.9	1.6	1.8	1.9	1.6	1.8	1.7	1.8	1.7
	Accuracy (%)	56.3	61.8	52.6	40.6	46.7	40.9	40.6	46.7	40.9	45.8	41.0	45.7
	Max. Error	38.2	24.7	37.7	11.1	8.5	9.7	11.1	8.5	9.7	6.9	10.1	6.5

Note: 1) Accuracy is defined as the complementary of the mean relative error; 2) Low demand stations imply less than 2 requests or returns per hour.

As seen in Table 3, the accuracy increases when the average number of expected bike movements is higher. Stations with large demand, peak hours, and working days are easier to predict than stations with low demand, weekends, and rainy days, which suffer higher variability in relative terms. In addition, requests are almost always predicted with more accuracy than returns. All three methods provide a similar accuracy. RF provides the best results on many cases, but suffers an accuracy drop when the sample for training is small (e.g. holidays). GB behaves similarly than RF, with slightly worse results in general, and suffering larger accuracy drops for the same contexts. Finally, NN is less affected by the size of the training database, and yields lower maximum errors. Given these results, and considering the marginal gains of the different methods in particular contexts, the overall conclusion is that any of them would be a good option for predicting the inventory level at bike-sharing stations in order to use it as an input for relocation operation algorithms.

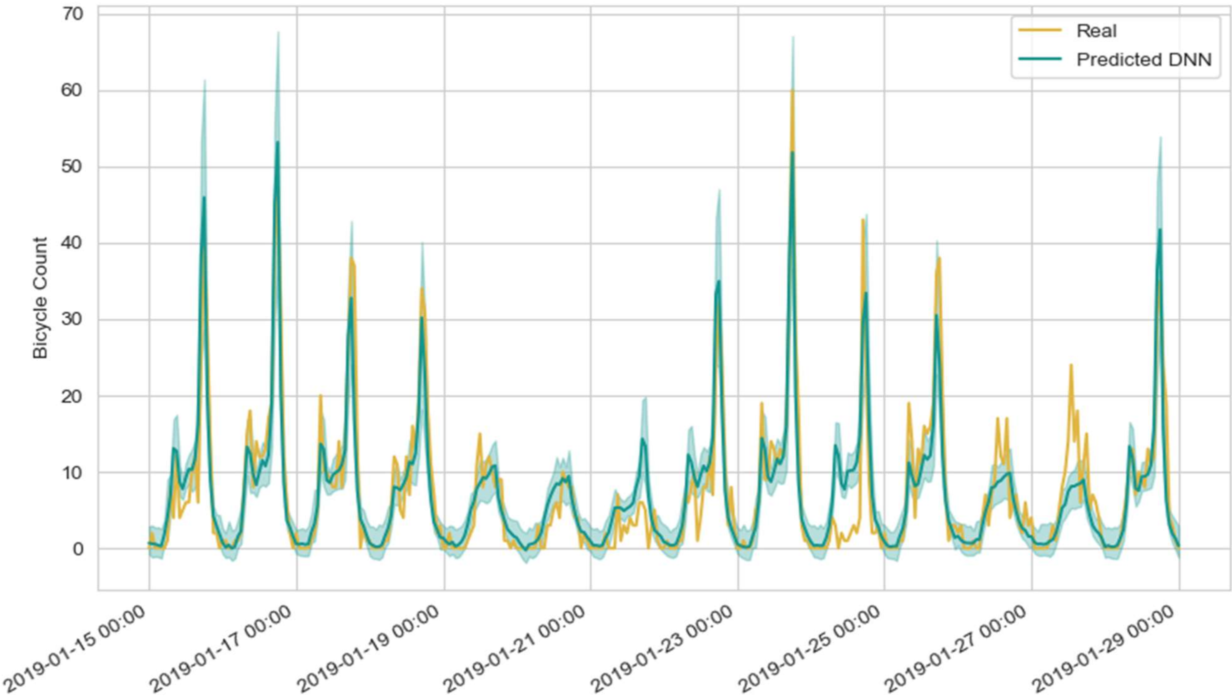

Fig. 5. Prediction of requests at Station 402 (high demand) from 2019/01/15 to 2019/01/29 using NN.

4.1. Effects of the time step duration in the prediction accuracy

Overall, the achieved accuracy of the predictions is approximately of 60%, at best. This drops to 40% at stations with low demand (even though the error represents a few trips in absolute terms) and grows to 70% during peak periods. In summary, the accuracy level achieved, with even the best calibration of the machine learning algorithms considered, is not very high.

The accuracy of the prediction grows (in relative terms) as it grows the demand at stations, due to the reduction of the totally random statistical variability. This means that considering longer time-steps (i.e. longer temporal data aggregation) would yield a better accuracy of the predictions. Fig. 6 shows the accuracy improvement resulting from an increase of the time-step from the defaults (i.e. 1h for regular stations and 3h for low demand stations) and up to 6h. In all cases, the accuracy improvement is significant.

The conclusion is that independently of the machine learning method used, the time-step for the prediction should be as large as the applicability of results allows. For instance, in case of static repositioning where rebalancing operations take place only when the system is closed at night hours, the time-step should include all the daily trips in the system. In contrast, for dynamic repositioning while the system is in operation the time-step should be of a few hours, depending of the repositioning algorithm considered.

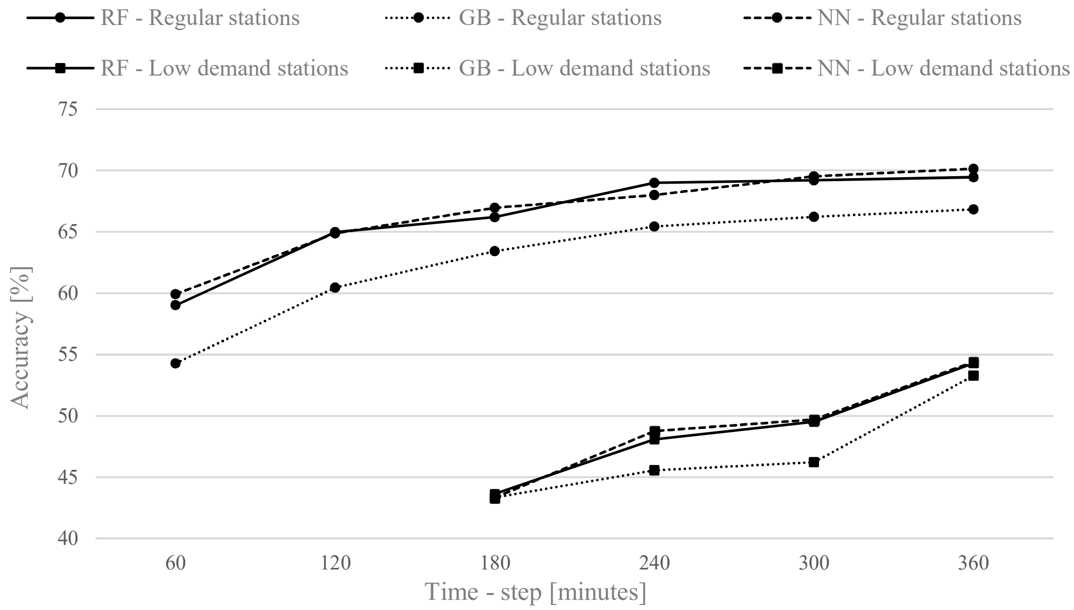

Fig. 6. Effects of the duration of the time-step in the accuracy of the prediction at the station level.
(Note: Low demand stations imply less than 2 requests or returns per hour)

4.2. Effects of spatial clustering in the prediction accuracy

Besides the temporal usage data aggregation, spatial aggregation could also contribute to increase the accuracy of usage predictions. This is not only due to a higher sample of usage data, which reduces its statistical variability, but also to the fact that, at the station level, usage demand has a significant random part not explained by the considered explanatory variables. Typically, the density of stations is large in bike-sharing systems, and users may choose one or another between nearby stations depending on its bicycle availability. This means that, creating clusters of nearby stations with a similar aggregate behavior may yield a more predictable aggregate number of requests and returns. This aggregate prediction would still be precious for the operating agency, as the repositioning planning could be performed at the cluster level and executed later on at the station level, as the repositioning team would have information on the real-time inventory of every station in the cluster.

The k-means clustering technique with Euclidean distance has been used to group similar stations in the proposed case study. The variables considered to compute the clustering "distance" between stations have been the UTM coordinates of the station location (i.e. to group nearby stations) and the overall number of requests and returns between 0-12am and 0-12pm for the different types of day considered (i.e. weekdays and weekends & holiday). These last variables intend to group stations with a similar aggregated demand pattern.

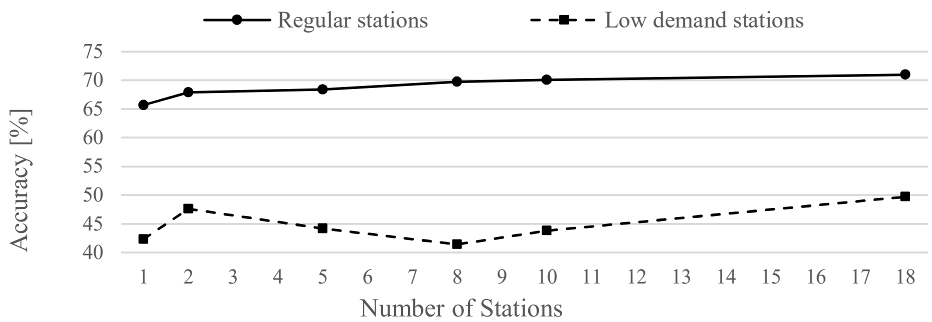

Fig. 7. Effects of stations' clustering in the prediction accuracy. (Note: Time-step is 3h for both stations' type).

Results in Fig. 7 show the effect of clustering a different number of similar stations on the accuracy of the aggregate prediction of their requests and returns. The accuracy improvements are below 10% in all cases. For regular stations, clusters of 8 stations are enough to achieve most of the improvement, while for low demand stations, clusters larger than 10 stations would be required to achieve some improvement.

In general, considering larger clusters is not an option, due to its excessive geographical extension for being considered as a single unit in the repositioning operations framework. For other applications, where the spatial distribution of sharing vehicles is not relevant, predictions could be estimated at the city level (i.e. all the stations of the system together). Results in Fig. 8Fig. show that in such case, the accuracy of the predictions can reach 80%.

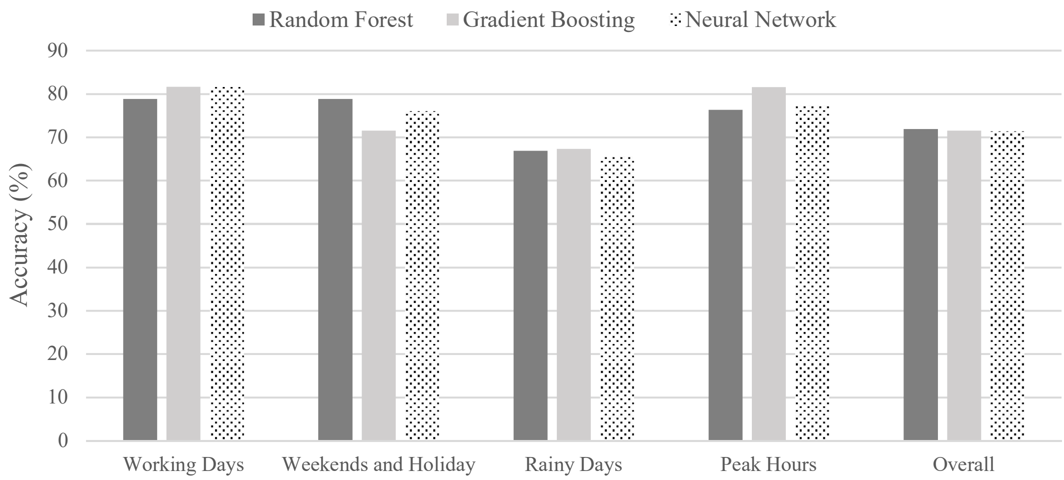

Fig. 8. Accuracy of the prediction at the city level using different machine learning methods.

5. Conclusions and further research

The paper presents a comparison of demand forecasting methods for bike-sharing systems based on machine learning algorithms. Three methods have been analyzed: Random Forest (RF), Gradient Boosting (GB), and Neural Networks (NN). They are calibrated and applied to the Citi Bike NYC case study to test their feasibility and accuracy.

The prediction of the inventory level at bike-sharing stations is an important input for the planning of repositioning operations. Considering this potential application of the results, the time-step of the prediction algorithms (i.e. the time-aggregation of data) has been selected so that it yields significant changes in the number of bicycles at stations and provides an adequate response time for the repositioning operations. A one-hour time-step was selected, although for very low demand stations, the time-step was extended to three hours to increase the significance of the results.

Results indicate that differences are small between the accuracy of the calibrated algorithms. In such context, the simple Random Forest method is an advisable option when a quick simple prediction is required. Having said that, Neural Networks use a Bayesian approach and it is the only of the three methods analyzed which is able to provide confidence intervals on the prediction. If this is a requirement in the application of the method (i.e. in the repositioning optimization model considered), then NN is the only feasible option.

The accuracy obtained for the predicted usage of bike-sharing stations with hourly time-steps is below 60%. Extending the time-step up to 6h can improve the accuracy to approximately 70%. Similar accuracy improvements could be obtained by predicting the aggregate usage of clusters of 8-10 nearby stations. Such temporal or spatial aggregated predictions would still fulfill the requirements for being feed to most of the dynamic rebalancing optimization algorithms. If static rebalancing at night hours is the objective, daily predictions would suffice. In such case, prediction accuracy might reach 80%.

Improvements of the regression methods used could include a more adequate definition of the calendar variables (e.g. position of a working day between holidays could be used instead of day of the week), or a more refined discretization of the continuous variables. Also, more specific neural networks (e.g. Long Short-Term Memory NN, Convolutional NN, Residual NN, Transformer NN, or graph-based NN) could be applied and analyzed. Finally, the effect of the amount of training data could also be analyzed. In the present case study, a whole year of data has been used to predict the next coming 15 days. It is possible that with much less data the accuracy would have not been affected.

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Paper V

Agent-based simulation of mixed vehicle-sharing systems

Submitted to *Journal of Simulation*, 2023

Agent-based simulation of vehicle-sharing systems

Enrique Jiménez and Francesc Soriguera

Abstract— Agent-based simulation is a powerful tool that allows emulating the complex behavior of individuals within a system in order to assess its global performance. Agent-based simulation has been applied successfully to address the design and operation of vehicle-sharing systems, in which the individual behavior of users might be difficult to model in aggregated terms. In spite of this, to the authors' knowledge, there is no specific simulation tool suiting the main needs of researchers and practitioners in the field of vehicle-sharing systems. This paper addresses this gap by developing a modular agent-based simulation framework, which includes the required fundamental capabilities for being used in any vehicle-sharing simulation. The modular structure of the simulator implies that each aspect is treated separately, so that each module can be replaced or modified without affecting the others. In this way, the simulator can be used to analyze specific problems such as the optimal location of stations, the design of repositioning strategies, or different vehicle usage policies, by focusing only on the relevant module. The simulation framework proposed has been implemented and tested in the analysis of two different systems: a station-based bike-sharing system and a mixed (i.e. station-based and free-floating) car-sharing initiative. The paper shows some of the results obtained from such implementations as an example of its possible applications.

Keywords— vehicle-sharing, bicycle-sharing, car-sharing, agent-based simulation (ABS).

