Universitat
Autònoma
de Barcelona

Facultat de Ciències

Departament de Física

Accurate predictive model for twisted nematic liquid crystal devices. Application for generating programmable apodizers and Fresnel lenses

Andrés Márquez Ruiz

Memoria presentada para optar al grado de
Doctor en Ciencias Físicas

Bellaterra, Junio 2001

Doña María Josefa Yzuel Giménez, Catedrática de Óptica de la Universidad Autónoma de Barcelona, y Don Juan Campos Coloma, Profesor Titular de Óptica de la Universidad Autónoma de Barcelona,

CERTIFICAN

que Don Andrés Márquez Ruiz, licenciado en Ciencias Físicas, ha realizado bajo su dirección y en el Departamento de Física de la Universidad Autónoma de Barcelona, el trabajo *"Accurate predictive model for twisted nematic liquid crystal devices. Application for generating programmable apodizers and Fresnel lenses"*, que se recoge en esta memoria para optar al grado de Doctor en Ciencias Físicas.

Y para que conste, de acuerdo con la legislación vigente, firman este certificado en Bellaterra, 21 de junio de 2001.

Prof. M. J. Yzuel Prof. J. Campos

A mis padres Andrés y Carmen
que se lo merecen todo

Agradecimientos / Acknowledgments

Muchas personas con sus ideas, enseñanzas y apoyos de todo tipo han contribuido a que mis primeros balbuceos en el terreno de la investigación hayan dado lugar a esta tesis doctoral. Intentaré no dejarme a nadie en el tintero:

A la Dra. María Josefa Yzuel Giménez y al Dr. Juan Campos Coloma por su excelente labor de dirección y por su impagable dedicación a lo largo de estos intensos años.

A todos los coautores de los artículos que forman la base de esta tesis, Dr. Jeffrey A. Davis, Dr. Ignacio Moreno, Dr. Juan Carlos Escalera, Dr. Claudio Iemmi, Dra. Silvia Ledesma, Alfonso Moreno, y Arnau Robert. Por lo mucho que me han enseñado y por los buenos momentos que he pasado con ellos.

A todos los miembros del Grupo de Óptica por el buen ambiente de trabajo: en especial a los compañeros del laboratorio Dr. Asticio Vargas, Dr. Abdelaziz Attaleb, Octavi López, Quim Barbé, Josep Nicolás y Carlos Cazorla.

Al Dr. Augusto Beléndez y a la Dra. Inmaculada Pascual por orientar mi rumbo hacia Alicante.

A todos los miembros del Departamento de Física, Ingeniería de Sistemas y Teoría de la Señal, de la Escuela Politécnica Superior de la Universidad de Alicante por su amable acogida.

A Celia y Cristina por soportarme en los pocos cafés compartidos en las últimas fechas. A Roque y Dani por acogerme en su seno y por su buen humor.

A Celia Martínez, María Teresa Castro, Rafael Moraira y Javier Hellín por darme el soporte necesario en las tareas administrativas y técnicas.

A Adolfo, Javi, Roberto, José, Carlos, y respectivas, por decir tantas tonterías y por su enorme cultura.

A mi hermana Loli y a Tomás por darme su apoyo y cariño incondicional.

A Niloo por corregir el inglés de esta memoria; sobre todo por compartir su felicidad conmigo.

This work has been financed by Dirección General de Enseñanza Superior del Ministerio de Educación y Cultura (Spain), project PB96-1134-C02-01, and by Ministerio de Ciencia y Tecnología (Spain), project BFM2000-0036-C02-01.

A. Márquez thanks the Comissionat per a Universitats i Recerca de la Generalitat de Catalunya for the grant 1997FI00470 in the program FI-FIAP/DGR from the 1rst of January, 1997 to the 31rst of January, 2000.

List of the papers of this Ph.D. thesis

The work performed in this thesis has been extracted for publication in a number of international journals in the field of Optics. A considerable percentage of the results of the thesis have already been published. This fact has been the main reason to decide to present the thesis as a compilation of papers. Along the text of this thesis we provide the interconnection between the thesis work and the work presented in the different papers. Furthermore, we intend to clarify the coherence and the unity existing between the different papers. In this text we have also decided to expand the explanations and the results presented in the papers in order to provide a deeper and wider insight that is not possible in the short extension of a paper.

This thesis is based on the following papers:

[PAPER A] *A. Márquez, J. Campos, M.J. Yzuel, I. Moreno, J. A. Davis, C. Iemmi, A. Moreno, and A. Robert, "Characterization of edge effects in twisted nematic liquid crystal displays", Opt. Eng.* ***39****, 3301-3307 (2000).*

In this paper we present a model to more accurately describe the optical properties of a twisted nematic liquid crystal display (LCD). In particular, we study the optical properties of molecules near either edge of the LCD that are unable to twist and tilt under application of an external electric field. The properties of these layers can be deduced from the intensity transmittances measured using different configurations of the external linear polarizers. The agreement between the theoretical and the experimental transmittances is excellent, thus permitting the prediction of the actual modulation for any configuration of the polarizers.

[PAPER B] *A. Márquez, C. Iemmi, I. Moreno, J. A. Davis, J. Campos, and M. J. Yzuel,"Quantitative prediction of the modulation behavior of twisted nematic liquid crystal displays based on a simple physical model", accepted for publication in Optical Engineering.*

A new method to perform a predictive search for a given amplitude and phase modulation response in twisted nematic liquid crystal displays is presented. The algorithm is based on a physical model that we recently proposed and that considers the effect of liquid crystal layers located in the vicinity of the edges, which are not capable to tilt. This model was demonstrated to explain accurately the experimental transmittance modulation curves. Here the model is applied to perform a predictive search for an optimized modulation by changing the input and output polarization configuration. A generalised configuration to generate and detect elliptically polarized light is proposed. The method is applied for searching two different configurations useful for optical image processors: phase-only modulation and amplitude-only modulation. The excellent agreement with the experimental measurements validates the potentiality of the proposed method.

[PAPER C] *J. A. Davis, J. C. Escalera, J. Campos, A. Márquez, M. J. Yzuel, and C. Iemmi, "Programmable axial apodizing and hyperresolving amplitude filters using a liquid crystal spatial light modulator", Opt. Lett.* ***24****, 628-630 (1999).*

Amplitude transmitting filters for apodizing and hyperresolving applications can be easily implemented using a two dimensional programmable liquid crystal spatial light modulator operating in a transmission-only mode. Experimental results are in excellent agreement with theoretical predictions. This approach can allow the analysis of different filter designs and can allow the filters to be rapidly changed for modifying the response of an optical system.

