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Trade-off Issues in 2DoF PID Control Design

Performance, Robustness, and Fragility

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CERTIFIES

That the thesis entitled **"Trade-off Issues in 2DoF PID Control Design: Performance, Robustness, and Fragility"** by Víctor M. Alfaro Ruiz, presented in partial fulfillment of the requirements for the degree of Doctor Engineer, has been developed and written under his supervision.

Dr. Ramon Vilanova i Arbós
Bellaterra, September 2012

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To María Amanda and to
Alejandro, Carolina, and Sebastián.

Contents

1 Introduction

A natural way to adjust or correct the behavior over time of a dynamic system output, the controlled variable, is by using an actuating input computed on the basis of the comparison of the actual output with its desired value: the feedback error. This is, by means of closed-loop control.

To compute the control action information of the feedback error is required. Normally its current value, its past evolution, and a prediction of its future behavior are used. This constitutes the control algorithm.

The feedback control structure has been used for a long time, but if we constraint ourselves to the industrial process control area, the proportional (present) integral (past) derivative (future) (PID) control algorithm age starts in 1940 with the introduction of the Taylor Fullscope 100 pneumatic PID controller.

The original simple three-term PID control algorithm has evolved to the actual four- or five-term two-degree-of-freedom (2DoF) PID implementations.

To guarantee a stable and successful operation of the control system the controller must be matched (tuned) to the controlled process, using information of the dynamic characteristics of the process usually represented by a low-order linear model.

At the beginning, controller tuning took into consideration only the control system *performance*, the output signal dynamic characteristics, to step changes in its inputs. It was noticed that if only performance is considered in the design it leads to control systems with very low robustness, its capability to deal with changes in the controlled process dynamic characteristics. Then, *robustness* was introduced into the controller design.

The performance/robustness trade–off in PID control system design is a well-know issue. Even in case that this trade-off is resolved at the design stage, it is important to evaluate the controller *fragility*, the effect of a change in the controller parameters, at its final fine-tuning.

Considering than in industrial control applications *regulatory control* operation, rejection of disturbance inputs, prevails over *servo control* operation, tracking of set-point changes, we are mainly interested in use 2DoF PI and PID controllers.

On the other hand, we have a variety of controlled processes dynamics, from the most common self-regulating over damped to integrating and unstable processes.

Nowadays the proportional integral and proportional integral derivative is the most used control algorithm in the industry. Although, it is reported elsewhere that there are many loops with very poor performance, badly tuned or not tuned at all. Considering the huge amount of PI and PID controllers in service at present in the process industry any improvement in their performance will produce big overall revenue.

In this thesis, a general design procedure for 2DoF PI/PID controllers is proposed based in the specification of the different control system closed-loop transfer functions that include parameters that affect the *performance/robustness trade-off*. Considering than the control system robustness requirement, is controlled process dependent, and more important, that it cannot be avoided, the robustness level, measured with the maximum sensitivity, is used as the design parameter.

The specification of the closed-loop transfer functions also takes into account obtaining smooth *control effort*, controller output, signals.

One of the purposes of the work has been develop a design methodology for robust control systems independent on the controller, PI or PID, and the controlled process avoiding appealing to *ad hoc* design procedures for each particular case (controller/process combination). The controlled process model specialties are incorporated only into the closed-loop target response specifications.

The proposed controller design methodology denoted as *Model Reference Robust Tuning* (MoReRT) is applied to tune 2DoF PI controllers for first- and second-order over damped, integrating, inverse response, and unstable controlled processes.

An extension of the MoReRT to 2DoF PID controllers is also considered.

The accomplishment of the robustness target level for all the controlled process models considered (over damped, integrated, and unstable) is one of the distinctive characteristics of the proposed design method.

List of publications

The thesis consists of the following papers, listed in descending order of publication:

1. Book chapters:
    - Alfaro, V. M. and Vilanova, R. (2012) - "Fragility Evaluation of PI and PID Tuning Rules", in *PID Control for the Third Millennium: Lessons Learned and New Approaches*, R. Vilanova and A. Visioli (Eds.), Springer-Verlag London Limited.

2. Journal papers:
   - Alfaro, V. M. and Vilanova, R. (2012) - "Model-reference robust tuning of 2DoF PI controllers for first- and second-order plus dead-time processes", *Journal of Process Control*, Vol. 22, 359-374.
   - Alfaro, V. M. and Vilanova, R. (2012) - "Performance/Robustness Trade-off Design Framework for 2DoF PI Controllers", *Studies in Informatics*, Vol. 21 (1), 75-83.
   - Vilanova, R. and Alfaro, V. M. (2012) - "Control PID robusto: una visión panorámica", *Revista Iberoamericana de Automática e Informática Industrial*, Vol. 8, 141-158.
   - Alfaro, V. M. and Vilanova, R. (2012) - "Simple Robust Tuning of 2DoF PID Controllers from a Performance/Robustness Trade-Off Analysis", *Asian Journal of Control* (submitted).
   - Alfaro, V. M. and Vilanova, R. (2012) - "Robust Tuning and Performance Analysis of 2DoF PI Controllers for Integrating Controlled Processes", *Ind. Eng. Chem. Res.* (submitted).
3. International Conference papers:
   - Alfaro, V. M. and Vilanova, R. (2012) - "Robustness-Based Tuning of Two-Degree-of-Freedom Proportional Integral Control for Unstable Processes", *16th International Conference on Systems Theory, Control and Computing* (ICSTCC 2012), October 12-14, Sinaia, Romania.
   - Alfaro, V. M. and Vilanova, R. (2012) - "Two-Degree-of-Freedom Proportional Integral Control for Inverse Response Second-Order Processes", *16th International Conference on Systems Theory, Control and Computing* (ICSTCC 2012), October 12-14, Sinaia, Romania.
   - Alfaro, V. M. and Vilanova, R. (2012) - "Conversion Formulae and Performance Capabilities of Two-Degree-of-Freedom PID Control Algorithms", *17th IEEE International Conference on Emerging Technologies & Factory Automation* (ETFA 2012), Sep. 17-21, Kraków, Poland.
   - Alfaro, V. M. and Vilanova, R. (2012) - "Set-Point Weight Selection for Robustly Tuned PI/PID Regulators for Over Damped Processes", *17th IEEE International Conference on Emerging Technologies & Factory Automation* (ETFA 2012), September 17-21, Kraków, Poland.

- Alfaro, V. M. and Vilanova, R. (2012) - "Model Reference Robust Tuning of 2DoF PI Controllers for Integrating Controlled Processes", *IEEE 20th Mediterranean Conference on Control and Automation* (MED 2012), July 3-6, Barcelona, Spain. (Best MED 2012 paper Award).
- Alfaro, V. M. and Vilanova, R. (2012) - "Performance Analysis of Model-Reference Robust Tuned 2DoF PI Controllers for Over Damped Processes", *IEEE 20th Mediterranean Conference on Control and Automation* (MED 2012), July 3-6, Barcelona, Spain.
- Alfaro, V. M. and Vilanova, R. (2012) - "Fragility-Rings - A Graphic Tool for PI/PID Controllers Robustness-Fragility Analysis", *IFAC Conference on Advances in PID Control* (PID'12), March 28-30, Brescia, Italy.
- Alfaro, V. M. and Vilanova, R. (2012) - "Optimal Robust Tuning for 1DoF PI/PID Control Unifying FOPDT/SOPDT Models", *IFAC Conference on Advances in PID Control* (PID'12), March 28-30, Brescia, Italy.
- Alfaro, V. M. and Vilanova, R. (2011) - "Fragility and Robustness Level Accomplishment of Well Know PI/PID Robust Tuning Rules", *9th IEEE International Conference on Control & Automation* (ICCA'11), December 19-21, Santiago, Chile.
- Alfaro, V. M. and Vilanova, R. (2010) - "Sintonización de los controladores PID de 2GdL: desempeño, robustez y fragilidad", XIV Congreso Latinoamericano de Control Automático (ALCA 2010), Agosto 24-27, Santiago, Chile.

Outline

The thesis is organized in two parts. The first part includes seven chapters that describe the main aspects of the work done. In chapter 2 the characteristics of the robust performance optimized tuning are presented. Chapter 3 describes the bases of the proposed model reference robust tuning (MoReRT) design methodology. The use of the MoReRT to design two-degree-of-freedom proportional integral controllers is presented in chapter 4 to control over damped, integrating, inverse response, and unstable processes. In chapter 5 the fragility of MoReRT controllers is analyzed. The extension of the MoReRT methodology to design two-degree-of-freedom proportional integral derivative controllers for inverse response and unstable processes is described in chapter 6. Followed by the conclusions and possible future work in chapter 7. The second part of the thesis provides a copy of the listed publications.

2 Optimal robust tuning of PI and PID controllers

One common approach for PID controllers tuning is by optimizing the control system response to its inputs, set-point and disturbance, using an integrated error criteria.

The control system design procedure is normally based on the use of low-order linear models, obtained at the system normal operating point, to represent the non-linear controlled processes. Then, it is necessary to forsee the changes in the process characteristics when the operating point changes, assuming certain relative stability margins or robustness requirements for the control system. Therefore, the *performance/robustness* trade-off is an ever-exiting issue.

A general analysis of the performance/robustness/fragility relation in 2DoF PI/PID controllers is presented in Alfaro and Vilanova (2010) and a revision of the robustness metrics used for robust control design in Vilanova and Alfaro (2011).

For the robust performance optimized tuning approach presented in the following consider the closed-loop control system in figure 2.1 where $P(s)$ and $C(s)$ are the controlled process model and the controller transfer functions respectively. In this system, $r(s)$ is the set-point, $u(s)$ the controller output signal, $d(s)$ the load disturbance, and $y(s)$ the process controlled variable. It is assumed that the disturbance enter at process input (load-disturbance).

The controlled process is represented by a second-order plus dead-time (SOPDT) model given by the general transfer function

$$P(s) = \frac{Ke^{-Ls}}{(Ts+1)(aTs+1)}, \ \tau_L = \frac{L}{T}, \tag{2.1}$$

where K is the gain; T, the main time constant; a, the ratio of the two time constants

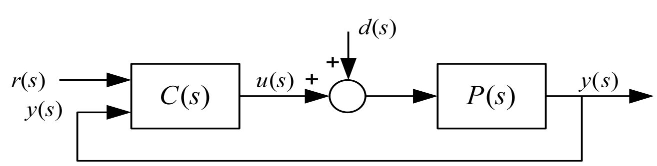


Figure 2.1: Closed-loop control system

($0 \leq a \leq 1.0$); L, the process apparent dead-time; and τ_L, the *normalized dead time*. The model transfer function (2.1) allows the representation of first-order plus dead-time (FOPDT) processes ($a = 0$), over damped second-order plus dead-time (SOPDT) processes ($0 < a < 1$), and dual-pole plus dead-time (DPPDT) processes ($a = 1$).

The process is controlled with a 2DoF Standard PID controller whose output is as follows (Åström and Hägglund, 1995):

$$u(s) = K_p \left\{ \beta r(s) - y(s) + \frac{1}{T_i s} \left[r(s) - y(s) \right] - \left(\frac{T_d s}{\alpha T_d s + 1} \right) y(s) \right\}, \tag{2.2}$$

where K_p is the controller *proportional gain*; T_i, the *integral time constant*; T_d, the *derivative time constant*; β the *proportional set-point weight*; and α, the *derivative filter constant*. Usually, $\alpha = 0.10$ (Corripio, 2001). Then, the controller parameters to tune are $\theta_c = \{K_p, T_i, T_d, \beta\}$. For 1DoF controllers $\beta = 1$ is used in (2.2).

The *performance* of the closed-loop control system is evaluated using the integrated absolute error cost functional given by

$$J_e \doteq \int_0^\infty |e(t)| \, \mathrm{d}t = \int_0^\infty |y(t) - r(t)| \, \mathrm{d}t, \tag{2.3}$$

for disturbance (J_{ed}) and set-point (J_{er}) changes.

The peak magnitude of the sensitivity function is used as an indicator of the system *robustness* (relative stability). The maximum sensitivity for the control system is defined as

$$M_S \doteq \max_\omega |S(\mathrm{j}\omega)| = \max_\omega \frac{1}{|1 + C_y(\mathrm{j}\omega)P(\mathrm{j}\omega)|}, \tag{2.4}$$

where $C_y(s)$ is the controller (2.2) aspect applied to the controlled variable (feedback signal).

If the system robustness is not taken into account for the design, the controller parameters θ_c may be optimized to maximize the system performance for set-point changes or load disturbance changes, $J_e^o = J_e(\theta_c^o)$, where θ_c^o are the optimal controller parameters.

Using an optimization procedure based on a *performance degradation factor* defined as

$$F_p \doteq \frac{J_e^o}{J_e}, \; F_p \leq 1, \tag{2.5}$$

the control system robustness M_S can be increased, by reducing F_p, while its performance decrease (J_e increase).

From the optimization results it is noted that for a given model increasing the control system robustness results in a substantial reduction in the controller gain (K_p). However, this increase in the robustness has negligible effects on the controller integral time (T_i) and derivative time (T_d), except in the case of models with a very low normalized dead-time (when high robustness is required).

Based on this observation, equations *that are independent of the target robustness level* can be obtained for the controller integral time constant and derivative time constant, as follows:

$$T_i = \mathbf{F}(T, \tau_L, a), \;\; T_d = \mathbf{G}(T, \tau_L, a). \tag{2.6}$$

With these equations at hand, the controller proportional gains are readjusted to match a target robustness M_S^t to obtain equations given by the following expression

$$K_p = \mathbf{H}(K, \tau_L, a, M_S^t). \tag{2.7}$$

For the 2DoF controllers the optimization results also show that for a given robustness level, the proportional set-point weight β is affected more by τ_L than by a. Then, an expression for β can be obtained as follows:

$$\beta = \mathbf{Q}(\tau_L, M_S^t). \tag{2.8}$$

The proposed *optimal robust tuning* equations for 1DoF PI and PID controllers ($uSORT_1$) are:

- Regulatory control operation:

$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_L^{a_2}, \tag{2.9}$$

$$\tau_i \doteq \frac{T_i}{T} = b_0 + b_1 \tau_L^{b_2}, \tag{2.10}$$

$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_L^{c_2}. \tag{2.11}$$

- Servo control operation:

$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_L^{a_2}, \tag{2.12}$$

$$\tau_i \doteq \frac{T_i}{T} = \frac{b_0 + b_1 \tau_L + b_2 \tau_L^2}{b_3 + \tau_L}, \tag{2.13}$$

$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_L^{c_2}. \tag{2.14}$$

The value of the constants a_i, b_i, and c_i in (2.9) to (2.14) for FOPDT and SOPDT models with τ_L in the range from 0.1 to 2.0 and four robustness design levels, $M_S^t \in \{1.4,\ 1.6,\ 1.8,\ 2.0\}$, are presented in Alfaro and Vilanova (2012e).

The corresponding equations for 2DoF PI (PI_2) and PID (PID_2) controllers ($uSORT_2$) are the ones for regulatory control (2.9) to (2.11) with an additional relation for the proportional set-point weight given by

$$\beta = d_0 + d_1 \tau_L^{d_2}. \tag{2.15}$$

Constants d_i for (2.15) are presented in Alfaro and Vilanova (2012m) for robustness levels $M_S^t \in \{1.4,\ 1.6,\ 1.8,\ 2.0\}$.

The proposed tuning provides a unifying design procedure for 1DoF and 2DoF PI and PID controllers for FOPDT and SOPDT controlled processes models that allows adjusting the control system robustness varying only the controller proportional gain.

The designer may select the model (first- or second-order) that best represents the over damped controlled process and the control system required robustness, selecting from four levels, according with the expected variations in the controlled process dynamics.

As an example of the application of the proposed $uSORT_2$ tuning, consider the SOPDT model given by the transfer function:

$$P_1(s) = \frac{1.2\mathrm{e}^{-1.5s}}{(2s+1)(s+1)}, \quad a = 0.5, \quad \tau_L = 0.75. \tag{2.16}$$

The PI_2 and PID_2 controllers parameters obtained with the proposed tuning relations, the resulting control system robustness, and the regulatory and servo control performance are listed in table 2.1 for four robustness design levels (M_S^t).

From the table data, the performance/robustness trade-off is evident. It is also noted that for same robustness level the MoReRT PID_2 controllers bring more regulatory control performance (lower J_{ed}) than the PI_2 with very similar servo control performance (J_{er}).

Control system responses with extreme and intermediate performances are shown in figure 2.2.

Comparison with other tuning rules, explicitly omitted here for space reasons, are presented in Alfaro and Vilanova (2012e,m).

Table 2.1: $uSORT_2$ control example, performance and robustness

M_S^t	2.0	1.8	1.6	1.4	2.0	1.8	1.6	1.4
	PI_2 controller				PID_2 controller			
K_p	0.838	0.740	0.613	0.461	1.037	0.951	0.801	0.620
T_i	3.743				2.454			
T_d	–				1.108			
β	1.00	1.18	1.44	1.82	0.68	0.76	0.89	1.16
M_S	2.03	1.83	1.62	1.41	1.93	1.79	1.60	1.41
$J_{ed}/\Delta d$	4.466	5.059	6.102	8.098	2.848	3.094	3.605	4.456
$J_{er}/\Delta r$	4.359	4.236	4.115	4.052	4.325	4.396	4.534	4.702

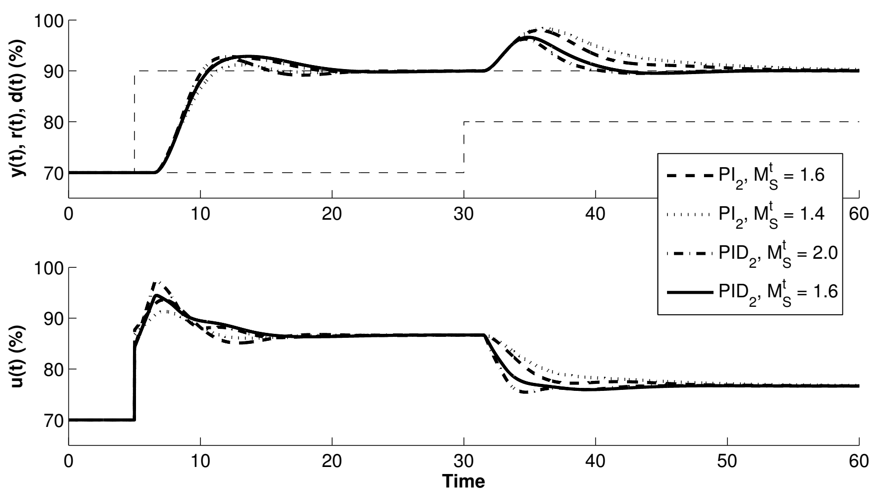


Figure 2.2: $uSORT_2$ control example, control system responses

Chapter remarks

Tuning equations for one- and two-degree-of-freedom (1DoF, 2DoF) proportional integral (PI) and proportional integral derivative (PID) controllers for first- and second-order over damped plus dead-time (FOPDT, SOPDT) controlled process models are presented. These are based on a performance/robustness trade-off analysis.

Performance was optimized by using the integrated absolute error for disturbance and set-point changes. Robustness was measured with the control system maximum sensitivity.

A performance degradation factor was used to increase robustness while performance was decreased.

Tuning equations for 1DoF PI and PID controllers are presented in Alfaro and Vilanova (2012e) and corresponding for 2DoF PI and PID controllers are presented in Alfaro and Vilanova (2012m).

On the proposed tuning method the controller integral time constant T_i and derivative time constant T_d depend only on the controlled process model parameters, and the controller gain K_p depends on the controlled process model parameters and the target robustness level. Hence, for a given controlled process model, T_i and T_d are fixed and the control system robustness M_S is adjusted by varying only K_p. This made the control system robustness adjustment a “one knob” variation process.

A detailed performance/robustness trade-off analysis for 1DoF and 2DoF PI and PID controllers with over damped controlled process models (first- and second-order) is presented in Alfaro and Vilanova (2012n).

3 Model reference robust tuning (MoReRT) design methodology

The optimal robust tuning procedure presented in chapter 2 takes into consideration the closed-loop control system *robustness* using the maximum sensitivity as the design parameter, and its *performance* by optimizing the integrated absolute error to step changes in its inputs, disturbance and set-point. However, the designer does not have control of the control system output and control effort signals shapes.

One possible approach to state the closed-loop system behavior is by using an analytically deducted tuning method (Alfaro, 2006). In this type of approach, for the deduction of the controller parameters tuning relations, the controlled process model dead-time needs to be approximated by a series in s or by a Pade transfer function. The use of this approximation affects the actual control system responses.

An alternative approach is by specifying the closed-loop poles and zeros location by stating the desired control system closed-loop transfer functions ("dominant pole design") and obtaining, by using an optimization procedure, the controller parameters that provide the best match to the desired responses while a target closed-loop control system robustness is obtained (Alfaro et al., 2010). A design methodology of this type is used in this thesis.

The controller design is based on the use of closed-loop transfer functions targets to obtain robust control systems, named *Model Reference Robust Tuning* (MoReRT), as outlined in Alfaro and Vilanova (2012g).

Consider the closed-loop control system in figure 2.1. Its output $y(s)$ as a function of its inputs $r(s)$ and $d(s)$ is

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \tag{3.1}$$

where

$$M_{yr}(s) = \frac{C_r(s)P(s)}{1+C_y(s)P(s)}, \tag{3.2}$$

is the *servo control* closed-loop transfer function, and

$$M_{yd}(s) = \frac{P(s)}{1+C_y(s)P(s)}, \tag{3.3}$$

the *regulatory control* closed-loop transfer function.

In (3.2) and (3.3) $C_r(s)$ is the controller aspect that operates on the set-point $r(s)$, the *set-point controller*, and $C_y(s)$ the controller aspect that operates on the controlled variable $y(s)$, the *feedback controller*.

The controlled process model and the feedback controller transfer functions are expressed as a quotient of polynomials in s as follows

$$P(s) = \frac{N_p^-(s)N_p^+(s)}{D_p(s)}, \tag{3.4}$$

$$C_y(s) = \frac{N_{cy}(s)}{D_{cy}(s)}, \tag{3.5}$$

where $N_p^+(s)$ is the controlled process model non-minimum phase part (dead-time and/or right-half plane zero).

Replacing $P(s)$ and $C_y(s)$ in (3.3) by (3.4) and (3.5) the regulatory control closed-loop transfer function can be expressed by

$$M_{yd}(s) = \frac{D_{cy}(s)N_p^-(s)N_p^+(s)}{D_{cy}(s)D_p(s) + N_{cy}(s)N_p^-(s)N_p^+(s)}. \tag{3.6}$$

The feedback controller $C_y(s)$ design methodology made use of a regulatory control closed-loop transfer function target for (3.6), $M_{yd}^t(s)$, that depends on the controlled process model non-minimum phase components, the rest of model parameters, the feedback controller parameters, and the control system *design parameters*, θ_d, with the following general form:

$$M_{yd}^t(s) = \mathscr{M}_d(N_p^+(s), \theta_p, \theta_{cy}, \theta_d, s). \tag{3.7}$$

Then, the regulatory control response target is

$$y_d^t(s) = M_{yd}^t(s)d(s). \tag{3.8}$$

Using (3.7) in (3.2) the servo control closed-loop transfer function target is given by the following general function:

$$M_{yr}^t(s) = \mathscr{M}_r(N_p^+(s), \theta_p, \theta_c, \theta_d, s) = C_r(s)M_{yd}^t(s). \tag{3.9}$$

Then using (3.7) and (3.9) in (3.1) the global system output target, $y^t(s)$, is computed as follows:

$$y^t(s) = M_{yr}^t(s)r(s) + M_{yd}^t(s)d(s), \tag{3.10}$$

and in the time domain as follows:

$$y^t(t) = y_r^t(t) + y_d^t(t), \tag{3.11}$$

where $y_r^t(t)$ is the servo control step response target and $y_d^t(t)$, the regulatory control step response target.

To obtain the controller parameters, that best match the target response (3.11) in the *least-squares sense*, a minimization procedure is used based on the differences between the target responses and the actual ones.

For the regulatory control model reference response optimization the cost functional to be optimized is defined as follows:

$$J_d(\theta_p, \theta_{cy}, \theta_d) \doteq \int_0^\infty \left[y_d^t(\theta_p, \theta_{cy}, \theta_d, t) - y_d(\theta_p, \theta_{cy}, t) \right]^2 \mathrm{d}t, \tag{3.12}$$

where $y_d^t(\theta_p, \theta_{cy}, \theta_d, t)$ is the regulatory control closed-loop step response target and $y_d(\theta_p, \theta_{cy}, t)$ is that of the regulatory control system with controlled process model $P(s)$ and the feedback controller $C_y(s)$.

In a similar way, the servo-control cost functional to be optimized is defined as follows:

$$J_r(\theta_p, \theta_c, \theta_d) \doteq \int_0^\infty \left[y_r^t(\theta_p, \theta_c, \theta_d, t) - y_r(\theta_p, \theta_c, t) \right]^2 \mathrm{d}t, \tag{3.13}$$

where $y_r^t(\theta_p, \theta_c, \theta_d, t)$ is the step response of the servo-control closed-loop transfer function target and $y_r(\theta_p, \theta_c, t)$ is that of the servo-control system with the controlled process $P(s)$ and the controller $C(s)$.

For the 2DoF controllers design, the following overall cost functional is optimized:

$$J_T(\theta_p, \theta_c, \theta_d) \doteq J_r(\theta_p, \theta_c, \theta_d) + J_d(\theta_p, \theta_{cy}, \theta_d). \tag{3.14}$$

Using (3.14) the controller parameters θ_c^o are obtained such that

$$J_T^o \doteq J_T(\theta_p, \theta_c^o, \theta_d) = \min_{\theta_c} J_T(\theta_p, \theta_c, \theta_d), \tag{3.15}$$

for design parameters θ_d selected in such a way that the control system robustness matches a target value (robust design) measured using the maximum sensitivity, M_S, defined by (2.4).

The performance/robustness trade-off in PI/PID controller design is a well-know issue and the M_S has become the *de facto* robustness measure. Robustness requirements are related with the expected changes in the controlled process dynamics from the nominal model used for controller tuning. An *a priori* evaluation of this requirement is needed to state the minimum robustness level desired for the designed control system. Then, if robustness is a must the design is focused on the obtainable performance and/or control effort characteristics, under the selected metrics, for the stated robustness.

The optimization of (3.14) is a *closed-loop model matching problem*, instead of a control system performance optimization problem as in the traditional PI/PID controllers optimization procedures. The control system behavior is stated by the closed-loop regulatory and servo control transfer functions targets and the design parameters θ_d. The optimization procedure searchs for the controller parameters θ_c that achieves the best match between the actual overall control system response and the target one.

The complete set of controller parameters θ_c are obtained when considering the regulatory control response and the servo control response at once.

A target robustness M_S^t is used as a *soft constraint*. For a given process the resulting control system robustness depends on the design parameters then, it is evaluated after each closed-loop model reference response optimization and the design parameters adjusted to met this target.

For the optimization a direct search Nelder-Mean (Lagarias et al., 1998) simplex-based algorithm (Mathews and Fink, 2004; The Mathworks, Inc., 2011) is used.

The proposed MoReRT design for 2DoF controllers is applied in chapter 4 (PI_2) and chapter 6 (PID_{2F}) to control a diversity of models representative of common industrial controlled processes.

Chapter remarks

The groundings of the proposed model reference robust tuning (MoReRT) method are presented.

It starts with the selection of regulatory control and servo control closed-loop transfer functions targets $M_{yd}^t(s)$ and $M_{yr}^t(s)$, respectively. These transfer functions state the desired shapes for the disturbance and set-point step responses $y_d^t(t)$ and $y_r^t(t)$.

The responses targets specification must take into account the controlled process model characteristics, particularly its non-minimum phase components and order. They depend also on the design parameters θ_d that affect the control system performance, control effort, and robustness.

Controller parameters θ_c are obtained by optimizing a cost functional that evaluate the difference between the target total response and the one obtained with the controller to tune. Design parameters are adjusted to obtain a robustness level target, M_S^t.

4 MoReRT design for 2DoF PI controllers

In chapter 3 the proposed model reference robust tuning design methodology is described. In the following, this procedure is applied to obtain robust PI controllers to control over damped, integrating, and unstable processes.

The controller has a 2DoF PI control algorithm whose output is as follows (Åström and Hägglund, 1995):

$$u(s) = K_p \left\{ \beta r(s) - y(s) + \frac{1}{T_i s}\left[r(s) - y(s)\right] \right\}, \tag{4.1}$$

where K_p is the controller *proportional gain*; T_i, the *integral time constant*; and β, the *proportional set-point weight*. Then, the controller parameters to tune are $\theta_c = \{K_p, T_i, \beta\}$.

For the PI controller (4.1) the regulatory control closed-loop transfer function target (3.7) is

$$M_{yd}^t(s) = \frac{(T_i/K_p)sN_p^+(s)}{D_M(\theta_p, \theta_{cy}, \theta_d, s)}, \tag{4.2}$$

where $D_M(\theta_p, \theta_{cy}, \theta_d, s)$ is the denominator of all the control system closed-loop transfer functions with $D_M(s=0) = 1$. The number of closed-loop poles depends on the controlled process model order.

Using the controller (4.1) aspect applied to the set-point and (4.2) into (3.9) the servo control transfer function target is

$$M_{yr}^t(s) = \frac{(\beta T_i s + 1)N_p^+(s)}{D_M(\theta_p, \theta_{cy}, \theta_d, s)}. \tag{4.3}$$

4.1 Over damped controlled processes

Most of the industrial processes are self-regulated and over damped and usually represented by a first- or second-order plus dead-time (FOPDT, SOPDT) linear model. The application of the 2DoF PI MoReRT methodology to this models is introduced in Alfaro and Vilanova (2012g) and fully explained in Alfaro and Vilanova (2012c).

The over damped controlled process (first- and second-order) are represented by a linear model given by the transfer function

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{(Ts+1)(aTs+1)}, \tag{4.4}$$

where K is the model gain, T the main time constant, a the ratio of the two time constants ($0 \leq a \leq 1.0$), and L the process apparent dead-time. The controlled process parameters are $\theta_p = \{K, T, a, L\}$,

Using the controlled process model gain K, and time constant T, as well as the transformation $\hat{s} \doteq Ts$, the controlled process model (4.4) can be expressed in normalized form as follows:

$$\hat{P}(\hat{s}) = \frac{\mathrm{e}^{-\tau_L \hat{s}}}{(\hat{s}+1)(a\hat{s}+1)}, \tag{4.5}$$

where $\tau_L \doteq L/T$ is its *normalized dead-time*.

The over damped second-order plus dead-time (SOPDT) normalized model (4.5) has two parameters, $\hat{\theta}_p = \{a, \tau_L\}$. For the particular case of the first-order plus dead-time (FOPDT) model ($a = 0$) it has only one, $\hat{\theta}_p = \tau_L$.

For FOPDT and SOPDT models the controller (4.1) parameters K_p and T_i can be expressed in normalized form as follows:

$$\kappa_p \doteq KK_p, \quad \tau_i \doteq \frac{T_i}{T}. \tag{4.6}$$

For (4.4) the PI closed-loop control system is of third-order.

Over damped closed-loop response target

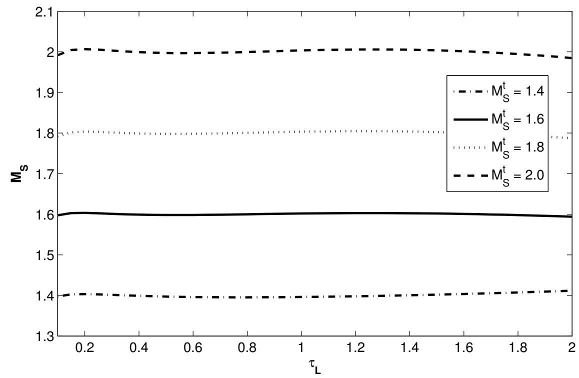
In order to obtain a non-oscillatory controlled variable and, as a side effect, a smooth controller output the closed-loop transfer functions targets (4.2) and (4.3) are stated such that the global control system output target $y^t(s)$ (3.10) is as follows:

$$y^t(s) = \frac{\mathrm{e}^{-Ls}}{(\tau_c Ts+1)(a\tau_c Ts+1)} r(s) + \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c Ts+1)^2(a\tau_c Ts+1)} d(s), \tag{4.7}$$

where τ_c is a dimensionless design parameter, which is an indication of the closed-loop system response speed in relation to the controlled process speed.

Using (4.7) with (3.12) and (3.13) for optimizing (3.14) the closed-loop response relative speed τ_c is adjusted to obtain a target robustness level M_S^t. The resulting controller

Figure 4.1: MoReRT PI controllers robustness accomplishment for first-order plus dead-time models



normalized parameters (4.6) can be expressed as functions of the controlled process model (4.5) parameters and the design robustness target M_S^t as follows:

$$\kappa_p = \frac{a_0 + a_1\tau_L}{a_2 + a_3\tau_L + a_4{\tau_L}^2 + a_5{\tau_L}^3}, \tag{4.8}$$

$$\tau_i = \frac{b_0 + b_1\tau_L}{b_2 + b_3\tau_L + b_4{\tau_L}^2 + b_5{\tau_L}^3 + b_6{\tau_L}^4}, \tag{4.9}$$

$$\beta = c_0 + c_1\tau_L + c_2{\tau_L}^2 + c_3{\tau_L}^3. \tag{4.10}$$

The a_i, b_i and c_i constants in expressions (4.8) to (4.10), are presented in Alfaro and Vilanova (2012c) for four robustness target levels ($M_S^t \in \{1.4,\ 1.6,\ 1.8,\ 2.0\}$); and models with six time constants ratios ($a \in \{0.0,\ 0.1,\ 0.25,\ 0.50,\ 0.75,\ 1.0\}$) and normalized dead-time τ_L in the range from 0.1 to 2.0.

The robustness obtained with (4.8) and (4.9) for FOPDT models are shown in figure 4.1. Robustness accomplishment for SOPDT models is presented in Alfaro and Vilanova (2012c).

Under damped closed-loop response target

In order to analyze if it is possible to modify the control system performance to a load-disturbance and set-point step changes without affecting its robustness a new global control system output target $y^t(s)$ with two under damped dominant poles is selected

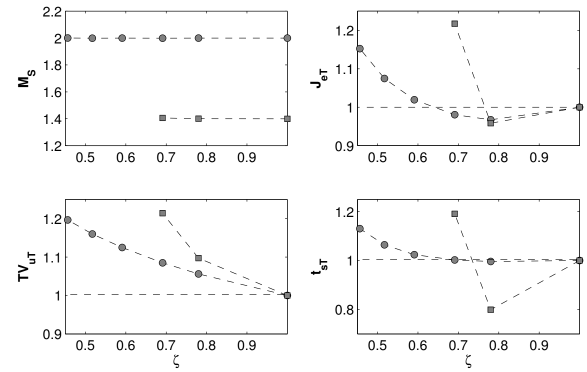


Figure 4.2: Damping ratio ζ effect over total performance, control effort variation, and settling-time for model with ($a = 0.75$, $L = 0.50$)

in Alfaro and Vilanova (2012f) and computed as

$$y^t(s) = \frac{(\tau_c T s + 1)\mathrm{e}^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(a\tau_c T s + 1)} r(s) + \frac{(T_i/K_p) s e^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(a\tau_c T s + 1)} d(s). \quad (4.11)$$

Now the control system has two design parameters, the *closed-loop poles relative speed* τ_c, and its *damping ratio* ζ. The effect of the damping ratio ζ over the closed-loop control systems performance and control effort characteristics is analyzed in Alfaro and Vilanova (2012f).

The regulatory control response indices evaluated are the integrated absolute error (J_{ed}), the controller output total variation (TV_{ud}), the maximum error (E_{max}), the time to reach the maximum error (t_{max}), and the settling time ($t_{5\%Emax}$). For the servo control response the indices evaluated are the integrated absolute error (J_{er}), the controller output total variation (TV_{ur}), the rise time (t_r), the control effort maximum value (U_{maxr}), the controller output instant change (ΔU_{0r}), and the settling time ($t_{5\%\Delta y}$). These indices are defined in Alfaro and Vilanova (2012f).

The analysis made use of damping ratios ζ in the range from 1.0 to 0.456 and robustness levels M_S from 2.0 to 1.40. Figure 4.2 shows the indices obtained for the model with ($a = 0.75$, $L = 0.5$). Case analysis is presented in Alfaro and Vilanova (2012f).

The analysis shows that, for same robustness, all the controllers obtained with the original non-oscillatory response target ($\zeta = 1.0$) provide the smoothest control efforts with an integrated absolute error performance similar to the lower obtainable value for the corresponding robustness level target.

An improvement in the control system performance (integrated absolute error and settling time); specially for the servo control; may be obtained if the closed-loop transfer

Table 4.1: Over damped model example, $M_S^t = 2.0$, effect of ζ over the control system performance and control effort

ζ	1.0	0.78	0.591	0.456
J_{ed}	1.406	1.410	1.495	1.697
TV_{ud}	1.627	1.648	1.683	1.737
J_{er}	2.158	2.038	2.139	2.411
TV_{ur}	2.111	2.345	2.590	2.774
U_{maxr}	1.327	1.442	1.560	1.642
ΔU_{or}	0.726	0.811	0.885	0.907
J_{eT}	3.564	3.448	3.634	4.108
TV_{uT}	3.738	3.993	4.273	4.511

function poles design damping ratio is selected in the range from 0.7 to 0.8 but adversely affecting the control effort characteristics.

Based on the analysis for over damped controlled processes it is recommended to use $\zeta = 0.8$ for MoReRT 2DoF PI design when under damped target responses are specified.

As an example of the effect of the damping ratio ζ over the control systems performance and control effort characteristics, consider the second-order plus dead-time normalized model given by

$$P_2(s) = \frac{e^{-0.5s}}{(s+1)(0.75s+1)}. \tag{4.12}$$

The regulatory and servo control performance J_e (measured with the integrated absolute error) and control effort total variation TV_u, maximum value U_{max}, and instant change ΔU_o for a robustness level target $M_S^t = 2.0$ and four damping ratios are listed in table 4.1.

The table data confirms that, keeping the control system robustness constant, the servo control performance can be improved, obtaining lower J_{er}, if the closed-loop poles damping ratio is decreased a bit, but affecting the control effort characteristics, TV_{ur}, U_{maxr}, ΔU_{or}. It also shows that very low closed-loop poles damping ratios deteriorate all control system characteristics.

Control system responses for $M_s^t = 2.0$ and four damping ratios are shown in figure 4.3.

Further evaluations of MoreRT tuning (4.8) to (4.10) and comparison with other robust tuning methods for over damped models are presented in Alfaro and Vilanova (2012c).

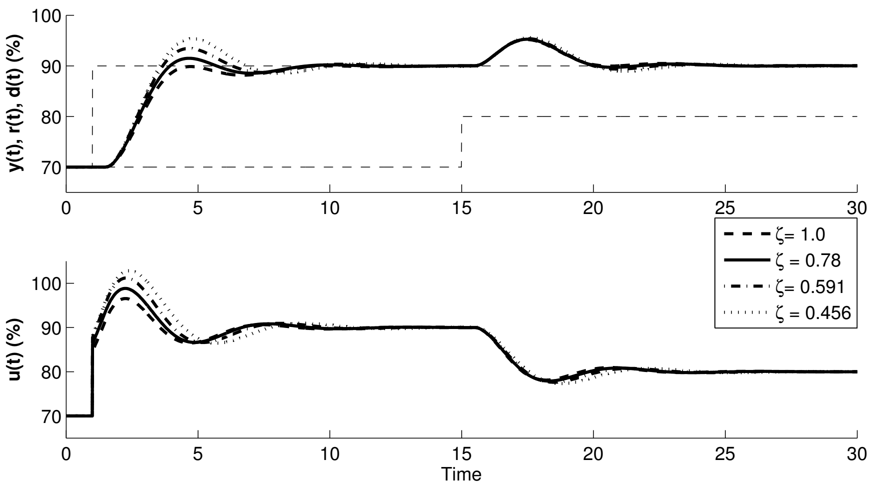


Figure 4.3: Over damped model example, control system responses ($M_S^t = 2.0$)

4.2 Integrating controlled processes

Even though most of the controlled processes found in the process industry are self-regulating, i.e. the process output seeks a stable operating point under a constant input, there are others that under a constant input their output is unbounded, rise or decrease without limit. These non-self-regulated processes are named integrating or unstable if their model transfer functions have a pole at the s-plane origin or at its right-half plane, respectively. Stable processes with very long time constants may also be approximated by integrating models. Integrating and unstable processes may be operated only under closed-loop automatic control and their controller tuning needs a special treatment.

The extension of the MoReRT methodology to integrating plus dead-time (IPDT) and integrating second-order plus dead-time (ISOPDT) models is presented in Alfaro and Vilanova (2012d).

Integrating second-order plus dead-time models

The integrating second-order plus dead-time (ISOPDT) model is given by the following transfer function:

$$P(s) = \frac{K e^{-Ls}}{s(Ts+1)}, \tag{4.13}$$

were K is the gain, T the time constant and L the dead-time. The controlled process parameters are $\theta_p = \{K, T, L\}$.

Using the controlled process parameters as well as the transformation $\hat{s} \doteq Ts$, the controlled process (4.13) and the PI controller parameters can be expressed in a normalized form as follows:

$$\hat{P}(\hat{s}) = \frac{e^{-\tau_L \hat{s}}}{\hat{s}(\hat{s}+1)}, \tag{4.14}$$

where $\tau_L \doteq L/T$ is the model normalized dead-time and

$$\kappa_p \doteq K_p\, K\, T, \quad \tau_i \doteq \frac{T_i}{T}, \tag{4.15}$$

are the *normalized gain* and *normalized integrating time* of the controller, respectively. Normalized model (4.14) has only one parameter, τ_L .

For (4.14) the PI control system is of third-order.

For the ISOPDT models the non-oscillatory global control system output target $y^t(s)$ (3.10) is computed as:

$$y^t(s) = \frac{e^{-Ls}}{(\tau_c Ts+1)^2} r(s) + \frac{(T_i/K_p)s e^{-Ls}}{(\tau_c Ts+1)^3} d(s). \tag{4.16}$$

The PI_2 controller parameters obtained following the optimization procedure stated in chapter 3 are used to fit the 2DoF PI controller normalized parameters equations as functions of the controlled process model (4.14) parameters and the design robustness M_S^t specified and given by

$$\kappa_p = \frac{a_0 + a_1 \tau_L}{a_2 + a_3 \tau_L + \tau_L{}^2}, \tag{4.17}$$

$$\tau_i = b_0 e^{b_1 \tau_L} + b_2 e^{b_3 \tau_L}, \tag{4.18}$$

$$\beta = c_0 + c_1 \tau_L + c_2 \tau_L{}^2. \tag{4.19}$$

The a_i, b_i and c_i constants for relations (4.17) to (4.19) for four robustness target levels ($M_S^t \in \{1.4,\ 1.6,\ 1.8,\ 2.0\}$) are presented in Alfaro and Vilanova (2012d).

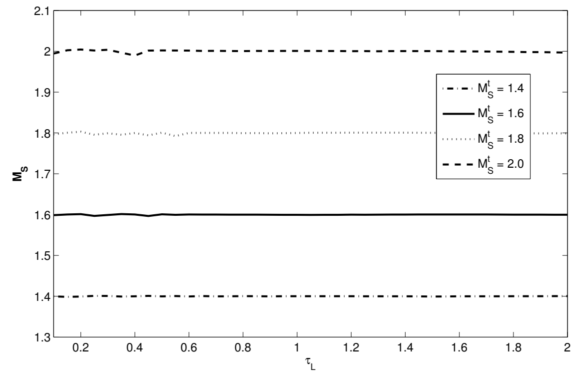


Figure 4.4: MoReRT PI controllers robustness accomplishment for integrating second-order plus dead-time models

The robustness obtained with (4.17) and (4.18) for ISOPDT models is shown in figure 4.4.

It is important to note that other available robust tuning rules for ISOPDT models do not produce control systems with a constant robustness level across their applicability range. Robustness evaluations are presented in Alfaro and Vilanova (2012d).

Integrating plus dead-time models

Integrating and over damped processes with very large time constants can also be approximated by an integrating plus dead-time (IPDT) model given by the following:

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{s}, \tag{4.20}$$

were K is the gain and L the dead-time. The controlled process parameters are $\theta_p = \{K, L\}$.

In this case the PI control system is of second-order and the over damped closed-loop control controlled variable target (3.10) is selected as

$$y^t(s) = \frac{\mathrm{e}^{-Ls}}{\tau_c Ls + 1} r(s) + \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c Ls + 1)^2} d(s), \tag{4.21}$$

where τ_c is the design parameter.

Using the controlled process parameters θ_p as well as the transformation $\tilde{s} \doteq Ls$, the controlled process (4.20) and the PI controller parameters can be expressed in a normalized form as follows:

$$\tilde{P}(\tilde{s}) = \frac{\mathrm{e}^{-\tilde{s}}}{\tilde{s}}, \tag{4.22}$$

$$\kappa_p \doteq K_p\, K\, L, \quad \tau_i \doteq \frac{T_i}{L}, \tag{4.23}$$

where κ_p and τ_i are the *normalized gain* and *normalized integrating time* of the controller, respectively.

As the normalized controlled process model (4.22) does not have any variable parameter just one optimization run is required for each robustness target level.

In the same way as with the ISOPDT model, during the optimization processes, the closed-loop relative speed parameter τ_c is selected in such a way that the robustness level of the resulting closed-loop system met a specific target (M_S^t) in the range from 1.4 to 2.0. The controller parameters are obtained directly as functions of only the closed-loop control system robustness.

The MoReRT PI_2 tuning equations for the IPDT models are as follows:

$$\kappa_p = a, \tag{4.24}$$

$$\tau_i = b, \tag{4.25}$$

$$\beta = c. \tag{4.26}$$

Constants a, b and c for (4.24) to (4.26) for four robustness target levels ($M_S^t \in \{1.4,\ 1.6,\ 1.8,\ 2.0\}$) are presented in Alfaro and Vilanova (2012d). Relations (4.24) to (4.26) are valid for IPDT models with any non-zero dead-time.

Under damped closed-loop response targets

For the analysis of the effect of using a closed-loop under damped response target for the integrating processes, IPDT and ISOPDT models, a new global control system output target $y^t(s)$ is computed in Alfaro and Vilanova (2012i) as

$$y^t(s) = \frac{\mathrm{e}^{-Ls}}{\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1} r(s) + \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(\tau_c T s + 1)} d(s), \tag{4.27}$$

replacing (4.16) for the integrating second-order plus dead-time models, and as

$$y^t(s) = \frac{(\tau_c L s + 1)\mathrm{e}^{-Ls}}{\tau_c^2 L^2 s^2 + 2\zeta\tau_c L s + 1} r(s) + \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{\tau_c^2 L^2 s^2 + 2\zeta\tau_c L s + 1} d(s), \tag{4.28}$$

replacing (4.21) for the integrating plus dead-time models.

From the analysis of the closed-loop responses obtained with (4.27) and (4.28) and taking into account the performance (integrated absolute error) to control effort (total variation) trade-off, target responses with damping ratio $\zeta = 0.8$ are selected to optimize the 2DoF PI controllers for integrating processes (IPDT and ISOPDT).

Tuning equations of the form (4.17) to (4.19) are obtained for ISOPDT models and of the form (4.24) to (4.26) for IPDT models. The new constants for these equations are presented in Alfaro and Vilanova (2012i).

As an example of the MoReRT PI tuning for integrating processes, consider the integrating third-order transfer function:

$$P_3(s) = \frac{0.833\mathrm{e}^{-0.2s}}{s(0.833s+1)(0.1s+1)} \tag{4.29}$$

For tuning purposes the controlled process model (4.29) is approximated by following ISOPDT and IPDT models:

$$P_3(s) \approx \frac{0.833\mathrm{e}^{-0.353s}}{s(0.780s+1)} \approx \frac{0.833\mathrm{e}^{-1.133s}}{s} \tag{4.30}$$

The MoReRT PI_2 controller parameters for $M_S^t \in \{1.4,\ 1.6\}$ obtained with (4.30) models are listed in table 4.2 and the control system responses to a 20% set-point step change followed by a -5% disturbance step change are shown in figure 4.5.

Table 4.2, includes the robustness (M_S) resulting with the controlled process (note that this value, in practice, can not be obtained). From the table it can be seen that the robustness of the control systems with the controllers tuned by using the ISOPDT model are closer to the design robustness level than the ones obtained with the controllers tuned by using the IPDT model. This is interpreted as an indication that the ISOPDT model provides a better representation of the controlled process dynamics than the IPDT model.

The performance (J_e) and control effort (TV_u) indices shown on table 4.2 were obtained with unitary set-point and disturbance step changes.

At both robustness levels the regulatory performance of the controllers tuned with the ISOPDT model are higher that the ones obtained with the IPDT models and with similar servo control performance. On the other hand, the control effort total variation is slightly higher for the controllers tuned with the ISOPDT model.

Comparison of MoReRT controllers for ISOPDT and IPDT models with other tuning rules is presented in Alfaro and Vilanova (2012d), for relations (4.17) to (4.19) obtained with over damped response targets, and in Alfaro and Vilanova (2012i), for relations obtained with under damped response targets.

Table 4.2: Integrating models example, controller parameters, robustness, performance and control effort indices

Model	IPDT	IPDT	ISOPDT	ISOPDT
M_S^t	2.0	1.6	2.0	1.6
K_p	0.600	0.440	0.769	0.506
T_i	5.441	7.044	4.925	6.191
β	0.477	0.516	0.358	0.364
M_S	1.724	1.487	1.945	1.582
J_{ed}	9.068	16.067	6.404	12.235
TV_{ud}	1.847	1.649	2.065	1.726
J_{er}	3.195	4.007	3.165	4.011
TV_{ur}	0.781	0.555	0.874	0.513
J_{eT}	12.263	20.074	9.569	16.246
TV_{uT}	2.629	2.205	2.939	2.239

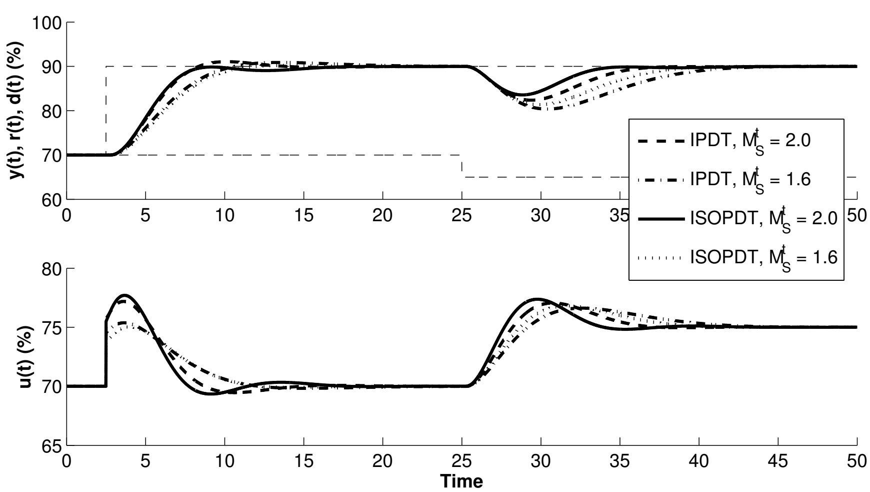


Figure 4.5: Integrating model example, control system responses

4.3 Inverse response controlled processes

The inverse response characteristic, i.e. the controlled process initial response to a step change is in the opposite direction to that of the steady-state direction, originated by two parallel competing dynamics it is present in industrial processes as distillation columns and chemical reactors. This non-minimum phase characteristics impose severe limits to the achievable closed-loop control system robustness.

The application of the proposed MoReRT tuning procedure to inverse response processes is presented in Alfaro and Vilanova (2012o).

The controlled processes with inverse response will be represented by a second-order plus right-half plane zero (SOPRHPZ) model given by the transfer function

$$P(s) = \frac{K(-bTs+1)}{(Ts+1)(aTs+1)}, \tag{4.31}$$

where K is the model gain, T the time constant, a the ratio of the two poles time constants ($0.1 \leq a \leq 1.0$), and b the relative position of the right-half plane zero.

For the SOPRHPZ model the PI control system is of third-order and the global control system output target $y^t(s)$ (3.10) is computed as

$$y^t(s) = \frac{-bTs+1}{(\tau_c Ts+1)(a\tau_c Ts+1)} r(s) + \frac{(T_i/K_p)s(-bTs+1)}{(\tau_c Ts+1)^2(a\tau_c Ts+1)} d(s), \tag{4.32}$$

where τ_c is the dimensionless design parameter.

Using the controlled process parameters and the transformation $\hat{s} \doteq Ts$, the controlled process and the 2DoF PI parameters can be expressed in a normalized form as follows:

$$\hat{P}(\hat{s}) = \frac{(-b\hat{s}+1)}{(\hat{s}+1)(a\hat{s}+1)}, \tag{4.33}$$

$$\kappa_p \doteq K_p K, \quad \tau_i \doteq \frac{T_i}{T}. \tag{4.34}$$

Normalized model (4.33) has two dimensionless parameters, a and b.

The controller normalized parameters (κ_p, τ_i, β) are obtained for right-half plane zero relative positions b in the range from 0.25 to 4.0 and time constants ratios a from 0.1 to 1.0.

It was noted that the position of the right-half plane zero affects the robustness level that may be obtained. Roughly $M_S = 2.0$ may be obtained up to $b \approx 4.0$, $M_S = 1.8$ up to $b \approx 3.5$, $M_S = 1.6$ up to $b \approx 2.25$ and $M_S = 1.4$ only up to $b \approx 1.25$. However at

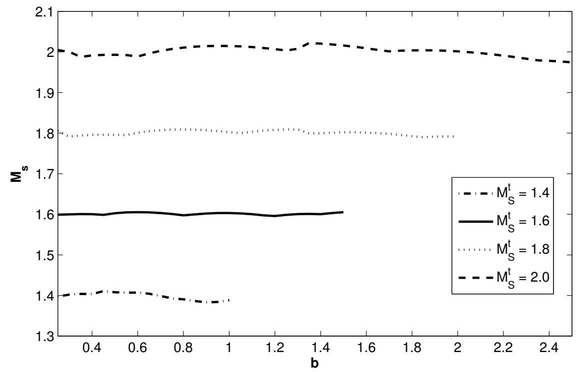


Figure 4.6: MoReRT PI controllers robustness accomplishment for second-order plus right-half plane zero models ($a = 0.25$)

the upside of all the b ranges the controller gains turn very low, with up to a 20 : 1 ratio between the highest and lowest values. Then, to obtain the controller tuning relations, the zero relative position b was selected from 0.25 to 1.0, 1.5, 2.0, and 2.5 for $M_S^t = 1.4$, 1.6, 1.8, and 2.0, respectively.

The PI_2 controller parameters obtained are used to fit the MoReRT equations for the normalized controller parameters and the proportional set-point weight for each one of the five time constants ratios a considered, given by:

$$\kappa_p = a_0 + a_1 b^{a_2}, \tag{4.35}$$

$$\tau_i = \frac{b_0 + b_1 b}{b_2 + b_3 b + b_4 b^2 + b_5 b^3 + b_6 b^4}, \tag{4.36}$$

$$\beta = c_0 + c_1 b + c_2 b^2 + c_3 b^3. \tag{4.37}$$

Constants a_i, b_i and c_i for expressions (4.35) to (4.37) are presented in Alfaro and Vilanova (2012o) for four robustness target levels ($M_S^t \in \{1.4,\ 1.6,\ 1.8,\ 2.0\}$) and five model time constants ratios ($a \in \{0.1,\ 0.25,\ 0.5,\ 0,75,\ 1.0\}$).

The robustness obtained with (4.35) and (4.36) for SOPRHPZ models is shown in figure 4.6 for the particular case of models with a time constant ratio $a = 0.25$ and right-half plane zero positions b in the ranges where tuning relations are valid for. Robustness accomplishment for other a values are shown in Alfaro and Vilanova (2012o).

As an example of the MoReRT control of an inverse response process, consider the SOPRHPZ model given by the transfer function:

$$P_4(s) = \frac{-0.80s + 1}{(s+1)(0.4s+1)}. \tag{4.38}$$

Table 4.3: Inverse response model example, PI_2 controller parameters

M_S^t	K_p	T_i	β
1.4	0.297	1.006	1.471
1.6	0.472	1.243	1.188
1.8	0.588	1.340	1.183
2.0	0.672	1.388	1.173

Table 4.4: Inverse response model example, robustness, performance and control effort indices

M_S^t	M_S	$J_{er}/\Delta r$	$J_{ed}/\Delta d$	$TV_{ur}/\Delta r$	$TV_{ud}/\Delta d$
1.4	1.40	2.923	3.756	1.000	1.236
1.6	1.61	2.401	3.013	1.117	1.377
1.8	1.82	2.044	2.676	1.541	1.486
2.0	1.99	1.988	2.471	1.957	1.682

The PI_2 controller parameters for (4.38) are shown in table 4.3.

Table 4.4 shows the robustness (M_S), performance (J_e), and control effort total variation (TV_u) obtained for four target robustness levels (M_S^t) to a set-point step change (Δr) and a disturbance step change (Δd). These data show the trade-offs between the control system *robustness*, its *performance*, and the *control effort*. An increment in the control system target robustness (reducing M_S^t) reduces its performance (increases J_{er} and J_{ed}) but made the control effort smoother (reduces TV_{ur} and TV_{ud}).

The corresponding closed-loop responses to a 20% set-point followed by a 10% disturbance step change are shown in figure 4.7.

4.4 Unstable controlled processes

Open-loop unstable processes are presented in chemical industrial systems and are know to be difficult to control particularly if they include dead-time.

The MoReRT methodology is applied to unstable controlled processes in Alfaro and Vilanova (2012l).

The unstable processes are represented by a first-order model with a pole at the right-half plane plus dead-time (UFOPDT) given by the following:

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{Ts-1}, \tag{4.39}$$

were K is the gain, T the time constant, and L the process apparent dead-time. The controlled process parameters are $\theta_p = \{K, T, L\}$.

For (4.39) the PI control system is of second-order.

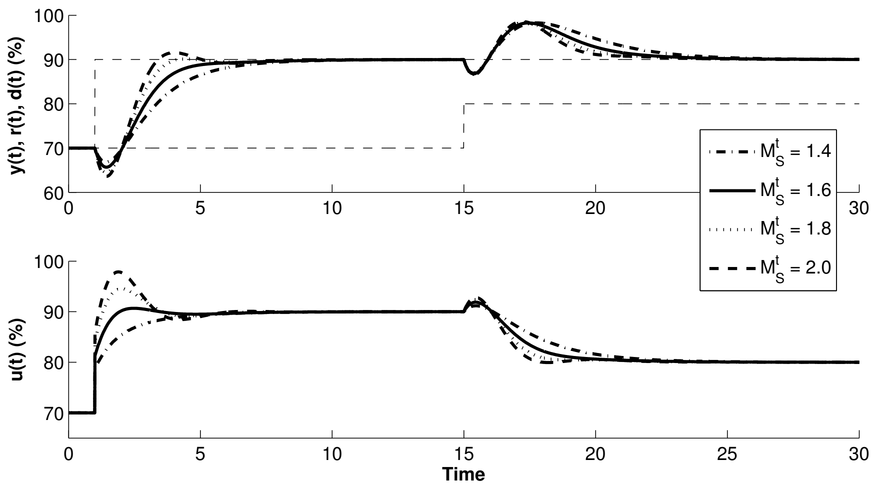


Figure 4.7: Inverse response model example, control system responses

In this case due to the limitations imposed by the unstable process it is not possible to obtain a first-order dynamics for the servo-control response by forcing $\beta \to \tau_c T/T_i$ as was made for first-order stable models. Then, in the UFOPDT model case the global control system output target $y^t(s)$ (3.10) is computed as

$$y^t(s) = \frac{(\beta T_i s + 1)\mathrm{e}^{-Ls}}{(\tau_c T s + 1)^2} r(s) + \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c T s + 1)^2} d(s). \tag{4.40}$$

Using the controlled process model parameters as well as the transformation $\hat{s} \doteq Ts$, the controlled process (4.39) and the 2DoF PI controller parameters can be expressed in a normalized form as follows:

$$\hat{P}(\hat{s}) = \frac{\mathrm{e}^{-\tau_L \hat{s}}}{\hat{s} - 1}, \tag{4.41}$$

$$\kappa_p \doteq K_p\, K, \quad \tau_i \doteq \frac{T_i}{T}, \tag{4.42}$$

where κ_p and τ_i are the *normalized gain* and *normalized integrating time* of the controller, respectively.

The normalized controlled process model (4.41) has only one dimensionless parameter, τ_L. For a given τ_L during the optimization procedure the closed-loop relative speed parameter τ_c is selected in such a way that the robustness level of the resulting closed-loop system met a specific target (M_S^t).

From the optimization results it is found that, the dead-time of the unstable processes imposes a severe limitation on the control system achievable robustness level. A closed-loop control systems with high robustness of $M_S^t = 1.4$ may only be obtainable for UFOPDT models with $\tau_L \leq 0.10$, the robustness $M_S^t = 1.6$ for $\tau_L \leq 0.15$, the $M_S^t = 1.8$ for $\tau_L \leq 0.20$, and the robustness $M_S^t = 2.0$ for models with $\tau_L \leq 0.26$. Robust control systems may be obtained for a very limited range of unstable models.

Although the usual control system minimum robustness level for stable processes corresponds to $M_S = 2.0$ in the unstable processes case the main control system purpose is to stabilize the process. Then, for the UFOPDT models the robustness level target is relaxed and the controller parameters obtained for M_S^t in the range from 2.0 to 6.0. The optimization also shows that for all cases the set-point weight $\beta = 0$.

Then, the controller parameters are obtained as functions of the model (4.41) parameter and of the closed-loop control system robustness target M_S^t with following relations:

$$\kappa_p = a_0 + a_1 {\tau_L}^{a_2}, \tag{4.43}$$

$$\tau_i = \frac{b_0 + b_1 \tau_L}{b_2 + b_3 \tau_L + {\tau_L}^2}, \tag{4.44}$$

$$\beta = 0. \tag{4.45}$$

The a_i and b_i constants for expressions (4.43) and (4.44) for five robustness levels ($M_S^t \in \{2.0,\ 3.0,\ 4.0,\ 5.0,\ 6.0\}$) are presented in Alfaro and Vilanova (2012l).

The robustness obtained with the MoReRT tuning equations (4.43) and (4.44) are shown in figure 4.8.

As an example of MoReRT PI control of unstable processes, consider the UFOPDT model

$$P_5(s) = \frac{e^{-0.2s}}{s-1}. \tag{4.46}$$

Table 4.5 shows the robustness (M_S), performance (J_e), and control effort total variation (TV_u) obtained for five target robustness levels (M_S^t) to a set-point step change (Δr) and a disturbance step change (Δd) of the MoReRT PI_2 control of the unstable model (4.46). Figure 4.9 shows the corresponding responses to a 20% set-point step change followed by a 10% load-disturbance step change for four robustness levels.

Comparison of MoReRT (4.43) to (4.45) for UFOPDT models with other tuning rules is presented in Alfaro and Vilanova (2012l).

Figure 4.8: MoReRT PI controllers robustness for unstable first-order plus dead-time models

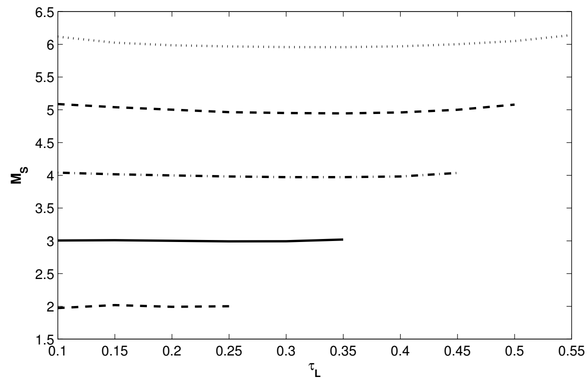


Table 4.5: Unstable model example, robustness, performance and control effort indices

M_S^t	M_S	$J_{ed}/\Delta d$	$TV_{ud}/\Delta d$	$J_{er}/\Delta r$	$TV_{ur}/\Delta r$
2.0	1.99	1.101	2.633	1.749	1.670
3.0	3.00	0.389	3.187	1.020	2.653
4.0	4.00	0.300	4.365	0.845	4.109
5.0	5.00	0.311	5.620	0.782	5.893
6.0	5.99	0.337	6.864	0.783	7.704

Chapter remarks

The model reference robust tuning (MoReRT) design is apply to obtain tuning relations for 2DoF PI controllers (PI_2) and models representing common dynamics found in industrial processes.

For self-regulated over damped processes represented by first- and second-order plus dead-time (FOPDT, SOPDT) models tuning relations are obtained in Alfaro and Vilanova (2012c) by using closed-loop over damped responses targets. These tuning relations guarantee the accomplishment of four robustness levels for models with normalized dead-times τ_L in the range from 0.1 to 2.0.

The four robustness design levels gone from the normal minimum corresponding to $M_S^t = 2.0$ to a high robustness design with $M_S^t = 1.4$.

Performance comparison analysis show that for a high-order controlled process more performance is obtained with PI_2 controllers if they are tuned using a process SOPDT model approximation in lieu of a simpler FOPDT model.

In Alfaro and Vilanova (2012f) it is shown that an improvement of the control system performance, measured with the integrated absolute error, can be obtained if the design

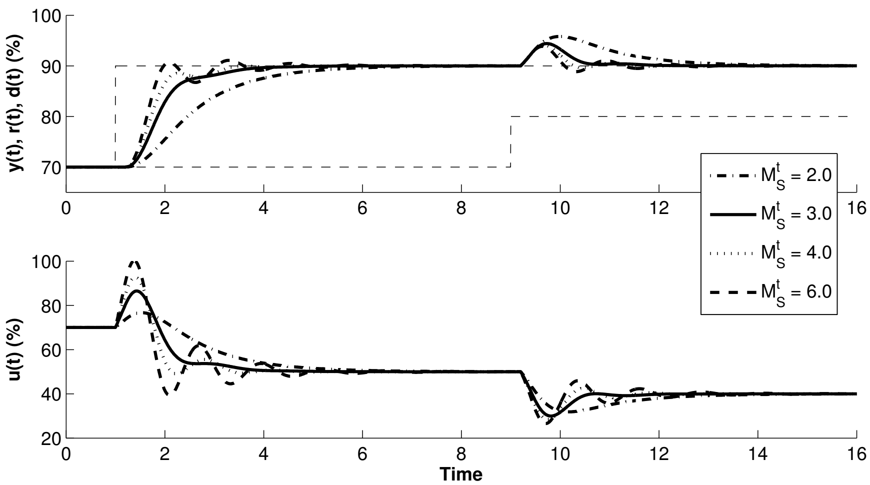


Figure 4.9: Unstable model example, control system responses

is made using closed-loop transfer functions targets with two dominant under damped poles with damping rations in the range from 0.7 to 0.8. The reduction of the dominant poles damping ratio increases the control effort total variation.

For process with integrating characteristics represented by integrated plus dead-time (IPDT) and integrated second-order plus dead-time (ISOPDT) models tuning relations are obtained in Alfaro and Vilanova (2012d) using over damped target responses and four robustness levels.

Using closed-loop under damped response targets an analysis of the closed-loop poles damping ratio influence over the performance/control effort trade-off is made in Alfaro and Vilanova (2012i). Improvements in performance obtained reducing the damping ratio depend on the control system robustness turned smaller as the robustness level target is increased.

Tuning relations for IPDT and ISOPDT are obtained in Alfaro and Vilanova (2012i) with the recommended damping ratio, $\zeta = 0.8$. These are valid for ISOPDT models with normalized dead-times τ_L from 0.1 to 2.0. Relations for IPDT models are valid for any non-zero dead-time L.

Performance comparison using an integrating third-order process show that more performance is obtained with PI_2 controllers tuned using the ISOPDT model approximation than using the IPDT model.

Controller tuning relations for inverse response processes represented by SOPRHPZ models are obtained in Alfaro and Vilanova (2012j). The non-minimum phase zero position impose limitations to the achievable robustness. These limitations turn higher as the zero moves toward the origin (the model zero relative position b increases).

Tuning relations were obtained for model non-minimum phase zero relative position b ranges that depend on the robustness target level M_S^t selectable from 2.0 to 1.4.

For unstable processes represented by an unstable first-order plus dead-time (UFOPDT) model PI_2 tuning relations are presented in Alfaro and Vilanova (2012k). In this case tuning relations for robustness corresponding to $M_S^t = 2.0$ can only be obtained for models with a very limited model normalized dead-time τ_L range. Robustness requirements were relaxed and tuning relations were obtained for five M_S^t levels in the range from 6.0 to 2.0.

Tuning relations for all controlled process models considered, over damped, integrating, inverse response, and unstable, were obtained following the proposed *model reference robust tuning* (MoReRT) design procedure. All of them guarantee achievement of the robustness target level selected.

5 Controllers fragility

One of the main characteristics of the MoReRT design methodology applied in chapter 4 for 2DoF PI control of over damped, integrating, and unstable controlled processes is to obtain control systems with the design M_S^t robustness level for all the models considered.

Even if the control system robustness level is guarantee, there is another consideration that must be taken into account in the design process: this is the effect of the variation of the controller parameters over the control system robustness and performance, know as the *controller fragility*.

The concept of controller *robustness fragility* as a measure of the control system loss of robustness is introduced in Alfaro (2007). It is used in conjunction with control system robustness evaluation in Alfaro and Vilanova (2011). The controller fragility analysis is extended to its *performance fragility* and used for evaluation of PI and PID tuning rules in Alfaro and Vilanova (2012h).

5.1 Delta 20 fragility indices

For the fragility analysis, the controlled process will be represented by a nominal model of the fixed parameters, θ_p^o, obtained at the control system normal operation point. This model is used for tuning the controller; then, the controller nominal parameters are θ_c^o.

Controller robustness fragility

The controller *Delta 20 robustness-fragility index* relates the control system loss of robustness to its nominal robustness when a change of up to 20% occurs in one or more of the nominal controller parameters values and is given by the following:

$$RFI_{\Delta 20} \doteq \frac{M_{S\Delta 20}^m}{M_S^o} - 1, \tag{5.1}$$

where $M_{S\Delta 20}^m$ and M_S^o are the control system delta 20 extreme and the nominal maximum sensitivity, respectively.

A controller will be considered *robustness-fragile* if the control system loses more than 50% of its robustness when all its parameters change up to 20%; otherwise, it is *robustness-non-fragile*. In addition, a controller will be *robustness-resilient* if the control system does not lose more than 10% of its robustness when its parameters change up to 20%.

The relative influence of a change on a single controller parameter p_i over its robustness can be obtained with the *Parametric delta 20 robustness-fragility index* $RFI_{\delta 20}^{p_i}$.

Controller performance fragility

The controller *Delta 20 performance-fragility index* relates the control system loss of performance to its nominal performance when a change of up to 20% occurs in one or more of the nominal controller parameters values given by the following:

$$PFI_{\Delta 20} \doteq \frac{J_{e\Delta 20}^{m}}{J_e^o} - 1, \tag{5.2}$$

were $J_{e\Delta 20}^{m}$ and J_e^o are the delta 20 extreme and the nominal performance, respectively, measured by the integrated absolute error.

A controller will be considered *performance-fragile* if the control system loses more than 50% of its performance when all its parameters change up to 20%; otherwise, it is *performance-non-fragile*. In addition, a controller will be *performance-resilient* if the control system does not lose more than 10% of its performance when its parameters change up to 20%.

The controller performance fragility must be evaluated for the servo control response, $PFI_{r\Delta 20}$, and for the regulatory control response, $PFI_{d\Delta 20}$.

The relative influence of a change on a single controller parameter p_i over its performance can be obtained with the *Parametric delta 20 performance-fragility index* $PFI_{\delta 20}^{p_i}$.

5.2 MoReRT controllers fragility

The robustness-fragility plots of PI_2 controllers tuned with MoReRT relations (4.8) to (4.10) for FOPDT models are shown in figure 5.1. The controllers are all robustness-non-fragile but none is robustness-resilient. For the same robustness level, the controllers become more robustness fragile as τ_L increases but all are balanced-robustness controllers. It can also be seen that the controller robustness-fragility index $RFI_{\Delta 20}$, and the control systems target robustness M_S^t are inversely related.

For performance the regulatory control performance-fragility plots of the MoReRT controllers for FOPDT models are shown in figure 5.2 and the servo control performance-fragility plots in figure 5.3.

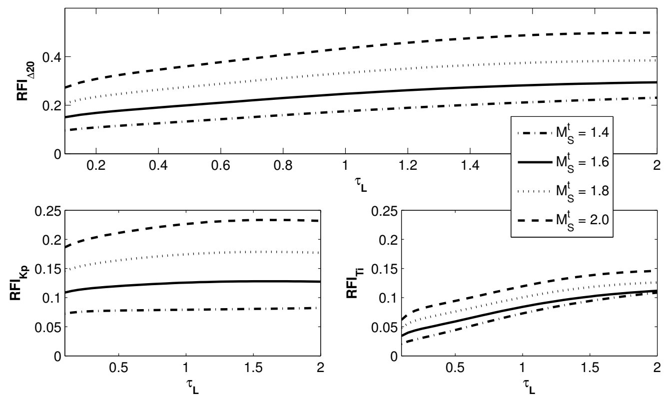


Figure 5.1: MoReRT PI controllers robustness-fragility, first-order plus dead-time models

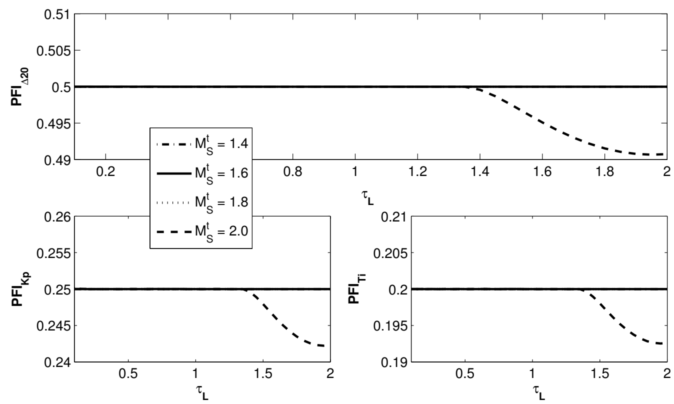


Figure 5.2: MoReRT PI controllers regulatory control performance-fragility, first-order plus dead-time models

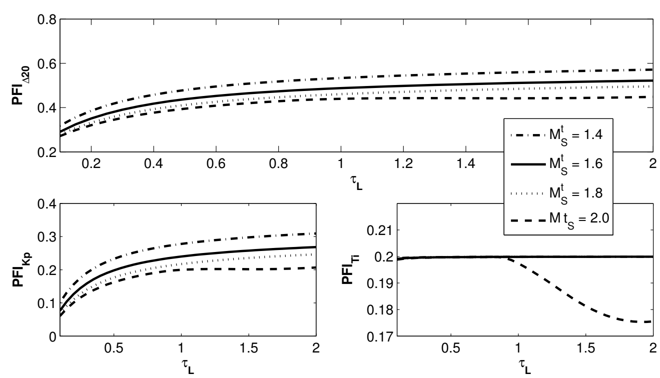


Figure 5.3: MoReRT PI controllers servo control performance-fragility, first-order plus dead-time models

For regulatory control, all controllers are at the border of the performance-fragile condition, losing 50% of the nominal performance if the controller parameters are changed by $\pm 20\%$, $PFI_{d\Delta 20} = 0.50$, while their parametric performance fragility indices are $PFI^{Kp}_{d\delta 20} = 0.25$ and $PFI^{Ti}_{d\delta 20} = 0.20$ with the exception of controllers tuned for lower robustness ($M^t_S = 2.0$) and particularly for $a \geq 0.50$. This is a result of the regulatory control non-oscillatory target response selected in (4.7) as deduced in Alfaro and Vilanova (2012c,h).

For servo control, the controller performance-fragility depends on the robustness level and the model normalized dead-time. The controller integral time parametric delta 20 performance-fragility is constant for all robustness levels and dead-times, but the controller gain has a variable influence over the loss of performance.

Similar robustness- and performance-fragility analysis of MoReRT PI_2 controllers for SOPDT models is presented in Alfaro and Vilanova (2012c).

It can be said that high controlled-process model normalized dead-times adversely affect the MoReRT control system robustness-fragility and its servo control performance-fragility but have no effect over its regulatory control performance-fragility.

The *Delta 20 robustness-fragility rings* plot introduced in Alfaro and Vilanova (2012h) is a simple graphic tool developed for robustness-fragility analysis. It uses the control system open-loop transfer function, $L(\mathrm{j}\omega)$, Nyquist curve of the nominal, and delta 20 perturbed controllers to provide an indication of the control system robustness-fragility. It shows the areas in the $L(\mathrm{j}\omega)$ plane that define when a controller is a robustness-resilient controller (RRC), a robustness-non-fragile controller (RNFC), or a robustness-fragile controller (RFC).

The fragility-rings plot is used in Alfaro and Vilanova (2012b) for robustness evaluation and comparison of several tuning rules.

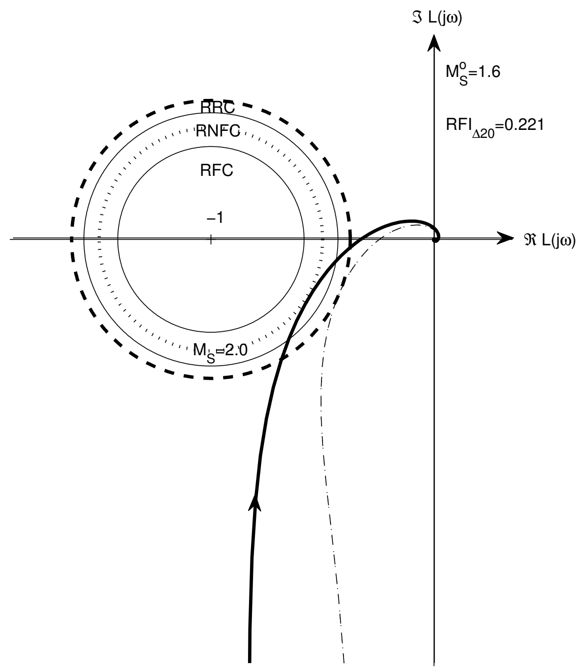
As an example of the fragility-rings plot use, consider a second-order plus dead-time model given by the transfer function:

$$P_6(s) = \frac{\mathrm{e}^{-0.277s}}{(0.876s+1)(0.719s+1)} \tag{5.3}$$

The MoReRT PI controller parameters for a robustness level $M^t_S = 1.6$ are $K_p = 1.14$ and $T_i = 1.465$.

Figure 5.4 shows the controller robustness-fragility rings plot for this control system. It can be seen that the Nyquist curve corresponding to the extreme maximum sensitivity, that gives the robustness-fragility index, enters the robustness-non-fragility controller (RNFC) ring. The controller is robustness-non-fragile with $RFI_{\Delta 20} = 0.221$.

Figure 5.4: MoReRT PI controller robustness-fragility rings plot example



It is also noticed that this Nyquist curve does not cross the $M_S = 2.0$ circle. The control system is robust ($M_S < 2.0$) even in case of a change of up to $\pm 20\%$ in all controller parameters.

Chapter remarks

At the closed-loop control system design stage robustness requirements are stated selecting the robustness target level to use with the tuning rules.

However, due to inaccuracies of the model used to represent the controlled process in the controller tuning procedure, the computed controller parameters should be taken only as a first approximation, requiring a final fine-tuning.

The effect of a $\pm 20\%$ change on the controllers parameters over the control systems robustness and the regulatory control and servo control performance can be foresee using the Delta 20 robustness-fragility and performance-fragility indices described in Alfaro and Vilanova (2012h).

Delta 20 controller robustness and performance fragility indices were used to define when a controller is considered robustness or performance fragile, non-fragile, or resilient.

Even more, the relative influence of a particular controller parameter can also be evaluated using the parametric delta 20 robustness fragility and parametric delta 20 performance fragility indices. These can be used to evaluate the controller fragility balance (robustness and performance).

Fragility analysis of controller tuned with several robust tuning rules are presented in Alfaro and Vilanova (2011, 2012h).

Robustness and performance fragility of MoReRT PI_2 controllers for FOPDT and SOPDT models is analyzed in deep in Alfaro and Vilanova (2012c).

The robustness-fragility rings plot in Alfaro and Vilanova (2012b,h) provides a simple visual indication of the controller robustness-fragility degree.

Controller fragility analysis provides important complementary information at the control system design stage on how controller parameters influence its robustness and performance.

6 MoReRT design for 2DoF PID controllers

The model reference robust tuning (MoReRT) design procedure presented in chapter 3 was applied in chapter 4 to tune two-degree-of-freedom proportional integral controllers (PI_2) to control over damped, integrating, inverse response, and unstable controlled process.

For PI_2 controllers the more control demanding models were those that represent processes with inverse or unstable responses. Then, the proposed design methodology is extended here to design 2DoF proportional integral derivative controllers (PID_2) to control such type of processes. Characteristics of the obtained PID_2 control systems are compared with the corresponding PI_2 ones.

While the PI control algorithm is implemented in the same way by manufactures this is not the case with the PID algorithm. The original three-term pneumatic “series” PID control algorithm implementation (Babb, 1990; Bennett, 2000) has evolved into the actual four- or five-term digital two-degree-of-freedom PID (Åström and Hägglund, 2006).

An analysis and conversion factors between 2DoF PID control algorithms implementations, “standard”, “parallel”, “series” or “industrial”, and “ideal with filter” is presented in Alfaro and Vilanova (2012a). Conversion factors take into consideration the controller filter.

It is shown that from these PID control algorithms the 2DoF Ideal PID with filter (PID_{2F}) given by the following output relation:

$$u(s) = K_p^* \left(\beta^* + \frac{1}{T_i^* s} \right) r(s) - K_p^* \left(1 + \frac{1}{T_i^* s} + T_d^* s \right) \left(\frac{1}{T_f s + 1} \right) y(s), \qquad (6.1)$$

is the more general one. Listed by their capabilities in descending order the 2DoF PID control algorithms are: ideal with filter, standard/parallel, and series/“industrial”.

The PID_{2F} controller (6.1) parameters to tune are $\theta_c^* = \{K_p^*,\ T_i^*,\ T_d^*,\ T_f,\ \beta^*,\ \gamma^* = 0\}$ and is implemented as shown in figure 6.1 for use in the following.

For the PID controller (6.1) the regulatory control closed-loop transfer function target (3.7) takes the form

$$M_{yd}^t(s) = \frac{(T_i^*/K_p^*)s(T_f s + 1)N_p^+(s)}{D_M(\theta_p, \theta_{cy}^*, \theta_d, s)}, \qquad (6.2)$$

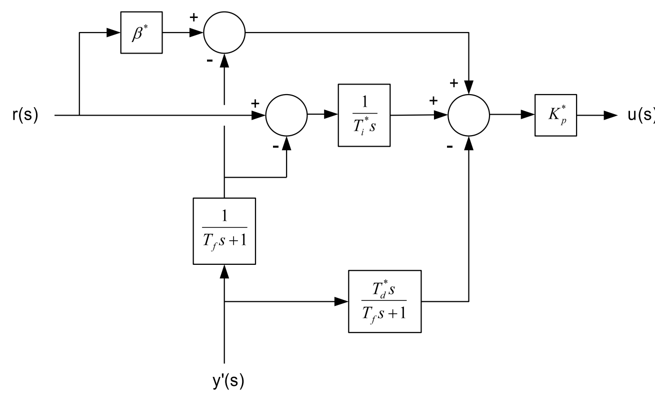


Figure 6.1: 2DoF Ideal PID with filter (PID_{2F}) control algorithm implementation

where $D_M(\theta_p, \theta_{cy}^*, \theta_d, s)$ is the denominator of all the control system closed-loop transfer functions with $D_M(s=0)=1$.

Using the controller (6.1) aspect applied to the set-point and (6.2) in (3.9) the servo control transfer function target is

$$M_{yr}^t(s) = \frac{(\beta^* T_i^* s + 1)(T_f s + 1)N_p^+(s)}{D_M(\theta_p, \theta_{cy}^*, \theta_d, s)}. \tag{6.3}$$

6.1 Inverse response controlled processes

The application of MoReRT design to 2DoF Ideal PID with filter controllers for inverse response processes is presented in Alfaro and Vilanova (2012j).

For the inverse response model given by the SOPRHPZ, transfer function

$$P(s) = \frac{K(-bTs+1)}{(Ts+1)(aTs+1)}, \tag{6.4}$$

with the PID_{2F} controller the closed-loop transfer functions denominator is of fourth-order. Taking into consideration the results in section 4.3, the regulatory control closed-loop transfer function target (6.2) is selected with two under damped dominant poles given by

$$M_{yd}^t(s) = \frac{(T_i^*/K_p^*)s(T_f s+1)(-bTs+1)}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(\tau_c T s + 1)(a\tau_c T s + 1)}, \tag{6.5}$$

where the closed-loop poles target damping ratio ζ and relative speed τ_c are the design parameters.

Selecting $T_f = \tau_c T$ in order to cancel the slower closed-loop pole reduces (6.5) to the following third-order regulatory control closed-loop transfer function target

$$M_{yd}^t(s) = \frac{(T_i^*/K_p^*)s(-bTs+1)}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c Ts+1)(a\tau_c Ts+1)}. \tag{6.6}$$

Using (6.6) the obtained servo control closed-loop transfer function target (6.3) is

$$M_{yr}^t(s) = \frac{(\beta^* T_i^* s+1)(-bTs+1)}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c Ts+1)(a\tau_c Ts+1)}. \tag{6.7}$$

To locate the controller zero $(\beta^* T_i^* s+1)$ to the left of the under damped closed-loop poles the proportional set-point weight is selected as $\beta^* = \tau_c T/T_i^*$. Then, the servo control closed-loop transfer function target is given by

$$M_{yr}^t(s) = \frac{(\tau_c Ts+1)(-bTs+1)}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c Ts+1)(a\tau_c Ts+1)}. \tag{6.8}$$

Using (6.6) and (6.8) the closed-loop control system total response target is stated as

$$y^t(s) = \frac{(\tau_c Ts+1)(-bTs+1)}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c Ts+1)(a\tau_c Ts+1)} r(s) + \frac{(T_i^*/K_p^*)s(-bTs+1)}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c Ts+1)(a\tau_c Ts+1)} d(s). \tag{6.9}$$

The performance analysis of the control system responses obtained using target damping ratios ζ from 1.0 to 0.6 shows that the highest performance improvement, measured with the integrated absolute error J_{eT}, is obtained, as previously with PI controllers, using $\zeta \approx 0.8$ but with a deterioration of the control effort total variation TV_{uT}.

Using the controlled process model (6.4) parameters the PID_{2F} controller parameters θ_c^* are normalized as

$$\kappa_p^* \doteq K_p^* K, \;\; \tau_i \doteq \frac{T_i^*}{T}, \;\; \tau_d^* \doteq \frac{T_d^*}{T}, \;\; \tau_f \doteq \frac{T_f}{T}. \tag{6.10}$$

For $\zeta = 0.8$ and each robustness target level the controller dimensionless parameters (6.10) are found for right-half plane zero positions b in the range from 0.1 to 3.0 and model time constants ratio a in the range from 0.1 to 1.0.

From the optimization data, is noted that the position of the right-half plane zero constrains the robustness level that may be achieved. Additionally, high b values made the controller derivative time to drop fast towards zero. Taking this into account, for the analysis, the position of the model right-half plane zero is taken in the range from 0.1 to 2.6 for robustness target level $M_S^t = 2.0$ and from 0.1 to 1.15 for $M_S^t = 1.6$.

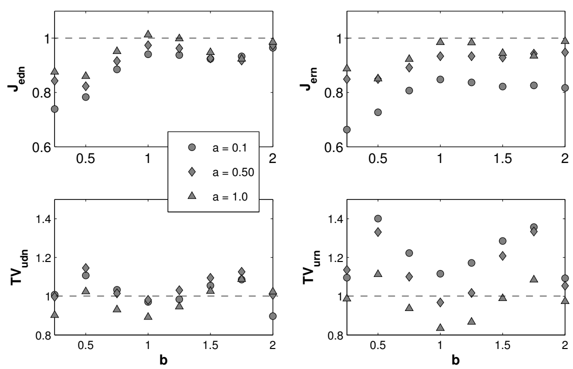


Figure 6.2: PID_{2F}/PI_2 performance and control effort total variation relation for $M_S^t = 2.0$. $J_{en} = J_{ePID}/J_{ePI}$, $TV_{un} = TV_{uPID}/TV_{uPI}$

Using a surface fitting tool the obtained controller parameter are used to fit the PID_{2F} MoReRT equations for SOPRHPZ models for the target robustness level $M_S^t = 2.0$ given by

$$\{\kappa_p^*, \tau_i^*, \tau_d^*, \tau_f, \beta^*\} = c_0 + c_1\, a + c_2\, b + c_3\, a\, b + c_4\, b^2 + c_5\, a\, b^2 + c_6\, b^3, \qquad (6.11)$$

and for $M_S^t = 1.6$ given by

$$\{\kappa_p^*, \tau_i^*, \tau_d^*, \tau_f, \beta^*\} = c_0 + c_1\, a + c_2\, b + c_3\, a\, b + c_4\, b^2. \qquad (6.12)$$

The c_i constants in (6.11) and (6.12) are presented in Alfaro and Vilanova (2012j). Tuning relations (6.11) ($M_S^t = 2.0$) are valid for zero relative position b in the range from 0.25 to 2.0 and tuning relations (6.12) ($M_S^t = 1.6$) for b from 0.25 to 1.0.

The characteristics of the 2DoF PID controller with filter (PID_{2F}) tuning presented above for second-order inverse response controlled processes is compared with the corresponding to the 2DoF PI controller (PI_2) tuning described in section 4.3 for the same processes.

Figure 6.2 shows the PID_{2F} to PI_2 performance, measured with the integrated absolute error, and control effort total variation relations for three SOPRHPZ model time constants ratios a and robustness target level $M_S^t = 2.0$. Comparison for $M_S^t = 2.0$ with other model time constants ratios a and for robustness target level $M_S^t = 1.6$ are presented in Alfaro and Vilanova (2012j).

Although indices depend on the model parameters a and b, the control system operation (servo or regulatory control), and the robustness level it can be said that, in general, the PID_{2F} controllers produce control systems with higher performance, under the integrated absolute error metric, but at the same time with more variability in their control signal.

For the $M_S^t = 2.0$ level the PID_{2F} controllers provide an average of 7.8% (regulatory control) and 10.5% (servo control) performance increase with 2.3% and 11.8% higher total variation of control effort, respectively. For the $M_S^t = 1.6$ level the corresponding average figures are 8.8% and 13.7% more performance with 1.04% and 19.4% more control effort variability.

If a general comment needs to be given, and based on the performance indices only, the use of the PID_{2F} controllers is recommended in all cases for the inverse response models considered.

Comparative analysis of MoReRT PID_{2F} controllers with PID controllers tuned with other methods is presented in Alfaro and Vilanova (2012j).

6.2 Unstable controlled processes

For unstable processes the extension of the MoReRT tuning for the PID_{2F} controllers is presented in Alfaro and Vilanova (2012k).

The controlled process is represented by an unstable first-order plus dead-time model given by the following transfer function:

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{Ts-1}. \tag{6.13}$$

For the 2DoF Ideal PID with filter controller design, the closed-loop is of third-order and the response target (6.2) is selected as

$$y^t(s) = \frac{(\beta^* T_i^* s+1)(T_f s+1)\mathrm{e}^{-Ls}}{(\tau_c T s+1)^3} r(s) + \frac{(T_i^*/K_p^*)(T_f s+1)s\mathrm{e}^{-Ls}}{(\tau_c T+1)^3} d(s). \tag{6.14}$$

The controller normalized parameters are

$$\kappa_p^* \doteq K_p^* K,\ \tau_i^* \doteq \frac{T_i^*}{T},\ \tau_d^* \doteq \frac{T_d^*}{T},\ \tau_f \doteq \frac{T_f}{T}. \tag{6.15}$$

In Alfaro and Vilanova (2012l) it was reported that in the PI_2 case the dead-time of the unstable process models impose severe limitations on the control system achievable robustness level and that a robust control system, $M_S \leq 2.0$, may only be obtained for a very limited range of unstable processes.

A comparison of the maximum robustness level attainable for unstable controlled processes using PI_2 controllers with target control system output (4.40) and PID_{2F} controllers with target control system output (6.14) is shown in figure 6.3. As can be seen

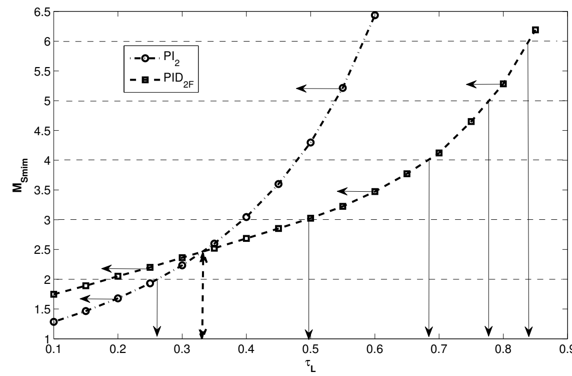


Figure 6.3: PI_2 and PID_{2F} controllers maximum attainable robustness for unstable first-order plus dead-time models

for models with low normalized dead-time, $\tau_L < 0.33$, the PI controller is capable to produce more robust control systems than the PID. On the other hand, for normalized dead times $\tau_L \geq 0.33$ the PID controller expands the range of models that can be tuned with the same robustness level.

For the selection of the tuning parameters the performance of the PID_{2F} controller tuned to obtain the maximum possible robustness (M_S minimum) and three constant robustness target levels ($M_S^t \in \{3.0,\ 4.0,\ 6.0\}$) is analyzed.

The results show that, if the control system over damped output profile in (6.14) is maintained, the unstable model normalized dead-time does not adversely affect the performance, and more relevant, that in this case there is not a performance/robustness trade-off. Increasing the closed-loop control system robustness the performance, measured with the integrated absolute error, increases (J_e decreases), but the control effort total variation increases. Then, the highest performance is obtained with the more robust design.

The optimization also shows that for all the analyzed cases the proportional set-point weight $\beta^* \rightarrow 0$. Then, for the unstable processes the PID_{2F} output (6.1) reduces to

$$u(s) = K_p^* \left\{ \left(\frac{1}{T_i^* s} \right) [r(s) - y(s)] - \left(\frac{T_d^* s + 1}{T_f s + 1} \right) y(s) \right\}. \tag{6.16}$$

Therefore, the resulting controller only has four adjustable parameters. For the development of the proposed tuning rule for UFOPDT models the PID_{2F} controller parameters that provide the maximum obtainable robustness are found.

The controller parameters obtained from the optimization procedure are used to fit the controller parameter equations of the proposed MoReRT approach for PID_{2F} controllers applied to unstable first-order plus dead-time controlled processes.

