Universitat Autònoma de Barcelona

Essays on Networks, Social Ties, and Labor Markets

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Essays on Networks, Social Ties, and Labor Markets

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Introduction

This thesis explores network theory and its applications to labor market. In particular, I examine how the stability and efficiency of networks depend on network externalities and linking restrictions, how firms use their employees' social networks to acquire information about the abilities of unemployed workers (employee referrals as a screening mechanism), and the aggregate influence of employee referrals on labor market outcomes.

In chapter 1 (or part I) of the thesis, *Listen Before You Link: Optimal Consent Rules for Network Formation in the Presence of Externalities*, I consider how communities (such as families) influence the formation of social networks (for instance, the marriage network) through social pressures. I study environments in which individuals are restricted to form or break certain relationships/links by members fo their communities/groups. I show that the restrictions can help reconcile the tension between stability and efficiency which often exists in the absence of such constraints due to the presence of network externalities. Firstly, I characterize consent rules (group structures and consent requirements) that can optimally lead to the formation of efficient networks. In the optimal consent rules, I find that the size of groups and the consent requirement (group's influence) are positively related. For instance, the optimal consent rules can be big groups with high consent requirement or small groups with low consent requirements. Secondly, I show how the optimal consent rules depend on the form of network externalities. In environments with negative externalities and where the payoff comes from multiples paths, one needs big groups with high consent requirements to stabilize efficient networks. On the other hand, in environments with negative externalities and where the payoff comes from shortest paths, one needs small groups with low consent requirements to stabilize efficient networks.

In chapter 2 (or part II), *The Weakness of Weak Ties in Referrals: An Obstacle for the Upwardly Mobile Black Men in the Private Sector*, I study how firms use their employees' social networks to acquire information about the abilities of unemployed workers (employee referrals as a screening mechanism). I build a model of employee referrals with two main features: unemployed workers choose which employed workers to ask for referrals based on the types of ties (weak or strong) they have with them, and firms try to infer some information about the abilities of the unemployed workers through the recommendations of its employees. The model predicts that the returns to using a tie vary with the unemployed worker's ability, the tie strength, and the proportion of workers who have access to different types of ties. I then develop two applications of this model. (1) There is significant evidence suggesting that the black-white wage gap widens as one moves up the wage hierarchies of the private sector in the US. The model shows that the lack of access to strong ties for black can be behind this empirical finding. (2) In the second application, I explore some implications of the employee referrals for job search. The model can explain (i) the mixed evidence about the use of different types of ties in job search, and (ii) the mixed evidence about the wage differentials between workers who found jobs through referrals and workers who found jobs by formally applying to firms. Although these predictions do not emerge in the existing models of employee referrals, they are consistent with existing empirical evidence, suggesting that the tie selection and the strategic recommendation are important aspects of employee referrals and are useful for understanding job search.

Finally, in chapter 3 (or part III), *A Survey on the Models of Employee Referrals: Search Frictions and Screening*, I discuss the aggregate influence of employee referrals on labor market outcomes by going over some of the key models in this literature. These models can be classified by two main functions of employee referrals: (1) reducing search frictions, and (2) screening. I will show how these models can address many important issues, including the following: Why is there a positive correlation between employment status of individuals who live in the same neighborhood, and/or have the same ethnicity, and race? Why do labor market participation rates differ across groups such as whites and blacks? How can one explain the persistent inequality in wages between blacks and whites? Why does the black-white wage gap widen as one moves up the wage hierarchies of the private sector? Why is there mixed evidence about the wage differentials between workers who found jobs through referrals and workers who found jobs by formally applying to firms? What determines the efficient use of employee referrals when other search methods are available? How useful are social contacts when workers can also use their educational degrees to signal their own abilities? Why is there mixed evidence about the use of different types of contacts in job search? I will also discuss various modeling choices, open questions and provide some possible avenues for future research.

Contents

Part I

Listen Before You Link: Optimal Consent Rules for Network Formation in the Presence of Externalities

1 Introduction

In many situations, individuals are restricted to form or break certain relationships/links by members of their communities/groups, such as their family, friends or colleagues. Sometimes these restrictions arise from social pressures. For instance, consider the case of high school cliques (geeks, jocks, popular, etc). Even if two individuals from different cliques want to become friends, they may not do so, in fear of being ostracized by members of their own cliques. Similarly, consider the most personal decision of finding a partner for marriage. In many cultures, couples require the consent of their family members to marry; otherwise, they would be ex-communicated by them. In other cases, restrictions on link formation are of a legal nature. A European Union member country cannot freely make a trade deal with a non communitarian country. The European Commission negotiates trade agreements between the European Union and other countries taking into account the interests of all the European Union member countries. However, the size of these groups and the influence of group members can vary. For instance, the size of European Union (twenty eight members primarily located in Europe) is much bigger than other custom unions such as Andean Community (four members including Bolivia, Colombia, Ecuador, and Peru) or Southern African Customs Union (five members including Botswana, Lesotho, Namibia, South Africa and Swaziland). In many eastern cultures, family members have stronger influence over marriage decisions than in many western cultures. Similarly in the case of high school cliques, the influence of clique members can vary by the school environment (freedom to select seats in a classroom, number of elective courses, etc).[1] Given the prevalence of such situations, it is important to understand the effect that these restrictions on link formation will have on the resulting network relationships. When should one expect the size of these groups to be big? When should one expect stronger influence of group members? And how do these features depend on the network environment? These considerations lead me to study a strategic model of link formation with group consent requirements.

I consider environments where each player belongs to a group (given by a partition p of the

[1]See McFarland et al (2014) for a recent study and some discussion on this subject. They argue that in environments with more elective courses, freedom to select seats in a classroom, the influence of group members can be strong in that few links are eventually formed between clique members.

players set N), and he needs the consent of at least a proportion q of his group members to form new links or to break his existing links. I will refer to a partition and consent requirement pair as a consent rule hereafter. Links represent bilateral relationships. There is a direct cost and a direct benefit from forming a link, and it generates externalities since indirect connections also add value. I assume throughout that the costs of link formation are the same for every link and that they are linearly increasing in the number of links. For a given consent rule (p, q), I define *contractually pairwise-stable* networks as those in which no pair of players want to deviate by forming a new link and no player wants to deviate by deleting one of his existing links, when each player also needs the consent of at least a proportion q of his group members to deviate.[2] This notion will be used to predict the networks that can be formed in equilibrium. Note that the pairwise stability notion of Jackson & Wolinsky (1996) is a special case of contractual pairwise stability in which either each individual is in a group by himself ($p = \{1, 2, ...n\}$) and/or the consent requirement is zero ($q = 0$). Observe that the consent requirement measures the influence that group members have over an individual's actions. Under no consent requirement, group members have no influence over the decisions of an individual. On the other hand, if consent requirement is unanimity, then each group member has veto power over the decisions of an individual.

In the presence of externalities and without the group consent requirement, a tension often arises between individual incentives to form or to break links and efficiency from an overall societal perspective. Self-interested individuals will be tempted to form or to break links without taking into account how their choices affect the welfare of others. As a result, the networks which are formed turn out to be inefficient from an overall societal point of view. The main goal of this paper is to consider how the group consent requirement can help reconcile some of this tension between stability and efficiency. I characterize the set of partitions that can stabilize the efficient networks. For each of these partitions, I look for consent requirements that can optimally stabilize efficient networks in the following sense: among the set of consent requirements which can stabilize efficient networks under a given partition, they are the ones that stabilize the least number of inefficient networks (in set inclusion terms). I will refer to such partitions and optimal consent requirement pairs as the optimal consent rules. My interest in looking for the optimal consent requirement comes from the fact that there may be some consent requirement that stabilizes efficient networks but also stabilizes many inefficient networks in the process. Thus, one can do better by looking for the consent requirement which not only stabilize efficient networks but also stabilize least number of inefficient networks.

I first characterize optimal consent rules for several stylized models. This allows me to understand how the optimal consent rules depend on the form of network externalities. In environments

[2]Caulier, Mauleon, and Vannetelbosch (2013) proposed the concept of *contractual stability* which is similar to contractual pairwise stability in that each player belongs to a group and he is required the consent of at least a proportion q of his group members to deviate. However, their notion considers multiple link deviations and allows deviation by more than two players. See section 3.6 for a discussion on this and other notions of stability.

with negative externalities and where the payoff comes from multiple paths, one needs big groups with high consent requirements to stabilize efficient networks. On the other hand, in environments where the payoff comes from the shortest path, small groups with low consent requirements can stabilize efficient networks. In particular, in environments where the payoff comes from the shortest path and externalities are negative, one can stabilize efficient networks even without any group consent requirements. I then characterize optimal consent rules more generally, and I find that the size of groups and the consent requirement (group's influence) that can optimally lead to the formation of efficient networks are positively related. For instance, the optimal consent rule can be big groups with high consent requirement or mid-size groups with at least a medium consent requirement or small groups with at least a low consent requirement. More precisely, if at most K individuals will give consent (because they are made weakly better off) to any deviations from any of the efficient networks, then any partition $p = \{S_1, S_2, ..., S_m\}$ with at least $K + 2$ number of individuals in each group (each player has $K + 1$ players in his group excluding himself) with consent requirement of $q_p = \frac{K+1}{min\{\#S_1,\#S_2,...,\#S_m\}-1}$ will stabilize all the efficient networks. Any consent requirement lower than q_p will not stabilize all the efficient networks, and any consent requirement higher than q_p can stabilize more of the inefficient networks. Therefore, the minimal consent requirement which stabilizes all the efficient networks is the optimal consent requirement because it also stabilizes the least number of inefficient networks (in set inclusion sense).

1.1 Outline

The rest of the paper is organized as follows. I will end this section by discussing closely related literature. In section 2, I present the general model and provide some definitions. In section 3, I examine several stylized versions of the general model. For each of these models, I characterize the optimal consent rules and compare them to see how the optimal consent rules depend on the form of externalities. In section 4, I characterize the optimal consent rules for the general model and some interesting extensions of the general model. These extensions include more general consent rules that allow each group to have its own consent requirement, and environments with heterogeneous individuals. In section 5, I conclude and provide some avenue for future research. All proofs are available in the appendix.

1.2 Related Literature

This work is related to a number of papers which study the tension between stability and efficiency in network formation. The issue was first considered by Jackson and Wolinsky (1996). Their analysis begins with several simple models that are tractable enough to examine directly the stability of efficient networks. An extensive literature took their approach to study formation of various networks, including free-trade, buyer-seller, oligopolistic firms, R&D collaborations and political alliances (see Jackson (2003), and Mauleon and Vannetelbosch (2015) for two excellent surveys). From looking at simple models one can gain some insight about how the particular structure of the externalities matters in determining which networks one can expect to form and whether these are efficient. However, the models in the existing literature differ in too many dimensions to exactly disentangle the impact of different forms of externalities. An alternative approach is to construct a model which has flexible enough structure so that one can include different forms of externalities as special cases. This approach was already taken by Goyal and Joshi (2006) and Currarini (2007). But their models are only flexible with respect to one dimension of externalities, as it allows for both positive and negative externalities. Yet, the form of externalities can vary with respect to other dimensions as well, and I consider them.

There is also a large body of literature considering various ways to reconcile this tension between stability and efficiency. A first strand of literature considers reconciliation through transfers across players. This approach was first taken by Jackson and Wolinsky (1996), and then by Dutta & Mutuswami (1997). A second strand of literature considers environments where value allocation and network formation are part of the same bargaining process. This approach was taken by Currarini and Morelli (2000), Mutuswami and Winter (2002), and Bloch and Jackson (2007). A third and recent strand of literature considers environments where players are partitioned into groups, and in order to add or delete links, players need some consent from some members of his/her group. This approach was taken by Caulier, Mauleon, and Vannetelbosch (2013), and Caulier, Mauleon, Sempere-Monerris and Vannetelbosch (2013).

Since my work is closely related to the third strand of this literature, I will now provide some further comparisons with the preceding papers. They also consider specific consent requirements of majority or unanimity as means to stabilize the efficient networks in some classical models of network formation. My analysis differs from theirs in several important respects. Firstly, it is focused on finding optimal consent rules, while they only intend to show that it is possible to stabilize efficient networks under some natural consent rules. Secondly, my objective is to understand how the optimal consent rules depend on the form of externalities. My analysis sheds some light on how the size of groups and the influence of group members depend on the network externalities. Finally, I extend the analysis by allowing each group to have its own consent requirements, considering environments with ex-ante heterogeneous individuals, and considering different notions of stability.

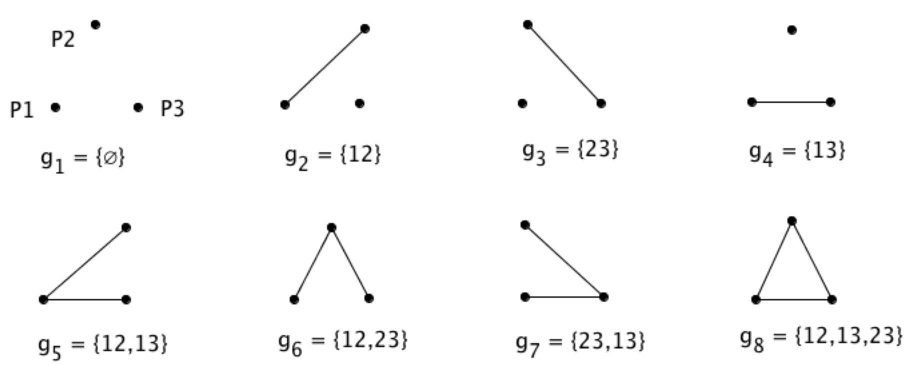
2 Environment

2.1 Networks

Let $N = \{1, ..., n\}$ be a finite set of players who are ex-ante identical. Network relationships are reciprocal and the network is thus modeled as an *undirected* graph. Players are the nodes in the graph and links indicate bilateral relationships between the players.

A network g is simply a list of which pairs of players are linked to each other with $ij \in g$ indicating i and j are linked under the network g. The *complete* network, denoted g_c, is the set of all subsets of N of size 2. The set of all possible networks on N is $G = \{g \,|\, g \subseteq g_c\}$. The phrase "unique network" means unique up to a renaming of the agents. For instance, consider the case of $n = 3$ below. Each network is a subset of the complete network $g_c = g_8$, and networks $\{g_2, g_3, g_4\}$ all have the same structure (a line and an isolated player[3]).

Fig. 1 Set of All Networks Among Three Players



Let $g + ij$ denote the network obtained by adding link ij to the existing network g, and $g - ij$ denote the network obtained by deleting link ij to the existing network g (i.e., $g + ij = g \cup \{ij\}$ and $g - ij = g \backslash \{ij\}$). A *path* connecting i and j in a network $g \in G$ is a sequence of players $i_1, ..., i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, ..., K-1\}$ and $i_1 = i$ and $i_k = j$. A *cycle* in a network $g \in G$ is a sequence of players $i_1, ..., i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, ..., K-1\}$ and $i_1 = i_K$. The *neighborhood* of a player i is the set of players that player i is linked to, i.e. $N_i(g) = \{j \,|\, ij \in g\}$. For any network g, let $N(g) = \{i \,|\, \exists\, j \text{ s.t. } ij \in g\}$ be the set of players who have at least one link in the network g. A non-empty network $g' \subset g$ is a *component* of g, if for all $i \in N(g')$ and $j \in N(g')$, $i \neq j$, there exists a path in g' connecting i and j, and for any $i \in N(g')$ and $j \in N(g)$, $ij \in g$ implies that $ij \in g'$.

2.2 Contractual Pairwise Stability

The payoff to a player i is represented by a function $u_i : G \to \mathbb{R}$, where $u_i(g)$ represents the net benefit that i receives if network g is in place. Given the payoffs to players as a function of the network $(u_1, ..., u_n)$, I can then define the notion of stability to predict the networks that are going to be formed at equilibrium. I build my notion of contractual pairwise stability by adding group

[3] A player is *isolated* if he has no links.

consent requirements to the pairwise stability notion of Jackson and Wolinsky (1996).[4] The basic idea behind pairwise stability is that a player can unilaterally break a link, but consent of both players is required to add a link. It also assumes that a player gives consent if the deviation makes him weakly better off.

I now consider an environment where each player belongs to a group given by the partition p of the players set N. A partition $p = \{S_1, S_2, ..., S_m\}$ of the player set N is such that $S_k \cap S_l = \emptyset$ for $k \neq l$, $\cup_{k=1}^{m} S_k = N$ and $S_k \neq \emptyset$ for $k = 1, ..., m$. Let $S(i) \in p$ be the group to which player i belongs, and P denotes the finite set of partitions. The role of groups in this environment is to eventually constrain the choices that players can make. In addition to requiring that a player gets the consent of the player he wants to form a new link with, he also needs the consent of at least a proportion q of his group members to form new links or to break his existing links.

Definition 1 A network g is *contractually pairwise stable* with respect to a payoff rule $u = (u_1, ..., u_n)$ and a consent rule (p, q) if

(i) for all $ij \in g$, either (a) $u_i(g) \geq u_i(g - ij)$ and $u_j(g) \geq u_j(g - ij)$, or

(b) for all $k \in \{i, j\}$ such that $u_k(g - ij) > u_k(g)$,
there exists $\widehat{S} \subseteq S(k)$ with $u_m(g - ij) < u_m(g)$ for all $m \in \widehat{S}$ and $\#\widehat{S} > (1 - q) * (\#S - 1)$.

(ii) for all $ij \notin g$, either (a) $u_i(g + ij) > u_i(g)$, then $u_j(g + ij) < u_j(g)$, or

(b) there exists $k \in \{i, j\}$ such that $u_k(g + ij) > u_k(g)$,
there exists $\widehat{S} \subseteq S(k)$ with $u_m(g + ij) < u_m(g)$ for all $m \in \widehat{S}$ and $\#\widehat{S} > (1 - q) * (\#S - 1)$.

The first part of this definition requires that no player wishes to delete a link that he is involved in or if they do, they do not get sufficient consent from their group members. The second part of the definition requires that if some link is not in the network and one of the involved players would benefit from adding it, then it must be that the other player would suffer from the addition of the link, or one of these players does not get sufficient consent from their group members.

There are many other notions of stability to which one could add group consent requirements. To keep the presentation neat, I will first go through the analysis using contractual pairwise stability and leave the discussion on other notions of stability to section 3.

2.3 Notions of Efficiency

I consider two commonly used notions of societal welfare. The first way of evaluating social welfare is via a utilitarian principle, i.e. the "best" network is the one which maximizes the total utility of the society. This notion was referred to as "strong efficiency" by Jackson and Wolinsky (1996), but I will simply refer to it as efficiency as in much of the subsequent literature.

[4] As in Dreze and Greenberg (1980) the word "contractual" is used to reflect the notion that groups/coalitions are contracts binding all members and subject to revision only with the consent of group members.

Definition 2 A network g is *efficient* relative to a profile of payoff rules $(u_1(g), ..., u_n(g))$ if $\sum_{i \in N} u_i(g) \geq \sum_{i \in N} u_i(g')$ for all $g' \in G$.

For the remainder of this paper, I will focus on this notion of efficiency since it is the standard notion of efficiency in this literature. However, all the results for the general model in section 4 are ordinal in nature, and therefore also apply to the pareto efficiency notion of welfare. Thus, I formally define it now.

Definition 3 A network g is *Pareto efficient* relative to a profile of payoff rules $(u_1(g), ..., u_n(g))$ if there does not exist any $g' \in G$ such that $u_i(g') \geq u_i(g)$ for all i with strict inequality for some i.

2.4 Optimal Consent Rules

Let $G(p, q)$ be the set of networks that are contractually pairwise stable under consent rule (p, q). Given a notion of efficiency, I can then define the optimal consent rules as follows. Let G^* be the set of efficient networks. I consider a consent rule to be optimal if (1) it stabilizes all the efficient networks and (2) among the set of consent requirements which can stabilize efficient networks under a given partition, the optimal consent requirement is the one that stabilizes the least number of inefficient networks (in set inclusion terms). More formally,

Definition 4 A consent rule (p, q_p) is *optimal* with respect to a profile of payoff rules $(u_1(g), ..., u_n(g))$ if

(i) $G^* \subseteq G(p, q_p)$, and

(ii) for all $\widehat{q} \neq q_p$, $G^* \subseteq G(p, \widehat{q})$, then $G(p, q_p) \subseteq G(p, \widehat{q})$.

My interest in looking for the optimal consent requirement comes from the fact that there may be consent requirements that stabilize efficient networks but also stabilize many inefficient networks in the process. Thus, one can do better by looking for the consent requirement which not only stabilize efficient networks but also stabilize the least number of inefficient networks.

Alternatively, one could choose to stabilize only some of the efficient networks and in return stabilize fewer inefficient networks. But destabilization of which inefficient networks makes it worthwhile to forgo stability of some efficient networks? It is unclear how to pick and choose among the efficient networks, and I refrain from making such comparisons here.

2.5 Breakers Away From Efficiency

I will now define a concept which will be crucial in characterizing the optimal consent rules. Observe that under the notion of contractual pairwise stability, a player can deviate by either adding or deleting a single link, and a player gives consent to a deviation if he is weakly better-off by the deviation. Given a profile of payoff rules $(u_1(g), ..., u_n(g))$, I can then define the breakers away from efficiency as follows.

Definition 5 The number of *breakers away from efficiency* K with respect to a profile of payoff rules $(u_1(g), ..., u_n(g))$:

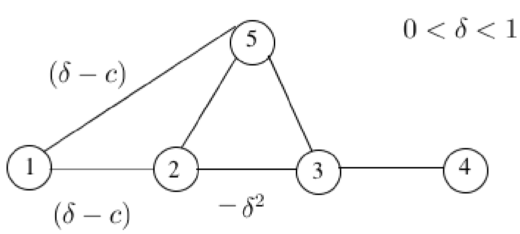
(i) is the highest number of players that will give consent to a player to deviate from one of the efficient networks G^*, and

(ii) if no players can be made strictly better-off by deviating from any of the efficient networks G^*, then $K = -1$.

3 Four Stylized Models and Form of Externalities

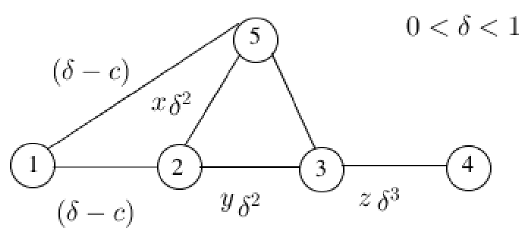
I begin by analyzing four stylized versions of the general model described in the last section. In all of these stylized models, $\delta \in (0, 1)$ is the benefit and c is the cost of maintaining a direct link. The first model is called *popularity by connections* (PBC) model. In this model, the indirect connections bring value from one of the shortest paths that is at most two distances away. Figure 2 below shows player 1's payoff in an example with five players. Player 1 gets a direct net benefit of $(\delta - c)$ from linking with player 2. He also gets an indirect benefit of δ^2 because player 2 is then linked with player 3. He does not get indirect benefit from player 4 because they are more than two distances away from each other. Player 1 is directly linked to player 5 and indirectly linked to player 5 through players 2, and 3. The key idea is that the shortest path between players 1 and 5 is through direct link, thus player 1 also gets a direct net benefit of $(\delta - c)$ from player 5. I call this the *popularity by connections* model because it can be seen as a model of popularity among individuals. Indirect cost represents the existence of people who don't recognize the individual (not directly linked) but recognize their connections (indirectly linked). For instance, in figure 2 below player 1 gets negative externality from player 3 because they are not directly connected but player 3 is directly connected to at least one of player 1's connections (players 2 and 5). Thus, player 1 is envious of players 2 and 5 for knowing player 3.

Fig. 2 Popularity By Connections Model



The form of externalities in this model has three dimensions: (1) externalities are negative, (2) indirect connections bring value from only one of the shortest paths, and (3) indirect connections bring value from paths that are at most two distances away. The other three models are variants of this model, and each varies by only one externality dimension relative to the popularity by connections model. The first variant model has positive externalities ($x = 0, y = -1, z = 0$ in figure 3 below). In the second variant model, connections bring values from multiple paths ($x = -1, y = -1, z = 0$ in figure 3 below). In the third variant model, indirect connections of any finite distances can bring value, i.e. ($x = 0, y = -1, z = -1$ in figure 3 below). This allows me to compare and understand precisely the effect of each of these dimensions of externalities on the optimal consent rule.

Fig. 3 Form of Externalities



3.1 Popularity By Connections Model

In the popularity by connections model, player i's payoff from a network g is given by

$$u_i(g) = \sum_{j\,:\,ij \in g} (\delta - c) - \sum_{k: ij \in g, jk \in g, ik \notin g} \delta^2$$

The set of efficient networks and stable networks coincide in this model. The intuition is as follows. If the net benefit from direct connections is negative, then the empty networks are efficient. They are also pairwise stable because it is in the best interest of each player to form no links. If net benefit from direct connections is positive, then the complete networks are efficient. They are also pairwise stable because it is in the best interest of each player to form links with everyone. Note that indirect connections bring negative values. If the net benefit from forming a direct connection is positive, then individuals will never want to have indirect connections and can form a direct link with any of their indirect connections to strictly improve their payoff. In this case, any consent rule will stabilize all the efficient networks. However, the optimal consent requirement is zero ($q = 0$) because it stabilizes the least number of inefficient networks (in set inclusion terms).

Proposition 1 - In the popularity by connections model, no players can be made strictly better-off by deviating from any of the efficient networks ($K = -1$) and the optimal consent rules are such that the optimal consent requirement for each partition p is $q_p = 0$.

3.2 Truncated Symmetric Connections Model

I now consider a model which is exactly the same as the popularity by connections model except that externalities are positive, i.e. $(x = -1, y = -1, z = 0)$ in figure 3. This is a truncated version of *the symmetric connections* model of Jackson & Wolinsky (1996). Player i's payoff from a network g is given by

$$u_i(g) = \sum_{j\,:\,ij\in g} (\delta - c) + \sum_{k:ij\in g, jk\in g, ik\notin g} \delta^2$$

Finding optimal consent rules can be a difficult task, but I have developed a method (series of steps) which easily identifies them. To illustrate this methodology, I will continue to consider the truncated symmetric connections model. First, I identify all efficient networks that are not pairwise stable in the model. Note that the pairwise stability is a special case of the contractual pairwise stability in which either each individual is in a group by himself ($p = \{1, 2, ...n\}$) and/or the consent requirement is zero ($q = 0$). In figure 4 below, I have summarized all efficient networks in this model. If the net benefit from direct connections is strictly above benefit from indirect connections, then complete networks are efficient.[5] The complete networks are also pairwise stable because it is in the best interest of each player to form links with everyone. If the net benefit from direct connections is strictly below benefit from indirect connections, then *star* networks are efficient. A star network is simply a network in which all players are linked to one central player and there are no other links. Such network gives $n-1$ players their maximum payoff (which leads to it being efficient) and the central player doesn't want to delete links as long they give him positive payoffs (which leads to it being pairwise stable as well). At some point when cost becomes too high (payoff from direct connections is negative), the star network is still efficient but it is not pairwise stable because the central player will always want to delete links.[6]

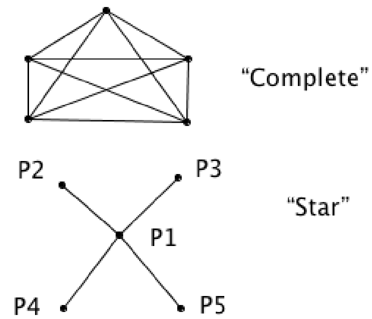
[5] A *complete* network is simply a network in which all players are directly linked to each other

[6] An *empty* network is simply a network in which no players are directly linked to each other. If cost is even higher, $\delta + ((n-2)/2)\delta^2 < c$, then empty network is efficient and pairwise stable.

Fig. 4 Pairwise Stability of Efficient Networks

$(\delta - c) > \delta^2$

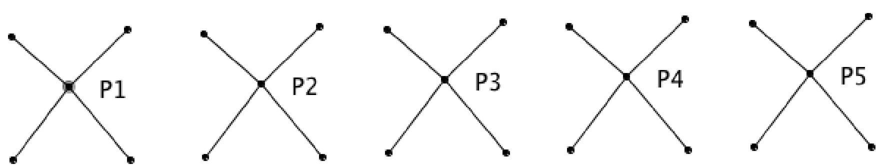
$(\delta - c) < \delta^2$



$(\delta - c) < 0$ Star is efficient but not stable!

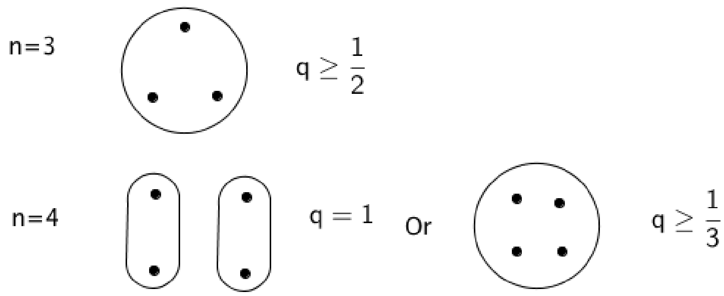
I then notice that the only deviation from unstable efficient networks (star networks) is that the central player wants to delete a link. This deviation makes everyone else worse-off because all the other players get their maximal payoff in the star network. Observe that the number of breakers away from efficiency is zero in this model, because no player would give consent to any deviations from the efficient networks. Thus, as long as the central player is in a group with some other player and is required his consent to form or to break links, star network will be contractually pairwise stabilized. For example, if player 1 is the central player, then consent rule ($P = \{\{1,3\},\{2\},\{4\},\{5\}\}, q = 1$) will be able to contractually pairwise stabilize the star network. However, there are permutations of star network where some player besides player 1 can be central player. See figure 5 below for all possible permutations of star network. To stabilize all efficient networks, one cannot use the identity of players. The only way to insure that central player is in a group with some other player is to put every player in a group with at least one other player. Moreover, to stop central player from deviating, each player should be required the consent of at least one of their group members.