[PAPER D] *M. J. Yzuel, J. Campos, A. Márquez, J. C. Escalera, J. A. Davis, C. Iemmi, and S. Ledesma, "Inherent apodization of lenses encoded on liquid crystal spatial light modulators", Appl. Opt.* **39***, 6034-6039 (2000).*

Diffractive optical elements encoded in liquid crystal spatial light modulators present an inherent apodizing of the focused spot due to the pixelated nature of these devices and the finite extent of each pixel. In this paper we present a theoretical explanation and experimental evidence for this effect. We also show an experimental procedure to measure the apodization and a method to compensate for this effect.

[PAPER E] *A. Márquez, C. Iemmi, J. C. Escalera, J. Campos, S. Ledesma, J. A. Davis, and M. J. Yzuel, "Amplitude apodizers encoded onto Fresnel lenses implemented on a phase-only spatial light modulator", Appl. Opt.* ***40****, 2316-2322 (2001).*

In this paper we show that both a lens and a non-uniform amplitude transmission filter can be simultaneously encoded onto a twisted nematic liquid crystal spatial light modulator (SLM) working in the phase-only mode. The inherent apodization due to the pixelated structure of the SLM is compensated. In addition, different types of non-uniform transmission pupils like transverse apodizing, transverse hyperresolving and axial hyperresolving (multifocusing) filters are implemented. The excellent agreement between numerical and experimental results shows the capability of this method.

CONTENTS

List of the papers of this Ph.D. thesis

Contents

Papers of this Ph.D. thesis

1 INTRODUCTION

1.1 Spatial light modulators and diffractive optics

This thesis mainly deals with two research topics: spatial light modulators and diffractive optics. The first one, spatial light modulators is by now a well established technology that generates a very important volume of business every year. It is a highly interdisciplinary field where materials science, optics, organic chemistry and electronics have combined to produce highly sophisticated devices that are the critical component of a variety of technologies. Diffractive optics on the other hand is a field whose scientific foundations are very well established. It has been in the recent years that diffractive optics has become an emerging technology, mature enough to pass from the research labs to the industrial environment. The research activity is very intensive in both lines, spatial light modulators and diffractive optics, in order to make possible more demanding devices and applications.

As its name implies, spatial light modulators (SLMs) are optical modulators constructed so as to spatially modulate, according to a prescribed input, a readout optical beam [Efr95,Ser99]. Although the output of the device is by definition optical, the input signal may be introduced either optically or electrically. The spatial modulation may be one-, two- or even three-dimensional. The most extended application is as a two-dimensional display for visualization purposes. The SLMs play also a critical role in optical data processing when real time operation is needed. In communication technologies SLMs provide a means to switch between multiple channels very effectively. In adaptive optics they are required as an active element to compensate for the aberrations induced on the wave front by the atmosphere. In active diffractive optics SLMs are programmable high-resolution devices that emulate mechanically moving optical elements to steer a beam, or to shape the profile of a laser beam. In holographic data storage they are the elements where the data page is displayed to modulate the object beam incident on the recording material.

Two important parameters characterize the SLMs operation [Efr95]: the spatial resolution and the frame rate. In general, the fast SLMs have a low spatial resolution, whereas the SLMs with a high resolution are relatively slow. Display application requires a relatively large resolution (>1000 x 1000 picture elements or pixels) with a relatively low (60 Hz) frame rate need. Adaptive optics application requires only a modest spatial resolution (≈100 x 100 elements) but imposes more demanding frame rates (>1 KHz). Optical data processing applications require both high resolution (1000 x 1000 elements) and ultrafast frame rate with a goal of over 1 MHz. Although SLMs in general rely on a variety of materials, devices, and processing technologies, there are two critical ones, namely, optical modulating materials and driving circuitry, which are the main focus of the developing efforts.

Different SLM technologies exist depending on the material and on the physical effect on which they are based [Efr95]:

- Liquid crystal SLMs (LCSLMs). Several physical principles for modulating light can be applied. Among them we are interested in a particular electro-optic effect, the so-called field-induced director axis reorientation: the liquid crystal is a uniaxial anisotropic material, in which the orientation of the molecules varies with the applied electric field.

- Magneto-optic SLMs (MOSLMs). They take profit of the Faraday effect, that produces the rotation of the polarization plane of the light depending on the magnetic field applied.

- Deformable mirror devices (DMDs). Each pixel is a mirror whose orientation can be electrostatically changed, then reflecting the beam at a different angle. There are also membrane deformable mirrors with no pixelation.

- Multiple-quantum-well (MQW) SLMs. They take advantage of certain quantum-mechanical effects, in particular new absorption peaks, associated with the fabrication of optoelectronic devices consisting of a large number of extremely thin layers (≈10 nm) of different semiconductor materials. These absorption peaks depend on the applied voltage.

- Electro-optic SLMs (EOSLMs). They are based on the Pockels effect or on the Kerr effect: the index of refraction of the material changes with the electric field (Pockels effect), or with the square of the electric field (Kerr effect).

- Acousto-optic SLMs (AOSLMs). They benefit from the elasto-optic effect, in which the index of refraction of the material is modulated with acoustic waves.

In this thesis the research is focused on the liquid crystal spatial light modulators (LCSLMs). Nowadays LCSLMs are substituting the traditional cathode ray tube (CRT) displays and they dominate the flat panel display (FPD) market [Chi99]. Their massive use in the display industry, where they are the well known liquid crystal displays (LCDs), makes possible to obtain low cost LCDs of a very high quality that can be applied for other SLM purposes.

On the other hand, diffractive optics has also taken its own place among the daily technological artifacts that surround us. As Jari Turunen and Frank Wyrowski comment [Tur97]*“diffraction, a fundamental property of wave motion, has had astonishingly few important technical applications in optics until recently”*. As they say *“one had to wait for the relatively recent emergence of high-performance computing facilities and microlithographic fabrication technology to fully realize the promise of diffractive optics. These enabling* *technologies have led to an explosion of applications of diffraction in optics, simultaneously establishing diffractive optics as one of the most rapidly advancing areas of current research in optical engineering*". It is in the present moment that many of the possibilities that A. W. Lohmann envisaged in the mid-1960's, when he produced the first computer-generated hologram [Loh67], have become a practical reality with industrial applications [Hep@,RPC@].

The optical elements whose performance is based on the diffraction properties of light propagation are the so-called diffractive optical elements (DOEs). They provide certain advantages with respect to their conventional counterparts, based on the refraction and reflection properties of light, for certain applications [Goo96,Tur97]. Diffractive optical elements generally have much less weight and occupy less volume than their refractive or reflective counterparts. They may also be less expensive to manufacture and in some cases may have superior optical performance (e.g. a wider field of view). Examples of applications of such components include optical heads for compact disks, beam shaping for lasers, grating beamsplitters, optical interconnects and reference elements in interferometric testing. In certain applications DOEs can be made to perform functions that would be difficult or impossible to achieve with more conventional optics (e.g., a single diffractive optical element can have many different focal points simultaneously, an optical correlation filter can act simultaneously as a convergent lens). At the same time it is possible to combine in the same set-up DOEs and conventional optical elements.