The normalized PID_{2F} controller parameters can be obtained with the following equations:

$$\begin{aligned}
\kappa_p^* &= 3.611 - 2.603\ \tau_L^{0.5343}, &(6.17)\\
\tau_i^* &= 2.886 + 50.09\ \tau_L^{4.663}, &(6.18)\\
\tau_d^* &= 0.345\ \tau_L^{0.9933}, &(6.19)\\
\tau_f &= \frac{0.4337 - 0.2068\ \tau_L}{6.061 - 27.39\ \tau_L + 100\ \tau_L^2}, &(6.20)\\
\beta^* &= 0, &(6.21)\\
\gamma^* &= 0. &(6.22)
\end{aligned}$$

Relations (6.17) to (6.22) are valid for UFOPDT models with $0.1 \leq \tau_L \leq 0.85$.

The obtainable robustness with tuning parameters (6.17) to (6.20) can be estimated using the following relation:

$$M_S = \frac{101.2 + 107.5\ \tau_L - 219.3\ \tau_L^2}{72.07 - 79.71\ \tau_L + \tau_L^2}. \tag{6.23}$$

Using the PID_{2F} parameters and the conversion relations in Alfaro and Vilanova (2012a) the equivalent 2DoF Standard PID (PID_2) controller parameters are obtained and presented in Alfaro and Vilanova (2012k). It is found that for models with normalized dead-time $\tau_L < 0.22$ the equivalent controller derivative filter constant α turns negative and that for $0.22 \leq \tau_L < 0.25$ it is very high. This reduces the range of unstable models that can be robustly controlled with the equivalent PID_2 controller.

Comparative analysis of MoReRT PID_{2F} controllers for unstable models with other tuning rules for PID_2 controllers is presented in Alfaro and Vilanova (2012k).

Chapter remarks

The extension of MoReRT design to proportional integral derivative (PID) controllers has been started with the more control demanding process dynamics.

The 2DoF PID control of inverse response processes is presented in Alfaro and Vilanova (2012j). It is shown that using a 2DoF Ideal PID controller with filter (PID_{2F}) more performance, measured with the integrated absolute error, is obtained in comparison with the performance obtained with a 2DoF PI controller.

Tuning relations were obtained for two robustness levels, $M_S^t \in \{2.0,\ 1.4\}$.

For unstable controlled processes MoReRT tuning of PID_{2F} is presented in Alfaro and Vilanova (2012k). For UFOPDT models with normalized dead-time $\tau_L \geq 0.33$ PID_{2F} controller allows to obtain control systems with higher robustness level than with the PI_2 counterpart. It also expands the normalized dead-time range of unstable models that can be controlled with certain robustness level.

For UFOPDT models the PID_{2F} design was based in obtain a control system with the highest possible robustness using over damped target responses.

7 Conclusions and future work

The *proportional integral derivative* (PID) is, with no doubt, the control algorithm most applied at the industry been most of the time used as *proportional integral* (PI) control. Despite of its simplicity and long history the appropriate selection of the PID adjustable parameters, controller tuning, is not a simple task. It must take into consideration the controlled process dynamic characteristics; the control system main operation mode; the design specifications related with its relative stability, controlled variable behavior, and control effort use and changes; and the effect of a variation in its own parameters.

Conclusions

The thesis presents a PI and PID design procedure that explicitly takes into account relevant aspects that influence the controller tuning and the behavior of the obtained control system.

The diverse industrial *controlled process dynamic characteristics* are taked into consideration by using approximated low order linear models. These include stable over damped first- and second-order plus dead-time transfer functions for self-regulated processes; integrating and integrating second-order plus dead-time transfer functions for processes with mass or energy storage capabilities; second-order plus a right-plane zero transfer functions, for inverse response processes; and first-order right-half plane pole plus dead-time transfer functions for runaway processes.

These controlled process linear models are obtained at the closed-loop control system normal operating point. Due to the non-linear characteristics of the industrial processes, it is necessary to consider the expected changes in the process dynamics, assuming certain stability margins, or *robustness* requirements, for the control system. Robustness is taked into account in the design by using the peak of the magnitude of the sensitivity function.

The *performance* of the control system, understanding this as the characteristics of the controlled variable evolution to a change in its inputs, set-point and load disturbances, is incorporated into the design selecting the controlled variable signal shapes by specifying closed-loop transfer functions targets.

The controller output signal, *control effort*, is the command to the final control element. To avoid extreme and excessive movement of the final control element smooth control signals are obtained by specifying non-oscillatory, or with small oscillations, control system controlled variables.

As in industrial process control applications the *regulatory control* operation, rejection of load disturbances, prevails over the *servo control* operation, tracking of a changing set-point, but that at some extent both operating modes are required the use of *two-degree-of-freedom* (2DoF) controllers is promoted. The design considers the regulatory and servo control operations in a unified way obtaining, at the same time, the complete set of controller parameters.

As the robustness requirement for the control system is controlled process dependent and, even more important, a specification that can not be avoided it is selected as the design parameter for the control system. The target control system robustness level, measured with maximum sensitivity, is stated by the designer.

The effects of controller parameters perturbation on a controller final fine-tuning is evaluated with the Delta 20 *robustness-fragility* and *performance-fragility* indices.

In brief, a general design procedure for robust two-degree-of-freedom (2DoF) proportional integral (PI) and proportional integral derivative (PID) controllers has been developed based on the use of regulatory and servo control transfer functions targets denoted as *Model Reference Robust Tuning* (MoReRT).

Tuning relations are obtained for 2DoF PI (PI_2) controllers that depend on the controlled process model parameters and *only one design parameter*: the control system robustness level.

For all models, except unstable ones, target robustness can be selected from the standard minimum robustness corresponding to $M_S = 2.0$ to a very high robustness level of $M_S = 1.4$. In the special case of unstable models robustness requirements need to be relaxed and its target has to be selected between $M_S = 6.0$ and $M_S = 2.0$. The applicability range of the tuning relations depends on the model characteristics and normalized parameters.

The achievement of the target (design) robustness for all the models considered is one of the main characteristics of the developed MoReRT relations.

The proposed design method is extended to tuning 2DoF Ideal PID with filter (PID_{2F}) controllers particularly for the control of difficult processes such as the inverse response and unstable ones. The PID_{2F} controllers are capable to bring more performance, measured with the integrated absolute error, than the PI_2 for the inverse response models, and more robust control systems in the unstable models case.

Future work

The MoReRT methodology provides a PI/PID controller design strategy suitable to obtain robust control systems for a wide variety of controlled process models that can be extended to addressed other important aspects for PID tuning. Some of the possibilities are outlined below.

To apply the proposed design methodology to tuning PID controllers that are capable to provide additional useful characteristics in industrial process control such as high frequency measurement noise attenuation and lack of any control output step change to a set-point change. For this, feedback signal and set-point filters can be added and included in the design process. This is a work in progress and promising initial results using controllers with two input filters, PI_{2IF} and PID_{2IF}, have been already obtained.

The evaluations and comparison tests made revel that the control system actual performance depends on the model order used for controller tuning. In the over damped processes case the use of second-order models turns out in control systems with more performance. At the same time, the model extra parameter made more difficult to obtain simple tuning rules that guarantee achievement of the design robustness level for models within a wide range of normalized parameters. Under this condition, a question arises: are the controller tuning relations required nowadays? or, is it more useful to have computer simulation and design software tools (or portable "apps") to resolve a particular PID control problem?

The control system design must take into consideration several conflicting specifications like performance, robustness, control effort use, and others. It is really a multi-objective optimization problem. One possible approach, in lieu of resolve a priory, at the development of the tuning method, the trade-offs between the specifications, is by using a multiobjective optimization method to obtain evenly distributed Pareto optimal solutions (the Pareto frontier) that allow the designer to decide over the more suitable controller for the particular control problem faced.

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Publications

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Sintonización de los controladores PID de 2GdL: desempeño, robustez y fragilidad

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Resumen: En este artículo se plantean los aspectos a considerar en el momento de sintonizar un controlador proporcional integral derivativo (PID) de dos grados de libertad (2GdL): el *desempeño* ante los cambios en la perturbación de carga y en el valor deseado de la variable controlada, la *robustez* del lazo de control ante los cambios en las características dinámicas del proceso controlado y la *fragilidad* del controlador ante la variación de sus propios parámetros. Mediante un ejemplo se muestran y analizan las relaciones entre estos tres índices. Su análisis integral durante el proceso de diseño, permite estimar de una mejor manera el funcionamiento del lazo de control.

Keywords: Controladores PID, dos grados de libertad, robustez, fragilidad

1. INTRODUCCIÓN

Desde su introducción comercial en 1940 [Babb (1990)] el controlador *proporcional integral derivativo* (PID), ha sido sin duda, la opción más utilizada en el control de los procesos industriales. Su popularidad se debe principalmente a la simpleza de su estructura, que la hace fácil de entender por parte del ingeniero de control, en contraste con otros esquemas de control más avanzados.

Desde que Ziegler and Nichols (1942) presentaron sus reglas de sintonización para los controladores PID, se han desarrollado un gran número de procedimientos como lo demuestra la recopilación hecha por O'Dwyer (2006). Algunas de ellas consideran solamente el desempeño del sistema de control, como las reglas clásicas de López et al. (1967) y Rovira et al. (1969), su robustez como las de Åström and Hägglund (1984) y Ho et al. (1995), o una combinación de desempeño y robustez como se observa en Voda and Landau (1995).

En las aplicaciones de control industrial, normalmente el valor deseado de la variable controlada o referencia, permanece constante y se requiere una buena eliminación del efecto de las perturbaciones o sea, se necesita un buen control regulatorio. Sin embargo, eventualmente podría ser necesario cambiar el valor deseado de la variable controlada, en cuyo caso se requiere un buen seguimiento a esos cambios esto es, un buen servo control. Debido a que estos dos funcionamientos no pueden ser satisfechos en forma simultanea con un controlador de solo un grado de libertad (1GdL), la utilización de un controlador de dos grados de libertad (2GdL), permite sintonizarlo considerando el desempeño del control regulatorio y la robustez del lazo de control, y utilizar luego el parámetro extra que caracteriza al segundo grado de libertad para mejorar el desempeño del servo control.

El procedimiento de diseño del sistema de control, usualmente, tiene como base el uso de modelos lineales de orden reducido para representar al proceso controlado, identificados estos en el punto de operación normal del sistema de control. Debido a las no linealidades encontradas en la mayoría de los procesos industriales, es necesario considerar los cambios esperados en las características del proceso, estableciendo algún margen de estabilidad relativa o requerimiento de robustez, para el sistema de control.

Por lo tanto, el diseño del sistema de control de lazo cerrado con controladores PID, debe considerar el *compromiso* de dos criterios en conflicto, por un lado el *desempeño* ante los cambios en la referencia y las perturbaciones y por el otro, la *robustez* ante cambios en las características del proceso controlado.

Si solamente se toma en consideración el desempeño, utilizando por ejemplo criterios de error integral (IAE, ITAE o ISE) o características de la respuesta temporal (sobrepaso, tiempo de levantamiento o tiempo de asentamiento), como en Huang and Jeng (2002) y en Tavakoli and Tavakoli (2003), el sistema de control de lazo cerrado resultante posiblemente tenga una robustez baja. Por otro lado, si el sistema se diseña para que tenga una robustez alta como en Hägglund and Åström (2002) y no se evalúa el desempeño del sistema resultante, el diseñador no tiene ninguna indicación del *costo* que le significa el tener ese sistema altamente robusto. Tanto el desempeño como la robustez son considerados por Shen (2002) y Tavakoli et al. (2005) optimizando el desempeño con los criterio IAE e ITAE, pero solo para el nivel usual de robustez mínima.

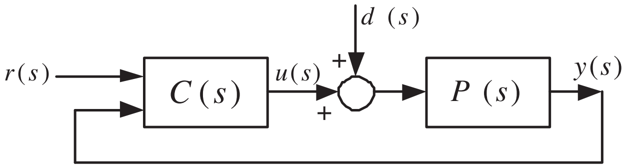

Figura 1. Sistema de control realimentado

Una vez obtenido un desempeño robusto del lazo de control, es importante estimar la *fragilidad* del controlador, esto es, la sensibilidad de la robustez del lazo de control a cambios en los parámetros del propio controlador, ya sean producto de inexactitudes en su implementación o por el *ajuste fino* final de los parámetros del controlador por parte del diseñador. Un controlador con un índice de fragilidad bajo, permitirá realizar la puesta a punto del controlador, sin tener que preocuparse por provocar una degradación importante en la robustez del mismo.

El resto de este artículo está organizado de la siguiente manera: en la Sección 2 se presenta el controlador PID de dos grados de libertad, en la Sección 3 los índices de desempeño, robustez y fragilidad a considerar en el diseño, en la Sección 4 mediante un ejemplo se ilustra el compromiso entre el desempeño y la robustez y su relación con la fragilidad, terminando con recomendaciones para el diseño de los sistemas de control.

2. CONTROLADORES PID DE 2GDL

Considérese el sistema de control realimentado de la Fig. 1, donde $P(s)$ es la función de transferencia del modelo del proceso controlado, $C(s)$ la función de transferencia del controlador, y $r(s)$ el valor deseado, $d(s)$ la perturbación de carga y $y(s)$ la variable controlada.

La ecuación de salida del controlador está dada por [Åström and Hägglund (1995)]

$$u(t) = K_c \left\{ e_p(t) + \frac{1}{T_i}\int_0^t e_i(\tau)d\tau + T_d \frac{de_d(t)}{dt} \right\} \quad (1)$$

con

$$e_p(t) = \beta r(t) - y(t) \quad (2)$$
$$e_i(t) = r(t) - y(t) \quad (3)$$
$$e_d(t) = \gamma r(t) - y(t) \quad (4)$$

donde K_c es la ganancia del controlador, T_i el tiempo integral, T_d el tiempo derivativo y β y γ los factores de peso del valor deseado.

El parámetro γ se utiliza usualmente como un "selector" (con valores 1 o 0), para la aplicación o no del modo derivativo a la señal de valor deseado r. Para evitar un salto instantáneo alto en la salida del controlador, cuando el valor deseado se cambia abruptamente, γ usualmente se hace cero. En este caso, la salida del controlador (1) se puede expresar como

$$u(s) = K_c \left\{ e_p(s) + \frac{1}{T_i s} e_i(s) + \frac{T_d s}{T_d/N s + 1} e'_d(s) \right\} \quad (5)$$

con

$$e_p(s) = \beta r(s) - y(s) \quad (6)$$
$$e_i(s) = r(s) - y(s) \quad (7)$$
$$e'_d(s) = -y(s) \quad (8)$$

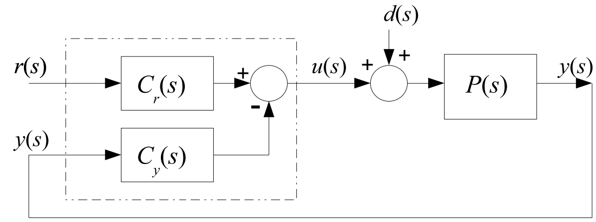

Figura 2. Control de dos grados de libertad (2GdL)

donde N es la *constante del filtro derivativo* (usualmente $N = 10$ [Visioli (2006)]).

La ecuación (5) se puede reordenar entonces como

$$u(s) = K_c \left(\beta + \frac{1}{T_i s} \right) r(s) \quad (9)$$
$$- K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{0,1 T_d s + 1} \right) y(s) \quad (10)$$

o en la forma más compacta mostrada en la Fig. 2 como

$$u(s) = C_r(s)r(s) - C_y(s)y(s) \quad (11)$$

donde $C_r(s)$ es la parte del controlador aplicada al valor deseado, la función de transferencia del *controlador de valor deseado* y $C_y(s)$ la parte aplicada a la señal realimentada, la función de transferencia del *controlador de realimentación*.

De la Fig. 2 se obtiene que la respuesta del lazo de control a un cambio en cualquiera de sus entradas, está dada por

$$y(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)} r(s) + \frac{P(s)}{1 + C_y(s)P(s)} d(s) \quad (12)$$

o en una forma más compacta por

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s) \quad (13)$$

donde $M_{yr}(s)$ es la función de transferencia desde el valor deseado hasta la variable controlada, la función de transferencia de lazo cerrado del *servo control*, y $M_{yd}(s)$ es la función de transferencia desde la perturbación hasta la variable controlada, la función de transferencia de lazo cerrado del *control regulatorio*.

La mayoría de los controladores PID comerciales no incluyen el factor de peso de valor deseado proporcional β (lo que equivale a tener $\beta = 1$). En este caso, como todos los parámetros de $C_r(s)$ son idénticos a los correspondientes en $C_y(s)$, no es posible especificar un desempeño dinámico del sistema de control ante cambios en el valor deseado, en forma independiente del desempeño ante un cambio en la perturbación. Si los grados de libertad de un sistema de control, se definen en función del número de funciones de transferencia de lazo cerrado que se pueden seleccionar en forma independiente [Horowitz (1963)], se tiene en este caso un sistema de control de *un grado de libertad* (1GdL).

La restricción anterior, obliga al diseñador a utilizar una regla de sintonización desarrollada específicamente para el funcionamiento más frecuente del lazo de control (servo control o control regulatorio), encontrándose entonces en la literatura reglas específicas para el control regulatorio en Cohen and Coon (1953) y Ziegler and Nichols (1942), para el servo control en Martin et al. (1975) y Rivera et al. (1986) y las que incluyen reglas separadas para ambas aplicaciones en Kaya (2004) y Sung and Lee (1999), para citar solo algunas.

En forma alternativa, se puede lograr que un sistema de control con solo un grado de libertad proporcione un *desempeño balanceado* respecto a ambos modos de operación, empleando el *control de dos grados de libertad implícitos* presentado en Arrieta and Vilanova (2007).

Para tener una mayor flexibilidad en el diseño de los lazos de control, un segundo grado de libertad fue introducido en el algoritmo de control PID en Araki (1984), mediante los factores de peso de valor deseado β y γ.

En este caso, dado un modelo del proceso controlado $P(s)$, se pueden seleccionar los parámetros del controlador de realimentación $C_y(s)$ (K_c, T_i, T_d), para lograr el desempeño especificado del control regulatorio $M_{yd}(s)$ y asegurar, al mismo tiempo, la estabilidad relativa o robustez del lazo de control. Y utilizar luego el factor de peso de valor deseado (β) del controlador de valor deseado $C_r(s)$, para modificar el desempeño del servo control $M_{yr}(s)$.

Bajo la definición anterior de los grados de libertad, se tiene ahora un sistema de control con dos grados de libertad (2GdL). Esta opción permitió el desarrollo de métodos de sintonización para controladores de 2GdL como las encontradas en Alfaro et al. (2008); Åström and Hägglund (2004); Gorez (2003); Hägglund and Åström (2002).

Respecto a la implementación comercial del algoritmo de control PID, se encuentra que usualmente esta es de solo 1GdL [Foxboro (1998); Honeywell (2007); Rockwell (2005)] y solo unas pocas tienen las capacidades de 2GdL [Mitsubishi (2002)]. En particular el controlador PID de 2GdL en Emerson (2008), permite que los dos factores de peso de valor deseado en (1) a (4), β y γ, se puedan ajustar en el ámbito 0 a 1.

En los controladores PID comerciales de 2GdL, se tiene la restricción de que el factor de peso de valor deseado proporcional, se puede ajustar solo en el ámbito $0 \leq \beta \leq 1{,}0$. Esto, posiblemente producto del interés de reducir a una fracción la contribución proporcional en la salida del controlador ante un cambio en el valor deseado, evitando así respuestas con sobrepasos excesivos o con mucha oscilación, cuando el controlador ha sido sintonizado para optimizar el desempeño ante cambios en la perturbación. Sin embargo, cuando se incorporan en el procedimiento de diseño los requerimientos de robustez del lazo de control, esto conduce a una reducción en el valor de la ganancia del controlador, produciendose entonces respuestas lentas del servo control. Como lo demuestran Alfaro et al. (2009a), en estos casos se puede logar un incremento considerable en el desempeño del servo control, si se emplean factores de peso $\beta > 1$.

3. COMPROMISOS PARA EL DISEÑO

Como se indicó anteriormente, la sintonización de los controladores PID de 2GdL debe tomar en consideración, el compromiso existente entre el *desempeño* ante cambios en el valor deseado y la perturbación de carga y la *robustez* del lazo de control ante cambios en las características dinámicas del proceso controlado, el cual debe complementarse con información de la *fragilidad* del controlador ante cambios en sus propios parámetros.

3.1 Desempeño y esfuerzo de control

Aunque evidentemente el tipo de comportamiento dinámico requerido del lazo de control, está íntimamente ligado a la aplicación particular del mismo, sin perder generalidad, se establecerá aquí un criterio para la medición del desempeño del lazo de control.

Se considerará como el *desempeño* o comportamiento dinámico requerido del lazo de control, el que este sea óptimo respecto a la funcional de costo que penaliza la integral del error absoluto, definida como

$$J_e \doteq \int_0^{\infty} |e(t)|\, dt = \int_0^{\infty} |r(t) - y(t)|\, dt \qquad (14)$$

la cual debe evaluarse tanto para cambios en el valor deseado, J_{er}, como en la perturbación de carga, J_{ed}. Estas proveerán una indicación del *desempeño de la salida* del sistema de control.

Los parámetros del controlador para el desempeño óptimo $\bar{\theta}_{co}$ serán tales que

$$J_{eo} \doteq J_e(\bar{\theta}_{co}) = \min_{\bar{\theta}_c} J_e(\bar{\theta}_c, \bar{\theta}_p) \qquad (15)$$

donde $\bar{\theta}_c$ y $\bar{\theta}_p$ son los parámetros del controlador y del modelo del proceso controlado respectivamente.

Aunque el objetivo principal sea optimizar el desempeño, no debe dejarse de lado el analisis de las características dinámicas del esfuerzo de control, la señal de salida del controlador, ya que es conveniente, para evitar el deterioro prematuro del elemento final de control, que los cambios del mismo no sean bruscos ni extremos. La evaluación de las variaciones del esfuerzo de control se puede realizar mediante un índice de su variación total, definido por

$$TV_u \doteq \sum_{k=1}^{\infty} |u_{k+1} - u_k| \qquad (16)$$

el cual debe ser lo menor posible. Este es una indicación de la *suavidad de la señal de control.*

También deben considerarse los valores extremos de la señal de control $|U_{\text{máx}}|$ y $|U_{\text{mín}}|$.

3.2 Robustez del lazo de control

Si la *estabilidad absoluta* del lazo de control es indispensable, su *estabilidad relativa* es importante para garantizar el funcionamiento estable del sistema de control, debido a las diferencias y variaciones de las características dinámicas del proceso controlado, respecto a las del modelo nominal de parámetros fijos utilizado para el diseño del controlador.

Como una medida del grado de estabilidad relativa o *robustez* del lazo de control, se ha empleado tradicionalmente el margen de ganancia junto con el margen de fase (A_m, ϕ_m). Sin embargo, en la actualidad este par se ha sustituido por la utilización de la sensibilidad máxima, definida por

$$M_s \doteq \max_{\omega} |S(j\omega)| = \max_{\omega} \frac{1}{|1 + C_y(j\omega)P(j\omega)|} \qquad (17)$$

cuyo valor recomendado usual, se encuentra en el ámbito de 2,0 a 1,2 .

El uso de la sensibilidad máxima como medida de robustez tiene la ventaja de que esta, aparte de ser un único índice, garantiza valores mínimos de los márgenes de ganancia y fase dados por [Åström and Hägglund (2006)]

$$A_m > \frac{M_s}{M_s - 1} \tag{18}$$

y

$$\phi_m > 2sen^{-1}\left(\frac{1}{2M_s}\right) \tag{19}$$

El *desempeño robusto* del lazo de control se logrará entonces combinando el índice de desempeño como el dado por (14) junto con el cumplimiento de una robustez mínima medida con (17).

3.3 Fragilidad del controlador

Mientras que la robustez del lazo de control es una indicación de cuanto pueden variar las características del proceso controlado sin que el sistema se vuelva inestable, su *fragilidad* está relacionada con la pérdida de estabilidad del lazo, debido a la variación de los parámetros del propio controlador.

La fragilidad de los controladores fue puesta de relieve por Keel and Battacharyya (1997), cuando analizaron el efecto de la variación de los parámetros de los controladores de orden alto, diseñados utilizando métodos de control óptimo y robusto. Ellos encontraron que en esos casos, pequeñas variaciones de los coeficientes del controlador hacían inestable el sistema. Posteriormente Ho (2001) y Silva et al. (2005) establecieron lineamientos para el diseño de controladores PID que fueran no frágiles.

Si bien la fragilidad de los controladores PID se relacionó inicialmente con la pérdida de la estabilidad absoluta del lazo de control, desde un punto de vista práctico, es más conveniente la utilización del *índice de fragilidad delta épsilon* $FI_{\Delta\epsilon}$ definido por Alfaro (2007). Este relaciona la pérdida de robustez del lazo de control cuando se varían los parámetros del controlador una cantidad determinada, con la robustez nominal del lazo de control y está definido como

$$FI_{\Delta\epsilon} \doteq \frac{M_{s\Delta\epsilon m}}{M_{so}} - 1 \tag{20}$$

donde la *sensibilidad máxima extrema* $M_{s\Delta\epsilon m}$ representa la mayor pérdida de robustez del sistema de control cuando todos sus parámetros se varían una cantidad $\delta_\epsilon = \pm\epsilon$, y M_{so} es la *sensibilidad máxima nominal*.

En particular el *índice de fragilidad delta 20* ($FI_{\Delta 20}$) permite definir el grado de fragilidad de los controladores PID como [Alfaro (2007)]:

- *Un controlador PID es frágil*, si su Índice de fragilidad delta 20 es mayor que 0,50, ($FI_{\Delta 20} > 0,50$).
- *Un controlador PID no es frágil*, si su Índice de fragilidad delta 20 es menor o igual a 0,50, ($FI_{\Delta 20} \leq 0,50$).
- *Un controlador PID es elástico*, si su Índice de fragilidad delta 20 es menor o igual a 0,10, ($FI_{\Delta 20} \leq 0,10$).

Por lo tanto, se considerará un controlador como frágil si el lazo de control pierde más de un 50 % de robustez cuando todos sus parámetros son perturbados hasta en un 20 %, de lo contrario se dirá que no es frágil. Además, un controlador será elástico, si el lazo de control no pierde más de un 10 % de robustez cuando sus parámetros son perturbados hasta en un 20 %. En este último caso, el controlador permitiría un afinamiento final de sus parámetros (hasta en un ±20 % de sus valores nominales), sin temor a que el sistema tenga una pérdida apreciable de robustez (esta será a lo más de un 10 %).

Cuadro 1. PID 1GdL, regulador óptimo

K_c	T_i	T_d	β
10,728	0,492	0,273	1

Cuadro 2. PID 1GdL, robustez y desempeño

M_s^r	J_{ed}	TV_{ud}	J_{er}	TV_{ur}
3,963	0,0067	0,4245	0,164	11,193

La influencia de cada uno de los parámetros del controlador en su fragilidad, se puede investigar utilizando el *índice de fragilidad paramétrica delta epsilon*, definido como

$$FI_{\delta\epsilon p} \doteq \frac{M_{s\delta\epsilon p}}{M_{so}} - 1 \tag{21}$$

La medición de la fragilidad con el índice $FI_{\Delta 20}$ junto con la utilización de los *gráficos de fragilidad* [Alfaro et al. (2009b)], permite complementar el diseño de controladores con un desempeño robusto.

4. EJEMPLO

Mediante la utilización de un ejemplo, se mostrará el compromiso existente entre el desempeño y la robustez del lazo de control, y su efecto sobre la fragilidad del controlador.

Considérese el proceso controlado de cuarto orden, cuya función de transferencia es

$$P(s) = \frac{1}{(s+1)(0{,}4s+1)(0{,}16s+1)(0{,}064s+1)} \tag{22}$$

Utilizando el método de identificación de tres puntos *123c* [Alfaro (2006)], se obtuvo la siguiente aproximación de segundo orden más tiempo muerto para representarlo

$$P_m(s) = \frac{e^{-0{,}147s}}{(0{,}856s+1)(0{,}603s+1)} \tag{23}$$

Primero se consideró en el diseño solamente el desempeño del sistema de control, determinando los parámetros del controlador PID de 1GdL que minimizan (14) ante un cambio en la perturbación (control regulatorio óptimo), indicados en el Cuadro 1. La robustez y los índices de desempeño y del esfuerzo de control obtenidos con ese controlador se muestran en el Cuadro 2. En esta y todas las demás evaluaciones se consideró $\Delta r = 20\,\%$ y $\Delta d = 10\,\%$.

Como se observa en el Cuadro 2, el sistema de control con el desempeño optimizado tiene una robustez muy pobre, no alcanza la especificación de robustez mínima usual que establece que $M_s \leq 2,0$. Por otra parte, se espera que en este caso el desempeño del control regulatorio medido con la integral del error absoluto J_{ed}, sea el mejor posible.

Para incorporar la robustez en el diseño del controlador, se determinaron los parámetros del PID de 2GdL utilizando el *método de sintonización analítica robusta* ART_2 [Alfaro et al. (2009c)], para cinco valores específicos de la robustez del lazo de control, los cuales se muestran en el Cuadro 3.

Cuadro 3. PID 2GdL, parámetros ART_2

M_s^t	τ_c	K_c	T_i	T_d	β
2,0	1,00	5,243	1,633	0,407	0,191
1,8	1,20	4,028	1,846	0,471	0,248
1,6	1,51	2,756	2,100	0,573	0,363
1,4	1,99	1,576	2,299	0,750	0,635
1,2	2,80	0,556	1,852	1,248	1,293

Cuadro 4. PID 2GdL, robustez y desempeño

M_s^t	M_s^r	J_{ed}	TV_{ud}	J_{er}	TV_{ur}
2,0	2,16	0,031	0,179	0,327	0,548
1,8	1,90	0,046	0,150	0,369	0,466
1,6	1,65	0,076	0,126	0,420	0,415
1,4	1,42	0,144	0,109	0,460	0,374
1,2	1,21	0,315	0,097	0,558	0,256

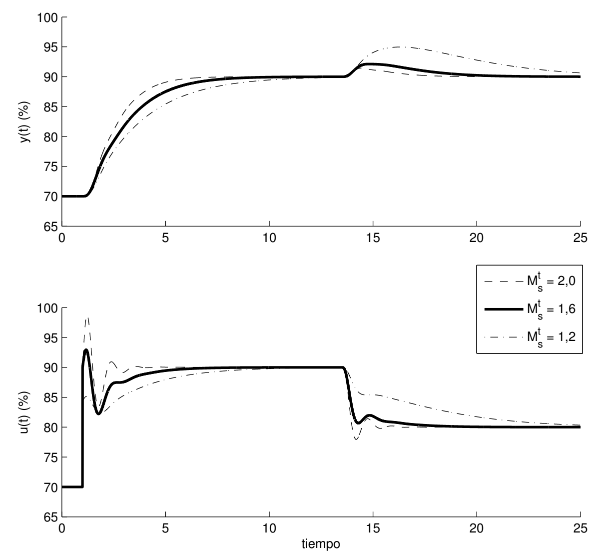

Figura 3. Ejemplo - Respuestas del sistema de control

La robustez y los índices del desempeño y del esfuerzo de control resultantes con el controlador de 2GdL robusto, se muestran en el Cuadro 4. Debe hacerse notar, que la robustez obtenida M_s^r, en todos los casos fue determinada utilizando el modelo como planta, ya que en la práctica esta no se puede determinar con el proceso real, mientras que los índices de desempeño se determinaron con el proceso a partir de las correspondientes señales.

En la Fig. 3 se muestra la respuesta del sistema de control con el controlador de 2GdL, a los cambios $\Delta r = 20\,\%$ y $\Delta d = 10\,\%$, para tres niveles de robustez diferentes.

Si se comparan los índices del regulador óptimo en el Cuadro 2, con los correspondientes a $M_s = 2,0$ en el Cuadro 4, se nota que este nivel mínimo de robustez, se logra a expensas de una pérdida importante en el desempeño.

Además, de los índices del Cuadro 4 es evidente el compromiso entre el desempeño y la robustez. Al exigirse un mayor grado de robustez (M_s^t), se deteriora el desempeño tanto del control regulatorio (J_{ed}), como del servo control (J_{er}), mientras que la variación del esfuerzo de control se torna más "suave", disminuyen TV_{ud} y TV_{ur}.

Cuadro 5. PID 2GdL, Fragilidad

M_s^t	$IF_{\delta 20K_c}$	$IF_{\delta 20T_i}$	$IF_{\delta 20T_d}$	$IF_{\Delta 20}$
2,0	0,250	0,0019	0,210	0,681
1,8	0,190	0,0011	0,166	0,486
1,6	0,133	0,0007	0,121	0,323
1,4	0,084	0,0004	0,078	0,194
1,2	0,042	0,0003	0,037	0,089

En el caso correspondiente a $M_s^t = 1,2$ el factor de peso de valor deseado determinado con el método ART_2 es $\beta = 1{,}293$. Si en vez de este se utiliza $\beta = 1$, el índice de desempeño del servo control sería $J_{er} = 0{,}666$, aproximadamente un 20 % mayor, lo que muestra claramente la ventaja de no restringir el factor de peso de valor deseado a valores $\beta \leq 1$, tal como sucede con los controladores comerciales actuales.

En relación con la fragilidad de los controladores, en el Cuadro 5 se muestran los índices de fragilidad paramétrica ($FI_{\delta 20p}$) de los tres parámetros y el índice de fragilidad delta 20 ($FI_{\Delta 20}$) del controlador. Con base en las definiciones dadas en la Sección 3.3, el controlador diseñado con el nivel mínimo de robustez ($M_s = 2,0$) es frágil, mientras que los de robustez intermedia ($1{,}4 \leq M_s \leq 1{,}8$) son no frágiles y el de robustez alta ($M_s = 1,2$) es elástico.

Además, considerando la definición dada en Alfaro et al. (2009b), los controladores diseñados con el método ART_2 son controladores con una *fragilidad desbalanceada*. La sensibilidad de la robustez a los cambios en K_c y T_d es similar, mientras que esta muestra ser prácticamente insensible a los cambios en T_i.

5. CONCLUSIONES

Las características de los controladores PID de dos grados de libertad (2GdL), permiten realizar su diseño en dos etapas, considerando primero el *desempeño del control regulatorio* y el *comportamiento del esfuerzo de control*, junto con los requerimientos de *robustez del sistema de control*, para determinar los parámetros (K_c, T_i, T_d), y luego, el mejoramiento del *desempeño del servo control* utilizando el parámetro β.

Por su parte, la determinación del grado de *fragilidad* del controlador resultante, le permitirá al diseñador efectuar el *afinamiento final* de la sintonía del controlador, teniendo a mano una estimación de los efectos que la variación de los parámetros del controlador pudiera tener sobre la robustez con que el sistema se diseñó, establecida esta en virtud de las variaciones esperadas de las características dinámicas del proceso controlado.

Por lo tanto, los métodos modernos de sintonización de controladores PID de 2GdL, deberían tener siempre en consideración la tripleta de índices {*desempeño*, *robustez* y *fragilidad*}.

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Fragility and Robustness Level Accomplishment of Well Known PI/PID Robust Tuning Rules

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***Abstract*— The robustness of a control system is by now a well established issue. Especially within the PI/PID domain, this is a requirement that is included as part of the design specifications by some of the new design approaches. In this paper we analyze how recognized robust PI/PID tuning rules accomplish with respect to their robustness specification. A concrete analysis is conducted where the chosen PI/PID controller tuning rules are based on a precise quantitative robustness specification such as the Maximum Sensitivity; M_S; value. It is reported that the claimed robustness specification in most of the cases is not accomplished for some set of plants, on other cases it is accomplished with excessive margin therefore providing a not satisfactory robustness/performance trade-off. In addition, even some of the PI/PID tuning rules do not accomplish with the specified robustness level, a fragility analysis is conducted to evaluate how they perform with respect to the achieved robustness. The conclusion drawn from this study is that even if the robustness issue has been incorporated in the new approaches for tuning and design of PI/PID controllers, this does not suffice. In order to appropriately tackle the robustness/performance trade-off, the achieved robustness level should not only be an integral specification but its accomplishment should be checked.**

I. INTRODUCTION

Since Ziegler and Nichols [25] presented their PID controller tuning rules, a great number of procedures have been developed as revealed in O'Dwyer's review [16]. Some of them consider only the system performance such as the classical tuning methods [15], [18], its robustness by authors such as in [4] and [11], or a combination of performance and robustness, as seen in [12], [22], [23].

The control-system design procedure is usually based on the use of low-order linear models identified at the desired closed-loop control system normal operation point. Due to the non-linear characteristics found in most of the industrial processes, it is necessary to consider the expected changes in the process characteristics assuming certain relative stability margins, or robustness requirements, for the control system. Therefore the design of the closed-loop control system with PI and PID controllers must consider the *trade-off* of two conflicting criteria, the time response *performance* to set-point changes and load disturbances[1], as well as the *robustness* to changes in the controlled process characteristics. If only the system performance is taken into account, using by example an integrated error criteria (IAE, ITAE or ISE) or a time response characteristic (overshoot, rise-time or settling-time) as in [13], [21], the resulting closed-loop control system probably will have a very low robustness. On the other hand, if the system is designed to have high robustness as in [9] and if the performance of the resulting system is not evaluated, the designer will not have any indication of the *cost* of having such highly robust system.

The need for such a *trade-off* arises since the introduction of robustness as a *de facto* specification to be included into any design or tuning approach, and it is this aspect that this paper deals about. Being stated the need for robustness, during the last years there has been a shift of perspective with respect to the robustness considerations. First of all there is the change from the classical Gain and Phase Margin measures to a single quantitative more general measure of robustness such as the Maximum of the Sensitivity function magnitude, otherwise called the M_S value or the Sensitivity peak. Being M_S a single-value measure of robustness, the introduction of M_S into the design phase was raised; usually as an inequality or equality constraint for the M_S value; in order to get tuning rules trying to assure a certain robustness level. The achievement of such robustness level, however, has never been tested. The importance of this observation comes from the previously mentioned performance/robustness trade-off. As the robustness is usually tackled from an inequality constraint point of view, the resulting tuning rule may provide control systems that are excessively conservative. This is to say too robust. Therefore with the corresponding performance reduction. Our claim is that if robustness were attained on a more tight way there would be more room for performance.

On the basis of the previous observations, on this paper, several tuning approaches that appeared in the literature during the last years are analyzed from the following point of view: Which was the M_S value the tuning rule was conceived for and which the achieved one? The analyzed tuning rules are the Kappa-Tau tuning of [5]; the AMIGO method proposed in [6]; the method of Tavakoli [21]; and the more flexible (from the M_S specification point of view) NORT rule [3] where the M_S itself becomes the tuning parameter. The common point shared by all these approaches to PI/PID tuning is that they are formulated for a First-Order-Plus-Dead-Time (FOPDT) model. Therefore a similar framework can be established and a plant-wide analysis can be conducted.

This analysis is complemented by introducing an assessment of the fragility that the corresponding robustness

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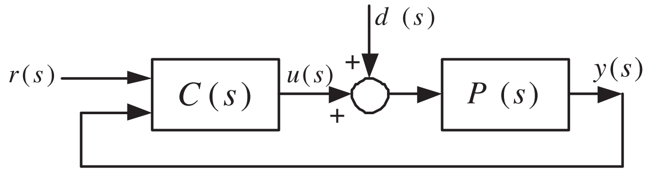

Fig. 1. Closed-Loop Control System

levels are achieved. Fragility of PID controllers is another important aspect that attempts to measure the sensibility of the achieved robustness with respect to a change in the controller's parameters. The analysis conducted here follows the indices proposed in [1]. In this respect the fragility analysis is conducted for those tuning rules that better achieve the designed robustness level, showing how the fragility depends on the process normalized dead-time, being worse as it increases.

The ultimate purpose of the paper is not to provide new tuning rules but an analysis of the robustness specification when facing the design of a PI/PID. It is the authors opinion that this analysis will show up a check point that is necessary when stating a robust tuning. This check point will help to compare tuning rules and approaches from both points of view: robustness (this one should be achieved for the set of plants of interest) and performance (with the same robustness level which one outperforms).

The rest of the paper is organized as follows. Next section presents the control setup by stating the controller and process equations as well as the robustness and fragility measures to be used. Section III conducts the robustness analysis whereas in Section IV fragility of the selected tuning rules is performed. The paper ends with some conclusions and suggestions for future research.

II. Formulation

The considered tuning rules are those that introduce as an explicit design constraint to achieve certain robustness level. All these tuning rules are based on the usual First-Order-Plus-Dead-Time (FOPDT) model for the controlled process:

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{Ts+1},\ \tau_o = L/T, \tag{1}$$

where τ_o is the model *normalized dead-time*. Sometimes the controllability index $\tau = L/(L+T)$; that is related to τ_o: $\tau = \tau_o/(\tau_o+1)$; is also used.

On the basis of the closed-loop control depicted in Fig. 1 and as we will only be concerned on robustness, we use the One-Degree-of-Freedom Standard PI/PID form. Therefore:

$$u(s) = C(s)[r(s) - y(s)], \tag{2}$$

with

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right), \tag{3}$$

where K_p is the controller gain, T_i the integral time constant and T_d the derivative time constant.

A. Robustness

As an indication of the system *robustness* (relative stability) the Sensitivity Function peak value will be used. The control system Maximum Sensitivity is defined as:

$$M_S \doteq \max_\omega |S(j\omega)| = \max_\omega \frac{1}{|1 + C(j\omega)P(j\omega)|}, \tag{4}$$

and recommended values for M_S are typically within the range 1.2 - 2.0 [7].

The use of the maximum sensitivity as a robustness measure, has the advantage that lower bounds to the gain, A_m, and phase, ϕ_m, margins [7] can be assured according to:

$$A_m > \frac{M_S}{M_S - 1}, \tag{5}$$

$$\phi_m > 2\sin^{-1}\left(\frac{1}{2M_S}\right). \tag{6}$$

Therefore, to assure $M_S = 2.0$ provides what is commonly considered a minimum robustness requirement that translates to $A_m > 2$ and $\phi_m > 29^o$, for $M_S = 1.4$ we have $A_m > 3.5$ and $\phi_m > 41^o$.

However, note that once a desired value for M_S is specified, any value lower than this one will provide the robustness requirement. Being an inequality based constraint it is not surprising that sometimes the resulting control system provides an M_S value smaller than the specified one. In fact, there is nothing wrong with this fact if we just look at the robustness requirement. However, this will introduce an unnecessary loose of performance. To quantitatively measure this constraint on the achieved performance is rather difficult mainly because each tuning rule is conceived on the basis of a different performance specification (or either a combination of). However it has to be acknowledged that if the robustness requirement could be achieved on a more tight way there would be more room for performance. The main reason on pointing out this fact is that, on the authors' opinion, tuning methods should be oriented towards such tight robustness accomplishment.

B. Fragility

Controller's fragility refers to the effect of a variation on the controller's parameters. In some sense this is a controller's counterpart to robustness against model uncertainty: if robustness of the control loop indicates the margin of variation in which the plant characteristics with a fixed controller may vary, the controller's fragility has a similar meaning in terms of the variation of its own parameters.

The study on controller's fragility was motivated by the fact that a certain degree of uncertainty inevitably exists in the controller parameters due to the tolerance of its components if these are analogue. In its digital implementation, there are inaccuracies because the use of fixed-length words and rounded errors of numerical calculations [24]. Within this context Keel and Bhattacharyya [14] documented the fragility of certain controllers designed with modern optimum and robust techniques. Datta, Ho, and Bhattacharyya [8], Ho [10] as well as Silva, Ho, and Bhattacharyya [19] have analyzed the fragility of the PID controllers and proposed design procedures to obtain less fragile controllers. Among the different proposals the one that better complements the robustness accomplishment analysis pursued here is the *delta epsilon fragility index* proposed in [1].

Alfaro's *delta epsilon fragility index*; $FI_{\Delta\epsilon}$; is an indication of the control system's loss of robustness measured with the maximum sensitivity M_S when the controller parameters perturb a $\delta_\epsilon = \pm\epsilon$ quantity from their design values, and is defined as

$$FI_{\Delta\epsilon} \doteq \frac{M_{S\Delta\epsilon m}}{M_{So}} - 1, \qquad (7)$$

where $M_{S\Delta\epsilon m}$ is the *extreme maximum sensitivity* given by

$$M_{S\Delta\epsilon m} = \max_{\Delta_\epsilon \bar{p}} \left\{ M_{S\Delta\epsilon}; \Delta_\epsilon \bar{p} = [\pm\epsilon K_{po}, \pm\epsilon T_{io}, \pm\epsilon T_{do}] \right\}, \qquad (8)$$

M_{So} is the control loop *nominal maximum sensitivity* and $\bar{p}_o = [K_{po}, T_{io}, T_{do}]$ the *controller nominal parameters*. As introduced in [1], the index is computed with a value $\epsilon = 20\%$: $FI_{\Delta 20}$.

The relative influence of a change in each one of the controller's parameters over its fragility can be obtained with the *parametric delta epsilon fragility index*

$$FI_{\delta\epsilon p} = \frac{M_{S\delta\epsilon p}}{M_{So}} - 1. \qquad (9)$$

On this respect, next section presents, the analysis conducted on the achieved M_S values; with respect to a set of plants characterized by $\tau_o \in (0, 2]$; provided by a collection of tuning rules that include the M_S value as a design specification.

III. Robustness Analysis

The robust tuning rules that can be found on the literature consider up to three different specifications for M_S. They range from the considered minimum robustness; $M_S = 2.0$; to a high robustness; $M_S = 1.4$; and also considering an intermediate value such as $M_S = 1.6$. The considered methods are the following:

- AMIGO method [6] provides tuning for a PI and a PID controller with a design specification of $M_S = 1.4$.
- Kappa-Tau (KT) method [5] provides tuning for a PI and a PID controller with a design specification of $M_S = 1.4$ and $M_S = 2.0$.
- Tavakoli method [21] provides tuning for a PID controller with a design specification of $M_S = 2.0$.
- SIMC method [20] provides tuning for a PI controller that provides a robustness of $M_S = 1.59$.
- NORT method [3] provides tuning for a PI controller with M_S being the design parameter. Here the controllers resulting for the three considered values of M_S; 1.4, 1.6 and 2.0; are analyzed.

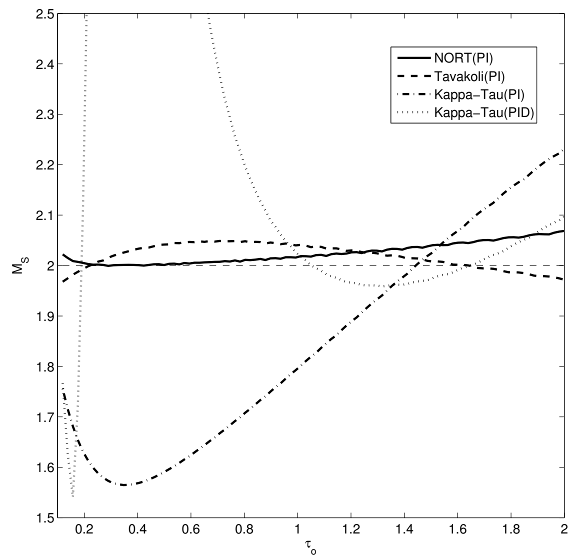

Fig. 2. M_S behaviour for an $M_S = 2.0$ specification

TABLE I

M_S constraint accomplishment. Design value *vs.* mean and standard deviation along the set of plants

Tuning Method	M_S	Mean	St.Dev.
AMIGO (PID)	1.4	1.43	0.01
AMIGO (PI)	1.4	1.28	0.05
KT (PID)	1.4	2.14	4.35
KT (PI)	1.4	1.30	0.02
NORT (PI)	1.4	1.41	0.04
S-IMC (PI)	1.6	1.59	0.00
NORT (PI)	1.6	1.61	0.00
Tavakoli (PI)	2.0	2.02	0.02
KT (PID)	2.0	3.26	5.61
KT (PI)	2.0	1.85	0.21
NORT (PI)	2.0	2.02	0.02

Figures 2 to 4 show the achieved M_S levels for a design specification of $M_S = 2.0$, 1.6 and 1.4 respectively, whereas Table I provides a numerical comparison. It can be seen that, for the $M_S = 2.0$ case, the Tavakoli and NORT methods are clearly superior to the KT, showing this one large excursions from the desired value. Both methods; Tavakoli and NORT; provide a more or less uniform value for M_S, however this value is above the desired one for almost all plants considered. It is the authors' opinion that this is a bit surprising because the robustness level of $M_S = 2.0$ is the less restrictive one and it seems it would be easier to be achieved. On the contrary, by looking at Fig. 3, it can be seen that this more severe robustness level is achieved on a more satisfactory way. Even for the $M_S = 1.4$ case, although some deviation is observed, the achieved levels are; in general terms; more robust than the specification.

It is remarkable the behavior of the two tuning rules that consider the $M_S = 1.6$ as a design specification; SIMC and NORT. On both cases the achieved robustness is

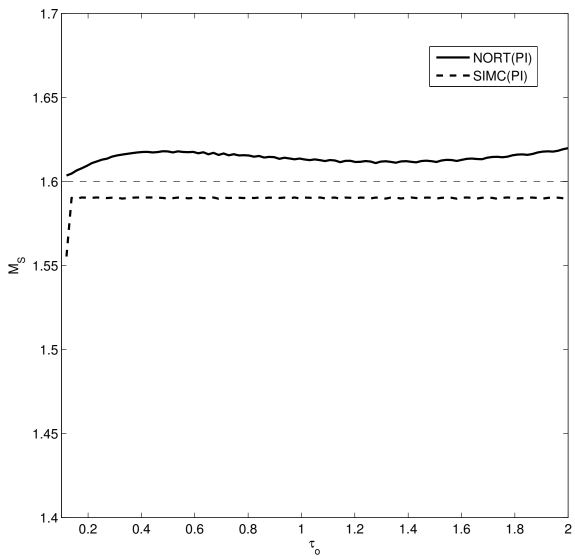

Fig. 3. M_S behaviour for an $M_S = 1.6$ specification

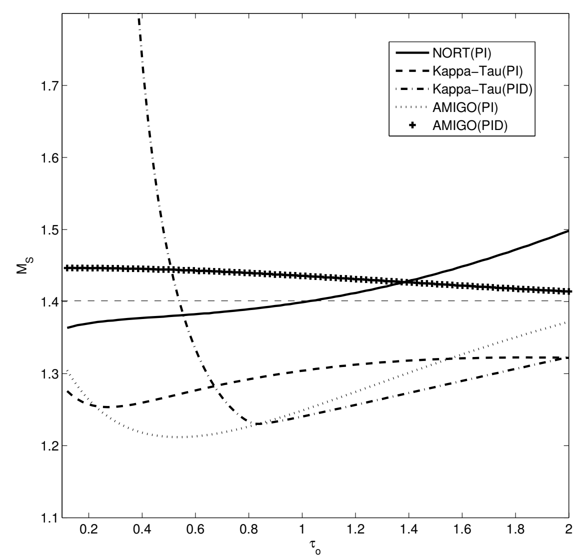

Fig. 4. M_S behaviour for an $M_S = 1.4$ specification

completely flat among the set of plants (except for very small normalized dead-times). Being the design philosophy of both approaches completely different (SIMC is analytically IMC-based whereas NORT is based upon an optimization procedure), this observation suggests that to work on this intermediate level may provide both, an easy achievement of the desired robustness level and more benefit from the point of view of the performance/robustness trade-off.

At this point it is worth to mention that the KT as well as the AMIGO method were conceived on the basis of a benchmark set of plants and the corresponding tuning rules adjusted in terms of the FOPTD model (1) parameters. The AMIGO method, that could be considered as a modern version of the KT based on non-convex optimization [17], provides a better fit for the considered case ($M_S = 1.4$). However, the question about the batch of plants used to conceive the tuning rules is raised: Is it better to work on a range of plants generated for $\tau_o \in (0, 2]$ or, instead, to use a test batch along the lines of [6]?

Another conclusion that can be drawn from Table I is that not by using a more powerful controller, such as a PID instead of a PI, we get a more robust control system. For the case $M_S = 1.4$, for example, the NORT design provides a PI that outperforms the AMIGO PID. Even the AMIGO and KT PI controllers provide more robustness than its PID counterparts. The same conclusion can be stated for the $M_S = 2.0$ case. The PI controllers provide more robustness, being the Tavakoli approach the one that better fits the target and, at the same time, exhibits less variation. Of course here we just consider the robustness viewpoint. How the corresponding controller performs in terms of performance is a more difficult aspect as each one of the tuning rules was designed on a different basis for performance (pole assignment, optimizing regulation performance, etc.). Of course, some generic, maybe well accepted, criteria for performance assessment could be used (IAE measures for tracking as well as regulation, control signal bandwidth and IAE, etc.). However in order to put more emphasis on the robustness aspect, the performance evaluation is not considered here.

IV. Fragility Analysis

From the preceding analysis, summarized in Table I, the approaches that better accomplish the designed robustness are the NORT(PI) and AMIGO(PID) for $M_S = 1.4$; NORT(PI) and SIMC(PI) for $M_S = 1.6$; and NORT(PI) and Tavakoli(PI) for a specification of $M_S = 2.0$. This section provides a fragility analysis on the achievement of such specification. It is worth to mention that this analysis does not measure how close the corresponding tunings are to instability [14], [8] but the deviation from the achieved robustness because of a change on the controller parameters. This deviation is computed at two levels by using the previously presented fragility measures. For each one of the mentioned tuning rules, the fragility is computed for the set of plants characterized by $\tau_o \in (0, 2]$. On this respect, what is important is the change of the fragility indices as τ_o increases. In order to get a more clear figure of the fragility characteristics of each tuning, each figure shows the results of a particular tuning.

As the NORT approach provides a good fit at the three designed levels, its fragility is commented first. Figure 5 provides the fragility figures associated to each one of the robustness levels. The first important thing to recognize is the correlation among robustness and fragility. As the system robustness increases, its fragility decreases. Therefore as the system is designed to be more robust, it will be less fragile to small controller parameter's adjustment. This is

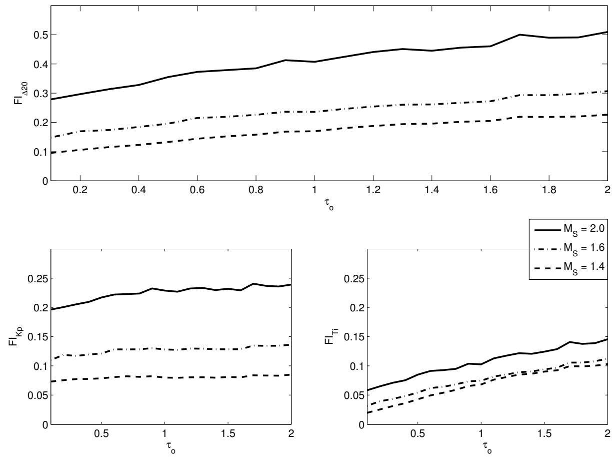

Fig. 5. NORT $FI_{\Delta 20}$ and $FI_{\delta \epsilon p}$

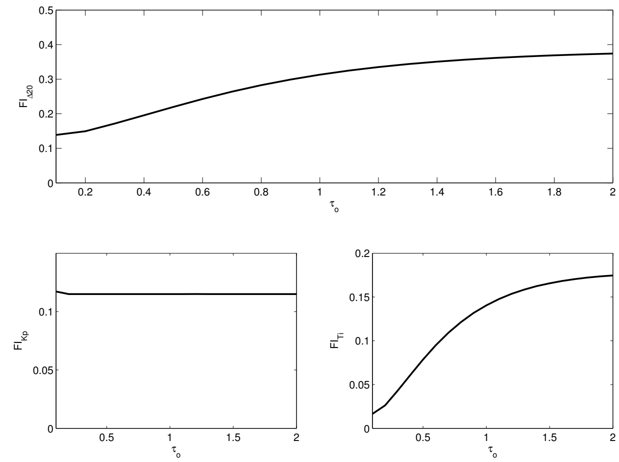

Fig. 7. SIMC $FI_{\Delta 20}$ and $FI_{\delta \epsilon p}$

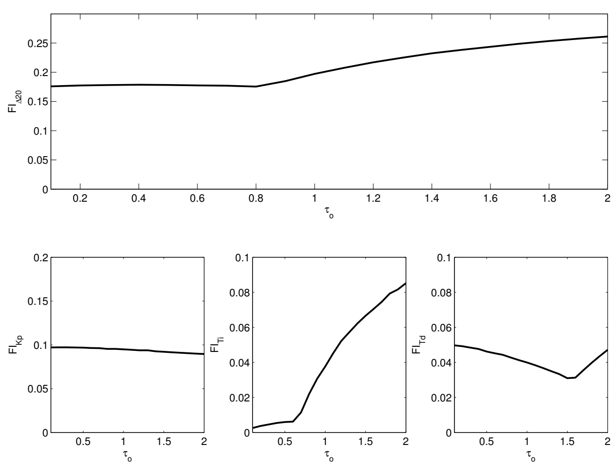

Fig. 6. AMIGO $FI_{\Delta 20}$ and $FI_{\delta \epsilon p}$

an important conclusion not noticed before on the literature. As it can be seen the $FI_{\Delta 20}$ index slightly increases with τ_o, being the integral time the major responsible of such increase (notice FI_{K_p} is almost constant). However, while there is a considerable loose of fragility when we move from $M_S = 1.6$ to $M_S = 2.0$, the fragility levels of both $M_S = 1.4$ and $M_S = 1.6$ are quite similar.

Figure 6 shows the $FI_{\Delta 20}$ for the AMIGO PID controller tuning. The fragility is almost the same over all the plant family range with a small increase for $\tau_o > 0.9$. As it is seen from the FI_{T_i} plot the integral time is the major responsible for such increase making the controller a fragility-unbalanced one [2]. Therefore showing special attention must be paid to the implementation (or fine adjustment) of the integral term. Comparing the fragility levels with the ones for the NORT approach for $M_S = 1.4$ we can see that the NORT PI results to be less fragile. Therefore, the NORT PI controller achieves a better accomplishment of the $M_S = 1.4$ robustness level and results less fragile than the AMIGO PID.

On the other hand, for the $M_S = 1.6$, see Fig. 7, even the SIMC tuning achieves the desired robustness almost perfectly, the controller results highly dependent on the integral time. As the τ_o increases, the fragility due to possible integral time adjustments augments considerably. The overall fragility is, again, higher than the NORT PI controller.

Comparison with the Tavakoli method for the less robust specification, $M_S = 2.0$, is shown in Fig. 8. In this case the results are highly similar with its counterpart, the NORT PI controller.

As a general conclusion of the above comparisons we can state that in almost all cases the provided tuning rules generate fragility-unbalanced controllers (specially for the AMIGO case). Showing the usual fine-tuning carried out at operator level once the controller is in place, should be performed very carefully or the achieved nominal robustness can be lost.

V. Conclusions

This paper has provided an analysis of the robustness accomplishment provided by several PI/PID tuning rules. The tuning rules appearing in the literature that are conceived for achieving a certain degree of robustness (specified in terms of its M_S value) show very different levels of accomplishment of such specification. More than this it is shown that the achieved robustness is plant dependent.

One conclusion drawn from this study is that the PI controllers result to exhibit a better accomplishment of the robustness specification than the PID controller. This opens the door to the search for more appropriate designs of PID controllers where the robustness is taken into account explicitly and further verified once the design is completed.

Even some of the analyzed methods do not provide the designed robustness, a fragility analysis has also been conducted. It has been confirmed that controllers' fragility

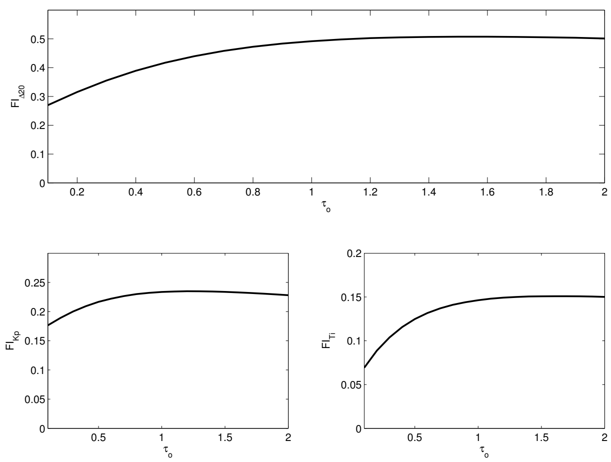

Fig. 8. $FI_{\Delta 20}$ and $FI_{\delta\epsilon p}$ for the Tavakoli PI controller

augments with the plant normalized time constant for all approaches. Moreover, the major responsible for the controller's fragility is the integral time term. This fact shows in almost all cases the tuning rules do generate fragility-unbalanced controllers.

From the aforementioned conclusions, it is the authors' opinion that new approaches for PI/PID controller design should be directed on attempting *flat* achievements for the robustness specification and trying to get out as much performance as possible once the specified robustness is achieved. This would constitute a more meaningful framework for controller design comparison as what is often compared are just the design values for the robustness specification.

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Tutorial

Control PID robusto: Una visión panorámica

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Resumen

En este trabajo se presenta una perspectiva general de los diferentes enfoques existentes, con que se ha afrontado el problema de obtener un controlador proporcional integral derivativo (PID) robusto. La estructura restringida y muy particular que impone el controlador PID, supone por un lado, un atractivo que ha sido la causa de su extenso uso en el sector industrial, pero por el otro, impone una serie de dificultades al plantear la incorporación de consideraciones de robustez en su diseño. Hoy en día, el abanico de posibilidades de diseño de un controlador PID es realmente amplio, pudiendo afrontarse con prácticamente cualquier enfoque, en concreto con cualquier enfoque de control robusto. A este respecto, es importante distinguir entre los métodos y las reglas de sintonía, siendo de especial interés en el caso del controlador PID, la generación de reglas de sintonía a la vez simples y que proporcionen ciertas garantías de robustez. Como paso previo a esta clasificación, conviene establecer de qué manera se representa y mide la robustez. Por lo tanto, de qué manera acaba formulándose como especificación de diseño. El trabajo repasa también, conceptos aparecidos recientemente en la literatura del control PID y relacionados con la robustez, como son el cumplimiento de la misma y la fragilidad del controlador. *Copyright © 2011 CEA.*

Palabras Clave:
Control PID, Robustez, Incertidumbre.

1. Introducción

Sin duda alguna, desde su introducción en 1940 (Babb, 1990; Bennett, 2000), los controladores PID (proporcional integral derivativo), son la opción más utilizada en las diferentes aplicaciones de control de procesos. Su éxito se debe principalmente, a la sencillez de su estructura (tres parámetros a sintonizar) y funcionamiento, que le permiten al ingeniero de control un entendimiento mejor y fácil, comparado con otras técnicas de control avanzadas. Este hecho ha motivado los continuos esfuerzos de investigación orientados a encontrar enfoques alternativos de diseño y nuevas reglas de sintonía, con el fin de mejorar el rendimiento de los lazos de control con base en controladores PID.

La literatura generada alrededor de las técnicas y las metodologías de diseño y sintonía de controladores PID es realmente vasta. Merece especial atención el *IFAC Workshop PID'00 Present and Future of PID control* que tuvo lugar en Terrassa, España, en abril de 2000, donde se proporcionó una visión del estado del arte en el tema, así como un buen augurio para el futuro de la investigación en el campo de los controladores PID. A este evento le sucedieron en enero de 2002 y febrero de 2006, sendos números especiales de las revistas *IEE Control Theory and Practice Part D* (ahora IET) e *IEEE Control Systems Magazine*. En estos números se presentan en considerable profundidad, muchos aspectos relacionados con el control PID, que van desde trabajos de control más especializados, hasta revisiones de productos tecnológicos, patentes, software, etc., confirmando de alguna manera, el interés que el controlador PID continúa teniendo, tanto en el ámbito académico como en el industrial.

La mayoría de los trabajos que han ido apareciendo a lo largo de los años, toman la forma de propuestas de diseño con base en modelos simples y, generalmente, dan lugar a reglas de sintonía que relacionan los parámetros del modelo del proceso, con los parámetros del controlador en una forma directa y sencilla. Así también encontramos diversos libros de ingeniería de control (algunos de ellos específicos de control PID) como son: Åström and Hägglund (2006), Johnson and Moradi (2005) y Visioli (2006), en los cuales se respalda la necesidad de disponer de diferentes técnicas y enfoques, reglas de sintonía simples, así como, cuando se requiera, de procedimientos más elaborados. No obstante, un punto común en todos ellos, es la

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necesidad de incorporar un buen conocimiento del problema de control y su relación con el modelado y el conocimiento del proceso a controlar.

En este sentido, el trabajo inicial que elaboraron Ziegler and Nichols (1942), constituye el primer intento para un diseño sistemático de un controlador PID, a partir de una información mínima del proceso. A este trabajo le siguió el también conocido trabajo de Chien et al. (1952), en el que se presenta una modificación del trabajo de Ziegler y Nichols y donde se plantea la necesidad de considerar métodos diferentes para la respuesta en referencia y para la atenuación de perturbaciones. La introducción de ideas sobre el diseño algebraico, dió lugar al llamado "λ-tuning" o método de Dahlin (1968). Este método, a su vez, está estrechamente relacionado con el predictor de Smith y el método de diseño con base en el control con modelo interno (IMC) (Rivera et al., 1986). De entre ellos, son de especial mención las consideraciones apuntadas, en las que se muestra que el control IMC toma la forma de un controlador PI o PID, dependiendo de la aproximación racional que se utilice del retardo. Estos enfoques, no obstante, utilizan la cancelación de los polos del proceso, lo que puede dar lugar a respuestas a una perturbación bastante indeseables, sobretodo para procesos con constantes de tiempo muy grandes. En Chien and Fruehauf (1990) se presenta una modificación que no cancela los polos del proceso, mientras que Skogestad (2003), presenta una variación del controlador IMC, aplicado a la sintonía de controladores PI y PID denominada SIMC, en la que se evita la cancelación mediante una redefinición del modo integral, para los casos de sistemas dominados por constantes de tiempo grandes.

Los métodos con base en la aplicación de técnicas de optimización, constituyen una alternativa a los métodos analíticos. La idea básica, es intentar capturar diferentes aspectos deseados del funcionamiento en lazo cerrado, bajo la firma de una determinada funcional de coste a minimizar. En Smith and Corripio (1985) y en Shinskey (1988), por ejemplo, se proporcionan controladores optimizados respecto a los criterios de error integral ISE, IAE e ITAE; y en Zhuang and Atherton (1993) se proporcionan reglas de sintonía simples, para diferentes variantes de criterios integrales. En Visioli (2001) se presenta la correspondiente versión para los sistemas inestables e integrantes.

Más recientemente, en parte gracias a la mayor accesibilidad a las rutinas de optimización, han aparecido enfoques de optimización multiobjetivo, como por ejemplo Toivonen and Totterman (2006) y Herreros et al. (2002), donde se plantea un enfoque genérico y se ejemplifica su aplicación al caso de un controlador PID. En un trabajo más reciente Reynoso et al. (2009) aplican esta estrategia al "benchmark" propuesto por el grupo de Ingeniería de Control de CEA-IFAC en el año 2008.

La aplicación de estas estrategias de optimización, aunque efectivas, recae en la utilización de técnicas numéricas bastante complejas y no resultan en reglas de sintonía. Mediante su apliación, se obtiene la sintonía del controlador como la solución al problema de optimización.

La disponibilidad efectiva de potentes herramientas de optimización, que posibilitan la resolución de problemas de control óptimo con relativa facilidad, pone encima de la mesa un pensamiento bastante común, en referencia a la necesidad o no, de disponer de reglas de sintonía. Si bien es cierto que es posible resolver en un tiempo más que razonable, problemas que hace unos años eran impensables, desde el punto de vista del control, la solución de estos problemas requiere de un modelo del proceso controlado que sea suficientemente fiable. La situación real, no obstante, es que la información disponible es normalmente tan solo marginal. De ahí la importancia de disponer de reglas de sintonía que requieran una mínima información del proceso a controlar. Una problemática adicional de los procedimientos de optimización, es la disponibilidad de una *buena* condición inicial que facilite o, en ocasiones, garantice la convergencia de los mismos. De esta forma, la aplicación de una regla de sintonía puede proporcionar esta condición inicial. Una constatación de la necesidad de disponer de reglas de sintonía de una complejidad limitada y que, a la vez, requieran una mínima información del proceso, son las propuestas que se pueden encontrar en la literatura y que contínuan hoy en día apareciendo.

Esta distinción, y porqué no clasificación, es de especial aplicación para los que se centran en la obtención de un controlador PID, con determinadas características de robustez. La robustez fue un aspecto no considerado durante mucho tiempo, como parte integrante de las consideraciones a contemplar en el diseño de un sistema de control. No únicamente en el caso del control PID, si no también desde un punto de vista completamente ámplio. No es hasta la formulación originada por Zames en el año 1981, del tratamiento de la incertidumbre mediante la norma infinito, que el control robusto entra de lleno en la teoría de control y da lugar a una doctrina hoy en día completamente desarrollada y madura, conocida como el control $\mathcal{H}_\infty$. Este enfoque ha quedado recogido en numerosos libros, que ofrecen una ámplia visión perspectiva sobre el tema y las diferentes variantes y planteamientos a que ha dado lugar (Grimble, 1994; Morari et al., 1988; Vidyasagar, 1985).

Las ideas emergidas del desarrollo del control robusto han acabado, por supuesto, siendo particularizadas al caso del controlador PID. Esta permeabilización ha originado diferentes planteamientos, de lo que se podría denominar *control PID robusto*. De esta forma, se distingue entre la obtención de un PID robusto; como resultado de la solución de un problema de control robusto aplicado a un controlador de estructura restringida; y las propuestas de reglas de sintonía simples; que incorporan en su concepción consideraciones de robustez. A pesar de que el resultado final es claramente diferente, en ambos casos, no obstante, es fundamental la descripción que se utiliza de la incertidumbre asociada al modelo del proceso a controlar. La manera en la que se incorpora la tolerancia del sistema de control a los errores de modelado, determina en una forma clara, las características y el planteamiento del problema de diseño.

De esta forma, por ejemplo, se tienen los métodos desarrollados con base en el control con modelo interno (Rivera et al., 1986), donde las reglas de sintonía quedan parametrizadas en términos de un parámetro directamente relacionado con la robustez del sistema. No obstante, esta robustez no queda vinculada directamente a un indicador o medida de robustez. Por otro lado, se tienen las conocidas estrategias de diseño estableciendo los márgenes de ganancia y fase, iniciadas en Åström and Hägglund (1984) y que han dado lugar a numerosas variantes y extensiones. En este caso, el parámetro de diseño es directamente la medida de robustez escogida para el sistema en lazo cerrado. Esta idea se extendió a la utilización del máximo de la función de sensibilidad (comúnmente denominado M_S), siendo una medida bastante común hoy en día. Aquí también se puede distinguir, entre los enfoques que se plantean conseguir un lazo cerrado que proporcione un determinado valor de M_S (Åström and Hägglund, 2004) y los enfoques más flexibles que proporcionan reglas de sintonía directamente parametrizadas por el valor de M_S deseado (Alfaro et al., 2009d).

Cuando no se habla de indicadores concretos de robustez; o medidas de robustez; el abanico de posibilidades se abre de una manera considerable. Aquí se incluirían estrategias que pretenden garantizar la estabilidad de un sistema descrito mediante varios modelos, correspondientes a diferentes puntos de trabajo posibles (Toscano, 2005); estrategias de diseño $\mathcal{H}_\infty$ o combinadas $\mathcal{H}_2/\mathcal{H}_\infty$ (Goncalves et al., 2008), donde el diseño es dirigido por funciones de peso en el dominio de la frecuencia, que caracterizan la incertidumbre asociada al modelo; formulaciones en el espacio de estados y estrategias de desigualdades matriciales lineales (LMI) (Ge et al., 2002); y aplicación de otros criterios de estabilidad en el dominio de la frecuencia (Hara et al., 2006), entre otros.

Mención aparte merecen los trabajos de Silva, Datta y Bhattacharyya, para la determinación del conjunto de controladores que estabilizan una familia determinada de plantas, presentados en forma comprensiva en (Silva et al., 2005). Estos establecieron otro enfoque para el diseño de sistemas de control robustos (Ho et al., 2001; Silva et al., 2003)

Tal como se ha comentado anteriormente, a pesar de la disponibilidad de herramientas numéricas avanzadas y la posibilidad de plantear diseños con base en la formulación de problemas de relativa complejidad, sigue existiendo una determinada predilección por la formulación del problema del ajuste de controladores PID, con base en reglas de sintonía; por poder ser estas de baja complejidad. A este respecto es interesante complementar la inclusión de especificaciones de robustez en el problema de diseño, con las siguientes consideraciones:

- Cumplimiento de las especificaciones de robustez: ¿Proporciona el controlador un lazo cerrado con la robustez indicada en las especificaciones?

- Compromiso entre la robustez y el rendimiento del sistema de control: ¿Se tiene una idea del precio que se paga (en términos de rendimiento), por la robustez especificada?

- Fragilidad del controlador: ¿Mantendrá el sistema de control la robustez obtenida, ante pequeñas variaciones en los parámetros del controlador (*ajuste fino*)?

De esta forma, en este trabajo se pretende dar un repaso a los enfoques del diseño de controladores PID, en los que se incorporan de manera explícita consideraciones de robustez. En la siguiente sección, se realiza una presentación del marco de trabajo, haciendo hincapié en las posibles medidas de robustez a utilizar, así como en las posibles maneras de describir la incertidumbre asociada al proceso. A continuación, en la sección 3, se presenta un breve repaso a las primeras formulaciones de diseños con base en especificaciones por márgenes de ganancia y fase. Seguidamente, en la sección 4, se presentan las reglas de sintonía con base en la incorporación de la sensitividad máxima como medida de robustez, distinguiéndolas de las reglas que incorporan consideraciones de robustez pero de una forma más genérica, sin vincularla a un indicador concreto, enfoques que se presentan en la sección 5. Obedeciendo a formulaciones completamente diferentes, en la sección 6, se ofrece una panorámica de la miscelánea de enfoques existentes, algunos especialmente particulares, con base en una adaptación de los métodos originados dentro del seno de la teoría de control robusto. El estudio se complementa con una serie de consideraciones, que son de especial aplicación para el caso de diseños con base en medidas de robustez. Estas consideraciones incluyen el reciente concepto de fragilidad de controladores; en especial su particularización al caso PID; así como observaciones necesarias en referencia al cumplimiento de las especificaciones de robustez y su relación con el conocido compromiso entre la robustez y el rendimiento del sistema de control.

2. La robustez: planteamientos y medidas

En lo que sigue, se utilizará el sistema de control monovariable representado en la figura 1. Cabe notar que este no es el diagrama de control más completo con que se puede encontrar, si no que obedece al propósito de facilitar la discusión que sigue. El proceso a controlar $P(s)$ se supone generalmente bajo la forma de la función de transferencia de un modelo de primer orden con retardo:

$$P(s) = \frac{Ke^{-Ls}}{Ts+1}, \tag{1}$$

donde se define el parámetro adimensional $\tau_o = L/T$, utilizado en ocasiones para parametrizar las relaciones de sintonía. El controlador $C(s)$ responde a un controlador PID en una de sus diversas formulaciones. No se entrará en la presentación de todas las posibles variaciones existentes puesto que, como se verá más adelante, no todos los enfoques de diseño utilizan la misma formulación del controlador PID y, además, de alguna manera las definiciones que siguen en lo referente a las medidas de estabilidad, son de carácter completamente general y no aplican tan solo al caso PID. No obstante, quizás la formulación más usual es la Estándar con filtro en la acción derivada, dada por:

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N}s + 1}\right). \tag{2}$$

De esta forma, en base a este diagrama quedan definidas las siguientes relaciones entre las señales de entrada y salida:

$$y(s) = T(s)r(s) + P(s)S(s)d(s), \tag{3}$$

$$u(s) = C(s)S(s)r(s) + T(s)d(s), \tag{4}$$

donde las funciones $S(s)$ y $T(s)$ son, respectivamente, las funciones de sensibilidad y sensibilidad complementaria, que quedan definidas como:

$$S(s) \doteq \frac{1}{1+P(s)C(s)}, \quad T(s) \doteq \frac{P(s)C(s)}{1+P(s)C(s)}, \tag{5}$$

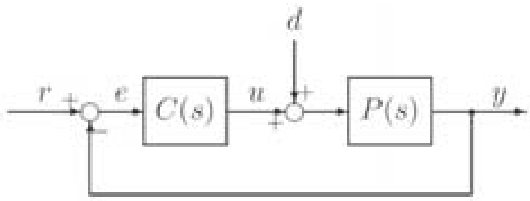

Figura 1: Diagrama de bloques sistema de control realimentado

y que, como se verá, juegan un papel fundamental en la determinación de las propiedades de estabilidad del sistema de control.

Al hablar de estabilidad del sistema de control, esta es usualmente entendida como estabilidad absoluta. Es decir, la función de transferencia resultante en lazo cerrado, siempre que no haya cancelación de modos inestables, tiene todos sus polos en el semiplano izquierdo. No obstante, si se desea introducir medidas de rendimiento del sistema respecto a la propiedad de estabilidad, entonces se necesita el concepto de estabilidad relativa. De alguna manera, la estabilidad relativa se utiliza para determinar cuán estable es el sistema o, dicho de forma alternativa, a qué distancia se encuentra el lazo cerrado de la inestabilidad. Esta idea de distancia a la inestabilidad, origina diferentes interpretaciones en tanto que se relaciona con las posibles fuentes de error en el modelo del proceso utilizado para el diseño del controlador: ¿Qué cantidad y qué tipo, de incertidumbre se puede permitir antes que el sistema de control se vuelva inestable? Ante esta interpretación, las diferentes medidas de estabilidad relativa se denominan *medidas de robustez*. De esta forma, la incorporación de estas medidas en los procedimientos de diseño y la concepción de las reglas de sintonía, da lugar a lo que se denomina como diseño de un controlador robusto. En nuestro caso, el de un controlador PID robusto.

2.1. *Medidas clásicas de robustez*

Las medidas de robustez han ido evolucionando a lo largo de los años, dando lugar, consecuentemente, a nuevos enfoques del control robusto. De esta forma, se conocen como medidas clásicas de robustez; o de estabilidad relativa; los márgenes de ganancia y fase. Estas medidas se apoyan en el criterio de estabilidad de Nyquist y consideran la posibilidad de variar el número de giros alrededor del punto crítico (-1, 0), bajo una variación en la ganancia o en la fase, respectivamente, del sistema.

Así, el *margen de ganancia*, A_m, es una especificación del punto en que, sobre el diagrama de Nyquist, la respuesta en frecuencia de la función de lazo debe cruzar el eje real negativo. Esta especificación determina un margen de ganancia, puesto que define un factor multiplicativo para la ganancia, para el cual el sistema se haría inestable. La figura 2 representa la geometría correspondiente al margen de ganancia. El valor de A_m se define a través de la condición:

$$A_m|C(j\omega_{-\pi})P(j\omega_{-\pi})| = 1, \tag{6}$$

de esta forma

$$A_m = \frac{1}{|C(j\omega_{-\pi})P(j\omega_{-\pi})|}. \tag{7}$$

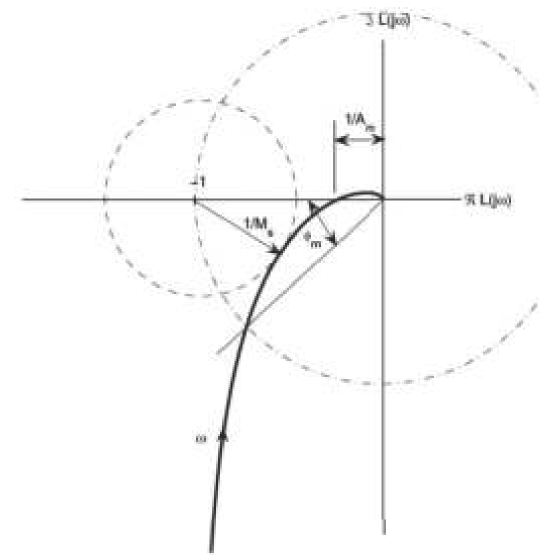

Figura 2: Definición de la sensibilidad máxima M_S y los márgenes de ganancia A_m y de fase ϕ_m

La frecuencia $\omega_{-\pi}$ en la que se calcula el margen de ganancia, es la frecuencia a la que la fase de la cadena directa vale -180^o. La interpretación del margen de ganancia como medida de robustez, nos indica que en el caso en que el modelo del sistema sea incorrecto, su ganancia estática puede aumentar en un factor A_m antes de que el sistema se haga inestable. Valores típicos de especificación del margen de ganancia son $2 \leq A_m \leq 5$.

Por otra parte, el *margen de fase*, ϕ_m, especifica en que cantidad puede retardarse la fase del sistema, de manera que la rotación que se produciría sobre la respuesta en frecuencia del sistema, en el diagrama de Nyquist, la llevaría a cruzar el punto crítico (-1, 0). La denominación margen de fase especifica pues, el desplazamiento que puede sufrir la fase del sistema antes de que el correspondiente lazo cerrado se haga inestable. La geometría correspondiente para el margen de fase, figura 2, conduce a la siguiente condición (en grados):

$$-\phi_m + \arg\{C(j\omega_1)P(j\omega_1)\} = -180^o, \tag{8}$$

de manera que

$$\phi_m = 180^o + \arg\{C(j\omega_1)P(j\omega_1)\}. \tag{9}$$

La frecuencia ω_1 en la que se calcula el margen de fase, es la frecuencia a la que la ganancia del sistema toma el valor uno. De manera análoga al margen de ganancia, la interpretación del margen de fase como medida de robustez, nos indica que en caso de que exista un error en el modelado del proceso, este puede llegar a sufrir un retraso de fase adicional de ϕ_m grados, a la frecuencia ω_1, antes de que el sistema se haga inestable. Valores típicos de especificación del margen de fase son $30^o \leq \phi_m \leq 60^o$.

2.2. *La función de sensibilidad. ¿Una medida moderna?*

La función de sensibilidad, $S(s)$, constituye hoy en día, la base para establecer la medida de robustez de referencia, dentro del ámbito del diseño de controladores PID. No obstante, su interpretación como función clave para determinar la tolerancia del lazo de control a variaciones en el proceso a controlar, es bien conocida prácticamente desde la formulación del lazo realimentado como estrategia de control. Efectivamente, si tenemos una función f que depende de un parámetro α, la sensibilidad de f a variaciones en α se denota por S_α^f y queda definida mediante:

$$S_\alpha^f \doteq \lim_{\Delta\alpha\to 0} \left.\frac{\Delta f/f}{\Delta\alpha/\alpha}\right|_{\alpha=\alpha_o} = \left.\frac{\alpha}{f}\frac{\partial f}{\partial\alpha}\right|_{\alpha=\alpha_o}, \qquad (10)$$

donde α_o es el valor nominal de α y Δf y $\Delta\alpha$ representan las desviaciones de α y f respecto a sus valores nominales. Se aplicará este concepto a la relación entre la señal de referencia y de salida de un sistema de control, para las versiones en lazo abierto y en lazo cerrado, dadas respectivamente por:

$$T_{la}(s) = P(s)C(s), \quad T_{lc}(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}. \qquad (11)$$

De esta forma, ante la presencia de incertidumbre en el proceso, $P(s) = P_o(s) + \Delta P$, las relaciones entrada salida anteriores sufrirán de esta incertidumbre y se verán desviadas respecto a los valores obtenidos en base a la aplicación del controlador a la planta nominal: $T_{la}(s) = T_{la,o}(s) + \Delta T_{la}$, $T_{lc}(s) = T_{lc,o}(s) + \Delta T_{lc}$. Aplicando la definición anterior, las sensibilidades de $T_{la}(s)$ y $T_{lc}(s)$ a variaciones en la planta $P(s)$ quedan determinadas como:

$$S_P^{T_{la}} = \lim_{\Delta P\to 0} \frac{\Delta T_{la}(s)/T_{la,o}(s)}{\Delta P(s)/P_o(s)} = 1, \qquad (12)$$

$$S_P^{T_{lc}} = \lim_{\Delta P\to 0} \frac{\Delta T_{lc}(s)/T_{lc,o}(s)}{\Delta P(s)/P_o(s)} = S(s). \qquad (13)$$

De esta forma, para la relación en lazo abierto, el error de modelado o cambio que pueda sufrir el proceso, se traslada en su totalidad a la relación entrada salida. Por otra parte, en una configuración en lazo cerrado, este cambio en el proceso queda multiplicado por la función de sensibilidad $S(s)$. De ahí que el controlador $C(s)$ pueda escogerse de manera que esta función, presente una magnitud pequeña dentro del rango de frecuencias en que pueda presentarse la incertidumbre. Este es reconocido como uno de los factores clave para el uso de la realimentación.

La función de sensibilidad tal como se ha presentado, es una función dependiente de la frecuencia y que, por tanto, no se puede utilizar de manera directa, como figura de mérito que nos proporcione una medida de robustez. No obstante, si vemos que $S(s)$ depende tan solo de la cadena directa del lazo de control, $L(s) = P(s)C(s)$, es posible una visualización geométrica de la misma en el plano de Nyquist. Efectivamente, puesto que $S(s) = [1 + L(s)]^{-1}$, se puede representar el valor complejo $1 + L(j\omega)$ como el vector que va desde el punto (-1, 0) hasta $L(j\omega)$. La función de sensibilidad será por lo tanto, menor que uno para aquellas frecuencias en las que la curva de Nyquist quede por fuera de una circunferencia con centro (-1, 0) y radio unitario. En base a esta interpretación se define el valor M_S como:

$$M_S \doteq \max_\omega |S(j\omega)| = \max_\omega \left|\frac{1}{1 + C(j\omega)P(j\omega)}\right|, \qquad (14)$$

$$= \frac{1}{\min_\omega |1 + L(j\omega)|}, \qquad (15)$$

siendo entonces la menor distancia desde la curva de Nyquist al punto crítico (-1, 0) igual a $1/M_S$. De esta forma, garantizando un determinado valor para M_S, se asegura que el lugar de Nyquist queda apartado del punto crtíco una distancia igual a $1/M_S$. Interesarán por lo tanto, valores pequeños de M_S para obtener una robustez más elevada. De esta forma, se establece M_S como una medida de robustez más generalizada que los anteriormente presentados márgenes de ganancia y de fase, siendo esta interpretación más amplia la que ha propiciado que haya quedado establecida como una medida de robustez estándar, dentro de los diseños que se proponen hoy en día. De hecho, es posible mostrar que un determinado valor de M_S nos garantiza, en forma simultánea, las siguientes cotas para los márgenes de ganancia y de fase:

$$A_m \geq \frac{M_S}{M_S - 1}, \quad \phi_m \geq 2\sin^{-1}\left(\frac{1}{M_S}\right). \qquad (16)$$

Otra implicación de garantizar que la curva de Nyquist permanezca fuera de un círculo con centro en el punto crítico y de radio $1/M_S$, es que se garantiza la estabilidad a pesar de que la ganancia de sistema se incremente en un factor $M_S/(M_S - 1)$ o se reduzca en un factor $M_S/(M_S + 1)$. Adicionalmente, se puede garantizar la estabilidad a pesar de insertar una nolinealidad estática , $f(x)$, en el lazo, siempre que se verifique que:

$$\frac{M_S}{M_S + 1} < \frac{f(x)}{x} < \frac{M_S}{M_S - 1}. \qquad (17)$$

Valores típicos de M_S oscilan en el intervalo de 1,4 a 2, implicando para el caso $M_S = 2$, un $A_m \geq 2$ y un $\phi_m \geq 29^o$. Para el caso $M_S = 1{,}4$ se garantiza un $A_m \geq 3{,}5$ y un $\phi_m \geq 41^o$. La ventaja que presenta el uso de M_S, es que determina el A_m y el ϕ_m de manera simultánea, mientras que estas dos cantidades son completamente independientes. A su vez, el M_S también aparece como referencia en las comparativas que se puedan realizar de la robustez conseguida por determinados enfoques, a pesar de que ésta no se contemplara explícitamente en la fase de diseño.

2.3. *Una medida de robustez generalizada*

Desde la perspectiva de la teoría de control robusto, la estabilidad del sistema de control en lazo cerrado se mantiene ante variaciones, $\Delta P(s)$, en el modelo nominal utilizado, $P_o(s)$, siempre que se verifique la condición:

$$|C(j\omega)\Delta P_o(j\omega)| < |1 + L(j\omega)|, \qquad (18)$$

expresión que puede reescribirse como

$$\left|\frac{\Delta P(j\omega)}{P_o(j\omega)}\right| < \frac{1}{T(j\omega)}. \qquad (19)$$

De esta forma, se pueden permitir variaciones relativas en el modelo del proceso siempre y cuando, en las frecuencias correspondientes, la función de sensibilidad complementaria tome valores pequeños. Esto conduce a considerar el valor máximo de $T(s)$ como una estimación de esta tolerancia:

$$M_T = \max_\omega |T(j\omega)| = \max_\omega \left|\frac{L(j\omega)}{1 + L(j\omega)}\right|. \qquad (20)$$

De esta manera, se puede formular un margen de robustez generalizado como en Kristiansson (2003) y Kristiansson and Lennartson (2006), GM_S, definido por:

$$GM_S = \text{máx}\{\|S\|_\infty, \alpha\|T\|_\infty\}, \qquad (21)$$

donde el parámetro α se define como $\alpha = M_S^d/M_T^d$ en base a los valores deseados para los máximos de las funciones de sensibilidad. Geométricamente, especificar un valor para M_T impone también un área prohibida para la curva de Nyquist, dada por un círculo con centro en $(-M_T^2/(M_T^2-1), 0)$ y de radio $M_T/(M_T^2-1)$. De esta forma, mediante la especificación para M_S y la especificación para M_T, se está imponiendo que el lugar de Nyquist se mantenga fuera de los correspondientes círculos. La propuesta de tener esta zona a evitar determinada por diferentes círculos, es presentada en detalle en Åström and Hägglund (2006).

3. Diseño por márgenes de ganancia y fase

El diseño con base en la especificación de unos determinados A_m y ϕ_m, se puede considerar como el primer planteamiento en el que se *específica* de manera explícita, la robustez en base a indicadores de robustez. La primera propuesta de diseño con base en A_m y ϕ_m se presenta en Åström and Hägglund (1984) para un A_m de 3 y un ϕ_m de 60^o. Así, las ecuaciones de diseño para un controlador PI y un sistema de primer orden con retardo que resultan son:

$$\frac{\pi}{2} + \tan^{-1}(\omega_p T_i) - \tan^{-1}(\omega_p \tau) - \omega_p L = 0, \qquad (22)$$

$$A_m K_p K = \omega_p T_I \sqrt{\frac{\omega_p^2\tau^2+1}{\omega_p^2 T_I^2+1}}, \qquad (23)$$

$$K_p K = \omega_g T_I \sqrt{\frac{\omega_g^2\tau^2+1}{\omega_g^2 T_I^2+1}}, \qquad (24)$$

$$\phi_m = \frac{\pi}{2} + \tan^{-1}(\omega_g T_i) - \tan^{-1}(\omega_g \tau) - \omega_g L. \qquad (25)$$

El problema que presenta el diseño en base a A_m y ϕ_m, es que las expresiones resultantes para el cálculo de los mismos son altamente no lineales. Para su solución se deben obtener, también, las mencionadas frecuencias de cruce, apareciendo en las ecuaciones de las mismas la función $\tan^{-1}$ que imposibilita su cálculo analítico. Debido a esta problemática básica, han aparecido diferentes enfoques con base en aproximaciones, los cuales proporcionan relaciones que posibilitan tanto, el cálculo de los A_m y ϕ_m asociados a una regla en particular, así como el diseño para conseguir unos A_m y ϕ_m determinados.

En Ho et al. (1996) por ejemplo, se proporcionan ecuaciones para la evaluación de los A_m y ϕ_m de un lazo de control, en base a un controlador PID y un sistema de primer orden con retardo. Estas ecuaciones se aplican luego para evaluar los A_m y ϕ_m, que proporcionan reglas de sintonía conocidas, como las clásicas de Ziegler-Nichols y Cohen-Coon, así como las derivadas de reglas óptimas (ISE, IAE, ITAE). Siendo los valores recomendados de entre 2 y 5 para el A_m y entre 30^o y 60^o para el ϕ_m. Ho et al. (1995b), Ho et al. (1995a) y Ho et al. (1996), mostraron que hay formulaciones que pueden proporcionar valores que están fuera de estos rangos. De esta forma, posteriormente, en Ho et al. (1999) se propone una sintonía que obedece a una especificación de A_m y ϕ_m, intentando obtener el mejor rendimiento en el sentido ISE. Así, para un A_m determinado, hay un ϕ_m que proporciona un ISE mínimo. Una especificación de ϕ_m mayor conlleva una degradación del rendimiento, existiendo por lo tanto un compromiso entre ambos. En cambio, para valores menores de ϕ_m, este compromiso no existe, pudiéndose especificar una mayor robustez sin perder rendimiento.

Un problema que presentan los A_m y ϕ_m, es que ambas medidas son independientes y que debe, por tanto, garantizarse el cumplimiento de ambas. Es por este motivo que en los últimos años, la especificación de robustez se ha orientado más hacia el uso de la función de sensibilidad, que al uso de estas dos medidas.

4. Diseño robusto con base en la sensibilidad máxima M_S

El uso de los márgenes de ganancia y fase como indicadores de robustez, ha quedado sustituido por el uso de un único indicador, el máximo de la función de sensibilidad, denotado por M_S. Sin embargo previo a esto, Gerry and Hansen (1987) propusieron como indicadores de robustez, la *Razón de ganancias* GR y la *Razón de retardos* DR, junto con el *gráfico de robustez*, retomados estos después por Shinskey (1990). El gráfico de robustez establece un área de restricción en torno al punto de sintonización del controlador, en el plano paramétrico del modelo, fuera de la cual debe encontrarse la curva límite de la estabilidad paramétrica del lazo de control. Esta restricción garantiza que la razón de ganancias y la razón de retardos, sean superiores a dos.

La utilización de la función de sensibilidad en el diseño de controladores PID, surge ligada a la sugerencia de optimizar el rechazo de perturbaciones, mediante la imposición de restricciones en la función de sensibilidad. El uso del máximo de la función de sensibilidad, M_S, fue propuesto por Persson (1992). No obstante, no es hasta la presentación de las reglas de sintonización *Kappa-Tau* por Åström and Hägglund (1995), que se difunde el desarrollo de reglas y métodos de sintonización que en forma implícita o explícita, consideran la robustez en el diseño del lazo de control utilizando el M_S.

A continuación se describen brevemente algunas de estas técnicas:

- Método *Kappa-Tau* (KT) (Åström and Hägglund, 1995)

  El método utiliza un diseño empírico con base en la localización de los polos dominantes de lazo cerrado, junto con una restricción en la robustez con el M_S, para obtener reglas de sintonización para controladores PI y PID Estándar de dos grados de libertad (2GdL). Emplea un conjunto particular de plantas de prueba, como representación de las diferentes dinámicas que se pueden encontrar en los procesos industriales, incluyendo procesos integrantes, caracterizadas por su ganancia K, su constante de tiempo dominante T, y su retardo aparente L, obtenidos estos a partir de la curva de reacción del proceso; o por sus parámetros críticos, ganancia K_{pu} y periodo de oscilación T_u . Establece dos niveles de robustez, una mínima $M_S = 2{,}0$ y una alta $M_S = 1{,}4$. El ámbito de aplicación del método es $0{,}1 \leq L/(L+T) \leq 0{,}85$.