1. Introduction

One-way vehicle-sharing mobility services allow users to pick-up one vehicle (e.g. a car, bike, motorbike, or scooter) in one location and return it in another location within a predefined service region. The popularity of such services has been continuously increasing over the last two decades, which has stimulated financial investing and new implementations worldwide, yielding scientific research thereafter. The main topics of interest in the scientific arena have been related to the optimization of the strategic design variables of the system (i.e. deciding on free-floating or station-based configurations; determining the optimal number of stations and the optimal vehicle fleet size, etc.). Also, the definition of vehicle repositioning algorithms, the estimation of attracted demand (either from a modal shift or induced demand), or the definition of pricing strategies, have received some amount of attention. These problems have been mainly addressed from a macroscopic perspective, through mathematical modelling or numerical optimization. We deem macroscopic models as those modelling approaches where the main trade-offs of the system are synthesized, allowing to obtain a first approach solution for strategical problems without getting blurred by the overwhelming details of operation. However, this type of models requires strong assumptions and the simplification of details. In contrast, the performance of vehicle-sharing systems at the operative level can be highly influenced by local effects or singular user decisions. Therefore, it is also important to complement macroscopic models with more detailed modelling or simulation approaches. Take as an example the stations' location problem. Typically, the macroscopic approach assumes a service area characterized only by its extension (i.e. [km^2]) and a uniform origin / destination demand characterized only by its density (i.e. [$trips/h \cdot km^2$]). In such context, the macroscopic solution would yield the optimal number of stations for the system. However, the inputs, and the model itself, lack the required level of detail in order to determine the stations' optimal locations. Such modeling "details" may include the specific candidate locations for the stations, the availability and the restrictions of public space, and the particular origin / destination locations of all the potential trips.

Trial and error are not rare in the implementation of vehicle-sharing systems. However, real-life tests require a lot of time and money. Moreover, the price to pay for a failure is high. In contrast, simulation represents an adequate tool that can be used to replicate any system, prior to implementation, in order to minimize the possibility of failures. In particular, agent-based simulation (ABS), is suitable to emulate vehicle-sharing systems in order to obtain results with a level of detail similar to that of experimental data from systems already in operation. In the ABS framework, each element of the system is an agent, with an autonomous individual behavior. This is especially useful in the simulation of vehicle-sharing systems, in which simple behaviors of individual agents generate complex and interrelated behavior of the overall subsystems. For instance, when users do not find an available vehicle or parking spot, they can take different decisions (e.g. to cancel the trip, to look for a different station, or to wait) depending on different circumstances, preferences or perceived benefit. The behavior of users in this context is governed by decision-making heuristics with a stochastic component. In turn, the users' autonomous decisions influence the accessibility and availability of vehicles within the service region, which needs to be rebalanced by repositioning teams (i.e. another type of agents in the simulation). Repositioning teams observe the status of the system and communicate each other in order to select the best task to be performed from a system wide perspective. Such task assignment process may experience a "learning" process and the adaptation to the particular distribution of vehicles over the service region. In this context, ABS is able to model this autonomous behavior of agents and their interrelationships obtaining overall performance results which are useful to support the planning process.

This paper describes a multi-purpose ABS tool, aiming to address a wide range of vehicle-sharing related problems by using simulation. In order to ease its applicability, the main characteristics of the simulator are the following:

- The ABS architecture is built in a modular fashion. This means that the different simulation procedures are defined as independent modules, with one or several internal functions. In this way, modules can be modified by adding, replacing, or improving such functions, without affecting the rest of the simulation processes. This is especially useful in the simulation setup and optimization processes, because it allows researchers and practitioners to add their own optimization models and methodologies. Examples of the different modules are i) the station location module, ii) the demand forecast module, or iii) the repositioning tasks assignment process.
- The ABS framework is valid for any kind of vehicle. Definition of agents are common for all types of vehicles (i.e. bike, car, scooter, motorbike) which can be analyzed indistinctly by just changing their related parameters.
- The ABS framework allows considering either station-based (i.e. where trips are from station to station), free-floating (i.e. where vehicles are parked on-street), or mixed systems (i.e. in which vehicles can be returned either on-street or at stations depending on the parking availability or existing regulations in the city).

The present paper describes in detail the agents, elements, inputs, and modules of the simulator with the purpose of allowing its replicability and adaptability. In addition, the paper describes the implementation of a new real-time reactive repositioning methodology, which is implemented as the default strategy in the corresponding simulator module.

The rest of the paper is structured as follows. Section 2 reviews the scientific state-of-the-art, analyzing those research works where vehicle-sharing problems are addressed through simulation. Sections 3 and 4 develop the simulator framework. Specifically, Section 3 introduces and defines the simulator agents and their features, and Section 4 develops the architecture of the simulator by explaining and detailing its modules, including all the actions that agents can perform. Next, Section 5 summarizes all the simulator inputs. Section 6 presents an overview of two different cases of study, corresponding to a station-based bike-sharing system and to a mixed car-sharing initiative. In both cases, the ABS is coded using the MATLAB R2021a coding platform. Finally, Section 7 analyzes the generalizability and scalability of the simulator. The paper ends with the conclusions, acknowledgements, and references sections.

2. Literature review

Vehicle-sharing systems are defined by four main elements: vehicles, stations, users, and repositioning staff (aka. repositioning teams). Users and repositioning teams are active elements, meaning that they are able to take decisions and perform actions. In contrast, vehicles and stations are passive elements. Passive elements do not perform actions, and their status is changed through the actions of the active elements.

Agent-based simulation is a suitable tool to model systems involving active elements with the highest level of detail. In spite of this, the status of the system might be determined only by that of passive elements (i.e. the status of vehicles and stations). If this was the only thing of interest in the simulation, users' demand and vehicle rebalancing could be modelled more simply as discrete events. Discrete event simulation would reduce the complexity of the model, but would miss the details of the behavior of the active elements. For example, if demand is modelled as a series of discrete events (i.e. requests and returns at stations), it would not be possible to determine the detailed metrics of the users, such as the increase of the walking access distance to secondary stations when vehicles are unavailable. Both simulation approaches (i.e. agent-based or event-based) are used in the related literature, depending on the level of detail desired to address their specific objectives. For instance, if the objective was only to evaluate rebalancing algorithms, users' details could be irrelevant and ignored. Table 1 summarizes the simulation alternatives chosen in the relevant related literature, and the characteristics of the system addressed.

Table 1. Summary of the relevant scientific literature

Reference	System*	Vehicle	e-vehicles	Users simulation	Repositioning simulation	Fleet	Stations	Demand [trips/day]
Alfian et al. (2014)	SB	Car	No	discrete-event	discrete-event	100	5	2000
Angelelli et al. (2020)	SB	Bike	No	discrete-event	agent-based	450	86	3000
Balac et al. (2017)	FF	Car	No	agent-based	N/A	7650	N/A	54070
Boyacı et al. (2017)	SB	Car	No	discrete-event	discrete-event	66	66	400
Boyacı et al. (2019)	SB	Car	No	discrete-event	discrete-event	60	60	500
Caggiani & Ottomanelli (2013)	SB	Bike	No	discrete-event	discrete-event	50	5	780
Caggiani et al. (2018)	FF	Bike	No	discrete-event	discrete-event	200	36	2100
Ciari et al. (2016)	SB/FF	Car	No	agent-based	discrete-event	N/A	N/A	N/A
Clemente et al. (2013)	SB	Car	Yes	agent-based	N/A	10	5	240
Fernandez et al. (2020)	SB	Bike	No	agent-based	discrete-event	1800	174	12296
Herrmann et al. (2014)	FF	Car	No	discrete-event	discrete-event	N/A	N/A	N/A
Huang et al. (2018)	SB	Car	No	discrete-event	discrete-event	500	354	2376
Huang et al. (2020)	FF	Car	No	discrete-event	discrete-event	8000	54	8000
Ji et al. (2014)	SB	Bike	Yes	agent-based	N/A	10	1	20
Jimenez & Soriguera (2023b)**	SB+FF	Car/Bike	Yes	agent-based	agent-based	4838	347	17369
Jin et al. (2022)	SB	Bike	No	agent-based	agent-based	19506	858	53425
Jorge et al. (2014)	SB	Car	No	discrete-event	discrete-event	390	69	1777
Kek et al. (2009)	SB	Car	No	discrete-event	discrete-event	N/A	12	41
Jian et al. (2016)	SB	Bike	No	discrete-event	discrete-event	9111	699	47100
Kuwahara et al. (2021)	SB	Car	Yes	agent-based	N/A	100	85	N/A
Lopes et al. (2014)	SB	Car	No	agent-based	agent-based	2000	181	29000
Nair & Miller-Hooks (2011)	SB	Car	No	discrete-event	discrete-event	94	14	43
Nourinejad et al. (2015)	FF	Car	No	discrete-event	discrete-event	250	24	200
Pfrommer et al. (2014)	SB	Bike	No	discrete-event	discrete-event	3708	354	14639
Romero et al. (2012)	SB	Bike	No	discrete-event	N/A	N/A	N/A	N/A
Yoon et al. (2019)	SB	Car	Yes	discrete-event	discrete-event	N/A	N/A	N/A
Wang et al. (2010)	SB	Car	No	discrete-event	discrete-event	48	4	2000

* SB = station-based; FF = free-floating. ** This refers to the present paper. Note that it is the only reference addressing mixed vehicle-sharing systems, where the FF and SB systems perform simultaneously.

Some of the works in Table 1 aim to develop a multi-purpose simulation framework, while others just use simulation as an auxiliary method to solve a particular issue. In the first group, the works of Lopes et al. (2014) for station-based car-sharing systems, and Fernandez et al (2020) and Angelelli et al. (2022) for station-based bike-sharing systems are remarkable. These works represent excellent starting points for the present research, since they describe in detail the elements of the proposed simulator. In particular, Lopes et al. (2014) develops a complete simulation framework for station-based car-sharing system, including users, repositioning activities, and maintenance activities. Moreover, their simulation framework includes setup modules for stations, vehicles, and O/D demand. In turn, Fernandez et al (2020) focuses on modelling the users' behavior, and develops an event-based simulator in which events correspond to the milestones in the users' life cycle. Different user types are defined, with different behavior (i.e. checking the app information or not, accepting the system recommendations or not) which are randomly generated. The model is applied to the simulation of the station-based bike-sharing system of Madrid. In this work, however, repositioning teams are not modelled as agents. Repositioning operations are considered as system inputs, with little margin to interact with the rest of the simulation or to test different repositioning algorithms. In contrast, the work of Angelelli et al. (2022) specifically focuses on the modelling of repositioning agents. Authors develop a complex algorithm in which repositioning teams decide which operations are necessary at each station based on the forecasted vehicles' inventory level. In this work, users are not defined as agents, and vehicle requests and returns at stations are generated as individual stochastic processes. In the present paper, user behavior has been modelled similarly to Fernandez et al (2020), and the idea of user life events was adapted to define user timers. Regarding the modeling of the repositioning tasks, it is based on the work by Jiménez & Soriguera (2023a), adapting the principles presented in Angelelli et al. (2022), in which teams reevaluate tasks in real-time, according to the inventory level forecasts.

Besides the previous multi-purpose simulation environments, other works use microsimulation as an auxiliary tool to address a specific problem. This is common in the literature, where the vast majority of these contributions address the optimization of repositioning tasks. This problem consists in determining which artificial movements are required in order to improve the performance of the system given a limited number of resources. Typically, the problem is addressed by estimating demand forecasts in order to plan repositioning movements in advance. In the context of bicycle sharing systems, Kek et al. (2009), Caggiani & Ottomanelli (2013), Caggiani et al. (2018), Jian et al. (2016), and Jin et al. (2022) are examples of the use of microsimulation to predict the number of requests and returns of bicycles at stations (or subzones of the whole service area) in order to optimize the repositioning operations. In turn, Wang et al. (2010), Nair & Miller-Hooks (2011), Clemente et al. (2013), Jorge et al. (2014), Nourinejad et al. (2015), and Boyacı et al. (2017, 2019) are examples of the same problem and similar methodologies, but applied to car-sharing systems. Note that in free-floating systems (where vehicles are not parked in stations, but they can be parked freely on-street over the service area), it is necessary to define subzones as singular elements which will act as virtual stations. In this regard, it is relevant the work by Caggiani et al. (2018), dealing with a free-floating bike-sharing system and including a clustering method to divide the service area in subzones. The work of Caggiani et al. (2018) is singular in the sense that clusters are redefined several times during the repositioning optimization process to better adapt to the repositioning needs. In general, and also in the present work, subzones are defined as static elements.

Simulation based approaches have also been used to address the strategical design of vehicle-sharing systems. Take as an example the works by Romero et al. (2012), and Huang et al. (2018, 2020), which develop a methodology to determine the fleet size, the number of repositioning personnel and the number of stations of bike-sharing and car-sharing systems, respectively. Or the works of Alfian et al. (2014), Ciari et al. (2014), and Herrmann et al. (2014) who assess the suitability of round-trip versus one-way services or free-floating versus station-based car-sharing configurations, by making decisions according to their simulation results. In turn, Martinez et al. (2012) focus on the stations' location optimization problem in bike-sharing systems, while Ji et al. (2014) evaluate, also using simulation, the convenience of installing battery recharging power banks for electric bicycles at stations and according to the expected needs. An analogous problem is addressed in Kuwahara et al. (2021), but in this case for car-sharing systems. As it can be seen, the recharging of electric vehicles is also a concerning issue commonly addressed by using simulation approaches.

The analysis of the demand attracted by vehicle-sharing systems has also been analyzed based on simulated environments. Simulation allows estimating the overall service provided, as in Ciari et al. (2016) and Yoon et al. (2019), or to evaluate specific design, parking, or pricing policies, as in Balac et al. (2017), Pfrommer et al. (2014), and Ciari et al. (2015), in all cases for car-sharing systems. This multimodal simulation allows evaluating the interaction between vehicle-sharing and alternative transportation modes and apply discrete mode choice models thereafter.

Despite the many differences in all the cited research works, some commonalities are found between them. First, almost all researchers rely and develop their own standalone simulators. The lone exceptions are the works of Wang et al. (2010) who use a commercial traffic simulator as the platform from where to simulate car-sharing trips, and Ciari et al. (2016), who build their model by using MATSim, an open-source framework for implementing large-scale agent-based transport simulations (MATSim, 2023). The main reason for this situation is that current commercial simulators are not well suited to address problems related to vehicle-sharing systems. This requires an important adaptation in the definition of processes and agents, which usually is seen as more challenging than creating an ad-hoc simulation environment from scratch. Commercial simulators have been only used when the required features are already implemented (i.e. routing, modal share) or include data sources that provide a bigger benefit.

Second, all works which develop ABS for vehicle-sharing systems do consider the same agents. These are users and repositioning teams as active agents, and vehicles and stations (or subzones) as passive agents. The only exception is the aforementioned work of Lopes et al. (2014), where repositioning and maintenance actions are differentiated and performed by two different types of agents. Recall that if the behavior of active agents is not simulated (i.e. users and repositioning are treated simply as discrete events), the framework becomes instead a discrete event simulation.

The third and last commonality shared by most of the previous works has to do with the user generation. In most of the vehicle-sharing ABS, user generation is modelled as a non-homogeneous Poisson process. This same approach has been used in the present simulation framework.