There are a number of technologies to produce DOEs. On one side, holographic techniques are possible [Sto91], which give rise to the holographic optical elements (HOEs) [Pas92]. Other technologies are based in the registration of the pattern calculated by means of computer-generated holography [Dal80], e.g. to produce optical interconnects [Fim94]. The so-called computer-generated holograms (CGHs) can be registered by means of a number of different technologies such as diamond turning, electron beam lithography, photolitography and laser beam writing. A complete review of the different techniques and the materials onto which CGHs are manufactured is done by Herzig [Her96]. Low cost techniques are also possible, such as the use of high resolution graphic devices [Mor95a,Pas00,Mar00]. The techniques mentioned generate static, non-reconfigurable DOEs. For certain applications we need the production of programmable DOEs, i.e. reconfigurable, as in active diffractive optics [McM93,Lau98,Ser99], in real time optical processing [Liu85,Jav86,Jav94, Ama90], or in adaptive optics [Kud97,Lov97]. The programmable DOEs are obtained by displaying the CGH on a SLM. The reconfigurable properties of the SLM are then exploited and the displayed DOEs can be changed simply by addressing a new CGH to the SLM. For example, in a zoom objective the focal length of the system is changed by mechanical movement. A lens displayed on a SLM can have its focal length changed without any mechanical movement. In imaging applications, apart from the lenses there are other elements, the non-uniform transmission filters, which are especially significant. These amplitude filters, also referred as apodizers, are inserted in an optical system to modify the point-spread function (PSF) (resolution, depth of focus,…). The programmability of these filters would constitute a very attractive property for imaging systems.

1.2 Liquid crystals for spatial modulation of light

1.2.1 Physics of liquid crystal devices

In this thesis we work with liquid crystal SLMs (LCSLMs). Actually, we use a particular class of LCSLMs: the so-called twisted nematic LCSLMs (TN-LCSLMs). Among the different SLM technologies the TN-LCSLM is the dominating one in the display industry [Chi99]. Its ready availability together with its good cost-quality relation have made the TN-LCSLM a common device in research activities where spatial modulation of a light beam is required. We mention their use in optical signal processing [Liu85,Gre86,Cam00b], in diffractive optics [Tam92a,Lau98], or in adaptive optics [Dou95].

A liquid crystal spatial light modulator consists of a very thin (≈ 2-10 μm) liquid crystal layer sandwiched between two parallel glass plates [Wu95,Kel01]. Transparent electrodes are patterned onto the inner surface of the glass plates to form the pixelated structure of the LCSLM. The LCSLM is made up of thousands of pixel elements, which can be considered at its turn as individual liquid crystal cells (LC cells) where a voltage is applied. The core element in LCSLMs is the liquid crystal material itself, whose electrooptical properties are employed. The LC layer is normally filled with nematic LC compounds. The term nematic designates one of the various phases in which we can find the liquid crystalline state.

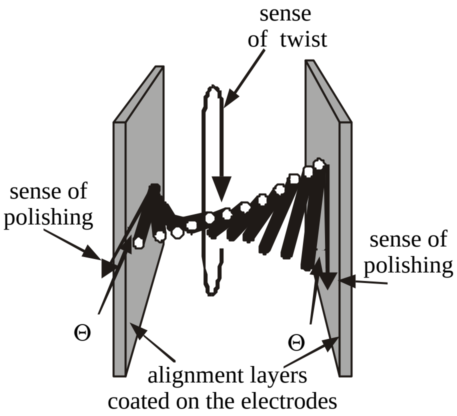

Figure 1.1. Scheme for the twisted nematic cell. The sense and orientation of the polishing of the alignment layer fixes the orientation of the anchoring of the LC molecules along the surface and the magnitude of the pretilt angle Q.

The usual working configuration is the so-called twisted nematic cell (TN-cell). The TN-cell has been since its origin in 1971 [Sch71] the working configuration that has focused more interest for LCSLM operation. We show in Figure 1.1 a scheme for the TN-cell where we can see the twisting of the nematic molecules from one glass plate to the other. In TN-cells the twist angle a from one surface to the other is in general about 90 degrees. We mention that higher values for the twist angle, between 180-270 degrees, are used in another working mode, the so-called supertwisted nematic (STN) [Yeh99,Kel01]. Alignment layers coated onto the electrodes are responsible for the anchoring of the LC molecules at a specific orientation at the surfaces. This layer undergoes a polishing process in a specific sense that both helps align the LC molecules along the surface and also establish a small angle of about 1-3 degrees, the pretilt angle Θ, with respect to the plane of the glass plate.

The nematic material is made up of rodlike molecules with no positional order and whose long axes tend to point in a specific direction, the so-called molecular director axis $\vec{n}$. The orientation of the director $\vec{n}$ across the TN-cell is specified by two angles, the so-called twist χ and tilt θ angles, which we illustrate in Figure 1.2. The LC molecules in the TN-cell twist around the direction perpendicular to the alignment layers, that is called the twist axis. We consider the Z axis of our reference frame parallel to the twist axis. At a given plane perpendicular to the Z axis, all the LC molecules have the same orientation. Hence, we can treat the distribution of the molecular director across the cell as a one dimensional problem: $\chi = \chi(z)$ and $\theta = \theta(z)$.

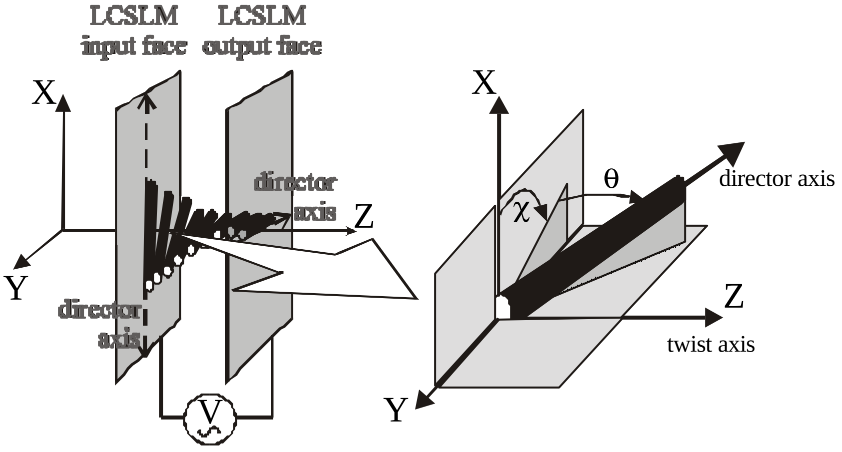

Figure 1.2. Twist χ and tilt θ angles of the molecular director $\vec{n}$.