- Método *AMIGO* (Hägglund and Åström, 2002, 2004; Åström and Hägglund, 2004)

  Toma como base el método *MIGO* ("M-constrained Integral Gain Optimization") de Åström et al. (1998), que optimiza la ganancia integral del controlador, K_i, e impone una restricción en la sensibilidad máxima, $M_S = 1{,}4$, aplicado a un conjunto ámplio de plantas de prueba, que incluye plantas de polo múltiple, procesos integrantes y con respuesta inversa. A partir de los parámetros óptimos, determina las ecuaciones para una sintonía robusta de controladores PI y PID Estándar de 2GdL. Una revisión del método AMIGO ("Approximated MIGO") se presenta en Åström and Hägglund (2006). Aunque el ámbito de estudio del método es $0 \leq L/(L+T) \leq 0{,}98$, para los controladores PID Estándar, solo se pueden obtener parámetros fiables para $L/(L+T) > 0{,}5$.

- Método $KL\kappa150$ (Kristiansson, 2003)

  Resolviendo un problema de optimización con restricciones de criterio múltple, en el que se incluye el desempeño ante perturbaciones J_d, la robustez con la sensitividad máxima generalizada GM_S , el esfuerzo de control J_u, y la sensitividad al ruido de medición de alta frecuencia, se determinan reglas de sintonización para controladores PI y PID Estándar de 1GdL, en función de la ganancia estática de la planta y de su ganancia a la frecuencia a la cual su fase es -150^o. Con este diseño se obtiene una robustez generalizada $GM_S \approx 1{,}7$ y un desempeño cercano al óptimo IAE. El método tiene la peculiaridad que considera la constante del filtro derivativo, como un parámetro ajustable más del controlador (Kristiansson and Lennartson, 2006).

- Optimización de objetivo múltiple (*MOO*) (Tavakoli et al., 2007)

  En un primer desarrollo (Tavakoli et al., 2005) se considera el desempeño óptimo IAE ante cambios en la consigna y la perturbación, junto con la restricción de robustez mínima $M_S \leq 2{,}0$, para controladores PI de 2GdL a partir de modelos de primer orden con retardo y modelos integrantes con retardo. Posteriormente para el método *MOO*, junto a lo anterior, se penaliza además la variación total del esfuerzo de control TV_u y se utiliza un algoritmo genético de objetivo múltiple para la optimización. Los controladores obtenidos con esta sintonización, producen sistemas de control con una robustez nominal $M_S \approx 1{,}7$ para modelos de primer orden con retardo con $0{,}1 \leq \tau_o \leq 2{,}0$.

  Las ecuaciones del MOO para los modelos de primer orden con retardo son:

$$\begin{aligned} K_pK &= \frac{1}{6} + \frac{5}{11\tau_o}, \\ \frac{T_i}{T} &= \frac{\frac{3}{14}\tau_o + \frac{7}{6}}{\tau_o + \frac{1}{5}}\tau_o, \\ \beta &= \frac{4}{9}\tau_o + \frac{1}{2}. \end{aligned} \tag{26}$$

  Para los modelos integrantes con retardo, las ecuaciones son:

$$\begin{aligned} K_pK &= \frac{5}{11\tau_o}, \\ \frac{T_i}{T} &= \frac{35}{6}\tau_o, \\ \beta &= \frac{1}{2}. \end{aligned} \tag{27}$$

- Método PI_{2Ms} (Alfaro et al., 2010)

  Partiendo de los parámetros óptimos obtenidos mediante el procedimiento de sintonía robusta no oscilatoria (NORT) (Alfaro et al., 2009c), para procesos de primer orden con retardo y $0{,}1 \leq \tau_o \leq 2{,}0$, se obtienen reglas de sintonización de controladores PI de 2GdL para cuatro niveles de robustez, $M_S \in \{1{,}4\ 1{,}6\ 1{,}8\ 2{,}0\}$. La característica no oscilatoria de la respuesta ante cambios escalón en la consigna y en la perturbación, impuesta en el diseño, garantiza además un comportamiento "suave" de la señal de control. La característica principal del método es el lograr los cuatro niveles la robustez de diseño, para todo el ámbito de aplicación del mismo. Las reglas de sintonía PI_{2Ms} para modelos de primer orden con retardo se obtienen ajustadas a una forma realmente simple:

$$\begin{aligned} \kappa_p \doteq K_pK &= a_0 + a_1{\tau_o}^{a_2}, \\ \tau_i \doteq \frac{T_i}{T} &= b_0 + b_1{\tau_o}^{b_2}, \\ \beta &= c_0 + c_1{\tau_o}^{c_2}, \end{aligned} \tag{28}$$

  donde el valor de las constantes $\{a_i, b_i, c_i\}$ se muestran en la Tabla 1.

- Otros procedimientos

  Diseño robusto de controladores PI con base en la sensibilidad máxima M_S , y el margen de ganancia A_m (Crowe and Johnson, 2002). Optimización simultanea del desempeño ante cambios en el valor de consigna y la perturbación, con una restricción en la robustez ($M_S \leq 2{,}0$), utilizando algoritmos genéticos para controladores PID Estándar de 2GdL (Shen, 2002).

5. Reglas de sintonía robustas

A pesar de la elevada cantidad de reglas de sintonía existentes (O'Dwyer, 2003), se puede decir que es mediante la formulación del IMC-PID (Rivera et al., 1986), que se elaboran

Tabla 1: Constantes para el método PI_{2Ms}

M_S	a_0	a_1	a_2
1,4	0,0674	0,3775	-0,9623
1,6	0,1687	0,4724	-0,9805
1,8	0,2118	0,5633	-0,9823
2,0	0,3208	0,5613	-1,0380
M_S	b_0	b_1	b_2
1,4	1,440	-0,1744	-0,659
1,6	8,672	-7,247	-0,04929
1,8	-3,952	5,426	0,08661
2,0	-2,105	3,595	0,1476
M_S	c_0	c_1	c_2
1,4	0,3803	0,7794	0,6851
1,6	0,2611	0,5763	0,4345
1,8	0,2296	0,4711	0,3588
2,0	0,2107	0,4043	0,3122

unas reglas de sintonía parametrizadas de tal manera, que se posibilita la obtención de los parámetros del PID atendiendo al grado de robustez deseado (obviamente con el correspondiente compromiso respecto al desempeño). La obtención de reglas de sintonía dentro del marco IMC, introduce dos ideas que han sido ámpliamente explotadas en los últimos años y que han dado lugar a multitud de formulaciones y reglas de sintonía: por un lado, el diseño analítico con base en un modelo del proceso; y por el otro, la incorporación de consideraciones de robustez ante discrepancias entre este modelo y el proceso real. Estas serían dos de las características que han hecho popular el enfoque IMC y que, por supuesto, se ha heredado para el caso de los controladores PI/PID.

Como factor distintivo de las reglas de sintonía que se han visto en las secciones anteriores, ubicamos aquí aquellas reglas que se plantean proporcionar un determinado grado de robustez, pero sin obedecer a ningún índice o indicador concreto. En otras palabras, hablaremos de incertidumbre no estructurada o robustez en el sentido amplio. Estos enfoques intentan, por otro lado, plantear la sintonía del controlador en términos de uno o dos parámetros, de manera que su elección constituye fijar el compromiso entre la robustez y el rendimiento del sistema de control.

De esta forma, dentro de los métodos analíticos es el IMC el que ha ganado más aceptación. En la formulación inicial de su aplicación al diseño de controladores PI/PID, en el trabajo de Rivera et al. (1986), se propone un conjunto de reglas para los diferentes tipos de modelos del proceso, en las que los tres parámetros del controlador resultante (PI/PID dependiendo de la estructura del modelo), quedan parametrizados en términos de las constantes del proceso y un parámetro de sintonía vinculado a la robustez de manera directa (a mayor valor, mejor es la robustez), pero inversamente al rendimiento. Así, para el caso de un modelo de primer orden con retardo (K, L, T), la sintonía que resulta toma la forma:

$$K_p = \frac{2T + L}{K(2\lambda + L)}, \quad T_i = T + L/2, \quad T_d = \frac{TL}{2T + L}. \tag{29}$$

Como se aprecia, las ecuaciones son de una complejidad baja y ofrecen una manera sistemática y simple de tratar la robustez; mediante el parámetro λ; aunque, como hemos dicho, sin ningún indicador relacionado con el parámetro. Este enfoque ha dado lugar a multitud de estrategias. Para un tratamiento bastante completo del diseño con base en el IMC y el correspondiente análisis de los efectos que tiene en la robustez y el rendimiento, se dispone del trabajo de Leva and Colombo (2004). Cabe decir que las estrategias con base en el diseño mediante IMC se caracterizan, básicamente, por la especificación de la relación entrada salida deseada. Siendo por lo tanto diseños orientados al seguimiento, más que a la regulación. En este sentido, es de destacar el trabajo de Chen and Seborg (2002), en el que se formula un diseño completamente analítico, en la línea propuesta por el IMC, pero especificando la relación deseada para el control regulatorio. Como resultado se genera un conjunto de reglas de sintonía, en función del modelo del proceso y la velocidad de respuesta para el rechazo de perturbaciones. En contraste con la regla IMC clásica de Rivera et al. (1986), en este caso las relaciones de diseño aparecen bastante más complejas, pero parametrizadas al igual que en el caso anterior, en término de las constantes del proceso y el parámetro de sintonía λ:

$$\begin{aligned} K_p =& \frac{1}{K}\frac{2TL + L^2/2)(2\lambda + L/2) - \lambda^3 - 3\lambda^2 L}{2\lambda^3 + 3\lambda^2 L + (L/2)(3\lambda + L/2)}, \\ K_i =& \frac{2TL + L^2/2)(2\lambda + L/2) - \lambda^3 - 3\lambda^2 L}{(2T + L)L}, \\ K_d =& \frac{3\lambda^2 TL + (TL^2/2)(3\lambda + L/2) - 2(T + L)\lambda^3}{2TL + L^2/2)(2\lambda + L/2) - \lambda^3 - 3\lambda^2 L}. \end{aligned} \tag{30}$$

No obstante, es importante remarcar que en ambos casos, la indicación de robustez viene dada, siempre de manera indirecta, por el valor del parámetro de sintonía λ. Esta situación es la misma para todas las variantes de diseños IMC, que han ido apareciendo en la literatura.

En la misma línea de planteamiento del diseño, recientemente ha habido un creciente interés en la derivación de reglas simples y que ofrezcan un nivel de robustez aceptable. En este sentido es de especial mención la regla SIMC planteada en Skogestad (2003), con base en un método de aproximación de modelos de orden alto ("the half rule") y un diseño orientado a mejorar el rendimiento en regulación. Generalmente, los métodos de sintonización desarrollados con base en las técnicas de control con modelo interno (IMC), utilizan cancelación de polos y ceros y por lo tanto producen respuestas muy lentas ante cambios en la perturbación, especialmente cuando se cancelan los polos lentos de la planta. Considerando esto, en este procedimiento se modifican las reglas de sintonía IMC clásicas de Rivera et al. (1986), para lograr respuestas rápidas ante cambios en la consigna y la perturbación. Para la sintonización de los controladores PI, se parte de un modelo de primer orden con retardo y para los PID de uno de segundo orden con retardo. En el desarrollo de estas reglas se utilizó un controlador PID "ideal", sin filtro derivativo. La sintonía SIMC para controladores PI y PID Serie resulta ser:

$$K_pK = \frac{1}{2\tau_o}, \quad \frac{T_i}{T} = \text{mín}\{1, 8\tau_o\}, \quad \frac{T_d}{T} = a. \tag{31}$$

Esta sintonía proporciona una robustez media $M_S = 1{,}59$ con controladores PI y PID de 1GdL.

Con un objetivo similar pero partiendo de un planteamiento completamente diferente, en Vilanova (2008) se proporciona un diseño de un controlador PID para plantas de primer orden con retardo, en base a una especificación de coincidencia de modelos en el sentido $\mathcal{H}_\infty$ y parametrizando las reglas en términos de dos parámetros relacionados con la robustez; z y la velocidad de respuesta T_M; (se debe notar que se proporciona también el valor de N para el filtro de la parte derivativa):

$$\begin{aligned} K_p &= \frac{T_i}{K(\rho + T_M)}, \\ T_i &= T + \chi_1 - T_M\frac{(\rho+z)}{(\rho+T_M)}, \\ \frac{T_d}{N} &= T_M\frac{(\rho+z)}{(\rho+T_M)}N, \\ N+1 &= \frac{T}{T_i}\frac{\rho}{L}\frac{(\rho+T_M)}{(\rho+z)}, \end{aligned} \tag{32}$$

con $\chi_1 = z + L - \rho$ y $\rho = L(L+z)/(L+T_M)$. En el mismo trabajo se proponen valores concretos para z y T_M, generando una sintonía altamente simple:

$$\begin{aligned} K_p &= \frac{T_i}{2{,}65KL}, \\ T_i &= T + 0{,}03L, \\ \frac{T_d}{N} &= 1{,}72L, \\ N+1 &= \frac{T}{T_i}, \end{aligned} \tag{33}$$

de manera que se garantiza un valor de $M_S \approx 1{,}4$, lo que significa una robustez considerable, a la vez que se mantiene un rendimiento aceptable.

Con base en este planteamiento, se han originado formulaciones similares que lo extienden con el objetivo de introducir consideraciones acerca de la mejora en la respuesta en regulación, así como un compromiso entre el rendimiento en regulación y seguimiento, a la vez que se garantiza una robustez determinada. De esta forma, en Alcántara et al. (2010a) se proponen dos problemas de coincidencia de modelos: uno para la relación entrada salida, especificando como relación deseada $T_d(s) = ((T_M - \gamma)s + 1)/(T_Ms + 1)$ y otro para la función de sensitividad, como $S_d(s) = \gamma s/(T_Ms + 1)$. Ambos problemas se solucionan de forma analítica y a continuación se analizan las repercusiones en la elección de los parámetros que especifican el problema. El valor de T_M se escoge de acuerdo a Vilanova (2008), mientras que el valor de γ es el que determina el compromiso entre los comportamientos de regulación y seguimiento. Por ejemplo, en la figura 3 se puede ver el rango sugerido

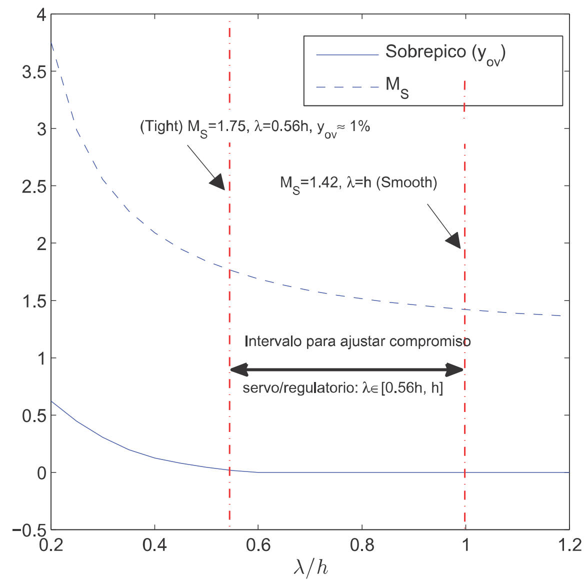

Figura 3: Determinación de la robustez y suavidad/agresividad del control en términos de λ/h

para el valor del parámetro λ (que juega el papel de filtro en el diseño del controlador IMC asociado al PID).

De esta forma, por ejemplo, evaluando un compromiso entre la perdida de rendimiento y la ganancia de robustez, se propone una regla para un PID. Una variante de esta extensión puede también encontrarse en Alcántara et al. (2010b), mientras que en Alcántara et al. (2011) se presenta esta metodología dentro del marco de diseño H_∞ y extendido al caso de sistemas inestables.

Por otro lado, es de resaltar el enfoque ART_2 ("Analytical Robust Tuning") presentado por Alfaro et al. (2009d) y que tiene como base, la formulación analítica del diseño para el control en regulación, a la vez que se vincula con una especificación de robustez en términos del valor conseguido de M_S. Este enfoque, por tanto, podría calificarse tanto de analítico como dirigido por M_S. Creemos que es importante enmarcarlo en esta sección, puesto que resulta de la evolución que han experimentado los enfoques analíticos, como resultado de añadirles consideraciones de robustez. En este trabajo se plantea el diseño de controladores PI en base a modelos de primer orden con retardo y de controladores PID en base a modelos de segundo orden con retardo. Así por ejemplo, para un controlador PI se obtiene el siguiente diseño:

$$\begin{aligned} \kappa_p \doteq K_pK &= \frac{2\tau_c - \tau_c^2 + \tau_o}{(\tau_c + \tau_o)^2}, \\ \tau_i \doteq \frac{T_i}{T} &= \frac{2\tau_c - \tau_c^2 + \tau_o}{1 + \tau_o}, \\ \beta &= \frac{\tau_c T}{T_i}, \end{aligned} \tag{34}$$

siendo $\tau_o = L/T$ y τ_c el parámetro adimensional de diseño (similar a la constante de tiempo λ del IMC), pero significando

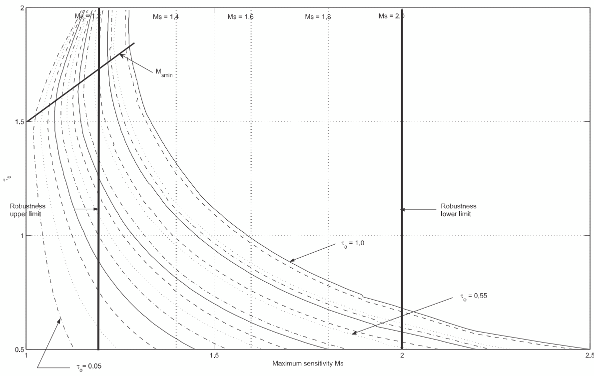

Figura 4: Relación entre la robustez, M_S, y el parámetro de diseño τ_c

la relación entre la constante de tiempo deseada para la respuesta de lazo cerrado y la constante de tiempo dominante del proceso controlado, para el cual se establece un límite inferior; $\tau_{cmin} \leq \tau_c$; si se quiere garantizar un determinado nivel de robustez (en términos de M_S). Así, el valor τ_{cmin} se ofrece parametrizado en términos de la robustez deseada:

$$\tau_{cmin} = k_{11} + \left(\frac{k_{21}}{k_{22}}\right)\tau_o, \quad (35)$$

$$k_{11} = 1{,}384 - 1{,}063M_S + 0{,}262M_S^2,$$

$$k_{21} = -1{,}915 + 1{,}415M_S - 0{,}077M_S^2,$$

$$k_{22} = 4{,}382 - 7{,}396M_S + 3{,}0M_S^2,$$

planteando por tanto, de una manera muy compacta, un compromiso entre robustez y rendimiento. El punto interesante de esta propuesta, es la de relacionar el efecto del parámetro de diseño (τ_c en este caso) con la medida de robustez M_S. De esta forma acaba siendo el mismo valor de M_S el que se convierte en un parámetro de diseño. En la figura 4 se puede ver esta relación para el caso del mencionado controlador PI y un sistema de primer orden con retardo.

Como puede verse, si se aumenta demasiado el valor de τ_c, se consigue el efecto inverso empezando a perder robustez, de ahí que el rango de valores para τ_c se limite también por arriba $\tau_{cmin} \leq \tau_c \leq 1{,}5+0{,}3\tau_o$. Tal como puede verse, en el fondo, este método obedece al mismo espíritu que el método de Síntesis Directa para regulación (DS-d) comentado anteriormente (Chen and Seborg, 2002). No obstante, para el caso del controlador PID las derivaciones no son exactamente las mismas y el hecho de que en Alfaro et al. (2009d) se presente la formulación para los controladores con dos grados de libertad (2GdL), el ART_2 proporciona un diseño global bastante mejor.

A modo comparativo, se presenta la aplicación de ambos métodos para el sistema de prueba:

$$P(s) = \frac{1}{(s+1)(0{,}4s+1)(0{,}16s+1)(0{,}064s+1)}, \quad (36)$$

modelado mediante la aproximación de segundo orden siguiente

$$P_2(s) = \frac{e^{-0{,}147s}}{(0{,}856s+1)(0{,}603s+1)}. \quad (37)$$

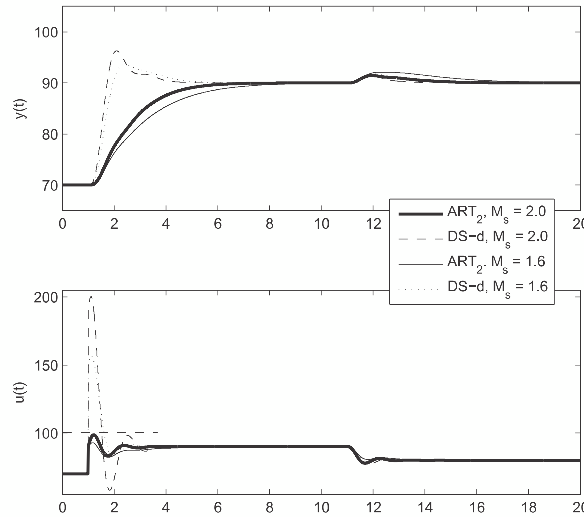

Figura 5: Comparación entre los métodos ART_2 y DS-d

En base a este modelo se aplican las relaciones de diseño de Alfaro et al. (2009d) y de Chen and Seborg (2002). Los resultados se muestran en la figura 5 para dos niveles de robustez distintos. Sin entrar en comparaciones mediante indicadores numéricos (para ello se puede consultar Alfaro et al. (2009d)), se puede constatar la mejora introducida por el ART_2.

El hecho de establecer un vínculo entre el parámetro de diseño y un indicador de robustez, abre las puertas a toda una serie de posibilidades en las que conducir extensiones de los métodos de diseño analíticos. De esta forma se podrá integrar de una manera mucho más explícita, el compromiso entre rendimiento y robustez.

6. Diseño de PID robustos: Enfoques alternativos

Los enfoques vistos hasta el momento se caracterizan por la elaboración, como resultado del planteamiento de diseño, de unas reglas de sintonía dirigidas a proporcionar las garantías de robustez planteadas (aunque esto no sea siempre cierto tal como se comenta en Vilanova et al. (2010)). Tal y como ya se ha comentado, este tipo de enfoques orientados a generar reglas de sintonía son de especial apreciación dentro del ámbito de los controladores tipo PI/PID. No obstante, la teoría de control robusto ofrece multitud de alternativas para hacer frente al problema de la robustez, posibilitando formulaciones de la incertidumbre más extensas.

Muchos de los enfoques con base en la aplicación de tratamientos procedentes del control robusto y orientados a la obtención de controladores de estructura restringida (en especial un PID), tienen como base técnicas de optimización. Los recientes progresos en técnicas numéricas de optimización, han conllevado el desarrollo de planteamientos realmente eficientes, con base en objetivos y restricciones convexas.

En trabajos iniciales Malan et al. (1994) y Grigoriadis and Skelton (1996), plantean una optimización H_∞ de controladores de orden y estructura prefijada, en base a algoritmos de optimización no lineales y no convexos. Esto implica una carga computacional muy alta, junto con el hecho que es usualmente difícil de ajustar las funciones de peso que definen el problema. Si bien es cierto que el uso de estos filtros y funciones de peso, es habitual dentro del contexto de la teoría de control robusto, usuarios de controladores tipo PID prefieren especificaciones más clásicas, en términos de indicadores como los presentados en las secciones anteriores. De ahí que planteamientos como los iniciados por Schei (1994), en el que se introducen restricciones en las funciones de sensitividad y sensitividad complementaria, hayan cuajado y hayan quedado establecidos hoy en día como medidas de robustez, alrededor de las cuales se han formulado diferentes enfoques y planteamientos. En concreto, esto es retomado por Panagopoulos et al. (2002) formulando un problema de optimizaición no convexo, en el que se optimiza el rechazo de perturbaciones juntamente con una restricción en la función de sensitividad. Este enfoque es revisado en mayor profundidad dentro de la presentación de las reglas MIGO/AMIGO, a las que dio lugar un posterior proceso de aproximación de los parámetros del controlador, para la solución del problema, para un conjunto de plantas de prueba (el denominado "test-batch").

Son de especial interés, no obstante, los planteamientos que conducen a problemas convexos, dado que esto permite la aplicación de herramientas de optimización potentes y eficientes. Dentro de estos planteamientos destacamos los dos siguientes: métodos con base en formulaciones LMI y que soportan una formulación multimodelo de la incertidumbre y, por otra parte, métodos con base en "loop-shaping" y que obedecen, a descripciones no estructuradas de la incertidumbre (algunas de ellas soportando, también, formulaciones multimodelo) (Grassi et al., 2001; Blanchini et al., 2004).

De esta forma, Ge et al. (2002) plantean el diseño en base a la solución de un problema de control óptimo cuadrático (LQR), mediante el uso de desigualdades matriciales lineales (LMI). Bajo esta formulación, el proceso a controlar se describe mediante una formulación en espacio de estados (A, B, C). La formulación acepta el uso de múltiples modelos, mediante una formulación incierta para las matrices del sistema $[A, B]$, que se suponen inciertas, pero pertenecientes a un politopo definido por el cubrimiento convexo:

$$\Omega = Cov\{[A_1, B_1], [A_2, B_2], ..., [A_{N_m}, B_{N_m}]\}, \tag{38}$$

donde N_m es el número de múltiples modelos. En este trabajo se supone un PID de la forma:

$$C(s) = K_p + \frac{K_i}{s} + K_d s, \tag{39}$$

y un proceso a controlar de segundo orden de la forma

$$P(s) = \frac{b_2}{s^2 + a_1 s + a_2}, \tag{40}$$

asumiendo sus parámetros una forma intervalar. De esta forma, si escribimos el sistema a controlar y el controlador en una descripción de espacio de estados como:

$$\begin{aligned} \dot{x} &= Ax + Bu + B_r r, \\ u &= -Kx + K_p r + K_d \dot{r}, \\ y &= Cx \end{aligned} \tag{41}$$

siendo y la salida del sistema, $x = [x_1, x_2, x_3]$ el estado, con

$$x_1 = y \;\; x_2 = \dot{x}_1, \;\; x_3 = -\int (r - y)dt, \tag{42}$$

y

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -a_2 & -a_1 & 0 \\ 1 & 0 & \end{bmatrix}, B = \begin{bmatrix} 0 \\ b_2 \\ 0 \end{bmatrix}, B_r = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \tag{43}$$

$$C = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, K = \begin{bmatrix} K_p, K_d, K_i \end{bmatrix}. \tag{44}$$

De esta forma, al obtener las ecuaciones de lazo cerrado el controlador PID se convierte en la ganancia, K, de una realimentación de estados. Esto posibilita encontrar sus parámetros en base a la solución de un problema de optimización con restricciones LMI. Partiendo de un problema LQR estándar:

$$J(u) = \int_0^\infty (x^T Q x + u^T R u) dt, \tag{45}$$

utilizando la formulación LMI, el problema se soluciona como un problema de optimización en $\hat{P}$ y $\hat{Y}$:

$$\min_{\hat{P}, \hat{Y}} x^T(0)\hat{P}^{-1}x(0), \tag{46}$$

sujeto a

$$\begin{bmatrix} A\hat{P} + \hat{P}A^T + BY + Y^T B^T & \hat{P} & Y^T \\ \hat{P} & -Q^{-1} & 0 \\ Y & 0 & -R^{-1} \end{bmatrix} \leq 0, \hat{P} > 0, \tag{47}$$

de esta forma, dada la solución óptima $(\hat{P}^*, Y^*)$ la ley de realimentación de estado óptima viene dada por:

$$K = -Y^*(\hat{P}^*)^{-1}. \tag{48}$$

Pudiéndose recuperar los parámetros del controlador PID a partir de este controlador óptimo.

Una ventaja de formular las restricciones en base a LMI, es que se pueden añadir otras especificaciones de diseño, siempre que estas sean expresables en este formato. De hecho, en el mismo trabajo Ge et al. (2002) muestran como incorporar consideraciones acerca de la energía de control, atenuación de perturbaciones, etc. De esta forma, por ejemplo, si se desea añadir una especificación de atenuación de perturbaciones en términos de la norma infinito, se considera primero el sistema en el que se añade la correspondiente entrada en perturbación w:

$$\dot{x} = Ax + Bu + B_w w, \tag{49}$$

$$y = Cx,$$

$$[A, B] \in \Omega, \tag{50}$$

$$\Omega = Cov\{[A_1, B_1], [A_2, B_2], ..., [A_{N_m}, B_{N_m}]\}. \tag{51}$$

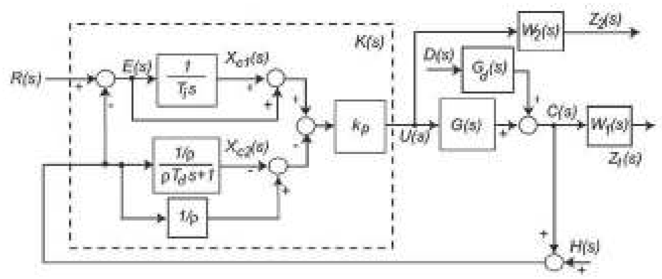

Figura 6: Diagrama de bloques para la optimización del controlador PID (Goncalves et al., 2008)

Una especificación del tipo $\|T_{wy}\|_\infty < \sigma$ (donde T_{wy} denota la función de transferencia en lazo cerrado desde w a y), se convierte en una restricción LMI del tipo:

$$\begin{bmatrix} A\hat{P} + \hat{P}A^T + BY + Y^T B^T & \hat{P}B_w & C^T \\ B_w^T \hat{P} & -\sigma I & 0 \\ C & 0 & -\sigma I \end{bmatrix} \leq 0. \quad (52)$$

De esta manera se pueden añadir restricciones adicionales, que serán tenidas en cuenta al solucionar el problema de optimización. Como resultado, acabamos obteniendo un conjunto relativamente complejo de LMI, que actúan como restricciones. No obstante, si hay algo que hace atractivo este enfoque, es la aceptación que ha ganado esta formulación y la existencia de software muy eficiente para solucionarla. Este enfoque tiene la ventaja de permitir la incorporación de un amplio abanico de especificaciones, no obstante, tal como está formulado en Ge et al. (2002), es de aplicación tan solo a un tipo de sistemas muy concreto y la formulación ideal del PID. En el mencionado trabajo, los autores presentan la aplicación del método a un reactor agitado, cuyo modelo se linealiza en tres puntos de operación diferentes, mostrando la efectividad de considerar el enfoque multimodelo.

Este enfoque, en el que se utiliza la formulación con base en LMI, así como la descripción de la incertidumbre en base a múltiples modelos, es retomada en Goncalves et al. (2008), donde se elabora un poco más en esta idea. Concretamente se plantea un método iterativo de dos pasos. Un primer paso en el que se resuelve un problema de optimización multiobjetivo no convexo y un segundo, de análisis con base en la utilización de LMI, en el que la solución multiobjetivo se verifica para el dominio incierto. En caso de que alguna de las restricciones se viole, se añaden nuevos puntos para iniciar una nueva iteración de la fase de diseño. Dos de las principales ventajas de este enfoque respecto al presentado anteriormente, es que no se supone ninguna estructura determinada para el modelo del proceso. A su vez, para el controlador PID, se supone una formulación de dos grados de libertad[1] en el formato Estándar, aspecto que proporciona al método un mayor grado de aplicabilidad y cercanía, a las formulaciones industriales de estos controladores.

[1]aunque así dicho por los autores, no es en realidad un PID de dos grados de libertad.

Aquí el problema de optimización se formula de la siguiente forma:

$$C^*(s) = \arg \min_{K(s)} \max_{\alpha} (\lambda_1 \|T_{z_1 d}(s,\alpha)\|_\infty, \lambda_2 \|T_{z_1 \nu}(s,\alpha)\|_2), \quad (53)$$

sujeto a

$$\begin{cases} \alpha \in \Omega_i, \\ C(s) \in \Gamma, \\ \|T_{z_1 d}(s,\alpha)\|_\infty \leq \epsilon_1, \\ \|T_{z_2 \nu}(s,\alpha)\|_2 \leq \epsilon_2, \end{cases} \quad (54)$$

donde Ω_i es el conjunto de plantas, Γ el espacio de búsqueda de los parámetros del controlador. Las funciones de transferencia $T_{z_1 d}(s,\alpha)$ y $T_{z_2 \nu}(s,\alpha)$, representan las funciones de lazo cerrado implicadas en la robustez y el rendimiento, respectivamente. De esta forma se plantea un compromiso entre ambas métricas (compromiso robustez rendimiento), a la vez que se intenta garantizar un valor mínimo para las mismas. El conjunto Ω_i contiene tan solo un conjunto finito de puntos del politopo Ω. De esta forma, en una segunda fase, una vez encontrada la solución, se considera el conjunto Ω para analizar si la función de coste toma su valor máximo fuera del conjunto Ω_i, o si alguna de las restricciones no se cumple. En ese caso, se añaden estos puntos a Ω_i y se soluciona de nuevo el problema de optimización. La iteración finaliza cuando las restricciones de verifican para todos los puntos de Ω y no hay posibilidad de minimizar la función de coste mediante la adición de nuevos puntos en Ω_i.

En este trabajo se considera también el mismo ejemplo que en Ge et al. (2002), mostrando que este enfoque proporciona mejores resultados. Vale la pena destacar que el sistema ejemplo del reactor encamisado, se trata también en el trabajo de Toscano (2005), en el que se formula también un problema de optimización de los parámetros del controlador, pero sin utilizar como base la formulación LMI. Este sistema se considera linealizado en tres puntos de operación diferentes, resultando en sendas funciones de transferencia.

Nuevas formulaciones expresan el controlador en forma parametrizada lineal. Así por ejemplo, en Hara et al. (2006) se presenta un enfoque con base en el lema KYP (Kalman-Yakoubovic-Popov) generalizado, que ajusta el lugar de Nyquist del sistema controlado. La idea es definir diferentes regiones convexas en el plano complejo (especificaciones para frecuencias altas, medias y bajas) y diseñar el controlador de manera que el lugar de Nyquist para cada rango de frecuencias, pase por la correspondiente región. A pesar de tener una formulación bastante potente y elegante, queda manifiesta su dificultad de aplicación, dentro de un marco como el asociado a los controladores PID en el que se prefieren especificaciones más *clásicas*.

Un planteamiento alternativo al anterior, lo encontramos en los trabajos Karimi et al. (2006, 2007), en los que se formula de una manera muy interesante el dar forma al lugar de Nyquist. La idea básica reside en definir un margen de estabilidad alternativo, que nos garantiza una cota inferior para los márgenes de ganancia y de fase, así como para el valor de M_S. La ventaja principal de este nuevo margen de estabilidad, es que resulta lineal respecto a los parámetros del controlador siempre que este obedezca a una parametrización lineal. De esta forma, el planteamiento de un rechazo de perturbaciones óptimo con restricciones sobre este margen de estabilidad, da lugar a un problema de optimización con restricciones lineales que puede ser resuelto mediante la aplicación de técnicas de programación lineal. Otro punto interesante de este enfoque, es que puede plantearse para el caso multimodelo, básicamente incrementando el número de restricciones.

Consideramos, por tanto, en primer lugar el controlador expresado en forma paramétrica lineal como:

$$K(s) = \rho^T \phi(s), \tag{55}$$

donde $\rho^T = [\rho_1, ..., \rho_n]$ son los parámetros del controlador y $\phi^T(s) = [\phi_1(s), ..., \phi_n(s)]$ un conjunto de funciones sin polos en el semiplano derecho. En el caso de un controlador PID, tenemos:

$$\rho^T = [K_p, K_i, K_d], \quad \phi^T(s) = \left[1, \frac{1}{s}, \frac{s}{1+T_f s}\right]. \tag{56}$$

De esta forma, cada punto del lugar de Nyquist, $K(j\omega)P(j\omega)$, puede expresarse de forma lineal en términos de los parámetros, ρ, del controlador:

$$K(j\omega_k)P(j\omega_k) = \rho^T \phi(j\omega_k)P(j\omega_k), \tag{57}$$

$$= \rho^T \mathcal{R}(\omega_k) + j\rho^T \mathcal{I}(\omega_k), \tag{58}$$

donde $\mathcal{R}(\omega_k)$ y $\mathcal{I}(\omega_k)$ son, respectivamente, las partes real e imaginaria de $\phi(j\omega_k)P(j\omega_k)$. Por otro lado, el rendimiento a optimizar se plantea como el rechazo a perturbaciones medido como la integral del error:

$$IE = \int_0^\infty e(t)dt, \tag{59}$$

valor que supone una buena aproximación al IAE para sistemas amortiguados. Este índice resulta igual a la inversa de la contante integral (hecho utilizado en Åström et al. (1998)) y, en el caso de un controlador parametrizado linealmente, $K(s) = (\rho_0 + \rho_1 s + ...\rho_n s^n)/R(s)$, donde $R(s)$ tan solo contiene un integrador ($R(s) = sR'(s)$), resulta igual a $1/\rho_0$. La aportación más importante de este trabajo es la introducción de lo que denominan el margen de robustez lineal, $l \in (0, 1)$. Este margen se define en base a una línea recta d_1 que cruza el eje real negativo dentro del intervalo (0, -1), con un ángulo $\alpha \in (0^o, 90^o]$. El valor de l se define como la distancia entre el punto crítico y esta recta d_1. La figura 7; tomada de Karimi et al. (2007); muestra gráficamente esta idea.

Mediante esta distancia l se pueden obtener las siguientes cotas inferiores para los márgenes de ganancia y fase, y la inversa de M_S

$$A_m \geq A_{ml} = \frac{1}{1-l}, \tag{60}$$

$$\phi_m \geq \phi_{ml}, \tag{61}$$

$$= \sin^{-1}\left((1-l)\sin^2\alpha + \cos\alpha\sqrt{1-(1-l)^2\sin^2\alpha}\right), \tag{62}$$

$$M_S^{-1} \geq M_l = l\sin(\alpha). \tag{63}$$

Dado un valor fijo del ángulo α, los valores A_{ml}, ϕ_{ml} y M_l son todos funciones crecientes de l, por lo que se utiliza l como

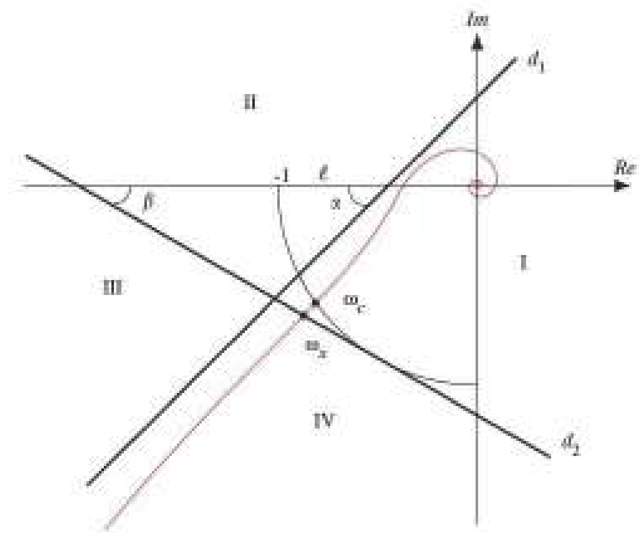

Figura 7: Definición del margen de robustez lineal y las correspondientes cotas para el margen de fase (Φ_l) e inversa de M_S (M_l) (Karimi et al., 2007)

medida de robustez. Por otra parte, la elección de α conlleva un compromiso entre el amortiguamiento del sistema y su ancho de banda. Valores típicos de l se sitúan entre 0,5 y 0,8, mientras que para el ángulo α tenemos entre $36{,}9^o$ y 90^o.

Por otra parte, la introducción de otra línea d_2 tangente al círculo unidad y que cruza el eje real negativo con un ángulo β, sirve para fijar una cota inferior ω_x a la frecuencia de corte ω_c. Cuanto mayor sea ω_x mayor será el desempeño, consiguiendo un rechazo de perturbaciones más rápido.

De esta forma, el problema de optimización que resulta depende de si queremos optimizar la robustez o el rendimiento. Así por ejemplo, para una frecuencia de corte deseada, se pueden encontrar los mejores márgenes de robustez. Así pues, las variables de diseño son ω_x, α y β, siendo el objetivo el maximizar el valor del margen de robustez lineal l. El problema de optimización toma la forma siguiente:

$$\text{máx}\, l, \tag{64}$$

sujeto a

$$\rho^T(\cot\alpha\mathcal{I}(\omega_k) - \mathcal{R}(\omega_k)) + l \leq 1 \quad para \quad \omega_k > \omega_x,$$
$$\rho^T(\cos\beta\mathcal{I}(\omega_k) + \mathcal{R}(\omega_k)) + l > -1 \quad para \quad \omega_k > \omega_x,$$
$$\rho^T(\cos\beta\mathcal{I}(\omega_k) + \mathcal{R}(\omega_k)) + l \leq -1 \quad para \quad \omega_k \leq \omega_x.$$

Otra posibilidad es, por ejemplo, optimizar el rendimiento en términos del índice IE introducido anteriormente, pero con unas determinadas restricciones de robustez (es decir, valores de α y l):

$$\text{máx}\, \rho_o, \tag{65}$$

sujeto a

$$\rho^T(\cot\alpha\mathcal{I}(\omega_k) - \mathcal{R}(\omega_k)) + l \leq 1 \quad \forall \quad \omega_k. \tag{66}$$

A modo ilustrativo, se reproduce un ejemplo extraído de Karimi et al. (2007) en el que se aplica este método para el

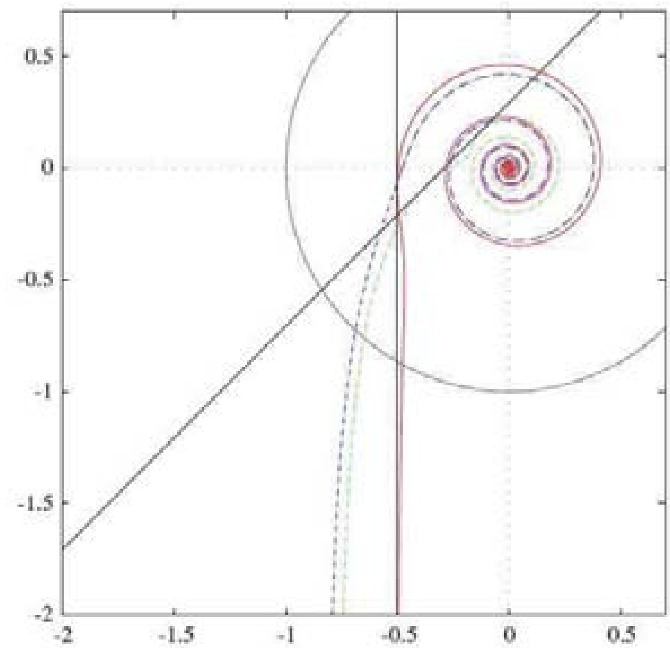

Figura 8: Curvas de Nyquist para el sistema de ejemplo $P_1(s)$. Método propuesto con $l = 0{,}5$ y $\alpha = 90^o$ (rojo), con $l = 0{,}707$ y $\alpha = 45^o$ (verde) y método en Panagopoulos (2002) (azul)

sistema

$$P_1(s) = \frac{1}{(s+1)^3} e^{-5s} \tag{67}$$

también utilizado en Panagopoulos et al. (2002). La respuesta en frecuencia de $P_1(s)$ se evalúa en $N = 8000$ puntos igualmente espaciados entre 0 $rad \cdot s^{-1}$ y 80 $rad \cdot s^{-1}$. En la figura 8 se pueden ver los resultados de dos diseños en los que se ha utilizado, por un lado $l = 0{,}5$ y $\alpha = 90^o$ y por otro $l = 0{,}707$ y $\alpha = 45^o$

7. Apuntes finales y perspectivas futuras

En las secciones anteriores se ha presentado todo un conjunto de diferentes opciones y alternativas, que se han ido formulando a lo largo de los años, con el fin de afrontar el problema de la obtención de un PID robusto. El común denominador a todas ellas, a pesar de utilizar tanto métricas de robustez como descripciones de incertidumbre diferentes, es que se toman en consideración, básicamente, las características de robustez. No obstante, es cada vez más patente la necesidad de mantener unos niveles de rendimiento altos a pesar de tener que garantizar una determinada robustez. En otras palabras: *minimizar la perdida de rendimiento debida a la introducción de consideraciones de robustez*. Asimismo, la robustez conseguida para el lazo de control puede resultar una característica frágil. Es decir, puede perderse ante pequeñas variaciones en los parámetros del controlador.

Por lo tanto, el diseño del sistema de control de lazo cerrado con controladores PID, debe considerar el compromiso de dos criterios en conflicto, por un lado el *desempeño* ante los cambios en la consigna y las perturbaciones y por el otro, la

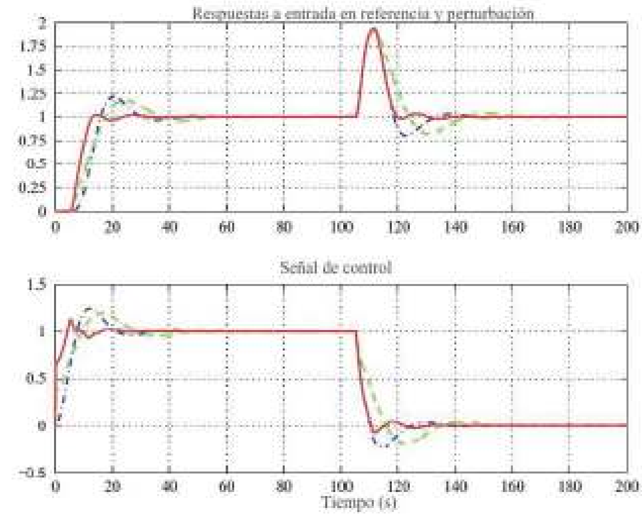

Figura 9: Respuestas temporales para el sistema de ejemplo $P_1(s)$. Método propuesto con $l = 0{,}5$ y $\alpha = 90^o$ (rojo), con $l = 0{,}707$ y $\alpha = 45^o$ (verde) y método en Panagopoulos (2002) (azul)

robustez ante cambios en las características del proceso controlado. Una vez obtenido un desempeño robusto del lazo de control, es importante estimar la *fragilidad* del controlador, esto es, la sensibilidad de la robustez del lazo de control a cambios en los parámetros del propio controlador, ya sean producto de inexactitudes en su implementación o por el *ajuste fino* final de los parámetros del controlador por parte del diseñador. Un controlador con un índice de fragilidad bajo, permitirá realizar la puesta a punto del controlador, sin tener que preocuparse por provocar una degradación importante en la robustez del mismo (Alfaro, 2007).

La posibilidad de que los controladores sean *frágiles* fue puesta en evidencia por Keel and Battacharyya (1997). Por su parte Alfaro (2007) asoció la fragilidad de los controladores PID a la pérdida de robustez del lazo.

Es en este sentido, que los autores creen que las estrategias de diseño deben plantearse de forma integral introduciendo consideraciones en estos tres ejes. Se debe establecer de manera cuantitativa la dependencia entre estas características y sus repercusiones en la sintonía final. A modo de ilustración, se presenta un ejemplo en el que se muestra el compromiso existente entre el desempeño y la robustez del lazo de control, y su efecto sobre la fragilidad del controlador. Considérese el proceso controlado de cuarto orden, cuya función de transferencia es:

$$P(s) = \frac{1}{(s+1)(0{,}4s+1)(0{,}16s+1)(0{,}064s+1)}. \tag{68}$$

Utilizando el método de identificación de tres puntos *123c* (Alfaro, 2006), se obtuvo la siguiente aproximación de segundo orden con retardo para representarlo:

$$P_m(s) = \frac{e^{-0{,}147s}}{(0{,}856s+1)(0{,}603s+1)}. \tag{69}$$

Al plantear el diseño se tiene en cuenta como *desempeño* o

Tabla 2: PID 1GdL, regulador óptimo

K_p	T_i	T_d	β
10,728	0,492	0,273	1

Tabla 3: PID 1GdL, robustez y desempeño

M_S^r	J_{ed}	TV_{ud}	J_{er}	TV_{ur}
3,963	0,0067	0,4245	0,164	11,193

comportamiento dinámico requerido del lazo de control, el que este sea óptimo respecto a la funcional de coste que penaliza la integral del error absoluto, definida como:

$$J_e \doteq \int_0^\infty |e(t)|\, dt = \int_0^\infty |r(t) - y(t)|\, dt, \tag{70}$$

la cual debe evaluarse tanto para cambios en el valor de consigna, J_{er}, como en la perturbación de carga, J_{ed}. Estas proveerán una indicación del *desempeño de la salida* del sistema de control. Los parámetros del controlador para el desempeño óptimo $\bar{\theta}_{co}$ serán tales que:

$$J_{eo} \doteq J_e(\bar{\theta}_{co}) = \min_{\bar{\theta}_c} J_e(\bar{\theta}_c, \bar{\theta}_p) \tag{71}$$

donde $\bar{\theta}_c$ y $\bar{\theta}_p$ son los parámetros del controlador y del modelo del proceso controlado, respectivamente.

Aunque el objetivo principal sea optimizar el desempeño, no debe dejarse de lado el análisis de las características dinámicas del esfuerzo de control, la señal de salida del controlador, ya que es conveniente, para evitar el deterioro prematuro del elemento final de control, que los cambios del mismo no sean bruscos ni extremos. La evaluación de las variaciones del esfuerzo de control, se puede realizar mediante un índice de su variación total, definido por:

$$TV_u \doteq \sum_{k=1}^{\infty} |u_{k+1} - u_k|\,, \tag{72}$$

el cual debe ser lo menor posible. Este es una indicación de la *suavidad de la señal de control*.

Primero se considera en el diseño solamente el desempeño del sistema de control, determinando los parámetros de un controlador PID de 1GdL que minimizan (70) ante un cambio en la perturbación (control regulatorio óptimo), indicados en la Tabla 2. La robustez y los índices de desempeño y del esfuerzo de control obtenidos con ese controlador se muestran en la Tabla 3. En esta y todas las demás evaluaciones se consideró $\Delta r = 20\,\%$ y $\Delta d = 10\,\%$.

Como se observa en la Tabla 3, el sistema de control con el desempeño optimizado tiene una robustez muy pobre, no alcanza la especificación de robustez mínima usual que establece que $M_S \leq 2,0$. Por otra parte, se espera que en este caso el desempeño del control regulatorio, medido con la integral del error absoluto J_{ed}, sea el mejor posible.

Para incorporar la robustez en el diseño del controlador, se determinaron los parámetros de un PID de 2GdL utilizando el *método de sintonización analítica robusta* ART_2 (Alfaro et al., 2009d), para cinco valores específicos de la robustez del lazo de control, los cuales se muestran en la Tabla 4.

Tabla 4: PID 2GdL, parámetros ART_2

M_S^t	τ_c	K_p	T_i	T_d	β
2,0	1,00	5,243	1,633	0,407	0,191
1,8	1,20	4,028	1,846	0,471	0,248
1,6	1,51	2,756	2,100	0,573	0,363
1,4	1,99	1,576	2,299	0,750	0,635
1,2	2,80	0,556	1,852	1,248	1,293

Tabla 5: PID 2GdL, robustez y desempeño

M_S^t	M_S^r	J_{ed}	TV_{ud}	J_{er}	TV_{ur}
2,0	2,16	0,031	0,179	0,327	0,548
1,8	1,90	0,046	0,150	0,369	0,466
1,6	1,65	0,076	0,126	0,420	0,415
1,4	1,42	0,144	0,109	0,460	0,374
1,2	1,21	0,315	0,097	0,558	0,256

La robustez y los índices del desempeño y del esfuerzo de control resultantes con el controlador de 2GdL robusto, se muestran en la Tabla 5. Debe hacerse notar, que la robustez obtenida M_s^r, en todos los casos fue determinada utilizando el modelo como planta, ya que en la práctica esta no se puede determinar con el proceso real, mientras que los índices de desempeño se determinaron con el proceso a partir de las correspondientes señales.

En la figura 10 se muestra la respuesta del sistema de control con el controlador de 2GdL, a los cambios $\Delta r = 20\,\%$ y $\Delta d = 10\,\%$, para tres niveles de robustez diferentes.

Si se comparan los índices del regulador óptimo en la Tabla 3, con los correspondientes a $M_S = 2,0$ en la Tabla 5, se nota que este nivel mínimo de robustez, se logra a expensas de una pérdida importante en el desempeño.

Además, de los índices de la Tabla 5 es evidente el compromiso entre el desempeño y la robustez. Al exigirse un mayor grado de robustez (M_S^t), se deteriora el desempeño tanto del control regulatorio (J_{ed}), como del servo control (J_{er}), mientras que la variación del esfuerzo de control se torna más "suave", disminuyen TV_{ud} y TV_{ur}.

En el caso correspondiente a $M_S^t = 1,2$ el factor de peso de valor deseado determinado con el método ART_2 es $\beta = 1{,}293$. Si en vez de este se utiliza $\beta = 1$, el índice de desempeño del servo control sería $J_{er} = 0{,}666$, aproximadamente un 20 % mayor, lo que muestra claramente la ventaja de no restringir el factor de peso de valor deseado a valores $\beta \leq 1$, tal como sucede con los controladores comerciales actuales (Alfaro et al., 2009a).

En relación con la fragilidad de los controladores, en la Tabla 6 se muestran los índices de fragilidad paramétrica ($FI_{\delta 20p}$) de los tres parámetros y el índice de fragilidad delta 20 ($FI_{\Delta 20}$) del controlador. Con base en las definiciones dadas en Alfaro (2007), el controlador diseñado con el nivel mínimo de robustez ($M_S = 2,0$) es frágil, mientras que los de robustez intermedia ($1{,}4 \leq M_S \leq 1{,}8$) son no frágiles y el de robustez alta ($M_S = 1,2$) es elástico.

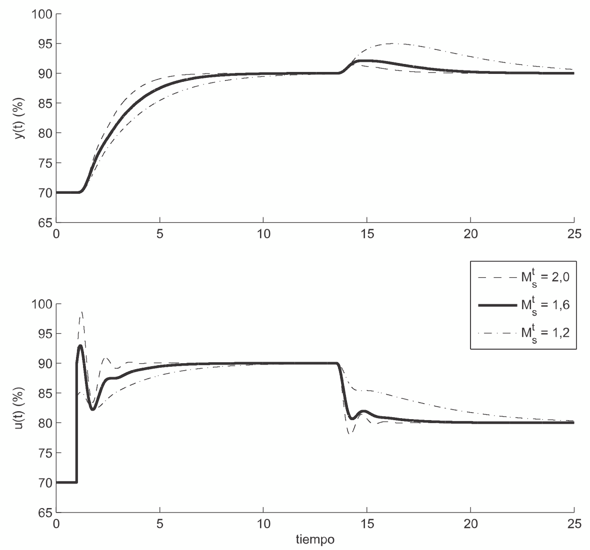

Figura 10: Ejemplo - Respuestas del sistema de control

Tabla 6: PID 2GdL, Fragilidad

M_S^t	$IF_{\delta 20K_p}$	$IF_{\delta 20T_i}$	$IF_{\delta 20T_d}$	$IF_{\Delta 20}$
2,0	0,250	0,0019	0,210	0,681
1,8	0,190	0,0011	0,166	0,486
1,6	0,133	0,0007	0,121	0,323
1,4	0,084	0,0004	0,078	0,194
1,2	0,042	0,0003	0,037	0,089

Además, considerando la definición dada en Alfaro et al. (2009b), los controladores diseñados con el método ART_2 son controladores con una *fragilidad desbalanceada*. La sensibilidad de la robustez a los cambios en K_p y T_d es similar, mientras que esta muestra ser prácticamente insensible a los cambios en T_i.

8. Conclusiones

La elevada cantidad de publicaciones que en los últimos años han ido apareciendo alrededor del diseño de controladores PID muestra, una vez más, el continuo interés en el tema. No obstante, es un común denominador de los nuevos enfoques su orientación hacia la inclusión de las consideraciones de robustez. Podemos por tanto decir que, de alguna manera, los diseños avanzados de PID que se nos auguran, serán diseños con unas características de robustez garantizadas.

En este sentido, son especialmente atractivos los enfoques que plantean un diseño dirigido por robustez utilizando alguna medida o métrica que nos ofrezca una cuantificación del grado de robustez del diseño. A su vez, esto posibilita analizar el compromiso existente entre la robustez y otras propiedades del lazo de control, como las apuntadas de desempeño y fragilidad. Obtener un diseño meramente robusto no es el objetivo. Al contrario, se debe tener una idea precisa del precio que se paga por exigir determinados niveles de robustez. En este sentido, tal como se ha apuntado en la sección anterior, deben encontrarse maneras que permitan analizar de una manera clara los compromisos existentes.

Son también de especial interés las formulaciones de medidas alternativas de robustez. Índices nuevos, más generales que los existentes hasta el momento y que permiten, por tanto, diseños más ámplios y generales utilizando un único parámetro para conducir el diseño.

English Summary

Robust PID Control: An Overview

Abstract

This paper presents a general overview of the existing approaches used to obtain a robust proportional integral derivative (PID) controller. The rigid and particular structure imposed by the PID controller, have been the main reason for its vast use in industrial applications but at the same time impose several constrains to include robustness considerations into its design. Nowadays, the spectrum of possibilities for the robust design of a PID controller is very wide and may be faced with practically any approach, specifically with any robust control approach. At this respect, it is important to distinguish between tuning methods and tuning rules been of interest, in the PID controller case, the development of simple tuning rules that at the same time guarantee its robustness. Previously to this classification, it is important to state how the robustness is measured and represented. Accordingly, how the design specifications are finally formulated. The paper also reviews other newer issues proposed in the PID literature and related with the control system robustness, as are its achievement and the controller's fragility.

Keywords:
PID Control, Robustness, Uuncertainty

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Performance/Robustness Trade-off Design Framework for 2DoF PI Controllers

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Abstract: The aim of this paper is to present a design framework for two-degree-of-freedom (2DoF) proportional integral (PI) controllers that allows to deal with the control system *performance/robustness trade-off*. It is based on the use of a model reference optimization procedure [illegible] target servo-control and regulatory-control closed-loop transfer functions for first- and second-order plus dead-time (FOPDT, SOPDT) models. A smooth [illegible] control performance is obtained by forcing both closed-loop transfer functions to perform as close as possible to target non-oscillatory dynamics. A comparison with other methods shows the effectiveness of the proposed design methodology.

Keywords: PI controllers, two-degree-of-freedom controllers, [illegible] reference control, performance/robustness trade-off.

1. Introduction

Since their introduction in 1940 [1,2] commercial *proportional integral derivative* (PID) controllers have with no doubt become the best option in industrial control applications. The success is mainly due to its simple structure and the meaning of the corresponding three parameters. This fact makes PID control easier to be understood by more control engineers than advanced control techniques. In addition, the performance of a PI or PID controller is satisfactory in most of industrial applications. See [3,4] as an example.

Since Ziegler and Nichols [5] presented the PID controller tuning rules, a great number of procedures have been developed, from the classic methods of Cohen and Coon [6], López et al. [7], and Rovira et al. [8], and modifications of the original tuning rules [9-11], to a variety of new techniques such as: analytical tuning [12,13]; optimization methods [14,15]; gain and phase margin optimization [14,16].

O'Dwyer [17] presents a collection of tuning rules for PI and PID controllers, which shows their abundance.

Among different approaches, the direct or analytical synthesis constitutes a quite straightforward approach to PI/PID controller design. The controller synthesis presented by Martin [18] made use of zero-pole cancellation techniques. Similar relations were obtained by Rivera et al. [19], applying the IMC concepts [20] to tune PI and PID controllers for low order process models. A combination of analytical procedures and the IMC tuning can be found in [13, 21-24].

A common characteristic of the analytically deducted tuning methods is that they include a *design parameter* usually related with the closed-loop control system speed of response. The selection of such design parameter will not only affect the system performance but also its relative stability.

In industrial process control applications, the set-point remains normally constant and a good load-disturbance rejection is required: regulatory control. In addition, due to process operation conditions, the set-point may eventually need to be changed and then a good transient response to this change is required, the so called servo-control. However, because these two demands can not be simultaneously satisfied with a one-degree-of-freedom (1DoF) controller, the use of a two-degree-of-freedom (2DoF) controller allows to tune the controller considering the regulatory control-loop performance and the robustness while using the extra parameter that is provided to improve the servo-control behaviour.

The control system design procedure is usually based on the use of low-order linear models identified at the closed-loop normal operation point. Due to the non-linear characteristics in most of the industrial processes, it is necessary to consider the expected changes in the process characteristics assuming certain relative stability margins, or robustness requirements for the control system. Therefore the design of the closed-loop control system with 2DoF PI controllers must take into account the trade-off between the system *performance* to load-disturbance and set-point changes and the *robustness* to variation of the controlled process characteristics [25].

If only the system performance is taken into account, using an integrated error criterion (integrated absolute error (IAE), integrated time-weighted absolute error (ITAE), or integrated squared error (ISE)) or a time response characteristic (overshoot, rise time, or settling time), as in [26,27], the resulting closed-loop control system will probably have very low robustness. On the other hand, if the system is designed to have high robustness, as in [10], and if the performance of the resulting system is not evaluated, the designer would have no idea of the *cost* involved in operating such a highly robust system. In some previous studies [28,29], the performance and robustness of the system were taken into account for optimizing the IAE or ITAE performance, but only the usual minimum level of robustness could be guaranteed.

Without considering the exception of [13,21,22] the analytically deducted and the IMC-PID tuning rules normally do not take into account the performance-robustness trade-off or provide a recommendation for the design parameter selection.

An alternative way for designing 2DoF PI controllers is presented in this case. What is presented in this paper is a *design framework* that allows considering all the previously commented aspects at once. The design is build up on a constrained model matching model reference optimization that allows resolving the performance-robustness trade-off with the selection of an appropriate design parameter for first- and second-order-plus-dead-time controlled process models.

As additional contribution the approach also provides a framework where different tuning rules can be evaluated and compared. An original way of establishing such comparison is addressed.

This paper is organized in the following way: the transfer functions of the controlled process model, the controller, and the control system are presented in Section 2; the proposed optimization procedure is described in Section 3; the optimization procedure is summarized in Section 4 and a comparison with other tuning methods is shown in Section 5. The paper ends with some conclusions.

2. Problem Formulation

Consider the closed-loop control system in Figure 1 where $P(s)$ and $C(s)$ are the controlled process model and the controller transfer functions respectively. In the system, $r(s)$ is the set point, $u(s)$ is the controller output signal, $d(s)$ is the load disturbance and $y(s)$ is the process controlled variable.

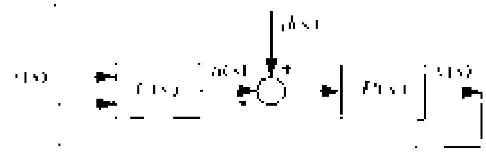

Figure 1. Closed-Loop Control System

The closed-loop control system output, $y(s)$, to a change at its inputs, $r(s)$ and $d(s)$, is given by

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \qquad (1)$$

where $M_{yr}(s)$ is the *servo control* closed-loop transfer function, and $M_{yd}(s)$ is the *regulatory control* closed-loop transfer function.

The regulatory control main objective is the *load disturbance rejection*, this is, to return the controlled variable to its set point if a disturbance enters to the control system. For the servo-control, it is intended to *follow a set-point change*, this is, to bring the controlled variable to its new set-point. These two different responses will depend on the closed-loop transfer functions in (1) and may not be independently selected if a 1DoF controller is used but may be selected with a constrained independence if a 2DoF controller is used.

The development of the proposed design approach for 2DoF PI controllers will take into account the closed-loop control system performance stating target responses for both the set-point and the load-disturbances step changes and measuring the control system performance with the integrated absolute error and the control effort total variation, and its robustness with maximum sensitivity.

2.1 2DoF proportional integral controller

The process will be controlled with a two-degree-of-freedom proportional integral (PI_2) controller [30] whose output is

$$u(s) = K_p\left\{\beta r(s) - y(s) + \frac{1}{T_i s}\left[r(s) - y(s)\right]\right\} \quad (2)$$

where K_p is the controller *proportional gain*, T_i the *integral time constant* and β the *set-point proportional weight*. The controller block diagram is shown in Figure 2.

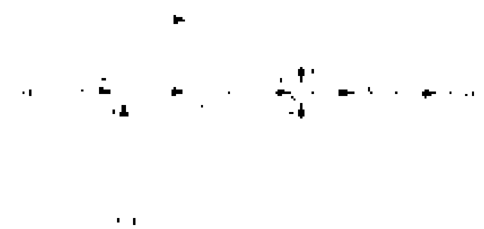

Figure 2. Two Degree of Freedom PI Controller

Controller output (2) will be rewritten for the analysis and for the implementation as:

$$u(s) = C_r(s)r(s) - C_y(s)y(s), \quad (3)$$

where

$$C_r(s) = K_p\left(\beta + \frac{1}{T_i s}\right), \quad (4)$$

is the PI_2 controller part applied to the set-point r, the *set-point controller* transfer function, and

$$C_y(s) = K_p\left(1 + \frac{1}{T_i s}\right), \quad (5)$$

is the PI_2 controller part applied to the feedback signal y, the *feedback controller* transfer function.

The servo-control and the regulatory control closed-loop transfer functions of (1) are now

$$M_{yr}(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)}, \quad (6)$$

and

$$M_{yd}(s) = \frac{P(s)}{1 + C_y(s)P(s)}, \quad (7)$$

which are related by

$$M_{yr}(s) = C_r(s)M_{yd}(s) \quad (8)$$

2.2 Over damped controlled process models

The over damped controlled processes will be represented by a linear model given by the transfer function

$$P(s) = \frac{Ke^{-Ls}}{(Ts+1)(aTs+1)}, \quad \tau = \frac{L}{T}, \quad (9)$$

where K is the model gain, T its main time constant, a the ratio of its two time constants ($0 \le a \le 1$), L the dead-time, and τ the model *normalized dead-time* ($0.1 \le \tau \le 2.0$).

Model transfer function (9) allows to represent first-order plus dead-time (FOPDT) processes ($a = 0$), over damped second-order-plus-dead-time (SOPDT) processes ($0 < a < 1$), and dual-pole-plus-dead-time (DPPDT) processes ($a = 1$).

The parameters of the controlled process model (9), $\theta_p = \{K, T, a, L, \tau\}$, may be identified from the process reaction curve [29].

2.3 Closed-loop target transfer functions

For the development of the proposed design method, it is important to have the lowest possible number of design parameters. The control system response target to a load-disturbance step change will have only one design parameter T_c (the closed-loop time constant). It is selected as non-oscillatory (for a smooth response) and with no steady-state error, by the following target transfer function

$$M_{yd}^{t}(s) = \frac{K_{cl}\, s e^{-Ls}}{(T_c s + 1)^2 (aTs + 1)}, \quad (10)$$

where K_{cl} and T_c are the regulatory control closed-loop transfer function static gain and time-constant respectively. In a PI regulatory control system the closed-loop transfer function gain K_{cl} in (10) is given by $K_{cl} = T_i/K_p$, then

$$M_{yd}^{t}(s) = \frac{(T_i/K_p)\, s e^{-Ls}}{(T_c s + 1)^2 (aTs + 1)}. \quad (11)$$

Using (11) and (4) in (8) the servo-control closed-loop transfer function results in:

$$M_{yr}^{t}(s) = \frac{(\beta T_i s + 1)e^{-Ls}}{(T_c s + 1)^2 (aT_c s + 1)} \quad (12)$$

Then, to have a response to a set-point step change without oscillation and no overshoot, and with no steady state error, the servo-control closed-loop target transfer function is selected as

$$M_{yr}^{t}(s) = \frac{e^{-Ls}}{(T_c s + 1)(aT_c s + 1)} \quad (13)$$

If T_c is expressed as a function of the controlled process model (9) main time constant ($T_c = \tau_c T$), then $\tau_c = T_c/T$ may be used as the dimensionless design parameter. The closed-loop performance specification will require only one parameter, τ_c, that the ratio of the closed-loop system response speed to the controlled process speed.

Using (13) and (11) in (1) the global target control system output $y^t(s)$ is computed as

$$y^t(s) = \frac{e^{-Ls}}{(\tau_c Ts + 1)(a\tau_c Ts + 1)} r(s) + \frac{(T_i/K_c)s e^{-Ls}}{(\tau_c Ts + 1)^2 (a\tau_c Ts + 1)} d(s) \quad (14)$$

In the particular case of the FOPDT models ($a = 0$) the control system target output is then

$$y^t(s) = \frac{e^{-Ls}}{\tau_c Ts + 1} r(s) + \frac{(T_i/K_c)s e^{-Ls}}{(\tau_c Ts + 1)^2} d(s) \quad (15)$$

3. Controller Design

Usually, the design of 2DoF PI controllers is made in two stages [10,29,32-34]. First, the parameters (K_c, T_i) of the feedback controller (5) required to obtain the desired regulatory control performance and/or a closed-loop control system with a specific robustness level are determined for a set of parameters of the controlled process model θ_p. After that and on a second step, the set-point controller (4) free parameter (β) is used to improve the servo-control performance.

Then a different approach is followed. The complete set of PI_2 controller parameters $\theta_c = \{K_c, T_i, \beta\}$ will be obtained when considering, the regulatory control and the servo-control performance at once, to obtain a controller with a target *servo/regulation combined performance* that will also produce a closed-loop control system with a specific *robustness* level.

The closed-loop control system target response (14) can be rewritten in the time domain as

$$y^t(t) = y_r^t(t) + y_d^t(t), \quad (16)$$

where y_r^t is the servo-control target step response and y_d^t the regulatory control target step response.

3.1 Regulatory control cost functional

For the regulatory control response the cost functional is defined as

$$J_d = \int_0^\infty \left| y_d^t(t) - y_d(t) \right| dt, \quad (17)$$

where $y_d^t(t)$ is the step response of the regulatory control target closed-loop transfer function (11) and $y_d(t)$ the corresponding one of the regulatory control system (7) with the controlled process (9) and the controller (5).

3.2 Servo-control cost functional

In a similar way the servo-control cost functional is defined as

$$J_r = \int_0^\infty \left| y_r^t(t) - y_r(t) \right| dt, \quad (18)$$

where $y_r^t(t)$ is the step response of the servo-control target closed-loop transfer function (13) and $y_r(t)$ the corresponding one of the servo-control system (6) with the controlled process (9) and the controller (4).

3.3 Controller optimization

For the 2DoF PI controller design an overall cost functional given by

$$J_T = J_d + J_r, \quad (19)$$

is optimized to obtain the controller optimum parameters $\theta_c^o = \{K_c^o, T_i^o, \beta^o\}$ such as

$$J_T^o = J_T(\theta_c^o) = \min_{\theta_c} J_T \quad (20)$$

Note that $\theta_c^o = \theta(\theta_p, \tau_c)$.

3.4 Performance and robustness evaluation

To allow the designer to select the appropriate design parameter τ_c, both the control system performance and robustness must be evaluated.

Performance

The control system output performance will be evaluated using the integrated-absolute-error cost functional given by

$$J_e = \int_0^\infty |e(t)|dt = \int_0^\infty |r(t) - y(t)|dt \tag{21}$$

This cost functional will be evaluated by regulatory (J_{ed}) and servo-control (J_{er}) operation.

On the other hand the *control effort total variation* will be evaluated by

$$TV = \sum_{i=1}^{\infty} |u_{i+1} - u_i| \tag{22}$$

that also will be evaluated by load disturbances (TV_d) and set-point (TV_r) changes.

Robustness

The closed-loop control system robustness will be computed using the maximum sensitivity M_s, defined as

$$M_s = \max_\omega |S(j\omega)| = \max_\omega \frac{1}{|1 + C_y(j\omega)P(j\omega)|} \tag{23}$$

3.5 Robust tuning of 2DoF PI controllers

Consider as a controlled process the four order model proposed as benchmark in [35] and given by the transfer function

$$P(s) = \frac{1}{\prod_{n=0}^{3}(\alpha^n s + 1)}, \quad \alpha = 0.5, \tag{24}$$

Using the three-point identification procedure *123c* [21] FOPDT and SOPDT models were obtained whose parameters are listed in Table 1.

Table 1. Controlled Process Models

K	T	a	L	τ_o
1	1.247	-	0.691	0.554
1	0.876	0.821	0.277	0.316

For the SOPDT model and following the proposed design procedure the parameters obtained for robust tuned 2DoF PI controllers are listed in Table 2.

Table 2. 2DoF PI Parameters

τ_c	K_p	T_i	β	M_s
0.7	1.709	1.449	0.519	2.013
0.9	1.183	2.487	0.601	1.626
1.1	0.805	1.405	0.735	1.412

Table 2 shows the existing performance/robustness trade-off. To increase the control system robustness M_s, its performance (speed) needs to be reduced (increasing τ_c).

4. 2DoF PI Controller Tuning

From the sections above the whole design procedure can be summarized as follows:

1. Obtain the model (9) from the controlled process reaction-curve or critical information.
2. Select a *design parameter* τ_c.
3. Optimize the cost functional J_e (19) to obtain the controller parameters $\theta = \{K_p, T_i, \beta\}$.
4. Evaluate the control system output performances, J_{ed} and J_{er} (21).
5. Evaluate the control effort total variations, TV_d and TV_r (22).
6. Evaluate the control system robustness, M_s (23).
7. Analyze the performance, control effort and robustness indicators and select a new design parameter (τ_c in step 2) if required.

5. Comparison with other Tuning Methods

For comparison purposes the following PI tuning methods that include a design parameter to deal with the performance/robustness trade-off were selected: the IMC based IMC-PID in [36] and the *Simple Control* (SIMC) [11] for 1DoF PI/PID controllers, and the *Analytical Robust Tuning* (ART) [21,22] for 2DoF PI/PID controllers.

The same four order controlled process (24) and models listed in Table 1, will bee used for the comparison.

In the particular process model used in this example, the IMC and SIMC tuning result in the same PI parameters.

Although the design parameter τ_c has the same meaning in all the compared methods its influence over the control system performance and robustness is different because the closed loop transfer functions used in the deduction of the methods were not necessarily the same. Considering this and to obtain a comparison on the same base, the design parameter used with each of the methods compared was selected to obtain a specific robustness level $M_S = \{2.0, 1.8, 1.6, 1.4\}$. Therefore, some iterations where needed when applying the selected tuning methods and the design method outlined in Section 4.

In this approach however all methods will provide the same robustness level, allowing to concentrate the analysis to the performance indicators.

The normalized (respect to the best one) performances, J_{ld}, J_{sp}, obtained from the above methods are shown in Figure 3 and the normalized control effort total variation in Figure 4.

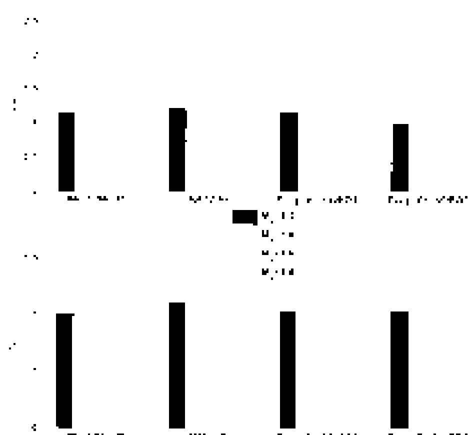

Figure 3. Example - Normalized Regulatory and Servo-control Performance

From the performance results it is noted the performance robustness trade-off. If the control systems robustness is increased, its performance decreases. It is also noted that for the same robustness level the PI controller obtained from the SOPDT model has better regulatory control performance without a reduction of its servo-control performance.

The control effort total variation has an inverse relation to the control system robustness. An increase in the control system robustness produces a *smoother* controller output.

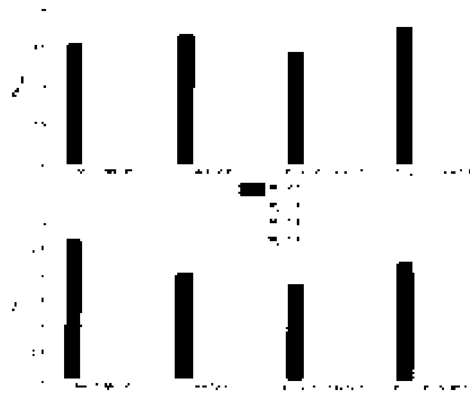

Figure 4. Example - Normalized Regulatory and Servo-control Total Variation

The quantitative indicators used (J, TV) must be complemented with some qualitative indicators obtained from the control system responses to step changes in the set point and load-disturbance.

The obtained closed-loop responses to a 20% set-point step change followed by a 10% disturbance step change are shown in Figure 5 for $M_S = 2.0$ and in Figure 6 for $M_S = 1.4$.

Figure 5. Example - Control Systems Responses $M_S = 2.0$

Given the results it is evident that the 1DoF PI controller (IMC, SIMC tuning) produces high changes in the controller output to a set-point step change that produces higher overshoots and more oscillating responses when a fast response (low robust system) is specified. The set-point weight factor of the 2DoF controllers allows a smooth controller output for all robustness levels.

Figure 6. Example 3 Control Systems Responses, $M_s = 1.4$

An overall evaluation of the control system characteristics: performance and control effort versus robustness, shows that the proposed PI controller design procedure provides the required flexibility to take into account several of the conflicting control system design criteria.

6. Conclusions

The proposed design framework for two-degree-of-freedom (2DoF) proportional integral (PI) controllers allows to deal with the control system performance/robustness trade-off selecting the design parameter τ_c to produce fast and nearly non-oscillating responses to a set-point or load-disturbance step change, and requiring excessive control effort variations.

From the comparison made with other tuning methods it is evident that the same robustness levels may be obtained with different sets of controller parameters, then some other quantitative and qualitative indicators are needed to evaluate the control system behaviour.

The use of the proposed design methodology may be used to obtain tuning rules that will produce robust control systems.

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Model-reference robust tuning of 2DoF PI controllers for first- and second-order plus dead-time controlled processes

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Fragility

ABSTRACT

The aim of this paper is to present a robust tuning method for two-degree-of-freedom (2DoF) proportional integral (PI) controllers. This is based on the use of a model reference optimization procedure with servo and regulatory target closed-loop transfer functions for first- and second-order plus dead-time (FOPDT, SOPDT) controlled process models. The designer is allowed to deal with the performance/robustness *trade-off* of the closed-loop control system by specifying the desired robustness level by selecting a maximum sensitivity in the range from 1.4 to 2.0. In addition, a *smooth servo/regulatory performance combination* is obtained by forcing both closed-loop transfer functions to perform as closely as possible to non-oscillatory dynamic targets. A unified set of controller tuning equations is provided for FOPDT and SOPDT models with normalized dead-times from 0.1 to 2.0 that guarantees the achievement of the design robustness level. The robustness of the control system is analyzed as well as the robustness–fragility and performance–fragility of the optimized controllers. Comparative examples show the effectiveness of the proposed tuning method. The exact achievement of the control system robustness target for all the controlled process models considered (first- and second-order) is one of the distinctive characteristics of the proposed *model reference robust tuning* (MoReRT) method.

1. Introduction

As has been widely reported [1–4], the proportional integral derivative (PID) controllers are still the type most extensively used in the process industry, with most actually used as proportional integral (PI) controllers. Their success is mainly due to their simple structure and low number of tuning parameters, which make them easier to understand than most other advanced control structures.

Since Ziegler and Nichols [5] presented tuning rules for the first commercially available pneumatic PID controller, the Taylor Fulscope 100 [6], a great number of other procedures have been developed, especially those using a low-order overdamped model representation of the controlled process, as revealed in O'Dwyer handbook [7]. Some of these, such as the classical tuning rules of López et al. [8] and Rovira et al. [9], consider only the performance of the closed-loop control system. Authors such as Åström and Hägglund [10] or Ho et al. [11] instead consider its robustness, while others [12–15] consider a combination of its performance and robustness.

In most of the industrial process control applications, the desired value of the controlled variable, or set-point, normally remains constant and a good load–disturbance rejection is required [16], which is usually known as *regulatory control*. However, due to variations in the process operating conditions, the controlled variable set-point may eventually need to be changed and then a good transient response to such change is required, which is known as *servo-control* operation. Satisfying these two operating conditions simultaneously is not possible by using a one-degree-of-freedom (1DoF) PI/PID controller, but using a two-degree-of-freedom (2DoF) PI/PID allows tuning of the controller in order to do so. The extra parameter it provides is used to improve its servo-control behavior while considering the regulatory control *performance* and the closed-loop control system *robustness*. This second degree of freedom; introduced by Araki [17–19]; is aimed at providing additional flexibility to control system design with PI/PID controllers [20,21].

The design procedure for a closed-loop control system is usually based on low-order linear models identified at the normal operating point. Due to the nonlinear characteristics found in most industrial processes, it is necessary to anticipate the changes in the process characteristics when the operating point changes, assuming certain relative stability margins or robustness requirements for the control system. Therefore, the design of a closed-loop control

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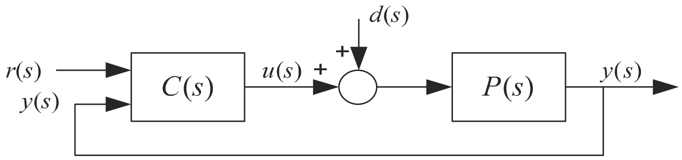

Fig. 1. Closed-loop control system.

system with 2DoF PI/PID controllers must take into account its performance to load–disturbance and set-point changes as well as its robustness to variations of the controlled process characteristics.

In addition, since no model can exactly reproduce the dynamic behavior of the controlled process, a last *fine-tuning* of the controller parameters is normally also required. To do this with confidence it is necessary to have an estimation of the controller *fragility*, which means the possible control system robustness and performance losses when the controller parameters are changed [22].

For over damped controlled processes there are methods such as kappa–tau [23] and AMIGO [4] that provide tuning rules by which to design 2DoF PI closed-loop control systems with a specific robustness, but only for one or two maximum sensitivity values (kappa–tau, $M_S \in \{1.4, 2.0\}$; AMIGO, $M_S = 1.4$). Besides, there is no guarantee the selected design robustness level will be obtained with either of these two tuning methods [24]. A fragility analysis of these and other tuning rules is presented in [25].

According to O'Dwyer handbook [7], most of the available PI tuning rules are based on first-order plus dead-time (FOPDT) models and very few on second-order plus dead-time (SOPDT) models. The SOPDT models have richer dynamics and therefore are better able to represent the controlled process, especially when of high-order. Note also that the PI tuning rules for SOPDT models are mainly just based on performance considerations.

An alternative tuning method for 2DoF proportional integral (PI_2) controllers is presented in this communication. The proposed approach explicitly considers the *trade-off* between the performance and robustness of a control system. The distinctive feature of the resulting tuning procedure is the incorporation of the desired robustness level as measured with the maximum sensitivity, M_S, which is the explicit and only design parameter. The proposed tuning method integrates in a single set of tuning equations the robust design of PI_2 controllers for first- and second-order plus dead-time controlled process models. Therefore, the designer may adopt the first- or second-order model that best represents the dynamics of the controlled process and select the desired robustness M_S level for the control system in the range from 1.4 to 2.0.

For evaluation of the proposed tuning in comparison with other available methods, three key aspects were considered: (1) the *performance* of the control system response to step changes in set-point and load–disturbance, (2) the *robustness* of the control system to changes in the controlled process characteristics, and (3) the *robustness–fragility* and the *performance–fragility* of the controller to changes in its own parameters.

The rest of the paper is organized as follows: Section 2 formulates the control problem; Section 3 describes the controller optimization procedure and the robustness of the proposed controllers; Section 4 analyzes the fragility characteristics of the obtained controllers; Section 5 demonstrates the new tuning relations; and the final section discusses the conclusions as well as considerations for future work.

2. Problem formulation

Consider a closed-loop control system, as shown in Fig. 1, where $P(s)$ and $C(s)$ are the controlled process model transfer function and the controller transfer function, respectively. In this system,

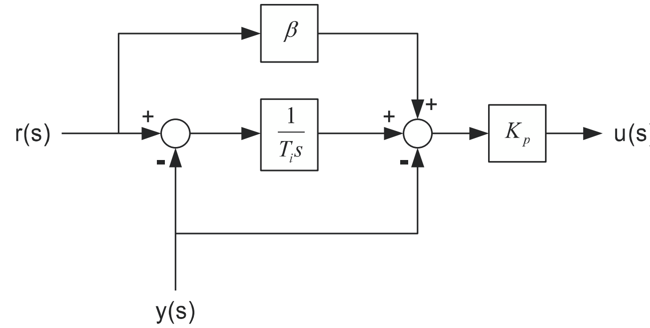

Fig. 2. Two-degree-of-freedom PI controller.

$r(s)$ is the set-point, $u(s)$ is the controller output signal, $d(s)$ is the load–disturbance, and $y(s)$ is the process controlled variable.

The closed-loop control system output, $y(s)$, in response to changes in its inputs, $r(s)$ and $d(s)$, is given by the following:

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \tag{1}$$

where $M_{yr}(s)$ is the transfer function from the set-point to the process controlled variable, and $M_{yd}(s)$ is that from the load–disturbance to the process controlled variable. These are known as the *servo-control* closed-loop transfer function and the *regulatory control* closed-loop transfer function, respectively.

The main objective of the regulatory control is *load–disturbance rejection*; that is, the controlled variable should be returned to its set-point when a disturbance enters the control system. For the servo control, the intention is to *follow a set-point change*; that is, the controlled variable is brought to its new set-point. These two different responses will depend on the closed-loop transfer functions in (1) and cannot be selected independently if a one-degree-of-freedom (1DoF) controller is used, but can be selected with constrained independence if a two-degree-of-freedom (2DoF) controller is used.

The development of the proposed tuning method for 2DoF PI controllers will take into account not only the closed-loop control system performance, stating target responses for step changes in the set-point and the load–disturbance, but also the control system robustness, measuring this with the maximum sensitivity, M_S.

2.1. 2DoF proportional integral controller (PI_2)

The process will be controlled with a two-degree-of-freedom proportional integral (PI_2) controller [23] whose output is expressed as follows:

$$u(t) = K_p \left\{ e_p(t) + \frac{1}{T_i} \int_0^t e_i(\tau) d\tau \right\}, \tag{2}$$

with

$$e_p(t) = \beta r(t) - y(t), \tag{3}$$

$$e_i(t) = r(t) - y(t), \tag{4}$$

or

$$u(s) = K_p \left\{ \beta r(s) - y(s) + \frac{1}{T_i s} [r(s) - y(s)] \right\}, \tag{5}$$

where K_p is the controller *proportional gain*, T_i is the *integral time constant*, and β is the *set-point proportional weight*. The block diagram of this controller is shown in Fig. 2.

For the purposes of analysis only, not implementation, the controller output (5) will be rewritten as follows:

$$u(s) = K_p \left(\beta + \frac{1}{T_i s} \right) r(s) - K_p \left(1 + \frac{1}{T_i s} \right) y(s), \tag{6}$$

and in a compact form as follows:

$$u(s) = C_r(s)r(s) - C_y(s)y(s), \quad (7)$$

where

$$C_r(s) = K_p\left(\beta + \frac{1}{T_i s}\right), \quad (8)$$

is the PI_2 controller aspect that operates on the set-point r, the *set-point controller* transfer function, and

$$C_y(s) = K_p\left(1 + \frac{1}{T_i s}\right), \quad (9)$$

is the PI_2 controller aspect that operates on the feedback signal y, the *feedback controller* transfer function.

The closed-loop transfer functions of the servo control and the regulatory control in (1) are then given by

$$M_{yr}(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)}, \quad (10)$$

and

$$M_{yd}(s) = \frac{P(s)}{1 + C_y(s)P(s)}, \quad (11)$$

which are related as follows:

$$M_{yr}(s) = C_r(s)M_{yd}(s). \quad (12)$$

2.2. Over damped controlled process models

The over damped controlled processes will be represented by a linear model in the form of the following general transfer function:

$$P(s) = \frac{Ke^{-Ls}}{(Ts+1)(aTs+1)}, \quad (13)$$

where K is the gain, T is the main time constant, a is the ratio of time constants ($0 \leq a \leq 1.0$), and L is the dead-time. The parameters of the controlled process model (13), $\overline{\theta}_p = \{K, T, a, L\}$, may be identified from the process reaction curve [26].

The transfer function (13) allows the representation of first-order plus dead-time (FOPDT) processes ($a = 0$), over damped second-order plus dead-time (SOPDT) processes ($0 < a < 1$), and dual-pole plus dead-time (DPPDT) processes ($a = 1$) modeling a wide range of self-regulating controlled processes [27,28].

2.3. Target closed-loop transfer functions

For the development of the proposed tuning method, it is important to have the smallest possible number of design parameters. Consequently, the desired control system response to a load–disturbance step change will involve only one design parameter, the closed-loop time constant T_c, and is selected with no steady-state error and non-oscillatory, for a smooth response. In general this can be expressed by a transfer function with the following form:

$$M^t_{yd}(s) = \mathcal{M}_d(T_c, \overline{\theta}_c, \overline{\theta}_p, s) = \frac{K_o s N_p^+(s)}{p(T_c, \overline{\theta}_p, s)}, \quad (14)$$

where $N_p^+(s)$ contains the non-minimum phase elements of the model, $p(T_c, \overline{\theta}_p, s)$ is the characteristic polynomial of the closed-loop control system, and $\overline{\theta}_c = \{K_p, T_i, \beta\}$ are the controller parameters.

From (14), taking into account the second-order controlled process model (13) and the first-order feedback part of the PI controller (9), the desired regulatory control closed-loop transfer function is obtained as the following third-order target transfer function:

$$M^t_{yd}(s) = \frac{K_o s e^{-Ls}}{(T_c s+1)^2(aT_c s+1)}, \quad (15)$$

where K_o and T_c are, respectively, the static gain and time-constant of the regulatory control closed-loop transfer function.

The regulatory control closed-loop transfer function (15) is similar but more general than the desired closed-loop transfer function specified in [29] for the direct synthesis (DS) disturbance rejection design. Specifying a non-oscillatory response for the control system it is pursued to obtain also a smooth controller output without abrupt changes or heavy oscillations to avoid a quickly deterioration of the final control element that it is usually a control valve.

For a PI controller, the regulatory control closed-loop transfer function gain in (15) was found as $K_o = T_i/K_p$; therefore,

$$M^t_{yd}(s) = \frac{(T_i/K_p)se^{-Ls}}{(T_c s+1)^2(aT_c s+1)}. \quad (16)$$

Using (16) and (8) in (12) the servo-control closed-loop transfer function is given by

$$M_{yr}(s) = \mathcal{M}_r(T_c, \overline{\theta}_c, \overline{\theta}_p, s) = \frac{(\beta T_i s+1)e^{-Ls}}{(T_c s+1)^2(aT_c s+1)}. \quad (17)$$

Then, in order to have a set-point step change response without oscillation, overshoot, or steady-state error, the servo-control target closed-loop transfer function is selected as follows:

$$M^t_{yr}(s) = \frac{e^{-Ls}}{(T_c s+1)(aT_c s+1)}. \quad (18)$$

The servo-control closed-loop transfer function (18) will provide first- or second-order over damped set-point step responses similar to the ones obtained with an IMC-based servo-control design [30–32].

If an analytical design of β is performed as in the ART_2 tuning method [33], satisfying (18) means selecting β to cancel one closed-loop pole. As will be seen in the next section, a different route is taken here by not imposing such a cancellation.

If T_c is expressed as a function of the main time constant of the controlled process model, then $\tau_c \doteq T_c/T$ may be used as the dimensionless design parameter. The closed-loop performance specification will require only one parameter, τ_c, which is an indication of the closed-loop system response speed in relation to the controlled process speed.

Using (18) and (16) in (1) the global control system output target, $y^t(s)$, is computed as follows:

$$y^t(s) = \frac{e^{-Ls}}{(\tau_c Ts+1)(a\tau_c Ts+1)}r(s) + \frac{(T_i/K_p)se^{-Ls}}{(\tau_c Ts+1)^2(a\tau_c Ts+1)}d(s). \quad (19)$$

For the particular case of FOPDT models ($a = 0$), the control system output target is given by

$$y^t(s) = \frac{e^{-Ls}}{\tau_c Ts+1}r(s) + \frac{(T_i/K_p)se^{-Ls}}{(\tau_c Ts+1)^2}d(s). \quad (20)$$

3. Controller design

Usually the design of 2DoF PI controllers is performed in two stages [20,34–37]. First, the parameters (K_p, T_i) of the feedback controller (9) required to obtain the desired regulatory control performance and a specific closed-loop control system robustness level are determined for a controlled process model parameters $\overline{\theta}_p$. Second, the set-point controller (8) free parameter (β) is used to improve the servo-control performance.

In what follows a different approach is taken. The complete set of PI_2 controller parameters $\overline{\theta}_c = \{K_p, T_i, \beta\}$ will be obtained considering, at the same time, the regulatory control and the servo-control performance, to obtain a controller with a targeted *servo/regulatory performance combination* that will also produce a closed-loop control system with a specific robustness level.

The closed-loop control system response target (19) can be rewritten in the time domain as follows:

$$y^t(t) = y_r^t(t) + y_d^t(t), \tag{21}$$

where $y_r^t(t)$ is the servo-control step response target and $y_d^t(t)$, the regulatory control step response target.

3.1. Regulatory control cost functional

For the regulatory control response, the cost functional to be minimized is defined as follows:

$$J_d(\tau_c, \overline{\theta}_c, \overline{\theta}_p) \doteq \int_0^\infty \left[y_d^t(\tau_c, \overline{\theta}_c, \overline{\theta}_p, t) - y_d(\overline{\theta}_c, \overline{\theta}_p, t)\right]^2 \mathrm{d}t, \tag{22}$$

where $y_d^t(\tau_c, \overline{\theta}_c, \overline{\theta}_p, t)$ is the step response of the regulatory control target closed-loop transfer function (16) and $y_d(\overline{\theta}_c, \overline{\theta}_p, t)$ is that of the regulatory control system (11) with the controlled process (13) and controller (9).

3.2. Servo-control cost functional

In a similar way, the servo-control cost functional to be minimized is defined as follows:

$$J_r(\tau_c, \overline{\theta}_c, \overline{\theta}_p) \doteq \int_0^\infty \left[y_r^t(\tau_c, \overline{\theta}_p, t) - y_r(\overline{\theta}_c, \overline{\theta}_p, t)\right]^2 \mathrm{d}t, \tag{23}$$

where $y_r^t(\tau_c, \overline{\theta}_p, t)$ is the step response of the servo-control target closed-loop transfer function (18) and $y_r(\overline{\theta}_c, \overline{\theta}_p, t)$ is that of the servo-control system (10) with the controlled process (13) and controller (8).