Fig. 5 Permutations of Star Network



To illustrate this idea, figure 6 below gives consent rules that can contractually pairwise stabilize all efficient networks for $n = 3$ and $n = 4$. For $n = 3$, the only way to ensure that each player is required to ask consent of at least one other player is to put all three players in a group (each player has at least one other player in their group) and then the consent requirement of at least one out of two group members. Similarly for $n = 4$, one way is to build two groups of size two with consent requirement of the other group member, and another way is to build one group of size four with consent requirement of at least one out of three group members.

Fig. 6 Consent Rules Which Stabilize All Efficient Networks



So far, I have described consent rules that can stabilize all efficient networks. However, the optimal consent rule also stabilizes the least number of inefficient networks (in set inclusion terms). Consider $n = 3$ and the partition in figure 6 above (all three players in the same group). Observe that any deviation that can be made by getting consent from more than one of group members ($q > \frac{1}{2}$) will surely be made by requiring consent from only one of group members ($q = \frac{1}{2}$). Thus, a lower consent requirement will stabilize less networks. Since any $q \geq \frac{1}{2}$ can stabilize efficient networks, consent requirement of $q = \frac{1}{2}$ is optimal consent requirement because it stabilizes least number of inefficient networks. This intuition can be generalized as I will show next.

Proposition 2 - In the truncated symmetric connections model, the number of breakers away from efficiency is $K = 0$ and the optimal consent rules are such that:

(1) Each partition $p = \{S_1, S_2, ..., S_m\}$ has at least two players in each group, and

(2) the optimal consent requirement for each partition p is $q_p = \frac{1}{min\{\#S_1, \#S_2, ..., \#S_m\} - 1}$.

3.3 Attention Based Utility Model

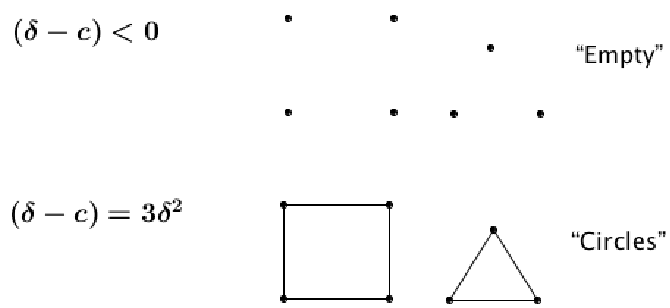
I now consider a model which is exactly the same as the popularity by connections model except that connections can bring values from multiple paths, i.e. $(x = -1, y = -1, z = 0)$ in figure 3. Player i's payoff from a network g is given by

$$u_i(g) = \sum_{j\,:\,ij \in g} (\delta - c) - \sum_{k : ij \in g, jk \in g} \delta^2$$

I call this the *attention based utility* model (ABU) because it can be seen as a model of attention seeking individuals. Being directly connected to a very connected individual brings lower value because each of his connections get lesser attention from him. For instance in figure 3, player 1 gets a net benefit of $(\delta - c)$ by having direct connection with player 2 but also faces indirect cost of $2\delta^2$ because player 2 is connected with players 3 and 5. Note that in the popularity by connections model, player 1 would only get a net benefit of $(\delta - c)$ from linking with player 5 because thats the shortest path. However, in the attention based utility model, player 1 also gets indirect cost of δ^2 from player 5 because he is connected to player 2. It is in this sense that I have changed the "control" that players have towards eliminating externalities on them. In the popularity by connections model, players could eliminate externalities by both adding and deleting links, and now they can only eliminate externalities by deleting links. In the popularity by connections model, player 1 didn't incur indirect cost of δ^2 from player 5 by having a direct connection with him.

In figure 8 below, I have only provided those efficient networks that are crucial to understanding the result. If the net benefit from direct connections is negative, then the empty networks are efficient. They are also pairwise stable because it is in the best interest of each player to form no links. If the net benefit from direct connections is positive and equal to three times the magnitude of indirect costs, then the efficient networks are such that each component is a *circle*. A *circle* is a network that has a single cycle and such that each node in the network has exactly two neighbors. In particular, there are efficient networks with multiple components (as shown below). To see that networks with multiple components can be efficient, consider the popularity by connections model. In that model, players could eliminate externalities by adding links. If the net benefit from forming a direct connection is positive, then it is never efficient to have players with indirect connections. Moreover, one could strictly increase the network value by forming a direct link between any two players with existing indirect connections. However in the attention based utility model, this is not the case. Even if the net benefit from forming a direct connection is positive, it may be efficient to have players with indirect connections. To see this, note that adding a link between any two players with existing indirect connections will not eliminate the externalities they have on each other. Thus, it is possible for efficient networks to be such that the net benefit from a direct connection is positive but not everyone is directly connected to each other. In particular, when $(\delta - c) = 3\delta^2$, then it is efficient for each player to have exactly two links, and such networks are circles.

Fig. 7 Pairwise Stability of Efficient Networks



Circles are efficient but not stable!

The net benefit of a direct link is $(\delta - c) = 3\delta^2$ and a player adding a link incurs an additional indirect cost of $2\delta^2$ since all players have two existing links. Therefore, each player has an incentive to add a link, and the efficient networks are not pairwise stable. In the efficient network described above, any two players adding a link in the circle encompassing exactly four players would strictly benefit from such a deviation, the two other players in the same circle would be strictly worse off since they incur additional indirect costs of $2\delta^2$ each, and all the other players are weakly better off since their payoffs are unchanged. Thus, consent requirement from at least $n - 2$ players is needed to stop this deviation ($K \geq (n-1) - 2$). However, to actually stabilize all the efficient networks, one may need consent requirement that is even higher (the only consent requirement higher is in fact the highest consent requirement of $(n - 1)$). Nonetheless, it is clear that externalities of this form require really high consent requirement to stabilize the efficient networks.

Proposition 3 - In the attention based utility model:

(i) For $n = 3$, the number of breakers away from efficiency is one ($K = 1$). The unique optimal consent rule is to have all three players in the same group ($p = \{\{1, 2, 3\}\}$) with optimal consent requirement of unanimity $q_p = 1$.

(ii) For $n = 6$, the number of breakers away from efficiency is $K \geq 1$. An optimal consent rule is to have all six players in the same group ($p = \{\{1, ..., 6\}\}$) with optimal consent requirement of $q_p = \frac{4}{5}$.

(iii) For $n = 4$, $n = 5$ and $n \geq 7$, the number of breakers away from efficiency is $K \geq (n - 3)$. The unique optimal consent rule is to have all the players in the same group ($p = \{\{1, ..., n\}\}$) with the optimal consent requirement of $q_p = \frac{n-2}{n-1}$.

3.4 Generalized Popularity By Connections Model

I now consider a model which is exactly the same as the popularity by connections model except that indirect connections of any finite distances can bring value, i.e. $(x = 0, y = -1, z = -1)$ in figure 3. In figure 3, the payoff structure is similar to the popularity by connections model except that player 1 also gets an indirect cost of δ^3 because player 2 is linked to player 3, and player 3 is then linked with player 4. In this model, player i's payoff from a network g is given by

$$u_i(g) = \sum_{j \neq i} \delta^{t(ij)} - \sum_{j\,:\,ij \in g} c$$

where $t(ij)$ is the number of links in the shortest path between i and j (setting $t(ij) = \infty$ if there is no path between i and j).

The set of efficient networks remain the same as in the popularity by connections model, and using the same arguments as before one can show that the optimal consent rules are also the same. The main idea is as follows. If the net benefit from forming a direct connection is positive, then individuals will never want to have any indirect connections (regardless of how far away). Moreover, they can form a direct link with any of their indirect connections to strictly improve their payoff. Similarly, in the symmetric connections model (which has positive externalities and allows for benefit to come from indirect connections that are any finite distance away), one can show that the set of efficient networks and optimal consent rules are the same as in the truncated symmetric connections model. To see this, note that the positive value from indirect connections that are two distances away is the maximal value one can get from any indirect connections ($\delta^2 > \delta^3 > ... > \delta^{n-1}$). The rest of the arguments then follow from the truncated symmetric connections model.

> **Proposition 4 -** In the generalized popularity by connections model, no players can be made strictly better-off by deviating from any of the efficient networks $(K = -1)$ and the optimal consent rules are such that the optimal consent requirement for each partition p is $q_p = 0$.

3.5 Comparison of the Four Models

I will now compare these four models to understand precisely the effect of each of these dimensions of externalities on the optimal consent rule. In the table below, the first column indicates the name of each model in abbreviated form. The second column indicates the form of externalities in these models along three dimensions: (1) negative or positive externalities, (2) whether the payoff comes from the shortest path or multiple paths, and (3) whether the payoff comes from indirect connections of only two distances away or indirect connections of any finite distances away. The third column indicates the number of breakers away from efficiency (indicated by K) which characterizes optimal consent rules.

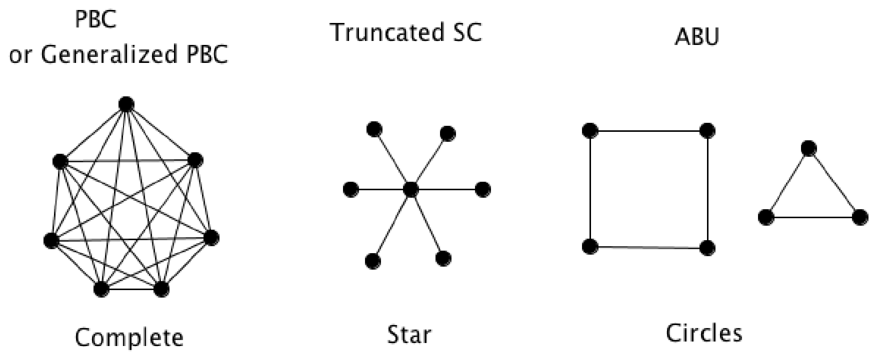
Models	Form of Externalities	K
PBC	Negative Externalities Shortest Path - Two Distances Away	-1
Truncated SC	Positive Externalities Shortest Path - Two Distances Away	0
ABU	Negative Externalities Multiple Paths - Two Distances Away	$K \geq n-3$ if $n \geq 7$
Generalized PBC	Negative Externalities Shortest Path - Any Finite Distances Away	-1

The first thing to note is that the attention based utility model (ABU) has the highest K. So the consent rules are most sensitive to whether players can eliminate externalities by only deleting links (payoffs come from multiple path) or by both adding and deleting links (payoff comes from the shortest path). To understand the reasoning behind this, consider the shapes of the key efficient networks when net benefit from direct connections is positive $(\delta - c) > 0$, given in figure 6 for $n = 7$.[7] The main difference between the ABU model and the other models is that the efficient network in the ABU model has multiple non-empty components.[8] Since a deviation in one component doesn't affect payoffs of players in other unaltered components, all the players from unaltered components would give consent to such deviation. This results in a large number of players that will give consent to deviations. Thus, in environments where players have less control over their externalities, then it is optimal to give them higher influence over the decisions of others.

[7] If the net benefit from direct connections is negative $(\delta - c) < 0$, then the efficient networks and the pairwise stable networks are the empty networks in all the models.

[8] The Co-author model of Jackson and Wolinsky (1996) is another example of a model where players can eliminate externalities by only deleting links. The efficient network consists of $n/2$ separate pairs for n even.

Fig. 8 Efficient Networks



The second thing to note is that the optimal consent rules depend on whether externalities are positive ($K = 0$ in Truncated SC) or negative ($K = -1$ in PBC). And finally, the optimal consent rules (and the efficient networks) do not depend on whether the payoff comes from indirect connections of only two distances away or indirect connections of any finite distances away.

3.6 Different Notions of Stability

I consider the robustness of my results with respect to the four models (the first column indicates the name of each model in abbreviated form). Recall, there are other notions of stability besides pairwise stability to which one could add group consent requirements. The table below indicates the number of breakers away from efficiency (which characterizes optimal consent rules) for each model and for four different notions of stability to which we add group consent requirements. The first notion is the pairwise stability of Jackson and Wolinsky (1996), which is the notion I used to build the contractual pairwise stability by adding group consent requirements. Similarly, I add group consent requirements to three other notions of stability to check the robustness of my results. Recall, pairwise stability allows for single link deviations and consent is given to deviations by players that are made weakly better off. The second notion is a weaker notion than pairwise stability and also introduced by Jackson and Wolinsky (1996). It allows for single link deviations and consent is given to deviations by players that are made strictly better off. The third notion is strong stability of Jackson and van den Nouweland (2005). It allows for multiple link deviations and consent is given to deviations by players that are made weakly better off. And finally, the fourth notion is strong stability of Dutta and Mutuswami (1997). It allows for multiple link deviations and consent is given to deviations by players that are strictly better off. I find that the results are robust to all these notions of stability for PBC, Truncated SC, and Generalized PBC models. For the attention based utility model, the optimal consent rules can depend on the notion of stability, but it always has higher number of breakers away from efficiency than the other three models. Thus, my main finding that the attention based utility model has the highest number of breakers away from efficiency is robust to the different notions of stability.

Models	JW (1996) Single/Weak	JW (1996) Single/Strict	JvdN(2005) Multiple/Weak	MD (1997) Multiple/Strict
PBC	-1	-1	-1	-1
Truncated SC	0	0	0	0
ABU	$K \geq n-3$ if $n \geq 7$	$K \geq 1$ if $n \geq 7$	$K \geq n-3$ if $n \geq 7$	$K \geq 3$ if $n \geq 11$
Generalized PBC	-1	-1	-1	-1

4 Characterization of Optimal Consent Rules

Using the intuition from the four stylized models, I will now characterize the optimal consent rules for the general model. I also consider several interesting extensions of the general model. Firstly, I consider more general rules that allow each group to have its own consent requirement. Secondly, I consider environments with heterogeneous players. And finally, I consider the case which allows for both, more general rules that allow each group to have its own consent requirement, and environments with heterogeneous players. After discussing the characterization of optimal consent rules for the general model, I will discuss the characterization of optimal consent rules for these extensions.

The optimal consent rules can be identified in three steps:

Step 1 - Consider efficient networks that are not pairwise stable

Step 2 - Consider all possible deviations from them

Step 3 - How many players will give consent to such deviation?

If at most K players will give consent (because they are made weakly better off) to any deviations from any of the efficient networks, then consent of at least $K+1$ will stabilize all the efficient networks. Any consent level lower than $K+1$ (such as the consent of at least K or the consent of at least $K-1$) will not stabilize all the efficient networks, and any consent level higher than $K+1$ (such as the consent of at least $K+2$ or the consent of at least $K+3$) can stabilize more of the

inefficient networks.[9] Therefore, the minimal consent requirement that will stabilize all the efficient networks is the optimal consent requirement because it also stabilizes the least number of inefficient networks (in set inclusion sense). For example, in the truncated symmetric connections model, no one will give consent to any deviations from any of the efficient networks ($K = 0$). So consent from at least one other player is the optimal consent requirement to build the efficient networks. Thus, any partition $p = \{S_1, S_2, ..., S_m\}$ with at least $K + 2$ number of players in each group (each player has $K + 1$ players in his group excluding himself) with optimal consent requirement of $q = \frac{K+1}{min\{\#S_1,\#S_2,...,\#S_m\}-1}$ will stabilize all the efficient networks.

> **Theorem 1 -** If K is the number of breakers away from efficiency with respect to a profile of payoff rules $(u_1(g), ..., u_n(g))$, then optimal consent rules are such that:
>
> (1) Each partition $p = \{S_1, S_2, ..., S_m\}$ has at least $K + 2$ number of players in each group, and
>
> (2) the optimal consent requirement for each partition p is $q_p = \frac{K+1}{min\{\#S_1,\#S_2,...,\#S_m\}-1}$.

Theorem 1 shows that the size of groups and the consent requirement (group's influence) that can optimally lead to formation of efficient networks are positively related. For instance, the optimal consent rule can be big groups with high consent requirement or mid-size groups with at least a medium consent requirement or small groups with at least a low consent requirement. More precisely, the bigger is the value of K, the bigger the size of groups and the higher the consent requirement (group's influence) needs to be to stabilize the efficient networks.

Observe that this result is robust to different notions of stability (besides contractual pairwise stability considered above) because none of the arguments in the proof depend on the type of deviations allowed. Moreover, this result also holds for the pareto efficient notion of efficiency (see proof of theorem 1 in the appendix).

4.1 Heterogeneous Consent Requirements

In the first extension of the model, I consider heterogenous consent requirements. A consent rule is now a pair (p, q) where $p = \{S_1,S_2,..., S_m\}$ is a partition of the set of players into groups, and $q = (q_1, q_2, ..., q_m)$ is a vector of consent requirements, one for each group. Suppose the highest number of players that will give consent to any deviations from any of the efficient networks is K. Then any partition $p = \{S_1, S_2, ..., S_m\}$ with at least $K + 2$ number of players in each group with consent requirement of $q_i = \frac{K+1}{\#S_i-1}$ for each group can stabilize the efficient networks.

[9] Let $G(K)$ be the set of networks that are stable under consent requirement from at least K players. Note that for $j \in \{1, 2, ..., n-1-K\}$, any deviation that can be made by getting consent from at least $K+j$ players will surely be made by requiring consent from only at least K players. Thus, a lower consent requirement refines stability, i.e. $G(K) \subseteq G(K+j)$. Therefore, the minimal consent requirement that will stabilize efficient networks is in fact optimal because it also stabilizes the least number of inefficient networks (in set inclusion sense).

Theorem 2 - If K is the number of breakers away from efficiency with respect to a profile of payoff rules $(u_1(g), ..., u_n(g))$, then optimal consent rules are such that:

(1) Each partition $p = \{S_1, S_2, ..., S_m\}$ has at least $K + 2$ number of players in each group, and

(2) the optimal consent requirement for each group in partition p is $q = (\frac{K+1}{\#S_1 - 1}, ..., \frac{K+1}{\#S_m - 1})$.

4.2 Ex-ante Heterogeneous Players

In the second extension of the model, I consider a society composed of $T < \infty$ types of players. Let N_t be the set of players belonging to type t, $t = 1, 2, ..., T$. The set of players is then $N = \cup_{t=1}^{T} N_t$. Consent requirement is now contingent on the type of individuals, $q = (q_1, q_2, ..., q_T)$. However, this vector of consent requirement is common across all groups. For a given partition $p = \{S_1, S_2, ..., S_m\}$, S_{it} denotes the subset of type t in group i. So each player requires consent from at least q_1 proportion of type 1 players in their group, at least q_2 proportion of type 2 players in their group, and so on.

Theorem 3 - If $K = (K_1, K_2, ..., K_T)$ is the vector of breakers away from efficiency for each type with respect to a profile of payoff rules $(u_1(g), ..., u_n(g))$, then optimal consent rules are such that:

(1) Each partition $p = \{S_1, S_2, ..., S_m\}$ has at least $K_t + 2$ number of players of type t, and

(2) the optimal consent requirement for each type in partition p is
$q = (\frac{K_1+1}{min\{\#S_{1t}, \#S_{2t}, ..., \#S_{mt}\} - 1}, ..., \frac{K_T+1}{min\{\#S_{1t}, \#S_{2t}, ..., \#S_{lt}\} - 1})$.

4.3 Heterogeneous Consent Requirements and Players

In the third extension of the model, I consider heterogenous consent requirements and heterogenous players. A consent requirement is now contingent on both the group and the type of players, i.e. q_{it} is the consent requirement from type t in group i. So each player in group i requires consent from at least q_{it} proportion of type t.

Theorem 4 - If $K = (K_1, K_2, ..., K_T)$ is the vector of breakers away from efficiency for each type with respect to a profile of payoff rules $(u_1(g), ..., u_n(g))$, then optimal consent rules are such that:

(1) Each partition $p = \{S_1, S_2, ..., S_m\}$ has at least $K_t + 2$ number of players of type t, and

(2) the optimal consent requirement from type $t \in \{1, 2, ..., T\}$ in group $i \in \{1, 2, ..., m\}$ in partition p is $q_{it} = (\frac{K_t+1}{S_{it}-1})$.

5 Conclusion

In many situations, one can see co-existence of groups and networks. In this paper, I analyzed how the two structures can influence each other when individuals need the consent of their group members to form/break links. Thus, I considered environments where each player belongs to a group (given by a partition p of the players set N), and he needs the consent of at least a proportion q of his group members to form new links or to break his existing links. I find that the size of groups and the consent requirement (group's influence) that can optimally lead to formation of efficient networks are positively related. For instance, the optimal consent rule can be big groups with high consent requirement or mid-size groups with at least a medium consent requirement or small groups with at least a low consent requirement. This result is robust and continues to hold even in environments where each group has its own consent requirement, and players are ex-ante heterogeneous. I also showed how the size of groups and the consent requirement that can optimally lead to the formation of efficient networks depend on the form of network externalities. In environments with negative externalities and where the payoff comes from multiple paths, one needs big groups with high consent requirements to stabilize efficient networks. On the other hand, in environments where the payoff comes from the shortest path, small groups with low consent requirements can stabilize efficient networks. In particular, in environments where the payoff comes from the shortest path and externalities are negative, one can stabilize efficient networks even without any group consent requirements.

In this paper, the role of groups has been to eventually constrain the choices that players can make. However, there are situations in which the groups can help enlarge the choices that players can make. For instance, in the case of the European Union, a member country may be able to form certain trade links because it belongs to the European Union. Otherwise, this country may not be able to form these links on its own. Similarly, in many eastern cultures, family members can help arrange marriages which wouldn't be possible otherwise. I considered group consent requirement as the mechanism through which groups can constrain the choices that players can make. However, it remains unclear the precise mechanisms through which the groups can help enlarge the choices that players can make. It seems to me that exploring such situations further is a fruitful avenue for future research.

Part II

The Weakness of Weak Ties in Referrals: An Obstacle for the Upwardly Mobile Black Men in the Private Sector

1 Introduction

It is well known that on average whites earn higher wages than blacks in the US.[10] A little less known fact is that the wage gap between whites and their black counterparts (same individual characteristics) widens as one moves up the wage hierarchies. Kaufman (1983) was the first to demonstrate that black men face the greatest disadvantage in labor market divisions (based on occupation and industry) at the high end of the wage hierarchy (based on mean wages). Using more contemporary data, Grodsky and Pager (2001) also explore the relationship between mean wages of occupations and black-white wage gaps. Although they do not find any relationship in the public sector, they do find a positive relationship in the private sector (on average, occupations with higher mean wages face wider black-white wage gaps). Huffman (2004) also confirms this finding for the wage hierarchy of local labor markets (labor market divisions based on occupation, industry, and metropolitan area). Despite the mounting evidence, it remains unclear which mechanisms are behind the increase in wage discrimination and inequality as one moves up the wage hierarchies.[11] Gaining a better understanding of these mechanisms is of paramount importance in forming public policy and the subject of this paper.

At this point, there is an empirical consensus, both in economics and in sociology, on the widespread use of employee referrals in the labor market.[12] About 50% of U.S. jobs are found through social networks and about 70% of firms have programs encouraging referral-based hiring.[13] In this paper, I show that the employee referrals can be behind the increase in wage penalty for blacks as one moves up the wage hierarchies. I build a model of employee referrals with three sorts

[10]For an excellent review of the black-white wage gap literature in economics, see Altonji and Blank (1999).

[11]This increase in black-white wage gaps is in both a relative and an absolute sense. Grodsky & Pager (2001) also demonstrate that their finding cannot be explained by the association between patterns of wage dispersion and average pay levels across occupations. In other words, they find that wages are not more dispersed in the highest-paying occupations. Similar to Grodsky & Pager (2001), Huffman (2004) also shows that his result is not an artifact of relatively high levels of wage variability at the upper end of the wage hierarchy. They show that the wage variability is largely independent of the average overall pay in a job.

[12]See Topa (2011), for a survey of the economics literature, and Marsden and Gorman (2001), for a survey of the sociology literature.

[13]Granovetter (1974) showed that roughly 50% of workers are referred to their jobs by social contacts, a finding that has been confirmed in more recent data (Topa 2011). A leading online job site estimates according to their internal data that 69% of firms have a formal employee referral program (CareerBuilder 2012).

of agents: firms, employed workers, and unemployed workers. The firm has no relationships/ties with the unemployed workers, but the employed workers have ties with them. As a result, the firm is indirectly connected to the unemployed workers through its employees. A tie between an employed worker and an unemployed worker can be either weak (acquaintance) or strong (close friends).[14] An unemployed worker is characterized by his ability level and the type of ties that he has with the employed workers. Each unemployed worker's ability level is his private information, but his ties (employed workers) observe a signal about his ability level. The type of signal that an employed worker receives depends on his relationship with the unemployed worker (weak ties have more noise in their signals). This model involves two main features: unemployed workers choose which employed workers to ask for referrals based on the type of ties (weak or strong) they have with them, and firms try to infer some information about the abilities of the unemployed workers through the recommendations of its employees. An employed worker receives some gratitude from the unemployed worker for providing him with a recommendation, and he faces reputation costs of providing an inaccurate recommendation. The value of this gratitude is strictly increasing in the wage offered to the unemployed worker, and the strength (weak or strong) of his relationship with the unemployment worker. The reputation costs depend on the distance between his recommendation and his expected value of the unemployed worker's ability. A firm can influence the reputation costs of its employees by choosing the magnitude of their penalty for providing an inaccurate recommendation. After presenting the main features of this model, I extend it to consider the races (blacks or whites) of unemployed workers and how their races can determine the type of ties they have access to. This extension allows me to explain the observed pattern of increasing black-white wage gap as one moves up the wage hierarchies.

The starting point of my analysis is to consider how the wage offered by a firm depends on the recommendations of the employed worker, and the type of tie between the employed worker and the unemployed worker. A strong tie is more informative than a weak tie, in that the interval of ability levels that a weak tie can infer through his signals $[0, \alpha_{weak})$ is included in the interval of ability levels that a strong tie can infer through his signals $[0, \alpha_{strong})$, i.e. $\alpha_{strong} > \alpha_{weak}$. This follows from the fact that an employed worker with a weak tie has more noise in his signals.[15] An employee's recommendations are strictly increasing in ability values until either the recommendations hit the

[14]The issues of strong versus weak ties have been considered before. For instance, see Granovetter (1974), Montgomery (1992, 1994), Karlan et al (2009) and Zenou (2013, 2015). Similar to my model, most works have focused on the ties between the unemployed and employed workers. However, Karlan et al (2009) consider the ties between the employed workers and firms.

[15]The underlying idea behind this is that the employed worker can learn about the unemployed worker's ability θ, but there is an upper-bound α (interpreted as the ability level of the employed worker) to how much he can learn. An employed worker with a strong tie receives a precise signal if the unemployed worker's ability is below his threshold ability level (for $\theta \leq \alpha$), and signals are noisy otherwise. An employed worker with a weak tie can learn less about the unemployed worker's ability. Therefore, he only receives a precise signal if the unemployed worker's ability is sufficiently below his threshold ability level (for $\theta \leq \alpha - \varepsilon$ where $\alpha \geq \varepsilon > 0$), and signals are noisy otherwise. Thus, an employed worker with a weak tie can infer smaller interval of ability levels, because he has more noise in his signals.

upper bound of recommendation values (the highest ability value) or the ability value reaches the informativeness level of the tie (α_{weak} for weak ties and α_{strong} for strong ties). For any ability levels above this point, the employee's recommendations are the same and firms will not be able to infer ability levels. Whether the firm can infer a bigger interval of ability levels from strong ties or weak ties depends on two opposing forces in the model. On the one hand, strong ties get higher gratitude value from providing referral, which leads them to send higher recommendation values. If a strong tie runs out of recommendation values that he can send before the signal reaches the informativeness level of the weak tie, then firms can infer smaller interval of ability levels from a strong tie. On the other hand, a weak tie is less informative than a strong tie, and therefore he will reach his informativeness level before a strong tie. At the equilibrium, firms maximize the number of signals they can infer by setting the highest penalty value for receiving an inaccurate recommendation from its employees. As a result, an employee's recommendation values will be influenced by his reputation costs more than his gratitude benefits, and firms will be able to infer a bigger interval of ability levels from a strong tie than from a weak tie. In particular, firms will be able to infer sufficiently high ability levels (above some threshold) from only strong ties. If some high ability workers don't have strong ties, then low ability workers (whose abilities the firm cannot infer using a weak tie) can get higher wage offers from using weak ties. This is because their use of weak ties leads to firm pooling them with these high ability workers who don't have strong ties. On the other hand, high ability workers can get better wage offers from using strong ties because this leads to firm infering that they are high ability workers.

I develop two applications of the employee referrals model. In the first application, I consider an extension of this model with two races (blacks and whites) and two occupations. I assume that fewer blacks have strong ties with employed workers than whites. This assumption follows from two empirical facts: employment differentials between blacks and whites in the US and the homophily feature of social networks (tendency to interact with others that have similar characteristics).[16] Since blacks have lower employment rates than whites, they have to rely on their cross-race ties more than whites do.[17] Such cross-race ties tend to be weaker because individuals tend to interact more often with others that belong to their own race.[18] There are two types of occupations, manual labor intensive occupation and human capital intensive occupation. In the manual labor intensive occupation, output is only increasing in ability up to some level, and having human capital beyond that doesn't add to output. In the human capital intensive occupation, output is always increasing

[16] See McPherson, Smith-Lovin and Cook (2001) for a survey on homophily.

[17] See Lang & Lehmann (2012) for a brief review of employment differentials between blacks and whites in the US.

[18] See Lin et al (1981) for a discussion on the extension of homophily principle which considers the strength of ties. See Homans (1950), Laumann (1966), Laumann and Senter (1976), Verbrugge (1979) for some references. See Tsui and O'Reilly (1989), and Thomas (1990) for some further evidence. Based on a national survey, Marsden (1987, 1988) reports that only 8 percent of people have any people of another race with whom they discuss important matters, which suggests the weakness of most cross-race ties. Jackson (2007) shows that in a network of the friendships in a high school from the Ad Health Data Set, most cross-race ties are weak.

in ability. In this extension of the model, I show that the lack of access to strong ties for blacks can explain the observed pattern of increasing black-white wage gap as one moves up the wage hierarchies. In human capital intensive occupation, firms will offer higher wages to high ability workers with strong ties because firms can infer that they are high ability workers. If only few blacks have strong ties, then both below and above average ability black workers enter higher earning occupations through weak ties, firm pools them together, and on average high ability black workers get relatively lower wage than their white counterparts (same ability levels).