A nematic LC material is an anisotropic medium which locally can be regarded as a uniaxial crystal whose optical axis is parallel to the molecular director $\vec{n}$. Electrically, the nematic LC is also a uniaxial anisotropic material whose symmetry axis is coincident with the director $\vec{n}$. Hence, the values for the dielectric permittivity ε and for the refractive index n are different when measured parallel ($\varepsilon_{//}, n_{//}$) or perpendicular ($\varepsilon_{\perp}, n_{\perp}$) to the director $\vec{n}$. The nematic materials used to fill the TN-cell have a positive dielectric anisotropy $\Delta\varepsilon$ and a positive difference of index Δn, i.e.,

$$\Delta\varepsilon = \varepsilon_{//} - \varepsilon_{\perp} > 0 \qquad (1\text{-}1)$$

$$\Delta n = n_{//} - n_{\perp} > 0 \qquad (1\text{-}2)$$

Next, we describe the field-induced director axis reorientation effect, which is the electrooptic effect used for TN-LCSLM operation [Wu95,Kel01]. The nematic LCs used in the TN-LCSLMs are nonconductive and nonpolar molecules. Under the application of an electric field along the Z axis an induced dipole moment appears along the long axis of the LC molecules that tends to align the director $\vec{n}$ with the Z axis. Therefore the optic axis, which is coincident with the director $\vec{n}$, is reoriented. The ordinary refractive index n_o is a constant, equal to $n_{\perp}$, independent of the propagation direction. The extraordinary refractive index n_e depends on the angle between the optical axis and the propagation direction of the light. For light propagating along the Z axis the extraordinary index n_e varies according to

$$\frac{1}{n_e^2(\theta)} = \frac{\cos^2\theta}{n_{//}^2} + \frac{\sin^2\theta}{n_{\perp}^2} \qquad (1\text{-}3)$$

where θ is the tilt angle expressed in Figure 1.2. Consequently, the difference of index in the medium depends on the tilt angle $\Delta n(\theta)$. For most LCs, the ordinary refractive index is about 1.5 and Δn is very large, ranging from 0.05 to 0.45.

A related parameter that is normally used to characterize the LC-cell is the birefringence β that is defined as

$$\beta = \frac{\pi}{\lambda}\int_{z=0}^{z=d} \Delta n(z)\,dz \qquad (1\text{-}4)$$

where d is the cell thickness, also called cell gap. λ is the wavelength of the incident light. If the difference of refractive index Δn is constant across the cell depth z the previous expression simplifies into

$$\beta = \frac{\pi d \Delta n}{\lambda} \tag{1-5}$$

We note that when talking about birefringence β we use the notation typically found in the literature related with LCDs and LCSLMs. From Eq. (1-4) the birefringence is defined as half of the retardance generated along a certain trajectory between the extraordinary and the ordinary rays. This is not the definition normally used in Optics, where birefringence is equivalent to the maximum value for the difference of index Δn in a uniaxial anisotropic material.

To analyse the field-induced axis reorientation mechanisms we have to take into account the mechanic properties, i.e. elastic and viscous behavior, of the LC media [Kel01]. The static director distribution at equilibrium is dependent on the three Frank elastic constants, k_{11}, k_{22} and k_{33} , which are associated with splay, twist, and bend deformations, respectively, shown in Figure 1.3. On the other hand, the dynamics of the LCs in response to a distorting force depends both on the elastic behavior and on the flow viscosity.

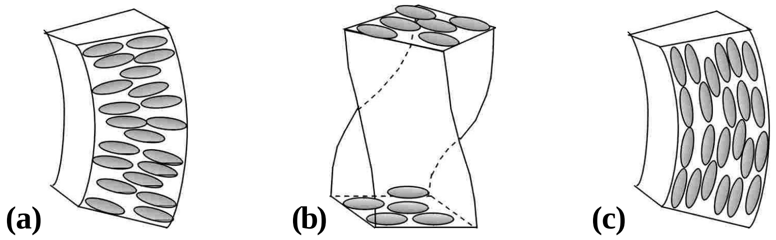

Figure 1.3. Schematic representation of the Frank elastic constants k_{11}, k_{22} and k_{33} for (a) splay, (b) twist and (c) bend in nematic LCs.

To get a deeper insight into LC compounds, working modes and devices, we recommend references [Jac92,Chi99,Yeh99,Kel01]. It is also interesting to take a look at the websites [LCH@,Plc@]. For an introduction in spanish we refer to [Oto90].

1.2.2 Theoretical treatment

The theoretical treatment normally used to study the liquid crystalline state is the continuum theory developed by Oseen [Ose33]. Frank added the elasticity theory to calculate the strains of molecular orientation [Fra58]. Finally, Leslie [Les68] and Ericksen [Eri61] formulated the laws that permit the theoretical study of the dynamics and time responses for LC devices. A lot of work has been done in the past thirty years on the understanding and modeling of the internal structures of the LC materials. Berreman [Ber74,Ber75] and van Doorn [Doo75] provided calculations for the dynamics response of the TN-cell. Chen *et al.* [Che97] presented a one-dimensional simulation methodology, based on the Frank-Oseen theory, for determining the director distribution in a LC-cell for arbitrary pretilt angles. For an electric field applied, it is important to distinguish if the director reorientation is produced under constant voltage or under constant charge conditions [Thu81]. Yeh [Yeh99] provides the equations and the methodology in order to calculate the static director distribution at equilibrium for a number of different LC-cell geometries, as the TN-cell, the supertwisted nematic cell (STN-cell), the electrically controlled birefringence cell (ECB-cell), and others. In general, the calculation of the director profile has to be done using numerical methods. No analytical function is obtained except in very specific situations.