3.3. Controller optimization

For the 2DoF PI controller design, the following overall cost functional is optimized:

$$J_T(\tau_c, \overline{\theta}_c, \overline{\theta}_p) \doteq J_r(\tau_c, \overline{\theta}_c, \overline{\theta}_p) + J_d(\tau_c, \overline{\theta}_c, \overline{\theta}_p), \tag{24}$$

to obtain the optimum controller parameters $\overline{\theta}_c^o = \{K_p^o, T_i^o, \beta^o\}$ such that

$$J_T^o \doteq J_T(\tau_c, \overline{\theta}_c^o, \overline{\theta}_p) = \min_{\overline{\theta}_c} J_T(\tau_c, \overline{\theta}_c, \overline{\theta}_p). \tag{25}$$

Note that $\overline{\theta}_c^o = \overline{\theta}_c^o(\overline{\theta}_p, \tau_c)$.

Moreover, for each $\overline{\theta}_c^o$ set obtained, the closed-loop control system robustness is measured using the maximum sensitivity M_S, which is defined as follows:

$$M_S \doteq \max_\omega |S(\mathrm{j}\omega)| = \max_\omega \frac{1}{|1 + C_y(\mathrm{j}\omega)P(\mathrm{j}\omega)|}. \tag{26}$$

In the proposed one-step optimization of (24) the control system target response is given by (19) where its performance, τ_c, is constrained by the minimum target robustness, M_S^t. The optimization problem stated is a *model matching problem* instead of a performance optimization problem as in the traditional two-step PI_2/PID_2 controller optimization procedure. Matching the selected M_{yd}^t (15) depends only on K_p and T_i but matching the selected M_{yr}^t depends on all the controller parameters (K_p, T_i, β). So, the one-step approach will provide a better overall optimization.

Using the controlled process model gain, K, and time constant, T, as well as the transformation $\hat{s} = Ts$, the controlled process (13) and the PI controller transfer functions (8) and (9) can be expressed in a normalized form as follows:

$$\hat{P}(\hat{s}) = \frac{e^{-\tau_o \hat{s}}}{(\hat{s}+1)(a\hat{s}+1)}, \tag{27}$$

$$\hat{C}_r(\hat{s}) = \kappa_p\left(\beta + \frac{1}{\tau_i \hat{s}}\right), \tag{28}$$

$$\hat{C}_y(\hat{s}) = \kappa_p\left(1 + \frac{1}{\tau_i \hat{s}}\right), \tag{29}$$

where $\tau_o = L/T$ is the model *normalized dead-time* and

$$\kappa_p = K_p K, \quad \tau_i = \frac{T_i}{T}, \tag{30}$$

are the *normalized gain* and *normalized integrating time* of the controller, respectively.

The normalized controlled process model (27) has two dimensionless parameters, τ_o and a. Thus, using process model time constant ratios in the range from 0 to 1.0 both first- and second-order models are taken into account under the same optimization design framework. In addition, the normalized dead-time of the model was selected in the range from 0.1 to 2.0 to consider both lag dominant and dead-time dominant controlled processes.

Starting first with a low closed-loop relative speed τ_c value, that produces a control system with fast responses but with low robustness (high M_S value), during the optimization process the design parameter (τ_c) was iteratively increased in such a way that the robustness level of the resulting closed-loop system was raised to a specific target M_S^t value selected in the range from 1.4 to 2.0.

From the optimization results, it is possible to obtain the normalized controller parameters and the resulting control system robustness as functions of the model parameters, $\overline{\theta}_p$, and the performance specification, τ_c. However, to simplify the design procedure, the controller parameters are expressed as direct functions of only the closed-loop control system robustness parameter, which is the maximum sensitivity, M_S. In this way, the control system robustness M_S replaces the relative speed τ_c as the design parameter.

The normalized controller parameters may therefore be expressed directly as functions of the model parameters and the closed-loop control system robustness as follows:

$$\{\kappa_{pk}, \tau_{ik}, \beta_k\} = \mathbf{f}(\overline{\theta}_{pk}, M_S^t), \tag{31}$$

where M_S^t is the control system robustness target, which is now the design parameter.

Fig. 3 shows the optimum normalized controller parameters (κ_p, τ_i, β) obtained in the particular case of a model with time constant ratio $a = 0.50$, normalized dead-time τ_o in the range from 0.1 to 2.0, and four robustness targets $M_S^t \in \{2.0, 1.8, 1.6, 1.4\}$.

This shows the influences of the controlled process normalized dead-time (τ_o) and the desired robustness (M_S^t) over the controller parameters required to meet the target step responses, for one model time–constants ratio (a).

It is important to note that if a control system with high robustness is required ($M_S^t = 1.4$) for processes with large normalized dead-time, τ_o, then the controller gains are reduced and proportional set-point weights $\beta > 1$ are required to improve the servo-control responses, as reported in [38].

3.4. Tuning equations

The controller parameters obtained from the optimization procedure were used to fit the controller parameter equations of the proposed model reference robust tuning (MoReRT) for each one of the six time constants ratios considered.

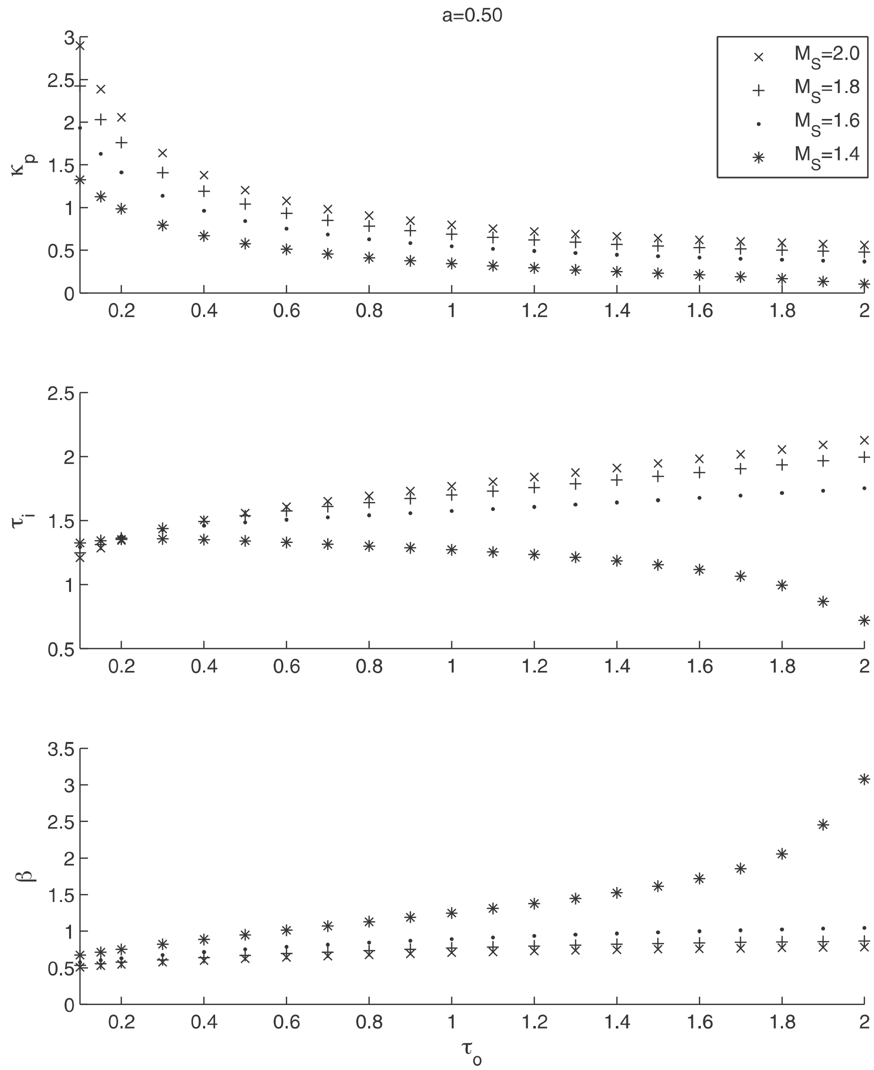

Fig. 3. PI controller parameters for SOPDT ($a = 0.50$).

The optimum normalized controller parameters can be obtained with the following equations:

$$\kappa_p = \frac{a_0 + a_1\tau_o}{a_2 + a_3\tau_o + a_4{\tau_o}^2 + a_5{\tau_o}^3}, \tag{32}$$

$$\tau_i = \frac{b_0 + b_1\tau_o}{b_2 + b_3\tau_o + b_4{\tau_o}^2 + b_5{\tau_o}^3 + b_6{\tau_o}^4}, \tag{33}$$

$$\beta = c_0 + c_1\tau_o + c_2{\tau_o}^2 + c_3{\tau_o}^3. \tag{34}$$

Tables 14–19 in the Appendix A list the a_i, b_i and c_i constants in expressions (32)–(34) for each of the four robustness levels.

Eqs. (32)–(34) provide a direct controller tuning for the FOPDT ($a = 0$) and the DPPDT ($a = 1$) models. In the case of the SOPDT models with $a \notin \{0.25, 0.5, 0.75\}$ the set of controller parameters must be obtained by linear interpolation between the two sets of parameters obtained with the adjacent a values used in the optimization. The six a points provided for each model normalized dead-time τ_o allows a good estimation of the controller parameters for the intermediate a values.

Consider for example the PI controller tuning with a design robustness $M_S^d = 1.6$ for the normalized SOPDT model given by:

$$\hat{P}(\hat{s}) = \frac{e^{-0.8\hat{s}}}{(\hat{s}+1)(0.4\hat{s}+1)}, \quad a = 0.40, \quad \tau_o = 0.8. \tag{35}$$

The controller parameters obtained with (32) and (33) for $M_s^d = 1.6$ are shown in Fig. 4 and their particular values for $\tau_o = 0.80$ with

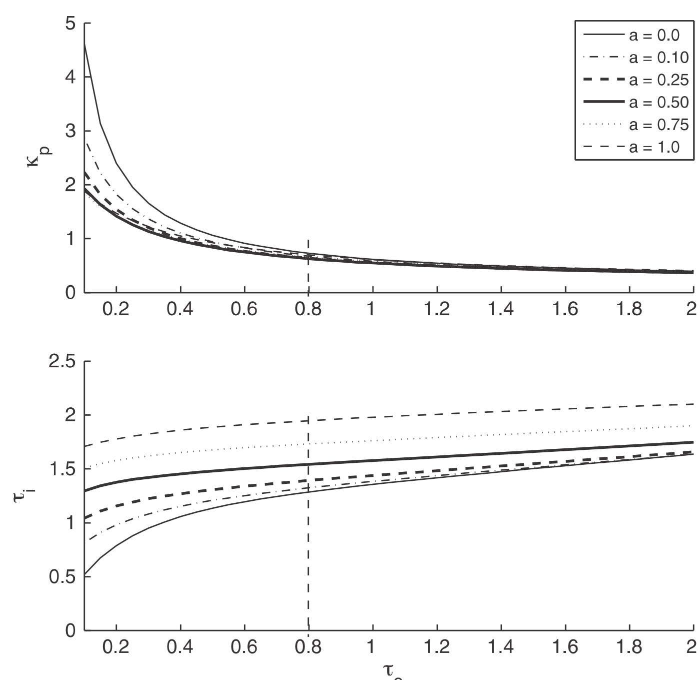

Fig. 4. PI controller parameters for $M_s = 1.6$.

Table 1
PI parameters for $M_s = 1.6$, $\tau_o = 0.80$.

	a		
	0.25	0.50	0.40
κ_p	0.6366	0.6282	0.6316
τ_i	1.3925	1.5421	1.4823
M_S	1.5998	1.5998	1.6062

$a = 0.25$ and $a = 0.50$ are listed in Table 1. As can be seen, the model time constants ratio a has a very small influence over the controller gain along the τ_o range, except for very low τ_o values, and a nearly constant influence over the integral time constant. In this particular case the controller parameters variation between $a = 0.25$ and $a = 0.50$ are $\Delta\kappa_p = -0.0336$ and $\Delta\tau_i = 0.1496$. Using linear interpolation the controller parameters for $a = 0.40$ were found and listed in Table 1. In this table the robustness obtained are also shown confirming that the design robustness is obtained even when parameter interpolation is used.

The designer may resolve the trade-off between performance and robustness by selecting the desired minimum robustness level for the control system according to the expected variation of the controlled process parameters. This gives the designer a closed-loop control system with the highest obtainable speed for the specified minimum robustness.

The MoReRT tuning given by (32)–(34) provides a unified approach to the design of robust control systems with 2DoF PI controllers for overdamped controller processes. This allows using the best model (FOPDT or SOPDT) to represent the controlled process dynamics, thus avoiding the constraint of using only FOPDT models, which may not satisfactorily represent the behavior of a high-order process under closed-loop control.

3.5. MoReRT control system robustness

The robustness obtained with (32)–(34) for each normalized dead-time in the range analyzed is shown in Fig. 5. As can be seen, the robustness profiles for all the six model time constants ratios considered are almost flat. This means that for a FOPDT or SOPDT model of a controlled process the MoReRT tuning guarantees that the robustness target is attained for all normalized dead-times in the range considered.

Because the robustness (M_S) is the design parameter of the MoReRT method, the designer may specify, according to the expected variations of the controlled process characteristics, the relative stability level required for the control system. Due to the achievement of such robustness by the proposed tuning method with the model,

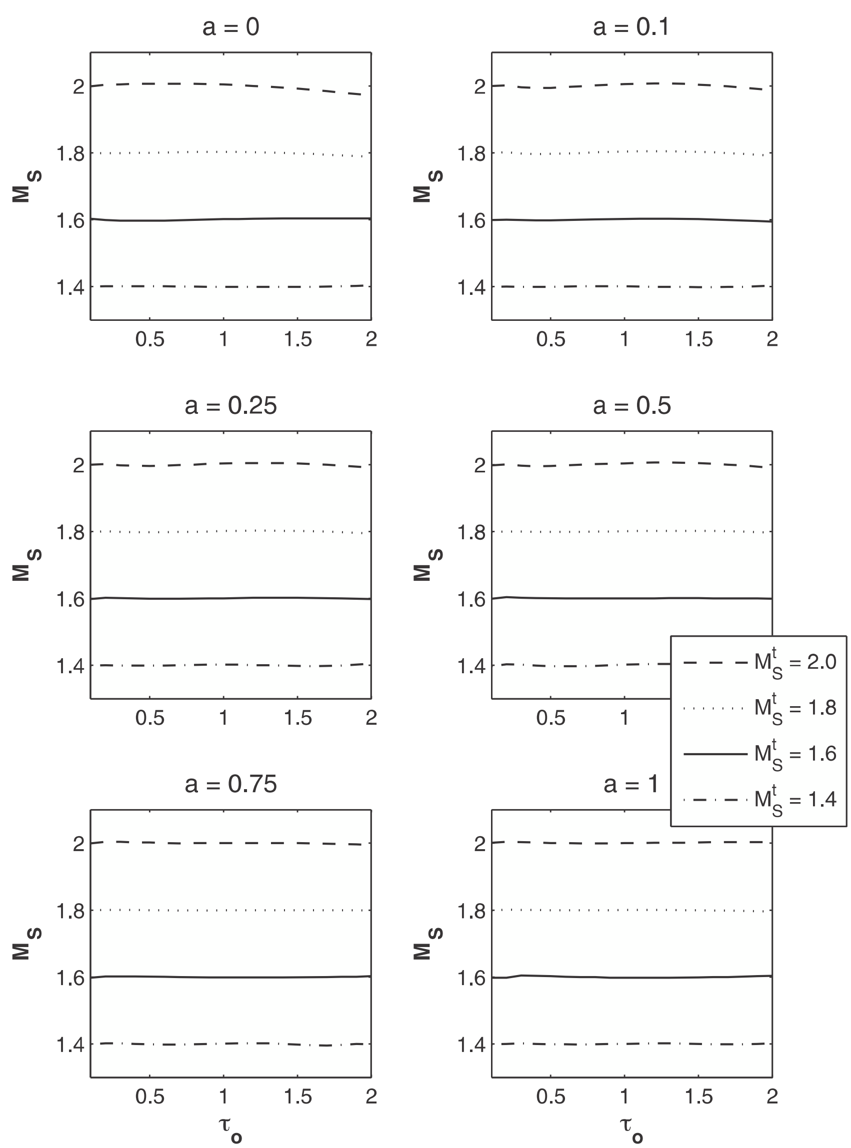

Fig. 5. Robustness of MoReRT controllers.

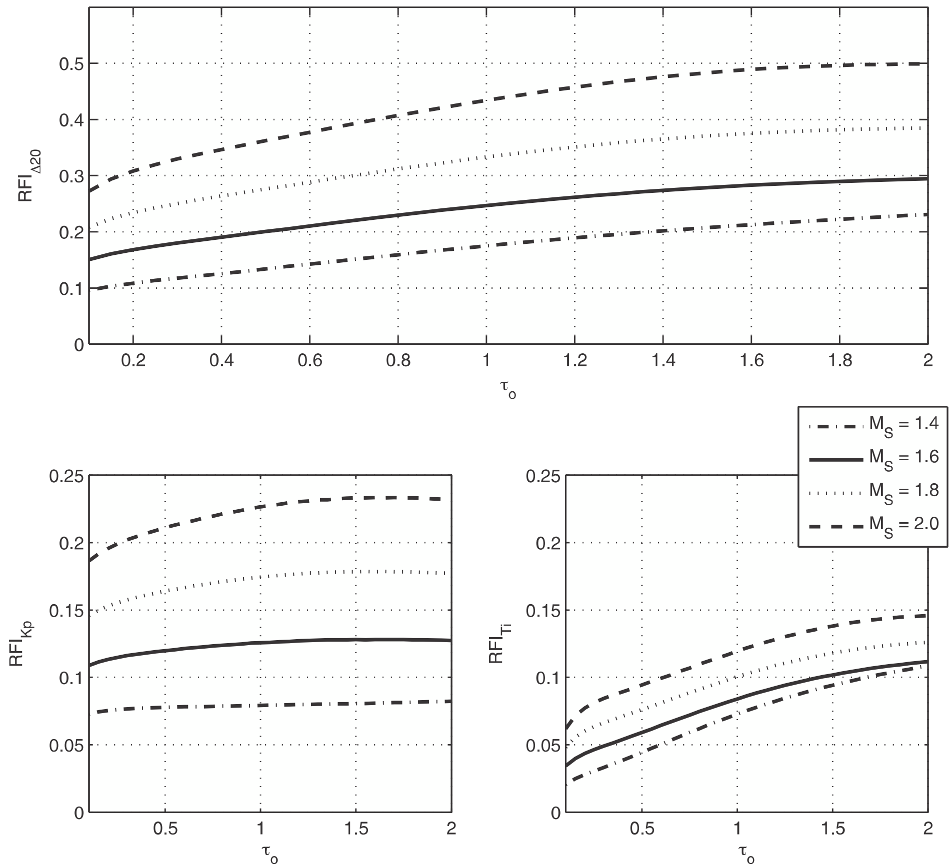

Fig. 6. Robustness–fragility of MoReRT controllers for FOPDT models.

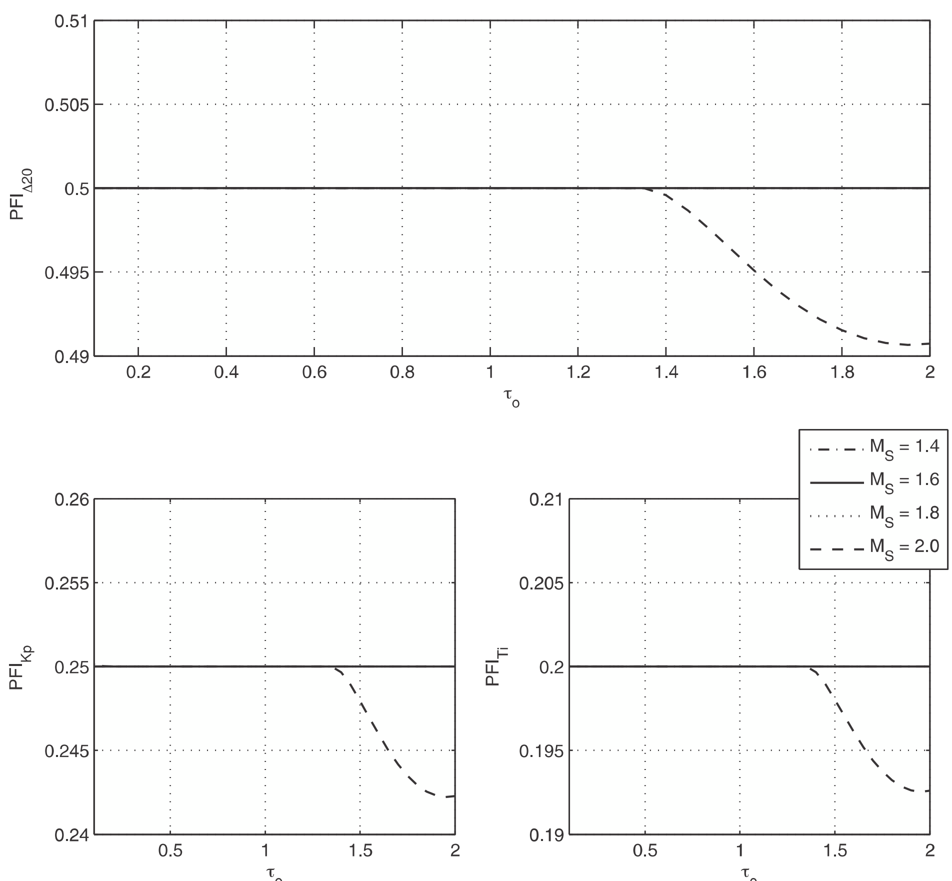

Fig. 7. Regulatory control performance–fragility of MoReRT controllers for FOPDT models.

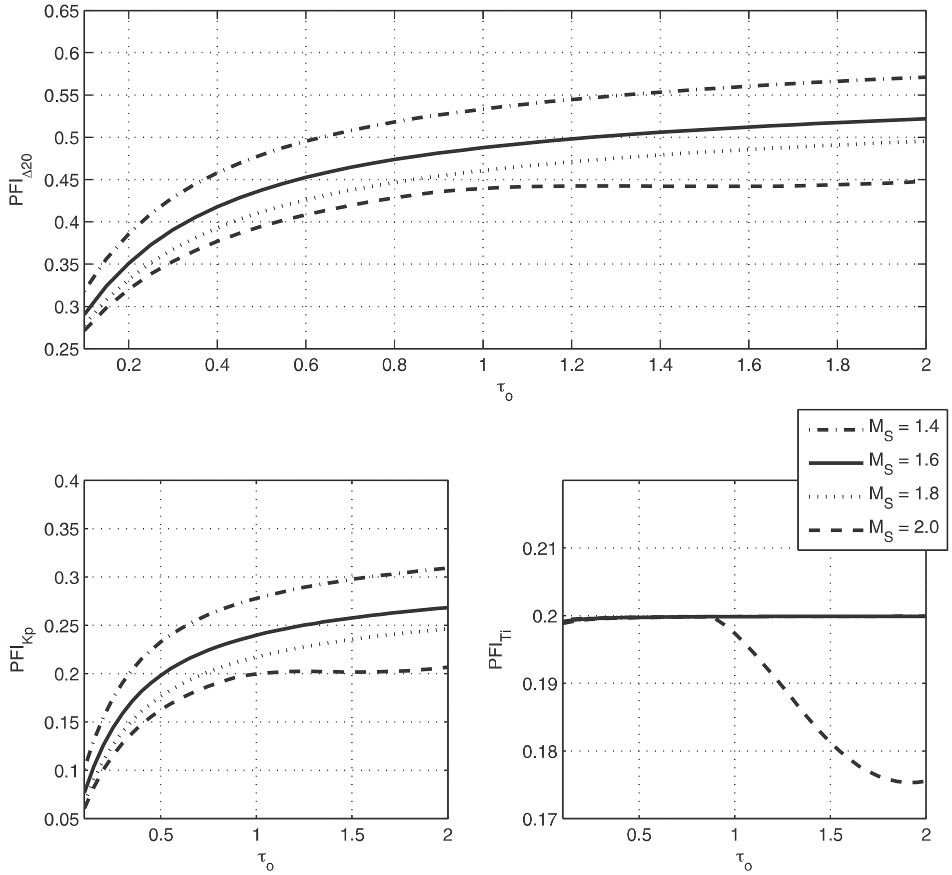

Fig. 8. Servo-control performance–fragility of MoReRT controllers for FOPDT models.

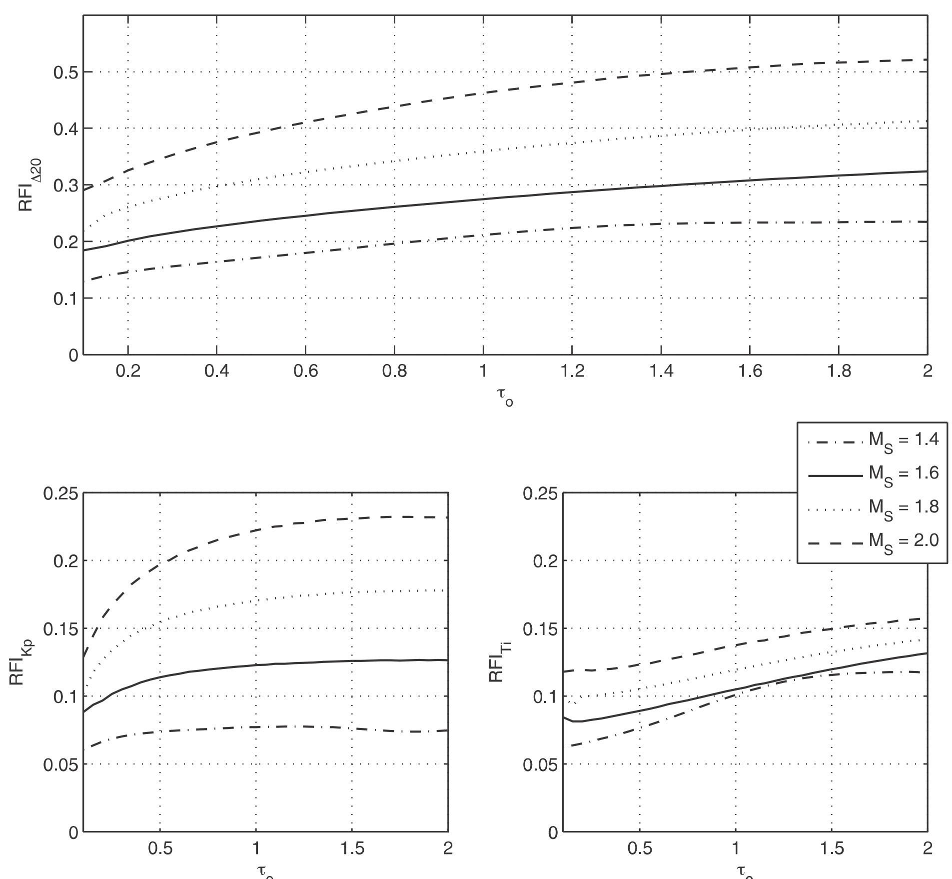

Fig. 9. Robustness–fragility of MoReRT controllers for SOPDT models with $a = 0.50$.

the designer could be sure that the required robustness will also be achieved with the controlled process. This is always constrained to the fact that the adopted model must adequately represent the dynamics of the controlled process.

The accomplishment of the robustness target for all the controlled process models considered (first- and second-order) is the distinctive characteristic of the proposed tuning method.

4. MoReRT controllers fragility

The delta–epsilon fragility index $FI_{\Delta\epsilon}$ and fragility plots of the controller, as defined in [22,39], are related with the loss of robustness in the closed-loop control system when the controller parameters change. Here, the controller fragility analysis will also include the loss of performance. First, the *robustness–fragility index* is redefined as follows:

$$RFI_{\Delta\epsilon} \doteq \frac{M^m_{S\Delta\epsilon}}{M^o_S} - 1 = \frac{\max\{M_S((1\pm\delta\epsilon)\overline{\theta}^o_c)\}}{M_S(\overline{\theta}^o_c)} - 1, \tag{36}$$

where $M^m_{S\Delta\epsilon}$ and M^o_S are, respectively, the extreme and nominal maximum sensitivities of the control system. Next, the *performance–fragility index* is defined as follows:

$$PFI_{\Delta\epsilon} \doteq \frac{J^m_{e\Delta\epsilon}}{J^o_e} - 1 = \frac{\max\{J_e((1\pm\delta\epsilon)\overline{\theta}^o_c)\}}{J_e(\overline{\theta}^o_c)} - 1, \tag{37}$$

where $J^m_{e\Delta\epsilon}$ and J^o_e are, respectively, the extreme and nominal performance that are evaluated with the integrated absolute error given by

$$J_e \doteq \int_0^\infty \left|e(t)\right| \mathrm{d}t = \int_0^\infty \left|r(t) - y(t)\right| \mathrm{d}t. \tag{38}$$

The relative influence of a change in each p_i controller parameter on the robustness fragility can be obtained with the *parametric delta–epsilon robustness–fragility index* given by

$$RFI^{p_i}_{\delta\epsilon} \doteq \frac{M^{p_i}_{S\delta\epsilon}}{M^o_S} - 1 = \frac{\max\{M_S((1\pm\delta\epsilon)p_i, \overline{\theta}^o_c)\}}{M_S(\overline{\theta}^o_c)} - 1, \tag{39}$$

while their relative influence on the performance fragility can be obtained with the *parametric delta–epsilon performance–fragility index* given by

$$PFI^{p_i}_{\delta\epsilon} \doteq \frac{J^{p_i}_{e\delta\epsilon}}{J^o_e} - 1 = \frac{\max\{J_e((1\pm\delta\epsilon)p_i, \overline{\theta}^o_c)\}}{J_e(\overline{\theta}^o_c)} - 1. \tag{40}$$

Further, a *fragility-balanced PI controller* is defined as one in which all the parametric delta–epsilon fragility indices (robustness and performance) are within a selected $\pm\sigma$% band (usually $\pm$25%) centered on its *average parametric fragility indices given by*

$$RFI^a_{\delta\epsilon} \doteq \frac{1}{2}\sum_{i=1}^{2} RFI^{p_i}_{\delta\epsilon}, \tag{41}$$

and

$$PFI^a_{\delta\epsilon} \doteq \frac{1}{2}\sum_{i=1}^{2} PFI^{p_i}_{\delta\epsilon}; \tag{42}$$

otherwise, the PI controller is considered a *fragility-unbalanced controller*.

Following the same criteria stated in [22], a controller will be considered *robustness–fragile* if its delta-20 robustness fragility index is greater than 0.50 ($RFI_{\Delta 20} > 0.50$), *robustness–nonfragile* if its delta-20 robustness fragility index is greater than 0.10 but less than or equal to 0.50 ($0.1 < RFI_{\Delta 20} \leq 0.50$), and robustness-resilient if its delta-20 robustness fragility index is less than or equal to 0.10 ($RFI_{\Delta 20} \leq 0.10$).

Fragility, as usually dealt and presented in the literature just considers robustness of the control system. However the same situation applies to the control system performance. Regarding to the loss of performance when the controller parameters are changed, a controller will be considered *performance–fragile* if its delta-20 performance fragility index is greater than 0.50 ($PFI_{\Delta 20} > 0.50$), *performance–nonfragile* if its delta-20 performance fragility index is greater than 0.10 but less or equal 0.50 ($0.1 < PFI_{\Delta 20} \leq 0.50$), and performance-resilient if its delta-20 performance fragility index is less or equal 0.10 ($PFI_{\Delta 20} \leq 0.10$).

The fragility of the MoReRT controller was analyzed using (36), (37), and (39)–(42).

4.1. Fragility analysis of MoReRT control for FOPDT models

The *robustness–fragility plots* of the MoReRT controllers for FOPDT models are shown in Fig. 6. As can be seen, the controllers are all robustness–nonfragile but none is robustness-resilient. For the same robustness level, the controllers become more robustness–fragile as τ_o increases but all are *balanced-robustness–fragility controllers* [39] with a slightly more influence of the controller gain on the loss of robustness. It can also be seen that the controller robustness–fragility index, $RFI_{\Delta 20}$, and the control system robustness level, M_S, are inversely related. The controllers tuned in order to obtain a highly robust control system are less robustness–fragile.

The *performance–fragility plots* of the MoReRT controllers for FOPDT models are shown in Fig. 7 for a load–disturbance step change and in Fig. 8 for a set-point step change. For the regulatory control operation, all the controllers are at the border of the performance–fragile condition, losing 50% of the nominal performance if all the controller parameters are changed by $\pm$20%, $PFI_{d\Delta 20} = 0.50$, while their parametric performance fragility indices are $PFI^{Kp}_{d\delta 20} = 0.25$ and $PFI^{Ti}_{d\delta 20} = 0.20$. For the servo-control operation, the controller performance–fragility depends on the robustness level and on the model normalized dead-time. The integral time parametric delta-20 performance–fragility of the controllers is constant for all robustness levels and dead-times, but the controller gain has a variable influence over the loss of performance.

From the robustness– and performance–fragility analyses, it can be concluded that the delta-20 robustness–fragility index of the controller is given by the perturbations toward higher gain and lower integral time, while its delta-20 performance–fragility index (servo-control and regulatory control) are given by the opposite condition of perturbations toward lower gain and higher integral time.

4.2. Fragility analysis of MoReRT control for SOPDT models

The *robustness fragility plot* of the MoReRT controllers for SOPDT models in the particular case of $a = 0.50$ are shown in Fig. 9. As can be seen, in this case also the controllers are all robustness–nonfragile, except at high normalized dead-times ($\tau_o > 1.6$) when the robustness requirements are minimal ($M^t_S = 2.0$). None of the controllers is robustness-resilient.

For the same robustness level, the controllers become more fragile as τ_o increases. Moreover, the parametric robustness–fragility indices show that for the same model normalized dead-time the controller gain has more influence than the integral time on the loss of control system robustness.

The performance–fragility plots of the MoReRT controllers for SOPDT models with $a \in \{0.1, 0.5, 1.0\}$ are shown in Fig. 10 for a load–disturbance step change and in Fig. 11 for a set-point step change.

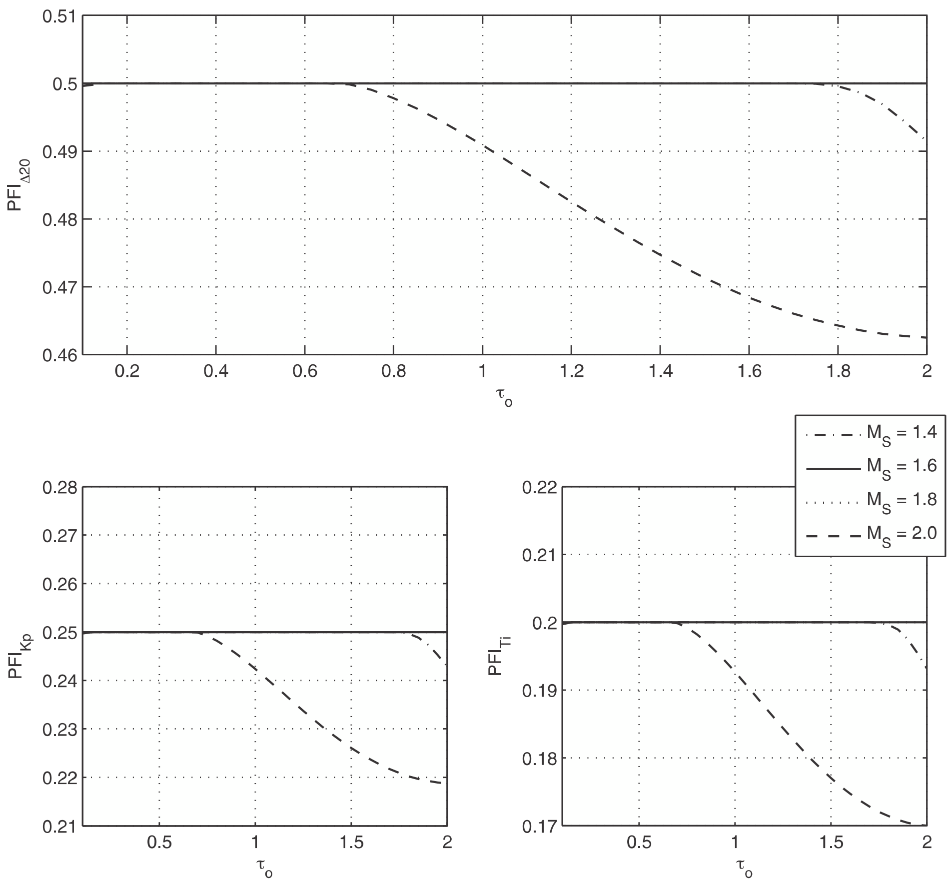

Fig. 10. Regulatory control performance–fragility of MoReRT controllers for SOPDT models with $a = 0.50$.

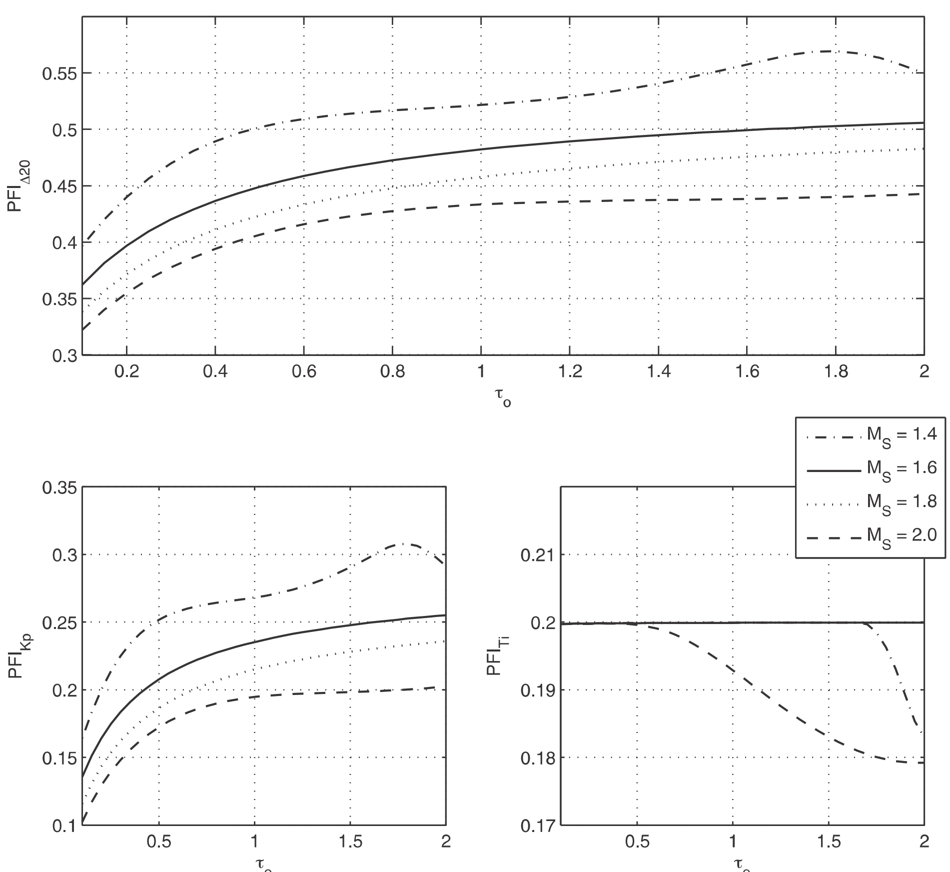

Fig. 11. Servo-control performance–fragility of MoReRT controllers for SOPDT models with $a = 0.50$.

Table 2
Example – $P_\alpha(s)$ FOPDT and SOPDT models.

α	K	T	a	L	τ_o
0.10	1	1.003	0	0.112	0.112
0.25	1	1.049	0	0.298	0.284
0.50	1	1.247	0	0.691	0.554
1.0	1	2.343	0	1.860	0.794
0.25	1	0.987	0.254	0.086	0.087
0.50	1	0.876	0.821	0.277	0.316
1.0	1	1.487	1.0	1.110	0.747

As in the FOPDT cases, the robustness and performance–fragility indices of the SOPDT models are given by the opposite corners of the controller delta-20 perturbed parameter matrix, thus reinforcing the existing trade-off between performance and robustness.

With the exception of controllers tuned for lower robustness ($M_S^t = 2.0$), and particularly for $a \geq 0.50$, the regulatory control performance–fragility indices of all controllers are $PFI_{d\Delta 20} = 0.50$, $PFI_{d\delta 20}^{Kp} = 0.25$ and $PFI_{d\delta 20}^{Ti} = 0.20$. These results may also be obtained analytically by considering that the regulatory control response target (15) is non-oscillatory. In such a case, the integrated absolute error (IAE) is equal to the integrated error (IE) and is given by the following [23]:

$$J_e \doteq \int_0^\infty \left| e(t) \right| \mathrm{d}t = \int_0^\infty e(t)\mathrm{d}t = \frac{T_i}{K_p}. \qquad (43)$$

Using (43) with (37) and (40) while considering a decrement in the controller gain ($\delta\epsilon_{K_p^-}$) and an increment in the controller integral time ($\delta\epsilon_{T_i^+}$), the delta–epsilon performance–fragility indices are obtained as follows:

$$PFI_{d\delta\epsilon}^{Kp} = \frac{\delta\epsilon_{K_p^-}}{1 - \delta\epsilon_{K_p^-}}, \quad PFI_{d\delta\epsilon}^{Ti} = \delta\epsilon_{T_i^+}, \qquad (44)$$

and

$$PFI_{d\Delta\epsilon} = \frac{\delta\epsilon_{K_p^-} + \delta\epsilon_{T_i^+}}{1 - \delta\epsilon_{K_p^-}}. \qquad (45)$$

Then, considering a 20% reduction in K_p and a 20% raise in T_i, from (44) and (45) there follows $PFI_{d\delta 20}^{Kp} = 0.25$, $PFI_{d\delta 20}^{Ti} = 0.20$, and $PFI_{d\Delta 20} = 0.50$.

This confirms that the regulatory control responses to a load–disturbance are non-oscillatory as designed.

5. Evaluation of the MoReRT tuning

The evaluation of the proposed MoReRT tuning and its comparison with other tuning rules will consider three main characteristics of the control system: its performance response to step changes in set-point and load–disturbance, its robustness to changes in the controlled process characteristics (model/plant mismatch), and its robustness– and performance–fragility to changes in the controller parameters.

Consider the fourth-order controlled processes proposed as a benchmark in [40] and given by the transfer function:

$$P_\alpha(s) = \frac{1}{\prod_{n=0}^{3}(\alpha^n s + 1)}, \qquad (46)$$

with $\alpha \in \{0.1, 0.25, 0.5, 1.0\}$.

Using the three-point identification procedure *123c* [26] FOPDT and SOPDT models were obtained whose parameters are listed in Table 2.

Table 3
Example – PI_2 from FOPDT models.

	M_S^t 1.4	1.6	1.8	2.0
$\alpha = 0.10$				
K_p	3.140	4.152	4.929	5.576
T_i	0.678	0.563	0.498	0.459
β	0.597	0.541	0.518	0.514
$\alpha = 0.50$				
K_p	0.725	0.976	1.175	1.336
T_i	1.445	1.458	1.438	1.413
β	0.935	0.765	0.682	0.635
$\alpha = 1.0$				
K_p	0.532	0.734	0.889	1.013
T_i	2.828	3.009	3.052	3.054
β	1.101	0.866	0.755	0.691

Tables 3 and 4 list the MoReRT controllers parameters for these models that result from application of the tuning Eqs. (32)–(34).

Because the MoReRT tuning equations for SOPDT models were deduced for models with time constant ratios $a \in \{0.1, 0.25, 0.50, 0.75, 1.0\}$, the controller parameters for the case with $\alpha = 0.25$ ($a = 0.254$) and $\alpha = 0.50$ ($a = 0.821$) were computed by linear interpolation between those corresponding to the two nearest a values.

For the evaluation of the control systems, the integrated absolute error J_e (38) cost functional will be used as a system performance measure and evaluated for step changes in the set-point, J_{er}, and load–disturbance, J_{ed}. The control signal total variation TV_u given by

$$TV_u \doteq \sum_{k=1}^{\infty} \left| u_{k+1} - u_k \right|, \qquad (47)$$

will be used as a measure of the smoothness of the control action for both input changes, TV_{ud} and TV_{ur}.

5.1. Regulatory control

Table 5 lists the control system robustness and the regulatory control performance obtained with the controllers tuned using the FOPDT models. The robustness levels (M_S) were obtained using the models while the performance indices were evaluated with the original controlled processes (46).

The robustness obtained with the controlled processes (M_S^p) (that cannot be obtained in a practical application) is included only to illustrate how well the models reproduce the dynamic characteristics of the processes under closed-loop control. One possible measure of how good a model is for control studies is related to how close M_S and M_S^p are.

Table 4
Example – PI_2 from SOPDT models.

	M_S^t 1.4	1.6	1.8	2.0
$\alpha = 0.25$				
K_p	1.690	2.366	2.937	3.445
T_i	1.088	1.015	0.953	0.911
β	0.664	0.592	0.555	0.537
$\alpha = 0.50$				
K_p	0.785	1.145	1.423	1.677
T_i	1.395	1.475	1.497	1.497
β	0.778	0.589	0.552	0.522
$\alpha = 1.0$				
K_p	0.482	0.731	0.917	1.065
T_i	2.494	2.882	3.038	3.117
β	0.891	0.684	0.606	0.566

Table 5
Example – PI_2 robustness, performance, fragility, FOPDT models.

M_S^t	1.4	1.6	1.8	2.0
$\alpha = 0.10$				
M_S	1.40	1.60	1.80	2.00
M_S^p	1.26	1.35	1.42	1.50
J_{ed}	0.216	0.136	0.101	0.082
TV_{ud}	1.171	1.312	1.419	1.514
$RFI_{\delta 20}^{Kp}$	0.073	0.110	0.147	0.188
$RFI_{\delta 20}^{Ti}$	0.022	0.037	0.052	0.067
$RFI_{\Delta 20}$	0.098	0.154	0.214	0.281
$\alpha = 0.50$				
M_S	1.40	1.60	1.80	2.00
M_S^p	1.35	1.50	1.63	1.78
J_{ed}	2.007	1.495	1.224	1.058
TV_{ud}	1.000	1.115	1.297	1.475
$RFI_{\delta 20}^{Kp}$	0.078	0.120	0.166	0.216
$RFI_{\delta 20}^{Ti}$	0.048	0.063	0.080	0.099
$RFI_{\Delta 20}$	0.139	0.206	0.285	0.376
$\alpha = 1.0$				
M_S	1.40	1.60	1.80	2.00
M_S^p	1.39	1.58	1.75	1.91
J_{ed}	5.313	4.099	3.433	3.112
TV_{ud}	1.000	1.123	1.343	1.556
$RFI_{\delta 20}^{Kp}$	0.074	0.115	0.156	0.201
$RFI_{\delta 20}^{Ti}$	0.089	0.099	0.116	0.135
$RFI_{\Delta 20}$	0.189	0.253	0.331	0.419

Table 6 lists the control system robustness and the regulatory control performance obtained with the controllers tuned using the SOPDT models.

It can be seen from Table 5 that the robustness targets were perfectly achieved with the MoReRT controllers for all the FOPDT models. Note also that the robustness eventually obtained with the processes will exceed the design robustness levels. The differences range from 1 to 5% for the $\alpha = 1.0$ case and from 10 to 25% with $\alpha = 0.1$. Although the process for $\alpha = 0.1$ is nearly first order while that for $\alpha = 1.0$ is fourth order with equal poles, and contrary to what might be expected, in this case the FOPDT model better represents the dynamic behavior of the processes under closed-loop control for higher α's, rather than for lower values.

Table 6
Example – PI_2 robustness, performance, fragility, SOPDT models.

M_S^t	1.4	1.6	1.8	2.0
$\alpha = 0.25$				
M_S	1.40	1.60	1.80	2.00
M_S^p	1.37	1.52	1.71	1.88
J_{ed}	0.644	0.429	0.325	0.264
TV_{ud}	1.091	1.330	1.568	1.797
$RFI_{\delta 20}^{Kp}$	0.062	0.084	0.114	0.136
$RFI_{\delta 20}^{Ti}$	0.049	0.064	0.088	0.104
$RFI_{\Delta 20}$	0.117	0.167	0.221	0.271
$\alpha = 0.50$				
M_S	1.40	1.60	1.80	2.00
M_S^p	1.40	1.59	1.80	2.00
J_{ed}	1.777	1.288	1.052	0.907
TV_{ud}	1.015	1.248	1.506	1.795
$RFI_{\delta 20}^{Kp}$	0.069	0.102	0.133	0.168
$RFI_{\delta 20}^{Ti}$	0.074	0.091	0.108	0.129
$RFI_{\Delta 20}$	0.161	0.221	0.282	0.356
$\alpha = 1.0$				
M_S	1.40	1.60	1.80	2.00
M_S^p	1.40	1.60	1.79	1.98
J_{ed}	5.173	3.942	3.315	3.051
TV_{ud}	1.000	1.151	1.395	1.642
$RFI_{\delta 20}^{Kp}$	0.074	0.115	0.156	0.201
$RFI_{\delta 20}^{Ti}$	0.089	0.099	0.116	0.135
$RFI_{\Delta 20}$	0.189	0.253	0.331	0.419

Table 7
Example – PI_2 servo-control performance, FOPDT models.

M_S^t	1.4	1.6	1.8	2.0
$\alpha = 0.10$				
J_{er}	0.489	0.394	0.341	0.305
TV_{ur}	1.435	2.131	2.839	3.562
$\alpha = 0.50$				
J_{er}	2.102	1.838	1.681	1.586
TV_{ur}	0.482	0.733	1.008	1.286
$\alpha = 1.0$				
J_{er}	5.029	4.502	4.191	4.111
TV_{ur}	0.502	0.719	0.974	1.241

From the standpoint of performance, the existence of a trade-off with robustness is clear. For a given model, if the control system robustness level is increased, the performance decreases, in that J_{ed} increases. On the controller output side, a robustness increment results in more smoothness, in that TV_{ud} decreases.

For the SOPDT models, Table 6 reveals not only the perfect achievement of the design robustness level with the MoReRT controllers for all models, but also a reduction in the gap between the robustness levels used in the design and those eventually realized with the processes. The differences are now lower than 1% for $\alpha = 1.0$ and no more than 6% with $\alpha = 0.25$.

The robustness fragility indices of the MoReRT controllers tuned with the FOPDT models are listed in Table 5, and those of the controllers tuned with the SOPDT models are listed in Table 6. From these data, it can be seen that the MoReRT controllers are all robustness–nonfragile and that their robustness fragility levels depend on the process dynamics (α), the model used (FOPDT or SOPDT), and the required robustness level (M_S^t). Note also that the controllers obtained from the SOPDT models are robustness–fragility-balanced controllers but the controllers from the FOPDT models are robustness–fragility-unbalanced controllers.

The regulatory control performance fragility analysis shows that, with the exception of the cases of $M_S^t = 2.0$ for processes with $\alpha \geq 0.75$, the MoReRT controllers, as expected, all have performance fragility indices $PFI_{d\Delta 20} = 0.50$ and $PFI_{d\delta 20}^{Kp} = 0.25$, $PFI_{d\delta 20}^{Ti} = 0.20$, due their non-oscillatory characteristics.

5.2. Servo-control

When a 2DoF controller is used, it is considered that the main function of the control system is to reject the load–disturbances, to maintain the regulatory control. Although the servo-control operation is not of primary importance, in the case of a set-point step change it is necessary to have a fast response with a smooth controller output.

Table 8
Example – PI_2 servo-control performance, SOPDT models.

M_S^t	1.4	1.6	1.8	2.0
$\alpha = 0.25$				
J_{er}	1.009	0.843	0.713	0.686
TV_{ur}	0.829	1.493	2.229	3.045
$\alpha = 0.50$				
J_{er}	2.087	1.894	1.723	1.612
TV_{ur}	0.557	0.879	1.263	1.728
$\alpha = 1.0$				
J_{er}	5.446	4.853	4.510	4.319
TV_{ur}	0.573	0.734	0.981	1.252

The servo-control performance obtained with the controllers tuned using the FOPDT models are listed in Table 7. From these data the trade-off between performance and robustness, J_{er} versus M_S, is evident for all α's considered. Note also that, as in the regulatory control case, the higher robustness systems have smooth controller outputs.

The servo-control performance obtained with the controllers tuned using the SOPDT models are shown in Table 8. From these data the trade-off between performance and robustness is also evident, in that J_{er} increases as the design robustness decreases (or M_S^t increases).

The servo-control performance fragility analysis shows that in this case the MoReRT controllers are less performance–fragile ($0.33 \leq PFI_{r\Delta 20} \leq 0.50$). The changes in the controller gain have less effect on the servo-control performance than on the regulatory control performance ($0.07 \leq PFI_{r\delta 20}^{Kp} \leq 0.26$), while the controller integral time has the same effect on all controllers ($PFI_{r\delta 20}^{Ti} = 0.20$).

Fig. 12 shows the MoReRT closed-loop responses to a 20% set-point step change followed by a 10% disturbance change for the controlled process with $\alpha = 0.50$.

Considering the above, for processes (46) with $\alpha \in \{0.25, 0.50, 1.0\}$ it is better to use the SOPDT models for the design of the 2DoF control system.

5.3. Performance, robustness, and fragility of other tuning methods

The same evaluation framework used with the MoReRT tuning will be used with the following PI tuning methods, which to some extent consider the control system robustness in the design procedure: the *kappa–tau* (KT) [23] that uses a closed-loop dominant pole design of 2DoF PI controllers for a batch of controlled processes and provides tuning relations for robustness levels of $M_S^t = 2.0$ and 1.4; the *simple control* (SIMC) [41] that is an IMC-based tuning for 1DoF PI controllers to obtain a good trade-off between speed of response, disturbance rejection, robustness ($M_S \approx 1.59$), and control effort requirements; the *approximated MIGO* (AMIGO) [34], which is based on the loop shaping MIGO method [42] that maximizes the controller integral gain for minimization of the integrated error for a step load–disturbance, subject to a robustness constraint ($M_S^t = 1.4$) for 2DoF PI controllers (in particular, the revised version of the AMIGO method in [4] will be used); and the *percent overshoot* (PO) method [43] that is an IMC-based tuning that uses a relation between the set-point step response overshoot and the closed-loop control system robustness to provide tuning relations for 1DoF PI controllers for tight control (POt) (10% overshoot, $M_S^t = 1.71$) and smooth control (POs) (0% overshoot, $M_S^t = 1.38$). All of these use a FOPDT model approximation for the controlled process. Controller parameters obtained with the above methods are listed in Table 9.

The robustness, the performance, the controller output variation and the robustness–fragility indices obtained with the KT method are listed in Table 10.

Note that with the KT method the design robustness is not obtained. In the $M_S^t = 1.4$ case the robustness ranges from 1.24 to 1.30 (1.28 average) and in the $M_S^t = 2.0$ case it ranges from 1.66 to 1.71 (average 1.69) with a consequent reduction in performance. For similar robustness levels, the KT values represent poorer performance than the MoReRT ones. They are also more robustness–fragile.

Table 11 lists the robustness, the performance, the controller output variation and the robustness–fragility indices obtained with the SIMC method.

With the SIMC method a constant robustness level of $M_S \approx 1.59$ is obtained. These controllers have a poorer regulatory control performance than the MoReRT ones (for $M_S^t = 1.6$) but a slightly better servo-control performance requiring higher changes in the control signal, especially for $\alpha = 0.1$.

The robustness, the performance, the controller output variation and the robustness–fragility obtained with the AMIGO method are listed in Table 12.

The average robustness of 1.25 obtained with the AMIGO method is very far from the 2.0 design value, which results in very poor performances and slow responses. Conversely, the robustness fragility is very low, yielding robustness-resilient controllers for low α's.

Table 9
Example – PI controller parameters.

α	0.1	0.50	1.0
KT, $M_S^t = 1.4$			
K_p	2.055	0.320	0.228
T_i	0.705	0.811	1.594
β	0.888	1.0	1.0
KT, $M_S^t = 2.0$			
K_p	4.901	0.673	0.489
T_i	0.705	0.811	1.594
β	0.474	0.549	0.569
SIMC			
K_p	4.464	0.903	0.630
T_i	0.896	1.247	2.343
β	1	1	1
AMIGO, $M_S^t = 2.0$			
K_p	2.475	0.368	0.280
T_i	0.639	1.159	2.270
β	0.0	0.0	0.0
POt, $M_S^t = 1.71$			
K_p	5.089	1.029	0.718
T_i	0.896	1.247	2.343
β	1	1	1

Table 10
Example – KT robustness, performance, fragility.

α	0.1	0.50	1.0
$M_S^t = 1.4$			
M_S	1.24	1.30	1.29
J_{ed}	0.344	2.600	6.991
TV_{ud}	1.114	1.034	1.000
J_{er}	0.502	2.653	6.991
TV_{ur}	1.199	0.742	0.772
$RFI_{\delta 20}^{Kp}$	0.044	0.046	0.049
$RFI_{\delta 20}^{Ti}$	0.019	0.096	0.093
$RFI_{\Delta 20}$	0.064	0.162	0.165
$M_S^t = 2.0$			
M_S	1.71	1.66	1.71
J_{ed}	0.144	1.561	4.375
TV_{ud}	1.247	1.454	1.471
J_{er}	0.515	2.162	5.403
TV_{ur}	2.040	1.390	1.341
$RFI_{\delta 20}^{Kp}$	0.138	0.105	0.121
$RFI_{\delta 20}^{Ti}$	0.026	0.186	0.211
$RFI_{\Delta 20}$	0.172	0.356	0.425

Table 11
Example – SIMC robustness, performance, fragility.

α	0.1	0.50	1.0
M_S	1.60	1.59	1.59
J_{ed}	0.201	1.381	3.823
TV_{ud}	1.124	1.177	1.198
J_{er}	0.268	1.589	4.223
TV_{ur}	4.014	0.915	0.855
$RFI_{\delta 20}^{Kp}$	0.116	0.115	0.115
$RFI_{\delta 20}^{Ti}$	0.016	0.088	0.121
$RFI_{\Delta 20}$	0.136	0.233	0.282

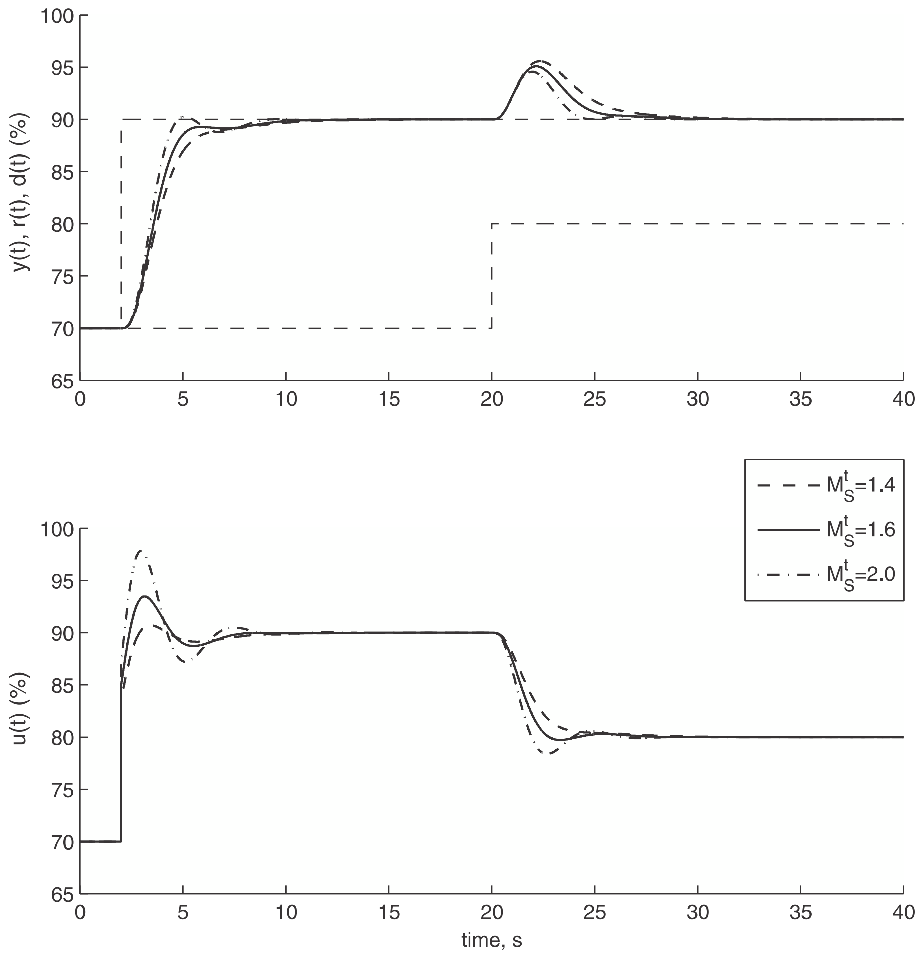

Fig. 12. MoReRT closed-loop responses for $\alpha = 0.50$ process.

Table 12
Example – AMIGO ($M_S^t = 2.0$) robustness, performance, fragility.

α	0.1	0.50	1.0
M_S	1.31	1.21	1.23
J_{ed}	0.259	3.150	8.094
TV_{ud}	1.172	1.000	0.999
J_{er}	0.899	4.309	10.372
TV_{ur}	1.687	1.000	1.000
$RFI^{Kp}_{\delta 20}$	0.055	0.039	0.042
$RFI^{Ti}_{\delta 20}$	0.025	0.055	0.062
$RFI_{\Delta 20}$	0.081	0.103	0.117

The robustness, the performances, the controller output variation and the robustness–fragility of the POt method are listed in Table 13.

Fig. 13 shows the closed-loop responses to a 20% set-point step change followed by a 10% disturbance step-change as obtained with several of the above tuning methods for the controlled process with $\alpha = 0.50$. These time responses can be compared with those obtained for the same process with the MoReRT tuning, as shown in Fig. 12, to confirm the advantages of using the proposed MoReRT tuning over other methods.

Table 13
Example – POt robustness, performance, fragility.

α	0.1	0.50	1.0
M_S	1.72	1.72	1.71
J_{ed}	0.176	1.220	3.567
TV_{ud}	1.167	1.295	1.355
J_{er}	0.250	1.557	4.210
TV_{ur}	4.908	1.178	1.087
$RFI^{Kp}_{\delta 20}$	0.143	0.142	0.142
$RFI_{\delta 20 T_i}$	0.018	0.100	0.140
$RFI_{\Delta 20}$	0.167	0.283	0.346

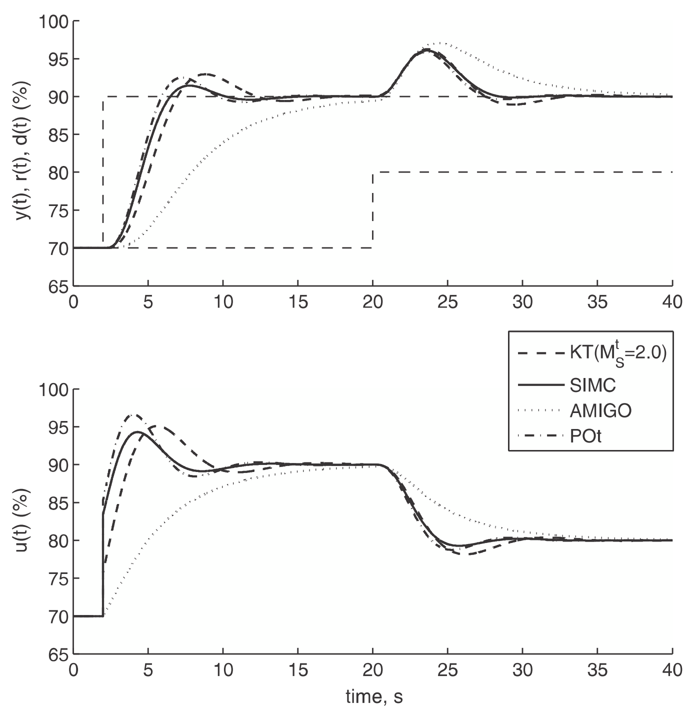

Fig. 13. Closed-loop responses for $\alpha = 0.50$ process.

6. Conclusions

The proposed MoReRT tuning method for 2DoF proportional integral (PI_2) controllers guarantees the design robustness level for first- and over damped second-order plus dead-time processes using only one design parameter, which is the required closed-loop control system robustness as measured with the maximum sensitivity M_S.

Tuning equations were obtained for $M_S \in \{1.4, 1.6, 1.8, 2.0\}$, allowing the designer to select the required robustness level by taking into account the expected variations in the process parameters. The models considered include six time constants ratios $a \in \{0.0, 0.1, 0.25, 0.50, 0.75, 1.0\}$ and normalized dead-times in the range from 0.1 to 2.0.

The robustness and performance fragility analyses presented for the controller provide useful information to perform with confidence, if required, a last fine-tuning of the control system.

The proposed tuning method provides a unified framework under which to design robust 2DoF PI control systems for over damped processes. For a given robustness, this yields performance similar or superior to that of other available tuning methods, but without their restrictions on the model used or the accomplishment of the design parameter.

Acknowledgments

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Appendix A.

MoReRT tuning rule constants for four robustness levels.

Table 14
MoReRT constants, FOPDT models ($a = 0.0$).

M_S^t	1.4	1.6	1.8	2.0
a_0	0.7253	0.4441	0.5249	0.5930
a_1	0.6505	0.1745	0.2281	0.2658
a_2	0.002337	0	0	0
a_3	2.143	1	1	1
a_4	1	0	0	0
a_5	0	0	0	0
b_0	−0.1606	−0.09742	0.1530	0.6088
b_1	47.67	83.72	115.5	154.9
b_2	4.166	10.71	18.67	29.32
b_3	30.23	51.35	68.28	88.39
b_4	7.973	3.948	−0.4553	−4.346
b_5	−4.738	−5.369	−4.952	−4.659
b_6	1	1	1	1
c_0	0.5049	0.4759	0.4706	0.4758
c_1	0.8330	0.5924	0.4360	0.3267
c_2	−0.1034	−0.1278	−0.09808	−0.07063
c_3	0	0	0	0

Table 15
MoReRT constants, SOPDT models with $a = 0.10$.

M_S^t	1.4	1.6	1.8	2.0
a_0	4.264	5.026	10.54	12.28
a_1	3.008	2.912	6.250	7.795
a_2	0.7672	0.6431	1.058	1.017
a_3	13.52	11.99	21.47	22.57
a_4	2.816	1	1	1
a_5	1	0	0	0
b_0	2.268	75.12	17.21	11.33
b_1	39.41	1426	265.2	151.1
b_2	3.965	165.9	41.16	28.29
b_3	27.77	1028	178.6	96.67
b_4	5.123	−110.40	−25.83	−16.01
M_S^t	1.4	1.6	1.8	2.0
b_5	−3.507	1	1	1
b_6	1	0	0	0
c_0	0.5565	0.5243	0.5123	0.5139
c_1	0.9507	0.6265	0.4547	0.3259
c_2	−0.3226	−0.2313	−0.1689	−0.1036
c_3	0.0872	0.03721	0.02538	0.01162

Table 16
MoReRT constants, SOPDT models with $a = 0.25$.

M_S^t	1.4	1.6	1.8	2.0
a_0	2.533	6.240	16.12	14.67
a_1	−0.1547	3.418	9.223	9.476
a_2	0.8599	1.441	2.857	2.084
a_3	7.432	15.02	33.12	27.52
a_4	−2.820	1	1	1
a_5	1	0	0	0
b_0	2.166	154.5	17.72	10.72
b_1	11.19	2042	203.4	109.2
b_2	2.230	196.9	24.89	15.86
b_3	6.897	1480	139.4	72.02
b_4	4.012	−152.1	−20.97	−12.87
b_5	−3.089	1	1	1
b_6	1	0	0	0
c_0	0.5796	0.5406	0.5151	0.5057
c_1	1.024	0.6162	0.4748	0.3758
c_2	−0.4927	−0.2497	−0.2081	−0.1633
c_3	0.1773	0.04321	0.03662	0.02808

Table 17
MoReRT constants, SOPDT models with $a = 0.50$.

M_S^t	1.4	1.6	1.8	2.0
a_0	3.998	5.072	31.07	13.96
a_1	−1.784	2.772	15.29	8.546
a_2	1.974	1.588	7.564	2.664
a_3	9.781	11.72	58.82	24.52
a_4	−6.350	1	1	1
a_5	1	0	0	0
b_0	16.33	188.6	174.4	33.74
b_1	−7.025	2668	1767	314.9
b_2	12.46	174.9	173.0	35.50
b_3	−7.889	1779	1096	187.3
b_4	5.904	−144.1	−128.5	−26.56
b_5	−4.141	1	1	1
b_6	1	0	0	0
c_0	0.4262	0.5252	0.4937	0.4777
c_1	1.994	0.5520	0.4335	0.3619
c_2	−2.060	−0.2216	−0.1896	−0.1616
c_3	0.8367	0.03796	0.0330	0.02835

Table 18
MoReRT constants, SOPDT models with $a = 0.75$.

M_S^t	1.4	1.6	1.8	2.0
a_0	5.774	13.09	780.7	1586
a_1	−2.612	4.900	304.2	671.0
a_2	3.256	4.764	214.0	350.6
a_3	12.27	25.71	1290	2340
a_4	−7.671	1	1	1
a_5	1	0	0	0
b_0	20.03	435.4	136.7	225.6
b_1	−8.585	4154	1262	1593
b_2	13.27	323.1	111.8	190.7
b_3	−7.615	2425	693.7	820.4
b_4	5.483	−144.7	−70.57	−92.06
b_5	−4.049	1	1	1
b_6	1	0	0	0
c_0	0.4223	0.4967	0.4631	0.4472
c_1	1.705	0.4609	0.3698	0.3115
c_2	−1.759	−0.1704	−0.1510	−0.1286
c_3	0.7198	0.02748	0.02498	0.0231

Table 19
MoReRT constants, SOPDT models with $a = 1.0$.

M_S^t	1.4	1.6	1.8	2.0
a_0	7.163	26.71	18.65	521.4
a_1	−2.794	8.032	7.737	199.0
a_2	4.118	10.02	5.215	117.7
a_3	13.68	45.76	27.97	684.5
a_4	−7.551	1	1	1
a_5	1	0	0	0
b_0	24.23	778.0	422.3	531.5
b_1	−7.143	41.60	261.7	3139
b_2	14.18	490.5	292.4	390.7
b_3	−6.404	2093	1242	1414
b_4	5.820	−88.86	−102.4	−135.1
b_5	−4.059	1	1	1
b_6	1	0	0	0
c_0	0.4986	0.4617	0.4307	0.4155
c_1	0.7797	0.3840	0.3130	0.2716
c_2	−0.4881	−0.1314	−0.1198	−0.1078
c_3	0.1978	0.02011	0.01928	0.01791

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Chapter 12
Fragility Evaluation of PI and PID Controllers Tuning Rules

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12.1 Introduction

The introduction in 1940 of the first commercially available *Proportional Integral Derivative* (PID) controller, the Taylor Fulscope 100 [18, 19], motivated Ziegler and Nichols to present in 1942 their well-known tuning rules [73]. Since that date, a great number of other tuning procedures have been developed for the PID controller and its variations, as revealed in O'Dwyer's handbook [54].

At the beginning, only the control system *performance* was taken into account in the controller design, considering a step change either in the set-point, *servo-control* operation, or in the load-disturbance, *regulatory control* operation, as in the classic tuning rules of Cohen and Coon [23], López et al. [48], and Rovira et al. [58], among others [21, 22, 38, 50, 57, 65], for One-Degree-of-Freedom (1DoF) PI and PID controllers.

Later, the consideration of the control system relative stability, its *robustness* to the changes in the controlled process characteristics, was introduced into the controller design, considering first the control-loop gain and phase margins (A_m, ϕ_m) as in [11, 27, 35, 36, 44, 47, 72]. More recently, these classic indicators of robustness have been replaced by a single value given by the maximum of the magnitude of the sensitivity function, denoted by M_S. This approach has been used in [5, 7, 12, 13, 17, 31, 46, 56, 65, 66].

The implementation of the Two-Degree-of-Freedom (2DoF) PID controllers, proposed by Araki [9, 10], allowed the separation of the control system design in

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two steps, considering in the first step, the control system stability and the regulatory control performance and in the second, the servo-control performance. See [3, 6, 16, 17, 30, 32, 64] and the references therein.

Up to this point, it is clear that a design procedure of a control system with PID controllers must make use of the capabilities provided by the 2DoF controllers, which must take into consideration several conflicting specifications: on the one hand, *performance*, or the response to the set-point and load-disturbance changes; and on the other hand, the system relative stability, or the *robustness* to the changes in the controlled process dynamics. The control signal variation and extreme values must also be taken into account. Therefore, the controller design is really a multi-objective problem [33, 41].

There is, however, another consideration that must be taken into account in the control system design process: the effect of the variation of the controller parameters over the control system stability and performance, known as the *controller fragility*. If the control system robustness is an indication of the margin of variation of the process characteristics with a fixed controller, then the controller fragility has a similar meaning but in terms of the variation of the controller parameters considering a fixed controlled process.

The fragility of certain controllers was documented by Keel and Battacharyya [45]. They found that many modern design techniques for optimum and robust controllers under the H_2, H_∞, and l_1 norms would produce extremely fragile, high-order controllers. They observed that in some cases, minimum variations of the parameters of these controllers would make the system unstable. A fragility analysis was included in the PID controller design by Datta et al. [25], Ho [34], and Silva et al. [62].

Although, in control system designs, the assumption is often made that the controller can be implemented exactly, a certain degree of uncertainty inevitably exists in the controller implementation. The controller fragility is affected by the tolerances of its analog components. In its digital version, there are inaccuracies because of the use of fixed-length words and rounded errors of numerical calculations [40, 69]. In addition, the controller must allow variations of its parameters around their design values, making it easy to *fine-tune* the controller when the control loop is placed in service. The latter is the most probable cause of major variations in the controller parameters from their design, or nominal, values. Effectively, most of the tuning approaches, either based on tuning rules or on optimization methods, provide precise values for the controller parameters, but due to the inaccuracies associated with the controlled process model used as part of the tuning procedure, normally these parameters should be taken only as a first approximation, and such final fine-tuning of the controller is normally required.

Considering the above, modern tuning rules for PI and PID controllers must take into account issues such as, the closed-loop servo- and regulatory control *performance*, the *control effort* requirements, the control system *robustness*, and the controller *fragility*.

The chapter is organized as follows. Section 12.2 introduces the controller fragility concept and its treatment in the control systems literature. Section 12.3

presents the framework used for its analysis. Section 12.4 states the fragility indices. In Sect. 12.5, the use of fragility graphic tools is introduced, and the fragility of several tuning rules is evaluated. The chapter ends with some conclusions and suggestions for future work.

12.2 Early Work on Controller Fragility

As indicated above, the fragility of high-order controllers designed with optimal and robust techniques was pointed out by Kell and Battacharyya [45]. As a measure of the controller fragility, they utilized the ratio of the l_2-norm of the perturbation vector of the controller transfer function coefficients that made the system unstable to the l_2-norm of its nominal parameter vector. If the parameter perturbations making the system unstable are "small", then the controller is considered *fragile*. Their fragility index is really a parametric stability margin measure around the nominal parameters of the designed controller. However, and as indicated by Mäkilä [49] and Paattilammi and Mäkilä [55] (see references therein), the high-order controllers fragility is not only a result of the optimization procedure used for their synthesis but also a result of the controllers implementation.

The "controller fragility" term is related more to high-order controller design [1, 26, 43] and realization [51, 52, 69] than with PID controllers. However, there are problems with the digital implementation of PID controllers with fixed-point arithmetic due to the Finite-Word-Length (FWL) effect, addressed in [41, 70].

The stability radii as measurements of the controller fragility were introduced by Whidborne [69]. If the controller nominal (design) parameters are $\bar{\theta}_c^o$ and $\Delta\bar{\theta}_c$ are the controller parameter perturbations, the l_2-norm controller *stability radius*, r_s, is the radius of the largest ball centered at $\bar{\theta}_c^o$ inside the controller parametric stability space and is given by the following:

$$r_s \doteq \max_{\Delta}\left\{\left\|\Delta\bar{\theta}_c\right\|_2 : \text{closed-loop control system is stable}\right\}. \tag{12.1}$$

Based on the results in [25], Ho [34], Ho et al. [37], and Xu [71] suggested a design method to obtain a robust ("non-fragile") PI (PID) controller by choosing the controller nominal parameters, $\bar{\theta}_c^o$, as the center of the circle (sphere) with the largest radius inscribed inside the controller stabilizing plane (three-dimensional space) for the particular controlled process model under consideration. This radius is the maximum l_2-norm parametric stability margin with respect to the perturbations of the controller parameters and the same as stated by (12.1). The particular case of the First-Order-Plus-Dead-Time (FOPDT) models is considered by Silva et al. [60]. It is important to note that this design method does not take into account any other design criteria, such as the control system performance with changes in the set-point or load-disturbances, the controller output requirements, or the control system robustness to changes in the controlled process characteristics.

To include some type of time-domain performance specifications into the design of non-fragile controllers, Silva et al. [62] considered only the controller parameter

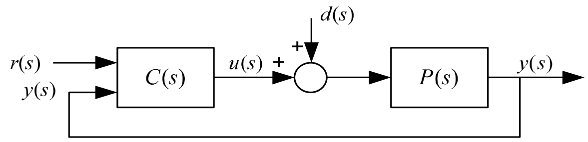

Fig. 12.1 Closed-loop control system

sets that lie inside a box of an arbitrary size defined in the parameter stabilizing space to mitigate to some extent the controller fragility problem and to perform a search inside this box analyzing the servo-control transient responses to select the controller parameter set that meets, or approximately meets, the performance specification. However, they do not provide any criteria for the selection of the box size, the allowed range of variation of the controller parameters, or a quantitative measure to establish when a controller can be considered non-fragile.

The concept of fragility of the PID controllers in the above-cited references is related to the minor variation of its parameters that make the control system unstable; therefore, it is more a stability margin measure. If we consider that a modern PID controller design method must take into account the closed-loop performance to changes in its inputs, set-point and load-disturbance, and its robustness to changes in the controlled process characteristics, then it is evident that from the designer point of view, it is very important that these characteristics be preserved when fine-tuning the controller. In addition, if this is not possible, then there should be at least some sort of information of how such changes in the controller parameters affect the control system robustness and performance. Taking this into account, the use of the PID controller fragility definition and tools introduced by Alfaro et al. [2, 4] are considered more suitable for the fragility analysis of the PI and PID controller tuning rules, as a measure of the control system loss of robustness and/or performance when the controller parameters change.

12.3 Problem Formulation and Framework

Consider the closed-loop control system of Fig. 12.1, where $P(s)$ and $C(s)$ are the *controlled process model* and the *controller* transfer function, respectively. In this system, $r(s)$ is the *set-point*, $u(s)$ is the *controller output signal*, $d(s)$ is the *load-disturbance*, and $y(s)$ is the *controlled process variable*. The parameters of the controlled process model transfer function, $P(s)$, will be considered constant for the fragility analysis.

We will consider the controlled processes represented by the First- or Second-Order-Plus-Dead-Time (FOPDT, SOPDT) models given by the transfer function:

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{(Ts+1)(aTs+1)}, \qquad \tau_o = L/T, \tag{12.2}$$

were K is the process static *gain*, T is the main *time constant*, a the *ratio of its two time constants* ($0 \leq a \leq 1.0$), L is the *dead-time*, and τ_o is the *normalized dead-time*.

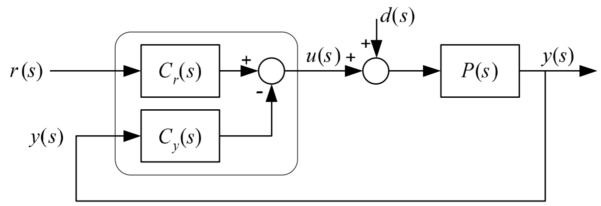

Fig. 12.2 Control system with a two-degree-of-freedom controller

Without the loss of generality, it is supposed that the controller is a Standard Two-Degree-of-Freedom PID (PID_2) controller [68] whose output is as follows:

$$u(t) = K_p \left\{ \beta r(t) - y(t) + \frac{1}{T_i} \int_0^t \left[r(\tau) - y(\tau) \right] \mathrm{d}\tau + T_d \frac{\mathrm{d}[\gamma r(t) - y(t)]}{\mathrm{d}t} \right\}, \quad (12.3)$$

or

$$u(s) = K_p \left\{ \beta r(s) - y(s) + \frac{1}{T_i s} \left[r(s) - y(s) \right] + \frac{T_d s}{\alpha T_d s + 1} \left[\gamma r(s) - y(s) \right] \right\}, \quad (12.4)$$

where K_p is the controller *proportional gain*, T_i is the *integral time constant*, T_d is the *derivative time constant*, β is the *proportional set-point weight*, and γ is the *derivative set-point weight*. In (12.4), α is the *derivative filter constant*, usually $\alpha = 0.10$ [24].

For the analysis, not for the implementation, the PID_2 controller output (12.4) will be rewritten as follows:

$$u(s) = K_p \left(\beta + \frac{1}{T_i s} + \frac{\gamma T_d s}{\alpha T_d s + 1} \right) r(s) - K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\alpha T_d s + 1} \right) y(s), \quad (12.5)$$

and in the compact form shown in Fig. 12.2 as follows:

$$u(s) = C_r(s) r(s) - C_y(s) y(s), \quad (12.6)$$

where

$$C_r(s) = K_p \left(\beta + \frac{1}{T_i s} + \frac{\gamma T_d s}{\alpha T_d s + 1} \right) \quad (12.7)$$

is the part of the PID_2 controller applied to r, the *set-point controller* transfer function, and

$$C_y(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\alpha T_d s + 1} \right) \quad (12.8)$$

is the part of the PID_2 controller applied to y, the *feedback controller* transfer function.

The closed-loop control system output, $y(s)$, to a change in its inputs, $r(s)$ and $d(s)$, is given by the following:

$$y(s) = \frac{C_r(s) P(s)}{1 + C_y(s) P(s)} r(s) + \frac{P(s)}{1 + C_y(s) P(s)} d(s), \quad (12.9)$$

and the closed-loop characteristic polynomial is as follows:

$$p(s) \doteq 1 + L(s) = 1 + C_y(s) P(s). \quad (12.10)$$

The control system stability depends on the controlled process model $P(s)$, with parameters $\bar{\theta}_p$, and on the feedback controller $C_y(s)$, with parameters $\bar{\theta}_{cy} = \{K_p, T_i, T_d\}$; hence, it is not affected by the controller set-point weights, β and γ.

12.3.1 Control System Robustness Evaluation

There are several quantitative measures of the control system *relative stability* that may be used for the robustness fragility definition, such as the classical *Gain Margin* and *Phase Margin* (A_m, ϕ_m) [29], that provide an indication of the distance from the open-loop transfer function, $L(j\omega)$, frequency response, or Nyquist curve, to the critical point $(-1, 0)$ on the open-loop polar graph. There is also the parametric *Gain Ratio* and the *Delay Ratio* of the *robustness plot* of Gerry and Hansen [28], which defines a parametric robustness region.

Another way to express the system robustness is by using the *Stability Margin*, which is the shortest distance from the Nyquist curve to the critical point [15]. This distance is the reciprocal of the maximum peak of the sensitivity function, or *Maximum Sensitivity* (M_S) [12], defined as follows:

$$M_S \doteq \max_{\omega} \left| S(j\omega) \right| = \max_{\omega} \left| \frac{1}{1 + C_y(j\omega)P(j\omega)} \right|. \tag{12.11}$$

The use of the maximum sensitivity as a robustness measure has the advantage that lower bounds to the gain and phase margins can be assured according to the following [12]:

$$A_m > \frac{M_S}{M_S - 1}, \qquad \phi_m > 2\sin^{-1}\left(\frac{1}{2M_S}\right). \tag{12.12}$$

The relations in (12.12) can be obtained from Fig. 12.3.

For the controller robustness fragility definitions, we will use the maximum sensitivity, M_S, as the indication of the closed-loop control system robustness.

12.3.2 Control System Performance Evaluation

The performance of the closed-loop control system may be evaluated with diverse indices such as those related to the integrated error, the difference between the controlled variable set-point and its real value, IAE, ITAE, ISE, and ITSE, and is given in general by the following:

$$J_{eg} \doteq \int_0^{\infty} t^m \left| e(t) \right|^n \mathrm{d}t, \tag{12.13}$$

or with other characteristics of the time response to a set-point or a load-disturbance change such as the overshoot, rise- or settling-time, and peak error, or decay ratio.

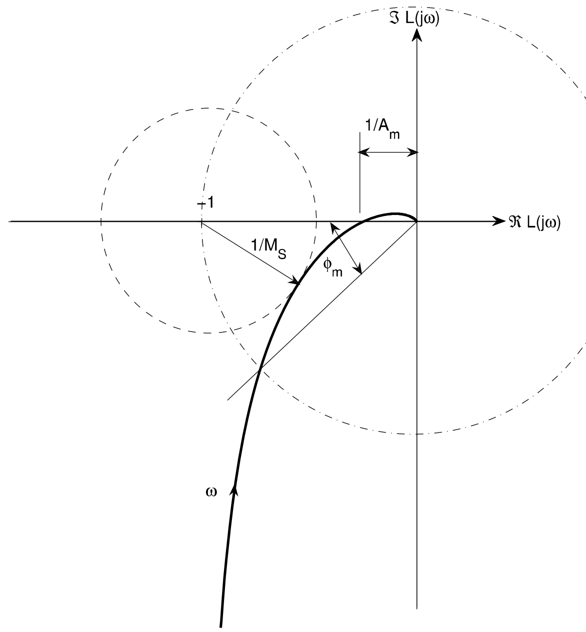

Fig. 12.3 Definition of the control system relative stability margins

For the controller performance fragility evaluation, it is desirable to use a performance indicator that takes into account economic considerations, or an economic performance measure, as does the integrated error (IE) [59]. Taking this into account, to avoid the cancellation of positive and negative errors, we will select, as the control system performance measure, the integrated absolute error, $m = 0$, $n = 1$ in (12.13); given by the following:

$$J_e \doteq \int_0^{\infty} \left|e(t)\right| \mathrm{d}t = \int_0^{\infty} \left|r(t) - y(t)\right| \mathrm{d}t. \tag{12.14}$$

12.4 Delta Epsilon Fragility Indices

The concept of PID controllers fragility, the Delta-Epsilon-Fragility Index $\mathrm{FI}_{\Delta\varepsilon}$ and their application to define when a controller is considered *fragile*, *non-fragile* or *resilient*, introduced in [2], are related to the closed-loop control system loss of robustness when the controller parameters are perturbed. These will be extended here to include the control system loss of performance. Then, in our context, the *PID controller fragility* is an indication of the reduction of the closed-loop control system robustness and/or performance when the controller parameters are perturbed, and not a measure of the system stability margin in the controller parameters space.

For the fragility analysis, the controlled process will be represented by a nominal model of the fixed parameters, $\bar{\theta}_p^o$, obtained at the control system normal operation point. This model is used for tuning the controller; then, the controller nominal parameters are $\bar{\theta}_c^o$ and their delta epsilon perturbations, $\delta\varepsilon$. In the following, $\delta\varepsilon$ denotes the variation of each individual controller parameter and $\Delta\varepsilon$ that all controller parameters are perturbed.

12.4.1 Robustness Fragility

The controller *Delta-Epsilon-Robustness-Fragility Index* relates the control system loss of robustness to its nominal robustness and is given by the following:

$$\mathrm{RFI}_{\Delta\varepsilon} \doteq \frac{M_{S\Delta\varepsilon}^m}{M_S^o} - 1 = \frac{\max\{M_S((1 \pm \delta\varepsilon)\bar{\theta}_c^o)\}}{M_S(\bar{\theta}_c^o)} - 1, \tag{12.15}$$

where $M_{S\Delta\varepsilon}^m$ and M_S^o are the control system extreme and the nominal maximum sensitivity, respectively.

The *extreme maximum sensitivity*, $M_{S\Delta\varepsilon}^m$, represents the highest loss of robustness of the control system when all the parameters, $\bar{\theta}_c$, of the controller have been perturbed by the same $\delta\varepsilon$ amount from their nominal values, $\bar{\theta}_c^o$, considering all the possible combinations of the perturbed parameters.

In the ideal case, for a completely delta epsilon robustness-resilient (or absolutely robustness-non-fragile) controller, $\mathrm{RFI}_{\Delta\varepsilon} = 0$, the controller would not lose robustness when its nominal parameters, $\bar{\theta}_c^o$, are perturbed by $\delta\varepsilon$.

The relative influence of a $\delta\varepsilon$ change in the controller parameter p_i over its robustness fragility can be obtained with the *Parametric-Delta-Epsilon-Robustness-Fragility Index* given by the following:

$$\mathrm{RFI}_{\delta\varepsilon}^{p_i} \doteq \frac{M_{S\delta\varepsilon}^{p_i}}{M_S^o} - 1 = \frac{\max\{M_S((1 \pm \delta\varepsilon)p_i, \bar{\theta}_c^o)\}}{M_S(\bar{\theta}_c^o)} - 1. \tag{12.16}$$

The final *fine-tuning* of the control-loop is considered the most probable cause of major variations in the controller parameters, for example, it is possible to see commissioning changes up to 10% or 20% in their values. Considering this, the *Delta 20 Robustness-Fragility Index* can be defined to measure the maximum loss of the control system robustness when a change of up to 20% occurs in one or more of the nominal controller parameters values and is given by the following:

$$\mathrm{RFI}_{\Delta 20} \doteq \frac{M_{S\Delta 20}^m}{M_S^o} - 1. \tag{12.17}$$

Based on the $\mathrm{RFI}_{\Delta 20}$, the controller robustness fragility degree is defined as follows:

Controller Robustness Fragility Definitions

- *Robustness Fragile PID controller*: a PID controller is robustness-fragile if its delta 20 robustness fragility index is higher than 0.50, $\mathrm{RFI}_{\Delta 20} > 0.50$.
- *Robustness Non-Fragile PID controller*: a PID controller is robustness-non-fragile if its delta 20 robustness fragility index is less than or equal to 0.50, $\mathrm{RFI}_{\Delta 20} \leq 0.50$.
- *Robustness Resilient PID controller*: a PID controller is robustness-resilient if its delta 20 robustness fragility index is less than or equal to 0.10, $\mathrm{RFI}_{\Delta 20} \leq 0.10$.

A controller will be considered *robustness-fragile* if the control system loses more than 50% of its robustness when all its parameters change up to 20%; otherwise, it is *robustness-non-fragile*. In addition, a controller will be *robustness-resilient* if the control system does not lose more than 10% of its robustness when its parameters change up to 20%. A controller with a low robustness-fragility will allow final fine-tuning without a significant reduction in the control system robustness.

The selection of a ±20% ($\Delta 20$) change in the controller parameters for the robustness fragility definition above considers a 10% reduction in the control system robustness as marginal and a 50% reduction as the maximum allowed limit because it will turn a highly robust system, with M_S lower than 1.4, into one with a minimally acceptable robustness, M_S of approximately 2.0. Although, using (12.15) and (12.16) it is possible to evaluate the effect of any other particular $\delta\varepsilon$ perturbation in one or more controller parameters.

12.4.2 Performance Fragility

The controller *Delta-Epsilon-Performance-Fragility Index* relates the control system loss of performance to its nominal performance and is given by the following:

$$\mathrm{PFI}_{\Delta\varepsilon} \doteq \frac{J^m_{e\Delta\varepsilon}}{J^o_e} - 1 = \frac{\max\{J_e(1 \pm \delta\varepsilon)\bar{\theta}^o_c\}}{J_e(\bar{\theta}^o_c)} - 1, \tag{12.18}$$

were $J^m_{e\Delta\varepsilon}$ and J^o_e are the extreme and the nominal performance, respectively, measured by the integrated absolute error (12.14).

The relative influence of a $\delta\varepsilon$ change in the controller parameter p_i over its performance fragility can be obtained with the *Parametric Delta-Epsilon-Performance-Fragility Index*, given by the following:

$$\mathrm{PFI}^{p_i}_{\delta\varepsilon} \doteq \frac{J^{p_i}_{e\delta\varepsilon}}{J^o_e} - 1 = \frac{\max\{J_e((1 \pm \delta\varepsilon)p_i, \bar{\theta}^o_c)\}}{J_e(\bar{\theta}^o_c)} - 1. \tag{12.19}$$

Considering the same 20% change in the controller parameters used in the robustness fragility definitions above, the *Delta 20 Performance-Fragility Index* could define the maximum loss of the control system performance when a change of up to 20% occurs in one or more of the nominal controller parameters values given by the following:

$$\mathrm{PFI}_{\Delta 20} \doteq \frac{J^m_{e\Delta 20}}{J^o_e} - 1. \tag{12.20}$$

Based on the $\mathrm{PFI}_{\Delta 20}$, the controller performance fragility degree is defined as follows:

Controller Performance Fragility Definitions

- *Performance Fragile PID controller*: a PID controller is performance-fragile if its delta 20 performance fragility index is higher than 0.50, $\mathrm{PFI}_{\Delta 20} > 0.50$.
- *Performance Non-Fragile PID controller*: a PID controller is performance-non-fragile if its delta 20 performance fragility index is less than or equal to 0.50, $\mathrm{PFI}_{\Delta 20} \leq 0.50$.
- *Performance Resilient PID controller*: a PID controller is performance-resilient if its delta 20 performance fragility index is less than or equal to 0.10, $\mathrm{PFI}_{\Delta 20} \leq 0.10$.

A controller will be considered *performance-fragile* if the control system loses more than 50% of its performance when all its parameters change up to 20%; otherwise, it is *performance-non-fragile*. In addition, a controller will be *performance-resilient* if the control system does not lose more than 10% of its performance when its parameters change up to 20%.

The controller performance fragility must be evaluated for the servo-control response, $\mathrm{PFI}_{r\Delta 20}$, and for the regulatory control response, $\mathrm{PFI}_{d\Delta 20}$.

In a similar way as was indicated for the robustness-fragility evaluation, using now (12.18) and (12.19), the controller performance-fragility may be evaluated for any other $\delta\varepsilon$ perturbation in one or more controller parameters.

12.4.3 Fragility Balance

To define when a controller is or is not a robustness- or performance-fragility-balanced controller, we must obtain first its *average parametric delta-epsilon-robustness-fragility index*:

$$\mathrm{RFI}^a_{\delta\varepsilon} \doteq \frac{1}{n}\sum_{i=1}^{n} \mathrm{RFI}^{p_i}_{\delta\varepsilon}, \tag{12.21}$$

and its *average parametric delta-epsilon-performance-fragility index*:

$$\mathrm{PFI}^{a}_{\delta\varepsilon} \doteq \frac{1}{n}\sum_{i=1}^{n}\mathrm{PFI}^{p_i}_{\delta\varepsilon}, \tag{12.22}$$

where the number of parameters is two for a PI ($n = 2$), and three for a PID ($n = 3$).

