

Chapter 3

Non-homogeneous turbulence

In recent years many efforts have been made to explain the intermittency phenomenon, particularly in homogeneous flows. In this context, many intermittency models have been proposed as discussed in chapter 2 and in Appendix A. In Contrast, in non-homogeneous flows which are more complex and have more practical interest, less attention has been given to the study of non-local dynamics which seems separated from intermittency and also seems to play an important role in non-homogeneous turbulence.

In the following we start by focusing our attention on the Landau remark (Frisch (1995)) concerning the energy dissipation rate, then we describe the Extended Self Similarity (ESS) technique which is a key way to analyze both homogeneous and non-homogeneous flows. Finally, we describe the Dubrulle (1994) and Babiano *et al.* (1997), hereafter BDF models (see also Babiano (2000)). The latter takes into account the non-uniformity in scale of the variance of transfer random field and it is based on the statistical properties of the absolute energy transfer at scale ℓ , related to the non-linear term of the Navier-Stokes equation. This theory will be used as the key model of non-homogeneous flows throughout this thesis.

3.1 Non-uniform energy dissipation random field

Kolmogorov (1962) and Oboukhov (1962) introduced the refined version of Kolmogorov's similarity hypothesis taking into account intermittency. They assumed that for locally homogeneous and isotropic turbulence, the energy dissipation field strongly fluctuates in both space and time. On the other hand, the average amplitude of the dissipation random field scales quite uniformly in both space and time.

In contrast, when the turbulence is non-homogeneous and non-isotropic, the dissipation random field is non-uniform in scale. This means that the fluctuations and the amplitude of the variance of the energy transfer are scale dependent quantities. In this case, the correction $\tau_{p/3}$ of relation (2.16) is associated with both the intermittency phenomenon linked to the rarest events and the anomalous dependence as a function of the length scale of transfer properties in the energy cascade scales. Therefore, the relation (2.16) is not valid, because ξ_p are anomalous and scale dependent. The methodology proposed in K62 is not valid for non-local dynamics on the velocity structure functions.

One can summarize, that in non-homogeneous and non-isotropic flows, we have two coupled anomalies.

The first is a deviation from a scale-independent quasi-Gaussian distribution of the largest but rarest events (intermittency) and the second is the non-uniformity in scale of the energy transfer random field (deviation from local homogeneity).

The non-uniform distribution of ϵ_ℓ , which is reflected in the even moments of the dissipation rate, can also be detected using the absolute value $\langle |\epsilon_\ell| \rangle$ even for $p = 3$, which is related to $\langle |\delta u_\ell|^3 \rangle$. Consequently, the statistical properties of odd order velocity structure functions is analyzed using the average of the absolute values of velocity increments using the concept of the Extended self similarity (ESS) introduced by Benzi *et al.* (1993).

Moreover, it is well known that in the framework of K41 and K62 theories, only the longitudinal component to the separation vector $\vec{\ell}$ is relevant (Frisch (1995)). In this case, the contribution to the energy transfer from the transversal to $\vec{\ell}$ velocity components and the pressure forces vanish statistically because of homogeneity. Concerning the contribution of the third-order velocity structure function to the statistical description of the non-homogeneous turbulence, the question is the following: is the longitudinal structure function the only main characteristic in such a description, as in the homogeneous case, or when the turbulence is non homogeneous we should use the total velocity structure function.

3.2 Extended Self similarity

Extended self similarity (ESS) is a property of velocity structure functions of homogeneous and non-homogeneous turbulence. It was introduced by Benzi *et al.* (1993) and has been extensively used in recent years, see for example the reviews of Camussi *et al.* (1996) and Sreenivasan and Antonia (1997). Instead of obtaining scaling exponents in the usual way by plotting structure functions of the absolute velocity increments $\langle |\delta u_\ell|^p \rangle$ against ℓ , they plot them against the third-order structure

function of the absolute velocity increment $\langle |\delta u_\ell|^3 \rangle$ (figure 3.1), then we have

$$\langle |\delta u_\ell|^p \rangle \sim \langle |\delta u_\ell|^3 \rangle^{\zeta_p/\zeta_3}, \quad (3.1)$$

where ζ_p/ζ_3 is a relative scaling exponent and ζ_p is defined by

$$\langle |\delta u_\ell|^p \rangle \sim \ell^{\zeta_p}, \quad (3.2)$$

where, ζ_p is now the *absolute scaling exponent* and may be different from ξ_p for odd values of p because absolute values of velocity increments are used. Using relation (3.1), many experimental analyses indicate that the relative scaling exponents ζ_p/ζ_3 tend to be scale independent for a large class quasi-homogeneous turbulent flows, even if both absolute scaling exponents ζ_p and ζ_3 depend on ℓ . This technique can be used in situations where an inertial range is absent such as in low Reynolds number experiments (Briscolini *et al.* (1994)) or in non-homogeneous turbulence (Babiano *et al.* (1997)) and also at high Reynolds number flows. Benzi *et al.* (1993) suggested that the ESS may extend the scaling range. Further, Stolovitzky and Sreenivasan (1993) found that the ESS works well only for low-order structure functions and they provided an alternative method for making dissipation range corrections to higher-order structure functions. Recently, Arneodo *et al.* (1996) used the ESS to analyze some 3D turbulent flows characterized by different Reynolds numbers and suggested that ESS works very well. Evidently, recovered scale-invariance for relative exponents when ζ_p and ζ_3 are scale-dependent cannot be easily interpreted with the models K41 and K62.