For the TN-cell the numerical calculation shows [Ber73] that the twist and the tilt angles vary with the cell depth z and with the voltage, thus $c=c(z,V)$ and $q=q(z,V)$. To calculate the director profile we need to know the details about the LC compound used in the LC-cell and some other parameters, as the cell gap, that are not provided by the device manufacturer. Thus, Taber [Tab90] considered analytical functions that reproduce in a qualitative way the realistic variations of the twist c and the tilt q angles shown in [Ber73]. Lu and Saleh [Lu90] presented another approach where they approximated the realistic director distribution in the field-on state with a simplified profile, explained in Section 2.1.2.2. This simplified model was used to calculate the analytical expressions for the Jones matrix of the transmissive [Lu90] and the reflective TN-LCSLMs [Lu91a,Lu91b]. The Lu and Saleh model has been extensively used by a number of authors [Ohk93,Sou94b,Gon95,Mor98b,Zhi98] because it is simple and we do not need to know the details about the nematic compound and the TN-cell. Nevertheless, the approximations made to obtain the model are coarse and the model is just able to provide a qualitative estimation. Coy *et al.* [Coy96] proposed a better approximation, that we explain in Section 2.1.2.2, to the realistic director distribution. However, their model is still not able to provide a high degree of agreement between the predicted modulations and the experimental measurements for the TN-cell, as we see in Section 2.2.5 and in [PAPER A]. We have developed a new model [PAPER A] able to provide highly accurate predictions for the modulations.

Optically, the TN-cell is an inhomogeneous uniaxial anisotropic material. To calculate the light propagation across the TN-cell we have to employ the profiles $c = c(z,V)$ and $q = q(z,V)$ and solve the Maxwell equations applied to an inhomogeneous anisotropic medium. Normally the LC-cell is sandwiched between two polarizers. If wide viewing angle properties are required the LC-cell can also be sandwiched between layers of birefringent material, the so-called compensating layers [Yeh99]. Both polarizers and compensating layers are added to the calculation of the light propagation.

In principle, we are interested in calculating the polarization state of the light at the output of the cell. Other interesting parameters are the intensity and/or the phase of the light at the output. Different formalisms can be applied. The classical method is the 4x4 matrix method proposed by Berreman [Ber72,Ber73]. Numerical integration of the so-called Berreman differential equation has to be done. The 4x4 matrix method is a rigorous application of the Maxwell equations. Multiple reflections, normal incidence and oblique incidence are included in the calculations. Thus, the results are totally correct, however, the computational cost is high and simpler methods can be applied that may also provide the required accuracy. Azzam and Bashara [Azz72] found that the evolution of the ellipse of polarization of light passing through an anisotropic medium can be described by a first order, second-degree, ordinary differential equation, the so-called Ricatti's equation. This approach was used by Gooch and Tarry [Goo75] to obtain the first analytical expression for the intensity transmission of a TN-cell in the field-off state. They considered the particular case of transmission axes of the input and output polarizers oriented respectively along the director axis at the input and output faces of the TN-cell. An approximate expression for the field-off state valid for arbitrary orientation of the transmission axes of the input and the output polarizers was calculated through a different approach by Gringberg and Jacobson [Gri76]. The Ricatti's differential equation is applied in the generalized geometrical-optics approximation (GGOA) developed by Ong [Ong88]. Ong calculated the analytical expressions for the complex amplitude in the field-off state for a wide range of twisted layer configurations [Ong88].

A simpler approach is to consider the LC-cell divided in a stack of thin uniaxial crystal layers, possessing a uniform optical axis direction within each sublayer. The direction of the optical axis changes by a small angle between adjacent layers, thus following the total director distribution in the cell. Each layer is represented by a Jones matrix, so that the Jones matrix for the system is found by multiplying all the matrices of the layers, together with the Jones matrices of the polarizers and/or compensating layers. This procedure was used by Taber [Tab90]. Yang included the effect of the multiple internal reflections [Yan00]. The Jones matrix formalism is a simple and powerful technique but it is only valid to calculate the propagation of light under normal incidence. However, a generalised formalism, the extended Jones matrix method [Lie90], is also available when we study oblique incident light.

As it was mentioned by Wu [Wu95] *"extensive efforts have been devoted to obtaining the analytical expressions and then optimizing the performance of various display modes"*. Analytic expressions are especially interesting in order to analyse the effect of the different parameters on the optical transmission [Ong88,Mor98b]. If we consider the realistic distribution of the director in the field-on state, the different methods discussed are not able to provide analytical expressions for the optical transmission of the cell. However, the simplified models [Lu90,Coy96] permit to obtain analytic expressions. The expressions provided until now by the simplified models are good to obtain a qualitative estimation, but they are not able to provide a high degree of accuracy for the optical transmission of the TN-cell [Ohk93,Mor98b,PAPER A]. The new model we have developed has both features: it is based on analytical expressions and it provides a high degree of accuracy [PAPER A].

1.2.3 Complex amplitude modulation capability

As we have commented, the available LCSLMs are normally the LCDs *cannibalised* from videoprojectors or pocket TVs. The suitability of the LCDs for SLM applications different from the display application has been analysed by several authors [Bor88,Liu89,Kir92,Ohk93] in terms of spatial resolution, time resolution, optical flatness. The LCDs are designed to produce intensity modulation. Nevertheless, in SLM applications with coherent light the important magnitudes are amplitude and phase modulations. Characterization of the amplitude modulation is obtained through measurements of intensity transmission. Different methods have been proposed to obtain accurate phase measurements. Mach-Zehnder [Kir92,Mor95b,Yam00], Michelson [Yam95b], and common-path [Mar01] interferometers have been used. Diffraction techniques are also possible [Zha94]. López *et al.* [Lop98] proposed an in situ characterization technique for LCDs in a real time correlator. In this thesis we have used an interferometric method based on the interference produced by two point sources on the LCSLM [Sou94a,Ber95]. This is a robust technique, easy to implement.

Both theoretically and experimentally, it is found that the amplitude and the phase modulations in TN-LCSLMs are coupled [Kon88,Lu90,Bou97]. In this sense, different authors [Gre92,Gon96] have proposed optical architectures with two TN-LCSLMs to control amplitude and phase modulation independently. In practice, this solution is difficult to implement. Other authors have studied the optimal implementation of complex valued correlation filters in a restricted coding domain [Jud93,Lau94]. The particular case of phase-only filters has been analysed by Moreno *et al.* [Mor95b] and by Labastida *et al.* [Lab00].

In Figure 1.4 we show some typical complex amplitude modulation domains. Ideally, we would need a SLM able to provide full complex amplitude modulation, illustrated in Figure 1.4(a). Actually, the different types of SLMs provide the restricted coding domains shown from Figure 1.4(b) to Figure 1.4(h). We illustrate the coupled amplitude-phase transmittance, typical of TN-LCSLMs, in Figure 1.4(h). Generally, we are interested in using one of the next specific modulation curves: amplitude-only modulation, Figure 1.4(b) or phase-only modulation, Figure 1.4(c). Usually, amplitude-only modulation is required by the scene in a correlator set-up, or by absorbing DOEs. There are several applications where phase-only modulation is required, e.g. phase-only DOEs [Dav89a] or phase-only correlation [Led98]. In principle, phase-only modulation enhances the energetic efficiency of the system.