Based on the parametric delta-epsilon-fragility indices, the controller fragility balance is defined as follows:

Controller Fragility Balance Definitions

- *Robustness-Fragility-Balanced PID controller*: a robustness-fragility-balanced PID controller is one in which all its parametric robustness delta-epsilon-fragility indices are within a selected $\pm\sigma\%$ band (usually $\pm 25\%$) centered on its average parametric delta-epsilon-robustness-fragility index, $\mathrm{RFI}^{a}_{\delta\varepsilon}$; otherwise, it is a robustness-fragility-unbalanced controller.
- *Performance-Fragility-Balanced PID controller*: a performance-fragility-balanced PID controller is one in which all its parametric performance delta-epsilon-fragility indices are within a selected $\pm\sigma\%$ band (usually $\pm 25\%$) centered on its average parametric delta-epsilon-performance-fragility index, $\mathrm{PFI}^{a}_{\delta\varepsilon}$; otherwise, it is a performance-fragility-unbalanced controller.

The robustness- or performance-fragility unbalance of a controller is caused by the controller parameter with the highest parametric robustness- or performance-fragility index.

12.5 Fragility of the PI and PID Tuning Rules

We will review the "fragility" evaluation of several tuning rules available in the control system literature before analyzing the robustness and performance fragility of several tuning rules with the fragility criteria stated in Sect. 12.4.

12.5.1 Assuring a Controller Stability Margin

Using the characterization of all the stabilizing PID controllers for First-Order-Plus-Dead-Time models, Silva et al. [60–62] analyzed several tuning rules for the controllers robustness with respect to small perturbations in their parameters. To obtain a good parametric stability margin, the controller integral gain value was forced to lie inside a box located 20% from the stabilizing integral gain boundaries for the fixed proportional and derivative gains provided by the tuning rule. As a result, the range of the model normalized dead-time that ensures the above controller robustness criteria for a tuning rule are as follows:

- Ziegler–Nichols [73]: $0 < \tau_o < 1.07$.
- Chien–Hrones–Reswick [22]: $0.37 < \tau_o$.
- Cohen–Coon [23]: $0 < \tau_o < 8.53$.
- Morari–Zafiriou IMC [53]: $0.37 < \tau_o$ $(\lambda/L = 0.25)$.

The controller robustness in this analysis is understood as a good l_2-norm parametric stability margin in the controller parametric space.

12.5.2 Tuning Rules for the Test

For the evaluation of the fragility of the PI and PID controllers tuning rules, a set of methods was selected ranging from the classic performance optimized rules to modern methods that allow dealing with the control system performance/robustness trade-off.

The selected tuning methods are the following:

1. Performance Optimization:
   - López et al. [48] [L_{IAE}, L_{ITAE}]—controllers: 1DoF PI and PID; design criteria: optimize the integrated absolute error (IAE) or the integrated time weighted absolute error (ITAE) to a load-disturbance step change; controlled process information: FOPDT model; application range: $0.1 \le \tau_o \le 1.0$.
   - Rovira et al. [58] [R_{IAE}, R_{ITAE}]—controllers: 1DoF PI and PID; design criteria: optimize the IAE or the ITAE error criteria to a set-point step change; controlled process information: FOPDT model; application range: $0.1 \le \tau_o \le 1.0$.
   - Taguchi and Araki [64] [*T&A*]—controllers: Two-Degree-Of-Freedom (2DoF) PI and PID; design criteria: two-step optimization of a weighted integrated error (1. d response, 2. r response); controlled process information: FOPDT model; application range: $0.1 \le \tau_o \le 1.0$.
2. From a Test Batch of Processes:
   - Åström and Hägglund Kappa-Tau [12] [*KT*]—controllers: 2DoF PI and PID; design criteria: dominant pole design with two robustness levels, $M_S \in \{1.4, 2.0\}$; controlled process information: K, T, L; application range: $0.1 \le \tau_o \le 6$.
   - Åström and Hägglund Approximated M constrained Integral Gain Optimization [14] [*AMIGO*]—controllers: 2DoF PI and PID; design criteria: maximize the controller integral gain subject to a constraint in the robustness, $M_S = 1.4$; controlled process information: K, T, L; application range: $0.001 \le \tau_o \le 50$.
3. Internal Model Control (IMC)-Based:
   - Skogestad Simple Control [63] [*SIMC*]—controllers: 1DoF PI and PID; design criteria: modified servo-control direct synthesis (IMC) for fast servo- and

regulatory control responses with a robustness $M_S = 1.59$; controlled process information: FOPDT model for PI tuning and SOPDT model for PID tuning; application range: $\tau_o \geq 0.1$.

- Ali and Majhi Percent Overshoot [8] [POS_S, POS_T]—controllers: 1DoF PI and PID; design criteria: set-point response percent overshoot specification, POS_S smooth control 0% OS ($M_S = 1.38$), POS_T tight control 10% OS ($M_S = 1.71$); controlled process information: FOPDT model for PI tuning and SOPDT model for PID tuning; application range: $\tau_o \geq 0.1$.

4. Multi-Objective Optimization:

- Tavakoli et al. [67] [MOO]—controllers: 2DoF PI; design criteria: two-step optimization considering servo- and regulatory control performance (IAE), control effort smoothness (TV_u), and robustness ($M_S \leq 2.0$); controlled process information: FOPDT model; application range: $0.1 \leq \tau_o \leq 2.0$.

5. Model Reference Design:

- Alfaro et al. [7] [PI_{2M_S}]—controllers: 2DoF PI; design criteria: two-step non-oscillatory servo- and regulatory control model reference optimization with a robustness restriction, $M_S \in \{1.4, 1.6, 1.8, 2.0\}$; controlled process information: FOPDT model; application range: $0.1 \leq \tau_o \leq 2.0$.

The above selected controller tuning rules will be used to illustrate the different robustness- and performance-fragility analysis that can be conducted. Although the intention of this chapter is not to present an exhaustive fragility analysis of all these tuning rules, the results allow evaluating and comparing their fragility issues.

12.5.3 Nominal Robustness and Performance of Tuning Rules

For the fragility analysis, a unit gain normalized controlled process model will be used, obtained by applying the transformation, $\hat{s} = Ts$, in (12.2), given by the following:

$$\hat{P}(\hat{s}) = \frac{e^{-\tau_o \hat{s}}}{(\hat{s}+1)(a\hat{s}+1)}, \quad 0 \leq a \leq 1.0. \tag{12.23}$$

With each tuning rule under evaluation, the controller nominal normalized parameters, $\hat{\theta}_c^o$, were found for the normalized dead-times, τ_o, in the range of application of the rule. For each one of these controllers, the nominal control system robustness, M_S^o, the nominal servo-control performance, J_{er}^o, and the nominal regulatory control performance, J_{ed}^o, were computed.

In all the evaluations, the PID controller derivative mode will be applied only to the feedback signal y (i.e., $\gamma = 0$), which will only affect the servo-control performance. In addition, the PID derivative mode will include a derivative filter with $\alpha = 0.1$, even for testing such tuning methods obtained using a non-proper, "ideal" PID controller.

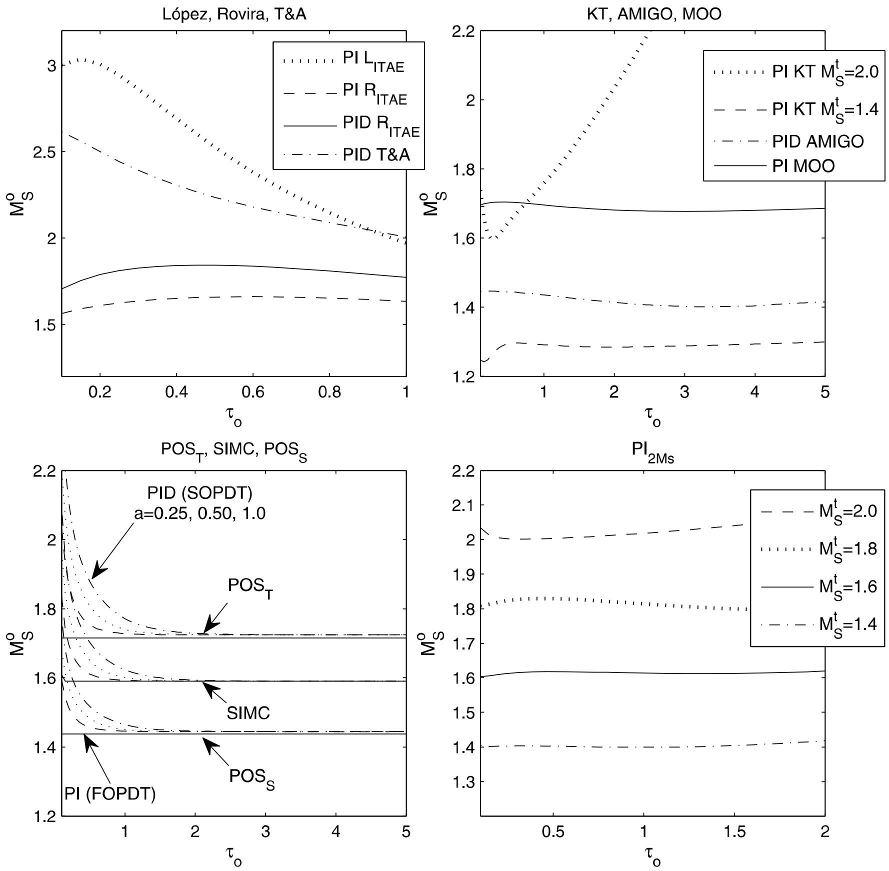

Fig. 12.4 Nominal robustness of the tuning rules

Nominal Robustness The nominal robustness, M_S^o, of the tuning rules under evaluation are shown in Fig. 12.4. As can be seen, the robustness of the control systems with the regulatory control optimized-performance controllers, PI L_{ITAE} and PID *T&A*, is very poor; in no case was the minimum robustness level of $M_S \leq 2.0$ obtained. The PI/PID L_{IAE}, PID L_{ITAE} and PI *T&A*, not shown, have even lower robustness. In the servo-control optimized-performance controller case, the PI/PID R_{ITAE} have middle-range robustness, but the R_{IAE} has robustness of up to 20% lower.

From the Kappa-Tau and *AMIGO* group of controllers, only the *KT* PI for high robustness with $M_S^o \approx 1.3$ and the *AMIGO* PID with $M_S^o \approx 1.42$ are near to their robustness target design level of $M_S^t = 1.4$. As has been reported elsewhere, the *KT* and *AMIGO* methods for minimum robustness ($M_S^t = 2.0$) did not achieve their design robustness criteria.

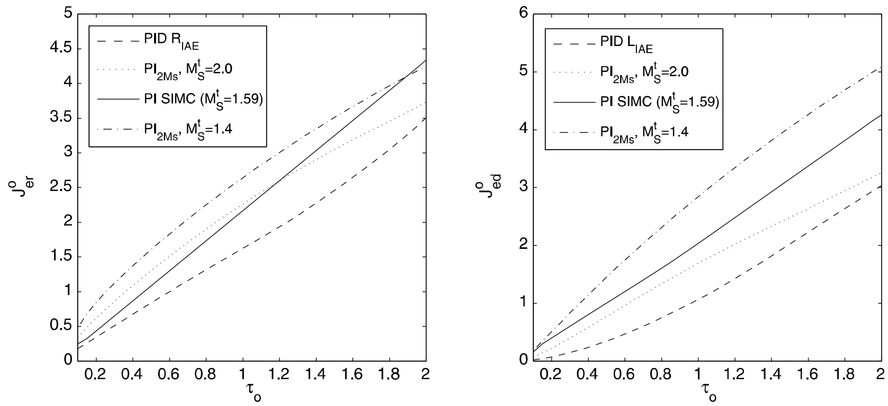

Fig. 12.5 Nominal servo- and regulatory control performance of the tuning rules

It is also noted from this figure that the Multi-Objective-Optimization Method (MOO) provides a nearly constant robustness level of $M_S^o \approx 1.7$ for models with normalized dead-times, even further than its range of application.

The IMC-based *SIMC*, POS_T and POS_S methods are designed to provide specific target robustness levels: *SIMC* $M_S^t = 1.59$, POS_T $M_S^t = 1.71$, and POS_S $M_S^t = 1.38$. The *SIMC* and POS_T target robustness are achieved without problems with a PI controller for FOPDT models, but the POS_S nominal robustness is $M_S^o = 1.44$ for FOPDT models. In the PID controller case tuned with the SOPDT models and due to these methods obtained using zero/pole cancellations with an ideal PID controller (without derivative filter), when they are applied to a "real" proper PID controller, the zero/pole cancellation does not take place, affecting the robustness of the control system. This is particularly severe for time-constant-dominant ($\tau_o < 1$) SOPDT models with time constants ratios of $a > 0.25$.

As seen in Fig. 12.4 with the PI_{2Ms} tuning method, it is possible to select four different robustness levels that are all accomplished successfully.

Nominal Performance For future use, the servo-control and regulatory control nominal performance of several of the evaluated tuning methods are shown in Fig. 12.5. As expected, at the performance evaluation side are the performance-optimized tunings of Rovira (servo-control) and López (regulatory control), which the methods showing the best performance to step changes in the set-point or the load-disturbance. Also noted in this figure is the constantly presented performance/robustness trade-off when the development of the tuning rule takes into account the control system robustness.

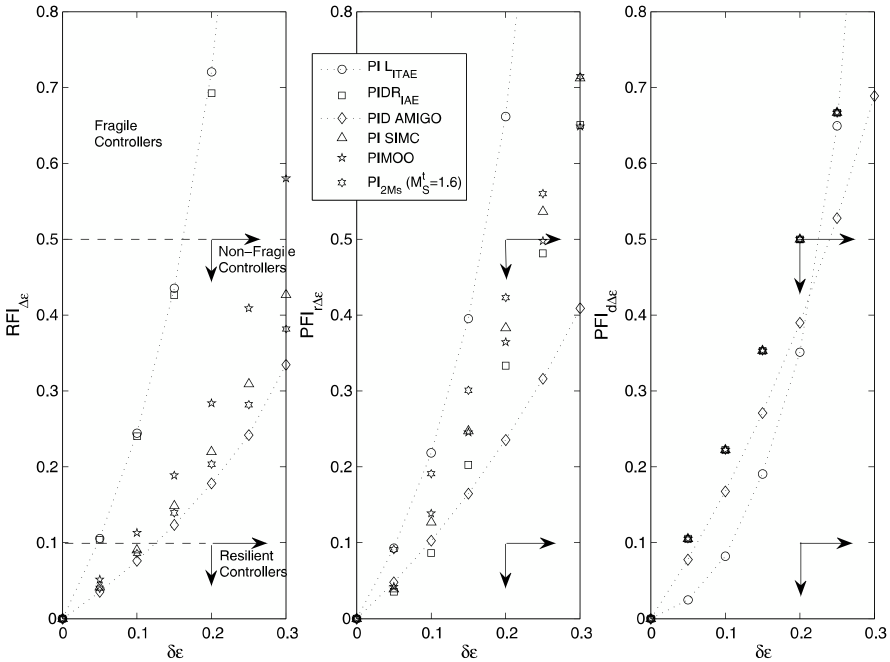

Fig. 12.6 Delta-epsilon-robustness- and performance-fragility

12.5.4 Delta-Epsilon and Parametric Robustness Fragility

The Delta-Epsilon-Fragility Indices, (12.15) and (12.18), can be used to analyze the effect of a change in the controller nominal parameters over the control system robustness and performance.

12.5.4.1 Robustness and Performance Delta-Epsilon Fragility

The Delta-Epsilon-Robustness- and Performance-Fragility Index Plots of Fig. 12.6 show the effect of the $\delta\varepsilon$ changes on the controller parameters and allow a comparison of the fragility of the tuning methods. In this particular case, the comparison was made using a normalized controlled process model (12.23) with $\tau_o = 0.50$.

As can be seen, the performance-optimized controllers (PI L_{ITAE}, PID R_{IAE}) lose robustness very quickly when the controller parameter are perturbed. Up to 25% of their robustness is lost with a 10% change in the controller parameters. At the other extreme, the controller tuned with the *AMIGO* method, with a high nominal robustness $M_S^o \approx 1.3$, loses only 8% of its robustness with the same 10% change in its parameters. For the particular case of the model considered, with the exception of the López and Rovira controllers, all are robustness non-fragile controllers, but none are robustness-resilient.

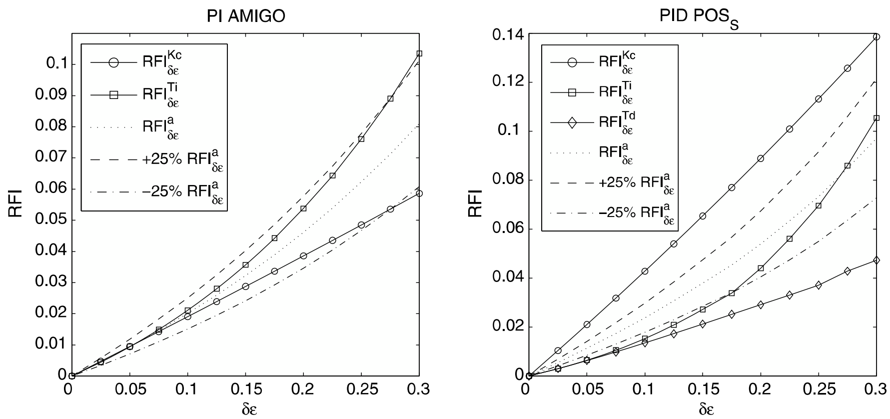

Fig. 12.7 Controller robustness-fragility balance

It is also noted that the servo-control performance fragility has the same behavior as shown by the robustness fragility. In addition, and excluding the PI L_{ITAE} and the PID *AMIGO* controllers, all the controllers have the same regulatory control performance fragility proportional to the $\delta\varepsilon$ change in the controller parameters.

12.5.4.2 Controller Fragility Balance

The parametric delta-epsilon-robustness-fragility indices can be used to evaluate the balance of controller fragility [4].

As can be seen in Fig. 12.7, the PI *AMIGO* is a robustness-fragility-balanced controller, while the PID POS_S is an unbalanced controller. It is also noted that in the PI *AMIGO* case, the control system robustness is affected more by a change in the controller integral time constant, T_i, than in its gain, K_p. In the PID POS_S case, the controller proportional gain, K_p, is the parameter that deteriorates more of the control system robustness and causes its fragility unbalance.

The controller final fine-tuning will be safer if the controller is fragility-balanced (robustness and performance); in such a case, each controller parameter variation will have a similar effect over the control system robustness and/or performance.

12.5.5 Robustness and Performance Fragility of the Tuning Rules

The Delta-Epsilon-Fragility Indices in the above sections showed the control system loss of robustness and performance for $\delta\varepsilon$ changes on the controller parameters from very small values up to values greater than the 20% change used to define when a controller is considered robustness- or performance *fragile*, *non-fragile*, or *resilient*.

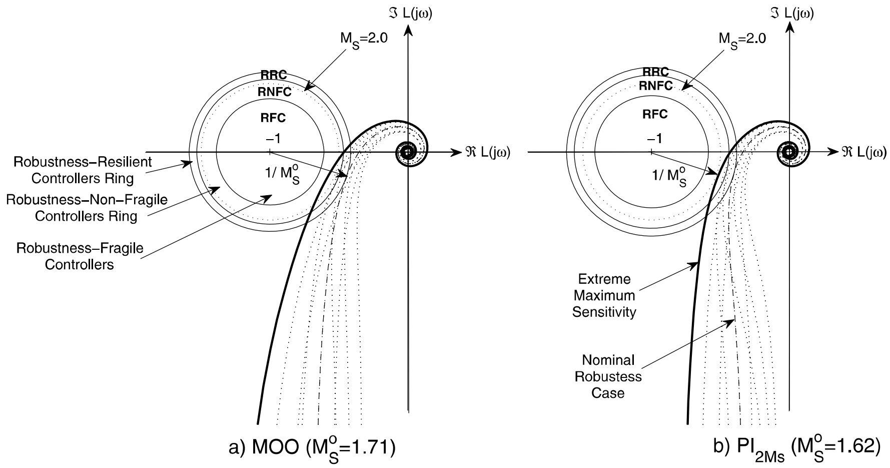

Fig. 12.8 Controller delta 20 robustness-fragility rings

In the following sections, the Delta 20 fragility is considered first to visually show the robustness fragility of a tuning rule for a specific controlled process model and, afterward, to analyze the robustness and performance fragility of a tuning rule as a whole. In the latter case, the evaluation will show if a tuning rule can be considered as *globally robustness-* and/or *performance-resilient*, *non-fragile* or *fragile*.

12.5.5.1 Delta 20 Robustness-Fragility Rings

We will introduce here the *Delta 20 Robustness-Fragility Rings*, which is a simple tool that uses the open-loop transfer function, $L(j\omega)$, Nyquist curve of the nominal, and Delta 20 perturbed controllers to provide an indication of the control system robustness-fragility. The plot includes the robustness-fragility rings to show the areas in the $L(j\omega)$ plane that define when the controller is a robustness-resilient controller (RRC), a robustness-non-fragile controller (RNFC), or a robustness-fragile controller (RFC).

Figure 12.8 shows the Delta 20 fragility rings of the *MOO* ($M_S^o = 1.71$) and PI_{2Ms} ($M_S^o = 1.62$) PI controllers for the $\tau_o = 0.5$ model case. As can be seen, both are robustness-non-fragile controllers; their perturbed open-loop Nyquist curves enter the robustness-non-fragile controller (RNFC) ring area. However, the *MOO* PI controller is more robustness-fragile ($\mathrm{RFI}_{\Delta 20} = 0.284$) than the PI_{2Ms} controller ($\mathrm{RFI}_{\Delta 20} = 0.202$), with an extreme maximum sensitivity of $M^m_{S\Delta 20} = 2.20$, which is higher than the normal minimal robustness level of $M_S = 2.0$. The extreme maximum sensitivity of the PI_{2Ms} controller is $M^m_{S\Delta 20} = 1.95$ in this case.

12.5.5.2 Robustness- and Performance-Fragility Plots of the Tuning Rules

In the following figures, the nominal robustness, M_S^o, the Delta 20 Robustness-Fragility Index $\mathrm{RFI}_{\Delta 20}$, and the Parametric Delta 20 Robustness-Fragility Indices, $\mathrm{RFI}_{\delta 20}^{K_p}$, $\mathrm{RFI}_{\delta 20}^{T_i}$, and $\mathrm{RFI}_{\delta 20}^{T_d}$, of two of the selected tuning rules are presented.

To provide useful information to the control system designer, the tuning rule-fragility evaluation must be conducted within the application range of the rule, covering the model normalized dead-time range for which the rule was designed. This will provide a picture of the global fragility of the tuning rule.

Simple Control—*SIMC* Although the IMC-based *SIMC* tuning rule does not impose any specific restriction to the model normalized dead-time, it will be evaluated in the $0.1 \leq \tau_o \leq 2$ range covering the time-constant-dominant and dead-time-dominant models.

The *SIMC* robustness-fragility indices are shown in Fig. 12.9. As can be seen, although the PI controller (FOPDT model) produces control systems with a constant nominal robustness of $M_S^o = 1.59$, its robustness-fragility is affected by the model normalized dead-time. The PI controller became more robustness-fragile as the normalized dead-time increased due to the raising integral time parametric robustness-fragility, while its gain parametric robustness-fragility remained constant.

For the PID controller (SOPDT model), it can be seen that the control system nominal robustness and the controller fragility depend not only on the model normalized dead-time, τ_o, but also on the model time-constants ratio, a. This is especially evident for lower values of τ_o and higher values of a.

The servo-control and the regulatory control nominal performance and the performance fragility of the *SIMC* method are shown in Figs. 12.10 and 12.11, respectively. The higher servo-control performance is obtained with the PI controller, but it is more performance-fragile. However, the servo-control Delta 20 and parametric performance fragility are almost constant; therefore, the expected adverse effect of a 20% change in the controller parameters over the servo-control performance could be predicted.

Regarding the regulatory control nominal performance, there is no significant difference between the PI and the PID controllers, and it decreased (J_{ed} increase) linearly with τ_o. It is also noted that, for $\tau_o \leq 0.8$, the regulatory control performance-fragility is constant: $\mathrm{PFI}_{d\Delta 20} = 0.50$, $\mathrm{PFI}_{d\delta 20}^{K_p} = 0.25$, $\mathrm{PFI}_{d\delta 20}^{T_i} = 0.20$, $\mathrm{PFI}_{d\delta 20}^{T_d} \approx 0$; these characteristics will be analyzed in more detail later. For higher τ_o values, the controller performance-fragility decreased.

Robust Tuning—PI_{2Ms} The PI_{2Ms} tuning addressed the performance/robustness trade-off, providing rules for four target robustness levels. Three are shown in Fig. 12.12: the normal minimum robustness level ($M_S^t = 2.0$), an intermediate level ($M_S^t = 1.6$), and a high robustness level ($M_S^t = 1.4$).

Although all the achieved robustness levels are nearly constant for $0.1 \leq \tau_o \leq 2.0$, the controller fragility increases with the model normalized dead-time. In addition, the low robustness controllers are more robustness-fragile than the high

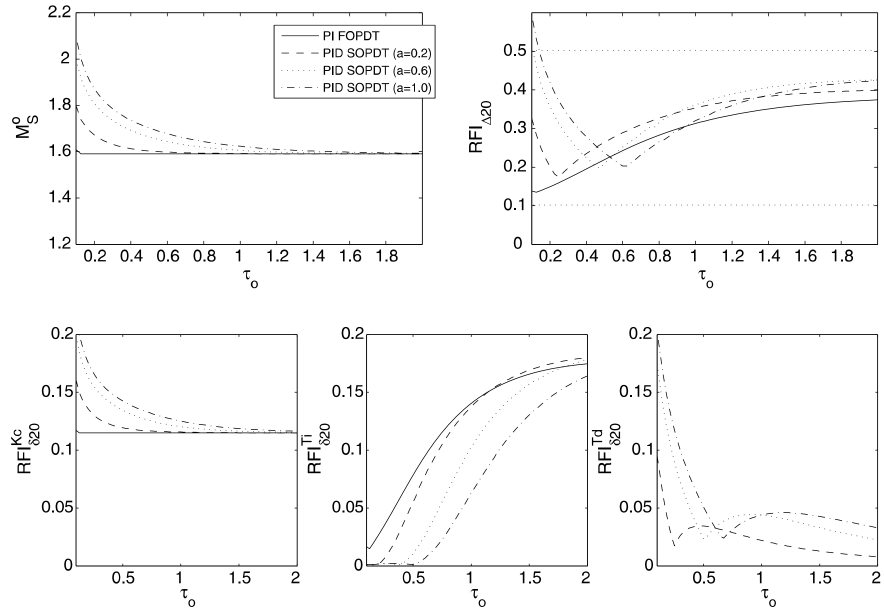

Fig. 12.9 *SIMC* nominal robustness and fragility

robustness ones. All the controllers are robustness-non-fragile, except for $M_S^t = 2.0$ and $\tau_o > 1.6$. It is also noted that the controller gain parametric fragility is nearly constant, except for $M_S^t = 2.0$, and the integral time constant parametric fragility increases linearly with τ_o.

The performance/robustness trade-off can be seen in Fig. 12.13 (servo-control) and Fig. 12.14 (regulatory control). If the control system nominal robustness increases, lower M_S^o, the system nominal performances decreases, higher J_{er}^o and higher J_{ed}^o.

In addition to the robustness-fragility, the servo-control performance-fragility increases with the model normalized dead-time, but in this case, the fragility/ robustness relation is inverted. The high robustness systems are more performance-fragile than the systems with low robustness. This rise in the performance-fragility is due to an increment in the gain parametric performance-fragility when the normalized dead-time increases. The integral time constant servo-control performance fragility is constant for all τ_o and M_S target levels, except for $M_S^t = 2.0$ and $\tau_o > 1.7$.

As noted in Fig. 12.14, the regulatory control performance is affected not only by the model normalized dead-time but also by the control system design robustness level. In this case, more robust control systems have a higher regulatory control performance, which is more evident for higher normalized dead-times.

With the exception of the controllers for $\tau_o \geq 1.5$ tuned for low robustness ($M_S^t = 2.0$), the regulatory control performance-fragility index of all the PI_{2Ms} controllers is $\mathrm{PFI}_{d\Delta 20} = 0.50$, and their parametric performance fragility indices are $\mathrm{PFI}_{d\delta 20}^{K_p} = 0.25$ and $\mathrm{PFI}_{d\delta 20}^{T_i} = 0.20$. For the PI_{2Ms} tuning rule, the regulatory per-

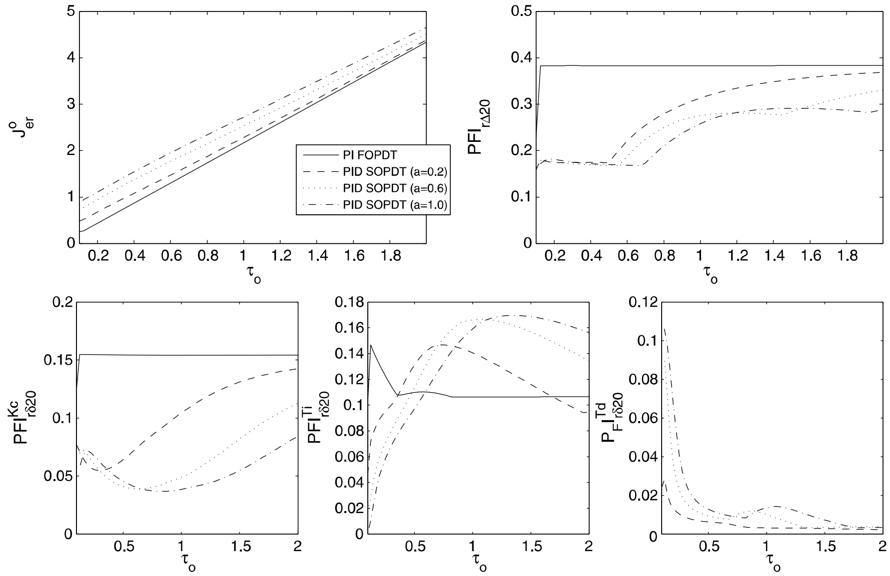

Fig. 12.10 *SIMC* servo-control nominal performance and fragility

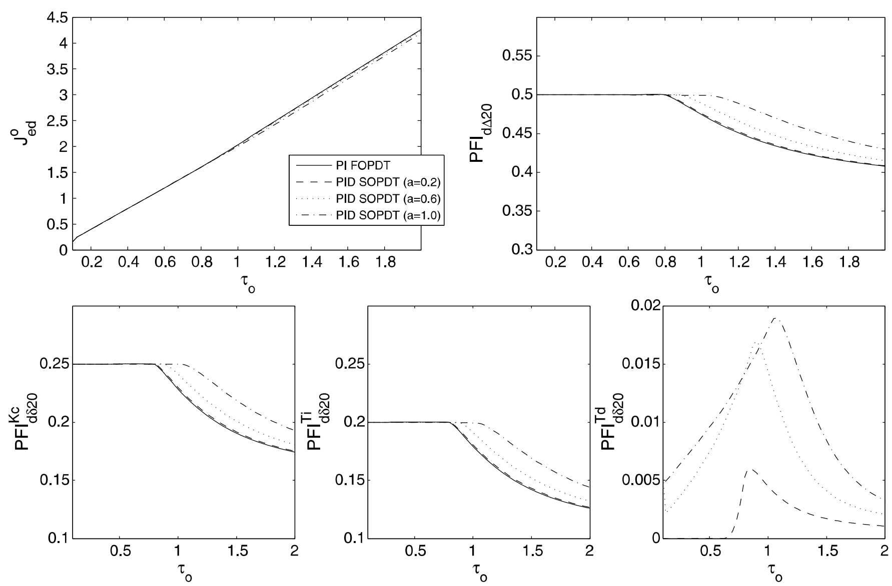

Fig. 12.11 *SIMC* regulatory control nominal performance and fragility

formance fragility is independent of the model normalized dead-time and of the robustness design criteria.

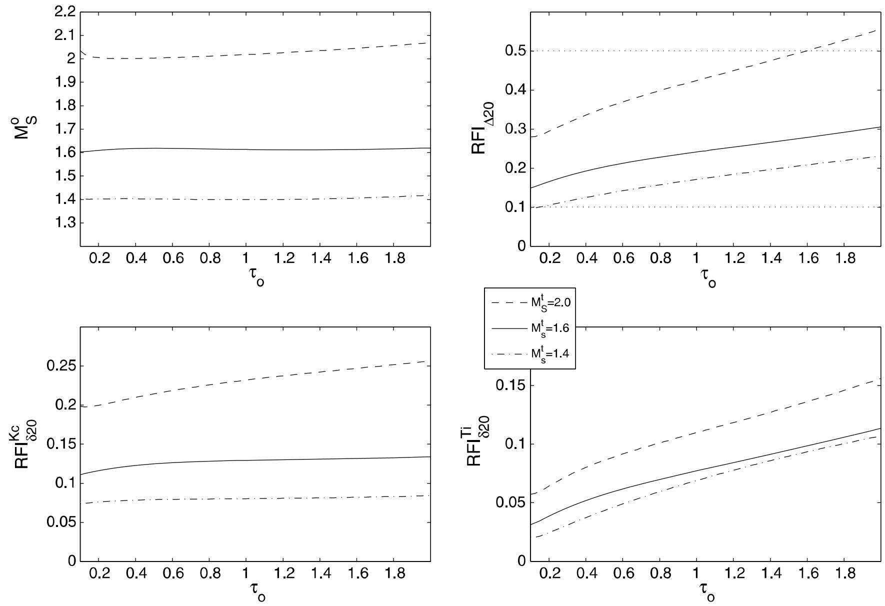

Fig. 12.12 PI_{2Ms} nominal robustness and fragility

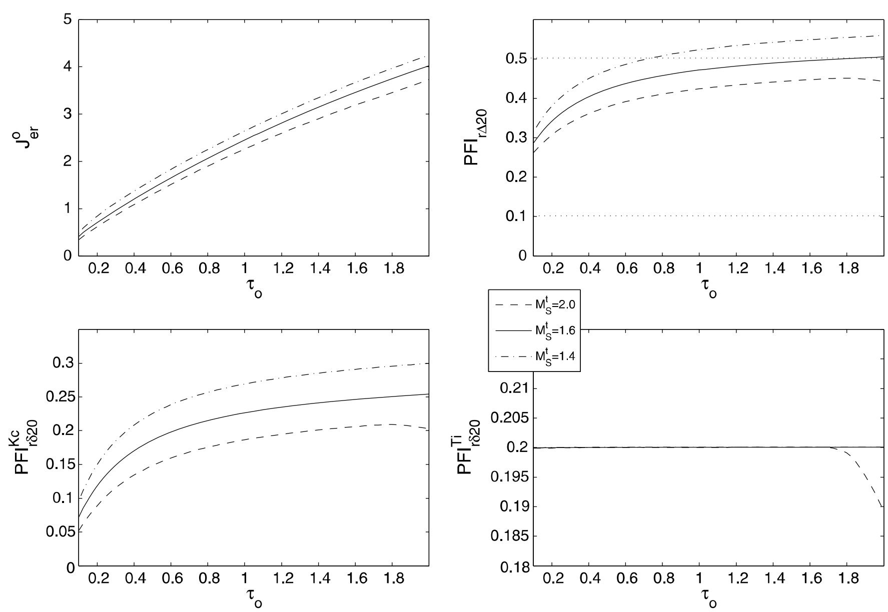

Fig. 12.13 PI_{2Ms} servo-control nominal performance and fragility

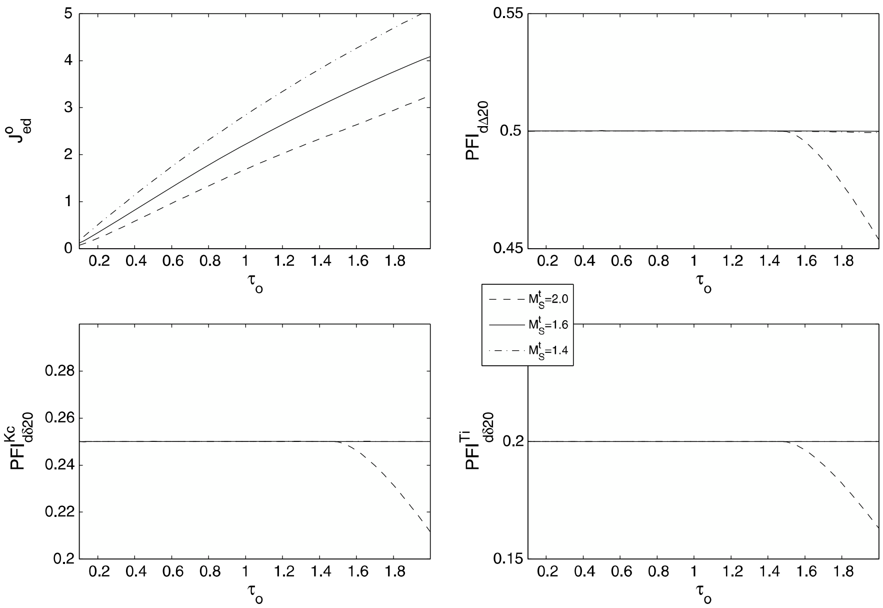

Fig. 12.14 PI_{2Ms} regulatory control nominal performance and fragility

The regulatory control design criterion of the PI_{2Ms} tuning is to obtain a non-oscillatory response to a load-disturbance step change with a specific robustness level. If the regulatory control step response is non-oscillatory, then its integrated absolute error, IAE, is equal to its integrated error, IE, and given by the following [12]:

$$J_{ed} = \int_0^\infty \left|e(t)\right| \mathrm{d}t = \int_0^\infty e(t)\, \mathrm{d}t = \frac{T_i}{K_p}, \tag{12.24}$$

which depends on the controller gain, K_p, and integral time constant, T_i, but not on its derivative time constant, T_d.

Using (12.24) with (12.18) and (12.19), and considering a decrement in the controller gain ($\delta\varepsilon_{K_p^-}$) and an increment in the controller integral time constant ($\delta\varepsilon_{T_i^+}$), the parametric delta-epsilon-performance-fragility indices for the regulatory control are as follows:

$$\text{PFI}^{K_p}_{d\delta\varepsilon} = \frac{\delta\varepsilon_{K_p^-}}{1-\delta\varepsilon_{K_p^-}}, \qquad \text{PFI}^{T_i}_{d\delta\varepsilon} = \delta\varepsilon_{T_i^+}, \qquad \text{PFI}^{T_d}_{d\delta\varepsilon} = 0, \tag{12.25}$$

and the delta-epsilon-performance-fragility index is the following:

$$\text{PFI}_{d\Delta\varepsilon} = \frac{\delta\varepsilon_{K_p^-} + \delta\varepsilon_{T_i^+}}{1-\delta\varepsilon_{K_p^-}} \tag{12.26}$$

Then, considering a 20% reduction in K_p and a 20% increment in T_i, from (12.25) and (12.26), $\text{PFI}^{K_p}_{d\delta 20} = 0.25$, $\text{PFI}^{T_i}_{d\delta 20} = 0.20$, $\text{PFI}^{T_d}_{d\delta 20} = 0$, and $\text{PFI}_{d\Delta 20} = 0.50$.

From this result it may concluded that the above regulatory performance-fragility figures reflect the non-oscillatory behavior of the PI_{2Ms} load-disturbance step response.

These characteristics were also noted with other tuning methods: PI/PID R_{IAE} and R_{ITAE}; PI *T&A* ($\tau_o \leq 1.6$); PID *KT* $M_S = 2.0$ ($\tau_o \geq 1.5$) and PI/PID *KT* $M_S = 1.4$ ($\tau_o \geq 1$); PI *AMIGO* ($\tau_o \geq 0.4$); PI/PID *SIMC* ($\tau_o \leq 0.8$); PI MOO ($\tau_o \geq 0.5$); PI/PID POS_S and POS_T ($\tau_o \leq 0.7$).

12.6 PID Controller Implementation Fragility

Although the PID controller fragility definitions and indices stated in Sect. 12.4 are independent of its implementation, in Sect. 12.5, the evaluation of the tuning rules fragility was made using a *standard* "non-interacting" PID controller (12.4) with $\gamma = 0$, which reduces to the following:

$$u(s) = K_p\left\{\beta r(s) - y(s) + \frac{1}{T_i s}\big[r(s) - y(s)\big] - \frac{T_d s}{\alpha T_d s + 1} y(s)\right\}. \tag{12.27}$$

However, this is not the only available implementation of the PID control algorithm [39, 42, 68]. In commercial industrial controllers, the *series* "interacting" PID is frequently presented by the following:

$$u(s) = K'_p\left\{(\beta - 1)r(s) + \left(\frac{T'_i s + 1}{T'_i s}\right)\left[r(s) - \left(\frac{T'_d s + 1}{\alpha_s T'_d s + 1}\right)y(s)\right]\right\}, \tag{12.28}$$

and in academic and research papers, the *parallel* or "three-gain" PID is often used:

$$u(s) = K_p\big[\beta r(s) - y(s)\big] + \frac{K_i}{s}\big[r(s) - y(s)\big] - \frac{K_d s}{\alpha_p K_d s + 1} y(s). \tag{12.29}$$

Applying parameter conversion factors [24], it is always possible to obtain a standard or parallel PID controller equivalent to a series controller, but only if $T_i \geq 4T_d$, then a series controller equivalent to a standard PID may be found. For PI controllers ($T_d = 0$), the standard and series implementations are equivalent.

The aim here is to evaluate how the PID controller robustness- and performance-fragility is affected by its implementation.

Since most of the tuning rules selected in Sect. 12.5.2 are for PI controllers and the selected rules for PID that achieve nearly constant robustness levels are based on zero/pole cancellation design techniques, the Brosilow and Joseph [B&J] [20] tuning was selected to evaluate the controller implementation influence over its fragility. This is an IMC-based PID tuning for FOPDT-controlled process models; the equations of which were found by expanding the IMC controller equation in a Maclaurin series and taking its first three terms. As IMC-based tuning, the B&J

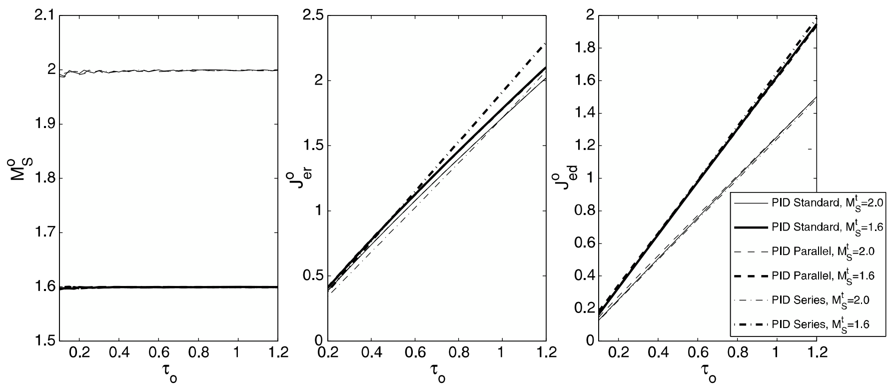

Fig. 12.15 B&J PID nominal robustness and performance

equations include a *tuning parameter*, the control system closed-loop time constant, which addresses the control system performance/robustness trade-off.

For the evaluation and changing of the design parameter, the parameters of a standard PID controller (12.27) were found for two target robustness levels, $M_S^t \in \{1.6, 2.0\}$. The model normalized dead-time was restricted to $0.1 \leq \tau_o \leq 1.2$ to obtain positive controller parameters. The obtained mean and [minimum, maximum] nominal robustness, M_S^o, values were 2.00 [1.99, 2.00] and 1.60 [1.60, 1.60], respectively.

Using conversion factors, the parameters of series (12.28) and parallel (12.29) PID controllers were found. The robustness obtained with these equivalent controllers was for the series PID: 1.97 [1.95, 1.98] and 1.61 [1.60, 1.62], which is very near to the target robustness, but the robustness was for the parallel PID: 2.09 [2.00, 2.51] and 1.61 [1.60, 1.72]. These differences between the target and the obtained nominal robustness with the equivalent series and parallel controllers were due to the influence of the controller derivative filter; in the robustness evaluation, the same derivative filter constants, $\alpha = \alpha_s = \alpha_p = 0.1$, were used. The control system robustness will be drastically reduced with the equivalent parallel PID controller, specially for lower τ_o.

To remove this unwanted side effect of the equivalent controllers, new sets of controller parameters were found for the series and parallel PIDs, using the B&J tuning joined with the conversion factors and controllers (12.28) and (12.29) to achieve the target robustness levels.

The nominal robustness and the nominal servo- and regulatory control performance, obtained with these new tunings, are shown in Fig. 12.15. It is important to remember that these are three different implementations of the PID control algorithm, standard, series and parallel, all tuned with the same rule but with three different sets of parameters that, as shown in this figure, all resulted in the same robustness and nearly the same performance for each target robustness level.

The robustness and performance fragility of the standard PID controller tuned with the B&J tuning method are shown in Fig. 12.16, corresponding to the series

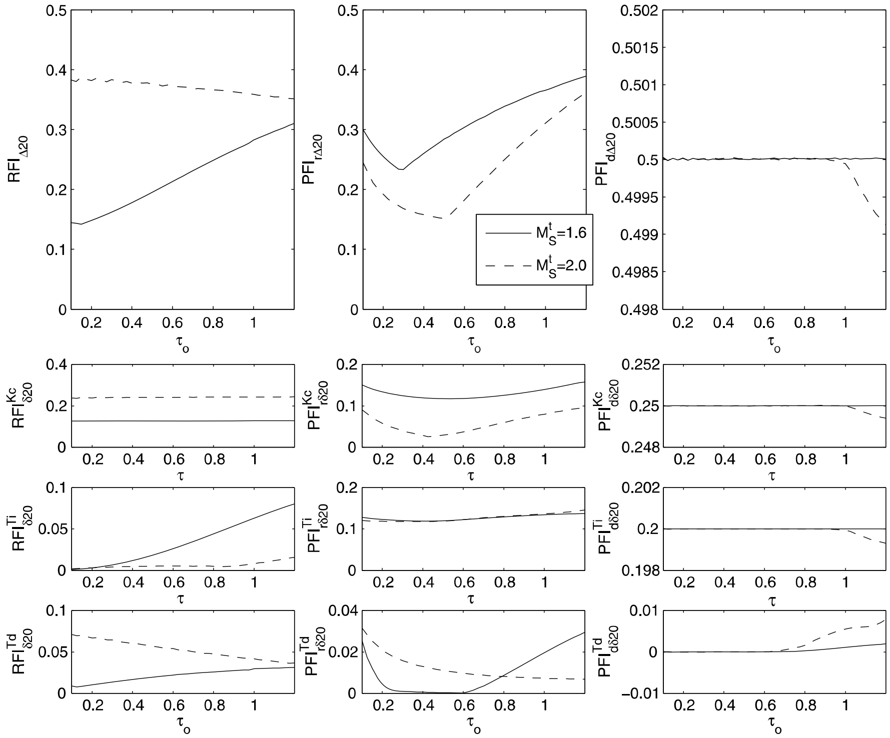

Fig. 12.16 B&J standard PID controller—robustness and performance fragility

PID in Fig. 12.17 and fragility indices for the parallel PID in Fig. 12.18. These figures will allow not only evaluating the tuning rule fragility but also the controller fragility due to its implementation.

Although the three controller implementations provide control systems with the same nominal robustness, and they all have in common that the low robustness controllers are more robustness-fragile than the high robustness ones, their robustness-fragility level and behavior are affected in different ways by the perturbation of the controller parameters and by the controlled process model normalized dead-time change.

It can be seen that the parallel implementation is less robustness-fragile, while the series is more robustness-fragile. It is also noted that the robustness-fragility of the parallel controller is more or less constant.

The controller gain parametric robustness-fragility, although it is the highest between the parametric-fragility indices, is nearly constant for all the controllers implementations, whereas the integral time parametric robustness-fragility increases with the model normalized dead-time, especially for the high robustness controllers.

With respect to the servo-control performance-fragility, it is the highest, but it is nearly constant in the series implementation.

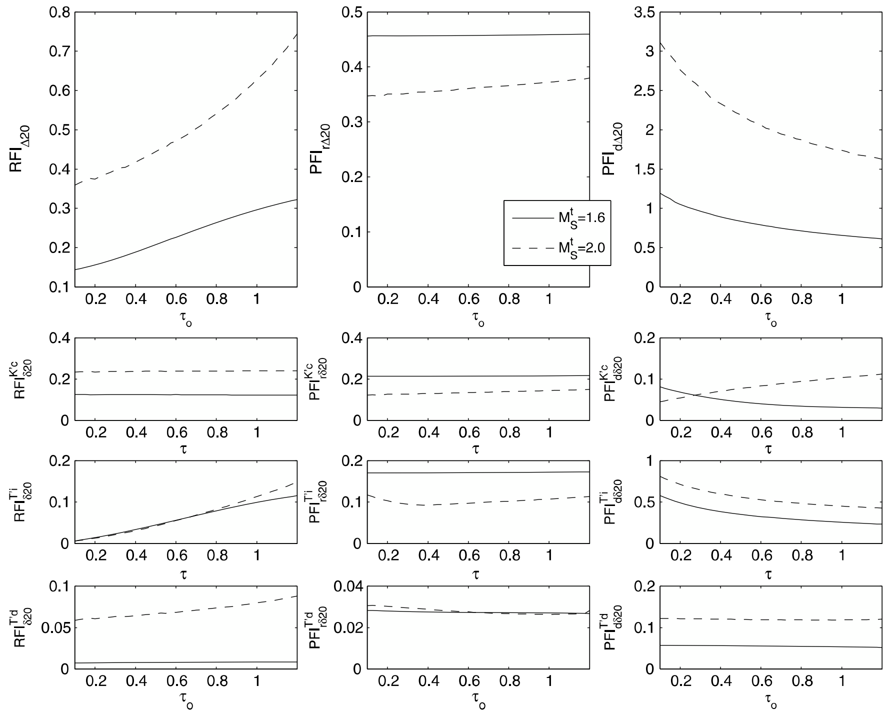

Fig. 12.17 B&J series PID controller—robustness and performance fragility

The regulatory control performance fragility of the standard implementation is completely flat, as are its gain and integral time constant parametric regulatory-control-fragility indices. Its regulatory performance-fragility is not affected by the derivative time constant.

The regulatory control performance of the series implementation of the PID control algorithm is extremely high and is very sensitive to the changes in the controller integral time constant.

In particular for the B&J tuning rule, if a predictable and flat behavior of the robustness- and performance-fragility indices is of interest, the parallel implementation of the PID control algorithm will be safer for performing the controller final fine-tuning.

12.7 Conclusions

Modern tuning methods for proportional integral derivative (PID) controllers must take into account the existing trade-offs between several conflicting design considerations: the *performance* with step changes in the set-point and load-disturbances, the *control effort* requirements, the control system *robustness* with the changing char-

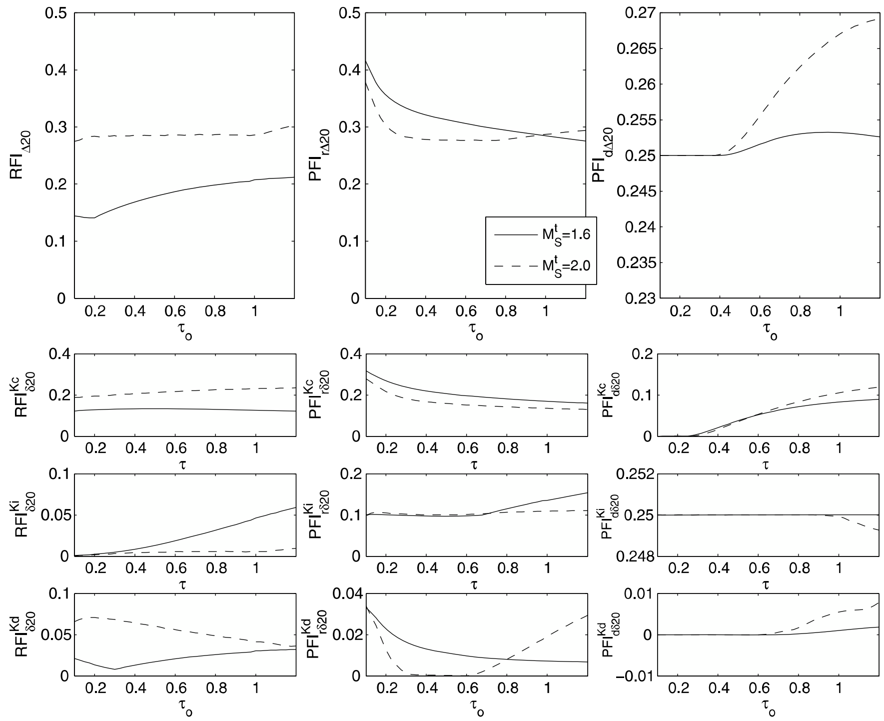

Fig. 12.18 B&J parallel PID controller—robustness and performance fragility

acteristics of the controlled process, and the controller robustness and performance *fragility* with the perturbation of its own parameters.

The fragility of a controller will depend not only on the tuning rule design considerations but also on the controller implementation, i.e., the PID control algorithm used.

The evaluation of the tuning rules presented in this chapter is a *worst-case analysis* for robustness and performance when all the controller parameters are perturbed by the same amount, a 20% change for our definition of the fragility indices. It may be expected that if there is a set of controller parameters that adversely affects the control system by reducing its robustness and/or performance, there should be another set of the controller parameters that may improve these characteristics of the closed-loop.

If the tuning rule can guarantee the control system design target robustness level, at least with the controlled process model used for tuning the controller, then it will be important to the control system designer that it be a robustness-non-fragile tuning rule. Preferably, it would be robustness-resilient, but not necessarily or even desirably that it also be servo- or regulatory performance-non-fragile, considering that, normally, the controller final fine-tuning has, as its main purpose, to modify in some way the transient response, and then the performance, of the control system.

A detailed analysis may be conducted in the controller parametric space to evaluate not only the control system reduction of the robustness and performance when they are perturbed but also the increment of these indicators. Using the perturbed controller extreme (maximum and minimum) robustness and performance measurements, the controller fragility may be redefined in such a way that the positive fragility indices will denote a loss of robustness or performance, and the negative fragility indices will denote an improvement in robustness or performance.

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Optimal Robust Tuning for 1DoF PI/PID Control Unifying FOPDT/SOPDT Models

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Abstract: The aim of the paper is to present tuning equations for one-degree-of-freedom (1DoF) proportional integral (PI) and proportional integral derivative (PID) controllers. These are based on a performance/robustness trade-off analysis with first- and second-order plus dead-time models. On the basis of this analysis a tuning method is developed for 1DoF PI and PID controllers for servo and regulatory control that allows designing closed-loop control systems with a specified M_S robustness that at the same time have the best possible IAE performance. The control system robustness is adjusted varying only the controller proportional gain.

Keywords: PID controllers, one-degree-of-freedom controllers, servo/regulatory control, performance/robustness trade-off.

1. INTRODUCTION

As it has been widely reported, proportional integral derivative (PID) type controllers are with no doubt, the controllers most extensively used in the process industry. Their success is mainly due to their simple structure, easier to understand by the control engineer than other most advanced control approaches.

In industrial process control applications, the set-point normally remains constant and good load-disturbance rejection (regulatory control) is required. There are also applications where the set-point following (servo-control) is the more important control task.

Although from their commercial introduction in 1940 (Babb, 1990) the original three-term PID control algorithm has evolved into the actual four- or five-term two-degree-of-freedom (2DoF) PID control algorithms the vast majority of the controllers still in use are of one-degree-of-freedom (1DoF) type.

Since Ziegler and Nichols (1942) presented their PID controller tuning rules, a great number of other procedures have been developed as revealed in O'Dwyer (2006) review. Some of them consider only the system performance (López et al., 1967; Rovira et al., 1969), its robustness (Åström and Hägglund, 1984), or a combination of performance and robustness (Ho et al., 1999).

There are tuning rules optimized for regulatory control operation (López et al., 1967) or optimized for servo-control operation (Tavakoli and Tavakoli, 2003). There are also authors that present separate sets of rules for each operation (Zhuang and Atherton, 1993; Kaya, 2004). For the servo-control operation there is an important group of tuning rules based on zero-pole cancellation, Internal Model Control (IMC), and direct synthesis techniques (Martin et al., 1975; Rivera et al., 1986; Alcántara et al., 2011).

Due to the constraints imposed by the 1DoF control algorithm it is necessary to develop separate tuning rules for servo and regulatory control. In addition, the control-system design procedure is usually based on the use of low-order linear models identified at the control system normal operation point. Due to the non-linear characteristics found in most industrial processes, it is necessary to consider the expected changes in the process characteristics assuming certain relative stability margins, or robustness requirements, for the control system.

Therefore, the design of the closed-loop control system with 1DoF PI and PID controllers must consider the main operation of the control system (*servo-control* or *regulatory control*) and the *trade-off* of two conflicting criteria, the time response *performance* to set-point or load-disturbances, and the *robustness* to changes in the controlled process characteristics. If only the system performance is taken into account, by using for example an integrated error criteria (IAE, ITAE or ISE) or a time response characteristic (overshoot, rise-time or settling-time) as in Huang and Jeng (2002), and Tavakoli and Tavakoli (2003), the resulting closed-loop control system probably will have a very low robustness. On the other hand, if the system is designed to have high robustness as in Hägglund and Åström (2002) and if the performance of the resulting system is not evaluated, the designer will not have any indication of the *cost* of having such highly robust system. Control performance and robustness are taken into account in Shen (2002), and Tavakoli et al. (2005) optimizing its IAE or ITAE performance but they just guarantee the usual minimum level of robustness.

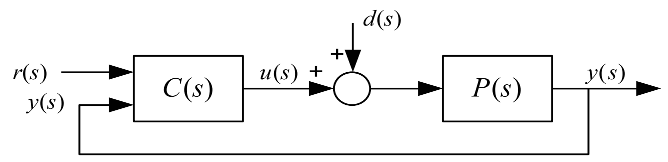

Figure 1. Closed-Loop Control System

To have an indication of the performance loss when the control system robustness is increased, using M_S as a measure, a performance/robustness analysis was conducted for 1DoF and 2DoF PI and PID control systems with first- (FOPDT) and second-order plus dead-time (SOPDT) models (Alfaro et al., 2010).

Based on this performance/robustness analysis, tuning rules are proposed for servo and regulatory 1DoF PI and PID controllers for four M_S robustness levels in the range from 1.4 to 2.0, to design robust closed-loop control systems that at the same time have the best possible performance under the IAE criteria. The presented tuning rules integrate in a single set of equations the tuning of controllers for first- and second-order plus dead-time process models.

The rest of the paper is organized as follows: the transfer functions of the controlled process model, the controller, and the closed-loop control system are presented in Section 2; the performance/robustness analysis is summarized in Section 3; the proposed *Optimal and Robust Tuning* is presented in Section 4 and particular examples of the performance/robustness trade-off are shown in Section 5. The paper ends with some conclusions.

2. PROBLEM FORMULATION

Consider a closed-loop control system, as shown in Fig. 1, where $P(s)$ and $C(s)$ are the controlled process model and the controller transfer function, respectively. In this system, $r(s)$ is the set point; $u(s)$, the controller output signal; $d(s)$, the load disturbance; and $y(s)$, the controlled process variable.

The controlled process is represented by an SOPDT model given by the general transfer function

$$P(s) = \frac{Ke^{-Ls}}{(Ts+1)(aTs+1)},\ \tau_o = \frac{L}{T}, \tag{1}$$

where K is the gain; T, the main time constant; a, the ratio of the two time constants ($0 \le a \le 1.0$); L, the dead-time; and τ_o, the *normalized dead time*. The model transfer function (1) allows the representation of FOPDT processes ($a = 0$), over damped SOPDT processes ($0 < a < 1$), and dual-pole plus dead-time (DPPDT) processes ($a = 1$).

The process is controlled with a 1DoF PID controller whose output is as follows (Åström and Hägglund, 1995):

$$u(s) = K_p\left\{\left(1+\frac{1}{T_i s}\right)e(s) - \left(\frac{T_d s}{\alpha T_d s+1}\right)y(s)\right\}, \tag{2}$$

where K_p is the controller *proportional gain*; T_i, the *integral time constant*; T_d, the *derivative time constant*; and α, the *derivative filter constant*. Then the controller parameters to tune are $\theta_c = \{K_p, T_i, T_d\}$. Usually, $\alpha = 0.10$ (Corripio, 2001).

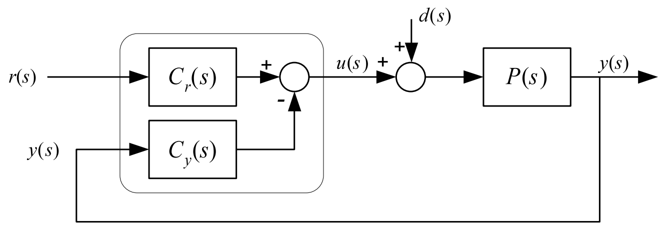

Figure 2. PID Closed-Loop Control System

Equation (2) may be rearranged, for analysis purposes, as follows

$$\begin{aligned} u(s) =& K_p\left(1+\frac{1}{T_i s}\right)r(s) \\ &- K_p\left(1+\frac{1}{T_i s}+\frac{T_d s}{0.1T_d s+1}\right)y(s), \end{aligned} \tag{3}$$

or in the compact form shown in Fig. 2 as

$$u(s) = C_r(s)r(s) - C_y(s)y(s), \tag{4}$$

where $C_r(s)$ is the *set-point controller* transfer function and $C_y(s)$ is the *feedback controller* transfer function.

The output of the closed-loop control system varies with a change in any of its the inputs as:

$$y(s) = \frac{C_r(s)P(s)}{1+C_y(s)P(s)}r(s) + \frac{P(s)}{1+C_y(s)P(s)}d(s), \tag{5}$$

or

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \tag{6}$$

where $M_{yr}(s)$ is the transfer function from the set-point to the controlled process variable and is known as the *servo control* closed-loop transfer function; $M_{yd}(s)$ is the transfer function from the load disturbance to the controlled process variable and is known as the *regulatory control* closed-loop transfer function.

The *performance* of the closed-loop control system is evaluated using the IAE cost functional given by

$$J_e \doteq \int_0^\infty |e(t)|\,\mathrm{d}t = \int_0^\infty |y(t)-r(t)|\,\mathrm{d}t. \tag{7}$$

The controller parameters in the servo-control closed-loop transfer function, M_{yr}, are the same than the controller parameters in the regulatory control closed-loop transfer function, M_{yd}. Therefore it is not possible to obtain a single set of controller parameters θ_c that optimize, at the same time, the control system response to a set-point step change and the control system response to a load-disturbance step change.

The performance (7) is evaluated for a step change in the set-point, J_{er} and in the load-disturbance, J_{ed}.

The peak magnitude of the sensitivity function is used as an indicator of the system *robustness* (relative stability). The maximum sensitivity for the control system is defined as

$$M_S \doteq \max_\omega |S(\mathrm{j}\omega)| = \max_\omega \frac{1}{|1+C_y(\mathrm{j}\omega)P(\mathrm{j}\omega)|}. \tag{8}$$

If the system robustness (8) is not taken into account for the design, the controller parameters may be optimized to maximize the system performance or to achieve the minimum value of the cost functional in (7), using M_{yr}

for set point changes (J^o_{er}) and M_{yd} for load disturbance changes (J^o_{ed}).

Because of the control system performance/robustness trade-off, if a robustness constraint is included into the design then, it is expected that the actual system performance will be reduced ($J_e \geq J^o_e$). Then, the *performance degradation factor* defined as

$$F_p \doteq \frac{J^o_e}{J_e},\ F_p \leq 1, \tag{9}$$

is used to evaluate the performance/robustness trade-off.

3. PERFORMANCE/ROBUSTNESS TRADE-OFF ANALYSIS

To evaluate the performance degradation when the system robustness is increased, the following steps, as they were presented in Alfaro et al. (2010), were followed.

3.1 1DoF Controllers Optimum Performance

For the 1DoF servo- and regulatory-control performance-optimized PI and PID controllers, the parameters $\theta^o_c = \{K^o_p, T^o_i, T^o_d\}$ were obtained using the cost functional (7) such that

$$J^o_e \doteq J_e(\theta^o_c) = \min_{\theta_c} J_e(\theta_c), \tag{10}$$

for (1) with $a \in \{0, 0.25, 0.5, 0.75, 1\}$ and ten τ_o in the range from 0.05 to 2.0, for set-point and load-disturbance step changes. The robustness of the control systems that deliver the optimal performance was evaluated by using M_S.

3.2 1DoF Controllers Degraded Performance

To increase the control-loop robustness, a target performance degradation factor, F^t_p, was included in the cost functional, as follows

$$J_{F_p} \doteq J(\theta_c, F^t_p) = \left| \frac{J^o_e}{J_e(\theta_c)} - F^t_p \right|, \tag{11}$$

for obtaining the PI and PID (servo and regulatory control) parameters θ^{o1}_c such that

$$J^o_{F_p} \doteq J_{F_p}(\theta^{o1}_c, F^t_p) = \min_{\theta_c} J_{F_p}(\theta_c, F^t_p). \tag{12}$$

When F^t_p was decreased, the control-system robustness was increased to the target level, M^t_S.

With starting point as the original unconstrained (from the point of view of robustness) optimal parameters θ^{o1}_c, a second optimization was conducted using the cost functional

$$J_{M_S} \doteq J(\theta_c, M^t_S) = \left| M_S(\theta_c) - M^t_S \right|, \tag{13}$$

in order to achieve the target robustness. The robust controller parameters, θ^{o2}_c, are such that

$$J^o_{M_S} \doteq J_{M_S}(\theta^{o2}_c, M^t_S) = \min_{\theta_c} J_{M_S}(\theta_c, M^t_S). \tag{14}$$

For the analysis, four target robustness levels were considered, $M^t_S \in \{2, 1.8, 1.6, 1.4\}$.

Finally, the performance degradation factor required for obtaining M^t_S in (14) was evaluated as follows

$$F_p(M^t_S) = \frac{J^o_e}{J_e(\theta^{o2}_c)}. \tag{15}$$

Therefore, the second optimization provided the controller parameters θ^{o2}_c required to formulate a system with the target robustness (8), M^t_S, and with the best performance allowed when using the IAE criteria (7), J_{er} or J_{ed}.

The performance/robustness analysis of the resulting in PI and PID closed-loop control systems pointed out the existing *trade-off* between them. As shown in Alfaro et al. (2010), in general performance optimized 1DoF PI controllers are more robust than the PIDs but their optimal performance is lower. The performance optimized regulatory control systems, for both PI and PID, are less robust than the servo-control ones, requiring also more performance degradation, lower *degraded performance factor*, to reach the same robustness level.

4. UNIFIED SIMPLE OPTIMAL ROBUST TUNING FOR 1DOF PI AND PID CONTROLLERS ($USORT_1$)

One of the purposes of this contribution is try to capture in a single set of equations the performance/robustness trade-off. This is with no doubt a novel feature as the first- and second-order models are considered at once, without forcing a distinction with respect to neither the model used nor the controller structure. The other purpose is that these robust tuning equations be as simple as possible.

Analysis of the regulatory and servo-control PI and PID controllers parameters shows that for a model with a given time constants ratio a, increasing the control system robustness by decreasing M^t_S, results in a substantial reduction in K_p. However, this increase in the robustness has negligible effect on T_i and T_d, except in the case of models with a very low τ_o (when high robustness is required).

On the basis of this observation, equations *that are independent of the target robustness level* can be obtained for the controller integral time constant and derivative time constant, as follows:

$$T_i = \mathbf{F}(T, \tau_o, a), \quad T_d = \mathbf{G}(T, \tau_o, a). \tag{16}$$

With these equations at hand, the controller proportional gains are readjusted to match a target robustness to obtain equations given by the following

$$K_p = \mathbf{H}(K, \tau_o, a, M^t_S). \tag{17}$$

For FOPDT and SOPDT models with τ_o in the range from 0.1 to 2.0 and four M^t_S values the normalized 1DoF PI and PID controller parameters can be obtained using the process model parameters, $\theta_p = \{K, T, a, L, \tau_o\}$, for servo-control and regulatory control from the following relations:

- Regulatory control operation:

$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_o^{a_2}, \tag{18}$$

$$\tau_i \doteq \frac{T_i}{T} = b_0 + b_1 \tau_o^{b_2}, \tag{19}$$

$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_o^{c_2}, \tag{20}$$

Table 1. Regulatory Control PI Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.265	0.077	0.023	-0.128	-0.244
a_1	0.603	0.739	0.821	1.035	1.226
a_2	-0.971	-0.663	-0.625	-0.555	-0.517
	Target robustness $M_S^t = 1.8$				
a_0	0.229	0.037	-0.056	-0.160	-0.289
a_1	0.537	0.684	0.803	0.958	1.151
a_2	-0.952	-0.626	-0.561	-0.516	-0.472
	Target robustness $M_S^t = 1.6$				
a_0	0.175	-0.009	-0.080	-0.247	-0.394
a_1	0.466	0.612	0.702	0.913	1.112
a_2	-0.911	-0.578	-0.522	-0.442	-0.397
	Target robustness $M_S^t = 1.4$				
a_0	0.016	-0.053	-0.129	-0.292	-0.461
a_1	0.476	0.507	0.600	0.792	0.997
a_2	-0.708	-0.513	-0.449	-0.368	-0.317
b_0	-1.382	0.866	1.674	2.130	2.476
b_1	2.837	0.790	0.268	0.112	0.073
b_2	0.211	0.520	1.062	1.654	1.955

- Servo-control operation:

$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_o^{a_2}, \quad (21)$$

$$\tau_i \doteq \frac{T_i}{T} = \frac{b_0 + b_1\tau_o + b_2\tau_o^2}{b_3 + \tau_o}, \quad (22)$$

$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1\tau_o^{c_2}, \quad (23)$$

The value of the constants a_i, b_i, and c_i in (18) to (23) are listed in Tables 1 to 4. As noted in these Tables only the a_i constants for K_p calculation depend on the robustness level M_S.

Equations (18) to (23) provide a direct controller tuning for the FOPDT ($a = 0$) and the DPPDT ($a = 1$) models. In the case of the SOPDT models with $a \notin \{0.25, 0.5, 0.75\}$ the set of controller parameters must be obtained by linear interpolation between the two sets of parameters obtained with the adjacent a values used in the optimization.

The performance/robustness analysis also shows that the PI controllers with performance optimized parameters for servo-control operation produce control systems with a robustness $M_S \approx 1.8$. Then, the minimum robustness level of $M_S = 2.0$ is exceeded in this case.

With a maximum absolute deviation from the target robustness M_S^t of 4.09% and an average deviation of only 0.70% the proposed $uSORT_1$ tuning may be considered as a *global robust tuning method* with levels $M_S^t \in \{2.0, 1.8, 1.6, 1.4\}$ for FOPDT and SOPDT models with normalized dead-times in the range from 0.1 to 2.0.

Equations (18) to (20) and (21) to (23) were obtained for tuning Standard PID controllers. It is know that an equivalent Serial PID controller only exists if $T_i/T_d \geq 4$. As can be seen from Fig. 3 for the $uSORT_1$ regulatory control $\tau_i/\tau_d < 4$, then there is no Serial PID equivalent in this case, and that for the $uSORT_1$ servo-control in general $\tau_i/\tau_d \geq 4$ for time constant dominant models ($\tau_o \leq 1.0$). In the particular case of FOPDT controlled process models the servo-control Serial PID equivalent exists for $\tau_o \leq 1.4$.

Table 2. Regulatory Control PID Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.235	0.435	0.454	0.464	0.488
a_1	0.840	0.551	0.588	0.677	0.767
a_2	-0.919	-1.123	-1.211	-1.251	-1.273
	Target robustness $M_S^t = 1.8$				
a_0	0.210	0.380	0.400	0.410	0.432
a_1	0.745	0.500	0.526	0.602	0.679
a_2	-0.919	-1.108	-1.194	-1.234	-1.257
	Target robustness $M_S^t = 1.6$				
a_0	0.179	0.311	0.325	0.333	0.351
a_1	0.626	0.429	0.456	0.519	0.584
a_2	-0.921	-1.083	-1.160	-1.193	-1.217
	Target robustness $M_S^t = 1.4$ †				
a_0	0.155	0.228	0.041	0.231	0.114
a_1	0.455	0.336	0.571	0.418	0.620
a_2	-0.939	-1.057	-0.725	-1.136	-0.932
	†Valid only for $\tau_o \geq 0.40$ if $a \geq 0.25$				
b_0	-0.198	0.095	0.132	0.235	0.236
b_1	1.291	1.165	1.263	1.291	1.424
b_2	0.485	0.517	0.496	0.521	0.495
c_0	0.004	0.104	0.095	0.074	0.033
c_1	0.389	0.414	0.540	0.647	0.756
c_2	0.869	0.758	0.566	0.511	0.452

Table 3. Servo-Control PI Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 1.8$				
a_0	0.243	0.094	0.013	-0.075	-0.164
a_1	0.509	0.606	0.703	0.837	0.986
a_2	-1.063	-0.706	-0.621	-0.569	-0.531
	Target robustness $M_S^t = 1.6$				
a_0	0.209	0.057	-0.010	-0.130	-0.220
a_1	0.417	0.528	0.607	0.765	0.903
a_2	-1.064	-0.667	-0.584	-0.506	-0.468
	Target robustness $M_S^t = 1.4$				
a_0	0.164	0.019	-0.061	-0.161	-0.253
a_1	0.305	0.420	0.509	0.636	0.762
a_2	-1.066	-0.617	-0.511	-0.439	-0.397
b_0	14.650	0.107	0.309	0.594	0.625
b_1	8.450	1.164	1.362	1.532	1.778
b_2	0.0	0.377	0.359	0.371	0.355
b_3	15.740	0.066	0.146	0.237	0.209

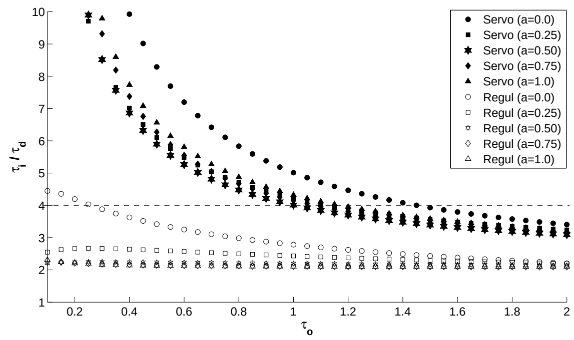

Figure 3. Servo and Regulatory Control τ_i/τ_d Ratio

Table 4. Servo-Control PID Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.377	0.502	0.518	0.533	0.572
a_1	0.727	0.518	0.562	0.653	0.728
a_2	-1.041	-1.194	-1.290	-1.329	-1.363
	Target robustness $M_S^t = 1.8$				
a_0	0.335	0.432	0.435	0.439	0.482
a_1	0.644	0.476	0.526	0.617	0.671
a_2	-1.040	-1.163	-1.239	-1.266	-1.315
	Target robustness $M_S^t = 1.6$				
a_0	0.282	0.344	0.327	0.306	0.482
a_1	0.544	0.423	0.488	0.589	0.622
a_2	-1.038	-1.117	-1.155	-1.154	-1.221
	Target robustness $M_S^t = 1.4$				
a_0	0.214	0.234	0.184	0.118	0.147
a_1	0.413	0.352	0.423	0.575	0.607
a_2	-1.036	-1.042	-1.011	-0.956	-1.015
b_0	1687	0.135	0.246	0.327	0.381
b_1	339.2	1.355	1.608	1.896	2.234
b_2	39.86	0.333	0.273	0.243	0.204
b_3	1299	0.007	0.003	-0.006	-0.015
c_0	-0.016	0.026	-0.042	-0.086	-0.110
c_1	0.333	0.403	0.571	0.684	0.772
c_2	0.815	0.613	0.446	0.403	0.372

Table 5. P_1 Servo-Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	-	0.778	0.646	0.482	-
T_i	-		2.546		-
M_S^r	-	1.81	1.61	1.40	-
$J_{er}/\Delta r$	-	2.947	3.282	4.392	-
	PID Controller				
K_p	1.132	1.003	0.846	0.642	1.174
T_i			3.022		3.085
T_d			0.495		0.589
M_S^r	2.0	1.80	1.60	1.40	2.21
$J_{er}/\Delta r$	2.458	2.512	2.976	3.918	2.481

5. EXAMPLES

For comparison of the performance and robustness obtained with the proposed $uSORT_1$ method we use the Madhuranthakam et al. (2008) [MEB] tuning rules for Standard PID controllers that optimize the IAE criteria for servo- and regulatory control operation.

First, we consider the FOPDT process given by

$$P_1(s) = \frac{1.2\mathrm{e}^{-1.5s}}{2s+1}.$$

The controller parameters and the control system performance and robustness for servo-control and regulatory control operation of P_1 are listed in Table 5 and Table 6, respectively.

As a second model we consider the SOPDT process given by

$$P_2(s) = \frac{1.2\mathrm{e}^{-1.5s}}{(2s+1)(s+1)}.$$

The controller parameters and the control system performance and robustness for servo-control and regulatory control operation of P_2 are listed in Table 7 and Table 8, respectively.

Table 6. P_1 Regulatory Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	0.885	0.779	0.651	0.500	-
T_i		2.576			-
M_S^r	2.01	1.81	1.61	1.42	-
$J_{ed}/\Delta d$	2.910	3.305	3.960	5.156	-
	PID Controller				
K_p	1.108	0.984	0.829	0.626	1.293
T_i		1.867			1.971
T_d		0.614			0.569
M_S^r	2.02	1.82	1.61	1.40	2.36
$J_{ed}/\Delta d$	1.969	2.215	2.593	3.303	1.666

Table 7. P_2 Servo-Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	-	0.711	0.590	0.441	-
T_i	-		3.421		-
M_S^r	-	1.83	1.62	1.41	-
$J_{er}/\Delta r$	-	4.311	4.831	6.469	-
	PID Controller				
K_p	1.110	0.989	0.839	0.625	1.497
T_i		4.264			5.121
T_d		0.921			0.812
M_S^r	1.98	1.79	1.61	1.40	2.78
$J_{er}/\Delta r$	3.385	3.596	4.234	5.687	3.798

Table 8. P_2 Regulatory Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	0.838	0.740	0.613	0.461	-
T_i		3.743			-
M_S^r	2.03	1.83	1.62	1.42	-
$J_{ed}/\Delta d$	4.466	5.059	6.102	8.098	-
	PID Controller				
K_p	1.037	0.951	0.801	0.620	1.539
T_i		2.454			2.971
T_d		1.108			0.883
M_S^r	1.93	1.79	1.60	1.41	2.94
$J_{ed}/\Delta d$	2.848	3.094	3.605	4.456	2.141

From Tables 5 to 8 it is noted that for same robustness design level (M_S^d) the PID controllers deliver more performance than the PI controllers. They also show the performance/robustness trade-off, an increment in control system robustness always reduces its performance. For example, to increase the robustness reducing M_S^d from 1.8 to 1.6 produces a 11 to 20% reduction in the control system performance.

It is also noted that the performance optimized MEB control systems have low robustness, $M_S > 2.0$ in all cases. Although the MEB controllers are performance optimized the servo-control $uSORT_1$ PID controllers for $M_S^d = 2.0$ produce control systems that are more robust and that at the same time have better performance.

The P_2 control system responses to a 10% set-point and load-disturbance step changes are shown in Fig. 4 and Fig. 5, respectively.

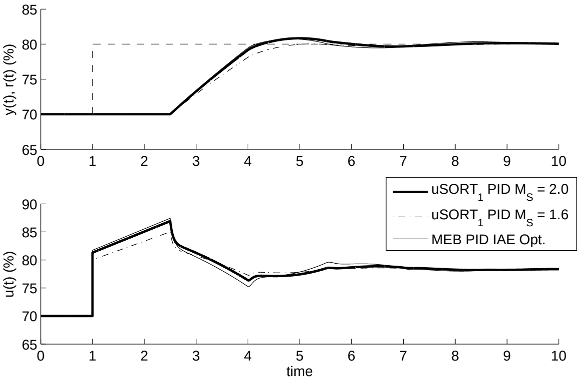

Figure 4. Model P_2 Servo-Control Responses

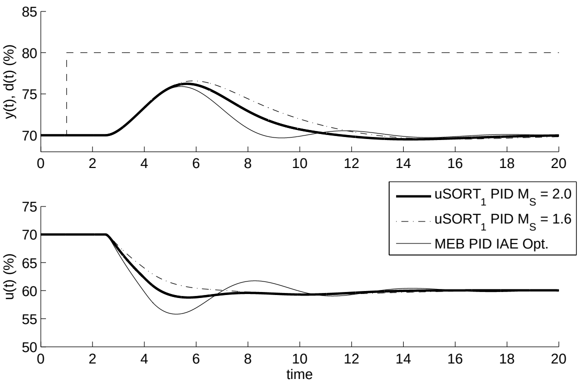

Figure 5. Model P_2 Regulatory Control Responses

6. CONCLUSIONS

Based on a performance (IAE) - robustness (M_S) analysis tuning relations are proposed that unifies the treatment of one-degree-of-freedom (1DoF) PI and PID controllers and the use of first- and second-order plus dead-time (FOPDT, SOPDT) models for servo- and regulatory control systems.

The proposed *Unified Simple Optimal and Robust Tuning* for 1DoF PI/PID controllers ($uSORT_1$) allows to adjust the control system robustness varying only the controller proportional gain.

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Fragility-Rings - A Graphic Tool for PI/PID Controllers Robustness-Fragility Analysis

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Abstract: The aim of the paper is to present the *Delta 20 Fragility-Rings* plot and its use for robustness-fragility analysis of proportional integral (PI) and proportional integral derivative (PID) controllers. Using the *Delta 20 Fragility Index* and the Nyquist plot it shows the areas on the $L(j\omega)$ plane corresponding to robustness-fragile, robustness-non-fragile and robustness-resilient controllers providing a visual aid for evaluation of the controller robustness-fragility when its parameters are perturbed.

Keywords: PID controllers, control system robustness, controllers fragility.

1. INTRODUCTION

Since Ziegler and Nichols (1942) presented their tuning rules, a great number of other tuning procedures have been developed for proportional integral (PI) and proportional integral derivative (PID) controllers, as revealed in O'Dwyer (2006) handbook.

At the beginning, only the control system *performance* was taken into account in the controller design, considering a step change either in the set-point, *servo-control* operation, or in the load-disturbance, *regulatory control* operation, as in the classic tuning rules of Cohen and Coon (1953), López et al. (1967), and Rovira et al. (1969), among others (Chien and Fruehauf, 1990; Rivera et al., 1986), for one-degree-of-freedom (1DoF) PI and PID controllers.

Later, the consideration of the control system relative stability, its *robustness* to the changes in the controlled process characteristics, was introduced into the controller design. Initially, considering the control-loop gain and phase margins (A_m, ϕ_m) as in Åström and Hägglund (1984); Fung et al. (1998) and Ho et al. (1995) . More recently, these classic robustness indicators have been replaced by a single value given by the maximum of the magnitude of the sensitivity function, denoted by M_S. This approach has been used in Alfaro et al. (2010); Åström and Hägglund (1995); Hägglund and Åström (2002) and Tavakoli et al. (2005).

There is, however, another consideration that must be taken into account when facing the design of control systems: the effect of the variation of the controller parameters over the control system stability and performance, known as the *controller fragility.* If the control system robustness is an indication of the margin of variation of the process characteristics with a fixed controller, then the controller fragility has a similar meaning but in terms of the variation of the controller parameters considering a fixed controlled process.

The fragility of certain controllers was documented by Keel and Battacharyya (1997). They found that many modern design techniques for optimum and robust controllers under the H_2, H_∞ and l_1 norms would produce extremely fragile, high-order controllers. They observed that in some cases, minimum variations of the parameters of these controllers would make the system unstable. A fragility analysis was included in the PID controller design by Datta et al. (2000), Ho (2000) and Silva et al. (2005).

Although, in control system designs, the assumption is often made that the controller can be implemented exactly, a certain degree of uncertainty inevitably exists in the controller implementation. The controller fragility is affected by the tolerances of its analog components. In its digital version, there are inaccuracies because of the use of fixed-length words and rounded errors of numerical calculations (Whidborne, 2000). In addition, the controller must allow variations of its parameters around their design values, making it easy to *fine-tune* the controller when the control loop is placed in service. The latter is the most probable cause of major variations in the controller parameters from their design, or nominal, values. Effectively, most of the tuning approaches, either based on tuning rules or on optimization methods, provide accurate values for the controller parameters, but due to the inaccuracies associated with the controlled process model used as part of the tuning procedure, normally these parameters should be taken only as a first approximation, and such final *fine-tuning* of the controller is normally required.

Considering the above, modern tuning rules for PI and PID controllers must take into account issues such as, the closed-loop servo and regulatory control *performance*, the

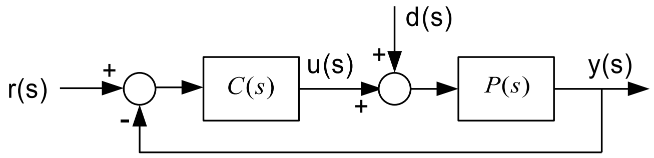

Figure 1. Closed-Loop Control System

control effort requirements, the control system *robustness*, and the controller *fragility*.

If the PID controller design takes into account the closed-loop performance to changes in its inputs, set-point and load-disturbance, and its robustness to changes in the controlled process characteristics, then it is evident that from the designer point of view, it is very important that these characteristics should be preserved regarding this fine-tuning of the controller. In addition, if this is not possible, then there should be at least some sort of information on how such changes in the controller parameters affect the control system robustness and performance.

The fragility-rings plot presented in this publication is based on the PID controller fragility definition introduced by Alfaro (2007) and Alfaro et al. (2009). It provides a graphical interpretation of the *Delta 20 robustness-fragility index* that is a measure of the control system loss of robustness when the controller parameters change.

The rest of the paper is organized as follows: in Section 2 the problem is formulated and the control system robustness and controller fragility indices are presented; the fragility-rings plot is described in Section 3 and its use to analyze the robustness-fragility of PI and PID controllers tuned with several tuning rules is shown in Section 4. The paper end with some conclusions.

2. PROBLEM FORMULATION

Consider the closed-loop control system of Fig. 1, where $P(s)$ and $C(s)$ are the *controlled process model* and the *controller* transfer function respectively. In this system $r(s)$ is the *set-point*, $u(s)$ is the *controller output signal*, $d(s)$ is the *load-disturbance* and $y(s)$ is the *controlled process variable*.

Without the loss of generality, it is supposed that the controller is a Standard PID controller (Visioli, 2006) whose transfer function is as follows:

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\alpha T_d s + 1}\right), \qquad (1)$$

where K_p is the controller *proportional gain*, T_i is the *integral time constant*, T_d is the *derivative time constant*. In (1), α is the *derivative filter constant*, usually $\alpha = 0.10$ (Corripio, 2001).

The closed-loop characteristic equation is as follows:

$$1 + L(s) = 1 + C(s)P(s) = 0. \qquad (2)$$

The control system stability depends on the controlled process model $P(s)$, with parameters $\bar{\theta}_p$, and on the controller $C(s)$, with parameters $\bar{\theta}_c = \{K_p, T_i, T_d\}$. The parameters of the controlled process model transfer function will be considered constant for the fragility analysis.

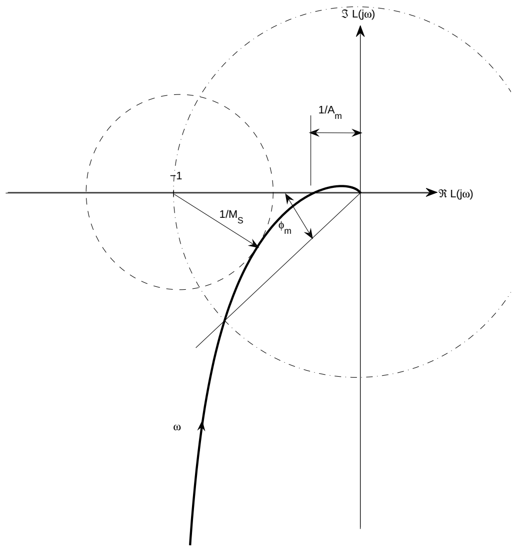

Figure 2. Definition of the Control System Relative Stability Margins

2.1 Control System Robustness Evaluation

There are several quantitative measures of the control system *relative stability* that may be used for the robustness fragility definition, such as the classical *Gain Margin* and *Phase Margin* (A_m, ϕ_m) (Goodwin et al., 2001), that provide an indication of the distance from the open-loop transfer function, $L(\mathrm{j}\omega)$, frequency response, or Nyquist curve, to the critical point (-1,0) on the open-loop polar graph.

Another way to express the system robustness is by using the *Stability Margin*, which is the shortest distance from the Nyquist curve to the critical point (Åström and Murray, 2008). This distance is the reciprocal of the maximum peak of the sensitivity function, or *Maximum Sensitivity* (M_S) (Åström and Hägglund, 1995), defined as follows:

$$M_S \doteq \max_{\omega} |S(\mathrm{j}\omega)| = \max_{\omega}\left|\frac{1}{1 + C(\mathrm{j}\omega)P(\mathrm{j}\omega)}\right|. \qquad (3)$$

The use of the maximum sensitivity as a robustness measure has the advantage that lower bounds to the gain and phase margins can be assured according to the following (Åström and Hägglund, 1995):

$$A_m > \frac{M_S}{M_S - 1}, \quad \phi_m > 2\sin^{-1}\left(\frac{1}{2M_S}\right). \qquad (4)$$

The relations in (4) can be obtained from Fig. 2.

For the controller robustness fragility definitions, we use the maximum sensitivity, M_S, as a measure of the closed-loop control system robustness.

2.2 Delta Epsilon Fragility Indices

The concept of PID controllers fragility, the Delta-Epsilon-Fragility Index $FI_{\Delta\epsilon}$ and their application to define when a controller is considered *fragile*, *non-fragile* or *resilient*, were introduced by Alfaro (2007). In our context, the *PID controller fragility* is an indication of the reduction of the closed-loop control system robustness and/or performance when the controller parameters are perturbed.

For the fragility analysis, the controlled process is represented by a nominal model of the fixed parameters $\bar{\theta}_p^o$, obtained at the control system normal operation point. This model is used for tuning the controller; then, the controller nominal parameters are $\bar{\theta}_c^o$ and their delta epsilon perturbations $\delta\epsilon$. In the following, $\delta\epsilon$ denotes the variation of each individual controller parameter and $\Delta\epsilon$ will be used when all controller parameters are perturbed.

The controller *Delta-Epsilon-Robustness-Fragility Index* relates the control system loss of robustness to its nominal robustness and is given by the following:

$$RFI_{\Delta\epsilon} \doteq \frac{M_{S\Delta\epsilon}^m}{M_S^o} - 1 = \frac{\max\{M_S((1 \pm \delta\epsilon)\bar{\theta}_c^o)\}}{M_S(\bar{\theta}_c^o)} - 1, \quad (5)$$

where $M_{S\Delta\epsilon}^m$ and M_S^o are the control system extreme and the nominal maximum sensitivity, respectively.

The *extreme maximum sensitivity*, $M_{S\Delta\epsilon}^m$, represents the highest loss of robustness of the control system when all the parameters of the controller, $\bar{\theta}_c$, have been perturbed by the same $\delta\epsilon$ amount from their nominal values, $\bar{\theta}_c^o$, considering all the possible combinations of the perturbed parameters.

In the ideal case, for a completely delta epsilon robustness-resilient (or absolutely robustness-non-fragile) controller, $RFI_{\Delta\epsilon} = 0$, the controller would not lose robustness when its nominal parameters, $\bar{\theta}_c^o$, are perturbed by $\delta\epsilon$.

The relative influence of a $\delta\epsilon$ change in the controller parameter p_i over its robustness fragility can be obtained with the *Parametric-Delta-Epsilon-Robustness-Fragility Index* given by the following:

$$RFI_{\delta\epsilon}^{p_i} \doteq \frac{M_{S\delta\epsilon}^{p_i}}{M_S^o} - 1 = \frac{\max\{M_S((1 \pm \delta\epsilon)p_i, \bar{\theta}_c^o)\}}{M_S(\bar{\theta}_c^o)} - 1. \quad (6)$$

The final *fine-tuning* of the control-loop is considered the most probable cause of major variations in the controller parameters, for example, in practice, it is possible to see commissioning changes up to 10% or 20% in their values. Considering this, the *Delta 20 Robustness-Fragility Index* can be defined to measure the maximum loss of the control system robustness when a change of up to 20% occurs in one or more of the nominal controller parameters values and is given by the following:

$$RFI_{\Delta 20} \doteq \frac{M_{S\Delta 20}^m}{M_S^o} - 1. \quad (7)$$

A controller is considered *robustness-fragile* if the control system loses more than 50% of its robustness when all its parameters change up to 20%; otherwise, it is *robustness-non-fragile*. In addition, a controller is *robustness-resilient* if the control system does not lose more than 10% of its robustness when its parameters change up to 20%. A controller with a low robustness-fragility will allow final fine-tuning without a significant reduction in the control system robustness. Therefore, based on the $RFI_{\Delta 20}$, the controller robustness fragility degree is defined as follows:

- *Robustness Fragile PID controller*: a PID controller is robustness-fragile if its delta 20 robustness fragility index is higher than 0.50, $RFI_{\Delta 20} > 0.50$.
- *Robustness Non-Fragile PID controller*: a PID controller is robustness-non-fragile if its delta 20 robustness fragility index is less than or equal to 0.50, $RFI_{\Delta 20} \leq 0.50$.
- *Robustness Resilient PID controller*: a PID controller is robustness-resilient if its delta 20 robustness fragility index is less than or equal to 0.10, $RFI_{\Delta 20} \leq 0.10$.

The selection of a ±20% ($\Delta 20$) change in the controller parameters for the robustness fragility definition above considers a 10% reduction in the control system robustness as marginal and a 50% reduction as the maximum allowed limit. Such maximum variation will turn a highly robust system, with M_S lower than 1.4, into one with a minimally acceptable robustness, M_S of approximately 2.0. However, using (5) and (6) it is possible to evaluate the effect of any other particular $\delta\epsilon$ perturbation in one or more controller parameters.

3. ROBUSTNESS-FRAGILITY RINGS PLOT

We will present here the *Delta 20 Robustness-Fragility Rings Plot*, which is a simple tool that uses the open-loop transfer function, $L(j\omega)$, Nyquist curve of the nominal and Delta 20 perturbed controllers to provide an indication of the control system robustness-fragility. It shows the areas in the $L(j\omega)$ plane that define when the controller is a robustness-resilient controller (RRC), a robustness-non-fragile controller (RNFC), or a robustness-fragile controller (RFC) as shown in Figure 3.

The plot includes the Nyquist curve of the nominal open-loop transfer function that defines the control system *nominal robustness level*, M_S^o, and states the robustness-fragility rings as follows:

- RRC Ring: $M_S^o \leq M_S \leq 1.1M_S^o$,
- RNFC Ring: $1.1M_S^o < M_S \leq 1.5M_S^o$,
- RFC Area: $1.5M_S^o < M_S$.