Some limitations of ESS were mentioned by Stolovitzky & Sreenivasan (1993) and by Benzi *et al.* (1994). They pointed out that the ESS does not seem to work when the shear is strong, such as in the shear behind a cylinder and in boundary layer turbulence. In contrast with these results, Babiano *et al.* (1997) and Gaudin *et al.* (1998) found that the ESS also works well in these situations. This suggests that the ESS may be also a specific and convenient tool to analyze non-homogeneous turbulence.

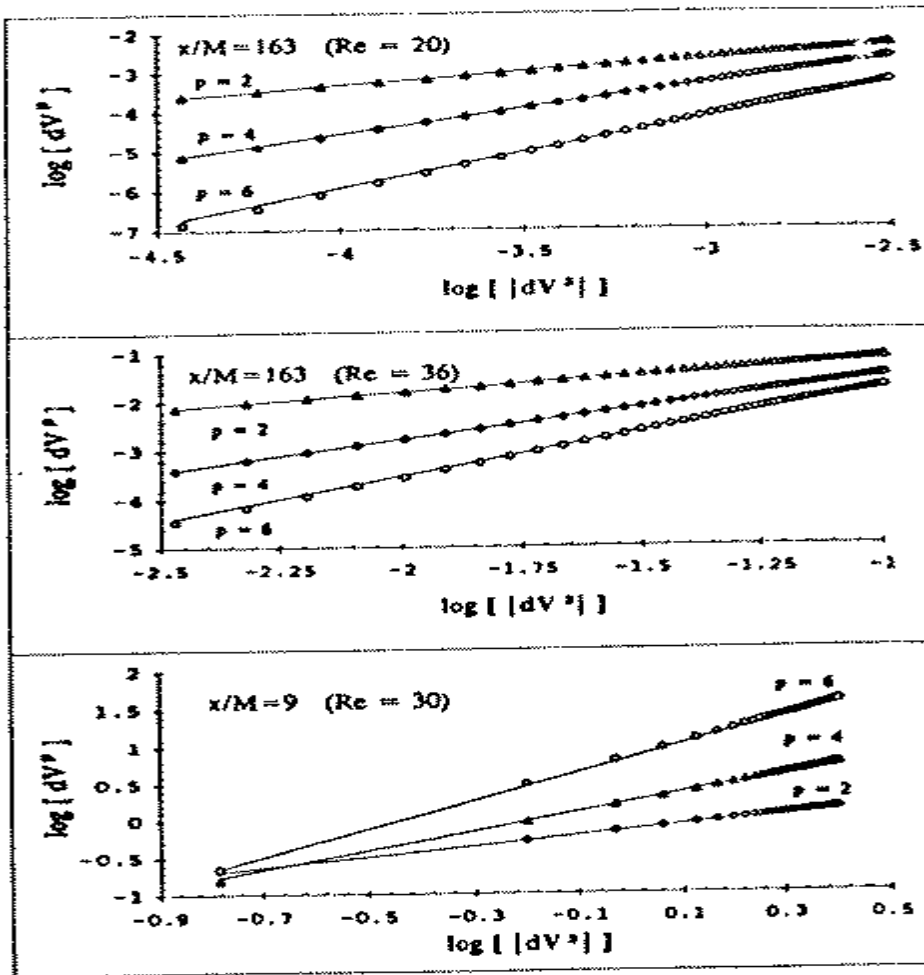


Figure 3.1: Extended Self Similarity scaling ranges for grid turbulence for different conditions (Camussi and Guj (1995)).

The use of $\langle |\delta u_\ell|^3 \rangle$ instead of $\langle \delta u_\ell^3 \rangle$ in ESS may be physically explained because it refers to the scale by scale absolute balance of transferred energy at a given scale ℓ (Babiano (2000)), and includes both energy transfers from larger to smaller scales (normal cascade) and the anomalous energy transfers from smaller to larger scales (inverse cascade). This fact suggests that the ESS relation (3.1) must be expressed in term of $|\delta u_\ell|^3$. Figure 3.2 shows the relative scaling exponents calculated using the ESS in homogeneous flows (Arneodo *et al.* (1996)).

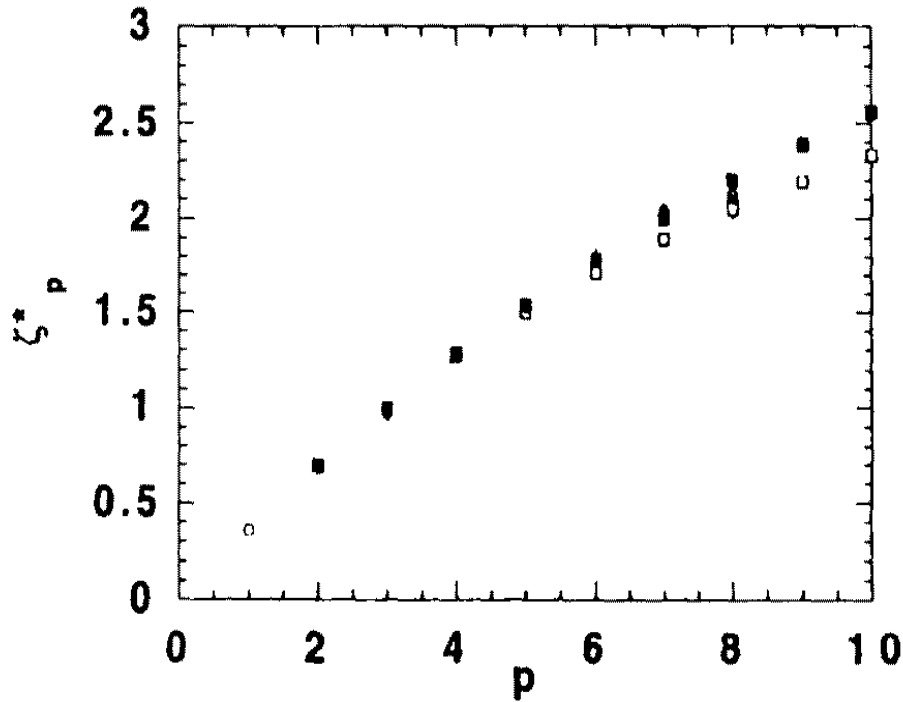


Figure 3.2: The relative scaling exponents in various flow configurations (Arneodo *et al.* (1996)).