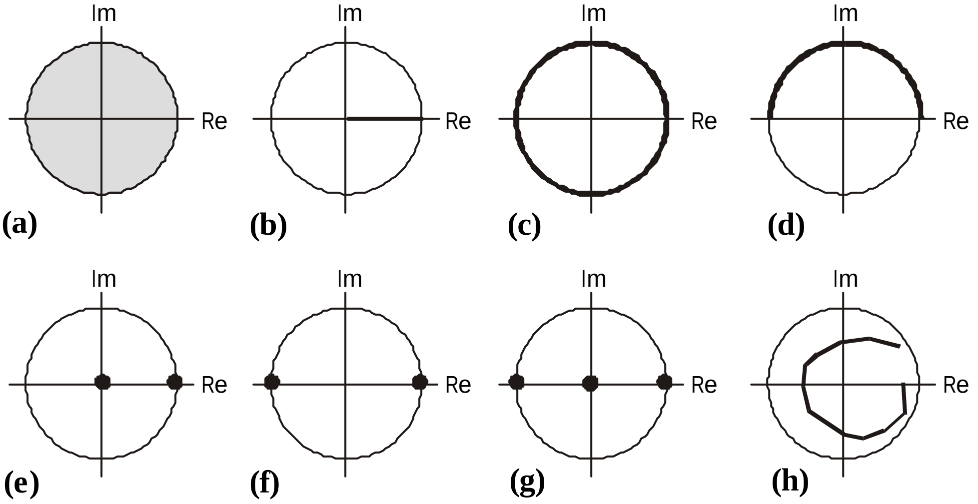

Figure 1.4. Typical complex amplitude modulation domains. (a) Ideal full range of modulation. (b) Amplitude-only. (c) Phase-only modulation with 2π depth. (d) Phase-only modulation with π depth. (e) Binary amplitude. (f) Binary phase. (g) Ternary. (h) Coupled amplitude-phase.

Several authors [Kon88,Bar89,Lu90] have studied under which conditions the TN-LCSLM is able to produce an amplitude-mostly modulation or a phase-mostly modulation. In principle, thick TN-LCSLMs operated at low voltage regime produce a phase-mostly regime [Kon88,Bar89]. The TN-LCSLM has to be inserted between two polarizers with their transmission axes parallel to the orientation of the molecular director at the input and the output faces respectively. Lu and Saleh [Lu90] determined operating conditions of voltage and orientation of the polarizers to obtain either amplitude-mostly or phase-mostly modulations. In all these cases, the TN-LCSLM is thick enough so that the twist angle a is smaller than the birefringence b at low voltages. This situation is called the Mauguin condition or adiabatic following. Under this condition incident light linearly polarized parallel to the director in the input face follows the helix described by the director axis and leaves the cell with a very low degree of ellipticity. The LC directors tilt collectively (but reserve the twist configuration) along the field direction (Z axis). Thus, the output beam is modulated in phase only, but not in amplitude. However, as voltage further increases, the twist continuity of the LC directors is interrupted and its polarization guiding effect is broken.

We note that the polarization guiding effect has been the main reason for the interest in TN-LCSLMs for display purposes because it provides a way to produce a good intensity contrast independent of the wavelength used and independent of thickness errors in the manufacturing process of the TN-LCSLM. Nevertheless, the need for LCDs with high resolution has caused the evolution to produce thinner LCDs. Thin LCDs have a faster time response, that permits a more effective addressing of the large number of pixels. However, the dynamic phase range is reduced due to the smaller cell thickness. Moreover, the Mauguin condition is no longer fulfilled in thin LCDs. New strategies for amplitude-only and phase-only modulation need to be found. In this sense, Pezzanitti and Chipman [Pez93] demonstrated the possibility of phase-only modulation by means of the average eigenvectors of the LCSLM. Davis *et al.* [Dav98] developed the theory for these eigenvectors and proposed the average eigenvectors in order to obtain phase-only modulation with thin LCSLMs. Several authors [Mor98a,Yam00,Mar01] have performed experimental measurements of the average eigenvectors. Following a different approach, Yamauchi and Eiju [Yam95a] performed a computer search to look for the orientation of the transmission axes of the polarizers that provides the best phase-only operation. They had to calculate, for varying voltages, the matrix elements for the Jones matrix of the TN-LCSLM. They considered the Lu and Saleh model in their calculations. Kelly and Munch [Kel98] studied the wavelength dependence of the phase modulation range in order to increase the phase modulation capability of thin TN-LCSLMs. Despite of all these efforts, none of the strategies have demonstrated the possibility to provide a phase modulation with 2π radians dynamic range, and with a low value of coupled amplitude modulation. Furthermore, no work has been done to look for amplitude-only modulation in the case of thin TN-LCSLMs. Both possibilities, amplitude-only and phase-only modulations with a thin TN-LCSLM have been demonstrated by us in [PAPER B].

1.3 Focusing elements and non-uniform pupils

1.3.1 Image formation

The image given by an optical system is not perfect even in the case when no aberrations are present. This is due to the diffraction produced by the finite extent of the pupil of the system. One of the first works dealing with this subject was done by Airy [Air35] in 1835, who found the analytical solution in the best image plane (BIP) for the diffraction pattern caused by an aberration-free system with circular pupil. Lord Rayleigh [Str79] in 1879 postulated that the image of the point given by an optical system with aberrations would not be significantly different from the image given by the aberration-free system if the wave front (non-spherical) is contained between two spherical wave fronts separated less than a quarter wavelength. Strehl [Str94] in 1894 indicated that the small aberrations reduce the intensity in the principal maximum. He proposed the ratio between the intensity at the maximum of the image of a point with and without aberrations as a ratio that describes the behavior of the optical system. This relation is called the Strehl ratio. The limit of a quarter wavelength in the wave front aberration corresponds to a value of 0.8 for the Strehl ratio. Generally, numerical methods are required to calculate the image given by an optical system. Particularly we mention the method proposed by Hopkins and Yzuel [Hop70] to evaluate the diffractional image of a point given by a system with arbitrary aberrations and circular pupil.

As pointed out by the works of Straubel [Str35] and Hopkins [Hop49], the introduction of non-uniform transmission filters on the pupil changes the response of the optical system without the need to modify other parts in the design. The first effect that was studied, the so-called apodization, consists in removing the secondary maxima in the diffractional image of a point object, or point-spread function (PSF). This effect is accompanied by an increase in the width of the central maximum, thus, diminishing the resolution. Jacquinot and Roizen-Dossier [Jac64] made a good compilation about the use of non-uniform transmission filters for apodization. By means of non-uniform transmission filters we can also obtain the opposite effect: the increase in resolution, i.e. the narrowing of the central maximum. This effect is called superresolution or hyperresolution. The increase in resolution is accompanied by the increase of the height of the secondary maxima. We mention the early studies of superresolution made by Toraldo di Francia [Tor52] with annular pupils. We note that the non-uniform transmission filters, no matter if they produce apodization or hyperresolution, are usually referred as apodizers. Along this thesis we will use indistinctly the name apodizer or non-uniform transmission filter.