The present paper, builds on these many examples of simulation approaches to vehicle-sharing systems, by proposing an ABS framework sharing the previous common attributes, in a modular and flexible simulation environment. For instance, to the authors knowledge, this is the first approach where mixed vehicle-sharing systems can be simulated, including a free-floating (FF) and station-based (SB) layouts performing simultaneously. Also, the simulation is not captive of a particular vehicle type, allowing the simulation of any kind of vehicle-sharing system (e.g. car-, bike-, scooter-sharing; possibility of electric vehicles). Different rebalancing strategies and algorithms can be easily implemented and tested in the proposed environment, where the level of service offered to users is tracked from different metrics. In conclusion, being a global simulator including all the sub-systems of vehicle-sharing systems, with enough flexibility to adapt to different geographic contexts, vehicle types, trip demands, operative layouts, or rebalancing algorithms, makes the proposed simulator an attractive tool for operating agencies and planners. The present paper addresses this research objective using an agent-based simulation framework, yielding the maximum level of detail for all the elements in the system.

3. Agents in the simulation

The present agent-based simulation framework uses five basic types of agents: users, vehicles (i.e. bikes, cars, or others), stations, free-floating zones, and repositioning teams. Each agent type has its own properties, possible actions, and characteristics. In this section, these agents are described and their properties are listed. Note, however, that to fully understand some of the properties it is needed to go through the description of the simulator modules, which will be presented in the coming sections. The properties of each agent include their coding name, the type of variable that they depict (i.e. ID, number, object, category, character) and the type of organizer used (i.e. single value, string, time array, list, array x list). These organization methods are defined as follows:

- Single: Only one numerical value.
- String: String of characters.
- Time array: One numerical value every time step, including the initial conditions at $t = 0$ and covering the whole simulation cycle. For instance, if the time step is set to 1 minute and the simulation cycle consist of 24 hours, the array will contain 24x60+1=1441 values. The +1 corresponding to the initial conditions.
- List: Several values of the same type (e.g. IDs of the vehicles inside a station). The size of lists is variable.
- Array x Lists: One list every time step, including $t = 0$ and covering the whole simulation cycle.

3.1. Stations

Stations are passive agents which do not perform actions. However, their properties are constantly being queried by other active agents in order to perform their own actions. For example, users (i.e. an active agent) check the vehicle list of the stations when searching for an available vehicle; or repositioning teams (i.e. another active agent) check the charger list of the stations when looking for a recharging dock for electric vehicles.

3.2. Free-floating zones

Free-floating zones play the role of virtual stations in free-floating systems. Therefore, most of their properties are common with stations. Note, however, that the default value for the property "numChargers" is null as it is assumed that there are not charging facilities on streets. In addition, free-floating zones have additional lists determining the number of stations contained in the zone and in its neighboring zones. These lists are used to speed-up the users' vehicle search process in the simulation by only considering vehicles and stations in the current zone and its neighbors. This reduces the computational time, especially in case of big vehicle fleets.

Table 2. Properties of the agent "station".

Property	Type	Organizer	Description
ID	ID (station)	Single	Station identifier
Name	Character	String	Name of the station (Optional)
X	Real	Single	X coordinate in UTM
Y	Real	Single	Y coordinate in UTM
Z	Real	Single	Z coordinate in UTM
zoneID	ID (zone)	Single	Identifier of the zone where the station belongs
capacity	Integer	Single	Maximum number of parking slots in the station
numChargers	Integer	Single	Number of electric chargers in the station
nearestCharger	ID (station)	Single	ID of the closest station with chargers
numVehs	Integer	Single	Current number of mechanical (non-electric) vehicles in the station
numEvehs	Integer	Single	Current number of electric vehicles in the station
accRequests	Real	Array	Predicted accumulated number of vehicle requests over time (not needed to be integer)
accReturns	Real	Array	Predicted accumulated number of vehicle returns over time (not needed to be integer)
optVehs	Integer	Array	Array of optimum number of vehicles in the station over time
listVehs	ID (vehicle)	List	List of vehicle IDs currently in the station
listCharging	ID (vehicle)	List	List of electric vehicle IDs currently connected to battery chargers in the station
vlistVehs	ID (vehicle)	Array x Lists	Array of vehicle lists IDs over time in the station
vlistCharging	ID (vehicle)	Array x Lists	Array of vehicle charging list over time in the station

Table 3. Properties of the agent "free-floating zone".

Property	Type	Organizer	Description
ID	ID (zone)	Single	Free floating zone identifier
X	Real	Single	X coordinate in UTM (centroid)
Y	Real	Single	Y coordinate in UTM (centroid)
Z	Real	Single	Z coordinate in UTM (centroid)
zoneArea	Real	Single	Area of the zone in km^2
capacity	Integer	Single	Maximum number of vehicles that can park in the zone (default value = 9999)
numChargers	Integer	Single	Number of electric chargers in the zone (default value = 0)
nearestCharger	ID (station)	Single	ID of the closest station with chargers
numStations	Integer	Single	Number of stations inside the zone
listStations	ID (station)	List	List of station IDs inside the zone
numStations_neig	Integer	Single	Number of stations inside neighboring zones
listStations_neig	ID (station)	List	List of station IDs inside neighboring zones
accRequests	Real	Array	Predicted accumulated number of vehicle requests over time (not needed to be integer)
accReturns	Real	Array	Predicted accumulated number of vehicle returns over time (not needed to be integer)
optVehs	Integer	Array	Array of optimum number of vehicles in the zone over time
listVehs	ID (vehicle)	List	List of vehicle IDs currently in the zone
vlistVehs	ID (vehicle)	Array x Lists	Array of vehicle lists IDs over time in the zone

3.3. Vehicles

Like stations and free-floating zones, vehicles are passive agents. However, they are mobile objects and can change its position over time. Vehicles do not perform actions by themselves, except reducing and increasing their battery levels if they are in use or in a charging list.

Table 4. Properties of the agent "vehicle".

Property	Type	Organizer	Description
ID	ID (vehicle)	Single	Vehicle identifier
X	Real	Array	Array of X coordinates in UTM over time
Y	Real	Array	Array of Y coordinates in UTM over time
Z	Real	Array	Array of Z coordinates in UTM over time
status	Category	Array	Array of vehicle status over time (0: idle, 1: reserved, 2: on-trip, 3: repositioning, 4: not enough battery)
isElectric	Category	Single	Determines if the vehicle is electric or not. (0: not electric, 1: electric)
batteryLevel	Real	Array	Array of battery levels (in percentage) over time

3.4. Users

Users are active agents. They are created randomly at every time step according to the input O/D demand matrices. On creation, each user tries to reserve a vehicle for his trip. If the reservation is made, a set of trip timers are calculated. These timers define when the user will perform the next action (i.e. pick the vehicle, park the vehicle). If the vehicle reservation cannot be made due to vehicle unavailability near the user's location, the user "dies" (i.e. it becomes inactive) and it is stored separately as demand lost.

Table 5. Properties of the agent "user".

Property	Type	Organizer	Description
X	Real	Single	Current X coordinate in UTM
Y	Real	Single	Current Y coordinate in UTM
Z	Real	Single	Current Z coordinate in UTM
XO	Real	Single	Origin of the trip X coordinate in UTM
YO	Real	Single	Origin of the trip Y coordinate in UTM
ZO	Real	Single	Origin of the trip Z coordinate in UTM
ZoneO	ID (zone)	Single	ID of the origin zone
VehZoneO	ID (zone)	Single	ID of the zone where it is located the reserved vehicle. This may be different from the origin zone
XD	Real	Single	Destination of the trip X coordinate in UTM
YD	Real	Single	Destination of the trip Y coordinate in UTM
ZD	Real	Single	Destination of the trip Z coordinate in UTM
ZoneD	ID (zone)	Single	ID of the destination zone
VehZoneD	ID (zone)	Single	ID of the zone where the vehicle will be returned. This may be different from the destination zone
VehID	ID (vehicle)	Single	ID of the reserved vehicle
StatO	ID (station)	Single	ID of the station where the reserved vehicle is located. If the reserved vehicle is on-street, then StatO = 0, and the previous VehZoneO is used instead.
StatD_min	ID (station)	Single	ID of the station where the vehicle is expected to be returned. If the vehicle is expected to be returned on-street, then StatD = 0, and VehZoneD is used instead.
StatD	ID (station)	Single	ID of the station where the vehicle was actually returned. It may be different from StatD_min, if the original selected station had no available parking spots upon arrival.
Xpark	Real	Single	X coordinate in UTM of the parking location if parked on-street.
Ypark	Real	Single	Y coordinate in UTM of the parking location if parked on-street.
tCreation	Integer	Single	Time step when the user was created
tO2Veh	Integer	Single	Time step when the user reaches the reserved vehicle at the origin of the trip.
tTrip	Integer	Single	Time step when the trip with the vehicle is finished and the vehicle has been parked and returned.
tVeh2D	Integer	Single	Time step when the user arrives to its final destination.
tParkAdd	Integer	Single	Additional trip time caused by the lack of available parking at destination.

3.5. Repositioning teams

Repositioning teams are active agents. They are created at the beginning of the simulation and they perform repositioning tasks. Tasks are defined in their basic properties by several lists. When repositioning tasks are assigned to them, some timers are created, setting when the task will be completed and triggering the next actions.

Table 6. Properties of the agent "repositioning team".

Property	Type	Organizer	Description
ID	ID (team)	Single	Repositioning team identifier
X	Real	Array	Array of X coordinates in UTM over time
Y	Real	Array	Array of Y coordinates in UTM over time
Z	Real	Array	Array of Z coordinates in UTM over time
status	Category	Array	Array of repositioning time status over time (0: idle, 1: moving on scooter -only car-sharing systems-, 2: carrying vehicles to a new location)
vehicles	ID (vehicle)	Array x Lists	Array of the vehicles' ID repositioned over time
capacity	Integer	Single	Maximum number of vehicles that the team can carry simultaneously.
taskStat	Category	List	List showing if the sequence of tasks performed by the vehicle took place at a station or at a free-floating zone (1: station, 2: free-floating zone)
taskList	ID (station/zone)	List	List showing the sequence of the ID of the station/zone where the tasks took place
taskType	Category	List	List showing the sequence of reasons for repositioning the vehicles (1: recharging, 2: relocation)
taskMovements	Integer	List	List showing the number of vehicles to move in each task. (>0: deploy vehicles; <0: take vehicles)
taskUtility	Real	List	List showing the utility of each task performed by the repositioning team
taskTime	Real	List	List showing the tasks ending times
taskCurrent	Integer	Single	Index of current task in the previous lists
vehID	ID (vehicle)	List	IDs of vehicles currently being repositioned by the repositioning team (for car-sharing systems the size of this list is one, as only one car can be repositioned simultaneously by a repositioning team.

3.6. Auxiliary object types

In addition to the five basic agent types described in the previous sections, the simulator includes two additional auxiliary objects: Tasks and City. Tasks are auxiliary objects which are created to ease the repositioning task assignment process. Potential tasks are created when the task assignment optimization process is run and deleted once the optimal task is assigned and stored as a property of the repositioning team. Potential tasks ease the process of classifying the tasks in several groups according to their priorities. Once one task is assigned to a repositioning team, its properties are added to its corresponding lists.

Table 7. Properties of the object "task".

Property	Type	Organizer	Description
taskID	ID (station/zone)	Single	ID of the station/zone where the task needs to place
X	Real	Single	X coordinate in UTM where the task needs to place
Y	Real	Single	Y coordinate in UTM where the task needs to place
Z	Real	Single	Z coordinate in UTM where the task needs to place
taskType	Category	Single	Reason for repositioning the vehicles (1: recharging, 2: relocation)
taskMovements	Integer	Single	Number of vehicles to move in the task. (>0: deploy vehicles; <0: take vehicles)
taskUtility	Real	Single	Utility of the task
taskTime	Real	Single	Task duration

In turn, City is an object which acts as a global container, and stores and organizes all the elements of the simulation, including its agents, auxiliary objects, inputs, and other elements such as the geometry of the service region and the O/D demand matrices. City is a special object which can contain other object types (e.g agents) in its lists.

Table 8. Properties of the object "city".

Property	Type	Organizer	Description
servArea	Object	List	Zonification of the service area defined as a list of polygons. Each polygon represents a single free-floating zone.
zoneNum	Integer	List	List containing the IDs of the zones composing the service area. It is used for a quicker search.
OD	Object	List	List containing the OD demand matrices. Demand is an input to the simulator. Each matrix holds for the same predefined time, which is input as a parameter of the simulation.
output	Object	Single	Postprocessing object. Defines the outputs to be obtained from the simulation.
numStations	Integer	Single	Total number of stations in the system.
vStations	Object	List	List containing all the stations in the system.
numFreeFloatZones	Integer	Single	Total number of free-floating zones in the system.
vFreeFloatZones	Object	List	List containing all free-floating zones in the system.
numCars	Integer	Single	Total number of cars in the system.
vCars	Object	List	List containing all the vehicles in the system.
minBatteryLevel	Real	Single	Minimum percentage of the battery level for electrical vehicles being set as available. Any vehicle with a battery level below this threshold will be set to the status "not enough battery" and will be unavailable for user reservations.
numRepoTeams	Integer	Single	Total number of repositioning teams in the system.
vRepoTeams	Object	List	List containing all the repositioning teams in the system.
numUsers	Integer	Single	Total number of active users in the system.
vUsers	Object	List	List containing all the active users in the system.
numFinishedUsers	Integer	Single	Total number of users who have finished their trips during the simulation.
vFinishedUsers	Object	List	List containing all the users who have finished their trips during the simulation.
vUnervicedUsers	Object	List	List containing all the not serviced users during the simulation.
vUsersGen	Object	List	List containing all the users generated.
usr_timer	Integer	Single	Global time counter for the whole simulation (including all warm-up cycles). It is used to record the time users were generated, or create to them when they come from a predefined list.

4. Architecture of the simulator

The proposed simulator is built in a modular fashion. Each process is defined by an independent module, which contains one or several internal functions and methods. Modules are classified in three groups according to when and how often they are run. These are “Setup” modules, “Time-step” modules and “Custom frequency” modules. Modules of the simulator are summarized in Table 9 and will be described in the next sections.

Time in the simulator is discretized in time steps. Processes and actions are performed at the start on any time step. The default value for the time step duration is set to 1 minute, which has been proven to provide enough detail in the results without incurring in excessive computational cost. In addition, the total simulation time can be divided in cycles of any duration (e.g. daily cycles). Cycles are repeated several times in order to produce as many warming up periods as desired. Results will only be stored during the last cycle of the simulation.

Table 9. Classification of the modules of the simulator

Setup modules	Time-step modules	Custom frequency modules
Zonification setup	User creation and vehicle assignment	Demand forecasting
O/D demand matrix setup	User movement	Optimal vehicle distribution
Stations’ location setup	Repositioning task assignment	Initialization tasks
Vehicle fleet setup	Movement of repositioning teams	
Repositioning teams’ setup	Electric vehicle recharging	

4.1. Setup modules

Setup modules include all the operations necessary in order to define and create the fixed elements of the simulation (i.e. service area, stations, demand input, vehicles, and repositioning teams). They are run only once before the simulation starts, and require the inputs defining the strategical variables of the vehicle-sharing system. In addition, most of the modules include several options in order to generate these inputs if they are not available. Finally, the setup modules return their results as readable outputs in order to replicate the same layout and demand in different simulation scenarios.

4.1.1. Zonification and O/D demand matrix setup

Zonification and O/D demand matrix are two setup modules with a strong relationship. These modules create a zonification of the service area and the O/D demand input for the simulator. The O/D matrix is a square matrix whose size is the number of zones in the zonification. The zonification module defines the agents called “free-floating zones” which are used for the management of the free-floating vehicle fleet (i.e. virtual stations). Although, the zonification for the user generation (i.e. O/D demand) and that of virtual stations for management purposes could be different, a single zonification is considered for the sake of simplicity.

