Appendix A

Notation

In general, uppercase boldface letters (**A**) denote matrices, lowercase boldface letters (**a**) denote (column) vectors and italics (a, A) denote scalars. In some occasions, matrices are also represented with calligraphic fonts (\mathcal{A}).

| $\mathbf{A}^{T}, \mathbf{A}^{*}, \mathbf{A}^{H}$ | Transpose, complex conjugate and transpose conjugate of matrix \mathbf{A} , respectively. |
|---|---|
| $\mathbf{A}^{-1}, \mathbf{A}^{\#}$ | Inverse and Moore-Penrose pseudoinverse of matrix $\mathbf{A},$ respectively. |
| $\mathbf{A}^{1/2}$ | Positive definite Hermitian square root of matrix \mathbf{A} , i.e. $\mathbf{A}^{1/2}\mathbf{A}^{1/2} = \mathbf{A}$. |
| $\mathbf{A}\left(\mathbf{B}\right)$ | Matrix \mathbf{A} is a function of the entries in matrix \mathbf{B} . |
| $\det\left(\mathbf{A}\right)$ | Determinant of matrix A . |
| ${\rm Tr}\left({\bf A} \right)$ | Trace of matrix A . |
| $\mathrm{vec}\left(\mathbf{A}\right)$ | Column vector formed stacking the columns of matrix ${\bf A}$ on top of one another. |
| $\operatorname{diag}\left(\mathbf{a}\right),\operatorname{diag}\left(\mathbf{A}\right)$ | Following the Matlab notation, diag (a) is the $N \times N$ diagonal matrix whose entries are the N elements of the vector a , and diag (A) is the column vector containing the N diagonal elements of matrix A . |
| $\mathrm{Dg}\left(\mathbf{A} ight)$ | An $N \times N$ diagonal matrix whose entries are the N elements in the diagonal of matrix \mathbf{A} , i.e., diag $([\mathbf{A}]_{1,1}, \ldots, [\mathbf{A}]_{N,N})$ or, equivalently, diag (diag (\mathbf{A})). |
| $\ \mathbf{a}\ $ | Euclidean norm of \mathbf{a} , i.e. $\ \mathbf{a}\ = \sqrt{\mathbf{a}^H \mathbf{a}}$. |
| $\left\ \mathbf{a}\right\ _{\mathbf{W}}$ | Weighted norm of a , i.e. $\ \mathbf{a}\ _{\mathbf{W}} = \sqrt{\mathbf{a}^H \mathbf{W} \mathbf{a}}$ (with Hermitian positive definite W). |
| $\left[\mathbf{A}\right]_{i,j}$ | The entry of matrix \mathbf{A} in the <i>i</i> -th row and the <i>j</i> -th column. |
| $[\mathbf{A}]_i$ | The i -th column of matrix A . |

- $[\mathbf{v}]_i$ The *i*-th element of vector \mathbf{v} .
- $\mathbf{A} \otimes \mathbf{B}$ Kronecker product between \mathbf{A} and \mathbf{B} . If \mathbf{A} is $M \times N$,

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} [\mathbf{A}]_{1,1} \mathbf{B} \cdots [\mathbf{A}]_{1,N} \mathbf{B} \\ \vdots & \ddots & \vdots \\ [\mathbf{A}]_{M,1} \mathbf{B} \cdots [\mathbf{A}]_{M,N} \mathbf{B} \end{bmatrix}$$

- $\mathbf{A} \odot \mathbf{B}$ Elementwise (Schur-Hadamard) product between \mathbf{A} and \mathbf{B} (they must have the same dimensions).
- $\mathbf{A} \geq \mathbf{B}, \mathbf{A} > \mathbf{B}$ The matrix $\mathbf{A} \mathbf{B}$ is positive semidefinite and positive definite, respectively.
- \mathbf{I}_N, \mathbf{I} The $N \times N$ identity matrix and the identity matrix of implicit size.
- $\mathbf{0}_{M \times N}, \mathbf{0}_M, \mathbf{0}$ An $M \times N$ all-zeros matrix, an M-long all-zeros vector and, an all-zeros matrix or vector of implicit size.
- $\mathbf{1}_{M \times N}, \mathbf{1}_M, \mathbf{1}$ An $M \times N$ all-ones matrix, an *M*-long all-ones vector and, an all-ones matrix or vector of implicit size.

$$\mathbf{d}_N$$
 The vector defined as $\mathbf{d}_N = [0, \dots, N-1]^T$

 \mathbf{e}_i Vector that has unity in its *i*-th position and zeros elsewhere.