Yzuel and Calvo [Yzu83] extended the Hopkins-Yzuel numerical method to include the calculation of non-uniform transmission pupils. The method can be applied to optical systems with any kind of residual aberrations. Mills and Thompson [Mil86] studied the effect of apodizers in aberrated systems with coherent illumination. Chung and Hopkins [Chu88] showed numerical results of the 3D monochromatic PSF for different non-uniform transmission filters. The non-uniform filters have been applied in very different fields. Hee [Hee75] used an apodized aperture for laser beam attenuation. In other cases, apodizing filters have been used to reduce the effect of aberrations [Yzu79,Cam89]. High resolution has also been sought through pupil filters in fields like scanning imaging [Heg86]. The influence of pupil plane filters in the axial response of an optical system has been investigated with monochromatic illumination [Oje86,She88], and with polychromatic illumination [Yzu88,Yzu90]. Normally, we look for an increase in the depth of focus or to produce a multifocus effect. Sometimes the pupil-filtering searches for a combined effect in the transverse and axial response, for instance in a photolithographic system, to produce both transverse superresolution and high focal depth [Fuk93,Hil95]. Other works have studied the symmetries in pupil filters to obtain identical axial response [Esc94,Cam00a]. A thorough revision of apodization and hyperresolution can be found in [Mil96].

Despite of the large amount of theoretical work dealing with non-uniform transmission filters, the experimental work has been minimum. Only annular filters (binary absorption) are easy to produce without the need of sophisticated methods. Nevertheless, most of the interesting non-uniform functions have a continuous amplitude profile. They can be fabricated by different techniques. Jacquinot and Roizen-Dossier [Jac64] described techniques of vaccuum deposition using rotating masks located between the evaporation source and the substrate for the filter. Sheppard and Hegedus [She88] generated the filters directly on photographic emulsion by using a light pen. The distribution of the random dots approximates the desired radial transmittance variation. The photographic emulsion is also the medium used by other authors [Hee75,Mil86] to generate the non-uniform transmission filters. All these methods exhibit serious drawbacks: in all the cases a continuous profile is difficult and expensive to fabricate accurately. Furthermore, the filter is registered on a static medium. Thus, the filter cannot be easily changed. In [PAPER C] we have proposed a technique to generate programmable apodizers by using a TN-LCSLM working in the amplitude-only mode.

1.3.2 Focusing diffractive optical elements

Advances in the technology of SLMs in general, and of LCSLMs in particular, make possible the implementation of DOEs whose characteristics can be changed dynamically. DOEs to steer a beam without mechanical movement have evident applications in scanning systems [McM93,Bro95], in optical networking and in holographic storage systems [Wan00]. Programmable DOEs are also interesting for adaptive wave front control in astronomy imaging systems [Lov94,Bol98]. Laser beam shaping for precision cutting and welding systems is another field for application of programmable DOEs [Ser99].

Among the different DOEs, the lens is likely to be the element which has received more attention. A programmable lens allows to change its focal length without the need of mechanical movement [Tam92a,Tam92b,Tak96]. This is a property that can be useful in a number of optical systems, e.g. in a self-focusing system. Ferstl and Frisch [Fer96] generated dynamic Fresnel zone lenses to collimate, focus and deflect individual light beams in a three-dimensional interconnection network. These dynamic zone lenses were made on a specifically designed liquid crystal SLM. Hain *et al.* [Hai01] investigated pixelated liquid crystal lenses to be used in a pickup head for digital versatile discs (DVDs), where a fast time response is of primary importance.

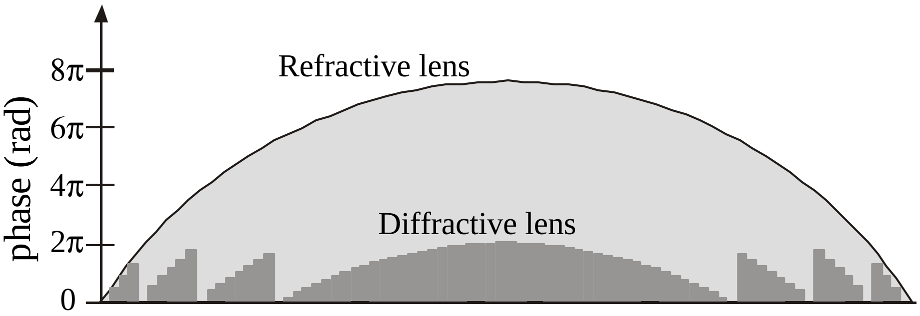

Figure 1.5. Comparison between the refractive lens and its diffractive counterpart.

The so-called diffractive Fresnel lens is the diffractive version of the usual refractive lens. In Figure 1.5 we show the phase profile for a diffractive Fresnel lens. We can see that the phase profile of the refractive lens is approximated by a stepped profile where the excess of material producing a phase larger than 2π radians has been removed. The action of this multilevel lens is governed by the diffraction properties of light propagation. The multilevel profile can be made by a number of fabrication techniques such as photolithography [Aur72,Fer94]. A review of techniques to produce both refractive and diffractive micro lenses can be found in [Nis87,Tur97]. In the case of subwavelength structures an accurate design requires the use of rigorous electromagnetic diffraction theory [Sch97].

We note that due to the pixelated nature of the SLMs, a lens displayed on the SLM has a stepped profile. Thus, the behavior of programmable lenses has to be studied under the theory of diffraction of light. As a diffractive element, the programmable lens can perform various functions simultaneously. Davis *et al.* [Dav89a] reported the capability for encoding binary Fresnel phase-encoded lenses multiplied by the two-dimensional Fourier transform of a desired output spatial pattern. They obtained the desired output at programmable distances depending on the focal length of the phase-encoded lens. This property is not possible with a refractive lens. In Ref. [Dav89b], Davis *et al.* described a technique in which a binary Fresnel lens encoded on a SLM reduces the false signals in an optical correlator. Cottrell *et al.* [Cot90] showed that Fresnel phase-encoded lenses can produce multiple spots when the focal length decreases. Thus, the SLM can produce multiple images of a desired pattern. This multiple imaging capability together with the fast refreshment of the SLM are attractive properties in optical image processing and in optical pattern recognition. Davis *et al.* [Dav92] demonstrated the anamorphic capability of programmable lenses. The work in the last four papers was done using a magneto-optic SLM (MOSLM). A number of authors have used parallel aligned LCSLMs [Tak96] and twisted-nematic LCSLMs [Tam92a,Tam92b] to display programmable lenses. Laude [Lau98] performed an extensive study dealing with the possibility of using a twisted-nematic SLM to display programmable lenses.