The analyst can input the zonification and the O/D demand matrix in two ways. First, directly from input files, if available. Note that O/D demand input files can only be introduced if the corresponding zonification file is also available. Alternatively, the operator can introduce some aggregated parameters of the service region and of the system demand and the simulator will generate the zonification and the O/D matrix.

Zonification input from an external file. The analyst introduces a shapefile (i.e. format .shp), which includes all the subzones that make up to the whole service area. It is advisable that subzones are rather homogeneous in its urban characteristics and geometrically “round” (i.e. not elongated) with an approximate average area of w^2, where w is defined as the maximum distance users are willing to walk to access the vehicle. This ensures that all the demand would be served if there is at least one available vehicle in each subzone. Subzones in the shapefile can have a property called SB_PRK, which depicts the fraction of users that will opt for station parking in the subzone, if available. This property is used as a proxy for the lack of on-street parking availability. If there is an O/D matrix available, this is the most advisable case, by using the shape file which matches the O/D zonification.

Custom zonification. The analyst defines the perimeter of the service region by introducing the coordinates of the vertices of a polygonal line. A square grid mesh is then created with elements of size w^2. This zonification can be exported as a shapefile, so that the analyst can make further modifications on it (e.g. edit the SB_PRKvalues).

O/D matrix input from external files. This input process is only available if the zonification was input from an external shape file. If this is the case a set of matrices, in the form of .csv files, can be used as an input. Each matrix holds for the same predefined time, which is introduced as a parameter in the simulation. All the matrices in the set must correspond to the same zone numbering as in the introduced zonification.

Custom O/D matrix. This process requires the input of several aggregated parameters, which define the potential demand. These parameters include the total number of potential trips, N, during the total time of the simulation (i.e. the demand cycle, T), and the spatial imbalance and temporal evolution factors. Temporal evolution is introduced through an array of temporal weights. The number of elements in the array defines the number of equal periods in which the demand cycle is divided. Each element is the relative weight of that period with respect the others in the whole cycle. For example, an array [1;2] means that the demand cycle is divided in two periods, and that in the second period the demand is the double than that of the first one. In turn, spatial imbalance is introduced by the aggregated imbalance parameters and a shape pattern. The aggregated imbalance parameters are three: *i)* The fraction of the service area in which there are more vehicle requests than returns, π_q (i.e. generating areas); *ii)* the fraction of the service area in which there are more vehicle returns than requests, π_t (i.e. attracting areas). Note that the sum of the two previous fractions must be less than or equal to 1; and *iii)* the average fraction of imbalanced trips, Φ, which determines the intensity of the spatial imbalance.

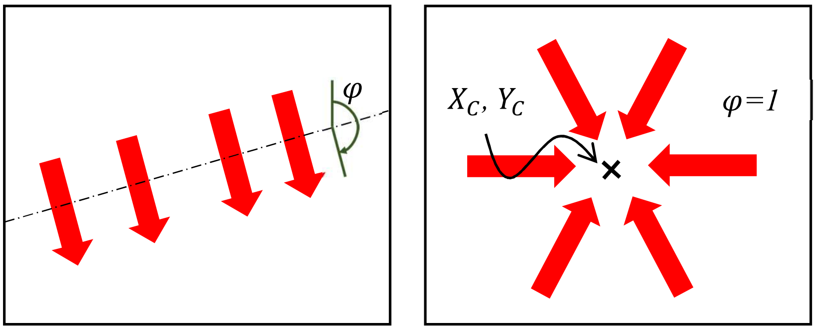

Fig. 1. Example of flat (left) and radial (right, with φ=1) imbalance shape patterns.

In addition, two different imbalance shape patterns are available as defaults: the radial and the flat imbalance patterns (see Fig. 1). On the one hand, in a radial pattern, generation/attraction areas are centripetally defined. On the other hand, in a flat pattern, generation/attraction areas are divided by a linear axis. The flat pattern implies that the imbalance has the same dominant direction over the whole service area. It is assumed that the dominant direction is perpendicular to the imbalance axis, defined as a straight line in the service region where the imbalance is null. If this pattern is selected, the dominant direction must be introduced as the positive angle (φ) created with the north direction. In contrast, the radial imbalance implies that imbalance has a focus, either attractive or generator. It is assumed that the imbalance is reduced linearly as one moves away from the focus. Therefore, the excess of requests around the focus area compensates the excess of returns in the perimeter (or vice versa). If this option is selected, the coordinates of the focus must be given as an input (X_C, Y_C), and the parameter, φ is set to $\varphi = 1$ if the focus is an attractive focus or set to $\varphi = -1$ if it is a generation focus.

4.1.2. Stations' location setup

This module creates the stations of the system. It takes as inputs the number of stations (S) and a list of station candidate locations. Stations' locations in that list must be ordered by the relative priority given to have a station at that point. This list can be empty if the analyst has no predefined locations.

From these inputs, the stations will be created as agents of the system according to the following criteria:

- If the number of stations, S, is smaller than the size of candidate locations, stations will be created at the first S locations.
- If the number of stations, S, is larger than the size of the list of candidate locations, all the stations in the list will be created. In addition, the remaining stations will be created at the centroid of empty subzones (i.e. without any station). Empty subzones will be prioritized to receive a station according to an objective function which aims to maximize the demand covered and to minimize the spatial imbalance created by the additional station. If at some point all subzones have at least one station, and the number of stations is still below S, then new stations will be created at a random point inside the subzone with less stations' density, in order to ensure homogeneity across the service region.

4.1.3. Vehicle fleet and repositioning teams' setup

Although these modules are independent, their options and procedures are analogous. The fleet size and number of repositioning teams are inputs to the simulator. n_{FF}, n_{SB}, n_k are respectively the number of free-floating vehicles, the number of station-based vehicles and the number of repositioning teams to be created in the simulation. The location of vehicles is set according to their optimal spatial distribution, which is determined according to the methodology proposed in Jiménez & Soriguera (2023a). In turn, repositioning teams are initially located at the centroid of the service area. Initially, all vehicles and repositioning teams are considered idle and empty, with full battery. The created agents can be exported in editable files.

Alternatively, it is also possible to create the vehicles and repositioning teams directly from input files (i.e. readable files with the same previous format) which include the following properties:

- Vehicles: location coordinates, battery level.
- Repositioning teams: location coordinates, capacity of the vehicle.

4.2. Time step modules

Time step modules are run at the beginning of every new time step.

4.2.1. User creation and vehicle assignment

Vehicle-sharing users are created every time step according to the O/D demand matrix for the current period. For each O/D pair, a random number of users are created following a Poisson distribution whose mean is the O/D average demand in one time step. Recall that the default simulation time step is of 1 minute.

For each user, the creation process is as follows:

- *Set the origin and destination locations*. Locations are generated as random points inside the origin and destination zones respectively, following a spatial uniform distribution. The user creation time, origin and destination zones, and location coordinates are stored. At the end of the simulation these data will be returned as an output, so that the analyst has the possibility of running different simulations with exactly the same set of generated users.
- *Check availability of vehicles at the origin*. Users will try to find an available vehicle near their origin location according to the algorithm shown in Fig. 2. This process is designed to work on either FF, SB or mixed systems. Note that if the FF or the SB system is missing, the algorithm will not find available vehicles for these options. If vehicles taken from stations are forced to be returned at stations, users will also check if there is any station within the maximum walking distance from their destination. If a vehicle

is assigned, the user becomes active, the vehicle is reserved, and the user location coordinates become the same as the vehicle ones. If not, the user will “die”.

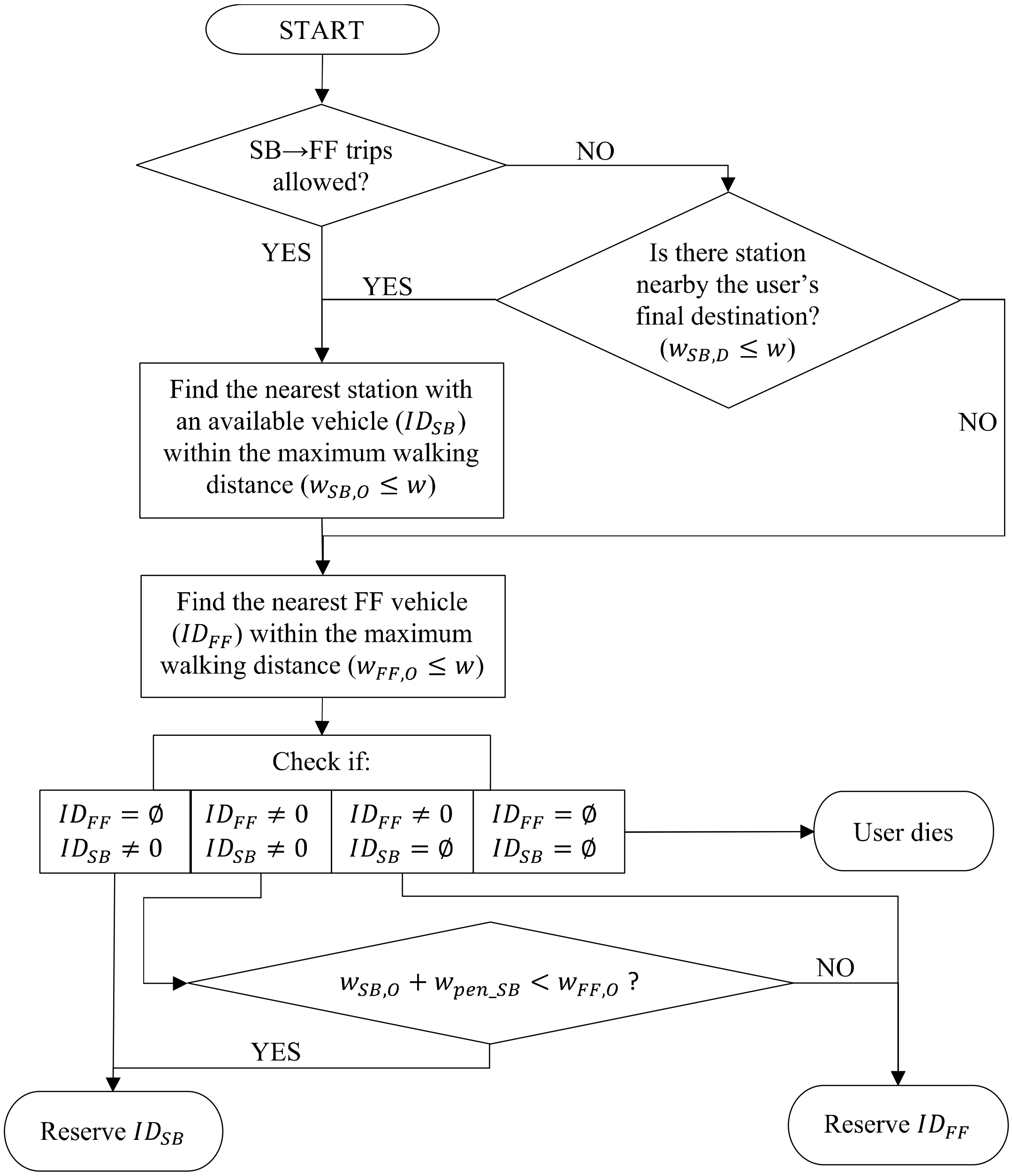

Fig. 2. Vehicle assignment process.

(Notation: $w_{SB,O}$ and $w_{FF,O}$ are the user's distance to the nearest available vehicle at the origin of the trip, in a station or free-floating, respectively. w is the maximum distance users are willing to walk to reach one vehicle. w_{pen_SB} is the access distance penalty if the vehicle is in a station. ID_{FF} and ID_{SB} are the IDs of the nearest available FF or SB vehicles, respectively.

- *Determine the expected parking location at destination*. The location where users expect to park at destination, and the time spent on the parking process, are determined according to the algorithm shown in Fig. 3. The model assumes that, in dense cities where vehicle sharing initiatives are generally deployed, on-street parking is somehow scarce. This might be due to the high on-street parking demand or due to parking regulations in the city. In any case, in general, users will not be able to park just in front of their final destinations. This situation may discourage on-street parking for some users, who will prefer to park at stations, if available. Users who finally park on-street will do so at a random location, different than their final destinations.

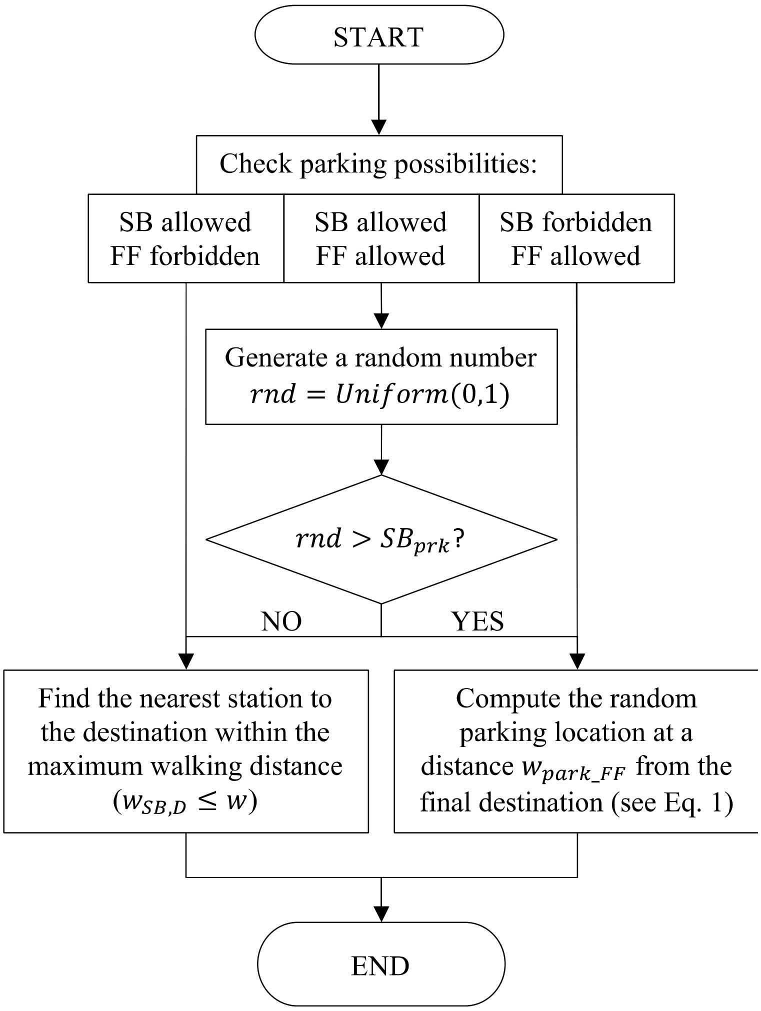

Fig. 3. Expected parking assignment process.