In the second application, I explore some implications of the employee referrals for job search. The model provides new insights about the employee referrals mechanism, which can help explain the empirical findings about the use of different types of ties in job search and the returns to ties. In the pioneering work of Granovetter (1974), he documents that a large proportion of jobs are found through weak ties. He argues that the weak ties to individuals with whom one has few common friends are most useful for job search, because they provide access to otherwise unobtainable information about job openings. This finding of the frequent use of weak ties led to the coining of the well-known phrase, the "strength of weak ties". However, the evidence about the use of weak ties is mixed. Many studies have found the frequent use of strong ties (Murray, Rankin, and Magill (1981), Bridges and Villemez (1986), Marsden and Hurlbert (1988)). Similarly, the empirical evidence about the returns to ties is mixed. Some studies show that workers who found their jobs through family, friends, and acquaintances earned more than those using formal and other informal job-search methods (Rosenbaum et al. (1999), Marmaros and Sacerdote (2002)). Others found no significant effect (Bridges and Villemez (1986), Holzer (1987), Marsden and Gorman (2001)) or even negative effect (Elliott (1999), Green, Tigges, and Diaz (1999)).

The employee referrals model provides a novel finding: returns to using a tie varies with the unemployed worker's ability, the tie strength, and the proportion of workers who have access to different type of ties. This finding can then simultaneously explain the mixed evidence about the use of different types of ties in job search, and the mixed evidence about the wage differentials between workers who found jobs through referrals and workers who found jobs by formally applying to firms. (1) The higher is the number of high ability workers who don't have strong ties, the higher are the returns from using weak ties, and the more workers use weak ties to pool with high ability workers. Contrary to the existing explanation, the frequent use of weak ties may not be due to its efficiency in matching workers and firms. When the access to strong ties is really scarce for high ability workers, many workers (even those with access to strong ties), use weak ties to pool with high ability workers. (2) The lower is the informativeness of the weak tie, the lower are the returns from pooling with high ability workers. This follows from the fact that really low ability workers are also included in the pool. Applying directly to the firm is equivalent to applying through a tie with the minimal informativeness level. If some high ability workers have access to ties, then the wage offer from pooling through direct application is below average ability. As a result, for above average ability workers, the returns to using a tie is always positive. However, for below average ability workers, the returns to using a tie can be small, insignificant, and even negative. Although these predictions do not emerge in the existing model of employee referrals, they are consistent with existing empirical evidence, suggesting that the tie selection and the strategic recommendation are important aspects of employee referrals and are useful for understanding job search.

Before presenting the model formally, let me mention some closely related literature. Firstly, there is a large literature on the employee referrals. The existing models in this literature either do not consider the incentives of the referrers (employed workers providing referrals) or they do not consider the type of ties selected by the unemployed workers.[19] As a result, these models are unable to explain the mixed evidence about the use of different types of ties in job search, and the mixed evidence about the wage differentials between workers who found jobs through referrals and workers who found jobs by formally applying to firms.[20] Secondly, there is a growing literature which considers social networks to explain racial inequality in labor market outcomes.[21] I contribute to this literature by using the employee referrals (a social networks based mechanism) to explain the widening of the black-white wage gap as one moves up the wage hierarchies. It is hard to explain this effect outside of my employee referrals model. In particular, there is a large literature on statistical discrimination.[22] In this literature, firms have uncertainty about the ability of the unemployed workers and they use the race of a worker as a signal of his ability level, which results in similarly skilled workers of different races to have different wages. Most mechanisms that can reduce this uncertainty would suggest shrinking of the wage discrimination as one moves up the wage hierarchies. For instance, the informational asymmetries between the firm and unemployed workers likely decrease for highly educated workers because education acts as a "signal" about their abilities (Spence (1973)).[23] At the same time, firms have a stronger incentive to invest in technologies (formal screening mechanisms such as aptitude tests and other attribute measurements) that accurately reveal the cognitive ability of workers in human capital intensive occupations. Since high earning occupations require higher education and they are more human

[19]There are two main classes of referral models: (i) models in which ties transmit information about job opportunities, and (ii) models in which ties transmit information about the productivity of workers. Calvo-Armengol (2004), Calvo-Armengol and Jackson (2004, 2007), Calvo-Armengol and Zenou (2005), Ioannides and Soetevent (2006), Fontaine (2008), Cahuc and Fontaine (2009), Bramoulle and Saint Paul (2009), and Gaelotti and Merlino (2014) belong to the first class of models. Montgomery (1991), Simon and Warner (1992), Arrow and Borzekowski (2004), Dustman et al (2011), and Galenianos (2012) belong to the second class of models. Both of these classes of referral models suggest that the returns to ties are positive, which is inconsistent with the mixed evidence about the returns to ties. See Saloner (1985) for a model which consider the incentives of the referrers. However, his model only focuses on the referrers and is therefore unable to provide any predictions about the unemployed workers (such as their returns to finding jobs through ties and their use of different type of ties).

[20]Loury (2006) has an explanation for the mixed evidence about the returns to ties but not the use of ties. Loury argues that workers with limited access to wage offers through other channels may rely on employee referrals as a last resort. Many of these workers would have lower rather than higher wages compared with those using other means to find jobs.

[21]See Ioannides & Loury (2004) for a survey.

[22]See Lang & Lehmann (2012) for a review of theories about race discrimination.

[23]See Lang & Manove (2011) for a detailed discussion and justification of this argument.

capital intensive, there is arguably lower statistical discrimination in high earning occupations relative to low earning occupations.

Thus, the contribution of this paper is to provide an employee referrals model which is consistent with the mixed findings about the use of ties and the returns to ties, as well as with the widening of the black-white wage gap as one moves up the wage hierarchies. The rest of the paper is organized as follows. Section 2 presents the employee referrals model. I will first present a motivating example and then provide the general results. In section 3, I extend the base model by considering multiple races and occupations to show the widening of the black-white wage gap as one moves up the wage hierarchies. In section 4, I explore some implications of the employee referrals for job search. I extend the base model by allowing workers to apply directly to the firm, and through ties. This extension helps explain the mixed findings about the returns to ties. Section 5 concludes with some policy implications of the model. All proofs are available in the appendix.

2 A Model of Employee Referrals

The model involves an environment with three sorts of agents: unemployed workers, employed workers, and a single firm. The firm has no relationships/ties with the unemployed workers, but the employed workers have ties with the unemployed workers. As a result, the firm is indirectly connected to the unemployed workers through its employees. A tie $t \in (t_{weak}, t_{strong})$ between an employed worker and an unemployed worker can differ by its strength (weak or strong). To keep the presentation neat, I assume that each employed worker has a tie (either weak or strong) with only one unemployed worker, and each unemployed worker has at most one tie of each type. Thus, an unemployed worker's type is a pair (θ, T), where θ is his ability level and the set T indicates the type of ties that he has with the employed workers. There is a continuum of ability levels $\theta \in [0, 1]$ and they are uniformly distributed $\theta \sim u[0, 1]$. I assume that each unemployed worker has access to a weak tie $Prob(t_{weak} \in T) = 1$, but only β of the unemployed workers have access to a strong tie $Prob(t_{strong} \in T) = \beta$. An employed worker's type is a pair (θ, t) where θ is the ability level of the unemployed worker he has a tie with, and t is the strength of his tie with this unemployed worker. I take the network of relationships/ties between the agents as given, and examine its influence on the wage determination process.

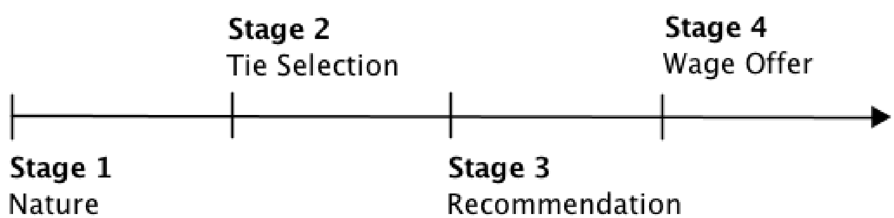
The role of network in this paper is to transmit information. An unemployed worker's ability level is his private information, but his ties receive a signal $s \in [0, 1]$ about his ability level, and the firm tries to infer some information about the ability level through the recommendations of the employed workers. The firm can only infer the lowest ability level, weak ties can infer ability levels up till $\alpha_{weak} \geq 0$, and strong ties can infer ability levels up till $\alpha_{strong} > \alpha_{weak}$. In this sense, both type of ties are more informative than firms about the ability levels of the unemployed workers, and strong ties are more informative than weak ties. Formally, employed ties and the firm receive the following signals about the ability levels of the unemployed workers.

$$s(\theta, \alpha_t) = \begin{cases} \theta & \text{if } \theta \in [0, \alpha_t) \\ x & \text{if } \theta \in [\alpha_t, 1] \end{cases} \tag{1}$$

where $x \sim u[\alpha_t, 1)$, $\alpha_t \in \{0, \alpha_{weak}, \alpha_{strong}\}$, $\alpha_{strong} > \alpha_{weak} \geq 0$, and $\alpha_t = 0$ for the firm.

My model consists of four stages, which are depicted in figure 1. I begin by describing the model and then discuss the economic content of my modeling assumptions.

Fig 1. Model Timeline



Stage 1: Nature. At the beginning of this stage, nature determines the ability levels of unemployed workers and the type of ties they have access to. Then, firms and employed workers receive signals about the ability levels of unemployed workers.

Stage 2: Tie Selection. At this stage, each unemployed worker (θ, T) chooses tie type $t \in T$ that maximizes his wage,

$$\pi_U(\theta, T) = w(\rho(s, t), t) \tag{2}$$

where wage depends on the tie's recommendation $\rho(s, t)$, and the tie's type t. Observe that the tie's recommendation $\rho(s, t)$ is also a function of the tie's type t and the signal s that he received. At equilibrium, the proportion of unemployed workers with ability level θ who chose tie type t is denoted by $f(t|\theta)$. I assume rational expectations in that employed ties and firm know $f(t|\theta)$ for each ability level θ and tie type t, and they use it to form their expectations.

Stage 3: Recommendation. An employed worker faces both gratitude benefits and reputation costs of providing a recommendation $\rho \in [0, 1]$. The value of gratitude is strictly increasing in the wage $w(\rho, t)$ offered to the unemployed worker. The reputation cost of providing an inaccurate recommendation is measured by the square of the difference between the recommendation and the employed worker's expected value of ability $(\rho - E[\theta|s, t])^2$. Given the signal s, and tie type t, each employed worker provides recommendation ρ that maximizes his payoff,

$$\pi_E(s, t) = w(\rho, t) - r(\rho - E[\theta|s, t])^2 \tag{3}$$

where $r > 0$ is the reputation costs parameter. The bigger is the reputation costs parameter, the lower are the incentives to provide inaccurate recommendations. Note that the employed worker's expected value of ability is equal to the signal if $s \in [0, \alpha_t)$ and is equal to the weighted average of ability values $E[\theta|s,t] = \int_{\alpha_t}^{1} \theta \frac{f(t|\theta)}{\int_{\alpha_t}^{1} f(t|\theta)d\theta} d\theta$ if $s \in [\alpha_t, 1]$.

Stage 4: Wage Offer. Given the recommendation ρ, and the tie type t, the firm forms its valuation $v(\rho, t) = E[\theta|\rho, t]$ of the unemployed worker's ability, and offers wage equal to its valuation $w(\rho, t) = E[\theta|\rho, t]$.

My model is a multistage sequential game, so I will derive its equilibrium through backward induction. In particular, stage 3 involves a signaling subgame. I will now define some key concepts of signaling games in the context of my employee referrals model. The basic problem in a signaling game is to analyze whether the receiver (firm) can infer the information (expected ability of unemployed worker) of the sender (employed worker) through his messages (recommendations).

Definition 1. An employed worker's recommendation strategy ρ *reveals his information* $E[\theta|s,t]$ if and only if:

(1) for $s \in [0, \alpha_t)$, $\{s' : \rho(s', t) = \rho(s, t)\} = \{s\}$, or

(2) for $s \in [\alpha_t, 1]$, $\{s' : \rho(s', t) = \rho(s, t)\} = [\alpha_t, 1]$.

For signals $s \in [0, \alpha_t)$, the employed worker knows the ability level and it is equal to the signal value. The first part of this definition says that if an employed worker received such a signal, then he is revealing his information if the firm can also infer the signal/ability value. For any signal in the interval $s \in [\alpha_t, 1]$, the employed worker expects the same ability value and it is equal to the weighted average of ability values in this interval. The second part of this definition says that if an employed worker received such a signal, then he is revealing his information if the firm can also infer that the signal belongs to this interval $[\alpha_t, 1]$.

An unemployed worker with ability level θ is *separating through an employed tie* t if the messages sent by the employed tie reveals his ability value to the firm. If the firm can't differentiate between a set of ability levels, then the firm *pools* unemployed workers with such ability values together. For this signaling game, I will focus on an appealing class of pure strategy Perfect Bayesian Equilibria (PBE), where employed workers reveal maximum information in the cheapest way possible. See appendix for a formal justification. I will now formally define this class of PBE and provide some intuition for its attractiveness.

Definition 2. An employed worker's recommendation strategy $\rho(s, t)$ is *cost efficient* if and only if:

(1) The initial value condition is $\rho(0, t) = 0$.

(2) If $\rho(s^*, t) = 1$, then for $s < min\{s^*, \alpha_t\}$, recommendations are increasing in signals $\rho_1(s, t) > 0$.

(3) For $s \geq min\{s^*, \alpha_t\}$, recommendations value minimizes reputation costs $\rho(s,t) \in argmin_{\rho \in [0,1]}(\rho - E[\theta|s,t])^2$ subject to $\rho(s,t) \neq \rho(s',t)$ for all $s' < min\{s^*, \alpha_t\}$.

A cost efficient recommendation strategy satisfies three conditions. The first condition is the "Riley Condition" of least costly separation in signaling games. For the lowest signal value, the employed worker can reveal the signal to the firm even by minimizing reputation costs. The second condition requires that the employed worker reveals information to the firm until he either runs out of recommendation values or the signal reaches his informativeness level. Since reputation costs depend on the distance between the recommendation value and the signal value, it is cheaper to reveal information by increasing recommendations as signal increases. Similar to the lowest signal value, for the set of signals at the top, the employed worker can reveal his information to the firm even by minimizing reputation costs. However, it may not be feasible to choose the recommendation value that minimizes reputation costs if it is already chosen for some lower signal value. In such case, the employed worker chooses the constrained minimum.

A cost efficient recommendation strategy is appealing in that (a) it maximizes the set of ability levels that a firm can infer, and (b) among all recommendation strategies that also maximize the set of ability levels that a firm can infer, the employed worker incurs lowest reputation costs by revealing information through a cost efficient recommendation strategy. I will focus on equilibria of the employee referrals model where recommendation strategies of the employed workers are cost efficient. I will refer to this class of equilibria as Cost Efficient Equilibria (CEE) hereafter.

2.1 Discussion of Modeling Assumptions

I now discuss some of the assumptions underlying my model.

Access to Ties. I made several assumptions regarding access to ties. Firstly, I assumed that each unemployed worker has access to weak ties. This assumption is quite natural in that everyone knows many acquaintances (weak ties). However, knowing acquaintances does not necessarily mean that they will provide referrals. Thus, I relax this assumption in section 4. I consider an extension of my model where some proportion of unemployed workers do not have access to weak ties. Secondly, I assumed that if an unemployed worker has access to a certain type of tie, then he has only one such tie. In section 2.4, I consider an extension of the model which relaxes this assumption. The main results of this paper are robust to both of these extensions (sections 4, and 2.4). Finally, I assume that each employed worker has a tie (either weak or strong) with only one unemployed worker. This assumption is based on the idea that an employed worker knows his most preferred unemployed tie. Thus, I define "an unemployed worker to have access to a tie" to mean that the employed worker will provide referral to this unemployed worker among all the unemployed workers that he has ties with.

Information Structure. The underlying idea behind the information structure is that the employed worker can learn about the unemployed worker's ability, but there is an upper-bound α (interpreted as the ability level of the employed worker) to how much he can learn. An employed worker with a strong tie receives a precise signal if the unemployed worker's ability is below his threshold ability level (for $\theta \leq \alpha$), and signals are noisy otherwise. An employed worker with a weak tie can learn less about the unemployed worker's ability. Therefore, he only receives a precise signal if the unemployed worker's ability is sufficiently below his threshold ability level (for $\theta \leq \alpha - \varepsilon$ where $\alpha \geq \varepsilon > 0$), and signals are noisy otherwise. Thus, an employed worker with a weak tie can infer smaller interval of ability levels, because he has more noise in his signals.

Firm's Role. So far I have assumed a passive role for the firm. The firm simply offers a wage equal to its valuation of the unemployed worker's ability. In the employee referral mechanism, a firm can not only choose the wage it offers to the unemployed worker, but it can also influence the reputation costs of its employees. In section 2.4, I consider an extension of the model which takes into account these two roles of the firm.

The fact that weak ties are less informative than strong ties does not necessarily imply that strong ties will provide more information to a firm through referrals. It depends on how much the employed worker values gratitude over reputation costs. The firm plays an important role here in that it can influence the reputation costs of its employees.[24]

Recommendation. I assumed that recommendation values are bounded $\rho \in [0, 1]$. The recommendation value is the ability value that an employed worker conveys to the firm. Thus, it is bounded by the values that ability variable can take $\theta \in [0, 1]$. Note that the firm does not blindly believe what the employed worker conveys to it. The firm tries to infer the signal that its employee received through his recommendation value, and forms its own beliefs.

The underlying idea behind the recommendation variable is as follows. An employed worker providing a referral puts some effort $e \in [e_{min}, e_{max}] \supseteq [0, 1]$ into recommending an unemployed worker. The more effort he puts into his recommendation, the higher the ability value he conveys to the firm $\rho'(e) \geq 0$. The recommendation function maps the employed worker's effort into recommendation value $\rho : [e_{min}, e_{max}] \to [0, 1]$. Instead of having an additional effort variable, I simply assume that the employed worker can choose the recommendation value. Note that this model belongs to the costly signaling class of model where recommendation (or effort value) is the costly message.

Reputation Costs. The recommendation value is the ability value that an employed worker

[24] Firms can influence the reputation costs of its employees in several ways. Firms can prolong the "probation period" of workers hired through employee referrals. This will allow firms to better learn about the abilities of workers hired through employee referrals, increases the likelihood that an employee will be caught if he provided an inaccurate recommendation, and therefore increases the expected reputation costs of employee for providing an inaccurate recommendation. Firms could worsen punishment for employees who provide inaccurate recommendations (see Heath (2013) and Beaman and Magruder's (2012)). The main idea here is that the firm can choose the reputation cost levels in the employee referral method of hiring, which is a key distinction from non-employee referrals. See Montgomery (1991) for some early references on reputation costs. See Fernandez & Mateo (2015) for some references and a discussion.

conveys to the firm. So reputation cost is the difference between the ability value than an employed worker conveys to the firm and the ability value that the employed worker actually believes $(\rho - E[\theta|s,t])^2$. Note that the reputation cost comes from recommendations that are lower and higher than the employed worker's expected value of ability. The results of this paper are robust to a variation of the model in which reputation costs only come from recommendations that are higher than the expected ability value (see remark at the end of appendix B).

It is clear that there are reputation costs to going over the expected ability value. By recommending higher than the ability value, the employed worker is putting his reputation as an employee at stake. The employed worker doesn't want to be responsible for recommending a bad worker. However, there are several reasons to believe that there are reputation costs of going under the expected ability value as well. By recommending lower than the expected ability value, the employed worker is putting his reputation as a "friend" (or whatever his relationship is with the unemployed worker) at stake. The employed worker doesn't want to be responsible for under-selling his "friend". Alternatively even as an employee, the employed worker will have a better reputation if he recommends accurately as oppose to always giving really low recommendations. The base model takes such considerations into account, and therefore the reputation costs come from both going under and over the expected ability value.

2.2 Motivating Example

I will first consider a simple example where weak ties have minimal information $\alpha_{weak} = 0$, strong ties have full information $\alpha_{strong} = 1$, and only half of the unemployed workers have access to strong ties $\beta = 0.5$. This example will provide intuition on how to characterize the equilibrium for the general case.

It is a multistage sequential game, so I will derive the equilibrium through backward induction. In stage 4, the firm sets wage equal to its valuation $w(\rho, t) = E[\theta|\rho, t]$. If the tie is weak, then the employee's signals do not provide any new information and therefore the firm's valuation does not depend on such employee's signals $w(\rho, t_{weak}) = E[\theta|t_{weak}] = \int_0^1 \theta \frac{f(t_{weak}|\theta)}{\int_0^1 f(t_{weak}|\theta)d\theta} d\theta$. If the tie is strong, then the firm's wage offer depends on whether it can infer its employees signals. In a cost efficient recommendation strategy, recommendations are (weakly) increasing in signal $\rho_1(s, t_{strong}) \geq 0$. For the interval of signals that the firm can infer, the firm sets wage equal to the signal value $w(\rho(s, t_{strong}), t_{strong}) = s$. Since recommendations are strictly increasing in signals $\rho_1(s, t_{strong}) > 0$ for such interval of signals, the firm's wage is strictly increasing in recommendation. Plugging this wage in the first order condition of the employed worker's maximization problem gives the following differential equation (DE).

$$\rho_1(s, t_{strong}) = \frac{1}{2r(\rho(s, t_{strong}) - s)} \quad \text{(DE)}$$

This differential equation then implies that employee recommends higher than his signal $\rho(s, t_{strong}) > s$. This follows from the fact that the firm's wage is strictly increasing in recommendation. As a result, the employed worker has an incentive to inflate his recommendations (recommend higher than his signal) in order to get more gratitude (which is increasing in wage). The reader can verify that the family of solutions to this differential equation is given by

$$\rho(s, t_{strong}) + c = -\frac{1}{2r} ln \left[\frac{1}{2r} + s - \rho(s, t_{strong}) \right]$$

where c is a constant which can be determined by the initial value condition $\rho(0, t_{strong}) = 0$. So I get $c = -\frac{1}{2r} ln \left[\frac{1}{2r} \right]$, and

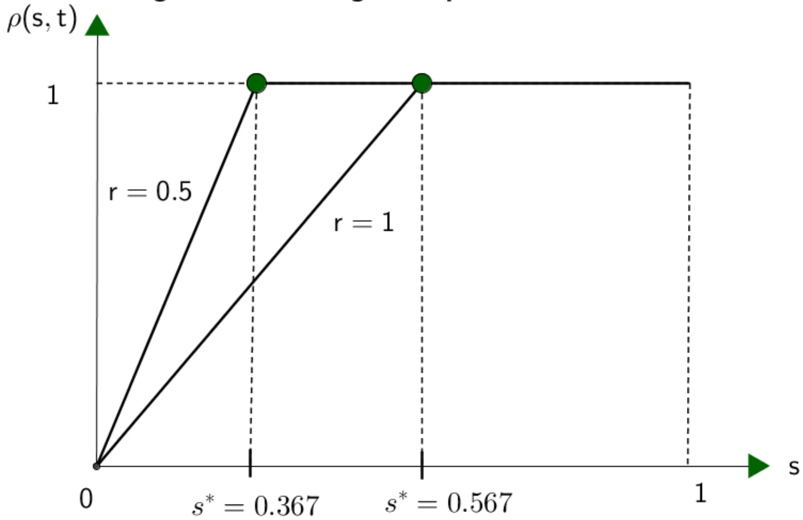
$$\begin{aligned} \rho(s, t_{strong}) &= \frac{1}{2r} ln \left[\frac{1}{2r} \right] - \frac{1}{2r} ln \left[\frac{1}{2r} + s - \rho(s, t_{strong}) \right] \\ \implies \rho(s, t_{strong}) &= \frac{1}{2r} ln \left[\frac{\frac{1}{2r}}{\frac{1}{2r} + s - \rho(s, t_{strong})} \right] \end{aligned} \quad (4)$$

For a given reputation parameter r, there exists a unique signal s^* such that recommendation is equal to one.

$$\begin{aligned} s^* &= \frac{1}{2r} \left[\frac{1 - e^{2r}}{e^{2r}} \right] + 1 \\ &= \frac{1}{2re^{2r}} - \frac{1}{2r} + 1 \end{aligned} \quad (5)$$

The reader can verify that as $r \to 0$, $s^* \to 0$ (using L'Hopital Rule), as $r \to \infty$, $s^* \to 1$, and the threshold signal is increasing in reputation cost parameter $s^{*'}(r) > 0$. Thus, for any finite reputation cost parameter, $s^* \in (0, 1)$. Figure 2a below depicts this.

Fig 2a. Motivating Example



So recommendations are strictly increasing in signal and inflated until they hit the upper bound of one. The employed worker's recommendations are equal to one $\rho(s, t_{strong}) = 1$ for any signal higher than this threshold signal $s \geq s^*$, which makes it impossible for firm to infer these signals from the employee's recommendations (firm can only infer signals below this threshold).

Similarly, wage offers from strong tie is strictly increasing in signal below threshold signal $s < s^*$, and equal to the average of signals $s \geq s^*$ otherwise.

$$w(\rho, t_{strong}) = \begin{cases} s & \text{if } s < s^* \\ E[\theta | s \geq s^*, t_{strong}] & \text{if } s \geq s^* \end{cases} \tag{6}$$

Observe that wage offers from weak ties is fixed, and wage offers from strong ties is increasing in signals. As a result, if $\widetilde{\theta}$ prefers strong tie, then all $\theta > \widetilde{\theta}$ prefer strong tie as well. Thus, the equilibrium involves a threshold ability θ^* such that $\theta < \theta^*$ prefer weak tie, and $\theta > \theta^*$ prefer strong tie. The value of threshold ability θ^* depends on threshold signal s^*. Let $\widehat{\theta} = \{\theta : \theta = E[\theta | t_{weak}]\}$ be the unemployed worker who is indifferent between choosing wage offer from a strong tie and revealing his ability or choosing wage offer from a weak tie. The reader can easily verify that $\widehat{\theta} = 0.41$. If threshold signal is high enough $s^* > \widehat{\theta}$, then threshold ability is $\theta^* = \widehat{\theta}$. If threshold signal is not high enough $s^* \leq 0.41$, then unemployed workers whose abilities are revealed with strong ties $\theta < s^*$ prefer weak tie. For unemployed workers with abilities $\theta \geq s^*$, they can get wage offer $E[\theta | s \geq s^*, t_{strong}] = \frac{1}{1-s^*} \int_{s^*}^{1} s ds$ by pooling with a strong tie or $E[\theta | t_{weak}] = \int_0^1 \theta \frac{f(t_{weak}|\theta)}{\int_0^1 f(t_{weak}|\theta) d\theta} d\theta$. The reader can easily verify that the wage offer from strong tie is strictly higher then any wage offer from weak ties $E[\theta|s \geq s^*, t_{strong}] > 0.5 \geq E[\theta|t_{weak}]$ since the threshold signal is positive $s^* > 0$.

$$\theta^* = \begin{cases} s^* & \text{if } 0 < s^* \leq 0.41 \\ 0.41 & \text{if } s^* > 0.41 \end{cases} \tag{7}$$

In this example, strong ties are more informative to the firm since the threshold signal is positive $s^* > 0$. As a result, low ability workers use weak ties to pool with high ability workers without strong ties, and high ability workers use strong ties to separate themselves. This example suggests that the cost efficient equilibrium will involve a threshold ability where strong ties are more informative to the firm. Is there always a cost efficient equilibrium with a threshold ability? If so, are strong ties always more informative to the firm?

2.3 Cost Efficient Equilibrium (CEE)

I will first characterize the cost efficient recommendation strategy for each type of employed worker. Similar to the motivating example, recommendations are strictly increasing in signal and inflated $\rho(s,t) > s$ until either the recommendations hit the upper bound of one or the signal reaches informativeness level of the tie, i.e. for $s < min\{s^*, \alpha_t\}$. If recommendations hit the upper bound of one, then recommendation value is equal to one for any higher signal, i.e. $\rho(s,t) = 1$ for $s \geq s^*$. If the signal reaches the informativeness level of the tie, then the employed worker chooses recommendation value that minimizes his reputation costs $\rho(s,t) = E[\theta|s,t]$ as long as it was not used for some lower signal. If it was used for some lower signal, then the employed worker chooses the lowest value that was not used for some lower signal. See figure 2b below for a case where $\alpha_{weak} < s^* < \alpha_{strong}$, and $\rho(s, t_{weak}) = lim_{s' \to \alpha_t} \rho(s', t)$ for $s \geq \alpha_{weak}$.

Proposition 1 - An employed worker's cost efficient recommendation strategy $\rho(s,t)$ satisfies:

(1) The initial value condition $\rho(0,t) = 0$.