3.3 Dubrulle model

Dubrulle (1994), was slightly modified the SL model by including the ESS property (Benzi *et al.* (1993)) and using $\langle \delta u_\ell^3 \rangle / \epsilon_\ell$ instead of the scale ℓ . The corresponding model was based on three hypotheses concerning the statistical properties of the non-dimensional energy dissipation π_ℓ ,

$$\pi_\ell = \frac{\epsilon_\ell}{\epsilon_\ell^{(\infty)}}, \quad (3.3)$$

where ϵ_ℓ is a scale independent quantity and $\epsilon_\ell^{(\infty)}$ is a normalization function, which has a scale-divergent behavior.

The three main hypotheses are the following:

i) Similarity:

$$\frac{\delta u_\ell^3}{\langle \delta u_\ell^3 \rangle} \stackrel{\text{law}}{=} \frac{\epsilon_\ell}{\langle \epsilon_\ell \rangle} = \frac{\pi_\ell}{\langle \pi_\ell \rangle}, \quad (3.4)$$

where $\stackrel{\text{law}}{=}$ indicates that the terms have the same scaling properties.

ii) Moment hierarchy:

$$\frac{\langle \pi_\ell^{p+1} \rangle}{\langle \pi_\ell^p \rangle} = A_p \left[\frac{\langle \pi_\ell^p \rangle}{\langle \pi_\ell^{p-1} \rangle} \right]^\beta, \quad (3.5)$$

where A_p are numerical constants and β is a positive constant smaller than or equal to 1. The straightforward development for $p = 0, 1, 2, \dots, p$ leads to the formula

$$\langle \pi_\ell^p \rangle \sim \langle \pi_\ell \rangle^{(1-\beta^p)/(1-\beta)}. \quad (3.6)$$

iii) Power-law intermittency:

$$\langle \pi_\ell \rangle = \frac{\langle \epsilon_\ell \rangle}{\epsilon_\ell^\infty} \sim \left[\frac{\langle \delta u_\ell^3 \rangle}{\epsilon_0 \eta} \right]^\gamma, \quad (3.7)$$

where η is the Kolmogorov scale and γ is an adjustable parameter characterizing the degree of heterogeneity of the transfer field and of the most intermittent structures participating in the transfers. The relation (3.7) holds irrespective of the geometry of the dissipative structures, which are characterized by γ (Dubrulle (1994)).

The combination of the three assumptions implies that the velocity structure functions follow the extended self-similarity relationship:

$$\langle \delta u_\ell^p \rangle = C_p \langle \delta u_\ell^3 \rangle^{\xi_p / \xi_3}, \quad (3.8)$$

with a relative scaling exponent given by:

$$\frac{\xi_p}{\xi_3} = \frac{p}{3} [1 - \gamma] + \gamma \frac{1 - \beta^{p/3}}{1 - \beta}. \quad (3.9)$$

Taking $\xi_3 = 1$ and $\gamma = \beta = 2/3$ leads to the She-Leveque formula (She and Leveque (1994)) for fully developed homogeneous turbulence. In the non-intermittent homogeneous situation corresponding to the K41 theory Kolmogorov (1941) we have $\beta = 1$.

The interest of this model is that the parameter γ determined by relation (3.7) also depends on the degree of non-uniform scale of $\langle \pi_\ell \rangle$ and $\langle \delta u_\ell^3 \rangle$. This behavior was shown in Babiano *et al.* (1995) in 2D turbulence. Taking the absolute value in relations (3.4) and (3.7) the Dubrulle model may be useful for non-homogeneous turbulence.

3.4 BDF non-homogeneous model

3.4.1 Energy transfer

In recent works, Babiano *et al.* (1997) and Babiano (2000) proposed a model for non-homogeneous turbulence characterized by residual intermittency. The tool applied is basically founded on the statistical properties of the *absolute energy transfer* at scale ℓ , related to the non-linear term of the Navier-Stokes equation rather than to the dissipation term. For a given length-scale ℓ , the instantaneous absolute value of the energy transfer due to the non-linear terms of the equation of motion of incompressible flow is given by

$$\sigma_\ell = \frac{1}{|\bar{V}_\ell|} \left| \int_{S_\ell} (u^2 + v^2 + P) u ds_\ell \right|, \quad (3.10)$$

where $u = \vec{V} \cdot \vec{n}$ is the velocity component along $\vec{\ell}$, which is also normal to the element ds_ℓ of surface S_ℓ containing the control volume defined at scale ℓ and centered at the space-position $\vec{x} + \vec{\ell}/2$, P is the pressure and v is the transversal component of \vec{V} , ($|\vec{V}|^2 = u^2 + v^2$).