Carcolé *et al.* [Car94a] developed a mathematical model to describe the behaviour of low-resolution Fresnel lenses encoded on SLMs. With this model they studied the effects of low resolution codification, such as the appearance of new secondary lenses. The new secondary lenses behave as if each lens were encoded through all the pixels of the device. The diffraction efficiency of these lenses [Car94b] and the way to optimize this diffraction efficiency [Car95] were also studied. In their formulation Carcolé *et al.* [Car94a] demonstrated that the image produced by a lens encoded onto a SLM appears to be convolved with the function that corresponds to the pixel shape. This convolution leads to an apodizing effect on the image produced by the lens. Arrizón *et al.* [Arr99] studied this apodizing effect. The apodizing effect is not a desired effect because it worsens the resolution of the optical system, i.e. it widens the width of the central maximum. In [PAPER D] we have demonstrated a method to measure the apodizing effect and to compensate for it.

We have explained the interest in generating programmable lenses and programmable apodizers. Therefore, we can imagine that the combination of both elements in a single programmable DOE would exhibit very attractive properties. This new programmable DOE, that we call programmable focusing apodizer, would be able to change both the focal plane and the PSF of a system. In [PAPER E] we have demonstrated the feasibility of this new DOE on a TN-LCSLM working in the phase-only regime.

1.4 Purpose and outline of this thesis

There are two main subjects that we face in the thesis. On one hand, we investigate a simplified description for the distribution of the molecular director $\vec{n}$ across the TN-cell. We demand that this description should be able to provide highly accurate quantitative predictions of the optical transmission of the TN-cell. On the other hand, our aim is to demonstrate the feasibility of generating on the LCSLM the next two programmable DOEs: first, apodizers, and second, the multipurpose DOE composed of Fresnel diffractive lens and programmable apodizer. We demand a good quality in performance of these DOEs, not only qualitatively but also quantitatively.

The first subject, the model of the director distribution and the possibility of accurate predictions of the optical transmission of the TN-LCSLM is related with the papers [PAPER A, PAPER B]. To achieve this goal we have to face different tasks in our work:

(1) We have to design a model for the molecular director distribution $\vec{n}$ with two features. On one side, the description should be simple enough so that the number of fitting parameters in the model is the minimum possible. We have to perform a reverse-engineering approach since the parameters of the LCSLM are not provided by the manufacturer. On the other side, the simplified description should be accurate enough so that the model may provide accurate prediction of the optical transmission, i.e. amplitude and phase modulations, of the TN-LCSLM.

(2) Together with the model we also have to provide some method that allows to calculate the values of the parameters of the model with a good accuracy.

(3) Using the proposed model we have to provide some procedure, if possible an analytical formula, to express the action of the LCSLM. We will use the Jones matrix formalism to develop the Jones matrix of the LCSLM.

(4) We have to calculate theoretically the expected light propagation through the LCSLM and through any other optical device which we may use in the experiments. We will use the Jones matrix formalism. If possible we will calculate analytical expressions for amplitude and phase modulations.

(5) We have to develop the appropriate experimental set-ups to characterize the amplitude and phase modulations of the LCD that we use as a SLM for non-display purposes. This is an important step because we need to evaluate the quality of the optical performance of the LCD panel for our particular application. The degree of agreement between these measurements and the theoretical predictions will decide if our theoretical work has been accurate enough.

(6) We have to study some suitable modifications in the set-up to obtain amplitude-only and phase-only regimes with a thin LCSLM. There are some possibilities such as using short wavelengths or adding new elements (e.g.

wave plates) to the set-up. Once the prediction capability of the model has been demonstrated, we design optimization procedures based on computer searchs. The computer search leads us to the optimum configurations providing the requested complex amplitude modulation.

This first subject is developed along the Chapter 2. In Section 2.1 we explain the different models and approaches which describe the molecular director distribution in a TN-LCSLM. Along Section 2.1 we mainly develop points (1), (2), (3) and (4) for the model we propose. In Section 2.2 we describe the set-ups to calibrate the amplitude and phase modulations of the LCSLM. We also fit the different parameters that need to be solved for a complete characterization of the LCSLM. Point (5) is mainly dealt in Section 2.2. In Section 2.3 we perform computer searchs to obtain the optimum configurations for amplitude-only modulation and phase-only modulation. In this sense, we propose a generalized optical set-up where wave plates are included. Section 2.3 deals with point (6).

The second subject of the thesis, the production of programmable apodizers and Fresnel lenses is connected with the work published in the papers [PAPER C, PAPER D, PAPER E]. This part of the work can be structured as follows:

(1) First, we choose the specific non-uniform transmission filters that we will implement on the LCSLM. From the existing literature we have chosen four different filters which allow to cover a wide range of possible modifications of the optical system response. We have to use numerical methods to calculate the theoretical performance of the apodizers.

(2) We have to develop the appropriate experimental set-up to display the programmable apodizers with the LCSLM in the amplitude-only regime. The agreement between experimental measurements of the PSF and the numerical predictions will tell us the quality of the programmable apodizers.

(3) The next goal is to combine in a unique DOE a programmable apodizer and a Fresnel lens on the LCSLM working in the phase-only

regime. This means that we have to develop some technique that makes possible the codification of complex information on a low resolution phase-only medium. In this sense, we will extend to the case of a quadratic phase carrier the technique that Davis *et al.* [Dav99d] proposed to encode complex amplitude information on a linear phase carrier.

(4) We analyse the imaging performance of diffractive Fresnel lenses on a pixelated device as the LCSLM. We have to develop some technique to measure and compensate for the distortions, the inherent equivalent apodizing effect, due to the pixelated nature of the LCSLM.

(5) We have to develop the experimental set-up to display the programmable focusing apodizers on the LCSLM working in the phase-only regime. The analysis of the experimentally measured PSFs will give us the final proof for the validity of this new DOE.

This second subject is the focus of Chapter 3. Point (1) is developed along Section 3.1 and Section 3.2. In Section 3.1, we also explain the fundamentals of diffraction theory to understand the action of non-uniform transmission filters located on the exit pupil of an optical system. Point (2) is developed in Section 3.3. Along Section 3.4., we generalise a method to encode complex amplitude information on a phase-only medium, i.e. point (3). In Section 3.5 we deal with point (4): we concentrate on Fresnel lenses generated on the LCSLM. In the Appendix we also deal with point (4): we develop an alternative approach to the one proposed in Section 3.5. In Section 3.6 we perform point (5).