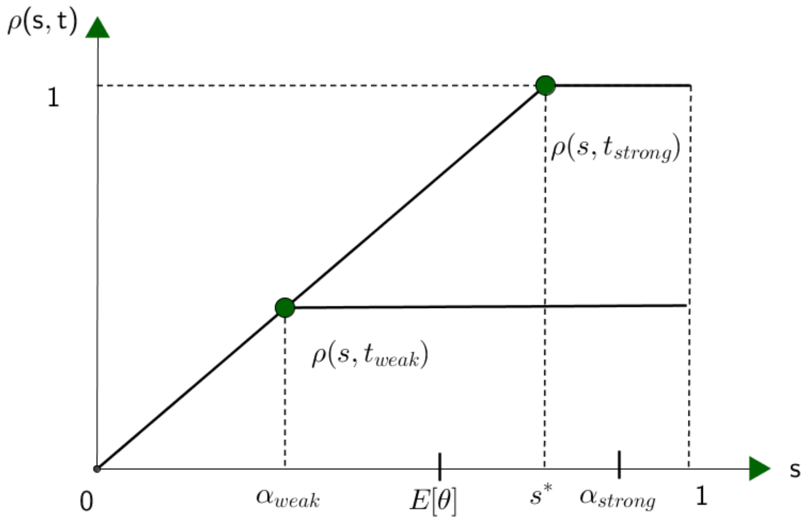
(2) For $0 < s < min\{s^*, \alpha_t\}$, $\rho(s,t) > s$, and

$$\rho(s,t) = \frac{1}{2r} ln \left[\frac{\frac{1}{2r}}{\frac{1}{2r} + s - \rho(s,t)} \right].$$

(3) For $s \geq min\{s^*, \alpha_t\}$,

$$\rho(s,t) = \begin{cases} 1 & \text{if } min\{s^*, \alpha_t\} = s^* \\ E[\theta|s,t] & \text{if } min\{s^*, \alpha_t\} = \alpha_t, \\ & \text{and } E[\theta|s,t] \neq \rho(s',t) \text{ for all } s' < \alpha_t \\ lim_{s' \to \alpha_t} \rho(s',t) & \text{if } min\{s^*, \alpha_t\} = \alpha_t, \\ & \text{and } E[\theta|s,t] = \rho(s',t) \text{ for some } s' < \alpha_t \end{cases}.$$

Fig 2b. Cost Efficient Referral Strategy



For the interval of signals that the firm can infer $s < min\{s^*, \alpha_t\}$, the firm sets wage equal to the signal value $w(\rho(s,t),t) = s$. Since recommendations are strictly increasing in signals $\rho_1(s,t) > 0$ for such interval of signals, the firm's wage is strictly increasing in recommendation. For the interval of signals that the firm can not infer $s \geq min\{s^*, \alpha_t\}$, the firm sets wage equal to the weighted average of these signal/ability values i.e. $w(\rho(s,t),t) = E[\theta|s \geq min\{s^*, \alpha_t\}, t] = \int_{min\{s^*,\alpha_t\}}^{1} \theta \frac{f(t|\theta)}{\int_{min\{s^*,\alpha_t\}}^{1} f(t|\theta)d\theta} d\theta$, where weights are determined by the the proportion of unemployed workers $f(t|\theta)$ with ability level θ who chose tie type t.

Proposition 2 - Given a recommendation ρ, and the tie type t, the firm offers wage

$$w(\rho,t) = \begin{cases} s & \text{if } s < min\{s^*, \alpha_t\} \\ E[\theta|s \geq min\{s^*, \alpha_t\}, t] & \text{if } s \geq min\{s^*, \alpha_t\} \end{cases}.$$

If the threshold signal is greater than the informativeness level of the weak tie $s^* > \alpha_{weak}$, then there is an interval of signals $s \in (\alpha_{weak}, min\{s^*, \alpha_{strong}\})$ where wage offers from weak ties is fixed, and wage offers from strong ties is increasing in signals. As a result, if $\widetilde{\theta} \in (\alpha_{weak}, 1)$ prefers strong tie, then all $\theta > \widetilde{\theta}$ prefer strong tie as well. Thus, the cost efficient equilibrium involves a threshold ability θ^* such that ability levels below the threshold ability $\theta < \theta^*$ (weakly) prefer weak ties ($\theta \in [0, \alpha_{weak})$ are indifferent between the two type of ties, and $\theta \in [\alpha_{weak}, \theta^*)$ prefer weak ties), and unemployed workers with ability levels above the threshold ability $\theta \geq \theta^*$ prefer strong ties.[25] The value of threshold ability θ^* depends on threshold signal s^* and the informativeness level of both type of ties α_{weak}, α_{strong}. Let $\widehat{\theta} = \{\theta : \theta = E[\theta | s \geq \alpha_{weak}, t_{weak}]\}$ be the unemployed worker who is indifferent between choosing wage offer from a strong tie and revealing his ability or choosing wage offer from a weak tie.

> **Theorem 1 -** If $s^* > \alpha_{weak}$, then the Cost Efficient Equilibrium exists and is characterized by a threshold ability $\theta^* = min\{\widehat{\theta}, s^*, \alpha_{strong}\}$ such that:
>
> Unemployed workers with ability levels below the threshold ability $\theta < \theta^*$ (weakly) prefer weak ties, and unemployed workers with ability levels above the threshold ability $\theta > \theta^*$ prefer strong ties.

Recall that the threshold signal is increasing in reputation cost parameter $s^{*'}(r) > 0$ (see equation 5). If the reputation cost parameter is low enough, then $s^* \leq \alpha_{weak}$ and both type of ties are equally informative to the firm. In such case, unemployed workers are indifferent between the two type of ties, and the cost efficient equilibrium does not exist. Moreover, if strong ties get more gratitude from providing recommendations (gratitude benefits parameter varies by tie strength $\gamma(t_{strong}) > \gamma(t_{weak})$), then weak ties can be even more informative to the firm.

To ensure that the equilibrium involves a threshold ability where strong ties are more informative to the firm, I need the reputation costs parameter to be high enough. In the next section, I show that if a firm can choose the reputation costs parameter for its employees, it will indeed set reputation costs parameter to be high enough. I consider an extension of the model in which there are two firms, strong ties get more gratitude from providing recommendations (gratitude benefits parameter varies by tie strength $\gamma(t_{strong}) > \gamma(t_{weak})$) and firms can influence the reputation costs of its employees (each firm $j \in \{1, 2\}$ chooses reputation costs parameter for its employees $r_j \in [0, r_{max}]$). After nature moves and before tie selection, firms compete by setting wages and reputation costs parameters. In this extension, each firm sets wage equal to its valuation and the reputation cost parameter to the maximum level. The maximum reputation cost level is such that strong ties are more informative to the firm $s^* > \widehat{\theta} \geq \alpha_{weak}$. Thus, I will focus on the case where the threshold signal $(\alpha_{strong} \geq s^* > \widehat{\theta})$ is high enough hereafter.

[25] If the highest ability level ($\theta = 1$) is also common knowledge, then it only changes the threshold equilibrium in that the highest ability workers will be indifferent between the two type of ties as well, i.e. $\theta \in [0, \alpha_{weak}) \cup \{1\}$ are indifferent between the two type of ties, $\theta \in [\alpha_{weak}, \theta^*)$ prefer weak ties, and $\theta \in [\theta^*, 1)$ prefer strong ties.

2.4 Firm's Role

I extend the base model by having two identical firms $j \in \{1, 2\}$. I assume that if an unemployed worker has access to a certain type of tie, then he has one such type of tie at each firm. As a result, these firms are competitive. Each firm can set reputation cost level for its employees $0 < r_j \le r_{max}$ where the maximum reputation cost level is such that strong ties are more informative to the firm $s^*_{strong} > \widehat{\theta} \ge \alpha_{weak}$. Stage 1 (nature) and stage 2 (tie selection) are the same as before, but a new stage (Stage 1.5) is introduced. After nature moves and before tie selection, firms compete by setting wages and reputation costs parameters.

Stage 1.5: Wage And Reputation Cost Setting. If an unemployed worker with ability level θ prefers wage offer from firm $j \in \{1, 2\}$ using tie $t \in T$, then $1_j(\theta, t) = 1$. If he is indifferent between the two firms, then $1_j(\theta, t) = 0.5$ and $1_j(\theta, t) = 0$ otherwise. In stage 2, I am assuming that if an unemployed worker is indifferent between the two firms, then he is equally likely to select any one of them. The set of unemployed workers who prefer weak ties is denoted by Θ_{weak}, and the set of unemployed workers who prefer strong ties is denoted by Θ_{strong}. Since only β of the unemployed workers have access to strong ties, $(1-\beta)$ of the unemployed workers who prefer strong ties have to choose weak ties. Firm's profit from hiring an unemployed worker with recommendation $\rho(s, t, r_j)$ and tie type t is $\theta - w(\rho, t)$. Observe that the wage $w(\rho, t)$ depends on the recommendation $\rho(s, t, r_j)$, and recommendation is a function of the reputation costs level r_j set by the firm. Each firm $j \in \{1, 2\}$ chooses wage $w(\rho, t)$ and reputation costs level $0 < r_j \le r_{max}$ that maximizes its profit

$$\begin{aligned}
\pi_j &= \int_{\theta \in \Theta_{weak}} 1_j(\theta, t_{weak}) \left[\theta - w(\rho, t_{weak})\right] d\theta \\
&+ \beta \int_{\theta \in \Theta_{strong}} 1_j(\theta, t_{strong}) \left[\theta - w(\rho, t_{strong})\right] d\theta \\
&+ (1-\beta) \int_{\theta \in \Theta_{strong}} 1_j(\theta, t_{weak}) \left[\theta - w(\rho, t_{weak})\right] d\theta.
\end{aligned} \tag{8}$$

Whether the firm can infer a bigger interval of ability levels from strong ties or weak ties depends on two opposing forces in the model. On the one hand, strong ties get higher gratitude value from providing referral, which leads them to send higher recommendation values. If a strong tie runs out of recommendation values that he can send before the signal reaches the informativeness level of the weak tie, then firms can infer less ability levels from a strong tie. On the other hand, a weak tie is less informative than a strong tie, and therefore he will reach his informativeness level before a strong tie.

Stage 3 is the same as before but now an employed worker's gratitude benefits are strictly increasing in the tie strength between the employed worker and the unemployed worker $\gamma(t)$. A strong tie gets more gratitude from providing recommendations than a weak tie, i.e. $\gamma(t_{strong}) > \gamma(t_{weak}) > 0$.

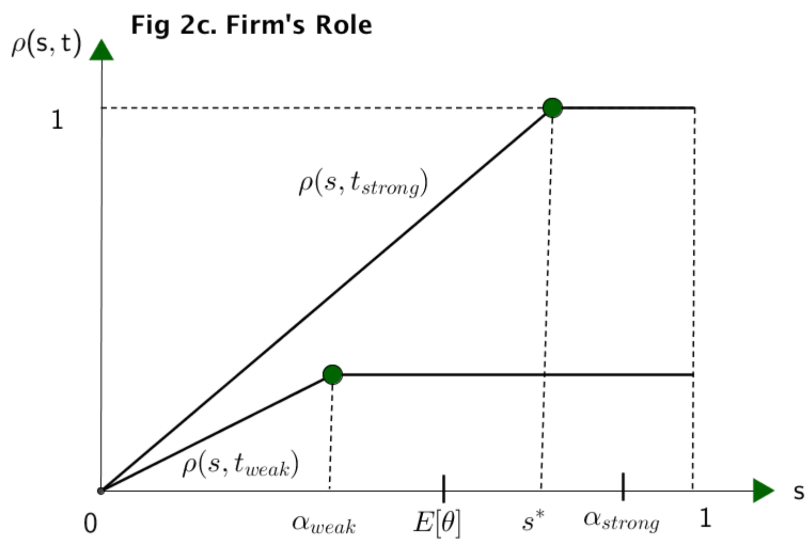
$$\pi_E = \gamma(t)w(\rho, t) - r_j(\rho - E[\theta|s, t])^2 \tag{9}$$

At stage 4, each firm $j \in \{1, 2\}$ forms its valuation $v(\rho, t)$ of the unemployed worker's ability, and offers the wage according to rule set in stage 1.5. At the equilibrium, firms maximize the number of signals they can infer by setting the highest penalty value for receiving an inaccurate recommendation from its employees. As a result, an employee's recommendation values will be influenced by his reputation costs more than his gratitude benefits, and firms will be able to infer a bigger interval of ability levels from a strong tie than from a weak tie. In particular, firms will be able to infer sufficiently high ability levels (above some threshold) from only strong ties. Figure 2c depicts this.

Proposition 3 - Each firm $j \in \{1, 2\}$ sets:

(1) reputation costs to the maximum level $r_j = r_{max}$, and

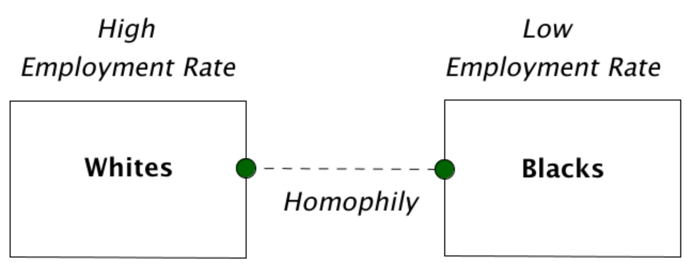
(2) wage equal to its valuation of the unemployed worker's ability $w(\rho, t) = v(\rho, t)$.



3 Wage Gap and Wage Hierarchies

In this section, I extend the employee referrals model by having two races (blacks and whites), and two types of occupations. I assume that fewer blacks have strong ties with employed workers than whites. This assumption follows from two empirical facts: employment differentials between blacks and whites in the US and the homophily feature of social networks (tendency to interact with others that have similar characteristics). Since blacks have lower employment rates than whites, they have to rely on their cross-race ties more than whites do. Such cross-race ties tend to be weaker because individuals tend to interact more often with others that belong to their own race (by homophily). Figure 3 below depicts this. Moreover, unemployed workers can choose the type of tie and the type of occupation they prefer. I then examine how the lower access to strong ties for blacks influences the black-white wage gap up and down wage hierarchies.

Fig 3. Access to Strong Ties



3.1 Races and Occupations

Now consider an environment with two groups $g \in \{black, white\}$ with blacks having less access to strong ties than whites, i.e. the proportion of unemployed blacks with access to a strong tie is less than the proportion of unemployed whites with access to a strong tie $\beta_{black} < \beta_{white}$. There is a single firm offering two type of occupations $occ \in \{occ_m, occ_h\}$, manual labor intensive occ_m and human capital intensive occ_h . In the manual labor intensive occupation, output is only increasing in ability up to some level. This ability level is below average $\theta_{low} < E[\theta]$, and having "human capital" (ability) beyond that doesn't add to output.

$$y_m = \begin{cases} \theta & \text{if } \theta \leq \theta_{low} \\ \theta_{low} & \text{if } \theta > \theta_{low} \end{cases} \tag{10}$$

In the human capital intensive occupation, output is always increasing in ability

$$y_h = \theta. \tag{11}$$

To work in human capital intensive occupation, an unemployed worker needs to pay a cost (getting education) k. However, it is "efficient" for above average ability workers to work in human capital intensive occupation in that their net value of working is higher $\theta - k > \theta_{low}$ for $\theta > E[\theta]$, and inefficient for below average ability workers to work in human capital intensive occupation in that their net value of working is lower $\theta - k < \theta_{low}$ for $\theta < E[\theta]$. I assume that strong ties can distinguish between below and above average ability workers and weak ties can't, i.e. $\alpha_{strong} \geq E[\theta] > \alpha_{weak}$.[26]

In this extension of the model, stages 1, 3, and 4 are same as described in section 2. However, stage 2 is modified in that each unemployed worker chooses the tie type and also the occupation type.

Stage 2': Tie Selection. At this stage, each unemployed worker (θ, T, g) chooses tie type $t \in T$ and the occupation type $occ \in \{occ_m, occ_h\}$ that maximizes his payoff,

$$\pi_u(\theta, T, g) = w(\rho(s,t), t, \beta_g, occ) - 1_{occ_h} k \tag{12}$$

where 1_{occ_h} is an indicator function which takes value of one if occupation is human capital intensive $occ = occ_m$ and zero otherwise. All the other aspects of the model are the same as before.

At stage 2', if an unemployed workers can get wage offer at least as high as the average ability $E[\theta]$, then he will choose the human capital intensive occupation. This follows from the fact that it is efficient for above average ability workers to work in human capital intensive occupation $\theta - k > \theta_{low}$ for $\theta > E[\theta]$, and inefficient for below average ability workers to work in human capital intensive occupation $\theta - k < \theta_{low}$ for $\theta < E[\theta]$. As a result, it is individually beneficial for an unemployed worker to get costly education if he can make firm believe that he is above average ability worker, even if he is actually not above average ability.

> **Proposition 4 -** An unemployed worker chooses the human capital intensive occupation if and only if $w(\rho(s,t), t, \beta_g, occ_h) \geq E[\theta]$ for some tie type $t \in T$.

Note that this result also implies that the mean wage of workers in the human capital intensive occupation is at least as high as average ability $\overline{w}_{occ_h} \geq E[\theta]$, and the mean wage of workers in the manual labor intensive occupation is below average ability $\overline{w}_{occ_l} \leq \theta_{low} < E[\theta]$. Thus, I can now show the stylized fact, i.e. the wage gap in occupation with higher mean wage (human capital intensive) is higher than the wage gap in occupation with lower mean wage (manual labor intensive). Wage gap in a given occupation is the average wage gap between whites and their black counterparts (same ability level). The stylized fact is satisfied whenever the wage gap in human capital intensive occupation is higher than the wage gap in manual labor intensive occupation, i.e. $\triangle WG = WG_h - WG_m > 0$.

[26]This assumption is quite natural. To keep the presentation neat, I have assumed that there are only two strength levels, weak and strong. If there were many different strength levels, then sufficiently strong ties will be able to distinguish below and above average ability workers. I can then partition the set of strength levels into two sets, the weak set includes the set of tie strengths which can't distinguish between below and above average ability workers, and the strong set includes the set of tie strengths which can distinguish between below and above average ability workers.

3.2 Motivating Example

I will first consider a simple example, which is an extension of the motivating example considered in section 2.1. Recall, the weak ties have minimal information $\alpha_{weak} = 0$, strong ties have full information $\alpha_{strong} = 1$. Suppose only half of the unemployed white workers have access to strong ties $\beta_{white} = 0.5$, and no black workers have access to strong ties $\beta_{black} = 0$.

Since the firm can infer the lowest ability level. All unemployed workers with the lowest ability level choose manual labor intensive occupation, and get wage equal to their ability values. For all the other ability levels $\theta \in (0, 1]$, an unemployed black worker will be offered wage equal to the average ability by choosing the human capital intensive occupation, i.e. $w(t_{weak}, \beta_{black} = 0, occ_h) = E[\theta]$. Since this wage is at least as high as the average ability $E[\theta]$, all unemployed black workers with ability levels above the lowest $\theta \in (0, 1]$ will choose the human capital intensive occupation. From section 2.1, I know that all unemployed white workers with ability levels $\theta \in (0, 0.41)$ will prefer weak ties. They will get wage lower than the average ability by choosing the human capital intensive occupation, i.e. $w(t_{weak}, \beta_{white} = 0.5, occ_h) = 0.41 < E[\theta]$. Thus, they will choose the manual labor intensive occupations. On the other hand, unemployed white workers with ability $\theta \in [0.41, 1]$ will prefer strong ties. Half of these workers without access to strong ties will choose manual labor intensive occupation for the same reason as $\theta \in (0, 0.41)$. For $\theta \in [0.41, 0.5)$ with access to strong ties, they will get wage lower than the average ability by choosing the human capital intensive occupation, i.e. $w(t_{weak}, \beta_{white} = 0.5, occ_h) = \theta < E[\theta]$. Thus, they will choose the manual labor intensive occupations. For $\theta \in [0.5, 1]$ with access to strong ties, they will get wage higher than the average ability by choosing the human capital intensive occupation, i.e. $w(t_{weak}, \beta_{white} = 0.5, occ_h) = \theta > E[\theta]$. Thus, they will choose the human capital intensive occupations.

Manual Labor Intensive Occupation	Whites	wage	Blacks	wage
$\theta = 0$	all	θ	all	θ
$\theta \in (0, 0.41)$	all	$min\{\theta_{low}, 0.41\}$	none	-
$\theta \in [0.41, 0.5)$	half	$min\{\theta_{low}, \theta\}$	none	-
$\theta \in [0.41, 1]$	half	$min\{\theta_{low}, 0.41\}$	none	-
$WG_m = 0$				

The table above indicates for each ability level and for each group, the proportions and the wages of unemployed workers that chose manual labor intensive occupation. In this occupation, only the lowest ability blacks and whites can be compared. The firm can infer this ability level and therefore black and white workers get the same wage. In this occupation, the wage gap between whites and their black counterparts is zero.

Human Capital Intensive Occupation	Whites	wage	Blacks	wage
$\theta \in (0, 0.5)$	none	-	all	$E[\theta]$
$\theta \in [0.5, s^*)$	half	θ	all	$E[\theta]$
$\theta \in [s^*, 1)$	half	$\frac{1+s^*}{2}$	all	$E[\theta]$
$WG_h = \frac{1}{0.5}\left[E[\theta \mid \theta \geq 0.5] - E[\theta \mid \theta \geq 0]\right] > 0$				

Similarly, the table above indicates for each ability level and for each group, the proportions and the wages of unemployed workers that chose human capital intensive occupation. Since the informativeness level of weak tie is minimal, and above average ability white workers with access to strong ties use their strong ties, firm expects a white worker with a weak tie to be below average ability. As a result, an above average ability white worker with access to a strong tie can separate himself from below average ability white workers, and only above average ability white workers enter the human capital intensive occupation through strong ties. On the other hand, both below and above average ability black workers enter the human capital intensive occupation through weak ties, and firm pools them together. On average, high ability black workers get relatively lower wage than their white counterparts because the ability distribution of whites with a strong tie first order stochastically dominates the ability distribution of blacks with a weak tie.

3.3 Main Results for Wage Gap

In this section, I will provide conditions under which the wage gap in occupation with higher mean wage (human capital intensive) is higher than the wage gap in occupation with lower mean wage (manual labor intensive). The results in this section are derived from the assumption that only strong ties can distinguish between below and above average ability workers $\alpha_{strong} \geq E[\theta] > \alpha_{weak}$.[27] If no black workers have access to strong ties, then some below average ability workers enter the human capital intensive occupation through weak ties because firm cannot infer their abilities. On the other hand, an above average ability white worker with access to a strong tie can separate himself from below average ability white workers if the proportion of whites with access to strong ties β_{white} is high enough (above some threshold β^*). If the proportion of whites with access to strong ties is high enough, then many above average ability workers will use strong ties. As a result, if the firm cannot infer the ability of a white worker with a weak tie, then the firm will give less "benefit of the doubt" to this worker, and will expect him to be below average ability. Thus,

[27] I have assumed that the set of ability levels that an unemployed worker can infer is downwardly biased $[0, \alpha_t)$, but the results will also hold if this interval was upwardly biased $(\alpha_t, 1]$. In either case, blacks don't have access to strong ties and only strong ties can distinguish between below and above average ability workers. As a result, some below average ability black workers $[\alpha_{weak}, E[\theta])$ will use weak ties to pool with higher ability black workers and enter human capital intensive occupation. This result follows from the assumption that only strong ties can distinguish between below and above average ability workers. Observe that this assumption is $\alpha_{strong} \geq E[\theta] > \alpha_{weak}$ if the interval is downwardly biased and it is reversed $\alpha_{weak} \geq E[\theta] > \alpha_{strong}$ if the interval is upwardly biased.

only above average ability white workers with strong ties will enter the human capital intensive occupation. Since blacks don't have access to strong ties, both below and above average ability workers enter human capital intensive occupation through weak ties, and the rest of the arguments follow from the example (see last four lines of section 3.1).

The higher is the informativeness level of a weak tie, the more of the lower ability levels the firm can infer through a weak tie. As a result, if the firm cannot infer the ability of a white worker with a weak tie, then the firm knows that his ability is sufficiently high. Thus, the higher is the informativeness level of a weak tie, the more benefit of the doubt the firm gives, and the higher the proportion of whites with access to strong ties needs to be. As long as the weak tie cannot distinguish between below and above average ability workers $\alpha_{weak} < 0.5$, there exists a threshold value $\beta^* < 1$ such that if the proportion of whites with access to strong ties β_{white} is above this threshold, then the Black-White wage gap is higher in the occupation with higher mean wage.

Proposition 5 - If no black workers have access to strong ties $\beta_{black} = 0$, and the informativeness level of weak tie is α_{weak}, then there exists a threshold value $\beta^* < 1$ such that:

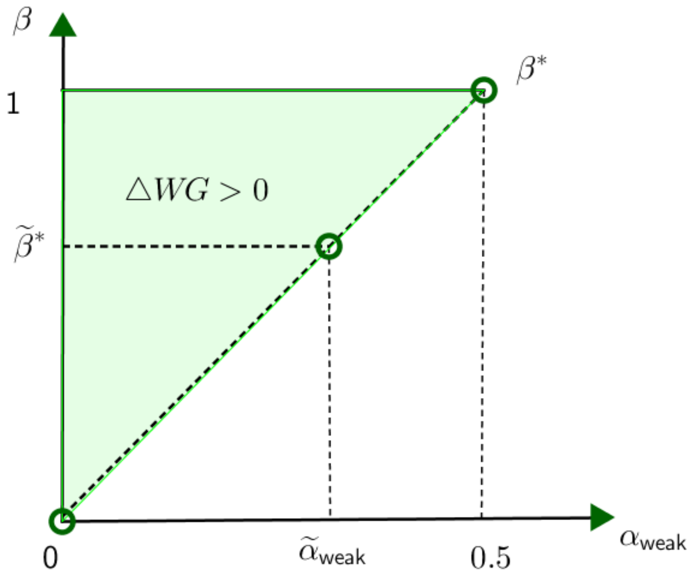
(1) If the proportion of whites with access to strong ties is higher than this threshold value $\beta_{white} > \beta^*$, then the Black-White wage gap is higher in the occupation with higher mean wage $\triangle WG = WG_h - WG_m > 0$.

(2) If $\alpha_{weak} = 0$, then $\beta^* = 0$.

(3) If $\alpha_{weak}^{'} > \alpha_{weak}$, then $\beta^{*'} > \beta^*$.

In the figure 3 below, the informativeness level of weak tie α_{weak} is indicated in the horizontal axis, and the proportion of white workers with access to strong ties is indicated in the vertical axis. The diagonal dotted line indicates the threshold value β^* for a given α_{weak}. Since the values strictly above the threshold value β^* leads to the Black-White wage gap to be higher in the occupation with higher mean wage, the diagonal line itself is not included. For convenience, a straight diagonal line has been drawn in the figure but it may not be a straight line. The shaded region indicates the values of α_{weak} and β_{white} for which the Black-White wage gap is higher in the occupation with higher mean wage $\triangle WG = WG_h - WG_m > 0$.

Fig 4. Wage Gap



If some blacks have access to strong ties, then some high ability blacks enter human capital intensive occupation with strong ties. As long as the proportion of blacks with strong ties is small enough ($\beta_{black} \leq \beta^*$), some below average ability black workers enter the human capital intensive occupation through weak ties because firm cannot infer their abilities. On the other hand, as long as the proportion of whites with strong ties is high enough ($\beta_{white} > \beta^*$), an above average ability white worker with access to a strong tie can separate himself from below average ability white workers. As a result, only above average ability white workers enter human capital intensive occupation. It can then be shown (with some alpha that the Black-White wage gap is higher in the occupation with higher mean wage.

> **Theorem 2 -** If the informativeness level of weak tie is α_{weak}, then there exists a threshold value β^* such that:
>
> For $\beta_{black} \leq \beta^* < \beta_{white}$, Black-White wage gap is higher in the occupation with higher mean wage $\triangle WG = WG_h - WG_m > 0$.

In the figure 3 above, if the informativeness level of weak tie is $\widetilde{\alpha}_{weak}$, the proportion of whites with access to strong ties is high enough (above the diagonal line $\beta_{white} > \widetilde{\beta}^*$) and the proportion of blacks with access to strong ties is high enough (above the diagonal line $\beta_{white} > \widetilde{\beta}^*$) and the proportion of blacks with access to strong ties is low enough (below the diagonal line $\beta_{black} \leq \widetilde{\beta}^*$), then the Black-White wage gap is higher in the occupation with higher mean wage $\triangle WG = WG_h - WG_m > 0$.

4 Use of Ties and Returns to Ties

In this section, I will explore some implications of the employee referrals model for job search. For instance, when should one expect weak ties to be used more than strong ties for job search? When should one expect higher returns from finding a job through a tie (weak or strong) than finding the same job through direct application to the firm? The employee referrals model has new insights about the use of different type of ties and the returns to ties, which can help answer these questions.

4.1 Use of Ties

It is well known that many jobs are found through social networks (see Ioannides and Loury (2004), and Topa (2011) for two surveys). Granovetter (1974) documents that a large proportion of jobs are found through weak ties. Granovetter (1973) argues that the weak ties to individuals with whom one has few common friends are most useful for job search, because they provide access to otherwise unobtainable information about job openings. This finding of the frequent use of weak ties led to the coining of the well-known phrase, the "strength of weak ties". However, the evidence about the use of ties is mixed. Studies in U.S. cities (Murray, Rankin, and Magill (1981), Bridges and Villemez (1986), Marsden and Hurlbert (1988)) find that both weak and strong ties are important for job search. In Japan, Watanabe (1987) documents that small business employers screen applicants using strong ties. In China, Bian (1997, 1999) argues that the guanxi system of personal relationships allocates jobs using strong ties and paths.

The employee referrals model can help explain the mixed evidences about the use of different type to ties. I will now provide conditions under which one type of tie will be used more often than the other type of tie to find jobs. To show this, I relax the assumption that each unemployed worker has access to a weak tie. Only β_{weak} of the unemployed workers have access to a weak tie, β_{strong} of the unemployed workers have access to a strong tie, and β_{both} of the unemployed workers have access to both weak and strong ties. As before, there is a threshold ability level, workers with abilities lower than this threshold prefer weak ties, and workers with abilities higher than this threshold prefer strong ties. Low (below threshold) ability workers use weak ties to pool with workers who don't have access to strong ties and have higher abilities than theirs. On the other hand, high ability workers use strong ties to reveal their abilities to the firm. The lower is the access to strong ties, the higher is the threshold ability, and the higher is the abilities of workers who prefer to pool with even higher ability workers through weak ties. If the access to strong ties is sufficiently low (β_{strong} and β_{both} are low or β_{weak} is high), then even above average ability workers will prefer to use weak ties to pool with even higher ability workers. In such case, the threshold ability is above average ability, and most jobs are found through weak ties. In the proposition 6 below, I state this result formally.