Defining the integrand of (3.10) as the differences at opposite sides of the control surface S_ℓ and maintaining all the non-homogeneous contributions it is easy to demonstrate that (3.10) is equivalent to:

$$\sigma_\ell = \frac{1}{2|\bar{V}_\ell|} \left| \int_{S_\ell} [u(\vec{x} + \vec{\ell})^3 - u(\vec{x})^3 - B - C] ds_\ell \right|, \quad (3.11)$$

or to:

$$\sigma_\ell = \frac{1}{2|\bar{V}_\ell|} \left| \int_{S_\ell} [\delta u_\ell^3 - (B - 3A) - C] ds_\ell \right|, \quad (3.12)$$

where the terms A , B and C are expressed as:

$$A = u(\vec{x} + \vec{\ell})^2 u(\vec{x}) - u(\vec{x})^2 u(\vec{x} + \vec{\ell}) \quad (3.13)$$

$$B = v^2(\vec{x}) u(\vec{x}) - v^2(\vec{x} + \vec{\ell}) u(\vec{x} + \vec{\ell}) \quad (3.14)$$

$$C = P(\vec{x}) u(\vec{x}) - P(\vec{x} + \vec{\ell}) u(\vec{x} + \vec{\ell}). \quad (3.15)$$

The positions \vec{x} and $\vec{x} + \vec{\ell}$ correspond to the opposite sides of the control surface S_ℓ . In homogeneous flows, the cross-correlation terms tend to be negligible quantities so $A = B = C = 0$. In this situation, the energy transfer (3.11) is related only to the third order longitudinal structure function of the velocity increments. In contrast, in non-homogeneous flows the situation is much more complicated since in general $A \neq B \neq C \neq 0$ in a statistical sense. Then both longitudinal and transversal terms are likely to be important.

3.5 Non-local dynamics

Continuing with equation (3.11), it is necessary to evaluate the dominant terms in the non-homogeneous case. Monin and Yaglom (1975) gave a relationship between the longitudinal $\langle \delta u_\ell^2 \rangle$ and transversal $\langle \delta v_\ell^2 \rangle$ second-order velocity structure function in the non-divergent case as a homogeneous case

$$\langle \delta v_\ell^2 \rangle = (1 + q\ell \frac{d}{d\ell}) \langle \delta u_\ell^2 \rangle. \quad (3.16)$$

The factor q characterizes the flow geometry. In 3D turbulence $q = 1/2$ while $q = 2$ in 2D turbulence. The equation (3.16) is called Karman's relation and is often used to characterize isotropy. It has an important role in our understanding of the transfer properties in incompressible turbulent flows. Dividing relation (3.16) by $\langle \delta u_\ell^2 \rangle$ one obtains:

$$\frac{\langle \delta v_\ell^2 \rangle}{\langle \delta u_\ell^2 \rangle} = 1 + q\ell \frac{d(\log \langle \delta u_\ell^2 \rangle)}{d \log \ell} = Q. \quad (3.17)$$

Taking $\langle \delta u_\ell^2 \rangle \sim \ell^{\xi_2}$, where ξ_2 is the scaling exponent of the second order velocity structure function, a simple form of (3.17) is derived as follows:

$$\frac{\langle \delta v_\ell^2 \rangle}{\langle \delta u_\ell^2 \rangle} \sim 1 + q\xi_2. \quad (3.18)$$

Defining $Q^* = \frac{\delta v_\ell^2}{\delta u_\ell^2}$, in terms of non-averaged quantities, we can write:

$$[v(\vec{x} + \vec{\ell}) - v(\vec{x})]\delta v_\ell \sim (1 + q\xi_2)[u(\vec{x} + \vec{\ell}) - u(\vec{x})]\delta u_\ell. \quad (3.19)$$

Multiplying (3.19) by factors $v(\vec{x} + \vec{\ell})v(\vec{x})$ and $u(\vec{x} + \vec{\ell})u(\vec{x})$ and combining the result using relations (3.13) and (3.14), we obtain:

$$B \sim Q^* A \left[1 - \frac{v(\vec{x} + \vec{\ell})v(\vec{x})}{Q^* A} \delta v_\ell - \frac{v^2(\vec{x} + \vec{\ell}) - v^2(\vec{x})}{A} \delta u_\ell \right], \quad (3.20)$$

An approximate relationship between A and B can be obtained from relation (3.13),

$$B \sim Q^* A [1 - T], \quad (3.21)$$

where, in terms of averaged quantities, T is given by:

$$T = \frac{1}{Q} \frac{\langle v^2(\vec{x}) \rangle}{\langle v^2(\vec{x} + \vec{\ell}) \rangle} \frac{R_2}{R_1} + \left(\frac{\langle v^2(\vec{x} + \vec{\ell}) \rangle}{R_1} \left[1 - \frac{\langle v(\vec{x} + \vec{\ell}) \rangle}{\langle v(\vec{x}) \rangle} \right] + \frac{\langle v^2(\vec{x}) \rangle}{R_2} \left[1 - \frac{\langle v(\vec{x}) \rangle}{\langle v(\vec{x} + \vec{\ell}) \rangle} \right] \right) \frac{1}{\delta v^2}, \quad (3.22)$$

where

$$R_1 = \frac{\langle u(\vec{x} + \vec{\ell})u(\vec{x}) \rangle}{\langle u^2(\vec{x} + \vec{\ell}) \rangle}, R_2 = \frac{\langle u(\vec{x} + \vec{\ell})u(\vec{x}) \rangle}{\langle u^2(\vec{x}) \rangle}. \quad (3.23)$$

Terms R_1 and R_2 refer to the velocity correlation coefficients respectively.