The parameter defining the preference for station-based parking is SB_{prk}, which represents the probability of users opting for parking at a station, if this is an available option at the destination zone. This parameter is a proxy for the difficulty to find on-street parking (i.e. $SB_{prk} = 0$, represents plenty of on-street parking availability, which might be the case for an unrestricted free-floating scooter-sharing system; $SB_{prk} = 1$ means that there is no on-street parking availability at all). According to the value of the SB_{prk} parameter, users who have both parking options will opt randomly for one of them. If user opts for parking at a station, the nearest station to the final destination with available parking slots is selected, and a fixed parking time, t_{park_SB}, is considered. t_{park_SB} is computed as the access penalty for parking at a station (i.e. users' parameter $penSB$ divided by the walking speed, v_w). Alternatively, if user opts for parking on-street, a random location is selected within a distance to the final destination determined by Eq. 1. This distance is never longer than three times the maximum walking distance, w. In turn, the time spent while parking, t_{park_FF}, is also randomly determined according to a negative exponential distribution whose mean is the average on-street parking time in the city weighted by the relative on-street parking availability at the destination zone (see Eq. 2).

$$w_{park\_FF} = min\left\{3w, \frac{w}{1 - SB_{prk}}\right\} \quad (1)$$

$$t_{park\_FF} \sim Exponential\left(\bar{t}_{park\_FF} \cdot \frac{SB_{prk}\ (in\ the\ zone)}{\overline{SB}_{prk}\ (in\ the\ service\ area)}\right) \quad (2)$$

- *Compute the users' timers*. This is to compute the specific time steps when the user will reach the vehicle at the origin of the trip, when she will reach the parking spot, and when she will eventually reach the final destination.
  - *Reaching the car at the origin of the trip*: This timer is computed by dividing the access distance by the users' walking speed, v_w. The access distance is determined between the origin of the user and the location of the assigned vehicle, considering an additional distance penalty, w_{pen} for SB vehicles. Distances in the service area are computed using L1 (i.e. Manhattan metric).
  - *Reaching the parking spot:* This timer is computed by dividing the trip distance between the initial vehicle location and the expected parking location by the vehicle's travelling speed, v, in the urban environment. The additional parking time is added considering a fixed time for parking at stations, and a random time for on-street parking, as described previously.
  - *Reaching the final destination*: This timer is computed by dividing the egress distance by the users' walking speed, v_w. The egress distance is determined between the parking location and the user's final destination, adding the SB penalty if the parking spot in a station.

4.2.2. Users' movement

This module compares the current simulation time step with the different timers of active users, and updates the status of users and assigned vehicles accordingly. It is assumed that users can reserve vehicles at the origin of the trip, but they are not allowed to reserve parking spots at destination. The process is summarized in Fig. 4, and the triggered actions for each step are the following:

- *User arrives to the assigned vehicle*: This happens when the user timer "tO2Veh" is reached. Then, the status of the vehicle changes from "reserved" to "in trip". The vehicle and the user's location coordinates change to "NaN". The vehicle is removed from the station/zone list and from the battery charging list (when applicable).
- *User reaches the parking location*: This happens when the user timer "tTrip" is reached. Then the user checks if there is any available parking spot to complete the return. If there is not, the user will run again the parking assignment process (see the "User creation" module) to find a new parking location with available parking spots. Timers will be updated accordingly. If no potential parking location is available, then the user will wait at the current location. In contrast, if there is an available parking spot at the location, the user returns the vehicle and the vehicle location coordinates change to those of the parking spot. Vehicle status also changes as follows:
  - If the assigned vehicle was electric and the battery level was below the minimum threshold, e_{min}, vehicle status changes to "discharged".
  - If the assigned vehicle is not electric or its battery level is still above e_{min}, vehicle status changes to "available".

- *Reaching the final destination*: This happens when the user timer "tVeh2D" is reached. Then, the user has arrived to his final destination. In such case, the user becomes inactive, and it is stored into the finished users' array.

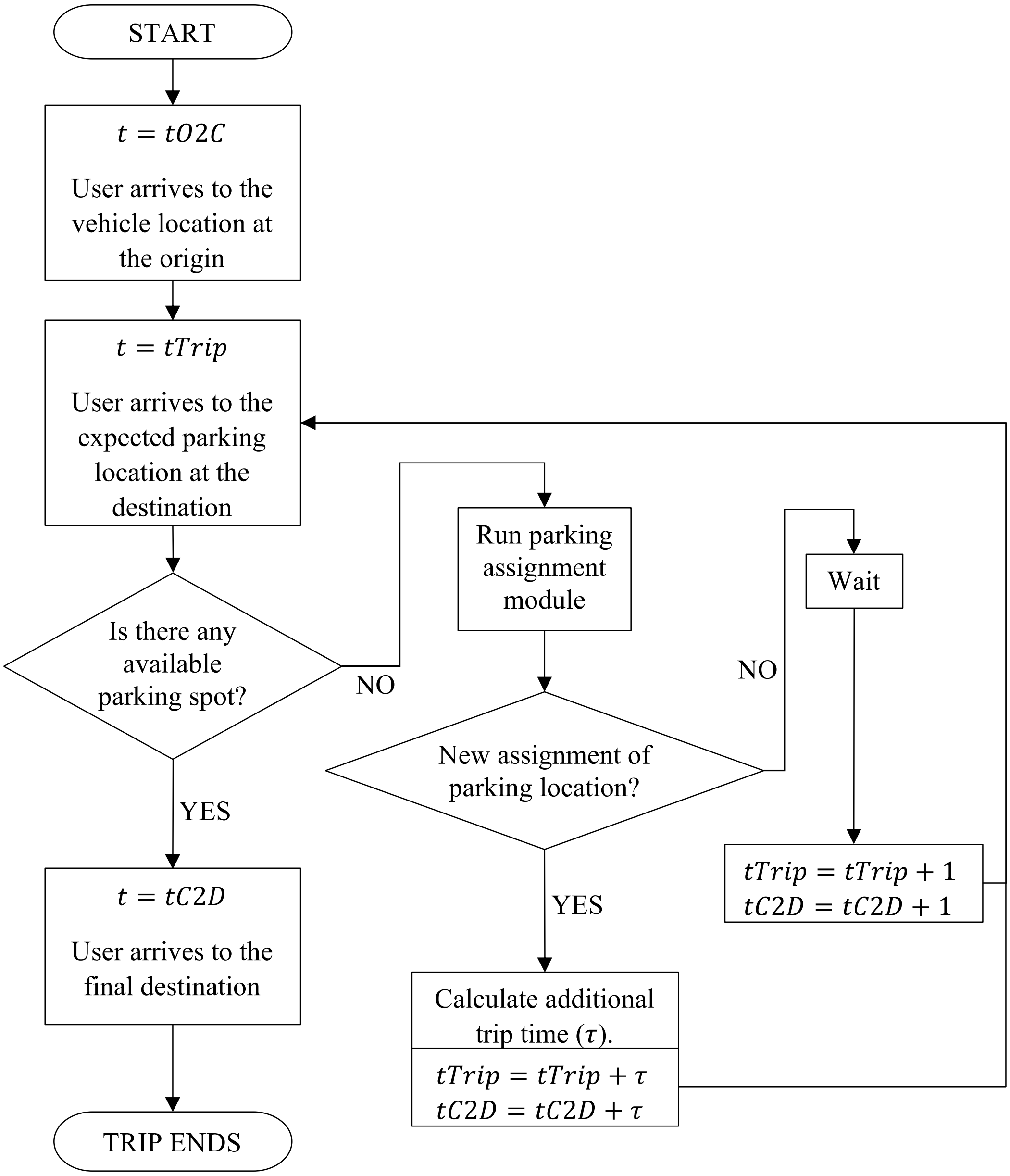

Fig. 4. User movement process for active users.

4.2.3. Repositioning task assignment

This module assigns tasks to idle repositioning teams. Since the optimization process of the task assignment is very dependent on the repositioning strategy considered and on the characteristics of the system under analysis, this module might require significant adaptations for any particular application. Variations may imply different types or repositioning teams (e.g. from a van carrying multiple bicycles in a bike-sharing system, to an employee moving with a scooter to reach the location of the next operation in a car-sharing system), different times of activation (e.g. static repositioning only during off-service periods or continuous dynamic repositioning while in operation; predefined daily task assignment or real-time task assignment), or different assignment strategies (e.g. organize repositioning tasks in different priority blocks).

The default method implemented in the simulator is the dynamic repositioning method based on the optimal real-time pairwise task assignment process described in Jiménez & Soriguera (2023a). This method has been proved to provide good performance with very low computational costs. When a repositioning team, k, becomes idle, the optimal vehicle pick-up & delivery locations are assigned to it. This optimization is achieved by maximizing the utility, $U_{i,j,t,k}$, of the repositioning movement performed by team k consisting in picking-up m vehicles from station (or zone) i, and delivering them to station (or zone) j, as expressed in Eq. 3.

$$U_{i,j,t,k} = U(-m)_{i,t} + U(+m)_{j,t} - C_t \left[\frac{d_{k,i}}{v_k} + \frac{d_{i,j}}{v} \right] \tag{3}$$

Where C_t is the unitary time cost of the repositioning team, $d_{k,i}$ is the distance between the repositioning team and the pick-up location, v_k is the average travelling speed of repositioning teams when travelling to the next task (e.g. with an scooter in case of car-sharing systems), $d_{i,j}$ is the distance between the pick-up and the delivery of the vehicle, and v is the average travelling speed of vehicles in the city. In turn, the utility of the pick-up task, $U(-m)_{i,t}$, and that of the delivery task, $U(+m)_{j,t}$, are computed by considering the improvement in the level of service offered and the increase in the revenues obtained by removing vehicles from station (or zone) i or by adding vehicles at j, respectively. We refer to Jiménez & Soriguera (2023a) for further details on the computation of these utility functions.

The previous process is run in the simulator sequentially in three priority blocks, until the idle repositioning team receives its optimal task assignment. Priority blocks restrict the available tasks for assignment as follows:

- *Priority 1: Battery recharging + balancing the system:* The first priority in repositioning operations is to move electrical vehicles with very low battery levels to a battery charging point. If while fulfilling this objective, the repositioning movement also contributes to balance the system (i.e. move vehicles from a location where they are in excess to a location where vehicles are scarce), then, even better. Therefore, Priority 1, considers only pick-up tasks of electrical vehicles whose battery level is below the minimum threshold, e_{min} (i.e. their status is “discharged”), and delivery locations will be composed only of stations with available battery chargers and with an inventory level below the optimum.
- *Priority 2: Battery recharging.* Priority 2 aims to move to battery charging points all the discharged electrical vehicles left over after Priority 1. Priority 2 considers again as pick-up tasks only the “discharged” electrical vehicles. In contrast, the delivery location will be the closest station with available battery chargers. Note that Priority 2 does not aim to balance the system, as this is not possible anymore after Priority 1 is completed. If Priority 2 tasks are completed, all recharging tasks have been assigned.
- *Priority 3: System rebalancing.* The third priority level only aims to balance the system. Pick-up tasks in Priority 3 are vehicles in stations/zones with an inventory level above the optimum. Delivery locations are stations/zones with a number of vehicles below the optimal inventory level. In addition, if the number of free-floating vehicles on-street is over the maximum threshold allowed by the policy makers, pick-up tasks will be restricted to free-floating vehicles, and delivery tasks will be only at stations.

4.2.4. Movement of repositioning teams

Properties of repositioning teams include lists defining the assigned and completed tasks, which are determined by a time counter, the ID of the station/zone where the task must be performed, and the number of movements, m (i.e. number of vehicles to be picked-up or delivered). When the simulation time step reaches any of these time counters, the repositioning team will perform the required actions. As a general rule, the action to perform depends on the sign of the “taskMovement” property. If negative, vehicles will be picked-up. If positive, vehicles will be delivered. General actions for pick-up and delivery tasks are described next:

- *Pick-up vehicles task.*
    - The repositioning team checks if the assigned number of pick-up movements, m, are still possible (i.e. if there are enough vehicles at the station/zone to pick-up). If there are, m vehicles will be taken by the repositioning team. If not, the task assignment process is run again
    - The vehicles taken by the repositioning team change their status to “in repositioning”, and their location coordinates to NaN. They are deleted from the station/zone list, and from the corresponding charging list, if applicable. They are input into the repositioning vehicle carry list.
    - Repositioning vehicle checks the following task. The current task counter advances, and the repositioning vehicle moves to the location of the delivery task.
- *Delivery vehicles tasks*:
    - The repositioning team checks if the programmed movements, m, are still possible (i.e. if there are enough parking slots to deliver the vehicles). If there are, the m vehicles in the repositioning vehicle will be moved to the station/zone. If not, the task assignment process is run again.
    - The delivered vehicles change their status. If the delivered vehicle is electric and the battery level is below the minimum, the vehicle status changes to “discharged”. If the delivered vehicle is not electric or its battery level is still above the minimum threshold, the vehicle status is changed to “available”. All the delivered vehicles change their position coordinates to that of the delivery station (or the centroid in case of free-floating zones), and are included in the station/zone list.
    - The current task counter of the repositioning team advances, and the repositioning vehicle becomes idle. The task assignment process is run again.

Note that the process can vary depending on the type of system and repositioning strategy considered. For example, in the case of car-sharing systems, $m = 1$ always.

4.2.5. Electric vehicle recharging

This module updates the battery level of all electric vehicles in the system. This process consists of three actions:

- *Battery consumption.* The battery level of all electric vehicles with “in trip” status is reduced by the equivalent percentage to one time-step of use (e.g. 1 minute). For car-sharing systems, battery consumption also is applied to vehicles with status “in repositioning”.
- *Battery recharge.* Battery level of all vehicles included in a recharging list is increased by the equivalent percentage to one time-step of recharge (e.g. 1 minute) to a maximum of 100%. All the vehicles in a recharging list must be in the status “idle” or “reserved”. This can be used as double-check.
- *Update the recharging list.* Recharging lists are cleaned by deleting all vehicles with battery level of 100%. After that, if there are free battery charging slots, vehicles in the corresponding station/zone that are not included in the recharging list and with battery level below 100% are added starting from the vehicle with the lowest battery level to the highest until reaching the maximum number of slots.

4.3. Custom frequency modules

Finally, there are custom frequency modules that can be run as frequently as the analyst considers it necessary.

4.3.1. Demand forecasting and optimal vehicle distribution

The demand forecasting module estimates how many vehicles are expected to be requested from, or returned to, each station/zone during the following operative period. This forecast is used as an input to the optimal vehicle distribution module which computes the optimal inventory level at each station/zone, based on the maximizing utility methodology described in Jiménez & Soriguera (2023a). Optimal inventory levels play a key role in the planning of the repositioning tasks.

The inputs to these modules are the O/D demand matrices. Generally, the input O/D matrixes do not change during the simulation, so that it is enough to run these modules at the beginning of the simulation process together with the setup modules. However, in advanced simulations, where demand might be continually updated from external real-data feeds, or maybe using a machine-learning predictive algorithm which considers historical data together with real time meteorological or calendar events, the model will become more accurate if the demand forecasting and optimal vehicle distribution modules are run periodically, and especially before the assignment of the repositioning tasks. Be aware that these implies additional computational costs.

4.3.2. Initialization tasks module

The initialization tasks module cleans the warm-up system data in order to only store data corresponding to the relevant simulation period. By default, the simulator stores results from one complete daily cycle, leaving to the operator the freedom to choose the number of warm-up daily cycles. Results from these warm-up cycles will be deleted at their end by the application of the initialization tasks module. The cleaning procedure consist in the following:

- *Users*:
  - The stored finished and dead users are deleted.
  - Timers of active users are recalculated by subtracting the previous cycle duration.
- *Vehicles*:
  - The arrays containing the status, battery level, and location coordinates of vehicles are reset. The first value of the array for the next cycle will be set to the last one of the previous cycle. For instance, if one vehicle ends the previous cycle “in trip”, will start the next cycle also “in trip”.
- *Stations/free-floating zones*:
  - The vehicle lists, recharge lists, and arrays containing the number of vehicles in the station/zone are reset. The first value for the next cycle will be set to the last one of the previous cycle. This implies that stations/zones must start the next cycle with the same vehicles as they ended the previous one.
- *Repositioning teams*:
  - The arrays containing the status, vehicles carried, and location coordinates of repositioning teams are reset. The first value of the array for the next cycle will be set to the last one of the previous cycle. For instance, if a repositioning team ended the previous cycle “idle”, it will start the next cycle also “idle”.
  - Timers for tasks are recalculated by subtracting the previous cycle duration. The finished tasks will result in timers lower than 1. These are deleted with all their associated properties.
  - If the repositioning team ended the last cycle while performing a task, its task counter is reset to 1. If it was idle and without any task assigned yet, the task counter will be reset to 0, and the repositioning team will start the next cycle idle waiting for a new task assignment.