Proposition 6 - If the access to strong ties is sufficiently low $\beta_{weak} > max\{\beta^*, \beta_{strong}\}$, then the threshold ability level is above average ability $\theta^* > E[\theta]$, and most jobs are found through weak ties $f(t_{weak}) > f(t_{strong})$.

This result provides an alternative explanation for the frequent use of weak ties (or the "strength of weak ties"). A worker will use the type of tie which gives him higher returns, if he has access to such type of tie. If many high ability workers don't have access to strong ties, then weak ties will be used more often in job searches because it allows below average ability workers and some above average ability workers to pool with even higher ability workers. Contrary to the existing explanations, the frequent use of weak ties may not be due to its efficiency in matching workers and the firm. Instead, when access to strong ties are really scarce, many workers use weak ties to pool with really high ability workers.

4.2 Returns to Ties

The empirical evidence about the returns to ties is mixed. Some studies show that workers who found their jobs through family, friends, and acquaintances earned more than those using formal and other informal job-search methods (Rosenbaum et al. (1999), Marmaros and Sacerdote (2002)). Others indicate that the initial wage advantage declined over time (Corcoran, Datcher, and Duncan (1980), Simon and Warner (1992)). Some analysts found no general initial or persistent wage effects (Bridges and Villemez (1986), Holzer (1987), Marsden and Gorman (2001)). In fact, some studies (Elliott (1999), Green, Tigges, and Diaz (1999)) show that those using contacts earned less than those using formal methods.

The employee referrals model can help explain the mixed evidences about the returns to ties. I will now consider a variant of the model where workers can directly apply to the firm "d" and through a tie "t". Applying directly to the firm is equivalent to applying through a tie with the lowest informativeness level $\alpha_d = 0$. Suppose the informativeness level of the tie is positive $\alpha_t > 0$. Thus, it can be seen as a special case of the model where weak ties have the lowest informativeness level. I will assume that everyone can apply directly to the firm, but only δ of these workers get the job through direct application, and only β of the unemployed workers have access to ties. Let $w_{t-d}(\theta < 0.5)$ denote the average returns to using ties than direct applications for below average ability workers.

Proposition 7 - (1) If the proportion of workers with access to ties is positive $\beta > 0$, then the threshold ability is below average $\theta^* < E[\theta]$. (2) In addition, if the informativeness level of the tie is above average ability $\alpha_t \geq E[\theta]$, the probability of finding jobs through direct application is $0 \leq \delta < 1$, then there exists a threshold value β^{**} such that:

(i) If $\beta \approx \beta^{**}$, then $w_{t-d}(\theta < 0.5) \approx 0$.

(ii) If $\beta < \beta^{**}$, then $w_{t-d}(\theta < 0.5) < 0$.

(iii) If $\delta' < \delta$, then $\beta^{**'} > \beta^{**}$.

As before, there is a threshold ability level. Workers with abilities lower than this threshold prefer to apply directly to the firm, and workers with abilities higher than this threshold prefer to use their ties. The wage offer from directly applying to the firm equals this threshold ability. Since the informativeness level of the firm is minimal $\alpha_d = 0$, workers can at most get a wage equal to the average ability by directly applying to the firm. This proposition states that (1) if the proportion of workers with access to ties is positive $\beta > 0$, then the threshold ability value is below average ($\theta^* < E[\theta]$). As a result, above average ability workers always get higher returns from ties than directly applying to the firm. This result is consistent with the empirical finding of Cappellari and Tatsiramos (2015). (2 (i), (ii)) The lower is the proportion of workers with access to ties, the higher is the threshold ability, and the lower are the returns to ties. If the proportion of workers with access to ties is sufficiently small ($\beta \leq \beta^{**}$), then for below average ability workers, the returns to tie can be small ($\beta \approx \beta^{**}$) and even negative ($\beta < \beta^{**}$). (2 (iii)) For workers who prefer to find jobs through direct applications, their returns to ties are negative. The lower is the probability of finding a job through a direct application, the more such workers will use ties to find jobs, and the lower are the average returns to ties. This result is consistent with the empirical finding of Loury (2004).

5 Conclusion

For many years, social scientists and policy makers have tried to understand mechanisms that determine the black-white wage gap in the US. There is significant evidence suggesting that the wage gap between blacks and their white counterparts (same individual characteristics) widens as one moves up the wage hierarchies of the private sector. This paper shows that the widespread use of employee referrals in the labor market, and the lack of access to strong ties for blacks (via black-white employment differentials and homophily) can be behind this empirical finding. The model predicts that low (below average) ability workers can get higher wages from finding jobs through weak ties. This is because their use of weak ties leads to firm pooling them with these high ability workers who don't have strong ties. On the other hand, high ability workers can get better wage offers from using strong ties because this leads to firm infering that they are high ability workers. Only high ability white workers enter higher earning occupations because they can separate themselves from low ability white workers through strong ties. As long as sufficiently low number of blacks have access to strong ties, both low and high ability black workers enter higher earning occupations through weak ties, firm pools them together, and on average high ability black workers get relatively lower wage than their white counterparts.

Similar to aptitude tests and other attribute measurements used by firms in formal hiring, the employee referrals mechanism is a useful device for screening job applicants because employees can convey some information to the firm about the abilities of their unemployed ties. However, the employee referrals mechanism is by its nature discriminatory in that not all unemployed workers have the same access to employed ties. Government can help level the playing field for the two races by subsidizing formal screening mechanisms in the higher earning occupations. Such policy suggestion seems counter-intuitive in that the firm already has strong incentives to use formal screening mechanisms in the higher earning (human capital intensive) occupations, and the informational asymmetries between the firm and unemployed workers are arguably lower in the such occupations. However, the lack of access to strong ties for black workers gives white workers an informational advantage in the higher earning occupations. The employee referrals mechanism reduces more asymmetric information between the firm and white workers than between the firm and black workers in the higher earning occupations. This informational advantage of whites at the high end of wage hierarchies is an obstacle for the upwardly mobile blacks towards obtaining equal wages to their white counterparts.

Part III

A Survey on the Models of Employee Referrals: Search Frictions and Screening

1 Introduction

Firms often encourage its employees to refer their social contacts (relatives, friends, and acquaintances) for job vacancies. At this point, there is an empirical consensus, both in economics and in sociology, on the widespread use of employee referrals in the labor market. About 50% of U.S. jobs are found through social contacts and about 70% of firms have programs encouraging referral-based hiring. Given the prevalence of employee referrals, it is important to understand their role in the labor market. Some researchers have argued that they are useful for reducing search frictions, because workers can find information about new job vacancies through their social contacts. Others have argued that they are useful to screen applicants, because firms can get some information about the abilities of applicants from the recommendations of its employees. The existing literature on employee referrals is mainly divided into these two views.[28] Both have interesting implications for labor market outcomes, and they have inspired development of various models.

In this survey, I will show how these models can address many important issues, including the following: Why is there a positive correlation between employment status of individuals who live in the same neighborhood, and/or have the same ethnicity, and race? Why do labor market participation rates differ across groups such as whites and blacks? How can one explain the persistent inequality in wages between these groups? Why does the black-white wage gap widen as one moves up the wage hierarchies of the private sector? Why is there mixed evidence about the wage differentials between workers who found jobs through referrals and workers who found jobs by formally applying to firms? What determines the efficient use of employee referrals when other search methods are available? How useful are social contacts when workers can also use their educational degrees to signal their own abilities? Why is there mixed evidence about the use of different types of contacts in job search? I will also use these models to discuss how the two views differ in their implications for labor market outcomes. Although these two views can have different implications, they also serve a common goal. They both help explain the widespread use of employee referrals, and the underlying incentives of workers and firms to use this mechanism. I will discuss how firms and workers can benefit from using employee referrals under each of these two views.

Before proceeding, I want to make clear what the scope of this survey will be. There is a large

[28] There are also other roles of employee referrals, besides its roles as a search frictions reducing mechanism and a screening mechanism. See section 4.2 for a discussion on some other views.

literature on the use of social contacts. In addition to finding jobs, this literature has considered how social contacts can be used to trade and exchange goods in non-centralized markets, to form mutual insurance agreements in developing countries, and to make various choices (including which products to buy, which career to choose, and who to vote for).[29] But here I will only cover its application in job search. Similarly, there is a rich and growing empirical literature of employee referrals, but I will not discuss all of the econometric issues and empirical findings here.[30] Instead, I will focus on some of the main models in the theoretical literature of employee referrals, and discuss whether the implications of these models are consistent with empirical findings. This survey is organized as follows. In section 2, I will discuss models in which employee referrals can help reduce search frictions. In section 3, I will discuss models in which employee referrals are used as a screening device. Finally, in section 4, I will conclude by discussing some open questions, and possible avenues for future research.

2 Search Frictions

Both economists and sociologists have considered the role of employee referrals as a mechanism to reduce search frictions. Some of the most influential research in this area was conducted by the sociologist, Mark Granovetter. I will begin by discussing models that were inspired by his ideas. Although the Granovetter inspired models are rich in several dimensions, they focus on the workers' side of the market, and so they only provide a partial equilibrium analysis. On the other hand, search and matching models were built to study job search in a general equilibrium framework (both workers and firms play a role), and economists have used them to study the impact of employee referrals as well.[31] I will conclude this section by discussing such models.

2.1 Worker's Perspective

Granovetter (1974) interviewed people in Amherst, Massachusetts, across a variety of professions, to determine how they found out about their jobs. He recorded not only whether they used social contacts in their employment searches, but also the strength of the social relationships as measured by frequency of interactions. Surprisingly, he found that a large proportion of jobs were found through "weak ties" (contacts with whom one has few interactions). This finding of the frequent use of weak ties led to the coining of the well-known phrase, the "strength of weak ties".

[29] See Goyal (2003), Vega-Redondo (2006), and Jackson (2008) for three excellent books on social networks. This literature has considered the influence of social contacts in a variety of situations. In particular, it is concerned with how the network of social contacts is formed and about its impact under different circumstances.

[30] See Topa (2011), for a survey in the economics literature, and Marsden and Gorman (2001), for a survey in the sociology literature.

[31] See Rogerson et al (2005) for a survey on the search and matching literature.

Granovetter's idea was that individuals involved in a weak tie were less likely to have overlap in their social contacts than individuals involved in a strong tie. Such ties then are more likely to form bridges across groups that have fewer connections to each other, and can thus play an important role in disseminating information. Researchers have tried to formalize these ideas by either considering models with different types of contacts (weak and strong ties) or modelling the network structure of social contacts. Next, I will discuss some of these works, and highlight some key insights that came out of them.[32]

2.1.1 Weak and Strong Ties

Boorman (1975) developed a model to understand the use of weak ties in job search. His model is based on the following situation. An individual has T units of time to spend on strong and weak ties. It takes one unit (numeraire) of time to maintain a weak tie, and $\lambda > 1$ units of time to maintain a strong tie. However, strong ties have priority in obtaining job information from social contacts. So the individual is faced with a tradeoff - having more ties, but weak ones, or fewer ties, but strong ones. If W is the number of weak ties that an individual has, and S is the number of strong ties, then they must satisfy:

$$W + \lambda S = T.$$

There are many identical individuals facing this situation. The timing of the model is as follows. Time evolves in discrete periods indexed by $t \in \{1, 2, ...\}$. In any period, with probability μ an individual needs a job, and with probability δ an individual hears about a job directly. These parameters (μ, δ) are the same across individuals and independent of history. If an individual needs a job, then he can find it in two ways. He can hear about a job directly, which happens at the exogenous rate δ. In that case, the individual takes the job. Or, he might not hear directly, but instead might have heard from an employed friend. In that case, the employed friend looks around at his strong ties and weak ties. If some of the strong ties are unemployed, then the employed friend passes the job to one of them uniformly at random. If all of the strong ties are employed, then the employed friend passes the job on to one of the weak ties uniformly at random. Thus, strong ties have a priority in hearing about a job.

Let Q_s and Q_w be the probability that one does not hear about a job through a given strong tie and weak tie respectively, when in need. The chance of getting a job when in need can then be written as

$$\delta + (1 - \delta)(1 - Q_s^S Q_w^W).$$

[32] In section 2.1, my presentation of the models (Boorman (1975) and Calvó-Armengol & Jackson (2004)) follow closely chapter 10 of Jackson's book (2008). However, my discussion differs in that I focus on the labor market implications of these models. Moreover, I also discuss other models such as Montgomery (1994) and some more recent works.

By the assumptions made above, the probability of not hearing about a job through a given strong tie is less than the probability of not hearing about a job through a given weak tie, i.e. $Q_s \leq Q_w$. So a given agent thus trades off the higher probability that strong ties lead to job information against being able to maintain fewer of them. One can derive the expressions for Q_s and Q_w as a function of the parameters of the model.

Boorman's analysis focuses on symmetric equilibria where every individual is choosing an optimal allocation, given the same allocation is selected by all the others. At equilibrium, the following conditions are satisfied:

(1) individuals are optimally choosing S, W given the anticipated Q_s and Q_w, and

(2) the anticipated Q_s and Q_w correspond to the ones generated by the choices that individuals have made concerning S, W.

This model is hard to solve, and it can involve multiple equilibria. Boorman analyzed it by running simulations for some parameter values, and these simulations provided him with the following results. First, as λ increases, the relative cost of strong versus weak ties goes up. As a result, the equilibrium involves fewer strong and more weak ties. Second, as μ decreases, so that one is less likely to need a job, the relative value of weak ties goes up. One only gets a job via a weak tie when all of the weak acquaintance's strong ties are employed, which is more likely when μ is low.

Discussion

Boorman (1975) can help explain why workers may choose to find jobs through weak ties.[33]

- When employment rate is high enough, the value of having many weak ties are higher than the value of having few strong ties. In such case, many jobs can be found through weak ties.

However, this model doesn't say much about the impact of weak ties on labor market outcomes. Montgomery (1994) was one of the first papers to explore this. He added a simple social structure and pattern of social interaction to a Markov model of employment transitions. In the model, society is composed of many small (two-person) groups. Unemployed individuals find jobs through strong ties (intra-group social interaction), weak ties (random intergroup interaction), and formal channels. Holding constant the total level of social interaction, the author examines how a change

[33]There are several interesting aspects of the Boorman's (1975) model. There is a growing literature on the formation of networks. Boorman's model is one of the first models in this literature. In particular, Calvó-Armengol (2004) uses Boorman's model to build his model of how social networks are strategically formed. Calvó-Armengol & Zenou (2005) use some elements of Calvó-Armengol (2004) to build their search equilibrium model, which allows us to consider the influence of employee referrals in a general equilibrium framework (see section 2.2.1 for some discussion about this model). On a different note, it is hard to construct a model that considers a rich social network structure, and can help explain its influence on labor market outcomes. Calvó-Armengol & Jackson (2004, 2007) were able to develop such a model by adding a rich social network structure to Boorman's model (see section 2.1.2 for some discussion about this model).

in the composition of social interaction (in particular, the use of weak ties) affects the steady-state equilibrium. He finds that an increase in weak-tie interactions reduces inequality, thereby creating a more equitable distribution of employment across groups. Furthermore, Calvó-Armengol et al (2007) and Zenou (2005) extended his model to consider the effects of unemployment benefits on crime, and the effects of weak-tie interactions on the employment rate, respectively.

2.1.2 Network Structure

Calvó-Armengol & Jackson (2004) developed a model to understand the influence of social networks on job search and aggregate labor market outcomes. They examine a model that is similar to Boorman's in having job information arrive directly and through social contacts. There are n workers or agents who are connected by a network, represented by the $n \times n$ symmetric matrix g, which has entries in $\{0, 1\}$. Time evolves in discrete periods indexed by $t \in \{1, 2, ...\}$.

The vector s_t describes the employment status of the agents at time t. If agent i is employed at the end of period t, then $s_{it} = 1$ and if i is unemployed then $s_{it} = 0$.

Period t begins with some agents employed and others unemployed, as described by the state s_{t-1}. Next, information about new job openings arrives. Each agent directly hears about a job opening with a probability $a \in [0, 1]$. This job arrival process is independent across agents. If an agent i is unemployed ($s_{i,t-1} = 0$) and hears about a job, then he takes that job and becomes employed. If an agent i is employed ($s_{i,t-1} = 1$) and hears about a job, then he picks an unemployed tie ($s_{j,t-1} = 1$) and passes the job information to that tie. If agent i has several unemployed ties, then the agent picks one of them uniformly at random. If agent $i's$ ties are all employed, then the job information is lost.

The probability of the joint event that agent i learns about a job and this job ends up in agent $j's$ hands is described by $p_{ij}(s_{t-1})$, where

$$p_{ij}(s_{t-1}) = \begin{cases} a & \text{if } s_{i,t-1} = 0 \text{ and i = j;} \\ \dfrac{a}{\sum_{k:s_{k,t-1}=0} g_{ik}} & \text{if } s_{i,t-1} = 0 \text{ and } g_{ij} = 1\text{; and} \\ 0 & \text{otherwise.} \end{cases}$$

At the end of a period some employed agents lose their jobs. This happens randomly according to an exogenous breakup probability, $b \in [0, 1]$, independently across agents.

Discussion

Calvó-Armengol & Jackson (2004) model provides several interesting insights.

- Employment is positively correlated across time and agents.

Observe that the better is the employment status of a given agent's contacts, the more likely it is that those contacts will pass information about a job opening to the agent. This sort of information passing leads to a positive correlation between the employment status of agents who are directly or indirectly connected in the network, within a period and across time.

- Unemployment exhibits duration dependence, i.e. the probability of obtaining a job decreases in the length of time that an agent has been unemployed.

A longer history of unemployment is more likely to come when the direct and indirect connections of an agent are unemployed. Thus, seeing a long spell of unemployment for some agent leads to a high conditional expectation that the agent's contacts are unemployed. This in turn leads to a lower probability of obtaining information about jobs through the social network.

They extend their framework to consider two groups with the same network structure, but with different initial employment rates. Moreover, individuals can choose to stay in the labor market or drop-out. They examine how the drop-out rates between these two groups depend on their initial employment rates, and they find the following.

- If staying in the labor market is costly and one group starts with a worse employment status, then that group's drop-out rate will be higher and their employment prospects will be persistently below that of the other group.

This can help explain why labor market participation rates differ across groups such as whites and blacks. Agents in the network with worse initial starting conditions have a lower expected discounted stream of future income from remaining in the network than agents in the network with better initial starting conditions. This is because employment is positively correlated across time and agents. This difference in expected discounted stream of future income might cause some agents to drop out in the worse network but remain in the better network. This dropping out has a contagion effect. When some of an agent's connections drop out, that agent's future prospects worsen since the agent's network is no longer as useful a source of job information. Thus, some agents connected to dropouts also drop out due to this indirect effect. This can escalate, so a slight change in initial conditions can lead to a substantial difference in drop-out decisions. As a larger drop-out rate in a network leads to worse employment status for those agents who remain in the network, they find that slight differences in initial conditions can lead to large differences in drop-out rates and sustained differences in employment rates.

Calvó-Armengol & Jackson (2007) developed a richer version of their model with heterogeneous jobs and multiple wage levels.[34] They used this model to study wage patterns and dynamics, and show that many of their results for employment can be applied to wages. In particular, this model can help explain the persistent inequality in wages between blacks and whites as follows.

[34] Bramoullé and Saint-Paul (2010) extended their model to consider situations where the network of social ties is not fixed.

- If staying in the labor market is costly and one group starts with a worse wage status, then that group's drop-out rate will be higher and there will be persistent differences in wages between groups according to the starting states of their networks.

2.2 Firm's Perspective

Models discussed above only analyze half of the market. That is, firms play no role in the analysis, as jobs and wages simply appear at an exogenous rate. I will now discuss models in the search and matching literature, which were built to study job search in a general equilibrium framework (both workers and firms play a role). There are two generations of such models. Those in the first generation consider almost no network structure. Diamond (1981), Holzer (1988), Montgomery (1992), and Mortensen & Vishwanath (1994) are some examples of such works. Models in the second generation consider symmetric networks (everyone is identical in their network position), and they examine the impact of network size on labor market outcomes. The basic environment in the second generation borrows from the matching model of Pissarides (2000). Ioannides and Soetevent (2006), Fontaine (2008), Kuzubas (2009), Cahuc & Fontaine (2009), Galenianos (2014), Galeotti & Merlino (2014) are some examples of second generation models. Next, I will discuss a model that belongs to the second generation.

2.2.1 Search and Matching

Calvó-Armengol & Zenou (2005) developed a job search model with symmetric network structure, described as follows. There is a continuum of ex-ante identical workers of mass n, and a continuum of ex-ante identical firms. Time evolves in discrete periods indexed by $t \in \{1, 2, ...\}$.

The timing of the model is as follows. At the end of period $t-1$, the unemployment rate is u_{t-1}, and period t begins with this unemployment rate. At the beginning of period, V_t vacancies are posted, and $v_t = \frac{V_t}{N}$ denotes the vacancy or job arrival rate. Let $h(s, u_{t-1}, v_t)$ denote the individual probability of finding a job through s social contacts in period t. At the end of each period, employed workers loose their jobs with probability δ. So the resulting employment rate at the end of period t is $(1-u_t) = (1-\delta)\left[(1-u_{t-1}) + u_{t-1}h(s, u_{t-1}, v_t)\right]$, where $u_{t-1}h(s, u_{t-1}, v_t)$ have been newly employed and $(1-u_{t-1})$ were already employed from last period. Period $t+1$ begins with the new employment rate, and so on.

At each period, each individual worker is in direct contact with a group of s workers. The network is symmetric (each worker has the same network position), and s can be interpreted as network size. At the beginning of each period, each worker draws his s contacts at random among the total population of workers. This implies that, on average, at each period, each worker meets with us unemployed workers and $(1-u)s$ employed workers. Consider one currently employed worker who is aware of some job slots available for the current time period. Then, this informed and employed worker passes the job information on to any of the us unemployed contacts he meets during the current period. The firm who has posted the vacancy then treats all these unemployed applicants to an equal footing, who all have the same probability to obtain the job. If an unemployed worker hears of two or more vacancies from two or more employed contacts, then he applies only for one randomly selected job.

To derive the individual probability of finding a job through social contacts, fix an unemployed worker i and consider some other worker j in direct contact with i for the time being. Worker j is employed and aware of a redundant job offer with probability $v(1-u)$. Redundant job information is transmitted to unemployed contacts. Worker i is the selected recipient for the information with probability:

$$\sum_{k=0}^{s-1} \binom{s-1}{k} \frac{1}{k+1}(1-u)^{s-k-1}u^k = \frac{1-(1-u)^s}{us}.$$

A random draw of s social contacts by worker j contains $0 \leq k \leq s-1$ additional unemployed workers, besides i, with probability $\binom{s-1}{k}(1-u)^{s-k-1}u^k$. The job information vacancy held by j is then assigned to any of these $k+1$ unemployed with uniform probability, and worker i receives such information with conditional probability $1/(k+1)$. Let

$$\tau(s,u,v) = v(1-u)\frac{1-(1-u)^s}{u}$$

Then, $\tau(s,u,v)/s$ is the probability of worker i finding a job from his direct contact j. So the individual probability of finding a job through social contacts,

$$P(s,u,v) = 1 - \left[1 - \frac{\tau(s,u,v)}{s}\right]^s.$$

Since $\tau(s,u,v)/s$ is the probability that one of the direct contacts of a given worker i transmits the job information, then $[1-\tau(s,u,v)/s]^s$ is the probability that none of his s direct contacts transmit this information to i and thus $P(s,u,v)$ is the complementary probability. Finding a job through word-of-mouth is thus a random experiment consisting of s repeated independent Bernoulli trials with a probability of success at each individual trial given by $\tau(s,u,v)/s$. So $P(s,u,v)$ depends on the current labor market conditions (u,v) and on the ongoing information transmission process, captured by s.

An unemployed worker can find a job directly with probability v or by gathering information about jobs through his social contacts with probability $P(s,u,v)$. So, he will find a job with probability

$$h(s,u,v) = v + (1-v)P(s,u,v).$$

There are u unemployed workers that find a job with probability $h(s,u,v)$. Since this probability is independent across different individuals, the rate at which job matches occur per unit of time is just $uh(s,u,v)$. Thus, the matching function for the labor market is

$$m(s,u,v) = u\left[v + (1-v)P(s,u,v)\right].$$

Discussion

Calvó-Armengol & Zenou (2005) highlight the importance of considering the network structure in these models.

- The properties of the matching function with respect to the unemployment rate and the vacancy are the same as in the earlier job matching literature; they differ only with respect to network degree, in which case it is not only not homogenous of degree one, as in Pissarides (2000), but not even monotonic.

When the network degree increases, on average, unemployed workers hear about more vacancies through their social network. At the same time, it is more likely that information about multiple vacancies reach the same unemployed worker. It is therefore important to see whether this non-monotonicity is present in the data. The properties of the matching function affect the equilibrium level of unemployment in the economy. It is increasing in the network size, for sparse networks, and decreasing for dense ones. Since aggregate labor market outcomes depend on the network structure, it is important to examine richer network structures in future research.

Granovetter (1974) argued that "a full understanding of matching requires also an assessment of when formal procedures have an advantage [over social networks], and when we may expect to find them. This is an important subject about which we know very little". To study this, Cahuc & Fontaine (2009) consider an environment in which unemployed workers and employers can be matched through social networks (employee referrals) and through other methods (private agencies and ads in newspapers), and they find the following.

- Due to coordination failures in search strategies of workers and firms, social networks can be over-utilized, with respect to an efficient allocation, in some circumstances and under-utilized in others.

When an agent chooses which methods to use, he considers which methods are used by the other side of the market. For example, it is not useful for a firm to put ads in newspapers if very few workers use newspapers to find a job. In the same way, it is not helpful for a worker to buy newspapers to find a job if no firm uses this hiring channel. More generally, in some cases, the choice of search strategies suffers from coordination failures. It would be efficient for the economy that the agents choose another search strategy. However, a change on only one side of the market is not profitable.

Hence, workers and firms must simultaneously modify their search strategies. In a decentralized equilibrium, such coordination of strategies may not be possible.

3 Screening

The role of employee referrals as a screening mechanism was mainly considered by models in the economics literature. In the seminal work of Albert Rees (1966), he identifies one key motivation for the use of informal hiring channels on both sides of the labor market. For the employer, relying on referrals from current employees may reduce the adverse selection problem he faces when trying to hire someone if there is uncertainty about the worker's or the match's quality. Similarly for the job seeker, there may be an information advantage in relying on a personal contact to gather information about the employer's characteristics or the prospective match quality.

I will begin by discussing models that formalize some of Rees' ideas. Although there is growing evidence which supports them, employee referrals are known to have biases, and these biases can reduce their informational advantages.[35] In particular, employees may recommend their friends not based on their quality, but because of nepotism. In the second half of this section, I will discuss models that consider the impact of nepotism.

3.1 Firm's Perspective

Rees argued that both employers and job seekers may prefer to use informal job matching methods rather than formal ones. On the employer side, "Employee referrals - the most important informal channel - usually provide good screening for employers who are satisfied with their present workforce. Present employees tend to refer people like themselves, and they may feel that their own reputation is affected by the quality of the referrals [they provide]". On the job seeker side, "... informal sources also have important benefits to the applicant. He can obtain much more information from a friend who does the kind of work in which he is interested than from an ad in the paper or a counselor at an employment agency, and he places more trust in it".

Researchers have formalized these ideas, but only from the employers perspective. In particular, they have considered the fact that employees tend to refer people like themselves (homophily), and reputation costs in providing referrals. Next, I will discuss some of the main models in this literature, and highlight some key insights that came out of these models.

[35] See Simon & Warner (1992), Arrow & Borzekowski (2004), and Dustmann et al (2016) for some discussions and findings that support Rees' ideas.

See Smith (2005, 2010), Beaman & Magruder (2012), Heath (2018), and Bramoulle & Huremovic (WP) for some discussions and findings on nepotism in referrals.

See Goldberg (1982) for nepotism in labor market hiring.

3.1.1 Homophily

A model where employees tend to refer people like themselves (homophily) was developed by Montgomery (1991). This model has two periods and there are a large number of workers and firms. The following assumptions are made on workers, firms, social structure (network), and what takes place in each of the two periods.

Workers. Each worker lives for one period, and there are equal numbers of workers in each period. There are two types of workers: high ability and low ability. High ability workers are more productive than low ability workers. In particular, the productivity of a high ability worker is one, and the productivity of a low ability is zero. Each worker knows his own ability (private information).

Firms. Firms are free to enter the market in either period. Each firm employs one worker in each period. The profit of a firm is equal to the productivity of the worker minus the wage which is paid to the worker (product price is exogenously determined and normalized to unity). Firms are uncertain about the ability of any particular worker.

Social Structure. Each period-1 worker knows at most one period-2 worker. Each period-1 worker has a tie (knows a period-2 worker) with probability τ. Conditional upon holding a tie, a period-1 worker knows a period-2 worker of his own type with probability $\alpha > 0.5$. Conditional upon holding a tie with a period-2 worker of some type (either same or different), a period-1 worker is equally likely to know any period-2 worker of this type. The social structure is thus defined by the two parameters, τ reflecting the density of social network and α reflecting the inbreeding bias in the social network. The assumption that $\alpha > 0.5$ reflects the idea that it is more likely that a worker knows someone with the same ability as himself (this feature is also known as homophily). Since ties are randomly assigned, it is possible that some period-2 workers have many ties while others have none. Although the assumed social structure is rather simplistic, it captures an important idea: some workers are "well connected", while others are not.

Timing. At the start of period 1, firms hire period-1 workers through the market, which clears at a wage given by w_{M1}. In period 1, production occurs. During the process of production, each firm learns the ability of its worker. Social ties are assigned between period-1 and period-2 workers. At the start of period 2, each firm decides whether to hire through employee referral or not. If firm i decides to hire through employee referral, then it offers a referral wage w_{Ri}. These are communicated via social contacts to workers in period 2, who then compare wage offers and accept one of the referral offers. If a worker rejects all offers, then he goes to the market. Similarly, if a firm's referral offer is rejected, then it goes to the market as well. The decentralized anonymous market in period 2 clears at a wage denoted by w_{M2}.