It was demonstrated in turbulent flows when the dynamic is non-local, that T is negligible compared to 1, except for the statistical parallelism between $u(\vec{x})v(\vec{x})$ and $u(\vec{x} + \vec{\ell})v(\vec{x} + \vec{\ell})$ (Babiano (2000)), consequently (3.21) can be estimated as

$$B - Q^* A \approx 0. \quad (3.24)$$

Accordingly, the energy transfer can be estimated as

$$\sigma_\ell = \frac{1}{2V_\ell} \left| \int_{S_\ell} [\delta u_\ell^3 - (Q^* - 3)A - C] ds_\ell \right|, \quad (3.25)$$

and the relationship (3.25) can be expressed as

$$\sigma_\ell = \frac{1}{2V_\ell} \left| \int_{S_\ell} [\delta u_\ell^3 + [Q^*(1 - T) - 3]A - C] ds_\ell \right|, \quad (3.26)$$

In 3D homogeneous turbulence, the energy flux in relation (3.11) is related only to the longitudinal third order structure function, because in relation (3.13) and (3.14)

the terms containing cross correlations are cancelled ($B = A = 0$). In this case, we have $\xi_2 = 2/3$ and by taking $q = 1/2$ we find $Q = 4/3$.

In the non-homogeneous case, the energy spectrum can be steeper than $k^{-5/3}$ and saturates to k^{-3} . This behavior is illustrated in figure 3.3, which shows clearly the transition from homogeneous and local dynamics $k^{-5/3}$ to non-local and non-homogeneous dynamics k^{-3} . On the other hand, the scaling exponent of the second-order velocity structure function ξ_2 shows an important deviation from Kolmogorov's $2/3$ prediction. In this case Q saturates to 3 for 2D turbulence and to 2 for 3D turbulence. This behaviour is illustrated in figures 3.4 and 3.5 for 2D and 3D turbulence, respectively.

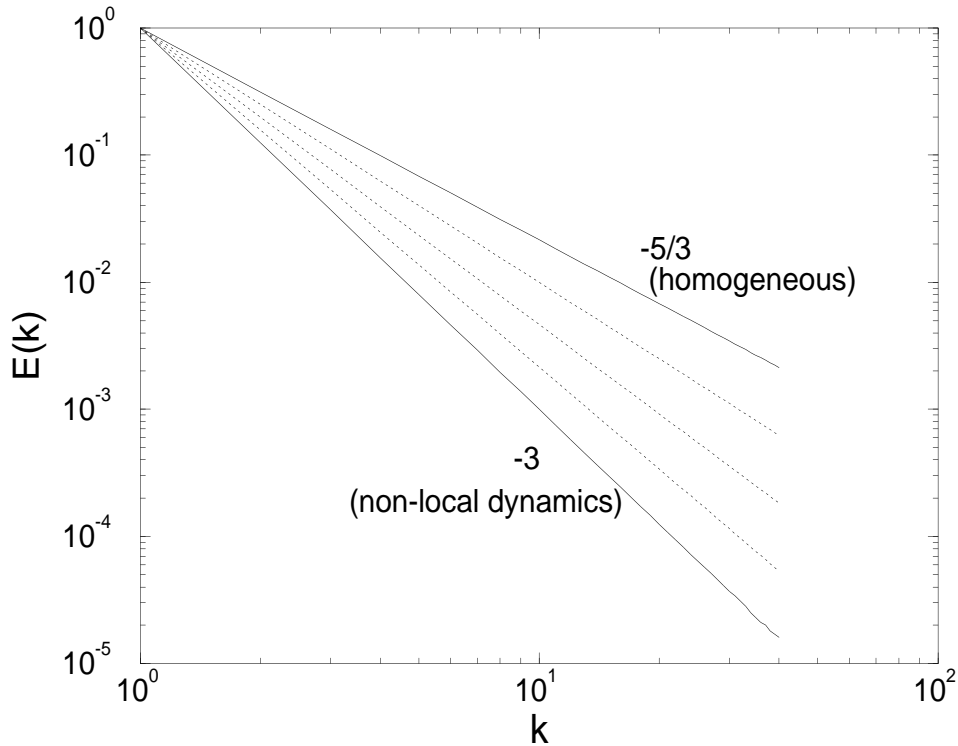


Figure 3.3: The evolution of energy spectrum $E(k)$ from non-local to local dynamics.

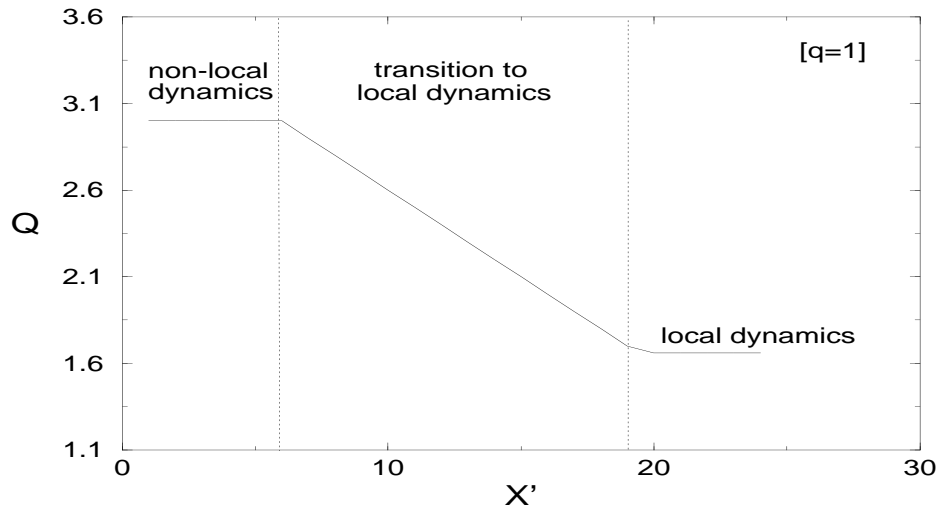


Figure 3.4: The behavior of Q as a function of non-dimensional distance X' for 2D turbulence.