5. Input parameters to the simulator

The following table summarizes all the inputs to the simulator. Note that not all them are necessary, as some only apply for particular option selections.

Table 10.a. Inputs to the agent-based vehicle-sharing simulator

	Property	Type	Description
SIMULATION	TotalTime	Integer	Total duration (T) of the simulation in minutes. It also defines the demand cycle.
	WarmUpCycles	Integer	Number of warming up cycles before computing the results. This allows reducing the effects of initial conditions.
	verbose	Option	Show messages during the simulation. • verbose = 0 => Messages disabled • verbose = 1 => Messages enabled
	rndSeed	String	Random seed code. Can be left blank. Allows the replication of results by introducing the random seed of a previous simulation.
ZONIFICATION	ServiceArea	Option	Selects the type of input to generate the zonification of the service area. • ServiceArea = 0 => Perimeter in an excel file (.xlsx) • ServiceArea = 1 => Full zonification in a shapefile (.shp)
	If ServiceArea = 1		
	ShapeFile	Path	Path to shapefile.
	If ServiceArea = 0		
	PerimeterFile	Path	Path to perimeter file.
	OutputShape	Path	Name of the output zonification shapefile if generated from perimeter. Stored in the simulation inputs recap folder.
O/D DEMAND MATRIX	OdmatKnown	Option	Selects the type of input for the O/D demand matrices. • OdmatKnown = 0 => Matrices not available. They will be estimated by the simulator from aggregated demand parameters. • OdmatKnown = 1 => Matrices available as .csv files.
	If OdmatKnown = 1		
	Odmat_prefix	Path	Path to the folder for the input O/D matrices and name of the set of matrices (e.g. "Data/folder/nameoftheset")
	TimeReDemand	Integer	Duration for which each input O/D demand matrix holds, in minutes.
	If OdmatKnown = 0		
	OutputODmat	Path	Name of the set for the output O/D matrices .csv files. They will be stored automatically in the simulation inputs recap folder.
	TotalTripsDay	Integer	Total number of potential trips, N, for the whole demand cycle, T (i.e. for the whole "TotalTime" duration) in the vehicle-sharing system.
	ImbalanceAvg	Real [0,1]	Average fraction of imbalanced trips, Φ. Average fraction of generated trips in a station/zone without an equivalent attracted trip. Must be between 0 and 1.
	areaRet	Real [0,1]	Fraction of the service area where attracted trips are higher than generated trips cars, π_t, resulting in a net accumulation of vehicles at the end of the day (i.e. attracting areas).
	areaReq	Real [0,1]	Fraction of service area where the generated trips are higher than the attracted trips, π_q, resulting in a net requirement of vehicles at the end of the day (i.e. generating areas).
	ImbalancePattern	Option	Average imbalance pattern within the service area • Radial => Imbalance is centered at a single point (focus), and evolves linearly in the radial direction from the focus. • Flat => Imbalance is defined by an axis (where the system is balanced) and evolves linearly in its perpendicular direction.
	If ImbalancePattern = Radial		
	ImbCentre	Array	The imbalance focus point. (X_C, Y_C) coordinates in UTM.
	ImbDirection	Option	Imbalance direction (φ): • ImbDirection = 1 => if vehicles' attraction grows towards the focus point (i.e. attractive focus). • ImbDirection = -1 => if vehicles attraction diminishes towards the focus point (i.e. generation focus).
	If ImbalancePattern = Flat		
	ImbDirection	Real	Positive angle (φ) between the North and the imbalance direction measured in sexagesimal degrees.
	TimeWeight	Array	Distribution of the total demand over the demand cycle, as an array of weight factors. The number of elements in the array defines the number of periods in which the demand cycle is divided. Each element is the weight of that period with respect to the others in the whole cycle. For example, an array [1;2] means that the demand cycle is divided in two periods, and in the second period the demand is the double than in the first one. By default, an array of typical 24 values is proposed, corresponding with the same hourly distribution of the example O/D matrices provided.
	UsersKnown	Option	Selects if the user generation is random or replicated from a previous simulation. • UsersKnown = 0 => Users randomly generated. • UsersKnown = 1 => Users created from a stored file.
	If UsersKnown = 1		
	UsersFile	Path	Path to the file of pre-generated users.

Table 10.b. Inputs to the agent-based vehicle-sharing simulator (continuation).

STATIONS	TotalStat	Integer	Total number of stations (S) used in the vehicle-sharing system.
	InputStationFile	Path	Path to the input stations' list. This file must exist, even if no station is to be used. In such case any default or empty file will suffice with no impact in the results.
	OutputStationFile	Path	Name of the MS Excel file with the output stations' list. This list includes only the stations finally used in the simulation. It will be stored automatically in the simulation inputs recap folder.
	defaultCapacity	Integer	Default number of available parking slots (i.e. capacity) at parking stations. Must be a strictly positive integer. It is only used when the capacity value at the stations' input file is void or for the stations generated within the simulator. Capacity values of 9999 or higher are considered as "infinite" for occupancy calculation purposes.
	defaultChargers	Integer	Default number of e-vehicle charging slots at parking stations. Must be a positive integer smaller or equal than the default capacity. It is only used when the corresponding value at the input file is void or for the generated stations within the simulator. Values of 9999 or higher are considered as "infinity" for occupancy calculation purposes.
VEHICLES	TotalVehsFF	Integer	Total number of vehicles initially deployed on-street (n_{FF}) (i.e. FF => free-floating).
	TotalVehsSB	Integer	Total number of vehicles initially deployed at stations (n_{SB}) (i.e. SB => station-based).
	streetFleetLimit	Integer	Maximum number of vehicles that can be parked on-street (e.g. maximum allowance by municipality regulations). It must be an integer value higher or equal than "TotalVehsFF".
	percEvehs	Real [0,1]	Fraction of electrical vehicles in the fleet. It must be a value between 0 and 1.
	BatteryConsume	Real	Electrical vehicle autonomy distance for 80% of the battery level [km]
	BatteryChargeTime	Real	Time required to charge 80% of the electrical vehicle battery [min]
	minBatteryPerc	Real	Minimum battery percentage (e_{min}) to consider an electrical vehicle as available for service.
USER BEHAVIOR	Wmax	Real	Maximum distance (w) that users are willing to walk to access a vehicle, at the origin or destination of the trip [m].
	WalkSpeed	Real	Average walking speed (v_w) [km/h]. A default value of 3 km/h is suitable in most cases.
	VehSpeed	Real	Average vehicle speed (v) while traveling within the city [km/h]
	avgParkTime	Real	Average time needed to park (i.e. return) the vehicle (t_{park_FF}) [min]
	Trips_SB2FF	Option	Types of trips allowed in the vehicle-sharing system: • Trips_SB2FF = 0 => SB to FF trips are NOT allowed. Segregated systems. • Trips_SB2FF = 1 => SB to FF trips ARE allowed.
	percParking	Real [0,1]	Fraction of users preferring to park at stations instead of on-street when both options are available (SB_{prk}). This represents the city average estimation. Recall that there exists the possibility of using different values of this parameter for the different zones of the service region. This zone-specific values are introduced through the zonification shapefile as the property "SB_PRK".
	penSB	Real	Additional access distance when using stations [m]. (Recall that $t_{park_{SB}} = penSB/v_w$)
	fullDest	Option	Reaction of users when return is not possible, due to lack of parking slots: • fullDest = 'change' => Users check other possible stations near destination. • fullDest = 'wait' => Users wait at the destination station until parking is possible.
	costLostFF	Real	FF Lost demand penalty. Perceived cost of not finding available vehicle on-street [€].
	costLostSB	Real	SB Lost demand penalty. Perceived cost of not finding available vehicle in stations [€].
REPOSITIONING	repoMethod	Option	Repositioning algorithm considered to assign tasks. • repoMethod = 0 => No repositioning considered. • repoMethod = 1 => Dynamic pairwise task assignment repositioning
	repoTeams	Integer	Number of repositioning teams working simultaneously (n_k).
	repoCapacity	Integer	Capacity of repositioning teams (i.e. number of vehicles that can be carried simultaneously).
	repoSpeed	Real	Average speed of repositioning teams (v_k) when moving between tasks [km/h]. This allows introducing a speed different than the average circulating speed in the city (e.g. scooter's speed).
	taskDuration	Real	Average duration of the fixed tasks for every repositioning operation [min]. This considers all the operations to be done by the repositioning employee when not moving (i.e. at the origin or destination of the repositioning task).
	costRepo	Real	Labor and equipment cost of repositioning teams prorated per hour [€/h].

6. Examples of application

The previous simulation architecture has been applied to two different cases of study. The first example is the simulation of a mixed car-sharing system including both free-floating cars on streets and also cars in stations, which can be requested and returned indistinctly at both locations. The second example simulates a station-based bike-sharing system. In both cases, the vehicle fleet is 100% electrical. Both simulations were run over layouts based on the city of Barcelona, Spain. These simulations could be used to test the design of the system and different operative strategies. However, since the purpose of the present work is not to evaluate these systems but to exemplify the application of the agent-based simulator, results are only aimed to illustrate its potential.

The main characteristics of the proposed simulations are summarized in Table 11. In both cases, results depict a 24-hour period. In the case of the car-sharing system, the warm-up period was set to 72h. For the bike-sharing simulation, since demand levels are higher, 48h were enough to produce enough bike trips to wash-out the effect of the initial ideal distribution of bicycles. Computational time was barely 10 minutes for each 24 hours of simulation in the case of bike-sharing and less than 5 minutes in the case of car-sharing. This difference is due to the smaller demand generated in the case of the car-sharing system.

Table 11. Summary of the simulation scenarios.

Property	Car-sharing scenario	Bike-sharing scenario
Fleet size ($n_{FF} + n_{SB}$)	250 cars	4838 bikes
Stations (S)	48 stations	347 stations
Fraction of fleet electrical	100%	100%
Capacity constraint at stations	No	Yes (12 to 54 bicycles)
Battery charging devices	All in station-based slots	All in station-based slots
Number of repositioning teams (n_k)	5 teams	13 teams
Total number of trips (in 24 hours) (N)	6481 trips	34840 trips
Average imbalanced trips [%] (ϕ)	22.6	11.8
Free-floating vehicle availability	Yes	No
Computational time (for 24h simulation)	4.6 min	10.3 min

For the car-sharing example, demand was introduced by means of available hourly O/D matrixes over an existing zonification. Then, the simulator generated random demands every time step of 1 minute using the Poisson generation process. For the bike-sharing example, only aggregate demand parameters where available (i.e. the total number of trips, the spatial imbalance and the temporal evolution factors). Then, the custom O/D demand was generated by the simulator considering a “flat” unbalance shape pattern (recall Fig. 1) with an imbalance axis of $\varphi \approx 45°$ to the north direction. This imbalance axis is parallel to the Barcelona coastline so that the bike-sharing trips tend to “precipitate” from the north-west to the south-east of the service region.

Fig. 5 depicts the difference between the aggregated number of trips in the O/D matrices and the actual randomly generated demand in the simulation (including served and unserved users). Note that the demand generated is close to the average expected values of O/D matrices, with a difference of 1.9% (car-sharing case) and 0.7% (bike-sharing).

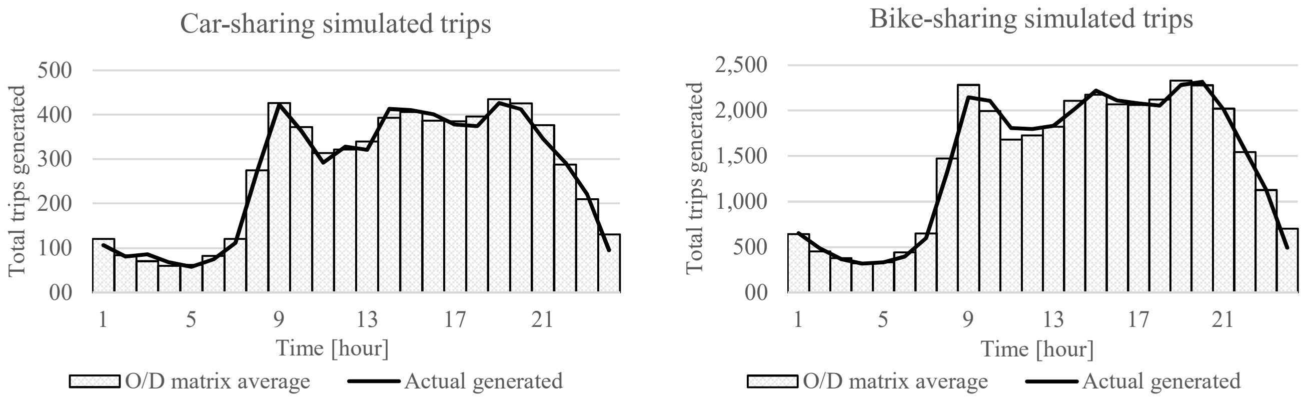

Fig. 5. Difference between the average expected demand and the actual generated demand for car-sharing (left) and bike-sharing (right) simulations.

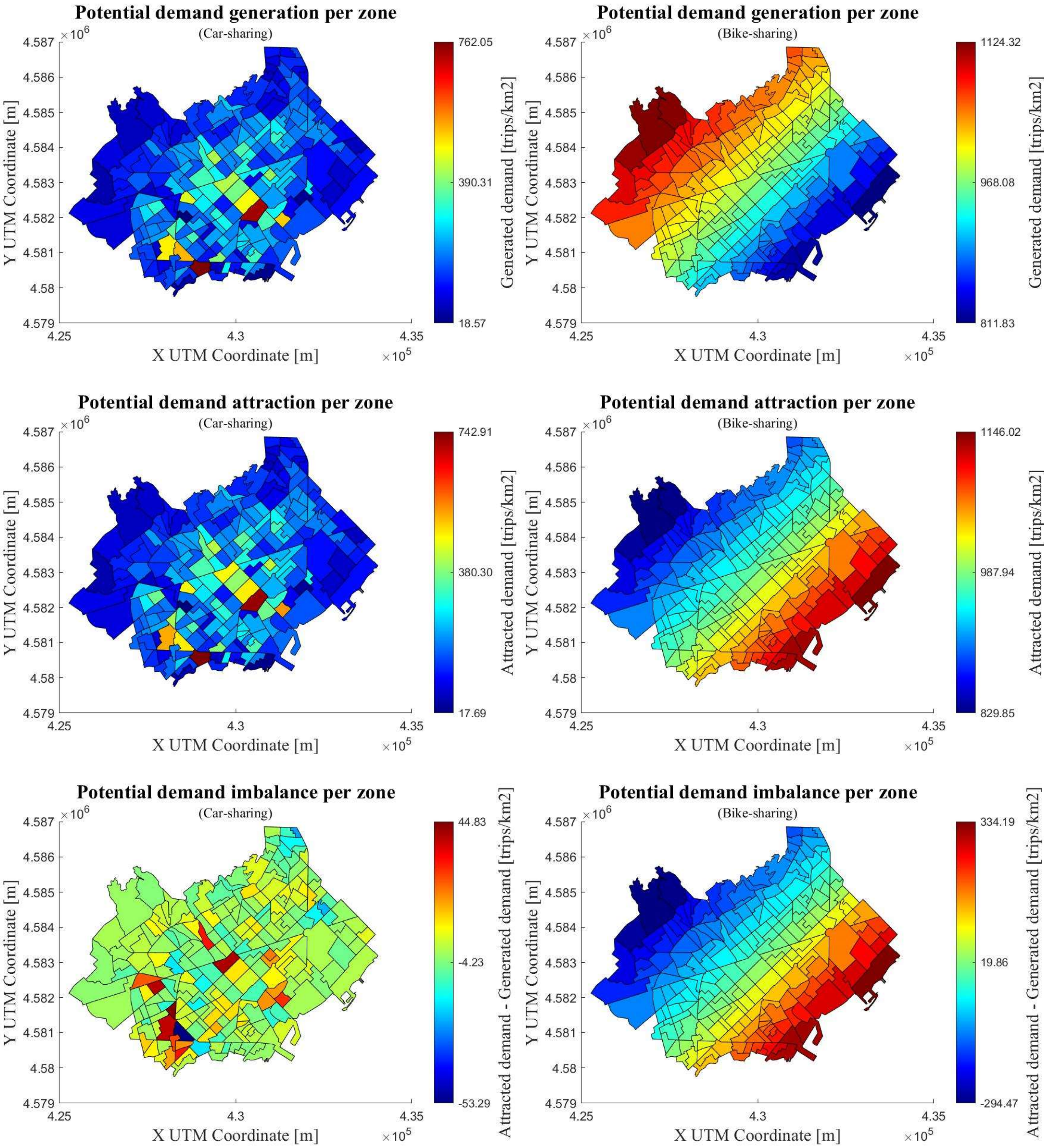

Fig. 6. Spatial distribution of demand density [trips/km^2] for car-sharing (left) and bike-sharing (right) simulations.