At equilibrium, no firms want to enter or exit the market, firms optimize given their information, and workers take the best offer they get. The equilibrium notion is a variation on a competitive equilibrium, since in offering a referral wage, a firm is entering an auction against other potential employers who might also be making a referral offer to the same worker. The analysis is concerned with how the market wages w_{M1}, w_{M2}, and the referral wages, w_{Ri}, relate to the parameters of social structure τ and α. It is useful to start by noting that in the absence of a social structure, learning about period 1 workers will give no information on period 2 workers, and so the two periods will be identical and separate. In such a world the probability that a firm hires a high type worker is equal to 0.5, and this will also be the market clearing wage. This is the benchmark to keep in mind when considering the influence of social structure (τ, α).

Discussion

Montgomery's (1991) model provides several interesting insights.

- Firms can earn positive profits by hiring through employee referrals.

In a world where $\tau > 0$ and $\alpha > 0.5$, learning about the period-1 worker gives the firm some information on the ability of a contact that its period-1 worker has. In particular, if period-1 worker has high ability, then the firm expects that a worker contacted via a referral is more likely to be a high type rather than a low type. The converse is true if the period 1 worker has low ability. Thus, a firm will want to hire via referral only if its period-1 worker is of high ability. So in period 2, a firm which has a high ability worker can make positive profits. This is because it will use referral wages, and there is imperfect competition between firms who use referrals.

- Workers hired through employee referrals earn higher wages than workers hired through the anonymous market (directly applying to the firms).

This is because every worker with ties has the option to find job through the anonymous market. So a period-2 worker will accept an offer through employee referral if he gets a wage offer at least as high as the market wage w_{M2}. A firm will offer such wage if its period-1 worker is of high ability. If this firm has at least one competitor (when period-2 worker has multiple ties), then it will offer wage strictly greater than the market wage w_{M2}. If this firm has no competitor (when period-2 worker has only one tie), then it will offer wage equal to the market wage w_{M2}.

- Homogenous workers can earn different wages.

This follows from the fact that workers differ in the number of social ties that they have. In equilibrium, a period-2 worker's wage is determined by the number and abilities of ties he holds. A low ability period-2 worker is likely to have ties mostly with low ability period-1 workers, and a high ability period-2 worker is likely to have ties mostly with high ability period-1 workers. Thus, a high ability period-2 worker is more likely to receive referral wage offers, and referral wage offers are at least as high as the market wage. Moreover, even among high type workers, those who have more links with period-1 high ability workers will receive more offers and therefore will end up getting higher wages. Thus, the social structure of contacts has powerful implications for wage inequality among period 2 workers.

In this model, a high ability worker has no way of signaling his ability. In labor markets, workers can use mechanisms such as educational degrees to communicate their abilities. This raises an important question: what is the role of social contacts when workers also have access to signaling mechanisms? Casella & Hanaki (2006) examine this question by using an extension of the Montgomery's (1991) model of referrals discussed above. The model contrasts signals and social contacts in the following way: a signal can be bought at a cost and it offers a proof of ability which is valid across all potential employers, while a social contact allows access to a single employer, and communicates ability via the assortative tie hypothesis (as in the referral model above). Their main result is as follows.

- If social contacts can be formed freely (no cost), then they are preferred to signaling by both firms and workers in most cases.

In a context where certificates are imperfect signals of ability, for signal to work well they must be costly to acquire. However, if they are costly to acquire, then social ties (which are cheap) become attractive and signals are not used. These contradictory pressures on signals imply that social networks are quite resilient even in the presence of "anonymous" mechanisms, such as educational certificates, which signal ability.

3.1.2 Reputation Costs

A model in which employees are concerned with how their own reputation is affected by the quality of the referrals they provide was developed by Saloner (1985). I will now discuss a simple version of his model in which there are only two referees in the network: referee 1 and 2. Each job applicant i of referee j, denoted $X_j^i \in \{0, 1\}$, is one of two quality types - high quality (equal to one) or low quality (equal to zero) - and the proportion of each is assumed to be the same for each referee.

While the referee is uncertain about the actual quality of the job applicant, he has some private information, y_j^i, on the basis of which he is able to form a subjective probability assessment, a_j^i, that the job applicant is a high-quality worker:

$$a_j^i = pr(X_j^i = 1 | y_j^i).$$

The standard positive screening assumption is made, namely that (in expectation) a higher proportion of job applicants are high quality the higher the referee's subjective probability assessment. This simply means that the referee's private information is useful in distinguishing between high and low quality job applicants. Thus, for example, a higher proportion of the persons that the referee subjectively believes to have a probability of 80% or more of being high quality are in fact high quality than of those that the referee believes have a probability of, say, 40% or more of being high quality. Thus,

$$b_j^i = pr(X_j^i = 1|a_j^i)$$

is the probability that job applicant X_j^i is high quality when the referee's assessment this is the case is a_j^i, then, by the positive screening assumption, $b_j(a_j^i)$ is a strictly increasing function of a_j^i. Moreover, $b_j(a_j^i)$ is continuous and $b_j(1) = 1$; that is, the referee is subjectively certain that an individual is high quality only if this is in fact the case. Let $f_j(a)$ be the number of job applicants to whom referee j assigns a subjective probability a of being high quality; that is, $f_j(a)$ is the frequency density function of subjective probability assessments of referee j.

The referee signals each job applicant as either high quality or low quality.[36] References of this kind correspond to the graduate/fail decision of an educational institution or a hire/don't hire reference type. So a strategy, r^j, for each referee j, is a function that describes whether the referee signals the job applicant of a particular type as high quality or low quality. Thus, $r_j : a_j^i \to \{0, 1\}$. The referee's strategy, for example, could be to signal as high quality all the job applicants, only those he considers to be the very best (or the very worst), a mixture of the best and worse, or those considered to have a moderate chance of being high quality. Potential employers have no private information at all about job applicants prior to their employment. However, if employed, the worker's true quality is later revealed. A referee's reputation is characterized by the average quality of group he signaled as high quality who are in fact high quality.

There is no conflict of interest between the user of the evaluation and any one evaluator. In other words, with complete information everyone would agree on the appropriate ranking of job applicants. Nonetheless, once there are multiple evaluators who are in competition with each other, one should expect the strategic use of private information when evaluations are made. For example, one may expect referees to make excessive claims about the workers they are recommending in order to have them placed ahead of the candidates being recommended by a rival referee.

The demand side of the market operates as follows. There is some number, D, of job openings. Since high-quality workers are more productive than low-quality workers, employers are assumed to want to maximize the expected average quality of the D workers hired to fill these openings. With D fixed, this is equivalent to maximizing the number of job applicants hired who are in fact high-quality workers.

Now consider the objective of the referee. Let T_i be the total number of referee $i's$ job applicants. In general the referee will prefer both that the job applicants he recommends find jobs and that the expected average quality of those job applicants be high. Let n_b to be the number of job applicants the referee recommends as high quality that are ultimately hired, and n_{bb} to be the expected number of the referee's recommended job applicants that are hired and that are in fact

[36] This means that referees are not able to give references that contain, for example, a subjective probability assessment. This assumption can be relaxed without altering the results.

high quality, then the referee's utility can be represented by the function $u(n_b, \frac{n_{bb}}{n_b})$. Moreover, $u(.)$ is concave, $u_1(.) \geq 0$ and $u_2(.) \geq 0$, where subscripts denote partial derivatives in the usual way.

The employer beliefs are represented by functions $B^j(s_j)$, $(j = 1, 2)$, where B^j is the employers' belief about the average quality of the group signaled as high quality by referee j when the number of job applicants signaled as high quality by that referee is s_j. The B^j functions trivially determine the employer beliefs about the average quality of the work seekers signaled as low quality, since the overall average quality of the job applicants is assumed to be known. Given the form of the employers' objectives above, the B^j functions also determine which job applicants will be hired. Employers face four groups of job applicants: those signaled as high quality and as low quality by referee 1 and similarly by referee 2. Clearly, they will attempt to satisfy their demand, D, by choosing workers first from the group with the highest average quality, next from the group with the second-highest average quality, and so on. Since the optimal employers' hiring rule is determined by their beliefs, the hiring rule is not made explicit in the definition of equilibrium that follows.

An equilibrium is a set of employers' beliefs and referees' strategies $\left[B^{1*}(s_1), B^{2*}(s_2), r_1^*, r_2^*\right]$ such that:

(1) given B^{1*} and B^{2*}, $r_1^*(r_2^*)$ is a best response to $r_2^*(r_1^*)$, and

(2) in expectation, the average quality of the s_j job applicants signaled as high quality by referee j using r_j^* is $B^{j*}(s_j)$.

Thus the equilibrium concept is Nash and has the usual rational expectations flavor to it. Given the beliefs of the employers, neither referee can alter his strategy to make himself better off. Further, the average quality of the worker seekers signaled as high quality using these strategies is such that the employers' beliefs are fulfilled in expectation.

If employers knew the actual quality of the job seekers, their problem would be trivial, as they would merely hire up to D of the high-quality workers. On the other hand, if they lacked this information but knew the referee's private assessments, a_j^i, and their probabilities of being correct, $b_j(a_j^i)$, they would be able to rank the job applicants by their expected quality and hire the D with the highest probabilities of being high quality. Instead, employers observe only the references given by the referees. Given that referees have only a binary choice of references, the employers will face at most four groups of workers: those signaled as high quality and as low quality by referee 1 and similarly by referee 2. By rational expectation, employers will learn the average quality of the work seekers in each of these groups. Clearly, if the average quality of the workers in those different groups differed, the employers would want to change their rule to something like the following: select workers first from the group that has had in the past the highest average quality, next from the group that has had in the past the highest average quality, next from the group that has had in the past the next-highest average quality, and so on until the demand of D has been satisfied. Recall, a referee's reputation is characterized by the average quality of group he signaled as high quality who are in fact high quality. Thus, referees will be concerned with how their own reputation is affected by their referrals. Given the hiring rule being used by the employers, the referees would want to adjust their strategies so as to have good reputation and maximize the number of their high-quality workers who are hired.

Discussion

Saloner (1985) provides an economic rationale for the existence of informal screening mechanisms in which intermediaries provide personal opinions about the likelihood of success of projects, investments, or personnel. The competition between the referees, coupled with the direction in which their incentives operate, induces them to reveal sufficient information to result in an optimal outcome for the employers.

- First, the intermediaries, or referees, act strategically, yet the result of their strategic action is a truthful ranking of the job applicants in the sense that no job applicant whom the referee believes is of a lower caliber than another is recommended above the other.

- Second, the equilibrium strategies involve "partial pooling"; that is, the referees divide the job applicants into only two distinguishable groups. Thus the strategies do not fully reveal the referees' private information. However, the same job seekers are hired as would have been hired had the employers had the same information as the referees.

3.2 Worker's Perspective

The models discussed above either do not consider the incentives of employees, or attribute to firms and employees the same objective.[37] However, employees may have their own biases, and these can result in important effects on the quality of referrals. In this section, I will consider models that allow for nepotism. In such models, employees may recommend their contacts not based on their quality, but because of personal or social values.

3.2.1 Nepotism

Karlan et al (2009) developed a model that allows for nepotism in employee referrals, but firms can choose which employees are allowed to provide referrals.[38] There is an employer t who needs to fill a vacancy. Potential employees are either high or low types; if hired, a high type generates total

[37] First, Montgomery's (1991) model ignores strategic communication (recommendations) between firms and its employees, each employee is connected with at most one job seeking worker and each employee must provide referral for his job seeking contact. Employees are more likely to be connected with people like themselves (homophily by ability), and firms can use this information to learn about applicants abilities. So employees don't play an active role in this model. Second, in Saloner's (1985) model, employees want to refer workers with highest abilities and firms want to hire workers with highest abilities. Thus, firms and employees have the same objective.

[38] More generally, they proposed a model where the social network influences how much agents trust each other. This employee referrals model is an application of their more general model.

value S_H and a low type generates S_L, where $S_H > S_L > 0$. In the formal labor market, worker types are unobservable, the proportion of high types is π_H, and the prevailing market wage rate is w. Thus, hiring from the labor market generates an expected surplus $\overline{S} = \pi_H S_H + (1 - \pi_H) S_L$, of which $\overline{S} - w$ accumulates to the employer. However, the employer may be able to hire a known high type through his social network G. If s is a high-type job candidate, and his type can be credibly communicated to the employer, then the surplus from hiring s versus hiring from the formal labor market is $S_H - \overline{S}$. Assuming that this surplus is divided by Nash bargaining, where the bargaining weight of the worker is α, the wage of s if hired is $w_H = w + \alpha * (S_H - \overline{S})$, and the excess profit of the firm relative to hiring from the labor market is $(1 - \alpha) * (S_H - \overline{S})$.

The type of worker s is only observed by himself and his direct friends, denoted $s_1, ..., s_k$. Although these friends can, in principle, provide recommendations, they face a moral hazard problem: a low-type worker s can bribe them to write good recommendations. Here bribes are interpreted broadly to include in-kind transfers, as well as being nice to the recommender. The amount candidate s is willing to spend on bribes is limited by the attractiveness of the job, $\alpha * (S_H - \overline{S})$; if he offers more, the bribes would exceed the profit from getting the job. This reasoning suggests that the network can only communicate worker type in a credible way when the employer's trust of recommenders, $s_1, ..., s_k$ exceeds the highest bribe that the worker can pay, $\alpha * (S_H - \overline{S})$.

The timing of the model is as follows. In stage 1, a set of agents including $s_1, ..., s_k$ and t, agree on a transfer arrangement that specifies transfer to be made in the event that $s_1, ..., s_k$ send recommendations, and s is hired and then turns out to be a low type. A side deal with bribes is a new transfer arrangement proposed by s to $s_1, ..., s_k$ at the beginning of stage 2, together with a set of bribes $b_1, ..., b_k$ that s pays to $s_1, ..., s_k$ in exchange for their recommendation. In stage 2, agents $s_1, ..., s_k$ choose whether to recommend s to the employer t. In stage 3, t decides whether to hire s or not; profits are earned, and the type of s is publicly revealed. In stage 4, if needed, the transfer arrangement is executed; and in stage 5, agents derive utility from their remaining relationships.

Discussion

In Karlan et al's (2009) model, when the network-based trust between the employer and recommenders exceeds the sensitivity of profits to worker's type, as measured by the term $\alpha * (S_H - \overline{S})$, the true type of the worker can be credibly communicated. There are several interesting implications of this result.

- Network-based trust should be more important for high-skilled jobs, where the employer's profits are more sensitive to worker's type.

This helps explain the mixed evidence about the strength of weak ties by showing that for high-skilled jobs strong ties should be more important. However, this result also implies that when filling high-skill vacancies, employers should search more through their networks. Although there is some empirical evidence supporting this finding, overall, the empirical evidence suggests the opposite - referrals are used more often for less educated and blue-collar workers.[39]

- Jobs obtained through the network should earn higher wages than jobs obtained in the market.

This follows from the fact that low-type workers are never hired through the network. Although there is some empirical evidence supporting this finding, overall, the empirical evidence is mixed.[40]

- Due to the increased importance of trust for high-quality jobs, the wage differential between network-based and market-based hires, $w_H - w = \alpha * (S_H - \overline{S})$, should be positively related to skill intensity.

If one assumes that fewer black workers have ties with employed workers than white workers, then this result can help explain the fact that black-white wage gap widens as one moves up the wage hierarchies of the private sector (Kaufman (1983), Grodsky & Pager (2001), Huffman (2004)). However, this result is based on a finding about wage that is inconsistent with the empirical evidence - the finding that wage differential between referral-based and market-based hires is positive. It is also based on a finding about the usage of referrals across jobs that is inconsistent with empirical evidence - the finding that referrals are used more for high-skill jobs.

3.2.2 Weak and Strong Ties

Safi (2018) developed a model in which employees are concerned with not only how their own reputation is affected by the quality of the referrals, but also by the gratitude benefits they receive from their social contacts for providing referrals. In addition, workers can choose which types of contacts to ask for referrals among the set of contacts they have access to. I will now discuss a simple version of my model in which weak ties (acquaintances) are completely un-informative about the abilities of the applicants, and strong ties (close friends) know the abilities of the applicants.

Social Structure. The model involves an environment with three sorts of agents: unemployed workers, employed workers, and a single firm. The firm has no relationships/ties with the un-

[39] Regarding the intensity of network search, Brown (1967) finds that among college professors, personal networks are more frequently used in obtaining jobs of higher rank, smaller teaching loads, and higher salaries and at more prestigious colleges. For these attractive jobs, reducing asymmetric information is likely to be more important, and hence, employers have a stronger preference for searching through their networks.

Ornsterin (1976), Corcoran et al (1980), Datcher (1983), Marx and Leicht (1992) all report higher usage for less educated job seekers. Elliot (1999) finds that informal contacts are more frequently used in high-poverty neighborhoods than in low-poverty ones. Similarly, Green et al (1995) report that poor job seekers in Atlanta were more likely to use friends and relatives than non-poor ones. Rees and Schultz (1990) and Corcoran et al (1980) both find that informal search methods are used more often for blue-collar than for white-collar occupations.

[40] Some studies show that workers who found their jobs through family, friends, and acquaintances earned more than those using formal and other informal job-search methods (Rosenbaum et al. (1999), Marmaros and Sacerdote (2002)). Others indicate that the initial wage advantage declined over time (Corcoran, Datcher, and Duncan (1980), Simon and Warner (1992)). Some analysts found no general initial or persistent wage effects (Bridges and Villemez (1986), Holzer (1987), Marsden and Gorman (2001)). In fact, some studies (Elliott (1999), Green, Tigges, and Diaz (1999)) show that those using contacts earned less than those using formal methods.

employed workers, but the employed workers have ties with the unemployed workers. As a result, the firm is indirectly connected to the unemployed workers through its employees. A tie $t \in (t_{weak}, t_{strong})$ between an employed worker and an unemployed worker can differ by its strength (weak or strong). An unemployed worker's type is a pair (θ, T), where θ is his ability level and the set T indicates the type of ties that he has with the employed workers. There is a continuum of ability levels $\theta \in [0,1]$ and they are uniformly distributed $\theta \sim u[0,1]$. I assume that each unemployed worker has access to a weak tie $Prob(t_{weak} \in T) = 1$, but only β of the unemployed workers have access to a strong tie $Prob(t_{strong} \in T) = \beta$. I take the network of relationships/ties between the agents as given, and examine its influence on the wage determination process.

Information Structure. The role of network in this paper is to transmit information. An unemployed worker's ability level is his private information, but his ties receive a signal $s \in [0,1]$ about his ability level, and the firm tries to infer some information about the ability level through the recommendations of the employed workers. Moreover, weak ties receive less precise signals than strong ties. For the purposes of this survey, I will consider the simple case where weak ties receive useless signals, $s = 0.5$ for each $\theta \in [0,1]$, and strong ties receive precise signals, $s = \theta$ for each $\theta \in [0,1]$.

Timing. My model consists of four stages. At the beginning of stage 1, nature determines the ability levels of unemployed workers and the type of ties they have access to. Then, firms and employed workers receive signals about the ability levels of unemployed workers. At the beginning of stage 2, each unemployed worker (θ, T) chooses tie type $t \in T$ that maximizes his wage,

$$\pi_U = w(\rho, t)$$

where wage depends on the tie's recommendation $\rho(s,t)$, and the tie's type t. Observe that the tie's recommendation $\rho(s,t)$ is also a function of the tie's type t and the signal s that he received. At equilibrium, the proportion of unemployed workers with ability level θ who chose tie type t is denoted by $f(t|\theta)$. I assume rational expectations in that employed ties and firms know $f(t|\theta)$ for each ability level θ and tie type t, and they use it to form their expectations. An employed worker faces both gratitude benefits and reputation costs of providing a recommendation $\rho \in [0,1]$. The value of gratitude is strictly increasing in the wage $w(\rho, t)$ offered to the unemployed worker. The reputation cost of providing an inaccurate recommendation is measured by the employed worker's expected value of the square of the difference between the recommendation and ability $\int_{\theta \in [0,1]} (\rho - \theta)^2 Prob(\theta | s, t) d\theta$. At the beginning of stage 3, given the signal s, and tie type t, each employed worker provides recommendation ρ that maximizes his payoff,

$$\pi_E = w(\rho, t) - r \int_{\theta \in [0,1]} (\rho - \theta)^2 Prob(\theta | s, t) d\theta$$

where $r > 0$ is the reputation costs parameter. The bigger is the reputation costs parameter, the lower are the incentives to provide inaccurate recommendations. At the beginning of stage 4, given the recommendation ρ, and the tie type t, the firm forms its valuation $v(\rho, t) = E[\theta|\rho, t]$ of the unemployed worker's ability, and offers wage equal to its valuation $w(\rho, t) = E[\theta|\rho, t]$.

If the tie is weak, then the wage offer doesn't depend on the recommendation since signals are not informative. Thus, if the tie is weak, then wage is $w = E[\theta|t_{weak}]$. If the tie is strong, then firm's wage offer depends on whether it can infer its employees signals. I will focus on an appealing class of pure strategy Perfect Bayesian Equilibria (PBE), where recommendations $\rho(s, t)$ are monotonically (weakly) increasing in signal with the initial value condition $\rho(0, t) = E[\theta|s = 0, t]$.[41] Due to gratitude benefits, recommendations are inflated in that recommendations are greater than the signal values $\rho(s, t) > s$. So recommendations are strictly increasing in signal and inflated until they hit the upper bound of one. The employed worker's recommendations are equal to one $\rho(s, t_{strong}) = 1$ for any signal higher than this threshold signal $s \geq s^*$, which makes it impossible for firm to infer these signals from the employee's recommendations (firm can only infer signals below this threshold). Similarly, wage offers from strong tie is strictly increasing in signal below threshold signal $s < s^*$, and equal to the average of signals $s \geq s^*$ otherwise.

$$w(\rho, t_{strong}) = \begin{cases} s & \text{if } s < s^* \\ E[\theta|s \geq s^*, t_{strong}] & \text{if } s \geq s^* \end{cases}$$

Observe that wage offers from weak ties are fixed, and wage offers from strong ties are increasing in signals. As a result, if $\widetilde{\theta}$ prefers strong tie, then all $\theta > \widetilde{\theta}$ prefer strong tie as well. Thus, the equilibrium involves a threshold ability θ^* such that $\theta < \theta^*$ prefer weak tie, and $\theta > \theta^*$ prefer strong tie.

Discussion

Safi (2018) provides several interesting insights.

- The use of different types of ties varies with worker's ability, and the proportion of workers who have access to different types of ties.

Sufficiently low ability (below threshold) workers use weak ties to pool with high ability workers without strong ties, and sufficiently high ability (above threshold) workers use strong ties to separate themselves. The value of threshold ability θ^* depends on the proportion of workers who have access to strong ties β. The higher is the number of high ability workers who don't have strong ties, the higher are the returns from using weak ties, and the more workers use weak ties to pool with high ability workers. This finding can help explain the mixed evidence about the use of weak ties. Contrary to the existing explanation, the frequent use of weak ties may not be due to its efficiency

[41] This class of PBE is appealing in that employed workers reveal maximum information in the cheapest way possible. See appendix of Safi (2018) for further details.

in matching workers and firms. When the access to strong ties are really scarce for high ability workers, many workers (even those with access to strong ties) use weak ties to pool with high ability workers. Unlike Karlan et al (2009), this explanation is consistent with the empirical finding that referrals are used more for low-skill jobs.

- The wage differential between referral-based and market-based hires varies with worker's ability, and the proportion of workers who have access to ties.

In the simple case considered here, applying directly to the firm is equivalent to applying through weak ties because weak ties are completely un-informative. So, rest of the arguments follow from what I discussed above about the use of different types of ties. This finding can help explain the mixed evidence about the wage differential between referral-based and market-based hired. In particular, the wage differential between referral-based and market-based hires is negative for sufficiently low ability (below threshold) workers, and it is positive for sufficiently high ability (below threshold) workers.

I then consider an extension of this model with two races (blacks and whites) and two occupations. I assume that fewer blacks have strong ties with employed workers than whites.[42] There are two types of occupations, manual labor intensive occupation and human capital intensive occupation. In the human capital intensive occupation, output depends on the worker's ability more. For each $i \in \{m, h\}$,

$$y_i(\theta) = \gamma_i * \theta + (1 - \gamma_i) * E[\theta]$$

where $0 \leq \gamma_m < 0.5$ and $0.5 < \gamma_m \leq 1$.

- Black-white wage gap widens as one moves up the wage hierarchies of the private sector.

Only employed workers with strong ties can reveal high abilities to firm. If most white workers have strong ties, then high ability white workers will enter high earning occupations through strong ties, and low ability white workers will enter low earning occupations. On the other hand, if only few black workers have strong ties, then two possible cases can arise. Either both below and above average ability black workers enter higher earning occupations through weak ties, firm pools them together, and on average high ability black workers get relatively lower wage than their white counterparts (same ability levels). Or both below and above average ability black workers enter lower earning occupations through weak ties, firm pools them together, and on average low ability black workers get relatively higher wage than their white counterparts. Both of these cases are

[42]This assumption follows from two empirical facts: employment differentials between blacks and whites in the US and the homophily feature of social networks (tendency to interact with others that have similar characteristics). Since blacks have lower employment rates than whites (Lang & Lehmann (2012)), they have to rely on their cross-race ties more than whites do. Such cross-race ties tend to be weaker because individuals tend to interact more often with others that belong to their own race (Marsden (1987, 1988), Tsui and O'Reilly (1989), and Thomas (1990)). See Safi (2018) for further discussion.

consistent with the empirical finding that the black-white wage gap widens as one moves up the wage hierarchies of the private sector. Moreover, unlike Karlan et al (2009), this explanation is also consistent with the mixed evidence about the wage differential between referral-based and market-based hired, and the finding that referrals are used more for low-skill jobs.

4 Conclusion

I survey the theoretical literature of employee referrals by discussing some of it key models. These models have many applications, but they have been particularly useful in the following three areas. First, there is a growing empirical literature on employee referrals, and these models can help explain some puzzles in this literature. For instance, they can help explain the mixed evidence about the use of different types of contacts in job search, and the mixed evidence about the wage differentials between referral-based hires and market-based hires. Second, these works have contributed to the search and matching literature. In particular, they have improved our understanding of the matching process by providing some micro-foundations for the aggregate matching function. Finally, these models have been useful for understanding inequalities in labor market outcomes. For instance, they help explain the persistent inequality in wages between blacks and whites, the widening of the black-white wage gap as one moves up the wage hierarchies of the private sector, among other characteristics of the labor market.

Although there are multiple factors that matter in employee referrals, this survey shows that the network of contacts, and the types of contacts are important in many of the applications. It seems to me that exploring these factors further are fruitful avenues for future research. For instance, some of the main results in the search frictions literature depend on the network structure, and so far, they have only considered symmetric networks (everyone is identical in their network position). It remains unclear how these results depend on richer network structures. Similarly, many results depend on the types of contacts, but the employee referrals literature has only focused on one characteristic of contacts, namely, their strength (weak and strong ties). Clearly, contacts differ in other ways - they can be personal or professional, employee or non-employee, and naturally formed or formed through networking at work related events. However, little is understood about the impact of these other characteristics.

Most of the models in the literature can be classified by two main roles of employee referrals: (1) reducing search frictions, and (2) screening. Recently, Galenianos (2013) considered the interaction between these two roles. However, little research has considered this interaction, and more work could be done on this front. Similarly, little is understood about other roles of employee referrals. For instance, Staiger (1990) argued that workers can use their contacts to gather information about the firm's characteristics or the prospective match quality, and Heath (2018) argued that firms can use referrals as a disciplining device to reduce moral hazard issues. Yet these areas remain largely unexplored, and they are exciting avenues to examine for future research.

Part IV

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Part V

Appendix

1 Appendix A - Listen Before You Link: Optimal Consent Rules for Network Formation in the Presence of Externalities

1.1 Four Stylized Models

Lemma 1 - In the popularity by connections model, the unique efficient network is

(i) the complete network if $c < \delta$, and

(ii) the empty network if $c > \delta$

Proof

(i) Indirect links only bring costs, and the total value always strictly increases by forming a link between any two players if $(\delta - c) > 0$.

(ii) Similarly, the total value always strictly decreases by forming a link between any two players if $(\delta - c) < 0$. ■

Proof of Proposition 1

(i) For $(\delta - c) > 0$, there are no possible deviations from the complete network because only possible deviations involve deleting a link, and deleting a link always strictly lowers the payoff of the players. Thus, the complete network is stable.

(ii) Similarly for $(\delta - c) < 0$, there are no possible deviations from the empty network because only possible deviations involve adding a link, and adding a link always strictly lowers the payoff of the players. Thus, the empty network is stable.

Since the efficient networks are always stable, the optimal consent requirement is zero ($q = 0$). ■

Remark 1 - In the truncated symmetric connections model, the unique efficient network is

(i) the complete network if $c < \delta - \delta^2$,

(ii) a star encompassing everyone if $\delta - \delta^2 < c < \delta + ((n-2)/2)\,\delta^2$, and

(iii) the empty network if $\delta + ((n-2)/2)\,\delta^2 < c$

The proof is available in Jackson and Wolinsky (1996).

Remark 2 - In the truncated symmetric connections model, pairwise stable networks are characterized as follows:

(i) A pairwise stable network has at most one (non-empty) component.

(ii) For $c < \delta - \delta^2$, the unique pairwise stable network is the complete network.

(iii) For $\delta - \delta^2 < c < \delta$, the star encompassing all players is pairwise stable, but not necessarily the unique pairwise stable network.

(iv) For $\delta < c$, any pairwise stable network which is non-empty is such that each player has at least two links and thus is inefficient.

The proof is available in Jackson and Wolinsky (1996).

Proof of Proposition 2

For $\delta < c < \delta + ((n-2)/2)\,\delta^2$, the star network is efficient but not stable. Recall, a star network is simply a network in which all players are linked to one central player and there are no other links.