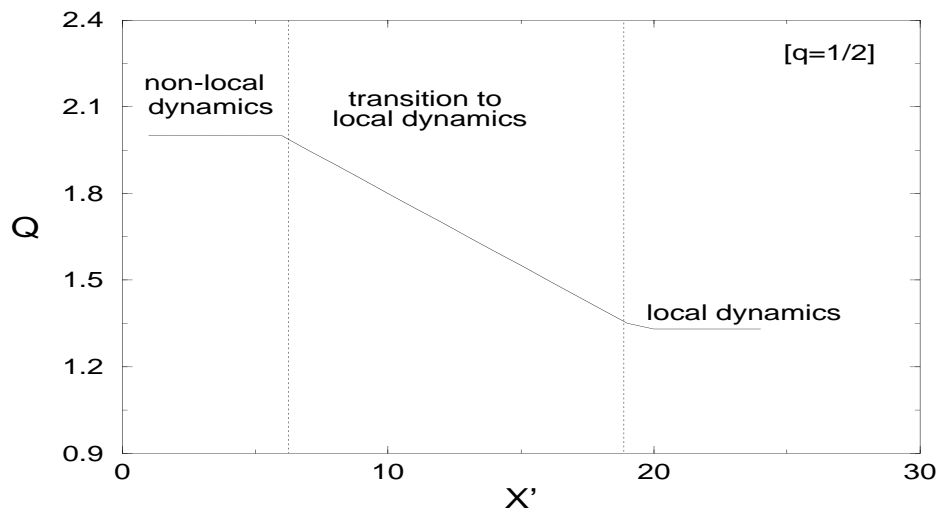


Figure 3.5: The behavior of Q as a function of non-dimensional distance X' for 3D turbulence.

One can conclude that in nonlocal and non-homogeneous turbulence, when ξ_2 deviates from Kolmogorov's $2/3$ prediction and Q deviates from $5/3$ for 2D and from $4/3$ for 3D, we have two cases:

i) $5/3 < Q < 3$ for 2D turbulence and $4/3 < Q < 2$ for 3D turbulence, which characterizes the deviation from locally homogeneous turbulence and from the equilibrium state. This implies that the transversal structure is transferred by the mean longitudinal velocity.

ii) $Q = 3$ for 2D turbulence and $Q = 2$ for 3D turbulence, this situation is called *transfer collapse* (Babiano (2000)), where $\xi_2 = 2$ and the second term $(Q - 3)A$ of relation tends to be a negligible quantity in 2D turbulence and tends to $-A$ in 3D turbulence. This indicates that the transversal energy component is accumulated at the scales at which it is generated. In other words, the transversal energy component is no longer transferred by the non-linear term of the Navier-Stokes equation and the contributions to the global flux σ_ℓ of the terms A and B tends to be inhibited. Consequently, the corresponding energy spectrum should be steeper than $k^{-5/3}$ and saturates with a dependence of k^{-3} .

Figure 3.6: The local energy transfer σ_ℓ^* as a function of time for cylinder wake turbulence (described in chapter 4) at the downstream distances $X/D = 2$ and $X/D = 20$ for different scales ℓ ($\ell_1 = 10\eta$, $\ell_2 = 20\eta$, $\ell_3 = 30\eta$).

Figure 3.6 shows the local energy transfer σ_ℓ^* for cylinder wake turbulence for different scales ℓ , at the downstream distances $X/D = 2$ and $X/D = 20$ from the cylinder. Due to the experimental limitations for transversal velocity, σ_ℓ^* is calculated only using the longitudinal velocity. The energy transfer will be discussed in more detail in chapter 5. The figure illustrates two different anomalies, the non-local dynamics related to the non-uniformity in scale of the energy transfer random field and the intermittency related to the largest but rarest events. The two anomalies are more pronounced at downstream distance $X/D = 2$ near the cylinder than at $X/D = 20$ due to the strong non-homogeneity near the cylinder.

3.6 Transfer hierarchy

Babiano *et al.* (1997) and Babiano (2000) defined the energy transfer hierarchy as:

$$H(p, \ell) = \frac{\langle \sigma_\ell^{p+1} \rangle}{\langle \sigma_\ell^p \rangle}. \quad (3.27)$$

It is bounded by two limits σ_ℓ^0 and σ_ℓ^∞ defined as

$$\sigma_\ell^0 = \lim_{p \rightarrow 0} H(p, \ell), \quad (3.28)$$

$$\sigma_\ell^\infty = \lim_{p \rightarrow \infty} H(p, \ell). \quad (3.29)$$

The quantity σ_ℓ^0 is equivalent to the mean absolute energy flux, while σ_ℓ^∞ characterizes the relative contribution of the most intermittent structures at scale ℓ . The numerical studies performed by Babiano *et al.* (1997) and Babiano (2000) show that the transfer hierarchy (3.27) saturates for a value of p of the order 10 in 2D and 3D numerical turbulence (figure 3.7). From the two limits of the transfer hierarchy we can define the following local scaling exponents:

$$\delta_0 = -\frac{d \ln \sigma_\ell^0}{d \ln \ell} \quad (3.30)$$

$$\delta_\infty = -\frac{d \ln \sigma_\ell^\infty}{d \ln \ell} \quad (3.31)$$

δ_0 and δ_∞ are the local scaling exponents, characterizing respectively the relative contribution of the least or most intermittent fluctuations of σ_ℓ at given scale ℓ . The measurement of δ_0 and δ_∞ provides interesting information about the internal structure and spatial distribution of the structures responsible for the energy transfer. δ_0 is a good indicator of the increase or decrease of the amplitude of the most frequent transfer fluctuations as a function of the length scale. In contrast, δ_∞ characterizes the scaling properties of the structures responsible for the largest but rarest transfer.