It also includes the Nyquist curve corresponding to the *extreme maximum sensitivity*, M_S^m, that gives the delta 20 robustness-fragility index, $RFI_{\Delta 20}$.

For example, the requirement for a robustness-non-fragile controller demands that the control system Nyquist curve does not enter the RFC area when all controller parameters are changed up to ±20%.

4. PI AND PID CONTROLLERS TUNING ROBUSTNESS-FRAGILITY EVALUATION

In the following the robustness-fragility of PI and PID controllers tuned with several well-know tuning rules will be analyzed using the fragility-rings plot.

Consider the following normalized first- and second-order plus dead-time controlled process models:

$$P_1(s) = \frac{e^{-0.3s}}{s+1}, \ \tau_o = 0.3 \quad (8)$$

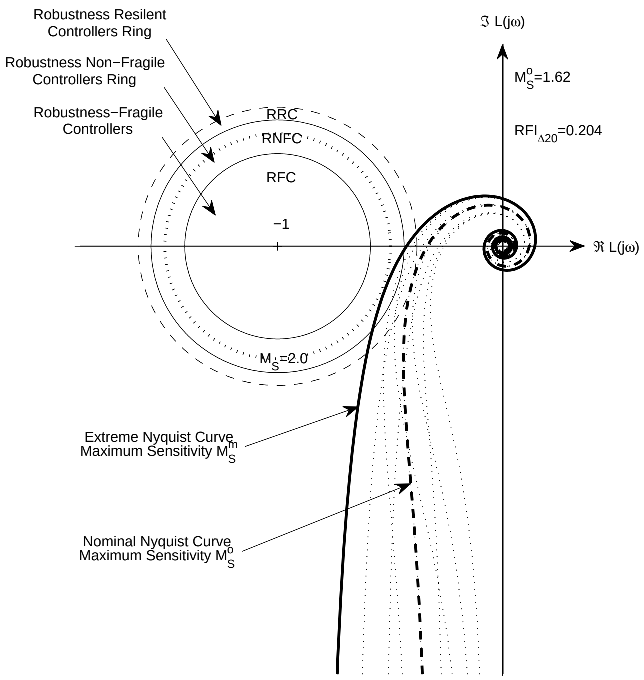

Figure 3. Controller Delta 20 Robustness-Fragility Rings Plot

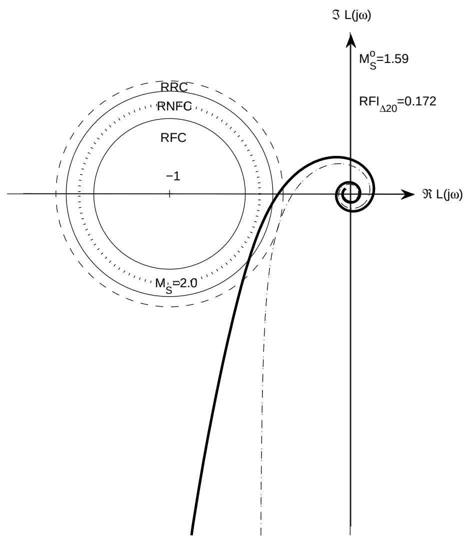

Figure 4. P_1 SIMC PI Robustness-Fragility

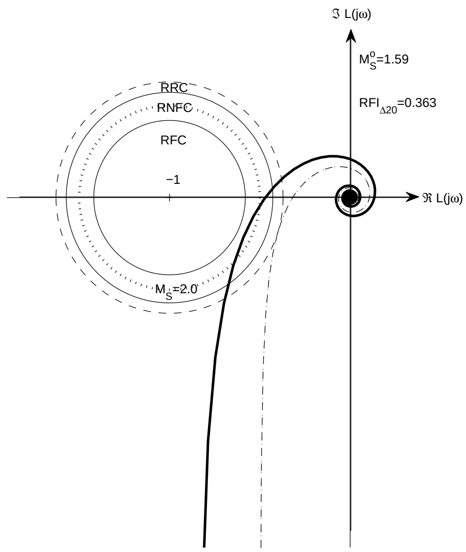

Figure 5. P_2 SIMC PI Robustness-Fragility

Table 1. Controllers Parameters

Method	Model	K_p	T_i	T_d
SIMC	P_1	1.667	1	0
SIMC	P_2	0.313	1	0
PO_S	P_3	2.430	1.800	0.444
PO_T	P_3	3.450	1.800	0.444
PI_{2Ms} $(M_S = 2.0)$	P_1	2.279	0.905	0
PI_{2Ms} $(M_S = 1.6)$	P_1	1.707	0.982	0
PI_{2Ms} $(M_S = 1.4)$	P_1	1.270	1.054	0
PI_{2Ms} $(M_S = 2.0)$	P_2	0.665	1.748	0
PI_{2Ms} $(M_S = 1.6)$	P_2	0.467	1.591	0
PI_{2Ms} $(M_S = 1.4)$	P_2	0.308	1.312	0

$$P_2(s) = \frac{\mathrm{e}^{-1.6s}}{s+1},\ \tau_o = 1.6 \tag{9}$$

$$P_3(s) = \frac{\mathrm{e}^{-0,3s}}{(s+1)(0.8s+1)},\ a = 0.8,\ \tau_o = 0.3 \tag{10}$$

where a and τ_o are the model time constants ratio and normalized dead-time, respectively.

Tuning rules to analyze include: *Simple Control* (SIMC) (Skogestad, 2003), *Percent Overshoot* (PO) (Ali and Majhi, 2009) and *Maximum Sensitivity-Based Robust Tuning* (PI_{2Ms}) (Alfaro et al., 2010). The controllers parameters obtained with these rules are listed in Table 1.

4.1 Simple Control (SIMC)

The PI SIMC tuning for FOPDT models produces control systems with a robustness $M_S = 1.59$ for all model normalized dead-times as shown in Figs. 4 and 5. The fragility rings plots show that both SIMC controllers are robustness non-fragile and that their robustness-fragility is affected by τ_o. For the P_1 model ($\tau_o = 0.3$) $RFI_{\Delta 20} = 0.172$ while for the P_2 model ($\tau_o = 1.6$) $RFI_{\Delta 20} = 0.363$. The controllers turn to more robustness-fragile as the model normalized dead-time increases requiring a more careful final *fine-tuning*.

4.2 Percent Overshoot (PO)

The PO method includes two design criteria for PI (FOPDT) and PID (SOPDT) controllers. A *smooth control* design (PO_S) for 0% OS ($M_S = 1.38$) and a *tight control* (PO_T) design for 10% OS ($M_S = 1.71$). As shown in Figs. 6 and 7 the target robustness are not obtained with the PID controllers. The main reason for this is that the PO method was obtained using an "ideal" non-proper

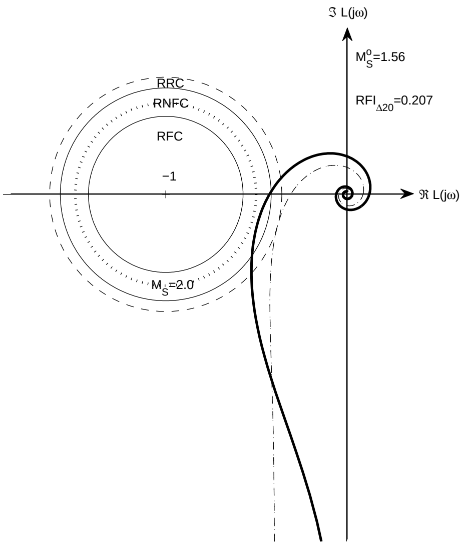

Figure 6. P_3 PO_S PID Robustness-Fragility

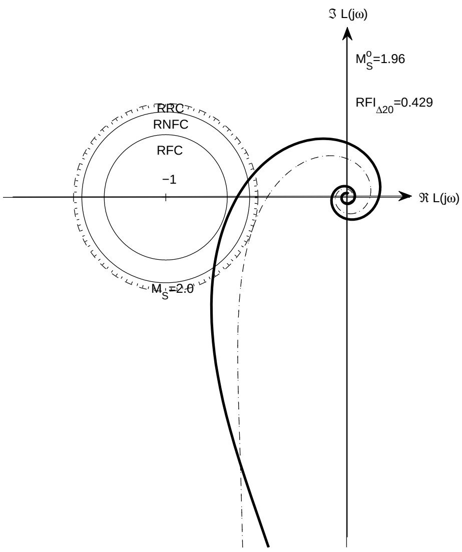

Figure 7. P_3 PO_T PID Robustness-Fragility

PID controller while we tested it using a "real" Standard PID controllers (with derivative filter). Both controllers are robustness-non-fragile but the PO_T controller is less robust and more fragile.

4.3 Maximum Sensitivity-Based Robust Tuning (PI_{2Ms})

The PI_{2Ms} uses a closed-loop model-reference design with a robustness constraint, $M_S \in \{1.4, 1.6, 1.8, 2.0\}$. In Figs. 8 and 9 two extreme cases are shown: a high robustness design ($M_S = 1.4$) for a process with low normalized dead-time (P_1) and a minimum robustness design ($M_S = 2.0$) for a process with high normalized dead-time (P_2).

In the first case $M_S^o = 1.4$ and the controller is nearly robustness-resilient with $RFI_{\Delta 20} = 0.116$. The control

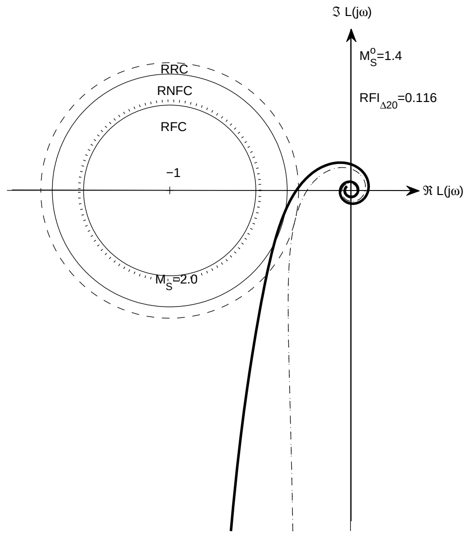

Figure 8. P_1 $M_S^t = 1.4$ PI_{2Ms} PI Robustness-Fragility

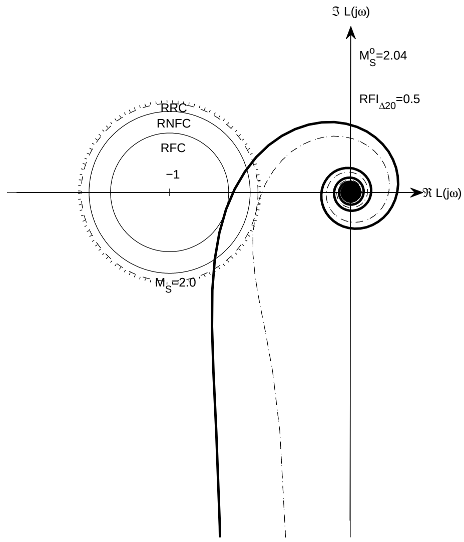

Figure 9. P_2 $M_S^t = 2.0$ PI_{2Ms} PI Robustness-Fragility

system is robust ($M_S < 2$) even in case of a change of up to ±20% in the controller parameters. In the second $M_S^o = 2.04$ and the controller is at the border of the robustness-fragile controllers area ($RFI_{\Delta 20} = 0.5$). The control system may turn non-robust if the controller parameters are perturbed.

5. CONCLUSIONS

Based on the Robustness-Fragility Indices the *Delta 20 Fragility-Rings* plot provides information of the controller robustness-fragility in case its parameters are changed as in the controller final fine-tuning. It is a valuable tool for control system design, which allows anticipating and quantifying the possible loss of robustness.

The delta 20 fragility-rings plot shows that even a tuning rule may provide control systems with same robustness for models with a wide range of normalized dead-times, τ_o, the controller robustness-fragility depends on τ_o turning more fragile as the model normalized dead-time increases.

The controller robustness-fragility depends not only on the model dead-time but also on the control system nominal robustness. Then, the selection of the design robustness level must consider, in addition to the expected changes in the controlled process dynamics, the controller robustness-fragility for the tuning rule used.

The PI and PID controller design should take into account not only the existing *performance/robustness* trade-off but also the controller *robustness-fragility*.

As an extension of the robustness-fragility-rings an analysis of the controller fragility in the frequency domain will provide an in-depth knowledge of its implication on the control system performance and robustness.

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Model Reference Robust Tuning of 2DoF PI Controllers for Integrating Controlled Processes

Víctor M. Alfaro and Ramon Vilanova

***Abstract*— The aim of this paper is to present a robust tuning method of two-degree-of-freedom (2DoF) proportional integral (PI) controllers for integrating controlled processes. This is based on the use of a model reference optimization procedure with servo and regulatory closed-loop transfer functions targets. The designer is allowed to deal with the performance/robustness *trade-off* of the closed-loop control system by specifying the desired robustness level through selecting a maximum sensitivity in the range from 1.4 to 2.0. In addition, a *smooth servo/regulatory performance combination* is obtained by forcing both closed-loop transfer functions to perform as closely as possible to non-oscillatory dynamic targets. Controller tuning equations that guarantee the design robustness level are provided for integrating second-order plus dead-time (ISOPDT) models with normalized dead-times from 0.1 to 2.0, and integrating plus dead-time (IPDT) models. The robustness of the control system is analyzed. Comparative examples show the effectiveness of the proposed tuning method.**

I. INTRODUCTION

Even though most of the controlled processes found in the process industry are self-regulating, i.e. the process output seeks a stable operating point under a constant input, there are others that under a constant input their output is unbounded, rise or decrease without limit. These non-self regulated process are named integrating or unstable if their model transfer functions have a pole at the s-plane origin or at its right-half plane, respectively. Stable processes with very long time constants may also be approximated by integrating models. Integrating and unstable processes may be operated only under closed-loop automatic control and their controller tunning needs a special treatment. Integrating characteristics may be found by example in tank level processes and communication networks [4], [8], [14].

For the integrating processes there are IMC-based tuning methods [4], [8] for one-degree-of-freedom (1DoF) proportional integral (PI) and proportional integral derivative (PID) controllers that include a design parameter, the closed-loop time constant, that can be used to deal with the control system performance/robustness trade-off. Kappa-Tau [5] and AMIGO [9], [10] methods provide tuning rules for two-degree-of-freedom (2DoF) PI and PID controllers for high robustness, $M_S = 1.4$.

Robustness was also included in the controller design for integrating processes using the gain and phase margins [16] and the maximum sensitivity [15].

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The IMC-based tuning SIMC [11] for 1DoF PI controllers is based on an integrated plus dead-time model and for 1DoF Ideal PID controllers on an integrating second-order plus dead-time model. The design parameter, the servo-control closed-loop time constant, is selected by a trade-off between fast speed of response, good disturbance rejection and robustness. With the SIMC controllers the control system has an intermediate robustness, $M_S = 1.70$.

An alternative tuning method of 2DoF proportional integral (PI_2) controllers for integrating controlled processes is presented in this communication. The proposed approach explicitly considers the *trade-off* between the performance and robustness of a control system. The distinctive feature of the resultant tuning procedure is the incorporation of the desired robustness level as measured with the maximum sensitivity, M_S, which is the explicit and only design parameter. Therefore, the designer may select the desired robustness M_S level for the control system in the range from 1.4 to 2.0.

II. PROBLEM FORMULATION

The controller design procedure described below for integrating controlled processes follows the model reference robust tuning (MoReRT) methodology proposed for over damped controlled processes [1], [2].

Consider a closed-loop control system, as shown in Fig. 1, where $P(s)$ and $C(s)$ are the controlled process model transfer function and the controller transfer function, respectively. In this system, $r(s)$ is the set-point, $u(s)$ is the controller output signal, $d(s)$ is the load-disturbance, and $y(s)$ is the process controlled variable.

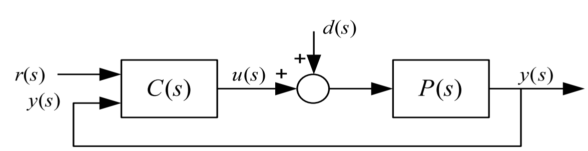

Fig. 1. Closed-Loop Control System

The closed-loop control system output, $y(s)$, in response to changes in its inputs, $r(s)$ and $d(s)$, is given by the following:

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \qquad (1)$$

where $M_{yr}(s)$ is the transfer function from the set-point to the process controlled variable, and $M_{yd}(s)$ is that from the load-disturbance to the process controlled variable. These are known as the *servo-control* closed-loop transfer function and the *regulatory control* closed-loop transfer function, respectively.

The development of the proposed tuning method of 2DoF PI controllers for integrating controlled process will take into account not only the closed-loop control system performance, by stating target responses for step changes in the set-point and the load-disturbance, but also the control system robustness, measured in terms of the maximum sensitivity, M_S.

A. 2DoF Proportional Integral Controller (PI_2)

The process will be controlled with a two-degree-of-freedom proportional integral (PI_2) controller [5] whose output is expressed as follows:

$$u(s) = K_p \left\{ \beta r(s) - y(s) + \frac{1}{T_i s}\left[r(s) - y(s)\right] \right\}, \quad (2)$$

where K_p is the controller *proportional gain*, T_i is the *integral time constant*, and β is the *set-point proportional weight*.

For the purposes of analysis only, not implementation, the controller output (2) will be rewritten as follows:

$$u(s) = C_r(s)r(s) - C_y(s)y(s), \quad (3)$$

where

$$C_r(s) = K_p\left(\beta + \frac{1}{T_i s}\right), \quad (4)$$

is the PI_2 controller aspect that operates on the set-point r, the *set-point controller* transfer function, and

$$C_y(s) = K_p\left(1 + \frac{1}{T_i s}\right), \quad (5)$$

is the PI_2 controller aspect that operates on the feedback signal y, the *feedback controller* transfer function.

The closed-loop transfer functions of the servo control and the regulatory control in (1) are then given by

$$M_{yr}(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)}, \quad (6)$$

and

$$M_{yd}(s) = \frac{P(s)}{1 + C_y(s)P(s)}, \quad (7)$$

which are related as follows:

$$M_{yr}(s) = C_r(s)M_{yd}(s). \quad (8)$$

III. Controller Design

Usually the design of the 2DoF PI controllers is performed in two stages [6], [9], [12]. First, as it is required to obtain the desired regulatory control performance and specific closed-loop control system robustness level, the parameters (K_p, T_i) of the feedback controller (5) are determined for a parameter set of the controlled process model $\overline{\theta}_p$. Second, the set-point controller (4) free parameter (β) is used to improve the servo-control performance.

In what follows a different approach is taken. The complete set of PI_2 controller parameters $\overline{\theta}_c = \{K_p, T_i, \beta\}$ will be obtained considering, at the same time, the regulatory control and the servo-control performance, to obtain a controller with a targeted *servo/regulatory performance combination* that will also produce a closed-loop control system with a specific robustness level.

A. Cost Functionals

For the regulatory control response, the cost functional to be minimized is defined as follows:

$$J_d \doteq \int_0^\infty \left[y_d^t(\tau_c, \overline{\theta}_c, \overline{\theta}_p, t) - y_d(\overline{\theta}_c, \overline{\theta}_p, t)\right]^2 \mathrm{d}t, \quad (9)$$

where $y_d^t(\tau_c, \overline{\theta}_c, \overline{\theta}_p, t)$ is the step response of the regulatory control closed-loop transfer function target, that will be defined later, and $y_d(\overline{\theta}_c, \overline{\theta}_p, t)$ is that of the regulatory control system $M_{yd}(s)$ (7) with the controlled process $P(s)$ and controller $C_y(s)$ (5).

In (9) τ_c is the dimensionless design parameter, the relative closed-loop system speed. Its role will become clear after the next section when describing the target transfer functions.

In a similar way, the servo-control cost functional to be minimized is defined as follows:

$$J_r \doteq \int_0^\infty \left[y_r^t(\tau_c, \overline{\theta}_p, t) - y_r(\overline{\theta}_c, \overline{\theta}_p, t)\right]^2 \mathrm{d}t, \quad (10)$$

where $y_r^t(\tau_c, \overline{\theta}_p, t)$ is the step response of the servo-control closed-loop transfer function target, defined later, and $y_r(\overline{\theta}_c, \overline{\theta}_p, t)$ is that of the servo-control system $M_{yr}(s)$ (6) with the controlled process $P(s)$ and controller $C_r(s)$ (4).

B. Controller Optimization

For the 2DoF PI controller design, the following overall cost functional is optimized:

$$J_T(\tau_c, \overline{\theta}_c, \overline{\theta}_p) \doteq J_r(\tau_c, \overline{\theta}_c, \overline{\theta}_p) + J_d(\tau_c, \overline{\theta}_c, \overline{\theta}_p), \quad (11)$$

to obtain the optimum controller parameters $\overline{\theta}_c^o = \left\{K_p^o, T_i^o, \beta^o\right\}$. Note that $\overline{\theta}_c^o = \overline{\theta}_c^o(\overline{\theta}_p, \tau_c)$.

Moreover, for each $\overline{\theta}_c^o$ set obtained, the closed-loop control system robustness is measured using the maximum sensitivity M_S, which is defined as follows:

$$M_S \doteq \max_\omega |S(\mathrm{j}\omega)| = \max_\omega \frac{1}{|1 + C_y(\mathrm{j}\omega)P(\mathrm{j}\omega)|}. \quad (12)$$

IV. Controlled Process Models and Closed-Loop Transfer Functions Targets

For the integrating processes two models are to be considered: the integrating second-order plus dead-time model, and the integrating plus dead-time model.

A. Integrating Second-Order Plus Dead-Time Models

We consider first the integrating second-order plus dead-time (ISOPDT) model given by the following:

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{s(Ts+1)}, \quad (13)$$

were K is the gain, T the time constant and L the dead-time. The controlled process parameters are $\overline{\theta}_p = \{K, T, L\}$.

Taking into account the second-order controlled process model (13) and the feedback part of the PI controller (5), the desired regulatory control closed-loop transfer function is obtained as the following third-order transfer function target:

$$M_{yd}^t(s) = \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(T_c s+1)^3}, \tag{14}$$

where T_c is the closed-loop time-constant.

Using (4) and (14) in (8) the servo-control closed-loop transfer function is given by

$$M_{yr}(s) = \frac{(\beta T_i s+1)\mathrm{e}^{-Ls}}{(T_c s+1)^3}. \tag{15}$$

In order to have a set-point step change response without oscillation, overshoot, or steady-state error, the servo-control closed-loop transfer function target is selected as follows:

$$M_{yr}^t(s) = \frac{\mathrm{e}^{-Ls}}{(T_c s+1)^2}. \tag{16}$$

The zero/pole cancellation required to obtain (16) ($\beta T_i = T_c$) will not be forced but taken into account by the optimization procedure to match the servo-control target close-loop target transfer function.

Then in the ISOPDT model case the global control system output target $y^t(s)$ is computed as:

$$y^t(s) = \frac{\mathrm{e}^{-Ls}}{(\tau_c T s+1)^2}r(s) + \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c T s+1)^3}d(s). \tag{17}$$

where $\tau_c \doteq T_c/T$ is the dimensionless design parameter, which is an indication of the closed-loop system response speed in relation to the controlled process speed.

Using the controlled process parameters $\overline{\theta}_p$ as well as the transformation $\hat{s} \doteq Ts$, the controlled process (13) and the PI controller transfer functions (4) and (5) can be expressed in a normalized form as follows:

$$\hat{P}(\hat{s}) = \frac{\mathrm{e}^{-\tau_o \hat{s}}}{\hat{s}(\hat{s}+1)}, \tag{18}$$

$$\hat{C}_r(\hat{s}) = \kappa_p\left(\beta + \frac{1}{\tau_i \hat{s}}\right),\ \hat{C}_y(\hat{s}) = \kappa_p\left(1 + \frac{1}{\tau_i \hat{s}}\right), \tag{19}$$

where $\tau_o \doteq L/T$ is the model *normalized dead-time* and

$$\kappa_p \doteq K_p\ K\ T, \quad \tau_i \doteq \frac{T_i}{T}, \tag{20}$$

are the *normalized gain* and *normalized integrating time* of the controller, respectively.

The normalized controlled process model (18) has only one dimensionless parameter, τ_o.

During the optimization procedure, the closed-loop relative speed parameter τ_c is selected in such a way that the robustness level of the resulting closed-loop system met a specific target M_S^t in the range from 1.4 to 2.0.

From the optimization results, it is possible to obtain the normalized controller parameters and the resulting control system robustness as functions of the model parameters, $\overline{\theta}_p$, and the performance specification, τ_c. However, to simplify the design procedure, the controller parameters are obtained directly as functions only of the closed-loop control system robustness parameter, which is the maximum sensitivity, M_S.

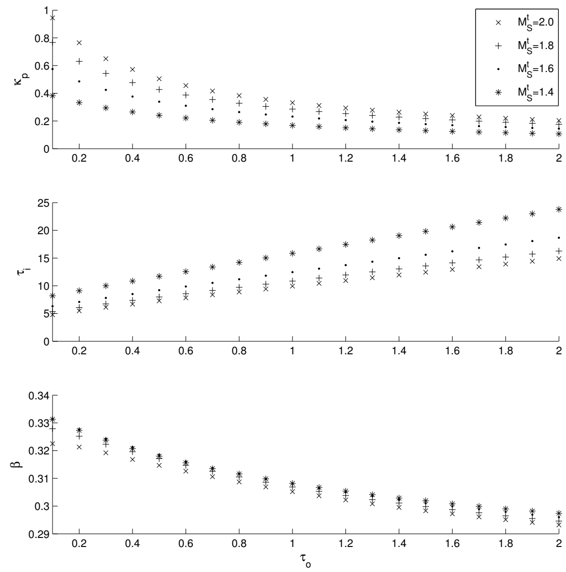

Fig. 2. PI Controller Parameters, ISOPDT Models

The controller optimum normalized parameters (κ_p, τ_i, β) obtained are shown in Fig. 2 along the entire dead-time range evaluated and the four robustness levels, $M_s^t = \{2.0, 1.8, 1.6, 1.4\}$.

This shows the influence of the controlled process dynamics (τ_o) and the desired robustness (M_S^t) over the controller parameters required to meet the target step responses.

The controller parameters obtained from the optimization procedure are used to fit the controller parameter equations of the proposed *Mo*del *Re*ference *R*obust *T*uning (*MoReRT*).

The normalized controller parameters can be obtained with the following equations:

$$\kappa_p = \frac{a_0 + a_1\tau_o}{a_2 + a_3\tau_o + {\tau_o}^2}, \tag{21}$$

$$\tau_i = b_0\mathrm{e}^{b_1\tau_o} + b_2\mathrm{e}^{b_3\tau_o}, \tag{22}$$

$$\beta = c_0 + c_1\tau_o + c_2{\tau_o}^2. \tag{23}$$

Table I lists the a_i, b_i and c_i constants for (21) to (23) for each of the four robustness levels.

B. Integrating Plus Dead-Time Models

Integrating and over damped processes with very large time constants can also be approximated by an integrating plus dead-time (IPDT) model given by the following:

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{s}, \tag{24}$$

were K is the gain and L the dead-time. The controlled process parameters are $\overline{\theta}_p = \{K, L\}$.

Using the same procedure described above the desired regulatory control closed-loop transfer function is obtained

TABLE I

MoReRT Constants, ISOPDT Models

	Target robustness M_S^t			
	1.4	1.6	1.8	2.0
a_0	0.4141	0.3781	0.3335	0.4156
a_1	0.3126	0.4085	0.4826	0.5484
a_2	0.9140	0.5324	0.3363	0.3317
a_3	2.402	1.864	1.519	1.567
b_0	18.38	12.77	10.27	9.123
b_1	0.2110	0.2408	0.2637	0.2735
b_2	-11.08	-7.192	-5.589	-4.978
b_3	-0.4811	-0.6446	-0.8042	-0.8900
c_0	0.3331	0.3336	0.3305	0.3257
c_1	-0.03254	-0.03273	-0.02941	-0.002441
c_2	0.007548	0.00713	0.005826	0.004094

as the following second-order target transfer function target:

$$y^t(s) = \frac{e^{-Ls}}{\tau_c Ls+1} r(s) + \frac{(T_i/K_p)se^{-Ls}}{(\tau_c Ls+1)^2} d(s), \qquad (25)$$

where τ_c is the design parameter.

Using the controlled process parameters $\overline{\theta}_p$ as well as the transformation $\tilde{s} \doteq Ls$, the controlled process (24) and the PI controller transfer functions (4) and (5) can be expressed in a normalized form as follows:

$$\tilde{P}(\tilde{s}) = \frac{e^{-\tilde{s}}}{\tilde{s}}, \qquad (26)$$

$$\tilde{C}_r(\tilde{s}) = \kappa_p \left(\beta + \frac{1}{\tau_i \tilde{s}}\right), \ \tilde{C}_y(\tilde{s}) = \kappa_p \left(1 + \frac{1}{\tau_i \tilde{s}}\right), \qquad (27)$$

where

$$\kappa_p \doteq K_p \ K \ L, \quad \tau_i \doteq \frac{T_i}{L}, \qquad (28)$$

are the *normalized gain* and *normalized integrating time* of the controller, respectively.

As the normalized controlled process model (26) does not have any variable parameter just one optimization run is required for each robustness level.

In the same way as with the ISOPDT model during the optimization processes, the closed-loop relative speed parameter τ_c is selected in such a way that the robustness level of the resulting closed-loop system met a specific target (M_S^t) in the range from 1.4 to 2.0 and the controller parameters are obtained directly as functions only of the closed-loop control system robustness.

The MoReRT tuning equations for the IPDT models are as follows:

$$\kappa_p = a, \qquad (29)$$

$$\tau_i = b, \qquad (30)$$

$$\beta = c. \qquad (31)$$

Table II lists the a, b and c constants for (29) to (31) for each one of the four robustness levels.

TABLE II

MoReRT Constants, IPDT Models

	Target robustness M_S^t			
	1.4	1.6	1.8	2.0
a	0.332	0.442	0.528	0.599
b	10.636	7.885	6.579	5.823
c	0.452	0.434	0.419	0.406

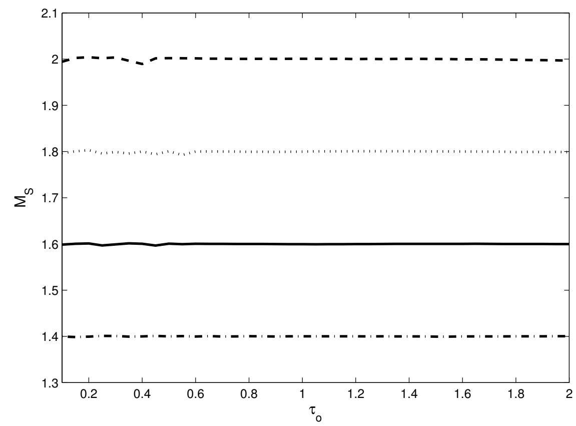

Fig. 3. MoReRT Robustness for ISOPDT Models

C. MoReRT Control System Robustness

The robustness obtained with (21) to (23) for each ISOPDT normalized dead-time in the range analyzed is shown in Fig. 3. As can be seen all the robustness profiles are nearly flat. This means that for an ISOPDT model of a controlled process the MoReRT tuning guarantees that the robustness target is attained for all normalized dead-times in the range considered.

It was also verified that the robustness obtained with (29) to (31) for any IPDT model dead-time L matches the target robustness as shown in Fig. 4.

The achievement of the robustness target for all the

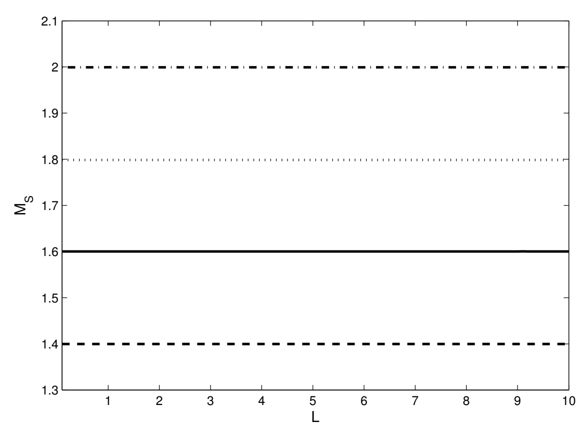

Fig. 4. MoReRT Robustness for IPDT Models

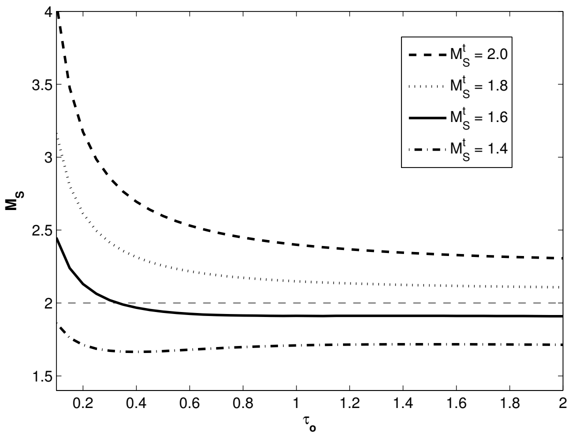

Fig. 5. ISOPDT Models - Wang & Cai Tuning Robustness

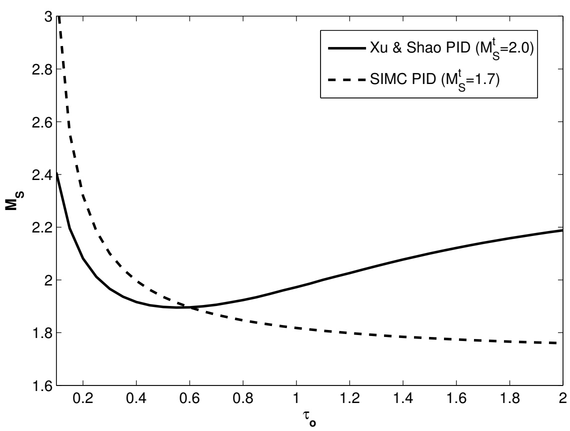

Fig. 6. ISOPDT Models - Xu & Shao and SIMC Tuning Robustness

integrating controlled process models considered (IPDT and ISOPDT) is one of the distinctive characteristics of the proposed tuning method.

V. Comparison with other Tuning Methods

For comparison purposes the target robustness accomplishment of several tuning rules for integrating process will be evaluated.

A. ISOPDT Models

For integrating second-order plus dead-time models available robust tuning rules [11], [15], [17] do not produce control systems with a constant robustness level across their applicability range as noted in Fig. 5 and Fig. 6.

B. IPDT Models

For integrating plus dead time models there are tuning rules that produce control systems with a constant M_S level along their entire applicability range but that are not robust like the PID performance optimized rules [13] that give $M_S = 16.24$ (ISE), $M_S = 8.55$ (ITSE), and $M_S = 6.42$

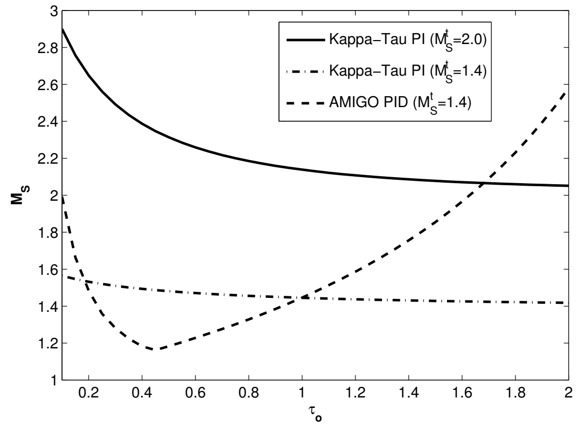

Fig. 7. IPDT Models - Kappa-Tau and AMIGO Robustness

TABLE III
EXAMPLE - IPDT PI CONTROL

Tuning Method	K_p	T_i	β	M_S	J_{eT}	TV_{uT}
MoReRT $M_S^t = 1.4$	0.66	5.32	0.45	1.40	0.69	0.14
AMIGO ($M_S = 1.41$)	0.70	6.70	1.0	1.41	0.72	0.22
MoReRT $M_S^t = 1.6$	0.88	3.94	0.47	1.60	0.46	0.17
SIMC ($M_S = 1.7$)	1.0	4.0	1.0	1.70	0.40	0.32

(ISTE), and the closed-loop transfer function polynomials matching method [7] that gives $M_S = 6.11$ (PI) and $M_S = 4.65$ (PID). The SIMC [11] PI tuning rule provides a constant intermediate robustness ($M_S = 1.7$) and the AMIGO [9] PI a constant high robustness ($M_S = 1.41$).

Other methods like Kappa-Tau [5] PI and AMIGO [10] PID do not provide a constant robustness level as shown in Fig. 7.

In contrast with the above methods the proposed robust tuning rule provides a perfect achievement of four robustness level targets for IPDT and ISOPDT models.

C. Control System Response Examples

Consider the IPDT model ($K = 1$, $L = 0.4$). The PI controller parameters with the proposed tuning method for $M_S^t \in \{1.4, 1.6\}$ and AMIGO and SIMC methods are listed in Table III. The Table also lists the obtained robustness and overall performance, the integrated absolute error (J_e) and the control effort total variation (TV_u). The control system response to a 10% set-point step change followed by a 5% disturbance step change is shown in Fig. 8.

For a high robustness level ($M_S \approx 1.4$) the MoReRT controller has better performance and smother control effort while at the medium robustness level ($M_S \approx 1.6$) the SIMC has better performance but with a big control output change to a set-point step.

As a second example consider the ISOPDT ($K = 1$, $T = 1$, $L = 1.2$). The PI controller is tuned with the Ali & Majhi (A&M) [3] method and with the proposed robust tuning. The control system response to set-point and disturbance unit step

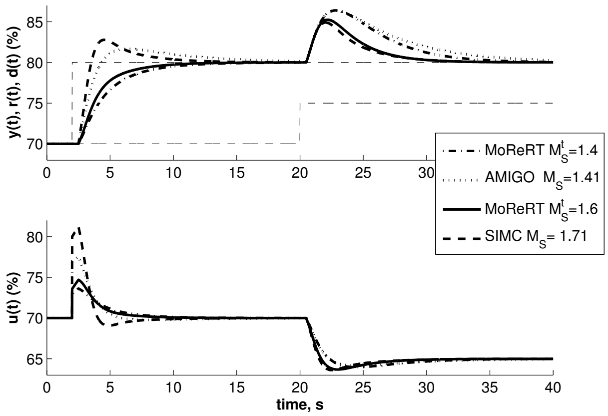

Fig. 8. Example - IPDT PI Control Responses

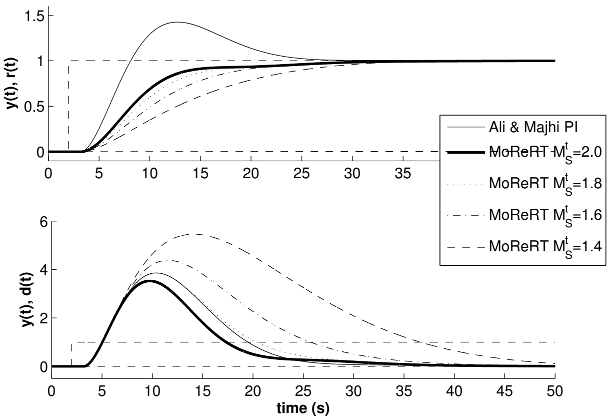

Fig. 9. Example - ISOPDT PI Control Responses

changes are shown in Fig. 9. The robustness and performance are listed in Table IV .

From this information is evident the robustness/performance trade-off and the flexibility of the proposed tuning that allows the designer to select the required robustness level while the A&M tuning produces a system with a robustness level that is model dependent.

VI. Conclusions

The proposed *MoReRT* tuning method for 2DoF proportional integral (PI_2) controllers guarantees the design robustness level for integrated second-order plus dead-time (ISOPDT) and integrated plus dead-time (IPDT) models using only one design parameter, which is the required closed-loop control system robustness as measured with the maximum sensitivity M_S.

Tuning equations were obtained for four robustness, $M_S \in \{1.4, 1.6, 1.8, 2.0\}$, allowing the designer to select the required robustness level by taking into account the expected variations in the process parameters. The integrating second-order plus dead-time models considered include normalized dead-times in the range from 0.1 to 2.0.

TABLE IV
Example - ISPDT PI Control

Tuning Method	M_S	J_{ed}	J_{er}	TV_{ud}	TV_{ur}
Ali & Mahji	1.83	43.34	7.93	1.85	0.67
MoReRT $M_S^t = 2.0$	2.00	37.27	7.65	1.95	0.29
MoReRT $M_S^t = 1.8$	1.80	47.18	8.33	1.79	0.23
MoReRT $M_S^t = 1.6$	1.60	66.55	9.55	1.65	0.19
MoReRT $M_S^t = 1.4$	1.40	116.76	12.41	1.54	0.13

Acknowledgments

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Optimal Robust Tuning for 1DoF PI/PID Control Unifying FOPDT/SOPDT Models

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Abstract: The aim of the paper is to present tuning equations for one-degree-of-freedom (1DoF) proportional integral (PI) and proportional integral derivative (PID) controllers. These are based on a performance/robustness trade-off analysis with first- and second-order plus dead-time models. On the basis of this analysis a tuning method is developed for 1DoF PI and PID controllers for servo and regulatory control that allows designing closed-loop control systems with a specified M_S robustness that at the same time have the best possible IAE performance. The control system robustness is adjusted varying only the controller proportional gain.

Keywords: PID controllers, one-degree-of-freedom controllers, servo/regulatory control, performance/robustness trade-off.

1. INTRODUCTION

As it has been widely reported, proportional integral derivative (PID) type controllers are with no doubt, the controllers most extensively used in the process industry. Their success is mainly due to their simple structure, easier to understand by the control engineer than other most advanced control approaches.

In industrial process control applications, the set-point normally remains constant and good load-disturbance rejection (regulatory control) is required. There are also applications where the set-point following (servo-control) is the more important control task.

Although from their commercial introduction in 1940 (Babb, 1990) the original three-term PID control algorithm has evolved into the actual four- or five-term two-degree-of-freedom (2DoF) PID control algorithms the vast majority of the controllers still in use are of one-degree-of-freedom (1DoF) type.

Since Ziegler and Nichols (1942) presented their PID controller tuning rules, a great number of other procedures have been developed as revealed in O'Dwyer (2006) review. Some of them consider only the system performance (López et al., 1967; Rovira et al., 1969), its robustness (Åström and Hägglund, 1984), or a combination of performance and robustness (Ho et al., 1999).

There are tuning rules optimized for regulatory control operation (López et al., 1967) or optimized for servo-control operation (Tavakoli and Tavakoli, 2003). There are also authors that present separate sets of rules for each operation (Zhuang and Atherton, 1993; Kaya, 2004). For the servo-control operation there is an important group of tuning rules based on zero-pole cancellation, Internal Model Control (IMC), and direct synthesis techniques (Martin et al., 1975; Rivera et al., 1986; Alcántara et al., 2011).

Due to the constraints imposed by the 1DoF control algorithm it is necessary to develop separate tuning rules for servo and regulatory control. In addition, the control-system design procedure is usually based on the use of low-order linear models identified at the control system normal operation point. Due to the non-linear characteristics found in most industrial processes, it is necessary to consider the expected changes in the process characteristics assuming certain relative stability margins, or robustness requirements, for the control system.

Therefore, the design of the closed-loop control system with 1DoF PI and PID controllers must consider the main operation of the control system (*servo-control* or *regulatory control*) and the *trade-off* of two conflicting criteria, the time response *performance* to set-point or load-disturbances, and the *robustness* to changes in the controlled process characteristics. If only the system performance is taken into account, by using for example an integrated error criteria (IAE, ITAE or ISE) or a time response characteristic (overshoot, rise-time or settling-time) as in Huang and Jeng (2002), and Tavakoli and Tavakoli (2003), the resulting closed-loop control system probably will have a very low robustness. On the other hand, if the system is designed to have high robustness as in Hägglund and Åström (2002) and if the performance of the resulting system is not evaluated, the designer will not have any indication of the *cost* of having such highly robust system. Control performance and robustness are taken into account in Shen (2002), and Tavakoli et al. (2005) optimizing its IAE or ITAE performance but they just guarantee the usual minimum level of robustness.

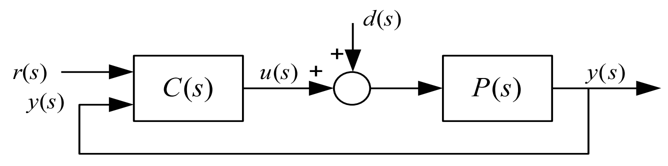

Figure 1. Closed-Loop Control System

To have an indication of the performance loss when the control system robustness is increased, using M_S as a measure, a performance/robustness analysis was conducted for 1DoF and 2DoF PI and PID control systems with first- (FOPDT) and second-order plus dead-time (SOPDT) models (Alfaro et al., 2010).

Based on this performance/robustness analysis, tuning rules are proposed for servo and regulatory 1DoF PI and PID controllers for four M_S robustness levels in the range from 1.4 to 2.0, to design robust closed-loop control systems that at the same time have the best possible performance under the IAE criteria. The presented tuning rules integrate in a single set of equations the tuning of controllers for first- and second-order plus dead-time process models.

The rest of the paper is organized as follows: the transfer functions of the controlled process model, the controller, and the closed-loop control system are presented in Section 2; the performance/robustness analysis is summarized in Section 3; the proposed *Optimal and Robust Tuning* is presented in Section 4 and particular examples of the performance/robustness trade-off are shown in Section 5. The paper ends with some conclusions.

2. PROBLEM FORMULATION

Consider a closed-loop control system, as shown in Fig. 1, where $P(s)$ and $C(s)$ are the controlled process model and the controller transfer function, respectively. In this system, $r(s)$ is the set point; $u(s)$, the controller output signal; $d(s)$, the load disturbance; and $y(s)$, the controlled process variable.

The controlled process is represented by an SOPDT model given by the general transfer function

$$P(s) = \frac{Ke^{-Ls}}{(Ts+1)(aTs+1)},\ \tau_o = \frac{L}{T}, \tag{1}$$

where K is the gain; T, the main time constant; a, the ratio of the two time constants ($0 \le a \le 1.0$); L, the dead-time; and τ_o, the *normalized dead time*. The model transfer function (1) allows the representation of FOPDT processes ($a = 0$), over damped SOPDT processes ($0 < a < 1$), and dual-pole plus dead-time (DPPDT) processes ($a = 1$).

The process is controlled with a 1DoF PID controller whose output is as follows (Åström and Hägglund, 1995):

$$u(s) = K_p \left\{ \left(1 + \frac{1}{T_i s}\right) e(s) - \left(\frac{T_d s}{\alpha T_d s + 1}\right) y(s) \right\}, \tag{2}$$

where K_p is the controller *proportional gain*; T_i, the *integral time constant*; T_d, the *derivative time constant*; and α, the *derivative filter constant*. Then the controller parameters to tune are $\theta_c = \{K_p, T_i, T_d\}$. Usually, $\alpha = 0.10$ (Corripio, 2001).

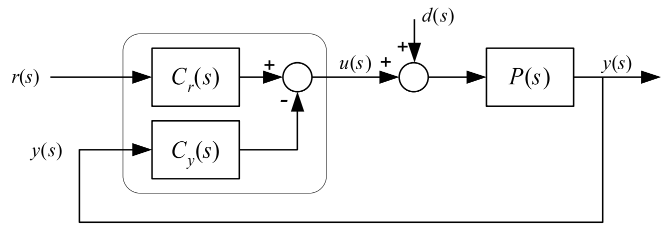

Figure 2. PID Closed-Loop Control System

Equation (2) may be rearranged, for analysis purposes, as follows

$$\begin{aligned} u(s) =& K_p \left(1 + \frac{1}{T_i s}\right) r(s) \\ &- K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{0.1 T_d s + 1}\right) y(s), \end{aligned} \tag{3}$$

or in the compact form shown in Fig. 2 as

$$u(s) = C_r(s) r(s) - C_y(s) y(s), \tag{4}$$

where $C_r(s)$ is the *set-point controller* transfer function and $C_y(s)$ is the *feedback controller* transfer function.

The output of the closed-loop control system varies with a change in any of its the inputs as:

$$y(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)} r(s) + \frac{P(s)}{1 + C_y(s)P(s)} d(s), \tag{5}$$

or

$$y(s) = M_{yr}(s) r(s) + M_{yd}(s) d(s), \tag{6}$$

where $M_{yr}(s)$ is the transfer function from the set-point to the controlled process variable and is known as the *servo control* closed-loop transfer function; $M_{yd}(s)$ is the transfer function from the load disturbance to the controlled process variable and is known as the *regulatory control* closed-loop transfer function.

The *performance* of the closed-loop control system is evaluated using the IAE cost functional given by

$$J_e \doteq \int_0^\infty |e(t)|\, \mathrm{d}t = \int_0^\infty |y(t) - r(t)|\, \mathrm{d}t. \tag{7}$$

The controller parameters in the servo-control closed-loop transfer function, M_{yr}, are the same than the controller parameters in the regulatory control closed-loop transfer function, M_{yd}. Therefore it is not possible to obtain a single set of controller parameters θ_c that optimize, at the same time, the control system response to a set-point step change and the control system response to a load-disturbance step change.

The performance (7) is evaluated for a step change in the set-point, J_{er} and in the load-disturbance, J_{ed}.

The peak magnitude of the sensitivity function is used as an indicator of the system *robustness* (relative stability). The maximum sensitivity for the control system is defined as

$$M_S \doteq \max_\omega |S(\mathrm{j}\omega)| = \max_\omega \frac{1}{|1 + C_y(\mathrm{j}\omega)P(\mathrm{j}\omega)|}. \tag{8}$$

If the system robustness (8) is not taken into account for the design, the controller parameters may be optimized to maximize the system performance or to achieve the minimum value of the cost functional in (7), using M_{yr}

for set point changes (J^o_{er}) and M_{yd} for load disturbance changes (J^o_{ed}).

Because of the control system performance/robustness trade-off, if a robustness constraint is included into the design then, it is expected that the actual system performance will be reduced ($J_e \geq J^o_e$). Then, the *performance degradation factor* defined as

$$F_p \doteq \frac{J^o_e}{J_e},\ F_p \leq 1, \tag{9}$$

is used to evaluate the performance/robustness trade-off.

3. PERFORMANCE/ROBUSTNESS TRADE-OFF ANALYSIS

To evaluate the performance degradation when the system robustness is increased, the following steps, as they were presented in Alfaro et al. (2010), were followed.

3.1 1DoF Controllers Optimum Performance

For the 1DoF servo- and regulatory-control performance-optimized PI and PID controllers, the parameters $\theta^o_c = \{K^o_p, T^o_i, T^o_d\}$ were obtained using the cost functional (7) such that

$$J^o_e \doteq J_e(\theta^o_c) = \min_{\theta_c} J_e(\theta_c), \tag{10}$$

for (1) with $a \in \{0, 0.25, 0.5, 0.75, 1\}$ and ten τ_o in the range from 0.05 to 2.0, for set-point and load-disturbance step changes. The robustness of the control systems that deliver the optimal performance was evaluated by using M_S.

3.2 1DoF Controllers Degraded Performance

To increase the control-loop robustness, a target performance degradation factor, F^t_p, was included in the cost functional, as follows

$$J_{F_p} \doteq J(\theta_c, F^t_p) = \left| \frac{J^o_e}{J_e(\theta_c)} - F^t_p \right|, \tag{11}$$

for obtaining the PI and PID (servo and regulatory control) parameters θ^{o1}_c such that

$$J^o_{F_p} \doteq J_{F_p}(\theta^{o1}_c, F^t_p) = \min_{\theta_c} J_{F_p}(\theta_c, F^t_p). \tag{12}$$

When F^t_p was decreased, the control-system robustness was increased to the target level, M^t_S.

With starting point as the original unconstrained (from the point of view of robustness) optimal parameters θ^{o1}_c, a second optimization was conducted using the cost functional

$$J_{M_S} \doteq J(\theta_c, M^t_S) = \left| M_S(\theta_c) - M^t_S \right|, \tag{13}$$

in order to achieve the target robustness. The robust controller parameters, θ^{o2}_c, are such that

$$J^o_{M_S} \doteq J_{M_S}(\theta^{o2}_c, M^t_S) = \min_{\theta_c} J_{M_S}(\theta_c, M^t_S). \tag{14}$$

For the analysis, four target robustness levels were considered, $M^t_S \in \{2, 1.8, 1.6, 1.4\}$.

Finally, the performance degradation factor required for obtaining M^t_S in (14) was evaluated as follows

$$F_p(M^t_S) = \frac{J^o_e}{J_e(\theta^{o2}_c)}. \tag{15}$$

Therefore, the second optimization provided the controller parameters θ^{o2}_c required to formulate a system with the target robustness (8), M^t_S, and with the best performance allowed when using the IAE criteria (7), J_{er} or J_{ed}.

The performance/robustness analysis of the resulting in PI and PID closed-loop control systems pointed out the existing *trade-off* between them. As shown in Alfaro et al. (2010), in general performance optimized 1DoF PI controllers are more robust than the PIDs but their optimal performance is lower. The performance optimized regulatory control systems, for both PI and PID, are less robust than the servo-control ones, requiring also more performance degradation, lower *degraded performance factor*, to reach the same robustness level.

4. UNIFIED SIMPLE OPTIMAL ROBUST TUNING FOR 1DOF PI AND PID CONTROLLERS ($USORT_1$)

One of the purposes of this contribution is try to capture in a single set of equations the performance/robustness trade-off. This is with no doubt a novel feature as the first- and second-order models are considered at once, without forcing a distinction with respect to neither the model used nor the controller structure. The other purpose is that these robust tuning equations be as simple as possible.

Analysis of the regulatory and servo-control PI and PID controllers parameters shows that for a model with a given time constants ratio a, increasing the control system robustness by decreasing M^t_S, results in a substantial reduction in K_p. However, this increase in the robustness has negligible effect on T_i and T_d, except in the case of models with a very low τ_o (when high robustness is required).

On the basis of this observation, equations *that are independent of the target robustness level* can be obtained for the controller integral time constant and derivative time constant, as follows:

$$T_i = \mathbf{F}(T, \tau_o, a), \quad T_d = \mathbf{G}(T, \tau_o, a). \tag{16}$$

With these equations at hand, the controller proportional gains are readjusted to match a target robustness to obtain equations given by the following

$$K_p = \mathbf{H}(K, \tau_o, a, M^t_S). \tag{17}$$

For FOPDT and SOPDT models with τ_o in the range from 0.1 to 2.0 and four M^t_S values the normalized 1DoF PI and PID controller parameters can be obtained using the process model parameters, $\theta_p = \{K, T, a, L, \tau_o\}$, for servo-control and regulatory control from the following relations:

- Regulatory control operation:

$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_o^{a_2}, \tag{18}$$

$$\tau_i \doteq \frac{T_i}{T} = b_0 + b_1 \tau_o^{b_2}, \tag{19}$$

$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_o^{c_2}, \tag{20}$$

Table 1. Regulatory Control PI Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.265	0.077	0.023	-0.128	-0.244
a_1	0.603	0.739	0.821	1.035	1.226
a_2	-0.971	-0.663	-0.625	-0.555	-0.517
	Target robustness $M_S^t = 1.8$				
a_0	0.229	0.037	-0.056	-0.160	-0.289
a_1	0.537	0.684	0.803	0.958	1.151
a_2	-0.952	-0.626	-0.561	-0.516	-0.472
	Target robustness $M_S^t = 1.6$				
a_0	0.175	-0.009	-0.080	-0.247	-0.394
a_1	0.466	0.612	0.702	0.913	1.112
a_2	-0.911	-0.578	-0.522	-0.442	-0.397
	Target robustness $M_S^t = 1.4$				
a_0	0.016	-0.053	-0.129	-0.292	-0.461
a_1	0.476	0.507	0.600	0.792	0.997
a_2	-0.708	-0.513	-0.449	-0.368	-0.317
b_0	-1.382	0.866	1.674	2.130	2.476
b_1	2.837	0.790	0.268	0.112	0.073
b_2	0.211	0.520	1.062	1.654	1.955

- Servo-control operation:

$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_o^{a_2}, \tag{21}$$

$$\tau_i \doteq \frac{T_i}{T} = \frac{b_0 + b_1 \tau_o + b_2 \tau_o^2}{b_3 + \tau_o}, \tag{22}$$

$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_o^{c_2}, \tag{23}$$

The value of the constants a_i, b_i, and c_i in (18) to (23) are listed in Tables 1 to 4. As noted in these Tables only the a_i constants for K_p calculation depend on the robustness level M_S.

Equations (18) to (23) provide a direct controller tuning for the FOPDT ($a = 0$) and the DPPDT ($a = 1$) models. In the case of the SOPDT models with $a \notin \{0.25, 0.5, 0.75\}$ the set of controller parameters must be obtained by linear interpolation between the two sets of parameters obtained with the adjacent a values used in the optimization.

The performance/robustness analysis also shows that the PI controllers with performance optimized parameters for servo-control operation produce control systems with a robustness $M_S \approx 1.8$. Then, the minimum robustness level of $M_S = 2.0$ is exceeded in this case.

With a maximum absolute deviation from the target robustness M_S^t of 4.09% and an average deviation of only 0.70% the proposed $uSORT_1$ tuning may be considered as a *global robust tuning method* with levels $M_S^t \in \{2.0, 1.8, 1.6, 1.4\}$ for FOPDT and SOPDT models with normalized dead-times in the range from 0.1 to 2.0.

Equations (18) to (20) and (21) to (23) were obtained for tuning Standard PID controllers. It is know that an equivalent Serial PID controller only exists if $T_i/T_d \geq 4$. As can be seen from Fig. 3 for the $uSORT_1$ regulatory control $\tau_i/\tau_d < 4$, then there is no Serial PID equivalent in this case, and that for the $uSORT_1$ servo-control in general $\tau_i/\tau_d \geq 4$ for time constant dominant models ($\tau_o \leq 1.0$). In the particular case of FOPDT controlled process models the servo-control Serial PID equivalent exists for $\tau_o \leq 1.4$.

Table 2. Regulatory Control PID Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.235	0.435	0.454	0.464	0.488
a_1	0.840	0.551	0.588	0.677	0.767
a_2	-0.919	-1.123	-1.211	-1.251	-1.273
	Target robustness $M_S^t = 1.8$				
a_0	0.210	0.380	0.400	0.410	0.432
a_1	0.745	0.500	0.526	0.602	0.679
a_2	-0.919	-1.108	-1.194	-1.234	-1.257
	Target robustness $M_S^t = 1.6$				
a_0	0.179	0.311	0.325	0.333	0.351
a_1	0.626	0.429	0.456	0.519	0.584
a_2	-0.921	-1.083	-1.160	-1.193	-1.217
	Target robustness $M_S^t = 1.4$ †				
a_0	0.155	0.228	0.041	0.231	0.114
a_1	0.455	0.336	0.571	0.418	0.620
a_2	-0.939	-1.057	-0.725	-1.136	-0.932
	†Valid only for $\tau_o \geq 0.40$ if $a \geq 0.25$				
b_0	-0.198	0.095	0.132	0.235	0.236
b_1	1.291	1.165	1.263	1.291	1.424
b_2	0.485	0.517	0.496	0.521	0.495
c_0	0.004	0.104	0.095	0.074	0.033
c_1	0.389	0.414	0.540	0.647	0.756
c_2	0.869	0.758	0.566	0.511	0.452

Table 3. Servo-Control PI Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 1.8$				
a_0	0.243	0.094	0.013	-0.075	-0.164
a_1	0.509	0.606	0.703	0.837	0.986
a_2	-1.063	-0.706	-0.621	-0.569	-0.531
	Target robustness $M_S^t = 1.6$				
a_0	0.209	0.057	-0.010	-0.130	-0.220
a_1	0.417	0.528	0.607	0.765	0.903
a_2	-1.064	-0.667	-0.584	-0.506	-0.468
	Target robustness $M_S^t = 1.4$				
a_0	0.164	0.019	-0.061	-0.161	-0.253
a_1	0.305	0.420	0.509	0.636	0.762
a_2	-1.066	-0.617	-0.511	-0.439	-0.397
b_0	14.650	0.107	0.309	0.594	0.625
b_1	8.450	1.164	1.362	1.532	1.778
b_2	0.0	0.377	0.359	0.371	0.355
b_3	15.740	0.066	0.146	0.237	0.209

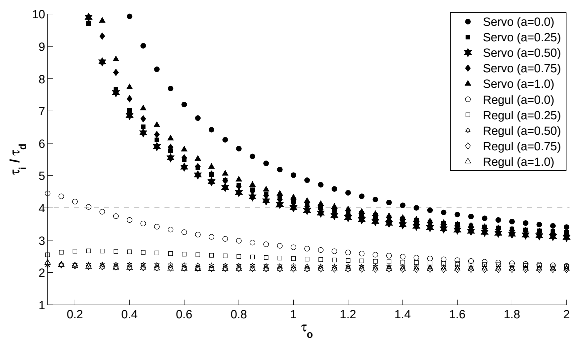

Figure 3. Servo and Regulatory Control τ_i/τ_d Ratio

Table 4. Servo-Control PID Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.377	0.502	0.518	0.533	0.572
a_1	0.727	0.518	0.562	0.653	0.728
a_2	-1.041	-1.194	-1.290	-1.329	-1.363
	Target robustness $M_S^t = 1.8$				
a_0	0.335	0.432	0.435	0.439	0.482
a_1	0.644	0.476	0.526	0.617	0.671
a_2	-1.040	-1.163	-1.239	-1.266	-1.315
	Target robustness $M_S^t = 1.6$				
a_0	0.282	0.344	0.327	0.306	0.482
a_1	0.544	0.423	0.488	0.589	0.622
a_2	-1.038	-1.117	-1.155	-1.154	-1.221
	Target robustness $M_S^t = 1.4$				
a_0	0.214	0.234	0.184	0.118	0.147
a_1	0.413	0.352	0.423	0.575	0.607
a_2	-1.036	-1.042	-1.011	-0.956	-1.015
b_0	1687	0.135	0.246	0.327	0.381
b_1	339.2	1.355	1.608	1.896	2.234
b_2	39.86	0.333	0.273	0.243	0.204
b_3	1299	0.007	0.003	-0.006	-0.015
c_0	-0.016	0.026	-0.042	-0.086	-0.110
c_1	0.333	0.403	0.571	0.684	0.772
c_2	0.815	0.613	0.446	0.403	0.372

Table 5. P_1 Servo-Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	-	0.778	0.646	0.482	-
T_i	-		2.546		-
M_S^r	-	1.81	1.61	1.40	-
$J_{er}/\Delta r$	-	2.947	3.282	4.392	-
	PID Controller				
K_p	1.132	1.003	0.846	0.642	1.174
T_i		3.022			3.085
T_d		0.495			0.589
M_S^r	2.0	1.80	1.60	1.40	2.21
$J_{er}/\Delta r$	2.458	2.512	2.976	3.918	2.481

5. EXAMPLES

For comparison of the performance and robustness obtained with the proposed $uSORT_1$ method we use the Madhuranthakam et al. (2008) [MEB] tuning rules for Standard PID controllers that optimize the IAE criteria for servo- and regulatory control operation.

First, we consider the FOPDT process given by

$$P_1(s) = \frac{1.2\mathrm{e}^{-1.5s}}{2s+1}.$$

The controller parameters and the control system performance and robustness for servo-control and regulatory control operation of P_1 are listed in Table 5 and Table 6, respectively.

As a second model we consider the SOPDT process given by

$$P_2(s) = \frac{1.2\mathrm{e}^{-1.5s}}{(2s+1)(s+1)}.$$

The controller parameters and the control system performance and robustness for servo-control and regulatory

Table 6. P_1 Regulatory Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	0.885	0.779	0.651	0.500	-
T_i		2.576			-
M_S^r	2.01	1.81	1.61	1.42	-
$J_{ed}/\Delta d$	2.910	3.305	3.960	5.156	-
	PID Controller				
K_p	1.108	0.984	0.829	0.626	1.293
T_i		1.867			1.971
T_d		0.614			0.569
M_S^r	2.02	1.82	1.61	1.40	2.36
$J_{ed}/\Delta d$	1.969	2.215	2.593	3.303	1.666

Table 7. P_2 Servo-Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	-	0.711	0.590	0.441	-
T_i	-		3.421		-
M_S^r	-	1.83	1.62	1.41	-
$J_{er}/\Delta r$	-	4.311	4.831	6.469	-
	PID Controller				
K_p	1.110	0.989	0.839	0.625	1.497
T_i		4.264			5.121
T_d		0.921			0.812
M_S^r	1.98	1.79	1.61	1.40	2.78
$J_{er}/\Delta r$	3.385	3.596	4.234	5.687	3.798

Table 8. P_2 Regulatory Control Operation

	$uSORT_1$ M_S^d				MEB
	2.0	1.8	1.6	1.4	IAE
	PI Controller				
K_p	0.838	0.740	0.613	0.461	-
T_i		3.743			-
M_S^r	2.03	1.83	1.62	1.42	-
$J_{ed}/\Delta d$	4.466	5.059	6.102	8.098	-
	PID Controller				
K_p	1.037	0.951	0.801	0.620	1.539
T_i		2.454			2.971
T_d		1.108			0.883
M_S^r	1.93	1.79	1.60	1.41	2.94
$J_{ed}/\Delta d$	2.848	3.094	3.605	4.456	2.141

control operation of P_2 are listed in Table 7 and Table 8, respectively.

From Tables 5 to 8 it is noted that for same robustness design level (M_S^d) the PID controllers deliver more performance than the PI controllers. They also show the performance/robustness trade-off, an increment in control system robustness always reduces its performance. For example, to increase the robustness reducing M_S^d from 1.8 to 1.6 produces a 11 to 20% reduction in the control system performance.

It is also noted that the performance optimized MEB control systems have low robustness, $M_S > 2.0$ in all cases. Although the MEB controllers are performance optimized the servo-control $uSORT_1$ PID controllers for $M_S^d = 2.0$ produce control systems that are more robust and that at the same time have better performance.

The P_2 control system responses to a 10% set-point and load-disturbance step changes are shown in Fig. 4 and Fig. 5, respectively.

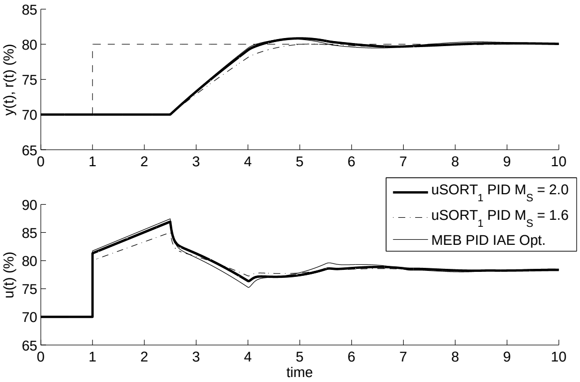

Figure 4. Model P_2 Servo-Control Responses

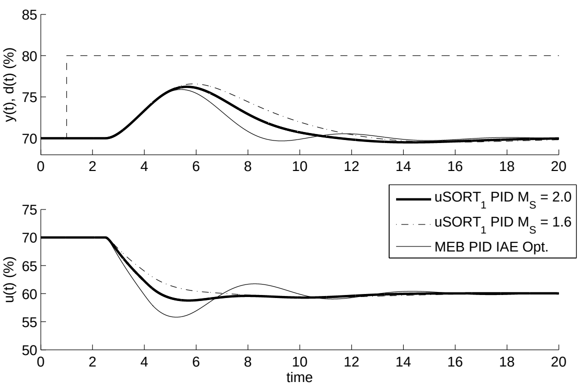

Figure 5. Model P_2 Regulatory Control Responses

6. CONCLUSIONS

Based on a performance (IAE) - robustness (M_S) analysis tuning relations are proposed that unifies the treatment of one-degree-of-freedom (1DoF) PI and PID controllers and the use of first- and second-order plus dead-time (FOPDT, SOPDT) models for servo- and regulatory control systems.

The proposed *Unified Simple Optimal and Robust Tuning* for 1DoF PI/PID controllers ($uSORT_1$) allows to adjust the control system robustness varying only the controller proportional gain.

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Conversion Formulae and Performance Capabilities of Two-Degree-of-Freedom PID Control Algorithms

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Abstract

The aim of the paper is to present two-degree-of-freedom (2DoF) proportional integral derivative (PID) control algorithms and conversion relations between their parameters. The Ideal PID with filter (PID_{2F}) is the more general PID controller. Restrictions to obtain equivalent 2DoF Standard or Series PID controllers are presented taking into account the derivative filter constant. Examples are used to illustrate when or when not equivalent controllers exist.

1 Introduction

As been widely reported, the proportional integral derivative (PID) control algorithm is the most extensively used for process control. Their success is mainly due to its simple structure that made it easier to understand by the control engineer than other most advanced control approaches.

The first commercially available PID controller, the Taylor Fullscope 100 [6], had a pneumatic "series" implementation of the PID control algorithm, a proportional integral (PI) controller in series with a proportional derivative (PD) controller.

From their commercial introduction in 1940 [7] the original three-term PID control algorithm has evolved into the actual four- or five-term two-degree-of-freedom (2DoF) PID control algorithms [4].

The PID control algorithm evolution and its technical implementation is well documented. See [6, 7, 8, 11, 14] and the references therein.

Usually, the control algorithm implementation is manufacturer dependent and not all of its variations are available in the same controller. In addition, it would be the case that a tuning rule of interest had been obtained using a control algorithm different from the one implemented in the controller to tune. In this case, controller parameters conversion is required, that will also indicate if the pursued equivalent controller exist.

In this paper, by using the same nomenclature, equations of 2DoF Standard, Parallel, Series and Ideal with filter PID control algorithms are presented and analyzed obtaining conversion relations in order to obtain equivalent controllers. These conversions take into account the controller derivative filter. This is a new feature of the conversions presented here not found in the literature.

Using an example the performance capabilities of different 2DoF PID control algorithms is illustrated.

2 Feedback Control Systems

Consider the closed-loop control system in Fig. 1 where $P(s)$ and $C(s)$ are the controlled process model and the controller transfer functions respectively. In this system, $r(s)$ is the set-point, $u(s)$ the controller output signal, $d(s)$ the load-disturbance, $y(s)$ the process controlled variable, $n(s)$ the measurement-noise and $y'(s)$ the noise contaminated feedback signal. It is assumed that the disturbance enters at the process input (load-disturbance).

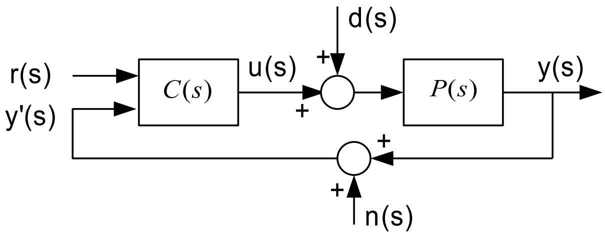

Figure 1. Closed-Loop Control System

In general, for two-degree-of-freedom (2DoF) controllers their output signal, the control effort, can be expressed as:

$$u(s) = C_r(s)r(s) - C_y(s)y(s), \qquad (1)$$

where $C_r(s)$ is the controller aspect operating over the set-point, the *set-point controller* transfer function and $C_y(s)$ is the controller aspect operating over the controlled variable, the *feedback controller* transfer function.

The closed-loop control system output to a change in its inputs is given by

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \tag{2}$$

where

$$M_{yr}(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)}, \tag{3}$$

is the transfer function from the set-point to the controlled process variable, the *servo-control* closed-loop transfer function, and

$$M_{yd}(s) = \frac{P(s)}{1 + C_y(s)P(s)}, \tag{4}$$

is the one from the load-disturbance to the controlled process variable, the *regulatory control* closed-loop transfer function which are related by

$$M_{yr}(s) = C_r(s)M_{yd}(s). \tag{5}$$

3 2DoF PID Control Algorithms

Although all the controllers with a proportional integral (PI) control algorithm are implemented in the same way, have the same transfer function, this is not the case with commercial controllers with proportional integral derivative (PID) control algorithms. Even more, the controller's manufacturers use different names for the same PID algorithm [9]. The diversity of the PID control algorithms is evident in [12].

Due to the fact that in most of the existing industrial process control applications the desired value of the controlled variable, or set-point, normally remains constant (regulatory control or disturbance rejection operation) but eventually may need to be changed (servo control or set-point tracking operation) we are mainly interested in the two-degree-of-freedom (2DoF) implementations of the PID control algorithm. The extra parameter that the 2DoF controllers provide is used to improve their servo-control behavior while considering the regulatory control performance and the closed-loop control system robustness [1, 2, 3, 16, 17].

The most widely used PID control algorithms are briefly described below.

3.1 2DoF Standard PID

- Parameters: $\theta_c = \{K_p, T_i, T_d, \alpha, \beta, \gamma\}$
- Equation:

The "textbook" 2DoF proportional integral derivative control algorithm is the Standard PID whose output is given by the following [5, 13, 15]:

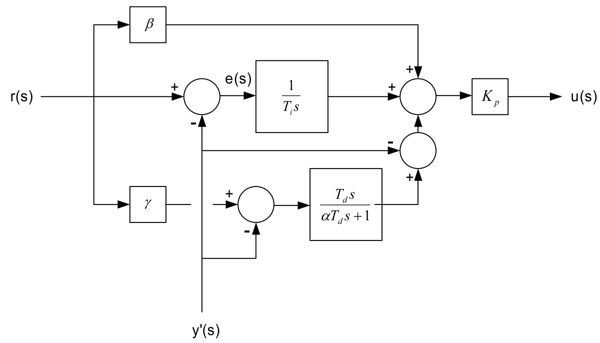

Figure 2. 2DoF PID Controller

$$u(t) = K_p\left\{e_p(t) + \frac{1}{T_i}\int_0^t e_i(\tau)\mathrm{d}\tau + T_d\frac{\mathrm{d}e_d(t)}{\mathrm{d}t}\right\}, \tag{6}$$

or

$$u(s) = K_p\left\{e_p(s) + \frac{1}{T_i s}e_i(s) + \frac{T_d s}{\alpha T_d s + 1}e_d(s)\right\}, \tag{7}$$

with

$$e_p(s) = \beta r(s) - y'(s), \tag{8}$$
$$e_i(s) = r(s) - y'(s), \tag{9}$$
$$e_d(s) = \gamma r(s) - y'(s), \tag{10}$$

where K_p is the *controller gain*, T_i the *integral time constant*, T_d the *derivative time constant*, β and γ the *set-point weights*, and α the *derivative filter constant*. The 2DoF PID block diagram is shown in Fig. 2

To avoid an extreme instantaneous change at the controller output signal when a set-point step change occurs normally γ is set to zero [18, 19]. In this case (7) reduces to

$$\begin{aligned} u(s) =& K_p\left(\beta r(s) - y'(s)\right) + \frac{K_p}{T_i s}\left(r(s) - y'(s)\right) \\ &- \left(\frac{K_p T_d s}{\alpha T_d s + 1}\right)y'(s), \end{aligned} \tag{11}$$

that will be denoted as PID_2. In addition, in the following it is supposed that the measurement noise is filtered, then $y'(s) \approx y(s)$.

According to (1) the controller output (11) may be rearranged, for analysis purposes, as follows

$$\begin{aligned} u(s) =& K_p\left(\beta + \frac{1}{T_i s}\right)r(s) \\ &- K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\alpha T_d s + 1}\right)y(s), \end{aligned} \tag{12}$$

Then, the controller parameters are $\theta_c = \{K_p, T_i, T_d, \alpha, \beta, \gamma = 0\}$.

Although the Standard form is the classical implementation of the PID control algorithm, the following forms are also found in the control literature [10, 19, 15]:

3.2 2DoF Parallel PID

- Parameters: $\theta_{cp} = \{K_p, K_i, K_d, \alpha_p, \beta_p, \gamma_p = 0\}$
- Equation:

$$u(s) = \left(\beta_p K_p + \frac{K_i}{s}\right) r(s) - \left(K_p + \frac{K_i}{s} + \frac{K_d s}{\alpha_p K_d s + 1}\right) y(s). \quad (13)$$

- From 2DoF Parallel PID to Standard PID:

A PID_2 controller (11) equivalent to the 2DoF Parallel PID (13) is found by using the following relations:

$$K_p = K_p, \quad (14)$$
$$T_i = \frac{K_p}{K_i}, \quad (15)$$
$$T_d = \frac{K_d}{K_p}, \quad (16)$$
$$\alpha = \alpha_p K_p, \quad (17)$$
$$\beta = \beta_p, \quad (18)$$
$$\gamma = \gamma_p = 0. \quad (19)$$

There is a direct relation between the Standard and Parallel PID algorithms then, this last one will not be further considered.

3.3 2DoF Series or "Industrial" PID:

- Parameters: $\theta_c' = \left\{K_p', T_i', T_d', \alpha', \beta', \gamma' = 0\right\}$
- Equation:

The 2DoF version of the series implementation of the PID algorithm is

$$u(s) = K_p'\left(\beta' + \frac{1}{T_i' s}\right) r(s) - K_p'\left(1 + \frac{1}{T_i' s}\right)\left(\frac{T_d' s + 1}{\alpha' T_d' s + 1}\right) y(s). \quad (20)$$

- From 2DoF Series PID to Standard PID:

It is possible to obtain a Standard 2DoF PID controller (11) equivalent to the 2DoF Series PID (20) by using the following relations:

$$K_p = F_c' K_p', \quad (21)$$
$$T_i = F_c' T_i', \quad (22)$$
$$T_d = \frac{(1 - \alpha' F_c') T_d'}{F_c'} \quad (23)$$
$$\alpha = \frac{F_c' \alpha'}{1 - \alpha' F_c'}, \quad \alpha' < 1 + \frac{T_i'}{T_d'} \quad (24)$$
$$\beta = \frac{\beta'}{F_c'}, \quad (25)$$
$$\gamma = \gamma' = 0, \quad (26)$$
$$F_c' = 1 + \frac{(1 - \alpha') T_d'}{T_i'}. \quad (27)$$

F_c' (27) is the *PID_{2s} to PID_2 conversion factor*. The conversion constraint in (24) usually holds then, we may say that there is a Standard PID equivalent to a Series one.

3.4 2DoF Ideal PID with filter

- Parameters: $\theta_c^* = \left\{K_p^*, T_i^*, T_d^*, T_f, \beta^*, \gamma^* = 0\right\}$
- Equations:

A commonly used PID implementation in Internal Model Control (IMC) based controller design is given by the following:

$$u(s) = K_p^*\left(\beta^* + \frac{1}{T_i^* s}\right) r(s) - K_p^*\left(1 + \frac{1}{T_i^* s} + T_d^* s\right)\left(\frac{1}{T_f s + 1}\right) y(s). \quad (28)$$

- From 2DoF Ideal PID with Filter to Standard PID:

A Standard 2DoF PID controller (11) equivalent to the Ideal PID with filter (28), denoted by PID_{2F}, can be obtained by using the following relations:

$$K_p = F_c^* K_p^*, \quad (29)$$
$$T_i = F_c^* T_i^*, \quad (30)$$
$$T_d = \frac{T_d^*}{F_c^*} - T_f, \quad T_d^* > F_c^* T_f, \quad (31)$$
$$\alpha = \frac{F_c^* T_f}{T_d^* - F_c^* T_f} \quad (32)$$
$$\beta = \frac{\beta^*}{F_c^*}, \quad (33)$$
$$\gamma = \gamma^* = 0, \quad (34)$$
$$F_c^* = 1 - \frac{T_f}{T_i^*}, \quad T_i^* > T_f. \quad (35)$$

F_c^* (35) is the *PID_{2F} to PID_2 conversion factor*.

In this case, an equivalent PID_2 controller can not always be obtained as shown in (31), and (35).

Using the conversion factors presented above, exact equivalent feedback ($C_y(s)$) and set-point ($C_r(s)$) controller transfer functions for a PID_2 (12) may be obtained for 2DoF PID controllers given by (13) , (20) and (28).

3.5 From 2DoF Standard PID to Series PID

In the other direction a 2DoF Series PID controller equivalent to a 2DoF Standard one can be found by using the following relations:

$$K'_p = F_c K_p, \quad (36)$$

$$T'_i = F_c T_i, \quad (37)$$

$$T'_d = \frac{(1+\alpha)T_d}{F_c} \quad (38)$$

$$\alpha = \frac{F'_c \alpha'}{F_c}, \quad (39)$$

$$\beta' = \frac{\beta}{F_c}, \quad (40)$$

$$\gamma' = \gamma = 0, \quad (41)$$

$$F_c = 0.5\left[1 + \frac{\alpha T_d}{T_i} + \sqrt{1 - \frac{(4+2\alpha)T_d}{T_i} + \frac{\alpha^2 T_d^2}{T_i^2}}\right]. \quad (42)$$

Due to the constraint in (42) there will not always exist a Series PID equivalent to a Standard PID. It will only exist if

$$\alpha^2\left(\frac{T_d}{T_i}\right)^2 - (4+2\alpha)\left(\frac{T_d}{T_i}\right) + 1 > 0 \quad (43)$$

If the PID_2 derivative filter constant is taken as $\alpha = 0.1$ there is a Series equivalent PID controller only if $T_i > 4.20\ T_d$. Fig. 3 shows that this constrain increase as α increases.

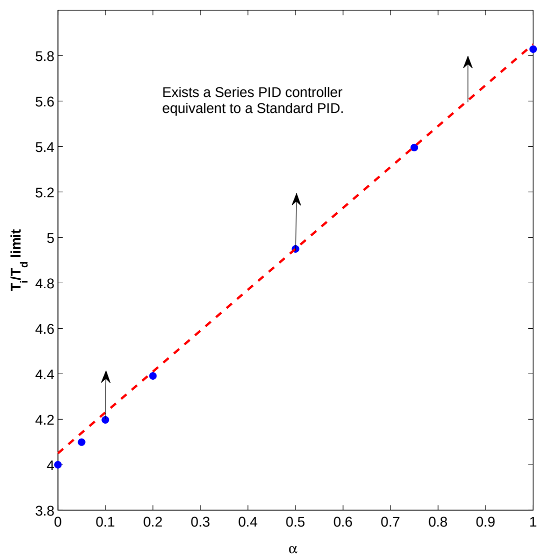

Figure 3. T_i/T_d **condition to obtain a Series PID Equivalent to a Standard PID**

Quadratic inequality (43) can be approximated by the following straight line for $0 \le \alpha \le 1.0$:

$$\frac{T_i}{T_d} > 4.05 + 1.80\ \alpha \quad (44)$$

3.6 From 2DoF Standard PID to Ideal PID with Filter

The PID_{2F} is a more general control algorithm and, as indicated above, not always an equivalent PID_2 controller may be obtained from the PID_{2F} but it is always possible to obtain a PID_{2F} control algorithm equivalent to the PID_2 by using the following relations:

$$K^*_p = F_{cf} K_p, \quad (45)$$

$$T^*_i = F_{cf} T_i, \quad (46)$$

$$T^*_d = \left(\frac{1+\alpha}{F_{cf}}\right) T_d, \quad (47)$$

$$T_f = \alpha T_d \quad (48)$$

$$\beta^* = \frac{\beta}{F_{cf}}, \quad (49)$$

$$\gamma^* = 0, \quad (50)$$

$$F_{cf} = 1 + \frac{\alpha T_d}{T_i}. \quad (51)$$

F_{cf} (51) is the *PID_2 to PID_{2F} conversion factor*.

To be realizable the 2DoF PID with filter PID_{2F} may be implemented as shown in Fig. 4.

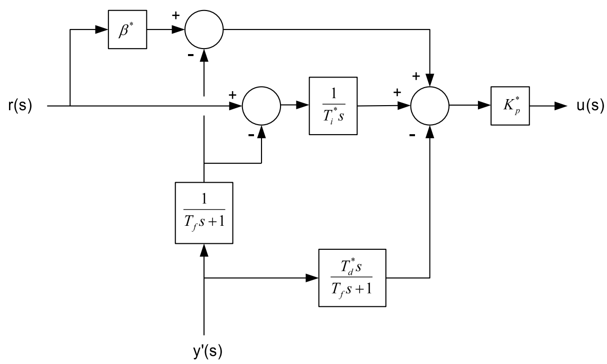

Figure 4. PID_{2F} **Controller Implementation**

Considering the above we may say that in the 2DoF PID controllers parametric space $\theta'_c \subset \theta_c \subset \theta^*_c$.

To summarize the above relations a 2DoF PID controllers conversion chart is shown in Fig 5. The solid arrows indicate directions on where there are always equivalent controllers and the dashed arrows the directions on where there are constraints to obtain equivalent controllers. As can be seen in this chart the 2DoF Ideal PID with filter is the most general proportional integral derivative control algorithm.

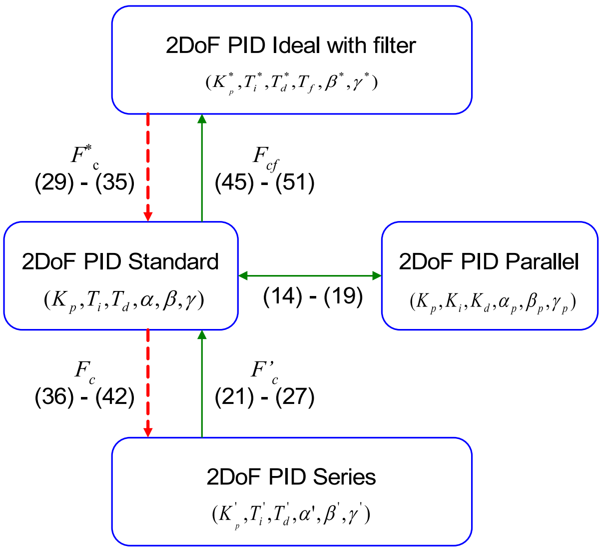

Figure 5. 2DoF PID Controllers Conversion

4 Comparative Analysis

For comparison of PID control algorithms the controlled process is represented by following transfer function:

$$P(s) = \frac{1.25\mathrm{e}^{-0.4s}}{(s+1)(0.5s+1)(0.25s+1)(0.125s+1)}. \tag{52}$$

4.1 Performance Optimized Series PID

The *performance* of the closed-loop control system is evaluated with the integrated absolute error (IAE) given by the following:

$$J_e \doteq \int_0^\infty |e(t)|\,\mathrm{d}t = \int_0^\infty |r(t) - y(t)|\,\mathrm{d}t. \tag{53}$$

The performance measure (53) will be evaluated for set-point and load-disturbance changes, J_{er}, J_{ed}, to obtain a combined performance cost functional

$$J_{eT} = J_{er} + J_{ed}. \tag{54}$$

First column of Table 1 lists the 2DoF Series PID controller parameters that optimize (54) on the process model (52). The controller performance index is $J_{eT} = 3.03$.

This table also includes the 2DoF Standard and 2DoF Ideal with filter equivalent PID controllers parameters. All three controllers will provide exactly the same closed-loop control system behavior with the test plant.

4.2 Performance Optimized Standard PID

Table 2 lists the 2DoF Standard performance optimized controller and 2DoF Ideal w/filter equivalent parameters. In this case the performance index (54) is $J_{eT} = 2.78$.

Table 1. PID Series Equivalent Controllers

	Series	Standard	Ideal w/filter
K_p	0.9345	1.5462	1.6142
T_i	1.0658	1.7635	1.8410
T_d	0.7752	0.3910	0.4488
T_f	-	-	0.0775
β	1.0280	0.6213	0.5951
α	0.10	0.1983	-
γ	0	0	0

Table 2. PID Standard Equivalent Controller

	Standard	Ideal w/filter
K_p	1.6649	1.7244
T_i	1.4721	1.5247
T_d	0.5259	0.5585
T_f	-	0.0526
β	0.5343	0.5159
α	0.10	-
γ	0	0

There is an 8.25% improvement over the performance obtained with the Series PID.

The 2DoF Standard and 2DoF Ideal w/filter controllers provide more performance than the Series one. It can be checked that there is no Series PID controller equivalent to the Standard and Ideal with filter PID controllers in Table 2.

4.3 Controllers High Frequency Gain

The high frequency gain of the different PID control algorithms are list in Table 3. These will establish the controller capabilities for high frequency measurement noise reduction.

Suppose now that a 2DoF Ideal PID with filter (PID_{2F}) is already tuned with following parameters: $K_p^* = 0.40$, $T_i^* = 1.50$, $T_d^* = 0.10$, $T_f = 0.50$, $\beta^* = 0.25$, and $\gamma^* = 0$, selected in order to have a controller with very low high frequency gain ($K_{PID2F\infty} = 0.08$) to reduce control effort variability.

The control system output and control effort to a 10% set-point step change followed by a 5% disturbance step change is shown in Fig. 6.

Table 3. PID High Frequency Gain

	K_∞
Standard (12)	$K_p\left(1 + \frac{1}{\alpha}\right)$
Parallel (13)	$\frac{1}{\alpha_p}$
Series (20)	$\frac{K_p'}{\alpha'}$
Ideal w/filter (28)	$\frac{K_p^* T_d^*}{T_f}$

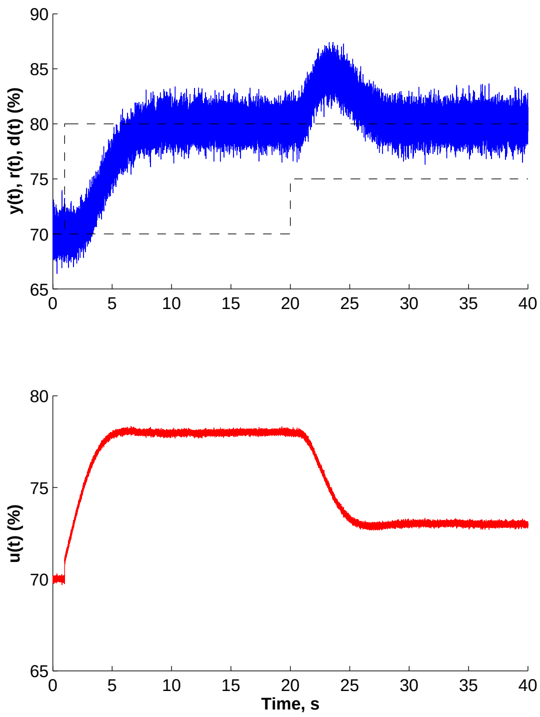

Figure 6. Control System Responses

In this case there are no equivalent Standard or Series PID controllers.

The 2DoF Ideal PID with filter control algorithm is able to provide performances and capabilities not attainable with other PID control algorithms.

5 Conclusions

Commonly used two-degree-of-freedom (2DoF) proportional integral derivative (PID) control algorithms were presented. Conversion factors and relations allow obtaining equivalent controllers and show when it is not possible to have and equivalent controller.

The Ideal PID with filter is the more general control algorithm followed by the Standard and Parallel PID been the Series PID algorithm the more restrictive one.

6 Acknowledgments

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Set-Point Weight Selection for Robustly Tuned PI/PID Regulators for Over Damped Processes

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Abstract

The aim of the paper is to present tuning equations for proportional integral (PI) and proportional integral derivative (PID) controllers with set-point weight (two degrees off freedom - 2DoF). These are based on a performance/robustness trade-off analysis with first- and second-order-plus-dead-time (FOPDT, SOPDT) models. On the basis of this analysis a tuning method was developed for 2DoF PI and PID controllers that allow designing closed-loop control systems with a specified M_S robustness that at the same time have the maximum IAE performance allowed. The control system robustness is adjusted varying only the controller proportional gain.

1 INTRODUCTION

Since their introduction in 1940 [11, 12] commercial *proportional integral derivative* (PID) controllers have been widely used in industrial control applications. This popularity is mainly due to their simple structure and the simple parameters that need to be used in these controllers.

Several methods for the design and tuning of PID controllers have been reported in the literature. Most of those methods were concerned with feedback controllers that are tuned either for disturbance rejection (regulatory control) [13, 17, 28] or for a well-damped fast response to a step change in the controller set point (servo control) [19, 22, 23]. A two-degree-of-freedom (2DoF) controller has been developed to meet both objectives. The second degree of freedom in this controller, obtained with the set-point weight, provides additional flexibility to the control system design. Examples of such a formulation can be found in the literature [5, 6], and the different tuning methods that have been developed over the last few years are examples of such formulations [3, 8, 9, 10, 15, 25].

The control system design is usually based on the use of low-order linear models; these models in turn are based on the normal operating point of the closed-loop control system. Because most industrial processes are non-linear, it is necessary to account for possible changes in the process characteristics by adopting certain relative stability margins or robustness requirements for the control system.

Therefore, in the design of a closed-loop control system with PI and PID controllers, we must consider the *trade-off* between two conflicting criteria: the time-response *performance* to the set point and load disturbances and the *robustness* to changes in the characteristics of the controlled process. If only the system performance is taken into account, using an integrated error criterion (IAE, ITAE or ISE) or a time response characteristic (overshoot, rise time, or settling time), as in [16] and [27], the resulting closed-loop control system will probably have very low robustness. On the other hand, if the system is designed to have high robustness, as in [15], and if the performance of the resulting system is not evaluated, the designer would have no idea of the *cost* involved in operating such a highly robust system. In some previous studies [16, 24, 26], the performance and robustness of the system were taken into account for optimizing the IAE or ITAE performance, but only the usual minimum level of robustness could be guaranteed. Servo/regulatory IAE optimization for two robustness levels is also available but only for first-order plus dead-time models [21].

To estimate the performance losses that occur when the control-system robustness is increased, a performance/robustness trade-off analysis was conducted on 1DoF and 2DoF PI and PID control systems with first- and second-order-plus-dead-time (FOPDT, SOPDT) models; maximum sensitivity, M_S, was used as a measure of the system robustness. Using this approach, the designer can estimate the performance losses that may occur when setting certain robustness levels [4].

The results of the trade-off analysis allowed to obtain robust-performance tuning rules for PI and PID controllers [1]. In addition, simplified servo and regulatory control tuning relations for one-degree-of-freedom (1DoF) PI and PID controllers for four robustness M_S levels are presented in [2].

In this paper one step further is progressed. The regulatory control tuning relations are extended to two-degree-of-freedom (2DoF) PI and PID controllers for four robustness M_S levels in the range from 1.4 to 2.0 and thus, facilitate the design of robust closed-loop control systems in which the robustness level is adjusted by varying the controller proportional gain alone.

After consulting O'Dwyer's [20] handbook, it was found that there are fewer tuning rules of PI and PID controllers for SOPDT models than that for FOPDT models and most of these rules have been devised only on the basis of the system performance.

The rest of the paper is organized as follows. The transfer functions of the controlled process model, controller, and closed-loop control system are presented in Section 2. The performance/robustness analysis is described in Section 3, and the proposed *Unified Simply Optimal and Robust Tuning* is presented in Section 4. An example of the use of the tuning rules and the performance/robustness trade-off is shown in Section 5. Finally, concluding remarks are provided.

2 Problem Formulation

Consider a closed-loop control system, as shown in Fig. 1, where $P(s)$ and $C(s)$ are the controlled process model and the controller transfer function, respectively. In this system, $r(s)$ is the set point; $u(s)$, the controller output signal; $d(s)$, the load disturbance; and $y(s)$, the controlled process variable.