Fig. 6 shows the spatial distribution of the trips served for both scenarios. For the car-sharing case study, this follows the spatial distribution of the mobility demand over the city of Barcelona, with some hot-spots (e.g. city center, universities area and City of Justice, on the west side). In turn, for the bike-sharing case study it can be seen the effect of the imbalance pattern simulated, with most of the trips running downhill from the mountain-side (i.e. north-west border) to the sea-side (i.e. south-east border). Note that the zones near the sea front show positive imbalance (i.e. more returns than request), while those in the mountain side experience the opposite behaviour (i.e. negative imbalance; more requests than returns).

Another typical issue of concern is the analysis of repositioning operations. Fig. 7 and Fig. 8 address the effects of repositioning in the car-sharing system and in the bike-sharing system, respectively. In both simulations it can be seen that, overall, the number of no-service events is reduced when repositioning takes place. This is more noticeable in the bike-sharing case study, as repositioning is more intense (i.e. more repositioning teams moving a larger number of bikes). It can also be observed that repositioning contributes to a more balanced distribution of vehicles over the service region, despite that in the car-sharing example the extremely high demand of a central zone cannot be accommodated. Repositioning prevents this zone from having an over-accumulation of vehicles, and this yields to a higher number of lost trips originated at that particular zone. For the bike-sharing example, note that in the absence of repositioning operations, stations on the lower side of the city would be permanently full, while those in the upper part would be generally empty.

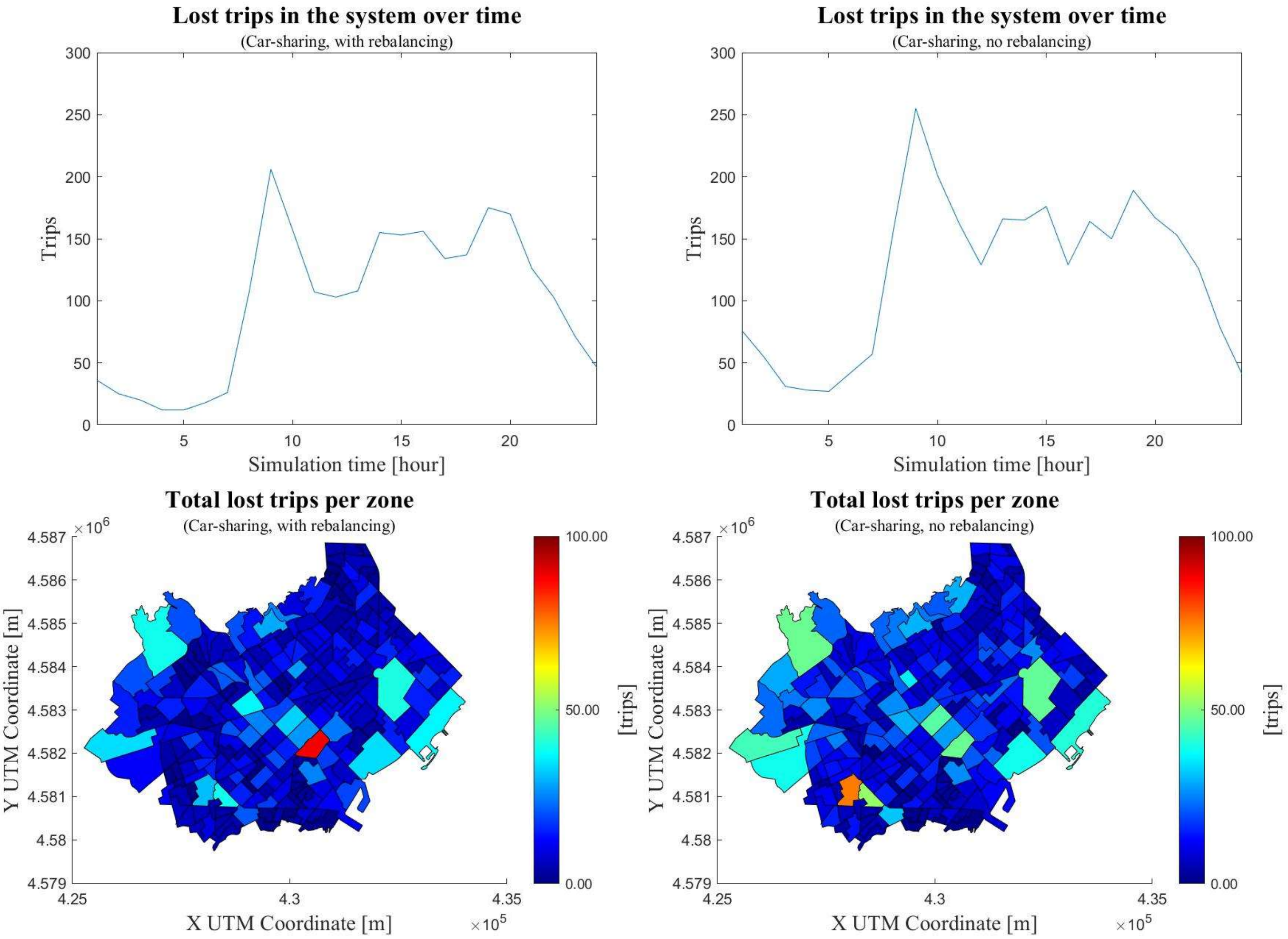

Fig. 7. Lost trips when repositioning is turned on (left) and off (right) in the car-sharing simulation.

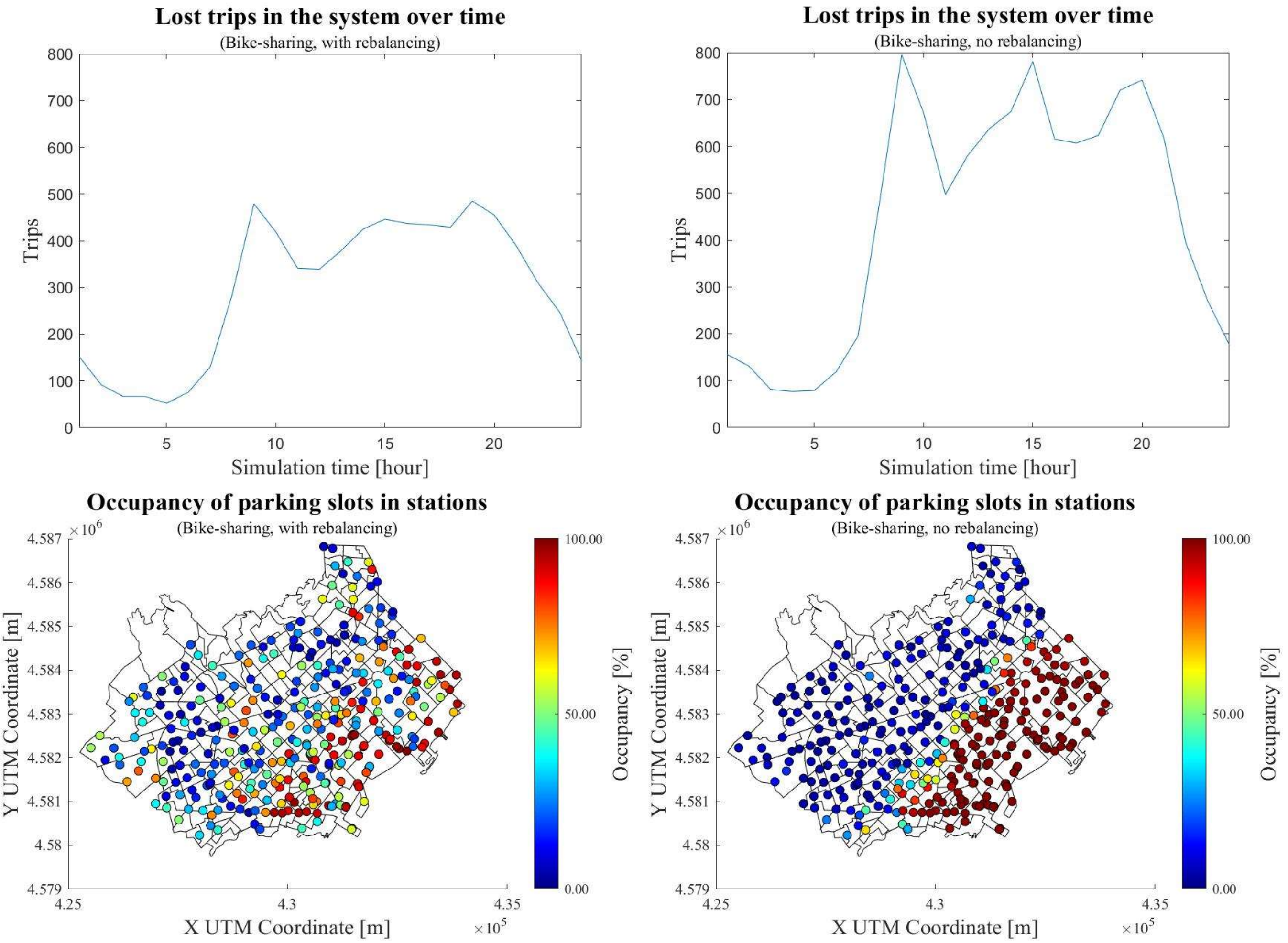

Fig. 8. Station occupancy when repositioning is turned on (left) and off (right) in the bike-sharing simulation.

Finally, battery consumption and recharging processes are analyzed. Fig. 9 shows how the average battery level evolves during several daily cycles. As expected, in the bike-sharing simulation the average battery level is much higher than in the car-sharing one. This is due to the fact that the bike-sharing system is station-based only, and all the idle bikes are parked at stations and recharging their batteries. In contrast, in the car-sharing simulation, part of the vehicle fleet is parked on-street, needing repositioning movements for being moved to a battery recharging spot in a parking station. In any case, the 100% electrical fleet is feasible for both vehicle-sharing systems, as the average battery level is enough to provide service in the long term, even in the peak-hour periods.

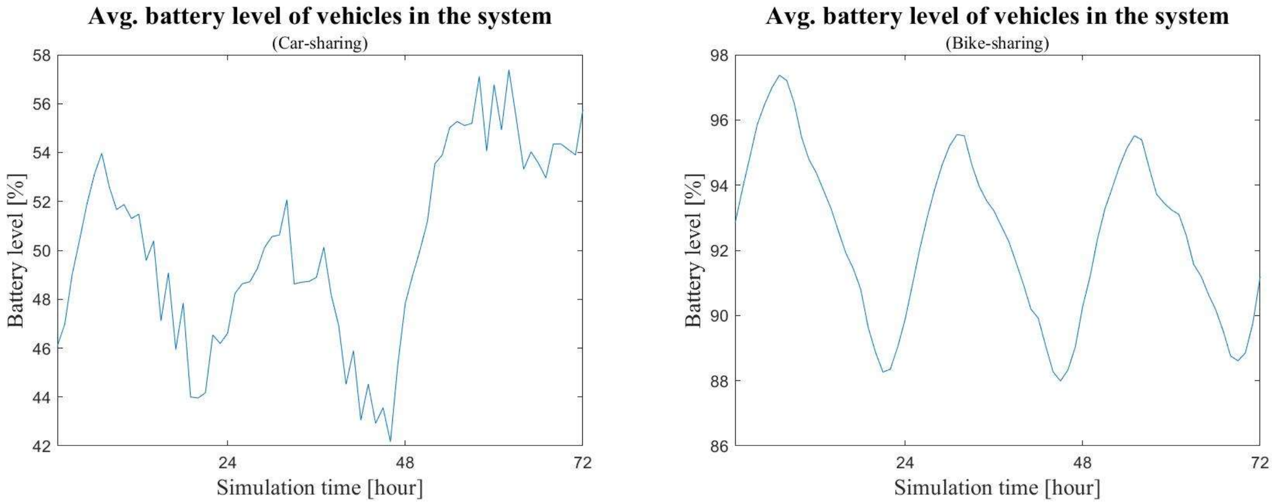

Fig. 9. Average vehicles' battery level for the car-sharing (left) and bike-sharing (right) simulations.

7. Generalizability and scalability of the proposed simulation model

The proposed ABS framework has been built with the objective of being a general and flexible model able to easily adapt to different vehicle-sharing systems. Customizability of the simulator is easily achieved by modifying the input parameters and variables (e.g. different geographical context, demand level, vehicle types, or operative settings). Still, this does not modify the underlying behavior of the agents in the different subsystems. To that end, the simulator has been built in a modularized fashion, so that the behavioral rules of particular agents can be modified within a particular subsystem, without affecting the rest of the components of the simulator. In spite of this, such modifications may require establishing new processes and functions within the affected module, whose complexity would be directly related to the complexity of the new agents' behavioral rules to implement.

This section of the paper clarifies the generalizability, flexibility and scalability of the proposed ABS framework, by describing the required modifications to the default simulator to address a different vehicle-sharing system, without affecting the basic behavioral rules of agents (see Section 7.1). In turn, an example of modifying the users' behavioral rules is also presented (see Section 7.2). Finally, the computational efficiency and the scalability of the simulator is analyzed (see Section 7.3).

7.1. Simulating a different vehicle-sharing system (without modifying the agents' underlying behavioral rules)

Changing the simulation scenario without modifying the agents' underlying behavioral rules is achieved by simply modifying the input parameters and variables which are fed to the simulator through its input files. Such modifications allow to change the geographical definition of the service region, the operative layout of the system (i.e. station-based, free-floating, mixed), the location of stations or free-floating zones, the type and characteristics of vehicles used (e.g. cars, bikes, scooter), the fraction of electrical vehicles, the size of the vehicle fleet, the number and efficiency of repositioning teams, the characterization of the system demand and the characterization of the users. Specifically, such modifications are achieved by only replacing the following input files:

- Geographical data of the service area, as a geometrical zone file or as its perimeter coordinates.
- Station list dataset, if applicable.
- Demand data for the system, either as O/D matrices or as a summary of aggregated values (e.g. total

expected potential demand per day and imbalance pattern).