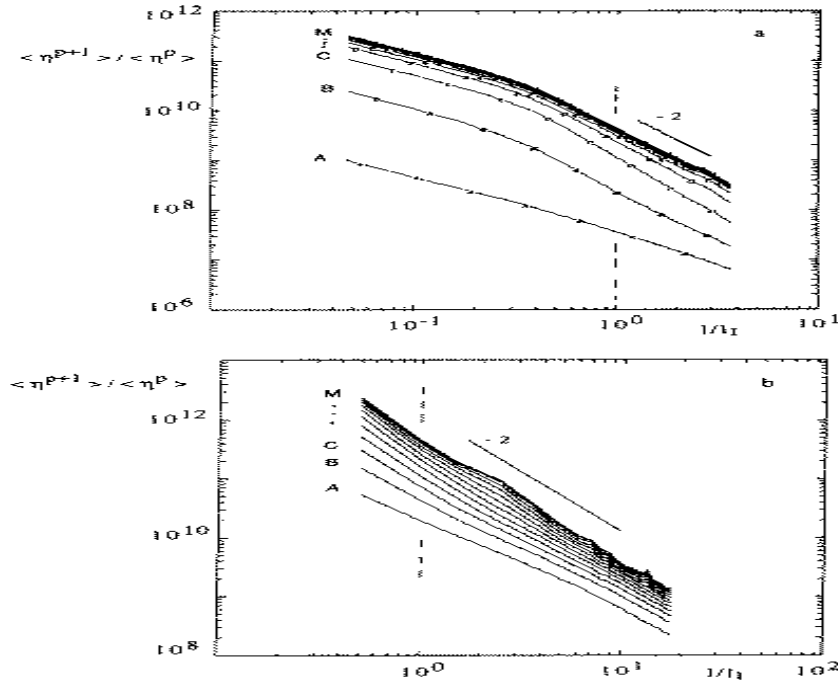


Figure 3.7: The transfer hierarchy as function of non-dimensional scale, (A): $p = 0$, (M): $p = 12$, (Babiano *et al.* (1997)).

It is important to point out that there are four different cases:

i) $\delta_0 = \delta_\infty = 0$; this means that energy transfer is uniform in scale and that the turbulence is non intermittent. This situation corresponds to Kolmogorov's theory K41 (Kolmogorov (1941)).

ii) $\delta_0 = 0$ and $\delta_\infty \neq \delta_0$; i.e. when the energy transfer variance is uniform in scale this corresponds to the homogeneous and intermittent case. This corresponds

to Kolmogorov's theory K62 (Kolmogorov (1962)).

iii) $\delta_0 \neq 0$ and $\delta_\infty = \delta_0$; this is the situation where the turbulence is non-homogeneous, but the degree of non-homogeneity does not increase with p . This is the non-homogeneous but non intermittent case.

iv) $\delta_0 \neq 0$ and $\delta_\infty \neq \delta_0$; this represent the maximal deviation from both K41 and K62 theories, because both non-homogeneity and intermittency prevail.

According to the relation (3.27), for any value of p , one can define the local scaling exponent δ_p as:

$$\delta_p = -\frac{d \ln H(p, \ell)}{d \ln \ell} = -\frac{d \ln \langle \sigma_\ell^{p+1} \rangle - d \ln \langle \sigma_\ell^p \rangle}{d \ln \ell}. \quad (3.32)$$

Assuming that $\delta_\infty > \delta_0$, and that δ_p obeys the following relation

$$\delta_p = \delta_\infty + (\delta_0 - \delta_\infty)h(p), \quad (3.33)$$

where $(\delta_\infty - \delta_0)$ is related to the maximum amplitude of the intermittency phenomenon in the turbulent flow and $h(p)$ is a monotonous decreasing positive function of p smaller or equal to 1. The consistency of relations (3.30), (3.31) and (3.33) requires that $h(p)$ tends to 1 when p tends to 0, and $h(p)$ tends to 0 when p tends to ∞ .

Babiano *et al.* (1997) gave, as a plausible approximation, one of the simplest functions which satisfies the above constraints:

$$h(p) = \exp(-ap), \quad (3.34)$$

where a is the factor characterizing the slope of $h(p)$ at $p = 0$, and is expressed as:

$$a = -h'(0) = \frac{\delta_p'(0)}{\delta_\infty - \delta_0}, \quad (3.35)$$

where $\delta_p'(0)$ is seen to be positive, because δ_p is a monotone increasing function.