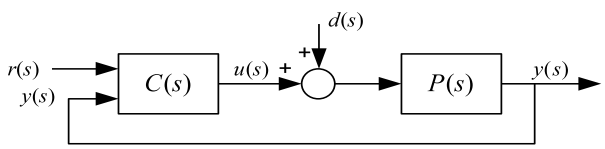

Figure 1. Closed-Loop Control System

The controlled process is represented by an SOPDT model given by the general transfer function

$$P(s) = \frac{Ke^{-Ls}}{(Ts+1)(aTs+1)}, \ \tau_o = L/T, \quad (1)$$

where K is the gain; T, the main time constant; a, the ratio of two main time constants ($0 \le a \le 1.0$); L, the dead-time; and τ_o, the *normalized dead time*. The model transfer function (1) allows the representation of FOPDT processes ($a = 0$), over damped SOPDT processes ($0 < a < 1$), and dual-pole-plus-dead-time (DPPDT) processes ($a = 1$).

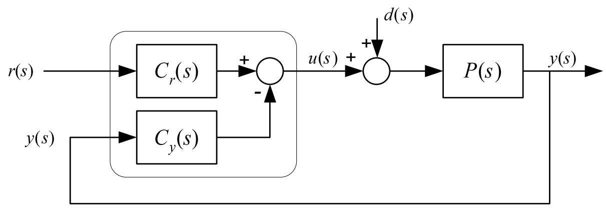

Figure 2. 2DoF Closed-Loop Control System

The process will be controlled with a 2DoF PID controller whose output is as follows [7]:

$$u(s) = K_p(\beta r(s) - y(s)) + \frac{K_p}{T_i s}(r(s) - y(s)) - \left(\frac{K_p T_d s}{\alpha T_d s + 1}\right) y(s), \quad (2)$$

where K_p is the controller *proportional gain*; T_i, the *integral time constant*; T_d, the *derivative time constant*; β, the proportional set-point weight; and α, the *derivative filter constant*. Then the controller parameters to tune are $\theta_c = \{K_p, T_i, T_d, \beta\}$. Usually, $\alpha = 0.10$ [14].

Equation (2) may be rearranged, but only for analysis purposes and not for implementation, as follows

$$u(s) = K_p\left(\beta + \frac{1}{T_i s}\right) r(s) - K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{0.1 T_d s + 1}\right) y(s), \quad (3)$$

or in the compact form shown in Fig. 2 as

$$u(s) = C_r(s) r(s) - C_y(s) y(s), \quad (4)$$

where $C_r(s)$ is the *set-point controller* transfer function and $C_y(s)$ is the *feedback controller* transfer function.

The output of the closed-loop control system varies with a change in any of its inputs as:

$$y(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)} r(s) + \frac{P(s)}{1 + C_y(s)P(s)} d(s), \quad (5)$$

or

$$y(s) = M_{yr}(s) r(s) + M_{yd}(s) d(s), \quad (6)$$

where $M_{yr}(s)$ is the transfer function from the set-point to the controlled process variable and is known as the *servo control* closed-loop transfer function; $M_{yd}(s)$ is the transfer function from the load disturbance to the controlled process variable and is known as the *regulatory control* closed-loop transfer function.

The *performance* of the closed-loop control system is evaluated using the IAE cost functional given by

$$J_e \doteq \int_0^\infty |e(t)| \, \mathrm{d}t = \int_0^\infty |y(t) - r(t)| \, \mathrm{d}t. \qquad (7)$$

The performance (7) will evaluated for a step change in the set-point, J_{er} and in the load-disturbance, J_{ed}.

The peak magnitude of the sensitivity function will be used as an indicator of the system *robustness* (relative stability). The maximum sensitivity for the control system is defined as

$$M_S \doteq \max_\omega |S(\mathrm{j}\omega)| = \max_\omega \frac{1}{|1 + C_y(\mathrm{j}\omega)P(\mathrm{j}\omega)|}. \qquad (8)$$

If the system robustness (8) is not taken into account for the design, the controller parameters may be optimized to maximize the system performance or to achieve the minimum value of the cost functional in (7), using M_{yr} for set point changes (J_{er}^o) and M_{yd} for load disturbance changes (J_{ed}^o).

Because of the control system performance/robustness trade-off, if a robustness requirement is included into the design then, it is expected that the actual system performance will be reduced ($J_e \geq J_e^o$). Then, the *performance degradation factor* defined as

$$F_p \doteq \frac{J_e^o}{J_e}, \ F_p \leq 1, \qquad (9)$$

is used to evaluate the performance/robustness trade-off.

3 Performance/Robustness Trade-off Analysis

To evaluate the performance degradation when the system robustness is increased, the following steps, as were presented in [4], were followed.

3.1 Controllers Optimum Performance

For the regulatory-control performance-optimized PI and PID controllers, the parameters $\theta_c^o = \{K_p^o, T_i^o, T_d^o\}$ were obtained using the cost functional (7) such that

$$J_{ed}^o \doteq J_{ed}(\theta_c^o) = \min_{\theta_c} J_{ed}(\theta_c), \qquad (10)$$

for (1) with $a \in \{0, 0.25, 0.5, 0.75, 1\}$ and ten τ_o in the range 0.05-2.0, for set-point and load-disturbance step changes. The robustness of the control systems that deliver the optimal performance was evaluated by using M_S.

3.2 Controllers Degraded Performance

To increase the control-loop robustness, a target performance degradation factor, F_p^t, was included in the cost functional, as follows

$$J_{F_p} \doteq J(\theta_c, F_p^t) = \left| \frac{J_e^o}{J_e(\theta_c)} - F_p^t \right|, \qquad (11)$$

for obtaining the PI and PID regulatory control parameters θ_c^{o1} such that

$$J_{F_p}^o \doteq J_{F_p}(\theta_c^{o1}, F_p^t) = \min_{\theta_c} J_{F_p}(\theta_c, F_p^t). \qquad (12)$$

When F_p^t was decreased, the control-system robustness was increased to the target level, M_S^t.

With starting point as the original unconstrained (from the point of view of robustness) optimal parameters θ_c^{o1}, a second optimization was conducted using the cost functional

$$J_{M_S} \doteq J(\theta_c, M_S^t) = \left| M_S(\theta_c) - M_S^t \right|, \qquad (13)$$

in order to achieve the target robustness. The robust controller parameters, θ_c^{o2}, are such that

$$J_{M_S}^o \doteq J_{M_S}(\theta_c^{o2}, M_S^t) = \min_{\theta_c} J_{M_S}(\theta_c, M_S^t). \qquad (14)$$

For the analysis, four target robustness levels have been considered, $M_S^t \in \{2, 1.8, 1.6, 1.4\}$.

Finally, the performance degradation factor required for obtaining M_S^t in (14) is evaluated as follows

$$F_p(M_S^t) = \frac{J_e^o}{J_e(\theta_c^{o2})}. \qquad (15)$$

Therefore, the second optimization provides the controller parameters θ_c^{o2} required to formulate a regulatory control system with the target robustness (8), M_S^t, and with the best performance allowed when using the IAE criteria (7), J_{ed}.

PI Regulatory Control

The analysis results for the regulatory control PI controllers for the particular case of FOPDT models ($a = 0$) are shown in Fig. 3. In this case, the performance degradation required to achieve a certain robustness level decreases as τ_o of the model increases.

PID Regulatory Control

The effect of the model time constants ratio a for this case are shown in Fig. 4 for the target robustness of $M_S^t = 1.6$.

3.3 2DoF Controllers Proportional Set-Point Weight

To extend the tuning procedure to 2DoF controllers the set-point weight is investigated.

Using the parameters for the robust performance of regulatory control PI and PID controllers, θ_c^{o2}, a third optimization is performed. The free β for the 2DoF PI and PID controllers is determined by optimizing (7) for a set-point step change (J_{er}).

4 Unified Simple Optimal and Robust Tuning for 2DoF PI and PID Controllers ($uSORT_2$)

Analysis of PI and PID controllers parameters for regulatory control showed that for a model with a given time

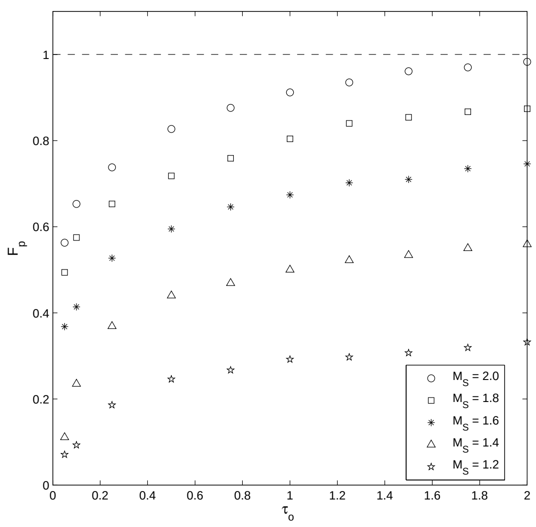

Figure 3. PI Regulatory Control Performance Degradation Factor ($a = 0.0$)

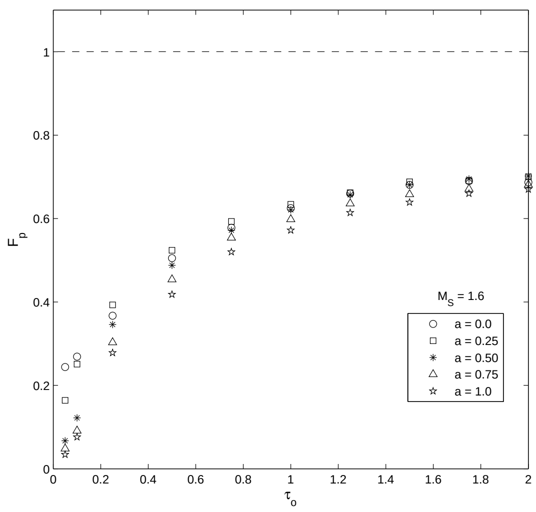

Figure 4. PID Regulatory Control Performance Degradation Factor ($M_S^t = 1.6$)

constants ratio a, increasing the control system robustness by decreasing M_S^t, results in a substantial reduction in K_p. However, this increase in the robustness has negligible effect on T_i and T_d, except in the case of models with a very low τ_o (when high robustness is required). An example of this phenomenon for a second-order model (1) with $a = 0.50$ is shown in Fig. 5.

On the basis of this observation, equations *that are independent of the target robustness level* can be obtained for the controller integral time constant and derivative time constant, as follows:

$$T_i = \mathbf{F}(T, \tau_o, a), \;\; T_d = \mathbf{G}(T, \tau_o, a). \tag{16}$$

With these equations at hand, the controller proportional gains were readjusted to match a target robustness to obtain equations given by the following

$$K_p = \mathbf{H}(K, \tau_o, a, M_S^t). \tag{17}$$

Relations (16) and (17) allow obtaining the regulatory control tuning rules [2].

The optimization results also show that for a given robustness level, the proportional set-point weight β is affected more by τ_o than by a. Then, it can be obtained as follows:

$$\beta = \mathbf{Q}(\tau_o, M_S^t). \tag{18}$$

Relation (18) allows to extend the tuning rules to 2DoF PI/PID controllers.

For FOPDT and SOPDT models with τ_o in the range 0.1-2.0 and four M_S^t values the normalized 2DoF PI and PID controller parameters can be obtained using the model (1) parameters, $\theta_p = \{K, T, a, L, \tau_o\}$, from the following relations:

$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_o^{a_2}, \tag{19}$$

$$\tau_i \doteq \frac{T_i}{T} = b_0 + b_1 \tau_o^{b_2}, \tag{20}$$

$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_o^{c_2}, \tag{21}$$

$$\beta = d_0 + d_1 \tau_o^{d_2}. \tag{22}$$

The value of the constants a_i, b_i, c_i,and d_i in (19) to (22) are listed in Tables 1 to 3.

With a maximum absolute deviation from the target robustness M_S^t of 4.09% and an average deviation of only 0.75% the proposed $uSORT_2$ tuning may be considered as a 2DoF PI and PID *global robust tuning method* with levels $M_S^t \in \{2.0, 1.8, 1.6, 1.4\}$ for FOPDT and SOPDT models with normalized dead-times in the range 0.1-2.0.

5 Examples

For comparison of the performance and robustness obtained with the proposed $uSORT_2$ method we will use the [18] [MEB] tuning rules for Standard PID controllers that optimize the IAE criteria for servo and regulatory control operation.

Consider the FOPDT and SOPDT processes given by:

$$P_1(s) = \frac{1.2\mathrm{e}^{-1.5s}}{2s+1}.$$

$$P_2(s) = \frac{1.2\mathrm{e}^{-1.5s}}{(2s+1)(s+1)}.$$

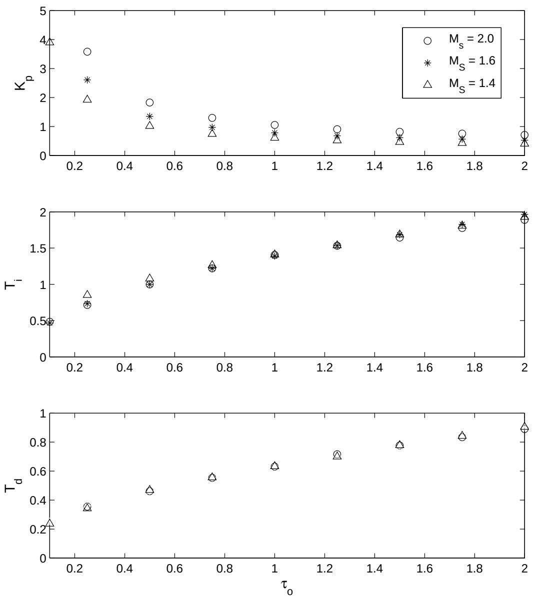

Figure 5. Parameters of the Regulatory Control Optimized PID Controller ($a = 0.50$)

Table 2. 2DoF PID Controller Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.235	0.435	0.454	0.464	0.488
a_1	0.840	0.551	0.588	0.677	0.767
a_2	-0.919	-1.123	-1.211	-1.251	-1.273
	Target robustness $M_S^t = 1.8$				
a_0	0.210	0.380	0.400	0.410	0.432
a_1	0.745	0.500	0.526	0.602	0.679
a_2	-0.919	-1.108	-1.194	-1.234	-1.257
	Target robustness $M_S^t = 1.6$				
a_0	0.179	0.311	0.325	0.333	0.351
a_1	0.626	0.429	0.456	0.519	0.584
a_2	-0.921	-1.083	-1.160	-1.193	-1.217
	Target robustness $M_S^t = 1.4$ †				
a_0	0.155	0.228	0.041	0.231	0.114
a_1	0.455	0.336	0.571	0.418	0.620
a_2	-0.939	-1.057	-0.725	-1.136	-0.932
	†Valid only for $\tau_o \geq 0.40$ if $a \geq 0.25$				
b_0	-0.198	0.095	0.132	0.235	0.236
b_1	1.291	1.165	1.263	1.291	1.424
b_2	0.485	0.517	0.496	0.521	0.495
c_0	0.004	0.104	0.095	0.074	0.033
c_1	0.389	0.414	0.540	0.647	0.756
c_2	0.869	0.758	0.566	0.511	0.452

Table 1. 2DoF PI Controller Tuning

	Controlled process time constants ratio a				
	0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.265	0.077	0.023	-0.128	-0.244
a_1	0.603	0.739	0.821	1.035	1.226
a_2	-0.971	-0.663	-0.625	-0.555	-0.517
	Target robustness $M_S^t = 1.8$				
a_0	0.229	0.037	-0.056	-0.160	-0.289
a_1	0.537	0.684	0.803	0.958	1.151
a_2	-0.952	-0.626	-0.561	-0.516	-0.472
	Target robustness $M_S^t = 1.6$				
a_0	0.175	-0.009	-0.080	-0.247	-0.394
a_1	0.466	0.612	0.702	0.913	1.112
a_2	-0.911	-0.578	-0.522	-0.442	-0.397
	Target robustness $M_S^t = 1.4$				
a_0	0.016	-0.053	-0.129	-0.292	-0.461
a_1	0.476	0.507	0.600	0.792	0.997
a_2	-0.708	-0.513	-0.449	-0.368	-0.317
b_0	-1.382	0.866	1.674	2.130	2.476
b_1	2.837	0.790	0.268	0.112	0.073
b_2	0.211	0.520	1.062	1.654	1.955

Table 3. 2DoF Controllers Set-Point Weight Constants

	Target robustness M_S^t			
	2.0	1.8	1.6	1.4
	PI Controller			
d_0	0.730	0.658	0.649	0.811
d_1	0.302	0.578	0.898	1.205
d_2	0.386	0.372	0.446	0.608
	PID Controller			
d_0	0.306	0.248	0.255	0.383
d_1	0.416	0.571	0.727	0.921
d_2	0.367	0.362	0.476	0.612

Table 4. P_1 **Control Performance and Robustness**

	$uSORT_2$ M_S^d 2.0	1.8	1.6	1.4	MEB IAE
	PI Controller				
K_p	0.885	0.779	0.651	0.500	-
T_i	2.576				-
β	1.00	1.18	1.44	1.82	-
M_S^r	2.01	1.81	1.61	1.42	-
$J_{ed}/\Delta d$	2.910	3.305	3.960	5.156	-
$J_{er}/\Delta r$	2.993	2.921	2.909	3.083	-
	PID Controller				
K_p	1.108	0.984	0.829	0.626	1.293
T_i	1.867				1.971
T_d	0.614				0.569
β	0.68	0.76	0.89	1.16	1.0
M_S^r	2.02	1.82	1.61	1.40	2.36
$J_{ed}/\Delta d$	1.969	2.215	2.593	3.303	1.666
$J_{er}/\Delta r$	3.036	3.113	3.231	3.412	3.091

Table 5. P_2 **Control Performance and Robustness**

	$uSORT_2$ M_S^d 2.0	1.8	1.6	1.4	MEB IAE
	PI Controller				
K_p	0.838	0.740	0.613	0.461	-
T_i	3.743				-
β	1.00	1.18	1.44	1.82	-
M_S^r	2.03	1.83	1.62	1.42	-
$J_{ed}/\Delta d$	4.466	5.059	6.102	8.098	-
$J_{er}/\Delta r$	4.359	4.236	4.115	4.052	-
	PID Controller				
K_p	1.037	0.951	0.801	0.620	1.539
T_i	2.454				2.971
T_d	1.108				0.883
β	0.68	0.76	0.89	1.16	1.0
M_S^r	1.93	1.79	1.60	1.41	2.94
$J_{ed}/\Delta d$	2.848	3.094	3.605	4.456	2.141
$J_{er}/\Delta r$	4.325	4.396	4.534	4.702	4.289

The $uSORT_2$ 2DoF PI and PID controller parameters and the control system performance and robustness for regulatory and servo-control operation of P_1 and P_2 are listed in Table 4 and Table 5, respectively. The MEB data in these tables corresponds to the 1DoF PID tuning for regulatory control operation.

From these tables it is noted that for same robustness design level (M_S^d) the PID controllers deliver more regulatory control performance than the PI controllers. They also show the performance/robustness trade-off, an increment in the control system robustness always reduces its performance. For example, to increase the robustness reducing M_S^d from the minimum robustness level of 2.0 to an intermediate robustness level of 1.6 reduces the regulatory performance of the control system in more than 30%.

It is also noted that the performance optimized MEB control systems have low robustness, $M_S > 2.0$ in both cases.

In case of a change in the set-point the 2DoF PI controllers have better performance than the PID controllers. In this case it is also noted that for same process and controller the $uSORT_2$ controller output "proportional kick", $\Delta u_{0+} = K_p \beta \Delta r$, is nearly the same for all design robustness levels. For these particular examples the PI_2 and PID_2 average normalized controller output instantaneous changes, $\Delta u_{0+}/\Delta r$, are 0.89 and 0.73, respectively, lower then the 1DoF MEB PID changes of 1.29 (P_1) and 1.54 (P_2). The set-point weight β of the 2DoF controllers speeds-up the servo-control responses when the controller gain K_p is low and reduces its overshoot when the controller gain is high.

The P_2 control system responses to a 10% set-point step change followed by a 5% load-disturbance step change are shown in Fig. 6. It clearly shows the existing trade-off between the regulatory control performance and the control system robustness and the advantages of using a 2DoF controller instead of a 1DoF controller tuned for regulatory control operation when a set-point change is made. In this latter case the response overshoot is reduced and the controller output is smoother and with a lower instantaneous change.

6 Conclusions

Based on a performance (*IAE*) - robustness (M_S) analysis simple tuning relations are proposed that unifies the treatment of two-degree-of-freedom (2DoF) PI and PID controllers and the use of first- and second-order-plus-dead-time (FOPDT, SOPDT) models.

The proposed *Unified Simple Optimal and Robust Tuning* for 2DoF PI/PID controllers ($uSORT_2$) allows adjusting the control system robustness varying only the controller proportional gain.

Acknowledgments

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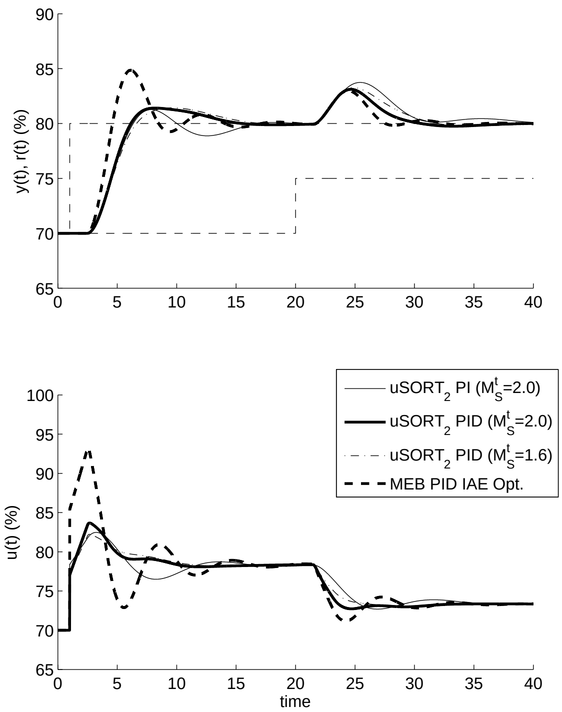

Figure 6. Example - P_2 Control System Responses

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Two-Degree-of-Freedom Proportional Integral Control of Inverse Response Second-Order Processes

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***Abstract*— The aim of this paper is to present a robust tuning method for two-degree-of-freedom (2DoF) proportional integral (PI) controllers for inverse response controlled processes modeled by a second-order plus a right-half plane zero (SOPRHPZ) transfer function. This is based on the use of a servo and regulatory control closed-loop model reference optimization procedure. The robustness of the control system is analyzed for model right-half plane zero relative positions in the range from 0.1 to 4.0. It is found that the non-minimum phase zero position affects the achievable control system robustness level. Controller tuning equations that guarantee the design robustness level are provided for inverse response second-order models with right-half plane zero relative positions in ranges that depend of the target robustness level. The designer is allowed to deal with the performance/robustness *trade-off* of the closed-loop control system by specifying the desired robustness level through selecting the desired maximum sensitivity in the range from 1.4 to 2.0. In addition, a *smooth servo/regulatory performance combination* is obtained by forcing both closed-loop transfer functions to perform as closely as possible to non-oscillatory dynamic targets. Comparative examples show the effectiveness of the proposed robust tuning method and its exact achievement of the control system robustness target for the inverse response controlled process models considered.**

I. INTRODUCTION

The inverse response characteristic, i.e. the controlled process initial response to a step change is in the opposite direction to that of the steady-state direction, originated by two parallel competing dynamics is present in industrial processes as distillation columns and chemical reactors [14]. This non-minimum phase characteristics impose severe limits to the achievable closed-loop control system performance [13].

IMC-based tunings of proportional integral (PI) and proportional integral derivative (PID) controllers for inverse response controlled processes are presented in [9], [10], [13]. All these methods include one design parameter, the closed-loop time constant, that is selected using only performance considerations although it also affects the control systems stability. A tuning method based on closed-loop transfer function matching is presented in [12]. Smith-predictor like control systems for inverse response processes can be found in [1], [18].

A direct synthesis design for disturbance rejection (*DS-d*) tuning of inverse response processes is proposed in [8]. This is based on a regulatory closed-loop transfer function target with a design parameter, the closed-loop system time constant. A relation of the *DS-d* design parameters and the control systems robustness for $M_S = 2.0$ is presented in [2].

While nowadays are more frequent the PI and PID robust design approaches for over damped processes there are no tuning relations based on a robustness specification for inverse response processes. For this reason it is not possible to include a comparative example with tuning rules that guarantee a constant robustness level for a set of inverse response processes as the proposed method does.

An alternative tuning method for two-degree-of-freedom (2DoF) proportional integral (PI) controllers for second-order plus right-half plane zero (SOPRHPZ) controlled processes is presented in this communication. This is based on the use of a model reference optimization procedure with servo and regulatory closed-loop transfer functions targets. The proposed approach explicitly considers the control system performance/robustness *trade-off*. The distinctive feature of the resulting tuning procedure is the incorporation of the desired robustness level as an explicit design parameter. Therefore, the designer may select the desired robustness M_S in the range from 1.4 to 2.0 for inverse response controlled process models.

The rest of the paper is organized as follows: the transfer functions of the controlled process model, the controller, and the closed-loop control system are presented in Section II; the general optimization procedure is described in Section III; the tuning equations of the proposed method in Section IV; and Section V shows the robustness and performance of the obtained control systems; ending with some conclusions.

II. PROBLEM FORMULATION

The controller design procedure described below for inverse response controlled processes follows the model reference robust tuning (MoReRT) methodology proposed in [5] used for over damped stable controlled processes [3] as well as for integrating processes [4].

Consider a closed-loop control system, as shown in Fig. 1, where $P(s)$ and $C(s)$ are the inverse response controlled process model transfer function and the PI controller transfer function, respectively. In this system, $r(s)$ is the set-point, $u(s)$ is the controller output signal, $d(s)$ is the load-disturbance, and $y(s)$ is the process controlled variable.

The closed-loop control system output, $y(s)$, in response to changes in its inputs, $r(s)$ and $d(s)$, is given by the following:

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \quad (1)$$

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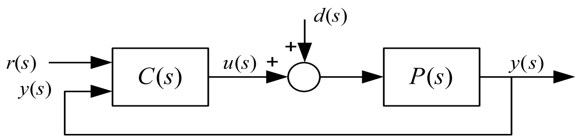

Fig. 1. Closed-Loop Control System

where $M_{yr}(s)$ and $M_{yd}(s)$ are the *servo control* (set-point tracking) and *regulatory control* (disturbance rejection) closed-loop transfer function, respectively.

The development of the proposed tuning method of 2DoF PI controllers for inverse response controlled process will take into account the closed-loop control system performance, stating target responses for step changes in the set-point and the load-disturbance, and also the control system robustness, measuring this with the maximum sensitivity, M_S.

A. 2DoF Proportional Integral Controller (PI_2)

The process will be controlled with a two-degree-of-freedom proportional integral (PI_2) controller [6] whose output is expressed as follows:

$$u(s) = K_p \left\{ \beta r(s) - y(s) + \frac{1}{T_i s} \left[r(s) - y(s) \right] \right\}, \quad (2)$$

where K_p is the controller *proportional gain*, T_i is the *integral time constant*, and β is the *set-point proportional weight*.

For the purposes of analysis only, not implementation, the controller output (2) will be rewritten as follows:

$$u(s) = K_p \left(\beta + \frac{1}{T_i s} \right) r(s) - K_p \left(1 + \frac{1}{T_i s} \right) y(s), \quad (3)$$

and in compact form as

$$u(s) = C_r(s) r(s) - C_y(s) y(s). \quad (4)$$

The block diagram of the PI_2 controller is shown in 2.

The closed-loop transfer functions of the servo control and the regulatory control in (1) are then given by

$$M_{yr}(s) = \frac{C_r(s) P(s)}{1 + C_y(s) P(s)}, \quad (5)$$

and

$$M_{yd}(s) = \frac{P(s)}{1 + C_y(s) P(s)}, \quad (6)$$

which are related as follows:

$$M_{yr}(s) = C_r(s) M_{yd}(s). \quad (7)$$

B. General Closed-Loop Transfer Functions Targets

For the development of the proposed tuning method, it is important to have the smallest possible number of design parameters. Consequently, the desired control system response to a load-disturbance step change will involve only one design parameter, initially the closed-loop time constant T_c, and is selected with no steady-state error and non-oscillatory, for a smooth response. In general this is expressed by the following:

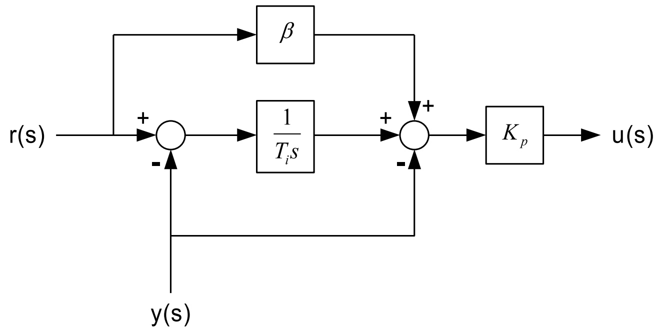

Fig. 2. Two-Degree-of-Freedom PI Controller

$$M_{yd}^t(s) = \mathcal{M}_d(T_c, \overline{\theta}_c, \overline{\theta}_p, s) = \frac{K_o s N_p^+(s)}{p(T_c, \overline{\theta}_p, s)}, \quad (8)$$

where $N_p^+(s)$ contains the non-minimum phase elements of the model, $\overline{\theta}_p$ are the controlled process model parameters, $\overline{\theta}_c$ are the controller parameters, and $p(T_c, \overline{\theta}_p, s)$ is the characteristic polynomial of the closed-loop control system.

Using (8) in (7) the servo-control closed-loop transfer function is given by

$$M_{yr}^t(s) = \mathcal{M}_r(T_c, \overline{\theta}_c, \overline{\theta}_p, s) = C_r(s) M_{yd}^t(s). \quad (9)$$

Then using (8) and (9) in (1) the global control system output target, $y^t(s)$, is computed as follows:

$$y^t(s) = M_{yr}^t(s) r(s) + M_{yd}^t(s) d(s), \quad (10)$$

and in the time domain as follows:

$$y^t(t) = y_r^t(t) + y_d^t(t), \quad (11)$$

where $y_r^t(t)$ is the servo-control step response target and $y_d^t(t)$, the regulatory control step response target.

III. Controller Design

Usually the design of the 2DoF PI controllers is performed in two stages [7], [11], [15], [16]. First, as required to obtain the desired regulatory control performance and specific closed-loop control system robustness level, the parameters (K_p, T_i) of the feedback controller $C_y(s)$ are determined for a parameter set of the controlled process model $\overline{\theta}_p$. Second, the set-point controller $C_r(s)$ free parameter (β) is used to improve the servo-control performance.

In what follows a different approach is taken. The complete set of PI_2 controller parameters $\overline{\theta}_c = \{K_p, T_i, \beta\}$ will be obtained considering, at the same time, the regulatory control and the servo-control performance, to obtain a controller with a targeted *servo/regulatory performance combination* that will also produce a closed-loop control system with a specific robustness level.

A. Cost Functionals

For the regulatory control response, the cost functional to be minimized is defined as follows:

$$J_d \doteq \int_0^\infty \left[y_d^t(\tau_c, \overline{\theta}_c, \overline{\theta}_p, t) - y_d(\overline{\theta}_c, \overline{\theta}_p, t)\right]^2 \mathrm{d}t, \quad (12)$$

where $y_d^t(\tau_c, \overline{\theta}_c, \overline{\theta}_p, t)$ is the step response of the regulatory control target closed-loop transfer function (8) and $y_d(\overline{\theta}_c, \overline{\theta}_p, t)$ is that of the regulatory control system (6) with the controlled process $P(s)$ and controller $C_y(s)$. In (12) τ_c is the dimensionless design parameter that will be defined latter.

In a similar way, the servo-control cost functional to be minimized is defined as follows:

$$J_r \doteq \int_0^\infty \left[y_r^t(\tau_c, \overline{\theta}_p, t) - y_r(\overline{\theta}_c, \overline{\theta}_p, t)\right]^2 \mathrm{d}t, \quad (13)$$

where $y_r^t(\tau_c, \overline{\theta}_p, t)$ is the step response of the servo-control target closed-loop transfer function (9) and $y_r(\overline{\theta}_c, \overline{\theta}_p, t)$ is that of the servo-control system (5) with the controlled process $P(s)$ and controller $C_r(s)$.

For the 2DoF PI controller design, the following overall cost functional is defined:

$$J_T(\tau_c, \overline{\theta}_c, \overline{\theta}_p) \doteq J_r(\tau_c, \overline{\theta}_c, \overline{\theta}_p) + J_d(\tau_c, \overline{\theta}_c, \overline{\theta}_p), \quad (14)$$

to obtain the optimum controller parameters $\overline{\theta}_c^o = \left\{K_p^o, T_i^o, \beta^o\right\}$ such that

$$J_T^o \doteq \min_{\overline{\theta}_c} \left\{J_r(\tau_c, \overline{\theta}_c, \overline{\theta}_p) + J_d(\tau_c, \overline{\theta}_c, \overline{\theta}_p)\right\}. \quad (15)$$

Note that $\overline{\theta}_c^o = \overline{\theta}_c^o(\overline{\theta}_p, \tau_c)$. Moreover, for each $\overline{\theta}_c^o$ set obtained, the closed-loop control system robustness is measured using the maximum sensitivity M_S, which is defined as follows:

$$M_S \doteq \max_\omega |S(\mathrm{j}\omega)| = \max_\omega \frac{1}{|1 + C_y(\mathrm{j}\omega)P(\mathrm{j}\omega)|}. \quad (16)$$

For J_T optimization a direct search Nelder-Mean simplex-based algorithm [17] is used. For a given controlled process model with parameters $\overline{\theta}_p$ a design parameters τ_c is selected, J_T is optimized to obtain the controller parameters $\overline{\theta}_c^o$, and the robustness of the resulting control system is evaluating using M_S. If required, the design parameter τ_c is readjusted accordingly to obtain the target robustness level. Controller parameters can be found as functions of the controlled process model and the target robustness M_S^t.

The optimization formulated with (14) is a *closed-loop model matching problem* instead of a traditional performance optimization problem. The control system behavior is stated by the selected servo and regulatory control closed-loop transfer function targets.

IV. Controlled Process Models and Closed-Loop Transfer Functions Targets

The controlled processes with inverse response will be represented by a second-order plus right-half plane zero (SOPRHPZ) model given by the transfer function

$$P(s) = \frac{K(-bTs + 1)}{(Ts + 1)(aTs + 1)}, \quad (17)$$

TABLE I

SOPRHPZ Models b Ranges

	Target robustness M_S^t			
	2.0	1.8	1.6	1.4
b_{min}	0.25	0.25	0.25	0.25
b_{max}	2.5	2.0	1.5	1.0

where K is the model gain, T the time constant, a the ratio of the two main poles time constants ($0.1 \leq a \leq 1.0$), and b the relative position of the right-half plane zero.

The desired control system response to a load-disturbance step change is given by the third order transfer function target

$$M_{yd}^t(s) = \frac{(T_i/K_p)s(-bTs + 1)}{(T_c s + 1)^2(aT_c s + 1)}, \quad (18)$$

and the corresponding one to a set-point step by the second order transfer function target

$$M_{yr}^t(s) = \frac{-bTs + 1}{(T_c s + 1)(aT_c s + 1)}, \quad (19)$$

where T_c is the regulatory control closed-loop transfer function time constant.

Then for the SOPRHPZ model the global control system output target $y^t(s)$ is computed as

$$y^t(s) = \frac{-bTs + 1}{(\tau_c Ts + 1)(a\tau_c Ts + 1)} r(s) + \frac{(T_i/K_p)s(-bTs + 1)}{(\tau_c Ts + 1)^2(a\tau_c Ts + 1)} d(s), \quad (20)$$

where τ_c is the dimensionless design parameter, $\tau_c \doteq T_c/T$.

A. PI_2 Controller Parameters

Using the controlled process models parameters, gain K and time constant T, the PI controller parameters were normalized as

$$\kappa_p \doteq K_p K, \quad \tau_i \doteq \frac{T_i}{T}. \quad (21)$$

The controller optimum normalized parameters (κ_p, τ_i, β) were obtained for right-half plane zeros relative positions b in the range from 0.25 to 4.0 and time constants ratios a from 0.1 to 1.0.

It was noted that the position of the right-half plane zero affects the robustness level that may be obtained. Roughly $M_S = 2.0$ may be obtained up to $b \approx 4.0$, $M_S = 1.8$ up to $b \approx 3.5$, $M_S = 1.6$ up to $b \approx 2.25$ and $M_S = 1.4$ only up to $b \approx 1.25$. However at the upside of all the b ranges the controllers optimal gains turn very low, with up to a $20 : 1$ ratio between the highest and lowest values. Taking this into account for fitting the tuning equations the position of the right-half plane zero will be constrained to the ranges shown in Table I.

The controller parameters for several of the model time constants ratio a analyzed and the reduced b ranges are shown in Figs. 3 to 5 for the four robustness levels, $M_S \in \{2.0, 1.8, 1.6, 1.4\}$.

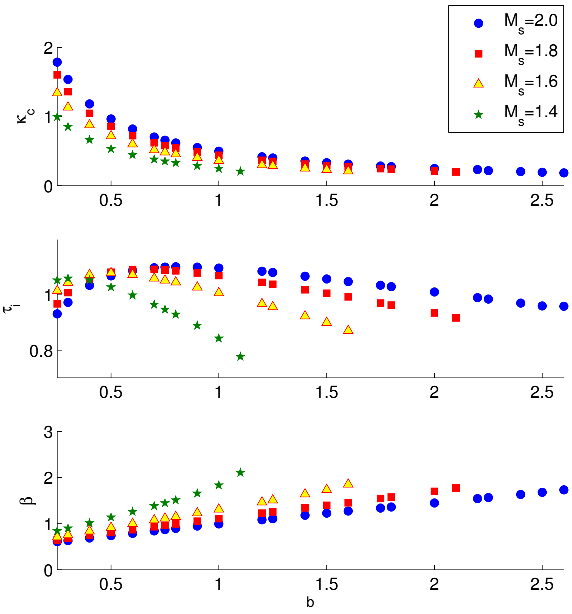

Fig. 3. PI Controller Parameters, SOPRHPZ ($a = 0.10$)

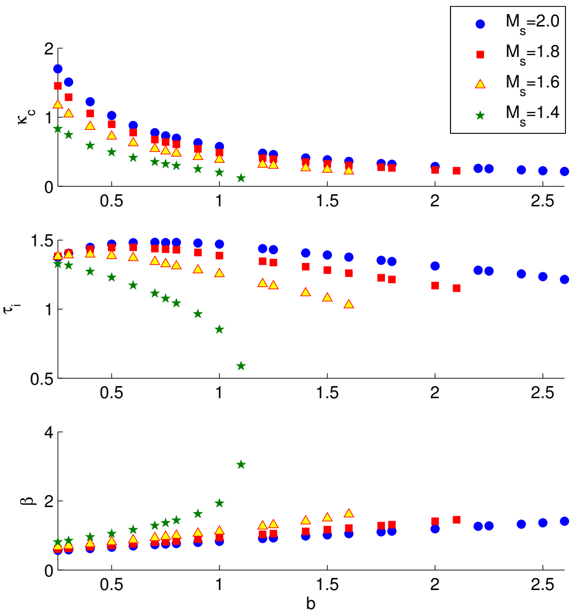

Fig. 4. PI Controller Parameters, SOPRHPZ ($a = 0.50$)

B. Tuning Equations

The controller parameter sets obtained were used to fit the *Model-Reference Robust Tunning* (MoReRT) equations for the normalized controller parameters and the proportional set-point weight for each one of the five time constants ratios a considered, given by:

$$\kappa_p \doteq K_p K = a_0 + a_1 b^{a_2}, \tag{22}$$

$$\tau_i \doteq \frac{T_i}{T} = \frac{b_0 + b_1 b}{b_2 + b_3 b + b_4 b^2 + b_5 b^3 + b_6 b^4}, \tag{23}$$

$$\beta = c_0 + c_1 b + c_2 b^2 + c_3 b^3. \tag{24}$$

Tables IV to VIII in Appendix show the a_i, b_i and c_i constants for expressions (22) to (24) for the four robustness levels. It must be kept in mind that these tuning equations may be used only for SOPRHPZ models with the right-half plane zero in a position in the ranges listed in Table I.

V. MoReRT Tuning Robustness and Performance

The robustness obtained with the MoReRT tuning equations (22) to (24) for $M_S \in \{1.4, 1.6, 1.8, 2.0\}$ are shown in Fig. 6. As can be seen all robustness curves except the ones corresponding to $M_S = 2.0$ are nearly flat.

The controller parameters for a SOPRHPZ model with $K = 1.0$, $T = 1.0$, $a = 0.40$ and $b = 0.80$ were computed interpolating the parameter obtained with the tuning equation for $a = 0.25$ and $a = 0.50$ and are shown in Table II. The corresponding closed-loop responses to a 20% set-point followed by a 10% disturbance step change are shown in Fig. 7.

Table III shows the robustness (M_S) and performance (J_e, TV_u) obtained for four target robustness levels (M_S^t) to a set-point step change (Δr) and a disturbance step change (Δd).

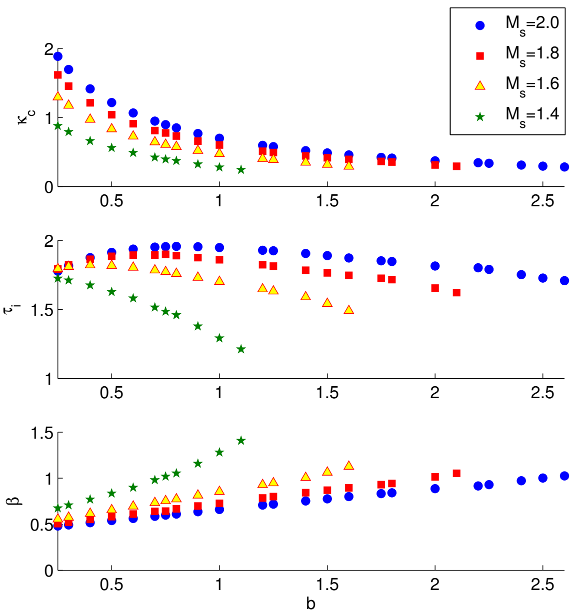

Fig. 5. PI Controller Parameters, SOPRHPZ ($a = 1.0$)

TABLE II
MoReRT Controller Parameters, SOPRHPZ Model ($a = 0.40$, $b = 0.80$)

M_S^t	K_p	T_i	β
1.4	0.297	1.006	1.471
1.6	0.472	1.243	1.188
1.8	0.588	1.340	1.183
2.0	0.672	1.388	1.173

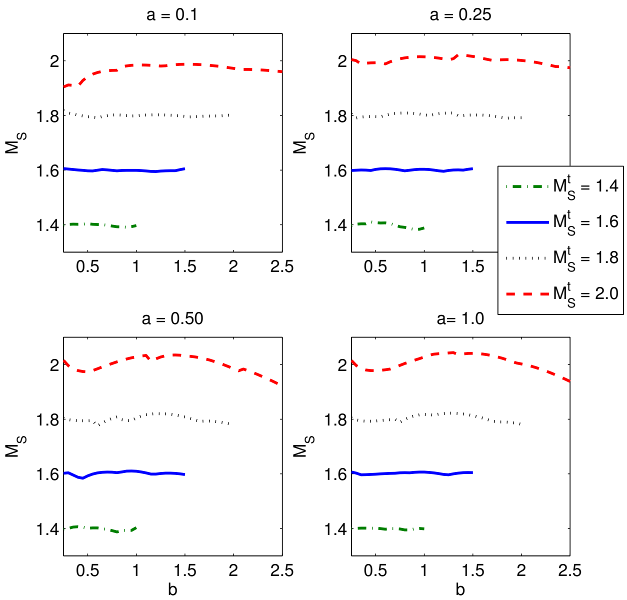

Fig. 6. MoReRT Robustness for SOPRHPZ Models

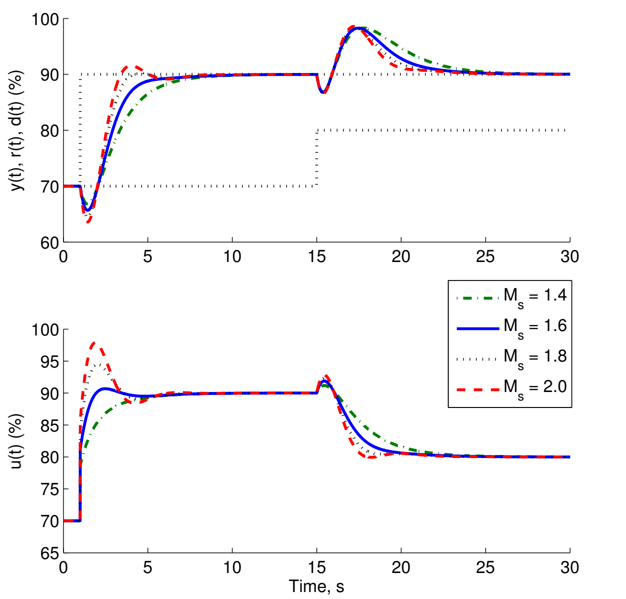

Fig. 7. Control system responses, SOPRHPZT model ($a = 0.40$, $b = 0.80$)

TABLE III
MoReRT Robustness and Performance, SOPRHPZ Model
($a = 0.40$, $b = 0.80$)

M_S^t	M_S	$\frac{J_{er}}{\Delta r}$	$\frac{J_{ed}}{\Delta d}$	$\frac{TV_{ur}}{\Delta r}$	$\frac{TV_{ud}}{\Delta d}$
1.4	1.40	2.923	3.756	1.000	1.236
1.6	1.61	2.401	3.013	1.117	1.377
1.8	1.82	2.044	2.676	1.541	1.486
2.0	1.99	1.988	2.471	1.957	1.682

These data show the trade-offs between the control system *robustness*, its *performance*, and the *control effort*. An increment in the control system target robustness (reducing M_S^t) reduces its performance (increases J_{er} and J_{ed}) but made the control effort more smooth (reduces TV_{ur} and TV_{ud}).

VI. Conclusions

A model-reference robust tuning of two-degree-of-freedom (2DoF) proportional integral (PI) controllers for second-order plus right-half plane zero (SOPRHPZ); inverse response; controlled processes was proposed in this paper. It guarantees the closed-loop control system robustness using only one parameter, the required robustness level measured with the maximum sensitivity M_S. Tuning equations were obtained for four robustness, $M_S \in \{1.4, 1.6, 1.8, 2.0\}$.

It was shown that the obtainable closed-loop control system maximum robustness is limited by the non-minimum phase zero position. Recommended applicability ranges of the proposed robust tuning rules are also included.

VII. Acknowledgments

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APPENDIX

Constants for MoReRT tuning equations (22) to (24).

TABLE IV

MORERT CONSTANTS, SOPRHPZ MODELS WITH $a = 0.1$

	Target robustness M_S^t			
	1.4	1.6	1.8	2.0
a_0	-0.2695	-0.1025	-0.06981	-0.06552
a_1	0.5152	0.4652	0.5104	0.5677
a_2	-0.6464	-0.8149	-0.8569	-0.8571
b_0	0.7881	-1.265	-2.702	-0.1015
b_1	-0.3003	12.93	92.63	8.507
b_2	0.8813	-0.7519	3.644	-0.1015
b_3	-1.358	10.5	66.85	6.066
b_4	2.784	0.8086	12.57	1
b_5	-2.728	1	1	0
b_6	1	0	0	0
c_0	0.3596	0.1708	0.464	0.4845
c_1	2.403	1.082	0.7584	0.5218
c_2	-2.442	-0.4081	-0.1539	-0.1766
c_3	1.537	0.1708	0.04236	0

TABLE V

MORERT CONSTANTS, SOPRHPZ MODELS WITH $a = 0.25$

	Target robustness M_S^t			
	1.4	1.6	1.8	2.0
a_0	-1.196	-0.2394	-0.1668	-0.1283
a_1	1.404	0.6029	0.6198	0.648
a_2	-0.2819	-0.6394	-0.7079	-0.7502
b_0	1.872	-1.061	-0.1773	0.7155
b_1	-1.511	7.798	33.98	7.639
b_2	1.705	-0.8034	1.196	1.10
b_3	-2.222	6.258	23.34	5.665
b_4	2.966	-0.2579	3.007	1
b_5	-3.002	1	1	0
b_6	1	0	0	0
c_0	0.4325	0.4974	0.4896	0.5023
c_1	2.144	0.9227	0.6414	0.4363
c_2	-2.434	-0.3466	-0.1187	0
c_3	1.827	0.1798	0.04156	0

TABLE VI

MORERT CONSTANTS, SOPRHPZ MODELS WITH $a = 0.50$

	Target robustness M_S^t			
	1.4	1.6	1.8	2.0
a_0	-1.46	-0.4883	-0.356	-0.2266
a_1	1.667	0.876	0.8524	0.799
a_2	-0.2298	-0.4642	-0.5458	-0.6399
b_0	9.525	7.481	1.364	1.293
b_1	-8.302	-2.218	5.781	9.085
b_2	7.048	5.899	1.247	1.349
b_3	-6.696	-4.889	2.889	4.738
b_4	4.844	6.401	1	1
b_5	-4.762	-4.198	0	0
b_6	1	1	0	0
c_0	0.3617	0.5068	0.4807	0.4812
c_1	2.419	0.6795	0.4604	0.3559
c_2	-3.262	-0.1772	0	0
c_3	2.401	0.1139	0	0

TABLE VII

MORERT CONSTANTS, SOPRHPZ MODELS WITH $a = 0.75$

	Target robustness M_S^t			
	1.4	1.6	1.8	2.0
a_0	-2.225	-0.4887	-0.3895	-0.3224
a_1	2.456	0.9172	0.9365	0.9593
a_2	-0.1607	-0.4531	-0.5159	-0.5687
b_0	0.6535	6.43	2.705	1.684
b_1	0.1825	8.407	7.026	12.12
b_2	0.4613	4.614	1.98	1.518
b_3	-0.2916	1.685	3.024	5.597
b_4	1.57	6.293	1	1
b_5	-1.934	-3.449	0	0
b_6	1	1	0	0
c_0	0.345	0.5041	0.4598	0.4558
c_1	2.137	0.4243	0.3662	0.2848
c_2	-2.837	0.05332	0	0
c_3	1.96	0	0	0

TABLE VIII

MORERT CONSTANTS, SOPRHPZ MODELS WITH $a = 1.0$

	Target robustness M_S^t			
	1.4	1.6	1.8	2.0
a_0	-2.486	-0.5199	-0.447	-0.406
a_1	2.765	0.9989	1.056	1.117
a_2	-0.141	-0.4339	-0.4864	-0.5237
b_0	11.54	-2.739	2.733	2.647
b_1	-2.237	18.01	10.61	16.11
b_2	6.451	-1.431	1.869	2.053
b_3	-0.446	9.672	4.295	6.579
b_4	0.1767	-0.3029	1	1
b_5	1	1	0	0
b_6	0	0	0	0
c_0	0.4901	0.423	0.4358	0.4281
c_1	0.8543	0.5706	0.2882	0.2282
c_2	-0.6064	-0.2504	0	0
c_3	0.5362	0.1038	0	0

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Robustness-Based Tuning of Two-Degree-of-Freedom Proportional Integral Controllers for Unstable Processes

Víctor M. Alfaro and Ramon Vilanova

***Abstract*— The aim of this paper is to present a robustness-based (M_S) tuning method of two-degree-of-freedom (2DoF) proportional integral (PI) controllers for unstable controlled processes. The design procedure uses a model reference optimization procedure with servo and regulatory closed-loop transfer functions targets. The unstable processes control system robustness with proportional integral (PI) controllers is analyzed. It is confirmed that the dead-time of the unstable processes imposes a constraint to the closed-loop control system achievable robustness. High robustness control systems may be obtained only for a reduced range of controlled process models. Using the proposed tuning method the designer is allowed to deal with the performance/robustness *trade-off* of the closed-loop control system by specifying the desired robustness level by selecting a maximum sensitivity in the range from 2.0 to 6.0. Controller tuning equations that guarantee the designed robustness level are provided for unstable first-order plus dead-time (UFOPDT) models with normalized dead-times in the range from 0.10 to 0.55. Comparative examples show the effectiveness of the proposed tuning method.**

I. Introduction

Even though most of the controlled processes found in the process industry are self-regulating, i.e. the process output seeks a stable operating point under a constant input, there are others that under a constant input their output is unbounded, rise or decrease without limit. These non-self regulated process are named integrating or unstable if their model transfer functions have a pole at the s-plane origin or at its right-half plane, respectively. Integrating and unstable processes may be operated only under closed-loop automatic control and their controller tuning needs a special treatment.

For unstable processes there are IMC-based PI/PID tuning methods [8], [13], [14], [24] or based on other procedures [10], [11], [18] that include a design control system performance parameter but not related with its robustness. There are also methods that optimize integrating error criteria [9], [20]. Relative stability is also considered using the gain and phase margins [6], [7], [12], [21], maximum sensitivity [22] or other robustness measures as design criteria [23].

A comparative study [25] shows that there are tuning methods for time constant dominant unstable first-order plus dead-time processes but only a few for dead-time dominant unstable processes.

In most of the industrial process control applications, the desired value of the controlled variable, or set-point, normally remains constant and a good load-disturbance rejection (*regulatory control*) is required [15]. However, due to variations in the process operating conditions, the controlled variable set-point may eventually needs to be changed and then a good transient response (*servo control*) is required. To satisfy these two operating conditions simultaneously is not possible using a one-degree-of-freedom (1DoF) controller, but using a two-degree-of-freedom (2DoF) controller it can be done with some limitations. The extra parameter that the 2DoF controller provides is used to improve its servo-control behavior while considering the regulatory control performance and the closed-loop control system robustness [4], [16], [17].

The design procedure for a closed-loop control system is usually based on linear models. Due to the non-linear characteristics found in most industrial processes, it is necessary to anticipate the changes in the process characteristics when the operating point changes, assuming certain robustness requirements for the control system. Therefore, the design of a closed-loop control system with 2DoF controllers must take into account its *performance* to load-disturbance and set-point changes as well as its *robustness* to variations of the controlled process characteristics.

An alternative tuning method of 2DoF proportional integral (PI_2) controllers for unstable controlled process is presented in this communication. The proposed approach considers the *trade-off* between the performance and robustness of the control system. The constrains imposed by the unstable system to the obtainable robustness are analyzed. The resultant tuning procedure incorporates as the only design parameter the desired robustness level as measured with the maximum sensitivity. Therefore, the designer may select the desired robustness M_S level for the control system in the range from 2.0 to 6.0.

The rest of the paper is organized as follows: the transfer functions of the controlled process model, the controller, and the closed-loop control system are presented in Section II; the general optimization procedure is described in Section III; its application to the unstable controlled process and the tuning equations of the proposed method are presented in Section IV; Section V shows the robustness and performance of the obtained control systems and Section VI its comparison with other tuning methods. The paper ends with some conclusions.

II. Problem Formulation

The controller design procedure described below for the unstable controlled processes has its roots in the model reference robust tuning (MoReRT) method proposed in [3]

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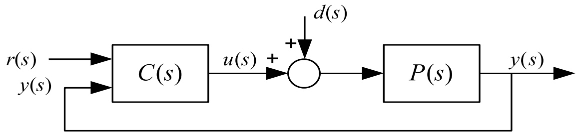

Fig. 1. Closed-Loop Control System

and used for 2DoF PI control of over damped [1] and integrating [2] processes.

Consider a closed-loop control system, as shown in Fig. 1, where $P(s)$ and $C(s)$ are the unstable controlled process model transfer function and the controller transfer function, respectively. In this system, $r(s)$ is the set-point, $u(s)$ is the controller output signal, $d(s)$ is the load-disturbance, and $y(s)$ is the process controlled variable.

The closed-loop control system output, $y(s)$, in response to changes in its inputs, $r(s)$ and $d(s)$, is given by the following:

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \tag{1}$$

where $M_{yr}(s)$ is the transfer function from the set-point to the process controlled variable, and $M_{yd}(s)$ is that from the load-disturbance to the process controlled variable. These are known as the *servo-control* closed-loop transfer function and the *regulatory control* closed-loop transfer function, respectively.

The main objective of the regulatory control is *load-disturbance rejection*; that is, the controlled variable should be returned to its set-point when a disturbance enters the control system. For the servo control, the intention is to *follow a set-point change*; that is, the controlled variable is brought to its new set-point.

The development of the proposed tuning method of 2DoF PI controllers for unstable controlled processes takes into account not only the closed-loop control system performance, stating target responses for step changes in the set-point and the load-disturbance, but also the control system robustness, measuring this with the maximum sensitivity, M_S.

A. 2DoF Proportional Integral Controller (PI_2)

The process is controlled with a two-degree-of-freedom proportional integral (PI_2) controller [5] whose output is expressed as follows:

$$u(s) = K_p \left\{ \beta r(s) - y(s) + \frac{1}{T_i s}\left[r(s) - y(s)\right] \right\}, \tag{2}$$

where K_p is the controller *proportional gain*, T_i is the *integral time constant*, and β is the *set-point proportional weight*.

For the purposes of analysis only, not implementation, the controller output (2) is rewritten as follows:

$$u(s) = C_r(s)r(s) - C_y(s)y(s), \tag{3}$$

where

$$C_r(s) = K_p\left(\beta + \frac{1}{T_i s}\right), \tag{4}$$

and

$$C_y(s) = K_p\left(1 + \frac{1}{T_i s}\right). \tag{5}$$

The closed-loop transfer functions of the servo control and the regulatory control in (1) are then given by

$$M_{yr}(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)}, \tag{6}$$

and

$$M_{yd}(s) = \frac{P(s)}{1 + C_y(s)P(s)}, \tag{7}$$

which are related as follows:

$$M_{yr}(s) = C_r(s)M_{yd}(s). \tag{8}$$

B. General Closed-Loop Transfer Functions Targets

The desired control system response to a load-disturbance step change is selected with no steady-state error and non-oscillatory, for a smooth response. In general this is expressed by the following:

$$M^t_{yd}(s) = \mathcal{M}_d(T_c, \overline{\theta}_c, \overline{\theta}_p, s) = \frac{K_o s N^+_p(s)}{p(T_c, \overline{\theta}_p, s)}, \tag{9}$$

where $N^+_p(s)$ contains the non-minimum phase elements of the model, $\overline{\theta}_p$ are the controlled process model parameters, $\overline{\theta}_c$ are the controller parameters, T_c the closed-loop apparent time constant, and $p(T_c, \overline{\theta}_p, s)$ is the characteristic polynomial of the closed-loop control system.

Using (9) in (8) the servo-control closed-loop transfer function is given by

$$M^t_{yr}(s) = \mathcal{M}_r(T_c, \overline{\theta}_c, \overline{\theta}_p, s) = C_r(s)M^t_{yd}(s). \tag{10}$$

Then using (9) and (10) in (1) the global control system output target, $y^t(s)$, is computed as follows:

$$y^t(s) = M^t_{yr}(s)r(s) + M^t_{yd}(s)d(s). \tag{11}$$

and in the time domain as follows:

$$y^t(t) = y^t_r(t) + y^t_d(t), \tag{12}$$

where $y^t_r(t)$ is the servo-control step response target and $y^t_d(t)$, the regulatory control step response target.

III. Controller Design

In what follows the complete set of PI_2 controller (2) parameters $\overline{\theta}_c = \{K_p, T_i, \beta\}$ is obtained considering, at the same time, the regulatory control and the servo-control performance, to obtain a controller with a targeted *servo/regulatory performance combination* that also produces a closed-loop control system with a specific robustness level.

Initially the dimensionless closed-loop apparent speed $\tau_c \doteq T_c/T$ is used as a design parameter.

A. Cost Functionals

For the regulatory control response, the cost functional to be minimized is defined as follows:

$$J_d(\tau_c, \overline{\theta}_c, \overline{\theta}_p) \doteq \int_0^{\infty} \left[y_d^t(\tau_c, \overline{\theta}_c, \overline{\theta}_p, t) - y_d(\overline{\theta}_c, \overline{\theta}_p, t)\right]^2 \mathrm{d}t, \tag{13}$$

where $y_d^t(\tau_c, \overline{\theta}_c, \overline{\theta}_p, t)$ is the step response of the regulatory control closed-loop transfer function target (9) and $y_d(\overline{\theta}_c, \overline{\theta}_p, t)$ is that of the regulatory control system (7) with the controlled process $P(s)$ and controller (5).

In a similar way, the servo-control cost functional to be minimized is defined as follows:

$$J_r(\tau_c, \overline{\theta}_c, \overline{\theta}_p) \doteq \int_0^{\infty} \left[y_r^t(\tau_c, \overline{\theta}_p, t) - y_r(\overline{\theta}_c, \overline{\theta}_p, t)\right]^2 \mathrm{d}t, \tag{14}$$

where $y_r^t(\tau_c, \overline{\theta}_p, t)$ is the step response of the servo-control closed-loop transfer function target (10) and $y_r(\overline{\theta}_c, \overline{\theta}_p, t)$ is that of the servo-control system (6) with the controlled process $P(s)$ and controller (4).

B. Controller Optimization

For the 2DoF PI controller design, the following overall cost functional is optimized:

$$J_T(\tau_c, \overline{\theta}_c, \overline{\theta}_p) \doteq J_r(\tau_c, \overline{\theta}_c, \overline{\theta}_p) + J_d(\tau_c, \overline{\theta}_c, \overline{\theta}_p), \tag{15}$$

to obtain the optimum controller parameters $\overline{\theta}_c^o = \left\{K_p^o, T_i^o, \beta^o\right\}$ such that

$$J_T^o \doteq J_T(\tau_c, \overline{\theta}_c^o, \overline{\theta}_p) = \min_{\overline{\theta}_c} J_T(\tau_c, \overline{\theta}_c, \overline{\theta}_p). \tag{16}$$

Note that $\overline{\theta}_c^o = \overline{\theta}_c^o(\overline{\theta}_p, \tau_c)$. Moreover, for each $\overline{\theta}_c^o$ set obtained, the closed-loop control system robustness is measured using the maximum sensitivity M_S, which is defined as follows:

$$M_S \doteq \max_{\omega} |S(\mathrm{j}\omega)| = \max_{\omega} \frac{1}{|1 + C_y(\mathrm{j}\omega)P(\mathrm{j}\omega)|}. \tag{17}$$

It is important to keep in mind that the optimization problem formulated (15) is a *model matching problem* instead of the traditional performance optimization problem. The control system behavior to both, disturbance and set-point step changes, is stated with the selection of the closed-loop transfer function targets $M_{yd}^t(s)$ and $M_{yr}^t(s)$, respectively.

C. Design Procedure

For a given controlled process model $P(s)$ with parameters $\overline{\theta}_p$ the design procedure can be summarized as follows:

- A design parameters τ_c is selected.
- The cost functional J_T (15) is optimized to obtain the controller parameters $\overline{\theta}_c^o$.
- The robustness of the resulting control system is evaluated using M_S (17).
- If required a new design parameter τ_c is selected to adjust the control system robustness.

For the optimization a direct search Nelder-Mean simplex-based algorithm is used [19].

IV. Controlled Process Models and Closed-Loop Transfer Functions Targets

We consider the unstable first-order plus dead-time (UFOPDT) model given by the following:

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{Ts - 1}, \tag{18}$$

were K is the gain, T the time constant, and L the dead-time. The controlled process parameters are $\overline{\theta}_p = \{K, T, L\}$.

From (9), taking into account the controlled process model (18) and the feedback part of the PI controller (5), the desired regulatory control closed-loop transfer function is obtained as the following second-order transfer function target:

$$M_{yd}^t(s) = \frac{K_o s\mathrm{e}^{-Ls}}{(\tau_c Ts + 1)^2}, \tag{19}$$

where $K_o = T_i/K_p$ and $\tau_c = T_c/T$ are the regulatory control closed-loop transfer function static gain and the dimensionless design parameter, respectively.

Using (19) and (4) in (8) the servo-control closed-loop transfer function target is given by

$$M_{yr}^t(s) = \frac{(\beta T_i s + 1)\mathrm{e}^{-Ls}}{(\tau_c Ts + 1)^2}, \tag{20}$$

In this case due to the limitations imposed by the unstable process it is not possible to obtain a first-order dynamics for the servo-control response by forcing $\beta \to \tau_c T/T_i$. Then, in the UFOPDT model case the global control system output target $y^t(s)$ is computed as

$$y^t(s) = \frac{(\beta T_i s + 1)\mathrm{e}^{-Ls}}{(\tau_c Ts + 1)^2} r(s) + \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c Ts + 1)^2} d(s). \tag{21}$$

A. Normalized transfer Functions

Using the controlled process parameters $\overline{\theta}_p$ as well as the transformation $\hat{s} = Ts$, the controlled process (18) and the PI controller transfer functions (4) and (5) can be expressed in a normalized form as follows:

$$\hat{P}(\hat{s}) = \frac{\mathrm{e}^{-\tau_o \hat{s}}}{\hat{s} - 1}, \tag{22}$$

$$\hat{C}_r(\hat{s}) = \kappa_p \left(\beta + \frac{1}{\tau_i \hat{s}}\right), \tag{23}$$

$$\hat{C}_y(\hat{s}) = \kappa_p \left(1 + \frac{1}{\tau_i \hat{s}}\right), \tag{24}$$

where $\tau_o = L/T$ is the model normalized dead time and

$$\kappa_p \doteq K_p\ K, \quad \tau_i \doteq \frac{T_i}{T}, \tag{25}$$

are the *normalized gain* and *normalized integrating time* of the controller, respectively.

The normalized controlled process model (22) has only one dimensionless parameter, τ_o. For a given τ_o during the optimization procedure the closed-loop relative speed parameter τ_c is selected in such a way that the robustness

TABLE I

UFOPDT MODELS τ_o RANGES

	Target robustness M_S^t				
	2.0	3.0	4.0	5.0	6.0
τ_{omin}	0.10	0.10	0.10	0.10	0.10
τ_{omax}	0.25	0.35	0.45	0.50	0.55

level of the resulting closed-loop system met a specific target (M_S^t). Then, the controller parameters are obtained directly as functions of the model parameters and of the closed-loop control system robustness.

B. Achievable Robustness Levels

From the optimization results it is found that the dead-time of the unstable processes impose a severe limitation on the control system achievable robustness level. A closed-loop control systems with high robustness of $M_S^t = 1.4$ may only be obtainable for UFOPDT models with $\tau_o \leq 0.10$, the robustness $M_S^t = 1.6$ for $\tau_o \leq 0.15$, the $M_S^t = 1.8$ for $\tau_o \leq 0.20$, and the robustness $M_S^t = 2.0$ for models with $\tau_o \leq 0.26$. Robust control systems may be obtained for a very limited range of unstable models.

C. Controller Parameters

Although the usual control system minimum robustness level for stable processes corresponds to $M_S = 2.0$ in the unstable processes case the main control system purpose is to stabilize the process. Then, for the UFOPDT models we relax the robustness level target and obtain the controller parameters for M_S^t in the range from 2.0 to 6.0.

The optimization also shows that for all cases the set-point weight $\beta = 0$. Then, for the unstable processes the PI_2 (2) reduces to

$$u(s) = K_p \left\{ -y(s) + \frac{1}{T_i s} \left[r(s) - y(s) \right] \right\}. \qquad (26)$$

The normalized controller parameters κ_p and τ_i for robustness target levels M_S^t in the range from 2.0 to 6.0 are shown in Fig. 2. These target robustness levels are achievable for models with normalized dead-times in the ranges listed in Table I.

D. MoReRT Tuning Equations

The PI_2 MoReRT normalized parameters can be obtained with the following equations:

$$\kappa_p \doteq K_p K = a_0 + a_1 {\tau_o}^{a_2}, \qquad (27)$$

$$\tau_i \doteq \frac{T_i}{T} = \frac{b_0 + b_1 \tau_o}{b_2 + b_3 \tau_o + {\tau_o}^2}, \qquad (28)$$

$$\beta = 0. \qquad (29)$$

Table II list the a_i and b_i constants for expressions (27) and (28) for the five robustness levels. It must be remember that these tuning equations may be used only for UFOPDT models with normalized dead-times in the ranges listed in Table I.

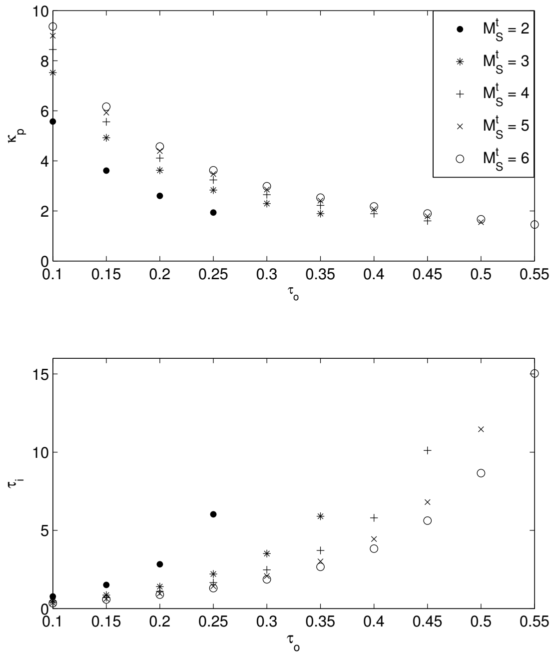

Fig. 2. PI_2 Parameters for UFOPDT Models

TABLE II

MORERT CONSTANTS

	Target robustness M_S^t				
	2.0	3.0	4.0	5.0	6.0
a_0	-1.149	-0.5287	-0.5091	-0.4010	-0.3995
a_1	0.9560	0.8898	0.9986	1.010	1.070
a_2	-0.8468	-0.9564	-0.9525	-0.9684	-0.9559
b_0	0.03242	0.004109	-0.03222	-0.01103	-0.0226
b_1	0.0	2.90	4.722	3.008	3.237
b_2	0.08534	0.8081	1.40	1.023	1.101
b_3	-0.5698	-2.166	-3.10	-2.285	-2.347

V. MORERT CONTROL SYSTEM ROBUSTNESS AND PERFORMANCE

The robustness obtained with the MoReRT tuning equations (27) to (29) for $M_S \in \{2.0, 3.0, 4.0, 5.0, 6.0\}$ are shown in Fig. 3. As can be seen all robustness curves are almost flat.

The achievement of the robustness target for all the unstable controlled process models considered is one of the distinctive characteristics of the proposed tuning method.

Fig. 4 shows the responses of the PI_2 MoReRT control system for a unstable model with parameters $\{K = 1, T = 1, L = 0.2\}$ to a 20% set-point step change followed by a 10% load-disturbance step change for four robustness levels.

Table III shows the robustness (M_S) and performance (J_e, TV_u) obtained for the five target robustness levels (M_S^t) to a set-point step change (Δr) and a disturbance step change (Δd).

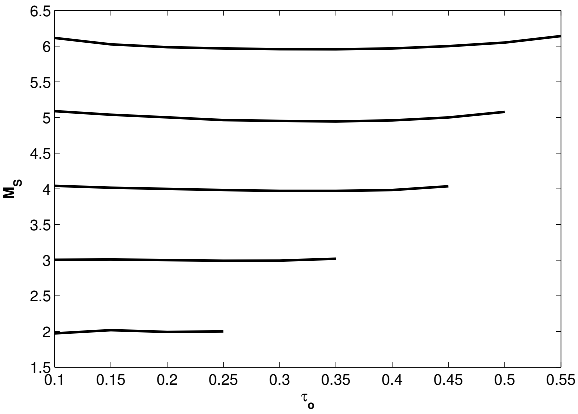

Fig. 3. MoReRT Controllers Robustness

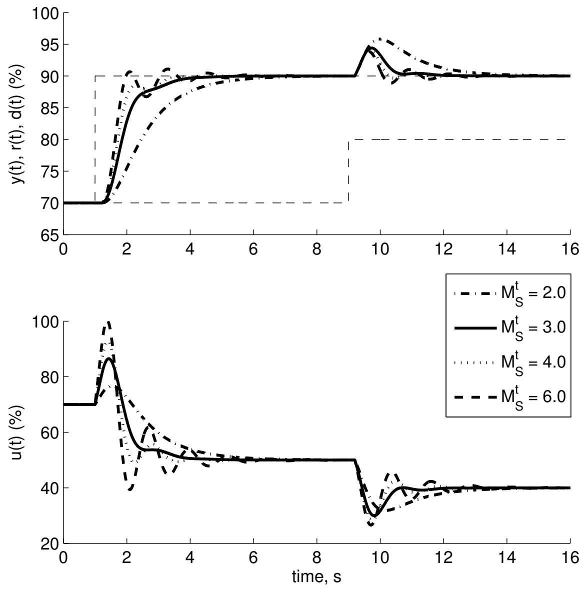

Fig. 4. MoReRT Closed-loop Responses ($\tau_o = 0.2$)

TABLE III
MoReRT Robustness and Performance

M_S^t	M_S	$\frac{J_{ed}}{\Delta d}$	$\frac{TV_{ud}}{\Delta d}$	$\frac{J_{er}}{\Delta r}$	$\frac{TV_{ur}}{\Delta r}$
2.0	1.99	1.101	2.633	1.749	1.670
3.0	3.00	0.389	3.187	1.020	2.653
4.0	4.00	0.300	4.365	0.845	4.109
5.0	5.00	0.311	5.620	0.782	5.893
6.0	5.99	0.337	6.864	0.783	7.704

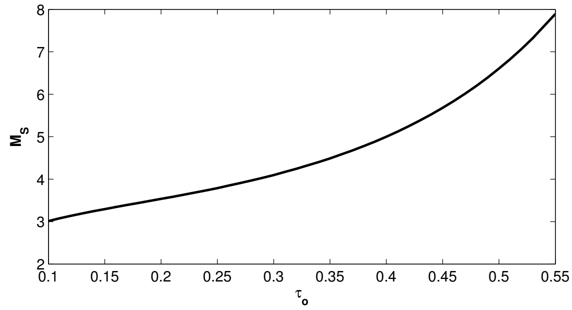

Fig. 5. Majhi & Atherton ISTE PI Robustness

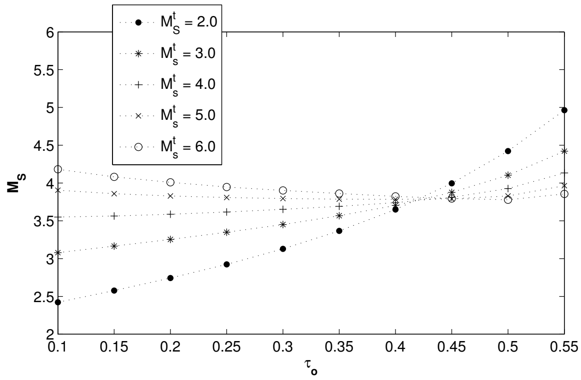

Fig. 6. Wang & Xu PID_2 Robustness

From this table is seen that from the regulatory control performance point of view for this UFOPDT model there is no sense to use a robustness $M_S^t > 4.0$.

VI. Comparison With Other Tuning Methods

The performance optimized (ISE, ITSE, ISTE) PID controllers in [20] for regulatory control have very low robustness and they decrease with the model normalized dead-time while their counterparts for servo-control have better robustness but also they are low and decease with τ_o. On the other hand, the servo-control ISTE performance optimized PI controllers in [9] are more robust but also their robustness is not constant as shown in Fig. 5.

The tuning method for unstable processes in [22] is based on a maximum sensitivity, M_S, specification but the obtained robustness do not achieve their target level as shown in Fig. 6.

For a comparison on the same base with the $\{K = 1, T = 1, L = 0.2\}$ unstable model we select the Wang & Xu PID_2 controller with $M_S^t = 6.0$ that yields $M_S = 4.01$ and the proposed PI_2 MoReRT with $M_S^t = 4.0$. The control system outputs with these two controllers are shown in Fig. 7.

The Wang & Xu control system have faster responses but produce a big output signal overshoot to a set-point step change with heavy variations on the controller effort that made it impractical to use.

The MoReRT controller output does not present any abrupt instant change, due to the use of $\beta = 0$, and it is very smooth.

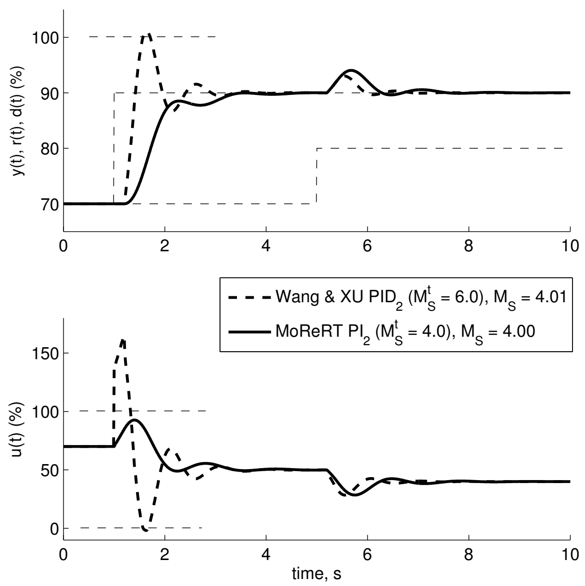

Fig. 7. Closed-loop Responses ($\tau_o = 0.2$)

VII. Conclusions

The proposed *MoReRT* tuning method for two-degree-of-freedom (2DoF) proportional integral (PI_2) controllers guarantees the design robustness level for unstable first-order plus dead-time (UFOPDT) models using only one design parameter, which is the required closed-loop control system robustness as measured with the maximum sensitivity M_S.

The unstable model dead-time made that control systems with robustness levels similar to the ones normally specified for stable processes may only be obtained for a very narrow normalized dead-time ranges. For unstable processes the robustness requirements need be relaxed.

Tuning equations were obtained for $M_S \in \{2.0, 3.0, 4.0, 5.0, 6.0\}$, allowing the designer to select the required robustness level by taking into account the expected variations in the controlled process parameters.

Using over damped servo and regulatory closed-loop transfer function targets a smooth control system response to both, set-point and disturbance step changes, is obtained.

Acknowledgments

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SIMPLE ROBUST TUNING OF 2DOF PID CONTROLLERS FROM A PERFORMANCE/ROBUSTNESS TRADE-OFF ANALYSIS

V.M. Alfaro and R. Vilanova

ABSTRACT

This paper presents the performance/robustness trade-off analysis of servo-control and regulatory-control systems. In this analysis, first-order-plus-dead-time (FOPDT) and second-order-plus-dead-time (SOPDT) controlled processes with one-degree-of-freedom (1DoF) and two-degree-of-freedom (2DoF) proportional integral (PI) and proportional integral derivative (PID) controllers are considered. The analysis shows that for the same process, performance-optimized 1DoF PI controllers are more robust to changes in the process characteristics, but the optimal performance of these controllers is inferior to that of the corresponding PID controllers. Regulatory-control performance-optimized PI and PID controllers are less robust than the corresponding servo-control systems, and a greater level of performance degradation is required in the former case to ensure that the robustness level becomes comparable to that in the latter case.

On the basis of this analysis, a *simply robust tuning* (SRT) method was developed for 2DoF PID controllers; the use of this procedure helps in achieving the desired level of robustness in closed-loop control systems. The proposed tuning method allows the use of a single set of rules for the robust tuning of PID controllers for FOPDT and SOPDT controlled processes

Key Words: PID control, performance, robustness, two degrees of freedom.

I. INTRODUCTION

As been widely reported, since their introduction in 1940 [1, 2] commercial *proportional integral derivative* (PID) controllers have been widely used in industrial control applications. This popularity is mainly due to their simple structure and the simple parameters that need to be used in these controllers. Hence, control engineers find PID control systems easier to understand than most of the other advanced control approaches.

Several methods for the design and tuning of PID controllers have been reported in the literature [3, 4, 5]. In particular, there is special mention of the International Federation of Automatic Control (IFAC) workshop "PID'00- Past, Present and Future of PID Control" held in Terrassa, Spain, on April 2000, where state-of-the-art PID control methods were discussed. Most of those methods were concerned with feedback controllers that are tuned either for disturbance rejection [6, 7, 8, 9] or for a well-damped fast response to a step change in the controller set point [9, 10, 11, 12]. A two-degree-of-freedom (2DoF) controller has been developed to meet both these objectives. The second degree of freedom in this controller provides additional flexibility to the control system design. Examples of such a formulation can be found in the literature [13, 14], and the different tuning methods that have been developed over the last few years are examples of such formulations [15, 16, 17, 18, 19, 20, 21]. The 2DoF

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formulation is found in the presented literature as well as in commercial PID controllers as the well-known set-point weight factor.

The control system design is usually based on the use of low-order linear models; these models in turn are based on the normal operating point of the closed-loop control system. Because most industrial processes are non-linear, it is necessary to account for possible changes in the process characteristics by adopting certain relative stability margins or robustness requirements for the control system.

Therefore, in the design of a closed-loop control system with PI and PID controllers, we must consider the *trade-off* between two conflicting criteria: the time-response *performance* to the set point and load disturbances and the *robustness* to changes in the characteristics of the controlled process. If only the system performance is taken into account, using an integrated error criterion (integrated absolute error (IAE), integrated time-weighted absolute error (ITAE), or integrated squared error (ISE)) or a time response characteristic (overshoot, rise time, or settling time), as in [22, 23], the resulting closed-loop control system will probably have very low robustness. On the other hand, if the system is designed to have high robustness, as in [19], and if the performance of the resulting system is not evaluated, the designer would have no idea of the *cost* involved in operating such a highly robust system. In some previous studies [22, 24, 25], the performance and robustness of the system were taken into account for optimizing the IAE or ITAE performance, but only the usual minimum level of robustness could be guaranteed.

To estimate the performance losses that occur when the control-system robustness is increased, a performance/robustness trade-off analysis was conducted on 1DoF and 2DoF PI and PID control systems with first- and second-order-plus-dead-time (FOPDT, SOPDT) models [21]; maximum sensitivity, M_S, was used as a measure of the system robustness. Using this approach, the designer can estimate the performance losses that may occur when setting certain robustness levels.

In this paper one step further is progressed. The results of the trade-off analysis allow us to obtain simple tuning relations for four robustness levels in the range $1.4 \le M_S \le 2.0$ and thus, facilitate the design of robust closed-loop control systems in which the robustness level is adjusted by varying the controller proportional gain alone. The proposed tuning procedure that allows the use of a single set of rules for the robust tuning of 2DoF PID controllers for FOPDT and SOPDT models is one of the main contributions of the paper.

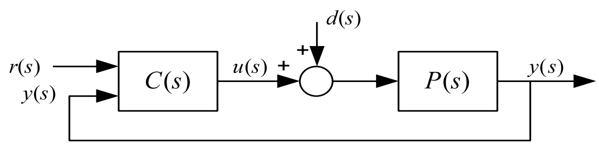

Figure 1. Closed-Loop Control System

The rest of the paper is organized as follows. The transfer functions of the controlled process model, controller, and closed-loop control system are presented in Section II. The performance/robustness analysis is described in Section III, and *simply robust tuning* (SRT) is presented in Section IV. An example of the use of the tuning rules and the performance/robustness trade-off is shown in Section V. Finally, concluding remarks are provided.

II. PROBLEM FORMULATION

Consider a closed-loop control system, as shown in Fig. 1, where $P(s)$ and $C(s)$ are the controlled process model and the controller transfer function, respectively. In this system, $r(s)$ is the set point; $u(s)$, the controller output signal; $d(s)$, the load disturbance; and $y(s)$, the controlled process variable.

The controlled process is represented by an SOPDT model given by the general transfer function

$$P(s) = \frac{Ke^{-Ls}}{(Ts+1)(aTs+1)},\ \tau_o = L/T, \tag{1}$$

where K is the gain; T, the main time constant; a, the ratio of two main time constants ($0 \le a \le 1.0$); L, the dead-time; and τ_o, the *normalized dead time*. The model transfer function (1) allows the representation of FOPDT processes ($a = 0$), overdamped SOPDT processes ($0 < a < 1$), and dual-pole-plus-dead-time (DPPDT) processes ($a = 1$).

The process will be controlled with a 2DoF PID controller [27], whose output is as follows:

$$u(s) = K_p \left\{ e_p(s) + \frac{1}{T_i s} e_i(s) - \frac{T_d s}{\alpha T_d s + 1} y(s) \right\}, \tag{2}$$

with

$$e_p(s) = \beta r(s) - y(s), \tag{3}$$
$$e_i(s) = r(s) - y(s), \tag{4}$$

where K_p is the controller *proportional gain*; T_i, the *integral time constant*; T_d, the *derivative time*

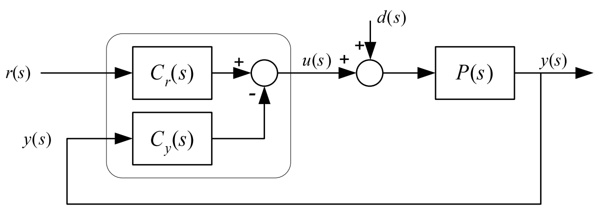

Figure 2. 2DoF Closed-Loop Control System

constant; β, the *proportional set-point weight*; and α, the *derivative filter constant*. Usually, $\alpha = 0.10$ [28].

To avoid the *derivative kick* at the controller output signal when a step change is made to the set point, in (2) the derivative term is applied only to the controlled process variable. This will only affect the servo-control response [29].

Equation (2) may be rearranged, but only for the analysis and not for implementation, as follows

$$u(s) = K_p\left(\beta + \frac{1}{T_i s}\right) r(s) - K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{0.1 T_d s + 1}\right) y(s), \quad (5)$$

or in the compact form shown in Fig. 2 as

$$u(s) = C_r(s)r(s) - C_y(s)y(s), \quad (6)$$

where $C_r(s)$ is the *set-point controller* transfer function and $C_y(s)$ is the *feedback controller* transfer function.

The output of the closed-loop control system varies with a change the input as

$$y(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)} r(s) + \frac{P(s)}{1 + C_y(s)P(s)} d(s), \quad (7)$$

or

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \quad (8)$$

where $M_{yr}(s)$ is the transfer function from the set point to the controlled process variable and is known as the *servo control* closed-loop transfer function; $M_{yd}(s)$ is the transfer function from the load disturbance to the controlled process variable and is known as the *regulatory control* closed-loop transfer function.

The *performance* of the closed-loop control system is evaluated using the IAE cost functional given by

$$J_e \doteq \int_0^\infty |e(t)|\, dt = \int_0^\infty |y(t) - r(t)|\, dt, \quad (9)$$

for a step change in the set point, J_{er} and that in the disturbance, J_{ed}.

The peak magnitude of the sensitivity function will be used as an indicator of the system *robustness* (relative stability). The maximum sensitivity for the control system is defined as

$$M_S \doteq \max_\omega |S(j\omega)| = \max_\omega \frac{1}{|1 + C_y(j\omega)P(j\omega)|}. \quad (10)$$

If the system robustness (10) is not taken into account for the design, the controller parameters may be optimized to maximize the system performance or to achieve the minimum value of the cost functional in (9), using M_{yr} for set point changes (J_{er}^o) and M_{yd} for load disturbance changes (J_{ed}^o).

Because of the control system performance/robustness trade-off, if a robustness requirement is included into the design then, it is expected that the actual system performance will be reduced ($J_e \geq J_e^o$). Then, the *performance degradation factor* defined as

$$F_p \doteq \frac{J_e^o}{J_e}, \; F_p \leq 1, \quad (11)$$

is used to evaluate the performance/robustness trade-off.

The F_p value will be an indication of the relative price to pay, in terms of control system performance reduction respect to its optimal performance, to have a robust control system.

III. PERFORMANCE/ROBUSTNESS TRADE-OFF ANALYSIS

To evaluate the performance degradation when the system robustness is increased, the following steps were followed [26].

3.1. 1DoF Controllers Optimum Performance

For the 1DoF servo- and regulatory-control performance-optimized PI and PID controllers, the parameters $\theta_c^o = \{K_p^o, T_i^o, T_d^o\}$ were obtained using the cost functional (9) such that

$$J_e^o = J_e(\theta_c^o) = \min_{\theta_c} J_e(\theta_c), \quad (12)$$

for (1) with $a \in \{0, 0.25, 0.5, 0.75, 1\}$ and ten τ_o in the range 0.05-2.0, for set point and load disturbance step changes. The robustness of the control systems that deliver the optimal performance was evaluated by using M_S.

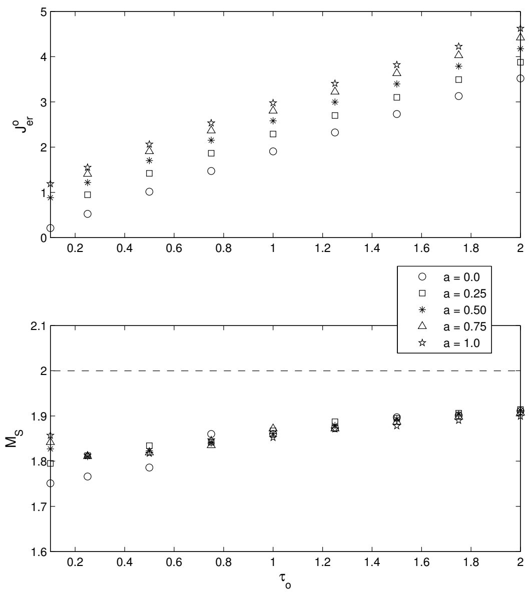

Figure 3. PI Servo-Control Optimum Performance and Robustness

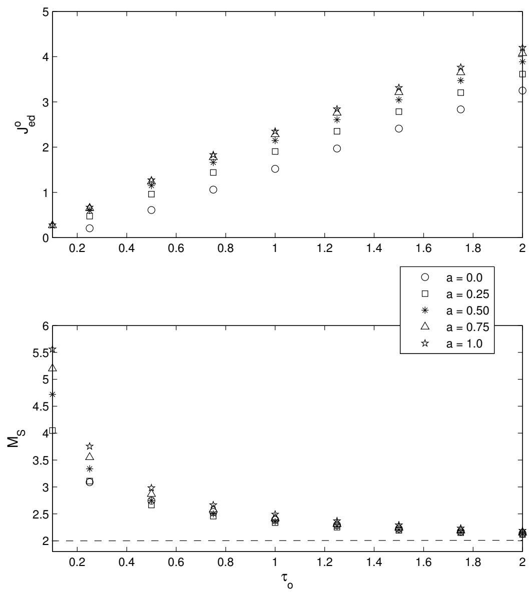

Figure 4. PI Regulatory Control Optimum Performance and Robustness

The performance and robustness of the servo-control performance-optimized PI controllers are shown in Fig. 3. The robustness of the resulting control systems, M_S, is in the range 1.75-1.9. The performance, J^o_{er}, obtained with the FOPDT ($a = 0.0$) models is higher than that obtained with the SOPDT models, but the robustness levels in the two cases are similar.

The performance and robustness of the regulatory-control performance-optimized PI controllers are shown in Fig. 4. In this case, the robustness levels of the resulting control systems, M_S, are very poor, especially for low τ_o. The performance, J^o_{ed}, obtained with the FOPDT models is higher than that obtained with the SOPDT models, but the robustness levels in the two cases are similar, except for low τ_o values.

As can be seen from Fig. 5, the performance of the servo-control performance-optimized PID controllers is higher than that of the corresponding PI controllers but the robustness is lower for PID controllers than the corresponding PI controllers. The minimum robustness level ($M_S \leq 2.0$) is obtained only for high τ_o.

From Fig. 6, it can be seen that the performance of the regulatory-control performance-optimized PID controllers is higher than that of the corresponding PI controllers but the robustness again is very low for PID controllers.

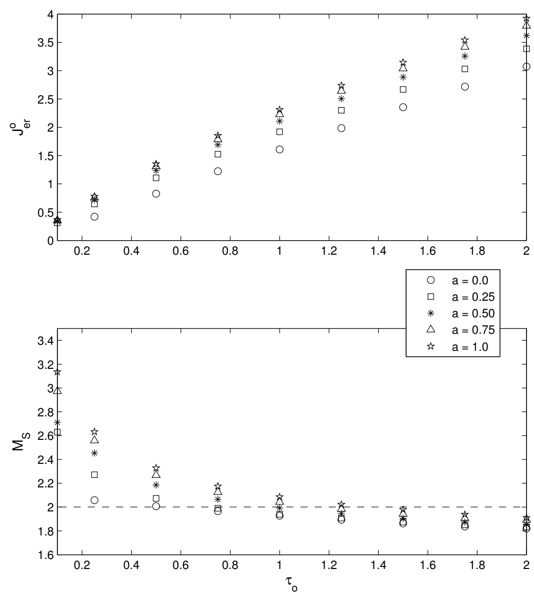

Figure 5. PID Servo-Control Optimum Performance and Robustness

Except for the case of set point PI controller, the robustness of the resulting control systems do not meet the allowed lower limit. Therefore, in such situations,

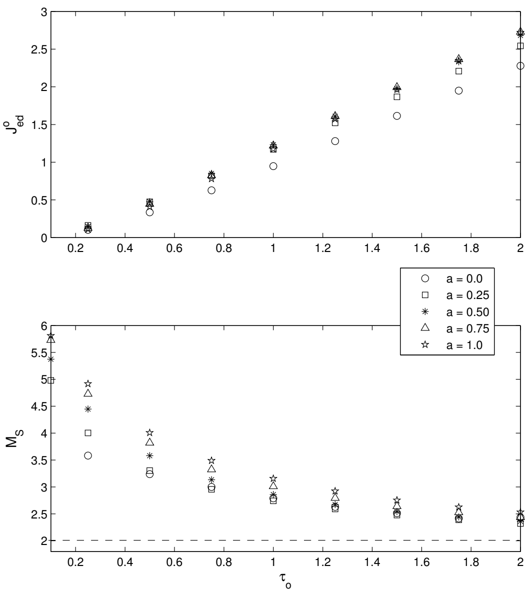

Figure 6. PID Regulatory Control Optimum Performance and Robustness

it would be desirable to increase the robustness of the control system. For this purpose, the performance must be degraded. The carried out performance degradation procedure is described in the following section.

3.2. 1DoF Controllers Degraded Performance

To increase the control-loop robustness, a target performance degradation factor, F^t_p, was included in the cost functional, as follows

$$J_{F_p} \doteq J(\theta_c, F^t_p) = \left| \frac{J^o_e}{J_e(\theta_c)} - F^t_p \right|, \qquad (13)$$

for obtaining the PI and PID (servo and regulatory control) parameters θ^{o1}_c such that

$$J^o_{F_p} = J_{F_p}(\theta^{o1}_c, F^t_p) = \min_{\theta_c} J(\theta_c, F^t_p). \qquad (14)$$

When F^t_p was decreased, the control-system robustness was increased to the target level, M^t_S.