 $\mathbb{R}^{M \times N}$, $\mathbb{C}^{M \times N}$ The set of $M \times N$ matrices with real and complex valued entries, respectively.

j Imaginary unit
$$(j = \sqrt{-1})$$
.

- $\operatorname{Re} \{A\}, \operatorname{Im} \{A\}$ The matrices containing the real and imaginary parts of the entries of A respectively.
- $\arg \{a\} \qquad \qquad \text{Angle of the complex number } a, \text{ i.e., } \arg \{a\} = \arctan \left\{\frac{\operatorname{Im}(a)}{\operatorname{Re}(a)}\right\}.$
- |a|, sign (a) Absolute value and sign of a real valued a.
- $\lceil a \rceil$ Smallest integer bigger than or equal to a.
- $\widehat{\mathbf{A}}$ Estimator or estimate of the matrix \mathbf{A} .
- $f_{\mathbf{A}}(\mathbf{A})$ Probability density function of the random matrix \mathbf{A} .
- $E\left\{\mathbf{A}\right\}$ Expectation of a random matrix \mathbf{A} .
- $E_{\mathbf{B}} \{ \mathbf{A} \}$ Expectation of a random matrix \mathbf{A} with respect to the statistics in \mathbf{B} .
- $\arg\min_{\mathbf{B}} f(\mathbf{B})$ Matrix **B** minimizing the scalar function $f(\mathbf{B})$
- $\arg \max_{\mathbf{B}} f(\mathbf{B})$ Matrix **B** maximizing the scalar function $f(\mathbf{B})$

 $\partial \mathbf{A}/\partial \mathbf{B}$ If **B** is $M \times N$, $\frac{\partial \mathbf{A}}{\partial \mathbf{B}}$ is a matrix formed as

$$\frac{\partial \mathbf{A}}{\partial \mathbf{B}} = \begin{bmatrix} \frac{\partial \mathbf{A}}{\partial \{\mathbf{B}\}_{1,1}} \cdots & \frac{\partial \mathbf{A}}{\partial \{\mathbf{B}\}_{1,N}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{A}}{\partial \{\mathbf{B}\}_{M,1}} \cdots & \frac{\partial \mathbf{A}}{\partial \{\mathbf{B}\}_{M,N}} \end{bmatrix}$$

In addition, for a given scalar b, $\partial \mathbf{A}/\partial b$ is the matrix containing the derivatives of the entries of \mathbf{A} with respect to b. If b is complex, we have $\frac{\partial}{\partial b} = \frac{\partial}{\partial \operatorname{Re}\{b\}} - j\frac{\partial}{\partial \operatorname{Im}\{b\}}$, see [Bra83].

 $\delta(i_1, \ldots, i_N)$ Multidimensional Kronecker delta defined as

$$\delta(i_1, \dots, i_N) = \begin{cases} 1 & i_1 = \dots = i_N \\ 0 & otherwise \end{cases}$$

 $\delta(\mathbf{x})$ Vectorial Dirac's delta defined as

$$\delta\left(\mathbf{x}\right) = \begin{cases} 1 & \mathbf{x} = \mathbf{0} \\ 0 & otherwise \end{cases}$$

- $\mathfrak{F}\left\{\cdot\right\}, \mathfrak{F}^{-1}\left\{\cdot\right\} \qquad \text{Direct and inverse Fourier transform for both the analog and discrete cases} \\ \text{defined as } \mathfrak{F}\left\{x(t)\right\} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \text{ and } \mathfrak{F}\left\{x[n]\right\} = \sum_{n=-\infty}^{\infty} x[n] e^{-j2\pi f n}, \\ \text{respectively.}$
- * Analog or discrete convolution defined as $x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau$ or $x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k]$, respectively.
- $x \in (A, B]$ The scalar x belongs to the interval given by x > A and $x \le B$
- sinc (x) Function defined as sinc (x) $\triangleq \begin{cases} \sin(\pi x) / (\pi x) & x \neq 0 \\ 1 & x = 0 \end{cases}$.

sup Supremum (lowest upper bound). If the set is finite, it coincides with the maximum (max).

- lim sup Limit superior (limit of the sequence of suprema).
- $O(\cdot), o(\cdot)$ Landau symbols for order of convergence.
- \triangleq Symbol used to define a new variable.
- \propto It stands for "proportional to" or sometimes "equivalent to".
- $\ln\left(\cdot\right)$ Natural logarithm.