- Parameters defining the vehicle-sharing system, such as the number of vehicles (in stations, street, electric), the total number of stations, repositioning teams and characteristics, etc.
- Parameters defining the users' characteristics, such as the walking speed, maximum access distance, parking preferences, etc.
- Simulation options, such as the warming up, replication of a former demand scenario, etc.

In summary, simulating a new vehicle-sharing scenario different than the ones presented here, can be achieve by simply adapting the input variables and parameters listed in Table . Note that despite the simplicity of such parameter tweaking, this already provides high flexibility in the simulator, which might fulfill most of its practical applications.

7.2. Modifying the underlying behavioral rules of the active agents in the simulation.

Changes in the behavioral rules of the active agents or modifications in the system setup calculations can be addressed by replacing or modifying the corresponding module in the simulator. For example, a different repositioning algorithm, affecting the task assignment procedure or the dynamics of the repositioning teams, would require to address the repositioning task assignment module, and, if necessary, the repositioning team movement module. The new implemented module alternatives can be stored in the simulator, and allow future users to choose which one they prefer to run. In fact, the current implementation already offers the possibility of activating or not the repositioning dynamics, two alternatives for the user behavior at full stations (i.e. wait when the destination station is full or find a new one nearby), or two alternatives for the user creation (i.e. random demand or creation from a trip list).

Another example affecting users' behavior could be illustrative. Suppose the objective is to study the impact of incentive-based user rebalancing schemes. This would require modeling the users' response to the incentives / recommendations provided. This could be implemented in the simulator by addressing the O/D demand matrix setup. Note that in the default implementation, demand is given and the O/D matrix is obtained either by its direct input or by its estimation from demand aggregated parameters. The modification of the module would require to implement how the O/D matrix is modified (e.g. in terms of its overall demand, or of the specific origin – destinations), according to the prevailing incentives or recommendations given. In turn, such incentives / recommendations could result from the observation of the system status (i.e. vehicle distribution) at the previous time step. The implementation of such modifications in the O/D matrix would allow to assess the effects of the user-based rebalancing schemes in the overall performance of the vehicle-sharing system. Note that the implementation in the simulator would be simple, and the complexity would reside in the specific modelling of the users' response to incentives and recommendations.

In summary, even if these changes represent a bigger difficulty than a simple change of scenario, the input-output structure of the module is kept. Therefore, the difficulty only lays in the complexity of the new behavior or alternate module to be implemented.

7.3. Scalability and computational performance

Finally, we present an analysis of the computational performance of the simulator, with special attention to its scalability (i.e. how the performance deteriorates with the scale of the simulation context). Note that all the simulations were run on MATLAB R2021a in an Intel(R) Core(TM) i7-10750H CPU @ 2.60GHz, with Windows 10 OS. Several runs were made for each different scenario sizes. Table 12 and Fig. 10 present the average performance results, showing that the computational time is not an issue, as it is admissible even for real-time simulation up to very large systems.

Table 12. Simulation performance [minutes per 24h of simulation]

System considered	**Demand** [24h]	**Scale factor*** x1	x2	x3	x4
Car-sharing simulation	6481 trips	4.6	12.6	15.3	23.3
Bike-sharing simulation	34840 trips	10.3	22.9	46.9	90.2

*Scale factor is applied to the fleet size, number of repositioning teams, and overall demand. These are the factors affecting the computational performance of the simulation.

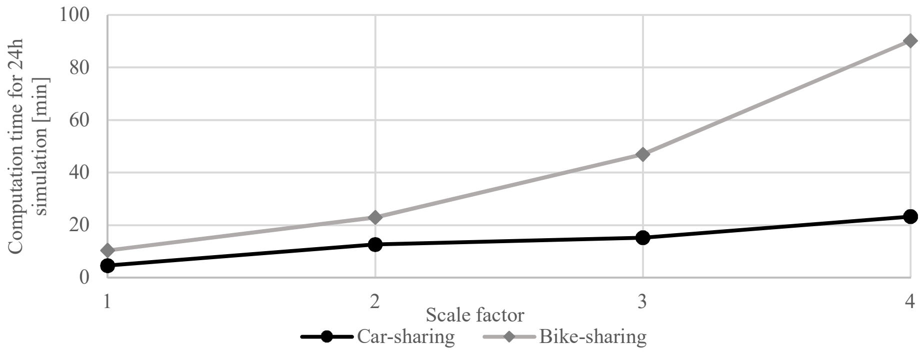

Fig. 10. Computational performance of the simulator.

8. Conclusions

A new agent-based simulation framework has been described and analysed. This framework is flexible enough to simulate different types of vehicle-sharing systems and different types of operational schemes. Two examples of application are presented to test the performance of the simulator, namely, a mixed car-sharing system (i.e. allowing free-floating and station-based vehicles) and a purely station-based bike-sharing system. In both cases the fleet is composed entirely of electrical vehicles. The modular architecture of the simulation framework makes simple the task of adapting the general features of the simulator to both cases, since only a few modules had to be modified.

Results show that the generation of the potential demand, the trips served, the trips lost, and the repositioning tasks, emulate well a real system with an affordable computational time. Therefore, the simulation framework can be used to analyse the performance of vehicle-sharing systems and support the decision-making process from the strategical design to the operative analysis, by performing efficiently a wide range of experiments.

Further development of the agent-based simulation is still possible. For instance, the setup modules could be adapted to read real-time inputs from web-based datasets monitoring stations' demand. Today, these datasets are easily obtained, as most of the station-based vehicle-sharing systems in the world provide web-based applications to monitor the status of stations in real time. With respect to the time-step and custom-frequency modules, many different managing strategies, operative methodologies, and user behaviour models could be implemented, in addition to the default ones included here. It would be particularly interesting adding vehicle maintenance operations as proposed in Jorge et al. (2014). This would require a new module which randomly generates vehicle failures and assigns maintenance task to the repositioning teams, or to other new agents (i.e. maintenance teams) if required.

Finally, the proposed agent-based vehicle-sharing simulator could be integrated into a comprehensive traffic and mobility simulation software, which includes other transportation modes. With such an integration, the potential demand for the vehicle-sharing system could be the result of a mode choice model, and it would be possible to examine the effects of vehicle-sharing systems in a whole multi-modal transportation network.

Acknowledgements

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Final remarks

Finally, even when each paper includes its own conclusions section, some final remarks about the thesis are discussed here, as a whole. As the first overall conclusion, it can be said that the main objective of the thesis has been fulfilled. The present work provides a set of tools and models to help in the implementation and management of mixed free-floating (FF) and station-based (SB) vehicle-sharing systems. Mixed systems imply that there are available vehicles on streets and in stations, working simultaneously and complementing each other. The FF layout helps to provide more accessibility to users and to reduce the parking renting costs. At the same time, the SB layout provides recharging infrastructure to electric vehicles, allows a better fleet control, and reduces street space occupancy. The macroscopic design model presented in Paper II and the agent-based simulator of Paper V are the culmination of this objective, considering two different stages of implementation. In particular, the model of Paper II yields the optimal design of the system in the planning stage, while the simulation framework in Paper V helps to manage the day-by-day operations of an already operating system. Up to date, these mixed FF+SB systems have never been modelled before. So, these models and tools are strong contributions by themselves.

For the achievement of these objectives, simplifications were made in order to construct parsimonious mathematical models. In this sense, this research stands out from previous works in literature. The present research does not intend to improve past models by developing more advanced and complex features. Instead, it analyzes how these models could be simplified without incurring in unacceptable accuracy losses. From there, the present research takes advantage of these simplification in order to achieve the modelling of a more complex system, (i.e. the mixed FF+SB) and to unveil significant insights.

1. Final remarks regarding the strategical design problem in the planning stage

From the conclusions obtained regarding the strategical design in the planning stage of vehicle-sharing systems, it is observed that its optimal design is achieved when:

i) vehicles achieve a high utilization rate.
ii) part of the potential demand is left unserved.
iii) the FF and SB weights in the overall mixed system are planned so that the repositioning tasks are minimized by its regular use.

The high utilization rate of vehicles, is a rather obvious finding, since it is expected that the profitability of the system relies on a higher utilization of the vehicle fleet. However, note that operators could be tempted to reduce fleets in order to increase the usage rate of vehicles. This could be a mistake based on the findings of Paper II about the sensitivity of system costs and profit to suboptimal fleet size designs. These findings show that an excess of fleet is preferable than vehicle scarcity, since the penalizations in the profits are smaller. The conclusion is that it would be always more convenient to deploy fleets in excess than by default. Note however, that deploying excessive fleets results in a higher occupation of public space, which implies negative externalities and an opportunity cost.

With respect to the second point, it is worth highlighting that in most of the analyzed scenarios, profit is maximized when 15-30% of the demand is left unserved. Even in the social optimum scenario (i.e. minimization of the generalized cost, including costly penalties for unserved demand), there is still a 15% of unmet trips. Trying to serve higher fractions of the potential demand, results in the over-dimensioning of the vehicle fleet size and in a high increase of repositioning operations, implying huge agency costs. Note, however, that the existence of a significant fraction of unserved demand also means that the service has a degree of unreliability that must be considered during the planning stages. For example, in car-sharing systems, it should be noted that unreliability would be an important drawback if the objective is to compete with private car. Probably less people would stop owning a car if the alternative is not available 100% of the time. However, if the vehicle-sharing system is planned as complementary to public transport, the unreliability effect would be less detrimental, because even if it is not available 100% of the time, it will improve the global service provided to users with an additional on-demand alternative, such as vehicle-sharing.

The third and last overall conclusion regarding the strategical design of vehicle-sharing systems, it proves the profitability of the mixed systems over dedicated SB or FF ones. In addition, it shows that the system is optimum in terms of repositioning operations when the total spatial imbalance created by users is equal and opposite to the main repositioning need. This can be easily understood with some examples. If a mixed vehicle-sharing system is working mostly as FF, and a significant number of the FF fleet must be moved daily to recharge (and an equivalent number of recharged vehicles must be made available on streets), the optimal design will be the one in which a similar number of SB vehicles must be relocated because they are not in an adequate location. Both repositioning issues are complementary and can be solved with minimum relocation movements. Analogously, if users park most vehicles in stations and at the end of the day a significant number of vehicles must be returned to streets in order to balance the FF to SB net movement, the optimal design will happen when a similar number of FF vehicles are expected to be out of position and require being relocated. Again, both repositioning needs are complementary, as it is possible to take these SB vehicles out of the stations and relocate them to the desired location on street to solve the FF spatial imbalance. These equilibrium points maximize the profit of the operating agency. In the absence of such complementarity between repositioning movements, only a transfer of costs takes place. For example, note that decreasing the fleet size in order to reduce the overall number of spatially imbalanced vehicles is not worthy, as captive repositioning movements should be made anyway, savings in repositioning costs will be minor, and an extra number of trips would be left unserved. Alternatively, if fleet size is increased, the total amount of spatially imbalanced vehicles grows, and extra inefficient repositioning movements will appear (e.g. moving one vehicle to a station for battery recharging, and afterwards moving it again once recharged to a different position on street).

Another interesting feature derived from the previous discussion regarding the design of mixed car-sharing systems, is that the total vehicle imbalance depends on four factors: i) the relative weights of FF and SB fleets, ii) the SB station coverage, iii) the average spatial demand imbalance (Φ), and the users' willingness to park at stations (PSB), which depicts the difficulty to park on street. Note that the first factor is directly controlled by the strategic design variables, and in fact its optimization is addressed in the model of Paper II. Also note that, PSB and Φ can be controlled through pricing strategies. This opens the door to study the implementation of pricing structures which incentivize users to park at strategic locations. In this way it could be possible to use other relative weights of the FF and SB fleet sizes and reach the equilibrium point by modifying the other two factors.

2. Remarks about the repositioning problem in the operative stage

Easiness of implementation is a desirable property of any strategy, model or tool. In order to improve the current performance of vehicle-sharing systems, research contributions do not only need to move forward the knowledge frontier regarding these systems, but also make the application of advanced research simple enough for being actually applied in practice, where operators have limited resources to implement the optimization models suggested. For this reason, the works of this block of the thesis were developed in a spirit of simplification. The performance of the proposed models and solutions should imply an advance in the state of the art, ant at the same time be easily applicable by operating agencies. This goal has been fulfilled in the case of the real-time pairwise repositioning task assignment, since results showed that this method provided better performance than the existing methods based on preemptive routing optimization, due to the inherent uncertainty of demand predictions for vehicle-sharing systems.

Avoiding unnecessary details leading to high computational burden also allowed to face a more complex system configuration, like the mixed SB+FF car-sharing system in the developed agent-based simulator. This simulator included a novel feature, which is the repositioning task assignment optimization considering different causes and priorities (i.e. spatial imbalance, limitation of the number of cars on street, and battery recharging needs). Generally, the analysis of vehicle-sharing repositioning operations in the scientific literature focusses only on solving the vehicle spatial imbalance. But in practice, there are other reasons for repositioning teams to visit stations, such as the maintenance and repair of vehicles and stations. The simplicity of the repositioning optimization models and its implementation is key to develop future works addressing these issues.

3. Future research lines for improvement

As an end note, the author wants to conclude with some ideas derived from the contents of this thesis which could be further developed in future research. The first one is the aforementioned inclusion of maintenance and repair tasks in the operative optimization. This problem has even more degrees of freedom than the repositioning optimization, because maintenance can be done on site or in a central workshop, and the options depends on the type of maintenance task to perform. In addition, mechanical failures and maintenance needs are spatially less predictable than the vehicle imbalance. So, simulation models, such as the ones presented here, could be a very good starting point to obtain insights and develop strategies to address this issue.

With respect to the strategic planning of vehicle-sharing systems, there is also room to further development of new ideas. Note that the design of such systems is strongly influenced by the vehicle repositioning tasks, mainly resulting from the user behavior. Both topics are rather difficult to model. In the macroscopic design model, presented in the first block of the thesis, the used modelling approach takes rather strong assumptions. For the modelling of the repositioning tasks, continuous approximations are used, despite being a discrete logistic problem. This allowed reducing the mathematical complexity of the model and the computational cost when dealing with its optimization. Consequently, the possibility of developing an optimizable and manageable mixed SB+FF vehicle-sharing model arose from the simplicity of the building models behind it. Regarding the modeling of the user behavior, and in particular the estimation of the potential demand, the present thesis considers exogenous and fixed demand, despite it is acknowledged that the fare and level of service provided can affect user behavior and trip decisions. For example, users may accept longer access distances if the fare is reduced, or the total potential demand may increase or decrease if the reliability of the service or the fare changes. To consider these relationships yields the endogenous modeling of potential demand with respect to the decision variables of the system. Such modelling approach has been disregarded in the present thesis as it is considered that the complexity added does not pay off with respect to the reliability and accuracy resulting from endogenous demand models. In spite of this, it is acknowledged that this possibility could be explored deeper in future works. The effects on potential demand of pricing strategies or competing transportation alternatives could be analyzed, using data from current pilot implementations (i.e. revealed preferences) or in stated preference surveying. In this way, demand and its characteristics (e.g. the maximum admissible access distance) could be treated as endogenous variables in the model.

Finally, spatial constraints also play an important role in the design of vehicle-sharing systems. Unfortunately, simplifications in the model lead to overlook the topology of the service area or any individual treatment of stations and subzones. Therefore, it would be advisable in the future to complement the present works with facility location models that could provide a way to optimize the definition of the service region and the location of stations, battery charging facilities, workshops and vehicle depots.