With starting point as the original unconstrained optimal (from the point of view of robustness) parameters θ^{o1}_c, a second optimization was conducted using the cost functional

$$J_{M_S} \doteq J(\theta_c, M^t_S) = \left| M_S(\theta_c) - M^t_S \right|, \qquad (15)$$

in order to achieve the target robustness. The robust controller parameters, θ^{o2}_c, are such that

$$J^o_{M_S} = J_{M_S}(\theta^{o2}_c, M^t_S) = \min_{\theta_c} J(\theta_c, M^t_S). \qquad (16)$$

For the analysis, five target robustness levels have been considered, $M^t_S \in \{2, 1.8, 1.6, 1.4, 1.2\}$.

Finally, the performance degradation factor required for obtaining M^t_S in (16) is evaluated as follows

$$F_p(M^t_S) = \frac{J^o_e}{J_e(\theta^{o2}_c)} \qquad (17)$$

Therefore, the second optimization provides the controller parameters θ^{o2}_c required to formulate a system with the target robustness (10), M^t_S, and with the best performance allowed when using the IAE criteria (9), J_{er} or J_{ed}.

Next, we analyze the optimal solution to the robust controller design problem for the servo-control and regulatory-control PI and PID controllers.

3.2.1. PI Servo-Control

Figs. 7-9 show the performance degradation factors required to achieve the robustness level at which the servo-control performance of a 1DoF PI controller can be optimized, for several a values. For carrying out analysis at an intermediate robustness level, the $M_S = 1.6$ case is considered; the performance degradation factor required for each model considered is shown in Fig. 10.

As can be seen from this figure, for a particular model, the required performance degradation increases with τ_o but is unaffected by a, i.e., the performance degradation for PI control systems with FOPDT and DPPDT models having the same τ_o must be similar so that the robustness of these controllers is increased to the same level.

An example showing the effect of varying the robustness is as follows. For a servo-control PI controller with FOPDT model (Fig. 7), increasing M_S to 1.8 yields a marginal cost, while decreasing M_S to 1.2 leads to a 60% decrease in the control system performance ($F_p \approx 0.4$).

The performance of the PI servo-control systems reduces up to 20% when M^t_S is 1.6.

3.2.2. PI Regulatory Control

The analysis results for the regulatory-control PI controller are shown in Figs. 11-14. Note that in this case, the performance degradation required to

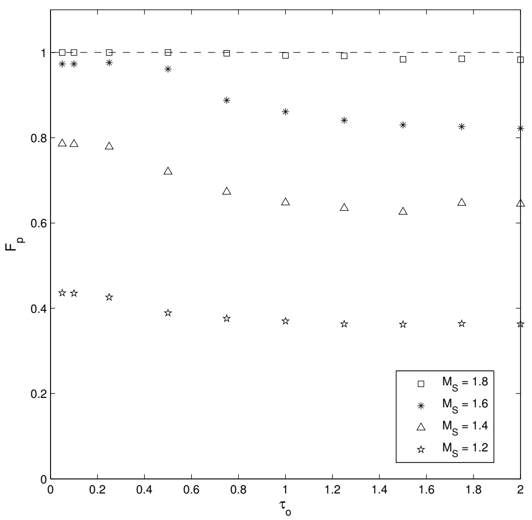

Figure 7. PI Servo-Control Performance Degradation Factor ($a = 0.0$)

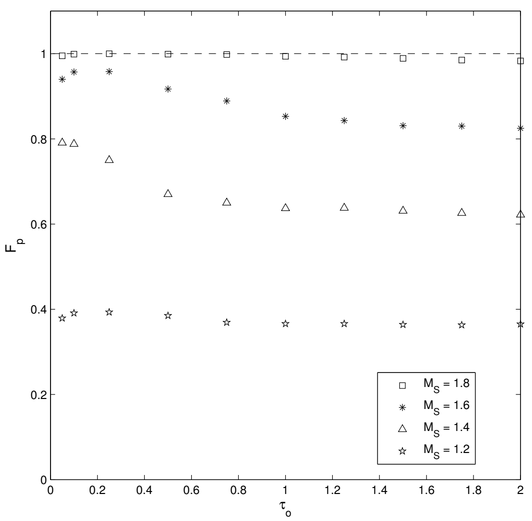

Figure 8. PI Servo-Control Performance Degradation Factor ($a = 0.50$)

achieve a certain robustness level decreases as τ_o of the model increases. In the regulatory control case, higher performance degradation is required to ensure that the robustness level is identical to that in the servo control case.

For an $M_S^t = 1.4$, while the servo-control PI controller for a SOPDT model with $a = 0.5$ and $\tau_o = 1$ (Fig. 8) requires an F_p of 0.64 (36% performance

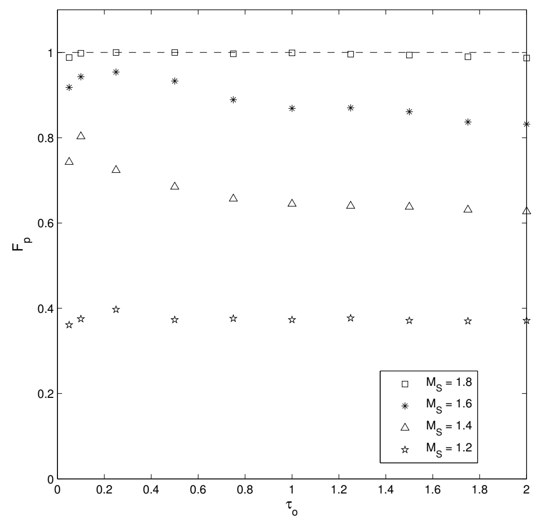

Figure 9. PI Servo-Control Performance Degradation Factor ($a = 1.0$)

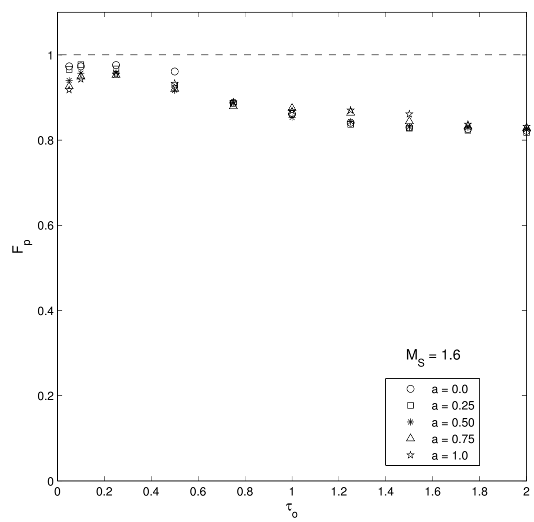

Figure 10. PI Servo-Control Performance Degradation Factor ($M_S^t = 1.6$)

reduction), the regulatory-control counterpart (Fig. 12) requires an $F_p = 0.51$ (49% performance reduction).

3.2.3. PID Servo-Control

Figs. 15-17 show the performance degradation required for the servo-control PID controller. On comparison of Fig. 17 ($M_S = 1.6$) with the corresponding

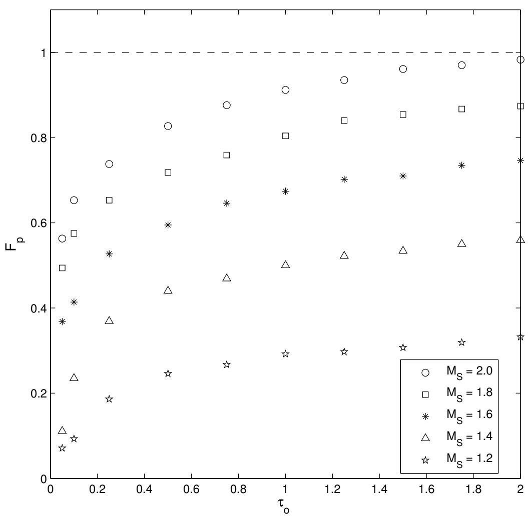

Figure 11. PI Regulatory Control Performance Degradation Factor ($a = 0.0$)

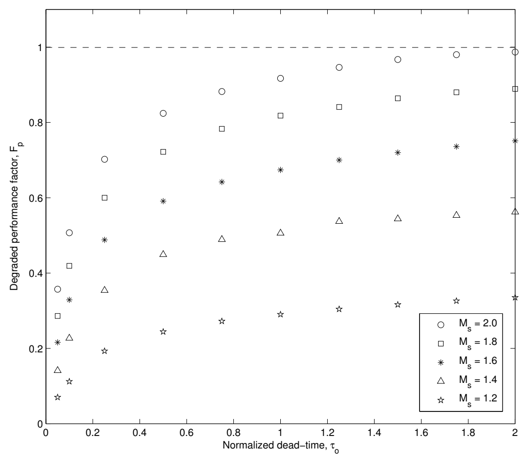

Figure 12. PI Regulatory Control Performance Degradation Factor ($a = 0.50$)

figure for the servo-control PI controller, Fig. 10, it is noticed that, except for a very large τ_o where the F_p values are similar, PI control systems require less performance reduction to achieve the same robustness as that of a PID based control systems.

In order to exemplify this, to reach a robustness level corresponding to $M_S^t = 1.6$, the servo-control PID controller by using a DPPDT ($a = 1.0$) model (Fig. 16 with $M_S = 1.6$) requires F_p values of 0.37 (reached

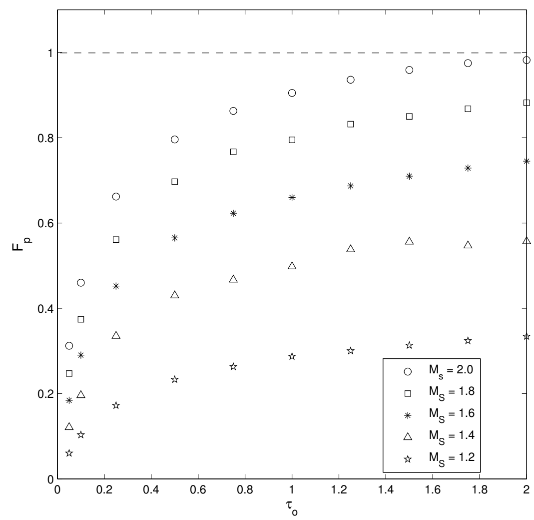

Figure 13. PI Regulatory Control Performance Degradation Factor ($a = 0.75$)

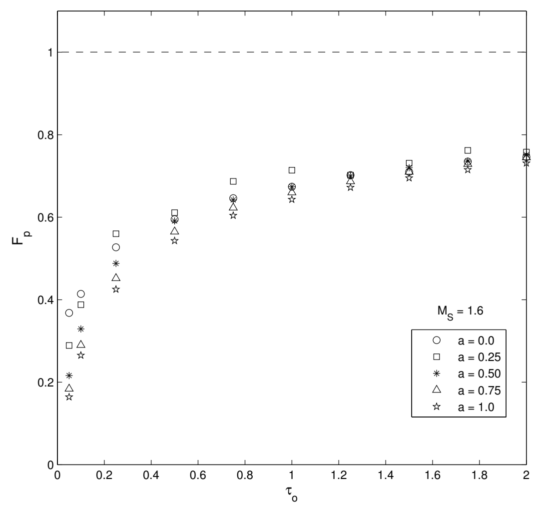

Figure 14. PI Regulatory Control Performance Degradation Factor ($M_S^t = 1.6$)

for $\tau_o = 0.10$) and 0.82 (reached for $\tau_o = 1.5$), while the corresponding PI controller (Fig. 9 with $M_S = 1.6$) requires F_p values of 0.94 ($\tau_o = 0.10$) and 0.86 ($\tau_o = 1.5$); therefore, the latter is better from the performance reduction to robustness increase ratio point of view.

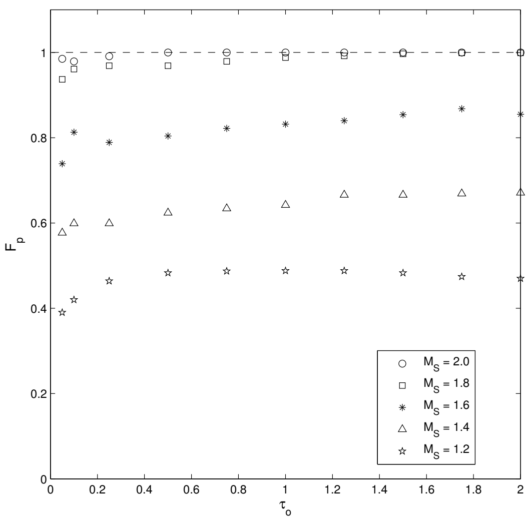

Figure 15. PID Servo-Control Performance Degradation Factor ($a = 0.0$)

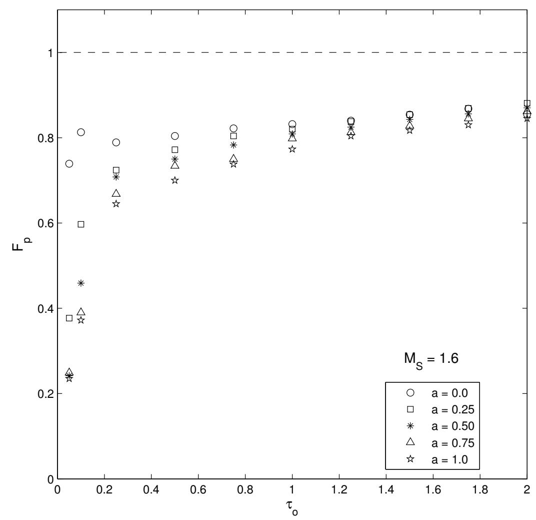

Figure 17. PID Servo-Control Performance Degradation Factor ($M_S^t = 1.6$)

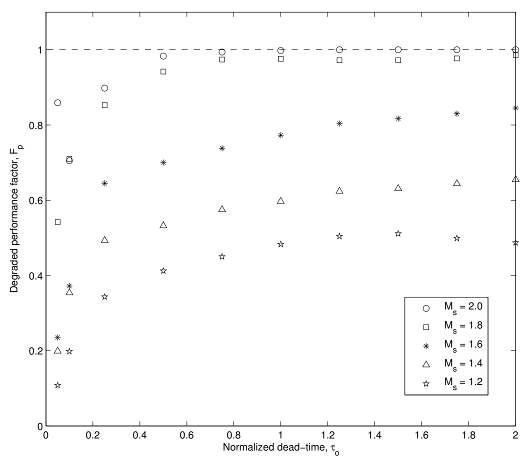

Figure 16. PID Servo-Control Performance Degradation Factor ($a = 1.0$)

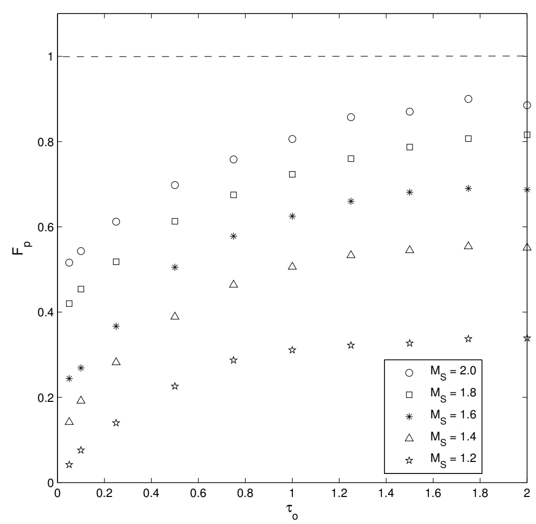

Figure 18. PID Regulatory Control Performance Degradation Factor ($a = 0.0$)

3.2.4. PID Regulatory Control

The results for this case are shown in Figs. 18-20. It can be seen that regulatory-control PID controllers are less robust than are regulatory-control PI controllers and servo-control PID controllers.

In general, performance-optimized PI control systems are more robust than the corresponding PID control systems, and servo-control PI and PID controllers are more robust than their regulatory control counterparts.

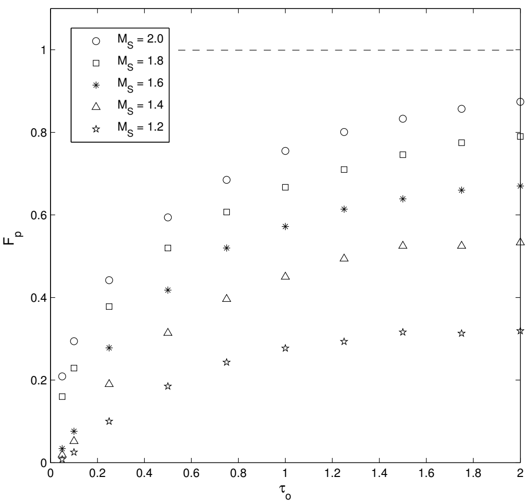

Figure 19. PID Regulatory Control Performance Degradation Factor ($a = 1.0$)

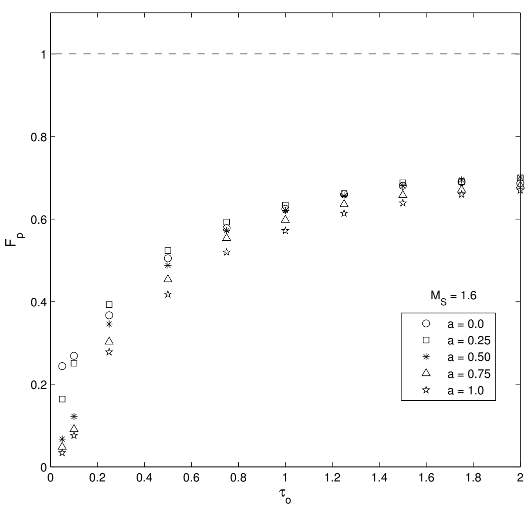

Figure 20. PID Regulatory Control Performance Degradation Factor ($M_S^t = 1.6$)

3.3. 2DoF Controllers Proportional Set-Point Weight

Using the parameters for the robust performance of 1DoF regulatory-control PI and PID controllers, θ_c^{o2}, a third optimization is performed. The free β for the 2DoF PI and PID controllers was determined by optimizing (9) for a set-point step change (J_{er}).

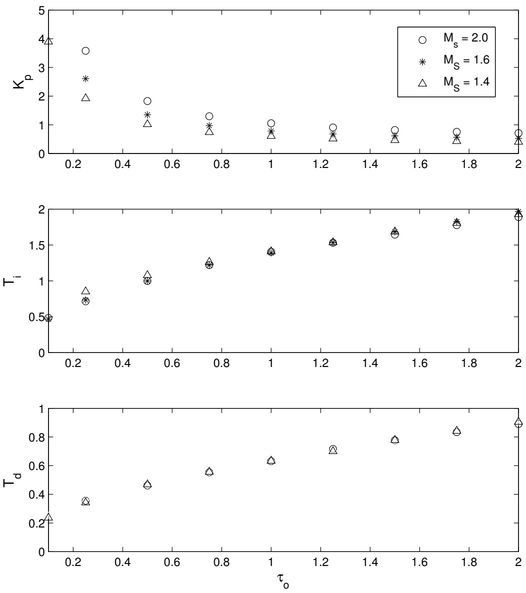

Figure 21. Parameters of the Regulatory Control Optimized PID Controller ($a = 0.50$)

For the same robustness level, the resulting 2DoF PI and PID controllers will deliver the same regulatory-control performance as do the corresponding 1DoF controllers; the servo-control performance will be lower than that of the corresponding 1DoF controllers.

IV. SIMPLE ROBUST TUNING OF 2DOF PID CONTROLLERS

The results of the performance/robustness trade-off analysis presented in the previous section show that for the same model and target robustness, the PID controllers show better IAE performance than do the PI controllers. Hence, for these controllers, it is desirable to devise tuning rules that can be used to optimize the performance and provide a robust control system.

Analysis of the regulatory-control PID controllers showed that for a given a, increasing the control system robustness by decreasing M_S^t, results in a substantial reduction in K_p. However, this increase in the robustness has negligible effect on T_i and T_d, except in the case of models with a very low τ_o (when high robustness is required). An example of this phenomenon for a second-order model (1) with $a = 0.50$ is shown in Fig. 21.

Table 1. Constants a_i for (20)

	Controlled process time constants ratio a 0.0	0.25	0.50	0.75	1.0
	Target robustness $M_S^t = 2.0$				
a_0	0.255	0.451	0.459	0.464	0.475
a_1	0.819	0.533	0.581	0.675	0.779
a_2	-0.938	-1.146	-1.216	-1.249	-1.254
	Target robustness $M_S^t = 1.8$				
a_0	0.227	0.392	0.403	0.410	0.421
a_1	0.726	0.482	0.521	0.600	0.690
a_2	-0.938	-1.128	-1.199	-1.232	-1.239
	Target robustness $M_S^t = 1.6$				
a_0	0.194	0.320	0.329	0.335	0.348
a_1	0.609	0.418	0.450	0.515	0.585
a_2	-0.940	-1.099	-1.168	-1.199	-1.212
	Target robustness $M_S^t = 1.4$ †				
a_0	0.149	0.176	0.229	0.151	0.195
a_1	0.462	0.394	0.365	0.506	0.521
a_2	-0.941	-0.952	-1.098	-0.951	-1.052
	†Valid only for $\tau_o \geq 0.25$ if $a \geq 0.50$				

Table 2. Constants b_i for (21)

	Controlled process time constants ratio a 0.0	0.25	0.50	0.75	1.0
b_0	-0.100	0.168	0.209	0.324	0.305
b_1	1.189	1.106	1.197	1.217	1.375
b_2	0.519	0.519	0.502	0.529	0.491

Table 3. Constants c_i for (22)

	Controlled process time constants ratio a 0.0	0.25	0.50	0.75	1.0
c_0	0.0	0.086	0.088	0.071	0.054
c_1	0.393	0.433	0.549	0.652	0.738
c_2	0.857	0.720	0.554	0.500	0.455

On the basis of this observation, equations *that are independent of the target robustness level* can be obtained for the controller integral time and derivative time, as follows:

$$T_i = \mathbf{F}(T, \tau_o, a), \quad T_d = \mathbf{G}(T, \tau_o, a). \qquad (18)$$

With these equations at hand, the controller proportional gains were readjusted to match a target robustness to obtain equations given by the following

$$K_p = \mathbf{H}(K, \tau_o, a, M_S^t). \qquad (19)$$

For FOPDT and SOPDT models with τ_o in the range 0.1-2.0 and four M_S^t values, the normalized controller parameters can be obtained using the model parameters, $\theta_p = \{K, T, L, \tau_o\}$, from the following equations:

$$\kappa_p \doteq K_p K = a_0 + a_1 \tau_o^{a_2}, \qquad (20)$$

$$\tau_i \doteq \frac{T_i}{T} = b_0 + b_1 \tau_o^{b_2}, \qquad (21)$$

$$\tau_d \doteq \frac{T_d}{T} = c_0 + c_1 \tau_o^{c_2}, \qquad (22)$$

The value of the constants a_i, b_i, and c_i in (20), (21), and (22) are listed in Tables 1, 2, and 3, respectively.

The controller gain depends on the model parameters and the target robustness level, but the integral time and derivative time depend only on the model parameters. Hence, for a given controlled process model, θ_p, T_i, and T_d, are fixed, and M_S^t, is adjusted by varying only K_p.

The robustness obtained with the SRT rules given in (20)-(22) is shown in Figs. 22 and 23 for the FOPDT ($a = 0.0$) and SOPDT ($a = 0.50$) controlled processes, respectively. As can be seen, all the M_S^t levels are achieved with good accuracy. The robustness profiles for other values of a show a similar behavior but these data are not shown owing to space constraints.

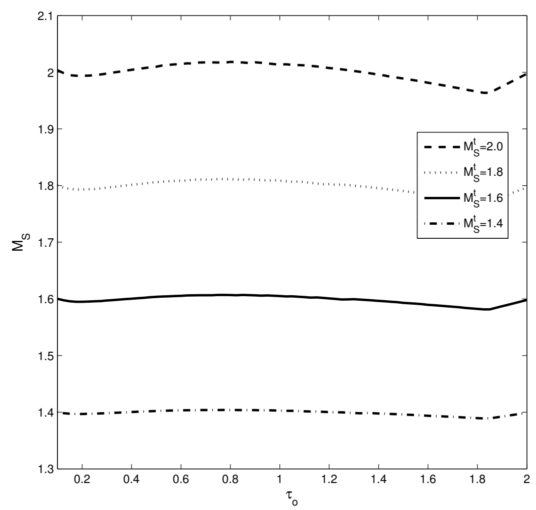

Figure 22. SRT Robustness Accomplishment for FOPDT Processes

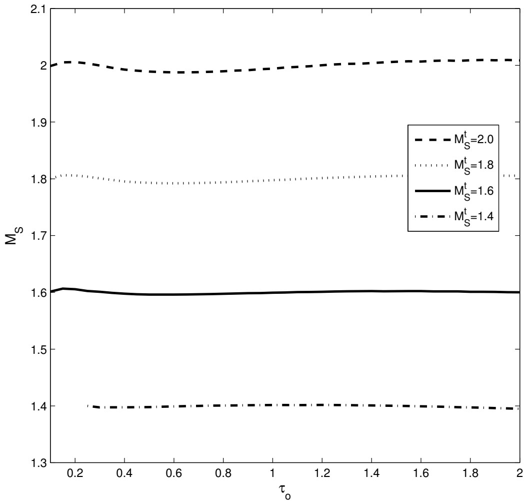

Figure 23. SRT Robustness Accomplishment for SOPDT ($a = 0.50$) Processes

Table 4. Constants d_i for (23)

	Target robustness M_S^t			
	2.0	1.8	1.6	1.4
d_0	0.439	0.339	0.315	0.441
d_1	0.278	0.476	0.664	0.858
d_2	0.560	0.440	0.524	0.658

The gain of a regulatory-control performance-optimized controller may be very high when low robustness is required for processes with a low τ_o and high a. On the other hand, it can be very low if high robustness is required for processes with a high τ_o. In both these situations, very poor set-point-following performance is observed. Then, to improve the servo-control performance, β is determined by optimizing (9) with J_{er}, as explained in section 3.3, taking advantage of the 2DoF PID controller.

The optimization results show that for a given robustness level, β is affected more by τ_o than by a. Then, for each M_S^t, a simple relation for β can be obtained as follows:

$$\beta^o = d_0 + d_1 \tau_o^{d_2}. \tag{23}$$

The values of d_i are listed in Table 4.

Even though (23) reduces the overshoot in the servo-control response when K_p is high, the instantaneous change in the controller output may become very large and this in turn may cause saturation of the controller output as exemplified in [26].

The instantaneous change in the controller output is related to the step set-point change as

$$\Delta u_r = K_p \beta \Delta e = K_p \beta \Delta r, \tag{24}$$

and hence, it is necessary to set a limit to it so that large instantaneous changes in the controller output are prevented.

The instantaneous change in the controller output can be limited to $\Delta u_r \leq 150\% \Delta r$ by selecting β of the 2DoF PID controllers as follows

$$\beta = min\left\{\frac{1.5}{K_p}, \beta^o\right\}. \tag{25}$$

The complete proposed SRT is given by (20)-(23), and (25).

V. EXAMPLE

To illustrate the use of SRT and the existing performance/robustness trade-off, we consider a process represented by a transfer function

$$P(s) = \frac{1}{(s+1)(0.4s+1)(0.2s+1)(0.1s+1)}. \tag{26}$$

Using the three-point identification method *123c* [30], FOPDT and SOPDT models were obtained using the following transfer functions

$$P_1(s) = \frac{e^{-0.584s}}{1.163s+1}, \; \tau_{o1} = 0.502, \tag{27}$$

$$P_2(s) = \frac{e^{-0.209s}}{(0.852s+1)(0.625s+1)}, \tag{28}$$
$$\tau_{o2} = 0.245, \; a = 0.734.$$

The controller parameters as well as the performance and robustness for the obtained closed-loop control systems are listed in Tables 5 and 6. For the SOPDT model, linear interpolation of the controller parameters obtained with the tuning equations was performed to account for the SOPDT model a value.

The control system robustness (M_S^m) were obtained using the corresponding models (27) and (28) (they cannot be obtained in a practical application with the controlled process) while the performance indices (J_{ed}, J_{er}) were evaluated with the original controlled process (26).

The abovementioned data, demonstrate the performance/robustness trade-off. If the control system

Table 5. SRT 2DoF PID tuned using the FOPDT model

	Design robustness M_S^d			
	2.0	1.8	1.6	1.4
K_p	1.818	1.612	1.358	1.032
T_i	0.851			
T_d	0.254			
β	0.628	0.691	0.778	0.986
M_S^m	2.01	1.81	1.60	1.40
$J_{ed}/\Delta d$	0.614	0.688	0.805	1.020
$J_{er}/\Delta r$	1.361	1.414	1.486	1.587

Table 6. SRT 2DoF PID tuned using the SOPDT model

	Design robustness M_S^d			
	2.0	1.8	1.6	1.4
K_p	4.326	3.764	3.084	2.068
T_i	0.764			
T_d	0.333			
β	0.347	0.399	0.486	0.725
β^o	0.566	0.596	0.633	0.781
M_S^m	2.01	1.80	1.60	1.40
$J_{ed}/\Delta d$	0.223	0.266	0.341	0.535
$J_{er}/\Delta r$	1.013	1.072	1.170	1.390

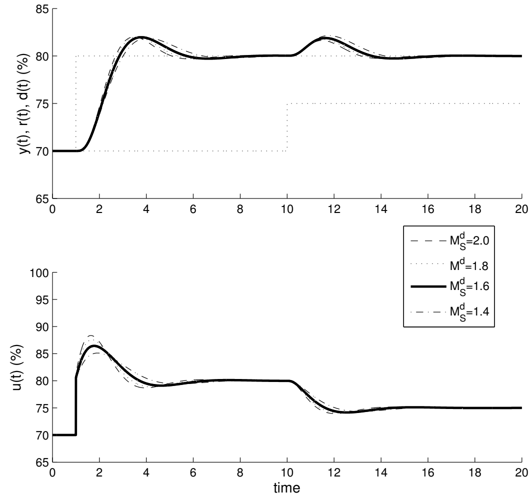

Figure 24. SRT 2DoF PID System Responses (FOPDT)

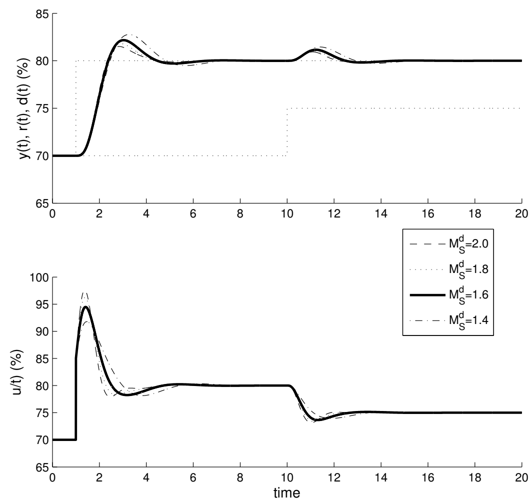

Figure 25. SRT 2DoF PID System Responses (SOPDT)

robustness is increased to take into account the expected dynamic changes in the controlled process, the obtained regulatory control performance decreases. For controller tuning with the FOPDT model, if the robustness is increased from the minimum level ($M_S = 2.0$) to a medium level ($M_S = 1.6$), the performance reduces by 24%; for an increase in the robustness to a high level ($M_S = 1.4$), the performance reduces by 40%. In a similar analysis for the SOPDT model, increasing the robustness to medium ($M_S = 1.6$) and high ($M_S = 1.4$) levels resulted in performance reductions of 35% and 58%, respectively.

Although the performance of the PID controller tuned using the second-order model reduces more with an increase in the control-loop robustness, the regulatory-control performance is 48% to 64% higher than the corresponding controllers tuned with the first-order model. Therefore, in this case for same robustness level the performance, J_{ed} and J_{er}, obtained for the controller is better for tuning using the second-order model parameters than for tuning using the first-order model parameters.

Fig. 24 shows the control system responses to a 10% set-point step change and a 5% load-disturbance step in the case of the SRT 2DoF PID controllers tuned with the FOPDT model parameters. Fig. 25 shows the corresponding responses for the controllers tuned with the SOPDT model parameters. It was assumed that during normal operation, all the system variables (r, u, and y) are at 70% of their normal operating range.

The higher regulatory control performance (IAE criterion) of the PID tuned using the SOPDT model is also significant for the lower maximum error and

Table 7. MEB 1DoF PID Controllers

Model	K_p	T_i	T_d	M_S^m	$J_{ed}/\Delta d$
FOPDT	2.131	0.873	0.217	2.295	0.531
SOPDT	4.933	0.763	0.419	2.784	0.195

shorter settling time achieved, but achieving this performance requires a more aggressive control effort.

To quantify the cost of use a robust tuning we will use for comparison 1DoF PID controllers tuned for regulatory-control operation with the IAE performance optimized (without robustness constraint) method presented in [31] denoted here as MEB tuning. The MEB controllers parameters and the control system robustness and performance obtained using models (27) and (28) are listed in Table 7.

As can be seen from this table the performance obtained when the robustness is not considered into the control system design is higher than the performance obtained with the proposed SRT tuning even with the lowest robustness design level but the robustness of such performance optimized control systems do not meet the minimum robustness requirement. In both cases MEB controllers have very poor robustness ($M_S > 2.0$), they are not robust controllers. It can also seen that the MEB controller tuned using the SOPDT model delivers more performance than the ones tuned using the FOPDT model but have lower robustness.

VI. CONCLUSIONS

Performance/robustness analysis of closed-loop systems with PI and PID controllers revealed the existing *trade-off* between performance and robustness. In general, performance-optimized 1DoF PI controllers are more robust but show a lower optimal performance than the corresponding PID controllers. The regulatory-control performance-optimized PI and PID systems are less robust than the corresponding servo-control systems; further, greater performance degradation (*lower performance degradation factor*) is required so that the robustness of the regulatory-control systems is comparable to that of the servo-control system.

The conducted analysis allowed the development of a *simplified robust tuning* (SRT) method that allows the use of a single set of rules for the design of 2DoF PID closed-loop control systems tuned with FOPDT and SOPDT models; the control systems showed a certain level of robustness, as measured in terms of M_S.

VII. ACKNOWLEDGMENTS

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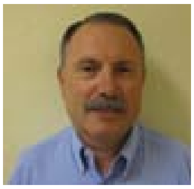

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Robust Tuning and Performance Analysis of 2DoF PI Controllers for Integrating Controlled Processes

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Abstract

The aim of this paper is to present a robust tuning method and a closed-loop performance analysis of two-degree-of-freedom (2DoF) proportional integral (PI) controllers for integrating controlled processes. A closed-loop model reference optimization method has been followed with non-oscillatory and under damped response targets. It has been found that non-oscillatory response targets produce smoother controller outputs and that the regulatory and servo-control performance (integrated absolute error and setting time) may be improved if lightly under damped responses are allowed. The proposed tuning method allows the designer to deal with the performance/robustness *trade-off* of the resulting closed-loop control system by specifying the desired robustness level through selecting a maximum sensitivity in the range from 1.4 to 2.0. Controller tuning equations that guarantee the design robustness level are provided for integrating second-order plus dead-time (ISOPDT) models with normalized dead-times in the range from 0.1 to 2.0, and integrating plus dead-time (IPDT) models. The robustness of the control system is analyzed. Comparative examples show the effectiveness of the proposed tuning method. The exact achievement of the control system robustness target for all the integrated controlled processes models considered is the distinctive characteristic of the proposed *Model Reference Robust Tuning* (MoReRT) method.

Introduction

Even though most of the controlled processes found in the process industry are self-regulating, i.e. the process output reaches a stable operating point under a constant input, there are others that under a constant input generate an unbounded output that rises or decreases without limit. These non-self regulated processes are named integrating or unstable if their model transfer function has a pole at the s-plane origin or at its right-half plane, respectively. Stable processes with very long time constants may also be approximated by integrating models. Integrating and unstable processes may be operated only under closed-loop automatic control and their controller tunning needs a special treatment. Integrating characteristics may be found by example in tank level and distillation column level control, and communication networks.[1–3]

For the integrating processes there are IMC-based tuning methods[1,2,4] for one-degree-of-freedom (1DoF) proportional integral (PI) and proportional integral derivative (PID) controllers that include a design parameter, the closed-loop time constant, that could be indirectly used to deal with the control system performance/robustness trade-off. Kappa-Tau[5] and AMIGO[6–8] methods

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provide tuning rules for two-degree-of-freedom (2DoF) PI and PID controllers for high robustness, with a maximum sensitivity of 1.4.

Robustness was also included in the controller design for integrating processes using the gain and phase margins,[9,10] and the maximum sensitivity.[11]

An analytically deducted (direct synthesis) tuning of 1DoF PI and Ideal PID controllers for integrating and integrating second-order plus dead-time models is presented by Chen and Seborg[12]. The design parameter is the regulatory control closed-loop time constant but its relation with the control system robustness is only illustrated with particular examples.

The IMC-based tuning SIMC[13] for 1DoF PI controllers is based on an integrated plus dead-time model and for 1DoF Ideal PID controllers on an integrating second-order plus dead-time model. The design parameter, the servo-control closed-loop time constant, is selected in order to provide a trade-off between fast response, good disturbance rejection and robustness. With SIMC controllers, the control system has an intermediate robustness corresponding to a maximum sensitivity of 1.7.

A revision of direct synthesis, IMC, and equating closed-loop transfer functions coefficients tuning methods for integrating controlled process is presented by Rao and Chidambaram[14]. Alternative control schemes as the model driven two-degree-of-freedom PID (MD TDOF PID)[15] has been proposed to control integrating processes.

Consulting O'Dwyer's handbook,[16] it was found that for integrating processes the most widely used representation is the simple integrating plus dead-time model for tuning both PI and 1DoF Ideal PID controllers. Most of these tuning rules only take into account performance considerations by using for example, an integrated error cost functional as the IAE, ITAE or ISE. For the integrating second-order plus dead-time models most of the controllers are 1DoF Ideal PID and, again, tuned by using performance specifications.

In most of the industrial process control applications, the desired value of the controlled variable, or set-point, normally remains constant and a good load-disturbance rejection is required,[17] which is usually known as *regulatory control*. However, due to variations in the process operating conditions, the controlled variable set-point may eventually need to be changed and then a good transient response is required, which is known as *servo-control* operation. Satisfying these two operating conditions simultaneously is not possible using a 1DoF controller, but using a 2DoF controller it can achieved within some constraints. The extra parameter that the 2DoF controller provides is used to improve its servo-control behavior while considering the regulatory control *performance* and the closed-loop control system *robustness*. This second degree of freedom introduced by Araki[18,19] is aimed at providing additional flexibility to control system design with PI/PID controllers.[20–22]

The design procedure for a closed-loop control system is usually based on linear models identified at the normal operating point. Due to the non-linear characteristics found in industrial processes, it is necessary to anticipate the changes in the process characteristics when the operating point changes, assuming certain relative stability margins or robustness requirements for the control system. Therefore, the design of a closed-loop control system with 2DoF controllers must take into account its performance to load-disturbance and set-point changes as well as its robustness to variations of the controlled process characteristics.

Following previous works of the authors,[23–25] in this paper the use of target models for the desired closed-loop dynamics is analyzed. Previously, the target reference models were taken without oscillations and completely determined by one single parameter (in fact directly related to the closed-loop speed). In this work, the use of more complexes, under damped, target reference models is proposed. The main goal of the paper is to carry out and present the performance/robustness analysis regarding the use of such target models in comparison to the over-damped ones in order to determine if it may be beneficial to specify a desired closed-loop response with some oscillation. From the analysis results a 2DoF PI controller tuning is proposed for integrating processes.

In the conducted analysis it is shown that it is possible to have closed-loop control systems with the same *robustness* level but with different *performance* (critical or under damped responses). The controller parameters can be adjusted to modify the servo and regulatory control performance keeping constant its robustness.

Based on the optimization data an alternative tuning method of 2DoF proportional integral (PI_2) controllers for integrating controlled processes is presented in this communication. The proposed approach explicitly considers the *trade-off* between the performance and robustness of a control system. The distinctive feature of the resulting tuning procedure is the incorporation of the desired robustness level as measured with the maximum sensitivity, M_S, which is the explicit and only design parameter. Therefore, the designer may select the desired robustness M_S level for the control system in the range from 1.4 to 2.0.

For evaluation of the proposed tuning in comparison with other available methods, the following aspects have been considered: 1) the *performance* of the control system response to step changes in set-point and load-disturbance, and 2) the *robustness* of the control system to changes in the controlled process characteristics.

Problem Formulation

The controller design procedure summarized below follows the model reference robust tuning (MoReRT) methodology[23] applied to overdamped[24] and integrating processes.[25]

Consider a closed-loop control system, as shown in Figure 1, where $P(s)$ and $C(s)$ are the controlled process model transfer function and the controller transfer function, respectively. In this system, $r(s)$ is the set-point, $u(s)$ is the controller output signal, $d(s)$ is the load-disturbance, and $y(s)$ is the process controlled variable.

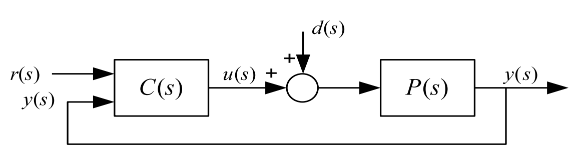

Figure 1: Closed-Loop Control System

The closed-loop control system output, $y(s)$, in response to changes in its inputs, $r(s)$ and $d(s)$, is given by the following:

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \tag{1}$$

where $M_{yr}(s)$ is the transfer function from the set-point to the process controlled variable, and $M_{yd}(s)$ is that from the load-disturbance to the process controlled variable. These are known as the *servo-control* closed-loop transfer function and the *regulatory control* closed-loop transfer function, respectively.

The main objective of the regulatory control is *load-disturbance rejection*; that is, the controlled variable should be returned to its set-point when a disturbance enters the control system. For the servo control, the intention is to *follow a set-point change*; that is, the controlled variable is brought to its new set-point. These two different responses will depend on the closed-loop transfer functions in Eq. (1) and can be selected with constrained independence if a 2DoF controller is used.

The development of the proposed tuning method of 2DoF PI controllers for integrating controlled process will take into account not just the closed-loop control system performance, stating target responses for step changes in the set-point and the load-disturbance, but also the control system robustness, measuring this with the maximum sensitivity, M_S.

2DoF Proportional Integral Controller (PI_2)

The process will be controlled with a two-degree-of-freedom proportional integral (PI_2) controller[5] whose output is expressed as follows:

$$u(s) = K_p \left\{ \beta r(s) - y(s) + \frac{1}{T_i s}\left[r(s) - y(s)\right] \right\}, \tag{2}$$

where K_p is the controller *proportional gain*, T_i is the *integral time constant*, and β is the *set-point proportional weight*. The block diagram of this controller is shown in Figure 2.

For the purposes of analysis only, not implementation, the controller output Eq. (2) will be rewritten as follows:

$$u(s) = C_r(s)r(s) - C_y(s)y(s), \tag{3}$$

where

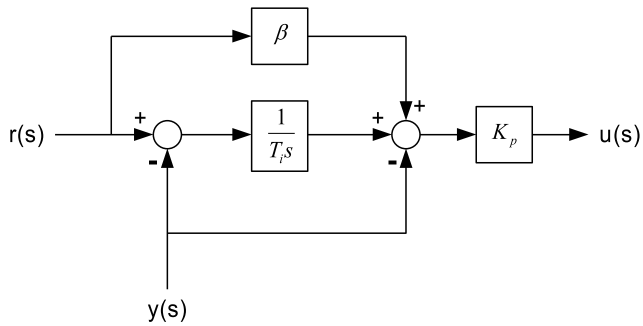

Figure 2: Two-Degree-of-Freedom PI Controller

$$C_r(s) = K_p\left(\beta + \frac{1}{T_i s}\right), \quad (4)$$

is the PI_2 controller aspect that operates on the set-point r, the *set-point controller* transfer function, and

$$C_y(s) = K_p\left(1 + \frac{1}{T_i s}\right), \quad (5)$$

is the PI_2 controller aspect that operates on the feedback signal y, the *feedback controller* transfer function.

The closed-loop transfer functions of the servo control and the regulatory control in Eq. (1) are then given by

$$M_{yr}(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)}, \quad (6)$$

and

$$M_{yd}(s) = \frac{P(s)}{1 + C_y(s)P(s)}, \quad (7)$$

which are related as follows:

$$M_{yr}(s) = C_r(s)M_{yd}(s). \quad (8)$$

General Closed-Loop Transfer Functions Targets

For the performance analysis and the development of the proposed tuning method, it is important to have the smallest possible number of design parameters. Consequently, the desired control system response to a load-disturbance step change involves only one design parameter, initially the closed-loop time constant T_c, and is selected with no steady-state error. In general, this is expressed by the following:

$$M_{yd}^t(s) = \mathscr{M}_d(T_c, \overline{\theta}_c, \overline{\theta}_p, s) = \frac{K_o s N_p^+(s)}{p(T_c, \overline{\theta}_p, s)}, \quad (9)$$

where $N_p^+(s)$ contains the delay and non-minimum phase elements of the model, $\overline{\theta}_p$ are the controlled process model parameters, $\overline{\theta}_c$ are the controller parameters, and $p(T_c, \overline{\theta}_p, s)$ is the characteristic polynomial of the closed-loop control system.

Using Eq. (9) in Eq. (8) the servo-control closed-loop transfer function is given by

$$M_{yr}^t(s) = \mathscr{M}_r(T_c, \overline{\theta}_c, \overline{\theta}_p, s) = C_r(s)M_{yd}^t(s). \quad (10)$$

Then using Eq. (9) and Eq. (10) in Eq. (1) the global control system output target, $y^t(s)$, is computed as follows:

$$y^t(s) = M_{yr}^t(s)r(s) + M_{yd}^t(s)d(s). \quad (11)$$

and in the time domain as follows:

$$y^t(t) = y_r^t(t) + y_d^t(t), \quad (12)$$

where $y_r^t(t)$ is the servo-control step response target and $y_d^t(t)$, the regulatory control step response target.

Controller Design

Usually the design of the 2DoF PI controllers is performed in two stages.[6,7,20,26] First, as it is required to obtain the desired regulatory control performance and specific closed-loop control system robustness level, the parameters (K_p, T_i) of the feedback controller Eq. (5) are determined for a parameter set of the controlled process model $\overline{\theta}_p$. Second, the set-point controller Eq. (4) free parameter (β) is used to improve the servo-control performance.

In what follows a different approach is taken. The complete set of PI_2 controller parameters $\overline{\theta}_c = \{K_p, T_i, \beta\}$ will be obtained considering, at the same time, the regulatory control and the servo-control performance, to obtain a controller with a targeted *servo/regulatory performance* *combination* that will also produce a closed-loop control system with a specific robustness level.

Cost Functionals

For the regulatory control response, the cost functional to be minimized is defined as follows:

$$J_d(\tau_c,\overline{\theta}_c,\overline{\theta}_p) \doteq \int_0^\infty \left[y_d^t(\tau_c,\overline{\theta}_c,\overline{\theta}_p,t) - y_d(\overline{\theta}_c,\overline{\theta}_p,t)\right]^2 \mathrm{d}t \tag{13}$$

where $y_d^t(\tau_c,\overline{\theta}_c,\overline{\theta}_p,t)$ is the step response of the regulatory control closed-loop transfer function target Eq. (9) and $y_d(\overline{\theta}_c,\overline{\theta}_p,t)$ is that of the regulatory control system Eq. (7) with the controlled process $P(s)$ and controller Eq. (5).

In Eq. (13) τ_c is the dimensionless design parameter that will be defined latter.

In a similar way, the servo-control cost functional to be minimized is defined as follows:

$$J_r(\tau_c,\overline{\theta}_c,\overline{\theta}_p) \doteq \int_0^\infty \left[y_r^t(\tau_c,\overline{\theta}_p,t) - y_r(\overline{\theta}_c,\overline{\theta}_p,t)\right]^2 \mathrm{d}t, \tag{14}$$

where $y_r^t(\tau_c,\overline{\theta}_p,t)$ is the step response of the servo-control closed-loop transfer function target Eq. (10) and $y_r(\overline{\theta}_c,\overline{\theta}_p,t)$ is that of the servo-control system Eq. (6) with the controlled process $P(s)$ and controller Eq. (4).

Controller Optimization

For the 2DoF PI controller design, the following overall cost functional is optimized:

$$J_T(\tau_c,\overline{\theta}_c,\overline{\theta}_p) \doteq J_r(\tau_c,\overline{\theta}_c,\overline{\theta}_p) + J_d(\tau_c,\overline{\theta}_c,\overline{\theta}_p), \tag{15}$$

in order to obtain the optimum controller parameters $\overline{\theta}_c^o = \left\{K_p^o, T_i^o, \beta^o\right\}$ such that

$$J_T^o \doteq J_T(\tau_c,\overline{\theta}_c^o,\overline{\theta}_p) = \min_{\overline{\theta}_c} J_T(\tau_c,\overline{\theta}_c,\overline{\theta}_p). \tag{16}$$

Note that $\overline{\theta}_c^o = \overline{\theta}_c^o(\overline{\theta}_p,\tau_c)$. Moreover, for each $\overline{\theta}_c^o$ obtained, the closed-loop control system robustness is measured using the maximum sensitivity M_S, which is defined as follows:

$$M_S \doteq \max_\omega |S(\mathrm{j}\omega)| = \max_\omega \frac{1}{\left|1 + C_y(\mathrm{j}\omega)P(\mathrm{j}\omega)\right|}. \tag{17}$$

Controlled Process Models and Closed-Loop Transfer Functions Targets

For the integrating processes two models are considered: the integrating second-order plus dead-time model, and the integrating plus dead-time model.

Integrating Second-Order Plus Dead-Time Models

We consider first the integrating second-order plus dead-time (ISOPDT) model given by the following:

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{s(Ts+1)}, \tag{18}$$

were K is the gain, T the time constant and L the dead-time. The controlled process parameters are $\overline{\theta}_p = \{K,T,L\}$.

In a previous work[25] the global control system output target, $y^t(s)$, was considered as:

$$y^t(s) = \frac{\mathrm{e}^{-Ls}}{(\tau_c Ts+1)^2}r(s) + \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c Ts+1)^3}d(s). \tag{19}$$

where $\tau_c \doteq T_c/T$ is the dimensionless design parameter, which is an indication of the closed-loop system response speed in relation to the controlled process speed.

The selection of the closed-loop transfer functions targets in Eq. (19) seeks also to obtain, as a side effect, a smooth controller output.

Now in order to analyze if it is possible to modify the control system performance to a load-disturbance and set-point step changes without affecting its robustness the regulatory control closed-loop transfer function target is selected with two under damped dominant poles given by

$$M_{yd}^t(s) = \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(\tau_c T s + 1)}, \tag{20}$$

Using Eq. (20) and Eq. (4) in Eq. (8) the servo-control closed-loop transfer function is

$$M_{yr}(s) = \frac{(\beta T_i s + 1)\mathrm{e}^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(\tau_c T s + 1)}. \tag{21}$$

Now, using $\beta = \tau_c T/T_i$ as in the analytically deducted PI_2 design method ART_2[27] the servo-control closed-loop transfer function target is selected as a second-order under damped system given by

$$M_{yr}^t(s) = \frac{\mathrm{e}^{-Ls}}{\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1}. \tag{22}$$

Then, the new global control system output target $y^t(s)$ is computed as

$$y^t(s) = \frac{\mathrm{e}^{-Ls}}{\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1} r(s) + \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(\tau_c T s + 1)} d(s). \tag{23}$$

The control system have now two design parameters, the *closed-loop relative speed* τ_c and the closed-loop dominant poles *damping ratio* ζ.

Closed-loop Performance Analysis

For analysis purposes, we select damping ratios in the range from 1.0 to 0.6.

During the optimization procedure, the closed-loop relative speed parameter τ_c is selected in such a way that the robustness level of the resulting closed-loop system met a specific target M_S^t in the range from 1.4 to 2.0.

For the entire controller parameter sets obtained both the control system robustness and performance are evaluated.

The regulatory control performance indices evaluated are the integrated absolute error (J_{ed}), the controller output total variation (TV_{ud}), the maximum error (E_{max}), the time to reach the maximum error (t_{max}), and the settling time ($t_{5\%Emax}$). For the servo-control the performance indices evaluated are the integrated absolute error (J_{er}), the controller output total variation (TV_{ur}), the rise time (t_r), the control effort maximum value (U_{max}), the controller output instant change (Δu_0), and the settling time ($t_{5\%\Delta y}$). These performance indices are defined in the Appendix.

As the model normalized dead-time τ_o is in the range from 0.1 to 2.0 and the control system robustness in the range from 2.0 to 1.4 we analyze some extreme and intermediate cases. Low and high robustness level targets, $M_S^t = 2.0$ and $M_S^t = 1.4$, and processes with $\tau_o \in \{0.1,\ 1.0,\ 2.0\}$.

Figure 3 and Figure 4 show the robustness and the normalized regulatory performance indices, as well as the normalized servo control performance indices, respectively, for the $M_S^t = 2.0$ cases. All the performance indices have been normalized using their corresponding values for the non-oscillatory target ($\zeta = 1.0$).

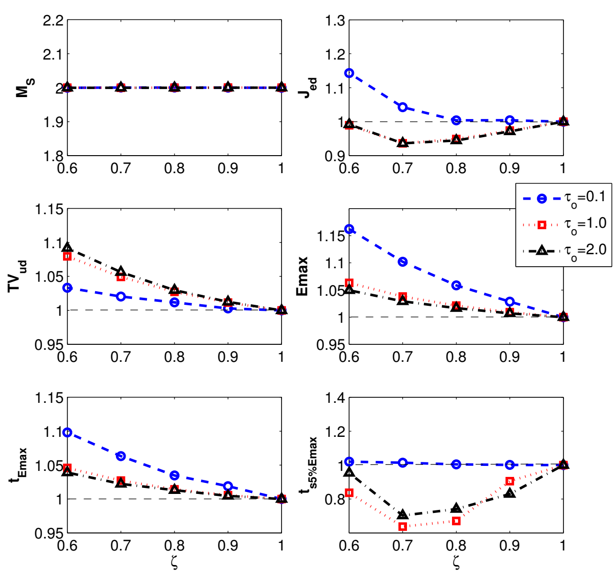

Figure 3: Robustness and Regulatory Control Performance, ISOPDT, $M_S^t = 2.0$

As can be seen from Figure 3 the robustness level target is perfectly accomplished in all cases. Allowing control system outputs with small oscillations ($0.70 \le \zeta \le 1.0$) it is possible to improve the regulatory control performance; a reduction on J_{ed} and $t_{s5\%Emax}$ values; but with a deterioration

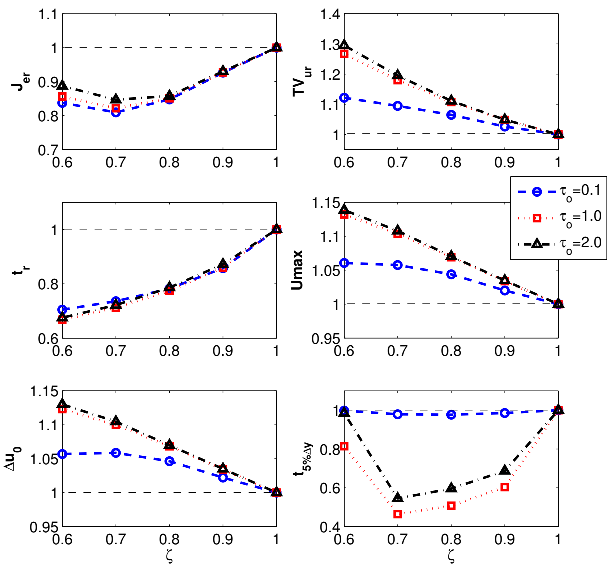

Figure 4: Servo Control Performance, ISOPDT, $M_S^t = 2.0$

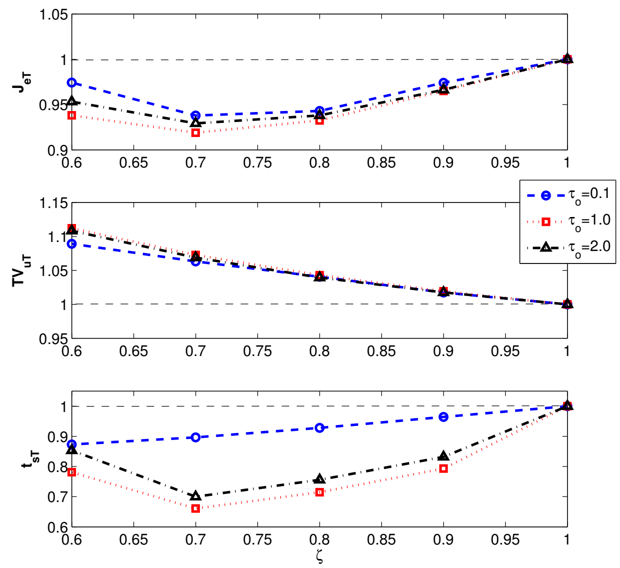

Figure 5: Servo/Regulatory Control Combined Performance, ISOPDT, $M_S^t = 2.0$

of the control effort smoothness (TV_{ud}), E_{max} and t_{Emax}.

At the servo-control side, shown in Figure 4, low values for the damping ratio will improve the J_{er}, t_r and $t_{s5\%\Delta y}$ indices, but will deteriorate the control effort characteristics, TV_{ur}, U_{max}, and Δu_0.

In order to take into account both responses (for servo and regulatory control), the normalized combined indices for the integrated absolute error ($J_{eT} = J_{er} + J_{ed}$), the controller output variation ($TV_{uT} = TV_{ur} + TV_{ud}$), and the settling time ($t_{sT} = t_{s5\%\Delta y} + t_{s5\%Emax}$) are computed and shown in Figure 5.

From the combined performance information it is seen that a good balance of the (J_{eT}, t_{sT}) versus TV_{uT} trade-off is obtained for damping ratios ζ in the range from 0.7 to 0.8.

Figure 6, Figure 7, and Figure 8 show the robustness and the normalized regulatory performance indices, the normalized servo control performance indices, and the combined performance normalized indices, respectively, for the $M_S^t = 1.4$ cases.

As can be seen from these figures the high robustness cases present a performance behavior similar to the corresponding of the low robustness cases but the performance improvements that can be obtained reducing the responses damping ratios are smaller.

Taking into account the previous analysis and considering that the performance improvement, J_{eT} reduction, obtained decreasing the damping factor ζ from 0.8 to 0.7 is lower that the increment in the control effort total variation (TV_{uT}) the closed-loop transfer functions targets are selected with a damping ratio $\zeta = 0.8$ to optimize the PI_2 controller parameters using the control system output target Eq. (23).

Integrating Plus Dead-Time Models

By considering the ISOPDT model of Eq. (18) it can be approximated by an integrating plus dead-time (IPDT) model given by the following:

$$P(s) = \frac{K e^{-Ls}}{s}, \tag{24}$$

were K is the gain and L the dead-time. The controlled process parameters are $\overline{\theta}_p = \{K, L\}$.

In a previous work[25] the global control system output target, $y^t(s)$, was computed as:

$$y^t(s) = \frac{e^{-Ls}}{\tau_c L s + 1} r(s) + \frac{(T_i/K_p) s e^{-Ls}}{(\tau_c L s + 1)^2} d(s), \tag{25}$$

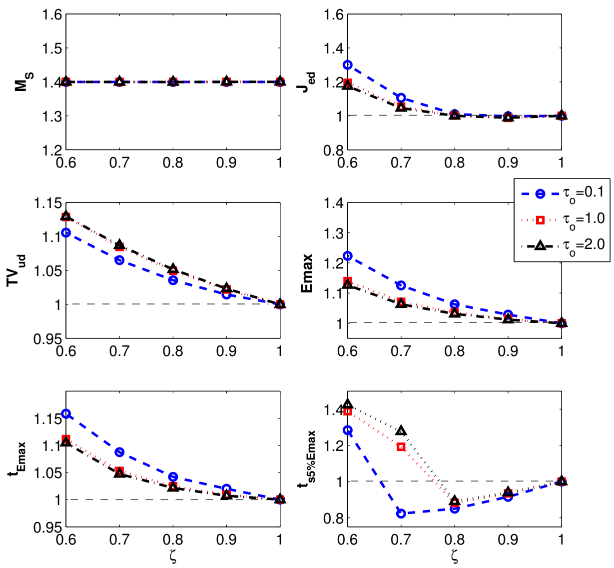

Figure 6: Robustness and Regulatory Control Performance, ISOPDT, $M_S^t = 1.4$

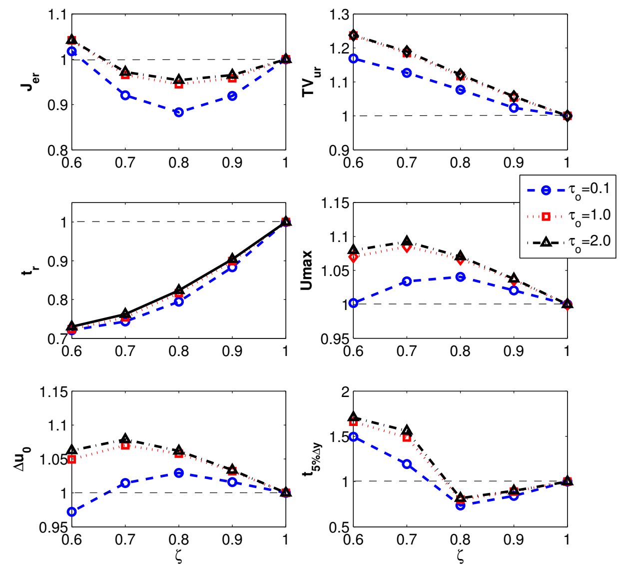

Figure 7: Servo Control Performance, ISOPDT, $M_S^t = 1.4$

where $\tau_c \doteq T_c/L$ is the dimensionless design parameter.

Now in order to analyze if it is possible to modify the control system performance without affecting its robustness the regulatory control closed-loop transfer function target is selected as a second-order under damped transfer function given by

$$M_{yd}^t(s) = \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{\tau_c^2 L^2 s^2 + 2\zeta\tau_c Ls + 1}, \qquad (26)$$

Using Eq. (26) and Eq. (4) in Eq. (8) the servo-control closed-loop transfer function is

$$M_{yr}(s) = \frac{(\beta T_i s + 1)\mathrm{e}^{-Ls}}{\tau_c^2 L^2 s^2 + 2\zeta\tau_c Ls + 1}. \qquad (27)$$

Now, selecting $\beta = \tau_c L/T_i$ the servo-control closed-loop transfer function target is selected as an under damped system given by

$$M_{yr}^t(s) = \frac{(\tau_c Ls + 1)\mathrm{e}^{-Ls}}{\tau_c^2 L^2 s^2 + 2\zeta\tau_c Ls + 1}. \qquad (28)$$

Then, the new global control system output target $y^t(s)$ is computed as

$$\begin{aligned} y^t(s) = & \frac{(\tau_c Ls + 1)\mathrm{e}^{-Ls}}{\tau_c^2 L^2 s^2 + 2\zeta\tau_c Ls + 1} r(s) \\ & + \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{\tau_c^2 L^2 s^2 + 2\zeta\tau_c Ls + 1} d(s). \qquad (29) \end{aligned}$$

The control system has again two design parameters, the *closed-loop relative speed* τ_c and the *damping ratio* ζ.

Closed-loop Performance Analysis

As with the ISOPDT for the IPDT model we select damping ratios in the range from 1.0 to 0.5 in order to analyze its effect over the control system performance. The same optimization and evaluation procedures as for the ISOPDT case are performed.

Figure 9, Figure 10, and Figure 11 show the robustness and the normalized regulatory performance indices, the normalized servo control performance indices, and the combined normalized performance indices, respectively.

From these figures information it can be seen that the IPDT processes output performance and control effort relation with the closed-loop transfer functions target damping ratio is similar to the relation presented by the ISOPDT processes.

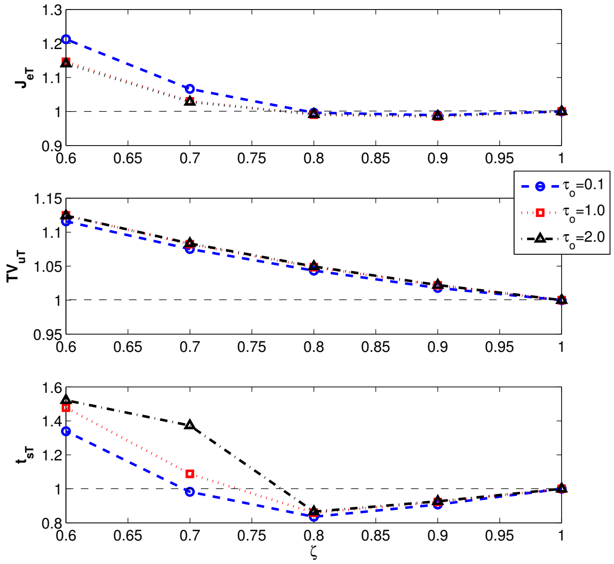

Figure 8: Servo/Regulatory Control Combined Performance, ISOPDT, $M_S^t = 1.4$

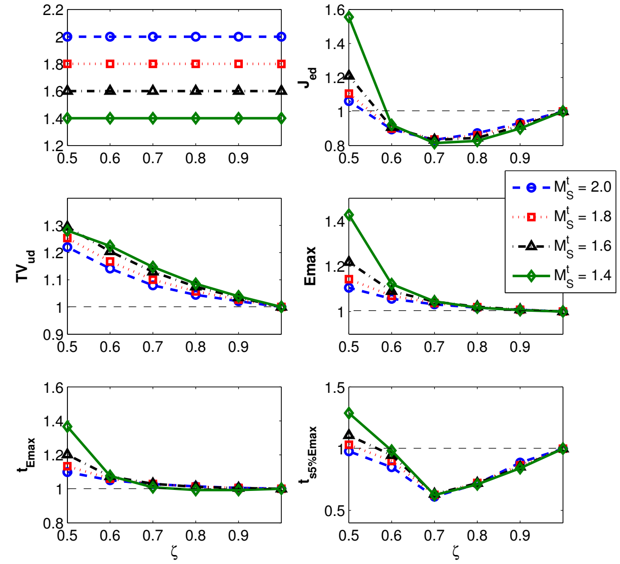

Figure 9: Robustness and Regulatory Control Performance, IPDT

For the IPDT process we also select the damping ratio $\zeta = 0.8$ for the closed-loop output target Eq. (29) used in the controller optimization process.

Controller Robust Tuning

Following the optimization procedure outlined before with the control system output targets Eq. (23) for the ISOPDT models and Eq. (29) for the IPDT models, both with $\zeta = 0.8$, the robust controller parameters are obtained.

Using normalized models and controllers a controller parameters optimization is performed for all sets of model parameters $\overline{\theta}_p$.

From the optimization results, it is possible to obtain the normalized controller parameters and the resulting control system robustness as functions of the model parameters, $\overline{\theta}_p$, and the performance specification, τ_c. However, to simplify the design procedure, the controller parameters are obtained directly as functions only of the closed-loop control system robustness parameter, which is the maximum sensitivity, M_S.

For a given model the normalized controller parameters $\overline{\theta}_c$ may therefore be expressed directly as functions of the model parameters $\overline{\theta}_p$ and the closed-loop control system robustness as follows:

$$\overline{\theta}_c = \left\{\kappa_p, \tau_i, \beta\right\} = \mathbf{f}(\overline{\theta}_p, M_S^t), \tag{30}$$

where M_S^t is the control system robustness target, which is the design parameter.

ISOPDT Models

Using the controlled process parameters $\overline{\theta}_p$ as well as the transformation $\hat{s} \doteq Ts$, the controlled process Eq. (18) and the PI controller transfer functions Eq. (4) and Eq. (5) can be expressed in a normalized form as follows:

$$\hat{P}(\hat{s}) = \frac{\mathrm{e}^{-\tau_o \hat{s}}}{\hat{s}(\hat{s}+1)}, \tag{31}$$

$$\hat{C}_r(\hat{s}) = \kappa_p \left(\beta + \frac{1}{\tau_i \hat{s}}\right), \tag{32}$$

$$\hat{C}_y(\hat{s}) = \kappa_p \left(1 + \frac{1}{\tau_i \hat{s}}\right), \tag{33}$$

where $\tau_o \doteq L/T$ is the model *normalized dead-time* and

$$\kappa_p \doteq K_p\, K\, T, \quad \tau_i \doteq \frac{T_i}{T}, \tag{34}$$

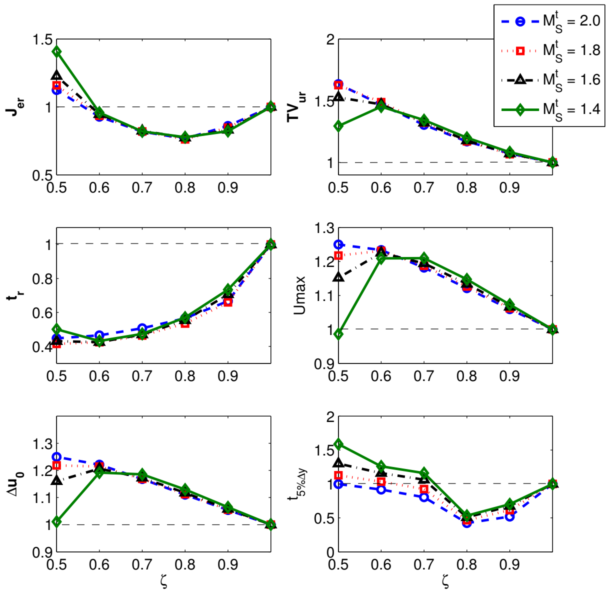

Figure 10: Servo Control Performance, IPDT

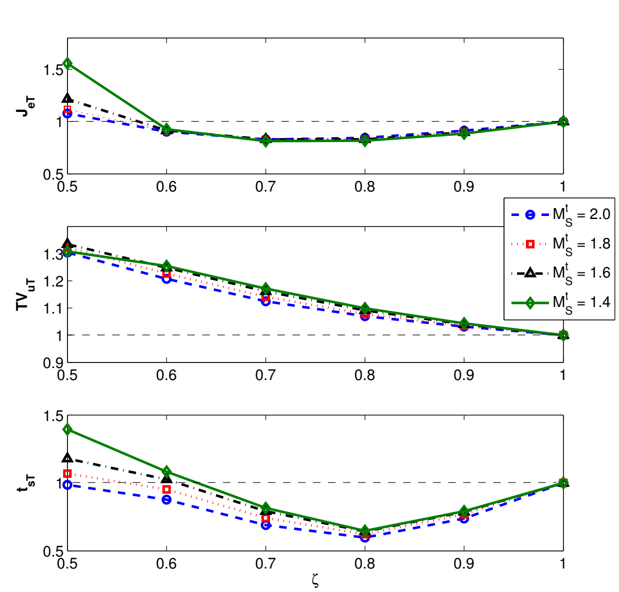

Figure 11: Servo/Regulatory Control Combined Performance, IPDT

are the *normalized gain* and *normalized integrating time* of the controller, respectively.

The normalized controlled process model Eq. (31) has only one dimensionless parameter, τ_o and it is selected in the range from 0.1 to 2.0.

The controller optimum normalized parameters (κ_p, τ_i, β) obtained are shown in Figure 12 along the entire dead-time range evaluated and the four robustness levels, $M_s^t = \{2.0, 1.8, 1.6, 1.4\}$.

This shows the influence of the controlled process dynamics (τ_o) and the desired robustness (M_S^t) over the controller parameters required to meet the target step responses.

The controller parameters obtained from the optimization procedure are used to fit the controller parameter equations of the proposed Model Reference Robust Tuning (MoReRT).

The normalized controller parameters can be obtained with the following equations:

$$\kappa_p = \frac{a_0 + a_1\tau_o}{a_2 + a_3\tau_o + {\tau_o}^2}, \quad (35)$$

$$\tau_i = b_0 e^{b_1\tau_o} + b_2 e^{b_3\tau_o}, \quad (36)$$

$$\beta = \frac{c_0 + c_1\tau_o}{c_2 + \tau_o}. \quad (37)$$

Table 1 lists the a_i, b_i and c_i constants for Eq. (35) to Eq. (37) for each of the four robustness levels.

Table 1: MoReRT constants, ISOPDT models

	Target robustness M_S^t			
	1.4	1.6	1.8	2.0
a_0	0.1040	0.1365	0.3886	1.0730
a_1	0.2800	0.3740	0.4670	0.5955
a_2	0.2539	0.2092	0.4455	0.9721
a_3	1.1970	1.1410	1.7280	3.3320
b_0	16.6700	10.9800	9.7360	8.3220
b_1	0.2070	0.2533	0.2477	0.2708
b_2	-10.0600	-5.9460	-5.4550	-4.5330
b_3	-0.4497	-0.6769	-0.6914	-0.8444
c_0	0.4673	0.5192	0.5853	0.6714
c_1	0.3093	0.3001	0.2927	0.2860
c_2	1.2150	1.3490	1.5360	1.7870

The designer may resolve the trade-off between performance and robustness by selecting the desired minimum robustness level for the control system according to the expected variation of the controlled process parameters. This gives the designer a closed-loop control system with the highest speed attainable for the specified minimum robustness.

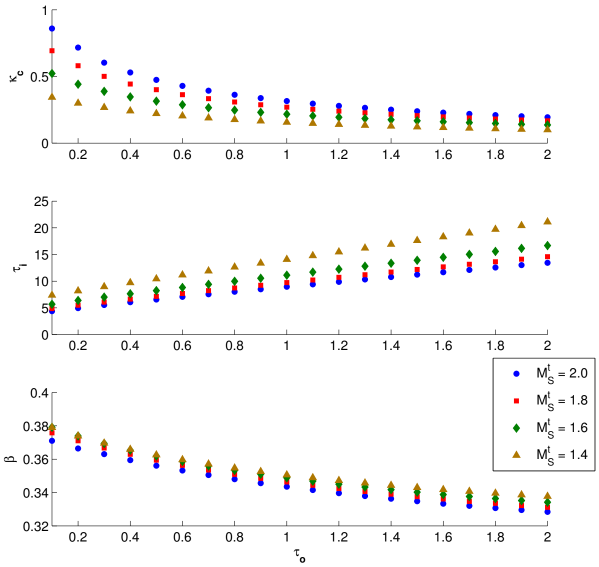

Figure 12: PI Controller Parameters, ISOPDT

IPDT Models

Using the controlled process parameters $\overline{\theta}_p$ as well as the transformation $\tilde{s} \doteq Ls$, the controlled process Eq. (24) and the PI controller transfer functions Eq. (4) and Eq. (5) can be expressed in a normalized form as follows:

$$\tilde{P}(\tilde{s}) = \frac{e^{-\tilde{s}}}{\tilde{s}}, \tag{38}$$

$$\tilde{C}_r(\tilde{s}) = \kappa_p \left(\beta + \frac{1}{\tau_i \tilde{s}}\right), \tag{39}$$

$$\tilde{C}_y(\tilde{s}) = \kappa_p \left(1 + \frac{1}{\tau_i \tilde{s}}\right), \tag{40}$$

where

$$\kappa_p \doteq K_p\, K\, L, \quad \tau_i \doteq \frac{T_i}{L}, \tag{41}$$

are the *normalized gain* and *normalized integrating time* of the controller, respectively.

As the normalized controlled process model Eq. (38) has no parameters only one optimization run is required for each robustness level.

In the same way as with the ISOPDT model during the optimization processes, the closed-loop relative speed parameter τ_c is selected in such a way that the robustness level of the resulting closed-loop system met a specific target (M_S^t) in the range from 1.4 to 2.0 and the controller parameters are obtained directly as functions only of the closed-loop control system robustness.

The MoReRT tuning equations for the IPDT models are as follows:

$$\kappa_p = a, \tag{42}$$
$$\tau_i = b, \tag{43}$$
$$\beta = c. \tag{44}$$

Table 2 lists the a, b and c constants for Eq. (42) to Eq. (44) for each one of the four robustness levels.

Table 2: MoReRT constants, IPDT models

	Target robustness M_S^t			
	1.4	1.6	1.8	2.0
a	0.312	0.415	0.498	0.566
b	8.086	6.217	5.320	4.802
c	0.544	0.516	0.495	0.477

MoReRT Control System Robustness

In what follows the nominal robustness obtained with the MoReRT tuning is verified.

Robustness Analysis of MoReRT control for ISOPDT Models

The robustness obtained with Eq. (35) to Eq. (37) for each normalized dead-time in the range analyzed is shown in Figure 13. As can be seen all the robustness profiles are nearly flat. This means that for an ISOPDT model of a controlled process the MoReRT tuning guarantees that the robustness target is attained for all normalized dead-times in the range considered.

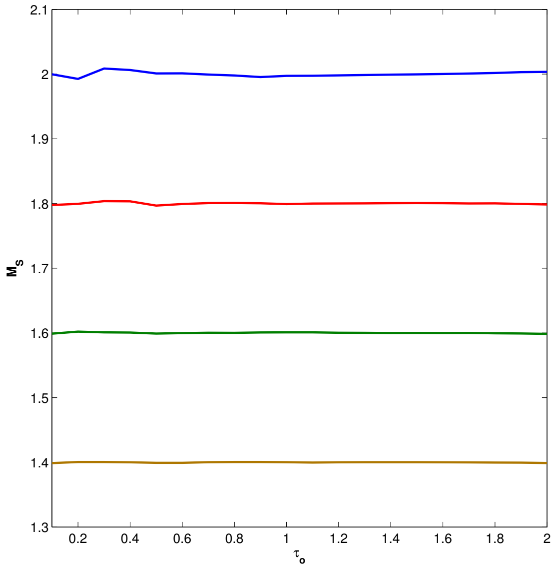

Figure 13: MoReRT Robustness for ISOPDT Models

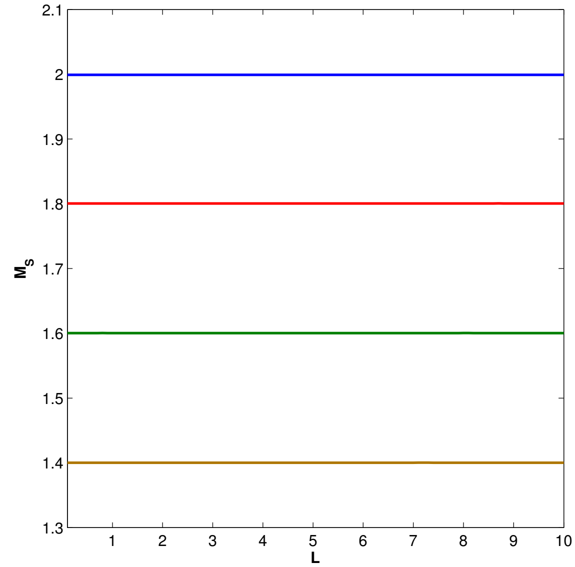

Figure 14: MoReRT Robustness for IPDT Models

Robustness Analysis of MoReRT control for IPDT Models

It is also verified that the robustness obtained with Eq. (42) to Eq. (44) for any model dead-time L matches the target robustness as shown in Figure 14.

Because the robustness level (M_S) is the design parameter of the proposed tuning method, the designer may specify the relative stability required for the control system, according to the expected variations in the controlled process characteristics, and be sure that the required robustness will be achieved, so long as the adopted model adequately represent the dynamics of the controlled process.

The achievement of the robustness target for all the integrating controlled process models considered (IPDT and ISOPDT) is one of the distinctive characteristics and contributions of the proposed tuning method.

Examples

As reported elsewhere[25] for integrating second-order plus dead-time models available robust tuning rules[11,13,28] do not produce control systems with a constant robustness level across their applicability range.

For integrating plus dead time models there are tuning rules that produce control systems with a constant M_S level along their entire applicability range but that are not robust like as PID performance optimized rules[29] and the closed-loop transfer function polynomials matching method.[30] The SIMC[13] PI tuning rule provides a constant intermediate robustness ($M_S = 1.7$) and the AMIGO[31] PI a constant high robustness ($M_S = 1.41$).

Other methods like Kappa-Tau[5] PI and AMIGO[31] PID do not ensure a constant robustness level.

In the first example the proposed tuning for IPDT models is compared with other methods, and due to the lack of robust tuning methods for integrating second-order plus dead time models, in the second example, the proposed tuning is used for comparing the performance obtained tuning the controller from the controlled process ISOPDT and IPDT model approximations and with a performance optimized tuning for 2Dof controllers.

Example 1

Consider first the distillation column bottom level control using the steam flow rate where the process model is given by the transfer function:[1,12]

$$P_1(s) = \frac{0.2\mathrm{e}^{-7.4s}}{s} \quad (45)$$

The PI controller parameters with the proposed tuning method for $M_S^t \in \{1.4\ 1.6\}$ and AMIGO[6–8] and SIMC[13] methods are listed in Table 3. It also lists the obtained robustness and the overall performance, the integrated absolute error (J_{eT}) and the control effort total variation (TV_{uT}). The control system responses to a 20% set-point step change followed by a -5% disturbance step change are shown in Figure 15.

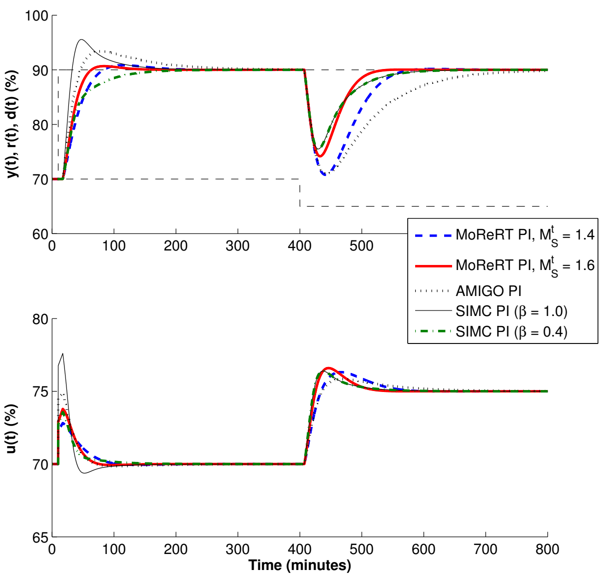

Figure 15: Example - IPDT PI Control Responses

The 2DoF PI controller set-point weight recommended for AMIGO method is $\beta = 1.0$ while the SIMC rule was derived for 1DoF controllers. In this latter case it is also considered the use of a set-point weight $\beta = 0.40$ as suggested in the literature[14,32,33] for integrating processes.

Comparing the performance (J_{eT}) and control effort (TV_{uT}) for similar robustness level control systems, MoreRT $M_S^t = 1.4$ with AMIGO and MoReRT $M_S^t = 1.6$ with SIMC ($\beta = 1.0$), in both cases the MoreRT tuning provides more performance and lower control total variation.

The use of a lower set-point weight with the SIMC tuning allows to reduce the servo control overshoot and the control effort maximum value but decreases the servo control performance (J_T increase) and turns the set point response slower.

It is not a good idea to use the same proportional set-point weight β of the 2DoF controllers with all integrating processes. As seen in Table 2 for IPDT models and in Figure 12 for ISOPDT models if the control system robustness needs to be increased, reducing M_S, the MoReRT controller proportional gain is reduced but, at the same time, its proportional set-point weight is increased in order to not deteriorate the servo control response. It is also noted that for ISOPDT models β depends on the model normalized dead-time τ_o.

Example 2

As a second example, consider the integrating third-order process with the transfer function:[34,35]

$$P_2(s) = \frac{0.833\mathrm{e}^{-0.2s}}{s(0.1s+1)(0.833s+1)} \quad (46)$$

For tuning purposes the process model Eq. (46) is approximated by following ISOPDT and IPDT models:

$$P_2(s) \approx \frac{0.833\mathrm{e}^{-0.353s}}{s(0.780s+1)} \approx \frac{0.833\mathrm{e}^{-1.133s}}{s} \quad (47)$$

Controlled process and models reaction curves are shown in Figure 16.

The MoReRT PI_2 controller parameters for $M_S^t \in \{1.4\ 1.6\}$ obtained with Eq. (47) models are listed in Table 4.

Table 4 includes also the robustness with the models (M_S^m) and the resulting with the controlled process (M_S^p), that in practice can not be obtained. From this table it can be seen that the robustness of the control systems with the controllers tuned with the ISOPDT mode is closer to the design robustness level than the one obtained with the controllers tuned with the IPDT model. This is interpreted as an indication that the ISOPDT model provides a better representation of the controlled process dynamics than the simple IPDT model.

The performance (J_e) and control effort (TV_u) indices on Table 4 were obtained with unitary set-point and disturbance step changes.

At both robustness levels the regulatory perfor-

Table 3: Example 1 - IPDT PI Control

	MoReRT $M_S^t = 1.4$	AMIGO	MoReRT $M_S^t = 1.6$	SIMC	SIMC†
K_p	0.211	0.227	0.280	0.338	0.338
T_i	59.836	99.160	46.006	59.200	59.200
β	0.544	1.0	0.516	1	0.40
M_S	1.401	1.391	1.599	1.704	1.704
J_{eT}	21.097	29.301	13.365	14.560	15.861
TV_{uT}	0.134	0.167	0.159	0.242	0.149

†Using $\beta = 0.4$[14,32,33]

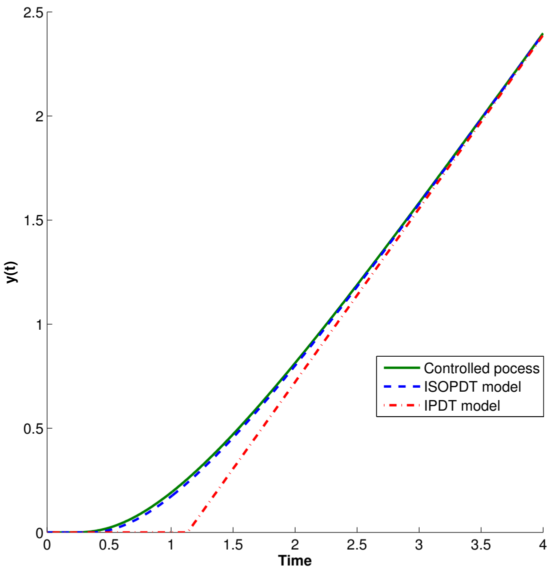

Figure 16: Example 2 - Controlled Process and Models Reaction Curves

Table 4: Example 2 - MoReRT PI Control

Model	IPDT	IPDT	ISOPDT	ISOPDT
M_S^t	2.0	1.6	2.0	1.6
K_p	0.600	0.440	0.769	0.506
T_i	5.441	7.044	4.925	6.191
β	0.477	0.516	0.358	0.364
M_S^m	2.000	1.601	2.000	1.600
M_S^p	1.724	1.487	1.945	1.582
J_{ed}	9.068	16.067	6.404	12.235
TV_{ud}	1.847	1.649	2.065	1.726
J_{er}	3.195	4.007	3.165	4.011
TV_{ur}	0.781	0.555	0.874	0.513
J_{eT}	12.263	20.074	9.569	16.246
TV_{uT}	2.629	2.205	2.939	2.239

mance of the controllers tuned with the ISOPDT model are higher that the ones obtained with the IPDT models and with similar servo-control performance. On the other hand, the control effort total variation is lightly higher for the controllers tuned with the ISOPDT model.

The table also shows the existing trade-off between performance and robustness. If the expected variation of the controller process dynamics forces to design for a high robustness control system an unavoidable performance reduction is expected.

For comparison purpose we consider the performance optimized (using integrated error cost functionals) tuning method denoted as T&A[20] that presents tuning relations for 2DoF PI controllers for IPDT and ISOPDT models.

For IPDT models the T&A tuned PI controllers produce control systems with only one low level robustness, $M_S = 2.82$, while the proposed tuning allows selecting between four robustness levels. For ISOPDT model the T&A PI control systems have very low and non-constant robustness as shown in Figure 17. The highest obtainable robustness is $M_S \approx 2.73(\tau_o = 0.70)$ and it turns very low for low and high model normalized dead-times. This contrast with the constant robustness labels obtainable with the proposed tuning as shown in Figure 13.

The controller parameters and performance obtained with the T&A tuning for Eq. (47) models

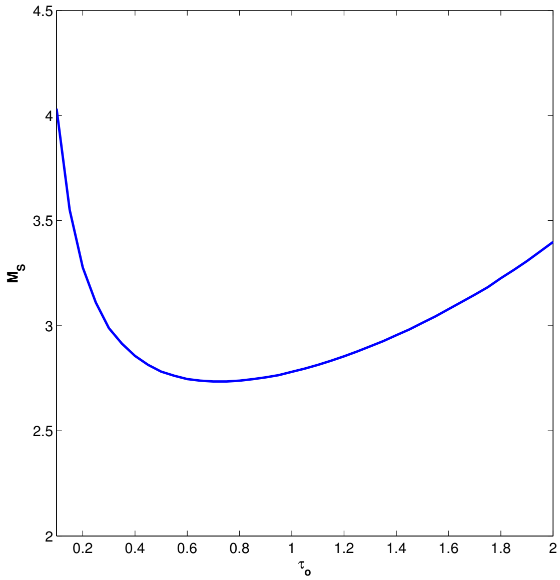

Figure 17: T&A PI robustness for ISOPDT models

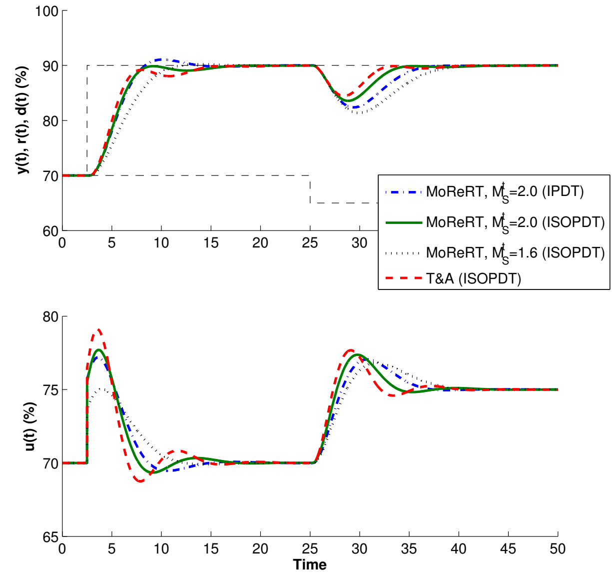

Figure 18: Example 2 - MoReRT and T&A PI_2 Control

are listed in Table 5.

Comparing the MoReRT controllers performance, Table 4, and T&A controllers performance, Table 5, these last ones have higher performance, especially for regulatory control, but more control effort variation and lower robustness.

Table 5: Example 2 - T&A PI Control

Model	IPDT	ISOPDT
K_p	8.812	0.965
T_i	4.635	4.732
β	0.319	0.336
M_S^m	2.818	2.315
M_S^p	2.032	2.207
J_{ed}	5.717	4.904
TV_{ud}	2.183	2.354
J_{er}	3.156	3.142
TV_{ur}	0.647	0.815
J_{eT}	8.873	8.046
TV_{uT}	2.830	3.169

Control system responses with MoreRT and T&A PI_2 controllers to a 20% set-point step change followed by a -5% disturbance step change are shown in Figure 15.

The above comparison highlights the MoReRT tuning that allows to design robust control systems for integrating processes with four robustness levels for either ISOPDT and IPDT models.

Conclusions

A performance analysis of robust tuned two-degree-of-freedom proportional integral controllers (PI_2) was conducted using a closed-loop model reference optimization design procedure with servo and regulatory control response targets with damping ratio in the range from 1.0 to 0.5 with robustness constrained to the range form 2.0 to 1.4.

It has been shown that all the controllers obtained with nos-oscillatory response targets ($\zeta = 1.0$) provide the smoothest control efforts with an integrated absolute error near the lower obtainable value for the corresponding robustness target level.

An improvement in the control system performance (integrated absolute error and settling time); especially for the servo-control; may be obtained if the closed-lop transfer functions damping ratio is selected in the range from 0.7 to 0.8 but this adversely affects the control effort characteristics.

Using servo and regulatory response targets with a damping ratio $\zeta = 0.8$ a robust tuning method for integrating models was developed.

The proposed *MoReRT* tuning method for 2DoF proportional integral (PI_2) controllers guarantees the design robustness level for integrated second-order plus dead-time (ISOPDT) and integrated plus dead-time (IPDT) models using only one design parameter, which is the required closed-loop control system robustness as measured with the maximum sensitivity M_S.

Tuning equations were obtained for four robustness, $M_S \in \{1.4, 1.6, 1.8, 2.0\}$, allowing the designer to select the required robustness level by taking into account the expected variations in the process parameters. The integrating second-order plus dead-time models considered include normalized dead-times in the range from 0.1 to 2.0.

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Appendix

For the controller *performance* evaluation, we select, as the main control system performance metric, the integrated absolute error (IAE) given by the following:

$$J_e \doteq \int_0^\infty |e(t)|\,\mathrm{d}t = \int_0^\infty |r(t) - y(t)|\,\mathrm{d}t. \qquad (48)$$

The performance measure Eq. (48) will be evaluated by set-point and load-disturbance changes, J_{er}, J_{ed}. As complementary performance metrics we will use the following indices shown in Figure 19, :

1. Servo control response
   - Rise-time, t_r (the time required to the control system output to change from $10\%\Delta y$ to $90\%\Delta y$).
   - Settling-time, $t_{s5\%\Delta y}$ (the time required to the control system output to rise and settle within a $\pm 5\%\Delta y$ band around its final value).
2. Regulatory control response
   - Peak error, E_{max} (the magnitude of the maximum error),
   - Time to the maximum error, $t_{E_{max}}$,
   - Settling-time, $t_{s5\%E_{max}}$ (the time required to the control system output to decrese and setttle within a $\pm 5\%E_{max}$ error band).

All the above indices can be obtained independently if the responses are over or under damped.

For the evaluation of the *control effort* the control signal total variation TV_u given by

$$TV_u \doteq \sum_{k=1}^{\infty} |u_{k+1} - u_k|, \qquad (49)$$

will be used as main indication of the *smoothness* of the control action for both input changes, TV_{ur} and TV_{ud}.

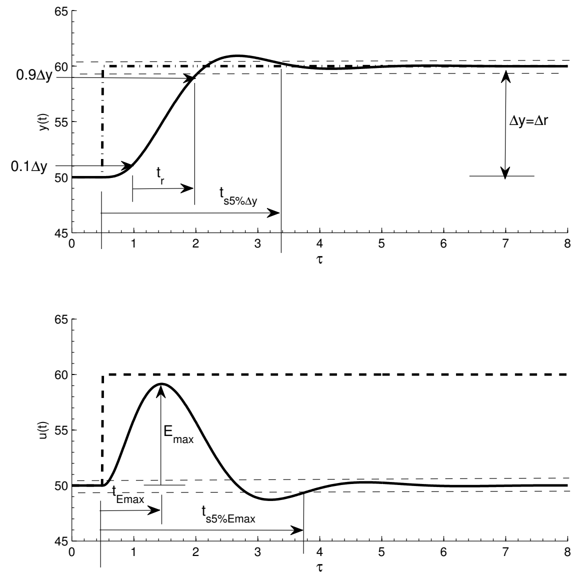

Figure 19: Servo and Regulatory Control Performance Indices

As complementary measurements of the control effort use we will consider the controller output instant change to a set-point step change (the "proportional kick") given by

$$\Delta u_0 \doteq \beta K_p \Delta r \qquad (50)$$

and the maximum control effort, U_{max}.