In the methodology proposed by Babiano *et al.* (1997), the scaling of $\langle \sigma_\ell^{p+1} \rangle$ is determined by the relation involving $\langle \sigma_\ell^p \rangle$ and $\langle \sigma_\ell^\infty \rangle$. Specifically, the ratio $\frac{\langle \sigma_\ell^{p+1} \rangle}{\langle \sigma_\ell^p \rangle}$ satisfies the relation:

$$\frac{\langle \sigma_\ell^{p+1} \rangle}{\sigma_\ell^\infty \langle \sigma_\ell^p \rangle} \sim \left[\frac{\langle \sigma_\ell^p \rangle}{\sigma_\ell^\infty \langle \sigma_\ell^{p-1} \rangle} \right]^{h(p)/h(p-1)}. \quad (3.36)$$

It is of interest to point out that this equation is compatible with the hierarchy proposed by She and Leveque (1994). It follows from (3.34) and (3.36) that,

$$\frac{h(p)}{h(p-1)} = \beta = \exp(-a), \quad (3.37)$$

where β is a characteristic parameter characterizing the degree of intermittency of energy transfers of the flow. As mentioned in chapter 2, She and Leveque (1994) showed that the parameter β is universal in homogeneous and isotropic turbulence with the value $\beta = 2/3$. In contrast, the numerical investigation made by Babiano *et al.* (1997) showed that the quantity $h(p)/h(p-1) = (\delta_p - \delta_\infty)/(\delta_{p-1} - \delta_\infty)$ depends on p . Therefore, the universality of β in the case of non-homogeneous and non-isotropic flows is not necessary.

From the hierarchy (3.27), the recurrent development for $p = 0, 1, 2, 3, \dots$ leads to the relationship between $\langle \sigma_\ell^p \rangle$ and $\langle \sigma_\ell \rangle^p$ as

$$\frac{\langle \sigma_\ell^p \rangle}{\langle \sigma_\ell \rangle^p} \sim l^{(\delta_\infty - \delta_0)[I(p) - p]}, \quad (3.38)$$

where

$$I(p) = \sum_{q=0}^{p-1} h(q). \quad (3.39)$$

We also consider that the generalized similarity hypothesis is valid in the case of non-homogeneous turbulence.

$$\frac{|\delta u_\ell^3|}{\langle |\delta u_\ell^3| \rangle} \stackrel{\text{law}}{=} \frac{\sigma_\ell}{\langle \sigma_\ell \rangle}, \quad (3.40)$$

where $\stackrel{\text{law}}{=}$ means that the terms have the same scaling properties; i.e., that the moments of the corresponding distribution are everywhere proportional, up to a (moment-dependent) numerical constant. Then, from (3.38) and (3.40) the relative scaling exponents ζ_p/ζ_3 , defined in the framework of the ESS relation (3.1), are given by

$$\zeta_p/\zeta_3 = p/3 + \Delta [I(p/3) - p/3], \quad (3.41)$$

where a new important parameter has been defined as

$$\Delta = \frac{\delta_\infty - \delta_0}{\zeta_3}. \quad (3.42)$$

Assuming the simple choice $h(p) = \exp(-ap)$, relation (3.41) converges to that proposed by She and Leveque (1994), provided that $\zeta_3 = 1$ and $\delta_0 = 0$. The function $I(p/3)$ is then related to the *intermittency factor* β as:

$$I(p/3) = \frac{1 - \beta^{p/3}}{1 - \beta}, \quad (3.43)$$

where $\beta = \exp(-a)$ (Babiano (2000)).

In locally homogeneous turbulence, $\zeta_3 = \xi_3 = 1$ and $(\delta_\infty - \delta_0 = \delta_\infty = 3/2)$; this provides $\Delta = 2/3$, which corresponds to the value proposed by She and Leveque (1994).

When the homogeneity is violated, the universality is recovered for Δ as defined in (3.42). The connection between $(\delta_\infty - \delta_0)$ and ζ_3 in a non-homogeneous case is an explicit consequence of (3.25) and of the average procedure taking the absolute values. Babiano (2000) assumed that δ_0 must be more sensitive to the change than δ_∞ ,

The numerical measurements made by Babiano (2000) showed that even when the localness of the flow dynamics changes in space or time the ESS works well in non-homogeneous flows.

This remarkable property was observed first in numerical 2D non-stationary and non-homogeneous inverse cascade of energy. It can explain the compensation effect which operates in (3.42), and consolidates the scale-invariance of the relative exponents, i.e the *ESS* property in non-homogeneous flow.

The observations, on the one hand, that Δ is scale-invariant and shows a constant universal behavior, as pointed out by Babiano *et al.* (1997) and, on the other hand, that the relative exponents may be spatially dependent in non-homogeneous flow, do not contradict each other.

The scale-invariance of Δ sustains the idea that the *ESS* basically concerns the most important anomalous behavior in the transfer dynamics, whereas $I(p/3)$ relates the residual intermittency, and must be interpreted in terms of a correction linked with largest but rarest anomalous transfers, persisting for a given underlying level of inhomogeneity. We assume that the framework of the theory described above is also valid in complex non-homogeneous three dimensional flows. Therefore, the main goal here is to explore the above physically sound conjectures on the basis of several laboratory experiments reported by Gaudin *et al.* (1998), Mahjoub *et al.* (1998) and Mahjoub *et al.* (2000a).

It is important to emphasize that relation (3.40) basically assumes that the statistical properties of both the energy transfer and the third-order longitudinal velocity structure functions are entirely connected, even in the absence of self-similarity

in the usual Kolmogorov sense. From (3.25) we see that this assumption is valid when the contribution to the energy transfer σ_ℓ coming from both the transversal velocity structure and pressure may be neglected. This happens when the turbulence is locally homogeneous, and in the opposite case, when the turbulence is non-homogeneous but the transfer dynamics is dominated by the non-local properties ($Q \rightarrow 2$ in 3D turbulence and $Q \rightarrow 3$ in 2D turbulence).