In this case, the player in the center of the star gets his lowest payoff across all networks since $(\delta - c) < 0$, and he is directly linked to all the other players. On the other hand, all the other players get the maximum payoff across all networks since they are only linked to one player in the center which gives him indirect benefit of δ^2 from all the remaining players, and δ^2 is the highest value a player could get from another player.

The only possible deviation from the star network is that the center player will delete a link which makes everyone else strictly worse off (number of breakers away from efficiency is $K = 0$). Thus, there is only one type of deviation which makes only one player better off, and requiring consent from at least one other player will stabilize the efficient networks. ■

Efficiency in the attention based utility model

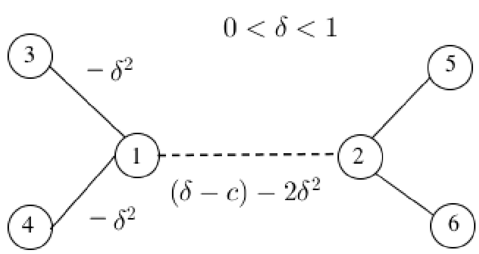
Let n_i be the number of direct links that player i has. The sum of utilities in terms of the net value that each player contributes is:

$$\begin{aligned}\sum_{i \in N} u_i(g) &= \sum_{i \in N} \sum_{j\,:\,ij \in g} \left[(\delta - c) - \sum_{k : k \neq i, jk \in g} \delta^2 \right] \\ &= \sum_{i \in N} n_i(\delta - c) - \sum_{i \in N} n_i(n_i - 1)\delta^2 \\ &= \sum_{i \in N} n_i \left[(\delta - c) - (n_i - 1)\delta^2 \right]\end{aligned}$$

The second equality follows from the fact that each player contributes $(\delta - c)$ units of direct benefit to all the players that he is linked with, and the sum of this gives the total value from direct links. Similarly, each player contributes $(n_i - 1)\delta^2$ units of negative externalities to all the players that he is linked with, and the sum of this gives the aggregate cost of negative externalities.

Maximizing the sum of utilities with respect to n_i, I get $(\delta - c) = (2n_i - 1)\delta^2$ for all $i \in N$. So it is efficient to add an additional link whenever the net benefit from adding the link $(\delta - c)$ is greater than the indirect costs incurred by all the players. To get some intuitive feeling for this expression, consider the example with six players in figure 13 below. Players 1 and 2 have two links each, and consider the change in the net value that player 1 contributes if he adds a link with player 2. Adding such a link generates $(\delta - c)$ units of direct benefit to player 2 and $2\delta^2$ units of negative externalities to him. In addition, players 3 and 4 also get an indirect externality cost of δ^2 each. The same argument applies to player 2 adding this link, since players 1 and 2 both have the same number of existing links. Thus, it is inefficient for players 1 and 2 to add a third link if $(\delta - c) < 4\delta^2$.

Fig. 9 ABU Model - Efficiency



Lemma 2 - In the attention based utility model:

(i) For $(\delta - c) < 0$, the unique efficient network is the empty network.

(ii) For n even and $(\delta - c) = \delta^2$, the unique efficient network consists of $n/2$ separate pairs.

(iiii) For $(\delta - c) = 3\delta^2$, a circle encompassing everyone is an efficient network structure, but not the unique efficient network structure.

(iv) For n even, efficient networks are such that each player has exactly $n_i = \frac{(\delta - c) + \delta^2}{2\delta^2}$ links whenever n_i is an integer and $n_i \leq n - 1$. In such case and for $n_i \geq 2$, a circle encompassing everyone and additional links increasing with n_i is an efficient network structure.

(v) For $n = 3$, and $\delta^2 < (\delta - c) < 2\delta^2$, the unique efficient network is the star network.

Proof

(i) If $(\delta - c) < 0$, then it is obviously efficient to have no links, and the empty network is the unique network structure where everyone has exactly no links.

$Max_{n_i} \sum_{i \in N} u_i(g) = \sum_{i \in N} n_i \left[(\delta - c) - (n_i - 1)\delta^2\right]$

FOC: $[n_i] \Longrightarrow$

$$\left[(\delta - c) - (n_i - 1)\delta^2\right] + n_i(-\delta^2) = 0$$

$$\Longleftrightarrow n_i = \frac{(\delta - c) + \delta^2}{2\delta^2}$$

(ii) It is easy to check that $n_i = 1$ when $(\delta - c) = \delta^2$. Note that $n/2$ separate pairs is the unique network structure where everyone has exactly one link and it is also feasible for n even.

(iii) It is easy to check that $n_i = 2$ when $(\delta - c) = 3\delta^2$. A circle encompassing everyone is a network structure where everyone has exactly 2 links. However, it is not unique. Consider a network in which all components are circles (of possible different sizes). In such network structures, everyone has exactly 2 links as well.

(iv) The maximization problem gives $n_i = \frac{(\delta - c) + \delta^2}{2\delta^2}$ for all $i \in N$. If n_i is not an integer, then it is not feasible to have such number of links. If $n_i > n - 1$, then it is also not feasible to have such number of links because $n - 1$ is the maximum number of links a player can have. Similarly for n odd, it may not be feasible to have each player with exactly the same number of links. For example, if $n = 5$, it is not possible to have each player with exactly three links.

For n even, n_i is an integer, $n_i \leq n - 1$, and for $n_i \geq 2$, a circle encompassing everyone and additional links increasing with n_i is a efficient network structure.

To see that this structure can characterize a efficient network, start from the case of $n_i = 2$. In such case, a circle encompassing everyone is a network structure where everyone has exactly two links. Now consider the case of $n_i = 3$, such network can be constructed from the previous network structure simply by linking each player with some player that they are not already linked with. Since there are even number of players, this can be done by forming $n/2$ new links. Similarly for the case of $n_i = 4$, and so on.

(v) For $n = 3$, and $\delta^2 < (\delta - c) < 2\delta^2$. A network with no links has value of zero. A network with only one link has value of $2(\delta - c)$. A star network has a value of $2(\delta - c) + \left[2(\delta - c) - 2\delta^2\right] > 2(\delta - c)$. And finally a complete network has a value of $6\left[(\delta - c) - \delta^2\right] = 2(\delta - c) + \left[2(\delta - c) - 2\delta^2\right] + [2(\delta - c) - 4\delta^2] < 2(\delta - c) + \left[2(\delta - c) - 2\delta^2\right]$ because $(\delta - c) < 2\delta^2$. ■

Proposition 3 - In the attention based utility model:

(i) For $n = 3$, the number of breakers away from efficiency is one ($K = 1$). The unique optimal consent rule is to have all three players in the same group ($p = \{\{1, 2, 3\}\}$) with optimal consent requirement of unanimity $q_p = 1$.

(ii) For $n = 6$, the number of breakers away from efficiency is $K \geq 1$. An optimal consent rule is to have all six players in the same group ($p = \{\{1, ..., 6\}\}$) with optimal consent requirement of $q_p = \frac{3}{5}$.

(iii) For $n = 4$, $n = 5$ and $n \geq 7$, the number of breakers away from efficiency is $K \geq (n - 3)$. The unique optimal consent rule is to have all the players in the same group ($p = \{\{1, ..., n\}\}$) with the optimal consent requirement of $q_p = \frac{n-2}{n-1}$.

Proof of proposition 3

(i) For $n = 3$, and $\delta^2 < (\delta - c) < 2\delta^2$. The unique efficient network is the star network, but it is not pairwise stable. The two players not in the center of the star will deviate by adding a link (either of these players can get a consent from one other player, so the number of breakers away from efficiency is $K = 1$). The only way to stop them is through the consent requirement of unanimity.

(ii) For $n = 6$, a circle encompassing everyone is the unique efficient network structure. Each player has an incentive to add a link. The two players adding a link would strictly benefit from such a deviation (either of these players can get a consent from one other player, so the number of breakers away from efficiency is $K = 1$), and four other players in the same circle would be strictly worse off since they incur additional indirect costs. Thus, requiring consent from at least two players is needed to stop this deviation.

(iii) For $(\delta - c) = 3\delta^2$, there is a efficient network with a deviation that makes $n - 2$ people weakly better off. For $n \geq 7$, there is a efficient network in which all components are circles, and some circle encompasses exactly four players. Note that it takes at least three players to construct a circle. If there are at least seven players, then I can always construct a circle with four players and another circle with all the remaining players. For $n = 4$ and $n = 5$, the following argument also applies.

At any efficient network, the net benefit of a direct link is $(\delta - c) = 3\delta^2$ and a player adding a link incurs an additional indirect cost of $2\delta^2$ since all players have two existing links. Therefore, each player has an incentive to add a link. Note that by having a circle encompassing exactly four players, the number of players who are made strictly worse off is only two. In any other circle of bigger size, such deviation would make more than two players strictly worse off. In the efficient network described above, any two players adding a link in the circle encompassing exactly four players would strictly benefit from such a deviation, the two other players in the same circle would be strictly worse off since they incur additional indirect costs of $2\delta^2$ each, and all the other players are weakly better off since their payoffs are unchanged. Thus, requiring consent from at least $n - 2$ players is needed to stop this deviation ($K \geq (n - 1) - 2$). ■

Proof of Proposition 4

Proposition 1 also holds for the generalized popularity by connections model using exactly the same arguments. Thus, the optimal consent requirement is the same in both models following the same arguments. ■

Observe that remarks 1 and 2 also hold for the symmetric connections model. Thus, the optimal consent requirement is the same in both models following the same arguments as in proposition 2.

1.2 Different Notions of Stability

Weaker notion of pairwise stability by Jackson and Wolinsky (1996)

I consider a notion of stability weaker than the notion of pairwise stability. In this notion, a player only gives consent to a deviator when the deviation makes him strictly better off. This notion was first considered by Jackson and Wolinsky (1996). Proposition 5 below shows that weakening the notion of stability can only decrease the number of breakers away from efficiency K since it allows for less deviations.

Proposition 5 - Under the weaker notion of pairwise stability by Jackson and Wolinsky (1996):

(i) $K = -1$ for popularity by connections model,

(ii) $K = 0$ for truncated symmetric connections model,

(iii) $K = -1$ for generalized popularity by connections model, and

(iv) $K \geq 3$ for attention based utility model if $n \geq 7$.

Proof of proposition 5

(i) Follows the same reasoning as discussed in proposition 2.

(ii) Consider the star network, and note that everyone besides the central player gets their maximal payoff. Therefore, the only possible deviation from the star network is that the center player will delete a link which makes everyone else strictly worse off. Thus, there is only one type of deviation which makes only one player better off, and requiring consent from more than one player will stabilize the efficient network.

(iii) Follows the same reasoning as discussed in proposition 2.

(iv) The attention based utility model requires consent from at least two players for $n \geq 7$ since $n - 4$ players are only weakly better off (not strictly) and now they will not give consent to the deviator. ■

Strong stability of Jackson and van den Nouweland (2005)

I consider a notion of stability which allows for multiple link deviations by more than two players. In order to consider multilateral deviations, one needs to investigate what are the possible changes in a network that can be made by a subset of players S? A network g' is obtainable from g via subset of players $S \subseteq N$ if they are the only players involved in adding or deleting links to existing network g. More precisely,

(i) $ij \in g'$ and $ij \notin g$ implies $\{i, j\} \subseteq S$, and

(ii) $ij \notin g'$ and $ij \in g$ implies $\{i, j\} \cap S \neq \emptyset$

Condition (i) requires that any new links that are added can only be between players inside S. This reflects the fact that consent of both players is needed to add a link. Condition (ii) requires that for deletion of a link, there must be at least one player belonging to S. This reflects the fact that either player in a link can unilaterally severe the relationship. Hence, it simply gives the possible resulting networks once coalition S has deviated from the existing network by adding or deleting some links to the existing network. Once one knows all the possible deviations that can be made, one can define the notion of strong stability as follows

Definition 6 A network g is strongly stable with respect to payoff rule u if for any $S \subseteq N$, g' obtainable from g via S and $i \in S$ such that $u_i(g') > u_i(g)$, there exists $l \in S$ such that $u_l(g') < u_l(g)$.

This notion of strong stability was introduced by Jackson and van den Nouweland (2005). Proposition 6 below shows that strengthening the notion of stability can only increase the number of breakers away from efficiency K since it allows for more deviations.

Proposition 6 - Under strong stability of Jackson and van den Nouweland (2005):

(i) $K = -1$ for popularity by connections model,

(ii) $K = 0$ for truncated symmetric connections model,

(iii) $K = -1$ for generalized popularity by connections model, and

(iv) $K \geq n - 3$ for attention based utility model if $n \geq 7$.

Proof of proposition 6

(i) Follows the same reasoning as discussed in proposition 2.

(ii) Consider again the star network, and note that everyone besides the central player gets their maximal payoff. Therefore, the only possible deviation from the star network is that the center player will delete one or more links which makes everyone else strictly worse off. Thus, there is only one type of deviation which makes only one player better off, and requiring consent from more than one player will stabilize the efficient network.

(iii) Follows the same reasoning as discussed in proposition 2.

(iv) The attention based utility model requires consent from at least $(n - 2)$ players for $n \geq 7$.

■

Strong stability of Dutta and Mutuswami (1997)

I also consider a notion of stability which allows for multiple link deviations by more than two players, but a player only gives consent to a deviator when the deviation makes him strictly better off. This last notion of stability is neither weaker nor stronger than pairwise stability but it gives one a complete sense of the robustness of the results to notions of stability along these two dimensions. This notion of strong stability was introduced by Dutta and Mutuswami (1997).

Proposition 7 - Under strong stability of Dutta and Mutuswami (1997):

(i) $K = -1$ for popularity by connections model,

(ii) $K = 0$ for truncated symmetric connections model,

(iii) $K = -1$ for generalized popularity by connections model, and

(iv) $K \geq 3$ for attention based utility model if $n \geq 11$.

Proof of proposition 7

(i) Follows the same reasoning as discussed in proposition 2.

(ii) Consider again the star network, and note that everyone besides the central player gets their maximal payoff. Therefore, the only possible deviation from the star network is that the center player will delete one or more links which makes everyone else strictly worse off. Thus, there is only one type of deviation which makes only one player better off, and requiring consent from more than one player will stabilize the efficient network.

(iii) Follows the same reasoning as discussed in proposition 2.

(iv) The attention based utility model requires consent from at least four players for $n \geq 11$. Note that there is a efficient network in which all components are circles, and at least two circles contain exactly four players. Note that it takes at least three players to construct a circle. If there are at least eleven players, then I can always construct at least two circles with four players and another circle with all the remaining players. In such case, consider the deviation where one new link is added in each of the circle containing exactly four players. Such deviation will make exactly two people adding the link strictly better off in each of the circle containing exactly four players. Since there are at least two such circles, I know that at least four players are made strictly better off. ■

1.3 General Model

Proof of Theorem 1

Recall that players are ex-ante identical. Hence, each network structure has many permutations with different players in different positions (see section 2.1 for an example with three players). To stabilize a network structure will mean to stabilize all the permutations of this network structure.

To stabilize all the permutations of some efficient network structure, one cannot use the identity of players, because each player takes a different position in different permutation. So the only robust information is the number of players that will give consent (because they are made weakly better off) to any deviations from any permutations of this efficient network structure.

Suppose the highest number of players that will give consent to any deviations from any of the efficient networks is $K < n-1$. Then any partition $p = \{S_1, S_2, ..., S_m\}$ with at least $K+2$ number of players in each group (each player has $K+1$ players in his group excluding himself) with consent requirement of $q = \frac{K+1}{min\{\#S_1, \#S_2, ..., \#S_m\}-1}$ will stabilize the efficient networks.

Note that the number of players that will give consent to any deviations from any of the efficient networks cannot be $n-1$, which guarantees that one can always stabilize the set of efficient networks. To see this, suppose that $K = n-1$. This only happens if one of the efficient network structure $g \in G^*$ has a deviation that gets consent from everyone. For this to be true, there must be some deviator who is made strictly better off and everyone else is made weakly better off. This contradicts that such network is efficient because it cannot maximize the sum of players payoff.

Observe that all these arguments also apply to pareto efficient networks. In particular, the number of players that will give consent to any deviations from any of the pareto efficient networks cannot be $n-1$, which guarantees that one can always stabilize the set of pareto efficient networks. To see this, suppose that $K = n-1$. This only happens if one of the pareto efficient network structure $g \in G^*$ has a deviation that gets consent from everyone. For this to be true, there must be some deviator who is made strictly better off and everyone else is made weakly better off. This contradicts that such network is pareto efficient. ■

Proof of Theorem 2

The only difference now is that each groups has its own consent requirement. Then any partition $p = \{S_1, S_2, ..., S_m\}$ with at least $K+2$ number of players in each group with consent requirement of $q_i = \frac{K+1}{\#S_i - 1}$ for each group can stabilize the efficient networks. The rest of the arguments follow from theorem 1. ■

Proof of Theorem 3

The phrase ‘‘unique network’’ now means unique up to a renaming of the agents with same type. Thus, there are still permutations for a given network structure. To stabilize all the permutations of some network structure, one cannot use the identity of players, because each player of the same type takes a different position in different permutation. So the only robust information is again the number of players that prefer to deviate from the given network structure from each type.

If $K = (K_1, K_2, ..., K_T)$ is the vector of breakers away from efficiency for each type with respect to a profile of payoff rules $(u_1(g), ..., u_n(g))$, then optimal consent rules are such that each partition $p = \{S_1, S_2, ..., S_m\}$ has at least $K_t + 2$ number of players of type t, and the optimal consent requirement for each type in partition p is $q = (\frac{K_1+1}{min\{\#S_{1t}, \#S_{2t}, ..., \#S_{mt}\}-1}, ..., \frac{K_T+1}{min\{\#S_{1t}, \#S_{2t}, ..., \#S_{lt}\}-1})$. The rest of the arguments follow from theorem 1. ■

Proof of Theorem 4

Similar to proof of theorem 3, the phrase "unique network" now means unique up to a renaming of the agents with same type. If $K = (K_1, K_2, ..., K_T)$ is the vector of breakers away from efficiency for each type with respect to a profile of payoff rules $(u_1(g), ..., u_n(g))$, then optimal consent rules are such that each partition $p = \{S_1, S_2, ..., S_m\}$ has at least $K_t + 2$ number of players of type t, and the optimal consent requirement from type $t \in \{1, 2, ..., T\}$ in group $i \in \{1, 2, ..., m\}$ in partition p is $q_{it} = (\frac{K_t+1}{S_{it}-1})$. The rest of the arguments follow from theorem 1. ■

2 Appendix B - The Weakness of Weak Ties in Referrals: An Obstacle for the Upwardly Mobile Black Men in the Private Sector

2.1 Lemmas, Propositions and Theorems

This appendix contains the formal arguments for the results in the text.

Proof of proposition 1

(1) The initial value condition $\rho(0, t) = 0$ follows from the definition of the cost efficient recommendation strategy.

(2) For $0 < s < min\{s^*, \alpha_t\}$, the employed worker's expected value of ability is equal to the signal for such interval of signals. Thus, an employed worker solves the following maximization problem.

$$Max_{\rho \in [0,1]} \left\{ w(\rho, t) - r(\rho - s)^2 \right\}$$

FOC:

$$[\rho] \Longrightarrow w_1(\rho, t) - 2r(\rho - s) = 0$$

Since recommendations are increasing in signals $\rho_1(s, t) > 0$, the employed worker's recommendations reveal his signal value to the firm $\{s' : \rho(s', t) = \rho(s, t)\} = \{s\}$ for all such signals. As a result, the firm offers wage equal to the signal value for such signals $w(\rho(s, t), t) = s \Longrightarrow w_1(\rho, t)\rho_1(s.t) = 1$. Plugging this expression in the first order condition (FOC) gives the following differential equation (DE).

$$\rho_1(s, t) = \frac{1}{2r(\rho(s, t) - s)} \qquad (DE)$$

Since recommendations are increasing in signals $\rho_1(s,t) > 0$, then the differential equation implies that the employee recommends higher than his signal $\rho(s,t) > s$. The reader can easily verify that the family of solutions to this differential equation is given by

$$\rho(s,t) + c = -\frac{1}{2r} ln\left[\frac{1}{2r} + s - \rho(s,t)\right]$$

where c is a constant which can be determined by the initial value condition $\rho(0,t) = 0$. So I get $c = -\frac{1}{2r} ln\left[\frac{1}{2r}\right]$, and

$$\begin{aligned} \rho(s,t) &= \frac{1}{2r} ln\left[\frac{1}{2r}\right] - \frac{1}{2r} ln\left[\frac{1}{2r} + s - \rho(s,t)\right] \\ \Longrightarrow \rho(s,t) &= \frac{1}{2r} ln\left[\frac{\frac{1}{2r}}{\frac{1}{2r} + s - \rho(s,t)}\right]. \end{aligned}$$

(3) If $min\{s^*, \alpha_t\} = s^*$, then recommendations hit the upper bound value of one for some signal. For any signal higher than this threshold signal, employee's payoff maximizing recommendation values are not feasible (greater than one). Thus, the employed worker chooses recommendation values as close to the payoff maximizing recommendation values as possible. The closest value to the payoff maximizing recommendation values for all such signals is one.

If $min\{s^*, \alpha_t\} = \alpha_t$, then the signal reaches the informativeness level of the employed worker. For any signal higher than the informativeness level, the employed worker chooses recommendation value that minimizes his reputation costs $E[\theta|s,t]$ as long as it was not used for some signal lower than the informativeness level. If it was used for some lower signal, then the employed worker chooses the lowest value that was not used for some lower signal, i.e. $lim_{s' \to \alpha_t} \rho(s',t)$. ■

Proof of proposition 2

(1) Since recommendations are increasing in signals for $s < min\{s^*, \alpha_t\}$, the employed worker's recommendations reveal his signal value to the firm $\{s' : \rho(s',t) = \rho(s,t)\} = \{s\}$ for all such signals. As a result, the firm offers wage equal to the signal value for such signals $w(\rho(s,t),t) = s$.

(2) For the interval of signals that the firm can not infer $s \geq min\{s^*, \alpha_t\}$, the firm sets wage equal to the weighted average of these signals $w(\rho(s,t),t) = E[\theta|s \geq min\{s^*, \alpha_t\}, t] = \int_{min\{s^*,\alpha_t\}}^{1} \theta \frac{f(t|\theta)}{\int_{min\{s^*,\alpha_t\}}^{1} f(t|\theta)d\theta} d\theta$, where weights are determined by the the proportion of unemployed workers $f(t|\theta)$ with ability level θ who chose tie type t. ■

Proof of theorem 1

(1) If the threshold signal is greater than the informativeness level of the weak tie $s^* > \alpha_{weak}$, then there is an interval of signals $s \in (\alpha_{weak}, min\{s^*, \alpha_{strong}\})$ where wage offers from weak ties is fixed, and wage offers from strong ties is increasing in signals. As a result, if $\widetilde{\theta} \in (\alpha_{weak}, 1)$ prefers

strong tie, then all $\theta > \widetilde{\theta}$ prefer strong tie as well. Thus, the cost efficient equilibrium involves a threshold ability θ^* such that ability levels below the threshold ability $\theta < \theta^*$ weakly prefer weak ties ($\theta \in [0, \alpha_{weak})$ are indifferent between the two type of ties, and $\theta \in [\alpha_{weak}, \theta^*)$ prefer weak ties), and unemployed workers with ability levels above the threshold ability $\theta > \theta^*$ prefer strong ties. At equilibrium, wage offer from a weak tie is

$$\begin{aligned}
E[\theta|s \quad &\geq \quad \alpha_{weak}, t_{weak}] = \int_{\alpha_{weak}}^{1} \theta \frac{f(t_{weak}|\theta)}{\int_{\alpha_{weak}}^{1} f(t_{weak}|\theta)d\theta} d\theta \\
&= \frac{1}{\int_{\alpha_{weak}}^{\theta^*} d\theta + \int_{\theta^*}^{1}(1-\beta)d\theta}\left[\int_{\alpha_{weak}}^{\theta^*} \theta d\theta + \int_{\theta^*}^{1} \theta(1-\beta)d\theta\right] \\
&= \frac{1}{2\left[\beta\theta^* + (1-\beta) - \alpha_{weak}\right]}\left[\beta(\theta^*)^2 + (1-\beta) - \alpha_{weak}^2\right]
\end{aligned}$$

The value of threshold ability θ^* depends on threshold signal s^* and the informativeness levels of both type of ties α_{weak}, α_{strong}. Let $\widehat{\theta} = \{\theta : \theta = E[\theta|s \geq \alpha_{weak}, t_{weak}]\}$ be the unemployed worker who is indifferent between choosing wage offer from a strong tie and revealing his ability or choosing wage offer from a weak tie. If the threshold signal and the informativeness level of strong ties are high enough $min\{s^*, \alpha_{strong}\} > \widehat{\theta}$, then threshold ability is $\theta^* = \widehat{\theta}$.

If $min\{s^*, \alpha_{strong}\} < \widehat{\theta}$, then the threshold ability is $\theta^* = min\{s^*, \alpha_{strong}\}$. This follows from the following two observations. Firstly, since $E[\theta|s \geq \alpha_{weak}, t_{weak}] > min\{s^*, \alpha_{strong}\}$ for all $\alpha_{weak} \leq \theta < min\{s^*, \alpha_{strong}\}$, unemployed workers whose abilities are revealed with strong ties will prefer weak ties. Secondly, unemployed workers who can pool with strong ties, they get higher returns from pooling with strong ties than pooling with weak ties.

$$\begin{aligned}
E[\theta|s &\geq min\{s^*, \alpha_{strong}\}, t_{strong}] = \int_{min\{s^*,\alpha_{strong}\}}^{1} \theta \frac{f(t_{strong}|\theta)}{\int_{min\{s^*,\alpha_{strong}\}}^{1} f(t_{strong}|\theta) d\theta} d\theta \\
&= \int_{min\{s^*,\alpha_{strong}\}}^{1} \theta \frac{\beta}{\int_{min\{s^*,\alpha_{strong}\}}^{1} \beta d\theta} d\theta \\
&= \frac{1}{1 - min\{s^*, \alpha_{strong}\}} \int_{min\{s^*,\alpha_{strong}\}}^{1} \theta d\theta \\
&> \frac{1}{1-\alpha_{weak}} \int_{\alpha_{weak}}^{1} \theta d\theta \\
&\geq \frac{1}{\int_{\alpha_{weak}}^{\theta^*} d\theta + \int_{\theta^*}^{1} (1-\beta) d\theta} \left[\int_{\alpha_{weak}}^{\theta^*} \theta d\theta + \int_{\theta^*}^{1} \theta (1-\beta) d\theta \right] \\
&= E[\theta | s \geq \alpha_{weak}, t_{weak}]
\end{aligned}$$

The second equality follows from the fact that the workers with abilities $\theta \geq min\{s^*, \alpha_{strong}\}$ get the same wage offers. Thus, any equilibrium involves all such workers preferring same type of tie. Suppose, all such workers prefer and use weak ties. They will get wage equal to the average of abilities $\theta \geq min\{s^*, \alpha_{strong}\}$,
i.e. $\frac{1}{1-min\{s^*,\alpha_{strong}\}} \int_{min\{s^*,\alpha_{strong}\}}^{1} \theta d\theta$. Then, $\theta > \frac{1}{1-min\{s^*,\alpha_{strong}\}} \int_{min\{s^*,\alpha_{strong}\}}^{1} \theta d\theta$ ability workers can profitably deviate by using a strong tie to get a higher wage. Thus, I consider equilibrium where all workers with ability levels $\theta \geq min\{s^*, \alpha_{strong}\}$ prefer strong ties. In such case, only those with access to strong ties will be able to use it. Then, the unemployed workers who can pool with strong ties $\theta \geq min\{s^*, \alpha_{strong}\}$ get wage equal to average of abilities $\theta \geq min\{s^*, \alpha_{strong}\}$ by choosing a strong tie. The first inequality (line 4) indicates that the wage that they can get from a strong tie is higher than the average of abilities $\theta \geq \alpha_{weak}$ because this includes some workers with strictly lower abilities $\theta \in [\alpha_{weak}, min\{s^*, \alpha_{strong}\})$. The second inequality (line 5) indicates that the average of abilities $\theta \geq \alpha_{weak}$ is weakly higher than any wage that workers can get from choosing a weak tie. This follows from the fact that everyone with abilities $\theta \in [\alpha_{weak}, \theta^*)$ choose weak ties and only β of workers with abilities $\theta \geq \theta^*$ choose weak ties.

Thus, the threshold ability is $\theta^* = min\{\widehat{\theta}, s^*, \alpha_{strong}\}$.

(2) If the threshold signal is lesser than the informativeness level of the weak tie $s^* \leq \alpha_{weak}$, then there is no Cost Efficient Equilibrium.

Workers with abilities $\theta \in [0, s^*)$ are indifferent between the two type of ties, and workers with abilities $\theta \geq s^*$ get same wage offers. Thus, any equilibrium involves all such workers $\theta \geq s^*$ preferring same type of tie. However, no such equilibrium exists. Suppose, all such workers prefer and use weak ties. They will get wage equal to the average of abilities $\theta \geq s^*$, i.e. $\frac{1}{1-s^*} \int_{s^*}^{1} \theta d\theta$. Then, $\theta > \frac{1}{1-s^*} \int_{s^*}^{1} \theta d\theta$ ability workers can profitably deviate by using strong ties to get a wage higher than the average of abilities $\theta \geq s^*$. Similarly, suppose all such workers prefer strong ties. Only those with access to strong ties will be able to use it. Then, the wage from weak and strong tie is the same, i.e. $\frac{1}{1-s^*}\int_{s^*}^{1}\theta d\theta$. As before, $\theta > \frac{1}{1-s^*}\int_{s^*}^{1}\theta d\theta$ ability workers can profitably deviate by using weak ties to get a wage higher than the average of abilities $\theta \geq s^*$. ■

Proof of proposition 3

(1) Each firm's weakly dominant strategy is to set $r_j = r_{max}$.

Each firm's weakly dominant strategy is to maximize the set of ability levels that it can infer. A firm's ability to infer ability levels is increasing in reputation cost level. To see this, note that the threshold signal (following similar arguments as in the motivating example) in this extension is

$$
\begin{aligned}
s^* &= \frac{1}{2\left(\frac{r_j}{\gamma(t)}\right)}\left[\frac{1-e^{2\left(\frac{r_j}{\gamma(t)}\right)}}{e^{2\left(\frac{r_j}{\gamma(t)}\right)}}\right]+1 \\
&= \frac{1}{2\left(\frac{r_j}{\gamma(t)}\right)e^{2\left(\frac{r_j}{\gamma(t)}\right)}} - \frac{1}{2\left(\frac{r_j}{\gamma(t)}\right)}+1
\end{aligned}
$$

And the threshold signal is increasing in reputation cost level,

$$
s^{*'}(r_j) = e^{2(r_j/\gamma(t))}\,(2/\gamma(t))\left(e^{2(r_j/\gamma(t))} - (2/\gamma(t))\,r_j - 1\right) > 0.
$$

This follows from the fact that $\left(e^{2(r_j/\gamma(t))} - (2/\gamma(t))\,r_j - 1\right) > 0$ for all $r_j > 0$. Thus, it is each firm's weakly dominant strategy to set its reputation cost to the maximum level $r_j = r_{max}$.

(2) If $r_1 = r_2 = r_{max}$, then there are two cases two consider.

Case 1 - For the interval of ability levels that firms can infer.

A firm incurs loss by setting wage above the ability level $w_j(\rho,t) > \theta$, so no firm will set such wage. I will first show that $w_1(\rho,t) = w_2(\rho,t)$. Suppose that $w_2(\rho,t) < w_1(\rho,t)$.

If $w_2(\rho,t) < w_1(\rho,t) = \theta$, then firm 1 can profitably deviate by setting wage $w_1(\rho,t) = w_2(\rho,t) < \theta$.

If $w_2(\rho,t) < w_1(\rho,t) < \theta$, then firm 2 can profitably deviate by setting wage $w_2(\rho,t) = w_1(\rho,t) < \theta$.

Thus, $w_1(\rho,t) = w_2(\rho,t)$. If $w_1(\rho,t) = w_2(\rho,t) \neq \theta$, then each firm can profitably deviate by offering slightly higher wage than the other firm.

Case 2 - For the interval of ability levels that firms cannot infer.

A firm incurs loss by setting wage above the expected ability level $w_j(\rho,t) > E[\theta|s \geq s^*, t]$, so no firm will set such wage. Using similar arguments as case 1, $w_1(\rho,t) = w_2(\rho,t) = E[\theta|s \geq s^*, t]$ for all such ability levels. ■

Proof of proposition 4

Each unemployed worker (θ, T, g) chooses tie type $t \in T$ and the occupation type $occ \in \{occ_m, occ_h\}$ that maximizes his payoff,

$$Max_{(t,occ)} \{w(\rho(s,t), t, \beta_g, occ) - 1_{occ_h} k\}$$

If $w(\rho(s,t), t, \beta_g, occ_h) \geq E[\theta]$ for some tie type $t \in T$, then unemployed worker can get higher payoff at human capital intensive occupation than the maximum payoff at manual labor occupation $w(\rho(s,t), t, \beta_g, occ_h) - k > \theta_{low}$. Thus, he will choose human capital intensive occupation.

If $w(\rho(s,t), t, \beta_g, occ_h) < E[\theta]$ for both tie types $t \in T$, then unemployed worker gets negative payoff by working at human capital intensive occupation $w(\rho(s,t), t, \beta_g, occ_h) - k < 0$, and non-negative payoff by working at manual labor intensive occupation $w(\rho(s,t), t, \beta_g, occ_m) \in [0, \theta_{low}]$. Thus, he will choose manual labor intensive occupation. ■

Following proposition 3, the threshold ability is $\theta^* = \widehat{\theta}$ where $\widehat{\theta} = \{\theta : \theta = E[\theta | s \geq \alpha_{weak}, t_{weak}]\}$. The next lemma describes four properties of $\widehat{\theta}$ which I use to prove proposition 5.

Lemma 1. $\widehat{\theta} = \{\theta : \theta = E[\theta | s \geq \alpha_{weak}, t_{weak}]\}$ satisfies the following five properties: (1) $\frac{\partial \widehat{\theta}}{\partial \alpha} > 0$, (2) $\frac{\partial \widehat{\theta}}{\partial \beta} < 0$, (3) $\widehat{\theta} < 0.75$, (4) if $\beta = 1$, then $\widehat{\theta} = \alpha_{weak}$, and (5) if $\alpha_{weak} = 0$ and $\beta = 0$, then $\widehat{\theta} = 0.5$.

(1) The first two properties follows directly from the equation which solves for $\widehat{\theta}$

$$\widehat{\theta} = \{\theta : \theta = E[\theta | s \geq \alpha_{weak}, t_{weak}]\} \Longleftrightarrow \widehat{\theta} = \frac{1}{2\left[\beta\widehat{\theta} + (1-\beta) - \alpha_{weak}\right]} \left[\beta(\widehat{\theta})^2 + (1-\beta) - \alpha_{weak}^2\right]$$

$$\Longleftrightarrow \beta(\widehat{\theta})^2 + 2\left[(1-\beta) - \alpha_{weak}\right]\widehat{\theta} - \left[(1-\beta) - \alpha_{weak}^2\right] = 0 \tag{13}$$

Applying the Implicit Function Theorem with $F(\widehat{\theta}, \alpha_{weak}, \beta) = \beta(\widehat{\theta})^2 + 2\left[(1-\beta) - \alpha_{weak}\right]\theta - \left[(1-\beta) - \alpha_{weak}^2\right] = 0$ gives the following.

$$\frac{\partial \widehat{\theta}}{\partial \alpha_{weak}} = -\frac{2\left[\alpha_{weak} - \widehat{\theta}\right]}{2\beta\widehat{\theta} + 2\left[(1-\beta) - \alpha_{weak}\right]} > 0$$

The numerator is negative since $\alpha_{weak} < \widehat{\theta}$. The denominator is positive as follows.

$$2\beta\widehat{\theta} + 2\left[(1-\beta) - \alpha_{weak}\right] > 2\beta\alpha_{weak} + 2\left[(1-\beta) - \alpha_{weak}\right] = 2(1-\alpha_{weak})(1-\beta) > 0$$

where the first inequality follows from $\alpha_{weak} < \widehat{\theta}$.

(2) Similarly,

$$\frac{\partial\widehat{\theta}}{\partial\beta} = -\frac{\left(\widehat{\theta}^2 - 2\widehat{\theta} + 1\right)}{2\beta\widehat{\theta} + 2\left[(1-\beta) - \alpha_{weak}\right]} < 0$$

The numerator is positive since $\widehat{\theta} \in [0,1]$. The denominator is positive as explained above.

(3) By property 1 and 2, $\widehat{\theta}$ is maximized by choosing maximum value of α_{weak} and minimum value of β. Since $\beta = 0$ is the minimum value and $\alpha_{weak} < 0.5$, then the equation (15) which solves for $\widehat{\theta}$ implies $\widehat{\theta} < 0.75$.

(4) Using quadratic formula, equation (13) implies

$$\widehat{\theta} = \frac{-(1-\alpha_{weak}-\beta) + \sqrt{1-\alpha_{weak}}\sqrt{1-\alpha_{weak}(1-\beta)-\beta}}{\beta} \tag{14}$$

If $\beta = 1$, then $\widehat{\theta} = \alpha_{weak}$.

(5) If $\alpha_{weak} = 0$ and $\beta = 0$, then equation (13) implies$2\widehat{\theta} - 1 = 0 \Longleftrightarrow \widehat{\theta} = 0.5$. ■

Proof of proposition 5

Let $\widehat{\theta}_{white}$ be the threshold ability for whites, $\widehat{\theta}_{black}$ be the threshold ability for blacks, and $s^* > \widehat{\theta}_{black} \geq \widehat{\theta}_{white}$ by proposition 3.

(1) All black workers with ability levels $\theta \geq \alpha_{weak}$ choose human capital intensive occupation. In the manual labor intensive occupation occupation, only workers with ability levels $\theta < \alpha_{weak}$ can be compared. The firm can infer these ability levels with strong and weak ties. Therefore, the black and white workers get the same wage, and the wage gap between whites and their black counterparts is zero.

$$WG_m = \frac{1}{\alpha_{weak}}\int_0^{\alpha_{weak}} \left[w_{white}(\theta,t) - w_{black}(\theta,t)\right] d\theta = \frac{1}{\alpha_{weak}}\int_0^{\alpha_{weak}} \left[\theta - \theta\right] d\theta = 0.$$

For a given informativeness level of weak tie α_{weak}, let β^* be the value for the proportion of white workers with access to strong tie such that $\widehat{\theta}_{white} = 0.5$. If $\alpha_{weak} = 0.5$, then $\beta^* = 1$ by property (4) of lemma 1. Since $\alpha_{weak} < 0.5$, property (1) of lemma 1 implies that $\widehat{\theta}_{white} < 0.5$ if $\beta_{white} = 1$. Then, property (2) of lemma 1 implies that there exists a threshold value $\beta^* < 1$ such that $\widehat{\theta}_{white} = 0.5$.

If $\beta_{white} > \beta^*$, then $\widehat{\theta}_{white} < 0.5$ follows from property (2) of lemma 1. If $\widehat{\theta}_{white} < 0.5$, then below average ability white workers will not be able to enter human capital intensive occupation through weak ties, and the Black-White wage gap is higher in the human capital intensive occupation (which is the occupation with higher mean wage) as follows.

$$
\begin{aligned}
WG_h &= \frac{1}{0.5}\int_{0.5}^{1}\left[w_{white}(\theta,t_{strong}) - w_{black}(\theta,t_{weak})\right]d\theta \\
&= \frac{1}{0.5}\left[\int_{0.5}^{s^*}\left(\theta-\widehat{\theta}_{black}\right)d\theta + \int_{s^*}^{1}\left(\frac{1+s^*}{2}-\widehat{\theta}_{black}\right)d\theta\right] \\
&= \frac{1}{0.5}\left[\frac{(s^*-0.5)}{2}\left(s^*+0.5-2\widehat{\theta}_{black}\right)+\frac{(1-s^*)}{2}\left(1+s^*-2\widehat{\theta}_{black}\right)\right] \\
&= \left(1-\widehat{\theta}_{black}\right)-0.5^2>0
\end{aligned}
$$

The first equality follows from the fact that only above average ability whites workers enter human capital intensive occupation through strong ties. In the human capital intensive occupation occupation, only above average ability workers can be compared. The second equality follows from the fact that for $\theta \in [0.5, s^*)$, firm can infer the ability levels of the white workers, and for $\theta \geq s^*$ firm cannot infer the ability levels of the white workers. The final inequality follows from the property (3) of lemma 1 $\widehat{\theta}_{black} < 0.75$.

(2) If $\alpha_{weak} = 0$, then $\beta^* = 0$ by property (5) of lemma 1.

(3) If $\alpha^{'}_{weak} > \alpha_{weak}$ and $\beta_{white} = \beta^*$, then property (1) of lemma 1 implies that $\widehat{\theta}^{'}_{white} > \widehat{\theta}_{white} = 0.5$. Then, property (2) of lemma 1 implies that $\beta^{*^{'}} > \beta^*$. ■

Proof of theorem 2

As before, all black workers with ability levels $\theta \geq \alpha_{weak}$ choose human capital intensive occupation. In the manual labor intensive occupation occupation, only workers with ability levels $\theta < \alpha_{weak}$ can be compared. The firm can infer these ability levels with strong and weak ties. Therefore, the black and white workers get the same wage, and the wage gap between whites and their black counterparts is zero.

$$
WG_m = \frac{1}{1-\alpha_{weak}}\int_{0}^{\alpha_{weak}}\left[w_{white}(\theta,t)-w_{black}(\theta,t)\right]d\theta = \frac{1}{1-\alpha_{weak}}\int_{0}^{\alpha_{weak}}\left[\theta-\theta\right]d\theta = 0.
$$

For the given informativeness level of weak tie α_{weak}, if $\beta_{black} \leq \beta^* < \beta_{white}$, then $\widehat{\theta}_{white} < 0.5$ and $\widehat{\theta}_{black} \geq 0.5$. As a result, below average ability white workers will not be able to enter human capital intensive occupation through weak ties, but below average ability black workers will enter human capital intensive occupation through weak ties. Thus, the Black-White wage gap is higher in the human capital intensive occupation (which is the occupation with higher mean wage) as follows.

Observe that

$$
\begin{aligned}
& \sum_{(t_{white},t_{black})\in\{t_{weak},t_{strong}\}^2} \left[\int_{0.5}^{1} f(t_{white}|occ_h,\theta)f(t_{black}|occ_h,\theta)d\theta\right] \\
= & \left[\int_{0.5}^{1} f(t_{strong}|occ_h,\theta)f(t_{strong}|occ_h,\theta)d\theta\right] + \left[\int_{0.5}^{1} f(t_{strong}|occ_h,\theta)f(t_{weak}|occ_h,\theta)d\theta\right] \\
= & \left[\int_{\widehat{\theta}}^{1} \beta_{white}\beta_{black}d\theta\right] + \left[\int_{0.5}^{\widehat{\theta}_{black}} \beta_{white}(1)d\theta + \int_{\widehat{\theta}_{black}}^{1} \beta_{white}(1-\beta_{black})d\theta\right] \\
= & (0.5)\beta_{white}
\end{aligned}
$$

where the first equality follows from the fact that $f(t_{weak}|occ_h,\theta) = 0$ for whites.

$$
\begin{aligned}
WG_h = & \sum_{(t_{white},t_{black})\in\{t_{weak},t_{strong}\}^2} \int_{0.5}^{1} \left[w_{white}(\theta,t) - w_{black}(\theta,t)\right] \frac{f(t_{white}|occ_h,\theta)f(t_{black}|occ_h,\theta)}{(0.5)\beta_{white}} d\theta \\
= & \frac{1}{(0.5)\beta_{white}} \int_{\widehat{\theta}_{black}}^{1} \left[w_{white}(\theta,t_{strong}) - w_{black}(\theta,t_{strong})\right]\beta_{white}\beta_{black}d\theta \\
+ & \frac{1}{(0.5)\beta_{white}} \int_{0.5}^{1} \left[w_{white}(\theta,t_{strong}) - w_{black}(\theta,t_{weak})\right]\beta_{white}f(t_{weak}|occ_h,\theta)d\theta \\
= & \frac{1}{(0.5)\beta_{white}} \int_{0.5}^{1} \left[w_{white}(\theta,t_{strong}) - w_{black}(\theta,t_{weak})\right]\beta_{white}f(t_{weak}|occ_h,\theta)d\theta \\
= & \frac{\beta_{white}}{(0.5)\beta_{white}} \int_{0.5}^{1} \left[w_{white}(\theta,t_{strong}) - w_{black}(\theta,t_{weak})\right]f(t_{weak}|occ_h,\theta)d\theta \\
= & \frac{1}{(0.5)} \left[\int_{0.5}^{\widehat{\theta}_{black}} \left(\theta - \widehat{\theta}_{black}\right)(1)d\theta + \int_{\widehat{\theta}_{black}}^{s^*} \left(\theta - \widehat{\theta}_{black}\right)(1-\beta_{black})\,d\theta\right] \\
+ & \frac{1}{(0.5)} \left[\int_{s^*}^{1} \left(\frac{1+s^*}{2} - \widehat{\theta}_{black}\right)(1-\beta_{black})\,d\theta\right] \\
= & \frac{1}{(0.5)} \left[(1-\beta_{black})\left(1-\widehat{\theta}_{black}\right)^2 - \left(\widehat{\theta}_{black} - 0.5\right)^2\right] \\
> & \; 0
\end{aligned}
$$

In the human capital intensive occupation occupation, only above average ability workers can be compared. The third equality follows from the fact that both white and black workers get the same wages when they both use strong ties, i.e. $w_{white}(\theta,t_{strong}) = w_{black}(\theta,t_{strong})$. The final inequality is $WG_h > 0 \iff \widehat{\theta}_{black} < \frac{0.5+\sqrt{1-\beta_{black}}}{1+\sqrt{1-\beta_{black}}}$ and it follows from $\alpha_{weak} < 0.5$. To see this, note that property (1) of lemma 1 implies $\frac{\partial\widehat{\theta}_{black}}{\partial\alpha_{weak}} > 0$. So there is some $\widetilde{\alpha}_{weak}$ at which the left hand side (equation 16) of the inequality $\widehat{\theta}_{black} < \frac{0.5+\sqrt{1-\beta_{black}}}{1+\sqrt{1-\beta_{black}}}$ equals the right hand side. For $\alpha_{weak} < \widetilde{\alpha}_{weak}$, this inequality is satisfied. It is easy to verify that $\widetilde{\alpha}_{weak} = 0.5$, and this condition is satisfied from assumption that weak ties can't infer between below and above average ability workers $\alpha_{weak} < 0.5$. ■

Proof of proposition 6

At equilibrium, wage offer from a weak tie is

$$\begin{aligned} E[\theta|s & \geq & \alpha_{weak}, t_{weak}] = \int_{\alpha_{weak}}^{1} \theta \frac{f(t_{weak}|\theta)}{\int_{\alpha_{weak}}^{1} f(t_{weak}|\theta) d\theta} d\theta \\ & = & \frac{1}{\int_{\alpha_{weak}}^{\theta^*} (\beta_{weak} + \beta_{both}) d\theta + \int_{\theta^*}^{1} \beta_{weak} d\theta} \left[\int_{\alpha_{weak}}^{\theta^*} (\beta_{weak} + \beta_{both}) \theta d\theta + \int_{\theta^*}^{1} \theta \beta_{weak} d\theta \right] \\ & = & \frac{1}{2 \left[\beta_{both} \left(\theta^* - \alpha_{weak}^2 \right) + \beta_{weak} \left(1 - \alpha_{weak} \right) \right]} \left[\beta_{both} \left((\theta^*)^2 - \alpha_{weak}^2 \right) + \beta_{weak} \left(1 - \alpha_{weak}^2 \right) \right] \end{aligned}$$

As before, setting $\widehat{\theta} = \{\theta : \theta = E[\theta|s \geq \alpha_{weak}, t_{weak}]\}$ gives

$$\beta_{both}(\theta^*)^2 + 2\theta \left[\beta_{weak} \left(1 - \alpha_{weak} \right) - \beta_{both} \alpha_{weak} \right] - \beta_{weak} \left(1 - \alpha_{weak}^2 \right) - \beta_{both} \alpha_{weak}^2 \quad = \quad 0$$

Applying the Implicit Function Theorem with $F(\widehat{\theta}, \alpha_{weak}, \beta_{weak}, \beta_{both}) = \beta_{both}(\widehat{\theta}^*)^2 + 2\widehat{\theta} \left[\beta_{weak} \left(1 - \alpha_{weak} \right) - \beta_{both} \alpha_{weak} \right] - \beta_{weak} \left(1 - \alpha_{weak}^2 \right) - \beta_{both} \alpha_{weak}^2 = 0$ gives the following.

$$\frac{\partial \widehat{\theta}}{\partial \alpha_{weak}} = -\frac{2\widehat{\theta} \left[\beta_{weak} + \beta_{both} \right] - 2\alpha_{weak} \left[\beta_{both} - \beta_{weak} \right]}{2\widehat{\theta} \beta_{both} + 2 \left[(1 - \alpha_{weak}) \beta_{weak} - \alpha_{weak} \beta_{both} \right]} > 0$$

The numerator is negative as follows.
$2\widehat{\theta} \left[\beta_{weak} + \beta_{both} \right] - 2\alpha_{weak} \left[\beta_{both} - \beta_{weak} \right] < -4\alpha_{weak} \left[\beta_{weak} \right] < 0$
where the first inequality follows from $\alpha_{weak} < \widehat{\theta}$.
The denominator is positive as follows.

$$2\widehat{\theta} \beta_{both} + 2 \left[(1 - \alpha_{weak}) \beta_{weak} - \alpha_{weak} \beta_{both} \right] > -2\alpha_{weak} \left[\beta_{weak} \right] + 2\beta_{weak} = 2(1 - \alpha_{weak}) \beta_{weak} > 0$$

where the first inequality follows from $\alpha_{weak} < \widehat{\theta}$.
Similarly,

$$\frac{\partial\widehat{\theta}}{\partial\beta_{weak}} = -\frac{2\widehat{\theta}(1-\alpha_{weak})-(1-\alpha_{weak}^2)}{2\widehat{\theta}\beta_{both}+2\left[(1-\alpha_{weak})\beta_{weak}-\alpha_{weak}\beta_{both}\right]} > 0$$

The numerator is negative as follows.

$$2\widehat{\theta}(1-\alpha_{weak})-(1-\alpha_{weak}^2) = -2\widehat{\theta}(1-\alpha_{weak})\alpha_{weak} < 0$$

The denominator is positive as explained above.

For a given informativeness level of weak tie α_{weak}, let β^* be the value for the proportion of workers with access to weak tie such that $\widehat{\theta} = 0.5$. If $\beta_{weak} > max\{\beta^*, \beta_{strong}\}$, then $\widehat{\theta} > 0.5$, and

$$\begin{aligned} f(t_{weak}) &= \int_0^1 f(t_{weak}|\theta)d\theta \\ &= \int_0^{\widehat{\theta}} (\beta_{weak}+\beta_{both})d\theta + \int_{\widehat{\theta}}^1 \beta_{weak}d\theta \\ &= \beta_{weak}+\widehat{\theta}\beta_{both} \\ &> \beta_{strong}+(1-\widehat{\theta})\beta_{both} \\ &= \int_0^{\widehat{\theta}} \beta_{strong}d\theta + \int_{\widehat{\theta}}^1 (\beta_{strong}+\beta_{both})d\theta \\ &= \int_0^1 f(t_{strong}|\theta)d\theta. \quad \blacksquare \end{aligned}$$

Proof of proposition 7

(1) At a cost efficient equilibrium, the wage offer from direct application is equal to the threshold ability $w_d = E[\theta|d] = \theta^*$. It is easy to verify that the $E[\theta|d] = \frac{1}{2[\beta\theta^*+(1-\beta)]}\left[\beta\theta^{*^2}+(1-\beta)\right]$, which doesn't depend on δ. As a result, the threshold ability doesn't depend on δ, and it is characterized (with $\alpha_{weak} = 0$) as before $\beta\theta^{*2}+2(1-\beta)\theta^*-(1-\beta) = 0$. Thus, I can apply results obtained in lemma 1.

Since the firm has minimal informativeness level $\alpha_d = 0$, $\beta^* = 0$ by property (5) of lemma 1. If $\beta > 0$, then $\theta^* < E[\theta] = 0.5$. As a result, for above average ability workers, returns to ties are always positive,

$$w_{t-d}(\theta > 0.5) = [E[\theta|t, \theta > 0.5] - w_d] = \int_{0.5}^1 \theta\frac{f(t|\theta)}{\int_0^1 f(t|\theta)d\theta}d\theta - \theta^* > 0.$$

(2) (i) For below average ability workers, returns to ties depend on β.

$$
\begin{aligned}
w_{t-d}(\theta < 0.5) &= [E[\theta|t, \theta < 0.5] - w_d] \\
&= \int_0^{0.5} \theta \frac{f(t|\theta)}{\int_0^1 f(t|\theta) d\theta} d\theta - \theta^* \\
&= \frac{1}{\int_0^{\theta^*} \theta\beta d\theta + \int_{\theta^*}^{0.5} \theta\beta(1-\delta) d\theta} \left[\int_0^{\theta^*} \theta\beta d\theta + \int_{\theta^*}^{0.5} \theta\beta(1-\delta) d\theta \right] - \theta^* \\
&= \frac{1}{2\left[0.5 - \delta\theta^*\right]} \left[0.5^2 - \delta{\theta^*}^2 \right] - \theta^* \\
&= \frac{1}{2\left[0.5 - \delta\theta^*\right]} \left[0.5^2 + \delta{\theta^*}^2 - \theta^* \right].
\end{aligned}
$$

By property (2) of lemma 1, $\frac{\partial\theta^*}{\partial\beta} < 0$. Moreover,

$$
\begin{aligned}
\frac{\partial\left[E[\theta|t, \theta < 0.5] - w_d\right]}{\partial\theta^*} &= \frac{\partial E[\theta|t, \theta < 0.5]}{\partial\theta^*} \\
&= \frac{1}{\left(2\left[0.5 - \delta\theta^*\right]\right)^2} \left\{ \left[0.5^2 - \delta{\theta^*}^2 \right] (-\delta) - 2\delta\theta^* \right\} \\
&< 0.
\end{aligned}
$$

This follows from the fact that $\theta^* < E[\theta] = 0.5$. Thus, $\frac{\partial[E[\theta|t,\theta<0.5]-w_d]}{\partial\beta} = \frac{\partial[E[\theta|t,\theta<0.5]-w_d]}{\partial\theta^*}\frac{\partial\theta^*}{\partial\beta} > 0$.

Since $\theta^* < E[\theta] = 0.5$, then

$$
\begin{aligned}
\frac{\partial\left[E[\theta|t, \theta < 0.5] - w_d\right]}{\partial\delta} &= \frac{\partial E[\theta|t, \theta < 0.5]}{\partial\delta} \\
&= \frac{\theta^*\left(0.5 - \theta^*\right)}{\left(2\left[0.5 - \delta\theta^*\right]\right)^2} \\
&> 0.
\end{aligned}
$$

By property (2) of lemma 1, $\frac{\partial\theta^*}{\partial\beta} < 0$. If $\beta = 1$, then $\theta^* = 0$. Thus, $\theta^* \in [0, 0.5)$.

If $\delta = 0$, then $\theta^* = 0.25 \implies [E[\theta|t, \theta < 0.5] - w_d] = 0$. If $\delta = 1$, then $\theta^* = 0.5 \implies [E[\theta|t, \theta < 0.5] - w_d] = 0$.

Since $\theta^* \in [0, 0.5)$, $\frac{\partial[E[\theta|t,\theta<0.5]-w_d]}{\partial\delta} > 0$, then for each $0 \leq \delta < 1$, there exists a β^{**} such that $[E[\theta|t, \theta < 0.5] - w_d] = 0$. If $\beta \approx \beta^{**}$, then for below average ability workers, average returns to tie is small and insignificant $[E[\theta|t, \theta < 0.5] - w_d] \approx 0$.

(ii) If $\beta < \beta^{**}$, then the average returns to tie is negative. $[E[\theta|t, \theta < 0.5] - w_d] < 0$.

(iii) Since $\frac{\partial[E[\theta|t,\theta<0.5]-w_d]}{\partial\beta} > 0$ and $\frac{\partial[E[\theta|t,\theta<0.5]-w_d]}{\partial\delta} > 0$, if $\delta' < \delta$, then ${\beta^{**}}' > \beta^{**}$. ■

2.2 Cost Efficient Equilibrium

I focus on an appealing class of pure strategy Perfect Bayesian Equilibria, which I call the Cost Efficient Equilibria. This section of the appendix provides a formal justification for my focus on the Cost Efficient Equilibria.

I consider recommendation strategies which *reveal maximum information* in that starting from the lowest ability level, the employed worker reveals information to the firm until he either runs out of recommendation values or the signal reaches his informativeness level. Let $[0, \theta_{max}]$ be the interval of ability levels that a firm can infer from an employed worker who reveals maximum information. The next proposition shows that the cost efficient recommendation strategy reveals maximum information in the cheapest way possible.

> **Proposition 8.** If an employed worker's recommendation strategy ρ is cost efficient, then
>
> (a) it reveals maximum information $[0, \theta_{max}]$, and
>
> (b) for any other recommendation strategy which also reveals maximum information $\rho' \neq \rho$, an employed worker incurs lower reputation costs by revealing maximum information through the cost efficient recommendation strategy, i.e. $\left(\rho - E[\theta|s,t]^2\right) \leq \left(\rho' - E[\theta|s,t]^2\right)$ for all $s \in [0,1]$, and the inequality is strict for at least one $\widetilde{s} \in [0,1]$.
>
> **Proof of proposition 8**

(a) This follows from point (2) of cost efficient recommendation strategy's definition. Since recommendations are increasing in signals for $s < min\{s^*, \alpha_t\}$ where $\rho(s^*, t) = 1$, the employed worker's recommendations reveal his signal value to the firm $\{s' : \rho(s', t) = \rho(s, t)\} = \{s\}$ for all such signals. Thus, the employed worker reveals information to the firm until he either runs out of recommendation values or the signal reaches his informativeness level.

(b) For the lowest signal value, the employed worker incurs zero reputation costs.

The employed worker reveals information by increasing recommendation values as signal value increases. Since reputation costs depend on the distance between the recommendation value and the signal value, revealing information by increasing recommendation values minimizes reputation costs.

For the set of signals at the top, the employed worker reveals his information to the firm by minimizing reputation costs. ■

Remark on reputation costs functional form

Let $1_{\rho > E[\theta|s,t]}$ be an indicator function which takes value of one if recommendations are higher than the expected ability value and zero otherwise. If reputation costs only come from recommendations that are higher than the expected ability value $1_{\rho > E[\theta|s,t]}(\rho - E[\theta|s,t])^2$, then Cost Efficient Recommendation Strategy still reveals maximum information in the cheapest way possible. Thus, all results of this paper will continue to hold in such variation of the model.

To see this, observe that the initial value condition is still the cheapest way to reveal the lowest signal. For the interval of signals that firm can infer, firm sets wage equal to signal $w(\rho(s,t),t) = s$. If $\rho(s,t) \leq s$, then FOC of the employed worker's maximization problem implies $w_1(\rho,t) = 0 \Longleftrightarrow \frac{1}{\rho_1(s,t)} = 0$. Then, firm can't infer any signals above the lowest signal because $\rho_1(s,t) \to \infty$. Thus, recommendation strategy must have $\rho(s,t) > s$ for firm to infer signals above the lowest signal. As a result, FOC of the employed worker's maximization problem (and the corresponding differential equation) implies recommendations are in increasing in signals for such interval of signals. And the rest of the arguments follow from proof of proposition 6.